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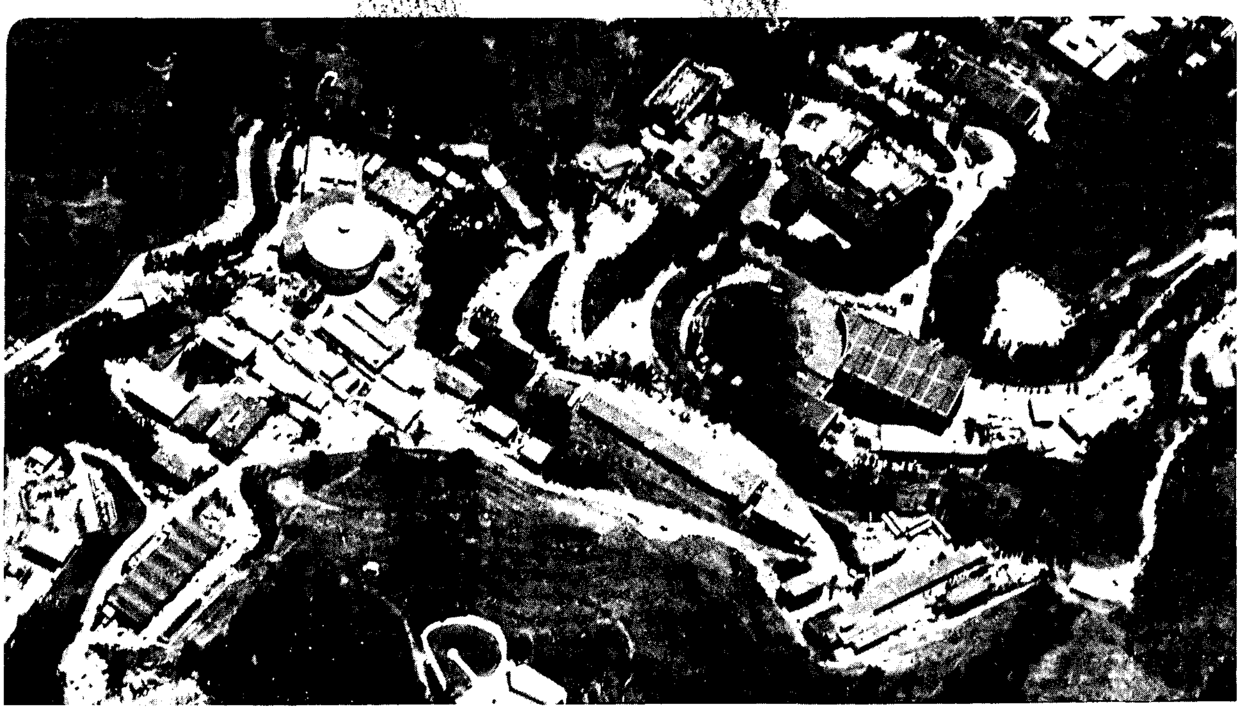

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DISCLAIMER

THE GGLP EFFECT FROM 1959 TO 1984*,†

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1. THE EMPIRICAL OBSERVATION

In 1959 I was involved in an experimental search for the ρ meson at the Bevatron. This search was inspired by a paper by Frazer and Fulco,[1] working in Berkeley at that time with Geoff Chew, who had just predicted that a pion-pion resonance with I = 1 and J = 1 should exist. We were working on a $\bar{p}p$ experiment in Wilson Powell's propane Bubble chamber at 1.05 GeV/c. We decided that the "hydron like" events would be a good place to look for the ρ meson. We thus calculated the invariant mass of pion pairs and realized that by comparing LIKE charged pairs with UNLIKE charged pairs we should be able to detect the ρ. As our statistics were not adequate enough, we were unable to observe the ρ meson in this experiment. However, we decided to also try comparing a simpler quantity: the cosine of the opening angle between a pion pair in the overall CM system. Here, to our surprise, we found a dramatic difference between LIKE and UNLIKE pion pairs. Thus we observed a clear deviation from phase space. The first empirical observation was thus the fact that the angular distributions for LIKE and UNLIKE charge pion pairs were distinctly different. We expressed this result quantitatively by quoting the ratio γ of the number of pion pairs with opening angles greater than 90° to those with angles less than 90°. Excerpts from this 1959 paper[2] are reproduced here in Figs. 1 and 2, which are given in the Appendix.

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*Talk presented by P. Mättig of DESY

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**Miller Professor, Miller Institute for Basic Research in Science, Berkeley, California (1984-85)

2. THE INTERPRETATION IN TERMS OF BOSE-EINSTEIN STATISTICS

2.1 A One Parameter Approach: An Average Radius

It took over a month to realize that we were observing the effect of Bose Einstein statistics for pions! Together with Abraham Pais we were able to obtain a quantitative fit to our data by doing a phase space calculation in which we symmetrized the two pion wave function for like pions.[3] Subsequently, as a result of this paper, this effect became known as the GGLP effect. It was only much later that we learned that Hanbury-Brown and Twiss had already proposed a similar effect of intensity interferometry for photons and had applied it to determine the diameter of stellar objects.[4]

In fact, for a considerable time—particularly after meson resonances had actually been observed by Alvarez and co-workers later in 1960—the interpretation of our result was frequently brought into question in that it was suggested that the effect we had observed in the comparison of LIKE and UNLIKE charge pion pairs was actually a reflection due to resonances in the UNLIKE pion pairs.[5]

From the first calculations[3] we worked with the negative of the invariant four momentum transfer squared Q^2. This implied that the entire effect was interpreted in terms of one invariant parameter: an average over the spatial and temporal radius of the pion source. Here $Q^2 = -(p_1 - p_2)^2 = M^2(12) - (m_1+m_2)^2$ where p_1, p_2 are the 4-momenta, M(12) is the invariant mass of particle 1,2 and m_1, m_2 are the rest masses. Excerpts from this 1960 paper[3] are reproduced here in Figs. 3-5 given in the Appendix.

2.2 The Question of Source Size and Shape

Over the next two decades several additional theoretical approaches were proposed, in particular by Kopylov and Podgoretsky (KP),[6] Shuryak,[7] Cocconi,[8] and later also by Gyulassy et al.,[9] which analyze the data in non-invariant terms and thus handle spatial and temporal parts separately. These take into account both the size of the source, radius r, and the thickness of the layer, $c\tau$, from which the pions are emitted. Cocconi, in particular, also suggested that the geometric shape of the source might be amenable to measurement.

2.3 New Variables and Correlation Coefficients

Needless to say this period saw a very considerable increase in the sophistication of both the experimental approaches and the theoretical analysis of the effect. It became clear that the strength of this effect is related to how close the absolute values of the momenta of the two pions were. Thus for example as higher statistical samples became available Bartke et al.[15] introduced the quantity $\delta = ||\vec{p}_1|-|\vec{p}_2||$ and found a considerable enhancement of the GGLP effect for low δ values. Firestone et al.[15] working on a 300 GeV/c pp experiment at FNAL cut on M, the mass of the multipion system, and noted a marked dependence on this variable.

Furthermore the experimental analyses began to shift from the angular variable we had first introduced[2] to correlation coefficients.[6,14] If we define $\rho(p_1) = \frac{1}{\sigma}\frac{d\sigma}{dp_1}$ and $\rho(p_1,p_2) = \frac{1}{\sigma}\frac{d^2\sigma}{dp_1 dp_2}$ as the one and two particle densities, then the 2-body correlation coefficient C_2 is given by:

$$C_2 = \frac{\rho(p_1,p_2)}{\rho(p_1)\rho(p_2)}$$

(sometimes also defined as $C_2' + 1$). Since the single particle densities $\rho(p_1)$, $\rho(p_2)$ are difficult to determine for a given reaction and experimental detector geometry, it has become customary to evaluate a ratio between an experimental 2-body density and that for a reference sample $\rho_0(p_1,p_2)$ which does not have any BE correlation.

Thus one defines:

$$R_0^L = \frac{\rho(p_1,p_2)}{\rho_0(p_1,p_2)} = \frac{C_2}{C_2^0}$$

where the superscript L stands for LIKE charges. The quantity $R_0^L - 1$ corresponds to the Fourier transform of the space time distribution of the particle source.[3,6] If expressed in invariant terms[3] this gives for a Gaussian source distribution:

$$R_0^L(Q^2) = 1 + e^{-Q^2 r^2}.$$

In terms of the KP variables,[6] where q_T is the component of the 3-momentum difference $\vec{p}_1 - \vec{p}_2$ perpendicular to $\vec{p}_1 + \vec{p}_2$ and $q_0 = |E_1 - E_2|$, one obtains

$$R_0^L(q_T,q_0) = 1 + [2J_1(q_T r)/q_T r]/[1+(q_0\tau)^2]$$

for independent sources uniformly distributed over a sphere of radius r and lifetime τ. Gyulassy, Kaufman and Wilson[9] (GKW) introduced an even simpler set of 2 variables, namely $|\vec{q}|$ and q_0, where $|\vec{q}| = |\vec{p}_1 - \vec{p}_2|$.

2.4 Control Samples

One of the difficulties in the experimental evaluation of the correlation coefficients is finding a suitable control sample.

a) The comparison between LIKE and UNLIKE pairs where the latter are the control sample is always open to the objection that some contribution to the observed effect might be due to the UNLIKE pairs.[5]

Other approaches have been:

b) Combining momentum vectors from different events.[6] This may be appropriate for low energy reactions with a more or less isotropic pion distribution but does not work well for high energy jets where the thrust axes for different events do not coincide or for detectors with limited angular acceptance. Even the rotating of the two thrust axes into each other, which I have attempted, does not yield a satisfactory result. Here it must also be noted that this method does not conserve momentum. Some of these objections have been overcome in the approach used in the TPC experiment[21] at PEP.

c) Random reshuffling of the transverse momenta relative to an axis within an event, while keeping the longitudinal momenta fixed, gives momentum conservation and approximate energy conservation. This method was suggested and tried by Deutschmann et al.[11]

d) Monte Carlo simulated events. Here care is needed to account for all the resonances in the data. A difficulty is that for statistically large experimental samples (e.g. 10^6 hadronic events at the J/ψ) similarly large MC event samples are needed.

2.5 A Chaotic Source or A Pion Laser?

Biswas, et al[10] realized that a factor was needed to reflect the fact that the BE effect appeared diluted in their data. In particular, Deutschmann, et al.[11] pointed out that while in our original calculation we assumed complete chaoticity—a fact which was very nearly true for the $\bar{p}p$ reaction we were studying at that time—one could modify the expression

$$1 + e^{-Q^2r^2}$$

to

$$1 + \lambda e^{-Q^2r^2}$$

where λ was a measure of the chaoticity and similarly for the KP variables. Fowler and Weiner[12], and also Gyulassy et al.,[9,13] pointed out that in principle the data could go from complete coherence, i.e., a "pion laser", with $\lambda = 0$, to complete chaoticity with $\lambda = 1$. In general the pion sources were not completely chaotic so that the λ parameter tends to be smaller than 1. Here it must be noted that other effects—such as intermediate state resonances (particles) which decay strongly e.g. ρ, ω, K^* and weakly e.g. D^0, D^+, F^+, B^0, B^-—can contribute to a reduction in the value of λ. Here I am assuming that "long lived" particles such as K^0 and Λ can be eliminated from the samples. Giovanni and Veneziano[14] pointed out that hadronic interactions and in particular $\bar{p}p$ annihilation should be extremely chaotic, while a much higher degree of coherence might be expected in the pion sources from jets produced in e^+e^- collisions. In this connection the J/ψ and perhaps also the Υ are of particular interest in that they are believed to decay into three gluons and hence should again represent a chaotic source even though produced in e^+e^- collisions.

3. PION INTERFEROMETRY

3.1 Experimental Goals

In principle, we now have a tool in the GGLP effect, to carry out detailed pion interferometry namely to study:

a) The degree of chaoticity λ in the pion source distribution.

b) The radius r of the pion source distribution.

c) The lifetime τ of the pion source distribution or the thickness of the region $c\tau$ from which the pions are emitted.

d) The geometrical shape of the region—considered as an ellipsoid—over which the pion sources are distributed.

In practice we are however still very far from achieving such a goal! There have been a large number of experiments attempting to apply these ideas and methods to study the pion source distributions.[15]

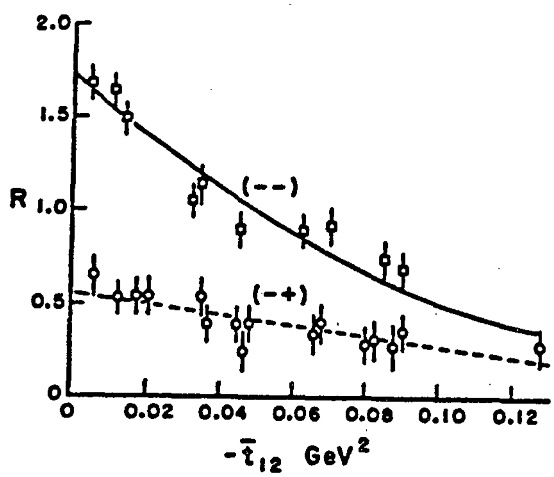

Fig. 6. $\pi^- p$ data at 200 GeV/c from Biswas et al.[10] (1976). Here R_U^L is given as function of Q^2 (called $-\bar{t}_{12}$) for limited Δy and ϕ intervals.

Table I Deutchman et al[11] results of the fits when using the standard background of pairs of unlike charge, N_U.

Incid. part.	No. of prongs	R, fm	cτ, fm	λ	C	χ^2/NDF
π^+	≥ 6	1.84 ± 0.06	1.08 ± 0.11	0.49 ± 0.02	0.89 ± 0.01	212/196
K^-	≥ 6	1.85 ± 0.10	0.96 ± 0.15	0.39 ± 0.03	0.92 ± 0.01	113/96
$\bar{p}$	4	1.88 ± 0.06	1.54 ± 0.14	1.20 ± 0.08	0.92 ± 0.01	105/96

Table II Deutchman et al[11] results of the fits when using as background the pairs of <u>unlike</u> charge, N_{UR}, constructed by reshuffling at random the transverse momentum components of the pions in each event.

Incid. part.	No. of prongs	R, fm	cτ, fm	λ	C	χ^2/NDF
π^+	≥ 6	1.45 ± 0.04	0.96 ± 0.05	0.88 ± 0.02	0.86 ± 0.01	142/96
K^-	≥ 6	1.36 ± 0.06	0.92 ± 0.07	0.83 ± 0.04	0.84 ± 0.02	79/96
$\bar{p}$	4	1.44 ± 0.09	1.27 ± 0.20	0.63 ± 0.05	0.92 ± 0.02	104/96

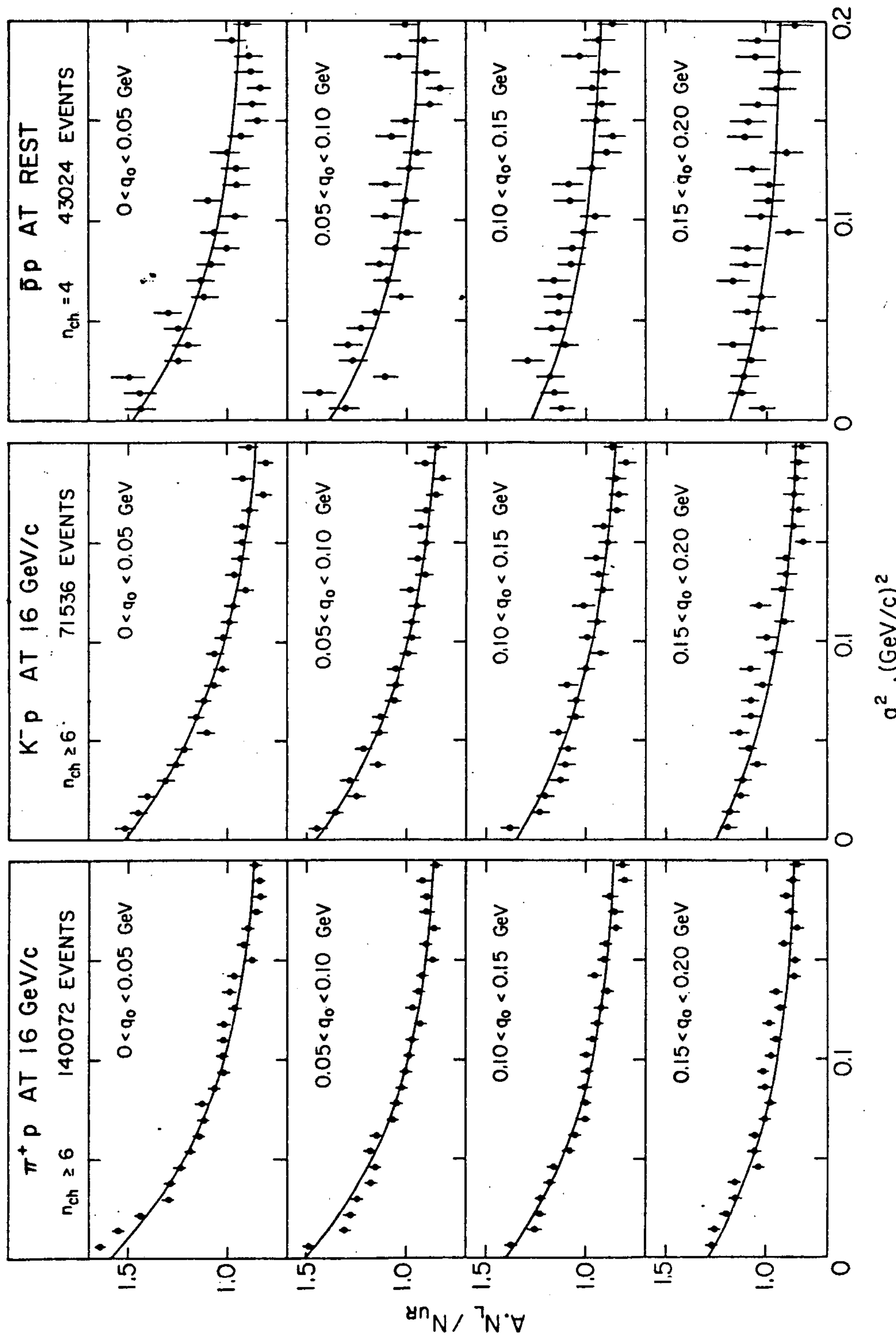

Fig. 7. Various hadronic reactions studied by Deutschmann et al.[11] (1982) using the reshuffling method. KP variables are used. Numerical values are given in Table II.

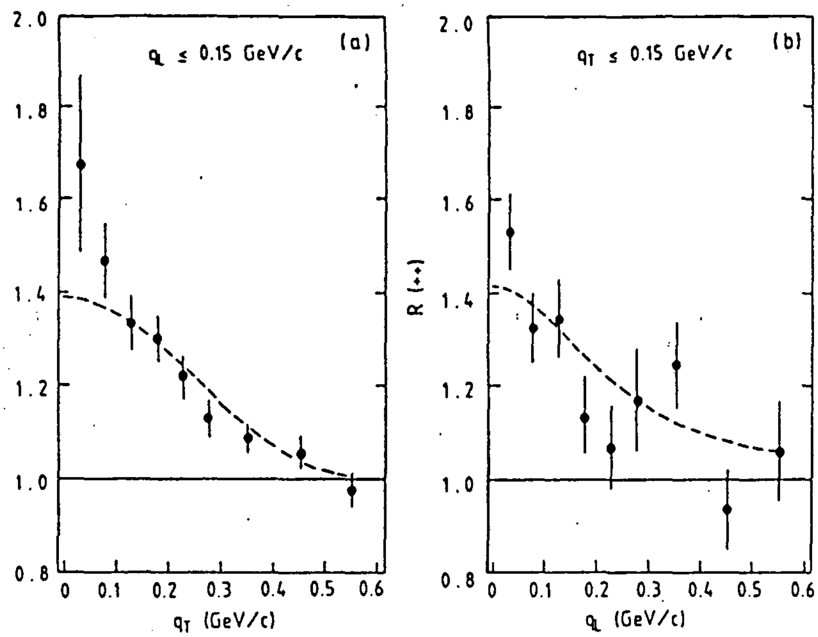

Fig. 8. pp data at 63 GeV from Akesson et al.[16] at the CERN ISR. The KP variables are used in a linear form. The results give $r(++) = (1.2 \pm 0.1)$ fm, $c\tau(++) = (0.9 \pm 0.2)$ fm and $\lambda(++) = 0.43 \pm 0.06$.

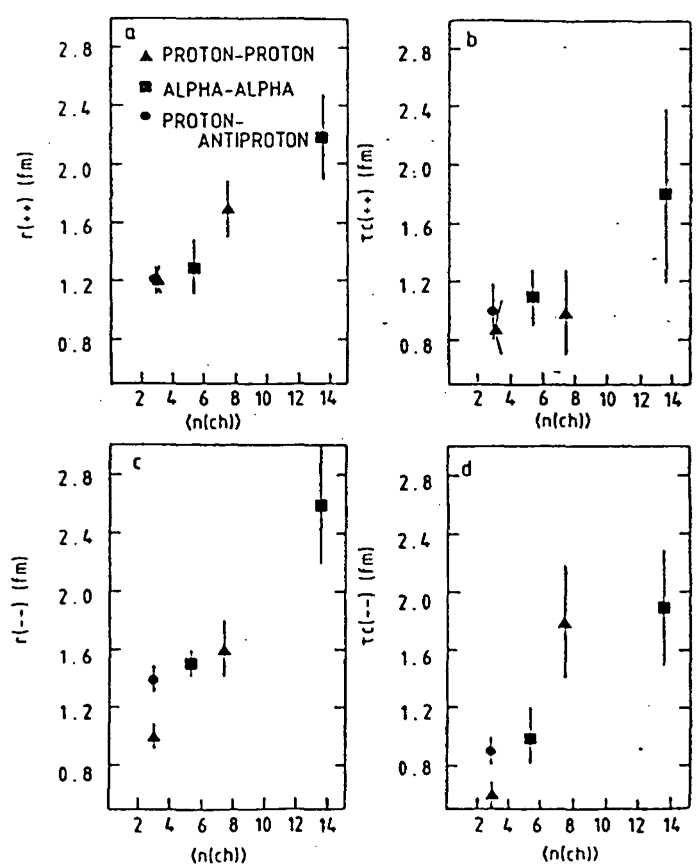

Fig. 9. CERN ISR data[16] for pp, $\bar{p}p$ and $\alpha\alpha$ showing the r and $c\tau$ parameters as function of the mean charged multiplicity for $\pi^+\pi^+$ and $\pi^-\pi^-$ separately.

3.2 Experimental Results: Hadron Reactions

In this section I present a few results from various hadronic experiments over the last decade.

Fig. 6 gives the result of Biswas et al.[10] for the 200 GeV $\pi^- p$ data analyzed in terms of Q^2 (called $-t_{12}$ in the figure).

Fig. 7 gives the results of Deutschmann et al.[11] using the KP variables and their "reshuffling" method for $\pi^+ p$ and $K^- p$ reactions as well as $\bar{p}p$ annihilations at rest. Their fitted results using unlike pions and reshuffled pions as control samples are given in tables I,II. A comparison of these 2 sets of variables illustrates how critical the choice of the control sample can be!

Fig. 8 gives pp results at $\sqrt{s} = 63$ GeV of Akesson et al.[16] who studied pp, $\bar{p}p$ and $\alpha\alpha$ reactions at the CERN ISR. These data are analyzed in terms of the KP variables where their $q_L \approx q_0$. They find typical λ values between 0.3 and 0.5. Furthermore, Åkesson et al. find an increase of the radius of the pion source with increasing multiplicity for all 3 reactions above. This is shown in Fig. 9. An interpretation of these data by S. Barshay[17] suggests that these larger radii correspond to a *decrease* of the impact parameters in the collisions giving higher multiplicities.

Fig. 10 gives a very recent result by Carlsson et al.[23] for the $\bar{p}p$ reaction at 9.1 GeV/c analyzed in terms of the KP variables. Here also the reshuffling method was used.

3.3 Experimental Results: e^+e^- Reactions

The J/ψ is a particularly good source for the study of the GGLP effect because large statistics are available, for example 1.3×10^6 events from the Mark II experiment at SPEAR, and furthermore the J/ψ is believed to decay via 3 gluons and hence is expected to have high chaoticity. I have given a preliminary report on this data[19], but work is still in progress together with Ivanna Juricic.

In Fig. 11 I present a qualitative illustration of what differences between LIKE and UNLIKE pions are occurring in the entire mass spectrum. Fig. 11a shows the difference between LIKE and UNLIKE mass distributions (normalized to equal numbers). Fig. 11b gives the corresponding ratio $R_U^L(M)$. Note that in these distributions, resonances in $\pi^+\pi^-$ states appear as dips rather than the peaks one sees in conventional mass plots. At low masses $M(\pi\pi) < 0.5$ GeV/c^2 we note a large peak—this is the GGLP effect. Right at threshold is a small dip, this is primarily the consequence of

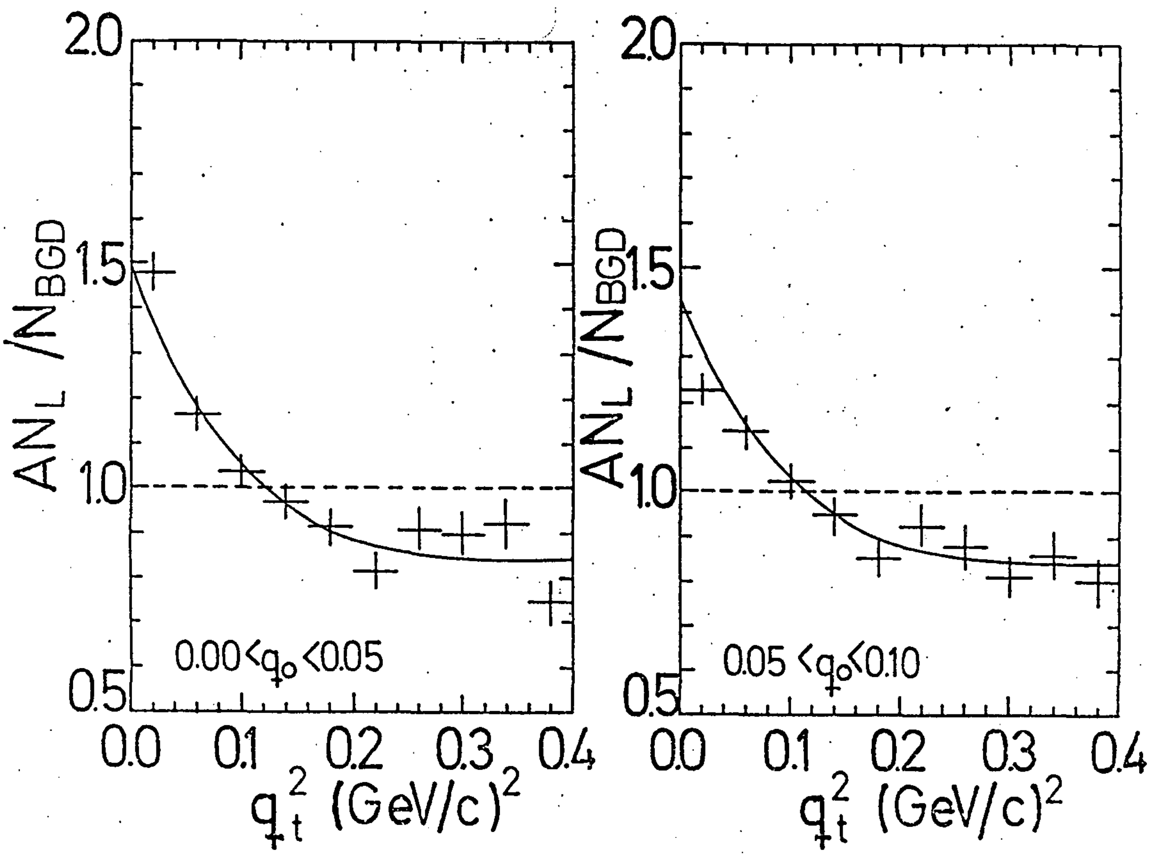

Fig. 10. $\bar{p}p$ data at 9.1 GeV/c from Carlsson et al.[23] The data is in terms of the KP variables and gives $r = 1.3 \pm 0.1$ fm, $c\tau = 0.9 \pm 0.2$ fm and $\lambda = 0.8 \pm 0.1$.

Table III. Two body correlations. Results from fits of $(1 + \alpha e^{-\beta Q^2})\gamma$ to the $\pi\pi$ data at the J/ψ for various δ regions. Mark II at SPEAR[19] data shown in Figs. 12-14.

δ (GeV/c)	α	β (GeV/c)^2	r (fermi)
< 0.1	0.94 ± 0.01	22.2 ± 0.2	0.93 ± 0.01
0.1, 0.2	0.71 ± 0.03	16.0 ± 1.2	0.79 ± 0.03
0.2, 0.3	0.55 ± 0.04	12.6 ± 1.7	0.70 ± 0.05
> 0.3	0.44 ± 0.05	10.5 ± 2.5	0.64 ± 0.07
all δ, $\pi\pi$	0.71 ± 0.03	18.7 ± 0.8	0.85 ± 0.02
all δ, $K\pi$	0.16 ± 0.04	10.0 ± 4.1	0.62 ± 0.12

TASSO preliminary (all δ) 34 GeV[20] data shown in Fig. 15

R_U^L	0.30 ± 0.08	22.5 ± 8.1	0.94 ± 0.17
MC normalized	0.72 ± 0.08	16.2 ± 2.4	0.79 ± 0.06

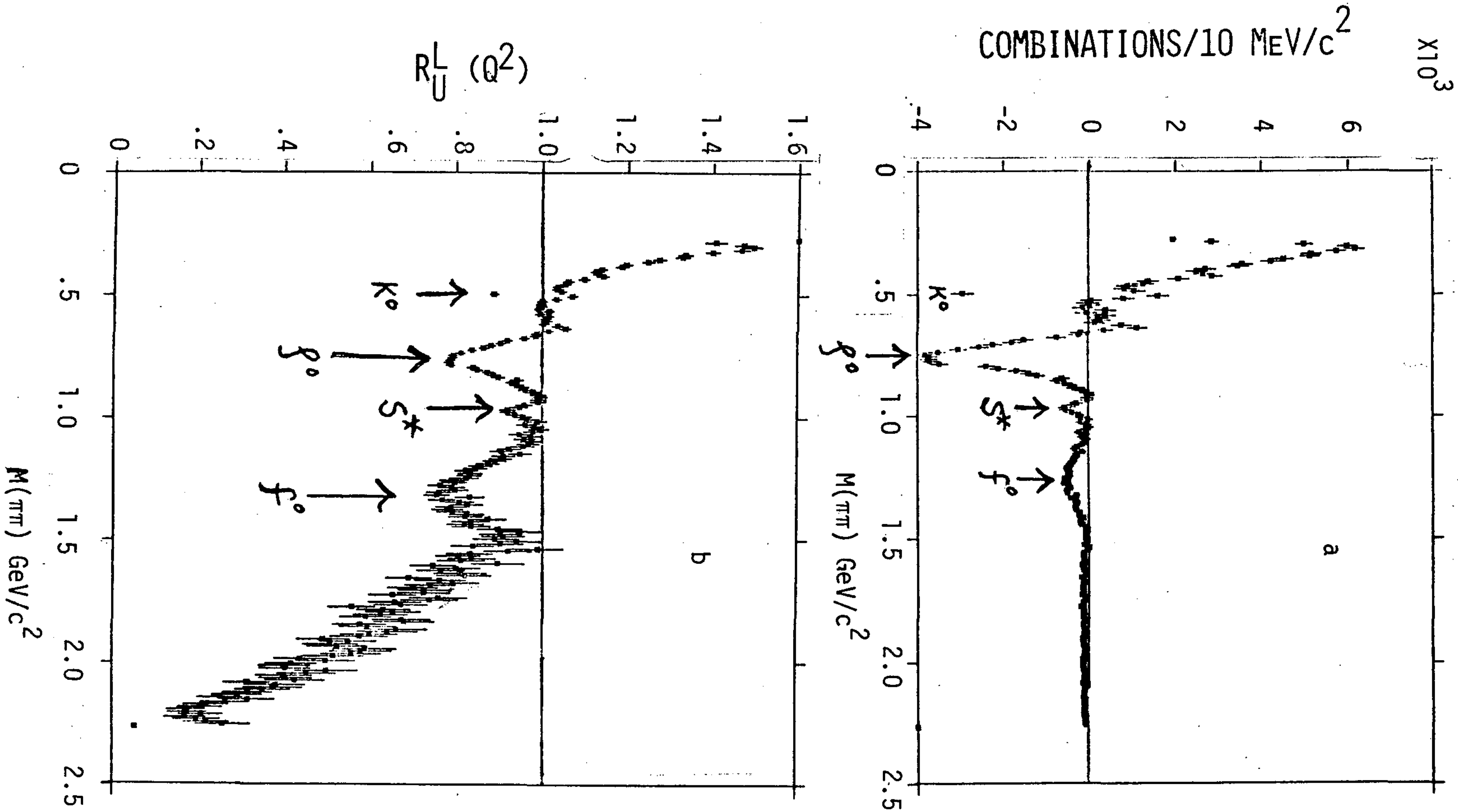

Fig. 11. e^+e^- data at the J/ψ from the Mark II at SPEAR.[19]

a) The difference in LIKE and UNLIKE $M(\pi\pi)$ distributions (normalized to equal areas).

b) The ratio $R_U^L(M)$. Both shown over the entire mass interval.

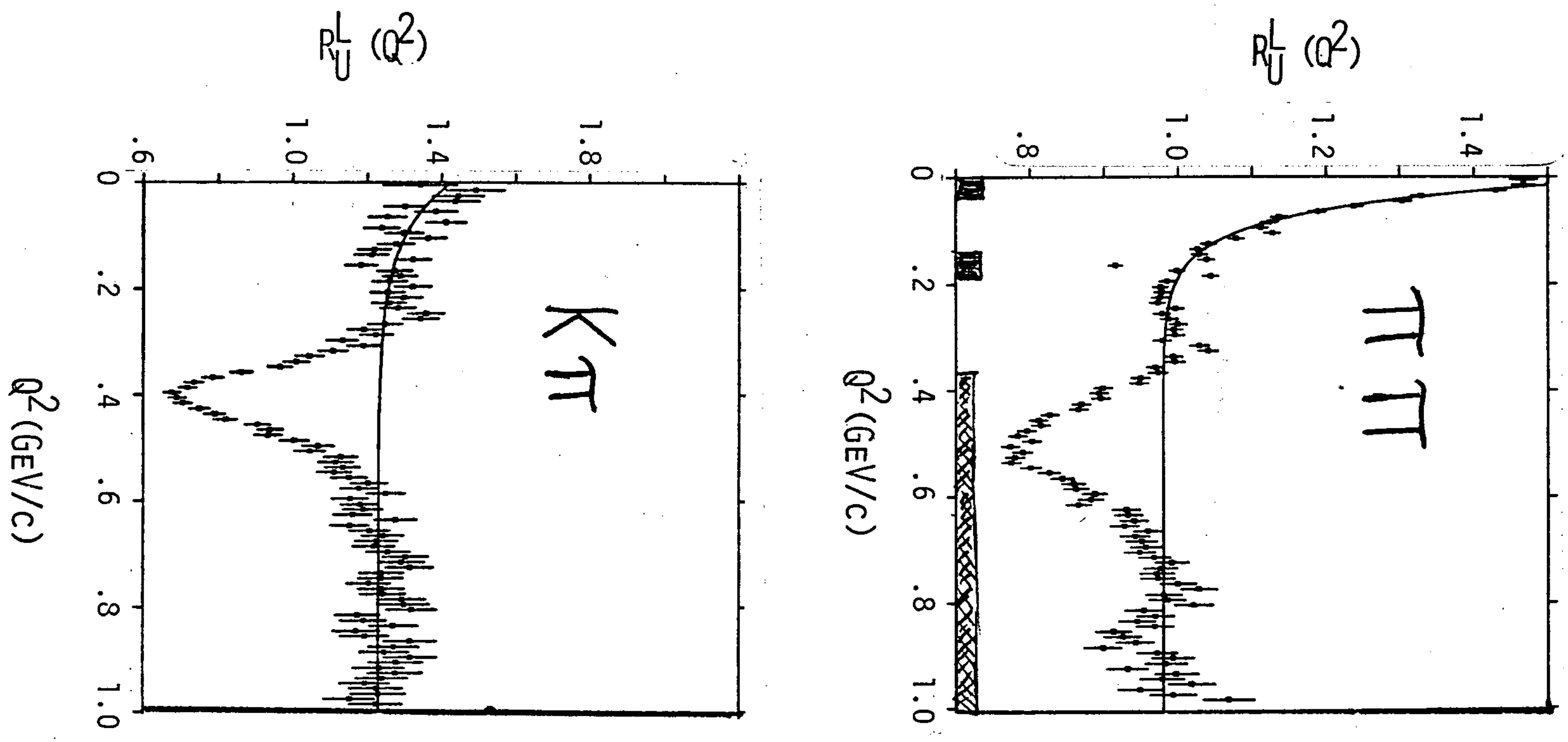

Fig. 12. e^+e^- data at the J/ψ.[19] The ratio $R_U^L(Q^2)$ in a comparison of identical $(\pi^+\pi^+)$ and non-identical $(K^+\pi^+)$ bosons. The shaded regions along the x-axis in (a) represent the regions left out in the fits both here and in Fig. 13. The curve is the result of a fit to $(1+\alpha e^{-\beta Q^2})\gamma$ in the unshaded Q^2 regions below the resonances. For the $K\pi$ case the K^* resonance was left out of the fit.

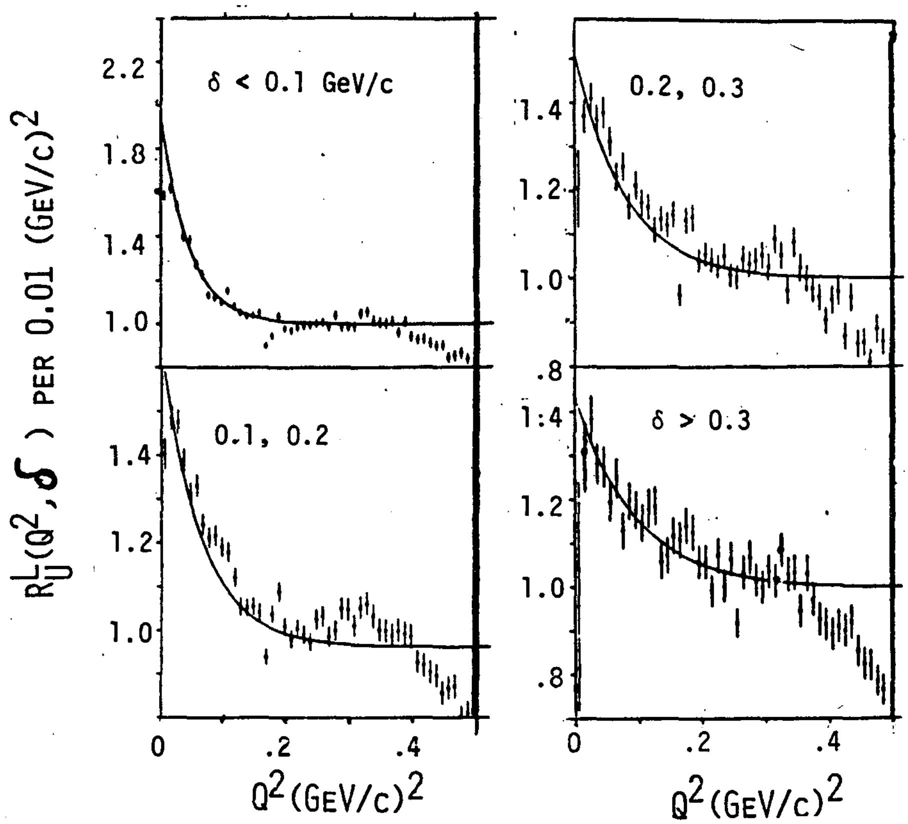

Fig. 13. e^+e^- data from the Mark II at the J/ψ.[19] The variation of the low Q^2 enhancement with δ. $R_U^L(Q^2,\delta)$ is shown for the δ intervals given in the figure. Table III and Fig. 14 give the numerical values of the fits to $(1+\alpha e^{-\beta Q^2})\gamma$.

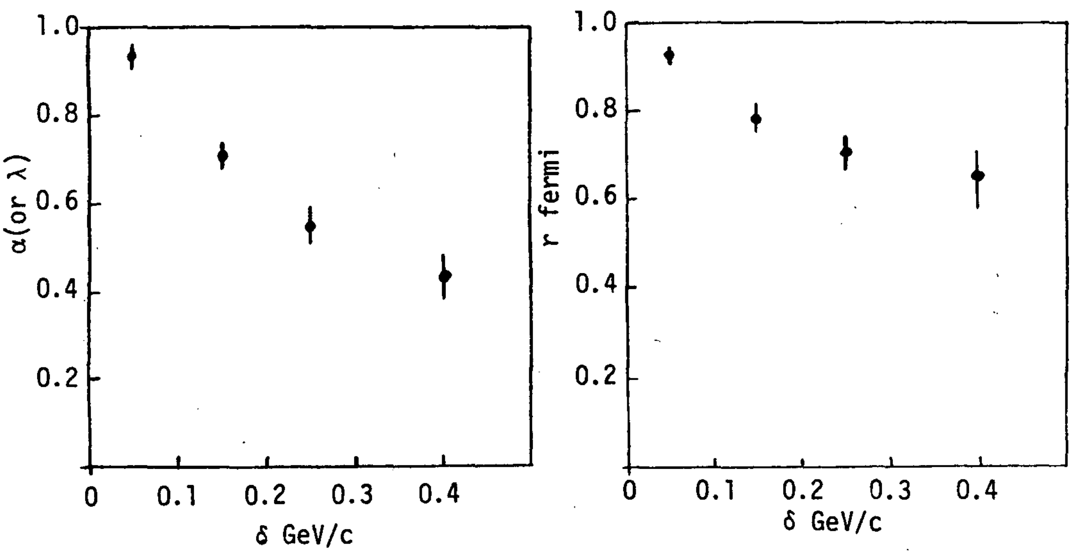

Fig. 14. e^+e^- data from the Mark II at the J/ψ.[19] The variation of the chaoticity and radius with δ.

an incomplete e^+e^- pair elimination. Then we note the K_s^0 (which was only partially eliminated), the ρ^0, s^* and f^0. Finally in Fig. 11b we note a large drop in R_U^L at high masses. This corresponds to the long range charge correlation, namely for an energetic positive forward pion there will be more energetic negative backward pions than positive ones. These pairs give the largest masses. There is one feature which is not understood at present—namely a small peak at $M(\pi\pi) \cong 0.6$ GeV.

The remainder of this discussion concerns itself with the GGLP effect only.

(i) *Demonstration of the BE nature of the GGLP effect.*

The GGLP effect occurs for LIKE charge pions, e.g. $\pi^+\pi^+$ but not for LIKE charge non-identical bosons, e.g. $K^+\pi^+$. This was first looked for by Eskreys et al.[15] and is being clearly demonstrated in our J/ψ data.[19] Fig. 12 gives the LIKE to UNLIKE charge ratio R_U^L for pion pairs and $K\pi$ pairs respectively. The fit to the expression:

$$R_U^L = (1+\alpha e^{-\beta Q^2})\gamma$$

is given in Table III. Where $\alpha \equiv \lambda$, $\beta \equiv r^2$ and γ is an overall normalization factor which, except for the $K\pi$ data, is very close to unity. Here the fit is made such as to exclude Q^2 regions corresponding to resonances. We note the large enhancement at low Q^2 values for the $\pi\pi$ data but not for the $K\pi$ data, while the resonance effects ρ^0 and K^{*0} respectively are clearly present in both data sets. The small α value for $K\pi$ is presumably due to K/π misidentification which allows a small feed through of $\pi\pi$ events.

(ii) *The J/ψ data compared with the 4-7 GeV data.*

In Fig. 13 I present the data at the J/ψ as a function of δ. Here $\delta = ||\vec{p}_1|-|\vec{p}_2||$ is essentially the same as the KP variable q_0. The corresponding fits are given in Table III and Fig. 14. We note that as $\delta \rightarrow 0$, α (i.e. λ) approaches 1, as expected for complete chaoticity. The radius r decreases as δ increases. On the other hand, the preliminary results for the 4-7 GeV region for $e^+e^- \rightarrow 3\pi^+ + 3\pi^- + x$ and $\delta < 0.2$ appear distinctly different with $\alpha = 0.52 \pm 0.06$, however with essentially the same radial value. See Table IV. This result implies evidence for coherence—associated with the onset of jets in e^+e^- annihilation as suggested by Giovannini and Veneziano.[14] Here however the qualifications on the interpretation of λ in section 2.5 should be borne in mind. To settle this question it would be very interesting to study the GGLP effect on and off the Υ resonance where,

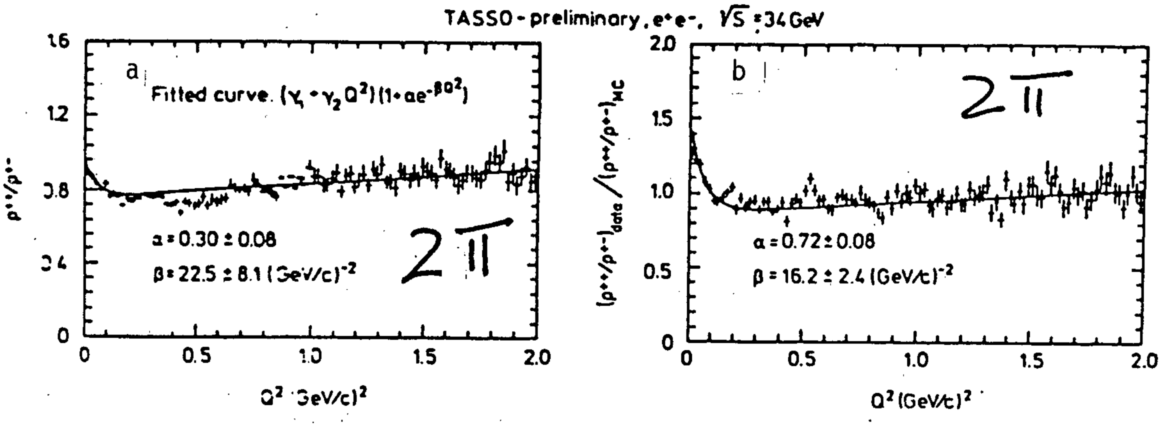

Fig. 15. e^+e^- data from TASSO (preliminary) at 34 GeV.[20] a,b) Two pion correlations as function of Q^2 without and with MC corrections respectively.

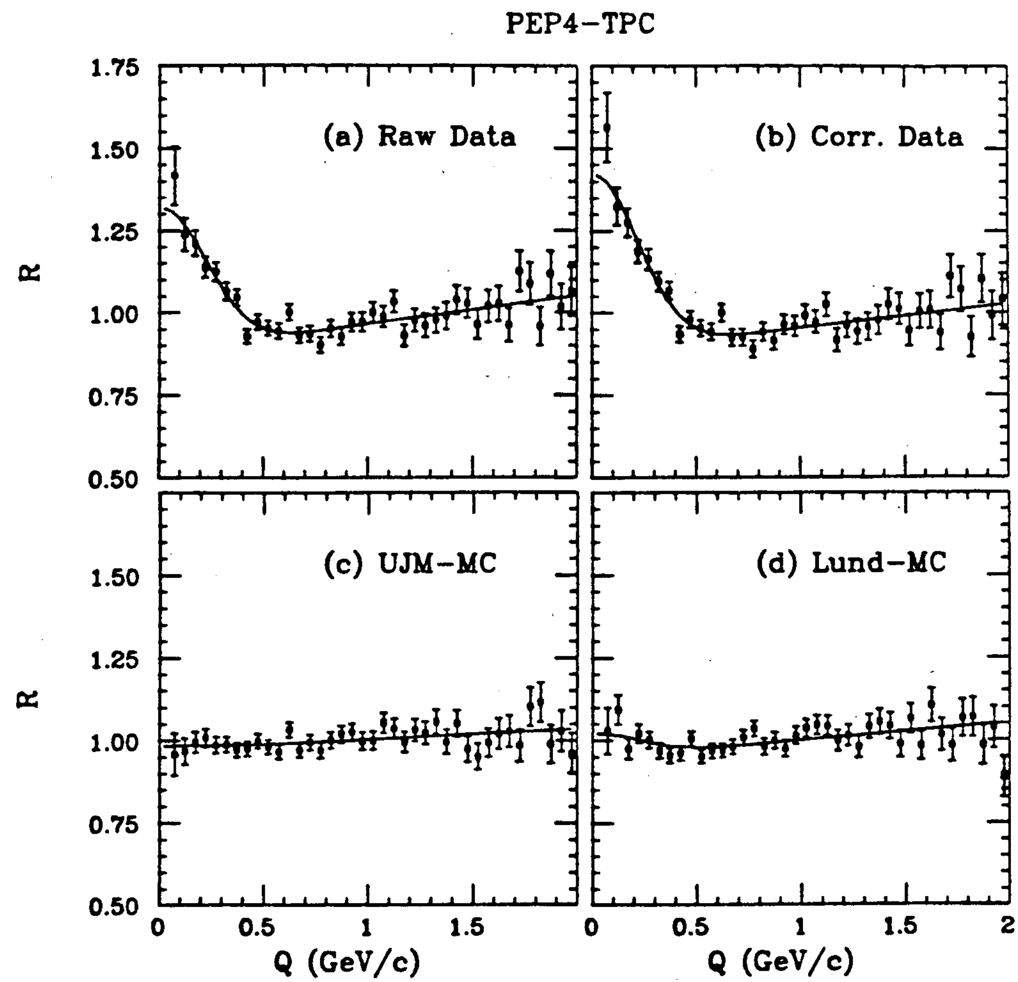

Fig. 16. e^+e^- data from PEP4-TPC at 29 GeV. a,b) Two body correlation as a function of $Q = \sqrt{Q^2}$. Reference sample comes from truncated event mixing. c,d) Correspond to the same technique applied to MC samples without explicit BE correlations for the "Uncorrelated Jet Model" and Lund Model respectively.

just as the J/ψ, a large degree of chaoticity is expected.

(iii) *The 29 GeV to 34 GeV region.*

W. Koch[20] has presented preliminary results from the TASSO experiment at 34 GeV. This data is sensitive to whether a normalization to UNLIKE data or to Monte Carlo calculations are used. See Fig. 15 and Table III. Very recent results have been presented by Aihara et al.[21] from the TPC experiment at PEP at an energy of 29 GeV. Fig. 16 shows this data. In this work a modified event mixing technique was used for the reference sample. The data was analyzed in terms of Q^2 and also the KP variables. Both methods give $\lambda \cong 0.6$ but rather different effective radii, 0.65 and 1.25 fermi respectively. In this experiment the power of the TPC is used to eliminate non-pion background. The authors also attempted to observe a shape dependent effect and stress that $\lambda < 1$ does not necessarily imply coherence.

3.4 Experimental Results: Relativistic Heavy Ion Collisions (RHIC)

The latest field which has applied the GGLP effect is that of heavy ion collisions.[22,24] Here there is considerable interest in attempting to find the size and shape of the interaction region or "hot spot" from which pions are emitted. The variables used are principally those of GKW,[9] namely $|\vec{q}|$ and q_0, the 3-momentum and energy differences respectively. In RHIC experiments typical λ values are from 0.4 to 1.0, radii are distinctly larger than for hadron-hadron or e^+e^- reactions namely 1 to 4 fermi and comparable $c\tau$ values.

Expressed as a function of A the atomic number of one of the projectiles, Zajc et al.[22] quote $r = (1.0 \pm 0.2)A^{1/3}$ fermi and $c\tau = (0.8 \pm 0.3)A^{1/3}$ fermi. Their results are shown in Fig. 17 after a correction for the Coulomb effect "Gamow correction" has been made. This approach may become of particular importance if and when a "quark-gluon plasma" is observed at very high nucleon densities.

3.5 Multiparticle Correlations

Since the BE statistics effects increase quadratically with the number of identical particles, one can also look for 3 and 4 pion correlations. Boesebeck et al.[18] have studied 3 and 4 pion correlations by extending our[3] definitions of two pion CM opening

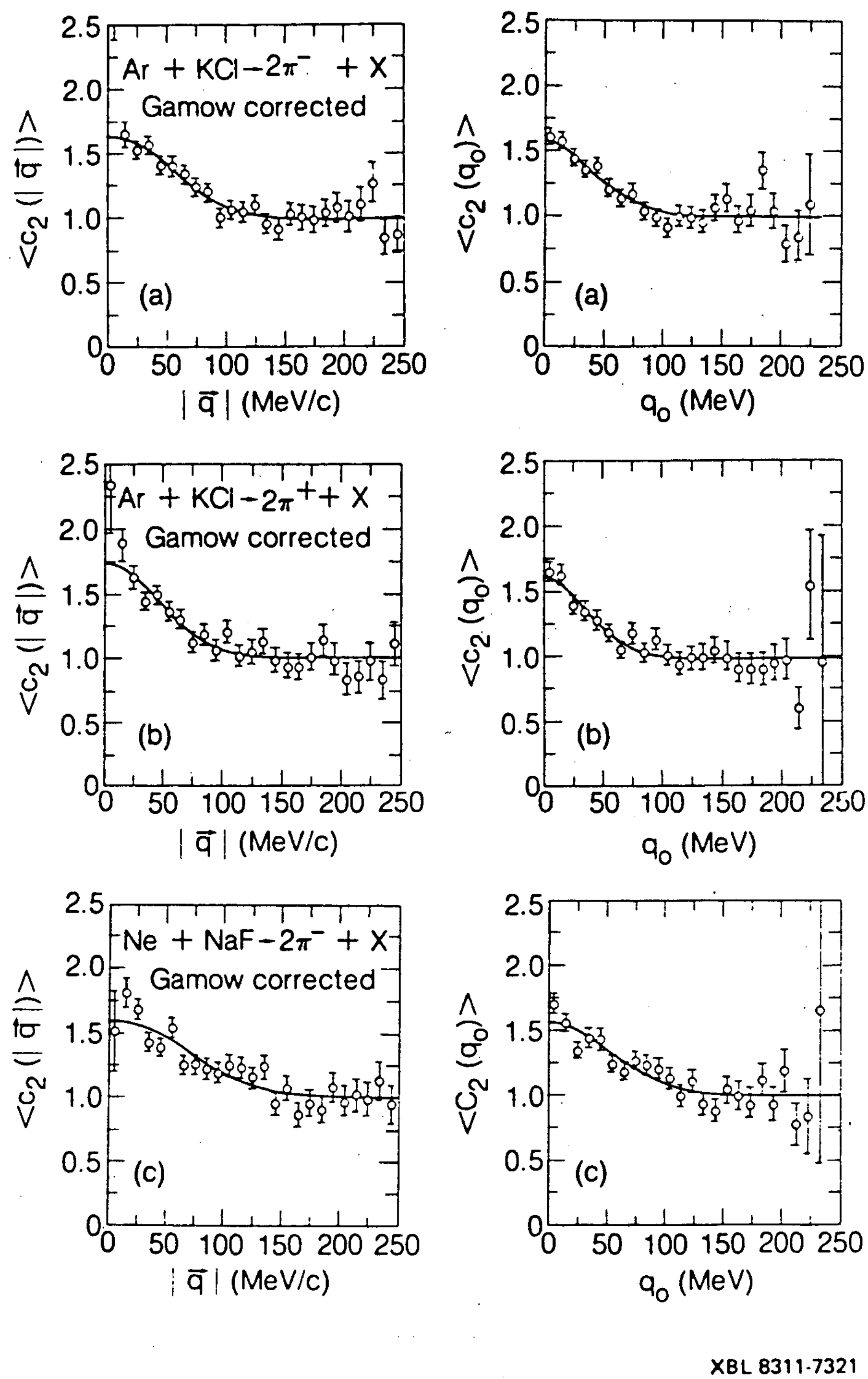

Fig. 17. Relativistic heavy ion collision data[20] in terms of GKW variables. Two pion correlation coefficients are given for the reactions shown in the figures.

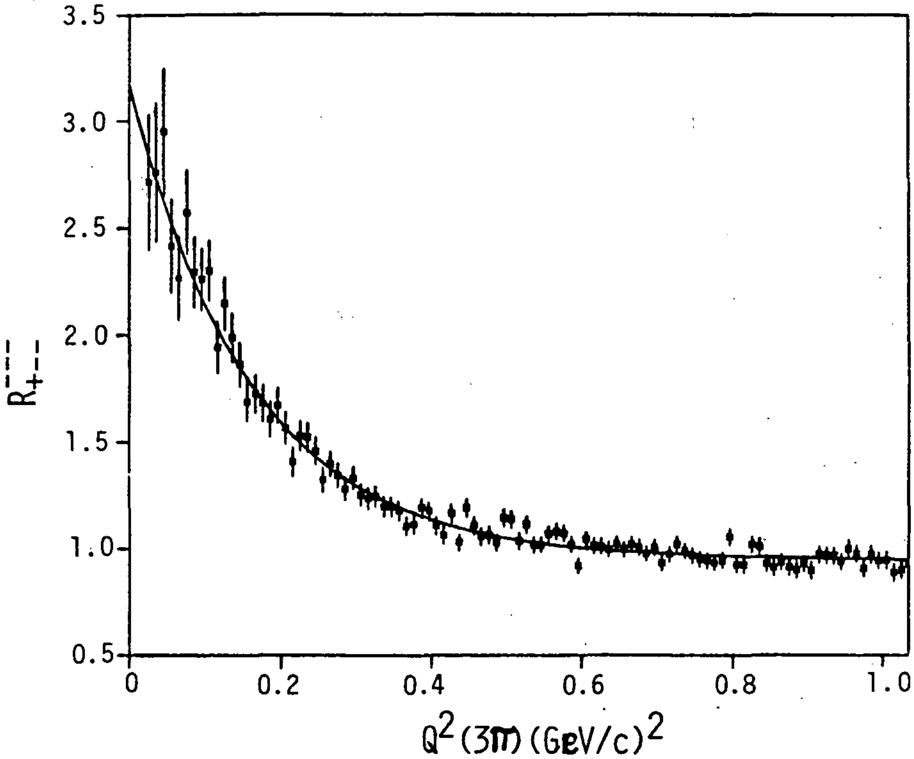

Fig. 18. e^+e^- data from the Mark II at the J/ψ.[19] Three pion correlations as a function of Q^2. The results of the numerical fits are given in Table IV.

Table IV. Results of fits to two and three pion correlations versus Q^2 for various E_{CM} regions and $e^+e^- \to 3\pi^+ + 3\pi^- + X$ final states.

	α_2'	r_2 (fermi)	α_3'	r_3 (fermi)
Mark II[19]				
J/ψ	0.89 ± 0.03	0.78 ± 0.02	2.33 ± 0.06	0.49 ± 0.01
4-7 GeV	0.52 ± 0.06	0.77 ± 0.08	1.09 ± 0.11	0.39 ± 0.04
TASSO[20] preliminary				
34 GeV	R_U^L		0.78 ± 0.12	0.47 ± 0.04
34 GeV	MC normalized		1.43 ± 0.22	0.53 ± 0.04

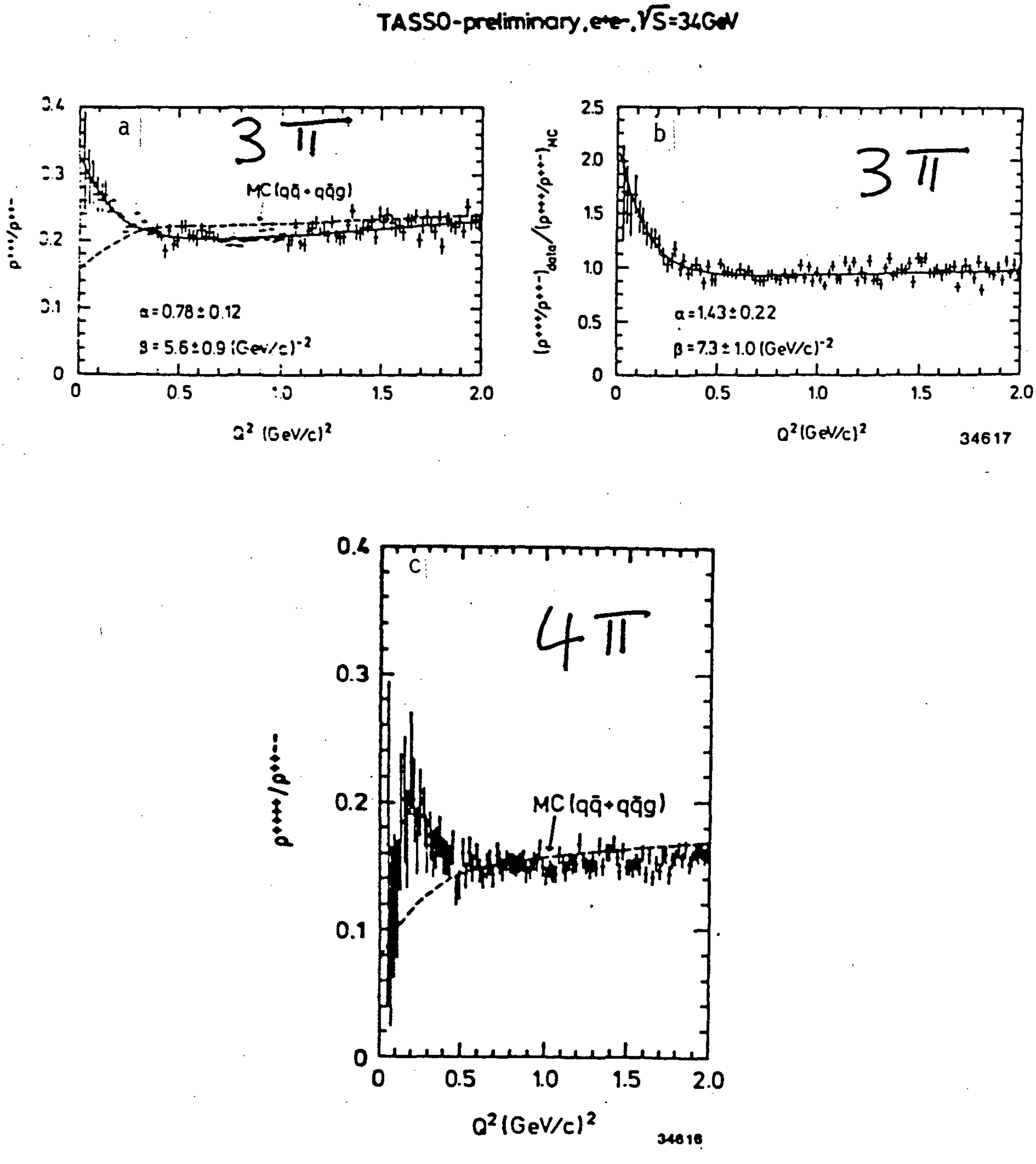

Fig. 19. e^+e^- data from TASSO (preliminary) at 34 GeV.[20] a,b) Three pion correlations as a function of Q^2 without and with MC corrections. c) First attempt to study four pion correlations.

angle to a larger number of pions. They concluded however that the 3 and 4 pion distribution could be interpreted as reflections of the two LIKE pion GGLP effect. Theoretical discussions of multiparticle correlations are given by Giovannini and Veneziano[14] and also by Biyajima et al.[25]

In the investigation of the GGLP effect in the Mark II detector at the J/ψ, and higher SPEAR energies $E_{CM} = 4\text{–}7$ GeV, we have also studied 3 LIKE pion effects. Here we define

$$Q^2(123) = M^2(123) - (\sum_{i=1}^{3} m_i)^2.$$

in analogy to $Q^2(12)$. In this case we can obtain a maximum value of $\alpha_3 = 5$ or $1 + \alpha_3 = 6$ as the maximum intercept. For 3 pions we can define the symbol R^{---}_{+-0} to correspond to a $\pi^-\pi^-\pi^-$ distribution with a $\pi^+\pi^-\pi^0$ distribution as the reference sample. Ideally we want to study R^{---}_{+-0} but this is not readily available experimentally. Instead I show R^{---}_{+--} from the Mark II experiment[19] at the J/ψ in Fig. 18 and data from TASSO[20] at 34 GeV in Fig. 19. Here the reference distribution $\pi^+\pi^-\pi^-$ itself has a BE enhancement effect. Thus $1 + \alpha_3 \leq (1 + \alpha_2')(1 + \alpha_3')$ could be as large as 6 at the J/ψ. Here α_2' corresponds to the two body correlation for events of the type $e^+e^- \to 3\pi^+ + 3\pi^- + x$ and α_3' is the factor from the fit to R^{---}_{+--}. Here again the 4-7 GeV region shows considerably lower values of α_3', see Table IV.

The enhancements are very striking but more work is needed to interpret these results, particularly we need to understand why the fits for the 3 LIKE charge pion states give considerably lower values for the radius, viz. ≤ 0.5 fermi, than the fits for two LIKE charge pion states.

Finally Koch et al.[20] have also attempted to look at 4 pion correlation. This preliminary TASSO result is shown in Fig. 19.

I wish to thank Mrs. Marian Golden and Valerie Heatlie for the careful typing and assembly of this paper.

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APPENDIX

Excerpts from the 1959 and 1960 papers on the GGLP effect from References 2 and 3

VOLUME 3, NUMBER 4 PHYSICAL REVIEW LETTERS AUGUST 15, 1959

PION-PION CORRELATIONS IN ANTIPROTON ANNIHILATION EVENTS*

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We have observed angular correlation effects between pions emitted from antiproton annihilation events. This experiment was carried out with a separated antiproton beam[1] of momentum $p_{\bar{p}}$ $=1.05$ Bev/c. A total of 2500 annihilation events were observed in 20 000 pictures taken with the Lawrence Radiation Laboratory 30-in. propane bubble chamber.

Pion pairs formed by the charged pions emitted in an antiproton-annihilation event can be considered in two groups: viz., like pairs (in the isotopic-spin state $I=2$) and unlike pairs (in the isotopic-spin states $I=0$, 1, or 2). We searched for correlation effects in these separate groups. Our results show that the distribution of the angles between pions of like charges is strikingly different from the distribution of the angles between pions of unlike charges. The angles between pion pairs were computed in the center of mass of the antinucleon-nucleon system.[2] The results shown in Fig. 1 were obtained from the analysis of the "hydrogenlike" events in which four and six charged pions, respectively, are emitted. We define as "hydrogenlike" those events giving rise to an equal number of positive and negative pions. Events showing visible evaporation prongs are excluded from this sample. The curves shown in Fig. 1 were calculated on the basis of the statistical model, expressed in the Lorentz-invariant phase-space[3] (LIPS) form, for pion production from a nucleon-antinucleon annihilation. This model imposes energy and momentum conservation, but no other constraints. The distribution of the pion-pair angles θ_{12} for an annihilation into n pions of mass μ is[4]

$$\phi_n(\cos\theta_{12}) = \iint p_1 p_2 F_{n-2}(W''^2) d\omega_1 d\omega_2,$$

with integration limits from $\omega_1 \geq \mu$, $\omega_2 \geq \mu$ to max

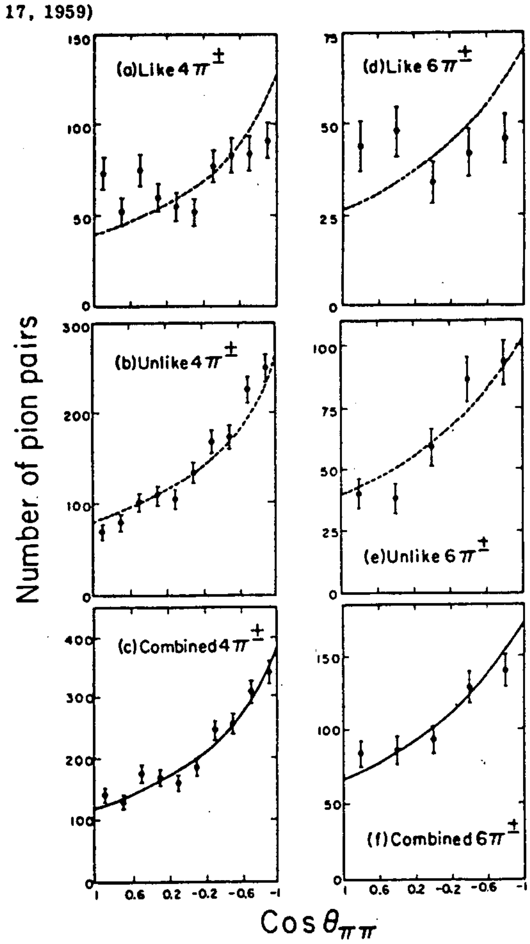

FIG. 1. Distribution of angles between pion pairs as a function of $\cos\theta_{12}$. The curves correspond to calculations on the Lorentz-invariant phase-space (LIPS) model. The deviations of the experimental distribution from the LIPS model are discussed in the text.

Fig. 1. Observation of difference between LIKE and UNLIKE charge pion pairs. Excerpt from Ref. 2.

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Table I. The ratio γ for like and unlike pion pairs and for the Lorentz-invariant phase-space (LIPS) model.[a]

$N_{\pi\pm}$	Like pions		Unlike pions		All pions combined		Statistical model
	No. of pairs	γ	No. of pairs	γ	No. of pairs	γ	γ
4	702	1.23±0.11	1404	2.06±0.12	2106	1.72±0.08	1.74
6	214	1.06±0.15	318	1.91±0.23	532	1.50±0.13	1.60

[a]The ratio γ is the number of pion-pair angles greater than 90° compared to those smaller than 90°. The errors quoted are the standard deviations based on the number of pairs observed.

values given by $W''^2=(n-2)^2\mu^2$. Here we define

$$W''^2=(W-\omega_1-\omega_2)^2-(\vec{p}_1+\vec{p}_2)^2;$$

$F_{n-2}(W''^2)$ is the Lorentz invariant phase space for $(n-2)$ pions, W is the total energy in the center of mass of the antinucleon-nucleon system, and we have

$$\cos\theta_{12}=\vec{p}_1\cdot\vec{p}_2/|p_1||p_2|.$$

To compare with the experimental distributions for events with $n_\pm$ charged pions, averages over ϕ_n values with $n \geq n_\pm$ are required. This takes into account the presence of additional neutral pions in the annihilation. We have used the frequency distribution of the pion multiplicity in annihilation events, as reported elsewhere,[5] for computing these averages.

In Table I we have expressed the distribution of pair angles in terms of the ratio, γ, of the number of pion-pair angles greater than 90° to the number smaller than 90°. As can be seen from Fig. 1 (c) and (f), the pion-pair distribution of like and unlike pions combined agrees very well with the LIPS model.[6] The distribution of angles between pions [Fig. 1 (a) and (d)] deviates distinctly from the LIPS model. The γ_{like} values for $4\pi^\pm$ and $6\pi^\pm$ differ from γ_{LIPS} by 5 and 3.4 standard deviations, respectively, in the direction of greater isotropy. The distribution of pion-pair angles for unlike pions appears to be slightly more asymmetric than the LIPS model predicts. In this case, the values of γ_{unlike} are 2 and 1.5 standard deviations, respectively, removed from the value given by the LIPS model.

We have also computed the invariant quantity

$$Q_{12}^2=(\omega_1+\omega_2)^2-(\vec{p}_1+\vec{p}_2)^2,$$

for each pion pair. Here Q is the total energy in the center of mass of the pion-pion system. These distributions are given in Fig. 2. Within statistical limits no significant difference between the Q^2 distribution of like and unlike pion pairs has been observed. Curves shown in Fig. 2 were also computed on the basis of the LIPS statistical model. The experimental Q^2 distributions show no marked

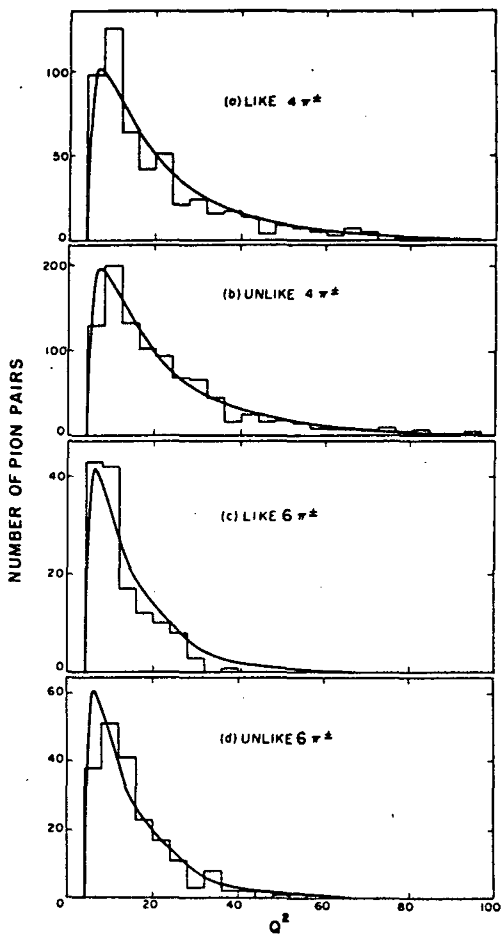

FIG. 2. The distribution of Q^2, the square of the total energy in the center of mass of the pion-pion system in units of μ^2.

Fig. 2. Experiment designed to search for ρ resonance. There were however insufficient statistics to find ρ. The ρ mass squared in units of μ^2 called Q^2 here is at 30.7. Excerpt from Ref. 2.

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Influence of Bose-Einstein Statistics on the Antiproton-Proton Annihilation Process*

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Recent observations of angular distributions of π mesons in $\bar{p}$-p annihilation indicate a deviation from the predictions of the usual Fermi statistical model. In order to shed light on these phenomena, a modification of the statistical model is studied. We retain the assumption that the transition rate into a given final state is proportional to the probability of finding N free π mesons in the reaction volume, but express this probability in terms of wave functions symmetrized with respect to particles of like charge. The justification of this assumption is discussed. The model reproduces the experimental results qualitatively, provided the radius of the interaction volume is between one-half and three-fourths of the pion Compton wavelength; the dependence of angular correlation effects on the value of the radius is rather sensitive. Quantitatively, there seems to remain some discrepancy, but we cannot say whether this is due to experimental uncertainties or to some other dynamic effects. In the absence of information on π-π interactions and of a fully satisfactory explanation of the mean pion multiplicity for annihilation, we wish to emphasize the preliminary nature of our results. We consider them, however, as an indication that the symmetrization effects discussed here may well play a major role in the analysis of angular distributions. It is pointed out that in this respect the energy dependence of the angular correlations may provide valuable clues for the validity of our model.

I. INTRODUCTION

RECENTLY a study has been made[1] in a propane bubble chamber of "hydrogenlike" annihilations of antiprotons of 1.05-Bev/c laboratory-system momentum, corresponding to an energy release of 2.1 Bev in the center-of-mass system. A hydrogenlike event is defined as one in which equal numbers of π^+ and π^- mesons are produced and in which no visible evaporation prongs appear.[2] The experiment indicates[1] that the distribution of the angle between pairs of pions (in the c.m.-system of $\bar{p}$-p) deviates from the prediction of the conventional statistical model. In particular it was found that there is a clear difference between the angular distribution for pion pairs of like charge and that for pairs of unlike charge. In the statistical model in its usual sense, there is no room for distinctions of this kind.

It is the purpose of this paper to indicate a simple refinement of the statistical model which could possibly explain the bulk of the effect, and which consists of taking into account the influence of the Bose-Einstein (BE) statistics for pions of like charge. As we show in what follows, such an interpretation appears to reproduce the experimental results qualitatively—provided, however, that the radius of the volume of strong interactions is about $\frac{3}{4}$ times the π Compton wavelength, which is a physically reasonable order of magnitude. The dependence of the angular effects on the interaction radius appears to be a sensitive one. Hence, it would seem that such effects may provide valuable information on the annihilation mechanism.

It should be stressed from the outset, however, that results of this study should not be construed to imply that detailed dynamical effects (such as, for example, π-π interactions) are definitely negligible in the discussion of the kind of phenomena considered here. The present stage of both our experimental and our theoretical knowledge of the annihilation process seems to us to be far too early to make such categorical statements. In the concluding remarks (Sec. IV), we briefly discuss the dependence of the BE effect on the available energy for annihilation. This gives one instance of how further experimental study may reveal whether or not the present considerations provide substantially the correct approach to the problem. It may directly be noted, however, that the symmetrization effects which we shall now outline are relevant regardless of whether π-π interactions are large or small.

For the statement of our ideas, it is helpful to recall first what the assumptions of the usual statistical model (SM) are. For definiteness, consider the system

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† Permanent address: Institute for Advanced Study, Princeton, New Jersey.
[1] G. Goldhaber, W. B. Fowler, S. Goldhaber, T. F. Hoang, T. E. Kalogeropoulos, and W. M. Powell, Phys. Rev. Letters 3, 181 (1959).
[2] All center-of-mass transformations were made on the assumption that the struck proton is at rest. From the known annihilation cross sections in carbon and hydrogen and from the π-multiplicity distribution, it was deduced that about 85% of the hydrogenlike events correspond to annihilations on hydrogen.

Fig. 3. Interpretation of the data in terms of B-E statistics. Excerpt from Ref. 3.

else.[8] But if we adhere to isotopic spin conservation and only consider equal weights for $I=0,1$ states, the number of projections of the charge partition (n_+,n_-,n_0) is in general smaller than n', and therefore some symmetries other than that of like-particle kind may remain.

Even so, the approximation is perhaps not too bad. In Appendix I we discuss this in a little more detail; there it is shown that for $N=4$ the assumption of equal weight for the projections of the charge partition (7) into the various isotopic spin states happens to give *exactly* the BE effect between like particles only. It is then shown, again for $N=4$, that the SM assumption of equal weight for the isotopic spin states [rather than for the projection (7)] leads to a small deviation from the pure like-particle-only effect. For the case of $N=5$, 6, no such detailed studies have been performed, but it is made plausible that there also the present picture may be a reasonable approximation.

Thus it would appear that, as a first orientation at least, the present assumption of BE symmetrization is not much less well-founded than any other aspect of statistical considerations in this domain. We repeat, however, that we consider this work as an orienting approach rather than as a definitive answer and wish to give one more reason for this reservation. Of course, an adequate model should at the same time give a reasonable account of all combined aspects of the annihilation process, especially also of the mean multiplicity. The usual SM needs a radius of ~ 2.5 $\hbar/\mu c$ to account for multiplicities.[9] Such a large radius is devoid of direct physical meaning. As we argue in Sec. IV, the inclusion of the BE effect tends to decrease this value of the radius, but at least in the way we proceed here, we cannot hope to fit the multiplicities with a value ~ 0.75 $\hbar/\mu c$ for the radius, which was quoted above in connection with the angular-correlation effect. Until this problem is resolved, our results must be considered as tentative. Possibly improved angular-momentum considerations may here bridge the gap, or, perhaps the presence of a π-π interaction is making itself felt.[10]

II. STATISTICAL MODEL WITH BE-CORRELATIONS

A. The Correlation Function

As an orientation, consider first the case of $N=2$ with two identical particles, having momenta $\mathbf{p}_1$, $\mathbf{p}_2$. The corresponding $P_2(\Omega)$ plays an important role in what follows and is denoted by $\psi(12)$. Thus we can write

$$\psi(12)=\int\int|\phi^S(1,2)|^2 d\mathbf{r}_1 d\mathbf{r}_2, \tag{9}$$

where we integrate twice over a sphere $\Omega=4\pi\rho^3/3$, and

$$\phi^S(1,2)=(1/2^{\frac{1}{2}}V)\{\exp[i(\mathbf{p}_1\cdot\mathbf{r}_1+\mathbf{p}_2\cdot\mathbf{r}_2)] +\exp[i(\mathbf{p}_2\cdot\mathbf{r}_1+\mathbf{p}_1\cdot\mathbf{r}_2)]\}. \tag{10}$$

Thus, on integration we obtain[11]

$$\psi(12)\approx 1+9\left(\frac{\cos t}{t^2}-\frac{\sin t}{t^3}\right)^2,\ t=|\mathbf{p}_1-\mathbf{p}_2|\rho,\ \text{(sphere)}. \tag{11}$$

Evidently $\psi(12)$ as defined by Eqs. (9) and (10) no longer depends only on the size of the interaction volume Ω but also on its shape. It is premature to discuss this shape dependence in any detail, but one point is of some computational interest, namely that $\psi(12)$ for a spherical model, given by Eq. (11), differs very little from $\psi(12)$ for a Gaussian-shaped volume:

$$\psi(12)=\int\int|\phi^S(1,2)|^2\exp[-(r_1^2+r_2^2)/2\lambda]d\mathbf{r}_1 d\mathbf{r}_2$$

$$\approx 1+\exp(-s^2),\ s=|\mathbf{p}_1-\mathbf{p}_2|\lambda^{\frac{1}{2}},\ \text{(Gaussian)}, \tag{12}$$

where we integrate twice over all space. This well-known property of the Fourier transform of a sphere relative to that of a Gaussian is shown in Fig. 1 where the two curves refer to a ratio of ρ to $\lambda^{\frac{1}{2}}$ given by

$$\rho=2.15\lambda^{\frac{1}{2}}. \tag{13}$$

The Gaussian model simplifies some computations to follow and therefore we shall adopt it from here on. However, we shall continue to refer to the "radius" ρ of the interaction volume—by which we mean the quantity related to λ by Eq. (13).

In one further respect we have used an argument of convenience to simplify the calculations as much as possible before reverting to numerical evaluation techniques. Instead of Eq. (12) we have actually used its relativistic counterpart,

$$\psi(12)=1+e^{-\lambda x_{12}}, \tag{14a}$$

where

$$x_{12}=(\mathbf{p}_1-\mathbf{p}_2)^2-(\omega_1-\omega_2)^2. \tag{14b}$$

This is indeed convenient because we have to deal with integrals of the type (5) but with a number of ψ functions—the "correlation functions"—entering into the integrand. Thus the relativistic scalar form of $\psi(x)$ makes it possible to make simplifying Lorentz transformations on the integrand. Of course, it must be asked how much difference it makes to use Eq. (14) as

[8] Proof: if all states have equal weight, we can as well choose a set of base states that have the following properties: (a) they have the desired BE symmetry to begin with; (b) they are orthogonal; (c) their number is just equal to n'. For an example of such a set of states for $N=4$, see Appendix *.

[9] See for example O. Chamberlain, G. Goldhaber, L. Jauneau, T. Kalogeropoulos, E. Segrè, and R. Silberberg, Phys. Rev. 113, 1615 (1959).

[10] E. Eberle, Nuovo cimento 8, 619 (1958); T. Gôto, Nuovo cimento 8, 625 (1958); and F. Cerulus, Nuovo cimento 14, 827 (1959).

[11] From here on we use the symbol $\approx$ to denote equality apart from such constant factors that do not affect the angular correlations under consideration.

Fig. 4. The symmetrized wave function in terms of the Lorentz invariant momentum transfer given here for both a spherical and Gaussian pion source distribution. Excerpt from Ref. 3.

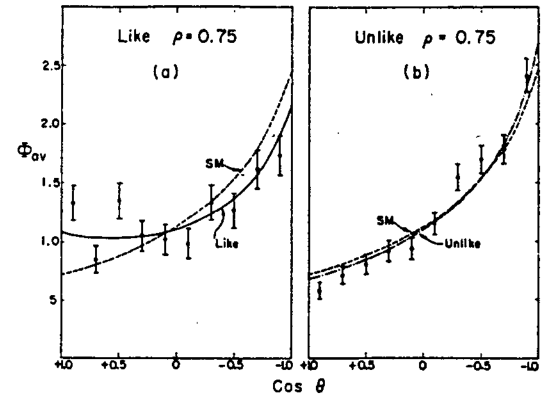

FIG. 6. The functions $\Phi_{av}(\cos\theta)$ computed at $\rho=0.75$ are compared with the experimental distribution of angles between pion pairs. Figures 6(a) and 6(b) give the distributions for like and unlike pions respectively. Also shown in each is the curve for $\Phi_{av}^{SM}(\cos\theta)$, the statistical distribution, without the effect of correlation functions. Here Φ_{av} represents an average of Φ_4, Φ_5, and Φ_6, weighted according to the individual charge channels. The experimental data comes from reference 1 (see also Table I, footnote a).

It is clear from this figure that the fit to the experimental data is improved for both like and unlike pions by the introduction of BE correlation functions.

In Tables I and II we give the experimentally determined values for γ together with a series of γ values calculated for various radii of interaction. An inspection of Table I, which lists also γ^{SM}, shows again that the bulk of the experimentally observed deviations from the SM can be accounted for by our calculations with a reasonable choice of ρ (i.e., ρ between $\frac{1}{2}$ to $\frac{3}{4}$ of $\hbar/\mu c$).[14] It cannot be concluded now whether the remaining discrepancy between experimental results and the SM including BE correlation effects, as evaluated here, is due to experimental uncertainty or to inadequacies of our model.

IV. CONCLUDING REMARKS

We have seen that the BE symmetrization leads to a fairly satisfactory possible interpretation of the observed angular distributions. We believe that this conclusion is of importance for the assessment of evidence for the existence of the strength of possible π-π interactions. The least the present results indicate is that if one wishes to extract information about such interactions from annihilation phenomena, such kinematic symmetry effects as here discussed must always be taken into account.

It may be asked whether further information can lead to arguments for or against the model here employed. Several possibilities exist for getting such information. In the first place one may study six- and higher-prong stars by the same method. Secondly, if the BE symmetrization is the major source for the deviations from the usual SM, this implies a specific dependence of quantities like γ^u, γ^l on the available annihilation energy, W. For the case $N=4$, this dependence is shown in Fig. 7. Here we have computed γ_4^l as a function of ρ for various values of W, the available energy in the center-of-mass system. We have chosen for W the energies 1.88, 2.5, and 4.4 Bev corresponding to $\bar{p}$ laboratory momenta of 0, 2.25, and 6 Bev/c, respectively. It can be seen from Fig. 7 that the correlation effects occur at smaller values of the radius as the energy increases. If a radius of interaction is a meaningful quantity for the annihilation and does not depend critically on the incident antiproton energy, it might be expected that the correlation effects due to BE statistics will decrease at higher bombarding energy. Studies of correlation effects as a function of W may thus be a test for the ideas discussed in this paper. Of course, with increasing W, the relative fraction of four-pion annihilations will decrease. It is therefore indicated that if one wishes to pursue the annihilation process in more detail, an unambiguous separation into the various individual multiplicities will become quite imperative. Only if this is done will curves like those of Fig. 7 and similar ones for other given N be of any use.

Finally, a comment may be made about the question of the mean pion multiplicity. It has been suggested by various people that the high ρ value obtained from the SM may be reduced by taking into account the

[14] It should be noted that the γ distribution, calculated by using the noninvariant form of the correlation function $\psi(x)$, will probably give a poorer fit to the experimental data than the invariant form. This is illustrated in Fig. 5.

TABLE I. Comparison between the experimental values for γ^l and γ^u and the corresponding values derived by use of the BE correlation functions for $\rho=0.5$ and 0.75. Also shown is the value for the usual Fermi SM. All the theoretical values have been averaged over the four-, five-, and six-pion distributions as discussed in the text.

	γ_{expt}	γ_{av} $\rho=0.5$	γ_{av} $\rho=0.75$	γ_{av}^{SM}
Like	1.23 ± 0.10[a]	1.41	1.38	1.80
Unlike	2.18 ± 0.12	1.95	1.91	

[a] The experimental data quoted in this paper is essentially the same as given in reference 1. A small improvement in the available data has, however, been incorporated involving (1) some additional events, namely a total of 752 like and 1504 unlike pion pairs coming from $(2^+,2^-,\pi^0)$ stars have been used, and (2) a complete recalculation of all the center-of-mass momentum and angle values making use of the known incident beam momentum $P_{\bar{p}}=1.05$ Bev/c rather than the measured value for each individual annihilation event.

TABLE II. List of computed $(\gamma^l)_{av}$ and $(\gamma^u)_{av}$ values. The values for $\rho=0.5$ and 0.75 are repeated here for clarity.

ρ $(\hbar/\mu c)$	$(\gamma^l)_{av}$	$(\gamma^u)_{av}$
0.3	1.57	1.91
0.5	1.41	1.95
0.75	1.38	1.91
1.0	1.44	1.87
2.0	1.66	1.79

Fig. 5. Fit to the experimental distribution using B.E. symmetrization. Excerpt from Ref. 3.

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