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## Level Dynamics: An Approach to the Study of Avoided Level-crossing and Transition to Chaos

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# Level dynamics: An approach to the study of avoided level-crossing and transition to chaos*

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The Dyson-Pechukas level dynamics has been reformulated and made suitable for studying avoided level-crossings and transition to chaos. The $N$-level dynamics is converted into a many-body problem of one-dimensional Coulomb gas with $N$-constituent particles having intrinsic excitations. It is shown that local fluctuation of the level distribution is generated by a large number of avoided level-crossings. The role played by avoided level-crossings in generating chaoticity in level dynamics is similar to the role played by short-range collisions in causing thermalization in many-body dynamics. Furthermore, the effect of level changing rates in producing avoided level-crossings is the same as particle velocities in causing particle-particle collisions. A one-dimensional su(2) Hamiltonian has been constructed as an illustration of the level dynamics, showing how the avoided level-crossings cause the transition from a regular distribution to the chaotic GOE distribution of the levels. The existence of the one-dimensional su(2) Hamiltonian with GOE level statistics makes it necessary to reconsider the chaoticity and non-integrability relation.

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## I. INTRODUCTION

Quantum Chaos, the quantum analogy of classical chaos, recently has become a hot topic and the subject of many publications [1–3]. Since classical chaotic motion is intimately related to the non-integrability [4], *i.e.*, the loss of constants of motion, its quantum analogy is naturally related to the loss of good quantum numbers. From the point of view of symmetry, a good quantum number implies a certain kind of dynamical symmetry, and, thus, loss of good quantum numbers means loss of dynamical symmetries[5]. From this point of view, one could say that quantum chaos pushes its way by breaking dynamical symmetry and by mixing states with different quantum numbers.

Dynamical symmetry is seriously broken at avoided level crossings. There even a tiny perturbation can cause a strong mixing. Due to the symmetry-breaking perturbation, at the crossing point, two levels with different quantum numbers are strongly mixed and the level crossing is avoided. Therefore, avoided level-crossings are the basic ingredients and the most effective way to break dynamical symmetry and to generate chaos.

To get a deep insight into the mechanism of how avoided level-crossings generate chaos, one needs a proper description of level dynamics (*i.e.*, to describe the level evolution of a quantum system), as the perturbation strength undergoes changing. To this end, we find that Dyson-Pechukas approach is very useful. Dyson[6] found a profound analogy between the random matrix ensembles and the canonical ensemble of a one-dimensional Coulomb gas. He considered the energy $E_i$ to be analogous to the position of a charge along an infinite line. If one places the line charges at random positions and lets the system evolve under the joint action of the potential $U = \frac{1}{2}\sum_i E_i^2 - \sum_{i<j} \ln|E_i - E_j|$ and a dissipa-

tive force which gives rise to a Brownian motion, then after equilibrium has been reached, the position $E_i$ at any instant will be described by the canonical ensemble appropriate to the "temperature" $1/\beta$ ($\beta$ =1,2, and 4 for GOE, GUE, and GSE respectively). Pechukas *et al.*[7–9] have generalized Dyson's idea from the equilibrium case to the non-equilibrium or dynamical case. They considered the level dynamics induced by a varying perturbation as the Hamiltonian dynamics of a one-dimensional Coulomb gas with "time-dependent" fluctuating "charges" produced by the intrinsic excitations of the particles of the Coulomb gas. Thus an $N$-level system is replaced by a system of one-dimensional Coulomb gas whose $N$ constituent particles have intrinsic structures and excitations. Recently, Yang *et al.*[10], have employed the Pechukas approach to study the level statistical property of a quantum chaotic system by solving the Pechukas equations with the molecular dynamical method. To our knowledge, nobody has so far used the level dynamics to study the avoided level-crossings and their effect on level statistics. It is the purpose of this work to reformulate the Dyson-Pechukas level dynamics, to explore its physical ingredients, and to make its formulas more suitable for the study of avoided level-crossings and transition to chaos.

## II. LEVEL DYNAMICS AS DYNAMICS OF A ONE-DIMENSIONAL COULOMB GAS

Consider a Hamiltonian $\hat{H}(t)$ consisting of an integrable part $\hat{H}(0)$ and a symmetry-breaking part $\hat{V}t$,

$$\hat{H}(t) = \hat{H}(0) + \hat{V}t, \tag{1}$$

where $t$ is the strength of the perturbation, assumed to be under change, a parameter of the problem. Thus one can consider $t$ as "time". Let us use the eigen representation of $\hat{H}(0)$ as working basis,

$$\hat{H}(0)\,|\,n(0)> = X_n(0)\,|\,n(0)> . \tag{2}$$

Since in this basis the diagonal part and the off-diagonal part of $\hat{V}$ play different roles, we regroup the total Hamiltonian as follows:

$$\hat{H}(t) = \hat{H}_0(t) + \hat{V}^{\text{off}}t, \tag{3}$$

where

$$\hat{H}_0(t) = \hat{H}(0) + \hat{V}^{\text{dia}}t, \tag{4}$$

and

$$\hat{V}^{\text{dia}} = \Big(< n(0)\,|\,\hat{V}\,|\,m(0) > \delta_{n,m}\Big)\,, \tag{5}$$

$$\hat{V}^{\text{off}} = \Big(< n(0)\,|\,\hat{V}\,|\,m(0) >, \quad m \neq n\Big)\,. \tag{6}$$

Since $[\hat{V}^{\text{dia}}, \hat{H}(0)] = 0$, $\hat{V}^{\text{dia}}$ preserves the dynamical symmetry, and $\hat{H}_0(t)$ has the same eigenfunctions as $\hat{H}(0)$,

$$\hat{H}_0(t)\,|n(0)> = \; X_0^n(t)\,|\,n(0)>, \tag{7}$$

$$X_0^n(t) = X_n(0) + P_n(0)\,t, \tag{8}$$

$$P_n(0) = < n(0)\,|\,\hat{V}\,|\,n(0) > . \tag{9}$$

In the following, we shall see that $\hat{V}^{\text{dia}}$ plays an important role in producing level crossings, while $\hat{V}^{\text{off}}$ is responsible for symmetry-breaking and level-mixing. In the language of the Coulomb gas, $\hat{V}^{\text{dia}}$ generates initial velocities of the particles, which in turn cause close particle-particle collisions, while $\hat{V}^{\text{off}}$ represents the interaction of the charges.

Let $|\,n(t)>$ be the eigenfunctions of $\hat{H}(t)$,

$$\hat{H}(t)\,|\,n(t)> = X_n(t)\,|\,n(t)>, \tag{10}$$

$$|\,n(t)> = \sum_m C_{nm}(t)\,|\,m(0)>, \tag{11}$$

and

$$P_n(t) = <n(t)\,|\,\hat{V}\,|\,n(t)>\,. \tag{12}$$

From Eqs.(10-12), we obtain the equations of motion for $X_n$, $P_n$ and $C_{nm}$ (assuming $\hat{H}(t)$ is real),

$$\frac{dX_n}{dt} = P_n, \tag{13}$$

$$\frac{dP_n}{dt} = 2\sum_{m\neq n}\frac{V_{nm}V_{mn}}{X_n - X_m}, \tag{14}$$

$$\frac{dC_{nm}}{dt} = \sum_{l\neq n} C_{lm}\frac{V_{ln}}{X_n - X_l}, \tag{15}$$

where

$$V_{mn}(t) \; = <m(t)\,|\,\hat{V}\,|\,n(t)> = \; \sum_{ll'} C_{ml}(t)C_{nl'}(t)V_{ll'}(0), \tag{16}$$

$$V_{ll'}(0) = <l(0)\,|\,\hat{V}\,|\,l'(0)>\,. \tag{17}$$

If one considers $X_n$ as a coordinate of the particle $n$, $P_n$ is the corresponding momentum or velocity since it determines the rate of changes of $X_n$. Eqs.(13-14) are in the form of Hamiltonian equations. If $V_{nm}$ is considered as the effective charge determining the interactions between particle $n$ and particle $m$, the effective Hamiltonian is

$$H_{\text{eff}} \; = \; \frac{1}{2}\sum_{n=1}^{N} P_n^2 - \sum_{n\neq m} V_{nm}V_{mn}\ln|X_n - X_m|. \tag{18}$$

Since each particle has intrinsic structure, $\hat{V}$ may also induce intrinsic excitations. The intrinsic state of the particle $n$ is described by $C_{nm}$, which obeys Eq.(15). Eq.(16) describes how intrinsic excitations alter the effective charges. Eqs.(13-17) are basic equations of level dynamics, which describe how the effective charges make the particles move and cause intrinsic excitations, and how the intrinsic excitations in turn modify the effective charges.

Thus the level dynamics has become a many-body problem: a one-dimensional Coulomb gas of composite particles with intrinsic structures. The information and knowledge accumulated in many-body physics can be employed to study the above Coulomb gas. For an interacting many-body system, the general feature is as follows: the short-range collisions are responsible for sharp fluctuations of the particle motions, while the long-range interactions produce a mean field which yields smoothly deformed paths for each particle. After the removal of the mean field, the residual interaction from the long-range interaction is rather small in comparison to that from the short-range interactions. Therefore, for an interacting many-body system, the local fluctuation of the particle motion is dominated by the short-range collisions. For the above Coulomb gas, the violent short-range collisions are the dominant ingredient to produce the local fluctuation of the particle motion. In the language of level dynamics, the short-range collisions mean the sharp avoided level-crossings. Therefore the avoided level-crossings are responsible for the local fluctuation of the levels.

In fact, the basic Eqs.(13-17) clearly reflect the mechanism of how the avoided level-crossings generate local fluctuations of the levels. According to Eqs.(14-15), as two levels get close to an avoided-crossing, the denominator, $X_n - X_m$ becomes very small. A small irregularity in the interaction $V_{mn}$ will be considerably amplified at this point and will produce a large local fluctuation. This kind of local fluctuation cannot be smeared out by the summation procedure in the equations, since other collisions are relatively weak long-range collisions and, when summed up, yield a smooth mean field, which in turn causes only a long-range deformation of the energy spectrum. Thus it is the avoided level-crossings which generate the local fluctuations in $P_n$, $X_n$, and $C_{nm}$. During the course of changing $t$, successive avoided level-crossings will produce a sequence of short-range collisions. As a result, more and more local fluctuations in $X_n$ and $C_{nm}$ are generated. In the language of level dynamics, the transition of a quantum system from regular motion to chaos just means the transition of the distribution of $X_n$ and $C_{nm}$ from the regular to the chaotic. This can only happen when a large number of local fluctuations are generated in $X_n$ and $C_{nm}$. This in turn needs a large number of avoided level-crossings or short-range collisions. In fact, according to our experience, for a Coulomb gas, thermalization can only be realized by a large number of short-range collisions. Thus the chaotization of level distribution is analogous to the thermalization of a Coulomb gas. The avoided level-crossings play a similar role as the short-range collisions in generating fluctuation and chaoticity.

To see how the avoided level-crossings generate chaos, it is instructive to examine the solutions,

$$X_n(t) = X_n(0) + P_n(0)\,t + 2\sum_{m\neq n}\int_0^t dt_1 \int_0^{t_1} dt_2 \frac{V_{nm}(t_2)V_{mn}(t_2)}{X_n(t_2) - X_m(t_2)}, \tag{19}$$

$$C_{nm}(t) = C_{nm}(0) + \sum_{l\neq n}\int_0^t C_{lm}(t')\frac{V_{ln}(t')}{X_n(t') - X_l(t')}dt'. \tag{20}$$

Eqs.(19-20) tell us that (i) The fluctuation of $X_n(t)$ and $C_{nm}(t)$ is generated by the Coulomb-like interactions. (ii) The most important fluctuations come from the avoided level-crossings or short-range collisions ($X_n \approx X_m$). (iii) The level crossings are induced by a certain kind of velocity distribution $P_n(0)$, and irregularly distributed $P_n(0)$ cause random short-range collisions. Therefore to make the level distribution chaotic, one needs the irregularity of $V_{mn}$ and randomly distributed velocities to produce a large number of avoided level-crossings.

Let us estimate the average number of avoided level-crossings and their effect on the wave functions. Let $D$ be the average level spacing of $X_n(0)$, $\epsilon$ be the average slope related to the average of $P_n(0)$ and $V_{mn}(t)$. If mixing is ignored, one has

$$X_n(t) \approx nD + (-1)^{\delta_n} \epsilon t. \tag{21}$$

Where $(-1)^{\delta_n}$ is the sign of the velocity of the level $n$. If two levels $X_n(t)$ and $X_m(t)$ with opposite signs of their slopes are crossing, and the total number of level crossings along the path from $X_n(0)$ to $X_n(t)$ and from $X_m(0)$ to $X_m(t)$ ($= X_n(t)$) is

$$\Delta m \equiv m - n = \frac{2\epsilon t}{D}, \tag{22}$$

then, at the perturbation $\hat{V}t$, on average, each level has experienced $\Delta m/2$ or $\epsilon$ $t/D$ level-crossings. As mixing is turned on, each crossing will mix two levels. When a level has experienced successively $\epsilon t/D$ level-crossings, it will mix with $\Delta N$ levels,

$$\Delta N = 2^{\epsilon t/D}, \tag{23}$$

which increases exponentially as a function of $t$.

From the above discussion, we come to the conjecture for the conditions of quantum chaotic spectrum:

(i) Non-integrable perturbation $\hat{V}t$ which breaks the dynamical symmetry, *i.e.*, $V_{mn} \neq 0 (m \neq n)$. It should be strong enough to cause a large level mixing width, *i.e.*, $\Delta m/2 = \epsilon t/D \gg 1$.

(ii) Irregularly distributed $P_n(0) (= V_{nn}(0))$ to produce a large number of level crossings.

(iii) Irregularity of $V_{mn}(m \neq n)$.

In the following section, we turn to a model investigation to illustrate the above general considerations and to confirm our conjecture.

## III. SU(2) MODEL

Since the level dynamics is formulated in a Hilbert space and only the dimension of the Hilbert space enters the formalism, the dimension of the configuration space and the number of degrees of freedom of the Hamiltonian are not directly relevant. This feature allows us to consider quite a simple model to illustrate the level dynamics without worrying about other restrictions. This is quite different from the classical case where the dimension of the configuration space and the number of degrees of freedom of the Hamiltonian are relevant.

We consider the su(2) model for illustration. The reasons for employing such a model are as follows:

- It is simple.
- It is solvable.
- It is of physical relevance, representing angular momentum or Lipkin two-level excitations.

- It is a one-dimensional model.

If it is possible to construct such an su(2) Hamiltonian which exhibits chaotic behavior, then one has a counter-example for the integrability-regularity relation. Since any one-dimensional system is classically integrable, its quantum analogy with chaotic behavior would be amazing. The simplest su(2) algebra of course causes problems. Since its structure is simple and its generators are few, one thus has very limited freedom to make a perturbation generate chaos. Hence it is rather difficult to construct an su(2) Hamiltonian with GOE (Gaussian Orthogonal Ensemble) statistics built in.

Now our task is to construct an su(2) Hamiltonian which can display the basic ideas of level dynamics. As is shown in the last section, the required Hamiltonian should consist of three parts: the integrable part $\hat{H}(0)$, the velocity part $\hat{V}^{\rm dia}t$ and the symmetry-breaking part $\hat{V}^{\rm off}t$. Let the su(2) algebra be su(2)$= \{J_0,\ J_+,\ J_-\}$, where $J_0$ is the Cartan operator, $J_+$ and $J_-$ are the raising and the lowering operators. The construction of the above three parts is as follows.

(i) $\hat{H}(0)$.

Since it is integrable, we choose it to be a function of $\hat{J}_0$. Therefore it has a good quantum number $m$. For GOE[11], after the local level fluctuation has been averaged out, the smooth level density follows the semi-circle law, which reflects the global deformation of the GOE level spectrum. In order to make our model Hamiltonian with GOE statistics, as the perturbation turns off, $\hat{H}(0)$ should produce the above global level deformation. Since it is rather difficult to express the semi-circle law by a simple Hamiltonian, we find an analytical expression to approximate it. It reads

$$\hat{H}(0) = 2j \sin\left[\frac{1}{3}\sin^{-1}\left(\frac{\hat{J}_0}{j}\right)\right], \tag{24}$$

which produces a parabolic level density to approximate a semi-circle, as shown in Fig.1.

(ii) $\hat{V}^{\mathbf{dia}}t$.

Since $\hat{V}^{\mathbf{dia}}$ commutes with $\hat{H}(0)$, it is also a function of $\hat{J}_0$. It represents a velocity operator and should have an irregular distribution that produces many level crossings. We simulate it by a two-component Fourier series with an alternation sign, namely,

$$\hat{V}^{\mathbf{dia}}t = (-1)^{\hat{J}_0}\left[\beta_1 \cos\left(2\pi k_1 \frac{\hat{J}_0}{j}\right) + \beta_2 \cos\left(2\pi k_2 \frac{\hat{J}_0}{j}\right)\right] t, \tag{25}$$

where $\beta_1$ and $\beta_2$ are adjustable strengths and $k_1/j$ and $k_2/j$ are adjustable frequencies.

(iii) $\hat{V}^{\mathbf{off}}t$ .

In level dynamics, we have distinguished the short-range collisions from the long-range interactions and noticed that the two different interactions play different roles. Here we specify the above notion by writing $\hat{V}^{\mathbf{off}}t$ as

$$\hat{V}^{\mathbf{off}}t = \sum_n \alpha_n (\hat{J}_+^n + \hat{J}_-^n)/j^n \cdot t. \tag{26}$$

Terms with $n \approx 1$ are associated with a short-range interaction, since it couples two nearest states, and $|X_{n+1} - X_n|$ is small. Terms with $n \approx 10$

are called "medium-range", since they couple states with $\Delta n \approx 10$, and $|X_{n+10} - X_n|$ is somewhat larger. The term with $n > 20$ is of "long-range", since it can only couple states with $\Delta n > 20$, and the corresponding energy difference is even larger. The parameters $\alpha_n$ describe the relative strength of the interactions with different ranges. The above expression is a rough description for the interactions, since there is no clear-cut division among the short-range, medium-range, and long-range interactions. However, by examining their relative contributions, one can get information about the roles played by the short-range and the long-range interactions.

With the above three terms specified, our total su(2) model Hamiltonian is just the summation,

$$\hat{H}(t) = \hat{H}(0) + \hat{V}^{\text{dia}} t + \hat{V}^{\text{off}} t. \tag{27}$$

With the above model Hamiltonian at hand, the level dynamical Eqs.(13-17) can be solved by the usual methods. Since for the Hamiltonian Eqs.(24-27) the matrix element of $\hat{H}(t)$ has an analytical expression, it is easier to solve the level dynamics by a direct diagonalization of $\hat{H}(t)$. We have solved the problem for a wide range of the parameter set ($\alpha_n$, $\beta_1$, $\beta_2$, $k_1$, $k_2$,) and keeping $t$ as a running parameter. The results are shown from Fig.1 to Fig.15, where the histograms or solid lines are the results of the calculations, while the dotted lines are the theoretical predictions.

(i) Two limits. The first limit is the case of no perturbation ($t = 0$). The level density $\rho(E)$ from $\hat{H}(0)$ is a parabola approximating the semi-circle (Fig.1). After unfolding, the level spectrum is nearly a harmonic oscillator spectrum. Therefore the nearest level spacing distribution $P(s)$ is nearly a delta function (Fig.2), the $\Delta_3$ is very small and remains constant. The other limit is reached as $t \longrightarrow \infty$. In this case, $P(s)$ and $\Delta_3$ approach Poisson distribution as shown by Figs.3 and 4. However there is no simple good quantum number except the Casimir operator in this limit in contrast to what the Poisson distribution should have. Conceivably, in between there must exist a chaotic region.

(ii) Chaotic region. For the set of parameters $j = 500$, $\beta_1 = 0.5$, $\beta_2 = -1.0$, $k_1/j = 8/500$, $k_2/j = 28/500$, $\alpha_1 = 1.0$, $\alpha_i = 0$ $(i \neq 1)$, and $t = 2.3$, $\hat{H}(t)$ produces a very nice GOE energy spectrum as $P(s)$ and $\Delta_3$ indicate in Figs.5 and 6. The amplitude distribution of the eigenfunctions of $\hat{H}(t)$ is shown in Fig.7. There is a mountain-like distribution. The number of fluctuating peaks is related to the number of avoided level-crossings. In the central region the number of avoided-crossings is about 6, and the spreading width is about 20. Off the central region, because of the finite boundary effect, the spreading width is getting narrower. The localization of the wave function is related to the tridiagonal (band) structure of the Hamiltonian $\hat{H}(t)$. The avoided level-crossings are shown in Fig.8. In the small $t$ region, the level distribution is similar to the harmonic oscillator; therefore, $P(s)$ follows a $\delta$-function. As $t$ becomes larger, the effective charges $V_{nm}(t)$ become very weak, since the overlap of the states $|n(t)>$ and $|m(t)>$ is very small. Therefore, the avoided level-crossings for larger $t$ become sharp like exact crossings and $P(s)$ approaches a Poisson distribution again. The chaotic GOE distribution occurs in the transition region ($t \approx 2$) where the avoided level-crossings are strong.

(iii) No velocity, no chaos. If in the above parameter set we let $\beta_i = 0$, $P(s)$ and $\Delta_3$ follow the same law as the first limit in case (i), as shown in Fig.9 . Thus, no velocity, no short-range collisions, no chaos.

(iv) No symmetry-breaking coupling, no chaos. In the parameter set of the case (ii), set $\alpha_1 = 0$, $P(s)$ and $\Delta_3$ follow the Poisson law as shown in Figs.10 and 11.

(v) The effect of long-range interactions. To study this effect, we make changes in the parameters of the case (ii). Firstly, we consider two cases: (1) set $\alpha_1 = 1.2$; (2) set $\alpha_1 = 1.0, \alpha_2 = 0.2$. We find that $P(s)$ and $\Delta_3$ have larger changes in case (1) than in case (2) (Figs.12 and 13). This means that as the interaction range increases, the contribution to the fluctuation decreases. Now let us consider the medium-range and the long-range interactions. We set $\alpha_1 = 1.0$ and $\alpha_{10} = 0.2$ for the medium-range case, $\alpha_1 = 1.0$ and $\alpha_{25} = 0.2$ for the long-range case. In these cases, $P(s)$ and $\Delta_3$ change slightly as shown in Figs.12 and 12. This is because the long-range interactions are mainly responsible for large scale deformation of the energy spectrum. After unfolding, its mean field effect has been removed, and its residual interaction leaves only a very small effect.

(vi) Dimensional stability in the Hilbert subspace. We have compared the results in the Hilbert spaces with $j = 200$, 500, 1000. The results are almost unchanged (Figs.14 and 15).

## IV. CONCLUSION AND DISCUSSION

In this paper, we have reformulated the Dyson-Pechukas level dynamics so that it is made suitable for studying avoided level-crossings and the transition to chaos. The $N$-level problem has been converted into a many-body problem, and the knowledge and concepts of many-body physics have been used to study the level dynamics. The perturbation is split into a "velocity" part, which is responsible for level crossings, and an interaction part, which produces level "collisions". It is shown that the level local fluctuation is generated by a large number of avoided level-crossings. The role played by avoided level-crossings in generating chaoticity in level dynamics is similar to the role played by particle-particle collisions in causing thermalization in many-body physics. In order to make the level distribution chaotic, one needs a large number of strong avoided level-crossings, which in turn requires a velocity field to produce. The chaotic level distribution is produced by a large number of successive avoided level-crossings. This requires that the average number of collisions, $\Delta m$, and thus according to Eq.(22), the ratio of the average perturbation to the average level spacing should be large, which is also noticed in [12]. From the analysis of level dynamics, one knows the conditions for level chaoticity: a velocity field to produce many level crossings, a sufficiently large symmetry-breaking perturbation to cause successive avoided level-crossings, and the irregularity of both.

Using the level dynamics as a guide, we constructed an su(2) model Hamiltonian which displayed the basic ideas of level dynamics and confirmed our conjecture about the conditions for level chaoticity. Furthermore, since the su(2) model is of one dimension, the existence of an su(2) Hamiltonian with the GOE level statistics built in has further implications. Since a one-dimensional Hamiltonian is classically integrable, the chaotic su(2) model makes it necessary to reconsider the relation between chaoticity and non-integrability in quantum case[13].

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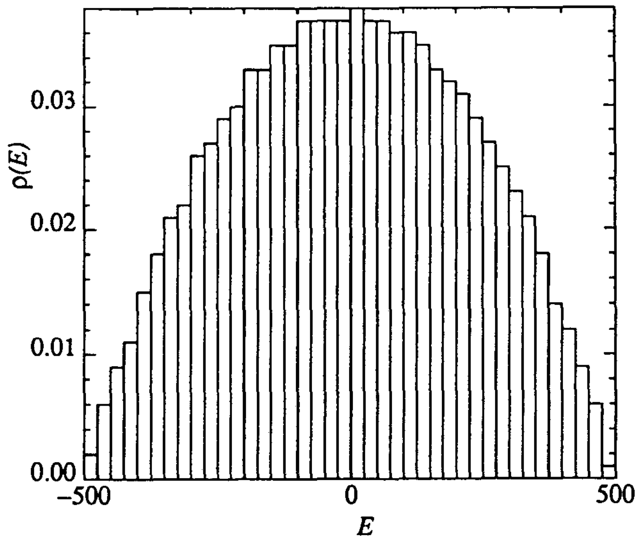


FIG. 1. Level density of the unperturbed Hamiltonian $\hat{H}(0)$ given by Eq.(24), with $j = 500$.

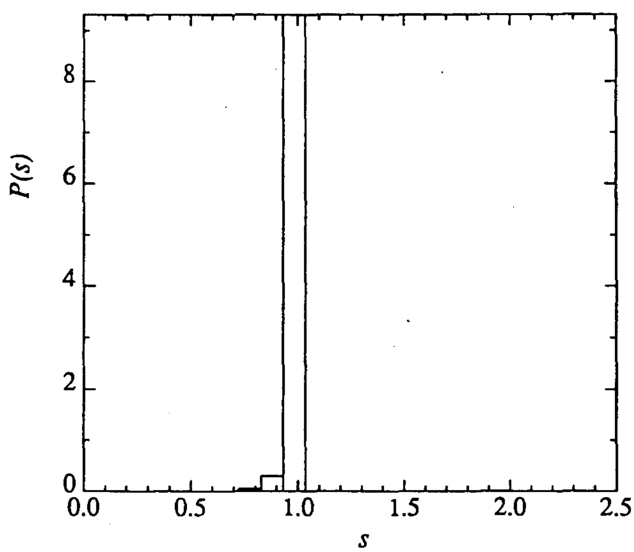


FIG. 2. The nearest level spacing distribution of the unperturbed Hamiltonian $\hat{H}(0)$, which is close to a delta-function centered at $s = 1$. This is the typical feature of the spectrum of one-dimensional Harmonic oscillator.

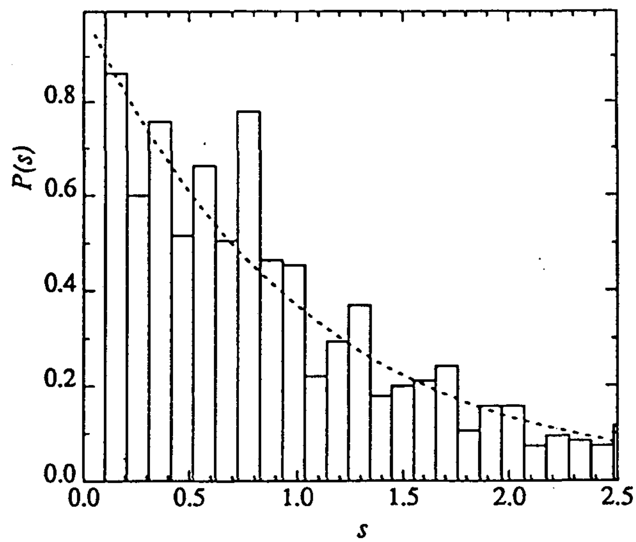


FIG. 3. The nearest level spacing distribution of the perturbation $\hat{V}t$, given by Eqs.(25,26) with $j = 500$, $\beta_1 = 0.5$, $\beta_2 = -1.0$, $k_1/j = 8/500$, $k_2/j = 28/500$, $\alpha_1 = 1.0$, $\alpha_i = 0$ $(i > 1)$. The histogram is the calculated data, and the dotted line is the Poisson distribution.

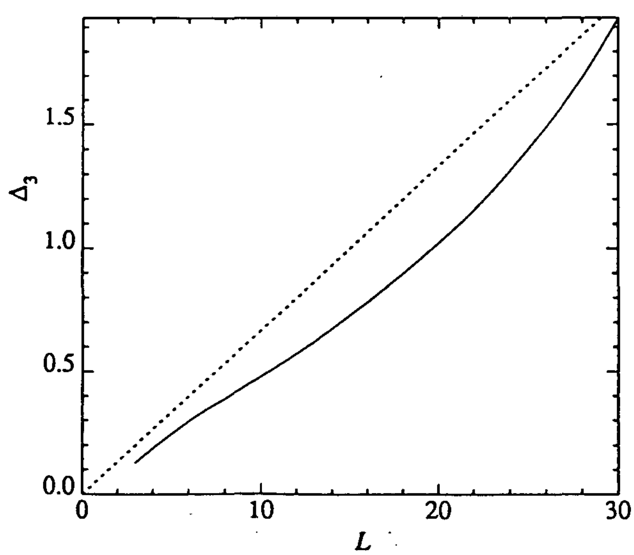


FIG. 4. The $\Delta_3$-statistics of $\hat{V}t$ with the same parameters as in Fig.3. The solid line represents the calculated result and the dotted line corresponds to Poisson distribution.

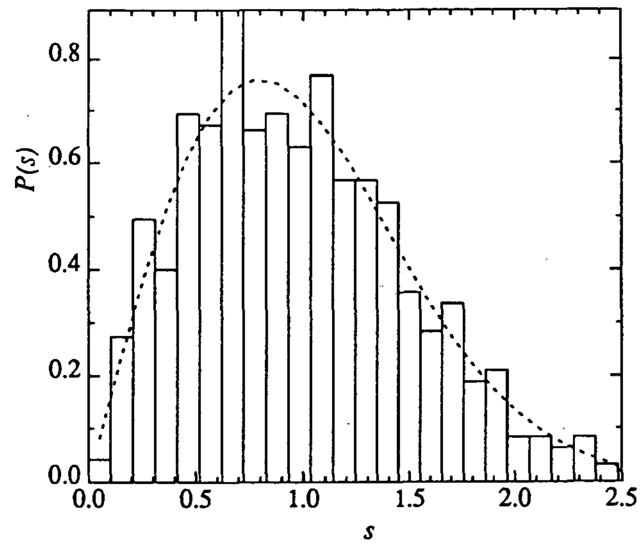


FIG. 5. The nearest level spacing distribution of the perturbed Hamiltonian $\hat{H}(t)$ given by Eq.(27) with $j = 500$, $\beta_1 = 0.5$, $\beta_2 = -1.0$, $k_1/j = 8/500$, $k_2/j = 28/500$, $\alpha_1 = 1.0$, $\alpha_i = 0$ $(i > 1)$, and $t = 2.3$. The dotted line shows the Wigner distribution.

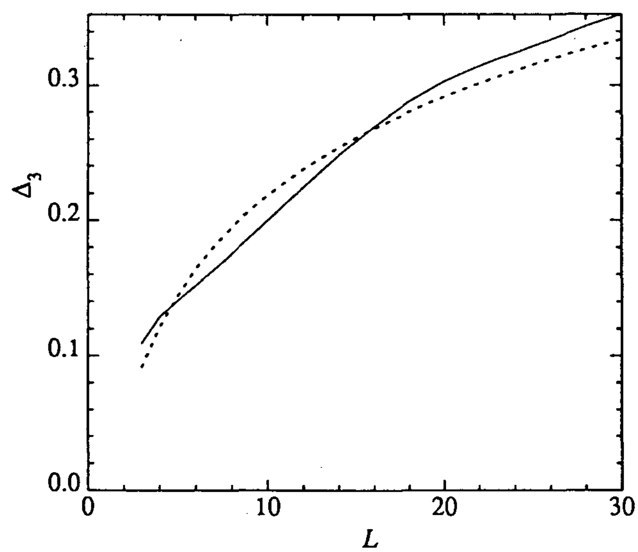


FIG. 6. The $\Delta_3$-statistics of the perturbed Hamiltonian $\hat{H}(t)$ with the same parameters as in Fig.5. The dotted line shows the case of GOE.

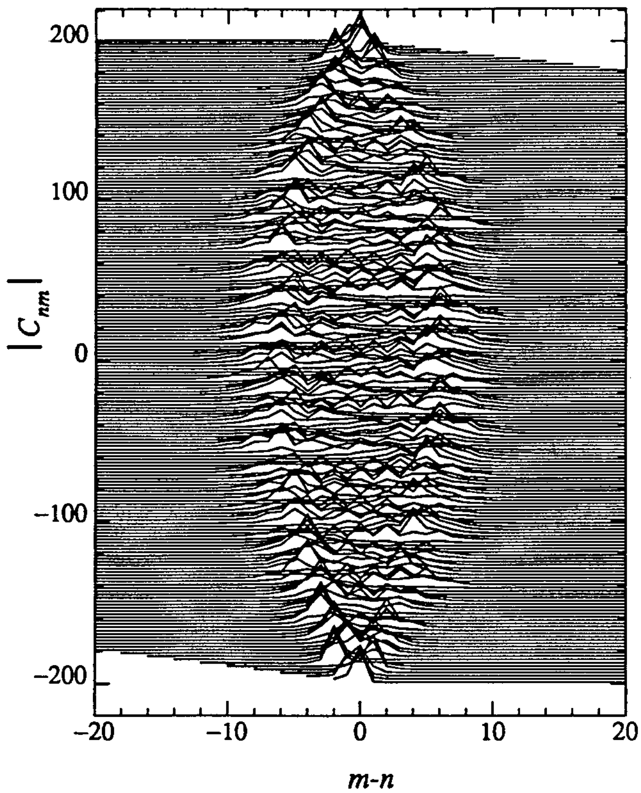


FIG. 7. The amplitude distribution of the eigenfunctions of $\hat{H}(t)$ with the same parameters as in Fig.5, except that $j$ is reduced to 200 for a faster calculation.

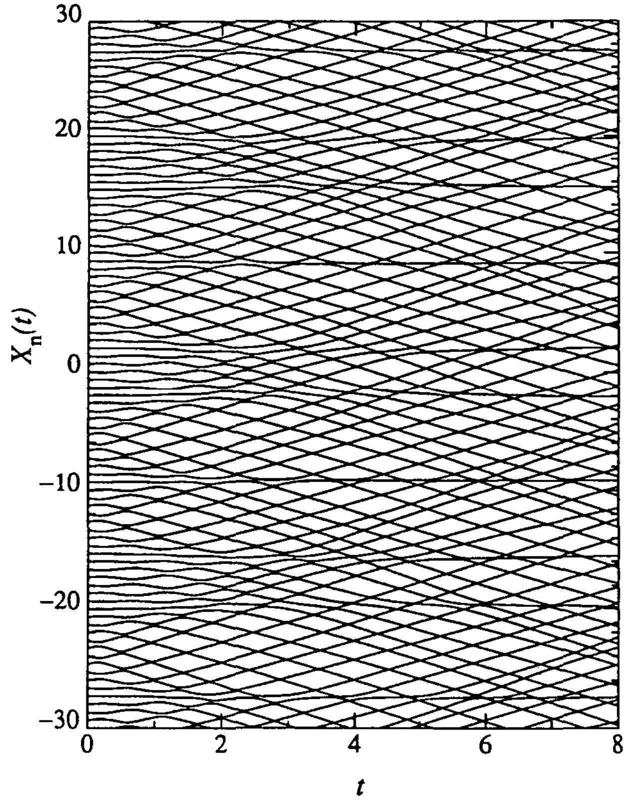


FIG. 8. The avoided level-crossing of $\hat{H}(t)$ with the same parameters as in Fig.7 except $t$ as a running parameter. Only 60 Levels are plotted.

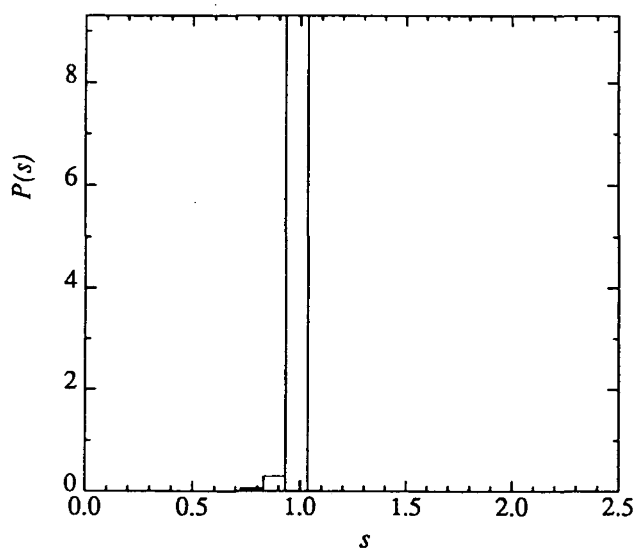


FIG. 9. The nearest level spacing distribution of the $\hat{H}(t)$ with $\beta_i = 0$ and other parameters are the same as in Fig.5.

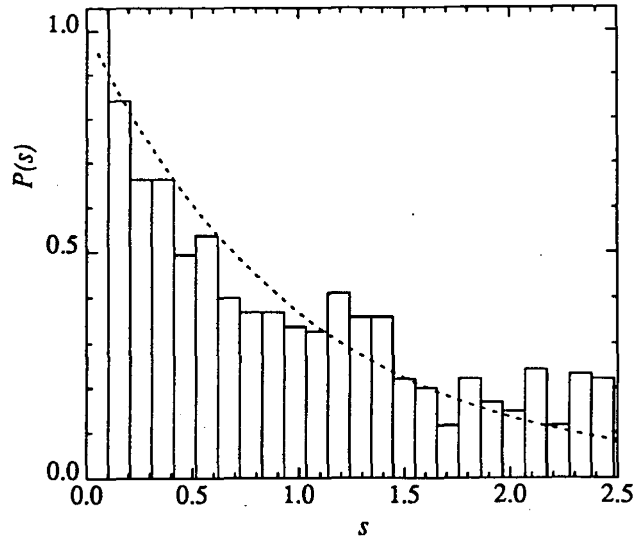


FIG. 10. The nearest level spacing distribution of the $\hat{H}(t)$ with $\alpha_i = 0$ and other parameters are the same as in Fig.5.

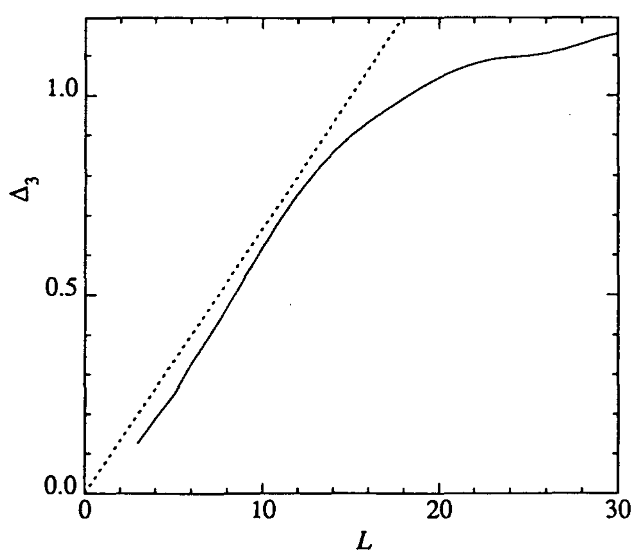


FIG. 11. The $\Delta_3$-statistics of $\hat{H}(t)$ with the same parameters as in Fig.10.

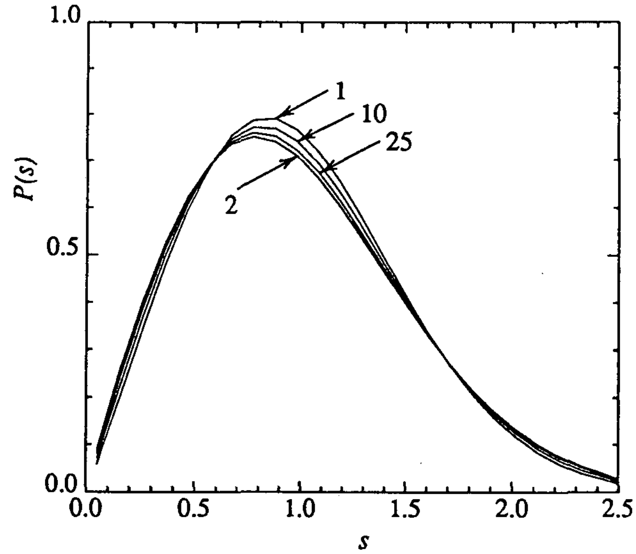


FIG. 12. The effect of the interactions with different ranges on the nearest level spacing distribution. The parameters of $\hat{H}(t)$ are the same as in Fig.5, except that curve 1: $\alpha_1 = 1.2$; curve 2: $\alpha_1 = 1.0$, $\alpha_2 = 0.2$; curve 10: $\alpha_1 = 1.0$, $\alpha_{10} = 0.2$; curve 25: $\alpha_1 = 1.0$, $\alpha_{25} = 0.2$.

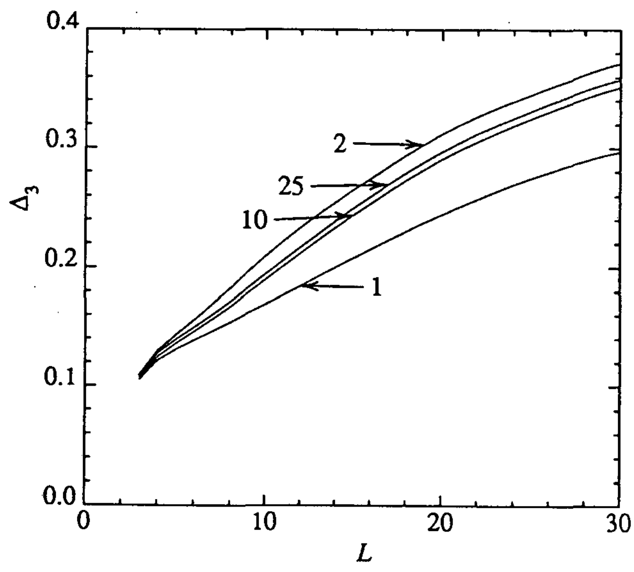


FIG. 13. The effect of the interactions with different ranges on the $\Delta_3$ statistics. The parameters are the same as in Fig.12.

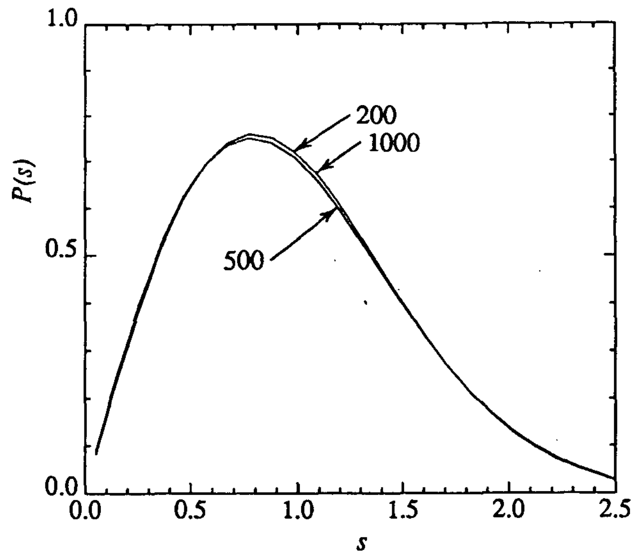


FIG. 14. The effect of the dimension of the Hilbert subspace on the nearest level spacing distribution. The parameters of $\hat{H}(t)$ are the same as in Fig.5, except that curve 200: $j = 200$; curve 500: $j = 500$; curve 1000: $j = 1000$. Only few cases are plotted here, the results with other $j$ have similar results.

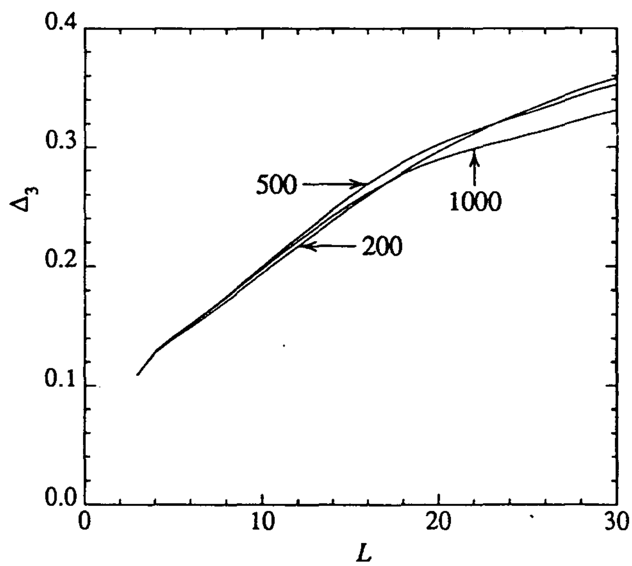


FIG. 15. The effect of the dimension of the Hilbert subspace on the $\Delta_3$ statistics. The parameters are the same as in Fig.14.

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