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New cosmological limit on neutrino mass

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We show that considerations of the equilibration of right- and left-handed neutrino seas in the early Universe and the well-known ^{4}He constraint on the number of relativistic degrees of freedom extant at the nucleosynthesis epoch lead to a new limit on the Dirac neutrino mass of $m_\nu \ll 300$ keV. This constraint would apply to any neutrino so long as it had a purely Dirac mass and a lifetime exceeding the nucleosynthesis time scale.

In this paper we describe a new astrophysical constraint on the neutrino mass which has its origin in the effect of neutrino degrees of freedom, or energy density, on the universal expansion rate during nucleosynthesis. The greater the number of relativistic degrees of freedom the faster will be the expansion rate and, ultimately, the larger will be the abundance of ^{4}He. The demand that ^{4}He not be overproduced relative to the primordial abundance inferred from observations yields a limit on the number of relativistic degrees of freedom. This is the origin of the well-known cosmological limit on the number of light-neutrino families,[1] which is in excellent agreement with the results from the Z^0 width experiments.[2] Consider now the effects of a "sterile" sea of right-handed (RH) neutrinos and left-handed (LH) antineutrinos in addition to the expected sea of normal LH (RH) neutrinos (antineutrinos). Hereafter when we refer to RH neutrinos we mean both RH neutrinos and LH antineutrinos. By "sterile" here we mean that RH neutrinos have much smaller interaction cross sections than their LH counterparts due to left-handed projection operators in the weak Lagrangian.

The extent to which the RH neutrino sea is populated can be constrained by the nucleosynthesis considerations discussed above. Indeed, related studies have used the consequences of a significant RH neutrino sea to limit electromagnetic,[3] oscillation,[4] and weak-interaction[5,6] properties of neutrinos. In these studies the RH neutrino sea was populated by electromagnetic interactions, *CP*-violating neutrino oscillations, or RH intermediate bosons.

However, if we make the assumption that neutrinos have mass then simple particle scattering populates the RH neutrino sea. In the case where neutrinos are Majorana particles the RH species is simply the antineutrino and no extra degrees of freedom are produced. Because our subsequent argument is based on the effect of added degrees of freedom during nucleosynthesis we must assume that neutrinos are purely Dirac particles. Since the amplitude for the helicity flip of Dirac neutrinos in a scattering event (or charged-current emission or absorption event) is proportional to the neutrino mass m_ν, the temperature in the early Universe at which the RH neutrino sea falls out of equilibrium with its LH counterpart depends on m_ν. Once these neutrino seas fall out of equilibrium any subsequent weakly interacting particle annihilations or phase transitions heat the LH neutrino sea but not the RH sea, lessening the relative energy density contribution of the RH sea in determining the expansion rate at nucleosynthesis. Using essentially the above arguments it has previously been noted that an *electron* neutrino of Dirac mass ~ 10 eV has a negligible effect on ^{4}He production.[7] Our study differs in that we go a step further, assume three neutrino families, and then turn the arguments around to give a limit on possible Dirac masses for ν_μ and ν_τ. The demand that the RH neutrino sea make a negligible contribution to the energy density at the nucleosynthesis epoch (to avoid overproduction of ^{4}He) means that the RH neutrinos must decouple prior to the dominant "annihilation" event, the confinement of quarks at temperatures in excess of 100 MeV. This sets an upper limit to the neutrino mass. We will discuss in turn each of the steps in this argument.

The RH neutrino sea will be in equilibrium with the LH sea only if the processes that flip neutrino helicity are very rapid compared to the universal expansion rate. One can show that in any scattering event the cross section for helicity-flip is suppressed[7,8] relative to that of the nonflip event by a factor of order $(m_\nu/2E_\nu)^2$, where E_ν is a characteristic neutrino energy. The time scale for helicity flipping, $\tau_{\rm flip}$, is

$$\tau_{\rm flip} \approx (n_w \sigma)^{-1} \approx \left[\frac{3}{16} \frac{\zeta(3)}{\pi^2} g_w G_F^2 m_\nu^2 T^3 \right]^{-1} , \qquad (1)$$

where n_w is the number density of weakly interacting particles, σ is a characteristic cross section for helicity flip, $\zeta(3)$ is the Riemann ζ function of argument 3, G_F is the Fermi constant, and g_w is the weakly interacting fermion statistical weight at temperature T. Since the crucial epoch for the decoupling of the RH neutrino sea is that of quark confinement[3-7] ($T > 100$ MeV) it is reasonable to assume that the number density of important weakly interacting targets is proportional to T^3 and that g_w includes electrons, muons, neutrinos, u and d quarks, and all associated antiparticles, which gives $g_w \approx 38$. The expansion time scale is, roughly, the inverse of the Hubble parameter H at temperature T with

$$H = \left[\frac{4\pi^3}{45} \right]^{1/2} m_{\rm Pl}^{-1} g^{1/2} T^2 , \tag{2}$$

and where $m_{\rm Pl}$ is the Planck mass and $g = g_b + \frac{7}{8} g_f$ is the statistical weight of relativistic particles including that in bosons, g_b, and that in fermions, g_f. At $T \approx 100$ MeV the inclusion of electrons, muons, photons, LH and RH neutrinos, u and d quarks, gluons, and all associated antiparticles yields $g \approx 56.5$. Although the exact temperature of decoupling must come from a solution of the Boltzmann equation[1,5] it suffices for the upper limit described here to assume that equilibrium between RH and LH neutrino seas obtains whenever $\tau_{\rm flip} \ll H^{-1}$. We can then conclude that for the RH neutrino sea to have decoupled *prior* to the Universe reaching temperature T the neutrino mass must be

$$m_\nu \ll \frac{4}{3[\zeta(3)]^{1/2}} \left[\frac{4\pi^7}{5} \right]^{1/4} (G_F^2 m_{\rm Pl})^{-1/2} \left[\frac{g^{1/4}}{g_w^{1/2}} \right] T^{-1/2} , \tag{3a}$$

or, adopting statistical weights relevant to the epoch prior to quark confinement,

$$m_\nu \ll 300 \text{ keV} \left[\frac{100 \text{ MeV}}{T} \right]^{1/2} . \tag{3b}$$

We emphasize that to be more exact in Eqs. (3a) and (3b) (especially near the upper end of the mass limit) would require a detailed calculation of the helicity-flip rates for all neutrino processes as well as an in depth treatment of phase-space considerations.

For both LH and RH relativistic neutrino seas the temperature is inversely proportional to the scale factor, but the proportionality constant can evolve differently for each once the RH sea decouples.[1] Subsequent to decoupling this proportionality constant remains unchanged for the RH sea, whereas particle-antiparticle annihilations and phase transitions can increase the product of scale factor and temperature for the LH sea. Since the proper entropy density contributed by interacting relativistic particles (in statistical equilibrium) is $(4\pi^2/90)(g_b + \frac{7}{8} g_f) T^3$ and the evolution of the Universe through phase-transition and particle-annihilation epochs is characterized by constant *comoving* entropy density we can write the ratio of the proportionality constants as

$$\frac{(RT)_2}{(RT)_1} = \left[\frac{g_1}{g_2} \right]^{1/3} , \tag{4}$$

where R is the scale factor, subscripts 1 and 2 refer to the times before and after the annihilation epoch, respectively, and g_1 and g_2 are the statistical weights of relativistic particles at these times. Since the product RT does not change for the decoupled RH neutrino sea the ratio in Eq. (4) gives the amount of heating of the LH sea relative to the RH sea: $T_{\rm LH}/T_{\rm RH} = (g_1/g_2)^{1/3}$. We now explain why the constraint in Eq. (3b) must be satisfied.

We can identify three relevant heating events prior to primordial nucleosynthesis: the electroweak transition; the QCD transition; and muon annihilation. Muon annihilation at $T \approx 100$ MeV is not very significant as the temperature difference implied by Eq. (4) is only of order 1%. In terms of the statistical weight the electroweak transition ($T \approx 100$ GeV) would be characterized by the $W^\pm$ and Z^0 annihilating, giving an insignificant temperature increment of about 4%. At a slightly lower temperature the τ lepton will similarly drop out of equilibrium with, again, a resulting insignificant temperature difference between coupled and decoupled neutrino species. By contrast the statistical weight changes by a factor of about 3 when the quarks annihilate and become bound in color singlets. Whether this takes place in a phase transition or in a process akin to ionization is irrelevant for the temperature change from Eq. (4) to our limit on m_ν. The order of any phase transition associated with the QCD epoch is also irrelevant for these quantities. The largest upper limit on m_ν will be obtained from the case where the quarks annihilate at the lowest possible temperature, which would be about 100 MeV.[9,10] Prior to confinement, as discussed above, the statistical weight in relativistic particles is $g_1 \approx 56.5$; whereas, after the quarks annihilate the only nearly relativistic strongly interacting particles will be pions, so that $g_2 \approx 17.25$ and $T_{\rm LH}/T_{\rm RH} \approx 1.44$. This implies that at the time of nucleosynthesis the relative energy densities in a LH and RH neutrino species will be $\rho_{\rm LH}/\rho_{\rm RH} \approx 4.3$, so that a sterile neutrino species counts as less than $\frac{1}{4}$ of an additional neutrino flavor. Muons and pions should annihilate or drop out of equilibrium at roughly $T \approx 100$ MeV, and if we add their statistical weight to the differential heating of the LH and RH seas then a RH neutrino species would count for only about 0.1 of an additional flavor.

Standard, homogeneous, big-bang nucleosynthesis (SBBN) predicts a ^{4}He mass fraction given by[11]

$$Y_p \approx 0.228 + 0.010 \ln\eta_{10} + 0.012(N_\nu - 3) + 0.185 \left[\frac{\tau_n - 889.8}{889.8} \right] , \tag{5a}$$

where η_{10} is the baryon-to-photon ratio in units of 10^{-10}, N_ν the number of neutrino families, and τ_n the neutron mean life in seconds. Assuming a lower limit of $\eta_{10} > 2.6$ from observations[11] of D+^{3}He, and assuming[11,12] $\tau_n \approx 898.8 \pm 4.4$ s, we can reexpress Eq. (5a) as a limit on the number of relativistic neutrino species in terms of the primordial ^{4}He mass fraction:

$$N_\nu \le 3.4 + 20 \left[\frac{Y_p - 0.240}{0.240} \right] . \tag{5b}$$

If we take primordial ^{4}He to be $Y_p = 0.23 \pm 0.01$ from observation[11,13] then $N_\nu \leq 3.4$. We stress here that our result will be sensitive to the adopted upper limit of $Y_p < 24\%$. A value of $Y_p \gtrsim 24.2\%$ would weaken our constraint. We know from the CERN LEP and SLAC Linear Collider (SLC) experiments[2] that $N_\nu = 3.05 \pm 0.2$, though it should be pointed out that nucleosynthesis counts only particles which are relativistic at the epoch of nucleosynthesis whereas the collider experiments count anything coupled to the Z^0 with a mass less than half of the Z^0 mass. In SBBN sterile neutrino states are not assumed to be present. If there are three light Dirac neutrinos and the RH components of one of them did not decouple until after the QCD epoch then those RH components would count for an extra 0.7 neutrino flavor. If the decoupling of the RH sea is after muon and pion annihilation then the RH components count for almost a full extra neutrino flavor. Even an extra 0.7 neutrino flavor is clearly not compatible with the limit $N_\nu \leq 3.4$; thus, the RH components of the neutrino (ν_μ or ν_τ) would have to decouple prior to the QCD epoch, yielding the Dirac mass limit in Eq. (3b).

At the upper end of the mass limit ($m_\nu \sim 300$ keV) the neutrino may be only mildly relativistic at some point during the nucleosynthesis epoch so that the effective number of neutrino flavors might be less than 3, and some extra contribution to the energy density from RH components would be acceptable. Also, annihilations for $m_\nu \gtrsim 20$ MeV will reduce the number density of massive neutrinos to a small fraction of the massless neutrinos. A detailed study of these possible loopholes is under way (see also Ref. 14).

The new constraint on the Dirac mass of the neutrino described here compliments and extends existing experimental and astrophysical limits. The present experimental bounds[15] on the mass of each neutrino species are $m_{\nu_e} \leq 18-32$ eV, $m_{\nu_\mu} < 250$ keV, and $m_{\nu_\tau} < 35$ MeV. More stringent constraints on the neutrino mass can be obtained from astrophysical considerations. For stable neutrinos the demand that the energy density of the Universe not exceed the critical value for closure yields constraints on the neutrino masses[16–18] of $m_\nu \gtrsim 2$ GeV or $m_\nu \leq 100$ eV h^2 (assuming no RH component, and where h is the present Hubble parameter in units of 100 km s^{-1} Mpc^{-1}). However, as stated above this constraint applies only if the neutrinos are stable. Radiative decays can be stringently limited by cosmic electromagnetic effects,[19–21] but nonradiative decays cannot be constrained over broad ranges of masses and lifetimes.[22] The Dirac mass limit described here will partially close this loophole for ν_μ and ν_τ.

The mass limit in Eq. (5) applies to ν_μ and ν_τ if they have purely Dirac masses and interactions, they are light and, to be precise, they (and their RH components) have lifetimes exceeding the nucleosynthesis time scale (≈ 100 s). We assume that LH and RH components have the same mass. If the decay mode of the neutrino is to a lighter neutrino and a relativistic weakly interacting particle (i.e., a Goldstone boson associated with the spontaneous breaking of a charge, lepton number, or family symmetry) then we only require that the lifetime exceed the weak decoupling time scale (≈ 1 s), because after this time the decay products will not thermalize with the plasma. We note, however, that most weak-interaction models[23] which produce these decay channels tend to give Majorana masses to neutrinos. Our limit would not apply to Majorana neutrino models in which two degrees of freedom are effectively removed, such as in a seesaw-type mechanism. Thus, our limit would most likely pertain to the simplest extensions[24] of the standard model which include Dirac masses for neutrinos. Finally, we stress that we also must assume that SBBN and the primordial ^{4}He abundance inferred from observations are correct.

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quate for our purposes. *Note added in proof*: A paper by R. Gandhi and A. Burrows [Phys. Lett. B **246**, 149 (1990)] has come to our attention, which improves on the Perez and Gandhi result by taking into account ν-nucleon scattering. Their Dirac mass limit is 14 keV.

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