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Essays on the Economics of Climate Change, Biofuel and Food Prices

by

Charles Séguin

A dissertation submitted in partial satisfaction of the
requirements for the degree of
Doctor of Philosophy

in

Agricultural and Resource Economics

in the

Graduate Division
of the
University of California, Berkeley

Committee in charge:

Professor Larry Karp, Co-Chair
Professor Christian Traeger, Co-Chair
Professor Wayne Getz
Professor David Zilberman

Spring 2012

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Abstract

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Doctor of Philosophy in Agricultural and Resource Economics

University of California, Berkeley

Professor Larry Karp, Co-Chair

Professor Christian Traeger, Co-Chair

Climate change is likely to be the most important global pollution problem that humanity has had to face so far. In this dissertation, I tackle issues directly and indirectly related to climate change, bringing my modest contribution to the body of human creativity trying to deal with climate change. First, I look at the impact of non-convex feedbacks on the optimal climate policy. Second, I try to derive the optimal biofuel policy acknowledging the potential negative impacts that biofuel production might have on food supply. Finally, I test empirically for the presence of loss aversion in food purchases, which might play a role in the consumer response to food price changes brought about by biofuel production.

Non-convexities in feedback processes are increasingly found to be important in the climate system. To evaluate their impact on the optimal greenhouse gas (GHG) abatement policy, I introduce non-convex feedbacks in a stochastic pollution control model. I numerically calibrate the model to represent the mitigation of greenhouse gas (GHG) emissions contributing to global climate change. This approach makes two contributions to the literature. First, it develops a framework to tackle stochastic non-convex pollution management problems. Second, it applies this framework to the problem of climate change. This approach is in contrast to most of the economic literature on climate change that focuses either on linear feedbacks or environmental thresholds. I find that non-convex feedbacks lead to a decision threshold in the optimal mitigation policy, and I characterize how this threshold depends on feedback parameters and stochasticity.

There is great hope that biofuel can help reduce greenhouse gas emissions from fossil fuel. However, there are some concerns that biofuel would increase food prices. In an optimal control model, a co-author and I look at the optimal biofuel production when it competes for land with food production. In addition oil is not exhaustible and output is subject to climate change induced damages. We find that the competitive outcome does not necessarily yield an underproduction of biofuels, but when it does, second best policies like subsidies and mandates can improve welfare.

In marketing, there has been extensive empirical research to ascertain whether there is evidence of loss aversion as predicted by several reference price preference theories. Most

of that literature finds that there is indeed evidence of loss aversion for many different goods. I argue that it is possible that some of that evidence seemingly supporting loss aversion arises because price endogeneity is not properly taken into account. Using scanner data I study four product categories: bread, chicken, corn and tortilla chips, and pasta. Taking prices as exogenous, I find evidence of loss aversion for bread and corn and tortilla chips. However, when instrumenting prices, the "loss aversion evidence" disappears.

To my wife, Anne, for her incredible support and great understanding throughout my degree.

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Preface

The starting point of this dissertation is the threat posed by climate change. The Intergovernmental Panel on Climate Change (IPCC), an organization established by the United Nations Environmental Programme and the World Meteorological Organization, provides the best scientific picture of climate change and its potential impacts. In its fourth¹ and most recent assessment report [Randall et al., 2007], the IPCC states that with current policies, the change in climate the planet experiences in the twenty-first century will be much more important than that experienced in the last century. The consequences of predicted changes in climate are far-reaching. Dry regions will become dryer, while both droughts and floods will become more frequent. Ecosystems will be transformed, and oceans are going to acidify. Coastal areas will be threatened by increased erosion and sea level rise. The negative impacts in poor regions will create migratory pressure, and many regions will see increased deaths from, for instance, vector-borne diseases. While some consequences of climate change may have positive impacts, all will require costly adaptation.

Since the publication of the IPCC fourth assessment report in 2007, a lot of attention has been devoted to tipping points and feedbacks in the climate system. The concern is that although climate has appeared to change gradually in response to greenhouse gas (GHG) emissions, it might get to a point where a threshold is crossed and climate or its consequences start to change more rapidly. Hence the optimal climate policy must incorporate these feedbacks. Once a policy is determined, instruments must be used to reach the policy objectives.

In the first chapter, I look at the impact of particular climate feedbacks on the optimal abatement policy. I develop a stochastic optimal pollution management model where the stock pollutant dynamics exhibit a non-convex feedback. I calibrate the model to represent carbon dioxide (CO₂) accumulation in the atmosphere and the mitigation of climate change.

One policy instrument that might be used to reach the optimal abatement targets is the promotion of biofuels as alternatives to fossil fuels. There is, however, a concern that such an instrument may have adverse consequences for the food market. To tackle the trade-off between reduced pollution costs and increased food prices, a co-author and I develop an optimal control model where the constraint on fossil fuel is not its exhaustible nature, but the damages its use creates by contributing to climate change. Two generations of biofuels can be used to reduce emissions, but they are competing with food for the fixed amount of land available.

Although the magnitude of the impact of biofuel production on food prices is still a matter of debate, it will likely increase food price volatility. In the third chapter, I look for evidence of loss aversion in demand for groceries. I consider a potential source of confoundedness in the measure of loss aversion: price endogeneity.

¹The fifth assessment report will be completed in 2013-2014.

Accounting properly for feedbacks in devising an optimal climate policy is crucial. Not only is most of the variation in the estimation of climate sensitivity attributable to differences in feedback modeling in global circulation models, but the non-linearity of some of these feedbacks makes the climate system even more difficult to predict.

Most economic literature on optimal climate policy has taken one of two approaches. The first focuses on linear feedbacks, which can be summarized in one parameter, referred to as climate sensitivity. An alternative approach introduces thresholds in the GHG stock which, when crossed, trigger either catastrophic negative payoffs or a change in the climate regime that exacerbates climate change. Nonlinear feedbacks have not yet found their way into economic models of optimal GHG emissions.

An important contributor of GHG emissions is the transport sector. Finding a cleaner substitute to gasoline could help reach abatement targets. Biofuels are one obvious alternative. There is however strong concerns that, as biofuel production compete with food production for land, there will be upward pressure put on food prices. This might be particularly harmful to the very poor, for whom price shocks can lead to malnutrition or even starvation.

Most of the literature on the trade-off between food and biofuel focuses on the exhaustibility of oil rather than on the abatement benefits of biofuel. It is, however, possible that the exhaustibility of fossil fuel reserves is of less concern than the effects of accumulated greenhouse gases (GHG) in the atmosphere. In such a case, biofuel production is used to mitigate GHG emissions.

In the marketing literature focusing on loss aversion, most models only estimate demand taking prices as given. Hence they introduce a simultaneous equation bias in the estimation. To justify the assumption that prices are exogenous, it is usually argued that prices for the products studied are determined in a global market, which is little impacted by the consumers under study because they represent only a small subset of that market.

This explanation is not entirely convincing. In practice, we know that store-level price adjustments (such as sales) exist, especially for groceries, which are the most studied market in this literature. In this case, there is a strong possibility that prices are somewhat endogenous to the purchasing decisions of customers. Proper estimation would therefore require the use of an instrument.

The model developed in chapter one is the standard stock pollution control problem with a non-convex feedback term and a stochastic term included in the pollution accumulation dynamics. The non-convex feedback introduced is similar to that found in an energy balance model. The resulting combined model, formally incorporating stochasticity, yields a non-convex dynamic optimization problem. It contributes to the climate change literature by formally incorporating non-convex feedbacks in the dynamics of the stock of GHG. It also contributes to the literature on non-convex pollution control models by setting the problem in a stochastic context and applying it to climate change.

In chapter 2, my co-author and I tackle the trade-off between reduced pollution costs and increased food prices. We develop an optimal control model where the constraint on fossil fuel is not its exhaustible nature, but the damages its use creates by contributing

to climate change. Two generations of biofuel can be used to reduce emissions, but they are competing with food for the fixed amount of land available. We derive conditions for the optimal allocation of land between food and biofuel production, and the optimal investment into second generation biofuels. Because of the GHG externality, the market outcome is suboptimal, and we consider different policies to improve upon it.

To assess the possibility of bias induced by price endogeneity in the context of loss aversion models, I look, in chapter three, at the impact of reference price preferences on the demand for four grocery product categories: bread, chicken, corn and tortilla chips, and pasta. I test for the presence of loss aversion, both at the extensive and intensive margins. I do this exercise both taking prices as given and instrumenting for them.

I use prices of commodities entering as inputs in the production of the relevant products as instruments. There is evidence that food commodity prices have little impact on regular shelf prices, but also that higher agricultural commodity prices reduce the frequency and depth of sale promotions, hence increasing the average net retail price. Therefore, commodity prices have the potential to be a good instrument for net retail prices.

In the first chapter, I find that the impact of non-convex feedback in a stochastic setting is to create an optimally controlled system with two basins of attraction and a control rule that is potentially discontinuous in between these basins.

The discontinuity of the optimal emissions policy depends on the steepness of the feedback function. A steep function, meaning that the onset of the feedback is sudden, leads to a discontinuous abatement policy function. A flatter feedback function, meaning that the onset of the feedback is more gradual, leads to a continuous abatement policy function. In both cases, the control rule is not monotonic in the stock of CO_2 .

Since the natural variability in the stock of CO_2 is relatively low, the distance between the basins of attraction is small. If this variability were to grow with the concentration of CO_2 in the atmosphere, the distance between the basins of attraction would grow, enlarging the set of stock values where it is uncertain in which basin of attraction the system will end up.

In the optimal biofuel production model, the competitive outcome overproduce fossil fuels. Results for food and both biofuels productions are in general ambiguous. Land scarcity plays a big role in assessing the results of the competitive outcome for food production. If land is abundant, food is also overproduced because its GHG emissions are not taken into account. However, that result does not follow through if land is scarce, because competition from biofuel production may drive up land value and reduce food production.

When appropriate, the optimal mandate for biofuel production is higher than the socially optimal quantities. This is to add extra competition to reduce fossil fuel overproduction and, if land is scarce, to also reduce food overproduction. The optimal subsidy for each biofuel generation is equivalent to using the optimal mandate. Each subsidy can be decomposed into four components, representing the incentive to reduce GHG emissions, to reduce fossil fuel production, to reduce food production and to compensate for lack of investment in second generation biofuels.

As for loss aversion, while standard estimation does not give strong evidence of loss aversion for chicken and pasta, it does for corn and tortilla chips and for bread. When instruments are used to correct for price endogeneity,, that effect disappears for corn and tortilla chips, and bread, while it still does not show up for chicken and pasta.

These results have two main implications. Empirical estimation of reference price dependent demand ought to pay careful attention to the issue of simultaneous equation bias. Otherwise, reported loss aversion could in fact just be confounded with the bias due to the endogeneity of prices. From a marketing perspective, it is therefore not clear whether supermarkets should pay attention to loss aversion in their pricing strategies. A lot of attention has been devoted to sales pricing and how it should be adjusted in light of reference price preferences. The absence of loss aversion would have considerable implications for this analysis.

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Chapter 1

Optimal Management of a Stochastic Stock Pollutant with Non-convex Feedback: An Application to Climate Change

1.1 Introduction

Optimal control of greenhouse gas (GHG) emissions is an ideal problem to apply the economic theory of stock pollutant management. The benefits from emissions, arising from consumption and production, must be traded off with the damages from changes in the climate, arising from the accumulation of GHG in the atmosphere. However, the GHG optimal control problem has the specificity that changes in atmospheric GHG concentrations as well as in climate lead to nonlinear feedbacks in the climate system. In addition, the problem is also stochastic in its nature, as climate has a natural variability and as emissions of GHG can only be imperfectly controlled by policy makers. To tackle these issues, I develop a stochastic optimal pollution management model where the stock pollutant dynamics exhibit a non-convex feedback.¹ I calibrate the model to represent carbon dioxide (CO₂) accumulation in the atmosphere and the mitigation of climate change.

Accounting properly for feedbacks in devising an optimal climate policy is crucial. Not only is most of the variation in the estimation of climate sensitivity attributable to differences in feedback modeling in global circulation models [Webb et al., 2006], but the non-linearity of some of these feedbacks makes the climate system even more difficult to predict. Examples of nonlinear feedbacks include the albedo-ice effect [Hansen and Nazarenko, 2004] and the thermohaline circulation [Rahmstorf et al., 2005].² Feedbacks between the cycle of carbon emissions and its sequestrations by natural sinks are also increasingly found to be inadequately modelled by linearization. Zickfeld et al. [2011] find that, in a coupled climate-carbon model, land and oceans carbon sinks are less effective than predicted by a linear combination of concentration-carbon and climate-carbon cycle feedbacks. Complex ecosystems also have the potential to generate nonlinear responses to climate feedbacks, which traditional carbon-cycle models render poorly [Heimann and Reichstein, 2008].

Nonlinear feedbacks in the climate system can lead to tipping points or thresholds in the climate system. Once a tipping point is reached, the climate system can start to change quickly without anthropogenic emissions needing to change quickly as well. Examples of tipping points include the melting of the Arctic ice sheet, the instability of the West Antarctic ice sheet, losses of permafrost and tundra, Atlantic deep water formation and others [Lenton et al., 2008].

Most economic literature on optimal climate policy has taken one of two approaches. The first focuses on linear feedbacks, which can be summarized in one parameter, referred to as climate sensitivity [Roe and Baker, 2007]. This linear formulation does not lead

¹While non-convex feedbacks are a particular type of non-linearity, it often corresponds to what the climate literature is describing as nonlinear feedbacks. Indeed, nonlinear climate or carbon feedbacks are often assumed to take-off at some point only to stabilize or level-off latter. In any case, the resulting non-linearity can be represented by a non-convex function, usually a convex-concave one.

²For a more detailed description of the relationship between the albedo-ice effect, the thermohaline circulation, and climate change see Clark et al. [1999].

to tipping points or thresholds in the resulting climate model. This is the approach of Ramsey-Cass-A models such as the DICE model [Nordhaus, 2008] and the model of Aaheim [2010]. An alternative approach introduces thresholds in the GHG stock which, when crossed, trigger either catastrophic negative payoffs, or a change in the climate regime that exacerbates climate change [Tahvonen and Withagen, 1996]. This approach has a stochastic analog that uses a hazard rate to represent the probability of crossing into the catastrophic regime, as in Gjerde et al. [1999] and Tsur and Zemel [2009]. In either the deterministic or stochastic setting, most thresholds introduced in climate change management models are exogenous, meaning the threshold, or the probability of crossing it, is independent of the decision maker's actions [Tsur and Zemel, 1996, 1998, Naevdal, 2006, 2007, Fisher and Narain, 2003]. Fewer models consider endogenous thresholds that depend on variables controlled by the decision maker [Lemoine and Traeger, 2010].

Nonlinear feedbacks have not yet found their way into economic models of optimal GHG emissions. Moreover, there is debate over how these feedbacks should be incorporated into climate models. Zaliapin and Ghil [2010] critique the one parameter climate sensitivity approach of Roe and Baker [2007] that has become widely used. The disagreement persists as Roe and Baker [2011] have rejected this critique, only to be challenged again by Zaliapin [2011]. However, problems with non-convexities have been tackled in other contexts. The seminal work of Skiba [1978], on the optimal growth of a one sector economy with a convex-concave production function, introduced the possibility of decision thresholds, since then referred to as "r points." In environmental economics, similar models were developed later to deal with stock pollutants that exhibited convex-concave dynamics. Tahvonen and Salo [1996] look at a general stock pollutant with a concave-convex decay function, while several articles on the so called "Shallow Lake Problem," including Carpenter et al. [1999] and Brock and Starrett [2003], looked at a similar problem but with a positive convex-concave feedback on the pollution stock. All these models share the conclusions that the optimal steady state is not necessarily unique, and that the control rule is likely non-monotonic and possibly discontinuous. All these conclusions are in striking contrast to the received wisdom on GHG optimal policy, a monotonic and continuous increase in abatement.

The model developed here is the standard stock pollution control problem with a non-convex feedback term and a stochastic term included in the pollution accumulation dynamics. The application to climate change adapts the framework developed by Rezai [2010] to fit an autonomous control problem. The model of Rezai [2010] is itself a modification of the DICE model [Nordhaus, 2008], where most of the climate module of DICE is replaced by a damage function expressed in terms of atmospheric CO₂ concentration. The non-convex feedback introduced is similar to that found in an energy balance model. Such models, presenting temperature as the equilibrium between incoming and outgoing radiations, have been little used in economic analysis of climate change. This is surprising given that they use the simplest representation of nonlinear feedbacks of any climate models. The resulting combined model, formally incorporating stochasticity, yields a non-convex dynamic optimization problem. It contributes to climate change literature

by formally incorporating non-convex feedbacks in the dynamics of the stock of GHG. It also contributes to the literature on non-convex pollution control models by setting the problem in a stochastic context and applying it to climate change.

The non-convexity of the model creates thresholds. There are no environmental threshold at a particular point of the stock of CO₂, but the non-convex feedback leads to a threshold region, where the feedback processes become significant. The threshold region in the climatic system is exogenous as it depends only on model parameters. It can lead to an endogenous decision threshold, i.e. a level of CO₂ concentration at which the level of optimal emissions changes discontinuously. The resulting discontinuous control rule determines the probabilities of entering different basins of attraction.

The rest of this paper is organized as follows. The generic model is presented in section 2. Section 3 presents the functional forms used to represent the problem of optimal CO₂ emissions and presents some analytical scenarios. The numerical approach used to solve the model, the calibration, and the results are presented in section 4. Section 5 concludes.

1.2 Model

The general problem involves a productive activity yielding utility that also generates pollution, which accumulates into a stock. This stock reduces utility either directly or indirectly by negatively affecting the productive activity.

This pollution model is fairly standard in environmental economics. However, if the dynamics of the stock pollutant display non-convex feedback, as in the case of climate change, the problem becomes much more complex and much less studied [Tahvonen and Salo, 1996, Carpenter et al., 1999, Brock and Starrett, 2003]. This paper contributes to that literature by setting the problem in a stochastic context and applying it to climate change.

The problem can be described as the maximization of the expected sum of discounted net benefits from the productive activity, subject to the dynamics of the pollution stock. Formally:

$$\max_{\{x_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \mathbf{E}_{\xi} u(x_t, S_t, \xi_t) \quad (1.2.1)$$

subject to

$$S_{t+1} = f(x_t, S_t, \xi_t) \quad (1.2.2)$$

$$x_t \in \Phi(S_t) \quad (1.2.3)$$

where:

- $x_t \in \mathcal{X} \subseteq \mathbb{R}$ is the productive activity (control variable) at time t ;
- $S_t \in \mathcal{S} \subseteq \mathbb{R}$ is the stock of pollution (state variable) at time t ;

- ρ is the positive discount rate;
- $\xi_t \in \Xi \subset \mathbb{R}$ is i.i.d. and Ξ is finite;
- $u : \mathcal{X} \times \mathcal{S} \times \Xi \rightarrow \mathbb{R}$ is the net benefit from productive activity x_t , pollution stock S_t and stochastic shock ξ_t at time t , where $\frac{\partial u}{\partial x} > 0$, $\frac{\partial^2 u}{\partial x^2} < 0$, $\frac{\partial u}{\partial S} < 0$, $\frac{\partial^2 u}{\partial S^2} < 0$;
- $f : \mathcal{X} \times \mathcal{S} \times \Xi \rightarrow \mathcal{S}$ is the transition function for the stock;
- $\Phi : \mathcal{S} \rightarrow 2^{\mathcal{X}}$ is the feasible action correspondence.

The existence of a solution to this problem depends on conditions that can be imposed on the one period reward function, the discount rate, the transition function and the feasible action correspondence. If $\mathbf{E}_\xi u(x_t, S_t, \xi_t)$ is real valued, continuous and bounded, $\rho > 0$, f is continuous and Φ is non-empty, compact-valued and continuous, then there exist a unique value function $V(S_t, \xi_t)$ that solves:³

$$V(S_t, \xi_t) = \max_{x_t} \left\{ u(x_t, S_t, \xi_t) + \frac{1}{1 + \rho} \mathbf{E}_\xi V(S_{t+1}, \xi_{t+1}) \right\} \quad (1.2.4)$$

None of the stated restrictions on the problem preclude the use of a non-convex feedback term in the pollution stock dynamics; the only additional restriction needed is that the overall behaviour of this stock be continuous. The next section shows that while the dynamics of the CO₂ stock is continuous, the location of steady states to the system can change discontinuously in the level of emissions.

Because of the infinite time horizon, the fact that the reward function does not depend on calendar time, and the distribution of ξ_t is time invariant, this is an autonomous dynamic programming problem. Hence the value function does not have time as an explicit argument. Such an autonomous problem has an optimal policy function that is stationary.

The first order condition of equation (1.2.4) gives a condition relating the marginal net benefit of the productive activity with the marginal cost imposed by the pollution stock along the optimal trajectory.⁴

$$u_x(x_t, S_t, \xi_t) = -\frac{1}{1 + \rho} \mathbf{E}_\xi V_S(S_{t+1}, \xi_{t+1}) f_x(x_t, S_t, \xi_t) \quad (1.2.5)$$

One interpretation of equation (1.2.5) is that for a given x_t, S_t, ξ_t combination to be optimal, it must be the case that the marginal utility derived from the productive activity is equal to the discounted expected marginal reduction in future utility due to extra pollution, weighted by the contribution of that marginal unit of productive activity to the future stock of pollution. Because this condition must hold for all time periods, there is

³The proof of that theorem is that of proposition 2 in chapter 6 of Bertsekas [1976].

⁴I use the notation u_x to denote the partial derivative of the function u with respect to variable x .

no opportunity for arbitrage. Hence, on an optimal path, it is impossible to rearrange the timing or total amount of productive activity such that the sum of discounted expected net benefits is increased.

1.3 Application to Climate Change

The application to climate change that I propose builds upon a modification of the DICE model⁵ developed by Rezai [2010]. While Rezai [2010] simplifies the climate module of the DICE model such that all stock dynamics and climate damages can be represented by a single stock variable (CO₂ concentration in the atmosphere), I also simplify the economy side of the model by assuming constant capital stock, constant savings, and constant production and abatement technology. These simplifications allow me to keep the model analytically and numerically tractable when I introduce non-convex feedback and uncertainty.

In the next subsection, I present the functional forms of the climate application of the model and discuss in more details how these depart from the original DICE model and its modification by Rezai [2010]. In the subsequent subsections, I present some further analytical results to establish what type of scenarios are possible given the choice of functional forms.

1.3.1 Applied model

While many models of climate change mitigation, such as DICE, are optimal growth models with exogenous technical change, I make the problem static in these two dimensions. This modification leads to an autonomous problem, which is more tractable for analytical optimal control and more stable when solving for the numerical control rule. In addition, dropping these exogenous changes does not fundamentally alter the problem at hand. Eliminating output growth reduces future wealth, which in turn reduces the incentive to shift abatement costs to future periods. In addition, on the technological side, eliminating the decline in abatement costs also reduces the incentives to shift abatement to the future. Part of these effects are counterbalanced by the constant business as usual level of emissions, which is due to the constant output and emissions intensity assumptions. Indeed since emissions are not expected to grow over time, there is less pressure to act now than in the standard DICE model.

The model presented in the previous section can be parametrized using the following functional forms to represent the mitigation of GHG emissions. The one period payoff, or reward function, is a utility function with constant elasticity of marginal utility, $u(m_t) = \frac{x(m_t)^{1-\eta}}{1-\eta}$. The choice variable, $m_t \in [0, 1]$, is the share of output devoted to the mitigation of CO₂ emissions. This variable completely determines per i consumption, $x(m_t)$, according

⁵The version of the model I will be referring to is DICE-07 [Nordhaus, 2008].

to the function $x(m_t) = \frac{\bar{y}d_t(1-m_t-\sigma)}{N}$, where $\bar{y} > 0$ is the constant gross world output,⁶ $d_t \in [0, 1]$ is the fraction of gross output remaining after climate damages have been accounted for, $\sigma \in [0, 1)$ is the constant savings rate, and $N > 0$ is the constant population level.

To be able to numerically solve the model, given the potential for a c point in a stochastic setting, the number of state variables must be kept to a minimum. Indeed the curse of dimensionality is quite acute in non-convex problems, as the concept of Skiba points must be expanded to Skiba lines, planes or hyperplanes, as the number of state variables grows to two, three or more. To avoid these complications, I limit the model to one state variable, which is the stock of CO₂ in the atmosphere, S_t , measured in parts per million volume (ppmv).⁷

Since CO₂ is the only state variable in the model, the climate damages must be expressed in terms of this stock. I use the formulation developed by Rezai [2010], where the fraction of output spared from climate damages is defined as:

$$d_t = \left(1 - \left(\frac{S_t - 280}{\bar{S} - 280} \right)^{1/\gamma} \right)^\gamma. \quad (1.3.1)$$

This damage function can lead to more or less damages than in the DICE model, depending on the value of $\gamma \in (0, 1)$. This parameter defines the curvature of the damage function, where damages get linear as $\gamma \rightarrow 1$. $\bar{S} > 280$ is the stock of CO₂, measured in ppmv, that leads to the complete destruction of world output.

I also adapted the formulation of the transition function from Rezai [2010]. Indeed, his recast-DICE model simplifies the climate module of DICE by replacing it by a transition function that depends only on the stock of CO₂. This approach bypasses most of the climate module of DICE, which eliminates several intermediate state variables such as carbon in upper and lower oceans, and atmospheric and oceanic temperatures. Such a streamlined formulation simplifies the analysis and lends itself more readily to numerical computations. The drawback is that some of the delays in the climate system are lost and the damage function must be specified in terms of CO₂ concentration instead of temperature.

The transition function, f , describing the evolution of the stock of CO₂ between each period is described by equation (1.3.2).

$$f(m_t, S_t, \xi_t) = (1 - \varepsilon)S_t + (\beta - M_t)y_t + g(S_t) + 280\varepsilon + \xi_t \quad (1.3.2)$$

Hence the stock of CO₂ next period depends on the current stock, S_t , which dissipates at rate $\varepsilon \in (0, 1)$, on the level of output net of climate damages, $y_t = \bar{y}d_t$, on a non-convex feedback term, $g(S_t)$, and on a stochastic term, ξ_t . Output has a constant CO₂ intensity, $\beta > 0$, which can be reduced through mitigation, M_t , defined as the per unit of output

⁶Measured in trillions of dollars.

⁷The pre-industrial concentration of CO₂ is fixed at 280 ppmv [Randall et al., 2007].

reduction in CO₂ emissions due to mitigation m_t . The relationship between M_t and m_t is represented by the function $M_t = \left(\frac{m_t}{\theta_1}\right)^{1/\theta_2}$, where θ_1 and θ_2 are positive parameters that affect the output cost of a given reduction in emission intensity as defined in DICE.

The new feature in this climate model is the feedback term, $g(S_t)$, in the transition function for the stock of CO₂. The functional form chosen for $g(S_t)$ is given in equation (1.3.3).⁸

$$g(S_t) = \mu(\tanh(\kappa(S_t - \hat{S})) + 1) \quad (1.3.3)$$

This function is convex-concave, where $\mu > 0$ is a scaling parameter of the feedback function such that $\max g(S_t) = 2\mu$, $\kappa > 0$ is a parameter affecting the slope of the feedback function, and $\hat{S} > 280$ is the stock of CO₂ at which the feedback function reaches its inflexion point. As will be illustrated in the next section with specific values, one can describe μ as the magnitude of the feedback, κ as the suddenness of the feedback, and \hat{S} as the midpoint of the region where the feedback takes off, or the onset region.

Defining this feedback term as an increasing convex-concave function can represent several climate scenarios. In one of its simplest representations, the climate system can be modelled as a zero dimensional energy balance system.⁹ The planet's temperature is the resulting equilibrium of the incoming energy absorbed from solar radiation and the outgoing energy emitted by the earth. As the temperature changes, the ice cover on the planet changes, which in turn changes the amount of solar radiation absorbed by the system, as more or less ice will reflect more or less radiation. This feedback effect makes the relationship between incoming energy and temperature non-convex.

Despite the model developed in this paper not being one of energy balance, a similar reasoning can be applied to a model of CO₂ concentration balance. As CO₂ concentration rises, temperature also rises, which sets in motion changes in carbon sources and sinks that will further increase the concentration of carbon. Such positive carbon cycle feedbacks have been found using recent data [Cox et al., 2000, Heimann and Reichstein, 2008, Randall et al., 2007] and also with statistical studies based on paleoclimatic data [Lemoine, 2010]. The evidence is mounting that these feedbacks are nonlinear and lead to non-convexities in the climate system [Zickfeld et al., 2011, Friedlingstein et al., 2001]. Since the model developed here is highly aggregated, like zero dimensional energy balance models, the specific functional form of $g(S_t)$ does not represent a specific climate process, but, as in Zaliapin and Ghil [2010], it is a smoothed version of the more usual step or piecewise linear functions used to describe non-convex dynamics.¹⁰

Finally, the last innovation in the model is the presence of uncertainty in the form

⁸This formulation is similar to that of Zaliapin and Ghil [2010], who use it in the context of an energy balance model.

⁹The dimensionality of energy balance models refers to their level of spatial aggregation. Zero dimensional models consider the planet surface temperature as uniform, one dimensional models distinguish temperature based on latitude, and two dimensional models distinguish both latitude and longitude.

¹⁰In energy balance models, the piecewise linear formulation is referred to as Sellers-type models [Ghil, 1976], while the piecewise constant formulation, or step function, is referred to as Budyko-type models [Held and Suarez, 1974].

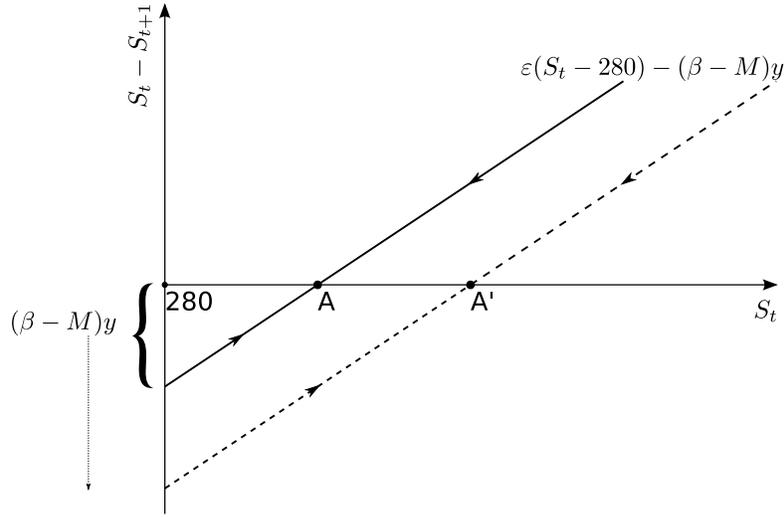


Figure 1.1: Linear CO₂ dynamics with constant emissions

of an additive random term ξ_t . By making the next period stock of CO₂, and therefore the damages, uncertain the model encapsulate risks of crossing thresholds, beyond which the carbon cycle will change significantly. Managing these risks is an important part of developing comprehensive policies to address the challenges posed by climate change.

1.3.2 Climatic thresholds in a deterministic setting

Thresholds are best introduced by first tackling the deterministic analog of the model, where ξ_t is always zero. In this subsection, as well as the subsequent ones, I use phase diagrams to represent the characteristics of different formulations of the model. In general, phase diagrams might not accurately represent the dynamics of a discrete time model, as discrete shifts in state and control variables can make the system jump around a point that would otherwise be monotonically reached. Appendix A shows that, in the deterministic version of the model, the stock of CO₂ is always monotonically approaching the stable steady states. Hence, phase diagrams provide an accurate rendition of the model dynamics.

On the purely climatic side, one must first understand the behavior of the stock of CO₂ for constant levels of emissions before moving to the optimally controlled climate-economy. Figure 1.1 presents this situation when there is no feedback. With constant emissions $(\beta - M)y$, the long run stock steady state is point A. If the initial stock, S_0 , is below A, the CO₂ concentration will increase to asymptotically reach point A, while it would decrease to asymptotically reach point A, if S_0 is above A. If M permanently decreases, inducing an increase in emissions, the long run steady state shifts proportionally to point A'.

In the nonlinear case, figure 1.2 presents the deterministic components of f . Intersections

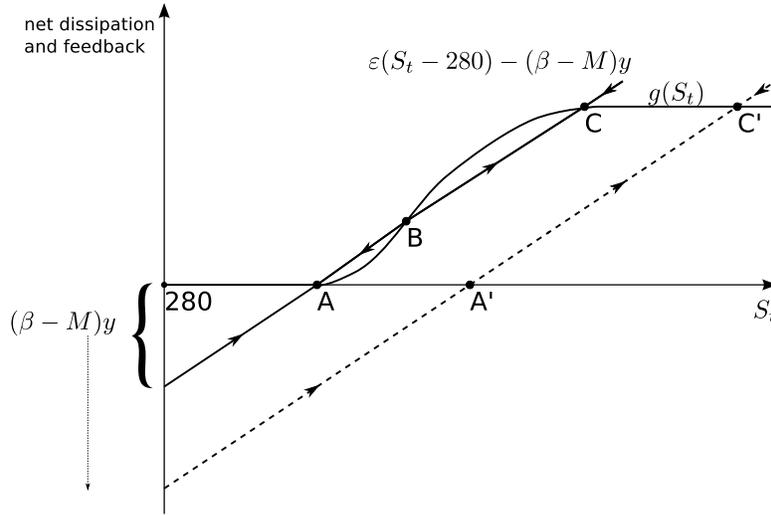


Figure 1.2: Nonlinear CO₂ dynamics with constant emissions

between the straight dissipation line and $g(S_t)$ now represent the steady states of the climate system. I will refer to the part of $g(S_t)$ to the left of its inflection point as the lower branch and the part to the right of that point as the upper branch. Points A and C are locally stable equilibria, while point B is an unstable one. Hence, for constant emissions $(\beta - M)y$, if S_0 is less than B , the stock of CO₂ would asymptotically converge to point A , while if S_0 is greater than B , the stock of CO₂ would converge to point C . As in the linear case, consider a permanent decrease in M . If that decrease is large enough, such that the shifted dissipation line now intersects only once with $g(S_t)$, only one steady state remains, C' , and it is now globally stable.

It is now easy to construct an example where a permanent change in emissions leads to a more than proportional change in the steady state stock of CO₂. Suppose that S_0 is less than B and emissions are $(\beta - M)y$. If M permanently decreases such that emissions increase as shown in figure 1.2, the steady state stock of CO₂ will not shift to A' as in the linear case, but to a much higher C' .¹¹ That is, the increase in emissions and their subsequent accumulation in the atmosphere have triggered feedbacks in the climate-carbon cycle such that the ultimate steady state of the CO₂ is much higher than it would have been without these feedbacks.

To understand how thresholds come to be in the climate system, an important distinction must be made between two potential cases that the functional form of $g(S_t)$ can represent. Those are depicted in figure 1.3. In the left panel, the maximum slope of $g(S_t)$ is greater than ε , while in the right panel it is smaller. In the left panel, there are two thresholds in the stock of CO₂. Starting from a stock of CO₂ below A , as emissions increase, the steady state stock moves along the lower branch of $g(S_t)$ up until point A' ,

¹¹Note that exactly the reverse example can be created if S_0 is above B and M increases enough.

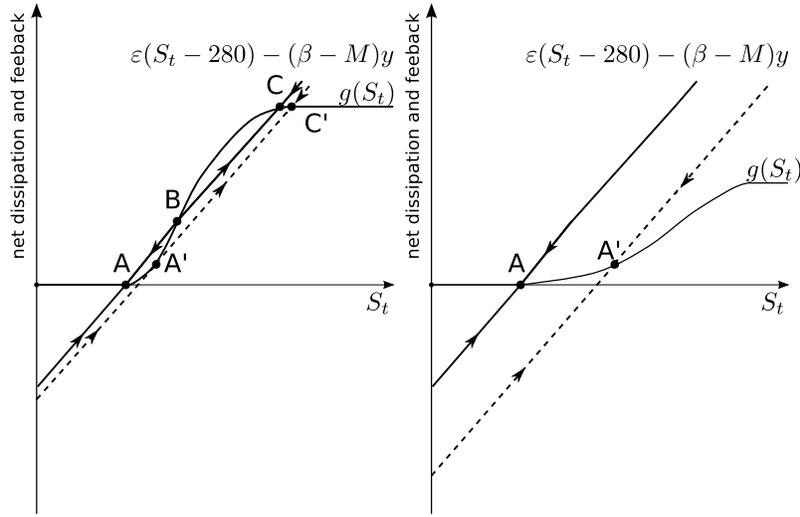


Figure 1.3: Nonlinear CO₂ dynamics with and without hysteresis

where the dissipation line and $g(S_t)$ are tangent. If emissions go beyond the intercept of the dotted line on the left panel of figure 1.3, then the steady state level of emissions would go beyond point C' on the upper branch of $g(S_t)$. In addition, if emissions would decrease back to the level where the steady state was previously A' , the steady state would not go back to A' but would stay at C' . This asymmetry in the relationship between emissions and stock steady state is referred to as hysteresis. Hence, the stock value corresponding to point A' is a threshold, below which the steady state moves along the lower branch of $g(S_t)$. There is an analogous point to A' , between B and C along $g(S_t)$, beyond which the steady state moves along the upper branch of $g(S_t)$. It is the second tangency point between the dissipation line and $g(S_t)$.

In the case shown in the right panel, no thresholds exist, as $g(S_t)$ is nowhere steeper than the dissipation line. As emissions increase, the stock steady state also increases along $g(S_t)$. Also, the absence of thresholds implies there is no hysteresis effect. Indeed, the steady state A' in the right panel of figure 1.3 can be reached from above or below monotonically.

I focus on the hysteretic case depicted by the left panel of figure 1.3. This case is likely the most accurate representation of the management of CO₂ emissions. Indeed, while ε is very small because carbon sinks retire CO₂ relatively slowly from the atmosphere, the maximum slope of $g(S_t)$ is likely to be very high, because nonlinear feedbacks are believed to kick in over a relatively short range of stock values. Budyko-type models use a step function to model such feedbacks, which leads to an infinite slope at the point of discontinuity.

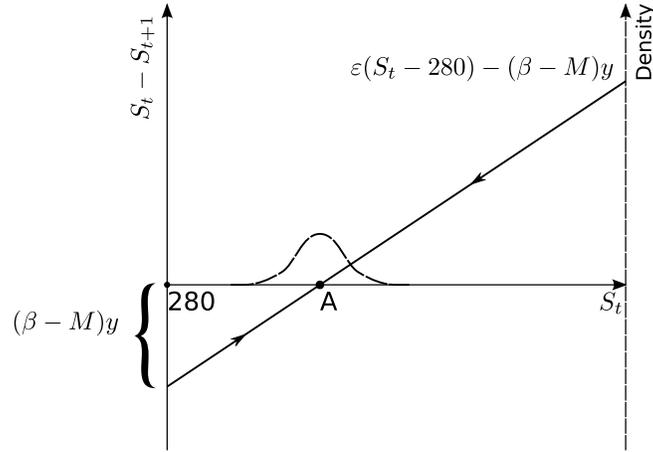


Figure 1.4: Linear stochastic CO₂ dynamics with constant emissions

1.3.3 Climatic thresholds in the stochastic setting

The introduction of the stochastic term into the function f changes the interpretation that can be given to climatic thresholds. To tackle this change, I build on the previous framework where emissions were kept constant over time, instead of being adjusted to follow an optimal policy. The behaviour of the problem with an optimized emissions path is addressed in the next subsection.

Because of the randomness of ξ_t , one must now think of steady state distributions instead of just steady states. The steady state distribution can be represented fairly easily in the linear case. Adding a vertical axis to the right of figure 1.1 to represent the probability density function of the steady state, this steady state distribution can be represented as in figure 1.4. The situation depicted assumes that ξ_t has expectation zero. In this case, A is the mean of the steady state distribution instead of being the steady state itself.

A similar exercise can be done with the nonlinear dynamics. Figure 1.5 shows how the density of the steady state distribution could look under two scenarios. Both scenarios assume that S_0 is below B , such that the deterministic steady state is point A . In the stochastic case, if the variance of the random term is low relative to the dissipation rate, the steady state distribution is clustered around A .¹² That is, no sequence of random shocks is strong enough to push the long run CO₂ stock towards C . In contrast, if the variance of the stochastic term is higher, then it is possible that a sequence of shocks pushes the CO₂ stock beyond B , where the deterministic part of the stock dynamics could keep pushing the stock in the direction of C . That is, even when S_0 is below B , there would be a non-zero probability that the long run stock of CO₂ is in the neighbourhood

¹²Note that the distribution is not symmetric anymore because the deterministic part of the stock dynamics is not symmetric.

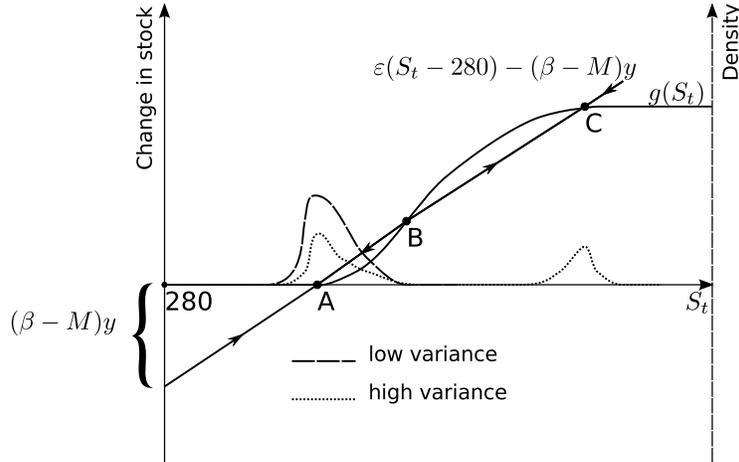


Figure 1.5: Nonlinear stochastic CO₂ dynamics with constant emissions

of C . In such a case, the steady state distribution is bimodal, as shown in figure 1.5.¹³

As the construction of these scenarios show, it is possible that by starting on either side of point B , the system would reach a different steady state distribution, gravitating towards either point A or point C . That is, the steady state distribution might not be unique. If it is non-unique, new thresholds arise in the model. With constant emissions and a deterministic stock, the dynamics of the model were quite simple. Start to the left of B , end up at A and start to the right of B , end up at C . Now with the stochastic term, there is some uncertainty when S_0 is in a certain neighbourhood of B as to the direction that the long run stock of CO₂ will take. However, if we are in the low variance case described before, it is possible that beyond certain stock values, away from B , it might be certain that the stock will not go back beyond B . Figure 1.6 illustrates such a situation. In this figure, D and E represent threshold stock values that mark the limit of a basin of attraction, i.e. a region that once the stock has entered, it will never leave. When the CO₂ concentration is between D and E , there is a positive probability that it will enter either of the basins of attraction. In the context of climate policy, these thresholds might be perceived as bounding risk regions. For example, with this constant emissions policy, the decision maker would know that if the stock gets above point D , there is a risk of ending up with a much higher CO₂ concentration, even if emissions remain unchanged.

¹³There is a third possibility that is not depicted in figure 1.5. The variance could be so big that the stochastic part of the stock dynamics completely overwhelms the deterministic part, such that the density of the steady state distribution is single peaked and encompasses both A and C . I however disregard this possibility as uninteresting, because it means that the system is so volatile that it can barely be controlled.

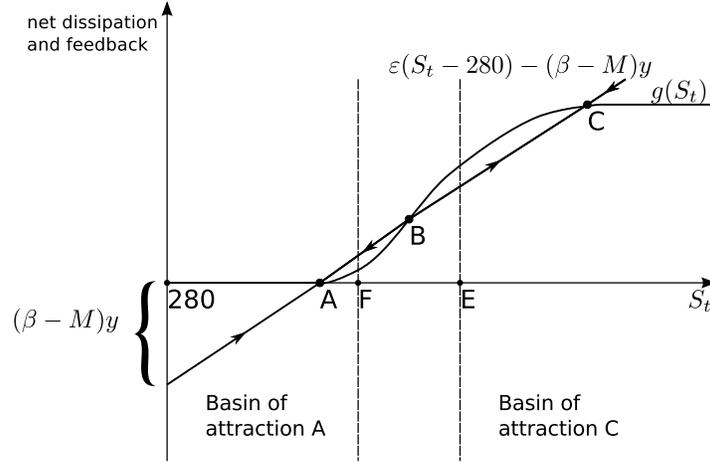


Figure 1.6: Basins of attraction for nonlinear stochastic CO₂ dynamics with constant emissions

1.3.4 Decision thresholds

Dropping the constant emissions assumption, I analyze the optimally controlled climate-economy. The optimal control of CO₂ emissions involves a policy rule where the level of emissions varies according to the observed stock of CO₂. As with the study of stock dynamics, it is useful to first look at a deterministic version of the model to highlight a few of its specificities before moving to the stochastic analog. The first step is to derive the two isoclines in the model, the stock and the mitigation isoclines. The stock isocline derivation requires only to equate f with S and then solve for M/β , the fraction of emissions abated.¹⁴ Hence the stock isocline is:

$$\frac{M}{\beta} = \frac{(280 - S)\varepsilon + g(S)}{\beta\bar{y}d} + 1 \quad (1.3.4)$$

To find the mitigation isocline, I use the first order condition. I rewrite the first order condition in equation (1.2.5) such that it represents the deterministic problem.

$$u_m(m_t, S_t) = -\frac{1}{1 + \rho} V_S(S_{t+1}) f_m(m_t, S_t) \quad (1.3.5)$$

I apply the envelope theorem to the deterministic value function to get a second marginal condition.

$$V_S(S_t) = u_S(m_t, S_t) + \frac{1}{1 + \rho} V_S(S_{t+1}) f_S(m_t, S_t) \quad (1.3.6)$$

¹⁴All the phase diagrams will be presented with M/β , the fraction of emissions abated, as a function of S . Technically M/β is not the choice variable, but it is a convenient transformation that facilitates interpretation.

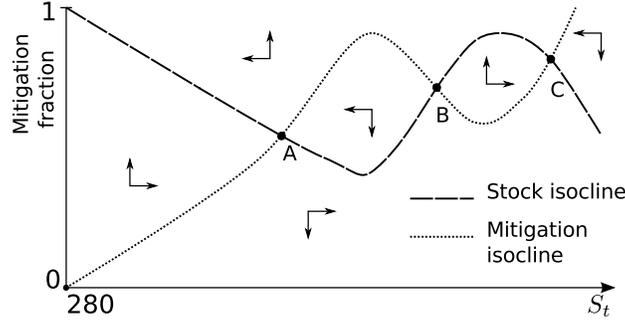


Figure 1.7: Phase diagram of the deterministic climate-economy

Combining equations (1.3.5) and (1.3.6) and dropping the time subscripts yields the condition for the mitigation isocline.

$$u_m = \frac{1}{1 + \rho} [f_S u_m - f_m u_S] \quad (1.3.7)$$

Substituting the appropriate partial derivatives by their parametric expressions, I obtain an implicit expression for the mitigation isocline, equation (1.3.8). This equation can be numerically solved for given parameter values.

$$\frac{1}{\theta_1 \theta_2} (1 - \sigma) \left(\frac{M}{\beta} \right)^{1 - \theta_2} + \left(1 - \frac{1}{\theta_2} \right) \frac{M}{\beta} = \frac{g_S - \varepsilon - \rho}{\beta \bar{y} d_S} + 1 \quad (1.3.8)$$

Both isoclines can be represented in a phase diagram like figure 1.7. It illustrates a situation where the optimally controlled climate-economy has three steady states; points A and C are stable, while B is unstable. In each sector delimited by the isoclines and the axes, directional arrows show qualitatively how the stock of CO_2 and the fraction of emissions abated evolve over time. This is only one possibility for the number of steady states. It could be the case that the isoclines only cross one time, either early at a point like A , or late at a point like C . In such cases, there would be only one steady state. I focus on the depicted case where there are three steady states, both because it is the most frequent in the numerical implementation and because it conceptually encompasses the other cases.

Using the three steady states phase diagram, I show there are three qualitatively different possibilities for the control rule. The first possibility is having a control rule that leads steady state A to be globally optimal. This situation is depicted in figure 1.8. In this case, regardless of the value of S_0 , the long run level of mitigation and CO_2 in the atmosphere will be point A . Note that such a control rule has to be non-monotonic in abatement, because for stock values between B and C , it must go through a region of the phase diagram where mitigation is increasing, while mitigation is decreasing in the two regions adjacent to the middle one. Conversely, it could be the case that steady state

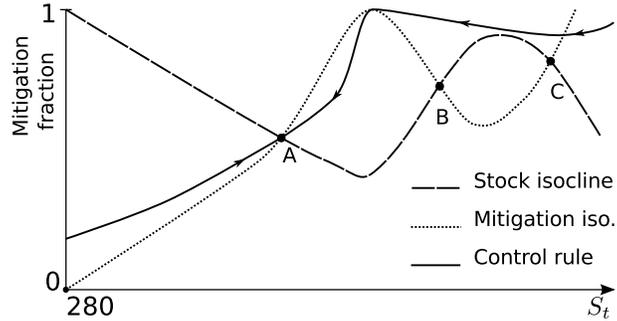


Figure 1.8: Control rule 1 (deterministic climate-economy)

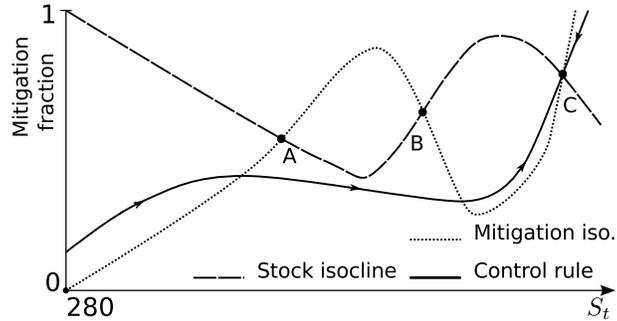


Figure 1.9: Control rule 2 (deterministic climate-economy)

C is globally optimal, resulting in a control rule that looks like that shown in figure 1.9. Once again, for a reasoning similar to that of the previous case, the control rule has to be non-monotonic in CO_2 concentration.

A third possibility can arise where both steady states A and C are locally optimal. This case is shown in figure 1.10. Here, the control rule is not only non-monotonic in abatement, but it is also discontinuous. The point at which the discontinuity occurs, point D , is the Skiba point. The Skiba point is a decision threshold. When S_0 is arbitrarily close to D , but smaller than it, it is optimal to abate a relatively large fraction of emissions in order to push the stock of CO_2 towards A in the long run. In contrast, when S_0 is arbitrarily close to D , but larger than it, it is optimal to abate a relatively smaller fraction of emissions and let the stock of CO_2 grow to its steady state value of C . That intuition confirms the mathematical property that at the Skiba point itself, the value function of either trajectory is the same, meaning that the decision maker is indifferent between following the mitigation path that leads to point A or the one that leads to point C .

Just from the shape of the isoclines, one can sometimes rule out certain possibilities. For example, the particular isoclines depicted in figure 1.8 do not allow for a control rule that makes point C the globally optimal steady state. Point C cannot be approached from an initial stock value below A , as the only phase region that allows the stock to grow from point A to point B also requires the mitigation fraction to decline. When the

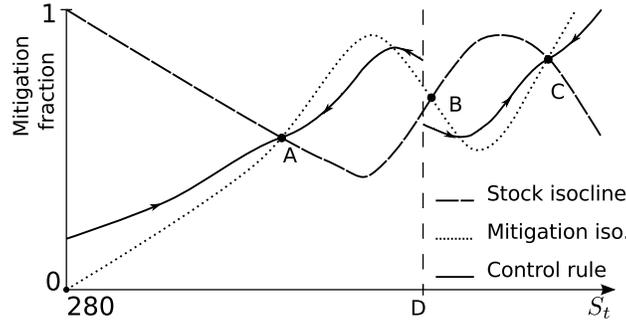


Figure 1.10: Control rule 3 (deterministic climate-economy)

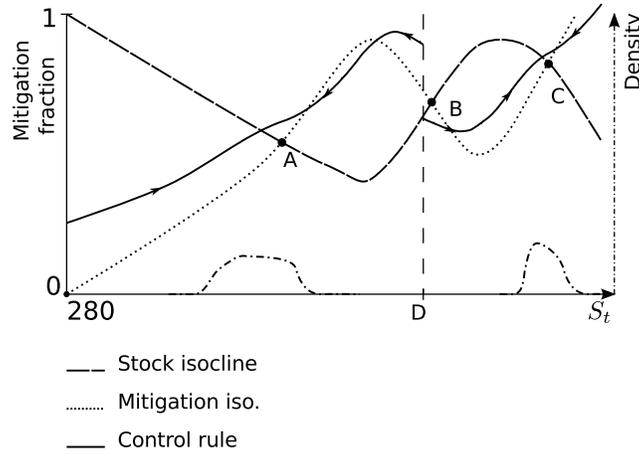


Figure 1.11: Optimally controlled stochastic climate-economy

stock would reach B , the mitigation fraction would be too low to let the system gravitate towards point C . Conversely, the example presented in figure 1.9 rules out the global optimality of steady state A . Using a similar reasoning as in the previous case, point A could not be reached from an initial stock value above C , as the mitigation fraction between point C and point B would become too high to enter the phase region leading to point A from above. However, the case with a Skiba point can never be ruled out just by the shape of the isoclines, whenever the optimally controlled climate-economy has three steady states.

Moving from the deterministic optimally controlled climate-economy to the stochastic one is similar to the previous discussion of the stochastic climate dynamics with constant emissions. To illustrate the impact of the addition of uncertainty to the deterministic optimally controlled climate-economy, I use the last example where the two stable steady states were optimal in the deterministic case.

Figure 1.11 is very similar to figure 1.10 with the addition of probability density functions for the steady state distribution. The control rule, however, no longer intersects

the deterministic steady states. This phase diagram keeps the deterministic isoclines only as a reference point for the new stochastic control rule. While the mitigation isocline does not retain a similar interpretation in the stochastic setting, the stock isocline is still meaningful in the stochastic setting as it represents the locus of points where the stock of CO₂ is constant in expectation. Hence one would expect the modes of the probability density function of the steady state distribution to gravitate around the intersections of the deterministic stock isocline and the stochastic control rule. Again, this steady state distribution may or may not be unique and there may be one or more basin of attractions over the state space.

The stochastic control rule depicted in figure 1.11 is arbitrary. There is very little analytical information that can be derived from the problem statement. In particular, the existence and location of a Skiba point cannot be analytically derived. Hence, to explore the influence of different parameters on the control rule and to quantify the climate policy implications of the present model, numerical solutions are necessary. The next section discusses numerical simulations performed with several calibrations of the model.

1.4 Numerical Implementation

In this section I use numerical techniques to solve for the optimal control rule to the problem posed in the previous section.¹⁵ In order for the results of this numerical exercise to be of interest, I calibrate the parameters of the model to represent the problem of climate change mitigation, within the limits of the current framework. The section begins with a description of the algorithm used to solve for the value function. Then, I present the calibration of reference, carefully explaining where the value of each parameter comes from. Finally, I do some comparative dynamics and robustness checks by varying the value of some of the key parameters.

1.4.1 Solution approach

To solve for the optimal control rule of the stochastic climate-economy, I use the collocation method. The possibility of a Skiba point complicates the search for this optimal control rule. Indeed, when such a point arises, the control rule is discontinuous at that point in the state space, while the value function is kinked at that same state location. Because it is easier to approximate kinked but continuous functions than discontinuous ones, I apply the collocation method by using the value function iteration. Once the value function has been solved, it is trivial to back out the control rule.

The collocation method approximates a function, here the value function, as the linear combination of basis functions over nodes forming a finite subset of the state space. If

¹⁵This implementation draws from ideas presented in Miranda and Fackler [2004] and some of the subroutines contained in the associated toolbox, as well as from notes from 2010 summer program from the Institute for Computational Economics at the University of Chicago.

the number of nodes is equal to the degree of the basis functions, the value function V , evaluated at the collocation nodes S_i , is exactly equal to the linear combination of the basis functions ϕ_j at these same collocation nodes:

$$V(S_i) = \sum_{j=1}^n c_j \phi_j(S_i) \quad \forall S_i \in \{S_1, S_2, \dots, S_n\} \quad (1.4.1)$$

where c_j represents each of the n coefficients of the linear combination that are to be solved for. Starting with an initial guess for the values of the coefficients, those are updated by using the equation

$$c = \Phi^{-1}v(c) \quad (1.4.2)$$

where $\Phi_{ij} = \phi_j(S_i)$ and $v_i(c) = \max_m \left\{ U(m, S_i) + \frac{1}{1+\rho} \mathbf{E}_\xi \sum_{j=1}^n c_j \phi_j(f(S_i)) \right\}$. That is, one performs the maximization implied by the value function at each of the evaluation nodes to find an approximate value function from which one can recover the implied coefficients. Those new coefficients are then compared to those of the previous round (or the original guess if this is the first round) to decide if the iteration process should be stopped. If the coefficient values derived from equation (1.4.2) are close enough to those of the previous round, the process is stopped, otherwise it gets repeated.

To address the particular challenges of approximating a kinked value function, I implement the collocation method with the following specification:

- cubic splines for basis functions;
- modified Chebyshev nodes;
- L_∞ norm to evaluate the distance between two sets of coefficients.¹⁶

Using splines to approximate the value function, repeated break points at some state node can be used to reduce the smoothness requirements of the spline approximation. Stacking n break points at a given node reduces the degree of continuity requirement of the function by $n-1$. By using cubic splines, having three breakpoints stacked at one point in the state space allows the approximated function to be kinked at that point, while the derivative of the value function must be continuous everywhere else. This approach is well suited to the current problem of approximating a potentially kinked value function. The main problem that remains is finding the location of the kink. Indeed, this location or even the existence of the Skiba point cannot be predicted theoretically. To this end, I develop an algorithm to update the location of the stacked break points in order to numerically find the kink. Chebyshev nodes are useful because they are denser at the boundaries of the state space, where there is less information to approximate the value function. In the current problem however, it is also good to have denser nodes in the vicinity of the kink to better pinpoint its exact location. To do that, I define Chebyshev nodes separately on

¹⁶The L_∞ norm takes the maximum distance between any two pairs of coefficients as the distance between the two sets.

the intervals on each side of the presumed kink. That way, the nodes are denser both at the boundaries of the state space but also in the presumed neighbourhood of the kink. Finally, the L_∞ norm is used because of the potential for a Skiba point. Small changes in the value function can lead to large implied changes in the control rule. As long as any of the coefficients keep changing significantly, the iteration process is allowed to keep going.

As for the algorithm that updates the location of the stacked break, theory says that the kink in the value function must occur at the same point in the state space where the Skiba point occurs in the control rule. If the stacked break points are misplaced, errors in the evaluation of the value function will generate successive spikes in the control rule that do not line up with the stacked break points location. These spikes emerge because I am trying to approximate a non-smooth function with a twice continuously differentiable combination of splines. In such a case, the stacked break points get moved toward the state value where the largest spike on the control rule has occurred, i.e. where the first derivative of the control rule has the greatest magnitude. The iterations are repeated until the kink in the value function and the discontinuity in the control rule occur at the same state node. Of course, one must keep in mind the possibility that there are no such kink and Skiba point in the problem at hand. In that case, putting stacked break point anywhere will not change the approximation and in the absence of discontinuities in the control rule, the algorithm will stop right away.

1.4.2 Calibration

I now present the values used for the different parameters of the model in what will be the benchmark specification of the numerical results. The time scale of the model is one period for one decade.

Most numbers are either those of the DICE model at the initial point in time or of their equivalent in Rezai [2010]. A few have been updated using the *CIA World Fact Book*. The innovation in terms of calibration is the value that is given to the three parameters of the feedback function.

- $\rho = (1 + 0.015)^{10} - 1$ is the per decade pure rate of time preference as defined in DICE (1.5% annual rate);
- $\bar{y} = 74 \times 10$ is ten times the 2010 world GDP in trillion of dollars [CIA, 2011];
- $\sigma = 0.2$ is the fixed savings rate;
- $N = 6.8$ is the 2010 world population in billion of people [CIA, 2011];
- $\bar{S} = 780$ is the CO₂ concentration in the atmosphere in ppmv that leads to the total destruction of world output (it is the number used by Rezai [2010]);
- $\gamma = 0.3$ is the elasticity of the damage function as defined in Rezai [2010];

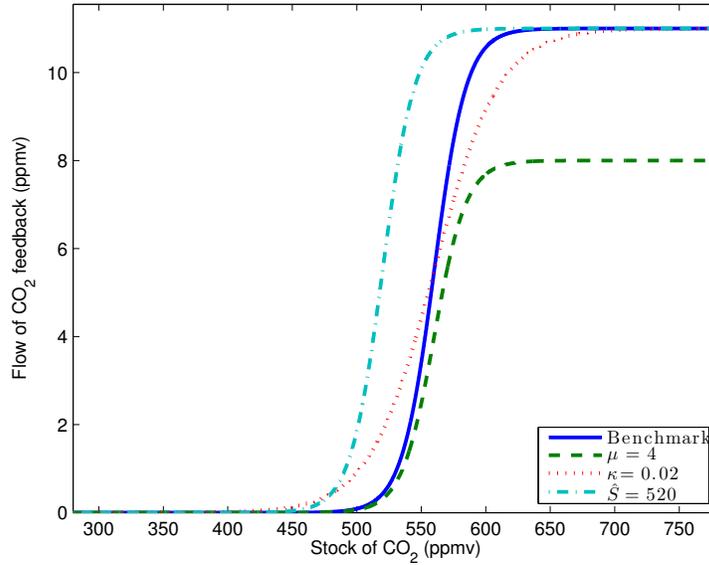


Figure 1.12: Feedback functions

- $\varepsilon = 0.036$ is the per decade dissipation rate of CO_2 [Rezai, 2010];¹⁷
- $\beta = \frac{0.063}{2.13}$ is the carbon intensity of output in ppmv per trillion dollars as initially defined in DICE;
- $\theta_1 = 0.051$ and $\theta_2 = 2.8$ as initially defined in DICE;
- ξ_t is the quadrature approximation of a random variable distributed $N(0, 0.5)$;¹⁸
- $\mu = 5.5$ is a feedback function parameter representing the magnitude of the feedbacks;
- $\kappa = 0.04$ is a feedback function parameter representing its maximum steepness;
- $\hat{S} = 560$ is a feedback function parameter setting the location of its inflection point,

The benchmark feedback function and some of its alternatives are represented in figure 1.12. The most important parameter of the feedback function for the extent of feedback mechanisms affecting climate change is μ . As it can be seen in figure 1.12, decreasing μ reduces the magnitude of the feedback in the climate dynamics. κ affects the steepness

¹⁷One cannot give a half life interpretation to the value of ε as carbon dioxide does not decay but cycles in between the atmosphere and terrestrial and oceanic reservoirs. Hence ε represents the net decadal rate at which these reservoirs absorb carbon dioxide from the atmosphere.

¹⁸This is the approximation of a definite integral by the weighted sum of function values at specified points. I use 5 points for this approximation.

μ		4			5.5			7		
\hat{S}		520	560	600	520	560	600	520	560	600
κ	0.02	58.8	56.5	54.2	80.9	77.7	75.6	103	98.9	94.8
	0.04	58.3	56.5	54.6	80.1	77.7	74.5	102	98.8	95.5

Table 1.1: Difference in year 2100 CO₂ concentration (ppmv) between models with and without feedbacks

of the feedback function, a measure of how quickly feedback processes kick in as the concentration of CO₂ in the atmosphere increases. As shown in figure 1.12, a lower κ leads to a more gradual feedback function, that starts increasing earlier than the benchmark one but that levels off later. Finally, \hat{S} , the inflexion point of the feedback function, represents the midpoint of the region where feedback processes take off. By changing \hat{S} , one can model feedbacks as arising earlier or later as the CO₂ concentration increases.

As it is most important to correctly calibrate the magnitude of the feedback phenomenon in the climate-carbon cycle, I spend the most time discussing the value of that parameter. While the value of \hat{S} is of quantitative importance, it is of relatively little qualitative importance, as changing its location affects the aggressiveness of the abatement schedule but not its fundamental shape. The case for κ is a little more subtle. As it has already been discussed, the steepness of the feedback function affects the likelihood of the control rule having a Skiba point. More careful robustness checks are therefore required for its value.

I calibrate the magnitude of the feedback function using a meta study of climate-carbon cycle feedbacks. Friedlingstein et al. [2006] apply 11 coupled climate-carbon cycle models to the IPCC¹⁹ A2 emissions scenario to evaluate the increase in atmospheric CO₂ concentration due to feedbacks by 2100. The A2 scenario is one with emissions increasing to very high levels, which is helpful in evaluating the magnitude of the feedback function at very high concentration levels (μ). To select an appropriate value for μ , I simulate the evolution of the CO₂ concentration using the predicted emissions from the A2 scenario and the deterministic climate dynamics (f) with and without the feedback function (g). I report the difference in CO₂ concentrations between the simulations with and without the feedback function for the year 2100 in table 1.1. What is obvious in this table is that parameters κ and \hat{S} play a relatively insignificant role in the contribution of the feedback function to the stock of accumulated CO₂. This small effect is due to the emissions path used for this simulation being exogenous. These two parameters can affect significantly the control rule and hence the CO₂ stock indirectly. As for μ , it is obviously the main parameter affecting the contribution of the feedback function to the CO₂ stock in the simulations.

The model runs from Friedlingstein et al. [2006] generate feedback contributions to CO₂ concentrations in 2100 between 20 and 200 ppmv. However, most model runs give

¹⁹Intergovernmental Panel on Climate Change.

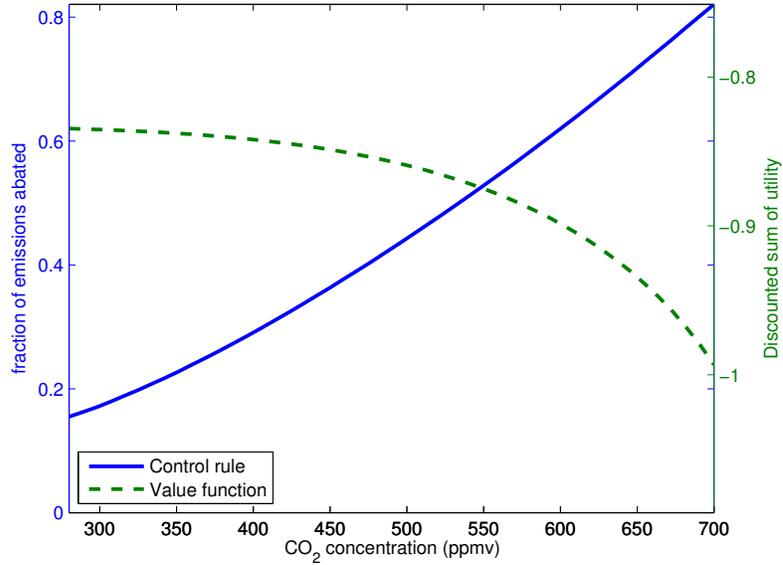


Figure 1.13: Control rule and value function without feedback and uncertainty

contributions located between 50 and 100 ppmv. That is why $\mu = 5.5$ is the benchmark value, as it corresponds roughly to the midpoint of this restricted range. I will also consider values of 4 and 7 for μ as those lead to approximately the lower and upper bounds of that most frequent range.

The final element to calibrate in the model is the stochastic term of the stock dynamics. It represents natural variations in atmospheric CO₂ concentrations. Doney et al. [2006] show that the natural variations in atmospheric CO₂ concentrations over a 1,000 year period has a range of 5 ppm for concentrations around pre-industrial levels and no anthropogenic emissions. To calibrate ξ , I generate 10,000 runs of 100 decades using the carbon dynamics represented by f , without emissions, and a starting value of 280 ppmv. With $\xi \sim N(0, 0.5)$, the median range of variation of the 10,000 runs is 5.44 ppmv and the ninety-fifth percentile is 7.87 ppmv. I hence use this calibration for the benchmark case, as it closely mimics the natural variability of a more complex 3-D global coupled carbon-climate model. However, as pointed out by Joos et al. [1999], data and models suggest that at current higher CO₂ concentrations, the variability is increased.

1.4.3 Results

I now present the results of the numerical simulation themselves. Before analyzing the benchmark calibration, it is useful to consider what the models yields without the feedback term. Figure 1.13 shows the control rule and the value function when the problem is solved without the feedback function, g , and the stochastic term ξ . The control rule in this case

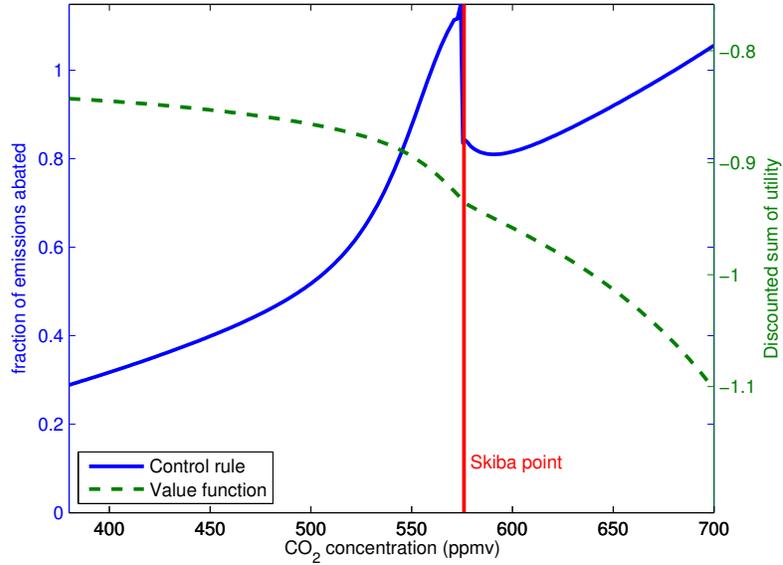


Figure 1.14: Control rule and value function for the benchmark parametrization

is a monotonically increasing function of the stock of CO_2 , while the value function is a decreasing one. This feedback free formulation highlights that any kink in the value function or discontinuity in the control rule will be attributable to the feedback term and not some idiosyncrasy of the model.

I now turn to the benchmark calibration of the model, including the feedback function and the stochastic term. The control rule and the value function are presented in figure 1.14. The benchmark calibration yields a discontinuous control rule. The Skiba point lies at 576 ppmv of CO_2 . One can see that, as expected, the discontinuity in the control rule is associated with a kink in the value function. Indeed, just before the Skiba point, the value function starts plunging quickly as very high abatement rates, some in excess of 100%,²⁰ are required to curb the stock of CO_2 to lower concentrations. However, as the stock passes the Skiba point, it is no longer optimal to try to decrease the stock. The sudden reduction in abatement reduces initially the rate at which the value function is falling.

To assess the impact of uncertainty on the control rule, I plot the benchmark control rule and its deterministic analog in figure 1.15. This figure also includes the isoclines of the deterministic problem. It shows that taking uncertainty into account leads to a more aggressive abatement policy, as the stochastic control rule lies almost everywhere above

²⁰The model constrains abatement expenses to be less than 100% of GWP, but that does not preclude abatement itself beyond 100%. Such a high level of abatement would imply increasing the carbon sinks beyond their natural levels. That could mean a variety of initiatives, going from reforestation to geo-engineering.

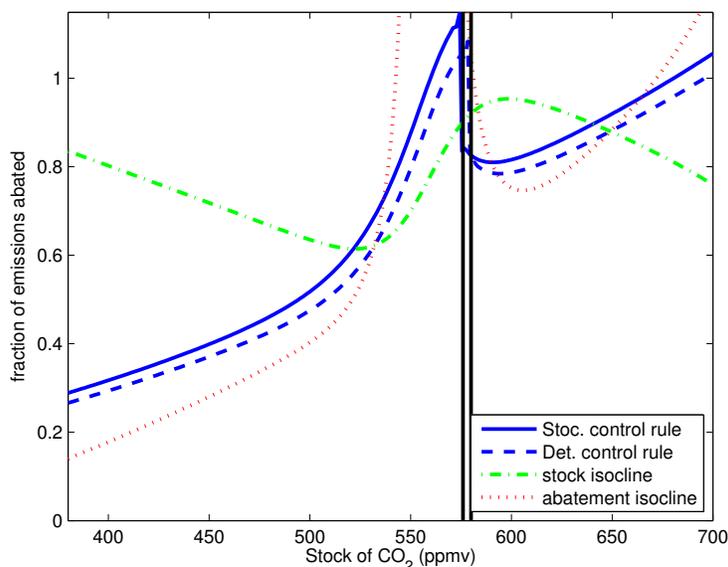


Figure 1.15: Deterministic vs. stochastic control rules, benchmark parametrization

the deterministic one. The only exception is between the Skiba points of each control rule. Uncertainty lowers the Skiba point, i.e. the threshold at which the stock is allowed to grow towards a higher basin of attraction falls. The intuition for these results is that the decision maker adopts a more aggressive abatement policy to reduce the likelihood of going to higher stocks, because she acknowledges that uncertainty reduces her control on the stock compared to the deterministic case. The extra cost of the more aggressive abatement policy is optimal because it reduces the likelihood of higher stock and hence higher marginal damages, since d is convex. However, the point at which it does not pay to prevent the stock from growing towards the upper basin of attraction is lower, because random shocks might push the stock over the previous threshold in spite of the high abatement effort (hence the lower Skiba point).

The steady state distribution is non-unique in the benchmark case. Using the state nodes as midpoints for bins, I discretize the state space to compute a Markov transition matrix representing the optimally controlled climate economy. The eigenvectors corresponding to the eigenvalues of value one give the steady state distributions of the system. In this benchmark case, there are two eigenvalues equal to one, hence two steady state distributions. The probability density function of each of these is shown in figure 1.16. Each of the distributions is centred around the intersection of the control rule and the expected stock isocline. The second distribution (higher in stock) is flatter than the first. This difference is not due to changes in the nature of the uncertainty, which is independent of the level of the stock, but to changes in the control rule. Indeed, the control rule being steeper in the neighbourhood of the first steady state distribution means that deviations

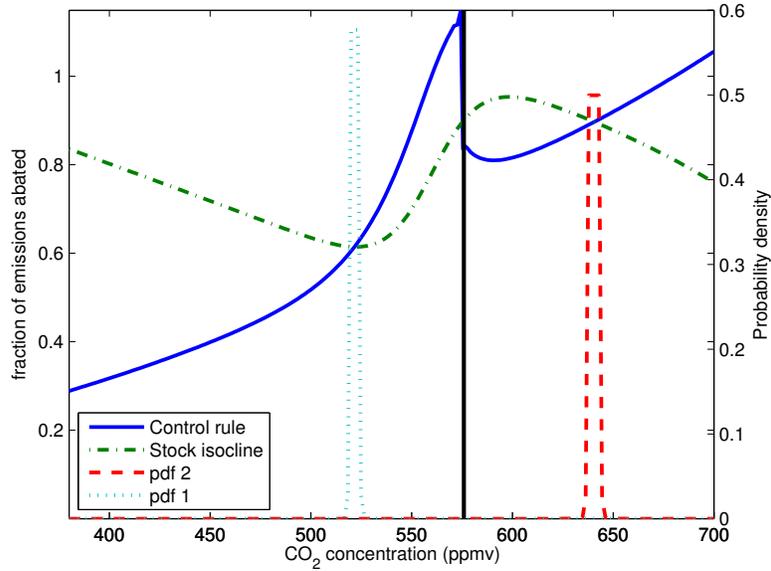


Figure 1.16: Steady state distributions for the benchmark parametrization

from the mean of that distribution are corrected faster. For example, if the stock is hit by a negative shock, it will be lower next period but so will the percentage of emissions abated. Hence the stock will be more likely to grow back to its mean steady state value in the subsequent period. Conversely, positive shocks will lead to rapid increases in abatement, which will contribute to curb back the stock. This corrective effect is weaker around the second steady state distribution, since the control rule is flatter there.

Each steady state distribution is contained within a basin of attraction. If the initial stock is far enough from the Skiba point, then starting below it would lead the stock distribution over time to converge towards the first steady state distribution, while it would converge to the second if it started above the Skiba point. Because the uncertainty is fairly small in the benchmark case, as shown by the tight steady state distributions, the space between the basin of attraction is also fairly small. Figure 1.17 shows the upper bound of the lower basin of attraction and the lower bound of the upper basin of attraction. These bounds are at approximately 574.5 and 577.5 ppmv of CO_2 . This interval is centred around the Skiba point (576 ppmv of CO_2). This symmetry is however not represented in probabilities. As shown in figure 1.18, the probability of ending in the lower basin of attraction decreases very quickly as one moves from the upper bound of that basin to the lower bound of the upper basin of attraction.

Because there is still so much uncertainty about the nature and extent of feedbacks in the climate system, it is important to understand how the control rule changes as the feedback function parameter values are varied. To look at the effect of each parameter, I plot the control rule for alternative parameter values and I compare these to the bench-

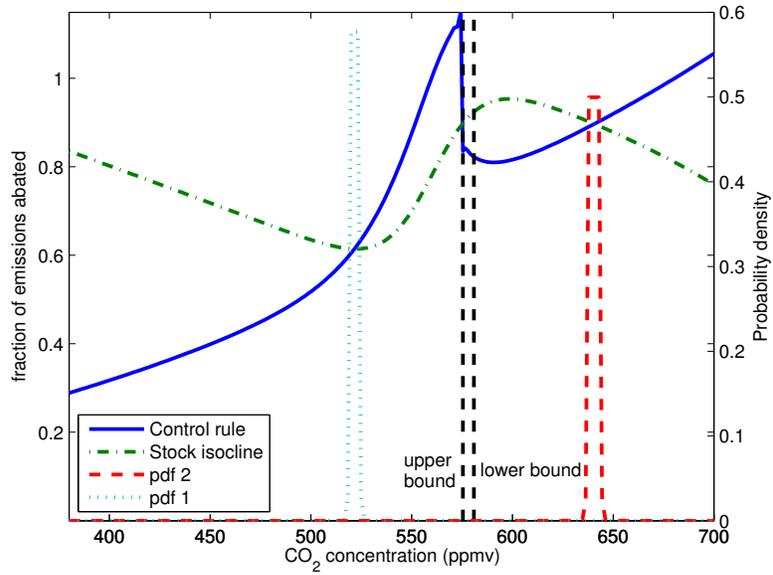


Figure 1.17: Basins of attraction for the benchmark parametrization

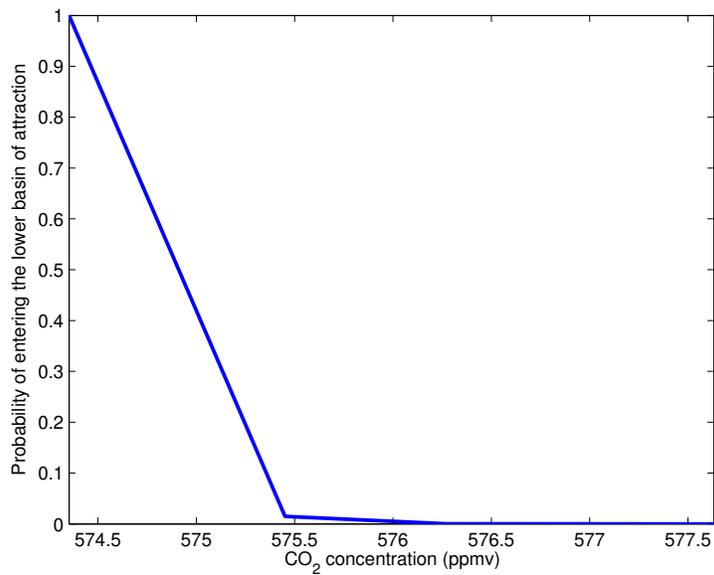


Figure 1.18: Probability of entering the lower basin of attraction in the inter bound range

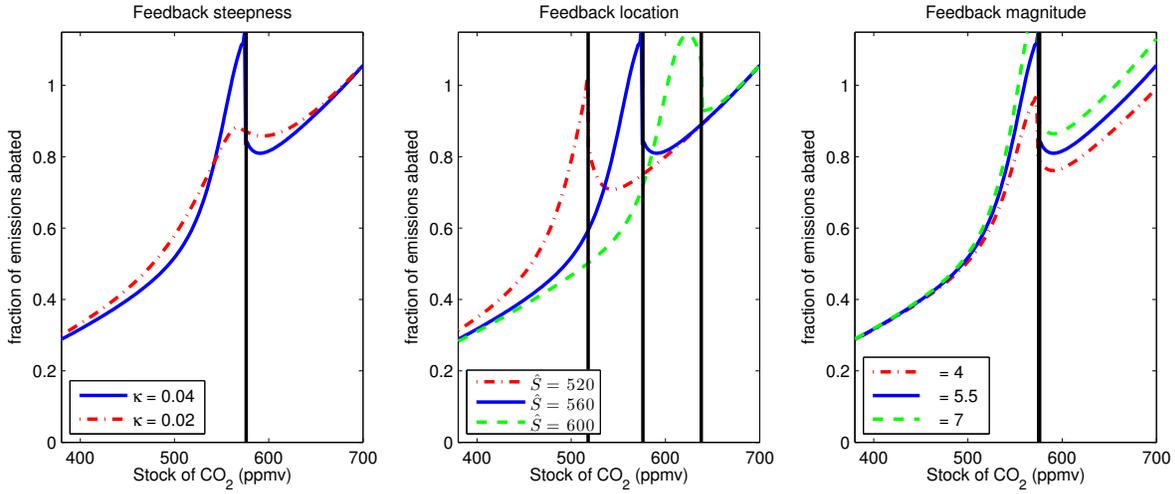


Figure 1.19: Alternative parameter values for the feedback function

mark. The three panels of figure 1.19 depict variations in each of the three parameters of $g(S_t)$. In all panels, the solid line represents the benchmark control rule. In the left panel, the value of κ is changed such that the steepness of the feedback function is reduced. Reducing the steepness of the feedback function makes the control rule much smoother. In fact, for $\kappa = 0.02$, the control rule is continuous, i.e. it does not have a Skiba point. This distinction means that the suddenness of the feedback onset is crucial to the presence of a discontinuity in the control rule. If feedback occurs moderately gradually (see figure 1.12 for the shape of the feedback function with $\kappa = 0.02$), then the control rule could become continuous. That is not to say that the steady state distribution in this case would become unique or unimodal. This change in the control rule is the deterministic equivalent of point B in figure 1.10 becoming an improper unstable node instead of an unstable spiral. In the center panel of figure 1.19, the location of the feedback²¹ is increased and decreased. The primary consequence on the control rule is to change the location of the Skiba point. Earlier onset of feedback implies a lower Skiba point and later onset implies a higher one. This relationship between the onset of feedback and the location of the Skiba point is not surprising as the Skiba point represents a decision threshold that occurs in response to the potential feedback in the stock if that stock is allowed to grow to a certain quantity. What is less obvious is that the location of the Skiba point can be below ($\hat{S} = 520$) or above ($\hat{S} = 560$ and $\hat{S} = 600$) the inflection point of the feedback function. This difference implies that if the feedback sets in earlier, the abatement policy should be very aggressive to prevent the feedback from really kicking in. However, if the feedback begins later, it would be optimal not only to start aggressive abatement later in the stock of CO_2 , but later along the feedback curve as well. This asymmetry between

²¹Technically, the inflection point of the feedback function.

the relative position of the Skiba point and the feedback onset is somewhat counterintuitive, as one would have expected the feedback to be more dangerous when it occurs with an already high stock, as damages would be higher. Finally, the right panel in figure 1.19 shows how the control rule changes as the magnitude of the feedback (parameter μ) increases or decreases. The striking result from this panel is that the feedback magnitude has virtually no impact on the location of the Skiba point. There is, however, an impact on the magnitude of the discontinuity of the control rule at the Skiba point. The higher the magnitude of the feedback, the bigger the discontinuity at the Skiba point. Overall the presence of the Skiba point depends on the suddenness of the onset of feedback, while its location depends on the location of the change in the feedback dynamics. The magnitude of the feedback does not however alter the existence or the location of the Skiba point.

1.5 Conclusion

The climate system and the carbon cycle are characterized by nonlinear feedbacks. I have modeled these feedbacks as a convex-concave CO_2 feedback function in the stochastic dynamics of the stock of pollution. The impact of this feedback is to create an optimally controlled system with two basins of attraction and a control rule that is potentially discontinuous in between these basins.

The discontinuity of the optimal emissions policy depends on the steepness of the feedback function. A steep function, meaning that the onset of the feedback is sudden, leads to a discontinuous abatement policy function. A flatter feedback function, meaning that the onset of the feedback is more gradual, leads to a continuous abatement policy function. In both cases, the control rule is not monotonic in the stock of CO_2 .

If there is a discontinuity in the control rule, its location in the state space depends on the location of the threshold region of the feedback function. If the onset of the feedback occurs at higher CO_2 concentrations, so does the threshold in the optimal abatement policy. The magnitude of this threshold depends on the magnitude of the feedback function. The higher the maximum contribution of feedbacks to the flow of CO_2 into the atmosphere, the higher is the discontinuous change in abatement at the decision threshold.

Stochasticity of the CO_2 stock dynamics reduces the level of the decision threshold. When the natural variability in the stock increases, it is optimal to increase the fraction of emissions abated everywhere, except in a neighborhood of the decision threshold. Since this decision threshold is moved to a lower stock, the fraction of emissions abatement goes down to its right until the previous decision threshold is reached. That is, the smaller the control of the decision maker over the stock, the lower the stock level at which it becomes optimal to let the stock grow toward the higher basin of attraction.

Since the natural variability in the stock of CO_2 is relatively low, the distance between the basins of attraction is small. If this variability were to grow with the concentration of CO_2 in the atmosphere, the distance between the basins of attraction would grow,

enlarging the set of stock values where it is uncertain in which basin of attraction the system will end up.

My simulations suggest that when the emissions path is exogenous, the magnitude of the feedback has the largest impact on equilibrium CO₂ concentrations, while suddenness and onset location have comparatively small contributions. This is in contrast to the optimal abatement policy that is most affected by feedback suddenness and onset location, while the magnitude has a comparatively minor impact. It seems that a good share of the empirical climate literature on feedbacks focuses on the magnitude because they consider only exogenous emissions paths. Optimal management of GHG emissions would therefore benefit from more empirical research on the suddenness and onset location of nonlinear feedbacks.

Finally, this framework could be used to tackle the uncertainty in the parameters themselves. Future work could incorporate parameter uncertainty, while also modeling how the decision maker learns about the uncertain parameters over time.

Chapter 2

Optimal Fuel Choice as a Food-Pollution Trade-off¹

¹In collaboration with David Zilberman

2.1 Introduction

As oil prices increase, biofuels have received more attention as a viable substitute to conventional gasoline. There is however strong concerns that, as biofuel production competes with food production for land, there will be upward pressure put on food prices [Headey and Fan, 2008]. This in turn might prove dramatic for the very poor, for whom price shocks can lead to malnutrition or even starvation.

Several studies have used different approaches to evaluate the impact of biofuel production on food prices. A good review on the topic is Chakravorty et al. [2009]. Corn prices have been especially studied [Collins, 2008, Glauber, 2008, Rajagopal et al., 2009], but results vary widely, with biofuels causing an increase in prices between 15% and 60%.

The literature on optimal biofuel production has often been tackled in an Hotelling framework [Hotelling, 1931]. One of the best example of that is Chakravorty et al. [2008]. In their model, production of biofuels is ramped up as fossil fuel reserves are depleted. Their approach also includes a Ricardian component, as it is possible that land constraints change biofuel production in discrete jumps.

It is, however, possible that the exhaustibility of fossil fuel reserves is a secondary level concern to the effects of accumulated greenhouse gases (GHG) in the atmosphere. In such a case, biofuel production is used to mitigate GHG emissions. Chakravorty et al. [2008] consider the impact of GHG accumulation only to the extent that there is a fixed exogenous total stock not to be exceeded. We propose to depart from their model to shift the focus away from the exhaustible nature of oil and towards the climate change benefits of biofuels.

To tackle the trade-off between reduced pollution costs and increased food prices, we develop an optimal control model where the constraint on fossil fuel is not its exhaustible nature, but the damages its use creates by contributing to climate change. Two generations of biofuels can be used to reduce emissions, but they are competing with food for the fixed amount of land available. The first generation as lower costs but provides less climate benefits than the second generation. We derive conditions for the optimal allocation of land between food and biofuel production, and the optimal investment into second generation biofuel. Because of the GHG externality, the market outcome is suboptimal, and we consider different policies to improve upon it.

2.2 Model

2.2.1 Social Optimum

Consider a social planner that is trying to maximize the sum of discounted utility over an infinite horizon by choosing at each instant in time the level of fossil fuel extraction, the quantity of land to be devoted to the production of food and of each biofuel type, the level of investment into second generation biofuels, and the amount of resources to be

devoted to GHG emissions abatement. The problem the social planner is solving is the following.

$$\max_{\substack{x_o(t), L_{b1}(t), L_{b2}(t), \\ L_f(t), v(t), a(t)}} \int_0^{\infty} e^{-\rho t} u(x(t), L_f(t), z(t)) dt \quad (2.2.1)$$

subject to:

$$x(t) = x_o(t) + f_1(L_{b1}(t)) + f_2(L_{b2}(t)) \quad (2.2.2)$$

$$\bar{L} \geq L_f(t) + L_{b1}(t) + L_{b2}(t) \quad \forall t \quad (2.2.3)$$

$$z(t) = \bar{y}(1 - d(S(t))) - c_o(x_o(t)) - c_b(L_{b1}(t), L_{b2}(t), K(t)) - c_f(L_f(t)) - v(t) - a(t) \quad (2.2.4)$$

$$\dot{S}(t) = g(x_o(t), L_{b1}(t), L_{b2}(t), L_f, a) - \varepsilon S(t) \quad (2.2.5)$$

$$S(0) \text{ given} \quad (2.2.6)$$

$$\dot{K}(t) = v(t) \quad (2.2.7)$$

$$K(0) \text{ given} \quad (2.2.8)$$

where:

- $u(\cdot)$ is the utility function of the representative agent, which is increasing and concave in its first three arguments and satisfy the Inada conditions;
- $\rho > 0$ is the rate of time preference;
- $x(t) \geq 0$ is the flow of energy produced at time t , which is the sum of fossil fuel ($x_o(t) \geq 0$) and the two generations of biofuel ($f_2(L_{b1}(t)) \geq 0$, $f_2(L_{b2}(t)) \geq 0$) energy;
- $\bar{L} > 0$ is the fixed amount of land available, which is divided between food production ($L_f(t)$) and biofuels production ($L_{b1}(t)$ and $L_{b2}(t)$) at each instant in time;
- $f_i(\cdot)$ is the production function for each biofuel generation i , for which $\frac{\partial f_i}{\partial L_{bi}} > 0$ and $\frac{\partial^2 f_i}{\partial L_{bi}^2} < 0$;
- $z(t)$ is the non-energy, non-food consumption flow at time t :
 - \bar{y} the fixed income flow;
 - $d(S_t)$ the proportion of income diverted by climate damages, where $\frac{\partial d}{\partial S} > 0$ and $\frac{\partial^2 d}{\partial S^2} > 0$;

- $c_o(x_o(t))$ is the cost of fossil fuel extraction, where $\frac{\partial c_o}{\partial x_o} > 0$;
 - $c_b(L_{b1}(t), L_{b2}(t), K(t))$ is the cost of biofuel production, where $\frac{\partial c_b}{\partial L_{b1}} > 0$, $\frac{\partial c_b}{\partial L_{b2}} > 0$, $\frac{\partial c_b}{\partial K} < 0$, $\frac{\partial^2 c_b}{\partial L_{b1} \partial K} = 0$ and $\frac{\partial^2 c_b}{\partial L_{b2} \partial K} < 0$;
 - $v(t)$ is investment second generation biofuels;
 - $a(t)$ is spending on emissions abatement.
- $S(t)$ is the stock of GHG in the atmosphere at time t which depends on $(\dot{S} = g(x_o, L_{b1}, L_{b2}, L_f, a) - \varepsilon S)$:²
 - $g(\cdot)$ is the growth rate of the pollution stock, for which $\frac{\partial g}{\partial x_o} > 0$, $\frac{\partial^2 g}{\partial x_o^2} > 0$, $\frac{\partial g}{\partial L_{b1}} > 0$, $\frac{\partial^2 g}{\partial L_{b1}^2} > 0$, $\frac{\partial g}{\partial L_{b2}} > 0$, $\frac{\partial^2 g}{\partial L_{b2}^2} > 0$, $\frac{\partial g}{\partial L_f} > 0$, $\frac{\partial^2 g}{\partial L_f^2} > 0$, $\frac{\partial g}{\partial a} < 0$ and $\frac{\partial^2 g}{\partial a^2} > 0$;
 - ε is the natural dissipation rate of the stock.
 - $K(t)$ is the capital stock in (cost reducing) second generation biofuel technology.

All functions in the problem are assumed to be continuously differentiable, while all variable are restricted to be non-negative real numbers. Under these conditions, the necessary conditions for optimality derived from the Hamiltonian are also sufficient.

Define the current value Hamiltonian as:

$$\begin{aligned}
 H(x(t), L_f(t), y(t), S(t)) &= u(x(t), L_f(t), z(t)) + \lambda(t)[g(t) - \varepsilon S(t)] + \mu(t)[v(t)] \\
 &\quad + \nu(t)[\bar{L} - L_f(t) - L_{b1}(t) - L_{b2}(t)]
 \end{aligned} \tag{2.2.9}$$

The necessary and sufficient conditions for optimality are:

$$\frac{\partial H}{\partial L_f} = \frac{\partial u}{\partial L_f} - \frac{\partial u}{\partial z} \frac{\partial c_f}{\partial L_f} + \lambda(t) \frac{\partial g}{\partial L_f} - \nu(t) = 0 \tag{2.2.10}$$

$$\frac{\partial H}{\partial L_{b1}} = \frac{\partial u}{\partial x} \frac{\partial f_1}{\partial L_{b1}} - \frac{\partial u}{\partial z} \frac{\partial c_b}{\partial L_{b1}} + \lambda(t) \frac{\partial g}{\partial L_{b1}} - \nu(t) = 0 \tag{2.2.11}$$

$$\frac{\partial H}{\partial L_{b2}} = \frac{\partial u}{\partial x} \frac{\partial f_2}{\partial L_{b2}} - \frac{\partial u}{\partial z} \frac{\partial c_b}{\partial L_{b2}} + \lambda(t) \frac{\partial g}{\partial L_{b2}} - \nu(t) = 0 \tag{2.2.12}$$

$$\frac{\partial H}{\partial x_o} = \frac{\partial u}{\partial x} - \frac{\partial u}{\partial z} \frac{\partial c_o}{\partial x_o} + \lambda(t) \frac{\partial g}{\partial x_o} = 0 \tag{2.2.13}$$

²We use the notation \dot{S} to represent $\frac{\partial S}{\partial t}$.

$$\frac{\partial H}{\partial v} = -\frac{\partial u}{\partial z} + \mu(t) = 0 \quad (2.2.14)$$

$$\frac{\partial H}{\partial a} = -\frac{\partial u}{\partial z} + \lambda(t)\frac{\partial g}{\partial a} = 0 \quad (2.2.15)$$

$$\frac{\partial H}{\partial S} = -\frac{\partial u}{\partial z} \frac{\partial d}{\partial S} - \lambda(t)\varepsilon = \rho\lambda(t) - \dot{\lambda}(t) \quad (2.2.16)$$

$$\frac{\partial H}{\partial K} = -\frac{\partial u}{\partial z} \frac{\partial c_b}{\partial K} = \rho\mu(t) - \dot{\mu}(t) \quad (2.2.17)$$

$$\frac{\partial H}{\partial \lambda} = g(t) - \varepsilon S(t) \quad (2.2.18)$$

$$\frac{\partial H}{\partial \mu} = v(t) \quad (2.2.19)$$

$$\left(\bar{L} - L_f(t) - L_{b1}(t) - L_{b2}(t)\right) \nu(t) = 0 \quad (2.2.20)$$

We can interpret these first order conditions in three groups. The first group includes the first four conditions, which pertain to the four flow variables directly influencing utility: land devoted to food production, land devoted to each biofuel type production, and fossil fuel production. For each of these variables, the first order condition has four components, except for fossil fuel that is lacking the land component. The first component is a benefit component, corresponding to the marginal utility of the variable. The second is the private production cost component, consisting of the marginal private cost of the variable in terms of utility. The third is the external cost component, consisting of the marginal contribution of the variable to the stock of GHG valued at the shadow price of that stock. The fourth is the private land cost component, corresponding to the shadow price of land. Hence for each of these variables we have the optimality condition that the marginal utility must be equal to the marginal private cost of production plus the marginal external cost plus the value of land.

First order conditions (2.2.14) and (2.2.15) apply to flow variables only affecting utility through their costs and their impact on a stock variable: investment in second generation biofuels and GHG abatement. For these two variables, optimality dictates that their marginal cost, which is just the marginal utility of money, must be equal to their marginal benefits. In the case of investment in second generation biofuels, the marginal benefit is just the shadow price of capital, while for abatement, it is the marginal reduction in GHG emissions valued at the shadow price of the GHG stock.

The information about these shadow prices is embedded into first order conditions (2.2.16) and (2.2.17). Equation (2.2.16) can be rearranged as:

$$\lambda(t) = \frac{-\frac{\partial u}{\partial z} \frac{\partial d}{\partial S} + \dot{\lambda}(t)}{\rho + \varepsilon}. \quad (2.2.21)$$

This equation shows that the shadow price of the stock of GHG is the infinite discounted sum of the marginal disutility from the GHG stock plus the change in that price. Note that the discount rate of that sum is not just ρ , but $\rho + \varepsilon$. This modified discounting takes into account the dissipation rate of the stock. A similar expression can be obtained for the shadow price of capital. Equation (2.2.17) can be rearranged as:

$$\mu(t) = \frac{-\frac{\partial u}{\partial z} \frac{\partial c_b}{\partial K} + \dot{\mu}(t)}{\rho}. \quad (2.2.22)$$

From this condition, $\mu(t)$ can be interpreted as the infinite discounted sum of the marginal benefit (marginal cost reduction valued at marginal utility) plus the change in that price.

Both interpretations of shadow prices are analogous to that of a share price. The price of a share represents the net present value of the stream of dividends (the marginal disutility of GHG for $\lambda(t)$ and the marginal benefit of capital for $\mu(t)$) plus a capital gain component, which is the appreciation of the share value (the $\dot{\lambda}(t)$ and $\dot{\mu}(t)$ terms).

The next to last two first order conditions only describe the equation of motion of the stock of GHG and the capital. The last condition is the complementary slackness condition on land availability. If all land is used, $\nu(t)$ is positive, but if some land is unused, $\nu(t) = 0$.

2.2.2 Competitive Outcome

In a competitive setting, agents maximize their private benefits. Hence they ignore the damages created by GHG, which are an externality. They also fail to invest in second generation biofuels, because the technology is here assumed to be a public good benefiting all agents. Because of the model complexity, we do not explicitly model the behavior of individual agents in the economy. Instead we rely on the results of the first welfare theorem. Here, the behavior of agents will be represented by the optimal decision of a social planner who would not account for climate damages and capital benefits. The competitive outcome can therefore be represented by modifying the first order conditions from the social optimum. Conditions (2.2.10) to (2.2.13), become:

$$\frac{\partial u}{\partial L_f} - \frac{\partial u}{\partial z} \frac{\partial c_f}{\partial L_f} - \nu(t) = 0, \quad (2.2.23)$$

$$\frac{\partial u}{\partial x} \frac{\partial f_1}{\partial L_{b1}} - \frac{\partial u}{\partial z} \frac{\partial c_b}{\partial L_{b1}} - \nu(t) = 0, \quad (2.2.24)$$

$$\frac{\partial u}{\partial x} \frac{\partial f_2}{\partial L_{b2}} - \frac{\partial u}{\partial z} \frac{\partial c_b}{\partial L_{b2}} - \nu(t) = 0, \quad (2.2.25)$$

$$\frac{\partial u}{\partial x} - \frac{\partial u}{\partial z} \frac{\partial c_o}{\partial x_o} = 0. \quad (2.2.26)$$

Since no abatement, nor any investment would take place in the purely competitive environment, conditions (2.2.14) and (2.2.15) are removed from the solution. The remaining equations stay the same.

To analyze the differences between the competitive outcome and the social optimum, it is useful to consider three cases based on the scarcity of land. If the land constraint is not binding in the social optimum nor in the competitive outcome, we will consider that case as abundant land. If the land constraint is binding in both the social optimum and the competitive outcome, we will consider that case as scarce land. Finally if land is binding only in the competitive outcome or only in the social optimum, we will consider that case as partially scarce land.

2.2.2.1 Case 1: Abundant Land

In this case, we assume that the land constraint would not be binding in the socially optimal situation, nor in the competitive outcome. If land is abundant, then the multiplier on its constraint is zero in both the social optimum and the competitive outcome.

Proposition 1. *When land is abundant, the competitive outcome yields an overproduction of fossil fuel and food, but has an ambiguous effect on both types of biofuel.*

For fossil fuel production, we compare first order conditions (2.2.13) and (2.2.26). The socially optimal fossil fuel quantity, x_o^* , solves $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial z} \frac{\partial c_o}{\partial x_o} - \lambda(t) \frac{\partial g}{\partial x_o}$. Since $\lambda(t) < 0$, GHG being a negative externality, it is the case that $\frac{\partial u}{\partial x} |_{x_o^*} > \frac{\partial u}{\partial z} \frac{\partial c_o}{\partial x_o} |_{x_o^*}$. Hence, because utility is concave in fuel and oil extraction costs are convex, $x_o^c > x_o^*$.³

For food production, we compare first order conditions (2.2.10) and (2.2.23). The socially optimal land use for food production, L_f^* , solves equation (2.2.10). Since the marginal external cost term is positive, it is the case that $\frac{\partial u}{\partial L_f} |_{L_f^*} > \frac{\partial u}{\partial z} \frac{\partial c_f}{\partial L_f} |_{L_f^*}$. Hence, since utility is concave in food and food production costs are convex, $L_f^c > L_f^*$.

For first generation biofuel production, we compare first order conditions (2.2.11) and (2.2.25). The socially optimal quantity of land devoted to first generation biofuel production, L_{b1}^* , solves equation (2.2.11). In the competitive outcome, the marginal external cost term vanishes, but the marginal utility of fuel term goes down, following the increase in fossil fuel production. The net effect is hence ambiguous. If the marginal external cost term is bigger than the change in the marginal utility of fuel, the marginal product of first generation biofuels would have to increase to satisfy equation (2.2.25), meaning

³We use * to denote socially optimal quantities and c for competitive ones.

$L_{b1}^c < L_{b1}^*$. However, the situation $L_{b1}^c > L_{b1}^*$ could arise if the change in the marginal utility of fuel is greater than the marginal external cost of first generation biofuel.

The analysis for second generation biofuels is similar to that for first generation, but it also incorporates the impact of underinvestment in technology. While the result is also ambiguous in general, it is more likely the case that $L_{b2}^c < L_{b2}^*$. This is because of both the lower marginal external cost of second generation biofuel and the technological effect. Since second generation biofuel is defined as cleaner than the first generation, we assume $\frac{\partial g}{\partial L_{b2}} < \frac{\partial g}{\partial L_{b1}}$. Therefore it is less likely for second generation biofuel to have its marginal external cost term larger than the change in the marginal utility of fuel. Hence, $L_{b2}^c < L_{b2}^*$ is more likely than the equivalent for first generation biofuel. This asymmetry is reinforced by the effect of underinvestment in technology in the competitive outcome. That effect raises the marginal private cost component in equation (2.2.25) compared to equation (2.2.12). This effect puts pressure for a higher marginal product of second generation biofuel, implying a lower land quantity devoted to second generation biofuel.

The apparent counter-intuitive result for biofuels production can be explain by decomposing the effect of moving from the social optimum to the competitive outcome in two parts. First, there is the pollution part. Since both type of biofuels also generate GHG, failing to account for that would increase their production in the competitive outcome. Second, there is the fuel mix part. Since the advantage of biofuels over fossil fuel is due to their lower emissions, failing to account for that would favor fossil fuel and hence reduce the optimal amount of land devoted to biofuels production. On top of that, the technology effect further reduces second generation biofuel production in the competitive outcome.

The relative impact of the pollution and fuel mix effects can be made unambiguous by one further assumption presented in the following proposition.

Proposition 2. *When land is abundant, if the marginal cost of fossil fuel production is constant and biofuels generate less GHG emissions than fossil fuel, then the competitive outcome yields an underproduction of both generations of biofuels.*

A constant marginal cost of fossil fuel extraction implies that the change in the marginal utility of fuel is equal to the marginal external cost of fossil fuel, i.e. $\Delta \frac{\partial u}{\partial x} = \lambda(t) \frac{\partial g}{\partial x_0}$. Since we assumed both biofuels are cleaner than the fossil fuel ($\frac{\partial g}{\partial L_{b2}} < \frac{\partial g}{\partial L_{b1}} < \frac{\partial g}{\partial x_0}$), it must be the case that the marginal utility of fuel decreases by more than the marginal external cost of either type of biofuels. In such a case, the marginal product of both biofuels has to go up, while its marginal private cost must go down to satisfy conditions (2.2.24) and (2.2.25). That is, both biofuels productions must be lower in the competitive outcome compared to the social optimum.

The intuition for this last proposition is that with constant marginal cost, fossil fuel is guaranteed to increased its share in the fuel mix to the extent that even an increased total fuel production would not lead to an increased biofuel production.

2.2.2.2 Case 2: Scarce Land

In this case, we assume that the land constraint would be binding in the social optimum and in the competitive outcome. That means $\nu(t) > 0$.

Proposition 3. *When land is scarce, the competitive outcome yields an overproduction of fossil fuel, but has an ambiguous effect on food and both types of biofuel.*

The analysis for fossil fuel is exactly the same as in the previous case, because the land constraint does not enter its production decision. For the three land related products, it is important to consider whether the competitive outcome will increase the shadow value of land compared to the social optimum or if it will decrease it.

Proposition 4. *When land is scarce, if the competitive outcome reduces land scarcity, it also yields an overproduction of food and an underproduction of at least the second generation biofuel.*

As a reduction in land scarcity implies a reduction in its shadow value, it is straightforward that land devoted to food production will increase in the competitive outcome. Indeed, not only does condition (2.2.23) lose its marginal external cost term compared to the social optimum equivalent, but $\nu(t)$ also goes down. Hence the marginal utility of land devoted to food production must go down as well, to preserve the equality, which implies that land area devoted to food production must increase. Since the land constraint was already binding in the competitive outcome, the expansion of food production must come at the expense of some other land use.

Second generation biofuel production is the first candidate despite the qualitative uncertainty surrounding the change in its first order condition. Indeed, because of its lower marginal external cost and of the lack of technological investment effect, it is more likely to see its land use reduced than first generation biofuel. For it, the change could go either way depending on the magnitude of the respective land use changes in food production and second generation biofuel.

Proposition 5. *When land is scarce, if the competitive outcome increases land scarcity such that the change in the shadow value of land is greater than that of its marginal external cost, then the competitive outcome yields an underproduction of food and an overproduction of at least the first generation of biofuel.*

The impact on food can be shown by using equation (2.2.10). Moving to the competitive outcome, this equation loses its marginal external cost term and sees the shadow value of land increase. Since this increase is assumed bigger than the marginal external cost, it must be that the marginal utility of food increases to preserve the equality, implying land use for food decreases. Since the constraint is still binding, some other land use must be increasing.

First generation biofuel is the first candidate to fill this role. Since its marginal external cost is higher than that of second generation biofuel, it would be more likely for first

generation biofuel that removing that cost from its first order condition, while increasing $\nu(t)$, requires a decrease in the marginal product of land and hence an increase in land used for its production. As for second generation biofuel, whether its land use goes up or down depends to the extent to which the change in land used for food is greater than the change in land used for first generation biofuel or not.

Proposition 6. *When land is scarce, if the competitive outcome increases land scarcity such that the change in the shadow value of land is smaller than that of its marginal external cost, then the competitive outcome yields an overproduction of food and an underproduction of at least the second generation of biofuel.*

The change in land used for food production can be explained as in the case for proposition 4. The only difference being that $\nu(t)$ increases instead of decreasing, but not enough to offset the elimination of $\lambda(t) \frac{\partial g}{\partial L_f}$. Hence the marginal utility of food must still decrease, which implies an increase in food production. That increase in land used for food must be offset by some other land use change.

Second generation biofuel is more likely to fill that role since its marginal external cost is smaller than that of first generation biofuel. Indeed, that implies that the elimination of $\lambda(t) \frac{\partial g}{\partial L_{b2}}$ in equation (2.2.12) is more likely to be offset by the increase in $\nu(t)$ than the equivalent change in equation (2.2.11). Again, the lack of technological investment reinforces this situation by increasing the marginal private cost of second generation biofuel compared to the social optimum. Such an offset can lead to an increase in the marginal product, which requires a decrease in land used. First generation biofuel land use will go up or down depending on how whether the increase in food land use is smaller or bigger than the decrease in second generation biofuel land use.

2.2.2.3 Case 3: Partially Scarce Land

In this case, the land constraint is binding in only one of the two outcomes, either in the social optimum or in the competitive economy. Qualitatively, the conclusions of this case are the same as for the second one. In general, there is overproduction of fossil fuel in the competitive outcome. If land scarcity is decreasing (from scarce in the social optimum to abundant in the competitive outcome) the results of proposition 4 apply. If, however, land scarcity is increasing (from abundant in the social optimum to scarce in the competitive outcome) the results of either proposition 5 or proposition 6 apply depending of the relative magnitude of the competitive shadow value of land and the socially optimal marginal external cost of food.

2.3 Dynamic Analysis

So far we have focused differences in levels between the social optimum and the competitive outcome on optimal paths. In this section, we change the focus to look more

closely at changes along the optimal paths, not only between them. To consider only a situation that is plausibly relevant, we will assume that the concentration of GHG in the atmosphere is increasing over time in both the social optimum and the competitive outcome.

This is the situation we are most certainly confronted to. At today's concentration, most models predict that the optimal path has GHG concentration increasing for some period of time. Within the model, this is possible if the initial stock of GHG ($S(0)$) is sufficiently low. There always exist such a low $S(0)$ as long as the steady state level of S is not zero. Since dissipation is proportional with the stock, this can only arise if all emissions are abated, a situation that is not very interesting given current technology. As in the previous section, we will consider three cases defined by their land scarcity.

2.3.1 Case 1: Abundant Land

When land is abundant, all the dynamics are determined by the evolution of each of the two stocks: GHG and capital. The GHG stock is increasing, because we are assuming the initial condition is below the steady state value. As for capital, it cannot decrease by definition. Since damages are convex in the stock of GHG, increases in that stock imply more negative shadow value of pollution over time, i.e. $\lambda(t)$ is decreasing. That decrease can be interpreted as an increase in the marginal external cost of all GHG emitting production processes.

Proposition 7. *In both the competitive outcome and the social optimum, if land is abundant, fossil fuel and food production are decreasing over time, while the change in biofuels production is ambiguous.*

In the competitive outcome, since there is no investment and climate damages are ignored, all the dynamics are driven by the reduction in wealth brought about by increasing damages to output. As output goes down, its marginal utility goes up, which means the opportunity cost of producing fuels and food goes up. Optimality dictates that this must be compensated by an increase in the marginal utility of these products. Increasing marginal utility of fuel and food means fossil fuel and food productions decrease over time. The ambiguous result for both types of biofuel comes from the opposing effects of the increase in the opportunity cost of production and the increase in the marginal utility of fuel. If the former dominates, biofuels production would decrease, but it could increase if the later effect dominates.

In the social optimum, results are driven by changes in $\lambda(t)$ and capital. A decreasing $\lambda(t)$ leads to increasing marginal external cost for all four GHG emitting products. Fossil fuel and food conditions (2.2.13) and (2.2.10) imply that the marginal utility of each must increase to preserve the optimality. That means the production of each must decrease. For each biofuel generation, since the increasing marginal utility of fuel interacts with the marginal product, we cannot unambiguously sign the change in production. For second generation biofuel however, the additional impact of an increasing capital will reduce its

marginal production cost and hence make it less likely that land used for its production would decrease over time.

Although the results are similar between the social optimal and the competitive outcome, it is for different reasons. In the competitive outcome, fossil fuel and food production are decreasing because climate damages are claiming a larger share of output. In the social optimum, fossil fuel and food production are decreasing over time as the emissions from their production are deemed more damaging due to the increasing stock of GHG.

Proposition 8. *In the competitive outcome, if land is abundant and the marginal cost of fossil fuel is constant, fossil fuel and food production decrease over time, while production of both generations of biofuel remains constant.*

A constant marginal cost of fossil fuel has no impact on food production and makes no qualitative difference to the path of fossil fuel production. The analysis supporting proposition 7 also applies here. For biofuels obviously, the analysis is different. Since the marginal cost of fossil fuel production is constant, the change in marginal utility of fuel must be proportional to the change in the marginal utility of output, in order for equation (2.2.26) to continue to hold as output decreases. That proportionality also applies to conditions (2.2.24) and (2.2.25), implying that there is no need for either marginal product to change to maintain the equality. Hence the land used for each generation of biofuel production must remain constant.

A simple arbitrage interpretation can be given to this result. Since the competitive outcome is not concerned by varying emission levels, the fuel mix solely depends on relative marginal costs and marginal products. With strictly increasing marginal costs, each generation of biofuel has a single production level that matches the constant marginal cost of fossil fuel. In an interior solution, any reduction of fuel production must be entirely borne by fossil fuel.

Proposition 9. *In the social optimum, if land is abundant and the marginal cost of fossil fuel is constant, fossil fuel and food production decrease over time, while production of both generations of biofuel increases.*

As in proposition 8, a constant marginal cost of fossil fuel has no impact on food production. For fossil fuel production, the constant marginal cost implies that the change in the marginal utility of fuel must equal the change in the marginal utility of output. That still implies fossil fuel production decreases over time. However, since biofuels are assumed cleaner than fossil fuel, the increase in their marginal external cost is smaller than the increase in the marginal utility of fuel. Hence the marginal product of both biofuels must decrease over time to keep equations (2.2.11) and (2.2.12) satisfied. This implies that production of each generation of biofuel must be increasing over time. The impact of the capital only reinforces that increase.

The arbitrage interpretation is here different than in the competitive outcome case, because the impact of GHG emissions is taken into account. As $\lambda(t)$ increases in magnitude, the wedge between the marginal external cost of fossil fuel and biofuels increases,

which creates an increasing marginal advantage for biofuels that is exploited by increased production.

2.3.2 Case 2: Scarce Land

In the abundant land case, only changes in the stock of GHG and in capital determine the dynamics of different productions. When land is scarce, changes in land scarcity also impact these dynamics. Changes in land scarcity impact the first order conditions through changes in the shadow value of land, $\nu(t)$.

From the abundant land case, we know that land demand may go up or down as there are many situations in which biofuel land use change is ambiguous. Hence in this subsection, we will consider situations where scarcity increases and others where it decreases.

Proposition 10. *When land is scarce and scarcity decreases, the change in food production over time is ambiguous in both the competitive outcome and the social optimum.*

In the competitive case, using condition (2.2.23), we can see the increase over time of the marginal utility of output is mitigated by the decrease in the shadow value of land. Since it is unclear which effect dominates, the change in the marginal utility of food and hence of food production itself is ambiguous.

In the social optimum, it is the marginal external cost term and the shadow value of land that change in opposite directions in condition (2.2.10). This fact also implies an ambiguous change in food production over time.

Proposition 11. *When land is scarce and scarcity increases, fossil fuel and food production decrease, while the production of at least one generation of biofuel must increase in both the competitive outcome and the social optimum.*

Note that land scarcity does not affect fossil fuel optimality conditions in either the competitive outcome or the social optimum. Hence any result for the time path of fossil fuel production in the abundant land case must carry to the scarce land case.

For food, the increase over time of the shadow value of land only adds to the increase in marginal utility of output in equation (2.2.23) or to the increase in marginal external cost in equation (2.2.10). In both situations, the impact on food is exacerbated such that food production is further decreasing.

For both biofuel types, an increasing shadow value of land does not remove the fundamental ambiguity of their time path. However, since the condition is here that land scarcity increases, knowing that food land use decreases, it must be that some other land use increases. In the social optimum, second generation biofuel must necessarily be increasing because of its lower marginal external cost and of the technological effect.

Proposition 12. *When land is scarce and scarcity decreases, if the marginal cost of fossil fuel is constant, results of proposition 9 apply in the social optimum.*

Just like in proposition 11, results for fossil fuel and food follow directly from the first order conditions of the social optimum. For biofuels, the decrease in $\nu(t)$ over time in equations (2.2.11) and (2.2.12) reinforces the dominance of the change in the marginal utility of fuel over that of the marginal external cost. This leads to a faster decline in both marginal products, meaning a faster increase in the production of both generations of biofuel.

2.3.3 Case 3: Partially Scarce Land

Partial scarcity consists in situation where the land constraint is not always binding. As the production of food and biofuels changes, so does the scarcity of the land. It is possible that such changes make abundant land that was previously scarce, or scarce land that was previously abundant.

In the competitive outcome, a transition from scarce to abundant land is possible if the system starts from conditions described by proposition 10. Such a system could eventually reach a point where land is abundant, in which case the results of proposition 7 would apply. On the other hand, if the initial condition of the system are such that land is abundant, i.e. satisfying proposition 7. Then it is possible, only under the condition that at least one biofuel land use is increasing, that land would become scarce and that we would observe results from proposition 11.

In the social optimum, both scenarios described under the competitive outcome are also possible. However, there is the possibility of a third scenario. If the marginal cost of fossil fuel is constant and the system is initially described by proposition 12, it is possible that at a certain point it enters the characteristics of proposition 9.

There are also transitions that are obviously not possible. Under the conditions of proposition 8, land can never become scarce, since land uses are all decreasing or constant. Under the conditions of proposition 11, land can never become abundant by assumption.

In the situations where there can indeed be a transition, that transition will always be continuous. That is the time path of all variables across a change in land scarcity is continuous, because changes in land scarcity are continuous. However, there may be kinks at the transition in some time paths as $\nu(t)$ goes from smoothly changing on the scarce side to always zero on the abundant one.

2.4 Second Best Policies

We now consider policies that could make the competitive outcome closer to the social optimum. The first best policies would imply a continuously variable emission tax equal to $\lambda(t)$ and a public provision of the technological investment, financed through a non-distortionary poll-tax. Such policies have however not been observed in practice. We will focus on two second best policies that derive from policies that have been implemented

in the United States to promote the production of corn based ethanol. These two policies are biofuel subsidies and biofuel mandate.

Biofuel subsidies will be assumed to reduce the marginal cost of producing either types of biofuels. The biofuel mandate will be assumed to require a binding minimum quantity of either type of biofuel to be produced. It is important to note that such policies always promote an increase in biofuel production. Hence, only in situations where the competitive outcome yields an under-provision of either biofuels will it be welfare improving to use these policies. That is, only conditions under which propositions 2, 4 and 6 apply will be considered. In such situations, where some biofuels are unambiguously under-provided by the competitive outcome, food is always over-provided. In addition fossil fuel is over-provided in all the possible cases of the competitive outcome.

2.4.1 Biofuel Mandate

We start with the biofuel mandate, because it is easier to conceptualize than the tax. The mandate involves choosing the level of production for each of the two generations of biofuels. With these two instruments, the policy maker attempts to correct five deficiencies of the competitive outcome: overproduction of food and fossil fuel, underproduction of both generations of biofuel, and underinvestment in second generation biofuel technology.

Proposition 13. *When both generations of biofuel are under-provided in the competitive outcome, the optimal mandate for each type of biofuel is larger than their socially optimal quantities, regardless of land scarcity.*

This is easily shown by considering a mandate for both biofuels at their socially optimal levels. In the abundant land case, the first order condition for first generation biofuel becomes: $\frac{\partial u}{\partial x} \frac{\partial f_1}{\partial L_{b1}} \Big|_{L_{b1}^*} = \frac{\partial u}{\partial z} \frac{\partial c_b}{\partial L_{b1}} \Big|_{L_{b1}^*}$. Comparing this to the socially optimal condition, $\frac{\partial u}{\partial x} \frac{\partial f_1}{\partial L_{b1}} \Big|_{L_{b1}^*} = \frac{\partial u}{\partial z} \frac{\partial c_b}{\partial L_{b1}} \Big|_{L_{b1}^*} - \lambda(t) \frac{\partial g}{\partial L_{b1}}$,⁴ we notice that the right hand side of the later is bigger than that of the former. Since the marginal products are equal, it must be the case that the marginal utility of fuel is lower with this mandate than in the social optimum. Since the same quantity of biofuel is produced by this mandate as in the social optimum, it must be that there is overproduction of fossil fuel. The mandate could therefore be improved upon by slightly raising the quantities of both biofuels, hence lowering the production of fossil fuel. This has to be welfare improving because the socially optimal quantity of biofuels has by definition a marginal benefit of zero, while that of an overproduction of fossil fuel must be negative.

A similar reasoning also applies when land is scarce. The policy marker would want to increase the optimal mandate beyond the social optimum to reduce the overproduction of fossil fuel. In addition land scarcity might also allow to tackle food overproduction. The first order condition for food with a mandate at the social optimum would be $\frac{\partial u}{\partial L_f} = \frac{\partial u}{\partial z} \frac{\partial c_f}{\partial L_f} + \nu(t) \Big|_{L_{b1}^*, L_{b2}^*}$. Comparing this to the social optimum, $\frac{\partial u}{\partial L_f} = \frac{\partial u}{\partial z} \frac{\partial c_f}{\partial L_f} - \lambda(t) \frac{\partial g}{\partial L_f} +$

⁴A similar argument applies to second generation biofuel.

$\nu(t) |_{L_{b1}^*, L_{b2}^*}$, the right hand side of the socially optimal condition must be higher than that of the mandate condition. Hence, the marginal utility of food must be lower with the mandate, which means there is overproduction of food. For a similar argument as for fossil fuel, it would be welfare improving to increase the biofuel mandate beyond the socially optimal level to bring food closer to its social optimum.

The combination of the fossil fuel and food effect when land is scarce is to push the optimal biofuel mandate beyond its optimal point when land is abundant.

2.4.2 Biofuel Subsidies

Identical results can be achieved with two generation specific biofuel subsidies as with biofuel mandates.⁵ The interesting point in considering the tax is that it highlights the role that each of the objectives of the policy maker plays in choosing the correct price (or quantity) for biofuel production.

Just like in the case of the mandate, the policy maker has two instruments to attempt to influence five variables: underproduction of each type of biofuel, overproduction of food and fossil fuel and underinvestment in second generation biofuel.

With biofuel subsidies, the first order conditions of the system remain the same as in the competitive outcome, except for the two biofuel conditions, which see a subsidy component added to them.

$$\frac{\partial u}{\partial x} \frac{\partial f_1}{\partial L_{b1}} = \frac{\partial u}{\partial z} \left(\frac{\partial c_b}{\partial L_{b1}} - s_1 \right) + \nu(t) \quad (2.4.1)$$

$$\frac{\partial u}{\partial x} \frac{\partial f_2}{\partial L_{b2}} = \frac{\partial u}{\partial z} \left(\frac{\partial c_b}{\partial L_{b2}} - s_2 \right) + \nu(t) \quad (2.4.2)$$

We decompose each per unit subsidy into four components to illustrate the multiple objectives:

$$s_i = s_{i1} + s_{i2} + s_{i3} + s_{i4} \quad \forall i = 1, 2. \quad (2.4.3)$$

For generation i , the per unit subsidy includes a GHG component, s_{i1} , which reflects the lower emissions of that biofuel compared to fossil fuel. The second element, s_{i2} , is the fossil fuel component, which improves biofuel competitiveness to reduce oil overproduction. Similarly, the food component, s_{i3} , improves biofuel competitiveness to reduce land overuse for food production. Finally, the technology component, s_{i4} , compensates for lack of investment in second generation biofuel in the competitive outcome.

The first restriction we can make is that since under investment only pertains to second generation biofuel, the technological term for first generation biofuel subsidy must be zero. Also, since the subsidy directly applies to the marginal cost of production, the

⁵This point of course ignores issues of heterogeneity between firms or assumes that firm specific mandates can be set.

technological component for second generation biofuel subsidy is of the same size as the change in marginal cost that capital would have brought, i.e. $s_{24} = \Delta \frac{\partial C_b}{\partial L_{b2}}$.

For both generations of biofuel, the key component of the subsidy is the GHG one. The magnitude of s_{i1} is an increasing function of the difference between fossil fuel emissions and each generation of biofuel emissions. The cleaner biofuels compared to fossil fuel the higher that component. In addition, it must be the case that $s_{11} < s_{21}$, since second generation biofuel is assumed to emit less GHG than first generation.

Components s_{i1} and s_{i4} of the optimal subsidies bring each generation of biofuel to its socially optimal production level. However, it has been shown in the case of the mandate that it is not sufficient to maximize welfare in the absence of first best policies.

When land is abundant, biofuel policy cannot affect food production. Hence, s_{i3} is always zero in this case. However, when land is scarce, biofuel subsidies can affect food production through increased competition for land. Since the difference in the marginal utility of food between the competitive case and a mandate at the socially optimal quantity of biofuels is determined by the marginal external cost of land used for food production, s_{i3} will be increasing with the marginal emissions due to food production. That is, dirtier food production will imply higher optimal subsidy of biofuels.

For the fossil fuel component, it does not matter whether land is abundant or scarce. The magnitude of the optimal s_{i2} will depend on the curvature of the marginal cost function of fossil fuel. The flatter the marginal cost function, the higher the subsidy, since the higher will be the response of oil production to that subsidy.

2.5 Conclusion

We have developed an optimal control model to look at the trade-offs between food production and biofuel production. Concerns over the exhaustibility of oil have been removed to focus on the impacts of climate change induced by GHG emissions from productive activities.

In general, the competitive outcome overproduce fossil fuels. Results for food and both biofuels production are in general ambiguous. Land scarcity plays a big role in assessing the impact the competitive outcome on food. If land is abundant, food is also overproduced because its GHG emissions are not taken into account. However, that result does not follow through if land is scarce, because competition from biofuel production may drive up land value and reduce food production.

In the competitive outcome, fossil fuel production is decreasing over time, but food production is only decreasing with certainty if land is abundant. The same conclusions apply to the social optimum.

If first best policies like emission taxes or cap and trade systems are not available, second best policies such as a biofuel mandate or biofuel subsidies can improve the competitive outcome. This is however only certain in cases where biofuels are unambiguously underproduced. It is the case when land is abundant but fossil fuel marginal cost of pro-

duction is constant. It can also be the case when land is scarce, but only if land scarcity under the competitive outcome is lower than under the social optimum. It could only be the case when land scarcity under the competitive outcome is greater than under the social optimum if that difference in land scarcity would be smaller than any marginal external cost.

When appropriate, the optimal mandate for biofuel production is higher than the socially optimal quantities. This is to add extra competition to reduce fossil overproduction and, if land is scarce, to also reduce food overproduction. The optimal subsidies are equivalent to using the optimal mandate. They can be decomposed in four components each, representing the incentive to reduce GHG emissions, to reduce fossil fuel production, to reduce food production and to compensate for lack of investment in second generation biofuels.

This is a highly stylized model, but we believe it is a useful tool to understand the interactions between climate policy, biofuel promotion, food concerns and land availability.

Chapter 3

Loss Aversion in Grocery Panel Data: The Confounding Effect of Price Endogeneity

3.1 Introduction

Psychology has had a growing influence in economics since the 1970s. In choice modeling, a landmark paper has been Kahneman and Tversky [1979], which introduced prospect theory. It revolves around the idea that when making decisions, people value gains and losses differently, i.e. not just to the extent that one is the opposite of the other.¹

This has led to the development of a literature, in the 1980s, attempting to incorporate different conceptions of price perception into empirical models. The bulk of the work has been done in marketing by incorporating reference price in the estimation of brand choice models (see among others Winer [1986] who looks at coffee, and Lattin and Bucklin [1989] who distinguish between the effect of regular prices and promotions on the reference price).

Studies on the topic continued to flourish in the 1990s. In the overwhelming majority, strong evidence was found to favor the importance of reference price formation and the existence of a loss aversion phenomenon, i.e. that losses (prices above the consumer reference point) are more salient than gains (prices below the consumer reference point) such that consumers purchases are more sensitive to them. Given the mounting evidence, Kalyanaram and Winer [1995] and Meyer and Johnson [1995] argued that reference price based decisions and loss aversion had become empirical generalizations.

This view has more recently been contested by Bell and Lattin [2000]. They show that, when heterogeneity in consumer price responsiveness is accounted for, the evidence of loss aversion disappears for many products.

In this paper, I consider another potential source of confoundedness in the measure of loss aversion: price endogeneity. Like in every market, prices are here simultaneously determined by supply and demand. Given that most models only estimate demand, taking prices as given might introduce simultaneous equation bias in the estimation. This is precisely what is done in the overwhelming majority of papers in this field. To justify the assumption that prices are exogenous, it is usually argued that the prices for the products studied are determined in a global market, which is little impacted by the consumers under study because they represent only a small subset of that market.

This explanation is not entirely convincing. There seems to be a strong possibility of store level price adjustments, such as sales, especially in the case of groceries, which are the most studied market in this literature. In that case, there is a strong possibility that prices are somewhat endogenous to the purchasing decisions of customers. Indeed, in a more general random utility model applied to scanner data, Villas-Boas and Winer [1999] find that prices are often endogenous in that context, which leads to significant estimation bias.

To assess this possibility in the context of loss aversion models, I look at the impact of reference price preferences on the demand for four grocery product categories: bread, chicken, corn and tortilla chips, and pasta. Using the theoretical framework developed

¹Earlier work on the psychological theory of adaptation level formation dates back to Helson [1964].

by Daniel Putler [1992], I test for the presence of loss aversion, both at the extensive and intensive margins. I do this exercise both taking prices as given and instrumenting for them.

I use prices of commodities entering as inputs in the production of the relevant products as instruments. Solis [2009] presents evidence that food commodity prices have little impact on regular shelf prices, but he also reports that higher agricultural commodity prices reduce the frequency and depth of promotions, hence increasing the average net retail price. Therefore, commodity prices has the potential to be a good instrument for net retail prices.

Initially, I find evidence of loss aversion for the bread and corn and tortilla chips categories. However, when instruments are used, most of that evidence disappears.

The next section presents the data used in the estimation, while section 3.3 and 3.4 describe respectively the model and the estimation strategy. Finally, results are presented in section 3.5.

3.2 Data Set

I use scanner data from a major U.S. supermarket chain. The data set includes all the purchases made from May 2005 to March 2007 at a single store located in California. The neighbourhood in which the store is located is relatively wealthy. The median family income for the subset of the sample for which income data is available is \$106,000. The sample is also overwhelmingly composed of Caucasian. The scanner data is combined with agricultural commodity prices obtained from Global Financial Data.

3.2.1 Scanner Data

As bar codes and scanner have become almost universal in grocery retailing, the use of scanner data by researchers has exploded. In most cases scanner data sets include a limited range of products and cover a varying number of locations.

The data set used for this paper is on the contrary very thorough. Every single product purchased during the time period is included. In all, it includes more than 18 million observations. An observation is one particular product bought by a given customer at a certain point in time.

Households are tracked over time with their customer fidelity cards. Overall, 96% of the 18 million observations are linked to a specific household through their fidelity card number. This represents 67,000 households making purchases at that supermarket over the two year period. For about 38,000 of those, there is information about their income, which will be useful in our estimation.

My analysis focuses on four product categories: bread, chicken, corn and tortilla chips, and pasta. Figure 3.1 presents the densities of the monthly purchases by product categories. Chicken, corn and tortilla chips, and pasta present very similar patterns. Each

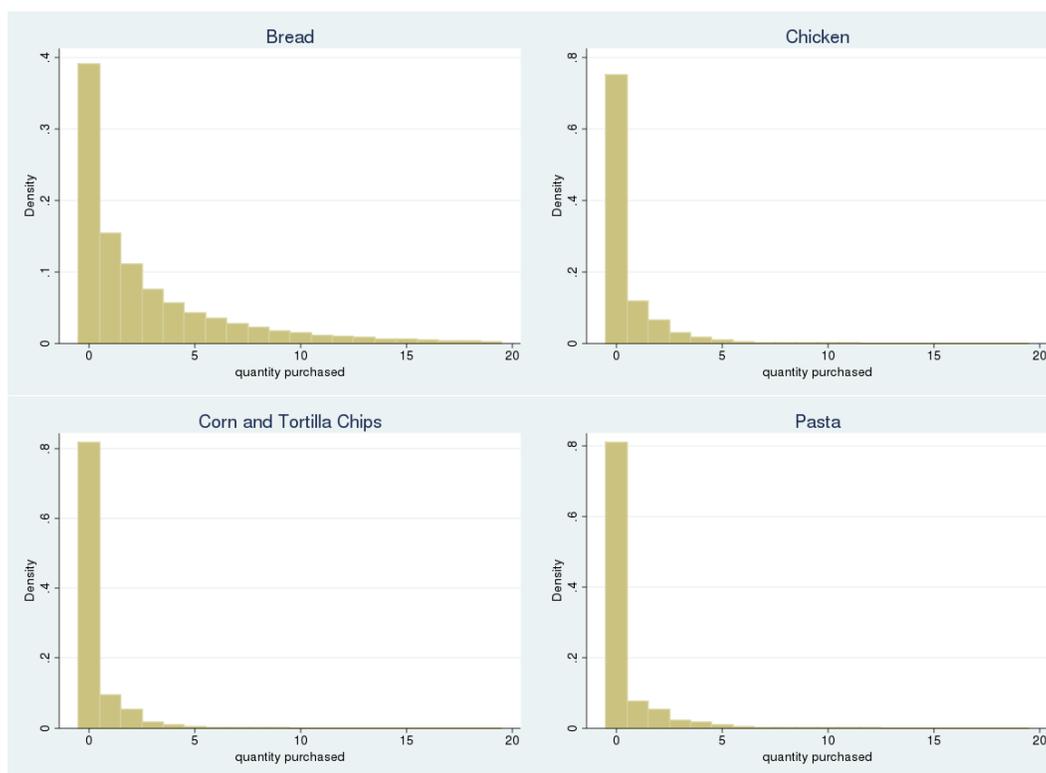


Figure 3.1: Densities of monthly product purchases

of the product is not bought in a given month by about 80% of the households, while very few customers buy more than 5 units in the month. The pattern for bread is markedly different. More people buy it and in greater quantities than for the other three products.

Nevertheless, all product categories display an important proportion of corner solutions. Indeed, a zero can be assimilated to a negative demand that a consumer would have had for that good at a particular purchase occasion. In the data, we define a purchase occasion a period (here a month) in which a household made at least one trip to the store.

Here I must emphasize again that this data is from a single store. The proportion of people who buy of a given category in a given month might seem low, but it does not take into account the fact that people are certainly buying at other stores also.

3.2.2 Commodity Prices

All the commodity prices needed to instrument the retail prices were available at the daily or monthly interval. Figure 3.2 presents the monthly evolution of those prices for the period 2005 - 2007. That group of commodities has been chosen because it represents

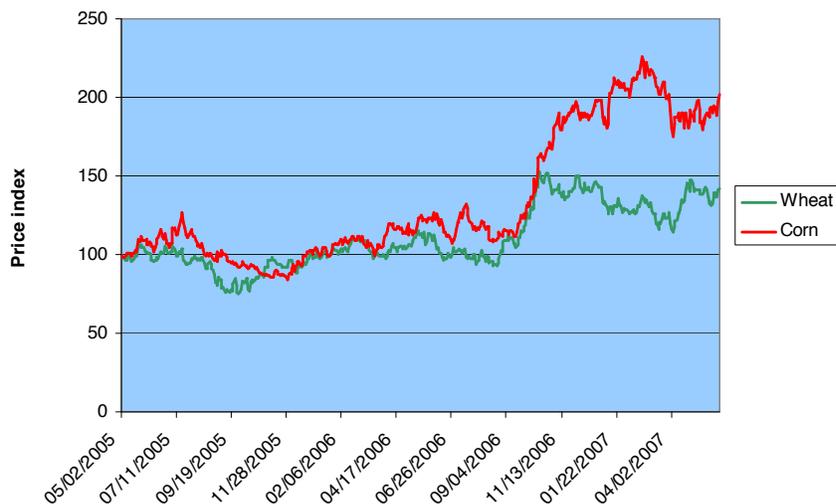


Figure 3.2: Commodity prices
January 2005 Price = 100

inputs in the production of the four product categories of interest as well as of some of their substitutes.

The period that is covered by the scanner data set has been characterized by very volatile commodity prices. That volatility reduces the possibility that some of the commodity prices are collinear. This is confirmed by figure 3.2 where we can see that both commodities do not systematically follow the price path of the other. There is also very rich variation in that data.

In addition, I never use all the commodities simultaneously as instruments. Relevant products sets are defined and only the appropriate subset of commodities is used.²

3.2.3 Aggregation Issues

The level of detail at which the data was recorded is very fine. For example, the pasta category counts almost 2,000 UPC codes. To make the data more usable in terms of the question addressed in this paper, I have had to aggregate it both across UPC codes and across time.

Scanners record the exact time of each transaction up to the exact second. There is therefore a complete latitude on the part of the researcher to aggregate the data at whatever level he wishes. The week is the natural time unit in which to conceptualize grocery purchases. Most people buy grocery every week. Aggregating data on a weekly basis gives me 96 periods to work with. However, further examination indicates that weekly aggregation might not be the best way to proceed in this case. Although most

²The relevant product sets and appropriate commodity subset are defined in the next section.

people buy groceries every week, some tend not to always shop at the same store. Kim and Park [1997] find that only 30% of grocery shoppers have a relatively high cost of switching store. Even when consumers visit the same store over and over, they do not buy exactly the same products every week. Since I am looking at a relatively small subset of products, this is an important issue. These considerations warrant the use of an alternative monthly aggregation level, which leaves the data set with 22 periods.³ Any further aggregation would seriously reduce the time dimension of the panel.

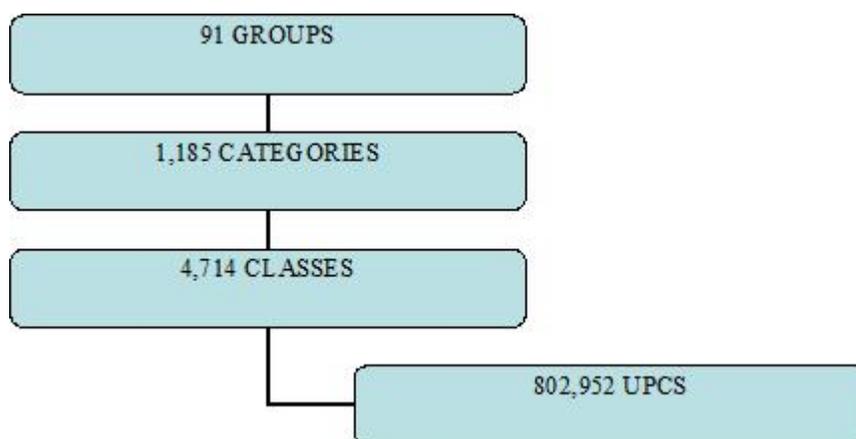


Figure 3.3: Levels of UPC aggregation

Product aggregation was done along the lines of a classification supplied by the supermarket. Figure 3.3 presents the different levels of aggregation of UPC codes. The product categories in this paper have been constructed at group and category levels.

Aggregation poses the problem of creating both price and quantity indices. In the data itself, there is actually no price per se. Each transaction recorded includes the quantity of the good purchased and the amount spent to acquire that good. At the UPC level, price can directly be computed by dividing amount spent by quantity. When purchases are aggregated by groups or categories, the creation of a price index is necessary for each aggregation unit. Equation (3.2.1) gives the simple formula of how this is done.

$$p_{it} = \frac{\sum_{c=1}^C \sum_{n=1}^{N_i} s_{ncit}}{\sum_{c=1}^C \sum_{n=1}^{N_i} q_{ncit}} \quad (3.2.1)$$

Where:

³May 2005 has to be dropped because the data set only contains 2 weeks of it.

p_{it}	is the price index for product category i at time t ;
s_{ncit}	is the total amount spent on product n (which is part of category i) by consumer c at time t ;
q_{ncit}	is the total quantity of product n (which is part of category i) purchased by consumer c at time t ;
C	is the total number of consumers
N_i	is the total number of products in category i .

This procedure is equivalent to creating a quantity weighted price average of all products within a category. Note that this price index also implicitly defines a quantity index, which is the denominator on the right hand side of equation (3.2.1).

3.3 Model

In this section, I present the theoretical model of reference price preferences. This is largely borrowed from Putler [1992] and a more complete exposition can be found in that paper.

The model is constructed around three assumptions. The first, known as temporal separability, implies that the consumer's actions in one period do not directly affect those in other periods. The second, referred to as perfect information, states that consumer are well informed about any product's quality and more specifically that prices are not perceived as conveying information on the level of quality. The last, reference price exogeneity, defines any given reference price as based on past price levels and is therefore exogenous at the time the consumer makes his decision.

The consumer maximizes his utility every periods

$$\max_x U(y, L, G) \tag{3.3.1}$$

subject to his budget constraint

$$\sum_{i=1}^I P_i y_i = M, \tag{3.3.2}$$

where:

y	is an I-vector of consumption levels;
L	is an I-vector of perceived losses;
G	is an I-vector of perceived gains;
P_i	is the price of good i ;

y_i is the consumption level of good i ;
 M is the predetermined level of expenditures for the current period.

Losses and gains for each individual product are defined as⁴

$$L_i = I_i(P_i - RP_i)y_i \quad (3.3.3)$$

$$G_i = (1 - I_i)(RP_i - P_i)y_i, \quad (3.3.4)$$

where:

I_i is an indicator that takes the value 1 if $P_i > RP_i$ and 0 otherwise;
 RP_i is the reference price for good i .

The maximization of (3.3.1) subject to (3.3.2) leads to Marshallian demand functions that depend not only on prices and budget level, but also on marginal gains and marginal losses.

3.4 Estimation Strategy

The estimation of the demands implied by the model poses several challenges. First is the choice of an appropriate functional form for the utility function. Because the product categories I look at are somewhat aggregated, a utility function that leads to Marshallian demands that can be easily aggregated over products seems best suited.

Second, consumers exhibit a lot of corner solution behaviour, i.e. the demand for a given product in a given period would be negative, but appears as zero in the data. Taking that into account not only improves the validity of the estimation, but also allows to look at the impact of reference price preferences on both the extensive and the intensive margins. At the extensive margin, households decide whether or not to buy the product. This can be represented as the probability of buying the product. At the intensive margin, households decide how much to buy given that they will buy. They is represented by the quantity purchased conditional on purchasing a positive amount.

Finally, because we are dealing with demands, careful attention must be paid to the endogeneity of prices. Preferably, instruments would be used to avoid any simultaneous equation bias.

⁴Putler discusses a possible monotone nonlinear transformation of the marginal loss (or gain) term.

3.4.1 Choosing functional forms

The first step of the estimation is to choose appropriate functional forms to be estimated. Functional forms must be chosen for both the reference price formation and the demand equation (through the appropriate choice of a utility function).

The reference price formation we consider is memory based. On any purchase occasion, a consumer compares current prices to previous prices of the same good at the last purchase occasion. We define a purchase occasion as a time period in which the consumer went to the store. By going to the store, the consumer learns about current prices and updates his reference point. If he does not go to the store in a given period, then his reference price does not change. More formally

$$RP_{cit} = S_{ct-1}P_{it-1} + (1 - S_{ct-1})RP_{cit-1}, \quad (3.4.1)$$

where:

RP_{cit}	is the reference price about good i of consumer c in period t ;
S_{ct}	is an indicator of store visit that takes the value 1 if consumer c visited the store in period t , and 0 otherwise;
P_{it}	is the price of good i in period t .

This is admittedly a very simple reference price concept. Nonetheless, it has advantages. Most importantly, because it does not depend on specific product chosen by the consumer, it avoids the confounding effects of price-response heterogeneity on estimates of loss aversion as noted by Bell and Lattin [2000].

As proposed by Putler [1992], we consider two different sets of preferences that lead to two distinct functional forms for demand estimation. The first group of preferences is characterized by a modified version of the well-known Klein-Rubin utility function and takes the form

$$U_{cit} = \sum_{i=1}^I \rho_i \log (y_{cit} - a_i - l_i L_{cit} - g_i G_{cit}) \quad (3.4.2)$$

where ρ_i , a_i , l_i and g_i are all parameters. This utility function translates into a demand function of the form

$$y_{cit} = \alpha_i^0 + \alpha_i^1 L_{cit} + \alpha_i^2 G_{cit} + \rho_i \frac{M}{P_i} + \sum_{j \neq i}^I \frac{P_j}{P_i} (\alpha_{ij}^3 + \alpha_{ij}^4 L_{cjt} + \alpha_{ij}^5 G_{cjt}) \quad (3.4.3)$$

where $\alpha_i^0 = (1 - \rho_i)a_i$; $\alpha_i^1 = (1 - \rho_i)l_i$; $\alpha_i^2 = (1 - \rho_i)g_i$; $\alpha_i^3 = \rho_i a_j$; $\alpha_i^4 = -\rho_i l_j$ and $\alpha_i^5 = -\rho_i g_j$.

Product categories	Other products in the group
Bread	Rice, Potatoes, Pasta
Chicken	Beef, Pork, Turkey
Corn and Tortilla Chips	Hard Bites, Potato Chips, Salty Snacks
Pasta	Bread, Potatoes, Rice

Table 3.1: Products included in the group of each relevant product categories

This demand specification allows for exact linear aggregation of goods into groups. It is also consistent with the representative consumer hypothesis. It is however inflexible. This inflexibility leads to a potential for confounding the reference price effect with the misspecification of the price response parameters. A more flexible functional form is therefore used to evaluate the potential for this problem to affect the results. It is the translog demand function which can be expressed as

$$y_{cit} = \gamma_i^0 + \sum_{j=1}^I \gamma_i^1 \log P_j + \frac{1}{2} \sum_{j=1}^I \sum_{k=1}^I \gamma_{ij}^2 \log P_j \log P_k + \sum_{j=1}^I \gamma_{ij}^3 L_{cit} + \sum_{j=1}^I \gamma_{ij}^4 G_{cit} + \gamma_i^5 \log M. \quad (3.4.4)$$

This demand specification also allows for exact linear aggregation, but it is not consistent with the representative consumer hypothesis.

To make the number of parameters to estimate manageable, we need to restrict the set of relevant commodities for each product category. Assuming preferences are weekly separable over groups, only prices of products within a given group are relevant for any product of that group (see Deaton and Muellbauer, 1980). Table 3.1 presents the relevant products for each of the product categories of interest. Those were chosen as obvious potential substitutes for the products analyzed.

3.4.2 Regression specifications

The equations estimated are (3.4.3) and (3.4.4), to which I add quarterly dummies, to control for seasonality, and an error term. Those two equations can therefore be rewritten for estimation as follows

$$y_{cit} = \alpha_i^0 + \alpha_i^1 L_{cit} + \alpha_i^2 G_{cit} + \rho_i \frac{M}{P_i} + \sum_{j \neq i}^I \frac{P_j}{P_i} (\alpha_{ij}^3 + \alpha_{ij}^4 L_{cjt} + \alpha_{ij}^5 G_{cjt}) + \sum_{h=2}^4 \alpha_{ih}^6 q_h + \epsilon_{cit}, \quad (3.4.5)$$

$$y_{cit} = \gamma_i^0 + \sum_{j=1}^I \gamma_i^1 \log P_j + \frac{1}{2} \sum_{j=1}^I \sum_{k=1}^I \gamma_{ij}^2 \log P_j \log P_k + \sum_{j=1}^I \gamma_{ij}^3 L_{cit} + \sum_{j=1}^I \gamma_{ij}^4 G_{cit} + \gamma_i^5 \log M + \sum_{h=2}^4 \gamma_{ih}^6 q_h + \epsilon_{cit}. \quad (3.4.6)$$

The length of a time period is defined as one month. As mentioned previously, this

appears as a good compromise. It is long enough such that each product is purchased by a reasonable proportion of households every period.⁵ It is also short enough such that short term variation in prices and reference price are not completely smoothed out and that the data retains a reasonable number of periods (22 complete months).

3.4.3 Estimation techniques

To tackle the issues of corner solutions and endogeneity mentioned at the beginning of this sections while taking into account the potential for simultaneous equations bias, I proceed in several steps.

Given that the data set is primarily composed of corner solutions, I express the general data generating process as

$$y_{cit}^* = \mathbf{X}_{cit}\beta + u_{cit}, \quad u_{cit}|\mathbf{X}_{cit} \sim \text{Normal}(0, \sigma^2) \quad (3.4.7)$$

$$y_{cit} = \max(0, y_{cit}^*) \quad (3.4.8)$$

where $\mathbf{X}_i\beta$ is the deterministic part of either equation (3.4.5) or (3.4.6), and y_i is the observed outcome. This formulation lends itself to a pooled Tobit estimation. This seems particularly appropriate to the problem, because it allows to estimate a global effect $\left(\frac{\partial E[y^*|\mathbf{X}]}{\partial x}\right)$, the extensive margin effect $\left(\frac{\partial Pr(y^* > 0|\mathbf{X})}{\partial x}\right)$ and the intensive margin effect $\left(\frac{\partial E[y|\mathbf{X}]}{\partial x}\right)$. Some caution against the use of the Tobit model to estimate all these effect noting that it forces the set of determinants of the extensive and intensive margins to be the same. In the present problem, this appears particularly plausible. The fact that prices and income dictate both whether or not someone buys a given product and if so how much that person buys is rational.

The normality and homoskedasticity assumptions are very important to the validity of the estimation. Violation of either assumptions makes the estimator inconsistent. It is unlikely that these assumptions are exactly satisfied, but in the next section, I present evidence that they are not significantly violated. Note however that the model allows for serial correlation of the error term across time within individuals. This is very convenient, because unobserved individual effects are most certainly creating serial correlation at that level.

There are good reasons to suspect that the price variables in equation (3.4.7) could be endogenous. To address this problem we use instrumental variables with the two step Newey's minimum chi-squared estimator [Newey, 1987]. This estimator is asymptotically consistent under the normality assumption. However, it does not allow to compute the effects on the intensive and extensive margin. In addition, it is relatively sensitive to the presence of instruments that are somewhat collinear.

The instruments used are prices of commodities entering as inputs in the production of the goods considered in the relevant group. Since those commodities are traded on

⁵For each category between 15% and 60% of households buy a product during a given month.

Product categories of interest in the relevant group	Commodities
Bread	Rice, Potatoes, Wheat
Chicken	Corn, Beef cattle, Hog, Live turkeys
Corn and Tortilla Chips	Corn, Rice, Potatoes, Wheat
Pasta	Wheat, Potatoes, Rice

Table 3.2: Commodities used as instruments in each group

world markets, consumption of their transformed consumer products in a local market is almost certainly not affecting them. In the other direction however, recent price spikes in the price of many commodities have had impact on the prices of the final goods in which they are production inputs (see among others Solis, 2009).

In addition to the price of commodities themselves, several lags are included in the specification. I do some sensitivity analysis with the number of lags, but beyond 3 or 4 issues of multicollinearity arise. Table 3.2 present the different commodities of which the prices are used in each group.

To complement the Tobit estimation, I will also report OLS estimates. Although OLS estimates are biased in this context, they can be useful in two settings. First and foremost, whether or not the Tobit model is correctly specified, a regression of y_{cit} on \mathbf{X}_{cit} for positive values of y_{cit} approximates the intensive margin effects near the mean values of the regressors.⁶ Second, it is possible all the OLS coefficients be inconsistent by the same multiplicative factor. Since I am mostly interested in the relative coefficients of Losses and Gains, it would be possible to evaluate that situation with OLS coefficients. However, the assumptions under which the previous result is valid are very restrictive, for example requiring the joint normality of the regressand and the regressors (see Wooldridge, 2002).

3.5 Results

In this section I present the results of the estimation of both the Klein-Rubin and translog demands. Since the question of interest concerns only the importance of reference price preferences and the prevalence of loss aversion, only the estimates for the coefficients on own losses (losses for the price of the product in question) and own gains are reported. Those correspond respectively to α_i^1 and α_i^2 in equation (3.4.5) for the Klein-Rubin utility and to γ_{ii}^3 and γ_{ii}^4 in equation (3.4.6) for the translog.

According to the theory, the coefficient on losses should be negative, while the one on gains should be positive. There is loss aversion if the coefficient on losses is of a greater magnitude than the one on gains.

Table 3.3 presents the results of the pooled regressions for the bread category. The striking result is that the coefficients on own gains has the wrong sign in all the speci-

⁶Given that all the second moments are finite.

VARIABLES				
Klein-Rubin	OLS	Tobit	ext.	int.
Own Losses	-9.167*** (0.367)	-16.81*** (0.876)	-0.914*** (0.0477)	-6.893*** (0.359)
Own Gains	-2.487*** (0.599)	-6.547*** (1.456)	-0.356*** (0.0792)	-2.684*** (0.597)
Observations	188905	188905	188905	188905
R^2	0.024			
Translog				
Own Losses	-9.852*** (0.425)	-17.94*** (1.036)	-0.979*** (0.0565)	-7.352*** (0.425)
Own Gains	-2.536*** (0.629)	-4.921*** (1.570)	-0.269*** (0.0857)	-2.017*** (0.644)
Observations	188905	188905	188905	188905
R^2	0.028			

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.3: Result of the pooled regressions of quantity on prices, gains and losses for Bread

cations. There is however strong evidence of loss aversion, almost too strong. Looking at the effect of losses on the probability to buy bread, we note the coefficient would mean that if the loss increased by one dollar, the probability to buy bread that month at that given store would decrease by 91%. Although we should expect more responsiveness from consumers given that they likely have the option to go to another store, this appears somewhat high.

Table 3.4 gives the results for chicken. The sign and magnitude of the coefficients are a little more plausible than for bread. Although the sign of coefficients for losses is almost always positive, it is never significant. There is no evidence of loss aversion. On the contrary, it seems that there might be something like “deal loving” going on.

As for corn and tortilla chips, for which the results are presented in table 3.5, it is probably the category that exhibits the prototypical expected results. Loss aversion is moderate but significant. For example, consider a one dollar increase in losses. This would reduce average individual monthly purchases by 1.4 units. This effect would materialize both at the extensive margin, by a reduction in the probability to purchase of 12%, and at the intensive margin, by a reduction of average monthly quantity purchased by those

VARIABLES				
Klein-Rubin	OLS	Tobit	ext.	int.
Own Losses	0.00620 (0.0154)	0.0536 (0.0827)	0.00507 (0.00783)	0.0133 (0.0206)
Own Gains	0.380*** (0.0281)	1.447*** (0.131)	0.137*** (0.0124)	0.360*** (0.0327)
Observations	188913	188913	188913	188913
R^2	0.018			
Translog				
Own Losses	-0.00401 (0.0153)	0.0323 (0.0832)	0.00306 (0.00787)	0.00803 (0.0206)
Own Gains	0.644*** (0.0312)	2.714*** (0.166)	0.257*** (0.0157)	0.674*** (0.0411)
Observations	188913	188913	188913	188913
R^2	0.020			

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.4: Result of the pooled regressions of quantity on prices, gains and losses for Chicken

VARIABLES				
Klein-Rubin	OLS	Tobit	ext.	int.
Own Losses	-0.198*** (0.0315)	-1.415*** (0.231)	-0.119*** (0.0193)	-0.306*** (0.0498)
Own Gains	0.0317 (0.0532)	0.377 (0.339)	0.0316 (0.0284)	0.0813 (0.0731)
Observations	188904	188904	188904	188904
R^2	0.014			
Translog				
Own Losses	-0.210*** (0.0334)	-1.534*** (0.251)	-0.129*** (0.0211)	-0.330*** (0.0541)
Own Gains	-0.00343 (0.0543)	0.160 (0.343)	0.0134 (0.0287)	0.0345 (0.0738)
Observations	188904	188904	188904	188904
R^2	0.015			

Robust standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 3.5: Result of the pooled regressions of quantity on prices, gains and losses for Corn & Tortilla Chips

who still buy the products of 0.3 units.

Note that for the first three categories, the results are very consistent across specifications. In addition, the OLS estimate is often very close to the intensive margin. This should not be surprising because both attempt to evaluate the impact of gains and losses on purchases for the sub-population of households that buy a positive amount of the product.

Pasta is the only group category for which the two demand specifications differ significantly.⁷ In table 3.6 we see that the coefficient on losses goes from negatively significant to insignificant when the demand specification switches from Klein-Rubin to translog. This could be due to the fact that the translog model is more flexible than the Klein-Rubin which may confound the loss term with something else. Note however that this discrepancy does not tell two different stories on the front of loss aversion. Even in the Klein-Rubin specification, the magnitude of the loss coefficients are much smaller than those of the gains. Hence, there is no evidence loss aversion in either cases for pasta.

Before proceeding to the IV estimation, I evaluate the validity of the estimation speci-

⁷Although it conserves the nice symmetry between OLS and intensive margin estimates.

VARIABLES				
Klein-Rubin	OLS	Tobit	ext.	int.
Own Losses	-0.264*** (0.0804)	-1.164** (0.500)	-0.0742** (0.0318)	-0.256** (0.110)
Own Gains	0.575*** (0.0797)	2.696*** (0.400)	0.172*** (0.0255)	0.594*** (0.0881)
Observations	188899	188899	188899	188899
R^2	0.016			
Translog				
Own Losses	-0.116 (0.0822)	-0.416 (0.526)	-0.0264 (0.0334)	-0.0913 (0.115)
Own Gains	0.517*** (0.0835)	2.623*** (0.480)	0.167*** (0.0305)	0.576*** (0.105)
Observations	188899	188899	188899	188899
R^2	0.017			

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.6: Result of the pooled regressions of quantity on prices, gains and losses for Pasta

fication for both demands. As suggested by Wooldridge [2002], I do a probit comparison of the tobit coefficients. This is not an exact test, but it allows to detect if the tobit model is clearly misspecified. The procedure consists in comparing the coefficients obtained from running a probit regression on equations (3.4.5) and (3.4.6) to the tobit coefficients rescaled by the standard error of their own regressions. If coefficients have different signs or magnitudes, then the tobit model is almost surely misspecified.

Results of this test for all coefficients are reported in the appendix B for both demands. In general there is very little concern for misspecification. Virtually all coefficient pairs have the same sign. The vast majority also are very similar in magnitude. The only specification for which some concern arises is the translog for bread. One coefficient pair in particular is very dissimilar. That could explain why some of the results for bread are somewhat surprising.

I now turn to the results of the IV estimations. If results change significantly when prices are instrumented for, it gives a good indication that there might be some simultaneous equation bias in the previous results. That is an important point, because most of the research in loss aversion never makes use of instruments. Because the data used is often very disaggregated, researchers just argue that prices are determined at a higher level and hence are exogenous.

In the case of bread, the peculiar results from the original estimation go away in the Klein-Rubin specification. Not only do gains no longer have a negative coefficient, but the magnitude of the coefficient on losses is reduced such that it is no longer significant. As one can see from table 3.7, the story is not as clear cut for the translog demand. The two stage least squares and two step Newey's minimum chi-squared estimators tell completely opposite stories, both coefficients being significantly positive in the first case and significantly negative in the second. That might be due to the fact that the translog demand in the case of bread is the most likely to be misspecified of all the specifications for all products (see the appendix B).

As for chicken, except in the case of the two stage least squares estimate for losses in the Klein-Rubin specification, all other coefficients are not significant. In the non-instrumented regressions, the gains coefficients were highly significant.

With the same exception for corn and tortilla chips, the significant loss aversion effect previously noted has now disappeared.

Finally, table 3.10 tells a similar story for pasta. While there was no loss aversion in the Klein-Rubin model but a significant negative coefficient on losses, it becomes insignificant when prices are instrumented. In the case of the translog model, two stage least squares give similar results as those of the Klein-Rubin. The two step Newey's minimum chi-squared estimators seems to give peculiar results just as in the case of bread. Note however two important differences. For pasta, both coefficients have the expected sign. In addition, even if the coefficient on losses is significant, it is not of a significantly greater magnitude than the coefficient on gains. Hence, there is no loss aversion in that case either.

Globally, there is moderate evidence of loss aversion when prices are not instrumented.

VARIABLES	Klein-Rubin		Translog	
	2SLS	Newey's two step	2SLS	Newey's two step
Own Losses	1.050 (0.946)	5.173 (3.397)	5.605*** (1.547)	-190.6*** (43.49)
Own Gains	5.779*** (1.254)	6.740 (4.524)	9.168*** (2.333)	-235.4*** (53.77)
Observations	188905	188905	188905	188905
R^2			0.020	

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 3.7: Result of the IV regressions of quantity on prices, gains and losses for Bread

VARIABLES	Klein-Rubin		Translog	
	2SLS	Newey's two step	2SLS	Newey's two step
Own Losses	-0.164*** (0.0359)	-0.290 (0.209)	-0.163 (0.103)	-0.0342 (1.469)
Own Gains	-0.0211 (0.0631)	-0.345 (0.359)	0.119 (0.161)	1.143 (3.632)
Observations	188913	188913	188913	188913
R^2			0.015	

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 3.8: Result of the IV regressions of quantity on prices, gains and losses for Chicken

VARIABLES	Klein-Rubin		Translog	
	2SLS	Newey's two step	2SLS	Newey's two step
Own Losses	0.196** (0.0981)	-0.509 (3.041)	0.760*** (0.137)	8.265*** (2.101)
Own Gains	0.108 (0.107)	11.78 (9.447)	-0.231* (0.127)	2.241 (1.421)
Observations	188904	188904	188904	188904
R^2	0.008		0.010	

Newey's two step

*** p<0.01, ** p<0.05, * p<0.1

Table 3.9: Result of the IV regressions of quantity on prices, gains and losses for Corn and Tortilla Chips

VARIABLES	Klein-Rubin		Translog	
	2SLS	Newey's two step	2SLS	Newey's two step
Own Losses	0.232 (0.161)	-23.62 (38.16)	-0.251 (0.250)	-30.91*** (6.994)
Own Gains	0.873*** (0.165)	11.33 (17.12)	1.412*** (0.299)	27.26*** (5.230)
Observations	188899	188899	188899	188899
R^2			0.015	

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 3.10: Result of the IV regressions of quantity on prices, gains and losses for Pasta

Bread and

corn and tortilla chips display evidence of loss aversion while chicken and pasta do not. However, when the models are estimated in an IV setup, almost all the loss aversion goes away, with the notable exception of the very imprecise two step Newey's minimum chi-squared estimator in the context of the translog specification for bread. This casts some doubt on the validity of the results of other research that finds strong evidence of loss aversion, but that does not account for the potential endogeneity of prices.

3.6 Conclusion

In this paper, I have proposed a novel explanation of why appearance of loss aversion in a reference price model might be confounded with other factors. If prices are endogenous, as it is often the case in demand estimation, the loss aversion parameters might just be picking up the bias in the estimation.

The results bring some evidence to support this hypothesis. While standard estimation does not give strong evidence of loss aversion for chicken and pasta, it does for corn and tortilla chips, and bread. When instruments are used to make the prices exogenous, that effect disappears for corn and tortilla chips, and bread, while it still does not show up for chicken and pasta.

These results have two main implications. Empirical estimation of reference price dependent demand ought to pay careful attention to the issue of simultaneous equation bias. Otherwise, reported loss aversion could in fact just be confounded with the bias due to the endogeneity of prices. From a marketing perspective, it is therefore not clear whether supermarkets should pay attention to loss aversion in their pricing strategies. A lot of attention has been devoted to sales pricing and how it should be adjusted in light of reference price preferences. Without loss aversion, it considerably modifies these analysis.

As such, results from this paper should be interpreted with care. Because the sample studied is relatively wealthy, it is possible that it displays less loss aversion than a the overall population. Also, because the extent of the market looked at here is relatively limited, it is not clear whether or not we should expect more or less loss aversion in a broader market.

Future research should off course pay particular attention to these issues. There is no question that behavioral scientist have found that many individuals display loss aversion in several contexts. Does that however necessarily transpose to the marketplace? And if so, does it depend on the extent of the market? Does it vary across individuals or products, and according to what characteristics? Overall, the debate is less about whether loss aversion exist, but whether it plays a significant role in some markets.

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Appendix A

Monotonic Approach of the Steady States

To ensure that the phase diagram exposition is correct in this discrete time setting, I check that the optimally controlled stock approaches stable steady states monotonically. Of course, such reasoning can only apply to the deterministic setting, as random shocks do shift the stock around stable steady states in the stochastic setting.

For the optimally controlled stock to approach the stable steady states monotonically, it must be the case that the function describing the future stock, $f(m_t^*, S_t)$, intersects the 45° line in the plane S_t, S_{t+1} with a positive slope. In the benchmark parametrization, it is the case that $f(m_t^*, S_t)$ has a positive slope everywhere, hence the condition for a monotonic approach of the steady states is satisfied. This is graphically represented in figure A.1, which depicts the change in stock, $f(m_t^*, S_t) - S_t$, as a function of the stock. Since the slope of the depicted function is everywhere greater than -1 , the stock approaches monotonically each steady state. The evolution of the stock over time is represented by the arrows in figure A.1. For any initial stock, one can read the change value on the function itself and report it as the stock next period by drawing a line of slope -1 between the point on the function and the horizontal axis.

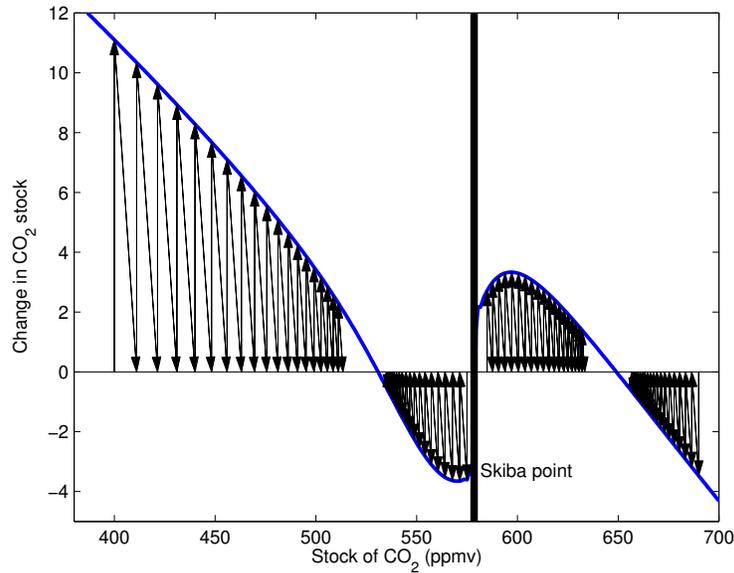


Figure A.1: Monotonicity of the steady state approach

Appendix B

Specification Test of Tobit Model

Coefficients	Bread		Chicken		Corn & Tort.		Pasta	
	Probit	Sc. Tobit	Probit	Sc. Tobit	Probit	Sc. Tobit	Probit	Sc. Tobit
α_i^1	-2.45	-2.34	0.00608	0.0154	-0.453	-0.431	-0.215	-0.262
α_i^2	-1.13	-0.911	0.375	0.417	0.141	0.115	0.599	0.607
ρ_i	0.00359	0.00367	0.00981	0.00990	0.00407	0.00406	0.00252	0.00257
α_{i1}^3	0.626	0.591	-0.496	-0.474	-0.876	-0.815	0.0371	0.0623
α_{i2}^3	-0.288	-0.193	-0.177	-0.159	-0.752	-0.743	-1.20	-1.15
α_{i3}^3	0.218	0.196	0.00604	0.00567	-0.150	-0.144	-0.786	-0.772
α_{i1}^4	-2.11	-1.66	0.656	0.645	2.46	2.54	-0.879	-0.789
α_{i2}^4	-0.426	-0.349	-0.362	-0.350	-0.486	-0.477	-0.175	-0.165
α_{i3}^4	0.258	0.150	-0.423	-0.406	0.534	0.617	0.150	0.116
α_{i1}^5	-0.383	-0.371	1.79	1.66	-0.115	-0.0919	0.604	0.555
α_{i2}^5	-0.953	-0.873	-0.259	-0.245	-0.0511	-0.0641	-0.228	-0.199
α_{i3}^5	0.842	0.619	-0.164	-0.159	-0.168	-0.175	0.0684	0.0651
α_{i2}^6	-0.0115	0.00858	0.153	0.164	0.164	0.172	-0.0561	-0.0592
α_{i3}^6	0.0925	0.0760	0.0570	0.0564	0.147	0.145	-0.0108	-0.0166
α_{i4}^6	0.140	0.114	0.00203	0.0160	-0.0407	-0.0320	0.0577	0.0450
α_i^0	1.46	0.887	-1.97	-1.92	-1.72	-1.86	-1.23	-1.29

Table B.1: Probit and scaled Tobit estimates for the Klein-Rubin model

Coefficients	Bread				Chicken				Corn & Tort.				Pasta	
	Probit	Sc. Tobit	Probit	Sc. Tobit	Probit	Sc. Tobit	Probit	Sc. Tobit	Probit	Sc. Tobit	Probit	Sc. Tobit	Probit	Sc. Tobit
γ_i^1	1.92	-0.768	22.7	20.7	1.32	0.783	-5.46	-5.47						
γ_{i1}^2	0.204	0.456	-1.57	-1.43	-0.378	0.345	0.653	0.638						
γ_{i2}^2	0.0222	0.361	-1.47	-1.33	-0.325	-0.233	0.660	0.675						
γ_{i3}^2	-1.06	-0.417	-1.62	-1.48	0.242	0.324	0.600	0.647						
γ_{i4}^2	-0.470	-0.0485	-1.32	-1.20	-0.124	-0.0364	1.17	1.13						
γ_{i1}^3	-2.58	-2.50	-0.00246	0.00935	-0.493	-0.469	-0.0450	-0.0940						
γ_{i2}^3	-0.530	-0.351	-0.469	-0.451	-0.734	-0.727	-1.93	-1.85						
γ_{i3}^3	-0.317	-0.266	-0.334	-0.328	-0.433	-0.434	-0.286	-0.271						
γ_{i4}^3	-0.525	-0.428	-0.294	-0.280	-0.177	-0.211	-0.547	-0.475						
γ_{i1}^4	-0.784	-0.687	0.747	0.785	0.0805	0.0489	0.582	0.593						
γ_{i2}^4	0.358	0.290	-0.00906	-0.0088	-0.142	-0.153	-1.30	-1.29						
γ_{i3}^4	0.146	0.129	-0.0505	-0.490	0.173	0.221	0.312	0.272						
γ_{i4}^4	0.587	0.464	-0.108	-0.103	-0.175	-0.181	0.0791	0.0714						
γ_i^5	0.254	0.264	0.250	0.251	0.255	0.257	0.290	0.296						
γ_{i2}^6	-0.0124	-0.00160	0.142	0.152	0.125	0.133	-0.0756	-0.0785						
γ_{i3}^6	0.0897	0.0705	0.0269	0.0234	0.121	0.119	-0.0269	-0.0320						
γ_{i4}^6	0.148	0.112	-0.0972	-0.0824	-0.0293	-0.0185	0.0358	0.0233						
γ_i^0	-6.21	-1.97	-88.7	-81.7	-6.16	-5.20	4.69	4.64						

Table B.2: Probit and scaled Tobit estimates for the Translog model