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Supergravity at colliders

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Abstract

We consider supersymmetric theories where the gravitino is the lightest superparticle (LSP). Assuming that the long-lived next-to-lightest superparticle (NSP) is a charged slepton, we investigate two complementary ways to prove the existence of supergravity in nature. The first is based on the NSP lifetime which in supergravity depends only on the Planck scale and the NSP and gravitino masses. With the gravitino mass inferred from kinematics, the measurement of the NSP lifetime will test an unequivocal prediction of supergravity. The second way makes use of the 3-body NSP decay. The angular and energy distributions and the polarizations of the final state photon and lepton carry the information on the spin of the gravitino and on its couplings to matter and radiation.

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1. Introduction

Deciphering hidden symmetries in nature has been one of the most exciting and challenging tasks in physics. Most recently, the discovery of the massive W and Z gauge bosons has established a spontaneously broken gauge symmetry as the basis of the electroweak theory. Here, we discuss how one may discover the massive gravitino, which would establish spontaneously broken local supersymmetry as a fundamental, hidden symmetry of nature.

If the theory underlying the standard model is supersymmetric, one may find superpartners of quarks, leptons and gauge bosons at the Tevatron, the LHC or a future linear collider. Even though an exciting discovery, this would still not answer the question how supersymmetry is realized in nature. To identify supersymmetry as an exact, spontaneously broken symmetry requires evidence for the goldstino. Only the discovery of the massive spin-3/2 gravitino, containing the spin-1/2 goldstino, would establish supergravity [1] with local supersymmetry as the fundamental structure.

In general it is difficult to detect gravitinos since their couplings are Planck scale suppressed. However, evidence for the gravitino may be obtained in collider experiments if it is the lightest superparticle (LSP). The gravitino mass may be of the same order as other superparticle masses, like in gaugino mediation [2] or gravity mediation [3]. But

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it might also be much smaller as in gauge mediation scenarios [4]. As we shall see, a discovery of the gravitino appears feasible for gravitino masses in the range from about 1 to 100 GeV. As LSP the gravitino is also a natural dark matter candidate.

We will assume that the next-to-lightest superparticle (NSP) is a charged slepton. This is a natural possibility with respect to the renormalization group analysis of supersymmetry breaking parameters. Scalar leptons may be produced at the Tevatron, the LHC or a linear collider. They can be directly produced in pairs or in cascade decays of heavier superparticles. The NSP lifetime is generally large because of the small, Planck scale suppressed coupling to the gravitino LSP.

The production of charged long-lived heavy particles at colliders is an exciting possibility [5]. Sufficiently slow, strongly ionizing sleptons will be stopped within the detector. One may also be able to collect faster sleptons in a storage ring. In this way it may become possible to study NSP decays. The dominant NSP decay channel is $\tilde{\ell} \rightarrow \ell + \text{missing energy}$, where $\tilde{\ell}$ and ℓ denote slepton and lepton, respectively.

In the following we shall study how to identify the gravitino as cause of the missing energy. First, one will measure the NSP lifetime. Since the gravitino couplings are fixed by symmetry, the lifetime is predicted by supergravity given the gravitino mass, which can be inferred from kinematics. In a second step spin and couplings of the gravitino or the goldstino can be determined from an analysis of the 3-body decay $\tilde{\ell} \rightarrow \ell + \psi_{3/2} + \gamma$.

2. Gravitino mass

To be specific, we focus in the following on the case where the scalar lepton $\tilde{\tau}$, the superpartner of the τ -lepton, is the NSP. It is straightforward to extend the discussion to the case where another scalar lepton is the NSP. As we shall see, phenomenologically particularly interesting is the case where the gravitino is not ultra-light, which implies a long NSP lifetime.

At LHC one expects $\mathcal{O}(10^6)$ NSPs per year which are mainly produced in cascade decays of squarks and gluinos [6]. The NSPs are mostly produced in the forward direction [7] which should make it easier to accumulate $\tilde{\tau}$ s in a storage ring. In a linear collider an integrated luminosity of 500 fb^{-1} will yield $\mathcal{O}(10^5)$ $\tilde{\tau}$ s [8]. Note that, in a linear collider, one can tune the velocity of the produced $\tilde{\tau}$ s by adjusting the e^+e^- center-of-mass energy.

A detailed study of the possibilities to accumulate $\tilde{\tau}$ NSPs is beyond the scope of this Letter. In the following we shall assume that a sufficiently large number of $\tilde{\tau}$ s can be produced and collected. Studying their decays will yield important information on the nature of the LSP. In the context of models with gauge mediated supersymmetry breaking the production of $\tilde{\tau}$ NSPs has previously studied for the Tevatron [9], for the LHC [10] and for a linear collider [11].

The NSP $\tilde{\tau}$ is in general a linear combination of $\tilde{\tau}_R$ and $\tilde{\tau}_L$, the superpartners of the right-handed and left-handed τ -leptons τ_R and τ_L , respectively,

$$\tilde{\tau} = \cos(\varphi_\tau) \tilde{\tau}_R + \sin(\varphi_\tau) \tilde{\tau}_L. \quad (1)$$

The interaction of the gravitino $\psi_{3/2}$ with scalar and fermionic τ -leptons is described by the Lagrangian [12],

$$\mathcal{L}_{3/2} = -\frac{1}{\sqrt{2}M_P} \left[(D_\nu \tilde{\tau}_R)^* \bar{\psi}^\mu \gamma^\nu \gamma_\mu P_R \tau + (D_\nu \tilde{\tau}_R) \bar{\tau} P_L \gamma_\mu \gamma^\nu \psi^\mu \right], \quad (2)$$

where $D_\nu \tilde{\tau}_R = (\partial_\nu + ieA_\nu) \tilde{\tau}_R$. Here A_ν denotes the gauge boson, and $M_P = (8\pi G_N)^{-1/2}$ is the reduced Planck mass. The interaction Lagrangian of $\tilde{\tau}_L$ has an analogous form.

The $\tilde{\tau}$ decay rate is dominated by the two-body decay into τ and gravitino,

$$\Gamma_{\tilde{\tau}}^{2\text{-body}} = \frac{(m_{\tilde{\tau}}^2 - m_{3/2}^2 - m_\tau^2)^4}{48\pi m_{3/2}^2 M_P^2 m_{\tilde{\tau}}^3} \left[1 - \frac{4m_{3/2}^2 m_\tau^2}{(m_{\tilde{\tau}}^2 - m_{3/2}^2 - m_\tau^2)^2} \right]^{3/2}, \quad (3)$$

where $m_\tau = 1.78$ GeV is the τ mass, $m_{\tilde{\tau}}$ is the $\tilde{\tau}$ mass, and $m_{3/2}$ is the gravitino mass. Neglecting m_τ , we arrive at

$$\Gamma_{\tilde{\tau}}^{2\text{-body}} = \frac{m_{\tilde{\tau}}^5}{48\pi m_{3/2}^2 M_{\text{P}}^2} \times \left(1 - \frac{m_{3/2}^2}{m_{\tilde{\tau}}^2}\right)^4. \quad (4)$$

For instance, $m_{\tilde{\tau}} = 150$ GeV would imply a lifetime of $\Gamma_{\tilde{\tau}}^{-1} \simeq 78$ s or $\Gamma_{\tilde{\tau}}^{-1} \simeq 4.4$ y for a gravitino mass of $m_{3/2} = 0.1$ GeV or $m_{3/2} = 75$ GeV, respectively. The crucial point is that the decay rate is completely determined by the masses $m_{\tilde{\tau}}$ and $m_{3/2}$, independently of other SUSY parameters, gauge and Yukawa couplings. The mass $m_{\tilde{\tau}}$ of the NSP will be measured in the process of accumulation. Although the outgoing gravitino is not directly measurable, its mass can also be inferred kinematically unless it is too small,

$$m_{3/2}^2 = m_{\tilde{\tau}}^2 + m_\tau^2 - 2m_{\tilde{\tau}} E_\tau. \quad (5)$$

Therefore, the gravitino mass can be determined with the same accuracy as E_τ and $m_{\tilde{\tau}}$, i.e., with an uncertainty of a few GeV.

Comparing the decay rate (3), using the kinematically determined $m_{3/2}$, with the observed decay rate, it is possible to test an important supergravity prediction. In other words, one can determine the ‘supergravity Planck scale’ from the NSP decay rate which yields, up to $\mathcal{O}(\alpha)$ corrections,

$$M_{\text{P}}^2(\text{supergravity}) = \frac{(m_{\tilde{\tau}}^2 - m_{3/2}^2 - m_\tau^2)^4}{48\pi m_{3/2}^2 m_{\tilde{\tau}}^3 \Gamma_{\tilde{\tau}}} \left[1 - \frac{4m_{3/2}^2 m_\tau^2}{(m_{\tilde{\tau}}^2 - m_{3/2}^2 - m_\tau^2)^2}\right]^{3/2}. \quad (6)$$

The result can be compared with the Planck scale of Einstein gravity, i.e., Newton’s constant determined by macroscopic measurements, $G_{\text{N}} = 6.707(10) \times 10^{-39}$ GeV⁻² [13],

$$M_{\text{P}}^2(\text{gravity}) = (8\pi G_{\text{N}})^{-1} = (2.436(2) \times 10^{18} \text{ GeV})^2. \quad (7)$$

The consistency of the microscopic and macroscopic determinations of the Planck scale is a crucial test of supergravity.

Furthermore, the measurement of the gravitino mass yields another important quantity in supergravity, namely the mass scale of spontaneous supersymmetry breaking,

$$M_{\text{SUSY}} = \sqrt{\sqrt{3} M_{\text{P}} m_{3/2}}. \quad (8)$$

This is the analogue of the Higgs vacuum expectation value v in the electroweak theory, where $v = \sqrt{2} m_W / g = (2\sqrt{2} G_{\text{F}})^{-1/2}$.

3. Gravitino spin

If the measured decay rate and the kinematically determined mass of the invisible particle are consistent, we already have strong evidence for supergravity and the gravitino LSP. In this section we analyze how to determine the second crucial observable, the spin of the invisible particle.

To this end, we consider the 3-body decay $\tilde{\tau}_{\text{R}} \rightarrow \tau_{\text{R}} + \psi_{3/2} + \gamma$, leaving final states with W - or Z -bosons for future studies. We only consider the diagrams of Fig. 1 and restrict ourselves to a pure ‘right-handed’ NSP $\tilde{\tau}_{\text{R}}$. Here, we have neglected diagrams with neutralino intermediate states, assuming that they are suppressed by a large neutralino mass.

In order to prove the spin-3/2 nature of the invisible particle, we compare the 3-body decay with final state gravitino with the corresponding decay involving a hypothetical neutralino λ . As an example, we consider the

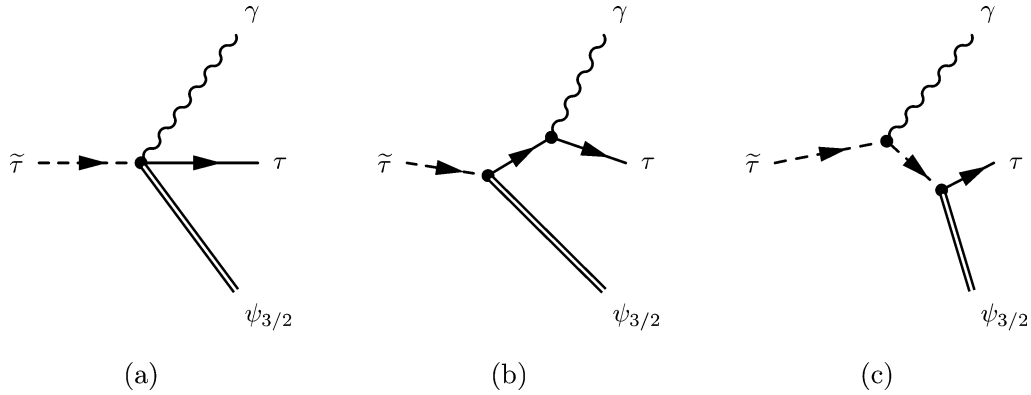


Fig. 1. Diagrams contributing to $\tilde{\tau} \rightarrow \tau + \psi_{3/2} + \gamma$ at tree level. We do not take into account the diagram with a neutralino intermediate state. It turns out that (a) is the crucial ingredient to prove the spin-3/2 nature of the gravitino.

Yukawa coupling,¹

$$\mathcal{L}_{\text{Yukawa}} = h(\tilde{\tau}_R^* \bar{\lambda} P_R \tau + \tilde{\tau}_L^* \bar{\lambda} P_L \tau) + \text{H.c.} \quad (9)$$

Accidentally, the coupling h could be very small, such that the $\tilde{\tau}$ decay rate would be consistent with the rate given in Eq. (3).

Also the goldstino has Yukawa couplings of the type given in Eq. (9). The full interaction Lagrangian is obtained by performing the substitution $\psi_\mu \rightarrow \sqrt{2/3} \partial_\mu \chi / m_{3/2}$ in the supergravity Lagrangian. Using the equations of motion one finds for the non-derivative form of the effective Lagrangian [14],

$$\mathcal{L}_{\text{eff}} = \frac{m_{\tilde{\tau}}^2}{\sqrt{3} M_{\text{P}} m_{3/2}} (\tilde{\tau}_R^* \bar{\chi} P_R \tau + \tilde{\tau}_L^* \bar{\chi} P_L \tau) - \frac{m_{\tilde{\gamma}}}{4\sqrt{6} M_{\text{P}} m_{3/2}} \bar{\chi} [\gamma^\mu, \gamma^\nu] \tilde{\gamma} F_{\mu\nu}, \quad (10)$$

where we have neglected a quartic interaction term which is irrelevant for our discussion. Note that the goldstino coupling to the photon supermultiplet is proportional to the photino mass $m_{\tilde{\gamma}}$. As a consequence, the contribution to $\tilde{\tau}$ -decay with intermediate photino is not suppressed for very large photino masses. As we shall see, this leads to significant differences between the angular distributions for pure Yukawa and goldstino couplings.

In the following we discuss two methods to determine the spin of the invisible particle. The first one is based on a double differential angular and energy distribution, the second one makes use of the angular distribution of polarized photons.

In $\tilde{\tau}$ -decay both, photon and τ -lepton will mostly be very energetic. Hence the photon energy E_γ and the angle θ between τ and γ can both be well measured (cf. Fig. 2(a)). We can then compare the differential decay rate

$$\Delta(E_\gamma, \cos\theta) = \frac{1}{\alpha \Gamma_{\tilde{\tau}}} \frac{d^2 \Gamma(\tilde{\tau} \rightarrow \tau + \gamma + X)}{dE_\gamma d\cos\theta}, \quad (11)$$

for the gravitino LSP ($X = \psi_{3/2}$) and the hypothetical neutralino ($X = \lambda$) with pure Yukawa coupling. Details of the calculation are given in Appendix A.

In the forward direction, $\cos\theta \geq 0$, bremsstrahlung (cf. Fig. 1(b)) dominates, and final states with gravitino and neutralino look very similar. In the backward direction, $\cos\theta < 0$, the direct coupling Fig. 1(a) is important, and the angular and energy distribution differs significantly for gravitino and neutralino. This is demonstrated by Fig. 3

¹ This interaction would arise from gauging the anomaly free U(1) symmetry $L_\tau - L_\mu$, the difference of τ - and μ -number, in the MSSM, with λ being the gaugino.

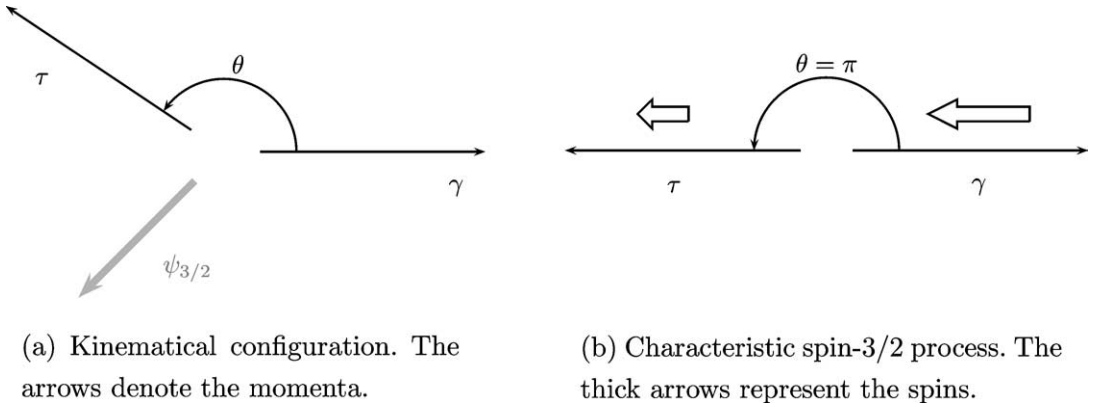


Fig. 2. (a) Shows the kinematical configuration of the 3-body decay. (b) Illustrates the characteristic spin-3/2 process: if photon and τ -lepton move in opposite directions and the spins add up to 3/2, the invisible particle also has spin 3/2.

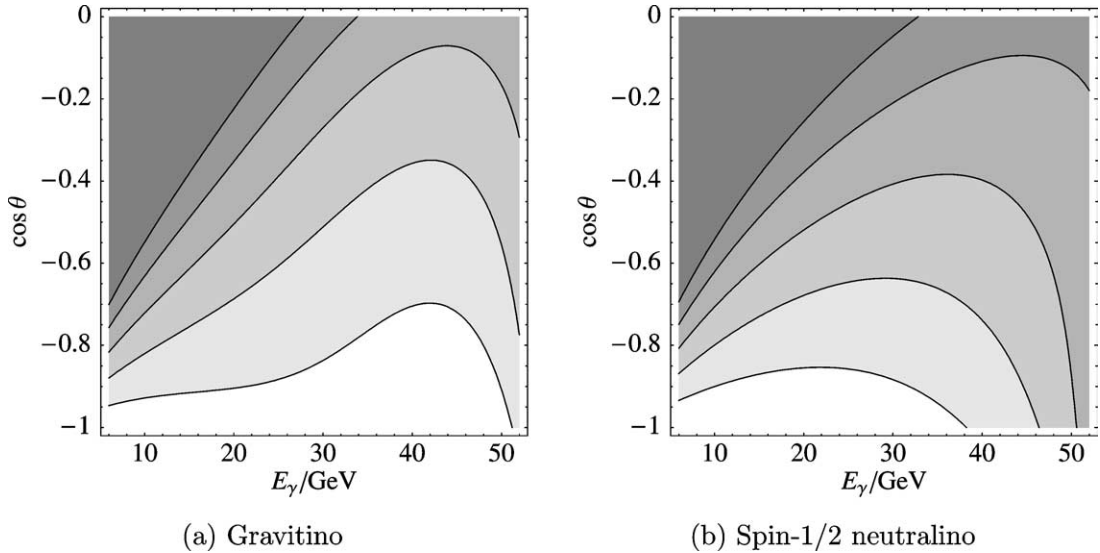


Fig. 3. Contour plots of the differential decay rates for (a) gravitino $\psi_{3/2}$ and (b) neutralino λ . $m_{\tilde{\tau}} = 150$ GeV and $m_X = 75$ GeV ($X = \psi_{3/2}, \lambda$). The boundaries of the different gray shaded regions (from bottom to top) correspond to $\Delta(E_\gamma, \cos\theta)[\text{GeV}^{-1}] = 10^{-3}, 2 \times 10^{-3}, 3 \times 10^{-3}, 4 \times 10^{-3}, 5 \times 10^{-3}$. Darker shading implies larger rate.

where $m_{\tilde{\tau}} = 150$ GeV and $m_{3/2} = 75$ GeV ($m_\lambda = 75$ GeV). The two differential distributions are qualitatively different and should allow to distinguish experimentally gravitino and neutralino.

Note that even for very small masses $m_{3/2}$ and m_λ , the differential decay rates Δ for gravitino $\psi_{3/2}$ and neutralino λ are distinguishable. In this small mass limit, $\psi_{3/2}$ can effectively be described by the goldstino χ (with the interaction (10)), and the differential decay rates for $\psi_{3/2}$ and χ essentially coincide. The discrepancy between λ and χ stems from the additional photino contribution, as discussed below (10). This makes it possible to discriminate the goldstino from the neutralino even for very small masses.

The second method to test the spin-3/2 nature is intuitively more straightforward though experimentally even more challenging than the first one. The main point is obvious from Fig. 2(b) where a left-handed photon and a right-handed τ move in opposite directions. Clearly, this configuration is allowed for an invisible spin-3/2 gravitino but forbidden for spin-1/2 neutralino. Unfortunately, measuring the polarizations is a difficult task.

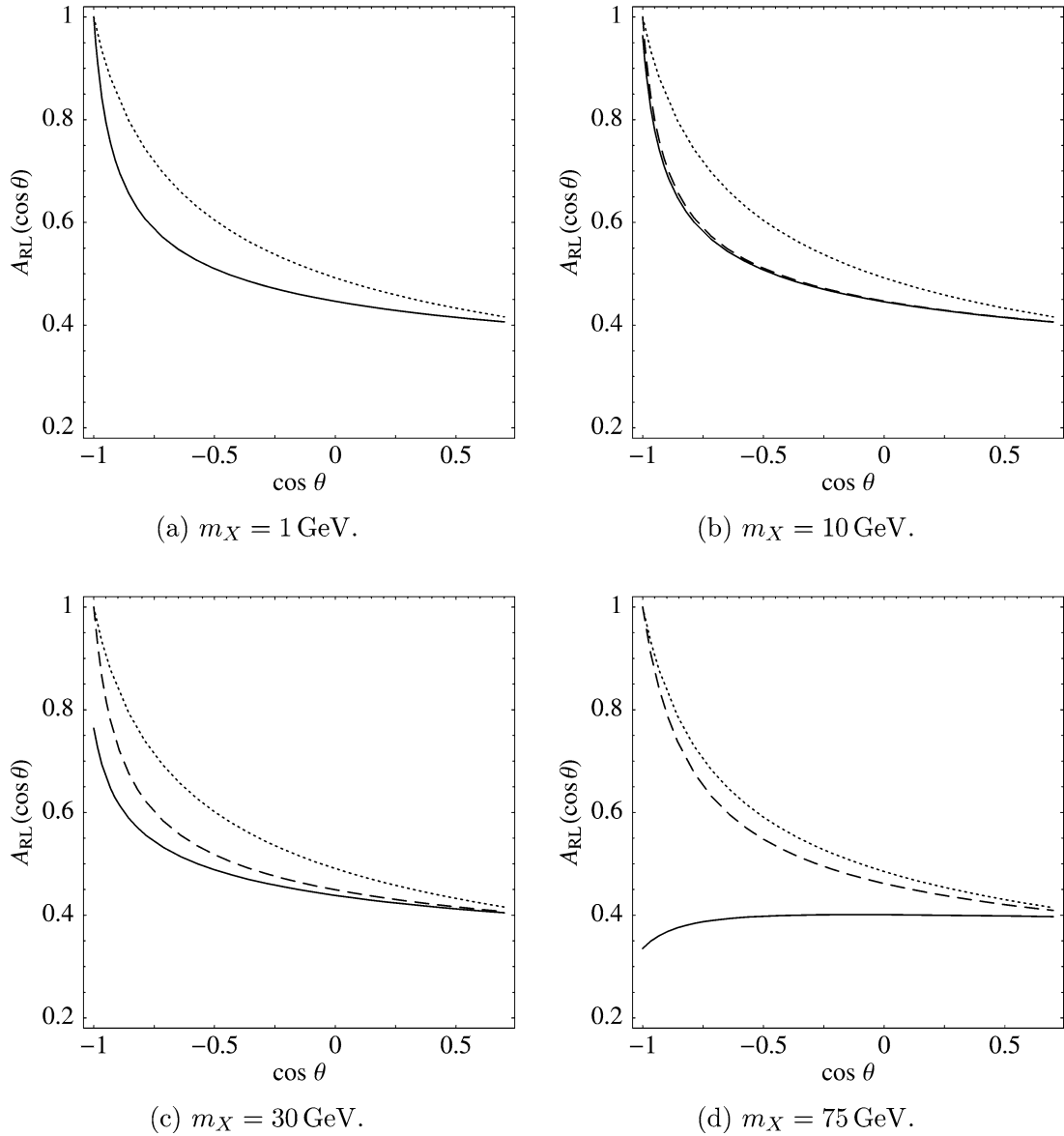


Fig. 4. Angular asymmetries for gravitino $\psi_{3/2}$ (solid curve), goldstino χ (dashed curve) and neutralino λ (dotted curve). We use $m_{\tilde{\tau}} = 150 \text{ GeV}$ and cut the photon energy below 10% of the maximal photon energy (cf. Appendix A). Note that the asymmetries only depend on the ratio $r = m_X^2/m_{\tilde{\tau}}^2$ ($X = \psi_{3/2}, \chi, \lambda$).

As Fig. 2(b) illustrates, the spin of the invisible particle influences the angular distribution of final states with polarized photons and τ -leptons. Again the difference between gravitino and neutralino is due to the direct coupling shown in Fig. 1(a) and most significant in the backward direction. An appropriate observable is the angular asymmetry

$$A_{RL}(\cos\theta) = \frac{(d\Gamma/d\cos\theta)(\tilde{\tau}_R \rightarrow \tau_R + \gamma_R + X) - (d\Gamma/d\cos\theta)(\tilde{\tau}_R \rightarrow \tau_R + \gamma_L + X)}{(d\Gamma/d\cos\theta)(\tilde{\tau}_R \rightarrow \tau_R + \gamma_R + X) + (d\Gamma/d\cos\theta)(\tilde{\tau}_L \rightarrow \tau_R + \gamma_L + X)}, \quad (12)$$

where X denotes gravitino ($X = \psi_{3/2}$) and neutralino ($X = \lambda$). Here, we also study the angular asymmetry for a pseudo-goldstino in the final state ($X = \chi$). Like a pseudo-Goldstone boson, the pseudo-goldstino has goldstino couplings and a mass which explicitly breaks global supersymmetry. Notice that, as mentioned above, the photino does not decouple in this case.

The three angular asymmetries are shown in Fig. 4 for $m_{\tilde{\tau}} = 150$ GeV and different masses of the invisible particle. As expected, the decay into right-handed τ and left-handed photon at $\theta = \pi$ is forbidden for spin-1/2 invisible particles (χ and λ), whereas it is allowed for the spin-3/2 gravitino. This is clearly visible in Fig. 4(c), (d); for small gravitino masses the goldstino component dominates the gravitino interaction as illustrated by Fig. 4(a), (b).

Our discussion is easily generalized to the case where the NSP is a linear combination of $\tilde{\tau}_R$ and $\tilde{\tau}_L$. One then needs further information on the left-right mixing angle φ_τ , which could be provided by a direct measurement of the τ -polarization or by the coupling to W -boson.

Let us finally comment on the experimental feasibility of the gravitino spin determination. The angular distribution of the 3-body decay is peaked in forward direction ($\theta = 0$). Hence, a large number of events is needed for the spin measurement. Compared to the 2-body decay, backward ($\cos\theta < 0$) 3-body decays are suppressed by $\sim 10^{-1} \times \alpha \simeq 10^{-3}$. Requiring 10, ..., 100 events for a signal one therefore needs 10^4 to 10^5 $\tilde{\tau}$ s, which appears possible at the LHC and also at a linear collider according to the discussion in Section 2.

4. Gravitino cosmology

The existence of gravitinos imposes severe constraints on the early history of our universe. If the gravitino is the LSP and stable, as assumed in our investigation, there are two important constraints which we shall briefly discuss in this section. The first one arises for large reheating temperatures T_R after inflation, which may lead to a relic gravitino abundance exceeding the observed cold dark matter density. This ‘overclosure’ constraint implies an upper bound on the reheating temperature [15]. Note, however, that there are several mechanisms which avoid this constraint and which, in addition, explain the observed cold dark matter in terms of gravitinos [16].

The second constraint concerns the decay of the long-lived NSP $\tilde{\tau}$. If it occurs during or after nucleosynthesis (BBN), it may spoil the successful predictions of BBN [15,17]. A recent detailed analysis [18] shows that the hadronic decay of a heavy particle during or after BBN imposes severe constraints on its abundance and lifetime. If the 3-body decay $\tilde{\tau} \rightarrow \psi_{3/2} + \tau + Z$ is allowed, one finds the upper bound on the $\tilde{\tau}$ -lifetime $(\Gamma_{\tilde{\tau}})^{-1} \lesssim 10^3$ s, or equivalently, $m_{3/2} \lesssim 0.4 \text{ GeV} (m_{\tilde{\tau}}/150 \text{ GeV})^{5/2}$. Note that in the case of non-zero left–right mixing φ_τ , processes involving W also have to be taken into account. On the other hand, if hadronic $\tilde{\tau}$ decays are sufficiently suppressed, which is the case for $m_{\tilde{\tau}} - m_{3/2} < m_Z$, only the effect of the electromagnetic NSP decay [19] has to be taken into account. The allowed mass range is then extended to $100 \text{ GeV} \lesssim m_{\tilde{\tau}} \lesssim 130 \text{ GeV}$ and $(m_{\tilde{\tau}} - m_Z) \lesssim m_{3/2} \lesssim 35 \text{ GeV}$ for a typical pair annihilation cross section $\sigma_{\tilde{\tau}}$ of $\tilde{\tau}$ s, and to $100 \text{ GeV} \lesssim m_{\tilde{\tau}} \lesssim 350 \text{ GeV}$ and $(m_{\tilde{\tau}} - m_Z) \lesssim m_{3/2} \lesssim 260 \text{ GeV}$, if $\sigma_{\tilde{\tau}}$ is enhanced by a factor 100 [20]. Note that for larger gravitino masses the spin determination is easier, as discussed in Section 3.

Finally, we should mention that the above BBN constraints disappear if there is sufficient entropy production between the decoupling of the NSP at $T_d \simeq \mathcal{O}(10 \text{ GeV})$ and BBN at $T_{\text{BBN}} \simeq \mathcal{O}(\text{MeV})$, or if the reheating temperature T_R is lower than T_d , $T_{\text{BBN}} < T_R < T_d$, so that there is no thermal production of $\tilde{\tau}$ s.

5. Conclusions

We have discussed how one may discover the massive gravitino, and thereby supergravity, at the LHC or a future linear collider, if the gravitino is the LSP and a charged slepton is the NSP. With the gravitino mass inferred from

kinematics, the measurement of the NSP lifetime will test an unequivocal prediction of supergravity. For gravitino masses $m_{3/2} \gtrsim \mathcal{O}(\text{GeV})$ also the determination of the gravitino spin appears feasible.

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Appendix A. 3-body slepton decays

This appendix provides some details of the calculations leading to the results of Section 3. For simplicity we take $m_\tau = 0$. The diagrams of Fig. 1 yield

$$\sum_{\text{spins}} |\mathcal{M}(\tilde{\tau}_R \rightarrow \tau_R + \gamma_L + \psi_{3/2})|^2 = \frac{2e^2 m_{\tilde{\tau}}^2}{3M_{\text{P}}^2} (1-r-2\eta) \frac{(1-r)^2(2-z) + 12rz\eta^2}{4rz\eta^2}, \quad (\text{A.1a})$$

$$\begin{aligned} & \sum_{\text{spins}} |\mathcal{M}(\tilde{\tau}_R \rightarrow \tau_R + \gamma_R + \psi_{3/2})|^2 \\ &= \frac{2e^2 m_{\tilde{\tau}}^2}{3M_{\text{P}}^2} \left\{ \frac{(1-r)^2(2-z)(1-r+2\eta)}{4rz\eta^2} + \frac{(1-r)^2[2-(1-r)z]}{rz(1-r-2\eta)} \right. \\ & \quad + \frac{12-10(1-r)z-2r(4-r)z^2+(1+r-2r^2)z^3}{rz^2(1-z\eta)} - \frac{[2-(1-r)z]^2}{rz^2(1-z\eta)^2} \\ & \quad \left. + \frac{[-8+6(1-r)z-(1-7r)z^2]-2\eta[2z-(1-4r)z^2]}{rz^2} \right\}, \quad (\text{A.1b}) \end{aligned}$$

where $r = m_{3/2}^2/m_{\tilde{\tau}}^2$, $\eta = E_\gamma/m_{\tilde{\tau}}$ and $z = 1 - \cos\theta$. The corresponding transition probabilities for the (hypothetical) spin-1/2 particle λ read (now with $r = m_\lambda^2/m_{\tilde{\tau}}^2$)

$$\sum_{\text{spins}} |\mathcal{M}(\tilde{\tau}_R \rightarrow \tau_R + \gamma_L + \lambda)|^2 = e^2 h^2 \frac{(2-z)(1-r-2\eta)}{2z\eta^2}, \quad (\text{A.2a})$$

$$\sum_{\text{spins}} |\mathcal{M}(\tilde{\tau}_R \rightarrow \tau_R + \gamma_R + \lambda)|^2 = e^2 h^2 \frac{(1-r)^2(2-z) + 4rz\eta^2}{2z\eta^2(1-r-2\eta)}. \quad (\text{A.2b})$$

In the case of pseudo-goldstino χ with interactions described by Eq. (10) one has to include the diagram with the photino intermediate state. We then obtain in the limit of a large photino mass $m_{\tilde{\gamma}}$ (with $r = m_\chi^2/m_{\tilde{\tau}}^2$)

$$\sum_{\text{spins}} |\mathcal{M}(\tilde{\tau}_R \rightarrow \tau_R + \gamma_L + \chi)|^2 = e^2 \frac{m_{\tilde{\tau}}^4}{3m_{3/2}^2 M_{\text{P}}^2} \frac{(2-z)(1-r-2\eta)}{2z\eta^2}, \quad (\text{A.3a})$$

$$\begin{aligned} & \sum_{\text{spins}} |\mathcal{M}(\tilde{\tau}_R \rightarrow \tau_R + \gamma_R + \chi)|^2 \\ &= e^2 \frac{m_{\tilde{\tau}}^4}{3m_{3/2}^2 M_{\text{P}}^2} \left\{ \frac{(1-r)^2(2-z) + 4rz\eta^2}{2z\eta^2(1-r-2\eta)} + \frac{2z\eta^2(1-r-2\eta)[2-(1-r)z]}{(1-z\eta)^2} \right. \\ & \quad \left. + \frac{2z(1-r-2\eta)}{1-z\eta} - 4(1-r) \right\}. \quad (\text{A.3b}) \end{aligned}$$

The limit $r \rightarrow 0$ in Eqs. (A.3) yields the results for massless goldstino. Indeed, as can be seen by straightforward calculation, they precisely reproduce the massless limits of gravitino transition probabilities, which are obtained by taking the limit $r \rightarrow 0$ in Eqs. (A.1) while keeping the SUSY breaking parameter $\sqrt{3}m_{3/2}M_{\text{P}}$ finite.

In order to present the angular distribution, we perform an E_τ integration,

$$\frac{d\Gamma}{d\cos\theta} = \frac{1}{128\pi^3} \int_0^{E_\tau^{\max}} dE_\tau \frac{E_\tau [m_{\tilde{\tau}}(m_{\tilde{\tau}} - 2E_\tau) - m_X^2]}{m_{\tilde{\tau}} [m_{\tilde{\tau}} - (1 - \cos\theta)E_\tau]^2} |\mathcal{M}|^2, \quad (\text{A.4})$$

where m_X denotes the mass of the invisible particle. Notice that the $|\mathcal{M}|^2$ s have singularities coming from soft photon ($1/E_\gamma \propto 1/\eta$), soft τ ($1/E_\tau \propto 1/[1 - r - 2\eta]$) and a collinear divergence ($1/[1 - \cos\theta] = 1/z$). The last two are not really divergent for finite m_τ . We remove the soft photons from the rates. This procedure is justified by the limited resolution of real detectors. The requirement that $E_\gamma \geq \delta E_{\max}$ (where $E_{\max} = m_{\tilde{\tau}}(1 - r)/2$ is the maximal energy of the photon) leads to

$$E_\tau \leq E_\tau^{\max}(\delta) = \frac{m_{\tilde{\tau}}(m_{\tilde{\tau}}^2 - m_X^2)(1 - \delta)}{2m_{\tilde{\tau}}^2 - \delta(m_{\tilde{\tau}}^2 - m_X^2)(1 - \cos\theta)}. \quad (\text{A.5})$$

In Section 3, $\delta = 0.1$ is used, i.e., the photons are cut below 10% of their maximal energy.

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