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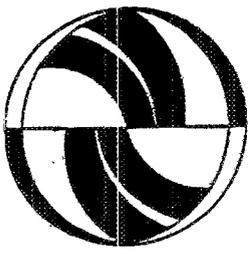
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**Development of a Pavement Rutting Model  
From Experimental Data**

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# **Development Of A Pavement Rutting Model From Experimental Data**

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# DEVELOPMENT OF A PAVEMENT RUTTING MODEL FROM EXPERIMENTAL DATA

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(Reviewed by the Highway Division)

**ABSTRACT:** Properly specified pavement deterioration models are an important input for the efficient management of pavements, the allocation of cost responsibilities to various vehicle classes for their use of the highway system, and the design of pavement structures. However, most empirical deterioration progression models developed to date have had limited success. This paper is concerned with the development of an empirical rutting progression model using experimental data. The data used in this paper comprise an unbalanced panel data set with more than 14,000 observations taken from the AASHO Road Test. The salient features of the model specification are (1) the model eschews conventional (predefined) axle load equivalencies and structural numbers in favor of relationships determined entirely by the data itself; (2) a thawing index variable has been incorporated to capture the effects of the environmental factors in the AASHO Road Test; and (3) the model predicts incremental changes in rut depth, which is particularly advantageous in a pavement management context. The specified model is nonlinear in the variables and the parameters and is estimated using both fixed-effects and random-effects specifications to account for unobserved heterogeneity. The estimation results show that the model replicates the pavement behavior well, that the inclusion of an environmental variable is important to avoid biases in other parameters, and that the size of the unobserved heterogeneity is significant. It is also found that interactions between some parameters in the nonlinear specification leads to significant differences between parameter estimates among the two wheel paths rutting models.

## INTRODUCTION

The main types of structural distress of asphalt concrete pavements are cracking and permanent deformation. The accurate prediction of their development is an essential input for the efficient management of pavement systems. In addition to their importance in maintenance and rehabilitation decision making, properly specified pavement deterioration models can be used to study the effect of different loading levels and thus in allocating cost responsibilities to various vehicle classes for their use of the highway system. Furthermore, such models can be used in the design of pavement structures. In particular, they can be used for evaluating different strategies for design, maintenance, and rehabilitation.

Rutting, loosely defined as longitudinal depressions in the wheel paths of asphalt concrete pavements, has historically been a primary criterion of structural performance in many pavement design methods. As pointed out by Paterson (1987), other types of permanent deformation are generally much less tractable for direct modeling because they depend to a larger degree on material properties, their local variations, and their interactions with the pavement's microclimate. Rutting is also a serious safety issue for road users. When water accumulates in the ruts, there is a potential for hydroplaning. The hydroplaning phenomenon consists of the buildup of a thin layer of water between the pavement and the tire and results in the tire losing contact with the surface, with the consequent loss of steering control (Yoder and Witczak 1975).

With increasing magnitudes and repetitions of loads and increased tire pressures, the rutting problem has become severe in many highway pavements (Haas et al. 1994). Considerable

research has been conducted over the years for developing models to predict the progression of rutting, but with limited success. This paper is concerned with the development of an empirical rutting progression model using experimental data. The data set used in this paper is taken from the AASHO Road Test [Highway Research Board (HRB) 1962]

The use of experimental data for model development, as opposed to field data collected from condition surveys of in-service pavements, has the following characteristics:

- *Advantages.* The main factors affecting rutting such as axle loads and layer thicknesses are carefully controlled; therefore, the researcher can capture their effects on rutting progression. This is hardly possible using field data alone. Field data generally involve a distribution of loads whose measurement is not very accurate. Discriminating the effect of each load level from a distribution of loads is a difficult problem, which is not present when each pavement is subjected to a known load level. Furthermore, in field data, the control of constructed layer thicknesses is of lower quality, and the design thicknesses are usually a function of traffic. The latter causes an econometric problem known as endogeneity. Endogenous variables are determined through the joint interaction with other variables within the model. In field data, layer thicknesses are endogenous because they are usually a function of predicted traffic. Estimation of parameters by ordinary least-squares regression results in biased and inconsistent results in the presence of endogenous variables (Greene 1997). This problem is avoided in experimental data.
- *Disadvantage.* The main disadvantage is that experimental data may not represent the true deterioration mechanism of in-service pavements. For example, material aging is not captured in accelerated pavement loading tests.

The salient features of the model specification in this paper are as follows:

- The model eschews conventional (predefined) axle load equivalencies and structural numbers in favor of relationships determined entirely by the data itself.
- A thawing index variable that captures the effects of the

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environmental factors in the AASHO Road Test is defined.

- The model predicts rut depths by adding predicted values of the increment of rut depth for each time period; this is particularly advantageous in a pavement management context where the engineer is interested in predicting changes in rut depth.

To estimate the model parameters, an unbalanced panel data set with more than 14,000 observations from the AASHO Road Test was used. An unbalanced panel data set consists of observations for different pavement units through time, where the numbers of observations for each pavement section are not necessarily the same. The model is nonlinear in the parameters and the variables, and so special routines had to be programmed to account for the nonlinearity of the model and the panel structure of the data.

The paper is organized as follows: the second section provides some background on the nature of the rutting problem; the third section briefly describes the AASHO Road Test, which is the data source used for model development; the fourth section explains the rationale for the specification of the model; the fifth section presents the results of the estimation of the model; and the last section concludes the paper.

## BACKGROUND

There is an extensive body of literature on the rutting of asphalt concrete pavements spanning many decades. Part of this literature is reviewed in the following subsections. First, we identify the factors affecting the rutting performance of asphalt concrete pavements. Then, we review some results from the mechanistic-empirical literature. The information in these two subsections is considered important for the development of a meaningful model specification. Finally, we review significant empirical models that have been developed to date.

### Factors That Affect Rutting

Rutting is strongly influenced by traffic loading, but climate can also have a large influence especially when the pavement subgrade undergoes seasonal variations in bearing capacity, or when bituminous courses are subjected to high temperatures [Organization for Economic Cooperation and Development (OECD) 1988]. Ruts develop within pavement layers when traffic loading causes layer densification and/or when stresses induced in the pavement materials are sufficient to cause shear displacements within the materials.

Research performed over several decades has shown that the susceptibility to rutting can be linked to the following material attributes: excessive asphalt content, excessive fine-grained aggregate, high percentages of natural sand, rounded aggregate particles, excessive permissible moisture in the mix or in granular materials and soils, temperature susceptible asphalt cement, and cold weather paving leading to low density. Other factors affecting rutting are temperature; precipitation; and the time, type, and extent of loading. The above factors when combined also determine measures such as Hveem and Marshal stability, complex modulus, resilient modulus, and deflection that are normally used for pavement distress modeling. Generally, only a few of these factors are measured in experimental data sets and thus can be used in an empirical model such as the one developed herein.

### Evidence from Mechanistic-Empirical Literature

The focus of this paper is the statistical estimation of models that relate rut depth trends to explanatory variables represent-

ing pavement structure, loading, and climate. These are described as empirical models in the pavement literature. Despite this focus, relevant results from the literature on the mechanistic-empirical approach to modeling pavement rutting are reviewed so as to identify suitable model forms.

Rutting is the result of the integration of the plastic strains over the pavement structure. Numerous models have been used to relate plastic strain accumulation to the number of load or stress repetitions. By far, the most common model found in the literature is of the form

$$\epsilon_p = aN^b \quad (1)$$

where  $\epsilon_p$  = permanent or plastic strain;  $N$  = number of stress applications; and  $a$  and  $b$  = estimated coefficients that are functions of applied stress and material characteristics.

The above model form has been proposed for subgrades and unbound materials (Monismith 1976; Diylajee and Raymond 1982; Vuong and Armstrong 1991; Behzadi and Yandell 1996) as well as for asphalt concrete mixes (Khedir 1986).

The following equation form is used by Kenis (1977) and by Ali et al. (1998):

$$\epsilon_p(N) = a'\epsilon_e N^{b'} \quad (2)$$

where  $\epsilon_e$  = elastic strain; and  $a'$  and  $b'$  = permanent deformation parameters. This equation is based on the proportionality between the plastic and elastic strains in a pavement structure under traffic loading. Given a level of elastic strain, this equation is equivalent to (1).

In the Texas Flexible Pavement System the permanent strain on asphalt concrete mixes is assumed to behave in essentially the same manner as above

$$\frac{\partial \epsilon_p}{\partial N} = a''\epsilon_e N^{b''} \quad (3)$$

where  $a''$  and  $b''$  = estimated parameters; and  $\epsilon_e$  = elastic strain (Button et al. 1990). Obviously, the form in (2) is obtained if this equation is integrated. In the Texas Flexible Pavement System analysis, the elastic strain is assumed to remain constant throughout the life of the pavement.

The coefficients  $a$  and  $b$  (and also  $a'$ ,  $a''$ ,  $b'$ , and  $b''$ ) are usually considered functions of applied stresses and material properties. Their estimates vary widely among researchers depending on the materials involved and test procedures. In general,  $a$  is influenced by these factors to a greater extent than  $b$ .

### Evidence from Empirical Literature

The most common relevant finding in the empirical literature is the concave shapes of rut depth with cumulative number of load repetitions. Such trends have been observed with heavy vehicle simulators by Maree et al. (1982) and by Harvey et al. (1997) and in other experiments such as the AASHO Road Test (HRB 1962). Furthermore, most developed models specify such a concave shape (Lister 1981; Paterson 1987).

Most rutting models developed to date have been limited to linear specifications [e.g., Saraf (1982)] and do not account for the effects of the environment [e.g., Saraf (1982) and Thompson and Nauman (1993)]. Paterson (1987) developed a nonlinear model with data from in-service pavements that included the effect of the environment. Unfortunately, in spite of its complexity, it produced a mediocre fit.

### AASHO ROAD TEST

To date, the AASHO Road Test remains the most comprehensive controlled experiment performed for evaluating the performance of pavements in the United States. The test was

$$RR = RD/N = A/N^B \quad (4)$$

where  $RR$  = rutting rate;  $RD$  = rut depth (in.);  $N$  = number of repeated load applications; and  $A$  and  $B$  = terms developed from field calibration data.

Furthermore, the concave trend of deformation with respect to the number of load applications appears in most empirical and accelerated test studies. Therefore, the same functional form will be the starting point for the model formulation in this section, but instead of a rutting rate as used by Thompson and Nauman (which was in fact a secant rate), rut depth will be used directly. Specifically, the following specification is used:

$$RD_{i,t} = \beta_{i,10} + a_i N_{i,t}^{b_i} \quad (5)$$

where  $RD_{i,t}$  = rut depth for section  $i$  at time  $t$  (mm);  $N_{i,t}$  = variable representing the cumulative number of load repetitions applied to pavement section  $i$  up to time period  $t$  (a more complete definition is given later);  $a_i$  and  $b_i$  = functions of the characteristics of pavement  $i$  such as layer thicknesses, gradations, etc.; and  $\beta_{i,10}$  = rut depth immediately after construction for pavement section  $i$  (the reason for using the subscript 10 will be apparent shortly).

For laboratory experiments that are usually carried out at a given stress level, the definition of  $N$  is straightforward, but this is not so for the AASHO Road Test where different pavement sections were subjected to various load levels and different load configurations (single or tandem axles). This is even more complex for actual pavement sections because each section is subjected to a distribution of loads and configurations. A possible solution to this is to use the cumulative number of equivalent single axle loads (ESALs). This is actually what many researchers have done in the past. The problem with this approach is that it is assumed that the load equivalency factors that were based on the serviceability index are appropriate in the case of rutting. This is unlikely, and consequently biases may be introduced in the estimation if this path is followed.

Nevertheless, the concept (if not the specific values) of axle load equivalencies is well-accepted in pavement engineering, and thus this concept can be used to define  $N_{i,t}$  as follows:

$$N_{i,t} = \sum_{s=1}^t \Delta V_{i,s} \left( \left( \frac{FL_s}{SAL} \right)^{\beta_5} + R_s \left\{ \left( \frac{AL1_s}{SAL} \right)^{\beta_6} + \left( \frac{AL2_s}{\beta_7 \cdot SAL} \right)^{\beta_6} \right\} \right) \quad (6)$$

where  $\Delta V_{i,s}$  = number of vehicle passes on section  $i$  during period  $s$  (in thousands);  $FL_s$  = load in front axle of truck used in section  $i$  [kN (lbs)];  $AL1_s$  = load in single load axle(s) [rear axle(s)] of truck used in section  $i$  [kN (lbs)];  $AL2_s$  = load in tandem load axle(s) [rear axle(s)] of truck used in section  $i$  [kN (lbs)];  $R_s$  = number of load axles in truck used in section  $i$  ( $R_s = 1$  or  $2$ );  $SAL = 80$  kN (18,000 lbs); and  $\beta_j$  = parameters to be estimated ( $j = 5, 6, 7$ ). These parameters determine the equivalencies between axle loads.

The divisions of  $FL_s$  and  $AL1_s$  by  $SAL$  in (6) result in the standardization of all the single loads to an equivalent 80-kN (18,000-lbs) single axle load, which is the standard practice in pavement engineering. Similarly, the division of  $AL2_s$  by  $(\beta_7 \cdot SAL)$  results in the standardization of tandem axles to a load of  $\beta_7 \cdot 80$  kN, which is the load of the standard tandem axle producing the same rutting as a single 80-kN axle. This definition of  $N_{i,t}$  makes it independent of the units being used.

We assumed that the exponent for the front axle load is the same as the exponent for single axle load ( $\beta_5$ ). The only difference between these axles is that the front axle had a single wheel, whereas the rear single axles had double wheels. Certainly, the rutting produced by these two different wheel configurations (for a given load) could be different, but the differences are mostly due to differences in stress distributions in

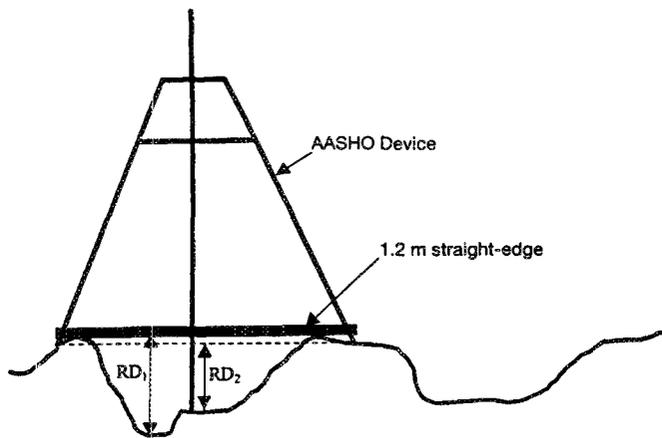


FIG. 1. Rut Depth Measurements with Straightedge ( $RD_1$ ) versus Rut Depth Measurements with AASHO Device ( $RD_2$ )

carried out between 1959 and 1962 near Ottawa, Ill., about 130 km (80 mi) southwest of Chicago.

All variables for pavement studies were concerned with pavement designs and loads within each of the sections. The sections were subjected to traffic for slightly more than 2 years. Twelve sets of sections were subjected to 12 different combinations of axle load and axle configuration. The climate of the road test area was temperate with an average annual precipitation of about 864 mm (34 in.). The average mean summer temperature was  $24.4^\circ\text{C}$  ( $76^\circ\text{F}$ ), and the average mean winter temperature was  $-2.8^\circ\text{C}$  ( $27^\circ\text{F}$ ). The soil usually remained frozen during the winter with alternate thawing and freezing of the immediate surface.

The experiment included a total of 234 structural sections or 468 test sections. A majority of the test sections in each of the 12 sets comprised a complete factorial experiment, the design factors of which were surfacing thickness, base thickness, and subbase thickness. These experiments were referred to as the main factorial designs. The data that will be used in this research consist of this main factorial design.

The information available for model estimation consists essentially of initial thicknesses of the asphalt concrete, base, and subbase layers and biweekly information on axle load and number of repetitions, mean maximum and minimum temperatures, and mean rut depth. The asphalt concrete, base, and subbase material characteristics were the same for all sections.

In the AASHO Road Test, the routine biweekly rut measurements were carried out with the device shown in Fig. 1. The distance between the legs of this device is 1.2 m (4 ft). This measurement methodology differs from the one specified in AASHTO (1993), which specifies that a 1.2 m (4-ft) straightedge should be laid across the rut and the maximum depth measured.

Nevertheless, for ruts where all the layers are contributing to rutting (i.e., the high and low points are far apart transversally and the low point is located near the middle of the rut), the difference should be negligible. According to cross sections shown in the AASHO Road Test report (HRB 1962), this seems to have been the case for most sections in the AASHO Road Test.

## MODEL SPECIFICATION

The literature revealed that the results of laboratory tests indicate that most materials are described by (1). Thompson and Nauman (1993), based on the above observation, argued that it was reasonable to assume that a pavement surface model would be of the same form. They proposed the following model:

the upper portions of the pavement. In the lower portions of the pavement the distributions of stresses are similar. The separation between tires of dual wheels is of the order of 0.3 m. Also, tire pressures are usually not equal because of temperature differences between the tires, road surface irregularities, bending of the axle, etc. (OECD 1998). Thus, although dual wheels distribute the load in a greater area of the pavement surface, the above factors diminish that advantage. Furthermore, the loads in front axles are usually smaller and thus of lower importance. In any case, although it would be desirable to obtain different coefficients for these two-wheel configurations, this was not possible because the ratio of rear single axle load (twin wheel) to front axle load (single wheel) varied only between 3 and 3.5 for the different pavement sections. Nevertheless, it is considered better to include the front axle loads into the model than to neglect them altogether as has been done in previous research.

The identification of a different exponent for tandem axles is considered important. The reason is that the axle separation in tandem axles is large (1.0–1.2 m). Thus, the differences in stresses between single and tandem axles are substantial at greater depths. Notice that  $\beta_6$  captures the equivalency between different load magnitudes for tandem axles.

The assumption of different exponents for single and tandem axles implicitly assumes that a considerable amount of rutting is originating at depth. However, some of the rutting originates in the asphalt concrete layer, which is not explicitly considered by the model. This is a limitation of the model that is the subject of current research by the writers. It should be noted, however, that after estimation, the model implicitly accounts for rutting in the asphalt concrete layer for stable mixes such as the ones used in the AASHO Road Test and the asphalt concrete layer thicknesses used in the test. The thicknesses in the AASHO Road test cover the most common thicknesses used in practice. Thus, in spite of this limitation, the model should give good rut depth estimates for stable asphalt concrete mixes.

### Specifications for $a_i$ and $b_i$

From the literature review in the second section, it seems that a plausible assumption for  $b_i$  is that it is relatively constant or, at most, varies linearly with pavement strength. In this study,  $b_i$  is assumed to be constant for all sections. On the other hand,  $a_i$  seems to vary widely with pavement strength.

After performing 192 regressions with the specification of (4), Thompson and Nauman (1993) observed low values of  $A$  for structural responses less than a certain value, but high magnitudes and erratic trends above that value. Based on these results, they concluded that their  $A$  term followed threshold type relations. However, this conclusion is indicative of the inability to estimate the true relation. It is more likely that the relation is rapidly varying near their threshold. The observed trends for weak pavements may be a consequence of their estimation approach, or it may be because the variance of the intercept increases with its mean value. Thompson and Nau-

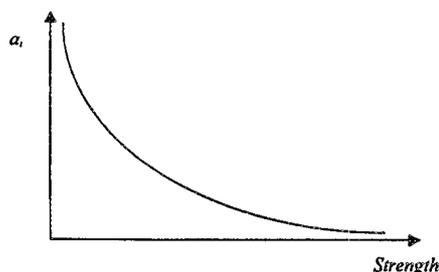


FIG. 2. Anticipated Relation between  $a_i$  Coefficient and Strength of Pavement

man's results indicate that as the structural response decreases (i.e., as the pavement strength increases) their  $A$  term decreases. This means that the  $a_i$  vary with strength in the manner illustrated in Fig. 2. This is also what our intuition would suggest. This simply says that the stronger the pavement, the less the accumulated rut depth for a given traffic.

The exponential function provides a way to obtain such a shape. To model pavement strength, a concept similar to the structural number defined in AASHTO (1993) is used. Specifically, the strength of the pavement is modeled as

$$RN_i = \beta_1 T_{i1} + \beta_2 T_{i2} + \beta_3 T_{i3} \quad (7)$$

where  $RN_i$  = resistance number for pavement  $i$  (although this is almost identical to the structural number, a different name is used to make explicit that this number is specific to rutting);  $T_{i1}$  = thickness of the asphalt concrete layer for pavement  $i$  (m);  $T_{i2}$  = thickness of the granular base layer for pavement  $i$  (m);  $T_{i3}$  = thickness of the subbase layer for pavement  $i$  (m); and  $\beta_j$  = contribution of the  $j$ th layer to the pavement resistance, where  $j = 1, 2, 3$  for asphalt concrete, base, and subbase, respectively.

The following expression is used to relate  $a_i$  to  $RN_i$ :

$$a_i = \beta_4 e^{-RN_i} = \beta_4 e^{-(\beta_1 T_{i1} + \beta_2 T_{i2} + \beta_3 T_{i3})} \quad (8)$$

The above equation admits the following interpretation. Assume that  $T_{i1} = T_{i2} = T_{i3} = 0$  [i.e., traffic loads move over the subgrade material (or over a thin wearing course that does not add structural resistance)]. In such a situation  $a_i = \beta_4$  represents the rut depth caused by the first standard axle load passage [see (5)]. Now if a thickness  $T_{i3}$  of subbase material is added to the pavement structure, then the rut depth caused by the first standard axle load passage is reduced in a proportion given by  $\exp(-\beta_3 T_{i3})$ . That is, the rut depth caused by the first standard axle load is now  $a_i = \beta_4 \exp(-\beta_3 T_{i3})$ . A similar reasoning for the base and asphalt concrete layers leads to (8). These  $\beta$  parameters are functions of the subgrade, subbase, base, and asphalt concrete materials. In summary,  $a_i$  represents the rut depth caused in the pavement structure by the first standard axle load.

This is a convenient interpretation because as the pavement becomes more resistant, rut depth approaches zero asymptotically. Of course, it also implies that rut depth could be reduced as much as one wants by using only a low quality subbase material, which is not realistic. But, as will be seen later, for common pavement structures, this specification produces reasonable results.

### Environmental Effects

Most sections in the AASHO Road Test showed an evident increment in the rate of rut depth progression during the spring months. In what follows, an environmental variable is defined with the information available.

The environmental information available in our database for the AASHO Road Test was very limited. Nevertheless, from the information about the maximum and minimum temperatures, a thawing index is computed with the following reasoning. Freeze will only accumulate when temperatures are below zero. Thus an accumulated freeze index for period  $t$  is computed as follows:

$$AccumFze_t = \max(0, -MeanMinT_t), \quad t = 1 \quad (9a)$$

$$AccumFze_t = \max(0, AccumFze_{t-1} - MeanMinT_t), \quad t = 2, \dots, T_i \quad (9b)$$

where  $MeanMinT_t$  = mean minimum temperature ( $^{\circ}\text{C}$ ) in the 2-week period  $t$  (in the AASHO Road Test there was no freezing in period 1, and so it is not necessary to worry about what

happened before that); and  $T_i$  = number of observations for section  $i$ .

Once the minimum temperature falls below zero, freezing starts to accumulate. At some point in time the minimum temperature again exceeds zero, thus reducing the freezing index. When there are enough periods with temperatures above zero the accumulated freeze index is exhausted and therefore the variable *AccumFze* becomes zero again.

The effect of thawing will be the greatest when there is considerable accumulated freeze from previous periods and the temperatures in the current period are substantially above zero. In such cases there will be large amounts of water in the pavement structure with consequent detrimental effects. Thus, a thawing index representing this interaction of cumulative freeze with temperatures above zero is defined as follows:

$$TI_t = AccumFze_t \cdot \max(MeanMaxT_t, 0) \quad (\text{with units of } ^\circ\text{C}^2) \quad (10)$$

where *MeanMaxT<sub>t</sub>* = mean maximum temperature ( $^\circ\text{C}$ ) in the 2-weeks period preceding  $t$ . This thawing index will be zero when the mean maximum temperature in the period is below zero or when there is no accumulated freeze. Thus, as illustrated in Fig. 3, when thawing starts, this variable starts increasing, reaches a maximum, and then returns to zero at the end of the thawing period. For freezing to occur in the pavement structure, there should be enough water available. Given the precipitations in the AASHO Road Test site and water table information, this seems to have been the case.

Having defined the thawing index, we now explain how it is incorporated in the model. Obviously, thawing alters the materials' properties so one could try to incorporate its effect in  $a_i$ . The problem is that (5) is not suitable for this kind of adjustment. The reason is that one would like to obtain a monotonic increasing function with traffic. If during thawing,  $a_i$  increases, then it is possible that the function decreases afterward when there is no more thawing.

Because the evidence in the literature suggests that (5) is a good approximation when the environmental conditions do not change, it is desirable to keep this functional form. Taking a first-order Taylor series approximation around the conditions in the previous time period gives.

$$RD_{it} \approx RD_{i,t-1} + a_i b_i \frac{\Delta N_{it}}{N_{it}^{1-b_i}} \quad (11)$$

where

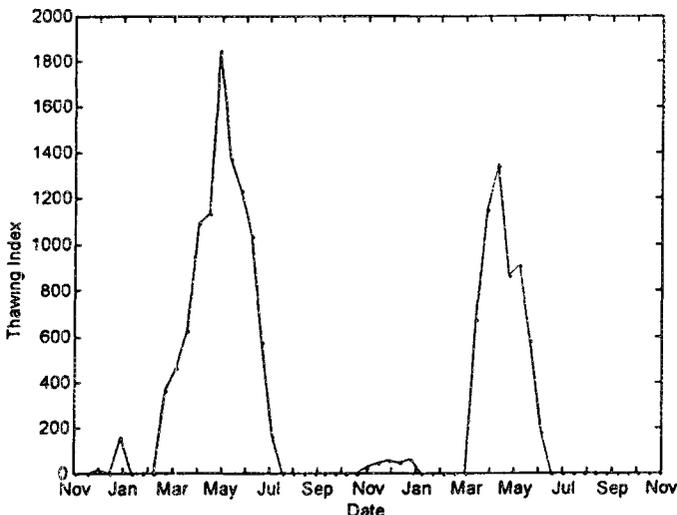


FIG. 3. Thawing Index Computed in AASHO Road Test

$$\Delta N_{it} = N_{it} - N_{i,t-1} = \Delta V_{it} \left( \left( \frac{FL_t}{SAL} \right)^{\beta_5} + R_t \left\{ \left( \frac{AL1_t}{SAL} \right)^{\beta_6} + \left( \frac{AL2_t}{\beta_7 SAL} \right)^{\beta_6} \right\} \right) \quad (12)$$

With this new formulation, introducing a correction factor for  $a_i$  when the environmental conditions change is done as follows:

$$RD_{it} \approx RD_{i,t-1} + a_i \exp \left[ \beta_8 \left( \frac{TI_t}{1000} \right) \right] \beta_9 \frac{\Delta N_{it}}{N_{it}^{1-\beta_5}} \quad (13)$$

where  $b_i$  has been replaced by  $\beta_9$ . Or, substituting successively the values of  $RD_{i,t-1}$ ,  $RD_{i,t-2}$ , etc.

$$RD_{it} \approx \beta_{10} + \sum_{s=1}^t a_i \exp \left[ \beta_8 \left( \frac{TI_s}{1000} \right) \right] \beta_9 \frac{\Delta N_{is}}{N_{is}^{1-\beta_5}} \quad (14)$$

Whenever the thawing index is zero, the new multiplicative factor  $[\exp(\beta_8 TI_t)]$  is 1, and whenever there is thawing the factor is greater than 1 implying that the pavement will rut faster during the corresponding period. Eq. (13), or equivalently (14), is the model specification that we used.

It is important to stress that the use of the thawing index is not intended to be a precise description of the freeze-thaw problem in more general cases. However, the intent is to capture most of the freeze-thaw effects in the AASHO Road Test so as not to cause bias in the estimation of the loading and resistance parameters.

## MODEL ESTIMATION RESULTS

Eqs. (13) and (14) are the expressions of the conditional expectation function of the rut depth for section  $i$  at time  $t$ ,

$E(RD_{it} | X_{it}, \beta)$ . This function gives expected rut depth conditional on the set of regressors  $X_{it} = (1, T_{i1}, T_{i2}, T_{i3}, \Delta V_{it}, \dots, \Delta V_{i1}, FL_t, R_t, AL1_t, AL2_t, TI_t)'$  and on the vector of parameters  $\beta = (\beta_1, \dots, \beta_{11})'$ . The model can be expressed as the following set of regression equations:

$$RD_{it} = E(RD_{it} | X_{it}, \beta) + \epsilon_{it}, \quad i = 1, \dots, S, \quad t = 1, \dots, T_i \quad (15)$$

where  $T_i$  = number of observations for section  $i$ ; and  $\epsilon_{it}$  = error term, which is assumed to have mean 0 and constant variance  $\sigma_\epsilon^2$ . As can be seen from either (13) or (14), this model is nonlinear in the variables and the parameters. Moreover, the vector  $X_{it}$  contains the whole history of loading through the  $\Delta V$  terms. All of these factors make the estimation of the model fairly complex.

When a data set consists of observations for different pavement units through time, several methods of pooling the data can be used. Such data sets are known as panel data sets. One could estimate separate cross-sectional regressions (each using observations for different pavement sections at the same point in time) or separate time-series regressions (each with observations for a single pavement section over time). However, if the model parameters are constant over time and over cross-sectional units more efficient parameter estimates (i.e., estimates with lower variance) can be obtained if all data are combined and a single regression is run. This is the case if all observations are the result of a single underlying deterioration process.

The simplest technique is to combine all cross-sectional and time-series data and perform ordinary least-squares regression on the entire data set. In the present context, this would mean performing a regression using (15) with  $E(RD_{it} | X_{it}, \beta)$  given by (14) and assuming that  $\beta_{10} = \beta_{10}$  is the same for all  $i$ . The problem with this procedure is that despite the reasonableness of the assumption that all of the observations are the result of a single underlying process, some unobserved heterogeneity

(unobserved and persistent pavement-specific factors) is still expected among different pavement sections.

Examples of unobserved heterogeneity are the initial cross-sectional profile and layer compaction. The former directly influences the intercept term in our model. Layer compaction can also influence the intercept term in a more subtle way. For example, a layer that has not been adequately compacted will densify rapidly with the first traffic loads on the wheel paths. This effect will show up mostly in the intercept term; afterward, the conditions are similar to the ones that would have been obtained with good compaction. These are examples of the kind of unobserved heterogeneity that we account for in our model.

The advantage of a panel data set over a cross-sectional data set is that it allows the researcher greater flexibility in modeling differences in behavior across individual units (Green 1997). The two most widely used frameworks for modeling unobserved heterogeneity are called fixed effects and random effects, respectively. Both approaches assume that the unobserved heterogeneity can be captured through the constant term. In the fixed-effects approach, the individual effect ( $\beta_{i,0}$ ) is taken to be constant over time and specific to the individual pavement section  $i$ . This approach always produces consistent results (consistent as the number of sections  $S$  approaches infinity), but it is costly in terms of the number of degrees of freedom lost, because a different intercept is required for each pavement section.

An alternative approach is the random-effects specification. Because the inclusion of different constant terms ( $\beta_{i,0}$ ) represents a lack of knowledge about the model, it is natural to view the section specific constant terms as randomly distributed across pavement sections. Specifically, it is assumed that  $\beta_{i,0} = \beta_{10} + u_i$ , where  $u_i$  is a random disturbance characterizing the  $i$ th section and is constant through time with mean  $E(u_i) = 0$  and constant variance equal to  $\sigma_u^2$ . With these assumptions, the random-effects specification is

$$RD_{it} = \beta_{10} + \sum_{s=1}^4 a_s \exp \left[ \beta_s \left( \frac{TI_s}{1000} \right) \right] \beta_9 \frac{\Delta N_{it}}{N_{it}^{1-\beta_9}} + u_i + \varepsilon_{it} \quad (16)$$

This approach is more appropriate if it is believed that the sampled cross-sectional units are drawn from a large population (Greene 1997), which is the case in the AASHTO Road Test. However it yields consistent parameter estimates only if the regressors are uncorrelated with the individual effects  $u_i$ . This can be tested using a Hausman specification test (Greene 1997).

Both approaches are used to estimate the model parameters in this paper. The estimation approach for linear models can be found, for example, in Greene (1997). The estimation of our model parameters is more complicated because our model

is nonlinear in the variables and the parameters, and the panel is unbalanced (i.e., there are different numbers of observations for different pavement sections). Therefore special routines had to be programmed for estimation of the model. The details of the estimation approach are given in Archilla (2000).

Initially, the model was estimated separately for the inner wheelpath (IWP) rutting and the outer wheelpath (OWP) rutting using both the fixed-effects and random-effects approaches. In all, 7,005 observations, corresponding to 244 pavement sections, were used for the IWP and 7,035 observations, corresponding to 247 pavement sections, were used for the OWP. For each wheel path, the parameter estimates were almost identical from both approaches. Although for both wheel paths, the fits were good and all the parameter estimates were significant (the smallest asymptotic  $t$ -statistics was 6.6), there were significant discrepancies between the parameters for both wheel paths. These differences were not surprising per se, as differences in the rate of rutting can be visually observed in plots of rutting trends over time. Furthermore, there are several reasons why this is the case. For example, the state of stresses caused by the same load is different in each wheel path because the boundary conditions are different. One would expect a higher degree of confinement in the IWP than in the OWP, and as is well known from soil mechanics, the shearing resistance of a soil is higher for higher confining pressures. Thus, the different states of stress and possibly different resistances can lead to different model parameters. For another example, consider the effect of thawing. If the pavement surface is not cracked, then after a thawing cycle it will take longer for the excess water to leave the pavement structure under the IWP than under the OWP (the drainage path is longer).

Even though differences in the parameter estimates for both wheel paths should be expected, the differences we obtained were suspiciously high. For example, the estimates for  $\beta_5$  were 2.16 for the IWP and 3.72 for the OWP, and the estimates for  $\beta_6$  were 2.92 for the IWP and 4.81 for the OWP. Furthermore, the estimate of  $\beta_9$  was 0.524 for the IWP and 0.377 for the OWP, which was inconsistent with the generally observed faster rutting rate for the IWP during periods without thawing.

A possible explanation for the above results is that the objective function is very flat near the optimum along some paths in the parameter hyperspace, and therefore the variation in performance between the two wheel paths can cause high differences in the parameter estimates. This may happen despite the high  $t$ -statistics, which are computed using partial derivatives of the predicted values of rut depth with respect to each of the parameters. The variation in one parameter alone may have a pronounced effect in the objective function, but the same variation in conjunction with variations from other pa-

TABLE 1. Model Estimation Results

Parameter (1)	Parameter description (2)	$H_0^*$ (3)	IWP		OWP	
			Parameter estimate (4)	Asymptotic $t$ -statistic (5)	Parameter estimate (6)	Asymptotic $t$ -statistic (7)
$\beta_1$	Asphalt concrete coefficient	0	3.34	10.10	5.43	17.50
$\beta_2$	Base coefficient	0	2.07	18.42	3.57	34.05
$\beta_3$	Subbase coefficient	0	2.36	25.36	2.87	32.06
$\beta_4$	Subgrade coefficient	0	0.90	10.18	1.89	11.73
$\beta_5$	Single axle exponent	0	2.98	42.57	2.98	42.57
$\beta_6$	Tandem axle exponent	0	3.89	35.90	3.89	35.90
$\beta_7$	Conversion to standard tandem axle	1	1.81	62.74	1.81	62.74
$\beta_8$	Thawing index coefficient	0	1.96	54.81	1.60	56.49
$\beta_9$	$N_{it}$ exponent	0	0.412	33.29	0.452	38.67
$\beta_{10}$	Constant	0	-0.449	-2.57	0.022	0.13

Note:  $\sigma_u^2 = 4.53$ ;  $\sigma_\varepsilon^2 = 6.17$ ; number of observations = 14,042.

\*Null hypothesis for which asymptotic  $t$ -statistics are computed.

parameters may have a negligible effect. For example, a high rutting rate could be attained with a high value for  $\beta_4$  (the subgrade coefficient), or high  $\beta_5$  and  $\beta_6$  (the load equivalence coefficients) with a low  $\beta_9$  (the loading exponent), or low  $\beta_5$  and  $\beta_6$ , but a high  $\beta_9$ , or some other combination. These interactions are believed to have caused the differences in the parameter estimates. Because the observations for both wheel paths show similar trends, but at the same time, some clear differences, combining them can help to reduce the uncertainty of common parameter estimates. This is similar in concept to the effect of the variance of the independent variables in linear regression models. In such models, other things being equal, the larger the variance in the independent variables, the smaller the variance of the parameter estimates.

From a practical point of view, one would like to summarize the effect of traffic in a single variable independent of wheel path. This is actually what is done by pavement engineers when they use the concept of an ESAL. Therefore the model was reestimated for both wheel paths simultaneously, restricting only  $\beta_5$ ,  $\beta_6$ , and  $\beta_7$  to be the same for both wheel paths. The fit was only slightly lower than when the model was estimated separately for each wheel path, thus confirming the suspicions mentioned above.

Table 1 shows the estimation results using the random-effects approach. The parameter estimates using the fixed-effects approach were practically the same. All the coefficients are statistically significant and have the expected signs. Examining the ratio  $\beta_1/\beta_2$ , the asphalt concrete layer is observed to be only 1.68 times more effective in reducing rutting than the base layer for the IWP and 1.53 times more effective for the OWP. For the OWP, the contribution of the base is 1.23 ( $\beta_2/\beta_3$ ) times the contribution of the subbase, but this result

is reversed for the IWP where the contribution of the base is 0.87 times the contribution of the subbase. The factors affecting the pavement performance on each wheel path mentioned above may play a role in this result. No comparisons are made between  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  for the different wheel paths because their values are related to the value of  $\beta_4$ . The estimate of  $\beta_4$  for the OWP is about twice the estimate for the IWP. This is in agreement with the hypothesis that subgrade material is more confined for the IWP.

The coefficient  $\beta_7 = 1.81$  indicates that a tandem axle load of 145.15 kN (32,659 lbs) has the same effect on rutting as an 80-kN (18,000-lbs) single axle load. This is in agreement with the assumption made at the AASHO Road Test (HRB 1962) that an 80-kN (18,000-lbs) single axle was equivalent to a 32,000-lbs tandem axle. Notice that the coefficients  $\beta_5$  and  $\beta_6$ , which indicate the equivalencies within axle configurations (single or tandem), are significantly different from each other. Furthermore,  $\beta_5 = 2.98$  is significantly different from 4.0. This illustrates the advantage of not having presupposed a four-power law for load equivalencies.

The significance of  $\beta_8$ , the coefficient of the thawing index, shows that the inclusion of the environmental effects is very important to avoid biasing the other parameters. These estimates are also in agreement with the hypothesis that thawing has a greater proportional effect on the IWP than on the OWP.

The values of  $\beta_9$  are consistent with the concave shapes reported in the literature. This is also a convenient result because, when using such models for prediction, traffic forecasts are usually subject to error, especially for longer planning horizons. Therefore, it is desirable for the model predictions to be robust with respect to traffic forecasting uncertainty. For the present model, a 20% underestimation of  $N_{it}$  causes only

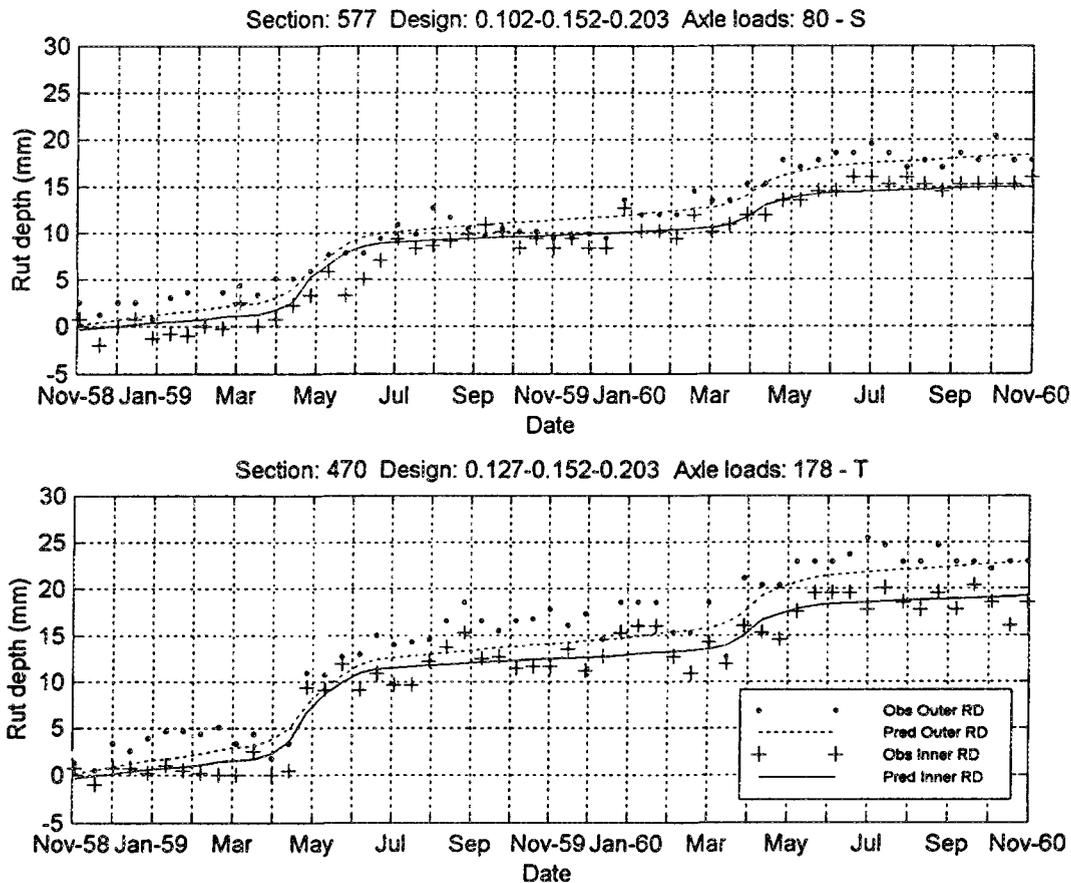


FIG. 4. Observed and Predicted Rut Depth versus Time for Two Sections Used to Estimate Model Parameters (Design Numbers Represent Thickness Values (in Meters) of Asphalt Concrete, Base, and Subbase, Respectively; Axle Loads Are Expressed in Kilonewtons; S and T Indicate Single or Tandem Axles, Respectively)

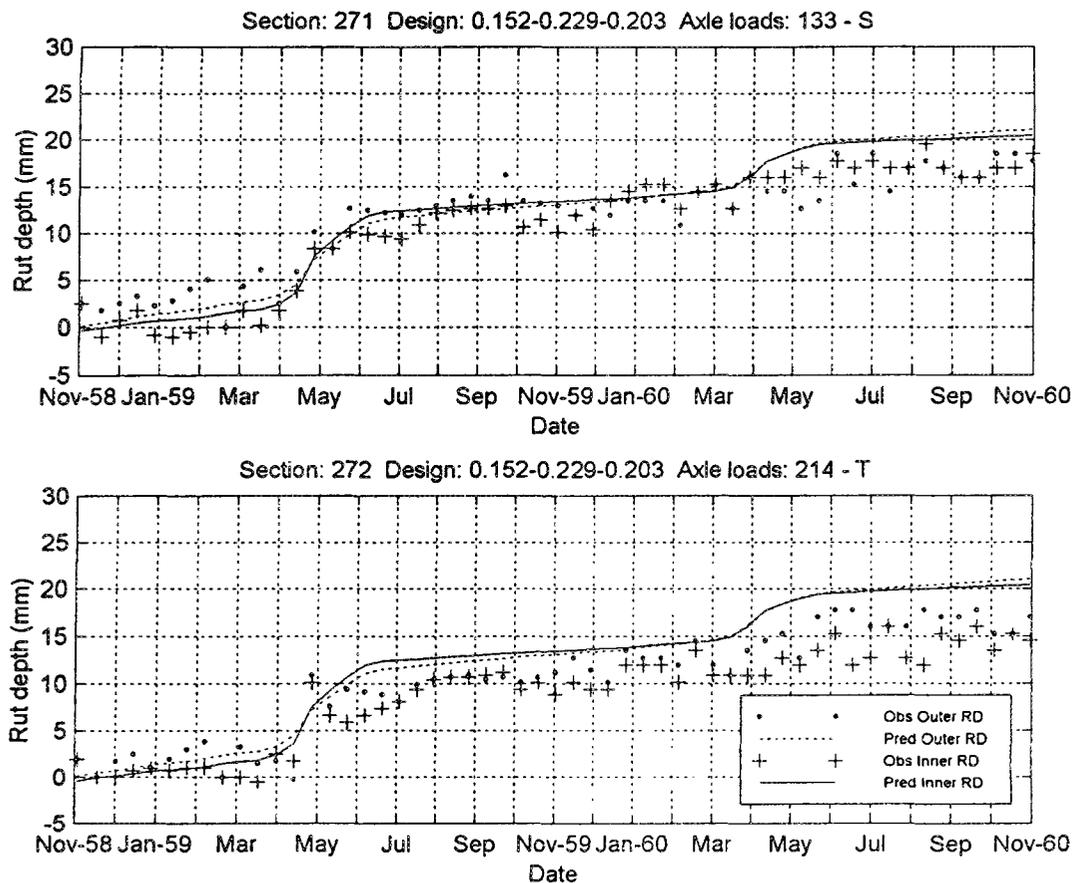


FIG. 5. Observed and Predicted Rut Depth versus Time for Two Sections Not Used to Estimate Model Parameters (Design Numbers Represent Thickness Values (in Meters) of Asphalt Concrete, Base, and Subbase, Respectively; Axle Loads Are Expressed in Kilonewtons; S and T Indicate Single or Tandem Axles, Respectively)

a 13% overestimation of  $\Delta RD_{i,t}$ , whereas a 20% overestimation of  $N_{i,t}$  causes a 10% underestimation of  $\Delta RD_{i,t}$ .

The estimates of  $\sigma_u^2$  ( $6.17 \text{ mm}^2$ ) and of  $\sigma_e^2$  ( $4.53 \text{ mm}^2$ ) indicate that the individual effects produce more than 50% of the variance. This shows that the size of the unobserved heterogeneity is significant.

Finally, the estimated standard error of the regression, 3.3 mm, is within the accuracy with which rut depth was measured in the AASHO Road Test. The result is even better in a pavement management context where the random effects are less important since previous observations of rut depth are used to predict the future observations. In this case the estimate of  $\sigma_e$  = 2.1 mm is more relevant.

Fig. 4 shows a comparison of the predicted rut depths to the observed rut depths for two of the sections in the estimation sample. As can be observed in the figure the pavement behavior of these sections is replicated quite well for both wheel paths. With a few exceptions, this was generally the case. This was further confirmed by a prediction test with a set of pavements not used for estimation. Fig. 5 shows two examples for these sections.

The above results indicate that the model assumptions seem to be generally valid. It should be noticed, however, that the residuals from some sections indicate some heteroskedasticity (variance increasing with thawing index), which leads to some estimation in efficiency.

## CONCLUSIONS

The goal of this paper was to develop a model of pavement rutting from the AASHO Road Test. A nonlinear model was specified and estimated. The model specification uses concepts that are familiar to pavement engineers such as load equivalence

and structural coefficients. However, the model in this paper is an improvement over other state-of-the-art empirical formulas for several reasons. The load equivalence parameters and the resistance parameters were allowed to vary freely during estimation. This is in contrast with previous research where these coefficients were prespecified. This is perhaps one reason for the lack of success in developing empirical models to date.

Another important difference with previous research is the introduction of a thawing index. This variable proved to be extremely important to capture the effect of the environment in the AASHO Road Test.

The model fits were good, especially considering the number of sections and observations that were used for their estimation. Both fixed-effects and random-effects specifications were used to account for unobserved heterogeneity. The results showed that the size of the unobserved heterogeneity was significant.

The specification of a nonlinear model allowed a good fit. However, it also called for a more careful analysis of the estimation results even when all the statistics indicated no problems. In particular, the present model contained several parameters that interacted so as to capture similar effects. By estimating the model parameters for both wheel paths jointly, we were able to reduce the uncertainty in these parameters' estimated values.

Finally, a prediction test with a set of pavements not used for estimation confirmed that the model replicates well the pavement behavior in the AASHO Road Test.

Despite the significant estimation results in this paper, the model has the following limitations. First, our model is limited to the materials used in the AASHO Road Test. Several factors affecting the rutting performance of asphalt concrete mixes, such as asphalt content, are not taken into account. Second,

the model cannot account for pavement rutting due to the instability of asphalt concrete mixes in high temperature environments. Finally, the experimental data may not represent the true deterioration mechanism of in-service pavements because of differences in factors such as traffic wander, traffic speed, and material aging.

It should be noted that most of the limitations mentioned above are common to all existing empirical models. For example, most models developed to date consider a very limited range of materials. Further, most of these models usually use the number of ESAL applications and the structural number as explanatory variables. However, the computation of ESAL and structural number is carried out with parameters that were either originally estimated or modified for serviceability models from the results in the AASHO Road Test or other tests. Therefore, all such models are implicitly more limited than the model presented here, particularly considering that the parameters were not estimated for rutting performance.

In addition, some of these limitations are not so severe for some situations. For example, if the asphalt concrete mix is stable, as was the case for the AASHO mixes, the first two limitations mentioned above are not relevant. Rutting of the mix is implicitly included in the model. Thus, even though the subgrade material in the AASHO Road Test is representative of weak subgrades, the model can be useful for the prediction of rut depth progression on pavements with weak subgrades, granular base and subbase, and stable mixes. According to results from special experiments in the AASHO Road Test, the model should also give reasonable results for pavements with asphalt-treated bases. For pavement with cement-treated bases, it may overestimate the rut depth by about 30–40%. The model should not be used for strong subgrade materials or when no source of water is present and/or there is no freeze-thaw cycles because in these cases it will grossly overestimate rut depths.

For the development of better pavement rutting models, it is necessary to overcome the above limitations, particularly the consideration of different sources of rutting (i.e., rutting originating in the asphalt concrete layer and rutting originating in the underlying layers). A promising approach is the use of joint estimation from different data sources (Ben-Akiva and Morikawa 1990). The objective of joint estimation is to yield a more reliable model of pavement rutting than those produced with either data source alone. This is the subject of ongoing research by the writers.

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