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Living Supply and Demand Curves

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## Living Supply and Demand Curves\*

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In a large introductory economics class, we dramatize the theory of supply and demand, with student actors forming living supply and demand curves, with dark-shirted students playing demanders and light-shirted students playing suppliers. Heights of demanders will represent their willingness to pay for a unit of the commodity demanded and heights of suppliers will represent their cost of providing a unit of the public good. Students enjoy watching and participating in this drama, and we believe it enables them to think creatively about the fundamental ideas of economics.

## The play's the thing

The actors: The instructor selects 10-12 student volunteers who are wearing dark-colored shirts and an equal number who are wearing light-colored shirts, to be the *players* in this drama. Results will more striking if there is greater variation in heights. Thus, we suggest that instructor try to get three or four students of each type of shirt of height 6 feet (182 cm) or greater and three or four of each type of shirt with height of 5 feet 2 inches (159 cm) or less. The instructor also selects two or three *student assistants*, who will help in organizing the performance.

The Equipment: Assistants should be provided with a laptop on which a bit of software supplied with this article has been installed.<sup>1</sup> In Act III we use a "limbo stick," which can either be a stick or a string or cord that is about 5 feet long.

## Act I: Supply and demand curves, trading and profits

The selected players come to the stage at the front of the room and report their heights to the assistants,<sup>2</sup> who record these heights on the Excel spreadsheet. The assistants' software will convert heights announced in feet and inches into height in centimeters. The assistant informs students of their centimeter heights.

**Describing players' roles:** The instructor explains the roles of the players as follows: The dark-shirted players are *demanders* who can consume either one or zero units of a good. The value of having a unit of the good to a dark-shirted player is a number of dollars equal to that player's height in centimeters. The light-shirted players are *suppliers*. A supplier can supply either zero or one unit of the good. The dollar cost to a light-shirted player of supplying a unit is equal to that player's height in centimeters.

<sup>\*</sup>We thank Ariel Gao for research assistance.

<sup>&</sup>lt;sup>1</sup>A spreadsheet with details on how to use it is available here: link to be added.

<sup>&</sup>lt;sup>2</sup>Alternatively, the assistants can use a smartphone (e.g. the measure app on an iphone) to directly measure each player's height.

Economic activity takes the form of partnerships between dark-shirted and light-shirted players, in which the light-shirted partner supplies a unit of the good to the dark-shirted partner in return for a payment that is at least as much as the light-shirted player's cost.

**Introductory discussion:** The instructor leads a class discussion about the profits made from trades. This discussion should lead to the conclusion that if a dark-shirted player of height H forms a partnership with a light-shirted player of height L and pays an amount p to the light-shirted player, the profits of the dark-shirted player will be H-p and the profits of the light-shirted player will be p-L. Thus, for any choice of p, the sum of profits of the two partners will be (H-p)+(p-L)=H-L, which is just the difference between their heights. These results should be projected on a screen or written on a blackboard.

The instructor now asks whether anyone has suggestions of a way to assign partners so that the total amount of profit made by all players is maximized. After some chatter, the instructor proposes the following method, which the players will act out:

Act I, Scene 1. Volunteers form two parallel lines, one with the dark-shirted volunteers and one with the light-shirted volunteers. Players in each line sort themselves according to height, going in opposite directions, with the tallest dark-shirted player and the shortest light-shirted player positioned on *stage right*.<sup>3</sup>

The instructor suggests that the two lines look like supply and demand curves, where the dark-shirted line is the *demand curve* and the light-shirted line is the *supply curve*.

**Act I, Scene 2.** The assistants find the "marginal matched demander," who is the shortest dark-shirted player that is taller than the adjacent light-shirted player, whom we will call the "marginal matched supplier." The instructor proposes a matching in which:

- 1. the marginal matched demander and all demanders who stand to the right of this player are matched with the adjacent suppliers
- 2. all players who stand to the left of the marginal matched demander and supplier remain unmatched. $^4$

The instructor then asks the matched players to assemble on stage right and the unmatched players to assemble on stage left. The assistants report the number of pairs who are matched and, using the Excel software, the sum of profits from these matches. This information is reported on the screen or blackboard.

#### Act II: Number of matches and total profits

The instructor can now ask the class to speculate about answers to the following questions:

Question 1: Is there any way to assign demanders to suppliers that results in greater total profits

 $<sup>^{3}</sup>$  "Stage right" is the right side of a person on stage facing the audience. Thus, stage right appears to the left from the viewpoint of an audience member.

<sup>&</sup>lt;sup>4</sup>With this procedure any pair of adjacent dark and light shirted players who are of equal heights will be left unmatched. Using the distribution of male and female heights in the population, we simulate the likelihood of obtaining an equilibrium with more than one pair of the same height. If there are ten students on each side of the market, the equilibrium involves more than one pair approximately nine percent of the times if height is measured in centimeters. We simulate having a less precise height measure by bunching people within a 3 centimeter range (e.g. height of 156, 157 and 158 cm are considered as 157 cm). In this case, having more than one equilibrium pair obtains approximately with a twenty percent frequency. If there are at least three students that are 6 feet (182 cm) or higher and three students of height 5 feet 2 inches (159cm) or lower, the frequency is reduced to less than two percent.

than that found by the matchings with living supply and demand curves?

Question 2: Is there a way to assign partnerships so that more partnerships are formed in which both the demander and supplier can make a profit? (i.e. where the dark-shirted partner is taller than the light-shirted partner.)

It turns out that the arrangement of partners determined by the living supply and demand curves maximizes total profits, but in general does not maximize the number of profitable sales. The appendix to this paper presents proofs of these propositions.

Before (or perhaps instead of) showing that these claims are true in general, we suggest that our actors perform the following routine, which we will denote as Routine IIA.

**Act II, Scene 1.** The dark and light shirted players (demanders and suppliers, respectively) again form parallel lines, but this time *both* lines are organized from tallest to shortest with the tallest on stage right.

Suppliers now stand next to demanders of the same rank. That is, the k-th tallest demander is next to the k-th tallest supplier.

In the next scene, partners will be rearranged as follows:

#### Act II, Scene 2.

- 1. Determine whether there is any any pair of participants such that the dark-shirted demander is shorter than the light-shirted supplier.
- 2. If there is any such pair, ask the *tallest* supplier *and* the *shortest* demander move to stage left. The remaining suppliers advance one place to form new partnerships.
- 3. This process is repeated until no dark-shirted demander is shorter than the adjacent light-shirted supplier.

The instructor now asks the matched players to assemble on stage right and the unmatched players to assemble on stage left. The assistants report the number of pairs who are matched and, using the Excel software, the sum of profits from these matches. This information is reported on the screen or blackboard and compared to the results of the procedure in Scene 2.

Act II, Scene 3. The instructor asks the tallest matched supplier and the shortest matched demander to come to center stage and stand next to each other. The class is asked who is taller. Almost certainly, the tallest light shirt (supplier) will be taller than the shortest matched dark shirt (demander).

The instructor points out that total profits of matched pairs will be higher if this supplier and demander are left unmatched.

#### Discussion: Comparing number of trades and total profits

The instructor displays the total number of partnerships and the total amount of profits found from each of the two methods of partnership formation. These will be calculated with the spreadsheet software. The second method will almost certainly result in more partnerships but less total profits.<sup>5</sup>

To show that maximizing the number of profitable trades does not maximize total profits, the

<sup>&</sup>lt;sup>5</sup>It is possible, but very unlikely, that the two methods produce equal results. Using simulations of ten students on each side of the market, the ratio of profits in Act II represents on average about two-thirds of the efficient outcome described in Act I.

instructor asks the class what would happen to total profits if the tallest matched supplier and the shortest matched demander are left unmatched. The answer is that total profits will rise. Why is this? Because total profits are equal to the sum of the heights of the demanders minus the sum of the heights of the suppliers. If the tallest matched supplier is taller than the shortest matched demander, then leaving both of them unmatched will reduce total costs by more than it reduces total willingness to pay.

A bit of discussion now should convince (most of) the class that if total profits are maximized, then every matched dark-shirted player must be at least as tall as every matched light-shirted player and also that every unmatched light-shirted player must be at least as tall as every unmatched dark-shirted player. We have shown that the first matching arrangements, those made with the living supply and demand curve, have this property.

We have now seen that profits are higher from the matching in Act I than in Act II, but in Act II, there is a larger number of profitable matchings. If profits are distributed only amongst the matched pairs, the second arrangement creates lower total profits, but distributes them among more people.

#### Act III: Prices and the limbo dance

So far we have shown that demand and supply curves can be used to sort out who should trade in order to maximize total gains. We have not yet considered the role of *prices* in this process.

Act III, Scene 1. The act begins with the dark-shirted suppliers on stage right and the light-shirted demanders on stage left. The marginal light-shirted player from the previous act is asked to stand in the middle of the stage. The two assistants now hold the "limbo stick" horizontally at a level of this player's chin. Light-shirted persons are asked to try to walk under the pole, without stooping. Those who can do so proceed to the right side of the stage. Next, dark-shirted persons are asked to try to walk under the pole while standing straight. Those who can do so proceed to the left side of the room. The assistants report the number of dark shirts and the number of light shirts who are now assembled on stage right.

Discussion of excess demand: The instructor reminds the class that a dark-shirted demander is willing to pay an amount up to his or her height in centimeters for a unit of the commodity and that the suppliers in white shirts would profit from selling a unit of the commodity for any price greater than their height. Since every demander on the right side of the stage is taller than the limbo stick and every supplier is shorter than the height of the limbo stick, any supplier and demander now on the right side of the stage could profit by trading at a price equal to the height of the limbo stick. We see that there are more demanders than suppliers on the right side, and so some demanders who would be willing to pay more than the height of the limbo stick do not find trading partners when all trades take place the at "limbo stick price."

The instructor can ask the class what one of the excluded demanders might do in order to attract a partner. When someone suggests that a demander might offer a higher price, the instructor can agree and suggest that for this reason, prices are not likely to stay at the equilibrium price, but will rise.

Act III, Scene 2. This scene proceeds much as Scene 1. This time the marginal dark-shirted player from Act I is asked to come to the center of the stage. The assistants now hold the limbo pole at 3 inches (8 cm.) above this person's head. As before, light-shirted players assemble on

<sup>&</sup>lt;sup>5</sup>If technology permits, the class might enjoy having "Limbo Rock" played in the background (available at https://www.youtube.com/watch?v=XgCHOrF5ryY)

stage left and dark-shirted players on stage right. Those light-shirted players who can walk under the limbo pole without stooping assemble on stage right, while those who cannot remain on the left side of the stage. Those dark-shirted players who can walk under the limbo pole without stooping assemble on stage left and those who cannot remain on stage right. The assistants report the number of dark shirts and the number of light shirts who are on stage right.

Discussion of excess supply: As in Scene 1, all of the dark-shirted suppliers on stage right are taller than the height of the limbo stick and hence are willing to pay more than this amount to buy a unit of the commodity. All of the light-shirted demanders are shorter than the limbo stick and hence would profit from selling a unit of the commodity at a price equal to its height. Thus, any demander and any supplier on the right side of the stage could make a mutually profitable deal. But there are more light-shirted suppliers than dark-shirted demanders on the right side. Hence, all of the demanders would be able to find partners for mutually profitable deals, but some suppliers who would profit from trading at the limbo price will not find trading partners.

The instructor now asks the students what such a supplier might do. When someone suggests that the supplier might offer to sell at a lower price, the instructor can suggest that if prices were at the level of this limbo stick, trades would start to occur at a lower price.

Act III, Scene 3. The marginal matched supplier from Act I is brought to center stage. The limbo stick is held at the level of the top of this player's head.

As before, light-shirted players assemble on the left side of the stage and dark-shirted players on the right. Players who can walk under the stick without stooping and without touching the stick are again asked to do so.

**Discussion of equilibrium:** We observe that this time, nearly equal numbers of dark-shirted demanders and light-shirted suppliers who wind up on the right side of the stage.<sup>6</sup> This means that if a price is set equal to the height of the limbo stick, the number of demanders who are willing to buy at this price is equal to the number of suppliers willing to sell at that price. At this price, no supplier could gain by offering to sell at a lower price and no demander could gain by offering to buy at a higher price. This price is known as the equilibrium price. There is no reason to expect the price to either rise or fall.

#### **Epilogue**

After the players take their bows and are dismissed, the instructor can use the Excel spreadsheet to show a graph of the demand and supply curve that were generated by the players' heights. On this graph, they can see the competitive equilibrium price and quantity as well as a depiction of the area representing total profits ("consumers' plus producers' surplus"). They can also be shown disequilibrium prices and excess demands or supplies corresponding to the non-equilibrium limbo dance prices. An example of a graph generated by the spreadsheet is provided in Figure 1.<sup>7</sup> In the example, five matchings result in equilibrium. The shaded area denotes the total profit that matchings generate, that is, the addition of the profit for each matching equals total consumers' plus producers' surplus.

<sup>&</sup>lt;sup>6</sup>If multiple players are very close to the height of the limbo stick, it may be that the number of dark and white shirts are not exactly equal.

<sup>&</sup>lt;sup>7</sup>The spreadsheet is also prepared in case the instructor in a future class may want to use the graph created with the living supply and demand curves to illustrate comparative statics or price controls.

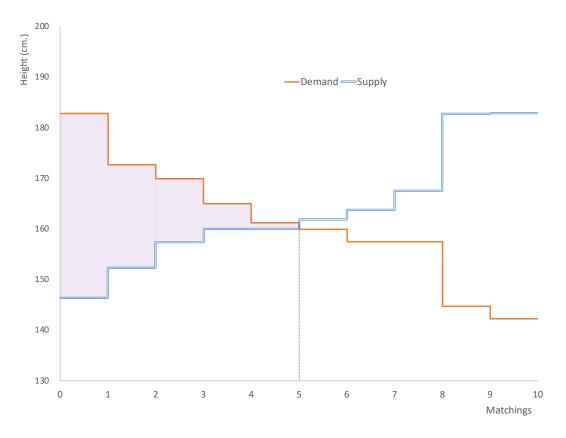


Figure 1: Example for Act I

Figure 2 shows the same participants but after Act II has been played out. The main differences between Figures 1 and 2 are that matched suppliers are ordered from tallest to shortest and unmatched participants are represented with single dots. The procedure maximizes the number of matchings with positive profit and Figure 2 shows that it is possible to generate eight such matchings. The green-dotted area represents total profits corresponding to the eight matchings.

Total profits when the number of matchings are maximized are lower than market profits. To see this, Figure 3 organizes matched suppliers from shortest to tallest while still presenting as dots unmatched participants. Profits for the eight matchings equal total profits under the five market matchings (light purple as in Figure 1) plus the profits from matchings six, seven and eight (red stripes). Notice that the profits from matchings six, seven and eight are negative as the supplier is taller than the demander. This means that adding the profits from these three additional matchings will result in total profits below market profits.<sup>8</sup>

<sup>&</sup>lt;sup>8</sup>That is, the green-doted area in Figure 2 is equal to the solid light purple area in Figure 3 plus the negative red-stripped area in Figure 3. The green-doted area in Figure 2 represents 80.2 percent of the solid purple are of Figure 3.

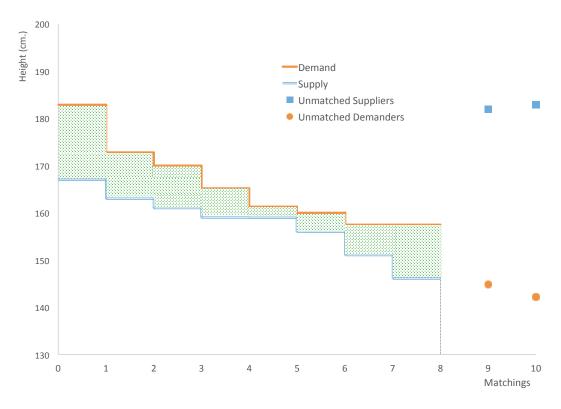


Figure 2: Example for Act II with suppliers ordered from tallest to shortest

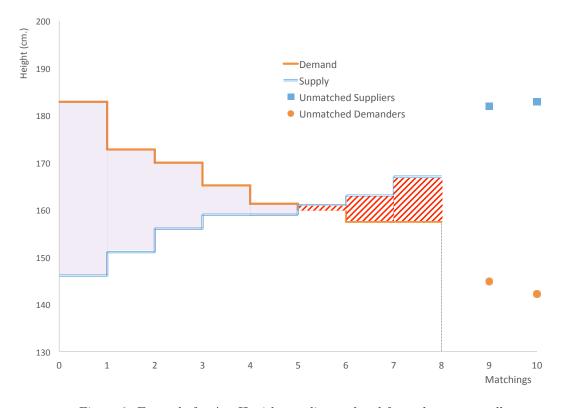


Figure 3: Example for Act II with suppliers ordered from shortest to tallest

## Appendix

We will refer to the routines used to assign trading partners in Act I as Routine I and the routines used in Act II, Scenes 1, 2, and 3 as Routines IIA, IIB, and IIC respectively.

#### Maximizing total profits

**Proposition 1.** The assignment of trading partners made by Routine I, maximizes total profits.

We prove this result, using a lemma and three remarks.

**Lemma 1.** Total profits depend only on who is matched and not on who they are matched with.

*Proof.* Let the matched pairs be numbered i = 1, ...k, where  $D_i$  and  $S_i$  are the heights of the demander and supplier in pair i. Total profits from this matching are  $\sum_{i=1}^{k} (D_i - S_i)$ . But it must be that

$$\sum_{i=1}^{k} (D_i - S_i) = \sum_{i=1}^{k} D_i - \sum_{i=1}^{k} S_i.$$
 (1)

The expression on the right side of Equation 1 depends only on the sum of the heights of the demanders and the sum of the heights of the suppliers who are matched. Thus total profits are determined by who is matched and is not changed by reassignment of partner among those who are matched.

**Remark 1.** There is no way of achieving higher total profits with the same number of trading pairs that is found in Act I.

*Proof.* The procedure ensures that every matched demander is at least as tall as any unmatched demander and every matched supplier is no taller than any unmatched supplier. The only way to achieve an assignment with the same number of players as the assigned matches in Act I is to swap some demanders and/or suppliers with assigned matches for demanders and/or suppliers who were not assigned matches. But such swaps could not increase the sum of heights of demanders or decrease the sum of heights of suppliers. Since total profits are the difference between the sum of the heights of demanders and the sum of heights of suppliers, such swaps cannot increase total profits.

**Remark 2.** There is no way of achieving higher total profits with a greater number of trading pairs than is found in Act I.

*Proof.* To increase the number of trading pairs, one must add equal numbers of suppliers and demanders from the set of unmatched players. But every unmatched supplier is taller than every unmatched demander, so this will decrease total profits.  $\Box$ 

**Remark 3.** There is no way of achieving higher total profits with a smaller number of trading pairs than is found in Act I

*Proof.* To reduced the number of trading pairs one needs to eliminate at least one supplier and at least one demander from the set of matched players. But every matched supplier is taller than every matched demander.

So there we are. Starting with the assignment of partners found in Act I, we cannot increase total profits while leaving the number of trades unchanged, nor by increasing the number of trades nor by decreasing the number of trades.

#### Maximizing the number of profitable transactions

**Proposition 2.** The assignment of partners made by Routine IIA in Act II Scene 1 maximizes the number of trades in which both trading partners profit.

Although the idea of the proof is pretty simple, proving it carefully probably requires more effort than one would want to impose on an introductory class.

The idea of the proof is as follows. Suppose that Routine IIA ends with an assignment of k demanders to k suppliers. At all previous steps, where we have k' > k suppliers and demanders: (i) the k' suppliers and demanders who remain at that step can not be profitably matched. (ii) No other set of k' suppliers and k' demanders selected from the original group of N suppliers and N demanders can be profitably matched. Claim (i) is obvious. It takes a bit of work to show Claim (ii). The idea is that the suppliers who have been eliminated at this stage are taller than all of remaining suppliers and the demanders who have been eliminated are shorter than all remaining suppliers. Since this is the case, there is no way to substitute unmatched for matched suppliers and demanders to obtain a group of k' suppliers and k' demanders that can be profitably matched.

To establish this result, it is useful to have the following definitions:

**Definition 1.** Let X and X' be two sets of N individuals, each of which is assigned a height. The set X height-dominates the set X' if there is a one-to-one matching such that each member X' of X' who is no taller than X. We say that X strictly height-dominates X' if X height-dominates X' and some element of X is taller than the matched element of X'.

Since total profits from a transaction are measured by the height difference between the demander and the supplier, we say that a set D of demanders and a set S suppliers can be profitably matched if there is a way to match each demander with a supplier who is shorter than he is.

**Definition 2.** A set D of N demanders can be profitably matched with a set S of N suppliers if the set D height-dominates the set S.

For any set of demanders or suppliers, we define the ordered vector of heights as a vector that lists the height of each member in descending order.

**Definition 3.** For any set X of N individuals with specified heights, the **ordered vector of heights** in X is the vector  $h(X) = (x_1, x_2, ..., x_n)$  sorted by height, so that  $x_i \ge x_{i+1}$  for i = 1...N-1.

We say that sets S of suppliers and D of demanders are profitably matched by rank order if for all  $k = 1 \dots N$ , the kth tallest demander is at least as tall as the kth tallest supplier.

**Definition 4.** A set of S of N suppliers and a set D of N demanders are **profitably matched by** rank order if  $h(D) \ge h(S)$ , where  $\ge$  is the standard vector ordering.

**Lemma 2.** If a set of S of suppliers and a set D of demanders can be profitably matched, then these sets are profitably matched by rank order.

Proof of Lemma 2. Suppose that the sets S and D are not profitably matched by rank order. Then for some k, the kth tallest supplier is taller than the kth tallest demander. Since demanders of rank  $k+1,\ldots N$ , are shorter than the demander of rank k, the only demanders who could possibly be taller than the k suppliers of rank k or higher are the k-1 demanders who are taller than the kth tallest demander. It follows that in any one-to-one matching of S and D, at least one of the tallest k suppliers must be matched with a shorter demander. Therefore if the sets S and D are not properly matched by rank order, there can be no profitable matching between these two sets.

**Lemma 3.** Suppose D,  $D^*$ , S, and  $S^*$  are sets of demanders and suppliers such that D height-dominates  $D^*$  and  $S^*$  height-dominates S. Then if  $D^*$  and  $S^*$  can be profitably matched, so can D and S.

Proof of Lemma 3. By assumption, the ordered vectors of heights of these sets satisfy the vector inequality  $h(D) \ge h(D*) \ge h(S*) \ge h(S)$ , where  $\ge$  represents vector inequality. Since the vector inequality is transitive, it follows that  $h(D) \ge h(S)$ , which means that the sets D of demanders and S of suppliers can be profitably matched.

The contrapositive of Lemma 3 can be stated as:

**Corollary 1.** If the sets D and S cannot be profitably matched and if D height dominates  $D^*$  and  $S^*$  height dominates S, then  $D^*$  and  $S^*$  cannot be profitably matched.

**Lemma 4.** Suppose that Routine IIA terminates with a matched set of k demanders and k suppliers, where k < N. For j such that  $N \ge j > k$ , let  $D_j$  and  $S_j$  be the sets of size j that remain after N - j iterations in which N - j demanders and N - j suppliers have been removed. If  $D_j$  and  $S_j$  can not be profitably matched, then no other subsets of D and S of size j can be profitably matched.

Proof. Suppose that  $D_j$  and  $S_j$  can not be profitably matched. Any other set of j suppliers and j demanders must be obtained by swapping individuals from  $D_j$  and  $S_j$  for members who have are excluded from  $D_j$  and  $S_j$ . But each of the N-j excluded suppliers is at least as tall as every supplier in  $S_j$  and each of the N-j excluded demanders no taller than any demander in  $S_j$ . It follows from Corollary 1 and the fact that  $D_j$  and  $S_j$  cannot be profitably matched that if  $D_j^* \subset D$  and  $S_j^* \subset$  have j matched pairs, then they cannot be properly matched.

Now we can prove Proposition 2.

*Proof of Proposition 2*. Routine IIA terminates with a profitable matching. From Lemma 4 it follows that if Routine IIA terminates with a matching of k demanders and k suppliers, then there is no profitable matching with more than k matched pairs.

While Routine IIA selects a matching of k suppliers and k demanders, there may be other sets of demanders and suppliers that also have k profitable trades. However, the set of suppliers and demanders selected by Routine IIA yields the highest total profits of any profitable matching of size k.

**Proposition 3.** Total profits yielded by the assignment from Routine IIA are at least as high as total profits from any other profitable assignment that has the same number of matched pairs.

*Proof.* Where  $D_k$  and  $S_k$  are the sets of k demanders and k suppliers who are matched by Routine IIA, any other matching of k demanders and k suppliers must be obtained by swapping some demanders and/or some suppliers who are not matched with some who are matched. But every excluded demander is no taller than any included demander and every excluded supplier is at least as tall as any included supplier. Therefore no such exchange can increase total profits.

#### Maximizing number of profitable transactions with the tallest possible suppliers.

Consider the following routine, which we will call Routine IIB. This routine leads to the same number of matches as Routine IIA. As in Routine IIA, the k tallest demanders are matched, but the matched suppliers include some suppliers who are taller than those chosen by IIA.

#### Routine IIB:

Step 1) Demanders and suppliers are lined up in rank order, with the kth tallest demander next to the kth tallest supplier.

Step 2) If possible, remove the tallest supplier who is taller than the corresponding demander and also remove the shortest demander from the demander line. Rematch the two lines according to rank order.

Step 3) If, after Step 2, no supplier is taller than the corresponding demander, terminate. If there remains a supplier who is taller than the corresponding demander, repeat Step 2.

**Proposition 4.** Routine IIB selects the largest possible number k of profitable trades between members of the sets D and S of n suppliers and n demanders.

*Proof.* . Let  $S_k$  and  $D_k$  be the sets of k demanders and k suppliers who are matched at the end of Routine IIB. Then  $D_k$  height-dominates  $S_k$  and so these sets can be profitably matched. We show that no larger subsets of D and S can be profitably matched.

Routine IIB terminates after n-k steps. For each j such that  $k < j \le n$ , in the n-jth step, one supplier and one demander is eliminated from the set of pairs to be matched. Let  $y_j$  and  $x_j$  denote the heights of the supplier and demander who are eliminated at step n-j. For each step, n-j, the supplier who was eliminated at the previous step is taller than the supplier eliminated at step n-j.

Since Routine IIB does not terminate at step n-j, it must be that the sets  $D_j$  and  $S_j$  can not be profitably matched. In fact, no set  $S^*$  of j suppliers from the set S can be profitably matched with  $D_j$ . To see this, notice that if  $S^*$  can be properly matched with  $D_j$ , it must replace the supplier of height  $y_j$  with a supplier who is shorter than  $y_j$ . But suppliers who have been eliminated earlier are taller than those who are eliminated later. Therefore there is no subset  $S^*$  of j members of S that can be profitably matched with with  $D_j$ .

Since the set  $D_j$  consists of the tallest j demander, it must be that  $D_j$  height-dominates any other j member subset of D. Since there is no j-member subset of S that is height-dominated by  $D_k$ , it follows from Corollary 1 that for j > k, there are no j-member sets  $S^* \subset S$  and  $D^* \subset D$  such that  $D^*$  height-dominates  $S^*$ .

**Corollary 2.** The number of trades resulting from Routine IIB is the same as that resulting from Routine IIA.

**Remark 4.** The assignment resulting from Routine IIB maximizes the sum of heights of suppliers who trade subject to the constraint that every trade is possible.

*Proof.* Routine IIB eliminates Supplier i only if every demander who is taller than supplier i is matched with a supplier who is taller than i. So there is no way to substitute unmatched for matched suppliers that would increase profits.

#### Maximizing number of profitable transactions with the shortest possible demanders.

Routine IIC parallels IIB. It also chooses the maximum possible number k of profitably matched pairs. But instead of matching the k tallest demanders to as tall a group of suppliers as possible, it finds a profitable match of the k shortest suppliers to as short a group of demanders as possible.

#### Routine IIC:

- Step 1) Demanders and suppliers are lined up in rank order, with the kth tallest demander next to the kth tallest supplier.
- Step 2) If possible, remove the shortest demander who is shorter than the corresponding supplier and also remove the tallest supplier from the supplier line. Rematch the two lines according to rank order.
- Step 3) If, after Step 2, no demander is shorter than the corresponding supplier, terminate. If there remains a demander who is shorter than the corresponding supplier, repeat Step 2.