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Bulk viscosity, decaying dark matter, and the cosmic acceleration

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We discuss a cosmology in which cold dark matter particles decay into relativistic particles. We argue that such decays could lead naturally to a bulk viscosity in the cosmic fluid. For decay lifetimes comparable to the present Hubble age, this bulk viscosity enters the cosmic energy equation as an effective negative pressure. We investigate whether this negative pressure is of sufficient magnitude to account for the observed cosmic acceleration. We show that a single decaying species in a $\Lambda = 0$, flat, dark-matter dominated cosmology can not reproduce the observed magnitude-redshift relation from Type Ia supernovae. However, a delayed bulk viscosity, possibly due to a cascade of decaying particles may be able to account for a significant fraction of the apparent cosmic acceleration. Possible candidate nonrelativistic particles for this scenario include sterile neutrinos or gauge-mediated decaying supersymmetric particles.

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I. INTRODUCTION

A significant challenge facing modern cosmology is that of understanding the nature and origin of both the dark energy responsible for the present apparent acceleration [1], and the dark matter [2] responsible for most of the gravitational mass of galaxies and clusters. The simplest particle physics explanation for the dark matter is, perhaps, that of a weakly interacting massive particle such as the lightest supersymmetric particle, an axion, or an electroweak singlet (e.g. "sterile" neutrino). The dark energy, on the other hand is generally attributed to a cosmological constant, a vacuum energy in the form of a "quintessence" scalar field possibly very slowly evolving along an effective potential, or even relativistic effects derived from the deviation of the present matter distribution from Friedmann homogeneity [3]. See [4] for a recent review.

In addition to these explanations, however, the simple coincidence that both of these unknown entities currently contribute comparable mass energy toward the closure of the universe begs the question as to whether they could be different manifestations of the same physical phenomenon. Indeed, many suggestions along this line have been made for so-called unified dark-matter. One possibility is a dark matter composed of a generalized Chaplygin gas [5] for which pressure depends upon density $p = -A/\rho^{\alpha}$, although it has been shown [6] that a generalized Chaplygin gas produces an exponential blow up of the matter power spectrum which is inconsistent with observations. There are also more exotic proposals such as the flow of dark matter from a higher dimension [7], or that the quintessence field itself can act as dark matter as in the Born-Infeld [8] model.

The possibility of particular interest for the present work, however, is that of a bulk viscosity within the cosmic fluid (e.g. [9,10]). Such a term resists the cosmic expansion and therefore acts as a negative pressure. Indeed, it has been shown [10] that for the right viscosity coefficient, an accelerating cosmology can be achieved without the need for a cosmological constant.

Although cosmic bulk viscosity is a viable candidate for dark energy, to date there has been no suggestion of how it could originate from known physics and known particle properties. In this paper we consider a simple mechanism for the formation of bulk viscosity by the decay of a dark-matter particle into relativistic products. Such decays heat the cosmic fluid and lead to an increase in entropy and are inherently dissipative in nature. Moreover, they lead to a cosmic fluid which is out of pressure and temperature equilibrium and can therefore be represented by a bulk viscosity. We propose a form for this viscosity and show that decay lifetimes comparable to the present Hubble time naturally produce an accelerating cosmology in the present epoch.

In the next section we summarize the general form for the bulk viscosity. Following that, we consider its effects on cosmology and suggest a specific form for the bulk viscosity induced by particle decay. In Sec. IV we discuss constraints on the properties of such particles and argue that several candidates exist. In Sec. V we compute the magnitude-redshift relation for Type Ia supernovae in this cosmology and show that a single decay does not reproduce these data. Only if decays are delayed, e.g. by a cascade of particle decays, can a significant fraction of the cosmic acceleration be explained in this scenario.

II. BULK VISCOSITY FROM DECAYING DARK MATTER

The fundamental problem that we want to address is the effect of the decay of nonrelativistic dark-matter particles into relativistic neutrinos. It has been proposed (e.g. [11]) that decaying dark matter could affect cosmic quintessence. Here instead we consider bulk viscosity as a way to introduce the effects of dark-matter decay directly on the equations for cosmic expansion.

The physical origin of bulk viscosity in a system can be traced to deviations from local thermodynamic equilibrium. This can be illustrated with a simple abstract example. Suppose that the energy-momentum tensor in an expanding volume has contributions from both a component of particles obeying nonrelativistic kinematics and a component following relativistic kinematics. Imagine that in a time step the system expands, but the momenta of the relativistic and nonrelativistic particles redshift (i.e. change) differently. In effect, this causes these two components to have different "temperatures" describing their energy-momentum distribution functions.

The second law of thermodynamics tells us [12,13] that the reestablishment of thermal equilibrium through (particle decay or) scattering of these component particles off each other or on another medium is a dissipative process that will generate entropy. This entropy generation can be related to the expansion rate or the local fluid velocity through a bulk viscosity term.

Thus, bulk viscosity arises any time a fluid expands too rapidly and ceases to be in thermodynamic equilibrium. The bulk viscosity, therefore, is a measure of the pressure required to restore equilibrium to a compressed or expanding system [14–16]. Hence, it is natural for such a term to exist in the cosmologically expanding universe anytime the fluid is out of equilibrium. Usually, in cosmology the restoration processes are taken to be so rapid that the establishment of equilibrium is almost immediate. However, there is a finite time for the system to adjust to the change of the equation of state induced by particle decays. For the cosmology proposed here, the attainment of equilibrium as the universe expands is delayed by the gradual decay of one or more species to another which occurs over $\sim 10^{10}$ yr. This leads to a nontrivial dependence of pressure on density as the universe expands, and therefore a bulk viscosity.

To see how this enters quantitatively in cosmology, we begin by summarizing the general treatment of imperfect fluids of Weinberg [13] (see also [17] for a generalization). It will provide further insight into the nature of the bulk viscosity.

When a fluid expands (or is compressed) and departs from thermodynamic equilibrium the processes that restore equilibrium are irreversible. Hence, they are in general accompanied by an increase in entropy which is evidenced in the dissipation of energy. For the case of interest here, the increase in entropy and dissipation is the heating and pressure produced by the particle decays. The existence of such dissipation leads to a modification of the perfect-fluid energy-momentum tensor,

$$T^{\mu\nu} = (\rho + p)U^\mu U^\nu + g^{\mu\nu}p + \Delta T^{\mu\nu}, \tag{1}$$

where ρ and p denote density and pressure while U^μ is the four velocity. Processes of heat flow and shear can play no role in the Friedmann-Lemaitre-Robertson-Walker (FLRW) homogeneous and isotropic conditions of interest here. Hence, the only possible nonadiabatic dissipative contribution $\Delta T^{\mu\nu}$ which guarantees translational and rotational invariance for a fluid in motion with four velocity U^ν is given by [13]

$$\Delta T^{\mu\nu} = -\zeta 3\frac{\dot{a}}{a}(g^{\mu\nu} + U^\mu U^\nu), \tag{2}$$

where a is the cosmic scale factor as specified below and ζ is the bulk viscosity coefficient. The total energy-momentum tensor can then be written

$$T^{\mu\nu} = \left(\rho + p - \zeta 3\frac{\dot{a}}{a}\right)U^\mu U^\nu + g^{\mu\nu}\left(p - \zeta 3\frac{\dot{a}}{a}\right). \tag{3}$$

From Eq. (3) it is obvious that the effect of bulk viscosity is to replace the fluid pressure with an effective pressure given by,

$$p_{\rm eff} = p - \zeta 3\frac{\dot{a}}{a}. \tag{4}$$

Thus, for large ζ it is possible for the negative pressure term to dominate and an accelerating cosmology to ensue. It is necessary, therefore, to clearly quantify the bulk viscosity for the system of interest.

III. COSMOLOGY WITH BULK VISCOSITY

To examine the effect of the bulk viscosity from particle decay on the cosmic acceleration, we analyze a flat ($k = 0, \Lambda = 0$) cosmology in a comoving FLRW metric,

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)[dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2]. \tag{5}$$

A comoving fluid element in this coordinate system will have $U^0 = 1$, $U^i = 0$, and $U^\lambda{}_{;\lambda} = 3\dot{a}/a$.

We consider a fluid with total mass-energy density ρ given by,

$$\rho = \rho_{\rm DM} + \rho_{\rm b} + \rho_{\rm h} + \rho_\gamma + \rho_{\rm l}, \tag{6}$$

where $\rho_{\rm b}$ is the baryon density, $\rho_{\rm DM}$ is the contribution from stable dark matter, $\rho_{\rm h}$ is the density in unstable decaying dark matter, $\rho_{\rm l}$ is the produced relativistic energy density from decay while ρ_γ is any other relativistic matter, i.e. photons and neutrinos from the big bang. Because of decay, neither the total energy density in relativistic particles $\rho_r = \rho_\gamma + \rho_l$ nor the pressure $p = \rho_r/3$ is negligible for this cosmology even at the present epoch.

In the FLRW frame, the energy-momentum tensor (Eq. (3)) then reduces to

$$T_{00} = \rho \tag{7}$$

$$T_{0i} = 0 \tag{8}$$

$$T_{ii} = \left(p - 3\zeta\frac{\dot{a}}{a}\right)g_{ii}, \tag{9}$$

where again, this last equation shows that the bulk viscosity enters as an effective negative pressure (i.e. dark energy) in the energy-momentum tensor.

The Friedmann equation does not depend upon the effective pressure and is exactly the same as for a nondissipative cosmology, i.e.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho, \tag{10}$$

where ρ is the total mass-energy density from matter and relativistic particles (Eq. (6)).

Although absent from the Friedmann equation, the bulk viscosity does appear in the conservation condition $T^{\mu\nu}{}_{;\nu} = 0$. To illustrate this consider a flat $k = 0$, $\Lambda = 0$ cosmology and ignore the small contribution from ρ_b and initial background radiation ρ_γ, so that

$$\rho = \rho_h + \rho_l. \tag{11}$$

The conservation equations can be solved to give the energy densities in matter and radiation:

$$\rho_h = \frac{1}{a^3}\rho_{m0}e^{-t/\tau}. \tag{12}$$

and

$$\rho_l = \frac{1}{a^4}\left[\rho_{l0} + \frac{\rho_{h0}}{\tau}\int_0^t e^{-t'/\tau}a(t')dt' + \rho_{BV}\right], \tag{13}$$

where ρ_{BV} is the dissipated energy in light relativistic species due to the cosmic bulk viscosity,

$$\rho_{BV} = 9\int_0^t \zeta(t')\left(\frac{\dot{a}}{a}\right)^2 a(t')^4 dt'. \tag{14}$$

The total density for the Friedmann equation will then include not only terms from heavy and light dark matter, but a dissipated energy density in bulk viscosity. This is the term that contributes to the cosmic acceleration.

Bulk viscosity coefficient

Bulk viscosity can be thought of [12–14] as a relaxation phenomenon. It derives from the fact that the fluid requires time to restore its equilibrium pressure from a departure which occurs during expansion. The viscosity coefficient ζ depends upon the difference between the pressure $\tilde{p}$ of a fluid being compressed or expanded and the pressure p of a constant volume system in equilibrium. Of the several formulations [14] the basic nonequilibrium method [18] is identical [13] with Eq. (4).

$$\zeta 3\frac{\dot{a}}{a} = \Delta p, \tag{15}$$

where $\Delta p = \tilde{p} - p$ is the difference between the constant volume equilibrium pressure and the actual fluid pressure.

In Ref. [13] the bulk viscosity coefficient is derived for a gas in thermodynamic equilibrium at a temperature T_M into which radiation is injected with a temperature T and a mean thermal equilibration time τ_e. The solution for the relativistic transport equation [19] can then be used to infer [13] the bulk viscosity coefficient. For this case the form of the pressure deficit and associated bulk viscosity can be deduced from Eq. (2.31) of Ref. [13] which we modify slightly and write as,

$$\Delta p \sim \left(\frac{\partial p}{\partial T}\right)_n (T_M - T) = \frac{4\rho_\gamma\tau_e}{3}\left[1 - \left(\frac{3\partial p}{\partial\rho}\right)\right]\frac{\partial U^\alpha}{\partial x^\alpha}, \tag{16}$$

where the factor of 4 comes from the derivative of the radiation pressure $p \sim T^4$ of the injected gas, and the term in brackets derives from the detailed solution to the linearized relativistic transport equation [13,19]. This term guarantees that no bulk viscosity can exist for a completely relativistic gas. In the cosmic fluid, however, we must consider a total mass-energy density ρ given by both nonrelativistic and relativistic components.

Here, as in Refs. [13,19], we also have a thermalized gas into which relativistic particles at some effective temperature are being injected. The deficit from equilibrium pressure, however, is due to the presence of unstable decaying nonrelativistic dark matter. At any time in the cosmic expansion the pressure deficit will be 1/3 of the remaining mass-energy density of unstable heavy particles. Hence, we replace $\rho_\gamma/3$ with $\rho_h/3$ in Eq. (16) and write,

$$\Delta p = \frac{4\rho_h\tau_e}{3}\left[1 - \left(3\frac{\partial p}{\partial\rho}\right)\right]\frac{\partial U^\alpha}{\partial x^\alpha}. \tag{17}$$

Here, the equilibration time τ_e is determined [14] from the particle decay time τ,

$$\tau_e = \int_0^\infty \frac{\Delta p(t)}{\Delta p(0)}dt = \frac{\tau}{[1 - 3(\dot{a}/a)\tau]} \tag{18}$$

where $\Delta p(0)$ denotes the initial pressure and the denominator results from approximating $H = \dot{a}/a \approx$ constant. Note, that this factor acts as a limiter to prevent unrealistically large bulk viscosity in the limit of a large τ.

Following the derivation in [13], and inserting Eq. (17) in place of Eq. (16), we infer the following ansatz for the bulk viscosity of the cosmic fluid due to particle decay,

$$\zeta = \frac{4\rho_h\tau_e}{3}\left[1 - \frac{\rho_l + \rho_\gamma}{\rho}\right]^2, \tag{19}$$

where the square of the term in brackets comes from inserting Eq. (17) into the linearized relativistic transport equation of Ref. [19]. Equation (19) implies a nonvanishing bulk viscosity even in the limit of long times as long as the total mass energy density is comprised of a mixture of relativistic and nonrelativistic particles. Hence, one should be cautious about using this linearized approximation in the long lifetime limit. Even so, a more general derivation has been made [16] which shows that, even in the limit of interest here of a long radiation equilibration time there is a nonvanishing bulk viscosity consistent with experimental determinations.

IV. DECAYING DARK MATTER CANDIDATES

Having postulated the existence of a decaying dark matter particle, it is important to briefly examine the constraints on such decays and whether such candidate particles could exist. To avoid observational constraints the decay products must have very little energy in photons or charged particles. The implied background in energetic photons with an energy density comparable to the present matter energy density would have been easily detectable. Hence, the decay products must be in some form which is not easily detectable. Neutrinos would be a natural candidate for such a background. In this case there are several decaying dark matter possibilities which come to mind.

A. Sterile neutrinos

Models with decaying neutrinos have been around for some time [20]. Perhaps the most realistic possibility is the decay of a sterile neutrino into light "active" neutrinos [21]. Models have been proposed in which singlet sterile neutrinos ν_s which mix in vacuum with active neutrinos (ν_e, ν_μ, ν_τ) provide warm and cold dark matter candidates [21–27]. In most of these models the sterile neutrinos are produced in the very early universe through active neutrino scattering-induced decoherence. This process could be augmented by medium enhancement stemming from a significant lepton number. In these sterile neutrino production processes there are two principal parameters: (1) the sterile neutrino mass m_s; and (2) the sterile neutrino's vacuum mixing angle θ with one or more of the active neutrino flavors. The net lepton number(s) of the universe could be regarded as an additional parameter. By virtue of the mixing with active neutrino species, the sterile neutrinos are not truly sterile and, as a result, can decay. For $m_s < 10$ MeV the dominant ν_s decay mode is into light, active neutrino species. The rate for this process is [21]

$$\Gamma_\nu \approx (8.7 \times 10^{-21}\ \mathrm{s}^{-1})\left(\frac{\sin^2 2\theta}{10^{-15}}\right)\left(\frac{m_s}{1\ \mathrm{MeV}}\right)^5. \tag{20}$$

Likewise, there is a subdominant ν_s decay branch into a light active neutrino and a photon with rate

$$\Gamma_{\nu\gamma} \approx (6.8 \times 10^{-23}\ \mathrm{s}^{-1})\left(\frac{\sin^2 2\theta}{10^{-15}}\right)\left(\frac{m_s}{1\ \mathrm{MeV}}\right)^5. \tag{21}$$

In this process the photon will be monoenergetic with an energy which is half the ν_s rest mass. Because the primary ν_s decay mode and the radiative branch scale the same way with m_s and $\sin^2 2\theta$, there is a fixed ratio of these rates.

The best particle candidates for a decay-induced bulk viscosity are those with a lifetime of order the Hubble time H_0^{-1} and rest masses $\sim$1 MeV. Setting $\Gamma_\nu = H_0$ we find that the relation between the ν_s rest mass and vacuum mixing angle is

$$m_s \approx 3.1\ \mathrm{MeV}\left(\frac{h}{0.71}\right)^{1/5}\left(\frac{10^{-15}}{\sin^2 2\theta}\right)^{1/5}, \tag{22}$$

where h is the Hubble parameter at the current epoch in units of 100 kms^{-1} Mpc^{-1} and we have scaled our result to $h = 0.71$, the WMAP best fit value. We conclude that one or a number of sterile neutrinos with rest masses in the $\sim$MeV range could provide a significant decay-induced bulk viscosity.

Regarding observational constraints, let us note that our bulk viscosity-selected range for m_s from Eq. (22) is relatively insensitive to the vacuum mixing angle. However, the radiative decay branch rate $\Gamma_{\nu\gamma}$ is linearly proportional to $\sin^2 2\theta$. Keeping $m_s \sim 1$ MeV, we can adjust $\sin^2 2\theta$ so that the diffuse decay photon flux is just at or below the observational limit [21,28,29] from the Diffuse Extragalactic Background Radiation (DEBRA). For this $m_s = 1$ MeV case, getting below the DEBRA limit would require $\sin^2 2\theta \le 10^{-15}$.

We conclude that it is possible to meet the bulk viscosity lifetime requirement and (barely) get under the DEBRA limit with sterile neutrinos as the decaying dark matter. We also note that sterile neutrinos with these parameters ($m_s \approx 1$ MeV, $\sin^2 2\theta \approx 10^{-15}$) could be produced in the early universe in the requisite relic densities (i.e., near closure) only in scenarios with large lepton number(s) and medium-enhanced decoherence [21,30], or with new neutrino couplings [26].

B. Decaying supersymmetric dark matter

For supersymmetric dark matter candidates, it is generally assumed [2] that the initially produced dark-matter relic must be a superWIMP in order to produce the correct relic density. Later, this superWIMP is then presumed to decay to a lighter stable dark-matter particle. One possible candidate for the scenario proposed here is therefore a decaying superWIMP with a lifetime comparable to the present Hubble time.

Alternatively, the light supersymmetric particle itself might be a candidate for decay. If the dark matter is a light unstable supersymmetric particle, then one might imagine an R-parity violating decay. In one scenario a particle might decay by coupling to right-handed neutrinos which then decay to normal neutrinos. Another possibility could be gauge-mediated supersymmetry breaking involving the decay of a supersymmetric sneutrino into a gravitino plus a light neutrino.

V. RESULTS

Having defined the cosmology of interest we now examine the magnitude-redshift relation for Type Ia supernovae (SNIa). The apparent brightness of the Type Ia supernova standard candle with redshift is given [31] by a simple relation for a flat $\Lambda = 0$ cosmology. The luminosity distance becomes,

$$D_L = \frac{c(1+z)}{H_0}\left\{\int_0^z dz'[\Omega_\gamma(z') + \Omega_l(z') + (\Omega_{DM}(z') + \Omega_b(z') + \Omega_h(z'))]^{-1/2}\right\}, \tag{23}$$

where H_0 is the present Hubble parameter. The Ω_i are the energy densities normalized by the critical density at each epoch, i.e. $\Omega_i(z) = 8\pi G\rho_i(z)/3H_0^2$. Ω_h is the closure contribution from the decaying heavy cold dark-matter particles. Their decay is taken here to produce light neutrinos Ω_l or other relativistic particles Ω_γ. Note that Ω_h, Ω_γ and Ω_l each have a nontrivial redshift dependence due to particle decays, while stable dark matter and baryons $\Omega_{DM}(z) + \Omega_b(z)$ obey the usual $(1+z)^{-3}$ dependence with redshift. Here, and in the following discussion we will define Ω_M as the present sum of nonrelativistic matter, i.e. $\Omega_M = \Omega_h(z=0) + \Omega_{DM}(z=0) + \Omega_b(z=0.)$

Figure 1 compares various cosmological models with some of the recent combined data from the High-Z Supernova Search Team and the Supernova Cosmology Project [1,32], while Table I summarizes the relevant parameters and reduced χ^2 goodness of fit. The lower figure shows the K-corrected magnitudes $m = M + 5\log D_L + 25$ vs redshift plotted relative to an open Ω_{DM}, Ω_B, $\Omega_\Lambda = 0$, $\Omega_k = 1$ cosmology.

The solid line on the upper and lower graphs in Fig. 1 shows the result of adding bulk viscosity from particle decay. The upper figure gives the distance-redshift relation while the lower figure shows the evolution of magnitudes relative to a fiducial $\Omega_k = 1/(a_0H_0)^2 = 1$ open cosmology, for which

$$D_L(\Omega_k = 1) = \frac{c(1+z)}{2H_0}\left[z + 1 - \frac{1}{(z+1)}\right], \tag{24}$$

and the relative distance modulus is given in the usual way $\Delta(m - M) = 5\log[D_L/D_L(\Omega_k = 1)]$.

From the lower graph of Fig. 1 we see that, although the bulk viscosity has indeed provided a negative pressure it does not reproduce the supernova distance-red shift relation. In fact it is much worse than the usual ΛCDM cosmology and is even worse than a pure matter dominated cosmology. The reason for this can be discerned from Fig. 2. Although the bulk viscosity is substantial, it scales

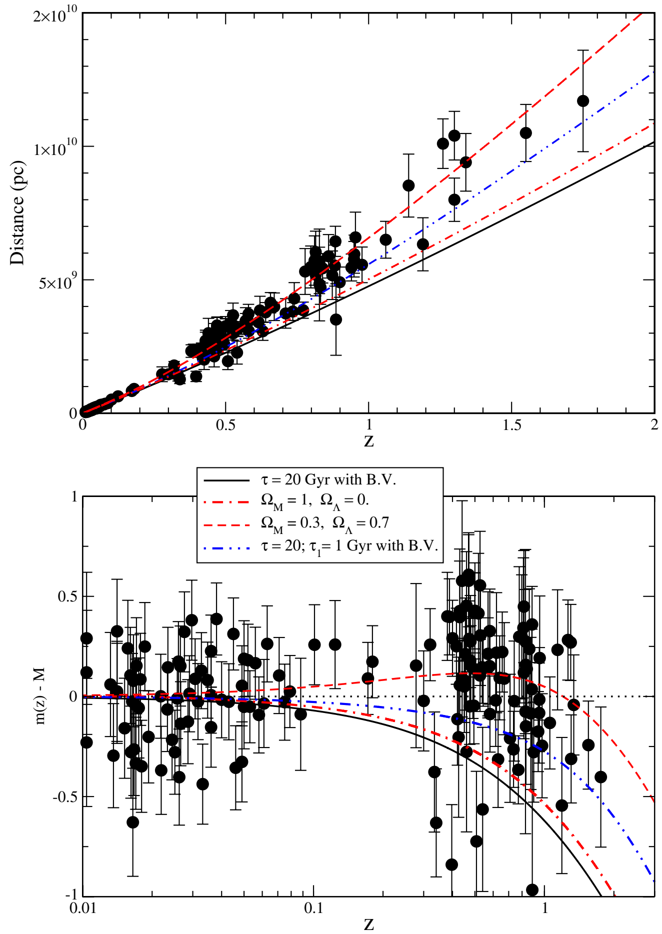

FIG. 1 (color online). Evolution of luminosity distance with redshift for a cosmology with bulk viscosity. Points are from the Gold data set of [32]. The upper figure shows the luminosity distance vs redshift. The lower figure shows the evolution of magnitudes relative to a fiducial $\Omega_k = 1$ open cosmology. In each figure the upper dashed line shows the evolution of a standard ΛCDM cosmology and the lower dot-dashed line shows the evolution of an $\Omega_M = 1$ cosmology. The solid line is for a illustrative decaying dark-matter model with $\tau = 20$ Gyr. The dash-dot-dot line illustrates the evolution of a cosmology in which a cascade of six particle decays each with a lifetime of $\tau_1 = 1$ Gyr is followed by a final radiative decay with $\tau = 20$ Gyr.

TABLE I. Parameter sets for various fits to the SNIa luminosity-redshift relation for $H_0 = 71$ km s^{-1} Mpc^{-1} and $\Omega_b = 0.044$. In the decaying (finite τ) models no stable dark matter was assumed (i.e. $\Omega_{DM} = 0$).

τ (Gyr)	Ω_M	Ω_Λ	χ_r^2
∞ (ΛCDM)	0.31	0.69	1.14
20	0.16	0.	3.93
1×6, 20	0.39	0.	1.96
∞ (CDM)	1.	0.	3.23

with the decaying dark matter which falls off faster with time than a^{-3} because of the decay. An accelerating cosmology requires a nearly constant value of $\rho_{\rm tot}$ with time.

A flattening of $\rho_{\rm tot}$ could be achieved in this context, however, if the onset of the bulk viscosity could be delayed until near the present epoch due to a cascading decay. In this possibility, the final decaying particle in the cascade would produce the relativistic products and the bulk viscosity. This final decay, however would need to be preceded by a series of decays to nearly degenerate states with shorter lifetime.

The cascade possibility might occur, for example, among sterile neutrinos. Another cascade possibility [2] is that the initially produced dark matter relic is a superWIMP. Later this superWIMP decays through a cascade of superWIMP states to a final unstable state or through a cascade of unstable light supersymmetric particle states.

Figure 3 illustrates a possible evolution of energy density in this scenario. Here, a cascade among six states each with a lifetime of $\tau_1 = 1$ Gyr is followed by the final decay of a long lived particle with $\tau = 20$ Gyr. The cascade is presumed to start with the initial population entirely in the first member of the cascade. The activity in the final decay product is delayed by the time needed to decay through the intervening states. The rates of the final and intermediate decays are given by a solution to the Bateman equation whereby the abundance of the final product is given by

$$\rho_{\rm h} = \Sigma h_j \exp(-\lambda_j t), \tag{25}$$

where $\lambda_j = \tau_j^{-1}$ is the decay rate of each species and the h_j are given by,

$$h_j = \Pi_{i\neq j}\left[\frac{\lambda_j}{(\lambda_i - \lambda_j)}\right]. \tag{26}$$

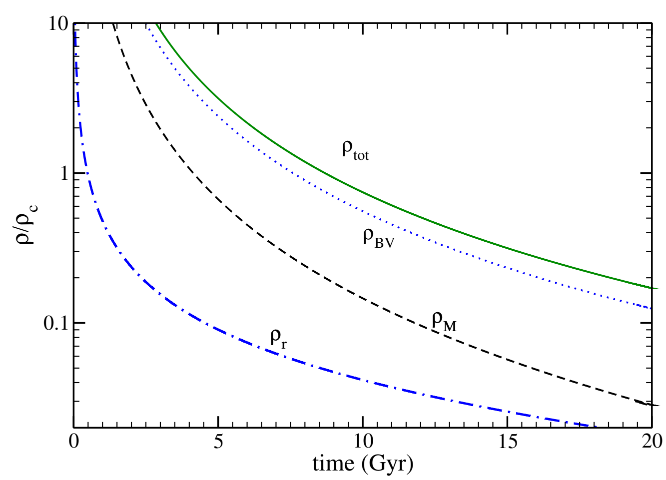

FIG. 2 (color online). Evolution of the quantities ρ_γ, ρ_M, $\rho_{\rm BV}$, and $\rho_{\rm tot}$ relative to the present critical density $\rho_c = (3H_0^2/8\pi G)$ as labeled for a cosmology in which the dark-matter decays with a lifetime of 20 Gyr

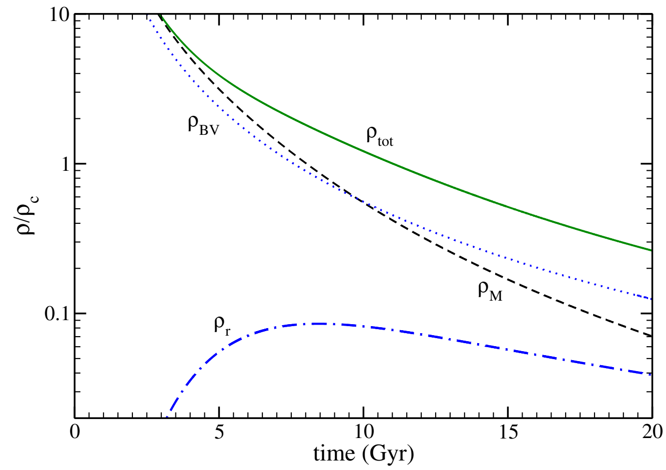

FIG. 3 (color online). Evolution of the quantities ρ_γ, ρ_M, $\rho_{\rm BV}$, and $\rho_{\rm tot}$ as labeled relative to the present critical density $\rho_c = (3H_0^2/8\pi G)$ in a cosmology in which decays among six nearly degenerate states occur with a lifetime of $\tau_1 = 1$ Gyr each is followed by the decay of a long lived particle with $\tau = 20$ Gyr.

For this possibility, we found that it was not possible in this way to account adequately for the cosmic acceleration, though a significant fraction could be obtained as illustrated by the dash-dot-dot lines on Fig. 1.

VI. CONCLUSION

We have considered models in which the apparent cosmic acceleration is affected by the bulk viscosity produced from the decay of a dark-matter particle to light relativistic species. An expression for the bulk viscosity is deduced and the implied redshift-distance relation has been computed.

As an illustrative example we considered the decay of dark matter with a lifetime of 20 Gyr in this cosmology. From the reduced χ_r^2 values in Table I, and the lines in Fig. 1 it is apparent that a flat $\Lambda = 0$ cosmology with bulk viscosity from decay of a single dark-matter species does not do better than a ΛCDM or a matter dominated cosmology. This is because the total mass-energy density does not become nearly constant with scale factor, but falls off more rapidly than even a simple matter dominated cosmology due to the combined effects of the decay of the dark matter and the existence of a high density of relativistic particles. We show, however that if the emergence of the bulk viscosity is delayed, then some, but not all, of the acceleration required by observations of Type Ia supernovae at high redshift can be explained. As we have outlined above, one mechanism for delaying the bulk viscosity could be a cascading decay process.

Obviously, however, one must decide whether the dilemma of a cosmological constant is less plausible than the dilemma of bulk viscosity produced by a delayed cascade of decaying dark-matter particles. Our goal here, however, has merely been to argue that the possibility exists. Having established that at least a possible paradigm exists, in future work we will examine the possible influence of this scenario on the CMB and the growth of large scale structure which will also constrain this possibility.

Indeed, a number of recent studies (e.g. [3]) suggest that changes in the extrinsic curvature due to changing relativistic gravity in an inhomogeneous cosmology can lead to cosmic acceleration. We have developed a computer model similar to [33] which includes the decay of heavy neutrinos. Our preliminary analysis indicates that the cosmic acceleration in a universe with this type of dark-matter decay is enhanced by its affect on large scale structure, i.e. decay of heavy neutrinos in a nonhomogeneous cosmology increases the expansion effect.

In brief, the decay produces a flow of light neutrinos from galactic clusters. Given a decay time τ and galactic clusters separated by a distance L this flow produces a momentum density of the order $s = \rho L/\tau$ (in units of $c = G = 1$). From the momentum constraint [Eq. (A4) of [33]], an enhancement of the extrinsic curvature of order $\rho L^2/\tau$ will occur. From the Hamiltonian constraint [Eq. (A3) of [33]] the trace of the extrinsic curvature will be reduced by a factor the order of $(\delta K/K)^2$. Since $\dot{a}/a \sim K/3$, the Hubble parameter, $\dot{a}/a$ becomes more nearly constant implying acceleration.

Another interesting aspect of this decaying dark-matter model is its possible effect on details of large scale structure. In our preliminary results based upon the planar inhomogeneous cosmology with dark-matter decay, the effects can be summarized as follows: 1) During the matter dominated epoch, the development of structure is indistinguishable from a standard ΛCDM cosmology; 2) The decay of dark matter, however, leads to a flattening of the of the dark-matter distribution (relative to that expected from simulations without decay) in the centers of galaxies and clusters. This is consistent with the dark-matter distribution inferred from observation (cf. [34]).

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