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Graviton emission from a soft brane

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Theories with compact extra spatial dimensions felt only by gravity are subject to direct experimental tests if the compactification volume is large. Estimates of high-energy collider observables induced by graviton radiation into extra dimensions are usually given assuming rigid branes. This may overestimate the accessibility of these theories. Brane fluctuations soften the coupling of graviton radiation, and reduce our ability to see effects in high-energy collisions. We calculate the size of this suppression on single jet plus gravitons at the LHC and single photon plus gravitons at an e^+e^- linear collider. We advocate the use of a brane softening variable as an additional parameter when comparing theory predictions to data.

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Introduction: Our universe may have δ large extra spatial dimensions felt by gravity [1,2]. This hypothesis has given many additional angles to attack the hierarchy problem, quantum gravity, and the cosmological constant problem. If the compactification volume of the δ extra dimensions is sufficiently large, it can have visible consequences in astrophysics, cosmology and particle physics experiments [3]- [15].

High-energy e^+e^- and pp collisions into photon or jet plus gravitons are some of the best ways that large extra dimensions can be experimentally confirmed. The gravitons escape the detector as a neutrino would, and are inferred as missing energy in the final state.

The reduced Planck scale $M_P \equiv 1/\sqrt{8\pi G_N}$ that we usually view as fundamental is now a derived quantity,

$$M_P^2 = M_D^{2+\delta} R^\delta, \quad (1)$$

where M_D is the fundamental scale of gravity, and R is the radius of toroidal compactification of the δ dimensions. For large enough compactification radius, $M_D \simeq 1$ TeV is allowed. The hierarchy problem, which concerns itself with why $M_P/m_W \simeq 10^{16}$ and how this ratio can be stable to quantum corrections, is now mapped to a potentially more tractable problem of how R can be stabilized to large enough values such that all fundamental scales are roughly equal, $M_D \simeq m_W$.

Similar to a particle in a box, the momentum of the $D = 4 + \delta$ dimensional massless graviton in the δ compactified dimensions is quantized,

$$p_\perp^2 = \frac{\vec{n} \cdot \vec{n}}{R^2}, \quad (2)$$

where

$$\vec{n} = (n_1, n_2, \dots, n_\delta) \quad (n_i = \text{integers}). \quad (3)$$

To an observer at low energies in the $3 + 1$ dimensional effective theory, each allowed momentum in the compact-

ified volume appears as a Kaluza-Klein (KK) excitation of the graviton with mass $m^2 = p_\perp^2$.

For any given KK graviton $G^{(n)}$, the production cross-section of $e^+e^- \rightarrow \gamma G^{(n)}$, for example, is

$$\frac{d\sigma_m}{dt} \propto \frac{1}{M_P^2}. \quad (4)$$

Summing over all possible accessible momenta (accessible KK graviton masses) in the extra dimensions requires integrating

$$\frac{d^2\sigma}{dt dm} = S_{\delta-1} \frac{M_P^2}{M_D^{2+\delta}} m^{\delta-1} \frac{d\sigma_m}{dt}, \quad (5)$$

where $S_{\delta-1}$ is the surface of a unit-radius sphere in δ dimensions. The M_P^2 factors cancel, and the final cross-section scales as $\sigma \propto s^{\delta/2}/M_D^{2+\delta}$. Signals can be discernible at colliders provided M_D is at the TeV scale or below.

Brane fluctuations: Usually calculations of final state graviton emission such as $e^+e^- \rightarrow \gamma G^{(n)}$ and $pp \rightarrow \text{jet } G^{(n)}$ assume a rigid brane. This implies the coupling of a massive KK graviton is precisely the same as a less massive KK graviton. A more complete formalism must ultimately be employed to describe scattering in energy domains that probe the flexibility of the brane. Without such a formalism to take into account brane fluctuations, unphysical results can arise in calculations, such as divergent virtual contributions of $e^+e^- \rightarrow G^{(n)} \rightarrow \gamma\gamma$ and non-unitary $pp \rightarrow \gamma G^{(n)}$ production cross-sections. Rigid cutoffs at M_D are used in many analyses to avoid these problems and to control the size of KK graviton effects in collisions.

In this letter, we incorporate brane fluctuations into the formalism at the beginning, as has been suggested in refs. [16,17]. The simplest way to do this is to promote

the extra dimensional coordinates $\vec{y} = (y_1, y_2, \dots, y_\delta)$ into fields $\vec{\phi}(x)$. The values of $\phi_i(x)$ represent the brane fluctuation in the y_i direction at the point x in our $3 + 1$ dimensional brane world. The fields $\vec{\phi}(x)$ are Nambu-Goldstone bosons from the spontaneous breaking of translation invariance in the D dimensions [18].

Expanding to lowest order in gravity, the induced metric on the 3-brane is

$$g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \vec{\phi}(x) \cdot \partial_\nu \vec{\phi}(x). \quad (6)$$

The kinetic terms of the $\vec{\phi}(x)$ fields are

$$-\tau \int d^4x \sqrt{-g} \Rightarrow \int d^4x \left(-\tau + \frac{\tau}{2} \partial_\mu \vec{\phi}(x) \cdot \partial^\mu \vec{\phi}(x) \right). \quad (7)$$

Canonically normalized fields $\vec{\pi}(x)$ are obtained by rescaling $\vec{\pi}(x) = \sqrt{\tau} \vec{\phi}(x)$.

The KK reduction of the graviton field $G_{\mu\nu}$ is now

$$G_{\mu\nu}(x, \vec{y}) \propto \sum_{\vec{n}} G_{\mu\nu}(x) \exp\left(i \frac{\vec{n} \cdot \vec{\pi}(x)}{\sqrt{\tau} R}\right). \quad (8)$$

Substituting this into the gravity-matter interaction Lagrangian, and normal ordering the exponential, one finds a suppressed interaction of gravity with matter,

$$\mathcal{L} \supset -\frac{e^{-\frac{1}{2} \frac{m^2}{\Delta^2}}}{M_P} G_{\mu\nu}^{(n)} T^{\mu\nu}, \quad (9)$$

where $m = |\vec{n}|/R$ is the mass of the KK graviton, and $\Delta^2 = 16\pi^2\tau/\Lambda^2$ is the ‘‘softening scale’’ derived from the brane tension τ and the cutoff scale Λ of a π_i -field loop. The magnitudes and relation of τ and Λ are expected to be related to M_D , but they are unknown. One should be prepared for Δ to take on almost any value.

As the KK graviton mass gets higher the coupling of the KK graviton to matter through the energy momentum tensor $T^{\mu\nu}$ reduces. The suppression caused by this exponential softening factor can significantly alter the observable effects of KK graviton production at high-energy colliders.

Insightful toy string models of large extra dimensions have been investigated recently [19–21]. The relative size of the gravitational scale to the string scale is dictated by the strength of the Yang-Mills gauge coupling on the D-brane, $M_D/M_S \sim 1/\sqrt{g_{\text{YM}}}$. For weakly coupled strings, the string scale is below the gravity scale, and string excitations play a significant role in the phenomenology. The string Regge excitations due to the finite size of the Standard Model (SM) particles can even overwhelm the signals of graviton KK excitations. In this circumstance, the cutoff that tames all high-energy observables comes

from the natural exponential suppressions inherent in Veneziano amplitudes [22,23] at energies near and above the string scale.

Nevertheless, brane fluctuation suppressions still can be important in string theories. Although the brane fluctuations are parametrically suppressed by an exponential factor of g_{YM}^2 [20], the other coefficients in the exponential are not precisely known. BPS brane tension calculations give $\tau_3 < M_S^4$, implying that the exponential suppression associated with brane fluctuations could be as significant as that from stringy effects. With broken supersymmetry, the tension is not presently calculable, but if it is significantly lower than the string scale, the brane fluctuation effects would be the most important suppression factor in graviton emission.

Furthermore, if the gauge coupling at the string scale is large $g_{\text{YM}}^2 \gtrsim 1$ there would not be a parametric suppression from brane fluctuations. In this case, M_D would be smaller than M_S , decreasing the string Regge excitations’ role in providing new-physics signatures at high-energy colliders. This may be the worst-case scenario for discovering and studying low-scale string theory, and it maps onto our field theory description of brane fluctuation suppressions with the identification $\Delta^2 \simeq \tau_3/M_S^2$. We therefore suggest that Δ is an additional useful free parameter in evaluating the phenomenology of large extra dimensions.

Collider sensitivity: Many studies have been published estimating the sensitivity to M_D and δ from virtual and external KK graviton effects with rigid branes. Brane fluctuation softening effects on virtual graviton exchange have been demonstrated in refs. [16,17]. Here we demonstrate the effects of brane fluctuations on the results of external KK graviton production.

In Fig. 1 we show the signal cross-section for jet + \cancel{E}_T at the LHC for $\delta = 4$ extra dimensions as a function of Δ . The jet is required to have $E_{T,\text{jet}} > 1$ TeV. The three lines from top to bottom correspond to $M_D = 2$ TeV, 4 TeV, and 6 TeV. The top line we terminate at $\Delta = 2$ TeV to be consistent with our expectation that $\Delta \lesssim M_D$. As expected the reduction of the signal is more than a factor of two below the rigid brane estimates ($\Delta \simeq M_D$) for reasonable values of $\Delta \lesssim M_D/2$. For even smaller Δ the rate drops off exponentially, making detection extremely difficult at the LHC.

In Fig. 2 we plot the signal cross-section for jet + \cancel{E}_T at the LHC for $M_D = 4$ TeV as a function of Δ . The three lines from top to bottom correspond to $\delta = 2, 4,$ and 6 . $\delta = 6$ is initially lower than $\delta = 4$ at lower values of Δ . The ratio of the higher δ value lines to lower δ value lines increases as Δ increases. This is because the density of states for higher allowed graviton masses increases as $(\Delta R)^\delta$. Taking into account that R is a function of M_D

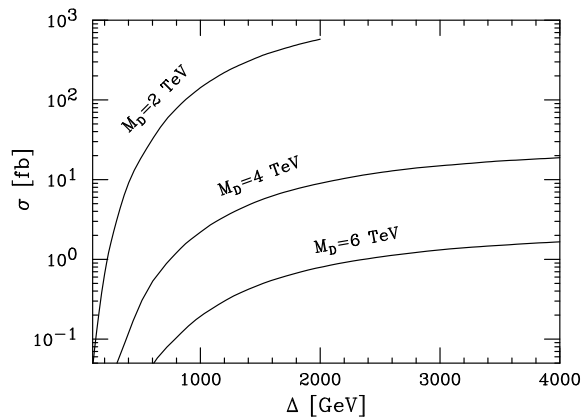


FIG. 1. Signal cross-section for jet + \cancel{E}_T at the LHC for $\delta = 4$ extra dimensions as a function of Δ , the exponential damping scale for KK graviton mode couplings to matter. The jet is required to have $E_{T,\text{jet}} > 1$ TeV.

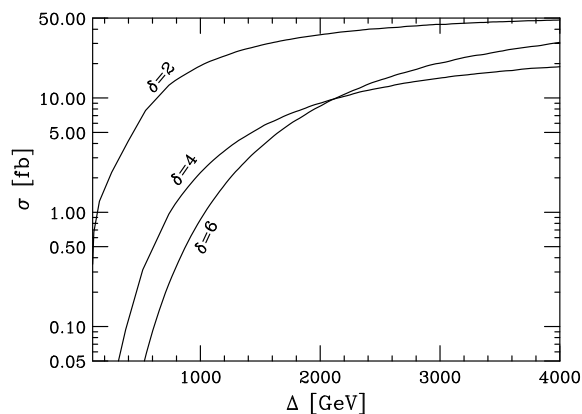


FIG. 2. Signal cross-section for jet + \cancel{E}_T at the LHC for $M_D = 4$ TeV as a function of Δ , the exponential damping scale for KK graviton mode couplings to matter.

and δ (see eq. 1), for high-enough Δ the $\delta = 6$ line can overcome the $\delta = 4$ line and cross, as it does in Fig. 2.

In Fig. 3 we plot the signal cross-section for jet + \cancel{E}_T at the LHC for $\Delta = 3$ TeV. The three lines from top to bottom at low M_D correspond to $\delta = 6, 4,$ and 2 . At higher M_D the three lines reorder themselves such that from top to bottom $\delta = 2, 4,$ and 6 . The reordering of these lines is again related to the multiplicity of available KK graviton states in the scattering process. When $M_D < \Delta$, the precise value of Δ is less important and the results are close to what are expected for the rigid brane assumption. As M_D increases above Δ the precise value of Δ is important again, and the multiplicity of gravitons with unsoftened interactions becomes $(\Delta R)^\delta \propto \Delta^\delta / M_D^{2+\delta}$. In this regime, the higher the number of extra dimensions δ the more suppressed the relative number of available states and so the larger δ lines dip below the smaller δ lines.

In Fig. 4 we plot the M_D sensitivity at the LHC with 100 fb^{-1} as a function of Δ . To be clearly above

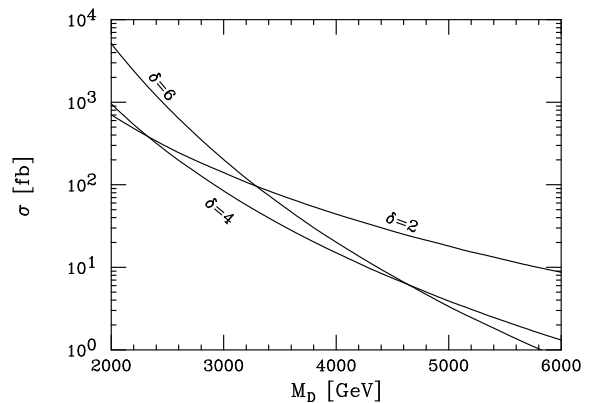


FIG. 3. Signal cross-section for jet + \cancel{E}_T at the LHC for $\Delta = 3$ TeV, where Δ is the exponential damping scale for KK graviton mode couplings to matter.

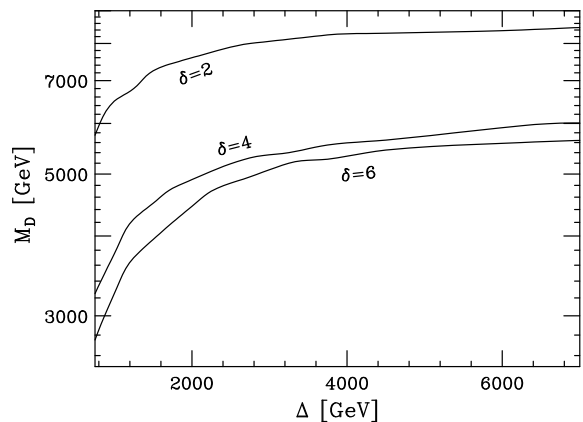


FIG. 4. M_D sensitivity (as explained in text) at the LHC with 100 fb^{-1} as a function of Δ , the exponential damping scale for KK graviton mode couplings to matter.

background we require at least 260 events with $E_{T,\text{jet}} > 1$ TeV [4]. The estimated M_D sensitivity with large Δ is similar to what is obtained for a rigid brane [4]. However, with lower values of Δ the M_D sensitivity of the LHC drops precipitously.

Finally, we plot the M_D sensitivity at a 1 TeV e^+e^- linear collider assuming 500 fb^{-1} as a function of Δ . This plot was obtained by requiring that the photon have transverse energy above 300 GeV and total energy below 450 GeV to overcome backgrounds [4]. We also assume 90% polarization of the e^- beam to further reduce the $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ background from WW fusion. Again, the sensitivity reduces dramatically for lower values of $\Delta < M_D$.

One advantage of the linear collider is the precise center-of-mass energy of the e^+e^- collisions. With data at several different center-of-mass energies, one could in principle fit the total rates to the exponential softening and determine the Δ mass scale. It would likely take careful measurements at more than two different center-of-mass energies to resolve unambiguously the values of

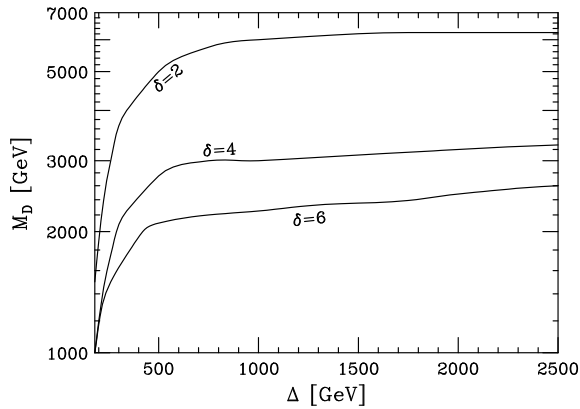


FIG. 5. M_D sensitivity (as explained in text) at a 1 TeV e^+e^- linear collider assuming 500 fb^{-1} as a function of Δ , the exponential damping scale for KK graviton mode couplings to matter.

Δ , M_D and δ . Angular distributions and photon energy distributions at a single energy are of limited value.

Conclusion: We have demonstrated that the ability to discover large extra dimensions via external graviton emission is diminished if the SM 3-brane has low tension compared to the fundamental gravity scale. This induces large brane fluctuations, softening the effective coupling between the extra-dimensional graviton and SM states living on the brane.

The softening of graviton couplings to matter only occurs for Kaluza-Klein modes with mass above the scale Δ , or in other words significant momentum into the extra dimensions. Astrophysical constraints [3,13,14] are derived from SM particle interactions with KK modes of very small mass compared to reasonable expectations for Δ . Therefore, astrophysical constraints are likely not affected much by brane fluctuations. Nevertheless, there exist more complex compactification schemes which are unconstrained by astrophysical experiments, and yet still can be probed effectively at high-energy colliders [15]. In these cases, we expect that a full complement of collider and astrophysical tests will be needed to understand the geometry of compactification, the properties of the SM brane, and the nature of the more fundamental theory.

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