

Lawrence Berkeley National Laboratory

Recent Work

Title

EQUIVALENT SPHERE ILLUMINATION AND VISIBILITY LEVELS

Permalink

<https://escholarship.org/uc/item/9xk113hg>

Author

Richardson, R.W.

Publication Date

1982-03-01



Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

ENERGY & ENVIRONMENT DIVISION

RECEIVED
LAWRENCE
BERKELEY LABORATORY
APR 16 1982
LIBRARY AND
DOCUMENTS SECTION

Submitted to Lighting Research and Technology

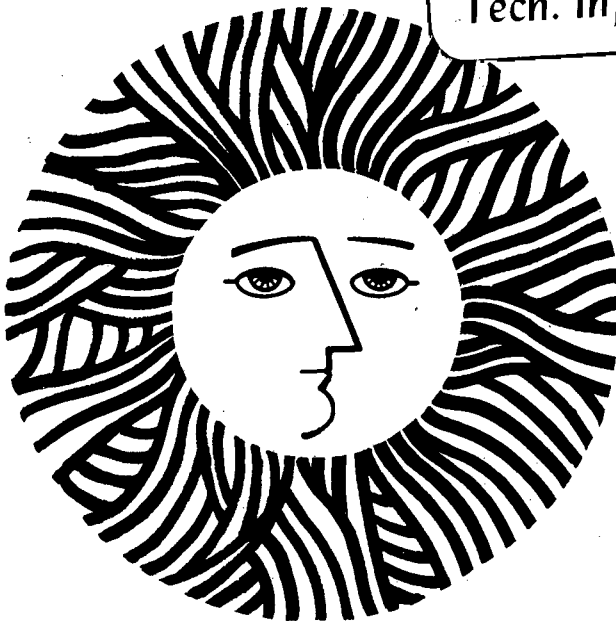
EQUIVALENT SPHERE ILLUMINATION AND VISIBILITY
LEVELS

R.W. Richardson and S.M. Berman

March 1982

TWO-WEEK LOAN COPY

This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 6782



LBL-9492 Rev.
c.2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Submitted to Lighting Research and Technology.

EQUIVALENT SPHERE ILLUMINATION AND VISIBILITY LEVELS

R.W. Richardson* and S.M. Berman

Energy and Environment Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

MARCH 1982

The work described in this paper was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Building Energy Research and Development, Building Equipment Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

*Permanent address: Physics Department, New York University,
4 Washington Place, New York, NY 10003.

This manuscript was printed from originals provided by the author.

ABSTRACT

The notion of ESI evolved from a series of investigations into visual processes. This report attempts to explain and criticize this notion by discussing both the "how" and the "why" of ESI. Section II concentrates on how ESI is determined; it paraphrases RQQ Report No. 4, which prescribes the determination of ESI. Section III interprets this prescription; it examines the empirical basis of ESI, its consequences and inconsistencies. This section relies upon C.I.E. reports. Section IV discusses two modifications to the method of determining ESI that are not in the RQQ Report. Section V summarizes methods used to determine ESI. Section VI considers the related notion of visibility levels and their relation to visual performance. Appendix A gives the illuminance distributions for several simple geometries.

EQUIVALENT SPHERE ILLUMINATION AND VISIBILITY LEVELS

R. W. Richardson* and S. M. Berman

Energy and Environment Division
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

1. PROLOGUE

1.1 Overview

Equivalent sphere illumination or ESI is intended to be a single figure of merit for a lighting system that takes into account both the quality and the quantity of the illumination produced by the system. This is done by relating the level of illumination in the actual environment to an equivalent level of illumination, ESI, in a reference environment. The reference environment is taken to be one which produces homogeneous, isotropic, unpolarized illuminance whose color temperature is 2856 K. Such an environment can be approximated by the interior of an integrating sphere and it is often called sphere illumination. The reference lighting environment can therefore be completely specified by the value of the intensity of the illumination. The notion of equivalence used in relating the two environments is that of equal visibility. The two environments are equivalent if they produce the same level of visibility, where visibility is defined in terms of performance of a prescribed visual task. Thus, the relative visibility produced by two lighting systems can be evaluated by comparing their ESI values. The advantage of comparing ESI's rather than visibilities is that ESI can be predetermined by calculation or it can

be measured by an instrument while visibility is a complex psychophysical quantity which is difficult to measure, much less calculate. Therefore, ESI can be used as a design tool or for the specification of standards.

The notion of ESI has evolved out of a long series of investigations into the visual process. In this report, we attempt to explain and criticize this notion by discussing both the "how" and the "why" of ESI. In Section II, we concentrate upon how ESI is determined. This section is essentially a paraphrasing of the RQQ Report No. 4¹ in which a prescription for determining ESI is given. We present an interpretation of this prescription in Section III. Here, we concentrate on the empirical basis of ESI and its consequences. This section relies very heavily upon three reports from the C.I.E.^{2,3,4} and the references cited therein. Two modifications of the method of determining ESI that are not included in the RQQ Report are discussed in Section IV. A brief review of the methods used for determining ESI is given in Section V. In Section VI, we consider the related notion of visibility levels and their relation to visual performance. The distribution of illuminance in several simple geometries is given in Appendix A. Technical terms that we use are defined in a Glossary. These terms are underlined when they are used in the text.

This review offers no new results which, in a fundamentally empirical field, can only come from new experiments. However, it is hoped that the analysis given here will aid newcomers to the field and perhaps stimulate some new thoughts in the oldtimers. It is best used as

an adjunct to the IES and CIE documents¹⁻⁴ since it does not stand alone. When used in this way, it will fill in some of the analytical steps that lie between the lines of those documents.

1.2 Introduction to the Literature

The literature in the field of lighting and visual performance is very large and, in many instances, self contradictory. In the absence of a consensus or a set of definitive sources, it is very difficult for the uninitiated to get a proper understanding of the important problems in the field. Here, we present some possible gateways into this literature as an introductory aid. We cite secondary sources such as reviews and books rather than the primary sources so as to keep our list down to a manageable size. No doubt, there are both sins of omission and sins of commission in our choice of references both in this section and in the main body of the text. We apologize to those authors we have slighted and qualify our list of references as being those we found to be helpful in preparation of this article.

The literature and the field are sharply divided into what we will call applied and basic subdivisions with very different points of view. Since the concerns of this article lie in the applied area, we will concentrate on this subfield and give only a few references to basic studies at the end of this section.

The early literature on lighting and visual performance is reviewed in a monumental study by Troland⁵ which includes about 800 references drawn from the period between 1738 and 1925. A bibliography of articles on lighting fundamentals and applications

that is limited to those published by the Illuminating Engineering Society in the period from 1920 to 1941 is given in Ref. 6. A very useful survey of the field as of 1944 has been given by Moon and Spencer.⁷ A review that contains a great deal of experimental data has been given by Crouch.⁸ The British point of view together with an extensive bibliography is presented in Ref. 9. A partial update of Ref. 6 is given in Ref. 10 which includes references through 1953. A further exposition of the British point of view together with a critique of the American I.E.S. methods has been given by Weston.¹¹ A Continental European point of view has been given by Bodmann¹² in a useful review. The C.I.E documents²⁻⁴ present a selection of more recent references. These documents have been amplified by Bodmann¹³ and Levy.¹⁴ Useful books on this subject have been written by Luckiesh and Moss,¹⁵ Moon,¹⁶ and Luckiesh.¹⁷ For an introduction to the basic side of vision research we refer the reader to the review of Ruddock¹⁸ and the books by Troland¹⁹ and Cornsweet.²⁰ These reviews plus the references contained therein should provide the interested reader with more than enough information.

We caution the reader to bring a great deal of scepticism to bear on whatever he or she chooses to read. As with all studies of humans, there are fundamental problems associated with both the experiments and their interpretation. With the experiments, one is never sure what aspects of the subject is being measured or how the observer is influencing the measurement. The Hawthorne experiments²¹ in which worker productivity increases when lighting levels were lowered as

well as when they were raised is an example of unknown extraneous variables that could dominate a measurement. A clearer example²² is the experiment in which two groups of people were asked to do arithmetic problems both with and without music playing in the background. One group was led to believe that the music would help their arithmetic while the other group was led to believe that it would be a hindrance. The results of the experiments were in accord with these beliefs and therefore the biases of the subjects were being measured and not a fundamental property of distraction. Given this kind of uncertainty in the experiments, it is no wonder that their interpretation can be the subject of considerable disagreement.

2. SYMBOL LIST

SYMBOL	MEANING	PAGE(s) INTRODUCED
A	Age of observer in years.	33
C	Contrast when treated as an independent variable.	21
C'	Contrast modified by a veiling luminance produced by glare sources.	41
C _{eq}	Equivalent Contrast.	53
C _{phy}	Physical contrast.	12
C _t	Threshold contrast.	27
CRF _{phy}	Physical Contrast rendering factor	13
CRF _{psy}	Psychophysical contrast rendering factor.	31
CS	Contrast sensitivity.	14
DGF	Disability glare factor.	38
d	Task detail size in arc minutes.	34
E _i (\vec{r}_i)	Total illumination incident at point r_i on the i^{th} interior surface of a room.	48
E _t	Total task illumination.	11
E _t (θ, ϕ)	Angular distribution of task illuminance.	49
E _{tr} (θ, ϕ)	Reflected component of E _t (θ, ϕ).	49
erf	Error function.	61
ESI	Equivalent sphere illumination.	15

SYMBOL	MEANING	PAGE(s) INTRODUCED
f_1, f_2	Visibility meter amplitudes.	21
$F(V)$	Visibility dependent factor in the factored form of CS.	24
G, H	Factors in the factored form of RV.	30
K	Unimportant constants.	29, 39, 63
$K_{ij}(\vec{r}_{ij})$	Radiation transfer factor.	49
L	Luminance.	29
L_b	Task background luminance.	11, 17
L'_b	Task background luminance in the presence of a veiling luminance.	21, 40
L_d	Task background luminance	12, 17
L'_d	Task detail luminance in the presence of a veiling luminance.	21, 40
L_e	Effective task background luminance used in the calculation of ESI.	16, 26
L_{et}	True effective task background luminance.	47
L_0	Normalization luminance for RCS functions. $L_0 = 100 \text{ cd/m}^2$.	28
$L_s(\theta, \phi)$	Distribution of luminance in the task surround.	17, 39
L_v	Veiling luminance.	21, 39
m	Visibility level multiplier	59
n	Normalization of the generalized RCS function.	34
$O(Z)$	Distribution function, $O(z) = 1/2[1 + \text{erf}(z/2)]$.	62
$p(S_t)$	Probability distribution of stimulus thresholds.	61

SYMBOL	MEANING	PAGE(s) INTRODUCED
$P(S)$	Cummulative probability distribution of stimulus thresholds.	61
P_1, P_2, P_3	Contributions of the three critical processes to RVP.	64
P_4	Contribution of the noncritical component to RTP.	69
$P_{4.1}, P_{4.2}$	Contributions to P_4 .	69
\vec{r}_i	Position on the i^{th} interior surface of a room.	48
\vec{r}_{ij}	$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$.	49
RCS	Relative contrast sensitivity.	15
RCS_{eff}	Effective relative contrast sensitivity	15
RCS_{ref}	Reference relative contrast sensitivity	15,30
RTP	Relative task performance.	68
RV	Relative visibility.	27
RVP	Relative visual performance.	64
s	Parameter in the generalized RCS function dependent upon A.	66
S	Parameter in the generalized RCS function dependent upon d, \bar{X} , and s .	34,65
$S(\vec{r}_i)$	Direct component of illumination at point r_i on the i^{th} interior surface of a room.	49
$S_t(\theta, \phi)$	Direct component of $E_t(\theta, \phi)$.	49
$S\tilde{F}$	Sensory field size in degrees.	34
t	Parameter in the generalized RCS function dependent upon A.	34,66
TAF	Transient adaptation factor.	44

SYMBOL	MEANING	PAGE(s) INTRODUCED
TP	Task performance.	68
TP _{max}	Maximum task performance.	68
V	Visibility.	20
\bar{V}	Visibility of the reference task under actual lighting conditions.	26
\bar{V}_{ref}	Visibility of the reference task under reference lighting conditions.	26
V _t	Threshold visibility, V _t = 50 percent.	24
VL	Visibility level.	52
VLM	Visibility level multiplier.	59
VP	Visual performance.	63
VP _{max}	Maximum visual performance.	63
w ₁ , w ₂ , w ₃	Weights for the three critical visual processes.	64
x _i	The elements of X.	19
X	The set of parameters needed to specify a visual task excluding L _b and C _{phy} .	19
\bar{X}	The set of parameters needed to specify the reference task under actual lighting conditions excluding L _b and C _{phy} .	25
\bar{X}'	The set of parameters needed to specify the reference task in an intermediate lighting environment, excluding L _b and C _{phy} , chosen to develop approximations for CRF _{psy} .	37
\tilde{X}	$\tilde{X} = 1/2 \tilde{SF}$	66
X ⁰	The set of parameters needed to specify a visual task excluding L _b , C _{phy} , and .	41
\bar{X}^0	The value of X ⁰ for the reference task under actual lighting condtions.	42

SYMBOL	MEANING	PAGE(s) INTRODUCED
X_{ref}	The value of X for the visibility reference task.	29
x_{ref}	The value of X for the visual task under reference lighting conditions	55
\bar{x}_{ref}	The value of X for the reference task under reference lighting conditions.	25
X_{ref}^0	The value of X^0 for the visibility reference task.	41
α	$\alpha = \log VL$.	63
$\bar{\alpha}_x$	Parameter in P_x with $x = 1,2,3,4.1,$ and 4.2.	65,67,69
β	Luminance factor.	12
$\beta_b(\theta_v, \theta, \phi)$	Bidirectional reflection coefficient for task background.	50
$\beta_d(\theta_v, \theta, \phi)$	Bidirectional reflection coefficient for task detail.	50
γ_x	Parameter in P_x with $x = 1,2,3,4.1,$ and 4.2	65,67,69
θ, ϕ	Polar angles measured from and about the observer's viewing axis.	39
θ_v	Observer's viewing angle measured from the normal to the task surface.	49
Λ	Glare parameter used in the determination of DGF.	39
Λ_{ref}	The value of Λ for the visibility reference task, $\Lambda_{ref} = 0.074$.	42
ρ_i	Reflectivity of the i^{th} interior surface of a room.	48
ϕ	See θ, ϕ .	

3. THE RQQ REPORT NO. 4 PRESCRIPTION

We present an overview of the I.E.S. method for calculating Equivalent Sphere Illumination, ESI, as put forth in RQQ Report No. 4.¹ This report gives a prescription for calculating the numerical value of ESI for a given environment but does not provide any interpretation of the method. We present the method in this section and an interpretation in the next one.

We consider the illumination of a specific visual task located at a particular point in a room whose lighting system is being evaluated. This task might be writing on a piece of paper and it is assumed that it has a specific orientation and that it is being viewed from a specified direction. The visual task is also assumed to be made up of small, dark task details, e.g., printed letters, on a brighter task background, e.g., white paper. The first step in determining the value of ESI for this situation is the evaluation of the total task illumination, E_t . This is the total luminous flux incident upon the task and is given in units of lux. It can be measured by a luminance meter or it can be calculated from the characteristics of the luminous environment (see Sec. V). We next determine the task background luminance, L_b . This quantity is the luminous flux emitted from the task background in the viewing direction and is given in units of candelas/meter². It can be measured by a luminance meter or calculated from the details of the task illuminance and the reflectivity of the task which depends upon both the direction of the incident illuminance and the viewing direction. Note that, due to

this latter dependence of the reflectivity, L_b will depend upon viewing direction. It is assumed that the task details are sufficiently small that L_b represents the adaptation luminance of the observers eyes.

We can calculate the luminance factor from the above two quantities,

$$\beta \equiv L_b/E_t \quad (2.1)$$

This quantity plays the role of a reflectance for the task but should not be literally identified with the reflectance of a diffusely reflecting surface since it will depend upon the details of the task illuminance as well as the viewing direction.

The next step towards evaluating ESI for this task is the determination of the contrast rendering factor, CRF, which was formerly called the contrast rendition factor. We must define the terms "contrast," "reference task" and "reference lighting" before we can describe how CRF is determined. For the task described above, we define physical contrast, C_{phy} to be

$$C_{phy} \equiv |L_b - L_d|/L_b \quad (2.2)$$

where L_d is the task detail luminance in the viewing direction.

From this definition, C_{phy} will depend upon the details of the task illuminance, the nature of the visual task, and the viewing

direction. It is, however, a purely physical quantity whose connection with vision must be firmly established (see Sec. III) since it plays a central role in this prescription. We have used the subscript "phy" for physical to emphasize this point. Note that C_{phy} will depend upon the details of the visual task, i.e., the reflectivities of the task detail and background, as well as the directional distribution of illuminance incident upon the task. It is independent of the overall intensity of the illuminance since both L_b and L_d are proportional to this intensity. The reference task is a standard visual task used to evaluate lighting environments. At present, the most completely specified reference task is that of reading pencil handwriting. Reference lighting is a standard lighting environment which is used to assess the quality of all other environments. It is a lighting environment in which the illuminance is isotropic, homogeneous, unpolarized and with a color temperature of 2856 K. In practice, such an environment can be realized inside a sphere with a diffusely reflecting surface. It is therefore also called sphere lighting or sphere illumination. Note that reference lighting is completely specified by the value of the intensity of the illumination. The physical contrast rendering factor is defined by

$$CRF_{\text{phy}} = \frac{C_{\text{phy}}(\text{reference task, actual lighting})}{C_{\text{phy}}(\text{reference task, reference lighting})} \quad (2.3)$$

where we have called the lighting environment being evaluated "actual lighting" and the contrasts are measured in the viewing direction. As

defined, CRF_{phy} is a purely physical parameter which only depends upon the details of the task illuminance (not including the overall intensity since the contrast, defined by (2.3), is independent of the intensity due to the linearity of the reflection process) and, of course, the choice of reference task. It could be taken as a measure of the quality of the lighting environment with higher values of CRF_{phy} connoting higher quality. However, it would be an incomplete measure due to its lack of dependence upon the intensity. ESI determined from CRF_{phy} and L_b below is intended to build in the proper dependence.

The function that is used to determine a value for ESI from the values of CRF_{phy} and L_b is called the relative contrast sensitivity function, RCS. Before specifying this function, we must discuss "threshold contrast" and "contrast sensitivity." A person performing a visual task can be scored as to the percentage of successful executions of that task, e.g., percent correct detection of presence or orientation of a target flashed briefly on a screen. If we vary the contrast while holding L_b fixed then the score will change and there will be some value of the contrast at which some threshold success rate is reached. The threshold rate is usually taken to be 50 percent of the maximum rate after corrections are made for lucky guesses. This value of contrast is called the threshold contrast and it is a decreasing function of L_b as well as being a complicated function of the nature of the task, lighting environment, and observer (see Sec. III). The contrast sensitivity, CS, is defined to be the

reciprocal of the threshold contrast. The relative contrast sensitivity, RCS , is obtained by multiplying $CS(L_b)$ by $1/CS(L_b = 100 \text{ cd/m}^2)$ so that RCS has the arbitrary value of one at $L_b = 100 \text{ cd/m}^2$.

We have just defined a whole family of RCS functions. To be more specific, we must specify the task, illumination, and observer. We obtain the reference relative contrast sensitivity function, $RCS_{ref}(L_b)$, by specifying these parameters as follows:

1. The visual task is the detection of presence of a disc with 4' angular diameter against a uniform background.
2. The illumination is reference or sphere illumination.
3. The observer is 20-30 years old with normal vision and one averages over many members of this group in order to eliminate large individual differences.

In this way one measures a reference contrast sensitivity, $CS_{ref}(L_b)$, and a reference relative contrast sensitivity, $RCS_{ref}(L_b)$. As indicated, they are still functions of the task background luminance L_b . Note that the visual task used to determine RCS_{ref} is very different from the one used to determine CRF_{phy} .

We have now set up the machinery for evaluating equivalent sphere illumination, ESI . One proceeds as follows:

1. Evaluate the reference relative contrast sensitivity function at the value of the task background luminance, $RCS_{ref}(L_b)$.
2. Calculate the effective relative contrast sensitivity,

$$RCS_{eff}(L_b) = CRF_{phy} \times RCS_{ref}(L_b) \quad (2.4)$$

3. Determine the effective task luminance L_e by finding the value of luminance for which RCS_{ref} equals RCS_{eff} , i.e., solve $RCS_{ref}(L_e) = RCS_{eff}(L_b)$ (2.5)
4. Use the luminance factor, Eq. (2.1), to relate ESI to L_e ,

$$ESI = L_e / \beta \quad . \quad (2.6)$$

This sequence of steps completes the determination of ESI. Note the attractive form of RCS_{eff} . It is the product of two factors one of which is determined by the quality of the lighting environment and the other determined by the magnitude of the task luminance.

The intent of the above procedure is to provide a single figure of merit for any lighting installation. Lighting environments with the same ESI are intended to be equivalent in terms of visibility. Since the lighting environment under reference lighting is determined by one parameter, the illumination, visibility in this environment will also be determined by this parameter. The above procedure uses the RCS function and CRF_{phy} to relate an actual lighting environment to the reference one which then yields a value for ESI. This part of the procedure is discussed in detail in the next section.

4. INTERPRETATION OF THE RQQ REPORT NO. 4 PRESCRIPTION

In the previous section, we concentrated on how a calculation of ESI is performed. The prescription given there is a bit Delphic and in this section we give an interpretation of the procedure and an indication of why the calculation is done as described.

We consider an experiment in visual performance in which an observer is asked to perform a visual task in a specific lighting environment. The visual task might be the detection of presence or orientation of a target that is flashed briefly on a screen or it might be the reading and/or comparing of lists of numbers. The essential feature of any visual task is that the dominant process be that of acquisition of information through the eyes. The lighting environment might be reference lighting or that of an actual lighting system which is being evaluated. The observer might be a member of the reference population of observers or he or she might be chosen so as to be an archtypical user of the actual system. Clearly, many parameters must be specified in order to determine the task, environment, and observer and thereby define the experiment. Two particularly important parameters, task background luminance and task contrast, have already been introduced in the preceding section and we will review their definitions before considering the other parameters.

It is assumed that the visual task can be characterized by three luminances--the task background luminance L_b , the task detail luminance L_d , and the task surround luminance L_s . The task background and detail occupy that part of the observers visual field that

illuminates his foveae (0° to about 1° off the visual axis) and the surround illuminates the rest of his retinae. For the purposes of this section, we assume that L_b is the adaptation luminance for the observers eyes. This would be the case if $L_s = L_b$ (see Sec. IV.A. for a small modification of this statement) and the task detail is small in spatial and/or temporal extent. In cases where the task details are not small, one must determine the observer's adaptation luminance in some, as yet, unspecified way. We shall only consider tasks with small details. The quantity L_s actually represents a distribution of luminance in the task surround and its effects are discussed in Sec. IV.A. For this section, we assume $L_s = L_b$ and neglect its effects. We use L_b and the physical contrast C_{phy} (2.2) to characterize the luminance distribution of the task. Note that the definition of contrast (2.2), which has the luminance difference $L_b - L_d$ within absolute value signs implicitly assumes that targets that differ only by contrast reversal are visually equivalent. However, it has been reported²³ that contrast reversal produces a 20 percent change in contrast sensitivity. In another study,²⁴ a rather complex relationship between visual performance and contrast reversal has been reported. They indicate that for small objects and low luminances black on white background leads to faster visual performance while the reverse is true for large objects at high luminances with a cross-over somewhere in between these two extremes. Also note that L_b and C_{phy} are best suited for use in a laboratory situation and they would be a very incomplete description of the luminance distribution from a realistic task.

In addition to L_b and C_{phy} there are many other parameters that must be specified in order to define the experiment. We call them other parameters and denote them by $X = \{x_1, x_2 \dots\}$, where the x_i are the individual parameters. These include the characteristics of the visual task (beyond L_b and C_{phy}), observer, and lighting environment. In general, we assume that the experimental results are averaged over a population of observers whose characteristics are included in X , e.g., the reference population of observers of 20 to 30 year olds with normal vision. This averaging reveals systematic trends in the data but loses important information on the spread of the distribution of results. Indeed, it is an almost universal criticism of experimenters in this field that they present average data without error bars or confidence intervals.²⁵ It is not possible to know from the published information whether the difference between two averages, obtained under different circumstances, is significant without this additional information.

During the course of a typical experiment, the observer is scored as to the percent correct executions of the task. This score, after being corrected for random guesses, will be called²⁶ the visibility V of the task. We will stick to this definition of the term even though it is used interchangeably with contrast which is taken to be the "measure of visibility" in the literature. We assume that we have been sufficiently detailed in our parameterization of the experiment that the outcome, i.e., visibility, is determined by the values of the parameters

$$V = V(L_b, C_{phy}; X) \quad (3.1)$$

This relation is meant to summarize symbolically all the data taken from the experiment and only assumes that there is a definitive relationship between the values of the parameters and the visibility. It has become customary to present data such as (3.1) in terms of standard functions called "log ogives" which, in more conventional terms, is an error function with a logarithmic argument. See Sec. VI for a discussion of visual performance as opposed to visibility.

A crucial feature of the above discussion is the treatment of the contrast C_{phy} as an independent variable. In general, this is not so. For artificial tasks, in which both background and detail are projected onto a screen independently, we can vary their luminances separately and contrast is a true independent variable. However, for more realistic tasks, the contrast will be determined by the physical properties of the task and the distribution of incident illuminance upon it and it will not be an independent variable. In order to emphasize this point, we have used C_{phy} to denote the physical contrast of the visual task. Then, according to the above discussion $C_{phy} = C_{phy}(X)$ for realistic tasks. The visibility meter²⁷ is a device that can be used to reduce the contrast of a realistic task without changing any of the other parameters characterizing the task. Thus, the existence of visibility meters makes contrast an independent variable for all tasks. Since visibility meters play a central role in what follows, we will discuss their operation before continuing our discussion of visibility.

For a realistic task (not one projected onto a screen) characterized by task background luminance L_b , physical contrast C_{phy} , and other parameters X , the physical contrast is a dependent variable, $C_{phy} = C_{phy}(X)$. A visibility meter is a device that enables the experimenter to reduce the value of the contrast without changing X . It, therefore, makes contrast an independent variable which we denote by C . Contrast, as defined by (2.2), is the difference between background and detail luminances divided by that of the background. A visibility meter changes the contrast of the visual task by superposing a uniform veiling luminance over the entire visual field, see Fig. 3.1. Luminance from the task, attenuated by a factor f_1 , is combined with the uniform veiling luminance L_v , attenuated by a factor f_2 , to form the visual signal. The resultant luminances of the task background L'_b and detail L'_d are then given by

$$\begin{aligned} L'_b &= f_1 L_b + f_2 L_v \\ L'_d &= f_1 L_d + f_2 L_v \end{aligned} \quad (3.2)$$

The resultant contrast is given by

$$C \equiv \frac{|L'_b - L'_d|}{L'_b} = \frac{f_1 |L_b - L_d|}{f_1 L_b + f_2 L_v}$$

we want to vary the contrast while holding the task background luminance fixed. We therefore require

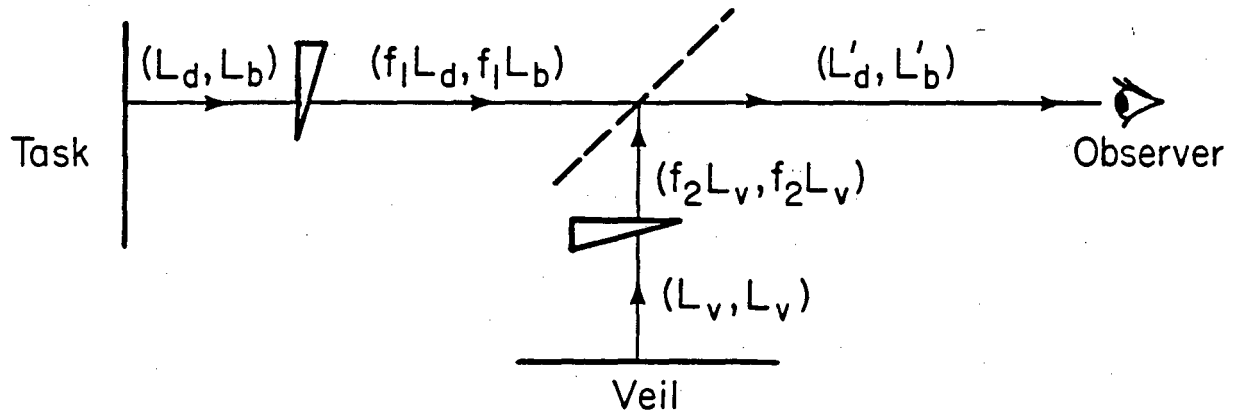


Fig. 3.1 Schematic diagram of a visibility meter. The symbols indicate (task detail luminance, task background luminance) at that point in the device where "detail" and "background" are interpreted as the corresponding parts of the observer's field of vision.

$$L_b' = L_b = f_1 L_b + f_2 L_v \quad (3.4)$$

In practice, this is achieved by setting $L_v = L_b$ and $f_2 = 1-f_1$.
The contrast is then changed to

$$C = f_1 C_{\text{phy}} \quad (3.5)$$

where C_{phy} is the physical contrast of the task. Thus, a visibility meter can be used to reduce the contrast of a given task while holding all other parameters fixed. This has the important mathematical effect of changing the status of contrast from that of a dependent variable to that of an independent variable. It also can be used to simulate important physical effects. For, if the dominant source of the X -dependence of V arises from the X -dependence of C_{phy} , then we can write $V \cong V(L_b, C_{\text{phy}}(X); X_0)$, where X_0 is some fixed set of parameters. The visibility meter enables us to replace $C_{\text{phy}}(X)$ by C , an independent variable in this expression. In this approximate fashion, use of a visibility meter simulates changes in the lighting environment, i.e., changes in X , by changes in C . Note that, for a task with physical contrast C_{phy} , the domain of the variable C is $0 \leq C \leq C_{\text{phy}}$.

The effective luminance L_e from which ESI can be calculated, see Eq. (2.6), is the luminance that the reference task must have under reference lighting conditions so that its visibility is the same as it has under the actual lighting conditions. This means that L_e is the solution to the equation

$$V(L_e, C_{\text{phy}}(\bar{X}^{\text{ref}}); \bar{X}^{\text{ref}}) = V(L_b, C_{\text{phy}}(\bar{X}); \bar{X}),$$

where \bar{X} and \bar{X}^{ref} are defined in Eq. (3.9) below. What follows is a series of assumptions about the form of the function V and approximations that render the solution of this equation practicable. Since we are unaware of any quantitative studies on the validity of the assumptions or the accuracy of the approximations, we will call the solution to this equation the true effective luminance L_{et} , see Eq. (5.2). This is done so as to distinguish it from L_e which is determined by the same equation after it has been transformed by the assumptions and approximations.

Visibility, defined by (3.1) is a difficult property to work with. Since it is a complex psychophysical quantity, one can not hope to calculate it or design a meter that will measure it. Furthermore, since it varies substantially from individual to individual, one can not use a single observer to measure it. We therefore seek to eliminate it from any assessment of a lighting environment. The first step towards this end is the reversal of the roles of C and V so that C is the dependent and V is an independent variable

$$C = C(L_b, V; X) \quad . \quad (3.6)$$

This relation can be obtained from the data summarized by (3.1) and should be interpreted as the value of contrast needed to provide the specified level of visibility for the task X when the task background luminance is L_b . Rather than working with contrast itself, it has become conventional to work with contrast sensitivity CS which is the reciprocal of C,

$$CS(L_b, V; X) \equiv 1/C(L_b, V; X) \quad (3.7)$$

and values of this function are used to summarize the data.

Study of the contrast sensitivity seems to indicate that it approximately factors into a function of V times one of L_b and X. A quantitative study of the accuracy of this approximate factoring does not seem to be available in the literature. Nevertheless, since this factoring enables us to eliminate the visibility from our assessment of a lighting environment, we shall proceed as if we knew it was accurate. We write the factoring as

$$CS(L_b, V; X) \equiv F(V) CS(L_b, V_t; X), \quad (3.8)$$

where $V_t = 50$ percent is the threshold value of V. This value is chosen since the data are most reliable for this value. From (3.8), we have $F(V_t) = 1$ and we expect that $F(V)$ will be a decreasing

function of V . See Sec. VI for the relationship between $F(V)$ and visibility level.

As in (2.3), a reference task is used together with the contrast sensitivity to relate the actual lighting environment to the reference lighting environment. This task is a standardized one used to simulate actual visual tasks which is also well characterized as to detail and background reflectivities. The pencil handwriting task seems to the best characterized reference task at present.²⁸ The reference task is to be placed in the actual lighting environment and we introduce some notation to label these two situations,

\bar{X} = reference task in actual lighting

\bar{X}^{ref} = reference task in reference lighting (3.9)

We use $C_{\text{phy}}(\bar{X})$ and $C_{\text{phy}}(\bar{X}^{\text{ref}})$ to denote the contrast of the reference task in these two situations. (As an aid to the reader, we review here our notational conventions. The other parameters of an arbitrary visual task are denoted by X . If this task is placed in reference lighting rather than actual lighting, then X is given a superscript "ref" as in (3.9). If the task is a particular reference task, then a bar is placed over X as in (3.9). These tasks should be distinguished from the particular visibility reference task which is denoted by X_{ref} .) We use (3.7) and (3.8) to write

$$\begin{aligned} \frac{1}{C_{\text{phy}}(\bar{X})} &= \text{CS}(L_b, \bar{V}; \bar{X}) \\ &\cong F(\bar{V})\text{CS}(L_b, V_t; \bar{X}) \end{aligned} \quad (3.10)$$

$$\begin{aligned} \frac{1}{C_{\text{phy}}(\bar{X}^{\text{ref}})} &= \text{CS}(L_e, \bar{V}^{\text{ref}}; \bar{X}^{\text{ref}}) \\ &\cong F(\bar{V}_{\text{ref}})\text{CS}(L_e, V_t; \bar{X}^{\text{ref}}) \end{aligned} \quad (3.11)$$

where L_b and \bar{V} and L_e and \bar{V}^{ref} are the task background luminance and visibility of the reference task in the actual and reference lighting environments respectively. Eqs. (3.10) and (3.11) are restatements of the basic assumption, (3.1), that visibility is determined by background luminance contrast, and the other parameters characterizing the task plus the factorization (3.8). That is, the first lines of (3.10) and (3.11) express the fundamental relationship between task background luminance and visibility when the other parameters of the task are fixed. The second lines of these equations are then obtained by using the factored form of CS given in (3.8).

We set up an equivalence between L_b and L_e in (3.10) and (3.11) by requiring $\bar{V} = \bar{V}^{\text{ref}}$, i.e., the luminances are equivalent if they produce the same level of visibility in the actual and reference lighting environments. The value of the visibility is not known but we do know that $F(\bar{V}) = F(\bar{V}^{\text{ref}})$ which, on substituting from (3.10) and (3.11), becomes

$$C_{\text{phy}}(\bar{X}^{\text{ref}})CS(L_e, V_t; \bar{X}^{\text{ref}}) = C_{\text{phy}}(\bar{X})CS(L_b, V_t; \bar{X}) \quad (3.12)$$

We clean up our notation a bit by introducing contrast threshold C_t defined by

$$C_t(L; X) \equiv C(L, V_t; X) = 1/CS(L, V_t; X) \quad (3.13)$$

Eq. (3.12) then becomes

$$\frac{C_{\text{phy}}(\bar{X}^{\text{ref}})}{C_t(L_e; \bar{X}^{\text{ref}})} = \frac{C_{\text{phy}}(\bar{X})}{C_t(L_b; \bar{X})} \quad (3.14)$$

This is the basic equation for L_e but many approximations must be made before it can be solved in a practical situation.

The quantities on each side of (3.14) are called² relative visibilities RV and they can be measured directly with a visibility meter. The relative visibility is defined by

$$RV(L; X) \equiv C_{\text{phy}}(X)/C_t(L; X) \quad (3.15)$$

and, from (3.5) we have

$$RV = 1/(f_1)\text{threshold} \quad (3.16)$$

Thus, RV can be obtained from an unambiguous psychophysical measurement. Eq. (3.14) can then be written as

$$\begin{aligned}
 RV(L_e; \bar{X}^{\text{ref}}) &= RV(L_b; \bar{X}) \\
 &= \frac{RV(L_b; \bar{X})}{RV(L_b; \bar{X}^{\text{ref}})} \times RV(L_b; \bar{X}^{\text{ref}}) \quad . \quad (3.17)
 \end{aligned}$$

The first form of (3.17) implies that two complex measurements of RV functions with different values of X must be done to obtain L_e while the second form suggests that we may be able to get away with one measurement if the first factor on the right-hand-side is simple.

We proceed by making approximations for the two factors on the right-hand-side of (3.17). We first consider the second factor, $RV(L_b; \bar{X}^{\text{ref}})$ which also appears on the left-hand-side. We note from (3.13) and (3.15) that RV is proportional to contrast sensitivity. Furthermore, if we divide both sides (3.17) by $RV(L_0, \bar{X}^{\text{ref}})$, where L_0 is arbitrarily chosen to be 100 cd/m^2 , then we can rewrite the equation in terms of relative contrast sensitivity RCS defined by

$$\begin{aligned}
 \text{RCS}(L; X) &\equiv RV(L; X) / RV(L_0; X) & (3.18) \\
 &= \text{CS}(L; X) / \text{CS}(L_0; X) \quad ,
 \end{aligned}$$

where we suppress the variable V in CS if $V = V_t$. Thus, RCS is the contrast sensitivity measured in units of its value at $L_0 = 100 \text{ cd/m}^2$. Using this definition, (3.17) becomes

$$\text{RCS}(L_e; \bar{X}^{\text{ref}}) = \frac{\text{RV}(L_b; \bar{X})}{\text{RV}(L_b; \bar{X}^{\text{ref}})} \times \text{RCS}(L_b; \bar{X}^{\text{ref}}) \quad (3.19)$$

The pencil handwriting task under reference lighting is a difficult one to use in a visibility measurement. We therefore approximate the RCS functions in (3.19) by one measured with a task more suitable for such a measurement. This task is called the visibility reference task and we will denote its parameters by X_{ref} . It is the detection of presence of a disk whose angular diameter is 4' and which is projected onto a diffusely reflecting surface of uniform luminance for a period of 1/5 of a second under reference lighting conditions. The observer is the average of the reference population of observers. We then make the approximation

$$\text{RCS}(L; \bar{X}^{\text{ref}}) \cong K \times \text{RCS}(L; X_{\text{ref}}), \quad (3.20)$$

where the constant K can depend upon X_{ref} and \bar{X}^{ref} but not on L .

A quantitative estimate of the validity of this approximation does not seem to be present in the literature. Nevertheless, we shall use it and proceed as if we knew that it is accurate. We tighten up the notation by introducing the reference RCS function, RCS_{ref} , defined by

$$RCS_{\text{ref}}(L) \equiv RCS(L; X_{\text{ref}}) \quad (3.21)$$

and write (3.19) as

$$RCS_{\text{ref}}(L_e) \equiv \frac{RV(L_b; \bar{X})}{RV(L_b; \bar{X}^{\text{ref}})} \times RCS_{\text{ref}}(L_b) \quad (3.22)$$

This completes the treatment of the second factor on the right-hand-side of (3.17).

There are many different approximations to the first factor in (3.17) in the literature. We will describe the simplest one here which leads to the prescription of RQQ Report No. 4. We discuss two sophistications of this approximation in Sec. IV. An assumption that is common to all practical approximations is that the factor is a constant independent of L_b . This means that we are assuming that $RV(L; X)$ factors into a function which depends on L but does not depend on the lighting environment times a function which does not depend on L but can depend on all the parameters X . That is, if we divide X into three parts describing the task X_{task} , the observer X_{obs} , and the environment X_{env} , then we are assuming that RV factors as

$$RV(L; X) \equiv G(L; X_{\text{task}}, X_{\text{obs}}) H(X). \quad (3.23)$$

This approximation is similar to (3.20), since (3.20) follows from (3.23). The validity of this assumption has not been quantitatively evaluated in the literature. However, some data²⁹ reported in the literature suggest that there can be as much as a 40 percent deviation from this factorization for simple tasks. To obtain the result of RQQ

Report No. 4, we make the more drastic approximation,

$$\frac{RV(L_b; \bar{X})}{RV(L_b; \bar{X}^{\text{ref}})} \cong \frac{C_{\text{phy}}(\bar{X})}{C_{\text{phy}}(\bar{X}^{\text{ref}})} \equiv \text{CRF}_{\text{phy}} \quad (3.24)$$

i.e., the ratio of the relative visibilities in the two environments is assumed to be equal to the ratio of the physical contrasts in the two environments. Note that the right-hand-side is CRF_{phy} introduced in (2.3). Also note that we should write $\text{CRF}_{\text{phy}}(\bar{X})$ since the value of CRF_{phy} has been determined using the reference task whose parameters are denoted by \bar{X} . In what follows, we will suppress the variables X in CRF_{phy} when they are equal to \bar{X} . With this approximation (3.22) becomes

$$\text{RCS}_{\text{ref}}(L_e) \cong \text{CRF}_{\text{phy}} \times \text{RCS}_{\text{ref}}(L_b) \quad (3.25)$$

which is the result of RQQ Report No. 4, see (2.4) and (2.5).

The approximation (3.24) is crucial for practical applications of this scheme. It replaces two complex psychophysical measurements, $RV(L_b; \bar{X})$ and $RV(L_b; \bar{X}^{\text{ref}})$, by the ratio of two simple physical contrasts that can be either calculated or measured with a meter. Since the ratio of RV's in (3.24) is a psychophysical quantity that plays the role of CRF in (2.5), we shall call it the "Psychophysical

CRF" and denote it by CRF_{psy} ,

$$CRF_{psy} \equiv \frac{RV(L_b; \bar{X})}{RV(L_b; \bar{X}^{ref})} \quad (3.26)$$

and the approximation (3.24) is $CRF_{psy} \cong CRF_{phy}$. We explore the meaning of this approximation by using the definition of RV (3.15) to write out CRF_{psy} in terms of contrasts,

$$CRF_{psy} = \frac{C_{phy}(\bar{X})}{C_t(L_b; \bar{X})} \times \frac{C_t(L_b; \bar{X}^{ref})}{C_{phy}(\bar{X}^{ref})}, \quad (3.27)$$

where recall that C_{phy} is the actual physical contrast and C_t is the threshold contrast of the task. Thus, the approximation is valid if the two threshold contrasts, under different lighting conditions, are the same. There does not seem to be any reason why this should be so and there is not any quantitative data in the literature that indicates that it is so. Furthermore, the effects considered in Sec. IV indicate that the value of CRF_{psy} can differ substantially from that of CRF_{phy} . The approximation should therefore be viewed with considerable suspicion.

Note that we should indicate the arguments in the psychophysical contrast rendering factor, i.e., $CRF_{psy}(L_b; \bar{X})$, in (3.26) since its value should depend upon the magnitude of the adaptation luminance

L_b and the choice of reference task \bar{X} . It is clear that the definition (3.26) can be generalized to an arbitrary task X giving $CRF_{phy}(L_b; X)$. However, unless specified otherwise, CRF_{psy} will refer to the reference task \bar{X} . In practice one hopes that $CRF_{psy}(L_b; X)$ is only weakly dependent upon L_b since it occurs only through the ratio of threshold contrasts for the same visual task under different distributions of illuminance and CRF_{psy} would be independent of L_b if the approximation (3.23) is valid. However, one does expect it to have a substantial X -dependence in any case.

The approximation of using RCS_{ref} instead of $RCS(L; \bar{X}^{ref})$ is also dictated by practical considerations. Some feeling for this approximation can be obtained from information in Refs. 2 and 3 which we now summarize. The function RCS_{ref} is given in a fitted analytic form in Ref. 3 and in tabular form, with a different normalization, in Ref. 2. We will use the fitted form,

$$RCS_{ref}(L) = 1.555 \left[1 + \left(\frac{1.639}{L} \right)^{0.4} \right]^{-2.5}, \quad (3.28)$$

L in cd/m^2 , which is a form originally proposed by Hecht.³⁰ The approximation (3.20) of neglecting the X -dependence of RCS is one of unknown accuracy. The dependence of RCS on the size of task detail, size of the field of visual search, and age of observer is discussed

in Ref. 3. We present these dependences and show that they can lead to unrealistic changes in the value of L_e .

The form for RCS given in Ref. 3 which includes the effects of the size of task detail d , size of the field of visual search SF , and age of the observer A is a generalization of (3.28),

$$RCS(L) = n \left[1 + \left(\frac{S}{tL} \right)^{0.4} \right]^{-2.5}, \quad (3.29)$$

where the parameter n is chosen so that $RCS = 1$ when $L = L_0 = 100 \text{ cd/m}^2$,

$$n = \left[1 + \left(\frac{S}{100t} \right)^{0.4} \right]^{2.5}. \quad (3.30)$$

The parameter S depends upon the size of the task detail d in arc minutes and the size of the field of visual search SF in degrees,

$$\log S = 0.5900 - 0.6235 \log d - 0.0990 \widetilde{SF} - s. \quad (3.31)$$

where the range of validity of this expression is $1' < d < 10'$ and $0^\circ < SF < 6^\circ$. The parameters t and s depend upon the age of the observing population A in years and are given in (6.21) and (6.22). While the parameters in RCS do not change greatly, the resultant changes in L_e are large and unrealistic since the small changes in

the parameters are amplified by the procedure which relates L_e and L_b and CRF_{phy} . We demonstrate this in a simple example.

We consider an example in which the observers are 40-year olds rather than the 25-year old reference population and all other parameters are the same. The RCS function representing the relative contrast sensitivity of the 40-year old population is given by³

$$RCS(L) = 1.620 \left[1 + \frac{2.089}{L} \right]^{0.4} - 2.5 \quad (3.32)$$

which should be compared with (3.28) for the reference population. We now have two possible courses of action. We could ignore the 40-year olds and use the 25-year olds to determine L_e . This would mean solving (3.25) for L_e . Alternatively, we could ignore the 25-year olds and solve (3.25) for L_e using (3.32) instead of RCS_{ref} . If we take $L_b = 1000 \text{cd/m}^2$ and $CRF_{phy} = 0.9$, then these two choices yield the values $L_e = 307, 334 \text{cd/m}^2$ for the 25- and 40-year-olds respectively. If we interpret the resultant values of ESI, which are proportional to L_e , as measures of visibility, then these numbers indicate that the 40-year-olds are better off than the 25-year-olds which is contrary to all experience. While this may not be a serious problem when comparing two lighting systems since the same method would be used to calculate L_e for both of them. It does however indicate that the actual values of ESI are highly elastic. This

problem is dealt with more explicitly when visibility levels (see Section VI) are used to assess lighting environments.

5. FURTHER CONSIDERATIONS: DISABILITY GLARE FACTOR AND TRANSIENT ADAPTATION FACTOR

We consider two more sophisticated approximations for CRF_{psy} , (3.26), that go beyond the simple replacement $CRF_{psy} \cong CRF_{phy}$ used in RQQ Report No. 4 in this section. These new approximations lead to the introduction of the disability glare factor, DGF, Sec. IV.A, and the transient adaptation factor, TAF, Sec. IV.B. The general methodology behind these new approximations is that of introducing task environments \bar{X}' that are intermediate between the actual environment \bar{X} and the reference environment \bar{X}^{ref} in order to separate out effects that are not accounted for by CRF_{phy} . This is done by multiplying and dividing the right-hand-side of (3.26) by $RV(L_b; \bar{X}')$ and writing

$$CRF_{psy} = \frac{RV(L_b; \bar{X})}{RV(L_b; \bar{X}')} \times \frac{RV(L_b; \bar{X}')}{RV(L_b; \bar{X}^{ref})} \quad (4.1)$$

The environment \bar{X}' should be chosen so that CRF_{phy} is a better approximation to the second factor than to CRF_{psy} itself. The first factor is then a modification of CRF due to the difference in the two environments \bar{X} and \bar{X}' . For example, if we want to account for the presence of glare sources in the task surround, then they would be present in the environment \bar{X} but not present in \bar{X}' . Under these circumstances, the first factor in (4.1) would be DGF and the second factor would be approximated by CRF_{phy} . This is clearly a better approximation to CRF_{psy} since the physical contrast of the task can

not account for the visual effects of glare. Similar considerations apply to the dynamic effects accounted for by TAF.

5.1 The Disability Glare Factor

The disability glare factor, DGF, accounts for the degradation of visibility due to glare sources in the task surround. According to the above discussion, it is given by

$$DGF = \frac{RV(L_b; \bar{X})}{RV(L_b; \bar{X}')} \quad , \quad (4.2)$$

where \bar{X} is the actual lighting environment, including glare sources, and \bar{X}' is the actual lighting environment modified so as to minimize the effect of glare on the second factor in (4.1). We note that glare is present in all environments since a dark task surround degrades visibility as well as a bright one. Thus, the environment \bar{X}' may be a theoretical construct rather than something realized in the laboratory. We also note that, with the introduction of DGF, the equation for the effective task luminance L_e becomes

$$RCS_{ref}(L_e) \cong DGF \times CRF_{phy} \times RCS_{ref}(L_b) \quad (4.3)$$

which is a more accurate approximation than (3.25). Reported values of DGF lie in the range $0.82 < DGF < 1.02$ which, when translated into values of L_e , make glare an important factor in determining ESI.

The degradation of visibility in the presence of glare sources is attributed to the scattering of light within the eye and/or possible interference effects which hinder the signal processing functions of

the retinae. In developing an expression for DGF, the assumption is made that the entire effect can be represented by the superposition of an equivalent veiling luminance L_v on the visual field of the observer.³¹⁻³⁶ This veiling luminance reduces the perceived contrast from the value extant in the task and therefore reduces contrast sensitivity. At the same time, however, it increases the adaptation luminance and this increases the contrast sensitivity. The expression for DGF is the result of competition between these two effects. In what follows, we review the method for calculating DGF as put forth in Ref. 2.

The observer's visual field is divided into two parts which we will call the task and the task surround. The task is roughly that part of the visual field that illuminates the observer's fovea and extends to about 1° off the visual axis. The task surround is the remainder of the observer's visual field and will be denoted by S . The luminance distribution of the task is characterized by the background and detail luminances L_b and L_d . The luminance distribution of the surround is given by $L_s(\theta, \phi)$ where θ and ϕ are polar angles measured from and about the observer's visual axis.

It is assumed that the effects of the glare source in the surround L_s can be subsumed in an equivalent veiling luminance L_v whose magnitude is given by $L_v = \Lambda L_b$, where

$$\Lambda = \frac{K}{L_b} \int_S d\Omega \frac{\cos\theta}{\theta^m} L_s(\theta, \phi) \quad (4.4)$$

where $d\Omega$ is the element of solid angle about the direction (θ, ϕ) from the observer's visual axis (measured in steradians), S is the range of the surround which is defined to be $1^\circ \leq \theta \leq 90^\circ$ and $0 \leq \phi \leq 350^\circ$, and K and m are adjusted to fit the data. The values reported in the literature are $K = 10$ and $m = 2$ for the reference population of observers with K being as much as three times larger for other populations. It should be pointed out that the reported values of K depend upon the rather peculiar convention of measuring $d\Omega$ in steradians but θ , in the denominator, in degrees. If a consistent measure of angles is used with $m = 2$, then K must be replaced by $(\pi/180)^2 K$. This expression for Λ is a fit to data that has a very large spread, about a factor of 10 about the mean, and should not be given any fundamental significance.³⁴ See Ref. 37 for an alternate set of experiments which show no dependence of visual performance upon glare.

We now express DGF in terms of Λ independent of the fit (4.4). In the presence of the veiling luminance L_v , the perceived contrast is reduced and the adaptation luminance is increased. The first effect decreases contrast sensitivity while the second increases it. For, L_v modifies the luminances of the task detail and background as

$$L_d \rightarrow L'_d = L_d + L_v, \quad L_b \rightarrow L'_b = L_b + L_v \quad (4.5)$$

and the perceived contrast becomes

$$C \rightarrow C' = \frac{1}{1+\Lambda} C \quad . \quad (4.6)$$

This expression for C' should be thought of as an effective contrast in the presence of glare and C is the actual contrast of the task. This means that the actual task contrast must be increased by a factor $1+\Lambda$ in order to have the same effect on the visual system that it would have had in the absence of glare. Thus, the contrast sensitivity is reduced by a factor $1+\Lambda$ by this effect. The increase in the adaptation luminance from L_b to $L_b = (1+\Lambda)L_b$ results in an increased contrast sensitivity since CS is an increasing function of adaptation luminance. It is useful to exhibit the Λ -dependence of CS explicitly. We therefore write $X = \{X^0, \Lambda\}$, where X^0 includes all the nonglare parameters. The above considerations then imply

$$CS [L; X^0, \Lambda] = \frac{1}{1+\Lambda} CS [(1+\Lambda)L; X^0, \Lambda = 0] \quad . \quad (4.7)$$

There is glare even in the reference environment X_{ref} according to (4.7) and the CS on the right-hand-side is not an observable since it does not correspond to any physically realizable situation. We denote X_{ref} by $\{X_{ref}^0, \Lambda_{ref}\}$ and use the inverse of (4.7) to obtain

$$\text{CS} [L; \bar{X}_{\text{ref}}^0, \Lambda = 0] = (1 + \Lambda_{\text{ref}}) \text{CS} \left[\frac{L}{1 + \Lambda_{\text{ref}}} ; \bar{X}_{\text{ref}} \right] . \quad (4.8)$$

This gives the unobservable CS function on the left-hand-side in terms of the observable one on the right-hand-side. The reference environment has $L_s = L_b$ and numerical integration of (4.4) then yields $\Lambda_{\text{ref}} = 0.074$.

We define DGF by choosing the intermediate environment \bar{X}' in (4.2). We denote \bar{X} by $\{\bar{X}^0, \Lambda\}$ and choose \bar{X}' to be $\{\bar{X}^0, \Lambda_{\text{ref}}\}$. This choice implies that both numerator and denominator in the second factor of (4.1) have the same amount of glare and one can hope that the ratio is insensitive to the presence of this glare. If this hope is fulfilled then CRF_{phy} may be a good approximation to this factor. Using this choice of \bar{X}' in (4.2), (3.15) for the RV's, and noting that C_{phy} is independent of Λ , i.e., $C_{\text{phy}}(\bar{X}) = C_{\text{phy}}(\bar{X}')$, we have

$$\text{DGF} = \frac{\text{CS}(L_b; \bar{X})}{\text{CS}(L_b; \bar{X}')} \quad (4.9)$$

We rewrite the numerator of (4.9), using (4.7), as

$$\text{CS}(L_b; \bar{X}) = \text{CS}(L_b; \bar{X}^0, \Lambda) = \frac{1}{1 + \Lambda} \text{CS}[(1 + \Lambda)L; \bar{X}^0, \Lambda = 0] \quad , \quad (4.10)$$

and make the approximation

$$CS[(1+\Lambda)L; \bar{X}^0, \Lambda = 0] \cong CS\left[(1+\Lambda)L; X_{\text{ref}}^0, \Lambda = 0\right] . \quad (4.11)$$

We then use (4.8) to obtain

$$CS(L_b; \bar{X}) \cong \frac{1+\Lambda_{\text{ref}}}{1+\Lambda} CS\left[\left(\frac{1+\Lambda}{1+\Lambda_{\text{ref}}}\right) L_b; X_{\text{ref}}\right] . \quad (4.12)$$

For the denominator of (4.9), we make the approximation

$$CS(L_b; \bar{X}') = CS(L_b; \bar{X}^0, \Lambda_{\text{ref}}) \cong CS(L_b; X_{\text{ref}}) \quad (4.13)$$

which is the same kind of approximation as (4.11). With these modifications (4.9) becomes

$$DGF \cong \frac{1+\Lambda_{\text{ref}}}{1+\Lambda} \frac{CS\left(\frac{1+\Lambda}{1+\Lambda_{\text{ref}}} L_b; X_{\text{ref}}\right)}{CS(L_b; X_{\text{ref}})} . \quad (4.14)$$

The CS functions in this expression can now be replaced by RCS_{ref} since the normalization cancels and we have

$$\begin{aligned}
 \text{DGF} &\cong \frac{1+\Lambda_{\text{ref}}}{1+\Lambda} \frac{\text{RCS}_{\text{ref}} \left[\frac{1+\Lambda}{1+\Lambda_{\text{ref}}} L_b \right]}{\text{RCS}_{\text{ref}}(L_b)} & (4.15) \\
 &\cong 1 - \frac{1}{1 + \left(\frac{1.639}{L_b} \right)^{0.4}} \left(\frac{\Lambda - \Lambda_{\text{ref}}}{1 + \Lambda_{\text{ref}}} \right) ,
 \end{aligned}$$

where the last expression is accurate to first order in $\Lambda - \Lambda_{\text{ref}}$ and (3.28) has been used for RCS_{ref} . This is the desired expression for DGF. In practice, the ratio of the two RCS functions is essentially one since the slope of RCS_{ref} is very small for realistic values of L_b .

In view of the very rough agreement between experiments and the relation $L_v = \Lambda L_b$ and the uncontrolled approximations used in (4.11) and (4.13) one can only expect (4.15) to give a very crude estimate of DGF.

5.2 The Transient Adaptation Factor

The discussion in the previous sections has dealt with static visual performance. The experiments are done in such a way that the observer knows exactly where the target will be presented and fixates his eyes on that spot before the presentation of the stimulus. Furthermore, the observers eyes are perfectly adapted to the task background luminance. In real life, an observer's eyes will move from the task to other regions of his environment according to a complicated schedule. In so doing, he exposed his eyes to a wide range

of luminances and his visual performance will be degraded because his eyes will not be adapted to the task background luminance being observed for some fraction of the time. The transient adaptation factor, TAF, is intended to account for this degradation.

A determination of TAF requires knowledge of the dynamic response of the eye to situations in which the adaptation luminance changes with time, the observers schedule of eye fixations, and the values of the luminances in the environment that the observer will fixate upon.³⁸ These three aspects of TAF can be investigated separately. The dynamic response of the eye can be measured in the time domain by measuring RCS as a function of time after an abrupt change in adaptation luminance. One would have to allow for a nonlinear amplitude dependence as well as the time dependence in such a measurement. However, the experiment would be a straightforward extension of those already done to measure RCS_{ref} . Experiments could also be done to characterize the schedule of fixations of an observer and their associated target luminances. This information would then be used to come up with an average value for the degradation of visibility in a dynamic viewing environment. Some work along these lines has been done^{39,40} but much more needs to be done before quantitative statements can be made. We refer the interested reader to the discussions given in Refs. 2 and 3 for more details.

6. METHODS FOR DETERMINING ESI

We now turn to the relatively straightforward task of determining the value of ESI for a given environment according to the prescription given in Sec. II. From (2.4) or (3.25), we see that there are two essential elements to this determination. One is a knowledge of the reference RCS function and we take this as given by (3.28). The other is the value of CRF_{phy} which depends upon the particular environment being considered. If we solve (3.25) for L_e using (3.28) for RCS_{ref} , we have

$$L_e = L_b \left\{ 1 + \left[1 + \left(\frac{L_b}{1.639} \right)^{0.4} \right] \left[(CRF_{phy})^{-0.4} - 1 \right] \right\}^{-2.5} \quad (5.1)$$

The value of ESI can be determined from L_e using (2.6), $ESI = (L_e/L_b)E_t$. Thus, we proceed to determine the task illumination E_t , and the task background and detail luminances L_b and L_d . These may be measured with luminance meter or they may be calculated from the characteristics of the environment and task. We will briefly describe the method of calculation after a discussion of the accuracy needed for sensible results in this section.

For the sake of discussion of accuracy, we assume that we seek a value for ESI that is within 10% of the "true" value. Here, the true value of ESI is the value of illumination on the reference task under reference conditions that produces the same level of visibility as the actual illumination on the reference task under actual conditions

does, i.e., it would be determined from the true effective task background luminance L_{et} which satisfies the equation

$$V(L_{et}, C_{phy}(\bar{X}^{ref}); \bar{X}^{ref}) = V(L_b, C_{phy}(\bar{X}); \bar{X}) . \quad (5.2)$$

Recall that contrast is not an independent variable in the real world. The various approximations that lead from (5.2) back to (5.1) are as follows:

1. Factor contrast sensitivity (3.8). The accuracy of this approximation is not known.
2. Replace RCS by RCS_{ref} (3.20). The accuracy of this approximation is not known.
3. Replace CRF_{psy} by CRF_{phy} (3.24). The discussion of DGF in Sec. IV suggests that this approximation is no better than 10% accurate.

If we assume that the first two approximations are within our 10 percent criterion for L_e then we have Eq. (5.1) with a 10 percent uncertainty in CRF from the third approximation. However, for realistic values of L_b , Eq. (5.1) amplifies errors in CRF. For example, (5.1) yields the values $L_e = 278, 307, \text{ and } 341 \text{ cd/m}^2$ for $L_b = 1000 \text{ cd/m}^2$ and $CRF = 0.89, 0.90, \text{ and } 0.91$ respectively. thus, there is about a factor of 10 amplification of errors in CRF which leads to a factor of 2 uncertainty in the value of ESI.

If we did not have the above problem with the third approximation, we would still have a problem calculating CRF_{phy} to 1 percent

accuracy. This requires that we calculate the illuminance incident on the task to 1 percent accuracy and that we know the reflectivities of the task detail and background to 1 percent accuracy. We doubt that the data⁴¹ are that accurate.

Numerical techniques for calculating CRF_{phy} are outlined in RQQ Report No. 5.⁴¹ We will describe the problem to be solved but in view of the above comments will not dwell on the subject. The problem can be split into three parts.

1. Calculate the distribution of illuminance in the room from the room characteristics and the distribution of light sources.
2. Determine the angular distribution of illuminance on the task from the above distribution and the task location and orientation.
3. Determine the task illumination, and task detail and background luminances from the angular distribution obtained in step 2, the reflectivities of the task, and the viewing angle.

Each of these steps is straightforward in principle but very messy to carry out in practice.

In order to calculate the distribution of illuminance in the room, we assume that its interior surface can be characterized by N plane, gray, diffusely reflecting surfaces. We denote the constant reflectivity of the i^{th} surface by ρ_i and the illumination at point \vec{r}_i on the i^{th} surface by $E_i(\vec{r}_i)$. The E_i are determined by the equations

$$E_i(\vec{r}_i) = S_i(\vec{r}_i) + \sum_{j=1}^N \frac{\rho_j}{\pi} \int_{A_j} d^2r_j K_{ij}(r_{ij}) E_j(r_j) \quad (5.3)$$

where $S_i(\vec{r}_i)$ is the direct illumination of point \vec{r}_i on the i^{th} surface due to the light sources in the room, A_j is the area of the j^{th} surface and K_{ij} is the radiation transfer factor

$$K_{ij}(\vec{r}_{ij}) = - \frac{(\vec{n}_i \cdot \vec{r}_{ij})(\vec{n}_j \cdot \vec{r}_{ij})}{r_{ij}^4}, \quad (5.4)$$

where \vec{n}_i is the inward pointing unit normal of the i^{th} surface, and $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ is the vector from point \vec{r}_j to point \vec{r}_i .

Eq. (5.3) may be solved using the numerical techniques of Ref. 41.

Having solved (5.3), the distribution of illuminance on the task $E_t(\theta, \phi)$ can be determined,

$$E_t(\theta, \phi) = S_t(\theta, \phi) + E_{tr}(\theta, \phi), \quad (5.5)$$

where the angles are defined in the figure 5.1

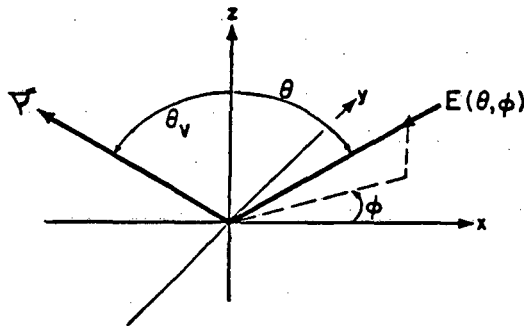


Fig. 5.1

The task illuminance is represented as the sum of a direct part S_t from the light sources in the room and a reflected part E_{tr} from the surfaces in the room. Explicit examples of illumination for simple geometries such as spherical cylindrical and planar rooms are given in the Appendix.

The task illuminance is given by integrating (5.5) over the upper hemisphere. The task background (detail) luminance is obtained by multiplying (5.5) by a background (detail) reflectivity $\beta_b(\theta_v, \theta\phi)$ ($\beta_d(\theta_v, \theta\phi)$) and then integrating over the upper hemisphere to obtain $L_b(L_d)$. These reflectivities are tabulated in Ref. 41. The resultant luminances are used to determine the task contrast which when divided by the task contrast under reference conditions, $C(X_{ref}) = 0.1675$, yields CRF_{phy} .

7. VISIBILITY LEVELS AND VISUAL PERFORMANCE

Thus far, we have described the method for determining the equivalent sphere illumination (ESI) of a given lighting environment. This number can be taken as a figure of merit for the lighting system which accounts for both the quantity and quality of the illumination. Since higher or lower values of ESI lead to higher or lower levels of visibility, it is a useful figure to use when comparing two systems. Furthermore, ESI has the advantage that it represents an easily visualized level of illuminance in a simple geometry. However, ESI does have the undesirable characteristic that the relationship between it and visual performance is complex. This relationship is usually taken into account through recommended levels of ESI for different visual tasks.

In this section, we discuss an alternative method for characterizing lighting environments which is done in terms of visibility levels (VL). The properties of VL compliment those of ESI. That is, on the one hand it does not represent an easily visualized physical characteristic of the illumination but on the other hand it is more directly related to visual performance. We emphasize that the use of VL rather than ESI brings nothing new to the picture since it is only a change of the variable used to describe the picture to one that is more natural for discussions of performance. In what follows, we first discuss visibility levels and the related concept of equivalent contrast. We then go on to present the very sophisticated CIE Model which relates visual performance to visibility level.

We can define visibility level (VL) directly from (3.6) and (3.8). For a given task, we can write the relationship between physical contrast, luminance, visibility, and other parameters (3.6) as

$$\begin{aligned} C_{\text{phy}}(X) &= C(L_b, V; X) \\ &\cong \frac{1}{F(V)} C_t(L_b; X) \quad , \end{aligned} \quad (6.1)$$

where we have used the approximate factorization (3.8) and recall that C_t is the threshold contrast, $C_t(L_b; X) \equiv C(L_b, V_t; X)$. One should be very careful to keep the proper interpretation of equations such as the first equality in (6.1) clearly in mind. The left-hand-side is a physical characteristic of the visual task that could be measured using a luminance meter. The right-hand-side is a psychophysical characteristic of the task and observer which can, in principle, be measured by psychophysical techniques. Since X is fixed on both sides of this equality, the equation provides a relationship between L_b and V for each value of X . We define visibility level as

$$\begin{aligned} \text{VL} &\equiv C_{\text{phy}}(X)/C_t(L_b; X) \\ &\cong 1/F(V) \end{aligned} \quad (6.2)$$

Thus, within the domain of validity of the factorization (3.8), VL is a function of visibility only. Furthermore, since we expect F to be a monotonically decreasing function of V , VL is a monotonically increasing

function of V with a unique inverse. We can therefore make a change of variables and use VL as an independent variable rather than V . Note that we have not introduced anything new since VL , when considered as a function of L_b and X rather than V , is just the relative visibility, (3.15). As such, it can be measured directly with a visibility meter. Also note that the use of VL as an alternative to V rests upon the unassessed validity of the factorization (3.8).

Equivalent contrast (C_{eq}) is a quantity that is closely related to VL . It is defined by

$$VL = C_{eq}/C_t(L_b; X_{ref}) \quad (6.3)$$

$$C_{eq} = C_t(L_b; X_{ref}) \frac{C_{phy}(X)}{C_t(L_b; X)} = C_{eq}(L_b; X) \quad ,$$

where we have emphasized the fact that the definition of C_{eq} implies that it is a function of both L_b and X . As with CRF_{psy} , (3.26), we expect that $C_{eq}(L_b; X)$ is only weakly dependent upon L_b but may have a substantial X -dependence. Note that we can write

$$C_{eq}(L_b; X) = \frac{RV(L_b; X)}{RCS_{ref}(L_b)} C_t(L_0; X_{ref}) \quad , \quad (6.4)$$

where we have used the definition of RV , (3.15), and RCS_{ref} , (3.21). The reason for introducing C_{eq} is that it can be used in a direct empirical determination of VL , see Eq. (6.7).

Recall the meaning of the various symbols in Eq. (6.3) and (6.4): The set of parameters X defines the actual task in the actual lighting environment and the set of parameters X_{ref} defines the visibility reference task (4' spot projected onto a screen for 0.2 seconds) which, by definition, is under reference lighting. As defined, C_{eq} is the contrast that the visibility reference task (recall that contrast is an independent variable for this task) must have if it is to be equal in visibility to the actual task at the specified level of background luminance L_b . For, translating this statement into an equation, we have C_{eq} defined by

$$V(L_b, C_{phy}(X); X) = V(L_b, C_{eq}(L_b; X); X_{ref}) \quad (6.5)$$

We then use the inversion Eq. (3.6) to express the contrasts in terms of V , L_b , and the X 's and the approximate factoring (Eq. (3.8)) on both sides of this equation

$$C_{phy}(X) = C(L_b, V; X) \cong \frac{C_t(L_b; X)}{F(V)},$$

$$C_{eq} = C(L_b, V; X_{ref}) \cong \frac{C_t(L_b; X_{ref})}{F(V)},$$

where we have used the notation C_t for contrast threshold that was introduced in Eq. (3.13). Solving these two equations for $F(V)$ and equating the result yields Eq. (6.3) for C_{eq} .

In using C_{eq} , one must be very careful to specify what lighting environment is used to determine its value. In order to define an approximation procedure for determining C_{eq} and hence VL , it is useful to introduce another equivalent contrast whose value is determined in reference lighting rather than actual lighting, then we determine C_{eq}^{ref} given by

$$C_{eq}^{ref} = C_t(L_b; X_{ref}) \frac{C_{phy}(X^{ref})}{C_t(L_b; X^{ref})} = C_{eq}(L_b; X^{ref}) \quad ,$$

where X^{ref} describes the actual task under reference lighting conditions. C_{eq}^{ref} is a measure of the intrinsic difficulty of the task, see (6.6), since all contrasts are evaluated under reference lighting.

We can relate C_{eq} to C_{eq}^{ref} in the following way. We solve (3.27), with \bar{X} replaced by X , for $C_{phy}(X)/C_t(L_b; X)$ and substitute the result into (6.3) and express $C_{phy}(X^{ref})/C_t(L_b; X^{ref})$ in terms of C_{eq}^{ref} defined above. We then have

$$C_{eq}(L_b; X) = CRF_{psy}(L_b; X) C_{eq}(L_b; X^{ref}) \quad , \quad (6.6)$$

where the second factor is C_{eq}^{ref} for the task X . The interpretation of this relationship is that the first factor gives the dependence of C_{eq} on the actual lighting environment while the second factor specifies the intrinsic difficulty of the visual task. However, this interpretation is only approximately true. While the second factor is independent of the actual lighting environment, it is not clear that the first factor is even approximately independent of task difficulty. Quite apart from this interpretation, we can use the various approximations for CRF_{psy} , e.g., $CRF_{psy}(L_b; X) \cong CRF_{phy}(X)$, that were treated in Sections III and IV when C_{eq} is factored as in (6.6) and in that way take into account the effects of the lighting environment.

In that approximation, C_{eq} can be determined by measuring a ratio of threshold contrasts under reference lighting conditions which is desirable since such ratios are reported to be rather insensitive to differences between observers and differences between methods of determining thresholds. In those cases in which the task cannot be placed under reference lighting, e.g., a highway scene, then C_{eq} must be determined directly without use of (6.6). Such measurements have the additional complicating feature of comparing the two tasks in different lighting environments which leads to additional uncertainties.

Thus far, the discussion has been in the context of the so-called "direct method" of determining thresholds (see discussion before Eqs. (3.1) and (3.20)). In this method, the observer is essentially

required to make a "yes" or "no" response to each presentation of the visual task. However, there are many visual tasks for which this method is inappropriate. For such tasks, the "indirect method" is the method of choice. In this method, Eq. (6.5) provides a basis for an empirical determination of C_{eq} or C_{eq}^{ref} if reference lighting is used. In order to use (6.5), we must first redefine what is meant by "equality of visibility" for the two, possibly very different, visual tasks represented by X and X_{ref} . This is done by bending the notion of visibility a bit and redefining the threshold visibility. Both tasks are assumed to equal in visibility when they are both at threshold, where threshold is now defined to be that combination of luminance and contrast in which the task is just "barely visible" to the observer. Here, "barely visible" means being just able to perform the task whatever it may be. The contrast on the right-hand-side of (6.5) is an independent variable and it is adjusted by the observer to the threshold value $C'_t(L_b; X_{ref})$, where the prime indicates the new definition of threshold. The contrast $C_{phy}(X)$ on the left-hand-side of (6.5) is reduced by the observer using a visibility meter to its threshold value $C'_t(L_b; X)$. Since C_{eq} , (6.3), scales linearly with the physical contrast of the task, we can use (6.5) with the new definition of equal visibility to write

$$C_{eq}(L_b; X) = \frac{C_{phy}(X)}{C'_t(L_b; X)} \times C'_t(L_b; X_{ref}) \quad .$$

The first factor in this expression is obtained from the setting of the visibility meter, see (3.16), and the second factor is known from the setting of the visibility reference task at the newly defined threshold. In practice, the contrast of the visibility reference task is varied by viewing it through a visibility meter also. The basic assumption in this method is that the ratio of threshold contrasts in this expression will be insensitive to the particular characteristics of the individual observer as well as the new definition of threshold. The evidence supporting this assumption seems to be very weak. The new method of determining thresholds has been reported⁴² to be unreliable and unduly influenced by extraneous variables. A direct comparison of much of the data reported in a study⁴³ which was intended to show that C_t^i is proportional to C_t can not be made. The data on threshold contrasts of photographs⁴³ which can be used in a comparison shows a typical cloud of data points with a range of about a factor of 10 around a linear relationship between the two threshold contrasts. These uncertainties show up in one study⁴⁴ in which the above system was compared with another visibility measurement system. The two procedures lead to recommended levels of illumination that differ by as much as a factor of 20 up or down in an arbitrary fashion dependent upon task.

We now turn to the problem of assigning a numerical value to VL. Although Ref. 4 gives several different expressions for VL, we will use here only (6.3) and express the threshold contrast in the

denominator in terms of RCS_{ref} and use (6.6) for C_{eq} to get the penultimate expression

$$VL = CRF_{psy}(L_b; X) C_{eq}(L_b; X^{ref}) \frac{RCS_{ref}(L_b)}{C_t(L_0; X_{ref})} \quad , \quad (6.7)$$

where $C_t(L_0; X_{ref})$ is the normalization factor for RCS_{ref} with the measured value of 0.0923. We proceed by using one of the approximate expressions for CRF_{psy} that were presented in Section IV, e.g., $CRF_{psy} \cong TAF \times DGF \times CRF_{phy}$. We next take advantage of the large amount of circularity in the above definitions for using (6.4), we see that the remaining factors in (6.7) are just $RV(L_b; X^{ref})$ and that the factor RCS_{ref}/C_t cancels a similar factor in the denominator of C_{eq}^{ref} , see (6.4). Thus, if we change both of these factors, we will not change the value of VL. We therefore replace RCS_{ref} by the generalized RCS function of (3.29) and, in order to account for an age-dependence in C_t , we replace $C_t(L_0; X_{ref})$ by $mC_t(L_0; X_{ref})$, where m is an age-dependent visibility level multiplier given by

$$\begin{aligned} m &= 1.00 + 0.00795 (A-20) \quad , \quad 20 < A < 42 \quad , \\ &= 1.175 + 0.0289 (A-42) \quad , \quad 42 < A < 64 \quad , \\ &= 1.811 + 0.1873 (A-64) \quad , \quad 64 < A < 80 \quad , \end{aligned} \quad (6.8)$$

where A is the observers age in years. Note that this replacement must be done both in (6.7) for VL and in (6.4) for C_{eq}^{ref} in

order to be consistent. This last modification would seem to have dubious value unless it leads to a situation in which the empirical determination of C_{ea}^{ref} is facilitated through the measurement of $RV(L_b; X)$ in (6.4). Putting these modifications together, we obtain the ultimate expression for VL

$$VL = TAF \times DGF \times CRF_{phy} \times C_{ea}^{ref} \times \frac{RCS}{0.0932m} \quad (6.9)$$

where the above text defines to the various factors.

We now turn to a discussion of visual performance and its relation to visibility levels. It has become customary to represent performance data in terms of error functions of the stimulus variable. These representations are referred to as "ogives" in the literature. We first present a possible justification for this representation. We then, in turn, discuss the visual performance reference task and visual performance in general.

A pseudojustification for the use of error functions to represent performance data can be based upon the assumption that performance thresholds are normally distributed in the population being tested. Consider a test in which a person is asked to give a response to some stimulus S . Let $P(S)$ be the average over the population of the probability that the testee, will give a correct response to the stimulus S , after correction for random guesses. For simplicity, we assume that S ranges from large negative values for no stimulus to large positive values for a strong stimulus and that $P(S)$ ranges from zero to

one over this range. These assumptions can be easily relaxed. We assume that the individual testee can be characterized by a threshold S_t below which there is no response and above which there is. We further assume that these thresholds are distributed normally over the population of testees. Thus,

$$p(S_t) = \frac{1}{\sqrt{2\pi} \sigma_t} e^{-\frac{(S_t - \bar{S}_t)^2}{2\sigma_t^2}} \quad (6.10)$$

is the probability density that a randomly selected individual would have a threshold between S_t and $S_t + dS_t$. In this expression \bar{S}_t is the average stimulus threshold and σ_t is its variance. In the test, all members of the population with thresholds less than S will respond to the stimulus S . Thus

$$\begin{aligned} P(S) &= \int_{-\infty}^S \frac{dS_t}{\sqrt{2\pi} \sigma_t} e^{-\frac{(S_t - \bar{S}_t)^2}{2\sigma_t^2}} \\ &= \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{S - \bar{S}_t}{\sqrt{2} \sigma_t} \right) \right] \quad , \end{aligned} \quad (6.11)$$

where

$$\operatorname{erf} \left(\frac{x}{\sqrt{2}} \right) = \int_{-x}^x \frac{dS}{\sqrt{2\pi}} e^{-S^2/2} \quad (6.12)$$

is the standard error function of probability theory. Therefore, a population with normally distributed thresholds will perform on a test

according to (6.11). This conclusion rests upon the assumption that individual thresholds were sharp, i.e., on or off, at S_t and this assumption is not borne out by the experimental observations. Individual response data for visual tests exhibit a shape similar to (6.11) but with values of \bar{S}_t and σ_t differing from individual to individual. Thus, all that can be said of (6.11) is that it provides a two parameter fit to the observed data. For future reference, we define the function

$$O(z) \equiv \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{z}{\sqrt{2}} \right) \right] \quad (6.13)$$

which will be used to represent data.

The visual performance reference task is an easily reproducible task that is intended to simulate realistic visual tasks. The target consists of 5 Landolt rings with one at the center and four more placed at the four points of the compass about the central one. The Landolt rings have an outer diameter of 20' angular size, are 4' wide, and have a 4' break. The four outer ones are located at a fixed angular distance from the center and this parameter can be varied. The breaks in the rings are located at one of the 8 major and minor points of the compass. The observer is told that no more than one of the outer rings will have its break at the same position of that of the central ring. The task is to identify which one of the outer rings, if any, has the same orientation as the central one. This can be done as a function of contrast, luminance, exposure duration, and

angular separation between the outer and central rings. The output of such an experiment is the probability of success as a function of these parameters and the characteristics of the observer.

The visibility level of the visual performance reference task can be established in a separate measurement. It turns out that the threshold contrast for this task is proportional to that of the visibility reference task (4' spot) so that, see (6.3),

$$VL = K \frac{C_{\text{phy}}(X)}{C_t(L_b, X_{\text{ref}})} \sim RCS_{\text{ref}}(L_b), \quad (6.14)$$

where K is independent of luminance and contrast but does depend upon the observer. The visibility level can therefore be varied in a known way by changing $C_{\text{phy}}(X)$ and/or L_b . However, see Eq. (6.18) below.

The measure of the stimulus is taken to be $\log VL$ and, following the notation in the literature, we define

$$\alpha \equiv \log VL. \quad (6.15)$$

Visual performance (VP) data are presented in terms of relative visual performance (RVP) as

$$VP = VP_{\text{max}} \times RVP, \quad (6.16)$$

where RVP ranges from zero to one as a function of α . The maximum value of VP, VP_{max} , may take on any value depending upon what has

been taken as the measure of performance. For example, if performance is taken as the probability of success in a fixed amount of time then VP_{\max} can be any positive number less than or equal to one. In another case, performance might be taken as the speed with which a given task can be accomplished and in this case, VP_{\max} would be an inverse time denoting the fastest speed.

Relative visual performance is expressed as the sum of contributions from three separate processes, (referred to as process 1, 2, or 3)

$$RVP = w_1 P_1 + w_2 P_2 + w_3 P_3 \quad , \quad (6.17)$$

where the w_i are weighting factors which are independent of α and the P_i are α -dependent functions. The three terms in (6.17) are intended to represent the contributions to RVP from the visual sensory process (subscript "1"), the maintenance of steady ocular fixation (subscript "2"), and the initiation and control of ocular saccades (subscript "3"). They are discussed below. While it is certainly true that these three processes are important parts of the successful execution of a visual task and it is claimed that they can be studied separately, there seems to be no justification for the form chosen in (6.17). A more conservative interpretation would have (6.17) as a functional form for RVP which is then to be fitted to experimental data.

Process 1 is the visual sensory process. Apparently by definition, its characteristics may be observed through experiments on the visual performance reference task. These experiments define P_1 , by taking $w_1 = 1$ and $w_2 = w_3 = 0$ in (6.17). The data are there summarized by the expressions

$$P_1(\alpha) = 0 \left(\frac{\alpha - \bar{\alpha}_1}{\gamma_1} \right), \quad (6.18)$$

where 0 is defined in (6.9) and $\bar{\alpha}_1$, and γ_1 are fitted parameters that depend upon the difficulty of the task, exposure time, and the characteristics of the observer. Study of the data indicates that $\bar{\alpha}_1$ and γ_1 are related by

$$\gamma_1 = 0.145 + 0.278(\bar{\alpha}_1 - 0.050) \quad (6.19)$$

where the quantity in the parenthesis is taken to be zero if the expression is negative. The threshold $\bar{\alpha}_1$ ($P_1(\bar{\alpha}_1) = 1/2$) is then fit to the data.

There is an additional $\bar{\alpha}_1$, dependence as well as a dependence on the age of the observer that is not indicated in (6.18). These modifications come about through the use of the generalized RCS function (3.29) rather than RCS_{ref} in (6.10). The parameter S is given by

$$\log S = 0.5900 - 0.6235 \log d - 0.1980 \tilde{X} - s, \quad (6.20)$$

where d is the task detail size in arc minutes, $\tilde{X} = 1/2 \tilde{S}F$ is the angular radius of the effective field of visual search, and s is a parameter that depends upon the age of the observer. Fits to data yield the following results

$$\log \tilde{X} = (\bar{\alpha}_1 - 0.550)/1.355$$

$$s = 0 \quad , \quad 20 < A < 44, \quad (6.21)$$

$$= 0.00406(A-44) \quad , \quad 44 < A < 64,$$

$$= 0.0812 + 0.00667(A-64) \quad , \quad 64 < A < 80,$$

where A is the observer age in years. The parameter t in (3.29) is given by

$$\log t = 0 \quad , \quad 20 < A < 30,$$

$$= -0.01053(A-30) \quad , \quad 30 < A < 44,$$

$$= -0.1474 - 0.0134(A-44) \quad , \quad 44 < A < 64, \quad (6.22)$$

$$= 0.4154 - 0.0175(A-64) \quad , \quad 64 < A < 80,$$

Thus, P_1 acquires an additional $\bar{\alpha}_1$ dependence through \tilde{X} in (6.20) and an age dependence through s and t when these parameters are used in the generalized RCS function (3.29) which is then used to determine VL (6.9).

Processes 2 and 3 come in to play when realistic visual tasks are studied. The data are fitted to functions of the form

$$P_i(\alpha) = 0 \left(\frac{\alpha - \bar{\alpha}_i}{\gamma_i} \right), \quad i = 2, 3, \quad (6.23)$$

with w_i , $\bar{\alpha}_i$, and γ_i fitted parameters the results of this fit are

$$w_1 = 1 - w_2 - w_3, \quad ,$$

$$\begin{aligned} w_2 = w_3 &= 0.100 + 0.0683 \bar{\alpha}_1, & \bar{\alpha}_1 < 0.6, & (6.24) \\ &= 0.141 + 0.627(\bar{\alpha}_1 - 0.600), & \bar{\alpha}_1 > 0.6, & \end{aligned}$$

$$\bar{\alpha}_2 = 0.107, \quad \gamma_2 = 0.180, \quad (6.25)$$

and

$$\begin{aligned} \bar{\alpha}_3 &= 0.107 + (0.678 \log \tilde{X} + 0.007) + (0.350 \log \tilde{X} - 0.050), \\ \gamma_3 &= 0.180, \quad (6.26) \end{aligned}$$

where only positive values of the quantities in parentheses in (6.23) are to be taken.

There remains the overall scale factor VP_{\max} in (6.16). This has been fitted to a function of $\bar{\alpha}_1$, when the measure of performance is taken to be probability of success. The result is

$$\log VP_{\max} = -.0408(\bar{\alpha}_1 - 0.284) - 0.175(\bar{\alpha}_1 - 0.6) \quad , \quad (6.24)$$

where again only positive values of the two expressions in parentheses are to be taken.

The three processes treated thus far are "critical" visual processes in that they are strongly dependent upon the visibility level. To obtain a measure of realistic task performance, one must include noncritical effects that are not so strongly affected. This leads to the notions of task performance (TP) and relative task performance (RTP) which are related by

$$TP = TP_{\max} \times RTP \quad (6.28)$$

where TP_{\max} is the maximum task performance and plays the same role as VP_{\max} in (6.16). The expression for RTP that is to be fit to data is

$$RTP = (1-w_4)RVP + w_4P_4, \quad (6.29)$$

where the first term, with RVP given by (6.17), represents the critical visual components of the task and the second term represents the noncritical components. In this expression w_4 is a parameter that is fitted to data. The function $P_4(\alpha)$ is given by

$$\begin{aligned}
 P_4 &= 0.3 P_{4.1} + 0.7 P_{4.2} \quad , \quad \alpha < 0.4 \quad , \\
 &= 1 \quad , \quad \alpha > 0.4 \quad , \quad (6.30)
 \end{aligned}$$

with the function $P_{4.i}$ given by

$$P_{4.i}(\alpha) = 0 \left(\frac{\alpha - \bar{\alpha}_{4.i}}{\gamma_{4.i}} \right) \quad , \quad i = 1, 2, \quad (6.31)$$

and the parameters given by

$$\begin{aligned}
 \bar{\alpha}_{4.1} &= -0.150 \quad , \quad \gamma_{4.1} = 0.145, \\
 \bar{\alpha}_{4.2} &= -0.700 \quad , \quad \gamma_{4.2} = 0.145. \quad (6.32)
 \end{aligned}$$

This concludes the description of parameterization of the data.

We review our presentation by considering the multitude of parameters that must be or have been fit to data. There are essentially five free parameters that must be fit to new data. They are: K which scales VL , see (6.14), (this could be measured independently), $\bar{\alpha}_1$, the process 1, threshold, see (6.18), TP_{\max} which scales TP , see (6.25), and w_4 which weights the critical and noncritical visual components. The expressions have often been fit to data that is represented by five or fewer data points. These fits are certainly not tests of the soundness of the model.

We count 53 fixed parameters in all and it is worthwhile to point out where they are. There are 3 parameters in (6.19) since it represents two straight lines with one common point (we shall use this

method of counting whenever a fit is a set of straight line segments). We then have 3 parameters in (6.20), 6 in (6.21), 6 in (6.22), 10 in (6.24), 2 in (6.25), 6 in (6.26), 4 in (6.27), 1 in (6.29), 3 in (6.30), 4 in (6.32), and 5 in (6.8).

One can only conclude from this section that this representation of visual performance data is excessively heavy. It is hard to believe that one must use a 58 parameter fit to represent smooth curves. At the very least, statistical tests should be run to test the significance of each and every one of these parameters.

8. EPILOGUE

The analytical description of visual performance data presented in the preceding sections must be described as being overly complex, incomplete in its choice of fundamental variables, and lacking a quantitative empirical basis even though it represents the current state of the art. We consider these three points in this concluding section of our review.

The complexity of the analytical description is due to two major causes. On the one hand, the choice of functions used to represent visibility appear to have changed with time leading to many different symbols for the same quantity. On the other hand the functions chosen to represent visual performance can be made more appropriate.

There are two basic functions that have been used to represent visibility in the analytic description. They are the physical contrast $C_{\text{phy}}(X)$ and the threshold contrast $C_t(L;X)$ of the task X . The first function is a purely physical characteristic of the task while the second one is a purely psychophysical characteristic of the task and observer. Recall from Sec. VI that C_t is a special case of contrast as a function of adaptation luminance, visibility, and visual task with a rather flexible definition of threshold. The six functions CRF_{phy} , RCS , RV , CRF_{psy} , VL , and C_{eq} are defined in terms of these two basic functions. The definitions are summarized in the following formulae:

$$\text{CRF}_{\text{phy}}(X) = C_{\text{phy}}(X)/C_{\text{phy}}(X^{\text{ref}}) \quad , \quad (7.1)$$

$$\text{RCS}(L;X) = C_t(L_0;X)/C_t(L;X) \quad , \quad (7.2)$$

$$\text{RV}(L;X) = C_{\text{phy}}(X)/C_t(L;X) = [C_{\text{phy}}(X)/C_t(L_0;X)] \times \text{RCS}(L;X) \quad , \quad (7.3)$$

$$\text{CRF}_{\text{psy}}(L;X) = [C_{\text{phy}}(X)/C_t(L;X)] \times [C_t(L;X^{\text{ref}})/C_{\text{phy}}(X^{\text{ref}})] \quad , \quad (7.4)$$

$$= \text{RV}(L;X)/\text{RV}(L;X^{\text{ref}}) \quad ,$$

$$= \text{CRF}_{\text{phy}}(X) \times [C_t(L_0;X^{\text{ref}})/C_t(L_0;X)] \times$$

$$\times [\text{RCS}(L;X)/\text{RCS}(L;X^{\text{ref}})] \quad ,$$

$$\text{VL}(L;X) = C_{\text{phy}}(X)/C_t(L;X) \quad , \quad (7.5)$$

$$= \text{RV}(L;X) \quad ,$$

$$= \text{CRF}_{\text{psy}}(L;X) \times \text{RV}(L;X^{\text{ref}}) \quad .$$

$$C_{\text{eq}}(L;X) = [C_{\text{phy}}(X)/C_t(L;X)] \times C_t(L;X_{\text{ref}}) \quad , \quad (7.6)$$

$$= \text{VL}(L;X) \times (C_t(L;X_{\text{ref}})) \quad ,$$

$$= \text{RV}(L;X) \times C_t(L;X_{\text{ref}}) \quad ,$$

$$= \text{RV}(L;X) \times \frac{C_t(L_0;X_{\text{ref}})}{\text{RCS}(L;X_{\text{ref}})} \quad ,$$

where recall that

- L = adaptation luminance of task,
 X = actual task in actual lighting,
 x^{ref} = actual task in reference lighting,
 χ_{ref} = visibility reference task.

It is clear from the many relationships among these functions that there are many new names for old quantities and, unless there is a pressing need for such a redefinition, one would be better off not introducing them. For example, a case can be made for the introduction of C_{eq} . This would be based upon the fact that it is a more readily measured characteristic of the task (due to cancellations in the ratio of the two C_t 's) than C_t itself. However, this claim must be substantiated with empirical evidence. If the analytical description were presented in terms of C_{phy} and C_t alone, it could be done in half the space used here and understood in perhaps a quarter of the time. At this point, the following quotation seems apt, "When the answer to a problem is too complicated, what we have is not an answer but a new problem." . . . David Rockefeller, chairman, Chase Manhattan Bank.

The principal variables describing the visual performance of a carefully defined visual task in a laboratory setting are time (or its reciprocal--speed of performance), luminance, contrast, and acuity.⁴⁵ In addition, the other parameters in X such as the age of

the observer population must be specified. The analytical description attempts to relate the first three principal variables, i.e., visual performance is related to contrast and adaptation luminance, and acuity is given a secondary role in the other parameters. There is some acuity-dependence in the generalized RCS function that is used to calculate VL, (6.9). However, as indicated in the discussion of that equation, if the various quantities are used consistently, then this dependence cancels out of the expression and all the acuity dependence lies in $RV(L_b; X)$ of (6.4). We feel that the proper relationship is one of performance as a function of luminance, contrast and acuity with the other parameters playing a secondary role. This relationship has not yet been fully explored experimentally.

For example, there is reported in the literature⁴⁶ the very interesting result that the acuity threshold is independent of luminance at constant visibility level. That is, if the luminance is reduced and the contrast increased in such a way that the visibility level remains fixed then the acuity threshold is a constant. This is reported to be true at visibility levels of 10 and 100. Since this result contradicts much of the previous work on acuity thresholds, it certainly suggests further studies along these lines.

The analytical description of visual performance with all its complexities discussed in Section VI indicates that an unnecessarily large number of functions are being used to represent the data. It is no vindication of this description to show that it can represent

various sets of smoothed experimental data. The basic question is one of the efficiency of the description in terms of the level of empirical knowledge. Here again it is clear that further experiments should be done in order to develop a more concise description.

The weak empirical basis of the analytical description is a failing that we have alluded to in many places in our review. By this we mean publicly available, published data⁴⁷ presented with proper and conventional statistical measures of precision such as standard errors and confidence intervals for parameters. Without such data, it is impossible to evaluate the various assumptions made in constructing the description. For example, to what extent does the contrast sensitivity factor as in (3.10). This is certainly not an exact statement and some indication of the magnitude of the error made in using it and its domain of validity would put the subsequent analysis on a firmer footing.

A quantitative understanding of this point along with the many other unquantified assumptions that have been employed in relating visual performance and psychophysical variables to the physical characteristics of the task and its environment is a fundamental step towards establishing this science. In the absence of a more convincing empiricism it is likely that decisions relating visual performance to physical variables will continue to be somewhat a matter of opinion.⁴⁸⁻⁵⁰

9. ACKNOWLEDGEMENTS

This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Building Energy Research and Development, Building Equipment Division of the U.S. Department of Energy under Contract No. DE-AC03-7600098.

We wish to thank Dr. Robert Clear for his many incisive and helpful comments.

10. REFERENCES

1. RQQ Report No. 4, A Method of Evaluating the Visual Effectiveness of Lighting Systems. Ill. Eng. 65, 504 (1970).
2. Publication CIE No. 19/1, A Unified Framework of Methods for Evaluating Visual Performance Aspects of Lighting. Technical report of CIE Committee TC-3.1, International Commission on Illumination, Paris (1972).*
3. Publication CIE No 19/2, An Analytic Model for Describing the Influence of Lighting Parameters upon Visual Performance. Technical report of CIE Committee TC-3.1, International Commission on Illumination, Paris (1980).*
4. Publication CIE (Draft No. 5), Implementation Procedures for Evaluating Visual Performance Aspects of Lighting. Technical report of CIE Committee TC-3.1, International Commission on Illumination, Paris (1976).*
5. L. T. Troland, An Analysis of the Literature Concerning the Dependency of Visual Functions upon Illumination Intensity. Trans. IES 26, 107 (1931).
6. Anon., Lighting Principles and Practice--A Bibliography of Papers Dealing with Lighting Application. Ill. Eng. 36, 577 (1941).

*Publications of the CIE can be obtained from: Dr. J. L. Tech, Room B-308 MET. Building, National Bureau of Standards, Washington, D.C. 20234. Telephone: (301) 921-3864.

7. P. Moon and D. E. Spencer, Visual Data Applied to Lighting Design. J. Opt. Soc. Am. 34, 605 (1944).
8. C. L. Crouch, The Relation Between Illumination and Vision. Ill. Eng. 40, 747 (1945); Discussion, Ill. Eng. 41, 565 (1946).
9. Anon., The Design of the Visual Field. Trans. IES (London) 18, 225 (1953).
10. Anon., Why we are where we are . . . Lighting Research. Ill. Eng. 51, 139 (1956).
11. H. C. Weston, Rationally Recommended Illumination Levels. Trans. IES (London) 26, 1 (1961).
12. H. W. Bodmann, Quality of Interior Lighting Based on Luminance. Trans. IES (London) 32, 22 (1967).
13. H. W. Bodmann, Visibility Assessment in Lighting Engineering. J. IES 2, 437 (1973).
14. A. W. Levy, The CIE Visual Performance System. Lighting Res. and Tech. 10, 19 (1978).
15. M. Luckiesh and F. K. Moss, The Science of Seeing. D. Van Nostrand Co., New York, 1937.
16. P. Moon, The Scientific Basis of Illumination Engineering. McGraw-Hill Book Co., Inc., New York, 1936. Reprinted by Dover Publications, Inc., New York, 1961.
17. M. Luckiesh. Light, Vision and Seeing. D. Van Nostrand Co., New York, 1944.
18. K. H. Ruddock, The Organization of Human Vision for Pattern Detection and Recognition. Reports Prog. Phys. 40, 603 (1977).

19. L. T. Troland, The Principles of Psychophysiology (3 vol.) D. Van Nostrand Co., New York, 1929-1932.
20. T. N. Comsweat, Visual Perception. Academic Press, New York, 1970.
21. F. J. Roethlisberger and W. J. Dickson, Management and the Worker. Harvard University Press, Cambridge, 1956.
22. D. K. Baker, Pre-Experimental Set in Distraction Experiments. J. Gen. Psychol. 16, 471 (1937).
23. H. R. Blackwell, Contrast Thresholds of the Human Eye. J. Opt. Soc. Am. 36, 624 (1946).
24. C. E. Ferree and G. Rand, Intensity of Light and Speed of Vision Studied with Special Reference to Industrial Situations--Part II. Trans. IES 23, 507 (1928).
25. See any text on statistics such as Introduction to the Theory of Statistics by A. M. Mood and F. A. Graybill, McGraw-Hill Book Co., Inc., New York, 1963, Chapter 11.
26. M. Luckiesh, General Considerations and Definitions of Glare and Visibility. Trans. IES 22, 542 (1927).
27. H. R. Blackwell, Development of Procedures and Instruments for Visual Task Evaluation. Ill. Eng. 65, 267 (1970).
28. H. R. Blackwell, Measurement of the Contrast Rendition Factor for Pencil Handwritten Tasks. Ill. Eng. 65, 199 (1970).
29. S. K. Guth and J. F. McNelis, Visual Performance: A Comparison in Terms of Detection of Presence and Discrimination of Detail. Ill. Eng. 63, 32 (1968).

30. S. Hecht, A Theory of Visual Intensity Discrimination. J. Gen. Physiology 18, 767 (1935).
31. L. L. Holladay, The Fundamentals of Glare and Visibility, J. Opt. Soc. Am. 12, 271 (1926); Action of a Light Source in the Field of View in Lowering Visibility. J. Opt. Soc. Am. 14, 1 (1927).
32. W. S. Stiles, The Effect of Glare on the Brightness Difference Threshold. Proc. Roy. Soc. 104B, 322 (1929); The Scattering Theory of the Effect of Glare on the Brightness Difference Threshold. Proc. Roy. Soc. 105B, 131 (1929).
33. P. W. Cobb and F. K. Moss, Glare and the Four Fundamental Factors in Vision. Trans. IES 23, 1104 (1928).
34. J. J. Vos and M. A. Bouman, Disability Glare: Theory and Practice. Comptes Rendus, 14th Session of the CIE, Vol. B, 298 (1959).
35. P. Moon and D. E. Spencer, The Specification of Foveal Adaptation. J. Opt. Soc. Am. 33, 444 (1943).
36. P. Moon and D. E. Spencer, the Visual Effect of Non-Uniform Surrounds. J. Opt. Soc. Am. 35, 233 (1945).
37. P. T. Stone and S. D. P. Groves, Discomfort Glare and Visual Performance. Trans. IES (London) 33, 9 (1968).
38. G. A. Fry, Measurement, Specification and Prediction of Transient States of Adaptation. Ill. Eng. 59, 453 (1964).
39. R. M. Boynton and N. D. Miller, Visual Performance Under Conditions of Transient Adaptation. Ill. Eng. 58, 541 (1963).

40. R. M. Boynton, E. J. Rinalducci, and C. Sternheim, Visibility Losses Produced by Transient Adaptational Changes in the Range from 0.4 to 4000 Footlamberts. *Ill. Eng.* 64, 217 (1969).
41. RQQ Report No. 5, The Predetermination of Contrast Rendition Factors for the Calculation of Equivalent Sphere Illumination. *J. IES* 2, 149 (1973).
42. H. R. Blackwell, Studies of Psychophysical Methods for Measuring Visual Thresholds. *J. Opt. Soc. Am.* 42, 606 (1952).
43. H. R. Blackwell, Further Validation Studies of Visual Task Evaluation. *Ill. Eng.* 59, 627 (1964).
44. A. A. Eastman and S. K. Guth, Comparison of Visibility Measurement Systems. *Ill. Eng.* 55, 176 (1960).
45. P. W. Cobb and F. K. Moss, Four Fundamental Factors in Vision. *Trans. IES* 23, 496 (1928).
46. H. R. Blackwell and O. M. Blackwell, The Effect of Illumination upon the Performance of Different Visual Tasks. *Ill. Eng.* 64, 143 (1968).
47. S. K. Guth and J. F. McNelis, Visual Performance-Subjective Differences. *Ill. Eng.* 64, 723 (1969).
48. E. Simonson and J. Brozek, Effects of Illumination Level on Visual Performance and Fatigue. *J. Opt. Soc. Am.* 38, 384 (1948).
49. M. A. Tinker, Brightness Contrast, Illumination Intensity, and Visual Efficiency. *Am. J. of Optometry and Arch. of Am. Acad. of Optometry* 36, 221 (1959).
50. P. R. Boyce, Is Equivalent Sphere Illumination the Future? *Ltg. Res. and Tech.* 10, 179 (1978).

11. GLOSSARY

Actual lighting. Referring to the lighting system under evaluation.

Adaptation luminance. Defined by the following cases: (1) Visual field of uniform luminance L_b except for spatially and/or temporally small details, then adaptation luminance equals L_b . (2) Visual field with details that are repeated with high spatial frequency then adaptation luminance equals the average luminance of the visual field. (3) In the general case it is not well defined. The text deals only with case 1 situations.

Candela/meter² Unit of luminance equal to 0.292 foot lamberts.

Color Temperature Characterization of the color of a light source by the temperature of a black body of equal chromaticity.

Contrast In a visual task that can be characterized by a task detail luminance L_d and task background luminance L_b , then contrast C is given by

$$C = |L_b - L_d| / L_b .$$

Contrast rendering factor, CRF The ratio of contrast of the reference task under actual lighting conditions to that under reference conditions.

Contrast sensitivity, CS The reciprocal of the contrast threshold.

Contrast threshold, C_t The value of contrast at which the visibility of a given task equals $V_{ref} = 50$ percent. C_t depends upon the task background luminance and the other parameters defining the task.

Disability glare factor, DGF The ratio of the relative visibility of a task in the presence of glare in the task surround to that in the absence of glare.

Effective relative contrast sensitivity, RCS_{eff} The value of the reference relative contrast sensitivity function after corrections have been made for actual lighting conditions, equals the right-hand-side of (3.19).

Effective task luminance The value, L_e , of the task background luminance under reference conditions that produces the same visibility as that produced by the actual lighting conditions.

Luminance factor, β The ratio of task background luminance to task illumination, see (2.1).

Lux The unit of illuminance = 1 lumen/meter², also called a meter-candle. 1 lux = 0.0929 foot-candle.

Other parameters, X The parameters other than task background luminance and contrast that define a visual task.

Physical contrast rendering factor, CRF_{phy} See contrast rendering factor.

Psychophysical contrast rendering factor, CRF_{psy} The ratio of the relative visibility of the reference task under actual lighting conditions to that under reference conditions.

Reference conditions The lighting conditions under which the illuminance is homogeneous, isotropic, unpolarized, and has a color temperature of 2856 K also called sphere lighting.

Reference population of observers The group of observers that are between 20 and 30 years old and who are classified as having normal vision.

Reference relative contrast sensitivity, RCS_{ref} The relative contrast sensitivity measured using the reference population of observers reference conditions, and where the visual task is the detection of presence of a disk whose diameter is 4 arc minutes which is flashed for 1/5 second on an otherwise uniform background.

Relative contrast sensitivity, RCS The contrast sensitivity normalized to a value of one at a task background luminance of 100 cd/m^2 .

Relative visibility, RV The ratio of the contrast of a visual task to its contrast threshold.

Task background luminance, L_d The luminance of the visual task that illuminates the fovea of the observer's eyes upon which the small task details are superimposed.

Task detail luminance, L_b The luminance of the small details of the visual task.

Task illumination, E_t The total luminous flux incident upon the task.

Task surround luminance, L The luminance of the observers' field of view that illuminates his retina not including the fovea.

Transient adaptation factor, TAF. The ratio of the relative visibilities of a visual task under dynamic and static viewing conditions.

Veiling luminance, L A uniform luminance that is superimposed over an observer's entire field of vision.

Visibility, V Percent correct executions of a given visual task under specified conditions.

Visibility meter A device for reducing the contrast of a visual task without altering the task background luminance or the other parameters.

Visual Performance The speed and accuracy with which a visual task is performed.

Visual task Any activity that has the acquisition of visual information as the dominant process.

12. APPENDIX A. ILLUMINANCE DISTRIBUTIONS IN SIMPLE GEOMETRIES

In this appendix, we present illuminance distributions calculated from Eq. (5.3) for several simple geometries. These distributions are useful in their own right and also serve as a check on numerical solutions which must be used for realistic geometry. The simple geometries that we treat here are characterized by their symmetry which is spherical, cylindrical, or plane. The case of spherical geometry is the simplest case and leads to sphere lighting which is the basis of ESI. The cases of cylindrical and plane geometry have sufficient symmetry that we can also write down a solution to (5.3). We first present the results of our calculations and then, in the latter half of this appendix, present some of the mathematical details of their derivations. In all cases, we will treat sources that have unit luminous intensity, and therefore they should be multiplied by the strength of the source for other situations.

For spherical geometry there is only one surface being illuminated--the interior of the sphere. We denote the radius of the sphere by R and assume that there is an isotropic point source of light located a distance a from the center of the sphere on the z -axis. The polar angle θ is measured from the z -axis. The distribution of illuminance on the surface of the sphere is then given by

$$E(\theta) = \frac{1}{4\pi R^2} \left\{ \frac{1 - \alpha \cos\theta}{(1 + \alpha^2 - 2\alpha \cos\theta)^{3/2}} + \frac{\rho}{1 - \rho} \right\}, \quad (\text{A.1})$$

where $\alpha = a/R$ and ρ is the reflectivity of the surface. The first term in (A.1) is the direct component from the light source, and the second term is the isotropic reflected component. Note that if the source is located at the center of the sphere, $\alpha = 0$, then the distribution is isotropic and the only effect of reflections is to amplify the intensity by a factor $1/(1-\rho)$.

A hemisphere with a perfectly black floor is a geometry that is just as simple as the full sphere but is certainly a closer approximation to a real room. We consider such a hemisphere of radius R with an isotropic point source of light located a distance a above the floor on the axis of symmetry of the hemisphere. The polar angle θ is measured from this axis. The distribution of illuminance of the hemispherical surface is given by

$$E_S(\theta) = \frac{1}{4\pi R^2} \left\{ \frac{1 - \alpha \cos\theta}{(1 + \alpha^2 - 2\alpha \cos\theta)^{3/2}} + \frac{\rho}{2 - \rho} \left(1 + \frac{\alpha}{1 + \alpha^2} \right) \right\}, \quad (\text{A.2})$$

where $\alpha = a/R$. Again, this is the sum of a direct part plus an isotropic reflected part that differs from (A.1) due to the fact that only half the sphere is reflecting. The distribution of illuminance on the black floor at a distance b from the center is given by

$$E_F(\beta) = \frac{1}{4\pi R^2} \left\{ \frac{\alpha}{(\alpha^2 + \beta^2)^{3/2}} + 2\rho \int_0^1 dx \frac{x(1 - \beta^2 + 2\beta^2 x^2)(1 - \alpha x)}{[(1 - \beta^2)^2 + 4\beta^2 x^2](1 + \alpha^2 - 2\alpha x)^{3/2}} + \frac{\rho^2}{2 - \rho} \left(1 + \frac{\alpha}{\sqrt{1 + \alpha^2}} \right) \right\}, \quad (\text{A.3})$$

where $\beta = b/R$. The three terms in this expression represent the direct, the once reflected, and the two or more times reflected components of the illuminance. The once reflected term is in the form of an integral that can be easily evaluated numerically. The illuminance on the floor at the center of the hemisphere, $b = 0$, is

$$E_F(0) = \frac{1}{4\pi R^2} \left\{ \frac{1}{\alpha^2} + \frac{2\rho}{3\alpha^2} \left[1 + 2\alpha^3 - (1 - 2\alpha^2)\sqrt{1 + \alpha^2} \right] + \frac{\rho^2}{2 - \rho} \left(1 + \frac{\alpha}{\sqrt{1 + \alpha^2}} \right) \right\} \quad (\text{A.4})$$

while the illuminance at the outer edge of the floor, $b = R$, is

$$E_F(1) = \frac{1}{4\pi R^2} \left\{ \frac{\alpha}{(1 + \alpha^2)^{3/2}} + \frac{\rho(2 + \rho)}{2(2 - \rho)} \left(1 + \frac{\alpha}{1 + \alpha^2} \right) \right\} \quad (\text{A.5})$$

Some representative values of $R^2 E_F(0)$ and $R^2 E_F(1)$ are given in Table A1. The above expressions should not be used for $\alpha = 1$ since the expression (A.3) is discontinuous at that value.

We now consider the illuminance incident upon a horizontal plane which is a distance c above the floor of the hemisphere. We consider a point on this plane a distance b from the axis of the hemisphere, and an isotropic point source of light is located on the axis a distance a from the center as in the case previously treated. The illuminance at this point is given by

$$\begin{aligned}
E = \frac{1}{4\pi R^2} & \left\{ \frac{\alpha - \gamma}{[(\alpha - \gamma)^2 + \beta^2]^{3/2}} \right. \\
& + 2\rho(1 - \gamma)^2 \int_0^1 dy \frac{y(a_0 - a_1 y + a_2 y^2)(c_0 - c_1 y)}{[(b_0 - b_1 y + b_2 y^2)(d_0 - d_1 y)]^{3/2}} \\
& \left. + \frac{\rho^2}{2 - \rho} \left(1 + \frac{\alpha}{\sqrt{1 + \alpha^2}} \right) \right\} \quad (A.6)
\end{aligned}$$

where

$$\begin{aligned}
a_0 &= (1 - \gamma^2)(1 - \beta^2 - \gamma^2) \quad , \quad b_0 = (1 - \beta^2 - \gamma^2)^2 \quad , \\
a_1 &= 3\gamma(1 - \gamma)(1 - \beta^2 - \gamma^2) \quad , \quad b_1 = 4\gamma(1 - \gamma)(1 - \beta^2 - \gamma^2) \quad , \\
a_2 &= 2(1 - \gamma)^2(\beta^2 + \gamma^2) \quad , \quad b_2 = 4(1 - \gamma)^2(\beta^2 + \gamma^2) \quad , \quad (A.7) \\
c_0 &= 1 - \alpha\gamma \quad , \quad d_0 = 1 + \alpha^2 - 2\alpha\gamma \quad , \\
c_1 &= \alpha(1 - \gamma) \quad , \quad d_1 = 2\alpha(1 - \gamma) \quad ,
\end{aligned}$$

and $\alpha = a/R$, $\beta = b/R$, $\gamma = c/R$. This expression reduces to (A.3) when $\gamma = 0$, and the interpretation of the three terms in (A.6) is the same as given there. The once reflected component which is the second term must be evaluated numerically except for on axis, $\beta = 0$, or on edge, $\gamma^2 = 1 - \beta^2$, where it can be expressed in terms of elementary functions.

For systems with cylindrical symmetry, we can consider systems with line sources or point sources. We first consider a volume bounded by a cylindrical surface of radius R and a line source of unit luminous intensity per unit length placed parallel to the axis of symmetry but offset a distance a from the center. If we measure the polar angle ϕ from the plane containing the axis of the cylinder and the line source, then the illuminance incident upon the cylindrical surface of reflectivity ρ is

$$E_c(\phi) = \frac{1}{2\pi R} \left\{ \frac{1 - \alpha \cos \phi}{1 + \alpha^2 - 2\alpha \cos \phi} + \frac{\rho}{1 - \rho} - \frac{\rho}{4} \sum_{n=1}^{\infty} \frac{\alpha^n \cos n\phi}{n^2 + \frac{1}{2}(\rho - \frac{1}{2})} \right\}, \quad (\text{A.8})$$

where $\alpha = a/R$. The first term in this expression is the direct component of the illuminance, the second term is the isotropic parts of the reflected component, and the last term is the anisotropic parts of the reflected component. The sum in this last term converges rapidly since $0 \leq \alpha < 1$ and, apart from the overall factor of ρ , is only weakly dependent upon the value of ρ since the denominators range from $n^2 - 1/4$, for $\rho = 0$, to $n^2 + 1/4$, for $\rho = 1$.

When we place a point source in the cylindrical volume, then the translational invariance along the z -axis of the system is broken as well as the rotational invariance about the cylinder axis if the source is off center. This system can still be solved by Fourier transform methods. However, the various transforms are not elementary

functions. We present the detailed analysis of this system below. We note in passing that the total illumination on the cylindrical surface is given by

$$\int_{-\infty}^{\infty} dz \int_{-\pi}^{\pi} R d\phi E_C(z, \phi) = \frac{1}{1 - \rho} \quad (\text{A.9})$$

regardless of the location of the source. In this sense, the "isotropic" component of the illuminance is amplified by reflections in much the same way as in a sphere.

A volume bounded by two plane surfaces--a floor and a ceiling--with either a line or point source of light can also be solved by Fourier transform techniques. We first consider a line source located a distance d below the ceiling which is a distance h above the floor. The reflectivities of the floor and ceiling are denoted by ρ_f and ρ_c . The illuminance incident upon the floor or ceiling at a horizontal distance x from the source is given by

$$E_{f,c}(x) = \int_0^{\infty} \frac{dk}{\pi} \tilde{E}_{f,c}(k) \cos kx \quad , \quad (\text{A.10})$$

with

$$\tilde{E}_f(k) = \frac{1}{2D} [e^{-k(h-d)} + \rho_c K(kh) e^{-kd}] \quad , \quad (\text{A.11})$$

$$\tilde{E}_c(k) = \frac{1}{2D} [e^{-kd} + \rho_f K(kh) e^{-k(h-d)}] \quad ,$$

and

$$D = 1 - \rho_f \rho_c k^2 (kh) \quad , \quad (A.12)$$

where

$$K(x) \equiv xK_1(x)$$

and where K_1 is the modified Bessel function of order one. We can obtain the total illuminance per unit length of source from the $k > 0$ limits of these expressions, i.e.,

$$\int_{-\infty}^{\infty} dx E_{f,c}(x) = \tilde{E}_{f,c}(k \rightarrow 0) = \frac{1 + \rho_{f,c}}{2(1 - \rho_{f,c})} \quad . \quad (A.13)$$

This result is similar to (A.9) and reduces to it for $\rho_f = \rho_c = \rho$.

Very similar results are obtained if we place a point source in the geometry described in the previous paragraph. We denote the illuminance incident upon the floor or ceiling at a distance $r = (x^2 + y^2)^{1/2}$ from the source by $E_{f,c}(r)$. We then have

$$E_{f,c}(r) = \int_0^{\infty} \frac{k dk}{2\pi} \tilde{E}_{f,c}(k) J_0(kr) \quad , \quad (A.14)$$

where J_0 is the Bessel function of order zero and $\tilde{E}_{f,c}(k)$ are given by (A.11) and (A.12). We also obtain a result similar to (A.13), where the left-hand side is integrated over the entire xy -plane rather than just x .

We now fill in some of the mathematical details behind the expressions given above. The solution to equations in spherical

geometry is well known and we present it here as an introduction to the general techniques used in all geometries. The source term in Eq. (5.3) for a point source of unit strength located at the position r_s is given by

$$S_i(\vec{r}_i) = \frac{1}{4\pi} \frac{\vec{n}_i \cdot (\vec{r}_s - \vec{r}_i)}{(\vec{r}_s - \vec{r}_i)^3} \quad (\text{A.15})$$

For the sphere, there is only one surface so we drop the index i . We place the source at the point $x_s = y_s = 0$, $z_s = a$, or $\vec{r}_s = a\hat{k}$, where \hat{k} is a unit vector along the z -axis. A general point \vec{r} on the spherical surface is given by $\vec{r} = R(\sin\theta\cos\phi\hat{i} + \sin\theta\sin\phi\hat{j} + \cos\theta\hat{k})$, where R , θ , ϕ are the spherical coordinates of the point and \hat{i} and \hat{j} are unit vectors in the x - and y -directions. The inward pointing normal to the surface at the point r is given by $\vec{n} = -(\sin\theta\cos\phi\hat{i} + \sin\theta\sin\phi\hat{j} + \cos\theta\hat{k})$. We then have

$$S(\vec{r}) = S(\theta) = \frac{(1 - \alpha \cos\theta)}{4\pi R^2 (1 + \alpha^2 - 2\alpha \cos\theta)^{3/2}} \quad (\text{A.16})$$

where $\alpha = a/R$.

We consider the reflected component of the illuminance at the point \vec{r} with coordinates R , θ , ϕ from the point \vec{r}' with coordinates R , θ' , ϕ' . The cartesian components of the various vectors needed to calculate the radiation transfer factor, (5.4), are given by \vec{r} and \vec{n} given above and \vec{r}' and \vec{n}' given by the same expressions with θ and ϕ replaced by θ' and ϕ' . We then have, from (5.4),

$$K(\vec{r} - \vec{r}') = \frac{-[\vec{n} \cdot (\vec{r} - \vec{r}')] [\vec{n}' \cdot (\vec{r} - \vec{r}')] }{(r - r')^4} = \frac{[R^2 - \vec{r} \cdot \vec{r}']^2}{4R^2 [R^2 - \vec{r} \cdot \vec{r}']^2} = \frac{1}{4R^2} \quad (\text{A.17})$$

which is independent of \vec{r} and \vec{r}' .

Substituting (A.16) and (A.17) into the radiation transfer equation (5.3) yields

$$E(\theta) = \frac{1}{4\pi R^2} \frac{1 - \alpha \cos \theta}{(1 + \alpha^2 - 2\alpha \cos \theta)^{3/2}} + \rho \int \frac{d\Omega'}{4\pi} E(\theta') \quad , \quad (\text{A.18})$$

where ρ is the reflectivity of the spherical surface and

$d\Omega' = d\phi' d(\cos \theta')$ is the element of solid angle at \vec{r}' . The solution to this equation which is given in (A.1) is obtained by multiplying (A.18) by $d = d\phi d(\cos \theta)$ and integrating it over the sphere.

For the hemispherical volume bounded by a hemispherical surface of reflectivity ρ and a perfectly black plane surface of reflectivity zero, the radiation transfer equation for the illuminance on the hemisphere (now denoted by $E_s(\theta)$) is again given by (A.18) but with the restriction that θ and θ' range between zero and $\frac{\pi}{2}$. The solution given in Eq. (A.2) is obtained by multiplying the equation by d and integrating over the hemisphere indicated by this range of θ .

In order to calculate the illuminance E_f incident upon plane surface in this volume, we must calculate the direct component coming from the source and the indirect component coming, by reflection, from the hemispherical surface. We consider a point located a distance b from the center of the hemisphere with coordinates $x = b, y = z = 0$. The direct component comes from (A.15) with $\vec{n}_f = \hat{k}, \vec{r}_f = b\hat{i}$, and

$\vec{r}_s = a\hat{k}$ and is given by the first term in (A.3). The radiation transfer factor from the point r on the hemisphere to the point b_i on the floor is

$$\frac{R\cos\theta(R - b\sin\theta\cos\phi)}{(R^2 + b^2 - 2Rb\sin\theta\cos\phi)^2} \quad (\text{A.19})$$

This is to be used with $E_s(\theta)$ to calculate the indirect component of the illuminance. Since E_s is independent of ϕ , we may integrate (A.19) with respect to ϕ . We then have the result given in (A.3).

We next consider the illuminance on a horizontal plane at a distance c above the floor and b from the axis of the hemisphere. The direct component is given by the first term in (A.6), and the radiation transfer factor from the point \vec{r} on the hemisphere to the point $b\hat{i} + c\hat{k}$ on the plane is given by

$$\frac{(R\cos\theta - c)(R - c\cos\theta - b\sin\theta\cos\phi)}{(R^2 + b^2 + c^2 - 2Rc\cos\theta - 2Rb\sin\theta\cos\phi)^2} \quad (\text{A.20})$$

This can be integrated with respect to ϕ . The range of θ is now from 0 to $\cos^{-1}\gamma$, $\gamma = c/R$. We then make the change of integration variables from $\cos\theta$ to

$$y = \frac{\cos\theta - \gamma}{1 - \gamma} \quad (\text{A.21})$$

and obtain the expression (A.6).

For a line source in a cylinder located a distance a from the center, the direct component of the illuminance incident upon the

cylinder surface at an angle ϕ from the plane through the cylinder axis and the line source are given by

$$\frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{(R - a \cos \phi) dz}{[R^2 + a^2 + z^2 - 2R a \cos \phi]^{3/2}} =$$

$$\frac{1}{2\pi R} \frac{1 - \alpha \cos \phi}{1 + \alpha^2 - 2\alpha \cos \phi}$$

which is the first term of (A.8). For the reflected component from the point $\vec{r}' = R(\cos \phi' \vec{i} + \sin \phi' \vec{j} + z' \vec{k})$ to the point $\vec{r} = R(\cos \phi \vec{i} + \sin \phi \vec{j})$, we have the radiation transfer factor given by

$$\frac{4R^2 \sin^4\left(\frac{\phi - \phi'}{2}\right)}{\left[4R^2 \sin^2\left(\frac{\phi - \phi'}{2}\right) + z'^2\right]^2} \quad (A.22)$$

This can be integrated over z' since the illuminance is independent of z to yield the equation

$$E_c(\phi) = \frac{1 - \alpha \cos \phi}{2\pi R(1 + \alpha^2 - 2\alpha \cos \phi)} + \frac{\rho}{4} \int_{-\pi}^{\pi} d\phi' \sin\left|\frac{\phi - \phi'}{2}\right| E_c(\phi') \quad (A.23)$$

for the illuminance at the point ϕ on the cylindrical surface. This equation can be solved using Fourier series with the result given in (A.8).

For the point source located at the point $z = 0$ and offset from the axis by a distance a , the radiation transfer equation can be obtained from the above factors and is

$$E_C(\phi, z) = \frac{R - a \cos \phi}{4\pi[R^2 + a^2 + z^2 - 2R a \cos \phi]^{3/2}} +$$

$$\frac{\rho}{\pi} \int_{-\infty}^{\infty} dz' \int_{-\pi}^{\pi} R d\phi' \frac{4R^2 \sin^4\left(\frac{\phi - \phi'}{2}\right) E_C(\phi', z')}{[4R^2 \sin^2\left(\frac{\phi - \phi'}{2}\right) + (z - z')^2]^2} \quad (\text{A.24})$$

When integrated over ϕ and z , this equation yields the result given in (A.9). This equation can be solved by Fourier transform techniques. However, the transforms of the various terms in (A.24) are not elementary functions. For example, if we transform the variable z and go to the mixed representation

$$\tilde{E}_C(\phi, k) = \int_{-\infty}^{\infty} dz e^{-ikz} E_C(\phi, z) \quad , \quad (\text{A.25})$$

then E_C satisfies the equation

$$\tilde{E}_C(\phi, k) = \frac{1 - \alpha \cos \phi}{2\pi R(1 + \alpha^2 - 2\alpha \cos \phi)} p_1 K_1(p_1) +$$

$$\frac{\rho}{4} \int_{-\pi}^{\pi} d\phi' \sin\left|\frac{\phi - \phi'}{2}\right| (1 + p_2) e^{-p_2} \tilde{E}_C(\phi', k) \quad (\text{A.26})$$

where

$$p_1 = k(R^2 + a^2 - 2R a \cos \phi)^{1/2} \quad ,$$

$$p_2 = 2 \left| k \left| R \sin\left|\frac{\phi - \phi'}{2}\right| \right| \quad ,$$

$\alpha = a/R$, and K_1 is the modified Bessel function of order one. It is clear that (A.26) reduces to (A.23) in the limit $k \rightarrow 0$ as must be the case. Therefore, $\tilde{E}_C(\phi, k = 0)$ is given by (A.8). For $k \neq 0$, this equation must be solved by numerical techniques.

We now turn to a volume bounded by two parallel plane surfaces. We assume that they are separated by a distance h and will call them "floor" and "ceiling" with reflectivities ρ_f and ρ_c . For a line source of light placed a distance d below the ceiling, the direct component of the illuminance on the ceiling at a distance x from the source is given by

$$\frac{1}{4} \int_{-\infty}^{\infty} dy \frac{d}{(x^2 + y^2 + d^2)^{3/2}} = \frac{1}{2\pi} \frac{d}{x^2 + d^2}.$$

The direct component for the floor is obtained from this expression by making the replacement $d \rightarrow h - d$. The radiation transfer factor for the reflection of light from the position (x', y') on the floor or ceiling to the position $(x, 0)$ on the ceiling or floor is given by $h^2 / [(x - x')^2 + y'^2 + h^2]^2$ which can be integrated with respect to y' since the illuminance is independent of y for a line source. Putting these factors together, we then have the equations

$$E_f(x) = \frac{h - d}{2\pi[x^2 + (h - d)^2]} + \frac{\rho_c}{2} \int_{-\infty}^{\infty} dx' \frac{h^2}{[(x - x')^2 + h^2]^{3/2}} E_c(x') \quad (\text{A.27})$$

$$E_c(x) = \frac{d}{2\pi[x^2 + d^2]} + \frac{\rho_f}{2} \int_{-\infty}^{\infty} dx' \frac{h^2}{[(x - x')^2 + h^2]^{3/2}} E_f(x')$$

These equations can be solved by Fourier transfer techniques with the result given by (A.11).

For a point source, we cannot do the y -integrations and the equations are

$$E_f(x,y) = \frac{h-d}{4\pi[x^2 + y^2 + (h-d)^2]^{3/2}} + \frac{\rho_c}{\pi} \int dx' dy' \frac{h^2}{[(x-x')^2 + (y-y')^2 + h^2]^2} E_c(x',y')$$

(A.28)

$$E_c(x,y) = \frac{d}{4\pi[x^2 + y^2 + d^2]^{3/2}} + \frac{\rho_f}{\pi} \int dx' dy' \frac{h^2}{[(x-x')^2 + (y-y')^2 + h^2]^2} E_f(x',y')$$

These equations can also be solved by Fourier transform techniques with the results given by (A.14).

Table A.1. Some representative values of $R^2 E_f(\beta)$ for $\beta = 0$ and 1, see Eqs. (A.4) and (A.5).

	0.5		0.6		0.7		0.8		0.9	
	0	1	0	1	0	1	0	1	0	1
0.0	4.00	0.36	2.78	0.38	2.04	0.38	1.56	0.38	1.23	0.37
0.5	5.16	0.96	4.05	1.01	3.43	1.04	3.07	1.06	2.87	1.06
0.6	5.48	1.16	4.40	1.22	3.80	1.26	3.47	1.29	3.29	1.30
0.7	5.84	1.41	4.78	1.48	4.22	1.53	3.91	1.56	3.76	1.58
0.8	6.25	1.71	5.22	1.79	4.69	1.85	4.41	1.90	4.29	1.93
0.9	6.72	2.07	5.74	2.18	5.23	2.25	4.99	2.31	4.90	2.35

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720