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### Publication Date

1993

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**CALIFORNIA PATH PROGRAM  
INSTITUTE OF TRANSPORTATION STUDIES  
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## **Longitudinal and Lateral Throughput on an Idealized Highway**

**Randolph W. Hall**

**UCB-ITS-PWP-93-15**

This work was performed as part of the California PATH Program of the University of California, in cooperation with the State of California Business, Transportation, and Housing Agency, Department of Transportation; and the United States Department of Transportation, Federal Highway Administration.

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OCTOBER 1993

ISSN 1055-1417

# **LONGITUDINAL AND LATERAL THROUGHPUT ON AN IDEALIZED HIGHWAY**

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## **ABSTRACT**

Highway automation has recently enjoyed renewed research interest as a method for solving highway congestion problems. To determine the feasibility of automated highways, it will be essential to estimate the automated highway benefits. A critical task in the analysis of automated highway benefits is to estimate the potential increase in highway capacity. This paper uses deterministic approximations to model highway throughput, accounting for both longitudinal requirements (i.e., lane flow) and lateral requirements (i.e., lane changing). The model also accounts for trip length distributions, and the effect of these distributions on the lane "flux" (i.e., the rate at which vehicles move between adjacent lanes, per unit length of highway). Based on these representations, the model identifies conditions under which lane changes have an appreciable effect on capacity, assuming certain idealized conditions are met. For typical highway parameters (mean trip length of 20 km, and mean speed of **30 m/s**), and an increased nominal capacity (i.e., not factoring in lane changes) of 7200 vehicles per hour, the incremental effect of lane changes only appears to be significant (i.e., cause a capacity reduction in excess of 20%) when the time-space requirement exceeds 1000 m-s. As an example, this requirement could imply a lane change maneuver that lasts 10 seconds and requires 100 m of lane-space during execution. This calculation does not factor in queueing effects. To ensure that all or most vehicles succeed in exiting at their desired locations, the actual capacity will have to be decreased somewhat further. The additional reduction is a subject of ongoing research.

## INTRODUCTION

Over the last two decades, the United States has experienced continued growth in urban and suburban highway traffic. Over the same period, there has been a marked slowdown in the construction of new highways and the expansion of old highways. As a result, highway congestion, and the associated loss in productivity, have grown.

One of the reasons for reduced highway construction has been the lack of suitable right-of-way along travel corridors. Within larger metropolitan areas, much of the potentially suitable land has been consumed for other purposes. In addition, highway externalities, especially noise and pollution, have made it difficult to construct new highways in all but the least developed areas. In response to this situation, the Federal Highway Administration (FHWA) and various state transportation agencies have embarked on a research program to increase the efficiency of existing facilities through the use of electronics, control and communication technologies (now called IVHS, intelligent vehicle highway systems).

A major component of the federal and State of California IVHS effort is the program to research and develop automated-highway-systems (AHSs). As presently conceived, an AHS would possess substantially higher lane capacity than conventional highways. As a result, existing highways might be modified to accommodate more traffic, or new high-capacity highways might be constructed in locations where they might otherwise be infeasible, due to construction costs or limited right-of-way availability.

One of the issues in **AHS** research is estimating the capacity increase. Resolving this question is crucial to ascertaining whether AHSs are a cost-effective investment. The usual approach for capacity analysis on conventional highways is highly empirical, and dependent on field measurements of either vehicle following behavior or traffic flows. However, given the current stage development, where only a few test vehicles have been prototyped, such an approach would be impossible for *AHSs*. Instead, capacity estimates must rely on models of theoretical vehicle behavior. While such models may be straight-forward to derive for a single lane of traffic, the effects of lane changes on capacity are not obvious. Even under the best control scenarios, each lane change would likely result in a capacity decrement. The capacity loss for each lane change, combined with the rate at which lane changes occur, strongly influence AHS capacity.

The objective of this paper is to develop a throughput model of a multiple-lane AHS with lane changes. The model is designed to account for trip-length distributions, and the effect of these distributions on the rate of lane changes between each pair of adjacent lanes. To illustrate fundamental principles, the model will be applied to an idealized highway operating under stationary conditions, both in time and space (concepts are easily extendible to non-stationary conditions). Parametric analysis will be used to study the effects of design parameters, pertaining to the execution of lane change maneuvers, on capacity. The primary parameter of interest is the time-space requirement for a lane change. If an **AHS** can be designed that requires no additional spacing during a lane change, then the theory presented in this paper would predict no

reduction in capacity. The throughput model is deterministic, meaning that transient and queueing conditions are not modeled. Queueing based representations are the subject of ongoing research.

The remainder of the paper is divided into five sections. First the literature on AHS capacity analysis is reviewed. Next, a model is developed to represent "workloads" on an AHS, accounting for both lateral (i.e., lane change) and longitudinal movement. In the third section, **trip** length distributions are converted into lane change rates. This is followed by a parametric analysis of highway capacity, and finally by conclusions.

## LITERATURE REVIEW

The earliest literature on automated highways dates from the 1950s and 1960s, when engineering aspects were studied at General Motors, General Electric and Ohio State University (Barwell, 1983; Bidwell, 1965; Black, 1975; Fenton, 1977; General Electric, 1968; Spreitzer, 1990). Example operating concepts can be found in Barton-Aschman (1968). These include: (1) dual-mode systems, where vehicles can either ride on tracks or ordinary roadways; (2) personal-rapid-transit, where standardized vehicles only travel on specialized roadways, (3) palletized systems, where ordinary vehicles are loaded onto trains, conveyors or pallet cars, and (4) full automated roadways, where automation is performed on ordinary pavement.

The current interest in automated roadways centers on the last concept, as does this paper. Within this category, Tsao, Hall and Shladover (1993) identify six alternative operating scenarios, along with four dimensions by which operating scenarios can be defined (separation of traffic, transitions between manual and automated driving, normal automated driving, and failure and emergency response). The alternative scenarios are defined along three dimensions: (1) separation of automated traffic from manual traffic, (2) separation between automated lanes, and (3) vehicle following rule. All of these factors have some impact on capacity.

The earliest systematic study of automated highway capacity appears to be the paper by Rumsey and Powner (1974), which examined a moving-cell operating concept. Recently, however, the interest in automated highways has focused more on a platooning concept introduced by Shladover (1979), in which vehicle separation is either very short (within platoon) or very large (between platoons). The advantage of this concept is that vehicle failures that cause rapid decelerations only result in low initial delta-velocity collisions. The drawback of platooning is that vehicle control rules need to be somewhat more complex, and that platooning can result in more frequent collisions, relative to operating strategies in which vehicles travel individually (Tsao and Hall, 1993). Shladover's work included capacity estimates as a function of various safety criteria, but did not calculate the effect of lane changing on capacity.

The capacity of automated highways with platooning and lane changing has recently been investigated by Rao et al (1993), Rao and Varaiya (1993) and Tsao et al (1993). All

of these papers utilize the SmartPath simulator developed by Eskafi and Varaiya (1992). SmartPath is highly microscopic, and models the system down to the level of exchange of messages between vehicles. However, as a consequence, computation time is relatively large, so it is difficult to simulate highways under a wide range of conditions. Tsao et al (1993) also includes stochastic/analytical models to represent the time to execute a lane change maneuver. By contrast, the deterministic models developed in this paper are somewhat simpler and, as a result, more clearly identify the conditions under which automated highways approach capacity.

## WORKLOAD/THROUGHPUT MODEL

This section is organized as follows. First, a traffic flow model is developed that relates lane flow to "lane flux" (i.e., rate at which traffic enters or leaves the lane, per unit distance). Next, the space occupied by an individual vehicle is modeled, both for a purely longitudinal mode and for a lane-change mode. These two models are then combined into a "workload model, which represents the space requirement within a lane as a function of the flow and flux. In the following section, the "workload" model will form the objective function for an optimization model, which seeks to maximize throughput through control of lane change behavior.

### Model for Lane Flow and Lane Flux

As illustrated in Figure 1, the AHS is divided into a finite number of lanes of infinite length. Each lane carries a constant, non-stationary (in time and space), flow at constant speed in the longitudinal direction. Traffic enters and leaves each lane from adjacent lanes with constant, non-stationary, "flux", with flux measured in vehicles/time per unit length of highway. To maintain stationarity, the flux between any pair of adjacent lanes must be identical in opposite directions. Without loss in generality, traffic is assumed to travel on the right-hand side of the highway, with lanes numbered in ascending order from right to left. Access and egress to the **AHS** are assumed to occur only through the right-hand lane (as a result, the model does not represent left lane exits at freeway-freeway interchanges). Exact entrance and exit locations are not modeled. Instead, access and egress are assumed to occur continuously over the entire length of the highway.

Upon entering the AHS, each vehicle is assigned to a lane. Vehicles immediately attempt to access their assigned lanes through lane-change maneuvers, beginning from the right-hand lane and progressing to the assigned lane, without backtracking. The process is reversed on approaching the destination. Consequently, lane flow has two components: (1) "**permanent**" flow represents flow of vehicles within their assigned lanes, and (2) "**temporary**" flow represents flow of vehicles in other than their assigned lanes, as they are moving into or out from their assigned lanes. (In some cases, it may be preferable to dynamically assign lanes based on prevailing conditions; when stochastic variations are large, this may improve performance.)

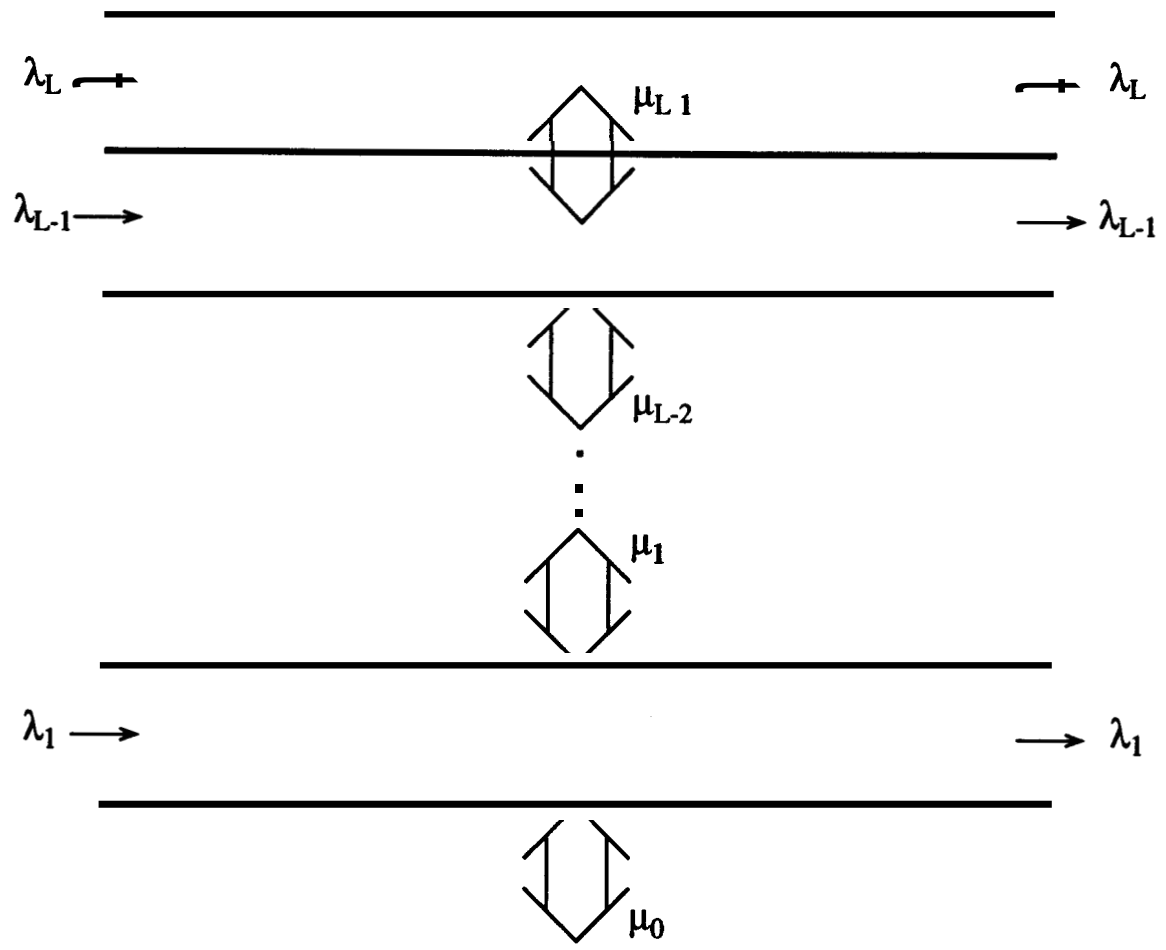


Figure 1. Idealized Highway Segment.



Let:

- $L$  = number of lanes in the **AHS**
- $\lambda$  = longitudinal flow across all lanes (vehicles/time)
- $\lambda_i$  = longitudinal flow of all vehicles assigned to lane  $i$  (vehicles/time)
- $\lambda'_i$  = temporary longitudinal flow adjustment in lane  $i$  (vehicles/time)
- $\mu_i$  = lateral flux from lane  $i$  to lane  $i+1$  (vehicles/time-distance, one directional)
- $\mu_0$  = flux entering/exiting the **AHS** (vehicle/time-distance, one directional)
- $\eta$  = average length of all trips
- $\eta_i$  = average length of trips assigned to lane  $i$

The temporary flow adjustment can be either positive or negative, and represents the net difference from what the longitudinal flow would be in the absence of temporary flow (i.e., lane changes take zero time). Specifically,  $\lambda'_i$  equals the temporary flow in lane  $i$  of vehicles assigned to lanes  $i+1, i+2, \dots$ , minus the temporary flow of vehicles assigned to lane  $i$  that occurs in lanes  $i-1, i-2, \dots$ . The adjustment is in all cases non-negative in the right-hand lane, and non-positive in the left-hand lane. Further, the sum of  $\lambda'_i$  across lanes must equal zero (i.e., longitudinal flow is conserved). Note that by definition,  $\mu_0$  is a measure of total AHS demand. Further,  $\mu_L = 0$  because there are no lane changes to the left of the left-most lane.

Little's formula defines the relationship between flow, flux and average trip length. Within the permanent flow class of vehicles in a lane  $i$ , the "arrival rate" equals  $(\mu_{i-1} - \mu_i)$ , and the mean trip length equals  $\eta_i$ . The product of these quantities defines the flow. A similar relationship applies to the highway as a whole

$$\lambda_i = \eta_i (\mu_{i-1} - \mu_i) \quad (1a)$$

$$\lambda = \eta \mu_0 = \sum_{i=1}^L \lambda_i = \sum_{i=1}^L \eta_i (\mu_{i-1} - \mu_i) \quad (1b)$$

### Space Occupied per Vehicle

The "workload" is defined as the total longitudinal space requirement within a lane (measured per length of highway), accounting for vehicles engaged in purely longitudinal movement, as well as vehicles engaged in lateral movement. Assume that vehicles travel in platoons of one or more vehicles, and that intra-platoon spacing is allowed to be less than inter-platoon spacing. Let:

- $v$  = vehicle velocity (length/time)
- $N$  = average number of vehicles in platoon
- $x_1$  = intra-platoon spacing (length of one vehicle and gap)
- $x_2$  = inter-platoon spacing (length of one vehicle and gap).

Then, under capacity conditions, the space required per vehicle engaged purely in longitudinal movement equals:

$$\begin{aligned} s_l &= \text{space per longitudinal vehicle (length)} \\ &= [(N-1)x_1 + x_2]/N \end{aligned} \quad (2)$$

The equation also applies to scenarios that do not employ platooning, by setting N to 1.

The following parameters apply to lane change maneuvers:

$$\begin{aligned} \tau &= \text{lane change duration (time)} \\ s' &= \text{time-averaged incremental space requirement, per } \textit{entering} \text{ lane change (length)} \\ s^\circ &= \text{time-averaged incremental space requirement, per } \textit{exiting} \text{ lane change (length)} \\ v &= \text{vehicle velocity (length/time)}. \end{aligned}$$

Accounting for both lane change duration, and incremental space, the following "occupancy" measure is defined:

$$\begin{aligned} o &= \text{space occupied per unit flux (time-length)} \\ &= \tau(s' + s^\circ) \end{aligned} \quad (3)$$

The precise value of  $o$  is an important research subject. In highly optimistic scenarios, under which the only incremental space requirement comes from simultaneously occupying two lanes,  $s'$  can be approximately  $s_l$ ,  $s^\circ$  can be approximately zero, and  $\tau$  can be approximately 5 seconds. Under highly pessimistic scenarios,  $s'$  and  $s^\circ$  can both be on the order of  $2x2$ , and  $\tau$  can be on the order of 30 seconds. Under high capacity scenarios,  $s_l$  has been estimated to be as small as 10-15 meters. If inter-platoon spacing is based on a "rick-wall" stopping standard, and vehicles decelerate at about 5 m/s, then  $x_2$  is on the order of 90m. This places  $o$  in the range between 50 seconds-meters and 5000 seconds-meters. Precisely resolving this factor-of-100 range is crucial to determining the ultimate capacity of an AHS, as will be demonstrated later. Moreover, calculation of  $s'$  and  $s^\circ$  is not straight-forward if platoons are allowed to change lane as a unit.

### Workload Model

Combining lateral and longitudinal movements, the total workload is defined as follows:

$$\begin{aligned} W_i &= \text{workload in lane } i \text{ (dimensionless)} \\ &= [(\lambda_i + \lambda'_i)/v]s_l + (\mu_{i-1} + \mu_i)o \end{aligned} \quad (4)$$

To simplify Eq. 4 the following dimensionless parameters are introduced:

- $p_i$  = proportion of total flow that is permanent and *occurs* in lane  $i$   
 =  $\lambda_i/\lambda$   
 $p'_i$  = proportional temporary flow adjustment for lane  $i$   
 =  $\lambda'_i/\lambda$   
 $r_i$  = ratio of average trip length in lane  $i$  to the average across all lanes  
 =  $\eta_i/\eta$

The fluxes in and out of lane  $i$  can be represented in terms of  $p_i$  and  $r_i$ , based on Little's formula. Substituting appropriate parameters results in the following:

$$W_i = \lambda[(p_i + p'_i)/v]s_i + (2\sum_{j=1}^L p_j/r_j - p_i/r_i)(o/\eta) \quad (5a)$$

$$= (\lambda s_i / v)[p_i + p'_i + (2\sum_{j=1}^L p_j/r_j - p_i/r_i)(o v / \eta s_i)] \quad (5b)$$

Introducing the following dimensionless parameters further simplifies the expression

- $a = \lambda s_i / v$  = longitudinal space requirement across all lanes  
 $\beta = o v / \eta s_i$  = ratio of incremental lateral requirement to longitudinal requirement, per unit flow

Then:

$$W_i = a[p_i + p'_i + \beta(2\sum_{j=1}^L p_j/r_j - p_i/r_i)] \quad (6)$$

### Derivation of $p'_i$

The parameter  $p'_i$  represents waiting time to execute lane maneuvers,. Under uncongested conditions, these values should be small. As congestion increases (i.e.,  $W_i$  approach one) queueing for lane changes will become significant. This inherently adds to the workload in the right-hand lanes (and reduce workload in left-hand lanes), because vehicles will have to spend a greater proportion of time in a "lane-change mode" and a small portion of time in their assigned lane. Nevertheless, the exact values of  $p'_i$  may have relatively little effect on the  $W_i$ , for the extra time spent in the right-hand lanes only amounts to an exchange of work between lanes, and not the creation of additional work (as would be the case if additional lane changes were needed).

Because queueing enters into the determination of  $p'_i$ , it is impossible to ascertain its exact value within the context of a workload model. Instead, a simplified representation will be used, such that the temporary longitudinal flow adjustment in

lane  $i$  is a linear function of the flux passing through lane  $i$  to lanes  $i+1, i+2, \dots$ , minus the flux assigned to lane  $i$  that must first pass through lanes  $i-1, i-2, \dots$ :

$$p'_i = 2\gamma \left[ \sum_{j=i+1}^L p_j / r_j - (i-1)p_i / r_i \right] \quad (7)$$

where  $\gamma$  is the waiting time to execute a lane-change, expressed as a proportion of the average trip travel time. Under any realistic scenario, this parameter would not exceed .1; otherwise, the highway would be severely over-crowded.  $\gamma$  will be treated as an additional parameter in the model, though it is in fact dependent on the workloads. However, we hope to show that the optimal workloads are largely independent of  $p'_i$ , even if the optimal assignment of traffic to lanes is dependent on these values. Substitution in Eq. 6 leads to the following expression for  $W_i$  :

$$W'_i = \alpha \left[ p_i + (2\beta + 2\gamma) \sum_{j=i}^L p_j / r_j - (\beta + 2i\gamma) p_i / r_i \right] \quad (8)$$

The form of Eq. 8 is such that the workload is the sum of the longitudinal requirement and adjustment factor representing the incremental workload for lane-change maneuvers. This incremental factor depends on:

- (1) incremental space requirement during the maneuver
- (2) duration of the maneuver
- (3) average trip length
- (4) total highway flow

Because the workload model has been reduced to three parameters, plus the variables  $p_i$  and  $r_i$  (defining how traffic is distributed across lanes), Eq. 8 is easily amenable to parametric analysis. Furthermore, as will be seen in the following section, the lane specific variables can be derived from the trip length distribution for the overall highway, which will simplify the model further.

## TRAFFIC DISTRIBUTION ACROSS LANES

Within this section, a methodology is presented for optimizing  $p_i$  and  $r_i$  as a function of the trip length distribution and the parameters  $\beta$  and  $\gamma$ . The objective function will be of the maxi-min type, with respect to  $W_i$ . In essence, this means that the objective is to equalize the workload across lanes and, in the process, equalize the congestion across lanes. It might be argued that a somewhat smaller workload is appropriate in the right-hand lanes than in the left-hand lanes, given the priority to minimize queueing delays of vehicles waiting to make lane changes. At this stage in research, the necessity for such a buffer is unclear. In the future, it would not be difficult to add lane-specific buffers to the formulation.

The exact formulation of the mathematical program is as follows:

$$\min\{\max\{W_i\}\} \quad (9a)$$

$$\text{s.t.} \quad \Sigma p_i = 1 \quad (9b)$$

$$\Sigma p_i/r_i = 1 \quad (9c)$$

In addition to the specified constraints, the assignment of traffic to lanes must also be in accordance with the trip length distribution, to be discussed later. An equivalent formulation is to optimize the mini-max of the ratio of  $W_i$  to the constant  $\alpha$ :

$$\begin{aligned} W'_i &= \text{ratio of workload in lane } i, \text{ to average longitudinal space requirement across} \\ &\quad \text{all lanes in the absence of lane-changes.} \\ &= W_i/\alpha. \end{aligned}$$

Substitution of  $W'_i$  for  $W_i$  in the objective function reduces the number of parameters to two ( $\beta$  and  $\gamma$ ), combined with parameters that define the trip length distribution.

Furthermore, the mini-max objective can be removed by introducing a new variable  $W$ , which represents the maximum of the  $W'_i$  :

$$\min \quad W \quad (10a)$$

$$\text{s.t.} \quad W'_i \leq W \quad (10b)$$

$$\Sigma p_i = 1 \quad (10c)$$

$$\Sigma p_i/r_i = 1 \quad (10d)$$

### Optimization of $W$

Optimization of Eq. 10 amounts to performing an assignment of traffic to lanes on the basis of trip length. This optimization is facilitated through the following theorem:

**Theorem:** For any instance of Eq. 10, there exists an optimal solution satisfying the following property: all trips assigned to lane  $i$  have equal to or greater length than all trips assigned to lane  $i-1$ , for all  $i$  greater than or equal to two.

**Proof:** The theorem can be proved by construction. Suppose that there exists a solution where a lane  $i$  contains some shorter trips than some of the trips on lane  $i-1$ . Then it would be possible to reassign some portion of the traffic that violates the theorem's property, such that the total longitudinal flow is conserved in the two lanes, and the theorem is obeyed. The new solution would not alter the longitudinal component of  $W'_i$  in either lane. However, by increasing the mean trip length in lane  $i$ , the **flux** value for lane  $i-1$  would decline. All other flux values would remain identical, because the

combined flow on lanes  $i$  and  $i-1$  remains identical, and all other flows remain identical. Therefore, for any solution that does not satisfy the property, it is possible to construct a solution that is at least as good through a simple flow transfer, thus completing the proof.

The significance of the theorem is that the solution to the math program amounts to a partition of trips by length, which can be defined as follows. Let:

- $z_i$  = the maximum length of trips assigned to lane  $i$  (with  $z_0 = 0$ )
- $y$  = the length of a given trip

Then a trip is assigned to lane  $i$  if and only if  $y \in (z_{i-1}, z_i]$ . Furthermore, specifying the optimal values of  $z_i$  uniquely defines the optimal values of  $p_i$  and  $r_i$ . Let:

- $f(y)$  be the p.d.f. for trip length (based on a random sampling from  $\mu_0$ )
- $F(y)$  be the cumulative distribution function for trip length. (based on same sample)

Then:

$$r_i = \frac{z_i}{z_{i-1}} \int_{z_{i-1}}^{z_i} y f(y) dy / \eta \tag{11a}$$

$$p_i = r_i [F(z_i) - F(z_{i-1})] \tag{11b}$$

The weighting factor is needed in the expression for  $p_i$  to convert a proportion based on trips to a proportion based on lane flow (i.e., longer trips result in proportionately larger lane flow).

Figure 2 illustrates the concepts of Eq. 11, based on the notion that an expectation for a non-negative random variable can be determined from integrating the complement of its distribution function. Within the figure,  $r_i$  represents the ratio of the arrow shown within the shaded region to the arrow representing the entire distribution.  $p_i$  represents the ratio of the shaded regions area to the entire area above the distribution function. By changing the cut-off point,  $z_i$ , between lanes, these two variables can be altered to obtain varying values of workload.

The optimization procedure will be based on one more assumption, which can only affect optimality under extreme conditions. That is, the procedure will seek to equalize workload across the lanes. While it is possible that such a solution does not minimize  $W$ , or that no such solution exists, this should only occur when the  $\beta$  and/or  $\gamma$  are extremely large, making it undesirable to utilize all of the lanes.

Solution to the math program is now reduced to finding a set of dividing points  $z_i$ , such that the workloads assigned to each lane are identical. Furthermore, if the distribution function is strictly increasing, it can be shown that the mathematical program possesses

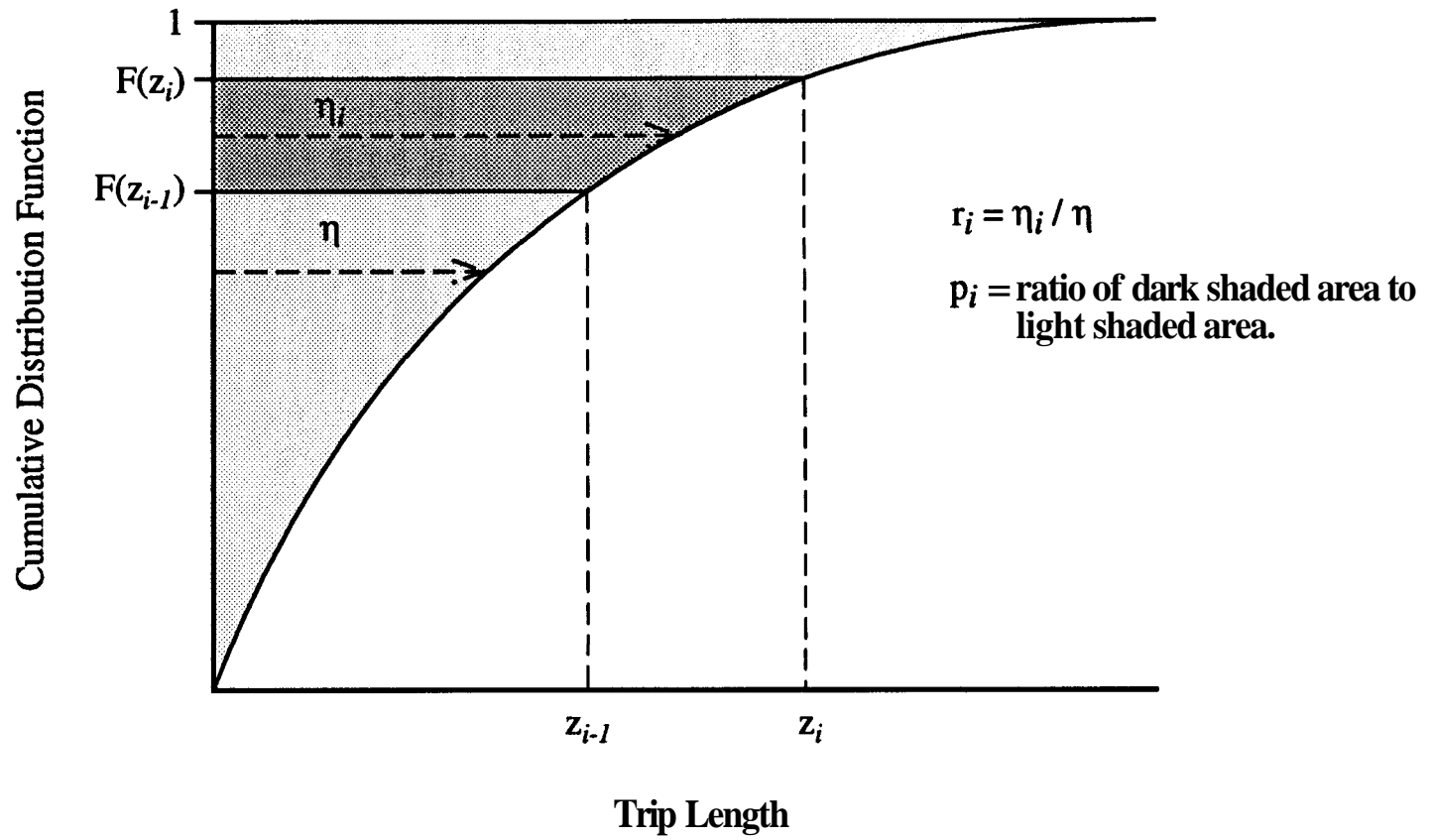


Figure 2. Graphical Representation of  $p_j$  and  $r_j$

a unique solution, by virtue of the fact that the workload function would also be strictly increasing for each lane. This reduces the problem to performing a series of one dimensional searches, performed sequentially on each of the L lanes, to equalize the workloads.

## PARAMETRIC ANALYSIS

In this section the optimization scheme coupled with parametric analysis, outlined above, is applied to sample trip length distributions. The deterministic and exponential distributions are used as upper and lower bounds on trip length "entropy". In the maximum entropy distribution (exponential), the variation in trip length is maximized among the lanes, resulting in a minimum number of lane changes for a given mean trip length (i.e., if left-hand lanes carry significantly longer trips, the number of lane changes is reduced significantly). In the minimum entropy distribution (deterministic), all trips are the same length. (In reality, highway trip lengths are much more similar to the exponential distribution than the deterministic distribution.)

The following provides expressions for  $r_i$  and  $p_i$  for the exponential distribution:

### Exponential Distribution

$$r_i = [e^{-z'i-1}(z'_{i-1} + 1) - e^{-z'i}(z'_i + 1)] / [e^{-z'i-1} - e^{-z'i}] \quad (12a)$$

$$p_i = [e^{-z'i-1}(z'_{i-1} + 1) - e^{-z'i}(z'_i + 1)] \quad (12b)$$

where:  $z'_i = z_i / \eta$

For the deterministic distribution  $r_i = 1$  for  $i=1,2,\dots,L$ . Furthermore, because the deterministic distribution can only produce a single value,  $p_i$  cannot be defined in terms of  $z_i$ . Instead, it is sufficient to simply assign a proportion of traffic to each lane. In either distribution, calculations can be performed independently of the distribution's mean (though the mean does enter into the equation for  $\beta$ ).

### Optimization Results

W was optimized for a set of problems defined by the following attributes:

- (1) Exponential and deterministic trip length distribution
- (2) Varying values of  $\beta$ , ranging from 0 upward.
- (3) Extreme values of  $\gamma$ , set to 0 or .05.
- (4) Varying number of lanes, with  $L = 1, 2, 3, 4,$  and  $5$ .

Instead of showing results in terms of  $W^*$ , its inverse was calculated. This represents the highway throughput, normalized relative to a highway where lane-changes do not decrease capacity (i.e., if  $\beta = 0$ , a 5-lane highway has a capacity of 5, a 4-lane highway



has a capacity of 4, etc.). This value is called "lane equivalence" in the figures. Results are plotted in Figures 3-6, as a function of  $\beta$  and L. The four figures represent the possible combinations of exponential/deterministic and  $\gamma$  (0 or .05).

The figures show that increases in  $\beta$  result in steadily decreasing capacity, with decreasing marginal effect. The figures also show that the curves for different values of L merge or approach at critical points. For instance, for  $\gamma = .05$  and the exponential distribution, a 5-lane highway and a 4-lane highway have the same capacity when  $\beta = .2$ . At this point, the lane-change requirement for moving vehicles all the way to lane 5 becomes so large that it is preferable not to use lane 5 at all. If  $\beta$  is very large, a 2-lane highway may have the same capacity as a 5-lane highway, again due to lane-change requirements.

The relationships for the two trip length distributions are similar, with the exception that the throughput is somewhat less for the deterministic case. This is due to the fact that trip lengths in the left-hand lanes are somewhat shorter for deterministic (it is impossible to assign longer than average trips to these lanes because all trips are the same length), which results in more lane changes. In addition, the throughput curves for different numbers of lanes only approach each other, rather than intersect.

Comparing the  $\gamma = .05$  to the  $\gamma = 0$  cases,  $\gamma$  appears to have a minor effect on throughput, as expected. However,  $\gamma$  does have a significant effect on the assignment of traffic to lanes (not shown), with much less traffic assigned to the right-hand lanes when  $\gamma = 0$ . Comparing the throughput figures, the most significant difference is that the throughput reaches a region of infeasibility much more quickly for the  $\gamma = .05$  cases. That is, for much smaller values of  $\beta$ , the results reach a point where zero traffic is assigned to lane 1. As a result, the curves for the different values of L do not continuously blend together.

For realistic values of  $\beta$ , of .1 or less, the effect of lane-changes is to reduce the throughput by 20% or less. Even  $\beta = .1$  may far exceed actual requirements. For a freeway where vehicles travel at 30 m/s, with average trip length of 20 km, and a nominal (non-adjusted) capacity of 7,200 vehicles/hour, this would imply a lane-change occupancy of 1000m-s.

Figure 7 illustrates how  $\beta$  affects the distribution of traffic across lanes. Taking the exponential distribution with  $\gamma = 0$  as an example, the figure shows, for lane 1, that both the lane flow percentage and trip percentage approach zero as  $\beta$  becomes large (the trip percentage always exceeds the flow percentage because lane 1 is always assigned the shortest trips).

# Normalized Throughput/ $\Gamma=0$ Deterministic Trip Length Distribution

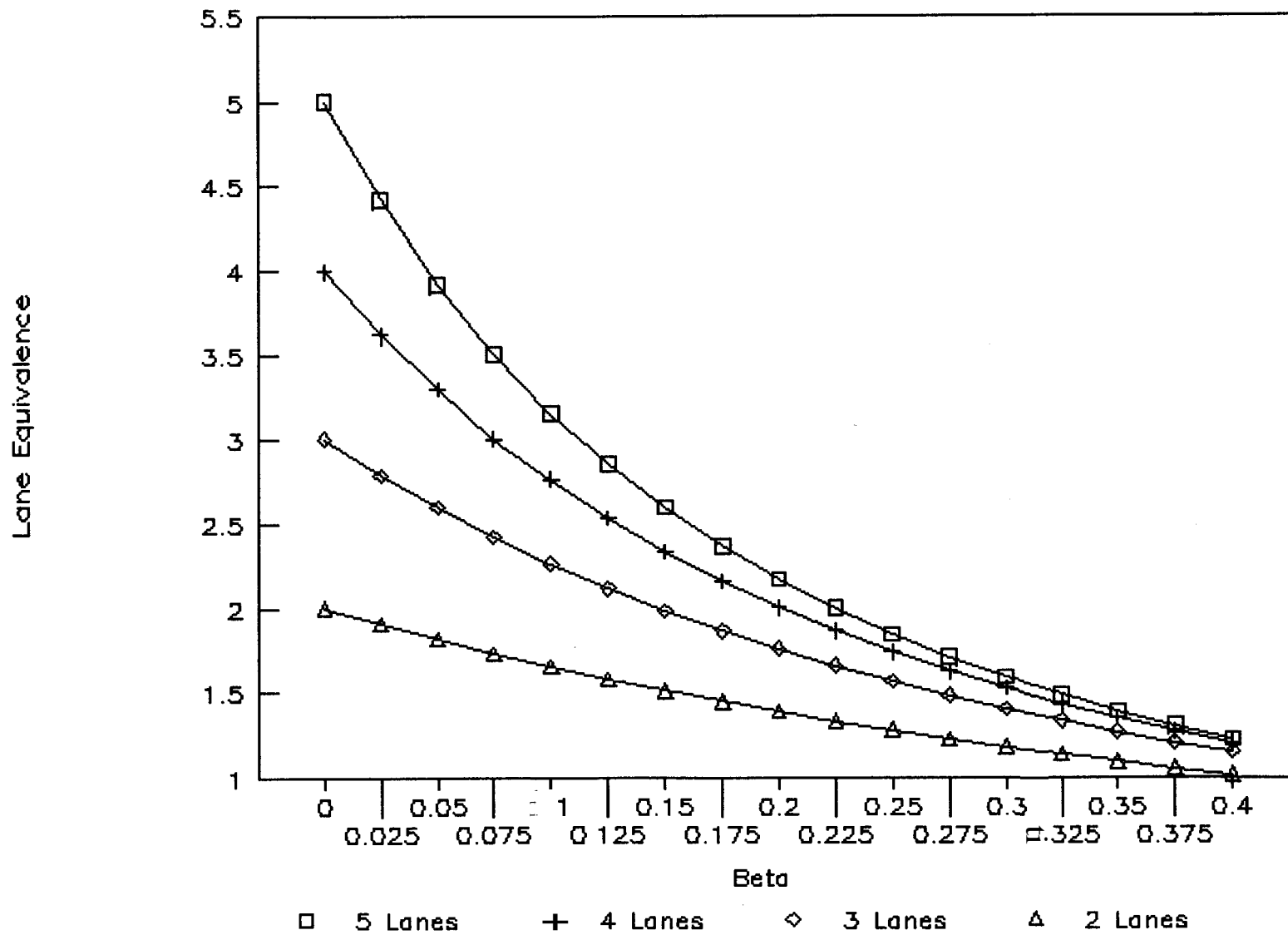


Figure 3. Lane Equivalence with  $\Gamma=0$  and Deterministic

# Normalized Throughput/ $\Gamma=0$ Exponential Trip Length Distribution

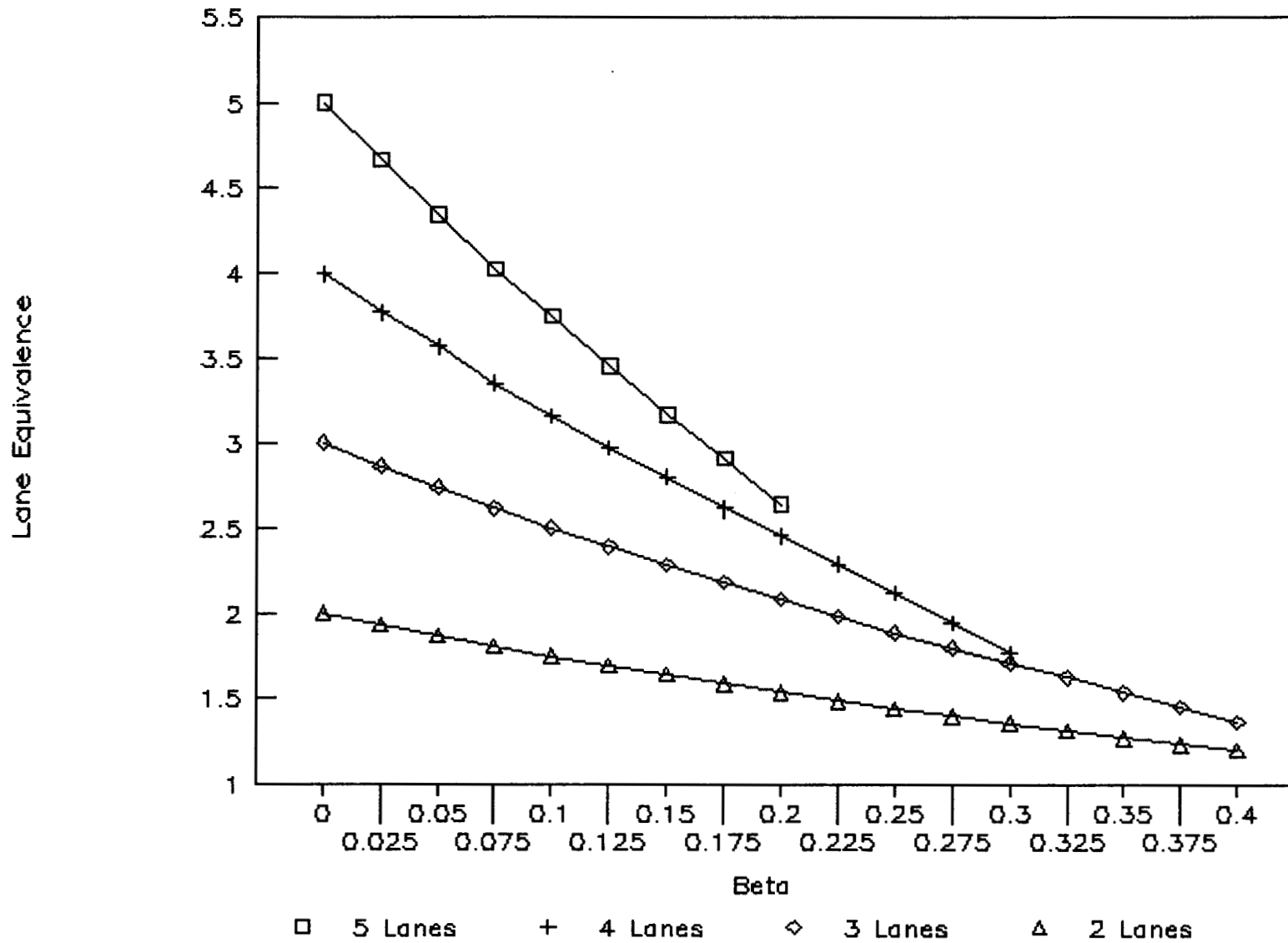


Figure 4. Lane Equivalence with  $\Gamma=0$  and Exponential

# Normalized Throughput/ $\Gamma=0.05$ Exponential Trip Length Distribution

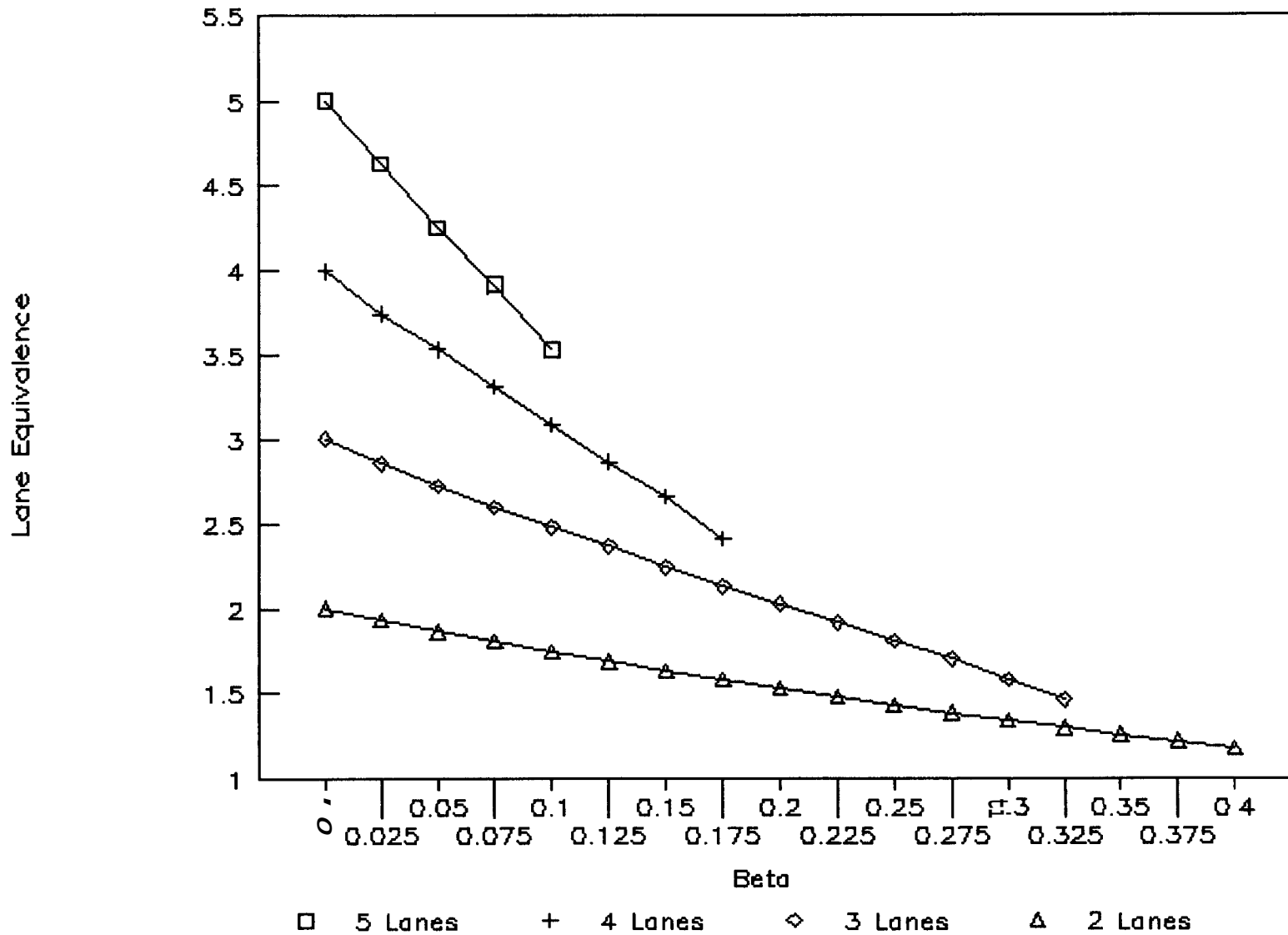


Figure 5. Lane Equivalence with  $\Gamma=0.05$  and Exponential

# Normalized Throughput/ $\Gamma=0.5$

Deterministic Trip Length Distribution

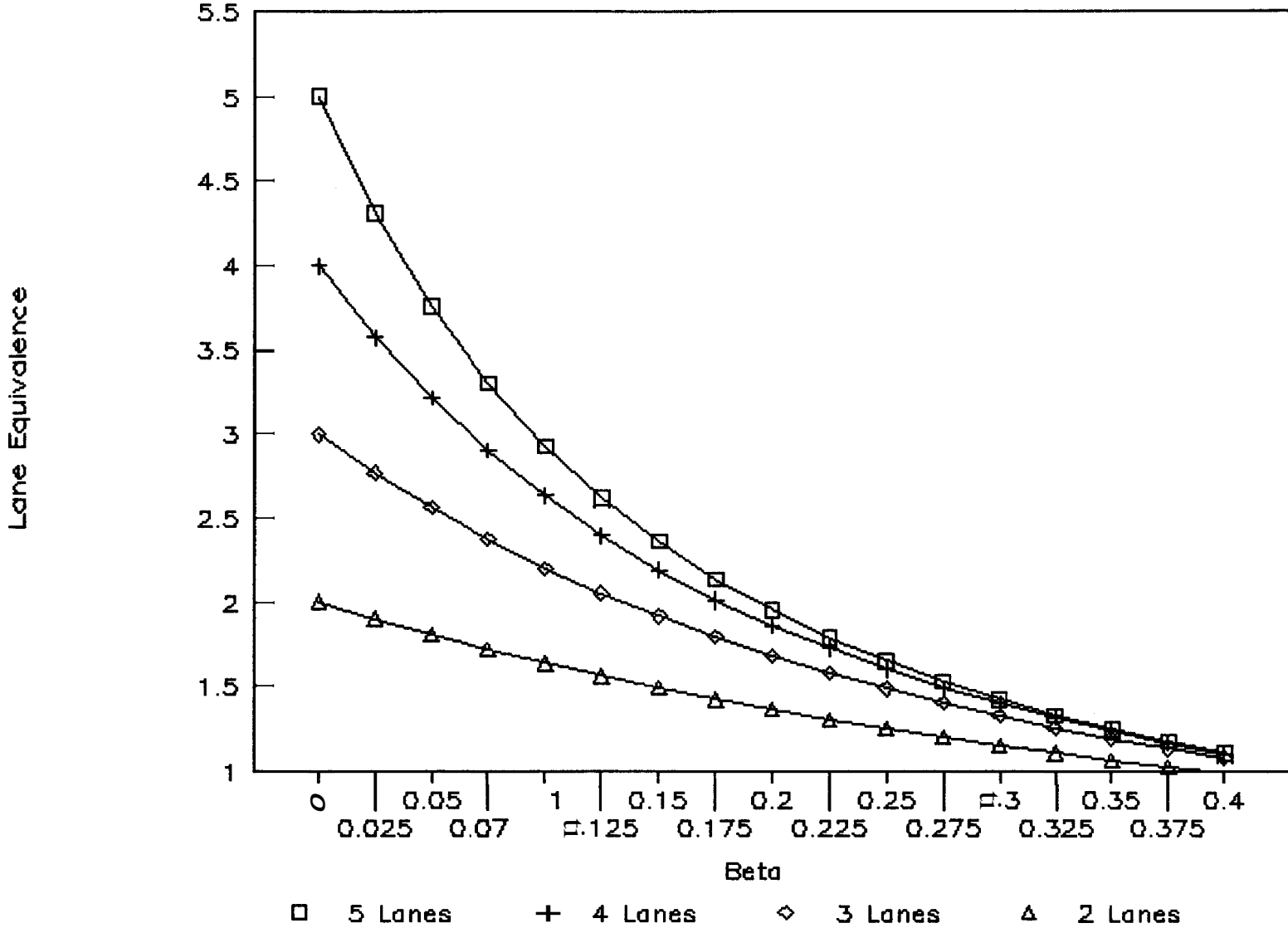


Figure 6. Lane Equivalence with  $\Gamma=0.5$  and Deterministic

# Traffic in Lane 1 -- 3 to 5 Lanes

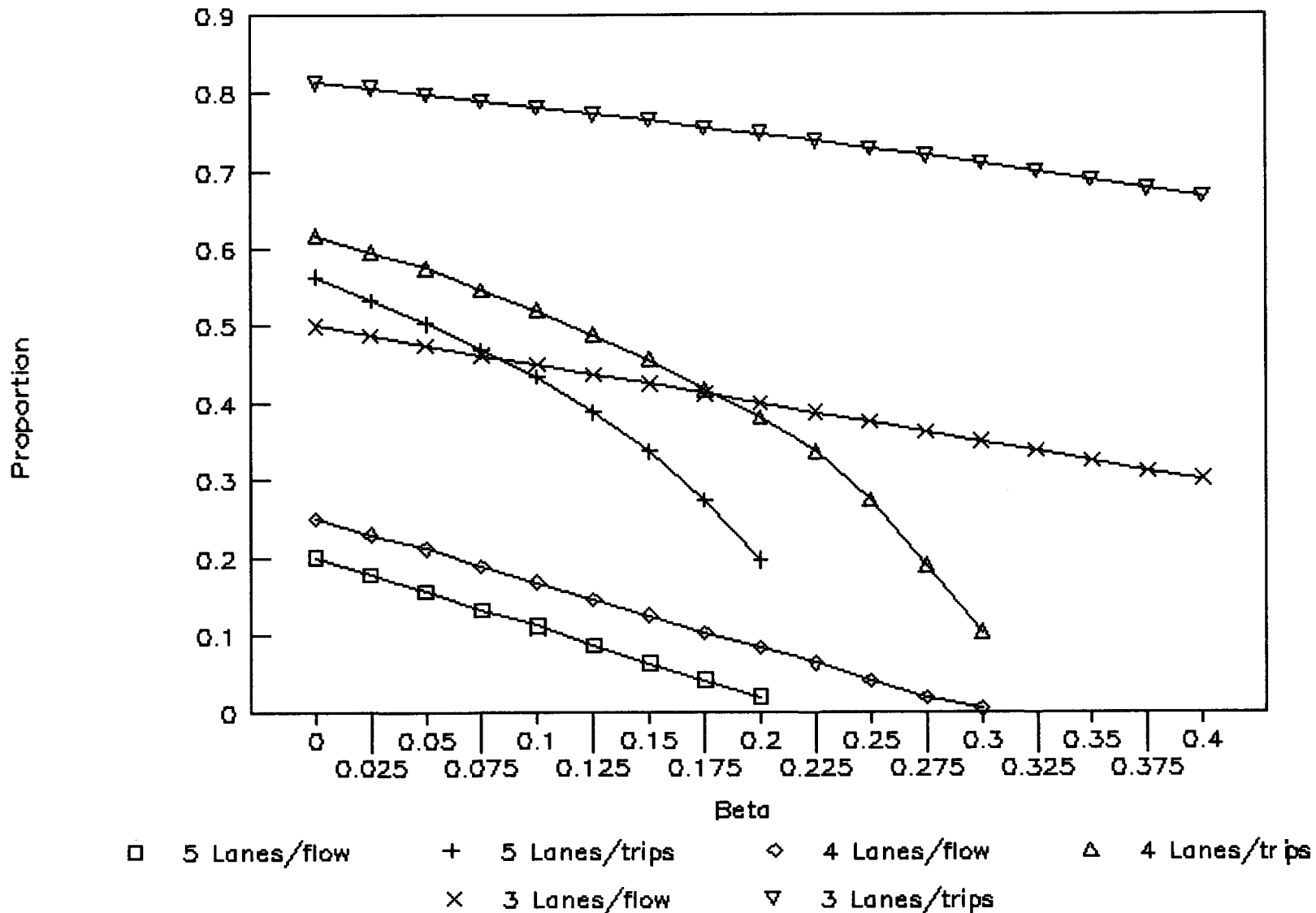


Figure 7. Proportion of Trips and Lane Flow Residing in Lane 1

## CONCLUSIONS

The idealized model has identified conditions under which lane changes result in a substantial decrement in capacity. This would occur when  $\beta$  is greater than .1, fairly independently of  $\gamma$ . This value of  $\beta$  roughly translates into a lane-change occupancy time of 1000 meter-seconds, based on an average trip length of 20 km, a velocity of 30 m/s, and a longitudinal space requirement of 15m (corresponding to a capacity of 7,200 vehicles/hour per lane, without lane changes). For values of  $\beta$  above .3, the capacity of a multi-lane AHS may become comparable to a conventional highway. This would occur for an occupancy of 3000 meter-seconds, for a highway with the cited attributes.

The throughput analysis does not estimate delays due to lane-change queueing. At the cited capacity values, queueing for lane changes would be substantial. This is certainly undesirable in any real system, for travelers need a high assurance that they can exit at desired locations. Therefore, it is essential to reduce the capacity values by a buffer margin. This is the subject of ongoing research.

The idealization does not account for spatial and temporal variations in flow and flux. To do so would require a site specific analysis, and modification of the objective function to reflect these site specific characteristic. If this is done, the math program could be modified to a non-linear multi-commodity flow formulation, where a commodity represents an origin-destination pair (i.e., a pair of highway entrance and exit), and links represent lane segments and lane-change flows. Optimization of such a model would specify the locations where lane-changes should occur. It would be possible, for instance, to eliminate most lane-changes within high-flow highway segments, shifting them to places where surplus capacity exists.

The model also doesn't represent the discrete nature of highway entrances and exits. At this point, it is unclear whether this discreteness substantially alters highway capacity. It's possible that the space requirement on the right-most lane is substantially higher than indicated, because lane changes are concentrated. The exact effect, will depend on the design of exit and entrance ramps. It may be that  $\beta\mu_0$  needs to be scaled upward in the expression for  $W'_1$  by a factor approximating the ratio of the length of the ramp running adjacent to the right lane, to the distance between ramps. This would reflect the concentration of work within a short segment. On a related subject, the model also does not account for restricting lane-changes to discrete "gates" (coupled with lane barriers, this has been promoted as a safety device).

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