Title
CP-violation and $ππ$-interaction in the radiative decays of K_L and K_S

Author
Bulanov, SS

Publication Date
2002-05-15

Peer reviewed

CP-violation in $K_{S,L} \to \pi^+\pi^-\gamma$ and $K_{S,L} \to \pi^+\pi^-e^+e^-$ decays

S. S. Bulanov

Institute of Theoretical and Experimental Physics,

B. Cheremyshkinskaya ul., 25, 117259 Moscow, Russia.

e-mail: bulanov@heron.itep.ru

Abstract

The dependence of $K_{S,L} \to \pi^+\pi^-\gamma$ decay probabilities on photon polarization is calculated. The phases of terms of amplitude that arise from the pion-pion interaction are obtained by using a simple realistic model of pion-pion interaction via virtual ρ-meson, instead of the ChPT. The results are compared with those of other authors and the origin of slight discrepancies is explained. The departure of the photon spectrum from pure bremsstrahlung due to the pion loop contribution to the direct emission amplitude is calculated. It is shown that the interference between the terms of amplitude with different CP-parity appears only when the photon is polarized (linearly or circularly). Instead of measuring the linear polarization, the angular correlation between the $\pi^+\pi^-$ and e^+e^- planes in $K_{S,L} \to \pi^+\pi^-e^+e^-$ decay can be studied.

1 Introduction

The theoretical and experimental study of the CP-violation in the radiative decays of the K_L and K_S has a long history. In view of future precise measurements of these decays we have recalculated the above effects. Generally our results are in agreement with the previous ones. A few discrepancies (see Conclusion) are caused by more realistic evaluation of pion loops in the present paper.

The pattern of the CP-violation in the $K_{L,S} \to \pi^+\pi^-\gamma$ decays was theoretically predicted in the 1960s. H. Chew [1] determined the amplitude structure of $K^0_{1,2} \to \pi^+\pi^-\gamma$ decays. He calculated the pion loop contribution to the direct emission amplitude and stated the possibility of the CP-violation in case the amplitude is a sum of terms with different CP-parity. G. Costa and P. K. Kabir [2] as well as L. M. Sehgal and L. Wolfenstein [3] studied the interference of the K^0_1 and K^0_2 in the decays into $\pi^+\pi^-\gamma$, identifying this effect with the CP-violation. They also qualitatively discussed the dependence of the decay probability on the photon polarization. A. D. Dolgov and L. A. Ponomarev [4] paid special attention to the K_L decay. They realized that the CP-violation effects in the K_L decay should be larger than in the K_S decay. They also found that the measurement of the photon polarization could enhance the signals of the CP-violation. They qualitatively discussed the measurement of the angular correlation between the $\pi^+\pi^-$ and e^+e^- planes in $K_L \to \pi^+\pi^-e^+e^-$ decay instead of measuring the linear polarization.

In the 1990s $K_{L,S} \to \pi^+\pi^-\gamma$ decays were thoroughly studied using ChPT (Chiral Perturbation Theory) with special emphasize on the K_L-meson decay. The K_L decay attracted special attention, because, as it is known experimentally, the contributions of the K_L decay amplitude terms with different CP-parity are of comparable magnitude and this makes the CP-violation to be distinctively seen. Contrary to this in the case of the K_S decay the CP-violation is difficult to detect due to the fact that the internal bremsstrahlung contribution shades the contribution of the direct emission.

G. D'Ambrosio and G. Isidori [5] and G. D'Ambrosio, M. Miragluiolo and P. Santorelli [6] presented a complete calculation of the direct emission contribution to the $K_{S,L} \to \pi^+\pi^-\gamma$ decay amplitude up to the 6th order of momenta in the framework of the ChPT. In view of future precise measurements and the new data on direct CP-violation obtained by KTeV collaboration (A. Alavi-Harati *et. al.*) [7] several authors addressed the problem of short distance contributions to the direct CP-violating observables in the radiative K-meson decays. X.-G. He and G. Valencia [8] studied the $s \to d\gamma$ transition. They described the long distance contribution in the framework of the ChPT and subtracted it from the physical amplitudes of the $K \to \pi\pi\gamma$ decays in order to constrain the new, short distance, interactions. They also illustrated two types of models in which the short distance interactions could be significantly enhanced with respect to the Standard Model, namely the left-right symmetric model and the supersymmetry.

G. Colangelo, G. Isidori and J. Portoles [9] analyzed the supersymmetric contributions to the direct CP-violating observables in $K \to \pi\pi\gamma$ decays induced by gluino-mediated magnetic-penguin operators. They found that the direct CP-violation could be substantially enhanced with respect to its Standard Model value especially in the scenario where the direct CP-violation is dominated by supersymmetric contributions.

J. Tandean and G. Valencia [10] also revisited the $K_L \to \pi^+\pi^-\gamma$ decay in order to study the possible contributions of $s \to d\gamma$ as well as gluonic, $s \to dg$, transitions to the direct CP-violating observables in the framework of two models: left-right symmetric model and the supersymmetry.

The experimental studies of the K_L decay were reported in Refs. [11], [12], [13]. In 1980 A. S. Carrol *et. al.* [11] first observed both the internal bremsstrahlung and the direct emission contributions in the K_L decay. E. J. Ramberg *et. al* [12] in 1993 presented more precise measurements of the pattern of the CP-violation in the K_L decay. S. Kettell [13] in his talk "Experimental Results on Radiative Kaon Decays" summarized the recent more precise results.

In the case of the K_S decay only the internal bremsstrahlung contribution was found by E. J. Ramberg *et. al* [12] and by H. Taureg *et. al* [14]. The latter established the upper bounds for the branching ratio of the interference of internal bremsstrahlung and electric direct emission. The theoretical study of the K_S decay was performed by G. D'Ambrosio, M. Miragliuolo and F. Sannino [15], where the electric direct emission amplitude was calculated in the framework of the ChPT. They also studied the photon spectrum departure from the pure internal bremsstrahlung expectation due to the interference of the internal bremsstrahlung and the electric direct emission. In Ref. [15] there was also noted that the measurement of this interference would provide a test of the proposed models.

In regard to the future higher precision experiments on the CP-violation it is important to study in more detail the properties of the phases caused by the $\pi\pi$-interaction in the $K_{L,S} \to \pi^+\pi^-\gamma$ decays. More precise calculation of these phases is the prerequisite for extracting the precise values of the CP-violating parameters in the K-meson decays.

In the present paper we calculate the probability of the $K_{S,L} \to \pi^+\pi^-\gamma$ decays, using instead of ChPT a simple realistic model of $\pi\pi$ interaction via virtual ρ-meson, proposed by B. W. Lee and M. T. Vaughn [16] for the purposes of studying the P-wave resonance in the $\pi\pi$ scattering. This model was elaborated in Ref. [1] for the purposes of describing the electric direct emission in the $K \to 2\pi\gamma$ decays. In the framework of this model we shall derive the phases of the amplitude terms, connected with the $\pi\pi$-interaction. According to the approach used in Ref. [1] we shall show that the phase of the electric direct emission amplitude is not equal to the phase of the $\pi\pi$ scattering in the P-wave, which could be expected according to the final state interaction theorem formulated by K. Watson [17], E. Fermi [18], S. Fubini and Y. Nambu [19]. This happens due to the fact that the interaction of pions occurs not in the final state, but in the intermediate state.

We shall calculate the departure of the photon spectrum from bremsstrahlung due to the pion loop contribution to the direct emission amplitude in the K_S decay. Then we shall compare our results on the photon energy dependence of the interference between the internal bremsstrahlung and the electric direct emission in the K_S decay with the results of Refs. [5], [15] obtained in the framework of ChPT. It will be proved that the "interference branching ratio" is 20% larger than the one obtained in Refs. [5], [15] (see Table 1). We shall show that the difference between these results appears due to the difference in models used to calculate the amplitude of $\pi\pi$ scattering in P-wave. We shall notice that this difference appears due to the fact that in Refs. [5], [15] the leading order of expansion in powers of momenta in ChPT was used, and the phase of the electric direct emission amplitude was assumed to be equal to the phase of $\pi\pi$ scattering in P-wave at the energy $\sqrt{s} = m_K$. Instead, we shall take into account the additional phase shift produced by pion loops and the energy dependence of the P-wave $\pi\pi$ scattering phase. We shall use the approach of Refs. [1], [16] in order to describe the phase energy dependence, which appears in the amplitude through the pion loop contributions to the ρ-meson propagator.

While studying the electric direct emission amplitude and the difference in the results for the "interference branching ratio" in the K_S decay we shall address the problem of $\pi\pi$ scattering in the P-wave. This problem is connected with the $\pi\pi$ interaction, which appears in the electric direct emission. We shall compare the experimental data with the results obtained in the framework of different models: the ChPT, the ChPT with ρ-meson contribution taken into account and the simple realistic model. We shall see that the results calculated within the framework of the ChPT coincide with the experimental data only for low energies, because the ChPT doesn't take into account the ρ-meson contribution. Then we shall see that the model of $\pi\pi$-interaction via ρ-meson and the ChPT with ρ-meson contribution show the same behavior of the P-wave $\pi\pi$ scattering phase.

In regard to the effects of CP-violation in the $K_{S,L} \to \pi^+\pi^-\gamma$ decays we shall note that in the case of the K_S decay the contribution of the CP-violating magnetic direct emission to the decay probability is negligibly small contrary to the case of the K_L decay, where the contributions of the CP-conserving magnetic direct emission and CP-violating internal bremsstrahlung to the decay probability are of comparable magnitude as it was theoretically predicted in Refs. [5], [6], [10], and experimentally confirmed in Refs. [11], [12], [13]. Therefore, in the case of the K_S decay we shall consider the interference between the amplitude terms with different CP-parity, due to the fact that the interference is the largest term in which the CP-violating effects reside. As we shall see one has to measure the polarization of photons to analyze the interference, because it is nonzero only when the polarization of the photon is observed. We note that this phenomenon was discussed qualitatively by G. Costa and P. K. Kabir [2], L. M. Sehgal and L. Wolfenstein [3], [4], A. D. Dolgov and L. A. Ponomarev [4], M. McGuigan and A. I. Sanda [20] for both K_S and K_L decays. The quantitative analysis of the K_L decay amplitude dependence on the photon polarization was performed by L. M. Sehgal and J. van Leusen [21], [22], [23].

According to the approach proposed in Ref. [21] for K_L decay, we shall examine the $K_S \to \pi^+\pi^-\gamma$ decay probability with the polarized photon, taking into account various cases of the photon polarization and we shall show that the measurement of the linear polarization in principle allows extraction of terms with opposite CP-parity. In the present paper we address the problem of studying the effects of different cases of the photon polarization in the case of the K_S decay. As it is known as an alternative to measuring the linear photon polarization, the angular correlation of $\pi^+\pi^-$ and e^+e^- planes in $K_{S,L} \to \pi^+\pi^-e^+e^-$ decay can be studied, as it was first suggested in Ref. [4]. The structure of the $K_L \to \pi^+\pi^-e^+e^-$ decay amplitude was studied by L. M. Sehgal, M. Wanninger [24] P. Heiliger, L. M. Sehgal [25] and J. K. Elwood, M. B. Wise [26], where the CP-violating asymmetry, arising from the angular correlation of $\pi^+\pi^-$ and e^+e^- planes, was also obtained. The predictions of Refs. [24], [25], [26] on the $K_L \to \pi^+\pi^-e^+e^-$ decay branching ratio and CP-violating asymmetry were confirmed by KTeV collaboration (A. Alavi-Harati *et. al.*) [27].

The problem of pion loop contribution in the $K_L \to \pi^+\pi^-e^+e^-$ decay was studied by J. K. Elwwod, M. B. Wise, M. J. Savage and J. W. Walden [28] in the framework of the ChPT. Their approach included both $\pi\pi \to \pi\pi$ and $\pi\pi \to \pi\pi\gamma^*$ rescattering. Previous estimates of the effect of pion loop contribution in Refs. [24], [25] used the measured pion phase shifts and neglected $\pi\pi \to \pi\pi\gamma^*$. They found that this effect could enhance the CP-violating asymmetry by about 45% over the estimates given in Ref. [26], resulting in the value of the asymmetry, which is in good agreement with the experimental data [27]. These results were summarized by M. J. Savage in his talk "$K_L^0 \to \pi^+\pi^-e^+e^-$ in Chiral Perturbation Theory" [29]. A more precise calculation of the $K_L \to \pi^+\pi^-e^+e^-$ decay branching ratio up to next-to-leading order in the framework of the ChPT was presented by H. Pichl [30]. G. Ecker and H. Pichl [31] updated the theoretical analysis of the CP-violating asymmetry in the $K_L \to \pi^+\pi^-e^+e^-$ decay using the ChPT and the most recent phenomenological information.

In the present paper we shall calculate the CP-violating asymmetry in the case of the $K_L \to \pi^+\pi^-e^+e^-$ decay in order to compare our results with those of Refs. [28], [29]. The result of Refs. [28], [29] for the CP-violating asymmetry is 14%, and we shall find that the asymmetry is $(13.4\pm0.9)\%$. The central values coincide within the accuracy of the calculation. We shall also calculate the CP-violating asymmetry in the case of the $K_S \to \pi^+\pi^-e^+e^-$ decay and we shall find it to be substantially smaller: $(5.1 \pm 0.4) \times 10^{-5}$. This could be qualitatively expected from the analysis of the $K_S \to \pi^+\pi^-\gamma$ decay, because the CP-violating asymmetry depends on the interference of the amplitude terms with opposite CP-parity. The CP-violating magnetic direct emission amplitude is very small in the K_S decay contrary to that of the K_L, where the CP-conserving magnetic emission and CP-violating internal bremsstrahlung are of comparable magnitude. In order to detect the asymmetry experimentally one will need more than 10^{10} $K_S \to \pi^+\pi^-e^+e^-$ decays, because one should have the statistical error to be smaller than the magnitude of the effect.

The paper is organized as follows. In Section 2 we discuss the structure of the $K_{S,L} \to \pi^+\pi^-\gamma$ decay amplitude. In Section 3 we present the simple realistic model of $\pi\pi$ scattering in the P-wave and derive the expression for pion loops contribution to the direct emission amplitude in the framework of this model. We discuss the spectrum departure from pure internal bremsstrahlung due to pion loops contribution in K_S decay in Section 4. We compare our results on the spectrum departure with the results obtained in the framework of ChPT in Section 5. In Section 6 we discuss the different models of the $\pi\pi$ scattering in P-wave and compare them with experimental data. In Section 7 we carry out the analysis of the dependence of the $K_{S,L}$ decay amplitude on the photon polarization in terms of Stokes vectors. We also discuss the angular correlation in $K_{S,L} \to \pi^+\pi^-e^+e^-$ decays and calculate the CP-violating asymmetry in $K_S \to \pi^+\pi^-e^+e^-$ decay in this section. Section 8 is devoted to the discussion of the main results and conclusion.

2 The amplitude structure

We start by labeling the momenta of the particles involved in the decay

$$K_{S,L}(r) \to \pi^+(p)\pi^-(q)\gamma(k,e), \tag{1}$$

where e is a 4-vector of photons wave function.

It is convenient to define three expressions:

$$T_B = \frac{pe}{pk} - \frac{qe}{qk}, \tag{2}$$

$$T_E = (pe)(qk) - (qe)(pk), \tag{3}$$

$$T_M = \varepsilon_{\mu\nu\rho\sigma}p_\mu q_\nu k_\rho e_\sigma. \tag{4}$$

The amplitudes of $K_{S,L} \to \pi^+\pi^-\gamma$ decays are made up of two components: the internal bremsstrahlung (B), proportional to T_B, and direct emission (D) [2], [3], [6]. In its turn, D is a sum of an electric term(E_D), proportional to T_E, and a magnetic term(M_D), proportional to T_M. We note that $T_E = T_B(pk)(qk)$; however, due to the different origin of the internal bremsstrahlung and electric direct emission it is convenient to treat them separately.

In accordance with the above, the amplitudes of the $K_{S,L} \to \pi^+\pi^-\gamma$ decays can be written as follows

$$A(K_S \to \pi^+\pi^-\gamma) = eAe^{i\delta_0^0}T_B + e(a+b)T_E + ie\eta_{+-}cT_M, \tag{5}$$

$$A(K_L \to \pi^+\pi^-\gamma) = \eta_{+-}eAe^{i\delta_0^0}T_B + e\eta_{+-}(a+b)T_E + i\ ecT_M, \tag{6}$$

where δ_0^0 is the S-wave pion scattering phase. Here the upper index is isospin, the lower – angular momentum and η_{+-} is the well known CP-violation parameter in the $K_L \to \pi^+\pi^-$ decay. The imaginary unit in front of the factor c stems from the hermiticity of the Hamiltonian describing direct emission, neglecting final state interaction. The factor $A \equiv A(K \to \pi^+\pi^-)$ is determined by the Low theorem for bremsstrahlung [32]. The term $(a+b)$ is the electric direct emission coupling, while c is the magnetic direct emission coupling.

As follows from Eqs. (5, 6) the electric direct emission coupling is divided into two terms. The first term (a) describes the loops of heavy particles. The second term (b) describes the loops of pions. Such subdivision is convenient because the pion loops contribution has an absorptive part and hence a phase, contrary to the contribution of the heavy particle loops.

The phases of direct emission couplings a and c are dictated by the final state interaction theorem [17], [18], [19], [33]. According to the law of conservation of angular momentum we have $J_\gamma = J_{\pi\pi} = 1$. Here J_γ is the total angular momentum of the photon, $J_{\pi\pi}$ is the total angular momentum of two pions. Since $J_{\pi\pi} = l_{\pi\pi}$, where $l_{\pi\pi}$ is the orbital momentum of the two pions, the spatial part of the two-pion wave function should be antisymmetric, the isospin part of the wave function should also be antisymmetric, according to the Bose generalized principles; i.e. $J_{\pi\pi} = 1(P\text{-wave})$, $T = 1$, where T is isospin of two pions. As a result we have for the phases

$$a = |a| e^{i\delta_1^1}, \quad c = |c| e^{i\delta_1^1},$$

where δ_1^1 is the pion P-wave scattering phase.

3 Contribution from pion loops

The phases of the pion loops and bremsstrahlung contributions in equations (5), (6) are defined by the strong interaction of pions. Let us describe the simple realistic model of the $\pi\pi$ interaction, mentioned above, which we shall use while considering the $\pi\pi$ scattering in the P-wave. The Lagrangian of this model [16] has the following form

$$L = -\frac{g}{\sqrt{2}} \varepsilon_{ijk} (\phi^i \partial_\mu \phi^j - \partial_\mu \phi^i \phi^j) B_\mu^k, \tag{7}$$

where i, j, k are isotopic indices, ϕ is the pion field, B_μ^k – ρ-meson field. The P-wave resonance in $\pi\pi$ scattering is due to the resonant structure of the ρ-meson propagator, the relevant part of which is

$$D^{\mu\nu}(k) = -D(k^2) g^{\mu\nu},$$
$$D(k^2) = [k^2 - m_\rho^2 - \Sigma(k^2)]^{-1}, \tag{8}$$

where $\Sigma(k^2)$ is the ρ self energy operator; in the "resonance approximation" in which we consider only the sum of the iterated bubble diagrams with pions running in the loop, $\Sigma(k^2)$ is given by the following expression

$$\Sigma(s) = J(s) - J(m_\rho^2) + i\; Im(\Sigma(s)), \tag{9}$$

with

$$Im(\Sigma(s)) = -\frac{g^2}{48\pi}\frac{(s-4m_\pi)^{3/2}}{s^{1/2}}\theta(s-4m_\pi), \tag{10}$$

$$J(s) = \frac{g^2}{48\pi}\left\{\frac{(s-4m_\pi)^{3/2}}{s^{1/2}}\ln\left[\frac{s^{1/2}+(s-4m_\pi)^{1/2}}{s^{1/2}-(s-4m_\pi)^{1/2}}\right] - \xi s\right\}, \tag{11}$$

$$\xi = \frac{m_\rho^2 - 4m_\pi^2}{m_\rho^2} + \frac{m_\rho^2+2m_\pi^2}{m_\rho}\frac{(m_\rho^2-4m_\pi^2)^{1/2}}{m_\rho}\ln\left[\frac{m_\rho + (m_\rho^2-4m_\pi^2)}{m_\rho - (m_\rho^2-4m_\pi^2)}\right]. \tag{12}$$

Here $\theta(x)$ is the step function $\theta(x)=0$ for $x<0$ and $\theta(x)=1$ for $x>0$.

In the case of internal bremsstrahlung contribution to the $K_{S,L}$ decay probability the interaction of pions can be described by the diagrams shown in Fig. 1. Though pions are in P-wave in the final state, as it was shown in the previous section, this group of diagrams results in the δ_0^0 phase of the amplitude, as it was assumed in Refs. [6], [5], [10], [15], [20], [21], [24]. It is due to the fact that the interaction of pions occurs not in the final state, but in the intermediate. The diagrams of Fig. 1 contribute to the $K\pi\pi$ vertex, these corrections are taken into account by using the the amplitude of the $K_S \to \pi^+\pi^-$ decay as the interaction constant and assuming it equal to its experimental value.

The emission of the photon from the loops of pions is governed by another group of diagrams shown in Fig. 2. Each of these diagrams is divergent but their sum is finite. The similar result holds in the ChPT. A straightforward calculation of this finite expression gives the result that is not gauge invariant. This effect arises from the cancellation of two 4-dimensional integrals proportional to l^2 and $l_\mu l_\nu$. The evaluation of these two integrals in the framework of the dimensional regularization scheme leads to a constant term that restores the gauge invariance [34].

Further we shall neglect the energy dependence of the $K\pi\pi$ vertex. We shall take the amplitude of the $K_S \to \pi^+\pi^-$ decay as the interaction constant, and use the experimental result for it.

We find for the matrix element arising from the diagrams, shown in Fig. 2:

$$E_D^{loop} = \frac{eg^2AD(s)}{(2\pi)^4}\left[\int \frac{4((r+l)e)(l(q-p))d^4l}{(l^2-m_\pi^2)((r+l)^2-m_\pi^2)((r+l-k)^2-m_\pi^2)}\right.$$

$$+\int \frac{4(le)(l(q-p))d^4l}{(l^2-m_\pi^2)((l+k)^2-m_\pi^2)((r+l)^2-m_\pi^2)}$$
$$-2\int \frac{((q-p)e)d^4l}{(l^2-m_\pi^2)((r+l)^2-m_\pi^2)}\Bigg] = \frac{eg^2A}{\pi^2}F(s)D(s)\frac{T_E}{rk} = ebT_E, \quad (13)$$

with

$$F(s) = \frac{1}{2} + \frac{s}{2rk}\left[\beta \mathrm{Arth}\left(\frac{1}{\beta}\right) - \beta_0 \mathrm{Arth}\left(\frac{1}{\beta_0}\right)\right]$$
$$-\frac{m_\pi^2}{rk}\left(\mathrm{Arth}^2\left(\frac{1}{\beta}\right) - \mathrm{Arth}^2\left(\frac{1}{\beta_0}\right)\right)$$
$$+\frac{i\pi}{2rk}\left(\frac{s}{2}(\beta-\beta_0) - 2m_\pi^2\left(\mathrm{Arth}\left(\frac{1}{\beta}\right) - \mathrm{Arth}\left(\frac{1}{\beta_0}\right)\right)\right), \quad (14)$$

where $s = (r-k)^2$, $\beta = \sqrt{1-4m_\pi^2/s}$, $\beta_0 = \sqrt{1-4m_\pi^2/m_K^2}$, g is the interaction constant of $\rho\pi\pi$. We take the amplitude of the $\rho \to \pi\pi$ decay as the interaction constant g and use the experimental result for it.

We note that since $F(s)$ is complex, the phase of the loop contribution is not equal to the pion P-wave scattering phase. The photon energy dependence of the b phase ($arg(b) = \delta_b$) is shown in Figure 3.

4 The spectrum departure from pure bremsstrahlung

Now we can use the results on the $K_S \to \pi^+\pi^-\gamma$ decay probability, obtained in the previous Section, to estimate the departure of the photon spectrum from the pure bremsstrahlung. It is plausible that the contribution of the heavy particle loops to the direct emission amplitude is small compared to that of the pion loop, and we can neglect the contribution of the heavy particles loops. Also we neglect the CP-violating magnetic direct emission amplitude. The calculations are carried out in the K_S-rest frame ($r = (m_K, 0, 0, 0)$, $k = (\omega, \omega, 0, 0)$), where

$$s = m_K^2 - 2m_K\omega, \quad \beta = \sqrt{1 - \frac{4m_\pi^2}{m_K^2 - 2m_K\omega}},$$
$$rk = m_K\omega.$$

So for the double differential decay width with unpolarized photon we obtain

$$\frac{d\Gamma(K_S\to\pi^+\pi^-\gamma)}{d\omega d\cos\theta} = \frac{2\alpha}{\pi}\frac{\beta^3}{\beta_0}\left(1-\frac{2\omega}{m_K}\right)\sin^2\theta\Gamma(K_S\to\pi^+\pi^-)$$
$$\times\left[\frac{1}{\omega(1-\beta^2\cos^2\theta)^2}+\frac{m_K^4|b|^2\omega^3}{16|A|^2}+\frac{Re(be^{-i\delta_0^0})\omega m_K^2}{2|A|(1-\beta^2\cos^2\theta)}\right]. \quad (15)$$

Here θ is an angle between photon and π^+ in the dipion rest frame. Integrating Eq. (15) over $\cos\theta$ between the limits $-1\le\cos\theta\le 1$ we obtain the following result for the differential decay width:

$$\frac{d\Gamma(K_S\to\pi^+\pi^-\gamma)}{d\omega} = \frac{2\alpha}{\pi}\frac{\beta^3}{\beta_0}\left(1-\frac{2\omega}{m_K}\right)\Gamma(K_S\to\pi^+\pi^-)$$
$$\times\left\{\frac{1}{\omega}\left[\frac{1+\beta^2}{2\beta^3}\ln\frac{1+\beta}{1-\beta}-\frac{1}{\beta^2}\right]+\frac{m_K^4\omega^3}{12}\frac{|b|^2}{|A|^2}\right.$$
$$\left.+\frac{\mathrm{Re}(be^{-i\delta_0^0})\omega m_K^2}{2|A|}\left[\frac{2}{\beta^2}-\frac{1-\beta^2}{\beta^3}\ln\frac{1+\beta}{1-\beta}\right]\right\}. \quad (16)$$

The second and the third terms in the brackets govern the departure of photon spectrum from the pure bremsstrahlung. We characterize the departure of spectrum by the ratio

$$R = \frac{\left.\frac{d\Gamma}{d\omega}\right|_{interf}}{\left.\frac{d\Gamma}{d\omega}\right|_{IB}}. \quad (17)$$

The ratio increases with the increase of ω, and varies from 0.1% at $\omega = 50$Mev to 1% at $\omega = 160$Mev (see Fig. 4).

5 Comparison with ChPT

Let us compare the results on the interference branching ratio in the K_S decay obtained in Refs. [5], [15] with the results presented above. In the framework of the ChPT the electric direct emission amplitude is a sum of the loop contribution and the counterterm contribution. The counterterms are needed in the ChPT to reabsorb divergencies arising from loops at each order in momenta, because the ChPT is a nonrenormalizable theory.

In general the loop contribution and the counterterm contribution are separately scale-dependent. However, in this case the counterterm contribution is scale independent and the loop contribution is finite, as it was shown in Refs. [5], [15]. The similar result for the loop contribution is obtained in the present paper (see equation (13)).

Counterterm contribution doesn't depend on the photon energy contrary to the loop contribution, as it is shown in Refs. [5], [15]

$$E_{ct} = \frac{eG_8 m_K^3}{4\pi^2 F_\pi} N_{E_1}, \tag{18}$$

where G_8 is the interaction constant of the $|\Delta S| = 1$ non-leptonic weak Lagrangian in the framework of the ChPT. The index 8 is due to the fact that the Lagrangian transforms under $SU(3)_L \times SU(3)_R$ as an $(8_L, 1_R)$ or $(27_L, 1_R)$. Only the octet part of the Lagrangian was taken into account in Ref. [15]. The value of G_8 was determined from the experimental data on the $K_S \to \pi^+\pi^-$ decay probability: $A(K_S \to \pi^+\pi^-) = 2G_8 F_\pi (m_K^2 - m_\pi^2)$, $|G_8| = 9 \times 10^{-6}$ Gev^{-2} . F_π is the constant of the pion leptonic decay: $F_\pi = 93.3$ Mev, N_{E_1} is a sum of the counterterm constants and should be fixed from the experimental results.

In Refs. [5], [15] the "interference branching ratio" was calculated for different values of $N_{E_1} = 1.15 k_f$, where $k_f = 0, \ \pm 0.5, \ \pm 1$. The results for $|k_f| = 0.5, \ 1$ show that the contribution of the counterterms is comparable with the contribution of the pion loops. However, we compare the results of the present paper with the result of Refs. [5], [15] with $k_f = 0$, due to the fact that the counterterm contribution corresponds to the heavy particle loop contribution in our model, which we neglected because of its smallness.

The pion loop contribution according to Refs. [5], [15] is equal to

$$E_{loop} = -\frac{eG_8 m_K (m_K^2 - m_\pi^2)}{8\pi^2 F_\pi \omega^2} \left\{ s \left[\beta \ln\left(\frac{1+\beta}{\beta-1}\right) - \beta_0 \ln\left(\frac{1+\beta_0}{\beta_0-1}\right) \right] \right.$$
$$\left. + m_K \omega + m_\pi^2 \left[\ln^2\left(\frac{1+\beta_0}{\beta_0-1}\right) - \ln^2\left(\frac{1+\beta}{\beta-1}\right) \right] \right\}. \tag{19}$$

We notice that the expression (19) is proportional to $(m_K^2 - m_\pi^2)$, i. e. to the weak vertex $K\pi\pi$ with pions on-shell in the framework of the ChPT. So the results obtained in the ChPT confirm the assumption we made while considering diagrams of Fig. 2. We took $A(K_S \to \pi^+\pi^-)$ as the interaction constant and considered pions on-shell.

In order to compare the results of the present paper on the "interference branching ratio" with the results obtained in the framework of the ChPT we present the inner bremsstrahlung and interference contributions to the branching ratios of the $K_S \to \pi^+\pi^-\gamma$ decay, for different values of the ω cut, along with the results of Refs. [5], [15] with $k_f = 0$ in Table 1. The result of the present paper for the interference contribution is 20% larger than that of Refs. [5], [15]. It is due to the fact that in Refs. [5], [15] ChPT leading order of expansion in powers of momenta was used for $\pi\pi$ scattering amplitude. The phase of the electric direct emission amplitude was taken to be $\delta_1^1(m_K)$, the phase of $\pi\pi$ scattering in P-wave at energy $\sqrt{s} =_K$. Instead we used the simple realistic model of $\pi\pi$ scattering via ρ-meson taking into account the energy dependence of the $\delta_1^1(s)$ phase.

6 Pion-pion scattering

In order to confirm our assumption that the simple realistic model for describing the $\pi\pi$ interaction in the P-wave is more appropriate than the standard ChPT approach, let us compare the results of different models for $\pi\pi$ scattering with experimental data. In framework of the ChPT the amplitude of $\pi\pi$ scattering to one loop takes the form [35]

$$A(s,t,u) = \frac{s - M^2}{F_\pi^2} + B(s,t,u) + C(s,t,u) + O(p^6), \tag{20}$$

where

$$\begin{aligned} B(s,t,u) &= (6F_\pi^4)^{-1}\left\{3(s^2 - M^4)K(s)\right. \\ &+[t(t-u) - 2M^2t + 4M^2u - 2M^4]K(t) \\ &\left.+[u(u-t) - 2M^2u + 4M^2t - 2M^4]K(u)\right\}, \end{aligned} \tag{21}$$

$$\begin{aligned} C(s,t,u) &= (96\pi^2F_\pi^4)^{-1}\left\{2\left(l_1 - \frac{4}{3}\right)(s - 2M^2)^2\right) \\ &+\left(l_2 - \frac{5}{6}\right)[s^2 + (t-u)^2] - 12M^2s + 15M^4\right\}, \end{aligned} \tag{22}$$

and

$$K(q^2) = \frac{1}{16\pi^2}\left(\sigma \ln \frac{\sigma - 1}{\sigma + 1} + 2\right),$$

$$\sigma = \left(1 - \frac{4M^2}{q^2}\right)^{1/2}.$$

This representation involves four constants: F_π, defined above, M, pion mass, $l_1 = 0.4 \pm 0.3$ and $l_2 = 1.2 \pm 0.4$, which are extracted from $\pi\pi$ data [36] and K_{e4} decay [37].

The two-loop representation of the scattering amplitude yields the first three terms in the chiral expansion of the partial waves [38]:

$$t_l^I(s) = t_l^I(s)_2 + t_l^I(s)_4 + t_l^I(s)_6 + O(p^8). \tag{23}$$

This representation involves 12 constants. The leading order contains F_π and M, the next-to-leading order – $l_1,\ l_2,\ l_3,\ l_4$. The contribution of the last two constants were not included to the scattering amplitude of Ref. [35], though they appear in the next-to-leading order. This fact is due to the smallness of the contributions proportional to these constants. The next-to-next-to-leading order generates six coupling constants $r_1,\ ...,\ r_6$.

The two different categories of these constants should be distinguished. First, the terms that survive in the chiral limit ($l_1,\ l_2,\ r_5,\ r_6$). They can be determined from the experimental data, as it was mentioned above for l_1 and l_2. The constants $r_5(M_\rho) = 3.8 \pm 1.0$, $r_6(M_\rho) = 1.0 \pm 0.1$ were calculated using the experimental data on $\pi\pi$ scattering and Roy equations in Ref. [41]. Second, symmetry breaking terms. The corresponding vertices are proportional to a power of the quark mass and involve the constants $l_3,\ l_4,\ r_1,\ r_2\ r_3,\ r_4$, which may be determined by using other than $\pi\pi$ scattering experimental information. The constant $l_4 = 4.4 \pm 0.2$ can be fixed by using experimental data on the pion scalar form factor. The contributions of l_3 to the scattering amplitude are very small and can be neglected, as it was shown in Ref. [41]. For $r_1,\ ...,\ r_4$ the theoretical estimates were used in Ref. [41].

The ρ-meson contribution is taken into account in the framework of the ChPT by considering the pole diagrams of $\pi\pi$-scattering via virtual ρ-meson. This procedure gives rise to an additional term in the expression for the amplitude of $\pi\pi$ scattering, as it was shown in Refs. [35], [39].

In order to compare the amplitude with the experimental data on P-wave $\pi\pi$ scattering [40] we should expand the combination with definite isospin in the s-channel

$$T^1(s,t) = A(t,u,s) - A(u,s,t) \tag{24}$$

into partial waves with different angular momenta

$$T^1(s,t) = 32\pi \Sigma_{l=0}^{\infty} (2l+1) P_l(cos\theta) t_l^1(s),$$

where θ is the scattering angle in the center-of-mass system of the initial pions. The partial amplitude is

$$t_1^1(s) = \frac{1}{64\pi} \int_{-1}^{1} T^1 P_1(\cos\theta) d\cos\theta. \tag{25}$$

In Fig. 5 we present the behavior of the phase δ_1^1 calculated in the framework of the ChPT in one loop approximation (with and without ρ) and the simple realistic model along with experimental data [40]. We also present the behavior of the phase δ_1^1 calculated in the framework of the ChPT with ρ in two-loop approximation in Ref. [41]. According to the standard approach used when considering phases of $\pi\pi$ scattering we utilize the elastic unitarity to determine δ_1^1 in the framework of the ChPT with and without ρ. Namely

$$Im(t_1^1) = \frac{2q}{\sqrt{s}} \left(Re(t_1^1)\right)^2,$$

where $q^2 = s/4 - m_\pi^2$. Then the phase takes the following form

$$\delta_1^1 = \arctan\left(\frac{2q}{\sqrt{s}} Re(t_1^1)\right). \tag{26}$$

As it is seen from Fig. 5 the result obtained in the framework of the ChPT without ρ is in strict disagreement with the experimental data. It is due to the fact that the ρ contribution was not taken into account. In fact the use of equation (26) leads to the imaginary part of bubble diagrams, which we considered above when calculating the pion loop contributions to the ρ propagator. Due to this fact the behavior of the phases calculated in the framework of the ChPT with ρ and in the framework of the simple realistic model should be similar, as it is shown in Fig. 5. However, the use of the unitarity to determine δ_1^1 in the framework of the ChPT with ρ in one loop as well as in two loop approximation leads to the imaginary part of the one-loop diagrams, which describe the "resonance" structure of the ρ-meson propagator. Instead, we considered the sum of the iterated bubble diagrams to calculate the pion loop contributions to the ρ propagator, as it is shown in Section 3. This fact leads to the difference between the result for the phase obtained in the framework of the ChPT with ρ contribution taken into account and the result of the present paper, as it can be seen in Fig. 6, which presents a magnified part of Fig. 5. The two-loop approximation gives the result for δ_1^1 that is in better agreement with the result of the present paper. It is worth mentioning that the ChPT without ρ at low energies gives values of the phase 15% lower than the result of the present paper as it can be seen from Fig. 6. This fact results in larger values of the "interference branching ratio" in the K_S decay in the framework of the simple realistic model.

The theoretical curves shown in Figs. 5, 6 are determined with some uncertainty due to the fact that we use the values obtained from the experimental data for the interaction constants and the masses. However, the experimental errors of the interaction constants are very small, i. e. $F_\pi = 93.3 \pm 0.3$, $g = 6.08 \pm 0.03$, resulting in the uncertainty near 0.5%. The experimental results for the amplitude and the phase of the $\pi\pi$ scattering have the uncertainty near 15%, therefore we do not show the errors of the theoretical values in Figs. 5, 6.

7 Analysis in terms of Stokes vectors

For further calculations we will need the magnetic direct emission coupling c. It can be estimated using the experimental data on the direct emission contribution to the $K_L \to \pi^+\pi^-\gamma$ decay [12], [13]. The corresponding double differential decay width for unpolarized photon is

$$\frac{d\Gamma(K_L \to \pi^+\pi^-\gamma)}{d\omega d\cos\theta} = \frac{2\alpha}{\pi}\frac{\beta^3}{\beta_0}\left(1 - \frac{2\omega}{m_K}\right)\frac{\Gamma(K_L \to \pi^+\pi^-)c^2 m_K^4}{16|A|^2|\eta_{+-}|^2}\sin^2\theta. \quad (27)$$

Identifying this expression with the direct emission rate given in [12], [13] we obtain $|c| = 0.76|A|$.

It is worth mentioning that there is no interference between the amplitude terms with opposite CP-parity, if photon polarization is not observed. However, the interference is nonzero when the polarization is measured, as it was repeatedly emphasized in Refs. [2], [3], [4], [20]. Therefore, any CP-violation involving interference of electric and magnetic amplitudes is encoded in the polarization state of the photon.

To determine the nature of this interference we write the $K_{L,S} \to \pi^+\pi^-\gamma$ decay amplitude more generally as

$$A(K_{S,L} \to \pi^+\pi^-\gamma) = ET_E + MT_M, \quad (28)$$

where for the K_S decay E and M have the form

$$E = \frac{eAe^{i\delta_0^0}}{(pk)(qk)} + eb \quad \text{and} \quad M = ie\eta_{+-}c, \tag{29}$$

and in the case of the K_L decay we have

$$E = \eta_{+-}\left[\frac{eAe^{i\delta_0^0}}{(pk)(qk)} + eb\right] \quad \text{and} \quad M = iec. \tag{30}$$

The photon polarization can be defined in the terms of the density matrix [42]

$$\rho = \begin{pmatrix} |E|^2 & E^*M \\ EM^* & |M|^2 \end{pmatrix} = \frac{1}{2}(|E|^2 + |M|^2)\left[1 + \mathbf{S}\tau\right], \tag{31}$$

where $\tau = (\tau_{\mathbf{1}}, \tau_{\mathbf{2}}, \tau_{\mathbf{3}})$ denotes Pauli matrices, $\mathbf{S}$ is the Stokes vector of the photon with components

$$S_1 = 2Re(E^*M)/(|E|^2 + |M^2|), \tag{32}$$

$$S_2 = 2Im(E^*M)/(|E|^2 + |M^2|), \tag{33}$$

$$S_3 = (|E|^2 - |M|^2)/(|E|^2 + |M^2|). \tag{34}$$

The effects of CP-violation reside in components S_1 and S_2, while the component S_3 measures the relative strength of the amplitude terms with opposite CP-parity. The component S_2 is the net circular polarization of the photon; it is proportional to the difference of $|E - iM|^2$ and $|E + iM|^2$, which are the probabilities for left-handed and right-handed polarization. The S_1 component appears as a coefficient of an interference term in case of linear polarization. If one chooses the polarization angle ϕ as the angle between $\mathbf{e}$, the polarization vector, and the unit vector $\mathbf{n}_\pi$ normal to the decay plane ($\mathbf{k} = (0, 0, \omega)$, $\mathbf{n}_\pi = (1, 0, 0)$, $\mathbf{p} = (0, p\sin\theta, p\cos\theta)$), then the decay amplitude will be proportional to the following expression

$$|A(K_{S,L} \to \pi^+\pi^-\gamma)|^2 \sim 1 - (S_3\cos 2\phi + S_1 \sin 2\phi). \tag{35}$$

It is obvious from (35) that the measurement of linear polarization in principle allows to extract the terms with opposite CP-parity (E and M).

$$|A(K_{S,L} \to \pi^+\pi^-\gamma)|^2 \sim |E|^2, \quad \phi = \pi/2 + \pi n,$$

$$|A(K_{S,L} \to \pi^+\pi^-\gamma)|^2 \sim |M|^2, \quad \phi = \pi n,$$

where n is integer.

In order to obtain a quantitative estimate of the CP-violation effects we study the photon energy dependence of the Stokes vector components. In figures 7 and 8 we show the photon energy dependence of the S_1 and S_2 components in the $K_{L,S} \to \pi^+\pi^-\gamma$ decay respectively. In figure 9 we demonstrate the S_3 component photon energy dependence in $K_{L,S} \to \pi^+\pi^-\gamma$ decays to obtain the estimate of the relative strength of the CP-violation effects in the decays under consideration. We see that the obtained results on the $K_L \to \pi^+\pi^-\gamma$ decay coincide with the results of Refs. [21], [22], [23]. Taking into account the physical meaning of the S_3 we conclude that the CP-violation effects in $K_S \to \pi^+\pi^-\gamma$ decay are substantially small. The reason is that the bremsstrahlung contribution shades the magnetic direct emission contribution even for the high photon energies.

It was suggested in Refs. [24], [25], [26] to use in place of $\mathbf{e}$ the vector $\mathbf{n_l}$ normal to the e^+e^- plane in the decay $K_L \to \pi^+\pi^-e^+e^-$. This can be achieved by replacing e_μ in the radiative amplitude (5, 6) by $e/k^2\overline{u}(k_-)\gamma_\mu v(k_+)$. This motivates the study of the distribution $d\Gamma/d\phi$ in the decays $K_{S,L} \to \pi^+\pi^-e^+e^-$, where ϕ is an angle between $\pi^+\pi^-$ and e^+e^- planes.

The distribution $d\Gamma/d\phi$ can be written in general form

$$\frac{d\Gamma}{d\phi} = \Gamma_1 \cos^2\phi + \Gamma_2 \sin^2\phi + \Gamma_3 \sin\phi\cos\phi. \qquad (36)$$

The last term changes sign under the transformation $\phi \to \pi - \phi$ and produces an asymmetry $A^{L,S}_{\pi\pi,\ ee}$ in the distribution of the angle ϕ between the vectors normal to the $\pi^+\pi^-$ and e^+e^- planes. The asymmetry is defined by the following expression

$$A^{L,S}_{\pi\pi,\ ee} = \frac{\left(\int_0^{\pi/2} - \int_{\pi/2}^{\pi} + \int_{\pi}^{3\pi/2} - \int_{3\pi/2}^{2\pi}\right) \frac{d\Gamma}{d\phi} d\phi}{\left(\int_0^{\pi/2} + \int_{\pi/2}^{\pi} + \int_{\pi}^{3\pi/2} + \int_{3\pi/2}^{2\pi}\right) \frac{d\Gamma}{d\phi} d\phi}. \qquad (37)$$

In the case of the K_L decay the contributions of the amplitude terms with different CP-parity to the decay probability are of comparable magnitude. This fact should result in the significant value of the asymmetry. This was demonstrated in Refs. [21], [22], [23], [24], [25], [26], [28], [29]. The $K_L \to \pi^+\pi^-e^+e^-$ decay probability and the CP-violating asymmetry from the $\pi^+\pi^-$ and e^+e^- planes correlation calculated in Refs. [21], [22], [23], [24], [25], [26], [28], [29]

$$Br(K_L \to \pi^+\pi^-e^+e^-) = 3.1 \times 10^{-7}, \quad |A^L_{\pi\pi,\ ee}| = 14\%, \qquad (38)$$

are in accordance with the recent experimental data published by KTeV [27] ($Br^{exp}(K_L \to \pi^+\pi^-e^+e^-) = (3.32 \pm 0.14 \pm 0.28) \times 10^{-7}$, $|A^L_{\pi\pi,\ ee}|^{exp} = (13.6 \pm 2.5 \pm 1.2)\%$).

The CP-violating asymmetry $A^{L,S}_{\pi\pi,\ ee}$ arises from the interference of the amplitude terms with different CP-parity. $A^{L,S}_{\pi\pi,\ ee}$ is proportional to the expression

$$\int d\cos\theta_\pi ds dk^2 \sin^2\theta_\pi \beta^3 X^2 \left(\frac{s}{k^2}\right) Re[ME^*], \tag{39}$$

where

$$X = \left[\left(\frac{m_K^2 - s - k^2}{2}\right) - sk^2\right]^{1/2},$$

θ_π is the angle between π^+ three-momentum and the K_L three-momentum in the $\pi^+\pi^-$ rest frame, $s = (p+q)^2$, $k^2 = (k_+ + k_-)^2$. The resulting CP-violating asymmetry in the $K_L \to \pi^+\pi^-e^+e^-$ decay is

$$|A^L_{\pi\pi,\ ee}| = (13.4 \pm 0.9)\%. \tag{40}$$

The value of the asymmetry is determined with some uncertainty due to the fact that we use the values obtained from the experimental data for interaction constants, masses and phase δ_0^0. The central values of the asymmetry obtained in the present paper and in Refs. [21], [22], [23], [24], [25], [26], [28], [29] coincide within the accuracy of the calculation.

According to the results of the present paper the CP-violating asymmetry in the $K_S \to \pi^+\pi^-e^+e^-$ decay is

$$|A^S_{\pi\pi,\ ee}| = (5.1 \pm 0.4) \times 10^{-5}. \tag{41}$$

This value of the asymmetry could be expected qualitatively from the analysis of the $K_S \to \pi^+\pi^-\gamma$ decay amplitude, where the CP-violating magnetic direct emission contribution is substantially small compared to the CP-conserving part of the amplitude.

8 Conclusions

In the case of the radiative K-meson decays we calculated the phases of amplitude terms using a simple realistic model of pion-pion interaction [16]. Also we calculated the pion loop contribution (E_D^{loop}) to the electric direct emission amplitude. The interference of the E_D^{loop} with the bremsstrahlung contribution is the main source of the departure of the photon spectrum from pure bremsstrahlung. To detect this effect the photon spectrum should be measured with the accuracy better than 1% for photon energies near 160 Mev and better than 0.1% for photon energies near 50 Mev.

We compared our results on the interference contribution with those of Refs. [5], [15] and found that the latter are 20% larger. This discrepancy arises from different models of $\pi\pi$ interaction and the fact that we took into account the energy dependence of the phases, while in Refs. [5], [15] the phases were taken at the energy $\sqrt{s} = m_K$. We compared with the experimental data the predictions for the phase of the $\pi\pi$ scattering in the P-wave obtained in the framework of different models. As seen from Fig. 5 the simple realistic model and the ChPT with ρ are in accordance with the experimental data. The ChPT without ρ shows strong disagreement with data. In Fig. 6 we present the behavior of the phase at low energies. The ChPT with ρ and the simple realistic model predictions differ due to the fact that actually the one-loop approximation was used to describe the ρ-meson contribution to the phase obtained in the framework of the ChPT with ρ, while we summed the iterated bubble diagrams to obtain the phase. Regarding the result of the ChPT without ρ we show in Fig. 6 that it is smaller than that of the present paper. This fact is one of the sources of the 20% discrepancy mentioned above.

Regarding the dependence of the K_S decay probability on photon polarization we found that the measurement of the linear polarization allowed in principle extraction of terms with opposite CP-parity.

We also studied the $K_{S,L} \to \pi^+\pi^-e^+e^-$ decays. The CP-violating asymmetry in the case of the K_L decay was found to be $(13.4\pm0.9)\%$. The central values of the asymmetry obtained in the present paper and in Refs. [21], [22], [23], [24], [25], [26], [28], [29] coincide within the accuracy of the calculation. We found that the CP-violating asymmetry in the case of the K_S decay is substantially smaller than in the K_L case being equal to $(5.1 \pm 0.4) \times 10^{-5}$, as it could be expected from the analysis of the $K_S \to \pi^+\pi^-\gamma$ decay.

Acknowledgments

The author appreciates L. B. Okun's scientific supervision and formulation of this problem. The author would also like to thank R. B. Nevzorov for fruitful discussions and G. D'Ambrosio, M. I. Vysotsky and E. P. Shabalin for valuable remarks. This work was supported by the RFBR (grant N 96-02-18010).

References

[1] H. Chew, Nuovo Cimento **26** 1109 (1962).

[2] G. Costa, P. K. Kabir, Nuovo Cimento **A51** 564 (1967).

[3] L. M. Sehgal, L. Wolfenstein, Phys. Rev. **162** 1362 (1967).

[4] A. D. Dolgov, L. A. Ponomarev, Yad Fiz. **4** 262 (1967).

[5] G. D'Ambrosio and G. Isidori, Z. Phys. **C65** 649 (1995).

[6] G. D'Ambrosio, M. Miragliuolo, P. Santorelli, Radiative non-leptonic decays: The DAΦNE Physics handbook: 265-313.

[7] A. Alavi-Harati *et al.*, Phys. Rev. Lett. **83** 22 (1999).

[8] X.-G. He, G. Valencia, Phys. Rev, **D61** 075003 (2000), hep-ph/9908298.

[9] G. Colangelo, G. Isidori, J. Portoles, Phys. Lett. **B470** 134 (1999).

[10] J. Tandean, G. Valencia, submitted to Phys. Rev. D, hep-ph/0008238.

[11] A. S. Carrol *et. al.*, Phys. Rev. Lett. **44** 529 (1980).

[12] E. J. Ramberg *et. al.*, Phys. Rev. Lett. **70** 2525 (1993).

[13] S. Kettell, To be published in the proceedings of 3rd Workshop on Physics and Detectors for DAΦNE (DAΦNE 99), Frascati, Italy, 16-19 Nov 1999. In 'Frascati 1999, Physics and detectors for DAΦNE' 449-459. BNL-67140, Feb 2000. hep-ex/0002009.

[14] H. Taureg *et. al.*, Phys. Lett. B**37** 92 (1976).

[15] G. D'Ambrosio, M. Miragliuolo, F. Sannino, Z. Phys. **C59** 451 (1993).

[16] B. W. Lee, M. T. Vaughn, Phys. Rev. Lett. **4** 578 (1960).

[17] K. Watson, Phys. Rev. **95** (1954).

[18] E. Fermi, Suppl. Nuovo Cimento **2** 58 (1955).

[19] S. Fubini, Y. Nambu, V. Wataghin, Phys. Rev. **111** (1958).

[20] M. McGuigan, A. I. Sanda, Phys. Rev. D**36** 1413 (1987).

[21] L. M. Sehgal, J. van Leusen, Phys. Rev. Lett. **83** 4933 (1999).

[22] L. M. Sehgal, Talk given at 1999 Chicago Conference on Kaon Physics (K 99), Chicago, IL, 21-26 Jun, 1999. hep-ph/9908338.

[23] L. M. Sehgal, J. van Leusen, hep-ph/0006336.

[24] L. M. Sehgal, M. Wanninger Phys. Rev. D**46** 1035 (1992).

[25] P. Heiliger, L. M. Sehgal, Phys. Rev. D**48** 4146 (1993).

[26] J. K. Elwood, M. B. Wise, Phys. Rev. D**52** 5095 (1995).

[27] A. Alavi-Harati *et al.*, Phys. Rev. Lett. **84** 408 (2000), hep-ex/9908020.

[28] J. K. Elwood, M. B. Wise, M. J. Savage, J. W. Walden, Phys. Rev. D**53** 4078 (1996).

[29] M. J. Savage, Talk presented at Kaon99, University of Chicago (1990).

[30] H. Pichl, Eur. Phys. J. C**20** 371 (2001).

[31] G. Ecker, H. Pichl, Phys. Lett. B**507** 193 (2001)

[32] F. E. Low, Phys. Rev. **110** 974 (1958).

[33] L. B. Okun, Leptons and Quarks (Amsterdam, "North-Holland", 1982).

[34] R. B. Nevzorov, private communication.

[35] J. Gasser, H. Leutwyler, Ann. of Phys. **158** 142 (1984).

[36] C. Riggenbach, J. Gasser, J. F. Donoghue and B. R. Holstein, Phys. Rev. D**43** 127 (1991).

[37] J. Bijnens, Nucl. Phys. B**337** 635 (1990).

[38] J. Bijnens, G. Colangelo, G. Ecker, J. Gasser and M. E. Sanino, Phys. Lett. B**374** 210 (1996).

[39] V. Bernard, N. Kaiser, U.-G. Meissner, Nucl. Phys. B**364** 283 (1991).

[40] E. A. Alekseeva *et. al.*, JETP **82** 1007 (1982).

[41] G. Colangelo, J. Gasser, H. Leutwyler, Nucl. Phys. B**603** 125 (2001).

[42] A. I. Akhieser, V. B. Berestetskii, Quantum Electrodynamics (New York, "Interscience", 1981).

Figure captions

Fig. 1. The interaction of pions in the case of the internal bremsstrahlung.

Fig. 2. The emission of the photon from the loops of virtual particles (D).

Fig. 3. The photon energy dependence of the pion loop phase δ_b ($b = |b|e^{i\delta_b}$), in degrees.

Fig. 4. The photon energy dependence of the ratio R defined in (17)

Fig. 5. The energy dependence of the $\pi\pi$ scattering phase $\delta_1^1(s)$ in the P-wave, in degrees. The experimental data are represented by stars. The ChPT without ρ-meson contribution – by diamonds. The ChPT with the ρ-meson contribution taken into account, according to the equations (3.11 – 3.14) of Ref. [39] – by filled triangles. The ChPT with ρ in two-loop approximation, according to the result of Ref. [41] – by empty triangles. The results of the present paper – by boxes.

Fig. 6. The energy dependence of the $\pi\pi$ scattering phase $\delta_1^1(s)$ in the P-wave at low energies, in degrees. Note that this figure is a magnified part of the previous figure. The notations are the same as in Fig. 5.

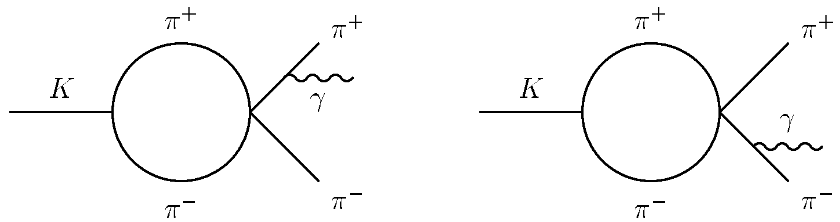
Fig. 7. Stokes parameters S_1 (upper curve) and S_2 (lower curve) for the $K_L \to \pi^+\pi^-\gamma$ decay.

Fig. 8. Stokes parameters S_1 (upper curve) and S_2 (lower curve) for the $K_S \to \pi^+\pi^-\gamma$ decay.

Fig. 9. Stokes parameter S_3 for the $K_S \to \pi^+\pi^-\gamma$ decay (straight line $S_3 = 1$) and for the $K_L \to \pi^+\pi^-\gamma$ decay (lower curve).

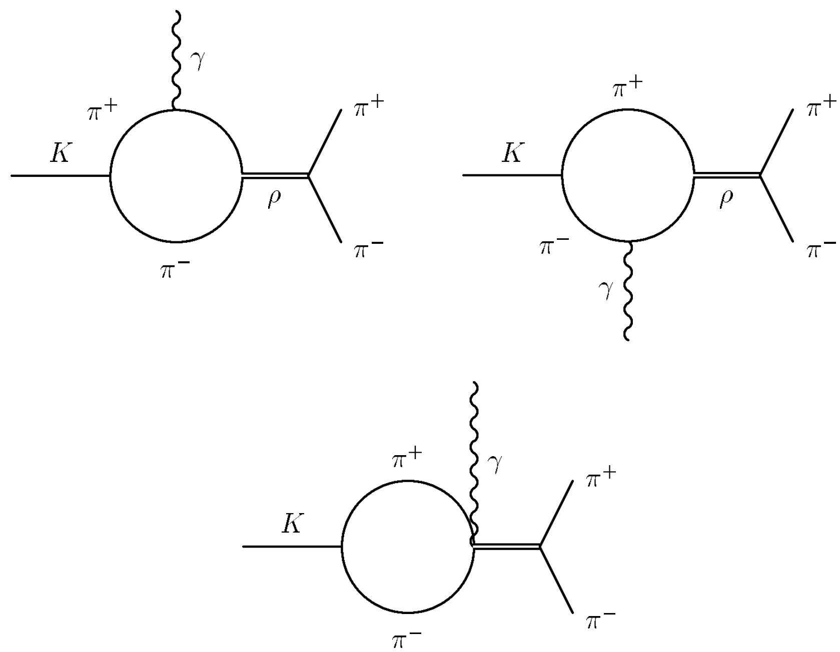
cut in ω	$\omega > 20$ Mev	$\omega > 50$ Mev	$\omega > 100$ Mev
10^3B(the present paper)	4.81	1.78	0.44
10^3B [15]	4.80	1.73	0.31
10^6 Interf	-6.5	-5.1	-2.0
10^6Interf($N_{E_1} = 0$) [15]	-5.5	-4.5	-1.9

Table 1
Internal Bremsstrahlung and interference contributions to the branching ratios of the $K_S \to \pi^+\pi^-\gamma$ decay for different ω cuts along with the results of Ref. [15]



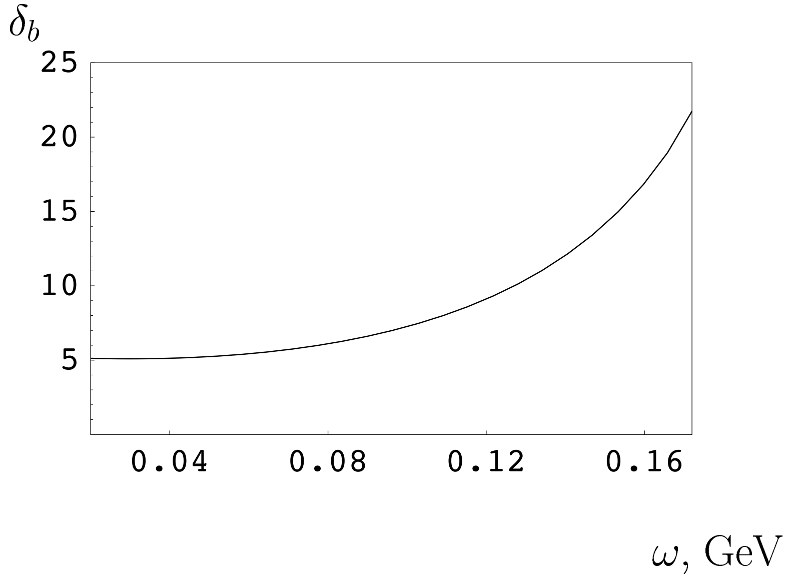
π^+
K
π^+
γ
π^-
π^-
π^+
K
π^+
γ
π^-
π^-

Fig. 1.



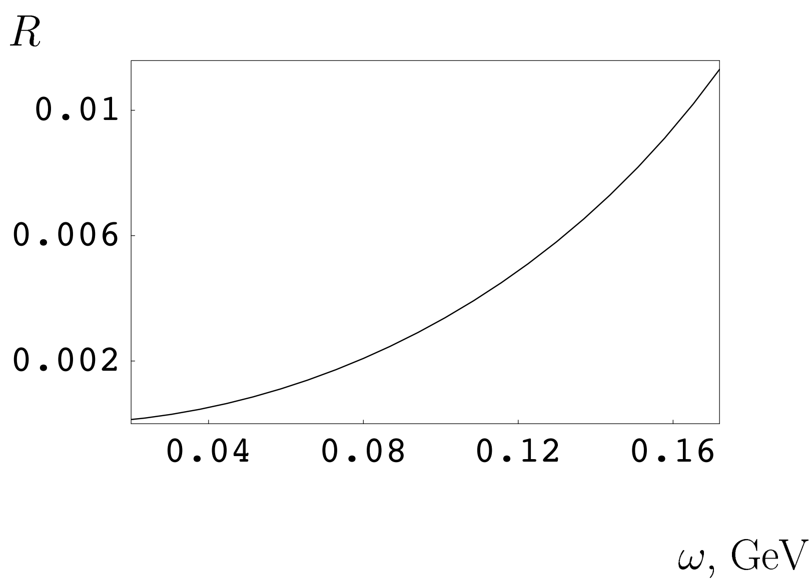
γ
π^+
K
π^+
ρ
π^-
π^-
π^+
K
π^+
ρ
π^-
π^-
γ
π^+
γ
K
π^+
π^-
π^-

Fig. 2.



δ_b
25
20
15
10
5
0.04
0.08
0.12
0.16
ω, GeV

Fig. 3.



R
0.01
0.006
0.002
0.04
0.08
0.12
0.16
ω, GeV

Fig. 4.

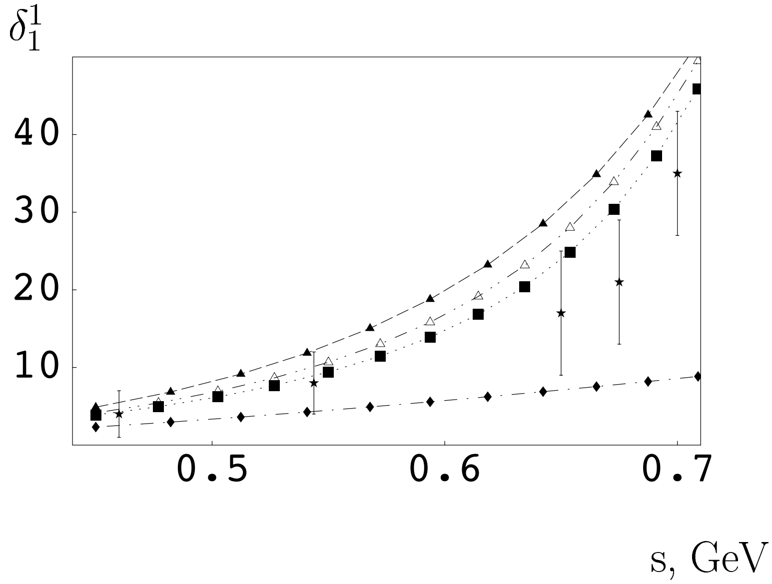


Fig. 5.

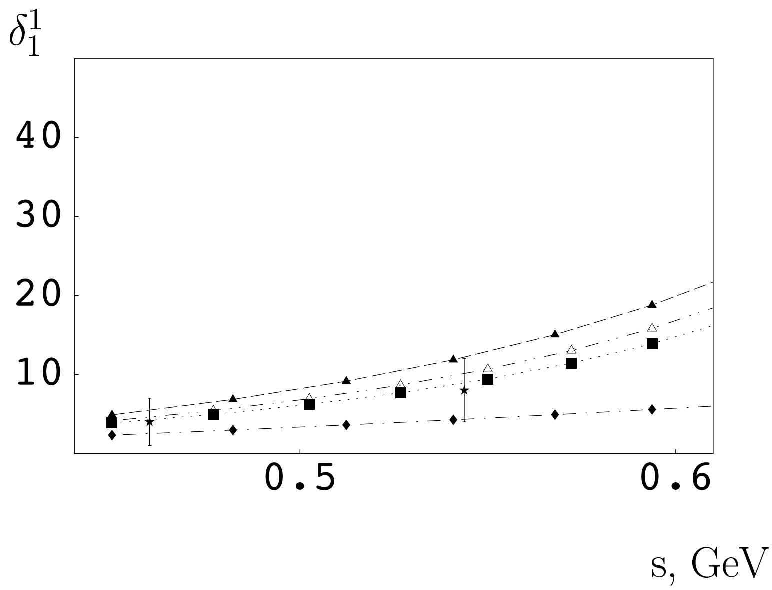
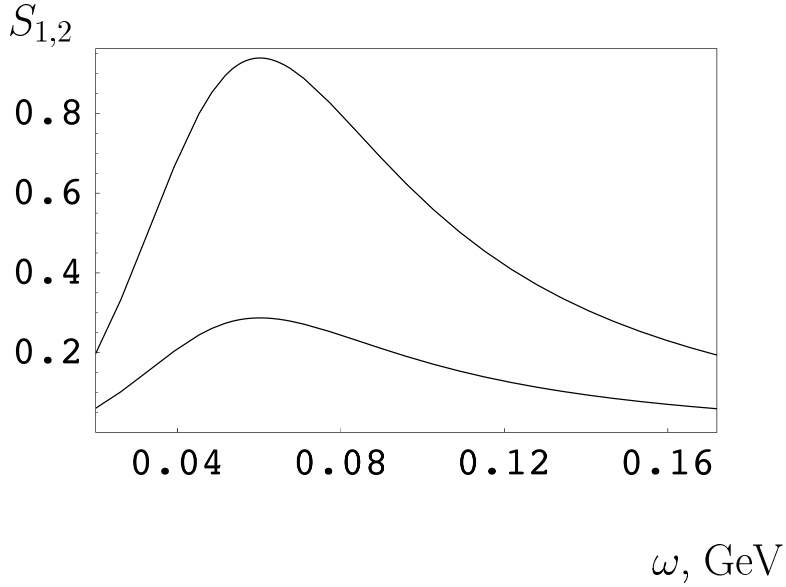
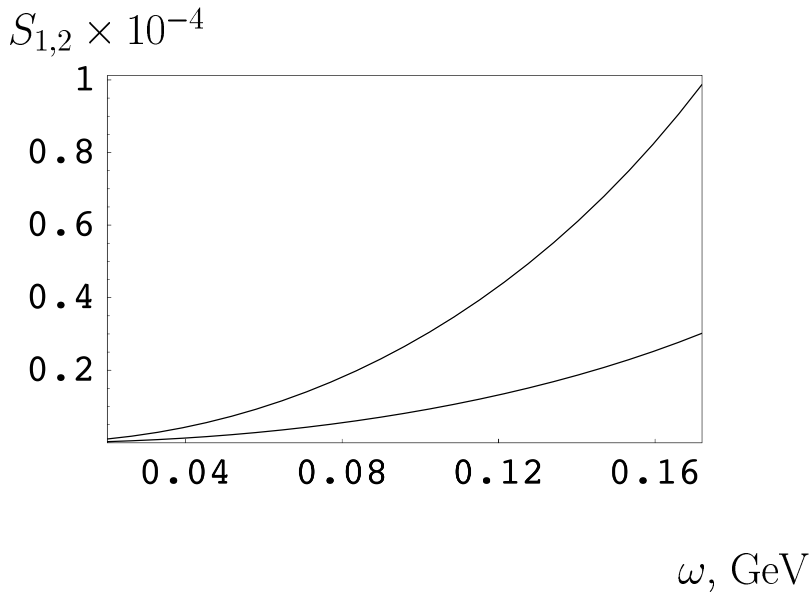


Fig. 6.



$S_{1,2}$
0.8
0.6
0.4
0.2
0.04
0.08
0.12
0.16
ω, GeV

Fig. 7.



$S_{1,2} \times 10^{-4}$
1
0.8
0.6
0.4
0.2
0.04
0.08
0.12
0.16
ω, GeV

Fig. 8.

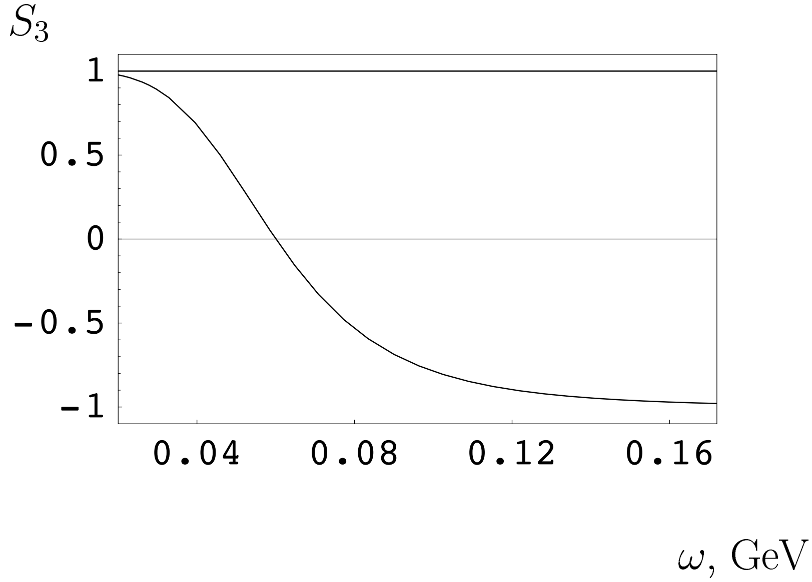


Fig. 9.