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Author
BARBOSA DE ALMEIDA, MAURO WILLIAM

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MAURO WILLIAM BARBOSA DE ALMEIDA
UNIVERSIDADE ESTADUAL DE CAMPINAS, SÃO PAULO, BRASIL
MWBA@UOL.COM.BR

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ON THE STRUCTURE OF DRAVIDIAN RELATIONSHIP SYSTEMS

MAURO WILLIAM BARBOSA DE ALMEIDA
UNIVERSIDADE ESTADUAL DE CAMPINAS, SÃO PAULO, BRASIL

Abstract : We propose a calculus for kinship and affinity relationships that generates the classification of Dravidian terminologies proposed by Dumont (1953 and 1958) in the form given to them by Trautmann (1981). This calculus operates on the language D of words for kinship and affinity, endowed with rules that select amongst the words in D* a sub-set of words in canonical Dravidian form. We prove that these rules generate uniquely the Dravidian structure (as in Trautmann's model B), and we demonstrate that that Trautmann's model B is the correct version of his model A. We discuss the meaning of the anti-commutative structure of D*, and finally point to a generalization of the proposed calculus allowing its rules to be seen in the more general Iroquois context.*

Introduction

This article deals with the formal theory of Dravidian kinship terminologies, and is particularly concerned with Louis Dumont's version of such systems (Dumont 1953). Dumont asserts in his influential 1953 article that "Dravidian kinship terminology ... can be considered in its broad features as springing from the combination ... of four principles of opposition: distinction of generation ..., distinction of sex, distinction of kin identical with alliance relationship, and distinction of age" (Dumont 1953:39). This statement contrasts with the prevailing point of view according to which distinctions based on sex and filiation alone are sufficient for the description of the relevant features of Dravidian terminologies, if we ignore distinctions concerning relative age. In opposition to this point of view, Dumont says that the opposition between kin (consanguineous) and allied (affine) is autonomous vis-à-vis filiation, both in a formal and in an empirical sense. He also claims that the choice for an adequate theory should take into account not only empirical validity, but also logical simplicity and elegancy as criteria: "Field acquaintance with Dravidian kinship terminology made me feel very strongly its systematic, logical character; I could not help thinking that it centered in marriage, and that it should be possible to express those two features in a simple formula" (Dumont 1953: 34).

In his classical book on Dravidian kinship, besides adding empirical credibility to Dumont's thesis on the special role of affinity in Dravidian culture, Thomas Trautmann has proposed a set of rules to calculate the affine/consanguineal character of a given kinship relation on the basis of its components (Trautmann 1981). Trautmann's analysis is a tour-de-force, and the present approach was developed essentially on the basis laid by it. We hope to have improved on his analysis, by showing how his set of 11 rules (with 11 x 4 sub-rules and 11 x 4 x 4 cases) follows from a smaller set of basic group-theoretical and transformation rules with a clear sociological and cognitive meaning. I show that this set is contained already in Trautmann's rules 1-7. In particular, we show that Trautmann's rules 8B-9B are the corrected form of his rules 8A-9A.

Tjon Sie Fat has analyzed and generalized Trautmann's rules in the form of multiplication tables in which crossness and not affinity is taken as the significant feature (Tjon Sie Fat 1989). A central feature of Tjon Sie Fat's algebraically oriented approach is the emphasis on non-associativity in order to account for multiple types of crossness. Although the relevance of non-associativity is undeniable – natural languages are in general non-associative in the sense that different ways of parsing convey distinct meanings - I make a case for keeping associativity as an essential part of an algebraic approach to kinship and affinity. On the other hand, I place a central role in our approach on anti-commutative relations.

Section I introduces a language of kinship and affinity words, which we call the D* language, and three sets of rules which are used to reduce terms of this language to a canonical form. These are what I call C-rules (classificatory rules), D-rules (Dravidian rules) and A-rules (affinization rules). C-rules describe the group-theoretical features of the universe of kinship and affinity words, A-rules characterizes Dravidian structures, and D-rules impose an affinized form on words in D*.

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In section II, we describe the structure resulting from the rules, and prove that they provide a complete and unambiguous classification of kinship and affinity words, which agrees with the classification of Dravidian terminologies proposed in Trautmann's model B.

Section III demonstrates that the D* language together with rules C-A-D is isomorphic to Trautmann's language together with a sub-set of his rules 1-7. This section deals specifically with the task of explaining why a "double classification" in Trautmann's model A does not occurs when Trautmann's language is written in an explicit way.

In section IV, we formulate a more general framework within which rules A and D are justified. In this more general framework, moreover, a comparison between Dravidian, Iroquois and Crow-Omaha systems may be pursued. This comparison is undertaken in a another paper,

The approach proposed here is more than a mere formal simplification of Trautmann's rules. The new notation and rules have been selected both for mathematical elegance and for substantive meaning. They contain sociological/cognitive information, and intended to encapsulate notions such as “elementary crossness” and “elementary affinity" which we show to be symmetrically defined, suggesting that there are no grounds for taking one or another as primary, apart from convention. This line of the argument is inspired by Viveiros de Castro's view on the role of affinity in South-American kinship systems, from which our interest in the subject arose originally (Viveiros de Castro 1998).

I am not a professional mathematician, and the text has no pretention to a standard mathematical style. Mathematics is here mainly a source for ideas and a model for the effort towards clarity and rigor. When a proof for a "theorem in kinship theory" is already available in books on group theory, we point to the fact and avoid proving it again in a different garb.

I. The Dravidian language

We start with the vocabulary of basic kinship and affinity words, or just *basic words.*

$D = \{e, s, f, f^{-1}, a\}$

The basic words will also be referred as letters. We may define D* as the set of strings formed by concatenation of basic words in D. A more explicit and constructive definition runs as follows:

Definition 0.

Words in D* are those formed as follows:

1. e, s, f, f^{-1} and a are words in D*.

2. If W is a word in D*, We, Ws, Wf and Wf^{-1} are words in D*.

We may interpret basic words as operators, categories (marked by letters), or as classes. Thus, we can think of the word s as referring, for male ego, to the category of sisters and, for female ego, to the category of brothers. Under this interpretation, the basic word e denotes, for male ego, the non-marked category of brothers and, for female ego, the non-marked category of sisters. In addition, the basic word f means, under this interpretation, the category of fathers for male ego, and, for female ego, the category of mothers, while the word f^{-1} refers, for male ego, to the category of sons, and, for female ego, to the category of daughters. Thus, e, f, f^{-1}, s can be translated as ♂B, ♂F, ♂S and ♂Z respectively, and as ♀Z, ♀M, ♀D and ♀B. In the Dravidian context, the words formed by means of e, f, f^{-1} and s will usually have an ego of unspecified sex.

This raises the issue of how to express, from a male point of view, the category of his mothers, and, for a female point of view, the category of her fathers. These relations are expressed by the words ♂sf and ♀sf, which literally mean, respectively, a male ego's sister's mother and a female ego's brother's father. Thus, sf is translated as ♂ZM and as ♀BF according to whether ego is male or female, under the understanding that ♂ZM = ♂M, and ♀BF = ♀F. In an analogous way, ego's opposite-sex children are denoted by $f^{-1}s$, an expression which can be translated as ♂SZ or ♀DB for male and female ego respectively, under the understanding that ♂SZ = ♂D and ♀DB = ♀S. Reading ♂sf as ♂M (and reading ♀sf as ♀F), and reading ♂$f^{-1}s$ as ♂D (respectively, ♀$f^{-1}s$ as ♀S) has a strong classificatory content. These conventions can be seen as extensions of Lounsbury's half-sibling rules.

The most significant fact about the universe of words in D* is the non-commutativity of word composition. We stress this fact by raising it to the rank of a principle:

Principle of Incest Prohibition, I: $fs \neq sf$

Principle of Incest Prohibition, II: $sf^{-1} \neq f^{-1}s$

Under the interpretation stated above, the principle of incest prohibition says in its form I that one's father's sister is not one's mother, and that one's mother's brother is not one's father. In the form II, the principle says that one's sister's son is not one's own son, and that one's brother's daughter is not one's own daughter.

One immediate consequence of the Principle of Incest Prohibition is that

$$s \neq f^{-1}sf$$

This says that sisters (♂s) are not wives (♂$f^{-1}sf$, a male ego's son's mothers) and brothers (♀s) are not husbands (♀$f^{-1}sf$, a male ego's daughter's fathers).

I.1 Classificatory rules (C-rules)

The universe of words in D* is clearly infinite. We start by classifying this universe by means of equations, which are used as rules to contract words. The contraction rules generate a smaller universe of words.

C-rules

C1. $ff^{-1} = e$, $f^{-1}f = e$, $ss = e$

C2. $eW = W$, $We = W$, where W stands for any of the basic words s, f, f^{-1}

This is how equations C1 (4 instances) and C2 (6 instances) are used as contraction rules. If a word W contains any syllable appearing at the left side of an equation C1-C2, replace it by the letter at the right side of the same equation. This operation is called an elementary contraction of the word W. A series of elementary contraction of a word W is called a contraction of W. We denote the action of contracting a word W into a word Z as W = Z without ambiguity since a contraction always reduces the length of a word. This convention helps to distinguish in the future the use of C-rules from the use of the remaining rules that will be indicated by the symbol "→ ".

It may be said that rules C1-C2, or C-rules, describe the more general structural features of classificatory systems of consanguinity in the sense of Lewis Morgan. Indeed, C-rules have the effect of collapsing lineal-collateral distinctions, and collapsing the distinction between full siblings and half siblings. This justifies labeling them as "classificatory equations" in the sense of Lewis Morgan, extended so as to include a generalized version of Lounsbury's half-sibling principle (Lounsbury 1969). Thus, the instance $f f^{-1} = e$ of rule C1 says that a father's son is a brother (for male ego), and a mother's daughter is a sister (for female ego). The instance $f^{1}f = e$ of C1 says that a son's father is a brother (for male ego), and a daughter's mother is a sister (for female ego). Rule C2 includes the assertions that (for male ego) a brother's father is a father ($ef = f$), and a father's brother is a father ($fe = f$). It also says that a brother's son is a son (ef^{-1}), just as a son's brother is again a son ($f^{-1}e = f^{1}$). Rule C1's especial case $ss=e$ asserts that there are two sex categories, one marked and another unmarked. It is equivalent to taking $s^{-1} = s$, under the understanding that whenever X Y = e, then Y is the inverse of X, a fact that we express by writing $Y = X^{-1}$.

Definition 1

A word W in K* is said to be in the contracted form, or just contracted, if and only if it cannot be changed by a C-rule.

We denote the effect of contracting a word W as [W]. The contracted form of a word is unique, and this fact justifies the use of the definite article when referring to [W] as the contracted form of W .[1] Thus, if a word W is already contracted, we write [W] = W . Words X and Y having the same contracted form are said to be equivalent. Thus, [X] = [Y] means that they can both be contracted into the same word and are equivalent. Clearly, the

[1] This assertion is proved in some books on group theory, while others assume it as obvious (Fraleigh 1967).

contracted form [X] of a word is the shortest word in its equivalence class. Since [X] is the unique representative of its equivalence class, an abuse of language justifies thinking of [X] both as the contracted form of X and as the equivalence class represented by [X]. We refer to [D*] as the set of contracted words in D*. The composition of contracted words [X] and [Y] in [K*] is defined as the contracted concatenation [XY]. In other words,

[X][Y] = [XY]

Since the contracted words are unique representatives of each equivalence class of words, we can think also of the composition law for contracted words as a composition law of classes of equivalence. Note also that if a word W is the concatenation of three words X, Y and Z, then [XYZ] = [[X] [XZ]] = [[XY] [Z]]. This follows from the fact that contraction is associative.

The classificatory structure allows for an infinite number of equivalence classes, represented by an infinite variety of contracted words. As an example, consider the sequence of words, all of them distinct from each other, and at the same generational level. This list also illustrates how to translate between the D* language and the kin type language.

♂e ≈ ♂B,	♀e ≈ ♀Z
♂*s* ≈ ♂Z,	♀*s* ≈ ♀B
♂fsf^{-1} ≈ ♂FZS,	♀ fsf^{-1} ≈ ♀MBD
♂$f^{-1}f$ ≈ ♂SF,	♀ $f^{-1}f$ ≈ ♀DM
♂$ffsf^{-1}f^{-1}$ ≈ ♂FFZDD,	♀ $ffsf^{-1}f^{-1}$ ≈ ♀MMBSS
♂$f(sf)s(f^{-1}s)f^{-1}$ ≈ ♂FMBDD,	♀ $f(sf)s(f^{-1}s)f^{-1}$ ≈ ♀ MFZSS [2]

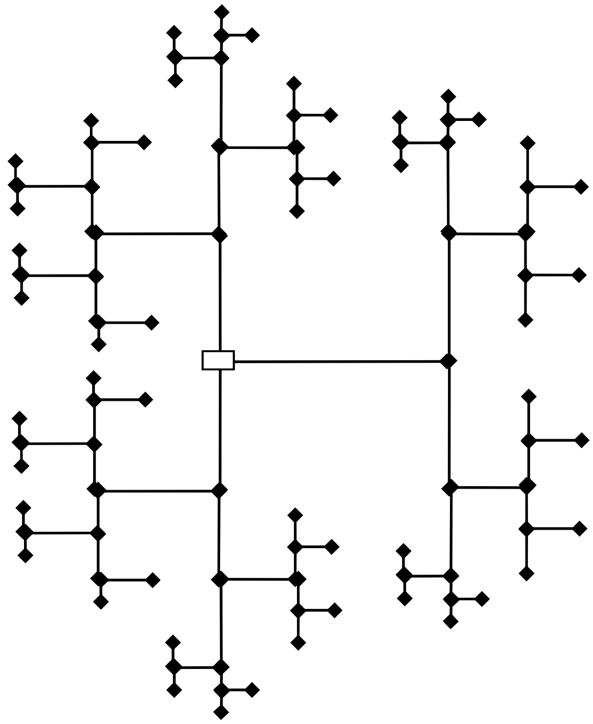
As said before, a word in D* with unspecified sex can be read ambiguously as preceded by ♂ or by ♀. In the Dravidian context, this ambiguity is not relevant, although the situation changes in other contexts.

Let us now focus our attention on the smaller vocabulary K = {e, s, f, f^{-1}} which differs from D for lacking the affinity symbol *a*. The set [K*] which results from contracting the words in K* can then be represented as the Cayley graph generated by s and *f* subject to the relation $s^2 = e$. Figure 1 below represents all words thus generated, of length 5 or less, under the following convention: all horizontal lines stand for the letter *s* (no matter the sense we travel along them), and all vertical lines stand for the letter *f* when followed upwards, and for the letter f^{-1} when read downwards..[3]

[2] The parentheses in *D** notation are used for the sake of clarity to distinguish letters that are translated by a single kin type (♂(*se*) ≈ ♂*M*). Parentheses will also be used with the same goal to specify the letters to which a rule is applied.

[3] A similar graph was introduced by Harrison C. White (1963, p. 14). While the graph in Figure 1 is generated by *f, s* with $s^2 = 1$, Harrison's graph is the graph of the free group generated by *f* , *a* subject to the relation a^2=1 where $a = f^{-1}sfs$ (see Section IV for this notation). I had not realized this possible influence of my reading White's book in the early seventies until reminded of it by Héran's recent book (2009) which Viveiros de Castro called to my attention.

Figure 1. A graph of contracted words in [K*] for words of length 5 or less generated by *f* (vertical lines) and *s* (horizontal lines).



Clearly, the classificatory axioms, while collapsing many genealogical positions into equivalence classes, results in an infinite universe (Figure 1 is a sample of this universe obtained by restraining the length of paths), still too big, even when ignoring the basic letter "*a*". Additional rules are needed to reduce this universe to a manageable size. This will be done with the help of what we call affinization rules and Dravidian rules.

I.2 Affinization rules (A-rules)

The affinization rules introduce a link between the language [K*] consisting of "kin" words alone, i.e. those words constructed with *f, s* and its inverses (together with the identity *e*), and the language [D*] with an added "affinity" term *a*. These transformations are called here *affinization* operations. Affinization operations have the form of *anti-commuting rules*: they replace "cross" relationships by a "consanguineal" relationship marked as affine, and a "spouse" relationship by an affine mark and a sex mark.[4]

[4] A justification for these rules and for the terminology is given in Section IV.

Affinization rules

A1. $fs \rightarrow sfa$ (affinization of ascending crossness)

A2. $sf^{-1} \rightarrow af^{-1}s$ (affinization of descending crossness)

A3. $f^{-1}sf \rightarrow sa$ (opposite-sex spouses are opposite-sex affines)

Definition 2

A word W in D* is in reduced form (or simply reduced) if and only it is in the normal form and cannot be changed by rules A1-A3.

We say that a sequence W0 … Wn of words is a reduction W0 → Wn if each word in the sequence is obtained from the previous one by one of the rules C, D and A, and Wn is in reduced form (this means that Wn cannot be further changed by any rule).
In the course of a reduction the use of contraction rules C is indicated by means of the sign "=", and the use of rules D and A is indicated by means of the sign "→". We express the effect of reducing a word W by writing ⟨W⟩. If W is a reduced word, ⟨W⟩ = W. Also, if W → V, we may write ⟨W ⟩ = ⟨V ⟩.

I.3 Dravidian Rules (D-rules).[5]

The next and final set of rules encapsulate the specific features of Dravidian rules, although not in an immediately obvious form. In section IV these rules will be given a more familiar appearance.

Dravidian rules

D1. $aa \rightarrow e$ (a-contraction rule: affinity is symmetrical)

D2. $aW \rightarrow Wa$ (a-commutation normalized to the right)

where W is any word in D*.

Definition 3

We say that a word is in normal (Dravidian) form (or just in normal form) if it is contracted under C-rules, is affinized and cannot be changed by D-rules

To put a word W in the normal form, regroup all occurrences of "*a*" in it to the right side of the remaining letters using rule D2, and contract (cancel all pairs "*aa*", "*ss*", "ff^{-1}" and "$f^{-1}f$", erase the resulting "*e*", and repeat while possible). The resulting normal word W' is now a contracted word not containing any "*a*", followed either by a single mark "*a*" or by no mark at all. Thus, the overall effect of rules C, A and D is to assign every word of W* to one of

[5] Section IV provides a justification for these rules.

two classes of words: non-marked words and marked words (those marked with an "*a*"). We may express describe a word in normal form with the following notation:

[W '] or [W ']*a* where W' has no occurrence of "*a*".

or

$[W' a^k]$ where $k = 0$ or $k=1$ and W' has no occurrence of "*a*".

This simple structure is not sufficient to generate the peculiarities of Dravidian classification. More specifically, we cannot obtain from it a composition law that yields the correct affine mark. To this end, we need the next and last set of rules.

II. The canonical form

We refer to the universe of words in D* together with the structure given to it by rules *C1-C2, D1-D2,* and *A1-A3* as the *calculus* D*. We write $\langle D^*\rangle$ to denote the set of reduced words in D*. In this section we prove that this calculus converts every word in *D** into a unique reduced word in $\langle D^*\rangle$. This is also called the set of words in *canonical form.* Our goal in now is to describe the structure of $\langle D^*\rangle$ and show that it is precisely the Dravidian classification according to Trautmann's model when restricted to generation levels G^{-2} to G^{+2}.

Definition 4

A word in the Dravidian canonical form (canonical form) if it can be represented in one of the following ways:

I. Ascending canonical form $[s^p f^k a^q]$

II. Horizontal canonical form $[s^p a^q]$

III. Descending canonical form $[f^{-k} s^p a^q]$

where there following conventions are used:

(i) $p \geq 0$, $q \geq 0$, $k > 0$.

(ii) $s^p = s \ldots s$ (p times), $a^q = a \ldots a$ (q times) , $a^0 = e$, $a^0 = a$, $p^0 = e$, $p^1 = p$.

(ii) $f^k = f \ldots f$ (k times), $f^{-k} = f^{-1} \ldots f^{-1}$ (k times).

We recall that [W] is the effect of contracting a word W. The contraction sign ensures that s^p is contracted to *e* for even p, and is contracted to *s* for odd p. Thus, $[s^4] = e$, $[s^3] = s$. The same convention holds for a^q, so that $[a^2] = e$ and $[a^1] = a$.

It is perhaps useful to add the following comments on the canonical form. The ascending canonical form comprises on one side the "consanguineal" or non-marked words f^k and sf^k, and on the other side the "affine", or marked, words $f^k a$ and $sf^k a$. Now, the non-marked status of the pair (f^k, sf^k) can be seen as a generalization for k>1 of the empirically observed "consanguineal" (non-marked) status of relationships ♂F and ♂M (translated as f and sf) in Dravidian societies. The marked pair ($f^k a$, $sf^k a$), in the same breath, can be seen as a generalization of the affine status assigned by Dravidian societies, according to Dumont, to fa and sfa, which translated ♂MB (♂sfs → $ssfa = fa$) and ♂FZ (♂fs→ ♂sfa).

Following the anthropological usage, the descending forms are the reciprocals of the ascending form. Thus, f^{-k} and $f^{-k}s$ are the reciprocals of "consanguineal" ascending forms (or their mathematical inverses), and can be interpreted as generalizations of the familiar consanguineal ♂S and ♂D kin types expressed as f^{-1} and $f^{-1}s$. Finally, the marked descending forms $f^{-k}a$ and $f^{-k}sa$ can be seen as generalizations of $f^{-1}a$ and $f^{-1}sa$ which are translations of ♂ZS ($sf^{-1}s \to f^{-1}sas = f^{-1}ssa = f^{-1}ea = f^{-1}a$) and ♂ZD ($sf^{-1} \to asf \to sfa$).

Summing it up, the canonical forms at G^{+1} are precisely f and sf (♂F, ♂M; ♀M, ♀F) and their marked copies fa and sfa (♂MB, ♂FZ; ♀FZ; ♀MB); the canonical forms at G^{-1} are f^{-1} and $f^{-1}s$ on the one hand (♂S, ♂D; ♀D, ♀S) and their marked copies $f^{-1}a$ and $f^{-1}sa$ on the other. This simple pattern is shown more clearly in the following form:

Definition 4' rearranged.

I. Ascending canonical forms	$[s^p]f^k$	$[s^p]f^k a$
II.Horizontal canonical form	$[s^p]$	$[s^p]\,a$
III. Descending canonical form	$f^{-k}[s^p]$	$f^{-k}[s^p]\,a$

(p = 0 or p = 1, k >0).

II.1 Reduced words have the canonical form

Our goal in this section is to show that reduced words have always the *canonical form*.[6] I use first the method of mathematical induction on the length of words, which in this case is the same as showing that words of length 1 are canonical, and, as words in D* of length n+1 are produced out of reduced words of length n (canonical by hypothesis), they inherit the canonical form.

Definition 5

Suppose m is the number of occurrences of "f " in W and n is the number of occurrences of "f^{-1}" in W. Then, the *generational depth* (or just depth) of W is m-n. If p is the number of occurrences of "s" in W, the *sex index* of W is 0 if p is even, and is 1 if p is odd.

[6] The route followed here has a historical reason: we first developed the calculus in terms of C-rules, D-rules and A-rules when seeking to express Trautmann's rules. One can see in Definition 0 that these rules could have been embedded in the recursive definition of words, thus generating directly the universe [D*].

The generational depth of any word W and its sex index are conserved through the reduction process. Therefore, both can obtained directly from W, by counting the symbols "f " and "f^{-1}" , and by counting the symbols "s" in the original word. This is not so with the affine character, which must be defined by means of the reduced form of W. This is indeed the whole point of reducing a word by means of rules A and D.

Definition 6

Suppose that the word W has the reduced form $\langle W\rangle = \langle W' \ a^{q}\rangle$. Then, the *affine character* of W is 0 if q is even, and 1 if q is odd.

We emphasize the point: the affine character of W is obtained by first transforming W to the reduced form $\langle W\rangle$, and then reading the resulting parity of a^q in $\langle W\rangle$, not in W. Now I proceed to show that the reduction process always results in a word in canonical form, and that the affine character thus obtained is unique.[7]

Proposition 1. If W is a word in D*, $\langle W\rangle$ has the canonical form.

We offer an inductive proof of Proposition 1 based on the length of words in D*. We must show first that (i) reduced words in D* of length 1 have the canonical form. As a second step we must show that, (ii) as words in D* of length n+1 are produced out of reduced words of length n (by hypothesis already in canonical form), they inherit a canonical form.

Since all words in D* are either of length 1 or are of length n+1 and obtained from words of length n by Definition 0, we can conclude from (i) and (ii) that Proposition 1 indeed holds for all words in D*.

The first step is obvious, for if a word W in D* has length 1, it is one of the basic words e, s, f, f^{1} or a. These words are of course reduced and canonical.

Assume now that, for every word W in D* of length n, its reduced form $\langle W\rangle$ is canonical. We must prove that, when we form the word Wx of length n+1 by adding one letter to it, the reduced word $\langle Wx\rangle$ is also canonical. In other words:

if $\langle W\rangle$ is canonical, $\langle Wx\rangle$ is canonical.

Lemma 1: $\langle Wx\rangle = \langle\langle W\rangle x\rangle$.

Lemma 1 asserts that we can always reduce a word by associating words from left to right, and it is really a consequence of associativity. Since composition is associative, we can apply rules as we produce a word without loss of generality. [8] Lemma 1 is useful in the proof

[7] The uniqueness of the affine character means that no "double classification" can occur within the *D** calculus.

[8] One could avoid invoking associativity by including contracting rules C, normalizing rules D and affinization rules D in the very construction process of *Dravidian words.* In this way, the Lemma would not be needed, since every Dravidian word would by definition be in reduced form.

because instead of calculating ⟨*Wx*⟩ from the start we need only to calculate ⟨⟨*W*⟩ *x*⟩, which has a very simple structure because ⟨*W*⟩ is already in canonical form.

We use the following notational devices:

Lemma 2a. ⟨[*W*]⟩ = ⟨*W*⟩ where *W* is any word in D*.

Lemma 2b. Suppose ⟨[*W*]⟩ = [*W*]. Then, then ⟨*W*⟩ = [*W*].

Lemma 2a says that reducing a contracted word is the same as reducing it since the reduction process implies contracting it. Lemma 2b says that, if reducing a contracted word leaves it unchanged, we can express the reduced form by means of the contracted form alone. Lemmas 1-2 will be used below without further mention.

Now we check the step from ⟨*W*⟩ to ⟨*Wx*⟩ in three cases. In case I, ⟨*W*⟩ has the ascending canonical form. In case II, ⟨*W*⟩ has the horizontal canonical form, and in case III, ⟨*W*⟩ has the descending canonical form.

Case I. Suppose ⟨*W*⟩ has the ascending canonical form $[s^{p} f^{k} a^{q}]$. We indicate this by writing $\langle {}^{+}W\rangle$. Then, $\langle {}^{+}Wx\rangle$ may be one of the following words: $\langle {}^{+}We\rangle$, $\langle + {}^{W}f\rangle$, $\langle {}^{+}Wf^{-1}\rangle$, $\langle {}^{+}Ws\rangle$. We reduce each of these instances, without mentioning the use of rule C except in a few select cases, and when a word is in the canonical form, we indicate this fact with the sign ■.

1. $\langle {}^{+}We\rangle = \langle\langle {}^{+}W\rangle e\rangle = \langle [s^{p} f^{k} a^{q}]\, e\rangle = [s^{p} f^{k} a^{q}]$. (C-rule) ■

2. $\langle {}^{+}Wf\rangle = \langle\langle {}^{+}W\rangle f\rangle = \langle [s^{p} f^{k} a^{q}] f\rangle = \langle s^{p} f^{k} a^{q} f\rangle$

$\langle s^{p} f^{k} a^{q} f\rangle \rightarrow \langle s^{p} f^{k} f a^{q}\rangle = [s^{p} f^{k+1} a^{q}]$ (rule A2). ■

3. $\langle {}^{+}W f^{-1}\rangle = \langle\langle {}^{+}W\rangle f^{-1}\rangle = \langle [s^{p} f^{k} a^{q}] f^{-1}\rangle = \langle s^{p} f^{k} a^{q} f^{-1}\rangle$
$\langle s^{p} f^{k} a^{q} f^{-1}\rangle \rightarrow \langle s^{p} f^{k} f^{-1} a^{q}\rangle = [s^{p} f^{k-1} a^{q}]$ (rule A2). ■

The next instance brings about a change in the affine character:

4. $\langle {}^{+}\mathrm{Ws}\rangle = \langle\langle {}^{+}\mathrm{W}\rangle \mathrm{s}\rangle = \langle \mathbf{[}s^{p} f^{k} a^{q}]\, s\rangle = \langle s^{p} f^{k} a^{q} s\rangle$

$\langle s^{p} f^{k} a^{q} s\rangle \rightarrow \langle s^{p} f^{k} s\, a^{q}\rangle$ (rule A2)

$\langle s^{p} (f^{k} s)\, a^{q}\rangle \rightarrow \ldots \rightarrow \langle s^{p} (s f^{k} a^{k})\, a^{q}\rangle$ (rule D1 and rule A2, applied k times)

In the last line, we have condensed *k* uses of rules D1 and A2. The overall effect of these *k* operations is to move the added "*s*" to the left of the *k* occurrences of "*f*", resulting in a new term a^{k}. (Here and in other instances, we have used parentheses for the sake of clarity.) The new affine term combined with the previous a^{q} results in the affine character:

$\langle s^{p} s f^{k} a^{q} a^{k}\rangle \rightarrow [s^{p+1} f^{k} a^{q+k}]$ ■

We see that the affine affine character of the reduced word depends both on affine character of the initial word (read in its canonical form) and on the generational depth k of the initial word.

Case II. Now, suppose $\langle W \rangle$ has the horizontal canonical form, that is to say, $\langle W \rangle = [s\ a^{q}]$. We check the sub-cases $\langle Wf \rangle$, $\langle Ws \rangle$, $\langle Wf^{-1} \rangle$.

5. $\langle Wf \rangle = \langle \langle W \rangle f \rangle = \langle [s^{p}\ a^{q}]\, f \rangle = \langle s^{p}\ a^{q}\ f \rangle$

$\langle s^{p}\ a^{q}\ f \rangle \rightarrow [s^{p}\ f\ a^{q}\ ]$ (rule A2). ■

6. $\langle Ws \rangle = \langle \langle W \rangle s \rangle = \langle [s^{p}\ a^{q}]\ s \rangle = \langle s^{p}\ a^{q}\ s \rangle \rightarrow \langle\ s^{p}\ s\ a^{q} \rangle = [s^{p+1}\ a^{q}]$ (A2). ■

In the next instance, we have again an instance of change in the affine character if the word involves a sex change:

7. $\langle W f^{-1} \rangle = \langle \langle W \rangle f^{-1} \rangle = \langle [s^{p}\ a^{q}]\, f^{-1} \rangle = \langle s^{p}\ a^{q}\ f^{-1} \rangle$

$\langle s^{p}\ a^{q}\ f^{-1} \rangle \rightarrow \langle s^{p}\ f^{-1}\ a^{q}\ \rangle$ (rule A2).

If p is even, $[s^{p}\ ] = e$ and this reduces to

$\langle\ f^{-1}\ a^{q}\ \rangle = \langle e\, f^{-1}\ a^{q}\ \rangle = \langle f^{-1}\ a^{q}\ \rangle$. (C-rule) ■

If p is odd, $[s^{p}] = s$ and we obtain:

$\langle (s\, f^{-1})\ a^{q}\ \rangle \rightarrow \langle (f^{-1} s\ a)\ a^{q} \rangle$ (rule A2)

$\langle f^{-1} s\ aa^{q} \rangle \rightarrow [f^{-1} s\ \ a^{q+1}]$ (rule D1). ■

Thus, in a horizontal word marked for sex, descending one-generation step change the affine character.

Case III. Finally, suppose $\langle {}^{-}W\ \rangle$ is in *descending form*. Then, $\langle {}^{-}W\ \rangle = [\,f^{-k}\ s^{p} a^{q}]$ with $k > 0$. We check $\langle {}^{-}Ws \rangle$, $\langle {}^{-}Wf^{1} \rangle$ and $\langle {}^{-}Wf \rangle$. The first instance is straightforward:

8. $\langle {}^{-}Ws \rangle = \langle \langle {}^{-}W \rangle\ s \rangle = \langle [f^{-k}\ s^{p}\ a^{q}]\ s \rangle = \langle f^{-k}\ s^{p}\ a^{q}\ s \rangle$

$\langle f^{-k}\ s^{p}\ a^{q}\ s \rangle \rightarrow \langle f^{-k}\ s^{p}\ s\ a^{q} \rangle = [f^{-k}\ s^{p+1}\ a^{q}]$ (Rule A2). ■

The next two cases in which a descending word is followed by a generation are straightforward when the word has an even sex index p (meaning that it involves no sex change). In these cases, $[s^{0}] = e$ and the result of adding a generation change does not affect the affine character.

$$\langle f^{-k}\ s^{0}\ f^{-1} a^{q} \rangle = \langle f^{-k}\, e\, f^{-1} a^{q} \rangle = \langle f^{-k-1} a^{q} \rangle = \langle f^{-(k+1)}\ a^{q} \rangle$$

$$\langle f^{-k} s^{0} f^{+1} a^{q}\rangle = \langle f^{-k} e f^{+1} a^{q}\rangle = \langle f^{-k+1} a^{q}\rangle = \langle f^{-(k-1)} a^{q}\rangle$$

We henceforth assume (for cases 9 and 10) that p is odd, and more specifically that $[s^{p}]= s$.

9. $\langle {}^{-}Wf^{-1}\rangle = \langle\langle {}^{-}W\rangle f^{-1}\rangle = \langle [f^{-k} s^{p} a^{q}] f^{-1}\rangle = \langle f^{-k} sa^{q} f^{-1}\rangle$

$\langle f^{-k} sa^{q} f^{-1}\rangle \rightarrow \langle f^{-k} sf^{-1} a^{q}\rangle$

$\langle f^{k} (sf^{-1}) a^{q}\rangle \rightarrow \langle f^{-k} (f^{-1} sa) a^{p}\rangle$ (rule D2)

$\langle f^{k} f^{-1} saa^{p}\rangle \rightarrow [f^{-(k+1)} sa^{p+1}]$ (rule A2). ■

Instance 9 says when a descending word involves a sex change, lowering a generation changes its affine character.

10. $\langle {}^{-}Wf\rangle = \langle\langle {}^{-}W\rangle f\rangle = \langle [f^{-k} sa^{q}] f\rangle = \langle f^{-k} sa^{q} f\rangle$ (we assume p is odd, as in 9).

$\langle f^{-k} s a^{q} f\rangle \rightarrow \langle f^{-k} s f a^{q}\rangle$ (rule A2).

$\langle f^{-k} s f a^{q}\rangle \rightarrow \langle f^{-k+1} (f^{-1} s f) a^{q}\rangle$ (this is a mere regrouping of terms)

$\langle f^{-k+1} (f^{-1} s f) a^{q}\rangle \rightarrow \langle f^{-k+1} (sa) a^{q}\rangle$ (rule D3)

$\langle f^{-k+1} saa^{q}\rangle \rightarrow [f^{-(k-1)} s a^{q+1}]$ (rule D1). ■

This concludes this tedious verification.

Corollary 1 to Proposition 1. An even number of ascending generation marks followed by a sex mark leaves the affine character invariant, and an odd number of ascending generations followed by a sex change changes the affine character. This result from case 4.

Example 1. The kin types ♂FF and ♀MM are clearly consanguineal, being translated in D* language as *ff* . According to Corollary 1, ♂FFZ is also consanguineal. In D* language, this is confirmed by the reduction

♂ffs = ♂$f(fs) \rightarrow$ ♂$f(sfa)$ = ♂$(fs)fa \rightarrow$ ♂$(sfa)fa \rightarrow$ ♂$sffaa$ = ♂sff ■ (rule A1)

The last word is in reduced-canonical form and has no affine mark. By the same token, ♂MMB is consanguineal, as one can check by reducing the word

♂$sffs$ = ♂$sf(fs) \rightarrow$ ♂$sf(sfa)$ = ♂$s(fs)fa \rightarrow$

$\rightarrow$ ♂$s(sfa)fa \rightarrow$ ♂$ssffaa$ = ♂$effe$ = ♂ff ■ (rule A1).

This is as predicted by Trautmann's Model B.

Corollary 2 to Proposition 1. A sex mark in a descending word, followed by an even number of generation marks, leaves the affine character unchanged, and a sex mark in a descending word followed by an odd number of generation marks changes the affine character. This corollary results from instances 9-10 that imply that, if a descending word ends with a sex change, adding one generation mark f^{-1} or f changes its affine character.

Example 2. Consider the kin word ♀DS. This is a consanguineal kin type, corresponding to the word $f^{-1}f^{-1}$s ending with a sex mark. Consider now adding two descending generations to it (♀DSSS):

$$♀f^{-1}f^{-1}sf^{-1}f^{-1} = ♀f^{-1}f^{-1}(sf^{-1})f^{-1} \rightarrow ♀f^{-2}(f^{-1}sa)f^{-1} \rightarrow$$

$$\rightarrow ♀f^{-3}sf^{-1}a \rightarrow ♀f^{-3}f^{-1}saa = ♀f^{-4}saa = ♀f^{-4} \ ■$$

The terminal expression is again consanguineal, as asserted in Corollary 2. We check now what happens when we add two *ascending* generations to the same word:

$$♀f^{-1}f^{-1}sff = f^{-1}(f^{-1}sf)f \rightarrow f^{-1}(as)f \rightarrow f^{-1}sfa \rightarrow as\,a \rightarrow saa \rightarrow se = s \ ■$$

This says that ♀DSFF is the same as ♀B.

We conclude this section with a note on the asymmetry between the two corollaries above. This is a consequence of two facts: words are formed from left to right in definition 1 and rules at the induction step are applied to the right:

$$W = (w_1 \ldots w_n)\, w_{n+1} = (w_1 \ldots w_n^{-1})(w_n\, w_{n+1})$$

Since ascending and descending canonical words are asymmetrical, applying the inductive process (which is the same as applying the rules) at the right side or at the left side of a word implies using different rules.

Words in D* could as well have been formed *from right to left,* and in that case natural parsing would have been

$$W = w_0(w_1 \ldots w_n) = (w_0\, w_1)(w_2 \ldots w_n)$$

What is the difference between a grammar that produces words from right to left as opposed to producing them from left to right? I think there is a difference worth mentioning. Adding a letter to the right of a word leaves ego invariant, and changes alter. It works as a translation in a coordinate frame. On the other hand, adding a letter to the left of a word transforms ego and leaves alter invariant, having the effect of a change of coordinates. This difference may have interesting empirical consequences.

II.2 Reducing a word by means of a permutation

In this section, we obtain the canonical form by means of a different and more direct method.

Proposition 2: The reduction of a word W in D^* to the canonical form is equivalent to a permutation followed by a contraction.

Suppose a word W in D* has p occurrences of "s", m occurrences of "f", n occurrences of "f^{-1}" and q occurrences of "a". Then, a form very near the canonical form may be immediately be obtained by means of two actions: a permutation of W, and a contraction.

- Ascending precanonical form: $W' a^t = [s^p f^{m-n} a^q]\, a^t$ if m>n,
- Descending precanonical form: $W'a^t = [f^{m-n} s^p a^q]\, a^t$ if m<n
- Horizontal precanonical form: $W'a^t = [s^p a^q]\, a^t$ if m = n.

We stress the fact that here the word W' within brackets is a permutation of the original word W. Observe however that we must add an additional term a^T in order to obtain the correct parity for the canonical form. This term is the result of the changes of parity resulting from using rules D1-D2 and D3. We now look for a way to determine the index T in a^T.

Call $W = W_0$. We may assume that W does not contain the term "a^q" (we may always reinsert the term "a^q" at the end of the calculation). Thus, W is a string of letters s, f and f^{-1}. Form the sequence

$$W = W_0 a^0 \Rightarrow W_1 a^1 \Rightarrow \ldots \Rightarrow W_T a^T = W' a^T$$

where each step $W^k a^k \Rightarrow W^{k+1} a^{k+1}$ is done as follows:

1. Suppose W is ascending. If every "s" in W^k is already at the left of every other letter, stop. If not, obtain $W^{k+1} a^{k+1}$ from W^k by looking for the first "s" at the right of an "f" or of an "f^{-1}" and shift it to the left by means of a transposition ($fs \rightarrow sf$), or means of a transposition ($f^{-1}s \rightarrow sf^1$), and substitute a^{k+1} for a^k. In other words, make a left transposition of an "s" and record the step by adding an affine mark.

2. Suppose W is descending. If W^k cannot be reduced by D-rules, stop. If not, $W^{k+1}a^k$ results from W^k by a transposition ($sf^{-1} \rightarrow f^{-1}$s) or by means of a transposition ($sf \rightarrow fs$) in W^k followed by an added term that changes a^k into a^{k+1}.

At the end of the series of t transpositions of an ascending word, $W^T = W'$ cannot be further reduced by D-rules: every "s" precedes every "f", and follows every "f^{-1}". We may write $W' = s^p f^{m-n}$ where m >n and p, m and n are the number of occurrences of "s", "f" and "f^{-1}" in W respectively, while a^T is a record of the number of transpositions of "s" that were performed. In a descending word, we obtain a word $W' = f^{m-n} s^p$, as well as a term a^t which is also a record of the number of transpositions performed.

We have thus found that the exponent T in a^T is obtained as the number of transpositions used in a reduction. Now, all left-transpositions of "s" can be effected either by means of rule D1 ($fs \rightarrow sfa$) or by means of its reverse rule D2' ($f^{-1}s \rightarrow sf^1 a$), and all right-transpositions of "s" can be performed either by rule D2 ($sf^{-1} \rightarrow f^{-1}$sa) or by its reverse rule D1' ($sf \rightarrow fsa$). At every use of these rules, the new term "a" added to the record is shifted by means of rule A2 to the rightmost side of the word. Thus, every step in the process described above can be justified by means of the rules. We used “reverse rules” in order to avoid the use of rule D3. However, they are equivalent to it. Recall that rule D3 is $f^{-1}sf \rightarrow sa$. This can be seen as either a left-transposition $f^{-1}sf \rightarrow sf^{-1}fa$ or as a right-transposition $f^{-1}sf \rightarrow f^{-1}fsa$, according to whether one deals with an ascending word or with a descending word. Therefore, we have indeed used all ruled D1-D3.

We now recall the following facts from elementary group theory.

Principle I. Any permutation can be represented as a product of transpositions.

Principle II. Either all the representations of a given permutation use an odd number of transpositions or they all use an even number of transpositions.

Principle I ensures that given a word *W,* it is always possible to put it in the pre-canonical form (i.e. a word that will become a canonical word after being contracted), by means of rules D1 and D2 used as transpositions (in their direct and converse forms). Principle II guarantees that the affine result is unique (1943, Satz 92; Fraleigh 1967, Theorem 5.2, and Garcia & Lequain 1988, Proposition IV.27, etc.).

II.3 Rules of thumb: Dravidian and Iroquois

The transposition approach to obtain the canonical form provides a rule for transforming any word into its canonical form without going through all derivations.

By a *left transposition* we mean either ($fs \rightarrow sf$) or ($f^{-1}s \rightarrow sf^{-1}$). By a *right transposition* we mean either ($sf \rightarrow fs$) or ($sf^{1} \rightarrow f^{1}s$). Using this language, we state the following rule.

Corollary 3 to Proposition 2. *Dravidian rule of thumb for affinity*

The affine character of a word W is the parity of the permutation that puts its W in the canonical order.

Special cases of Corollary 3:

1. The affine character of an ascending word is 0 or 1 according to whether the number of left transpositions that take all its "s" to the ascending canonical order is even or odd.

2. The affine character of a descending word is 0 or 1 according to whether the number of right transpositions that take all its "*s*" to the descending canonical order is even or odd.

Example 1. Consider ♂MFZS, written as ♂$sfsfsf^{-1}s$. We check that it is ascending because the generational terms can be grouped as $f^2 f^{-1} = f^{2-1} = f$.

Rearrange the letters in the ascending canonical order by a single permutation:

Perm $(sfsfsf^{-1}s) = ssssfff^{-1}$

We can contract this result to obtain *f*:

[**Perm** ($sfsfsf^{-1}s$)] = [$ssssfff^{-1}$] = f.

Check the parity of **Perm** by counting, for each "*s*" not already in the canonical position, how many "f" and "f^{-1}" must be transposed to reach the canonical (leftmost) position. In the above example, for the first "*s*" this number is 1; for the second "*f*" the number of 2, and for the third "*f*" this number is 3. The resulting number of transpositions is therefore 1 + 2 + 3 = 6. This has parity even, so that the affine character is 0.

$[T(sfsfsf^{-1}s)]\,[a^{1+2+3}] = [ssssfff^{-1}a^{6}] = [s^{4}f^{2-1}a^{6}] = f$

We conclude that ♂MFZS/♀FMBD (♂ $sf\,sf\,sf^{1}s$) is classified as a consanguineal relative and is equivalent to a ♂F/♀M.

An Iroquois rule of thumb

In an ascending word W, count the number of non-initial occurrences of "*s*" (shift every "*s*" to the left of every "f" and of every "f^{-1}", counting one operation for each "*s*"). W is cross or parallel according to whether this number is odd or even.

In a descending word, count the number of non-terminal occurrences of "*s*" (shift every "*s*" to the right of every "f" and of every "f^{-1}", counting one operation for each "s"). The word is again cross or parallel according to the parity of the number obtained.

In a horizontal word, the cross index is the number of non-initial, non-terminal occurrences of "*s*". In this case, either of the above shifting methods gives the same result. [9]

II.4 A characteristic function is impossible

One peculiarity of the Dravidian structure is its algebraic simplicity. This fact is most clearly evidenced by means of the signed notation. A signed word is a word in D* where "*e*" is replaced by "1" and "*a*" is replaced by "-1". Thus, words in signed version are concatenations of letters in $D = \{1, f, s, f^{-1}, -1\}$. The contracted version of a signed word is obtained by manipulating "1" and "-1" by using the usual properties of multiplication. This takes care automatically of the properties of "*a*", since now they follow properties of of "-1" in elementary algebra.

We list in a summary way the signed version of C rules, D rules and A rules. For any word w in Ds*:

[9] A justification of these rules of thumb, based on unified approach to Dravidian and Iroquois systems, is provided in another paper. As a matter of fact, the rules of thumb for the Iroquois system were obtained before the underlying rules were formulated.

C-rules in signed notation:

$ff^{-1} = f^{-1}f = 1,\ 1w = w,\ w1 = w,\quad ss = 1$

D-rules in signed notation:
$(-1)(-1) = 1,\quad (-1)w = w(-1) = -w$

A-rules in signed notation:

Rule A1.	fs	$\rightarrow -\ sf$
Rule A2	sf^{-1}	$\rightarrow -f^{-1}s$
Rule A3.	$f^{-1}sf$	$\rightarrow -\ s$

We use the signed notation to show why there is a reason why, in a certain sense, no characteristic function for affinity can be defined on the universe of kinship and affinity words D*.

Proposition 3.

There is no function $\varphi : [D^*] \rightarrow \{1, -1\}$ satisfying all the following conditions:

(1) $\varphi(f) = 1,\ \varphi(s) = 1$.

(2) $\varphi(fs) = -1,\ \varphi(sf^{-1}) = -1$

(3) $\varphi(wz) = \varphi(w)\varphi(z)$ for any words w and z.

Condition (1) says that φ should mark genitors and opposite-sex siblings as consanguineal. Condition (2) says that φ should mark cross-uncles/aunts, and cross-nephews/nieces, as cross. Condition (3) says that φ can be used to obtain the affinity of a composite word based on the affinity of is components. More precisely, condition (3) says that φ is an homomorphism of D* into {1, -1} where composition in D* is the contracted concatenation of words, and composition in {1, -1} is Boolean multiplication.

The proof of Proposition 3 is straightforward. For suppose that such a function φ exists. Then,

$\varphi(fs) = -1$ by (2)

but

$\varphi(fs) = \varphi(f)\,\varphi(s) = 1 \times 1 = 1.$ by (3 and 1).

This contradiction shows that no mapping φ can satisfy (1), (2) and (3).

III. Trautmann's model revisited

In this section I deal with the relationship between the system presented in the previous sections and Trautmann's theory of Dravidian systems. We show that Trautmann's calculus (rules 1-7) reproduces the reduction process of the *D** calculus, and we approach the issue of "double classification" involving his rules 8A-9A. This section, except for sub-section III.1, can be skipped without loss of continuity, for a reader who can convince himself that the rules presented in the two preceding sections are indeed equivalent to Trautmann's rules for Model B.

III.1. The canonical classification and model B are the same.

Recall from Definition 4' that canonical Dravidian form comes in three varieties:

I. Ascending canonical forms $[s^p] f^k$ $[s^p] f^k a$

II. Horizontal canonical form $[s^p]$ $[s^p]\, a$

III. Descending canonical form $f^{-k} [s^p]$ $f^{-k} [s^p]\, a$

For $p = 0$ or $p = 1$, $k > 0$, with $[s^1] = s$, $[s^0] = e$. We now take k in the interval $0 < k \leq 2$, and replace s^p with e or p.

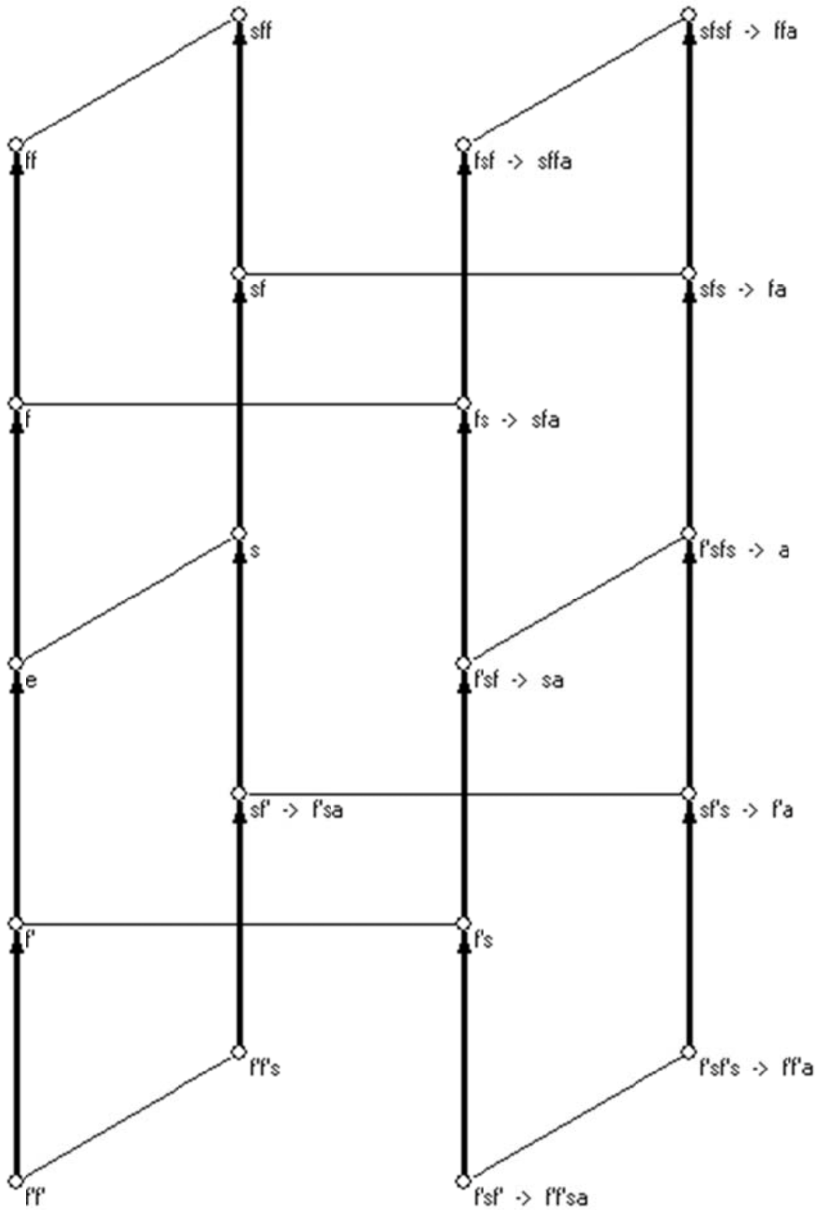
Table 1. Canonical form at five generation levels (Trautmann's Model B).

	non-affine		*affine*	
G^{+2}	f^2	sf^2	$f^2 a$	$sf^2 a$
G^{+1}	f	sf	$f\, a$	$sf\, a$
G^0	e	s	a	sa
G^{-1}	f^{-1}	$f^{-1} s$	$f^{-1} a$	$f^{-1} sa$
G^{-2}	f^{-2}	$f^{-2} s$	$f^{-2} a$	$f^{-2} s\, a$

In Figure 2 below, we show how the words in Table 1 are connected. This three-dimensional graph should be compared with the two-dimensional graph of Figure 1. The paths can be obtained by means of the following convention: ↑ stand for *f*, ↓ stand for f^{-1}, and ↔ stand for *s*. The vertices in the graph are labelled with the Dravidian *reduced* (canonical) form of words constructed with these two symbols. Thus, for instance, the vertice at the end of path *fs* is labeled as *sfa* because this is the canonical form of *fs,* and the vertice at the end of the path sf^{-1} is labelled $f^{-1}sa$.

Paths $f^{-1}sfs$ and $fsf^{-1}s$ have terminal vertices labelled as *a* and it can be seen that they indeed correspond to affine relations and cross-cousin relations, which are here the same path. It may be noted that "diagonal" imaginary lines connect affines to each other.

Figure 2. Trautmann's box (Model B) in *D notation (5 generations).**



The labels at each vertex shows path leading to them and the Dravidian reduced (canonical) form. As an example, the vertex labelled *fs* is labeled as $fs \rightarrow sfa$, meaning that *sfa* is the reduced form of *fs*. [10] Several paths can lead to the same vertex. Thus, $fsf^{-1}s$ (a cross-cousin path) and $f^{-1}sfs$ (an affine path) both lead to the vertex *sa*. In the above graph, the relation *a* is not explicitly represented as an edge.

[10] Cf. Harrison White (1963, p. 21 Fig. 1.13). In my notation, White's graph would generated by *f* and *a* alone, and therefore no sex difference is represented on it.

For comparison, this is Trautmann's paradigm for Dravidian Model B, together with kin type notation, its literal translation in D* language and the canonical form.

Table 2. A Dravidian paradigm (Hill Maria). Male ego.

	non-affine		affine	
G^{+2}	tado: ♂FF ♂ff ≈ $C_{=}^{+2}$	kako: ♂MM FF ♂sff ≈ $C_{\neq}^{+2}$	ako: ♂MF ♂$sfsf$→♂ffa ≈ $A_{=}^{+2}$	bapi: ♂FM ♂fsf→♂$sffa$ ≈ $A_{\neq}^{+2}$
G^{+1}	tappe: ♂F ♂f ≈ $C_{=}^{+1}$	talugh: ♂M ♂sf ≈ $C_{\neq}^{+1}$	mama: ♂MB ♂sfs → ♂fa ≈ $A_{=}^{+1}$	ato: ♂FZ, ♂MBW ♂fs → ♂sfa ≈ $A_{\neq}^{+1}$
G^{0}	dada: ♂eB, ♂FeBS ♂e ≈ $C_{=}^{0}$	akka: ♂eZ, ♀HeBW ♂s ≈ $C_{\neq}^{0}$	marriyo: ♂MBS, ♀FZS ♂$sfsf^{-1}$→ a ≈ $A_{=}^{0}$	mandari:♂MBD,♀FZS ♂$sfsf^{-1}s$ → sa ≈ $A_{\neq}^{0}$
G^{-1}	maghi: S ♂f^{-1} ≈ $C_{=}^{-1}$	miari:D ♂$f^{-1}s$ ≈ $C_{\neq}^{-1}$	♂bacha: ♂ZS, ♀BD $sf^{-1}s$ → $f^{1}a$ ≈ $A_{=}^{-1}$	♂bachi: ♂ZD, ♀BD ♂sf^{-1}→ $f^{-1}sa$ ≈ $A_{\neq}^{-1}$
G^{-2}	tamo:♂SS, ♀DS ♂f^{-2} ≈ $C_{=}^{-2}$	hellar: ♂SD ♂$f^{-2}s$ ≈ $C_{\neq}^{-2}$	ako: ♂DS, ♀SD ♂$f^{-1}sf^{1}s$ → $f^{-2}a$ ≈ $A_{=}^{-2}$	ako: ♂DD ♀SS $f^{1}sf^{-1}$ → $f^{-1}f^{1}sa$ ≈ $A_{\neq}^{-2}$

Source: Trautmann 1981, p. 196, Fig.3.32.

The agreement between the canonical form and Trautmann's Dravidian paradigm suggests that indeed both systems are equivalent. Indeed, it is easy to infer from the above table that every one of Trautmann's terminal expressions is in canonical form according to the following correspondence:

Canonical form I. $[\,s^{p} f^{m}\,] \approx C_{p}^{m}$ $\qquad [\,s^{p} f^{m} a] \approx A_{p}^{m}$

Canonical form II. $[\,s^{p}] \approx C_{p}^{0}$ $\qquad [\,s^{p} a] \approx A_{p}^{0}$

Canonical form III. $[f^{-n} s^{p}] \approx C^{p-n}$ $\qquad [f^{-n} s^{p} a] \approx A_{p}^{-n}$

where p stands for "1" or "0" in D* formulas, and p stands for "≠" or" =" in Trautmann's formulas.

Can we also reproduce within Trautmann's rules the reduction process described in the previous section? One reason for making this implausible at first sight is the number of rules in Trautmann's list (Trautmann 1981:179; Godelier, Trautmann and Fat 1998:18). There are

11 groups of rules numbered 1, 2, 3, 4, 5, 6, 7, 8A, 9A, 8B, 9B. Each group comprises four schemata, in a total of 44 schemata, and each schema generates according to the four rules according to the possible choices of sex indices. Thus, there are 44 x 4 = 176 sub-rules. Moreover, Trautmann's calculus has two different rules to calculate affinity for generations G^{+2} and G^{-2} (rules 8A-9A and rules 8B-9B). Rules 8A-9A are supposed to produce "double classification", and rules 8B-9B encompasses an infinite number of generations (special rules for generations G^{+2} and G^{-2} are therefore redundant) and no "double classification" arises.

Another apparent discrepancy is that the keystone of the D* calculus is non-commutativity in general and anti-commutativity in particular, while no such property appears to be present in Trautmann's rules. Another way of putting it is that while the proposed D* language emphasized the ordering of sex and generation in kinship and affinity words, Trautmann's symbols treat sex and generation as non-ordered dimensions of a semantic space.

Let us start with this last point. It would seem that Trautmann's symbols could be represented as unordered triplets

$$(A^{x}, C_{p}, C^{m})$$

where the x, p and m are indexes for affinity, sex and generation. This mode of representing Trautmann's symbols brings to the fore the fact that the notation employed by Trautmann represent relations is a space of three dimensions. Suppose the indices x and p may have values 0 or 1, and the index m is in the range -2 to +2. When we interpret A^1 as the absence of a mark A, the symbol C_1 as $C_{\neq}$, and if we interpret A^0 as the absence of a mark A, and C_0 as $C_{=}{}^{0}$, it becomes clear that this notation is equivalent to a triple in our notation:

$$(a^{x}, s^{p}, f^{m})$$

where x, p and m may have the values 0 and 1.

This unordered triple looks very much like the canonical Dravidian form – if ordered in the right way. The issue is precisely of how it should be ordered.

To see the importance of ordering, consider how two triples should be composed to form another word. One would expect the composition law * between unordered symbols to take the follow form.

$$(a^{x}, s^{p}, f^{m}) * (a^{y}, s^{q}, f^{n}) = (a^{z}, s^{r}, f^{t}) \quad \text{where } z = \varphi(x, y).$$

This equation has an immediate solution for the sex term and for the generation term, which is

$$r = |p+q| \ , \quad t = m+n$$

where $|p+q|$ is Boolean addition and m+n is ordinary addition. The whole difficulty lies in expressing the index z as a function of x and y, and in way consistent with the consanguineaffine structure of Dravidian terminologies. Proposition 3, asserting that no characteristic function exists, suggests that such as function is indeed not possible if it is required that it should depend only on *x* and *z*. I believe that this is the reason why, in order to solve the problem of determining the value of *z* in the above equation, Thomas Trautmann constructed a whole set of transformation rules. We argue that, in spite of their complex appearance, Trautmann's rules have a hidden, simple structure, which is made transparent – particularly in the signed version -- by the rules of the D* calculus to which Trautmann's calculus is indeed isomorphic. This is what we proceed to show: that at the bottom of Trautmann's rules there is a rock-solid group-theoretical foundation (classificatory rules) upon which anti-commutative rules (A rules and D rules) erect the Dravidian house.

III.2. The calculus D* is isomorphic to Trautmann's model B.

We start by providing a translation guide between the language D* and Trautmann's T-language, relying on Trautmann's own rules. Second, we show that a subset of Trautmann's rules 1-7 imply D* rules C, D and A. Thirdly, we show how rules C, D and A of the D* calculus imply the entire corpus of Trautmann's rules 1-7. As a corollary, rules 9A and 9B are shown to be a result of the previous rules.

Proposition 4. Trautmann's rules T1-T7 imply rules A, C, and D.

We start with the translation of Trautmann's symbols into D* language. This task will result in the conclusion that Trautmann's rules imply rules A1, A2 and A3. The other cases will follow in a straightforward manner.

Translating Trautmann's composed symbols

The unordered approach to the classification of Dravidian terminologies works without problems for those symbols we call *simple* or *pure,* involving at most one marked dimension (or at most a single non-zero index) each. These are:

Translation for Simple symbols

1. $C_{=}^{0} \approx e$ (a^0, s^0, f^0)

2. $C_{\neq}^{0} \approx s$ (a^0, s^1, f^0)

3. $C_{=}^{+1} \approx f$ (a^0, s^0, f^{+1})

4. $C_{=}^{-1} \approx f^{-1}$ (a^0, s^0, f^{-1})

5. $A_{=}^{0} \approx a.$ (a^1, s^0, f^0)

Translation for Composed symbols

All other symbols in Trautmann's notation involve two or more marked dimensions. We call them composite symbols or mixed symbols. This brings to the fore the issue of order. Consider the symbol $C_{\neq}^{+}$ with the two marked dimensions of sex and generation. It should be represented at first sight as the unordered triplet (a^0, s^1, f^1). Now, there are two symbols in D* notation involving the symbols *s* and *f*, namely, *sf* and *fs*. The apparent ambiguity is resolved once we realize that these orderings are well expressed in Trautmann's notation as the composition of simple symbols $C_{\neq}^{0}C_{=}^{+1}$ and $C_{=}^{+1}C_{\neq}^{0}$. The problem is now: how are these "composite symbols" expressed as a mixed symbol?

I show now how the composition simple symbols is governed by Trautmann's own rules, and this settles the issue. Indeed, rule $T2.1_{=\neq}$ says that $C_{=}^{+1}C_{\neq}^{0} \rightarrow A_{\neq}^{+1}$, while rule $T5.4_{\neq=}$ says that $C_{\neq}^{0}C_{=}^{+1} \rightarrow C_{\neq}^{+1}$. This implies that $C_{\neq}^{+1}$ is the mixed symbol for *sf* and $A_{\neq}^{+1}$ is the mixed symbol for *fs*. We record these translations as follows:

6. $C_{\neq}^{0}\, C_{=}^{+1} \rightarrow C_{\neq}^{+1} \approx$ *sf* ($T5.4_{\neq=}$)

7. $C_{=}^{+1}\, C_{\neq}^{0} \rightarrow A_{\neq}^{+1} \approx$ *fs* ($T2.1_{=\neq}$, translation of $A_{\neq}^{+1}$)

Lines 6 and 7 above imply that $C_{\neq}^{0}\, C_{=}^{+1} \neq C_{=}^{+1}\, C_{\neq}^{0}$. In our notation, lines 6 and 7 express the inequality $fs \neq sf$. Moreover, Lines 6 and 7, together with $A_{=}^{0} \approx a$, imply

7a. $C_{=}^{+1}\, C_{\neq}^{0} \rightarrow A_{=}^{0}\, C_{\neq}^{0}\, C_{=}^{+1}$.

This corresponds to our rule $fs \rightarrow asf$. Line 7a may be understood as the definition of $A_{\neq}^{+1}$ in simple symbols.

We repeat the argument with the mixed symbol $C_{\neq}^{-1}$, which seems to express the unordered triple (a^0, s^1, f^{-1}). There are two possible ordering: $C_{=}^{-1}C_{\neq}^{0}$ corresponding to $f^{-1}s$, and $C_{\neq}^{0}C_{=}^{-1}$, which is expressed as sf^{-1} in my notation. Again, Trautmann's rules settle the issue of which mixed symbol should correspond to this composition of simple symbols.

Rule $T4.1_{\neq=}$ says $C_{\neq}^{0}\, C_{=}^{-1} \rightarrow A_{\neq}^{-1}$, while rule $T7.4_{=\neq}$ says that $C_{=}^{-1}C_{\neq}^{0} \rightarrow C_{\neq}^{-1}$. And this settles without ambiguity the following renderings:

8 $C_{=}^{-1}C_{\neq}^{0} \rightarrow C_{\neq}^{-1} \approx f^{-1}s$ ($T7.4_{=,\neq}$)

9. $C_{\neq}^{0}\, C_{=}^{-1} \rightarrow A_{\neq}^{-1} \approx sf^{-1}$ ($T4.1_{\neq,=}$)

Thus, $f^{-1}s \neq f^{-1}sa$.
In addition, taking into account 8 and 9,

9a. $C_{\neq}^{0}\, C_{=}^{-1} \rightarrow C_{=}^{-1}C_{\neq}^{0}\, A_{=}^{0} \approx sf^{-1} \rightarrow f^{-1}sa$.

We are thus in the very core of the D* calculus, namely the affinizing rules A1 and A2. (The position of $A_{=}^{0}$ and of *a* is not relevant, since under Dravidian rules the affine term commute with all other terms.)

We move on to the composite symbol $A_{\neq}^{0}$. Here, the unordered interpretation suggests the triplet (a^1, s^1, f^0). This offers no real problem since in this case the possible orderings $A_{=}^{0}C_{\neq}^{0}$ and $C_{\neq}^{0}A_{=}^{0}$ or *as* and *sa* are identified under D2 rules.

Indeed, this identification, together with the commutativity $af = fa$ and $af^{-1} = f^{1}a$, are the real distinctive principles under the Dravidian system.[11] What we show now is that the composite symbol $A_{\neq=}$ can be seen as the composition of $C_{=}^{+1}\, C_{\neq}^{0}\, C_{=}^{-1}$ and also as the composition of $C_{=}^{-1}\, C_{\neq}^{0}\, C_{=}^{+1}$ under all possible associations. In addition, this is the same as asserting that $fsf^{-1} = f^{-1}sf = sa$, an assertion that which identifies opposite-sex cross cousins, opposite-sex parents of common children, and opposite-sex affines.

First, $fsf^{-1} \approx C_{=}^{+1}C_{\neq}^{0}C_{=}^{-1}$ by a letter-by-letter translation (cf. 2, 3 and 4 above). We associate the second expression to the left and use Trautmann's rules to fill in the sub-expressions within parentheses, using the definitions for composite symbols already obtained:

10. $(C_{=}^{+1}C_{\neq}^{0})C_{=}^{-1} = A_{\neq}^{+1}\, C_{=}^{-1}$	(Translation **7** above)
$A_{\neq}^{+1}\, C_{=}^{-1} \rightarrow A_{\neq}^{0}$	$(T3.1_{\neq=})$
$\approx (fs)f^{-1} \rightarrow (sfa)f^{-1} \rightarrow sff^{-1}a = sa$	(A1, D2, C)
11. $C_{=}^{+1}\,(C_{\neq}^{0}C_{=}^{-1}) \rightarrow C_{=}^{+1}\, A_{\neq}^{-1}$	(Translation **9** above)
$C_{=}^{+1}\, A_{\neq}^{-1} \rightarrow A_{\neq}^{0}$	$(T3.3_{=\neq})$
$\approx f\,(sf^{1}) \rightarrow f\,(f^{-1}sa) = esa = sa$	(A2, C)

This confirms that bilateral opposite-sex cross cousins are identified with opposite-sex affines.Next, consider the mixed symbols $C_{=}^{-1}C_{\neq}^{0}C_{=}^{+1}$ or $f^{-1}sf$.

12. $(C_{=}^{-1}C_{\neq}^{0})\, C_{=}^{+1} = C_{\neq}^{-1}\, C_{=}^{+1}$	(Translation **8** above)
$C_{\neq}^{-1}\, C_{=}^{+1} \rightarrow A_{\neq}^{0}$	$(T6.1_{\neq=})$
$(f^{-1}s)f \rightarrow sa$	(A3)
13. $C_{=}^{-1}(C_{\neq}^{0}C_{=}^{+1}) = C_{=}^{-1}\, C_{\neq}^{+1}$	(Translation **6** above)
$C_{=}^{-1}\, C_{\neq}^{+1} \rightarrow A_{\neq}^{0}$	$(T6.1_{=\neq})$
$f^{-1}(sf) \rightarrow sa$	(A3)

Finally,

14. $A_{=}^{0}\, C_{=}^{+1} \rightarrow A_{=}^{+1} \approx af = fa$	$(T5.1_{==})$
15. $A_{=}^{0}\, C_{=}^{-1} \rightarrow A_{=}^{-1} \approx af^{-1} = f^{-1}a$	$(T4.1_{==})$

[11] We leave this point for Section IV. This is the reason for calling them "Dravidian rules" (D-rules).

We conclude that Trautmann's symbol $A_{\neq}^{0}$ subsumes opposite-sex cross cousins and opposite-sex affines.

This completes task of justifying the translation of composite symbols in Trautmann's theory. We regroup in a table the translations obtained.

Table 3. Trautmann's simple symbols and *D* words.

Def.	*T symbols*	*D* words*	♂	♀
1	$C_{=}^{0}$	*e*	B	Z
2	$C_{\neq}^{0}$	*s*	Z	B
3	$C_{=}^{+1}$	*f*	F	M
4	$C_{=}^{-1}$	f^{-1}	S	D
5	$A_{=}^{0}$	*a*	WB	HZ

Table 4. Trautmann's composed symbols

Def.	*T symbols* →	*Composite symbols*	*D* words*	*Kin types* ♂	*Kin types* ♀	
6	$C_{\neq}^{0} C_{=}^{+1}$	$C_{\neq}^{+1}$	*sf*	ZM=M	BF=F	T5.4$_{\neq =}$
8	$C_{=}^{-1} C_{\neq}^{0}$	$C_{\neq}^{-1}$	$f^{-1}s$	SZ=D	DB=S	T7.4$_{= \neq}$
7	$C_{\neq}^{+1} A_{=}^{0}$	$A_{\neq}^{+1}$	*sfa*	MA	FA	T5.4$_{\neq =}$, T2.1$_{= \neq}$
9	$C_{\neq}^{-1} A_{=}^{0}$	$A_{\neq}^{-1}$	$f^{-1}a$	SA	DA	
10	$C_{\neq}^{0} A_{=}^{0}$	$A_{\neq}^{0}$	*sa*	ZA=W	BA=H	

Trautmann's rules imply C-rules and D-rules.

We proceed to show in a synoptic manner how Trautmann's rules (in fact, its sub-rules) imply C-rules, A-rules, as well as D-rules.First, C-rules are contained under Trautmann's rules T1, T2, T3, T4, T5, T6 and T7 (12 sub-cases in all)

C1. Inverses of *s* and f^{-1} (3 sub-rules)
$f^{-1}f = e$ ≈ $C_{=}^{-1} C_{=}^{+1} \to C_{=}^{0}$ (T6.2$_{==}$)
$ff^{-1} = e$ ≈ $C_{=}^{+1} C_{=}^{-1} \to C_{=}^{0}$ (T3.4$_{==}$)
$ss = e$ ≈ $C_{\neq}^{0} C_{\neq}^{0} \to C_{=}^{0}$ (T1.4$_{\neq\neq}$)
C2. Behavior of *e* as the identity (9 sub-rules)
$ae = a$ ≈ $A_{=}^{0} C_{=}^{0} \to A_{=}^{0}$ (T1.1$_{==}$)
$ea = a$ ≈ $C_{=}^{0} A_{=}^{0} \to A_{=}^{0}$ (T1.3$_{==}$)
$ee = e$ ≈ $C_{=}^{0} C_{=}^{0} \to C_{=}^{0}$ (T1.4$_{==}$)
$es = s$ ≈ $C_{=}^{0} C_{\neq}^{0} \to C_{\neq}^{0}$ (T1.4$_{=\neq}$)
$se = s$ ≈ $C_{\neq}^{0} C_{=}^{0} \to C_{\neq}^{0}$ (T1.4$_{\neq=}$)
$fe = f$ ≈ $C_{=}^{+1} C_{=}^{0} \to C_{=}^{+1}$ (T2.2$_{==}$)
$ef^{-1} = f^{-1}$ ≈ $C_{=}^{0} C_{=}^{-1} \to C_{=}^{-1}$ (T4.2$_{==}$)
$ef = f$ ≈ $C_{=}^{0} C_{=}^{+1} \to C_{=}^{+1}$ (T5.4$_{==}$)

$f^{-1}e = f^{-1}$ ≈ $C_{=}^{-1} C_{=}^{0} \rightarrow C_{=}^{-1}$ ($T7.4_{==}$)

Next, affinization rules A1-A3 are implied by sub-cases of Trautmann's rules T2, T4 and T6 (4 sub-cases in all).

Affinization rules

A1. $fs \rightarrow sfa$ ≈ $C_{=}^{+1} C_{\neq}^{0} \rightarrow A_{\neq}^{+1}$ ($T2.1_{=\neq}$)
A2. $sf^{-1} \rightarrow af^{-1}s$ ≈ $C_{\neq}^{0} C_{=}^{-1} \rightarrow A_{\neq}^{-1}$ ($T4.1_{\neq=}$)
A3. $(f^{-1}s)f \rightarrow sa$ ≈ $C_{\neq}^{-1} C_{=}^{+1} \rightarrow A_{\neq}^{0}$ ($T6.1_{\neq=}$)
$f^{-1}(sf) \rightarrow sa$ ≈ $C_{=}^{-1} C_{\neq}^{+1} \rightarrow A_{\neq}^{0}$ ($T6.1_{=\neq}$)

Finally, D-rules that contain the gist of the Dravidian system are implied by Trautmann's rules T1, T2, T5, T7 (6 sub-rules in all).

D1 rule (1 rule)
$aa \rightarrow e$ ≈ $A_{=}^{0} A_{=}^{0} \rightarrow C_{=}^{0}$ ($T1.2_{==}$)

D2 rule (with *s* as *W*)
$as \rightarrow sa$ ≈ $A_{=}^{0} C_{\neq}^{0} = C_{\neq}^{0} A_{=}^{0}$.

This results from the following two rules that together imply the equality shown above in Trautmann's notation:

$A_{=}^{0} C_{\neq}^{0} \rightarrow A_{\neq}^{0}$ ($T1.1_{=\neq}$) and $C_{\neq}^{0} A_{=}^{0} \rightarrow A_{\neq}^{0}$ ($T1.3_{\neq=}$).

We have normalized the position of the letter "*a*" by placing it at the right side. Therefore, our formulation has the form of a directed replacement rule.

D2 rule (with *f* as *W*)

$af \rightarrow fa$ ≈ $A_{=}^{0} C_{=}^{+1} = C_{=}^{+1} A_{=}^{0}$.

As an equality, this rhis results jointly from:

$A_{=}^{0} C_{=}^{+1} \rightarrow A_{=}^{+1}$ ($T5.1_{==}$, cf. 14) and $C_{=}^{+1} A_{=}^{0} \rightarrow A_{=}^{+1}$ ($T2.1_{==}$).

We have normalized the placement of "a", and instead of an identity used a directed replacement rule.

D2 rule (with f^{-1} as *W*)

$af^{-1} \rightarrow f^{1}a$ ≈ $A_{=}^{0} C_{=}^{-1} = C_{=}^{-1} A_{=}^{0}$. This results from:

$A_{=}^{0} C_{=}^{-1} \rightarrow A_{=}^{-1}$ ($T4.1_{==}$, cf. 15) and $C_{=}^{-1} A_{=}^{0} \rightarrow A_{=}^{-1}$ ($T7.3_{==}$).

As in the previous cases, Trautmann's rules imply in fact an identity. We have here used a directed rule instead of an identity.

The phrasing of D-rules show, the only substantive difference between the content of our rules and Trautmann's rules is that we have chosen, in an arbitrary way, a definite placement for the mark of affinity. The convention of placing this mark at the right of a word in canonical form is arbitrary, and a different convention could have been adopted. In Trautmann's notation the issue does not come up since the order of letters is not dealt with there. This feature of Trautmann's rules is reflected in our signed version, in which we can ignore the placement of the "-1" affine mark.

This completes the proof that Trautmann's rules (T-rules) imply rules C1-C2, A1-A3 and D1-D2. In the course of the above derivation, we never needed the full set of Trautmann's rules, but only 12 sub-cases of rules T1-T7 for our C-rules, 4 sub-cases of rules T2, T4 and T6 for our A-rules, and 6 sub-cases of rules T1, T2, T5 and T7 for our D-rules, in a total of 12+4+6 = 22 sub-cases of Trautmann's full set of rules.

This is far below the figure of 112 (=7 x 16) sub-cases of rules T1-T7. We have thus proved that Trautmann's rules imply the D* calculus.

Proposition 5. Rules C, A and D generate all of Trautmann's sub-rules (1-7, 9A and 9B).

In the following argument we use indices in order to abbreviate the notation, noting that several of Trautmann's rules can be expressed in a shorthand way. The letters *p, q, r, s* will be used to denote *same sex* or *opposite sex* in Trautmann's notation (where they may be replaced for "=" and "≠", and sometimes are used to denote *generation* also in Trautmann's notation. Thus, A_p^q subsumes $A_=^{+1}$ when p is replaces by "=" and q is replaces by "+1". In our notation, the same symbols are used as variables that may be replaced by 0 or 1, and are used as exponents. Thus, a^q may be $a^1=a$ or $a^0 = e$ according to whether q replaced with "0" or with "1". An important caveat: exponents of *s* and of *a* are are always summed modulo 2: thus, $s^1s^1=s^{1+1} = s^0 = e$, and $a^1a^1=a^{1+1} = a^0 = e$. On the other hand, when exponents are used in *f,* the usual arithmetical sum is used. [12] This tedious verification may be skipped without loss by a reader who is satisfied that indeed our rules imply Trautmann's rules T1-T7, and 9A-9B.

Trautmann's rule T1 results from our classificatory rules C and from rules D.

T1.1. $A_0\, C_0 \rightarrow A_0 \approx (s^p a)(s^q) \rightarrow s^{p+q}\, a$ (D2, C)

T1.2 $A^0\, A^0 \rightarrow C^0 \approx (s^p a)(s^q a) \rightarrow s^{p+q} aa \rightarrow s^{p+q}\, e = s^{p+q}$ (C)

T1.3 $C^0 A^0 \rightarrow A^0 \approx (s^p)\,(s^q a) = s^{p+q}\, a$ (C, def. $A_{\neq}^{0}$)

T1.4 $C^0\, C^0 \rightarrow C^0 \approx (s^p)\,(s^q) = s^{p+q}$ (C)

[12] This implies that we are not imposing any generational restriction on the system. These restrictions will be mentioned below. They can take the form of cancellation rules such as *ff* = *e* (Kariera, Cashinahua), or they can take the form of erasing rules such as *fff* > *ff* (as the Dravidian case seems to suggest).

All these cases can be gathered together as

T1. $(C_=^p A_=^q)(C_=^r A_=^s) \to C^{p+r} A_=^{q+r} \approx (s^p a^q)(s^r a^s) = s^{p+r} a^{q+s}$

Trautmann's rules T2 can be regrouped in two sets: rules resulting from our rules C-D and rules resulting from our rules C-D and A1:

T2.1'. $C_p^{+1} A_=^0 \to A^{+1}$ $(s^p f)(a) = s^p fa$ (C)
T2.2. $C_p^{+1} C_=^0 \to C^{+1}$ $(s^p f)(e) = s^p f$ (C)
T2.3'. $A_p^{+1} A_=^0 \to C^{+1}$ $(s^p fa)(a) = s^p faa \to s^p fe = s^p f$ (C,D)
T2.4 $A_p^{+1} C_=^0 \to A^{+1}$ $(s^p fa)(e) = s^p fa = s^p fa$ (C)

T2.1. $C_p^{+1} C_{\neq}^0 \to A^{+1}$ $(s^p f)(s) = s^p fs \to s^p (sfa) = s^{p+1} fa$ (A1, C)
T2.2'. $C_p^{+1} A_{\neq}^0 \to C^{+1}$ $(s^p f)(sa) = s^p fsa = s^p sfaa \to s^{p+1} sfe = s^{p+1} f$ (A1,C,D)
T2.3. $A_p^{+1} C_{\neq}^0 \to C^{+1}$ $(s^p fa)(s) = s^p fsa \to s^p sfaa = s^{p+1} fe = s^{p+1} f$ (A1,C,D)
T2.4'. $A_p^{+1} A_{\neq}^0 \to A^{+1}$ $(s^p fa)(sa) = s^p fs\, aa \to s^p sfaaa = s^p sfae = s^{p+1} fa$ (A1,C,D)

These two groups of rules may be summarized as follows:

T2.I. $(s^p fa^q)(a^s) \to s^p f a^{q+s}$ (C, D)

T2.II $(s^p f)(s^r a^q) \to s^p (s^r fa^r) a^q \to s^{p+r} fa^{q+r}$ (A1, C, D)

In short, Trautmann's rules T1 and T2 express the fact that both sex and affinity behave as they should under a "boolean" calculus.

Trautmann's Rules T3 are quite straightforward applications of rules C and D:

T3.1. $A^+C^- \to A^0 \approx (s^p fa)(f^{-1} s^r) \to$

$\to s^p ff^{-1} s^r a = s^p e s^r a = s^p s^r a = s^{p+r} a$ (C,D)

T3.2. $A^+A^- \to C^0 \approx (s^p fa)(f^{-1} s^r a) \to$

$\to s^p ff^{-1} s^r aa \to s^p es^r e = s^p s^r = s^{p+r}$ (C, D)

T3.3. $C^+A^- \to A^0 \approx (s^p f^{+1})(f^{-1} s^q a) \to$

$\to s^p f^{+1} f^{-1} s^q a = s^p es^q a = s^p s^q a = s^{p+q} a$ (C)

T3.4. $C^+C^- \to C^0 \approx (s^p f)(f^{-1} s^q) \to$

$\to s^p ff^{-1} s^q = s^p es^q = s^p s^q = s^{p+q}$ (C)

Trautmann's rules T4 fall in two groups: those involving only rules C and D, and those involving affinizing rules A2. The first group involves the straightforward application of contraction rules C, and commutation of "*a*":

T4.1' $A_=^0\ C^- \rightarrow A^- \approx (a)(f^{-1}s^{\,q}) \rightarrow f^{1}s^{\,q}\,a$ (D)

T4.2. $C_=^0\ C^- \rightarrow C^- \approx (e)\,(f^{-1}s^{\,q}) = ef^{1}s^{\,q} = f^{-1}s^{\,q}$ (C)

T4.3'. $A_=^0\ A^- \rightarrow C^- \approx (a)(f^{-1}s^{\,q}a) \rightarrow f^{-1}s^{\,q}aa = f^{-1}s^{\,q}e = f^{-1}s^{\,q}$ (D2, C)

T4.4. $C_=^0\ A^- \rightarrow A^- \approx (e)(f^{-1}s^{\,q}a) \rightarrow ef^{-1}s^{\,q}a = f^{-1}s^{\,q}a$ (C)

The second group demands the application of the affinizing rule A2:

T4.1 $C_{\neq}^0\ C^- \rightarrow A^- \approx (s)\,(f^{-1}s^{\,q}) \rightarrow$

$\rightarrow (s\,f^{-1})\,s^{\,q} \rightarrow (f^{-1}s\,a)\,s^{\,q} = f^{-1}s\,s^{\,q}\,a = f^{-1}s^{\,q+1}\,a$ (A2)

T4.2'. $A_{\neq}^0\ C^- \rightarrow C^- \approx (s\,a)(f^{-1}s^{\,q}) \rightarrow$

$\rightarrow (sf^{-1})s^{\,q}\,a \rightarrow (f^{-1}\,sa)s^{\,q}\,a \rightarrow f^{-1}\,ss^{q}\,aa = f^{-1}\,s^{q+1}$ (A2)

T4.3. $C_{\neq}^0\ A^- \rightarrow C^- \approx (s)(f^{-1}s^{\,q}a) \rightarrow$

$\rightarrow (s\,f^{-1})\,s^{\,q}a \rightarrow (f^{-1}s\,a)\,s^{\,q}a \rightarrow f^{-1}ss^{\,q}aa = f^{-1}s^{\,q+1}$ (A2)

T4.4'. $A_{\neq}^0\ A^- \rightarrow A^- \approx (sa)(f^{-1}s^{\,q}a) \rightarrow$

$\rightarrow sf^{-1})s^{\,q}aa \rightarrow (f^{-1}sa)s^{\,q} \rightarrow f^{-1}s^{\,q+1}a$ (A2,C)

Rule T5 follows in a straightforward from rules C and D.

T5.1. $A_p^0\ C_q^{\,+} \rightarrow A_{p+q}^{\,+1} \approx (s^{\,p}a)(s^{\,q}f) = s^{\,p}as^{\,q}f = s^{\,p}s^{\,q}fa = s^{\,p+q}fa$ (C, D)

T5.2. $A_p^0\ A_q^{\,+} \rightarrow C_{p+q}^{\,+1} \approx (s^{\,p}a)(s^{\,q}fa) = s^{\,p}as^{\,q}fa = s^{\,p}s^{\,q}faa = s^{\,p+q}f$ (C, D)

T5.3. $C_p^0 A_q^{\,+1} \rightarrow A_{p+q}^{\,0} \approx (s^{\,p})(s^{\,q}fa) = s^{p}s^{\,q}fa = s^{p+q}fa$ (C)

T5.4. $C_p^0\ C_q^{\,+1} \rightarrow C_{p+q}^{\,+1} \approx (s^{\,p})\,(s^{\,q}) = s^{\,p+q}$ (C)

Rules T.6 are partitioned in two groups: one using rules C-D, and the other using rules A3, which expresses the marriage relation as a relation between affines. This is the first group:

T6.1'. $C_p^{-1}A_p^{\,+1} \rightarrow A_=^{\,0} \approx (f^{-1}s^{\,p})(s^{\,p}fa) \rightarrow$

$\rightarrow f^{-1}s^{\,p}s^{\,p}fa = f^{-1}s^{\,p+\,p}fa = f^{-1}fa = ea = a$ (C)

T6.2. $C_p^{-1}\ C_p^{\,+1} \rightarrow C_=^{\,0} \approx (f^{-1}s^{\,p})(s^{\,p}f) \rightarrow f^{-1}s^{\,p+\,p}f = f^{-1}f \rightarrow e$ (C)

T6.3' . $A_p^{-1}A_p^{\,+1} \rightarrow C_=^{\,0} \approx (f^{-1}s^{\,p})(s^{\,p}fa) \rightarrow f^{-1}s^{\,p}s^{\,p}fa = f^{-1}fa = ea = a =$ (C)

T6.4' . $A_p^{-1}A_{p\neq}^{\,+1} \rightarrow A_{\neq}^{\,0} \approx (f^{-1}s^{\,p}a)(s^{\,p+1}fa) \rightarrow$

$\rightarrow f^{-1}s^{p}s^{p+1}faa = f^{-1}sfe = ase = as$ (C)

The second group follows:

T6.1. $C_p^{-1}C_{p\neq}^{+1} \rightarrow A_{\neq}^{0} \approx (f^{-1}s^{p})(s^{p+1}f) \rightarrow f^{-1}s^{p+\ p+1}f = f^{-1}sf \rightarrow as$ (D3, C)

T6.2'. $C_p^{-1}A_{p\neq}^{+1} \rightarrow C_{=}^{0} \approx (f^{-1}s^{p})(s^{p+1}fa) \rightarrow f^{-1}s^{p+\ p+1}fa = f^{-1}sfa \rightarrow asa = s$ (D3, C)

T6.3 . $A_p^{-1}A_{p\neq}^{+1} \rightarrow C_{\neq}^{0} \approx (f^{-1}s^{p})(s^{p+1}fa) \rightarrow f^{-1}s^{p}s^{p+1}fa = f^{-1}sfa \rightarrow asa = saa = s$ (D3,C)

T6.4 . $A_p^{-1}C_p^{+1} \rightarrow A_{=}^{0} \approx (f^{-1}s^{p}a)(s^{p}f) \rightarrow f^{-1}s^{p+p}fa = f^{-1}efa \rightarrow f^{-1}fa = ea = a$ (D3,C)

Rule T7 results from rules C-D of the D* calculus. We present a short version of them, in which we use A^0 and A^1 to stand for the presence/absence of the affine mark A.

T7. $(C_p^{-1}A^{k1})(C_q^{0}A^{k2}) \rightarrow C_{p+q}^{-1}A^{k1+k2}$ with $k1 \neq k2$
$\approx (f^{1}s^{p}a^{k1})(s^{q}a^{k2}) \rightarrow f^{1}s^{p}s^{q}a^{k2}a^{k1} = f^{1}s^{p+q}a^{k2+k1}$

This concludes the tedious verification that Trautmann's rules T1-T7 can all be derived from rules C, D and A. We must also conclude that Trautmann's rules for obtaining Dravidian affinity in generations G+2 and G-2 must be redundant or contradictory with rules T1-T7 since the rules of D* calculus generate the affine classification for any generation level, and so the sub-set of Trautmann's rules T1-T7 discussed above.

III.3 Double classification does not happen

I show now why T8A, when rewritten at the level of sub-rules does not produce "double classification". I recalculate the unique classification at the level of sub-rules, using the rules C, A and D (which we have shown to be equivalent to Trautmann's own rules T1-T7) and show that, after revised and regrouped, rules T8A are the same as rules T8B. The same reasoning can be applied to rules T9A and T9B.

We consider now rules 8A. We start with

T8.1A. $A^{+}C^{+} \rightarrow A^{++}$

We will use the translations indicated at the tables above when needed. In each case we give the rule in form given by Trautmann (as in the above example), and then rewrite them in all its sub-cases and revise the sub-cases, manner, using * to point out the sub-cases where our right-side conclusion differs from Trautmann's. The disagreement is of no significance, however, since (a) our derivations can be mimicked within Trautmann's own rules T1-T7, and (b) Trautmann himself gives the corrected version of rules T.8 in his rules T9.

Rule T8.1. $A^{+}C^{+} \rightarrow A^{++}$ (original form).

T8.1$A_{==}$. $A_{=}^{+}C_{=}^{+} \rightarrow A_{=}^{++} \approx (fa)f = faf \rightarrow ffa$ (rule D). This is OK.

T8.1$A_{\neq=}$. $A_{\neq}^{+}C_{=}^{+} \rightarrow A_{\neq}^{++} \approx (sfa)f = sfaf \rightarrow sffa$ (rule D1). OK

*T8.1$A_{=\neq}$ $A_{=}^{+}C_{\neq}^{+} \rightarrow C_{\neq}^{++} \approx (fa)(sf) \rightarrow (fs)fa \rightarrow (sfa)fa \rightarrow sffaa = sff$ (A1, D)

*T8.1$A_{\neq\neq}$. $A_{\neq}^{+}C_{\neq}^{+} \rightarrow C_{=}^{++} \approx (sfa)(sf) \rightarrow s(fs)fa \rightarrow s(sfa)fa \rightarrow ssffaa = ff$ (A1,C)

We conclude that in the last two cases, the transformation results in a non-affine term, not in an affine term. We use this example to show how our revision of the rule may be derived by means of Trautmann's own rules. The first step is to express $A_{=}^{+}C_{\neq}^{+}$ in its explicit components:

*T8.1$A_{=\neq}$. $A_{=}^{+}C_{\neq}^{+} \equiv (A_{=}^{0}C_{=}^{+1})(C_{\neq}^{0}C_{=}^{+1})$
(Translations 14 and 15, based on rules T5.1$_{==}$, T4.1$_{==}$)

We proceed by applying to $(A_{=}^{0}C_{=}^{+1})(C_{\neq}^{0}C_{=}^{+1})$ further translation rules:

$A_{=}^{0}(C_{=}^{+1}C_{\neq}^{0})C_{=}^{+1} \rightarrow A_{=}^{+}(A_{\neq}^{+1})C_{=}^{+1}$ (rule T2.1$_{=\neq}$)

$A_{=}^{0}(A_{\neq}^{+1})C_{=}^{+1} \rightarrow A_{=}^{0}(A_{=}^{0}C_{\neq}^{+1})C_{=}^{+1}$ (Def. 7, T2.1$_{=\neq}$)

$(A_{=}^{0}A_{=}^{0})(C_{\neq}^{+1}C_{=}^{+1}) \rightarrow (C_{=}^{0})(C_{\neq}^{+1}C_{=}^{+1}) = C_{\neq}^{+1}C_{=}^{+1}$ (T1)

$C_{\neq}^{+1}C_{=}^{+1} = C_{\neq}^{0}C_{=}^{+1}C^{+1} = C_{\neq}^{++}$ (the last step is a notational convention)

This derivation is not affected by a different choice of parentheses, as long as we respect the order of terms. We see that the source of discrepancy lies in the fact that in rule T8.1$A_{=\neq}$ the composition of two expressions containing an ascending generation term produces a cross term *fs* that must be affinized as *sfa*, thus introducing a second affine mark which cancels out the original one.

Let us continue the derivation for the other sub-rules, using for brevity our D* language from now on.

Rule T8.2A $A^{+}A^{+} \rightarrow A^{++}$ (original formulation).

*T8.2$A_{==}$ $A_{=}^{+}A_{=}^{+} \rightarrow C_{=}^{++} \approx af\,af \rightarrow ffaa = ffe = a$ (C, D)

*T8.2$A_{\neq=}$ $A_{\neq}^{+}A_{=}^{+} \rightarrow C_{\neq}^{++} \approx asf\,af \rightarrow sffaa = sffe = sff$ (D, C)

T8.2$A_{=\neq}$ $A_{=}^{+}A_{\neq}^{+} \rightarrow A_{\neq}^{++} \approx af\,asf \rightarrow (fs)faa \rightarrow (sfa)fa = sffaaa = sffae = sffa$ (A1,D,C)

T8.2$A_{\neq\neq}$ $A_{\neq}^{+}A_{\neq}^{+} \rightarrow A_{=}^{++} \approx asfasf \rightarrow s(fs)faa = s(sfa)faa \rightarrow ssffaaa = ffa$ (A1, D, C)

Rule T8.3A $C^{+}A^{+} \rightarrow A^{++}$ (original form)

T8.3A$_{==}$ $C_{=}^{+}A_{=}^{+} \rightarrow A_{=}^{++}$ $\approx$ *f af = ff a* (D)

T8.3A$_{\neq=}$ $C_{\neq}^{+}A_{=}^{+} \rightarrow A_{\neq}^{++}$ $\approx$ *sf af* $\rightarrow$ *sffa* (D)

*T8.3A$_{=\neq}$ $C_{=}^{+}A_{\neq}^{+} \rightarrow C_{\neq}^{++}$ $\approx$ *f asf* = (*fs*)*fa* $\rightarrow$ (*sfa*)*fa* $\rightarrow$ *sffaa* $\rightarrow$ *sff* (A1, D)

*T8.3A$_{\neq\neq}$ $C_{\neq}^{+}A_{\neq}^{+} \rightarrow C_{=}^{++}$ $\approx$ *sf asf* $\rightarrow$ *s*(*fs*)*fa* $\rightarrow$ *s*(*sfa*)*fa* $\rightarrow$ *ssffaa* = *effe* = *ff* (A1, D,C)

T8.4A $C^{+}C^{+} \rightarrow C^{++}$ (original version)

T8.4A$_{==}$ $C_{=}^{+}C_{=}^{+} \rightarrow C_{=}^{++}$ $\approx$ *f f* (notational convention)

T8.4A$_{\neq=}$ $C_{\neq}^{+}C_{=}^{+} \rightarrow C_{\neq}^{++}$ $\approx$ *sf f* (notational convention)

*T8.4A$_{=\neq}$ $C_{=}^{+}C_{\neq}^{+} \rightarrow A_{\neq}^{++}$ $\approx$ *fsf* = (*fs*)*f* $\rightarrow$ (*sfa*)*f* $\rightarrow$ *sffa* (A1, D1)

*T8.4A$_{\neq\neq}$ $C_{\neq}^{+}C_{\neq}^{+} \rightarrow A_{=}^{++}$ $\approx$ *sfsf* = *s*(*fs*)*f* $\rightarrow$ *s*(*sfa*)*f* $\rightarrow$ *ssffa* = *ff a* (A1, D1)

We can see that there is a pattern here: precisely half the sub-cases of T8A stand as they are, while half the sub-cases must undergo a revision (in the last example, we obtain affine terms, not consanguineal). We collect these rules in another order and check that the cases are the same as the cases under T8B:

*T8.4A$_{=\neq}$ $C_{=}^{+}C_{\neq}^{+} \rightarrow A_{\neq}^{++}$ T8.1B$_{=\neq}$ (A1, D1)

*T8.4A$_{\neq\neq}$ $C_{\neq}^{+}C_{\neq}^{+} \rightarrow A_{=}^{++}$ T8.1B$_{\neq\neq}$ (A1, D1)

T8.3Ai$_{==}$ $C_{=}^{+}A_{=}^{+} \rightarrow A_{=}^{++}$ T8.1B$_{==}$ (D)

T8.3A$_{\neq=}$ $C_{\neq}^{+}A_{=}^{+} \rightarrow A_{\neq}^{++}$ T8.1B$_{\neq=}$ (D)

T8.4A$_{==}$. $C_{=}^{+}C_{=}^{+} \rightarrow C_{=}^{++}$ T8.2B$_{==}$ (original notation)

T8.4A$_{\neq=}$ $C_{\neq}^{+}C_{=}^{+} \rightarrow C_{\neq}^{++}$ T8.2B$_{\neq=}$ (original notation)

*T8.3A$_{=\neq}$ $C_{=}^{+}A_{\neq}^{+} \rightarrow C_{\neq}^{++}$ T8.2B$_{=\neq}$ (A1, D)

*T8.3A$_{\neq\neq}$ $C_{\neq}^{+}A_{\neq}^{+} \rightarrow C_{=}^{++}$ T8.2B$_{\neq\neq}$ (A1, D,C)

*T8.1A$_{=\neq}$ $A_{=}^{+}C_{\neq}^{+} \rightarrow C_{\neq}^{++}$ T8.3B$_{=}^{\neq}$ (A1, D)

*T8.1A$_{\neq\neq}$. $A_{\neq}^{+}C_{\neq}^{+} \rightarrow C_{=}^{++}$ T8.3B$_{\neq\neq}$ (A1,C)

*T8.2A$_{==}$ $A_{=}^{+}A_{=}^{+} \rightarrow C_{=}^{++}$ T8.3B$_{==}$ (C, D)

*T8.2A$_{\neq=}$ $A_{\neq}^{+}A_{=}^{+} \rightarrow C_{\neq}^{++}$ T8.3B$_{\neq=}$ (D, C)

T8.1$A_{==}$. $A_{=}^{+}C_{=}^{+} \rightarrow A_{=}^{++}$ T8.4$B_{==}$ (D)

T8.1$A_{\neq=}$. $A_{\neq}^{+}C_{=}^{+} \rightarrow A_{\neq}^{++}$ T8.4$B_{=\neq}$ (D1)

T8.2$A_{=\neq}$ $A_{=}^{+}A_{\neq}^{+} \rightarrow A_{\neq}^{++}$ T8.4Bi (A1,D,C)

T8.2$A_{\neq\neq}$ $A_{\neq}^{+}A_{\neq}^{+} \rightarrow A_{=}^{++}$ T8.4Bii (A1, D, C)

Rules 91A follow the same pattern, with D* rules A2 replacing the role of rules A1.

We think that versions A and B of Trautmann's rules 8 and 9 reveal the depth of Trautmann's apprehension the inner logic of the system. After facing the apparent problem of ambiguity in his calculus for generations G^{+2} and G^{-2} under rules 8.A and 9.A, resulting in fact from a notational difficulty, he obtained rules 8.B and 9.B, which gives a non-ambiguous result. The only missing step was to check back both set of rules at the level of sub-rules to see that they contained the same left-side terms with the correct right-side terms given by the B version.

We skip a detailed revision of Trautmann's own argument on the existence of "double classification" (Trautmann 1981, p. 190, and Tjon Sie Fat 1998, p. 85). A detailed analysis shows that, in both derivations, at some point a *commutation* of terms *f* and *s* was introduced without the corresponding addition of an affine mark. For instance, we believe that Tjon Sie Fat's assertion that under a certain bracketing ♂MHF would be classified as "cross" depends on his commuting $C_{=}^{+1}C_{\neq}^{0}$ into $C_{=}^{+1}C_{\neq}^{0}$ instead of into $A_{=}^{0}\, C_{=}^{+1}C_{\neq}^{0}$. This point of course does not affect the more general implication of Tjon Sie Fat's argument in defense of the non-associative approach for the formal study of the structure of kinship terminologies.

III.4. Rules for limiting the generational depth

An implication of the previous argument is that "double classification" cannot be invoked to model the phenomenon of merging affines and consanguineals in generations G^{+2} and G^{-2} in systems similar to the Dravidian one. In order to obtain the intended merging what is needed are specifically generational rules. One form that such a rule can take is that of a *forgetting rule:*

Forgetting rule. In a canonical word, this rule collapses generations above a certain depth and also collapses the affine distinction at the boundary generations:

F1. $fff \rightarrow ff$

F2. $f^{-1}f^{-1}f^{-1} \rightarrow f^{-1}f^{-1}$

F3. $Xa \rightarrow X$ if X contains ff or a term $f^{1\text{-}1}$.

The effect rules F1 and F2 is to limit the number of generations to G^{-2}, G^{-1}, G^{0}, G^{+1} and G +2. The effect of rule F3 is to suppress the affine mark from every word in generations G^{+2}

and G^{-2} . In the proposed form, F-rule is applied on a word already in canonical form. This implies that *only after a word is reduced* according to the previous rules the forgetting rules are applied. However, this possible rule was not actually tested against other possible formulations (i.e. allowing the "forgetting" rules F1 and F2 to be applied in the course of reduction). One main reason for that is the lack of evidence on how deep genealogical connections are actually processed under Dravidian rules.

There is another family of depth-limitation rules:

Alternating generations rule: $f^2 \rightarrow e$

The obvious reason for the name is that, under this rule, grandparents are identified with siblings and with grandsons, as seems to be the case in the Cashinahua case (Kensinger 1984: 229, Figure 1).

As is well known by now (Dumont, Viveiros de Castro), Kariera and Dravidian systems do not classify relatives in the same way. One way of formulating the difference is to rephrase the affinization rules A1-A2 in the following way:

Kariera rules. C and D hold without change. Additionally:

A1(Kariera). $sf \rightarrow fsa$
A2(Kariera). $sf^{-1} \rightarrow f^{-1}sa$
A3(Kariera). $f^{-1}sf \rightarrow as.$
Kariera generation rule. $f^2 = e.$

Note the form of rule A1K. In this system, male ego should classify his mother as his father's sister's affine, and female ego should classify her father as her mother's brother's affine.

IV. A more general framework.

The above theory was phrased so as to simplify formally the rules, by taking advantage of the peculiarly symmetrical structure of the Dravidian terminologies. This simplification means in the first place that we were able to use a single symbol *a* for affinity and for crossness, which would otherwise demand a separate symbol, say **x**. In the second place, in the Dravidian context we could use a single symbol for *a* and for its inverse a^{-1} since both were identified under the assumption that $aa = e$. The same identification applies of course to the identification $\mathbf{x} = \mathbf{x}^{-1}$. Now, in order to apply the mathematical approach to a more general context, we need to drop these assumptions. We will show that these assumptions underly the Dravidian rules D1 and D2. We therefore enlarge our vocabulary, and rephrase Dravidian rules in a context that refers explicitly both to crossness and affinity, and that distinguishes "laterality", that is to say, matrilateral from patrilateral relations either of "crossness" or of affinity. This section is intended as an introduction to a future comparison with Iroquois and Crow-Omaha systems (in a forthcoming paper).

We begin by expanding our original vocabulary with the introduction of words for crossness and affinity. [13]

$K_{x,a} = \{e, s, f, f^{1}, \mathbf{x}, \mathbf{x}^{-1}\ a, a^{-1}\}$

$K_{x,a}*$ is the universe of words formed with the vocabulary $K_{x,a}$. Both the Dravidian vocabulary D and the enriched vocabulary $K_{x,a}$ can be regarded as extensions of a basic vocabulary $K = \{e, s, f, f^{-1}\}$.

Definition 7

Elementary cross words and elementary affine words are as follows:

K1. $fsf^{-1}s \equiv \mathbf{x}$ (direct cross cousin; isolateral cross; ♂FZS, ♀MBD)

K2. $sfsf^{-1} \equiv \mathbf{x}^{-1}$ (inverse cross cousin; anisolateral cross; ♂MBS,♀FZD).

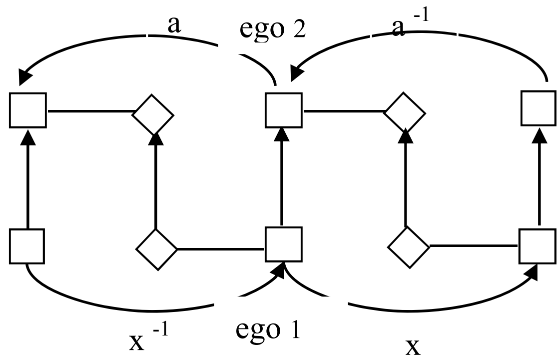
K3. $f^{-1}sfs \equiv a$ (donor affine, or direct affine; ♂WB, ♀HZ)

K4 . $sf^{-1}sf \equiv a^{-1}$ (receiver affine, inverse affine; ♂ZH; ♀BW)

The way cross symbols and affine symbols are defined is consistent with the notation employed for inverses, so that $\mathbf{xx}^{-1} = e$, $\mathbf{x}^{-1}\mathbf{x} = e$. The above identities are understood as definitions in the following sense. They allow rewriting words in the universe K* of basic kinship words as words in the enlarged universe $K_{x,a}*$ with added cross and affine symbols. These symbols are then interpreted as abbreviations of the defined cross words and affine words. We use a diagram to show how the language of $K_{x,a}*$ words is represented in a simple way, and to illustrate the connection between crossness and affinity. In this diagram we use symbols □ and ◊ to represent distinct sexes, although this is not really needed since our notation and the corresponding diagram is purely relational: only *changes* of sex and *changes* of generation are of interest.

[13] This whole section preceded the others in a previous version of this article. It introduces a framework to be used in another article dealing jointly with Dravidian, Iroquois and Crow-Omaha rules (or "classificatory systems of affinity and consanguinity").

Figure 3. Crossness and affinity



Upward oriented arrows represent f, and the same arrows read in the opposite sense represent f^{-1} (see also Figures 1 and 2 where the same convention applies). This representation is fully consistent with the use of mark a in Figure 2. The relations $\mathbf{x}$ and $\mathbf{x}^{-1}$ are represented from *ego 1*'s point of view, and relations a and a^{-1} are shown from *ego 2*'s point of view. The diagram in Figure 2 can be extended indefinitely both in the horizontal sense and in the vertical sense (as in suggested in the "fractal" form of Figure 1). This extension would be the complete graph-like representation of the free group generated by f and s subject to the relation $s^2 = e$, partially represented in Figure 1. Dravidian axioms can now be seen as symmetries imposed on the diagram represented in Figure 2.[14] They essentially identify the upper and lower parts of the diagram, and identify the left and right sides of the diagram.

We observe that Figure 2 is in a sense a version of what Lévi-Strauss once called "the atom of kinship": a combination of relations of affinity, crossness and filiation (Lévi-Strauss 1958, p. 54). Consider a male ego 1 in Figure 1. It is seen that his cross cousin (path ♂$\mathbf{x}$, ♂FZS) is also his father's inverse affine's son (♂$f\,a^{-1}f^{-1}$, ♂FZHS), and that his inverse cross-cousin ♂$\mathbf{x}^{-1}$ is his father's affine's son (♂$f\,a\,f^{-1}$, ♂FWBS). If we take a male ego 2's point of view, we see that his affine ♂a is also his son's inverse cross cousin's father (♂$f^{1}\,\mathbf{x}^{-1}f$, ♂S$\mathbf{X}^{-1}$F), and that his inverse affine ♂a^{-1} is also his son's cross cousin's father (♂$f^{-1}\mathbf{x}\,f$, ♂S$\mathbf{X}$F). We have used the "crossness" symbols $\mathbf{X}$ and $\mathbf{X}^{-1}$ in the kin type notation. In others words, the language of crossness and the languages of affinity are interchangeable in the sense that every expression involving cross terms in the sense of Definitions 7 (K1-K4) can be rewritten as an expression involving only affine terms.[15] Crossness and affinity are relative

[14] Although we have not made wide use of diagrams in this article, they were used in its development in parallel with the algebraic formulation. For the role of symmetries, cf. Weyl 1951, 1953 and Barbosa de Almeida 1991.

[15] The passage from "affine" to "cross" consists in a homomorphism that takes a word W to the word $f\,W f^{-1}$. This homomorphism takes e into e, s into fsf^{-1} = xs, f into f, f^{-1} into f^{-1}, and finally takes a into x^{-1} and

notions at this stage of reasoning. Different cultural systems will chose which one formulation is basic, and which one should be taken as derived. Or, in other words, different cultural systems may differ in the way they chose which idiom, consanguinity or affinity, is taken as given (or as the datum) and which is taken as constructed (Viveiros de Castro 2002: 403-407, 2009:17-18).

We now formulate the Dravidian Axioms in a more basic way.

Dravidian Axioms

DA 1. $a = a^{-1}$ (affinity is symmetrical)

DA 2. $\mathbf{x} = a$ (crossness and affinity are the same)

Axiom DA1 asserts the symmetrical character of affinity, and thus distinguishes Dravidian and similar systems from "generalized exchange" systems where $a \neq a^{-1}$, such as JingPahw. Axiom DA2 identify crossness and affinity, and thus distinguishes Dravidian and similar systems from systems of the general Iroquois type where affinity is not the same as crossness and so $\mathbf{x} \neq a$.

We note as

Corollaries to Axioms DA1-DA2:

1. $\mathbf{x} = a^{-1}$ (immediate from DA1 and DA2)

2. $s\mathbf{x}s = \mathrm{x}'$, $sas = a^{-1}$ (immediate from definitions K1 and K2)

3. $\mathbf{x}^{-1} = a$ ($\mathbf{x} = a^{-1} \Rightarrow s\mathbf{x}s = sa^{-1}s \Rightarrow \mathbf{x}^{-1} = a$)

4. $\mathbf{x}^{-1} = \mathbf{x}$ (from 3 and Axiom DA2)

Dravidian Axioms DA1-DA1 together with definitions K1-K4 implies the following commuting equations:

5. $as = sa$

$(sas)s = (sas)s$

$(a^{-1})s = sass = sa$ (K4, C-rules)

$a^{-1}s = sa \rightarrow as = sa$ (Axiom DA1)

6. $af = fa$

$af = (\mathbf{x}^{-1})f$ (Axiom DA2, Corollary 3)

$\mathbf{x}^{-1}f = (sf\,sf^{-1})f = sfse = sfs = f(f^{-1}sfs) = fa$ (Axiom D1)

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takes a^{-1} into x. The passage from "cross" to "affine" consists, in a similar way, in the homomorphism $f^{-1}Wf$.

7. $af^{-1} = f^{-1}a$
$af^{-1} = a^{-1}f^{-1}$ (Axiom DA1)
$a^{-1}f^{-1} = (sf^{-1}sf)\,f^{-1} = sf^{-1}se = sf^{-1}s$ (K4, C-rules)
$sf^{-1}s = f^{-1}(fsf^{1}s) = f^{-1}(\mathbf{x})$ (K1)
$f^{-1}(x) = f^{-1}a$ (Axiom DA2)

We observe that rules D1-D3 of the D* calculus result directly from definitions K1-K4 together with axioms DA1 and DA2. This shows that Axioms DA1 and DA2 together with definitions K1-K4, together with rules C, imply rules A1-A2 and rules D1-D3 of the D*-calculus. Since definitions K1-K4 hold for all systems in which "crossness" and "affinity" (in the sense of marriage) makes sense, and since C-rules characterize all "classificatory" systems, the distinctive features of Dravidian systems are contained in Axioms DA1 and DA2.

Classificatory systems, in the sense of systems in which axioms C hold, can be distinguished as follows:

Type I. General Iroquois. $\mathbf{x} \neq a$, $\mathbf{x}^{-1} \neq \mathbf{x}$, $a^{-1} \neq a$. Crossness is distinct from affinity, crossness is asymmetrical, affinity is asymmetrical. Type I is the structure underlying systems which, with added equations, lead to Iroquois and Crow and Omaha variants (this program is carried out in a forthcoming paper).

Type II. Generalized exchange. $\mathbf{x} = a$, $\mathbf{x}^{-1} \neq \mathbf{x}$, $a^{-1} \neq$ a. Crossness is affinized, crossness is asymmetrical and affinity is asymmetrical. Represented by among the systems dealt with by Lévi-Strauss under the "Kachin" label (Lévi-Strauss 1967: 273 ss.) .

Type III. Restricted exchange. $\mathbf{x} = a$, $\mathbf{x}^{-1} = \mathbf{x}$, $a^{-1} = a$. Crossness is affinized and crossness and affinity are symmetrical. These characterizes the Dravidian system and other "two line" systems when combined with "forgetting" rules that limit the generational depth and erase the mark "*a*" in G^{+1} and G^{-2}. These rules generate the identities relating relationship systems not only in Dravidian terminologies as described by Dumont (at three medial generations) and Trautmann (at five medial generations), but also in South-American cases such as the Piaroa relationship system (Kaplan1975: 199-200, Apendix A).

When combined with the equation $f^{2} = e$, and changes in the orientation of affinization rules, we obtain systems of the Kariera and Cashinahua type (

Type IV. Hypothetical. $\mathbf{x} \neq a$, $\mathbf{x}^{-1} = \mathbf{x}$, $a^{-1} = a$. Crossness is distinct from affinity, but crossness and affinity are both symmetrical. In this case, $as = sa$ and $\mathbf{x}s = sa$, but $fa \neq af$ and $fx \neq xf$. I can cite no empirical instances for Type IV.

V. Remarks on the literature used

Since I am not a specialist in the field of kinship (but having lectured on the topic at the University of Campinas in the last years) it is perhaps convenient to summarize the main sources of ideas and problems for this paper, which make no claims for including a review of the literature on the subject. This paper got started by 2005 as a result from reading Trautmann's work (1981) which in turn dealt with Dumont's (1953, 1971[1957]) and Lévi-Strauss's [1957[1949], 1958] theory of affinity and consanguinity, which goes back to Lewis Morgan (1997[1871]). The current mathematical approach combines ideas going back to A. Weil (1949), particularly in the version by Pierre Samuel (1967), as well as to Lounsbury (1956, 1969[1964]), Lorrain (1975) and Tjon Sie Fat (1990, 1998). Although I only realized this when revising this article, it betrays possibly the influence of reading Harrison White's pioneer book (1963) in the early 1970's, when I also read F. Lorrain's work containing a similar approach to "classificatory reduction". The general philosophical approach is influenced by chapter 1 of Weyl (1953[1939]), and several more accessible books by the same author.

Several of the rules implied by Trautmann's calculus and expressed in the D* calculus, and which formalize the notion of "classificatory systems" (Morgan 1871) have been formalized before, at least since Lounsbury. Thus, Lounsbury's merging rule and half-sibling rule express some features of the group-theoretical structure of the classificatory system. Kay's 1965 article contains a version of the odd-even rule, arrived at by a very different method and insufficiently phrased (Kay 1965a, 1965b). The notation employed here has an analogue in the dimensional notation proposed by Romney and d'Andrade 1963. The issues of commutativity, associativity and characteristic functions obviously reflect a running dialogue with Tjon Sie Fat's illuminating ideas (1990), and the approach to affinity is indebted to Viveiros de Castro (1998).

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