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OLIGOPOLISTIC LOCATION UNDER  
IMPERFECT CONSUMER INFORMATION

BY

KONRAD STAHL

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OLIGOPOLISTIC LOCATION UNDER IMPERFECT  
CONSUMER INFORMATION

BY

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# OLIGOPOLISTIC LOCATION UNDER IMPERFECT CONSUMER INFORMATION

KONRAD STAHL

## ABSTRACT

A simple N-person noncooperative game is developed in which firms react, by choice of location, to consumers' search for their most preferred among several substitutable alternatives. It is shown that noncooperative behavior always leads to some spatial concentration, if not concentration of all firms in one marketplace. The latter solution is never in the core, however. Thus, it can always be improved upon from the firms', if not from a general welfare, point of view.



## 1. INTRODUCTION

The model presented in this paper deals with sellers' reactions, in terms of choice of location, to consumers' search for their most preferred product. It should serve two purposes: one, in complementing the results of the so-called search literature on firms' pricing behavior in the face of imperfect consumer information by introducing many (more or less substitutable) products and concentrating on the locational choice of the firms merchandising these products; a second one in explaining and evaluating the observed concentration of some markets in space, relative to the spatial distribution of consumers. The markets discussed here involve products whose purchase requires personal search beyond search for the lowest price. Most suitable examples are antiques, used cars or fashion clothing; or more generally, commodities whose supply characteristics do change more frequently than purchases are made by the typical consumer.<sup>1</sup>

At this point, it does not seem warranted to review the vast literature in the Stigler [4] tradition on models involving consumer search and firms' response in terms of pricing policies. An assumption common to these models is that the cost of obtaining information about any seller's price quotations (or the product variety offered by him) is an exogenous constant. Firms' pricing behavior is analyzed in response to consumers' search. However, no account is given to the fact that the locational choice of suppliers influences consumers' search costs; comparing qualities or prices is easy at marketplaces where many of the relevant firms are located; it is more and more costly as these sellers disperse in space.<sup>2</sup>

This idea is made precise in the simple model developed below. It is formulated as an N-person noncooperative game, involving the locational choice of



N firms that sell at constant identical prices, one each of N differentiated commodities. Consumers located at a uniform density along an unbounded line, are differentiated by their tastes for these commodities. The typical consumer knows which commodities in principle are available, but does not know which one of the commodities is offered where. In order to find out, he has to visit a marketplace, defined as an assembly of one or more firms. If he does so and he finds commodities suiting his preferences, he purchases one unit of the most preferred commodity offered in that marketplace. The typical firm, knowing of consumers' preferences and behavior, chooses the location maximizing its sales under the Cournot assumption. The structure of the model thus maintains the informational asymmetry typical of the search models.

The explicit incorporation of space gives rise to an interesting variety of concepts and results. As it will turn out, the size of a marketplace as defined by the number of firms located or products offered there will determine the size of the market area within which consumers are willing to visit the marketplace: the larger a marketplace, the longer the distance the typical consumer is willing to travel in order to pay a visit, because the expected utility derived from visiting that place increases with its size.

As a result of this, a marginal seller choosing a profit maximizing location faces the following principle alternative: he may either establish a local monopoly at an isolated location, or he may join other firms in a competitive large marketplace. In the first case, he will fetch the demand of *all* consumers visiting his marketplace for whom his variety is acceptable. However, the *area* from which he attracts consumers will be relatively *small*. Conversely, in the second case, he will realize the demand from *only a small subset* of the *many* consumers patronizing the large marketplace. Putting things differently,

the change in our seller's market demand when joining that marketplace may be decomposed into two effects: a negative *substitution effect* generated from consumers who, confronted with several acceptable varieties beyond the one offered by our seller, will prefer another one, and a positive *market area effect* generated from the joint location of sellers. *Despite competition, our seller will join the larger marketplace if the reduction in the demand due to the substitution effect will be more than outweighed by the market area effect.* Furthermore, if joining the large market place that seller will, by increasing its attractiveness, confer an *external benefit* to the sellers already located there, at least to those that do not compete with him about consumers' demand. This external benefit is exclusively due to the fact that consumers are not informed as to which products are offered where. It is conferred even and especially if the demands for the different commodities provided in the marketplace are unrelated in the sense that no consumer derives utility from more than one variety of the commodities. This external benefit may be directly internalized by the typical seller in offering several varieties of the commodities or indirectly by a developer in providing an institutional framework for sellers' collusion. Some consequences of such arrangements will be discussed later.

The thrust of the results derived from the formal model is on a characterization of the equilibrium solutions of the noncooperative location game. After establishing the straightforward, yet fundamental result that the attractiveness of a market place varies with its size, we demonstrate firstly that there is no equilibrium involving separate locations for all firms, and secondly, that whenever consumers are choosy enough to consider acceptable only one variety of the commodities, the only equilibrium solution is joint

location at one marketplace. We then go on giving, under slightly more restrictive assumptions, sufficient conditions for a joint location at one marketplace to be in the set of equilibria even if consumers consider several varieties acceptable. Furthermore, conditions are derived under which firms, if confronted with the alternative of choosing an isolated location vs joining a large marketplace, will always find it profitable to do the latter. After thus having established that concentration of all firms in one marketplace is an equilibrium configuration very likely to arise, we show by a counterexample that it is *not* in the core. Thus *establishing separate marketplaces for suitable selections of firms is always preferable from the sellers' point of view.*

It might be of interest that the model presented here provides a specification - out of several possible - of the somewhat diffuse concept of "*agglomeration economies*" that is frequently employed in the literature on spatial economics and regional science. In addition, it is demonstrated that - at least within the present model - an exhaustion of these economies by firms acting noncooperatively may be *excessive* from the point of view of the firms and from a general welfare point of view. Inasmuch as this tendency is not more than offset by deglomerative forces not incorporated in the model (such as congestion), the latter result provides a rationale for location and land use controls to be exercised by public authorities.<sup>3</sup>

The results generated from this model appear to be similar to those derived by Hotelling in a celebrated paper [1]. However, it is well known that his results are very sensitive to the assumptions used, in particular as to the elasticity of consumers' demand and the number of sellers participating in the game.

The sequel of the paper is divided into four sections: section 2 contains the model description and comments on the relevant assumptions. The principal results are derived in section 3. In section 4, some further interpretations and extensions of the model are discussed informally. Section 5 offers remarks on future research.

## 2. THE MODEL

Consider an economy involving a finite number  $N$  of differentiated commodities  $i, i \in N, N = \{1, \dots, N\}$ , and a numeraire good whose quantities are denoted by  $x$ . There is a continuum of consumers making indivisible and mutually exclusive purchases of one of the  $N$  commodities. They are distinguished by preferences and income. More specifically, we assume that at zero prices for all commodities, consumer type  $i, i=1 \dots N$  derives positive utility from a selection  $a_i$  of the set  $N$  of differentiated commodities, and that he has a strictly transitive preference ordering over the commodities  $j_i \in a_i$ . Let that consumer's preferences be represented by the index  $u_i(j_i, x_i)$  where  $j_i$  refers to one unit of that consumer's  $j^{\text{th}}$  preferred alternative and  $x_i$  to the quantity of the numeraire good.  $u_i$  is assumed to satisfy the following properties:

(i)  $u_i$  is continuous and strictly increasing in  $x_i, x_i \geq 0$

(ii)  $u_i(j_i, .) \begin{cases} > u_i(0, .) & j_i \in a_i, \quad a_i \subset N \\ = u_i(0, .) & \text{otherwise} \end{cases}$

(iii)  $a_i = \{k | k = i + (j-1) \bmod N, j=1, \dots, m_i\}$ , with  $m_i = \#a_i$ ;  
 $u_i(j_i, .) > u_i(j_i+1 \bmod N, .), j_i = i+j, j=1, \dots, m_i-1.$

Without loss of generality we denote in the sequel by  $u_i(j, \cdot)$  the utility consumer  $i$  derives from consumption of his  $j^{\text{th}}$  preferred alternative. Let finally consumers type  $i$  be endowed with income  $R_i$ .

There are  $N$  firms,  $N > 2$ , with firm  $i$  selling commodity  $i$  at location  $l_i$  at zero cost and price,  $i = 1 \dots N$ . Let the set of all firms' locations be denoted by  $L$ .

The geographical space considered in the model is the real line  $(-\infty, +\infty)$  with  $y$  denoting a consumer's location. There is an infinite population of consumers. Consumers type  $i$  are distributed uniformly along  $(-\infty, +\infty)$ . Thus if  $\phi_i(y)$  denotes that distribution we assume that

$$(iv) \quad \phi_i(y) = \phi_i > 0, \quad y \in (-\infty, +\infty), \quad i = 1, \dots, N.$$

The numeraire good is available everywhere along the line, whereas commodity  $i$  is available only at location  $l_i$ .

We define a *market place*  $k$  to consist of a location with an assembly of one or more firms. There are  $K$  market places,  $1 \leq K \leq N$ . The *size of market place*  $k$ ,  $s_k$ , is defined as the number of firms assembled at that market place. Thus  $1 \leq s_k \leq N$ ,  $\sum_k s_k = N$ . Let  $S = \{s_k\}$ . The *composition of market place*  $k$ ,  $e_k$ , is the set of firms assembled in that market place. It follows that  $e_i \cap e_k = \emptyset$ ,  $i \neq k$ ;  $i, k = 1, \dots, K$  and that  $\sum_{k=1}^K \# e_k = N$ . Let  $E = \{e_k\}$  denote the partition that defines the composition of market places. Finally, let  $v_k$  denote the location of market place  $k$ ,  $v_k \in (-\infty, +\infty)$ . Thus, if firms  $i \in e_k, j \in e_k$ , then  $l_i = l_j = v_k$ . Let  $V = \{v_k\}$ .

Let us now turn to a description of consumer and firm behavior. Consumers are assumed to know the locations  $V$  and the sizes  $S$  of the market places, but not  $E$ , the composition of the market places. In order to find out about any  $e_k$ ,

they must visit  $k$ . In doing so they incur transportation costs to the marketplace linear in distance, at rate  $c$ . Each consumer visits, at most, one marketplace. If he does so, he samples all stores at no additional cost and decides either to purchase the commodity suiting his preferences best, or none if no acceptable commodity is available. His choice of a marketplace is based on expected utility maximization he assigns equal probability to finding any alternative in the marketplace. He decides not to visit any marketplace, however, if the maximal expected utility from visiting a marketplace falls short of the utility derived from staying put and spending all of his income on the numeraire good. Let  $V$ , and  $S$  be given. Let furthermore

$$\pi_j(s_k) = \text{Prob} \{1, 2, \dots, j-1 \notin e_k, j \in e_k\}$$

denote the probability that the first  $(j-1)$  preferred commodities are not, but the  $j^{\text{th}}$  preferred commodity indeed is offered in market place  $k$  of size  $s_k$ .

Then the first step in the consumer's maximization process is given by

$$\max_k \left[ \sum_{j=1}^{m_i} \pi_j(s_k) u_i(j, R_i - cd(v_k, y)) + (1 - \sum_{j=1}^{m_i} \pi_j(s_k)) u_i(0, R_i - cd(v_k, y)) \right]$$

where  $d(v_k, y)$  is the Euclidian distance function. Denoting by  $k^*$  his optimal choice, he does (not) visit market place  $k^*$  iff

$$\sum_{j=1}^{m_i} \pi_j(s_{k^*}) u_i(j, R - cd(v_{k^*}, y)) - \left[ 1 - \sum_{j=1}^{m_i} \pi_j(s_{k^*}) \right] u_i(0, R - cd(v_{k^*}, y)) \begin{matrix} > \\ < \end{matrix} u_i(0, R). \quad (1)$$

The firms considered in the model know  $V$ ,  $S$  and  $E$  and in addition are assumed to have full knowledge of consumers' preferences, endowments and behaviour. Let  $l_{-i}$  denote the collection of locations of all firms but  $i$ . Given  $l_{-i}$ , firm  $i$  selects at no cost the location  $l_i^* \in (-\infty, +\infty)$  maximizing its profits. Thus, letting  $\Pi_i(l_i, l_{-i})$  be the payoff (sales)  $i$  obtains at  $l_i$ , given  $l_{-i}$ , then  $\Pi_i(l_i^*, l_{-i}) \geq \Pi_i(l_i, l_{-i}) \forall l_i$ .

A noncooperative equilibrium is a set  $L$  of locations such that no firm considers it profitable to relocate on its own. Thus  $L^* = \{l^*_1, \dots, l^*_i, \dots, l^*_N\}$  is a noncooperative equilibrium iff

$$\Pi_i(l^*_i, l^*_{-i}) \geq \Pi_i(l_i, l^*_{-i}) \quad \forall i, i = 1 \dots N.$$

Let  $L^*$  denote the set of noncooperative equilibria.

An *imputation* is a vector of payoffs  $\Pi = (\Pi_1, \dots, \Pi_N) \in \mathbb{R}_+^N$  representing an outcome of the game considered here. It is *feasible* if there is a set of locations of the  $N$  sellers such that  $(\Pi_1, \dots, \Pi_N)$  is realized. A *coalition* is a subset  $C \subset N$ . An imputation  $\Pi$  is *feasible for C* if  $C$  can ensure its members  $j$  the payoffs  $\Pi_j$ . Furthermore, that imputation is *feasible* if it is feasible for  $C$  and for  $N \setminus C$ . The coalition  $C$  *blocks the imputation*  $(\Pi_1^0, \dots, \Pi_N^0)$  if there is an imputation  $(\Pi_1^1, \dots, \Pi_N^1)$  feasible for  $C$  such that  $\Pi_j^1 \geq \Pi_j^0 \quad \forall j \in C$  and  $\Pi_j^1 > \Pi_j^0$  for at least one  $j \in C$ . Finally, the *core* of the game consists of the set of feasible imputations which are not blocked by any coalition. This concludes the description of the model. The following remarks explain some of its features and justify the assumptions.

Remark 1: The game considered here has the property that the typical seller  $i$ 's payoff is discontinuous in his own strategy  $l_i$  at all  $l_j = l_i, j \neq i$ . This is an implication of the somewhat rigid but convenient assumption that consumers visit, at most, one marketplace. It follows that the existence of equilibria cannot be shown in general. However, since any location is feasible for any seller, existence follows from showing that for a certain  $L^*$ , each seller's decision  $l_i^*$  given  $l_{-i}^*$  dominates all of his alternative decisions.

The discontinuity of the payoff functions also implies that equilibrium configurations are influenced by the presumed initial locations of firms and the sequence in which firms are allowed to choose their optimal strategies. It

follows that a characterization of equilibria is not straightforward.

Remark 2: That consumers make indivisible and mutually exclusive purchases is an unnecessary but convenient and, in addition, a quite realistic assumption. For most, if not all commodities in the class of commodities considered here, consumption decisions are made that way. Assumptions (i) and (ii) are innocent. Assumption (iii), resting on the equality between the number of consumer types and number of brands available, specifies that consumer type  $i$ 's  $j$ th preferred commodity is variety  $i + j$ ,  $j = 1, \dots, m_i$ . Together with assumption (iv) restricting the distribution of consumer types in space, it imposes a quite restrictive regularity on the aggregate demand as perceived by the sellers.<sup>4</sup> Such regularity assumptions are standard in location theory and quite justifiable when they serve to generate paradigmatic results.

Generating results on the basis of arbitrary variations in (iii) or (iv) would not add to an analysis of the forces discussed here. In fact, we sometimes will use even more rigid assumptions standardizing consumer demand. They are

$$(i') \quad u_i(j, x) = u_i(j) + x$$

$$(ii') \quad u_i(j) = u_k(j) = u(j) \quad \forall i, k \in N \text{ and } j = 1 \dots m$$

together with the assumption

$$(iv') \quad \phi_i(y) = \phi \quad \forall i.$$

Of these, only (i') is not exclusively for expositional purposes. In defense of that assumption, one might argue that the income effects of the shopping trips considered here are negligible. The assumptions about consumer behavior specify a simple search process in which, due to zero cost of local search, all stores in the marketplace are sampled and recall is allowed for. The formulation has the advantage that the typical seller's demand can be determined explicitly as a function of  $L$ , i.e., all firms' locations.



Remark 3: That firms sell at constant (zero) cost and prices seems a very rigid assumption indeed. Standardizing costs contradicts the observation that rents vary considerably across space, assuming away price competition interferes with standard hypotheses of microeconomic theory. In the present paper, both assumptions are made for analytical convenience. There is a straightforward rationalization for the first assumption, however: if rents to fixed factors (such as land at given locations) are determined as residuals, then we first have to concentrate on the forces generating the surpluses which, in turn, may be absorbed in land rent. This is done here. As to the second assumption, it is not at all clear that its relaxation will lead to prices being driven down in large competitive marketplaces. In fact, if they do go down, then the land rents generated at such marketplaces would have to be zero, contradicting all empirical evidence. A further comment on this point is given in section 4.

Remark 4: The unboundedness of the geographical space considered here is convenient since by this we do away with spatial competition. This is because of the finiteness of  $N$ : each firm can move as far away from another as it wishes without changing the composition of its demanders. As will become obvious below, introducing bounds on consumer and firm locations does reinforce the results on the spatial concentration of sellers that constitute the thrust of the paper.

### 3. RESULTS

The results obtained from this model are organized in the following way: We first show the principal feature bringing about the concentration of markets in space, namely that the attractiveness of a market place measured in the size of its market area increases with its size. We then show that for

all practical purposes there cannot be a noncooperative equilibrium involving  $N$  market places. We continue by demonstrating that if consumers are very choosy, the only equilibrium configuration possible is the concentration of firms in one market place, and that furthermore, this proposition is likely to hold even if consumers are less choosy. The final result shows that such a configuration cannot be in the core.

The conditions under which equilibrium characterizations are given involve the parameters  $m_i$ ,  $i = 1 \dots N$ ;  $s_k$ ,  $k = 1 \dots K$ ; and  $N$ . The number of alternatives  $m_i$  acceptable to consumers is critical in determining the *share of consumers* to which a firm sells its commodity when locating in market places of alternative *structure*. The sizes of market places  $s_k$  are important because they determine the *number of consumers* of each type a firm sells its commodity to when locating in marketplaces of alternative *size*. All these parameters are related to  $N$ , the total number of commodities, consumers, and firms.

Before we turn to the first proposition, let us define by  $\tilde{z}_i(s)$  the maximal distance a consumer type  $i$  is willing to travel to a market place of size  $s$ . Thus,

$$\tilde{z}_i(s) = \left\{ z \mid \sum_{j=1}^{m_i} \pi_j(s) [u_i(j, R_i - cz) - u_i(0, R_i - cz)] + u_i(0, R_i - cz) = u_i(0, R_i) \right\} \quad (2)$$

Proposition (1): Suppose (i) through (iv) hold. Then  $\tilde{z}_i(s)$  is strictly increasing in  $s$ ,  $i = 1, \dots, N$ .

Proof: Using (2), it suffices to show that for given  $z$ ,  $s' > s$  implies

$$\sum_{j=1}^{m_i} (\pi_j(s') - \pi_j(s)) [u_i(j, R_i - cz) - u_i(0, R_i - cz)] > 0,$$

which is equivalent to showing that

$$\begin{aligned}
 & [\pi_1(s') - \pi_1(s)][u_i(1, R_i - cz) - u_i(2, R_i - cz)] \\
 & + [\pi_1(s') + \pi_2(s') - (\pi_1(s) + \pi_2(s))][u_i(2, R_i - cz) - u_i(3, R_i - cz)] \\
 & + \dots \\
 & + [\pi_1(s') + \dots + \pi_{m_i}(s') - (\pi_1(s) + \dots + \pi_{m_i}(s))][u_i(m_i, R_i - cz) - \\
 & \qquad \qquad \qquad - u_i(0, R_i - cz)] > 0
 \end{aligned}$$

Using (iii) this inequality is satisfied if for  $1 \leq \hat{m} \leq m_i$ ,

$$\sum_{j=1}^{\hat{m}} [\pi_j(s') - \pi_j(s)] > 0.$$

Observe that for any  $k$

$$\begin{aligned}
 \sum_{j=1}^{\hat{m}} \pi_j(s_k) &= \text{Prob} \{1 \in e_k\} + \text{Prob} \{1 \notin e_k, 2 \in e_k\} + \dots \\
 &+ \text{Prob} \{1, 2, \dots, \hat{m} - 1 \notin e_k, \hat{m} \in e_k\} \\
 &= \text{Prob} \{(1 \in e_k) \vee \dots \vee (\hat{m} \in e_k)\}.
 \end{aligned}$$

What we have demonstrated in proposition 1 follows rather straightforwardly from the conception of the model, but still is interesting on its own: the attractivity of a marketplace, and therefore its market area, increases with its size, i.e., the number of alternatives from which consumers may choose.

We now show that under the assumptions specified so far, there cannot be noncooperative equilibrium involving  $N$  marketplaces.

Proposition (2): Suppose that (i) - (iv) hold. Then  $L^*$  such that

$\#L^* = N$  is never in  $L^*$ .

The proof of this proposition is trivial. Consider two firms  $i$ , and  $j$ . Let  $j = (i-1)$ . Suppose to the contrary that there is a noncooperative equilibrium  $L^*$  with the property that  $l_i^* \neq l_j^*$ ,  $i \neq j$ . Then

$$\Pi_i(l_j^*, l_{-i}^*) > \Pi_i(l_i^*, l_{-i}^*)$$

because when joining  $j$ , firm  $i$  does not face any competition about consumers; but its market area increases, by proposition 1. Contradiction.

The result is obtained at special ease via assumption (iii). It should be obvious, however, that the conditions specified there are far from necessary. One might be tempted to ask at this point whether there are conditions guaranteeing an  $N$  marketplace equilibrium. Given that consumers are imperfectly informed, the only sufficient condition for spatial concentration never to take place is that, contrary to assumption (iii), *all* commodities are considered *perfect substitutes* by *all* consumers. For, if this does not hold for all consumers and for all commodities, proposition 1 demonstrates that some firms may gain from spatial association.

Turning now to conditions under which all firms will concentrate at one market place we show first a formally trivial, but nevertheless educative result that if all consumers are choosy enough to desire, at current prices, one and only one commodity type, then concentration at one market place will be the only noncooperative (and cooperative) equilibrium possible.

Proposition (3): Suppose that (i) - (iv) hold and in addition

that  $m_i = 1$ ,  $i = 1 \dots N$ . Then  $\# L^* = 1$ .

The proof of this proposition follows readily from proposition 1.<sup>5</sup> This result is not at all surprising: By assumption, the demands faced by the individual firms are *independent* in the sense that no consumer wants to

purchase more than one variety. The *interdependence between firms' payoffs that drives them together is due to consumers' imperfect information.* In fact, were the consumers fully informed about varieties offered at any market place, the firms would locate arbitrarily under the conditions of proposition 3. In this situation, the external benefit conferred upon each other appears most explicitly.

Now the condition that  $m_i = 1 \forall i$  is sufficient but obviously not at all necessary. Primarily for the reasons mentioned in remark 1, however, a straightforward generalization of proposition 3 to the case where  $m_i > 1$  is difficult. The following example therefore might support the intuition that  $m_i = 1$  is far too restrictive.

Example: Let the set  $N$  of consumer types be partitioned into two mutually exclusive subsets  $N_1, N_2$  of size  $n$  and  $N-n$  respectively. Suppose that for consumers  $i \in N_1, m_i = 1$  whereas for consumers  $i \in N_2, m_i = 2$ . Let (i'), (ii'), (iii), and (iv') hold for all consumers  $i \in N$ . We will show that there is an  $\hat{n} < N$  such that  $n \geq \hat{n}$  implies  $\# L^* = 1$ . Let firms  $i \in N_2$  be selected such that, if they are joined at one market place, no one faces competition with any other: More precisely that each firm  $i, i \in N_2$  will realize the demand from all consumer types  $q_i, q_i \equiv \{ k \mid u_k(i, \cdot) > u_k(0, \cdot) \}$ . By assumption,  $\# q_i = 2, i \in N_2$ . Recall from proposition 3 that all firms  $i \in N_1$  will locate jointly - at  $\ell_i = v_k, i \in N_1$ , say - and so do all firms  $i \in N_2$  - at  $\ell_i = v_{k'}, i \in N_2$ , say. The desired result will hold if for  $i \in N_2$

$$\Pi_i(v_k, \ell_{-i}) > \Pi_i(v_{k'}, \ell_{-i}) \quad \ell_j = v_k, j \in N_1; \quad \ell_j = v_{k'}, j \in N_2, j \neq i$$

Using (2) this will hold if for  $n > \hat{n}$

$$\sum_{j=1}^2 \pi_j^{(n+1)} u(j) > \sum_{j=1}^2 \pi_j^{(N-n)} u(j), \quad j \in N_2.$$

Observe that

$$\pi_j(s) = \frac{\binom{N-j}{s-1}}{\binom{N}{s}} \quad j \leq N-s+1$$

Hence the inequality is satisfied if

$$\frac{n+1}{N} u(1) + \frac{(n+1)(N-n-1)}{N(N-1)} u(2) > 2 \frac{N-n}{N} u(1) + 2 \frac{(N-n)n}{N(N-1)} u(2).$$

Taking account of (iii) it is easily checked that the inequality holds

if  $n \geq \frac{3N^2 - 5N + 2}{4N - 6}$ . Let  $n$  be the smallest integer satisfying the

latter inequality. ■

The example demonstrates that even under the extreme case that by chance all  $(N-n)$  firms would be selected and located jointly in a marketplace so that they would face no competition with each other, they would prefer to join the larger competitive marketplace provided that this place is large enough.

The following two propositions show first that for  $m_i$  not too large,  $i = 1 \dots N$ , concentration at one marketplace is in the set of noncooperative equilibria; and second, that under the same provision there is a level of concentration of firms such that, given any further concentration, there is no noncooperative equilibrium involving one large and many small marketplaces. As discussed in remark 2, the additional assumptions are introduced mainly for expositional purposes.

Proposition 4: Suppose (i'), (ii'), (iii), and (iv') hold.

There is a  $\hat{m}$ ,  $1 < \hat{m} < N$  such that whenever  $m < \hat{m}$ ,  $L^*$  such that  $\# L^* = 1$  is in the set of equilibria,  $L^*$ .

Proof: This follows from the construction of  $\hat{m}$ . We need to determine  $\hat{m}$

such that whenever  $m < \hat{m}$

$$\Pi_i(l_i, l_i = l_j, j \in N) > \Pi_i(l_i, l_i \neq l_j, j \in N).$$

From (ii') it follows that  $\# q_i = m \forall i$ , and from (iv') that the inequality holds if  $\tilde{z}(N) > m\tilde{z}(1)$ . Simplifying (2) according to (i'), this is implied by

$$u(1) > m \left( \frac{1}{N} \sum_{j=1}^m u(j) \right)$$

or

$$\frac{N}{m} > \frac{\sum_{j=1}^m u(j)}{u(1)}.$$

Using (iii) we obtain

$$m > \frac{\sum_{j=1}^m u(j)}{u(1)}$$

$$\text{Let } \hat{m} = \max \{ k \in \mathbb{N} \mid k < \sqrt{N} \}.$$

Thus, if the number of alternatives acceptable to the typical consumer is not too large relative to the total number of alternatives available, then a configuration in which all firms are concentrated at one market place will be an equilibrium.

Proposition 5: Suppose that (i'), (ii'), (iii) and (iv') hold. Consider

a market place  $k$  at location  $v_k$  that is of size  $s_k$ . Then for each  $m$ ,

$m = 2, \dots, \hat{m}$ ,  $\hat{m} = \max \{ k \in \mathbb{N} \mid k < \sqrt{N} \}$  there is an  $\hat{s}(m)$  such that

$s_k > \hat{s}(m)$  implies

$$\Pi_i(v_k, l_{-i}) > \Pi_i(l_i, l_{-i}) \quad l_i \neq l_j \quad i \neq j.$$

Furthermore,  $\hat{s}(m)$  is increasing in  $m$ .

Proof: This follows again from the construction of  $\hat{s}$ . Simplifying (2)

according to the assumption made and solving explicitly for  $\tilde{z}(s)$ , recalling furthermore that by the assumptions made  $\#q_i = m \quad \forall i$ , it follows that for  $s > \hat{s}$  the following inequality must be satisfied:

$$\sum_{j=1}^m [\pi_j(s) - m\pi_j(1)]u(j) > 0.$$

Since  $u(j) > 0$ ,  $j = 1 \dots m$ , this inequality holds if

$$\sum_{j=1}^m \pi_j(s) - m \sum_{j=1}^m \pi_j(1) > 0.$$

Observe that

$$\begin{aligned} \sum_{j=1}^m \pi_j(s) &= \text{Prob} \{ 1 \in e_k \vee 2 \in e_k \vee \dots \vee m \in e_k \} \\ &= 1 - \text{Prob} \{ 1 \notin e_k, 2 \notin e_k, \dots, m \notin e_k \} \\ &= 1 - \frac{\binom{N-m}{s}}{\binom{N}{s}} = 1 - \frac{\binom{N-s}{m}}{\binom{N}{m}} \end{aligned}$$

Hence the inequality is satisfied if for  $s > \hat{s}$

$$\frac{(N-s)!}{(N-s-m)!} < \frac{N-m^2}{N} \frac{N!}{(N-m)!}$$

or

$$(N-s)(N-s-1) \dots (N-m-s+1) < (N-m)^2 (N-1)(N-2) \dots (N-m+1).$$

This inequality holds always for  $s \geq m^2$ . Let  $\hat{s}(m) = m^2$ ,  $m = 2 \dots \hat{m}$ .

Thus, a sufficient condition for no firm establishing a marketplace on its own is that there is a marketplace large enough worth joining. The reason for this is that *the positive market area effect generated from the concentration of sellers in one marketplace may more than offset the negative substitution effect arising from the presence of*



competitors. In that case, firms seek competition rather than evade it.

In summarizing the results presented on the case where consumers consider several alternatives acceptable, we showed that if sellers sell imperfect substitutes (no matter how close) some concentration always occurs; that under rather mild conditions as to the range of alternatives acceptable to the typical consumer, a single marketplace configuration is a noncooperative equilibrium; that there is a threshold level on the size of a big marketplace such that if it is surmounted, no firm considers it profitable to establish a separate marketplace; and finally, that in an example involving two classes of consumers the single marketplace configuration is the only equilibrium configuration possible if the class of consumers to whom several alternatives are acceptable is not too large. All this should give sufficient support to the conjecture that noncooperative equilibria tend to involve the location of firms in a few market places only, and in the extreme in one marketplace.<sup>6</sup>

The last result now will demonstrate that for  $m_i \geq 2$  the latter equilibrium configuration is never in the core. Thus, there are always coalitions among firms under which it would be profitable to establish several market places. Again, the standardization assumptions are made for expositional purposes only.

Proposition 6: Suppose that (i'), (ii'), (iii) and (iv') hold. Let  $N$  be even. Let finally  $m \geq 2$ . Then a noncooperative equilibrium  $L^*$  such that  $\# L^* = 1$  is not in the core.

Proof: It is sufficient to show that there is a coalition  $C$  of firms blocking the imputation  $\Pi^*$  resulting from  $L^*$ . Let  $C = \{1, 3, \dots, N-1\}$ . Let  $j \in C$  and  $j \in N \setminus C$  be located in two separate marketplaces, respectively, and denote the resulting imputation by  $\Pi'$ . But  $\Pi' \gg \Pi^*$  iff, using (i'), (ii'), and (iv') and employing (2),

$$u(1) < 2 \sum_{i=1}^m \pi_i \left(\frac{N}{2}\right) u(i).$$

Since

$$\pi_1 \left(\frac{N}{2}\right) = \frac{\frac{N}{2}}{N} = \frac{1}{2}, \text{ we can rewrite this inequality as}$$

$$u(1) < u(1) + 2 \sum_{i=2}^m \pi_i \left(\frac{N}{2}\right) u(i).$$

$$\text{But } 2 \sum_{i=2}^m \pi_i \left(\frac{N}{2}\right) u(i) > 0 \quad \forall m = 2, \dots, N.$$

The example used in the proof of proposition 6 is quite intuitive: from proposition 1 we know that a separation of firms into two marketplaces of size  $\frac{N}{2}$  each will reduce the market area for each one firm in any one of the marketplaces. However, the structure of the coalitions suggested here induces the spatial separation of "close competitors." It thus guarantees that the loss in market demand due to the reduction in the market area is more than offset by the gain in sales to consumer types that in a single marketplace would prefer the competitor's variety.<sup>7</sup>

While the formal derivation of this result is trivial, its economic consequences are not: *individual sellers' interest in partaking of the "externalities" of the large marketplace is too strong despite the fiercer competition (in brand space) generated from spatial association.* This behavior is definitely inefficient from the sellers' point of view

if considered as a cooperative. It may not be inefficient in the Pareto sense. However, it is questionable whether the Pareto-criterion is useful in the present context.<sup>8</sup> If we are willing to accept as a substitute efficiency criterion, the maximum of the unweighted sum of consumers' surpluses, then it is easily checked that in an appropriately closed version of the model the concentration of all firms at one marketplace will be *inefficient* provided that  $[u_i(1) - u_i(2)]$  is sufficiently small for all consumer types  $i$ . It should finally go without further explication that such a concentration is *inequitable* no matter what equity criterion is applied. Within the confines of this model, we therefore are confronted with the interesting situation that *a policy designed to remove an inefficiency conforms with, rather than contradicts, a policy directed towards an equity objective.*

#### 4. SOME INFORMAL EXTENSIONS

We will informally discuss here the impacts of relaxing some of the more rigid assumptions used so far. Let us first consider relaxations that tend to reinforce the results on the spatial concentration of sellers, and then those tending to weaken them. Introducing *effective market boundaries* implies spatial competition between marketplaces in the sense of conventional location theory. It is easily seen that such competition tends to even more reduce the profitability of establishing small marketplaces. This especially reinforces the result established in proposition 5, saying loosely that the existence of a large marketplace precludes separate small marketplaces from coming into existence. For a sufficiently small geographical area, the resulting spatial concentration may cease to be excessive, however. It is also worth mentioning at this point that all

results generalize at ease to a two dimensional geographical space.

So far, we assumed consumers to search for one and only one out of N commodities. The results will also be reinforced if *search for several (nonsubstitutable) commodities on one shopping trip* is admitted. In fact, a seller of a commodity which is not a substitute to either one of the N commodities will *always* find it advantageous to locate in the largest market place.

It finally is worth observing that assuming more realistically *transportation schedules to be strictly concave* in distance will reinforce all the results discussed in section 3. However, this tendency may be offset by the effect of *congestion* on individuals' transportation schedules.

Another extension obviously weakening the tendency towards concentration is to allow for an *internalization* of the "external effect" generated from joint location: one form of internalization may be done by the typical seller in selling several varieties of the commodity; another form of internalization is performed by the typical developer of a shopping center. In both forms, internalization may lead to decentralization. It cannot be determined beforehand, however, whether those forces are sufficiently strong to remove the excessive concentration discussed before.

We finally should ask whether an incorporation of sellers' competitive price setting and of cost differentials due to land rent would substantially modify the results. In the model as developed so far, not only consumers' but, also, sellers' surplus is generated from spatial concentration. The latter surplus would be removed if competitive price setting in large marketplaces (offering close substitution) would indeed

drive down sellers' profits. This, in turn, would not allow landowners to absorb surplus in the form of rents which would contradict empirical evidence. The following plausibility argument might contribute to a resolution of that apparent contradiction between a standard hypothesis and empirical observations: suppose firms enter sequentially the sequence of entry being determined by maximal profitability (conditional upon the spatial arrangement of and price levels prevailing in the marketplaces). This initially will lead to a formation of marketplaces in which relatively poor substitutes are offered at relatively high prices.<sup>9</sup> In turn, this allows landlords to absorb some of producers' surplus in land prices. Consider now a marginal entrant intending to sell a close substitute to any one of the commodities already sold in the marketplace. Then, provided that landlords have some degree of monopoly power he will be faced with payments to the landowner not allowing him to compete down the incumbent sellers' asking prices. If this argument is correct, then *marginal landlords, by their pricing behavior, provide an effective protection to inframarginal sellers' (and landlords') profits insofar as they prevent price competition from becoming effective.* In that case, the landlords' quest for a maximal rent would not lead, as often purported, to an efficient allocation of that scarce resource.

##### 5. CONCLUDING REMARKS

In this paper, it is demonstrated how market places for commodities form whose purchase requires personal search. It is in particular shown why consumers prefer to visit, and sellers choose to locate in, a competitive market place. It is finally demonstrated that the resulting process

of concentration is excessive at least from the suppliers' point of view. The results do not imply that there is a general tendency for all markets to concentrate in space. We confined our analysis to a special, but important class of commodities. Other classes of commodities: in particular commodities purchased habitually and frequently have not been dealt with in this paper.<sup>10</sup> There are many other extensions possible that may be analyzed within the present framework. Many of them have been hinted at in the last section. A particularly interesting one concerns the endogenous choice of the product varieties and the nonprice competition induced thereby; another one, to include price setting by the firms; a third one, to admit search over several marketplaces; a fourth one, to consider alternative information structures on the part of consumers. What happens, for instance, if consumers don't know exactly what they want? Or conversely, if they truly anticipate that in large marketplaces a "broader" variety is offered? What happens if firms in a large marketplace are allowed to pursue advertising policies? A fifth extension is to consider sequential entry of sellers together with alternative assumptions as to the formation of entrants' expectations.

The paradigm discussed here is very simple. Many generalizations are possible. Yet it seems to provide an interesting approach to an analysis of some of the causes and welfare implications of the spatial concentration of economic activity.

1. The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that this is crucial for the company's financial health and for providing transparency to stakeholders.

2. The second part outlines the specific procedures for recording transactions, including the use of standardized forms and the requirement for double-checking entries. It also mentions the need for regular audits to ensure the accuracy of the data.

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4. The fourth part discusses the importance of timely reporting and the role of the finance department in providing regular updates to management. It also mentions the need for clear communication and collaboration between all departments.

5. The fifth part concludes by reiterating the company's commitment to high standards of financial integrity and transparency. It expresses confidence that these measures will ensure the company's long-term success and growth.

FOOTNOTES

- 1 Concentration phenomena also arise from the fact that consumer transportation outlays are decreasing in the quantities purchased per shopping trip. This phenomenon is analyzed in another forthcoming paper [3].
- 2 There is one interesting exception to this, namely Stuart [5] which was brought to my attention after this paper had been written. He addressed himself to that very phenomenon, concentrating on consumers' optimal search strategies. This allows him to generate interesting comparative statics about consumers' equilibrium search behavior. By contrast, the present model builds on a somewhat simpler representation of consumer behavior but emphasizes an analysis and evaluation of firms' equilibrium location patterns. Such an analysis is not done in the former model and probably is difficult to execute without stronger assumptions.
- 3 While a similar notion of agglomeration economies is developed in [5], no evaluation is given of the equilibrium location of firms resulting from them.
- 4 Note however that assumption (iv) may quite realistically be reinterpreted as follows: let sellers be uncertain about the spatial distribution of consumers, then (iv) may specify the typical seller's subjective prior to that distribution. All the results generated below will follow directly under the provision that sellers are risk neutral.
- 5 It should not be disturbing that, due to the unboundedness assumption on the space of locations, the location of that singular marketplace is indeterminate.
- 6 If we consider sequential entry of the  $N$  firms, the case for a spatial concentration of all firms will be much stronger even for  $m_i > 1$ : let the order in which firms enter be determined by the maximal<sup>1</sup> profit that may be obtained from selling some good, *given* the location decisions of inframarginal entrants. Then initially a marketplace will form in which firms don't compete. When competitors will enter, the marketplace will be sufficiently large to let entry be profitable even under competition.  
Note finally that all this follows if firms act myopically, and it will be even reinforced if firms upon entry anticipate further growth of the (large) marketplace.
- 7 We have not specified whether the core of this game is empty or not. However, this is immaterial for the interpretation of the present result.
- 8 As it may easily be observed, *every* allocation is Pareto-efficient in an appropriately closed version of this model.
- 9 In that situation, substantial parts of consumers' surplus may be absorbed by the sellers. The author has developed a companion model to the present



one in which Cournot equilibria in prices are derived for marketplaces of different sizes. It is shown that prices for commodities sold in both small and large marketplaces may be higher in the latter if the typical consumer's elasticity of substitution across commodities is not too large. The result can be strengthened considerably if the Cournot assumption is replaced by one which firms behave more strategically.

10 See, however, [3].

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