Title
MODEL FOR THE DECAY ;u (l66o)-^unn

Authors
Eberhard, Philippe
Pripstein, Morris.

Publication Date
1967-07-17

University of California

Ernest O. Lawrence Radiation Laboratory

MODEL FOR THE DECAY $\Sigma(1660) \rightarrow \Sigma\pi\pi$

Philippe Eberhard and Morris Pripstein

July 17, 1967

Berkeley, California

Research and Development

UCRL-17682
UC-34 Physics
TID-4500 (50th Ed.)

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

MODEL FOR THE DECAY $\Sigma(1660) \to \Sigma\pi\pi$

Philippe Eberhard and Morris Pripstein

July 17, 1967

Printed in the United States of America
Available from
Clearinghouse for Federal Scientific and Technical Information
National Bureau of Standards, U.S. Department of Commerce
Springfield, Virginia 22151
Price: Printed Copy $3.00; Microfiche $0.65

MODEL FOR THE DECAY $\Sigma(1660) \to \Sigma\pi\pi$

Philippe Eberhard and Morris Pripstein

Lawrence Radiation Laboratory
University of California
Berkeley, California

July 17, 1967

ABSTRACT

An explicit calculation of the probability distributions of the particles $\Sigma\pi\pi$ from $\Sigma(1660)$ decay has been performed. The model for the decay process is described and the result of the computation is given.

INTRODUCTION

A model for the decay of the $\Sigma(1660)$ into a Σ and two π's is described here. It has been constructed for use in an experimental study[1] of the decays

$$\Sigma(1660)^+ \rightarrow \Sigma^+\pi^+\pi^- \quad (1)$$

$$\Sigma(1660)^+ \rightarrow \Sigma^-\pi^+\pi^+. \quad (2)$$

The computation of the relevant distributions is an application of the Jacob and Wick formalism[2] and of Jackson's[3] expression for the shape of the resonances.

Only the relative distributions of the $\Sigma^{\pm}$, $\pi^{\mp}$, and π^+ particles from the $\Sigma(1660)^+$ decay are of interest here. The distributions of the $(\Sigma\pi\pi)^+$ configuration with respect to the other particles involved in the $\Sigma(1660)$ production reaction depend on the production process; therefore, their prediction would require further assumptions concerning the production process. Moreover, once a production process is assumed, the density matrix for the $\Sigma(1660)$ would most likely vary with production angle and beam momentum, and the analysis of our sample of events, obtained at different beam momenta and different production angles, would be complicated considerably. We limit ourselves to the distributions of the decay quantities, i.e., the decay particles with respect to each other; these distributions are independent of the $\Sigma(1660)$ density matrix and therefore independent of the production process, beam momentum, and production angle.[4] Our computation is made using a density matrix equal to unity.

BASIC HYPOTHESES

a) The $\Sigma(1660)$ spin is supposed to be 3/2 as determined from an Adair analysis[5] of our $\Sigma(1660)$ events produced in the most backward direction in the c.m. system.[1] The two cases of parity (positive and negative) are considered so that we could compare both predictions to the distribution of real events and determine the $\Sigma(1660)$ parity.

b) The decays (1) and (2) are assumed to occur via processes involving $(\Sigma\pi)^0$ states of spin-parity $1/2^-$ and $3/2^+$ only, namely,

I. The $\Lambda(1405)$, with[6] spin-parity $1/2^-$, has been shown[7] to play a dominant role in decay (1) and therefore in decay (2) by isospin arguments.

II. A non-resonant $(\Sigma\pi)$ state of isospin 1, spin-parity $1/2^-$, as seen by Humphrey and Ross,[8] can be expected to contaminate the $\Lambda(1405)$ resonance.

III. Finally, the $3/2^+$ $\Sigma(1385)$ is an established $(\Sigma\pi)$ state[9] and cannot be ignored.

More elaborate models can be considered but we limit ourselves only to well-established $(\Sigma\pi)$ systems for simplicity. It is found[1] that this model fits the data if the $\Sigma(1660)$ parity is negative; it does not fit if positive parity is assumed.

COMPUTATION OF THE TRANSITION MATRIX

Let m_{1660} and m_Σ be the z component of the spin and the $\Sigma(1660)$ and of the Σ particle, respectively. The matrix elements $T_{m_\Sigma, m_{1660}}$ form a 2×4 transition matrix T that is a function of the configuration of the particles. The probability per unit of phase space for a given configuration is proportional to

$$F = \frac{1}{4} \text{ Trace of } \{T\,T^{+}\} \qquad (3)$$

because we used a $\Sigma(1660)$ density matrix equal to unity.

Each process k corresponding to a given $(\Sigma\pi)^0$ state leads to the computation of a transition matrix T_k and

$$T = \sum_k T_k \,. \qquad (4)$$

Because there are two $(\Sigma\pi)^0$ states in decay (2) and only one in decay (1), there are twice as many states k to be considered in decay (2) as in decay (1). Also, a factor $1/\sqrt{2}$ must be introduced for Bose symmetry in decay (2).

Each matrix T_k can be computed by multiplying a function G_k, which includes the Breit-Wigner form and the momentum barrier terms,[3] by an angular correlation term derived from a direct application of the Jacob and Wick formalism.[2] For the process involving the decay $\Sigma(1660) \rightarrow \Sigma(1385) + \pi$, only the lowest allowed partial wave is considered, i. e., s-wave (p-wave) when negative (positive) parity is assumed for the $\Sigma(1660)$.

RESULT OF THE COMPUTATION FOR DECAY (1)

We first define the $\Sigma(1660)$ Breit-Wigner function

$$W_{1660} = \frac{1}{M - M_0 - i\,\frac{\Gamma}{2}} \,, \qquad (5)$$

where M refers to the mass of the $(\Sigma\pi\pi)^{+}$ system, M_0 the central mass value, and Γ the width of the $\Sigma(1660)$ resonance. Similarly we define W_{1405} and W_{1385} by the same formula (5), where M is now the $(\Sigma\pi)^0$

mass, M_0 the central mass value, and Γ the width of the $\Lambda(1405)$ and the $\Sigma(1385)$, respectively.

Then we define the momentum barrier terms, f_k:

$$f_I = f_{II} = p_0^+ \qquad \text{for parity minus} \tag{6}$$

$$f_I = f_{II} = (p_0^+)^2 \qquad \text{for parity plus}$$

and

$$f_{III} = p_-^- \qquad \text{for parity minus} \tag{7}$$

$$f_{III} = p_0^+ \, p_-^- \qquad \text{for parity plus}$$

where p_0^+ is the momentum of the π^+ in the $(\Sigma\pi\pi)^+$ system, and p_-^- the π^- momentum in the $\Sigma^+\pi^-$ system.

We then construct the functions

$$G_I = W_{1660} \; W_{1405} \; f_I \; A_I \tag{8}$$

$$G_{II} = W_{1660} \; f_{II} \; A_{II} \tag{9}$$

$$G_{III} = W_{1660} \; W_{1385} \; f_{III} \; A_{III} \tag{10}$$

$$G = G_I + G_{II} \, . \tag{11}$$

A_I, A_{II} and A_{III} are complex parameters representing the decay amplitudes for processes I to III. Their relative values have to be fitted to the data.

The probability per unit of phase space for a given configuration is proportional to the function

$$F^+ = |G|^2 + |G_{III}|^2 + 2\,\mathrm{Re}\,\{G^* G_{III}\} \cos\alpha_- \tag{12a}$$

in the case of parity minus, and

$$F^+ = |G|^2 + |G_{III}|^2 \, \frac{7 - 6\cos^2\alpha_-}{5} + 2\,\mathrm{Re}\,\{G^* G_{III}\} \, \frac{\cos\alpha_-}{\sqrt{5}} \tag{12b}$$

in the case of parity plus. In these equations α_- is the angle between both pions in the $\Sigma^+\pi^-$ system.

RESULT FOR DECAY (2)

For decay (2), there are two π^+ particles able to form a $(\Sigma\pi)^0$ system with the Σ^-. We label them π_a^+ and π_b^+. We compute, with the help of formulas (5) through (12), the functions G^a, G_{III}^a, and F^a as we would compute G, G_{III}, and F where now π_a^+ replaces the π^- and Σ^- the Σ^+. Also the coefficients A_{II} and A_{III} are changed in sign to take into account the isospin one attributed to the $(\Sigma\pi)^0$ system considered. The functions G^b, G_{III}^b, and F^b are computed in the same way, interchanging now the role of π_a^+ and π_b^+ in the definition of G^a, G_{III}^a, and F^a.

The probability per unit of phase space for a given configuration is then proportional to the function

$$
\begin{aligned}
F^- = {} & \frac{F^a + F^b}{2} + \mathrm{Re}\,\{G^{a*}G^b\} \cos\alpha_0 \\
& + \mathrm{Re}\,\{G^{a*}G_{III}^b\} \cos(\alpha_0 - \alpha_b) \\
& + \mathrm{Re}\,\{G_{III}^{a*}G^b\} \cos(\alpha_0 - \alpha_a) \\
& + \mathrm{Re}\,\{G_{III}^{a*}G_{III}^b\} \cos(\alpha_a + \alpha_b - \alpha_0) \qquad (13a)
\end{aligned}
$$

in the case of parity minus, and

$$
\begin{aligned}
F^- = {} & \frac{F^a + F^b}{2} + \mathrm{Re}\,\{G^{a*}G^b\} \frac{3\cos^2\alpha_0 - 1}{2} \\
& + \mathrm{Re}\,\{G^{a*}G_{III}^b\} \frac{\cos\alpha_b + 3\cos(2\alpha_0 - \alpha_b)}{4\sqrt{5}} \\
& + \mathrm{Re}\,\{G_{III}^{a*}G^b\} \frac{\cos\alpha_a + 3\cos(2\alpha_0 - \alpha_a)}{4\sqrt{5}} \qquad (13b) \\
& + \mathrm{Re}\,\{G_{III}^{a*}G_{III}^b\} \frac{13\cos(\alpha_a + \alpha_b) - 12\cos(\alpha_a - \alpha_b) + 3\cos(\alpha_a + \alpha_b - 2\alpha_0)}{20}
\end{aligned}
$$

in the case of parity plus. In both equations α_0, α_a, α_b are the angles between π_a^+ and π_b^+ in the $\Sigma^- \pi_a^+ \pi_b^+$, $\Sigma^- \pi_a^+$, $\Sigma^- \pi_b^+$ systems, respectively.

BACKGROUND

Processes other than the $\Sigma(1660)$ decay are in general also present in a sample of events carefully selected to be rich in $\Sigma(1660)$ decay. We consider three additional processes, not interfering[10] with processes I, II, and III.

IV. A process that gives a phase-space distribution in $\Sigma^+\pi^+\pi^-$ events

V. A process that gives a phase-space distribution in $\Sigma^-\pi^+\pi^+$ events

VI. A $\Lambda(1520) + \pi^+$ background distributed like phase space but then followed by a decay $\Lambda(1520) \rightarrow \Sigma^{\pm}\pi^{\mp}$ with a branching ratio equal to the one expected from the available phase space.

We define three more parameters A_{IV}, A_V, and A_{VI}, expressing the amounts of process IV, V, and VI, respectively. These parameters are real and are to be fitted to the data. We define also a $\Lambda(1520)$ Breit-Wigner function by formula (5), where now M_0 and Γ refer to the $\Lambda(1520)$ mass and width, and M to the mass of a given $(\Sigma\pi)^0$ combination. Therefore, there is one value of W_{1520} for the $\Sigma^+\pi^+\pi^-$ events and two, W^a_{1520} and W^b_{1520}, for the $\Sigma^-\pi_a^+\pi_b^+$ events. We define the distribution functions

$$F = F^+ + A_{IV} + A_{VI}\, W_{1520} \qquad \text{(for } \Sigma^+\pi^+\pi^- \text{ events)},$$

$$F = F^- + A_V + A_{VI}\, \frac{W^a_{1520} + W^b_{1520}}{2} \qquad \text{(for } \Sigma^-\pi^+\pi^+ \text{ events)}. \qquad (14)$$

PROBABILITY

We can finally express the probability of a configuration, τ, of an event in a data sample. Given the selection criteria for the sample, one considers the domain R included in the boundaries. The probability is then

$$P(\tau) = \frac{F}{\int_R F \, d\phi} , \qquad (15)$$

where $d\phi$ stands for an element of phase space.

Here $P(\tau)$ is a function of the configuration τ and of the selection criteria on one hand, and of the parameters A and the parity assumed for the $\Sigma(1660)$ on the other. For each case of parity, one has to fit the parameters A from which we obtain two expressions, $P^-(\tau)$ and $P^+(\tau)$, for each configuration, corresponding to parity minus and plus, respectively.

REMARKS ABOUT THE BREIT-WIGNER FUNCTIONS

The Γ appearing in the bottom of formula (5) is a variable Γ computed according to the rules given by Jackson.[3] The radius of interaction r_0, unknown in our case, is set to zero. The variation of the $\Lambda(1405)$ width is computed as if the decay produced a pion and a particle with a mass equal to the average between the Σ^+ and Σ^- mass. The variation of the $\Sigma(1385)$ width is approximated by assuming that all $\Sigma(1385)$ decay into $\Lambda + \pi$.

The $\Sigma(1660)$ width is approximated as being the sum of two partial widths. The first one is the decay of $\Sigma(1660)$ into a pion and a particle of mass equal to the central value M_0 of the resonance $\Lambda(1405)$ to approximate the $(\Sigma\pi\pi)$ decay mode; the second one is a decay into a pion and a particle of mass in the vicinity of the Σ mass, to approximate all other decay modes. One expects the second partial width to be important for the events with a $(\Sigma\pi\pi)$ mass close to the threshold of $\Lambda(1405) + \pi$ decay, where the correction for variable Γ makes the first partial width equal to zero.

ALTERNATE MODES

Whenever the process II and/or III is considered to be not a decay mode of the $\Sigma(1660)$ but rather the result of a non-resonant $(\Sigma\pi\pi)$ state with the same spin-parity as that of the $\Sigma(1660)$, one can modify the definition of G_{II} and/or G_{III} accordingly in (9) and/or (10), replacing the term W_{1660} by 1.

When data is combined from different beam momenta and production angles, the resulting distribution is of the form defined above, provided the importance of terms in $|G_{II}|^2$ and/or $|G_{III}|^2$ is small.[4]

This work was done under the auspices of the U. S. Atomic Energy Commission.

FOOTNOTES AND REFERENCES

1. P. Eberhard, M. Pripstein, F. T. Shively, U. E. Kruse, and W. P. Swanson, Determination of the Spin and Parity of the $Y_1^*(1660)$, submitted to Phys. Rev. Letters (UCRL-17590).
2. M. Jacob and G. C. Wick, Ann. Phys. 7, 404 (1959).
3. J. D. Jackson, Nuovo Cimento 34, 1644 (1964).
4. Of course this statement applies only to the distributions characteristic of the $\Sigma(1660)$ decay in principle. The relative amount of non $\Sigma(1660)$ background and its phase would be expected to vary with respect to the production angle of the $(\Sigma\pi\pi)^+$ system and beam momentum. However, the non-interfering background [i.e., the background of spin-parity different from that of the $\Sigma(1660)$] adds linearly to the square of the $\Sigma(1660)$ matrix element in the expression of the distributions. Therefore, combining different samples of data from different production conditions results in a distribution where the background terms are simple averages of all conditions. The same property applies to the interfering background [same spin-parity as that of the $\Sigma(1660)$] as long as it is small enough so that the square of the background matrix element plays no important role and the interference between background and $\Sigma(1660)$ can simply be averaged.
5. R. Adair, Phys. Rev. 100, 1540 (1955).
6. J. K. Kim, Phys. Rev. Letters 14, 29 (1965).
7. P. Eberhard, F. T. Shively, R. R. Ross, D. M. Siegel, J. R. Ficenec, R, I. Hulsizer, D. W. Mortara, M. Pripstein, and W. P. Swanson, Phys. Rev. Letters 14, 466 (1965).

8. W. E. Humphrey and R. R. Ross, Phys. Rev. <u>127</u>, 1305 (1962).

9. A. H. Rosenfeld et al., Data on Particles and Resonant States, UCRL-8030 and references quoted there.

10. These processes are made non-interfering with processes I, II, and III because most of the background is expected to occur in ($\Sigma\pi\pi$) systems of different spin-parity than that of the $\Sigma(1660)$, and we look only at the relative distributions of the Σ and π particles.