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Classification and asymptotic scaling of the light-cone wave-function amplitudes of hadrons

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Abstract. We classify the hadron light-cone wave-function amplitudes in terms of parton helicity, orbital angular momentum, and quark-flavor and color symmetries. We show in detail how this is done for the pion, ρ meson, nucleon, and delta resonance up to and including three partons. For the pion and nucleon, we also consider four-parton amplitudes. Using the scaling law derived previously, we show how these amplitudes scale in the limit that all parton transverse momenta become large.

1 Introduction

Although the hadron structure is believed to be described by the fundamental theory of strong interactions, quantum chromodynamics (QCD), the actual solution of the problem is notoriously difficult to achieve. Apart from numerically solving QCD on a spacetime lattice, there is no other systematic theoretical approach that has been very successful. The closest might be the light-front quantization approach in which the old-fashioned method of diagonalizing a hamiltonian is followed [1, 2]. The conceptual advantage here is obvious: Hadrons are described by light-cone wave functions which have a clear physical meaning and are very useful phenomenologically, whereas in lattice QCD the natural language is classical gluon configurations, such as instantons and monopoles, in Euclidean space. As to why light-cone quantization is superior compared to equal-time quantization, we just wish to point out that the vacuum structure in the former approach, which consists of just $k^+ = 0$ particles, can be easily separated from the part of the hadron structure consisting of $k^+ \neq 0$ particles. Moreover, in high-energy scattering hadrons travel near the speed of light, and light-cone coordinates appear naturally.

To be sure, there are many difficulties that one must clear before a realistic light-cone description of hadrons becomes possible. One of them is that hadrons are now described by an infinite number of light-cone Fock amplitudes, and there is no apparent reason why the amplitudes with 100 partons (quarks and gluons) are strongly suppressed relative to those with two or three partons. The answer, of course, depends on the choice of the gauge, and of ultraviolet and infrared cut-offs, and ultimately on the underlying QCD dynamics. One way to check is to truncate the Hilbert space first so as to include the partons to a maximum number n and then to determine how the solution changes when the Fock components with $n+1$ number of partons are included. The optimistic view has been that since the constituent quark models work so well phenomenologically, there must exist a light-cone description of hadrons in which only the Fock components with a few partons are necessary. In high-energy exclusive processes, we know for sure that only the wave-function components with a few partons are relevant.

In the light-front description of a hadron, the very first step is to classify independent wave-function amplitudes given a particular parton combination. To our knowledge, there has not been much systematic study in the literature along this direction. In [3], we have proposed an approach by writing down the matrix elements of a class of light-cone-correlated quark–gluon operators, in much the same way as has been used to construct independent light-cone distribution amplitudes in which the parton transverse momenta are integrated out [4]. In [5], we have applied the approach to the nucleon, finding that six amplitudes are needed to describe the three-quark sector of the nucleon wave function. However, using the approach to handle Fock states with more partons appears to be complicated.

In [6], we have developed a more direct method to write down the general structure of the light-cone wave function for n partons with orbital angular momentum projection l_z. We have also found a general power counting rule which determines the asymptotic behavior of the light-cone amplitudes when the transverse momenta of all partons become large. From the wave-function counting rule, we have derived the dimensional scaling law for high-

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energy exclusive processes including parton orbital angular momentum [7,8]. In this paper, we report further progress in this direction. In particular, we use the method to classify the higher Fock components of hadrons. We will consider in detail how the flavor (for quarks) and color degrees of freedom of the partons are systematically coupled. We will write down the amplitudes for the pion and proton up to four partons. We also work out the leading light-cone wave functions for the Δ and ρ meson, leaving more complicated cases for the readers.

Based on our work here, one can go on to parameterize the light-cone wave-function amplitudes and fit to the experimental data. Although the amplitudes thus determined are phenomenological, they can be made to obey the asymptotic behavior at large transverse momenta [6]. Therefore, our result can provide guidelines for phenomenological studies for exclusive processes [9–12]. By committing ourselves to the light-cone amplitudes, we are also committing ourselves to the light-cone gauges $A^+ = 0$ [13,14]. One subtlety about the light-cone gauge is that it requires additional gauge fixing [15–17]. Depending on whether the additional gauge condition is time-reversal invariant or not, the wave-function amplitudes are real or fully complex. In the latter case, the final state interaction effects might be included in the amplitudes [18–21]. A related issue is that the light-cone amplitudes have light-cone singularities at small x which require regularization.

Our plan of the presentation is as follows. We start in Sect. 2 by describing a general strategy to classify the independent wave-function amplitudes for a hadron state with a specific parton content. In Sect. 3, we apply this method to write down the amplitudes of π^+ for up to four-parton Fock components. We extend these discussions to ρ mesons in Sect. 4, where the amplitudes up to three-parton Fock components will be given. In Sect. 5, the proton wave-function amplitudes for three-quark and three-quark plus one-gluon Fock components will be presented. The leading results for the delta resonance will be given in Sect. 6. The final section contains a brief summary and outlook.

2 General strategy and symmetry constraints

In this section, we discuss our general strategy in classifying and enumerating the hadron light-cone amplitudes. The goal is to find a simple and general way to write down all possible light-cone amplitudes of a hadron once a parton content is specified. In Sect. 2.1, we explain our notation and conventions. In Sect. 2.2, we consider the helicity and angular momentum structure of a general Fock component. In Sect. 2.3 we make general comments about flavor and color structure. In Sects. 2.4 and 2.5, we consider the parity and time-reverse constraints on the light-cone wave-function amplitudes.

2.1 Notation

We work in the framework of light-cone (or light-front) quantization [1,14]. The light-cone time x^+ and coordinate x^- are defined as $x^\pm = 1/\sqrt{2}(x^0 \pm x^3)$. Likewise we define the Dirac matrices $\gamma^\pm = 1/\sqrt{2}(\gamma^0 \pm \gamma^3)$. The projection operators for the Dirac fields are defined as $P_\pm = (1/2)\gamma^\mp\gamma^\pm$. Any Dirac field ψ can be decomposed into $\psi = \psi_+ + \psi_-$ with $\psi_\pm = P_\pm\psi$. ψ_+ is a dynamical degree of freedom and has the canonical expansion

$$\begin{aligned}\psi_+(\xi^+ = 0, \xi^-, \xi_\perp) &= \int \frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\mathrm{d}k^+}{2k^+}\sum_\lambda \Big[b_\lambda(k)u(k\lambda)\mathrm{e}^{-\mathrm{i}(k^+\xi^- - \mathbf{k}_\perp\cdot\xi_\perp)} \\ &\quad + d^\dagger_\lambda(k)v(k\lambda)\mathrm{e}^{\mathrm{i}(k^+\xi^- - \mathbf{k}_\perp\cdot\xi_\perp)}\Big] , \qquad (1)\end{aligned}$$

where $b^\dagger(b)$ and $d^\dagger(d)$ are quark and antiquark creation (annihilation) operators, respectively. We adopt covariant normalization for the particle states and the creation and annihilation operators, i.e.,

$$\begin{aligned}\left\{b_\lambda(k), b^\dagger_{\lambda'}(k')\right\} &= \left\{d_\lambda(k), d^\dagger_{\lambda'}(k')\right\} \\ &= (2\pi)^3\delta_{\lambda\lambda'}2k^+\delta(k^+ - k'^+)\delta^{(2)}(\mathbf{k}_\perp - \mathbf{k}'_\perp) , \qquad (2)\end{aligned}$$

where λ is the light-cone helicity of the quarks which can take the values $+1/2$ or $-1/2$. We ignore the masses of the light up and down quarks. Later, to simplify the notation, we simply use $u^\dagger$ and $\overline{u}^\dagger$ to represent creation operators for up and anti-up quarks, respectively, and so on.

Likewise, for the gluon fields in the light-cone gauge $A^+ = 0$, $A_\perp$ is dynamical and has the expansion

$$\begin{aligned}A_\perp(\xi^+ = 0, \xi^-, \xi_\perp) &= \int \frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\mathrm{d}k^+}{2k^+}\sum_\lambda \Big[a_\lambda(k)\epsilon(k\lambda)\mathrm{e}^{-\mathrm{i}(k^+\xi^- - \mathbf{k}_\perp\cdot\xi_\perp)} \\ &\quad + a^\dagger_\lambda(k)\epsilon^*(k\lambda)\mathrm{e}^{\mathrm{i}(k^+\xi^- - \mathbf{k}_\perp\cdot\xi_\perp)}\Big] . \qquad (3)\end{aligned}$$

Implicitly, the gauge fields A^μ is a traceless 3×3 matrix with $A^\mu = \sum_a A^{a\mu}T^a$, where T^a are the $SU(3)$ Gell-Mann matrices satisfying $[T^a, T^b] = \mathrm{i}f^{abc}T^c$ and $\{T^a, T^b\} = \frac{1}{3}\delta_{ab} + d_{abc}T^c$. Again, we have the following covariant normalization for the creation and annihilation operators for the gluon:

$$\left[a_\lambda(k), a^\dagger_{\lambda'}(k')\right] = (2\pi)^3\delta_{\lambda\lambda'}2k^+\delta(k^+ - k'^+)\delta^{(2)}(\mathbf{k}_\perp - \mathbf{k}'_\perp) , \qquad (4)$$

Later, we simply use $g^\dagger$ to represent a gluon creation operator. ψ_- and A^- are dependent variables, which can be expressed in terms of ψ_+ and $A_\perp$ using the equations of motion [13].

2.2 Angular momentum structure

For a given parton content, i.e., a specification of quarks, antiquarks and gluons, the light-cone amplitudes of a hadron with helicity Λ can be classified in terms of the total parton light-cone helicity λ. The angular momentum conservation then demands that the partons have angular momentum projection $l_z = \Lambda - \lambda$. Let us find the angular momentum structure of the amplitudes satisfying these conditions [6].

Suppose a Fock component has n partons with creation operators $a_1^\dagger, \ldots, a_n^\dagger$, where the partons can either be gluons or quarks, and the subscripts label the partons' quantum numbers such as spin, flavor, color, momentum, etc. Assume all color, flavor (for quarks) indices have been coupled properly using Clebsch–Gordon coefficients (see the next subsection). The longitudinal momentum fractions of the partons are x_i $(i = 1, 2, \ldots, n)$, satisfying $\sum_{i=1}^n x_i = 1$, and the transverse momenta $\mathbf{k}_{1\perp}, \ldots, \mathbf{k}_{n\perp}$, satisfying $\sum_i^n \mathbf{k}_{i\perp} = 0$. We will eliminate $\mathbf{k}_{n\perp}$ in favor of the first $n-1$ transverse momenta. Assume the orbital angular momentum projections of the partons are $l_{z1}, \ldots, l_{z(n-1)}$, respectively, and let $l_z = \sum_{i=1}^{n-1} l_{zi}$; then

$$l_z + \lambda = \Lambda \ , \tag{5}$$

where $\lambda = \sum_{i=1}^n \lambda_i$ is the total parton helicity. Following the above procedure, we can eliminate all negative l_{zj}, a general $l_z > 0$ component in the wave function reads. A general structure for $l_z > 0$ component in the hadron light-cone wave-function amplitude reads [6]

$$\int \prod_{i=1}^n \mathrm{d}[i] \quad (k_{1\perp}^+)^{l_{z1}} (k_{2\perp}^+)^{l_{z2}} \ldots (k_{(n-1)\perp}^+)^{l_{z(n-1)}} \times \left(\psi_n(x_i, k_i, \lambda_i, l_{zi}) + \sum_{i<j=1|_{l_{zi}=l_{zj}=0}}^{n-1} \mathrm{i}\epsilon^{\alpha\beta} k_{i\alpha} k_{j\beta} \psi_{n(ij)}(x_i, k_{i\perp}, \lambda_i, l_{zi}) \right) \times a_1^\dagger a_2^\dagger \ldots a_n^\dagger |0\rangle \ , \tag{6}$$

where $\sum_i l_{zi} = l_z$ and $l_{zi} \geq 0$, and the sums over i and j are restricted to the $l_{zi} = 0$ partons. The above equation is our starting point to write down independent light-cone amplitudes.

2.3 Flavor and color structure

For a given quark content, we classify the amplitudes in terms of the flavor symmetry. For instance, for the pion state we need to project out the Fock component with the total isospin 1. The problem becomes more involved if a Fock state contains many quark–antiquark pairs because there is more than one way to construct the states with the definite isospin.

Our general strategy is as follows: we first consider all possible ways to construct the same isospin. We then use the freedom that the labels of the quark partons are arbitrary to shuffle the particles around. If after the shuffling, the flavor content of a combination is identical to the one considered before, the combination is ignored. For example, consider the $u\bar{d}q\bar{q}$ component of a π^+ particle. The $q\bar{q}$ can either couple to $I = 1$ or $I = 0$. It turns out that the combination coupled to $I = 1$ is not independent after reshuffling of the particle label.

All the hadrons are color neutral. Therefore, we couple all partons to the color singlet. All possible ways of making the coupling must be considered.

2.4 Parity

Consider a hadron moving in the z direction with helicity Λ, $|P\Lambda\rangle$. Under a parity transformation, the momentum changes direction, and the helicity changes sign. However, if we make an additional 180° rotation around the y axis, the original momentum is restored, and we have a state $|P - \Lambda\rangle$. According to Jacob and Wick [22], we have

$$(-1)^{s-\Lambda}\eta|P - \Lambda\rangle = \hat{Y}|P\Lambda\rangle \ , \tag{7}$$

where s is the total spin of a hadron state or a parton state. $\hat{Y}$ is a parity operation followed by a 180° rotation around the y axis, and η is the intrinsic parity of the hadron.

For a particle state with non-zero helicity, the above equation allows one to obtain the wave function of the state with helicity $(-\Lambda)$ from that with helicity Λ. On the other hand, for a particle of zero helicity, the above equation can be considered as a constraint on the wave function.

When $\hat{Y}$ acts on the individual partons, the transformation is

$$(-1)^{s-\lambda}\eta|k_x, -k_y, k_z, -\lambda\rangle = \hat{Y}|k_x, k_y, k_z, \lambda\rangle \ , \tag{8}$$

where the intrinsic parity for a quark is $+1$, an antiquark -1, and a gluon -1. For instance, for a u quark state,

$$\hat{Y}|u_\uparrow\rangle = |u_\downarrow\rangle, \quad \hat{Y}|u_\downarrow\rangle = -|u_\uparrow\rangle, \tag{9}$$

where we have omitted the momentum label. For a $\bar{d}$ quark state,

$$\hat{Y}|\bar{d}_\uparrow\rangle = -|\bar{d}_\downarrow\rangle, \quad \hat{Y}|\bar{d}_\downarrow\rangle = |\bar{d}_\uparrow\rangle, \tag{10}$$

because of the opposite intrinsic parity.

2.5 Time reversal

Time reversal usually provides constraints on the reality of the wave-function amplitudes. Under the transformation, however, the light-cone time and coordinate interchange. To preserve the original light-cone coordinates, we consider the combined time reversal and parity operations.

The light-cone gauge condition is invariant under the combined transformation. However, $A^+ = 0$ does not fix the gauge freedom completely; additional gauge fixing must be specified. Physically the additional gauge fixing corresponds to a choice of boundary conditions for gauge fields at $\xi^- = \pm\infty$. If one chooses the antisymmetric boundary condition, $A_\perp(\xi^- = -\infty) = -A_\perp(\xi^- = \infty)$, which is invariant under the combined transformation, then one can show that all light-cone wave-function amplitudes are real (principal-value prescription). On the other hand, if one chooses either advanced or retarded boundary conditions $A_\perp(\xi^- = \pm\infty) = 0$, the combined transformation is broken, and the wave-function amplitudes are complex.

3 Wave-function amplitudes for the pion

In this section, we classify the light-cone wave-function amplitudes for the π^+ meson up to and including four partons. The amplitudes for other isotriplet members can be obtained by using the isospin lowering operator. The pion is a pseudoscalar meson with spin $J = 0$ and parity $P = -1$. These quantum numbers are necessary constraints when the light-cone wave-function amplitudes are constructed.

A pion moving in the z direction has the following transformation under $\hat{Y}$:

$$\hat{Y}|\pi^+\rangle = -|\pi^+\rangle \ . \tag{11}$$

Every Fock component we write down must have this symmetry.

In the following subsections, we present the wave-function amplitudes of π^+ up to four-particle component Fock states, i.e., $u\bar{d}$, $u\bar{d}g$, $u\bar{d}gg$, and $u\bar{d}q\bar{q}$. Related studies on the light-cone distribution amplitudes for π mesons can be found in [14, 23–26].

3.1 The $u\bar{d}$ component

For this component, $n = 2$, and the total quark helicity λ can be either 0 or 1. The isospin does not provide any additional constraint. From (6) we can have two wave-function amplitudes, corresponding to $l_z = 0$ and $|l_z| = 1$. They have been discussed in [3] and many other references before. For completeness, we present the results here:

$$|\pi^+\rangle_{u\bar{d}}^{l_z=0} = \int \mathrm{d}[1]\mathrm{d}[2] \tag{12}$$
$$\times\ \psi^{(1)}_{u\bar{d}}(1,2)\frac{\delta_{ij}}{\sqrt{3}}\left[u^\dagger_{\uparrow i}(1)\bar{d}^\dagger_{\downarrow j}(2) - u^\dagger_{\downarrow i}(1)\bar{d}^\dagger_{\uparrow j}(2)\right]|0\rangle \ ,$$

$$|\pi^+\rangle_{u\bar{d}}^{|l_z|=1} = \int \mathrm{d}[1]\mathrm{d}[2] \tag{13}$$
$$\times\ \psi^{(2)}_{u\bar{d}}(1,2)\frac{\delta_{ij}}{\sqrt{3}}\left[k^-_{1\perp}u^\dagger_{\uparrow i}(1)\bar{d}^\dagger_{\uparrow j}(2) + k^+_{1\perp}u^\dagger_{\downarrow i}(1)\bar{d}^\dagger_{\downarrow j}(2)\right]|0\rangle \ ,$$

where i and $j = 1, 2, 3$ are the color indices, and $\uparrow$ and $\downarrow$ label the quark light-cone helicities $+1/2$ and $-1/2$, respectively. The color factor $\delta_{ij}/\sqrt{3}$ is normalized to 1. The amplitudes $\psi^{(1,2)}_{u\bar{d}}(1,2)$ are functions of quark momenta with argument 1 representing x_1 and $k_{1\perp}$ and so on. The dependence on the transverse momenta is of the form $\mathbf{k}_{i\perp}\cdot\mathbf{k}_{j\perp}$ only. Since the momentum conservation implies $\mathbf{k}_{1\perp} + \mathbf{k}_{2\perp} = 0$ and $x_1 + x_2 = 1$, $\psi^{(1,2)}_{u\bar{d}}(1,2)$ depend on the variables x_1 and $k^2_{1\perp}$ only. The integration in the above equation become

$$\int \mathrm{d}[1]\mathrm{d}[2] = \int \frac{\mathrm{d}^2k_{1\perp}}{(2\pi)^3}\frac{\mathrm{d}x_1}{2\sqrt{x_1(1-x_1)}} \ .$$

It is easy to check that the amplitudes $\psi^{(1,2)}_{u\bar{d}}(1,2)$ have the correct transformation behavior under $\hat{Y}$.

3.2 The $u\bar{d}g$ component

For this component, $n = 3$, and total parton helicity λ can be 0, 1, or 2. Therefore, the light-cone wave-function amplitudes must have $|l_z| = 0$, 1, or 2. Again isospin symmetry does not provide any constraint.

To satisfy the constraint from parity, we consider the $u\bar{d}$ pair with definite properties under the $\hat{Y}$ transformation

$$\begin{aligned}(u\bar{d})^\dagger_{S,0} &= u^\dagger_{\uparrow i}(1)\bar{d}^\dagger_{\downarrow j}(2) + u^\dagger_{\downarrow i}(1)\bar{d}^\dagger_{\uparrow j}(2) \ ,\\ (u\bar{d})^\dagger_{A,0} &= u^\dagger_{\uparrow i}(1)\bar{d}^\dagger_{\downarrow j}(2) - u^\dagger_{\downarrow i}(1)\bar{d}^\dagger_{\uparrow j}(2) \ ,\\ (u\bar{d})^\dagger_{A,1} &= u^\dagger_{\uparrow i}(1)\bar{d}^\dagger_{\uparrow j}(2) \ ,\\ (u\bar{d})^\dagger_{A,-1} &= u^\dagger_{\downarrow i}(1)\bar{d}^\dagger_{\downarrow j}(2) \ .\end{aligned} \tag{14}$$

It is clear that

$$\begin{aligned}\hat{Y}(u\bar{d})^\dagger_{S,\lambda_z}|0\rangle &= (u\bar{d})^\dagger_{S,-\lambda_z}|0\rangle,\\ \hat{Y}(u\bar{d})^\dagger_{A,\lambda_z}|0\rangle &= -(u\bar{d})^\dagger_{A,-\lambda_z}|0\rangle \ ,\end{aligned} \tag{15}$$

where we have neglected the transformation of the momentum labels. On the other hand, the one-gluon state transforms under $\hat{Y}$ as follows:

$$\hat{Y}|g_\lambda\rangle = -|g_{-\lambda}\rangle \ , \tag{16}$$

because the gluon is a vector particle.

There is only one way to couple the color indices. The quark and antiquark (with color indices i and j) couple to an octet which in turn couples to the octet gluon (with color index a) to yield a singlet. The coupling can be achieved with the SU(3) matrices T^a_{ij}.

When $l_z = 0$, the helicity of the quarks must be $\lambda_{u\bar{d}} = \pm 1$ because $\lambda_g = \mp 1$. From (6) we have two independent amplitudes,

$$\begin{aligned}|\pi^+\rangle^{l_z=0}_{u\bar{d}g} &= \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\\ &\times \frac{T^a_{ij}}{2}\Big\{\psi^{(1)}_{u\bar{d}g}(1,2,3)\left[(u\bar{d})^\dagger_{A,1}g^{a\dagger}_\downarrow(3) - (u\bar{d})^\dagger_{A,-1}g^{a\dagger}_\uparrow(3)\right]\\ &\quad + \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi^{(2)}_{u\bar{d}g}(1,2,3)\\ &\quad\quad \times \left[(u\bar{d})^\dagger_{A,1}g^{a\dagger}_\downarrow(3) + (u\bar{d})^\dagger_{A,-1}g^{a\dagger}_\uparrow(3)\right]\Big\}|0\rangle \ ,\end{aligned} \tag{17}$$

where $\alpha, \beta = x, y$ are the transverse indices. Again the color factor $T^a_{ij}/2$ is normalized to one. The above states obey the right transformation under $\hat{Y}$ because the wave-function amplitudes are invariant when all y components of the parton momenta change sign.

When $|l_z|=1$, the total quark helicity must be $\lambda_{u\bar{d}}=0$ again because $\lambda_g=\pm 1$. We can write down four independent wave-function amplitudes,

$$\begin{aligned}
|\pi^+\rangle_{u\bar{d}g}^{|l_z|=1} &= \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3] \\
&\times \frac{T^a_{ij}}{2}\Big\{\psi^{(3)}_{u\bar{d}g}(1,2,3) \\
&\quad\times\Big[k^+_{1\perp}(u\bar{d})^\dagger_{A,0}g^{a\dagger}_{\downarrow}(3)-k^-_{1\perp}(u\bar{d})^\dagger_{A,0}g^{a\dagger}_{\uparrow}(3)\Big] \\
&\quad+\psi^{(4)}_{u\bar{d}g}(1,2,3) \\
&\quad\times\Big[k^+_{2\perp}(u\bar{d})^\dagger_{A,0}g^{a\dagger}_{\downarrow}(3)-k^-_{2\perp}(u\bar{d})^\dagger_{A,0}g^{a\dagger}_{\uparrow}(3)\Big] \\
&\quad+\psi^{(5)}_{u\bar{d}g}(1,2,3) \\
&\quad\times\Big[k^+_{1\perp}(u\bar{d})^\dagger_{S,0}g^{a\dagger}_{\downarrow}(3)+k^-_{1\perp}(u\bar{d})^\dagger_{S,0}g^{a\dagger}_{\uparrow}(3)\Big] \\
&\quad+\psi^{(6)}_{u\bar{d}g}(1,2,3) \\
&\quad\times\Big[k^+_{2\perp}(u\bar{d})^\dagger_{S,0}g^{a\dagger}_{\downarrow}(3)+k^-_{2\perp}(u\bar{d})^\dagger_{S,0}g^{a\dagger}_{\uparrow}(3)\Big]\Big\}|0\rangle\ .
\end{aligned} \tag{18}$$

Finally, when $|l_z|=2$, the total quark helicity $\lambda_{u\bar{d}}=\pm 1$, and $\lambda_g=\pm 1$. We have three amplitudes:

$$\begin{aligned}
|\pi^+\rangle_{u\bar{d}g}^{|l_z|=2} &= \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3] \\
&\times \frac{T^a_{ij}}{2}\Big\{\psi^{(7)}_{u\bar{d}g}(1,2,3) \\
&\quad\times\Big[k^+_{1\perp}k^+_{1\perp}(u\bar{d})^\dagger_{A,-1}g^{a\dagger}_{\downarrow}(3)-k^-_{1\perp}k^-_{1\perp}(u\bar{d})^\dagger_{A,1}g^{a\dagger}_{\uparrow}(3)\Big] \\
&\quad+\psi^{(8)}_{u\bar{d}g}(1,2,3) \\
&\quad\times\Big[k^+_{1\perp}k^+_{2\perp}(u\bar{d})^\dagger_{A,-1}g^{a\dagger}_{\downarrow}(3)-k^-_{1\perp}k^-_{2\perp}(u\bar{d})^\dagger_{A,1}g^{a\dagger}_{\uparrow}(3)\Big] \\
&\quad+\psi^{(9)}_{u\bar{d}g}(1,2,3)\Big[k^+_{2\perp}k^+_{2\perp}(u\bar{d})^\dagger_{A,-1}g^{a\dagger}_{\downarrow}(3) \\
&\qquad-\ k^-_{2\perp}k^-_{2\perp}(u\bar{d})^\dagger_{A,1}g^{a\dagger}_{\uparrow}(3)\Big]\Big\}|0\rangle\ .
\end{aligned} \tag{19}$$

Summing up, we have a total of nine independent light-cone amplitudes for the pion component with three partons.

3.3 The $u\bar{d}gg$ component

For this component, the helicity of the two gluons can be $\lambda_{gg}=0,\ \pm 2$, and that for the quarks $\lambda_{u\bar{d}}=0,\ \pm 1$, and so $|l_z|=0,1,2$, or 3. To make the $\hat{Y}$ transformation simple, we combine the two gluons in the similar way as we did for $u\bar{d}$ in the last subsection,

$$\begin{aligned}
(gg)^\dagger_{S,0} &= g^\dagger_{\uparrow a}(3)g^\dagger_{\downarrow b}(4)+g^\dagger_{\downarrow a}(3)g^\dagger_{\uparrow b}(4)\ , \\
(gg)^\dagger_{A,0} &= g^\dagger_{\uparrow a}(3)g^\dagger_{\downarrow b}(4)-g^\dagger_{\downarrow a}(3)g^\dagger_{\uparrow b}(4)\ , \\
(gg)^\dagger_{S,2} &= g^\dagger_{\uparrow a}(3)g^\dagger_{\uparrow b}(4)\ , \\
(gg)^\dagger_{S,-2} &= g^\dagger_{\downarrow a}(3)g^\dagger_{\downarrow b}(4)\ ,
\end{aligned} \tag{20}$$

where the subscripts A and S indicate that there is a factor -1 and 1, respectively, under the $\hat{Y}$ transformation.

There are three different ways to couple the color indices of the two quarks and two gluons to form color singlets. If the color indices for the two quarks are i and j and those for two gluons are a and b, we have the singlet combinations $f_{abc}T^c_{ij}$, $d_{abc}T^c_{ij}$, and $\delta_{ab}\delta_{ij}$. The last two are symmetric in the color indices of the two gluons, while the first one is antisymmetric. In the following, we only present the results for the color coupling $\delta_{ab}\delta_{ij}$ (the quark pair and two gluons are both color singlet), and those for the other two couplings can be obtained similarly.

To maximally utilize Bose symmetry between the two gluons, we will eliminate the momentum of the up quark (labeled by 1 below) in favor of the momenta of the anti-down quark and the two gluons.

For $l_z=0$, the only possible parton helicity combination is $\lambda_{gg}=0$ and $\lambda_{u\bar{d}}=0$. In this case, we have six independent light-cone amplitudes following (6):

$$\begin{aligned}
|\pi^+\rangle_{u\bar{d}gg}^{l_z=0} &= \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4] \\
&\times\frac{\delta_{ij}\delta^{ab}}{\sqrt{24}}\Big\{\psi^{(1)}_{u\bar{d}gg}(1,2,3,4)(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{S,0} \\
&\quad+\psi^{(2)}_{u\bar{d}gg}(1,2,3,4)(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{A,0} \\
&\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi^{(3)}_{u\bar{d}gg}(1,2,3,4)(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{S,0} \\
&\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{3\alpha}k_{4\beta}\psi^{(4)}_{u\bar{d}gg}(1,2,3,4)(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{S,0} \\
&\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi^{(5)}_{u\bar{d}gg}(1,2,3,4)(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{A,0} \\
&\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{3\alpha}k_{4\beta}\psi^{(6)}_{u\bar{d}gg}(1,2,3,4)(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{A,0}\Big\}|0\rangle\ ,
\end{aligned} \tag{21}$$

where we have used the symmetry between two gluons ($3\leftrightarrow 4$) to reduce the number of independent amplitudes. For example, $\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{4\beta}$ can be obtained from $\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}$ by 3 and 4 exchange, and so the former is not independent. This property is general for all of the light-cone amplitudes of the $u\bar{d}gg$ component, and will be used throughout the following classification. Because of the (anti-) symmetric properties for the two gluons, the above amplitudes have the following symmetry: $\psi^{(1,6)}_{u\bar{d}gg}(1,2,3,4)=\psi^{(1,6)}_{u\bar{d}gg}(1,2,4,3)$ and $\psi^{(2,4)}_{u\bar{d}gg}(1,2,3,4)=-\psi^{(2,4)}_{u\bar{d}gg}(1,2,4,3)$.

For $|l_z|=1$, the parton helicity has two possible combinations: either $\lambda_{gg}=0$ and $\lambda_{u\bar{d}}=\mp 1$, or $\lambda_{gg}=\mp 2$ and $\lambda_{u\bar{d}}=\pm 1$. For the first case, we have eight independent amplitudes:

$$
\begin{aligned}
|\pi^+\rangle^{|l_z|=1}_{u\bar{d}gg} &= \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4] \\
\times \frac{\delta^{ij}\delta_{ab}}{\sqrt{24}} &\Big\{\psi^{(7)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{2\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,0} + k^-_{2\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,0}\right] \\
&+\psi^{(8)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{3\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,0} + k^-_{3\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,0}\right] \\
&+\psi^{(9)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{2\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{A,0} - k^-_{2\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{A,0}\right] \\
&+\psi^{(10)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{3\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{A,0} - k^-_{3\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{A,0}\right] \qquad (22) \\
&+\mathrm{i}\epsilon^{\alpha\beta}k_{3\alpha}k_{4\beta}\psi^{(11)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{2\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,0} - k^-_{2\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,0}\right] \\
&+\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{4\beta}\psi^{(12)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{3\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,0} - k^-_{3\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,0}\right] \\
&+\mathrm{i}\epsilon^{\alpha\beta}k_{3\alpha}k_{4\beta}\psi^{(13)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{2\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{A,0} + k^-_{2\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{A,0}\right] \\
&+\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{4\beta}\psi^{(14)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{3\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{A,0} + k^-_{3\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{A,0}\right]\Big\}|0\rangle\ ,
\end{aligned}
$$

where the $3 \leftrightarrow 4$ symmetry again plays an important role in reducing the number of independent amplitudes. For the second case, $\lambda_{gg} = \mp 2$ and $\lambda_{u\bar{d}} = \pm 1$, we find four independent amplitudes:

$$
\begin{aligned}
|\pi^+\rangle^{|l_z|=1}_{u\bar{d}gg} &= \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4] \\
\times \frac{\delta^{ij}\delta_{ab}}{\sqrt{24}} &\Big\{\psi^{(15)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{2\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,-2} + k^-_{2\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,2}\right] \\
&+\psi^{(16)}_{u\bar{d}gg}(1,2,3,4) \qquad (23) \\
&\times \left[k^+_{3\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,-2} + k^-_{3\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,2}\right] \\
&+\mathrm{i}\epsilon^{\alpha\beta}k_{3\alpha}k_{4\beta}\psi^{(17)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{2\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,-2} - k^-_{2\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,2}\right] \\
&+\mathrm{i}\epsilon^{\alpha\beta}k_{3\alpha}k_{4\beta}\psi^{(18)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{3\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,-2} - k^-_{3\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,2}\right]\Big\}|0\rangle\ .
\end{aligned}
$$

The Bose symmetry implies the following constraints:

$$\psi^{(7,13,15)}_{u\bar{d}gg}(1,2,3,4) = \psi^{(7,13,15)}_{u\bar{d}gg}(1,2,4,3)$$

and

$$\psi^{(8,11,17)}_{u\bar{d}gg}(1,2,3,4) = -\psi^{(8,11,17)}_{u\bar{d}gg}(1,2,4,3).$$

For $|l_z| = 2$, the parton helicity must be $\lambda_{gg} = \mp 2$ and $\lambda_{u\bar{d}} = 0$. We find 12 independent amplitudes:

$$
\begin{aligned}
|\pi^+\rangle^{|l_z|=2}_{u\bar{d}gg} &= \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4] \\
\times \frac{\delta^{ij}\delta_{ab}}{\sqrt{24}} &\Big\{\psi^{(19)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{2\perp}k^+_{2\perp}(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{S,-2} + k^-_{2\perp}k^-_{2\perp}(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{S,2}\right] \\
&+ \psi^{(20)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{2\perp}k^+_{3\perp}(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{S,-2} + k^-_{2\perp}k^-_{3\perp}(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{S,2}\right] \\
&+ \psi^{(21)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{3\perp}k^+_{3\perp}(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{S,-2} + k^-_{3\perp}k^-_{3\perp}(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{S,2}\right] \\
&+ \psi^{(22)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{3\perp}k^+_{4\perp}(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{S,-2} + k^-_{3\perp}k^-_{4\perp}(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{S,2}\right] \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{3\alpha}k_{4\beta}\psi^{(23)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{2\perp}k^+_{2\perp}(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{S,-2} - k^-_{2\perp}k^-_{2\perp}(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{S,2}\right] \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{4\beta}\psi^{(24)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{3\perp}k^+_{3\perp}(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{S,-2} - k^-_{3\perp}k^-_{3\perp}(u\bar{d})^\dagger_{A,0}(gg)^\dagger_{S,2}\right] \\
&+ \psi^{(25)}_{u\bar{d}gg}(1,2,3,4) \qquad (24) \\
&\times \left[k^+_{2\perp}k^+_{2\perp}(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{S,-2} - k^-_{2\perp}k^-_{2\perp}(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{S,2}\right] \\
&+ \psi^{(26)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{2\perp}k^+_{3\perp}(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{S,-2} - k^-_{2\perp}k^-_{3\perp}(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{S,2}\right] \\
&+ \psi^{(27)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{3\perp}k^+_{3\perp}(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{S,-2} - k^-_{3\perp}k^-_{3\perp}(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{S,2}\right] \\
&+ \psi^{(28)}_{u\bar{d}gg}(1,2,3,4) \\
&\times \left[k^+_{3\perp}k^+_{4\perp}(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{S,-2} - k^-_{3\perp}k^-_{4\perp}(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{S,2}\right] \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{3\alpha}k_{4\beta}\psi^{(29)}_{u\bar{d}gg}(1,2,3,4)
\end{aligned}
$$

$$\times \left[k^+_{2\perp}k^+_{2\perp}(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{S,-2} + k^-_{2\perp}k^-_{2\perp}(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{S,2}\right]$$
$$+ \mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{4\beta}\psi^{(30)}_{u\bar{d}gg}(1,2,3,4)$$
$$\times \left[k^+_{3\perp}k^+_{3\perp}(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{S,-2} + k^-_{3\perp}k^-_{3\perp}(u\bar{d})^\dagger_{S,0}(gg)^\dagger_{S,2}\right]\Big\} |0\rangle \ .$$

The Bose symmetry yields the following constraints:

$$\psi^{(19,22,25,28)}_{u\bar{d}gg}(1,2,3,4) = \psi^{(19,22,25,28)}_{u\bar{d}gg}(1,2,4,3)$$

and

$$\psi^{(23,29)}_{u\bar{d}gg}(1,2,3,4) = -\psi^{(23,29)}_{u\bar{d}gg}(1,2,4,3).$$

For $|l_z| = 3$, the parton helicity must be $\lambda_{gg} = \pm 2$ and $\lambda_{u\bar{d}} = \pm 1$. We find eight independent light-cone amplitudes:

$$|\pi^+\rangle^{|l_z|=3}_{u\bar{d}gg} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4]$$
$$\times \frac{\delta^{ij}\delta_{ab}}{\sqrt{24}} \Big\{\psi^{(31)}_{u\bar{d}gg}(1,2,3,4)$$
$$\times \left[(k^+_{2\perp})^3(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,-2} + (k^-_{2\perp})^3(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,2}\right]$$
$$+ \psi^{(32)}_{u\bar{d}gg}(1,2,3,4)$$
$$\times \left[(k^+_{3\perp})^3(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,-2} + (k^-_{3\perp})^3(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,2}\right]$$
$$+ \psi^{(33)}_{u\bar{d}gg}(1,2,3,4)\left[(k^+_{2\perp})^2k^+_{3\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,-2} + (k^-_{2\perp})^2k^-_{3\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,2}\right]$$
$$+ \psi^{(34)}_{u\bar{d}gg}(1,2,3,4)\left[(k^+_{3\perp})^2k^+_{2\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,-2} + (k^-_{3\perp})^2k^-_{2\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,2}\right] \quad (25)$$
$$+ \psi^{(35)}_{u\bar{d}gg}(1,2,3,4)\left[(k^+_{3\perp})^2k^+_{4\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,-2} + (k^-_{3\perp})^2k^-_{4\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,2}\right]$$
$$+ \psi^{(36)}_{u\bar{d}gg}(1,2,3,4)\left[k^+_{2\perp}k^+_{3\perp}k^+_{4\perp}(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,-2} + k^-_{2\perp}k^-_{3\perp}k^-_{4\perp}(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,2}\right]$$
$$+ \mathrm{i}\epsilon^{\alpha\beta}k_{3\alpha}k_{4\beta}\psi^{(37)}_{u\bar{d}gg}(1,2,3,4)\left[(k^+_{2\perp})^3(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,-2} - (k^-_{2\perp})^3(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,2}\right]$$
$$+ \mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{4\beta}\psi^{(38)}_{u\bar{d}gg}(1,2,3,4)\left[(k^+_{3\perp})^3(u\bar{d})^\dagger_{A,-1}(gg)^\dagger_{S,-2} - (k^-_{3\perp})^3(u\bar{d})^\dagger_{A,1}(gg)^\dagger_{S,2}\right]\Big\} |0\rangle \ .$$

The Bose symmetry implies the symmetry relations:

$$\psi^{(31,36)}_{u\bar{d}gg}(1,2,3,4) = \psi^{(31,36)}_{u\bar{d}gg}(1,2,4,3)$$

and

$$\psi^{(37)}_{u\bar{d}gg}(1,2,3,4) = -\psi^{(37)}_{u\bar{d}gg}(1,2,4,3).$$

Similarly, one can obtain the amplitudes when the quark pair and two gluons are in color-octet states. If the two gluons are symmetric in color, we have $\psi^{(i)}_{u\bar{d}gg}$ with $i = 39, \ldots, 76$, defined in the same way as the above equations except that the color factor is replaced by $\sqrt{\frac{3}{20}}d_{abc}T^c_{ij}$. When the two gluons are antisymmetric in color, we obtain $\psi^{(i)}_{u\bar{d}gg}$ with $i = 77, \ldots, 114$, again defined in the same way, except for the color factor $\sqrt{\frac{1}{12}}f_{abc}T^c_{ij}$. Note that there are sign changes for the symmetry relations derived from Bose symmetry.

Therefore, we have a total of $38 \times 3 = 114$ independent amplitudes for the Fock component $u\bar{d}gg$ in π^+.

3.4 The $u\bar{d}q\bar{q}$ component

We first consider the up and down sea-quark flavors. In this case, the following two flavor structures have total isospin $I = 1$:

$$u\bar{d}(u\bar{u} + d\bar{d}) \ ,$$
$$(u\bar{u} - d\bar{d})u\bar{d} - u\bar{d}(u\bar{u} - d\bar{d}) \ . \quad (26)$$

The first structure arises from the first quark pair coupled to isospin 1 and the second pair coupled to isospin 0. The second structure comes from both pairs coupled to isospin 1. However, after some rearrangements of the particle labels, the second structure can be reduced to the first one, and hence is not independent. Therefore, we consider only the first isospin structure with all possible color and spin combinations.

To simplify the $\hat{Y}$ transformation, we introduce the following combinations for the sea quark pair:

$$(q\bar{q})^\dagger_{S,0} = u^\dagger_{\uparrow k}(3)\bar{u}^\dagger_{\downarrow l}(4) + u^\dagger_{\downarrow k}(3)\bar{u}^\dagger_{\uparrow l}(4) + d^\dagger_{\uparrow k}(3)\bar{d}^\dagger_{\downarrow l}(4) + d^\dagger_{\downarrow k}(3)\bar{d}^\dagger_{\uparrow l}(4) \ ,$$
$$(q\bar{q})^\dagger_{A,0} = u^\dagger_{\uparrow k}(3)\bar{u}^\dagger_{\downarrow l}(4) - u^\dagger_{\downarrow k}(3)\bar{u}^\dagger_{\uparrow l}(4) + d^\dagger_{\uparrow k}(3)\bar{d}^\dagger_{\downarrow l}(4) - d^\dagger_{\downarrow k}(3)\bar{d}^\dagger_{\uparrow l}(4) \ ,$$
$$(q\bar{q})^\dagger_{A,1} = u^\dagger_{\uparrow k}(3)\bar{u}^\dagger_{\uparrow l}(4) + d^\dagger_{\uparrow k}(3)\bar{d}^\dagger_{\uparrow l}(4) \ ,$$
$$(q\bar{q})^\dagger_{A,-1} = u^\dagger_{\downarrow k}(3)\bar{u}^\dagger_{\downarrow l}(4) + d^\dagger_{\downarrow k}(3)\bar{d}^\dagger_{\downarrow l}(4) \ . \quad (27)$$

We use these as basic building blocks in the Fock expansion.

We can form two color-singlet structures from the four color indices i,j,k, and l: $\delta_{ij}\delta_{kl}$ and $\delta_{il}\delta_{jk}$, where we have implicitly assumed that the first and third are for quarks and second and fourth are for antiquarks. The first structure corresponds to the state in which the two quark pairs are both coupled to the color singlet, while the second corresponds to the state in which the two quark pairs are in a color octet. The wave-function amplitudes for both color combinations are similar.

The quark helicity has combinations $\lambda_{u\bar{d}} = 0,\ \pm 1$, and $\lambda_{u\bar{u}+d\bar{d}} = 0,\ \pm 1$. Therefore we can have three different orbital angular momentum projections, $|l_z| = 0, 1, 2$.

For $l_z = 0$, the quark helicity has the combination $\lambda_{u\bar{d}} = 0$ and $\lambda_{u\bar{u}+d\bar{d}} = 0$, or $\lambda_{u\bar{d}} = \pm 1$ and $\lambda_{u\bar{u}+d\bar{d}} = \mp 1$. Together, we find 12 independent amplitudes:

$$
\begin{aligned}
|\pi^+\rangle^{l_z=0}_{u\bar{d}q\bar{q}} &= \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4] \\
&\times \frac{\delta_{ij}\delta_{kl}}{3} \Big\{ \psi^{(1)}_{u\bar{d}q\bar{q}}(1,2,3,4)(u\bar{d})^\dagger_{A,0}(q\bar{q})^\dagger_{S,0} \\
&+\psi^{(2)}_{u\bar{d}q\bar{q}}(1,2,3,4)(u\bar{d})^\dagger_{S,0}(q\bar{q})^\dagger_{A,0} \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi^{(3)}_{u\bar{d}q\bar{q}}(1,2,3,4)(u\bar{d})^\dagger_{S,0}(q\bar{q})^\dagger_{S,0} \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi^{(4)}_{u\bar{d}q\bar{q}}(1,2,3,4)(u\bar{d})^\dagger_{S,0}(q\bar{q})^\dagger_{S,0} \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi^{(5)}_{u\bar{d}q\bar{q}}(1,2,3,4)(u\bar{d})^\dagger_{S,0}(q\bar{q})^\dagger_{S,0} \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi^{(6)}_{u\bar{d}q\bar{q}}(1,2,3,4)(u\bar{d})^\dagger_{A,0}(q\bar{q})^\dagger_{A,0} \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi^{(7)}_{u\bar{d}q\bar{q}}(1,2,3,4)(u\bar{d})^\dagger_{A,0}(q\bar{q})^\dagger_{A,0} \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi^{(8)}_{u\bar{d}q\bar{q}}(1,2,3,4)(u\bar{d})^\dagger_{A,0}(q\bar{q})^\dagger_{A,0} \\
&+ \psi^{(9)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[(u\bar{d})^\dagger_{A,1}(q\bar{q})^\dagger_{A,-1} - (u\bar{d})^\dagger_{A,-1}(q\bar{q})^\dagger_{A,1}\right] \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi^{(10)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[(u\bar{d})^\dagger_{A,1}(q\bar{q})^\dagger_{A,-1} + (u\bar{d})^\dagger_{A,-1}(q\bar{q})^\dagger_{A,1}\right] \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi^{(11)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[(u\bar{d})^\dagger_{A,1}(q\bar{q})^\dagger_{A,-1} + (u\bar{d})^\dagger_{A,-1}(q\bar{q})^\dagger_{A,1}\right] \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi^{(12)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[(u\bar{d})^\dagger_{A,1}(q\bar{q})^\dagger_{A,-1} + (u\bar{d})^\dagger_{A,-1}(q\bar{q})^\dagger_{A,1}\right]\Big\} |0\rangle\ .
\end{aligned} \quad (28)
$$

Note that δ_{ij} implicitly contracts the color indices in the $u\bar{d}$ pair and δ_{kl} contracts the $q\bar{q}$ pair.

For $|l_z| = 1$, the quark helicity can either be in the combination $\lambda_{u\bar{d}} = 0$ and $\lambda_{u\bar{u}} = -1$, or $\lambda_{u\bar{d}} = -1$ and $\lambda_{u\bar{u}} = 0$. Taking these together, we find 24 independent amplitudes:

$$
\begin{aligned}
|\pi^+\rangle^{|l_z|=1}_{u\bar{d}q\bar{q}} &= \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4] \\
&\times \frac{\delta_{ij}\delta_{kl}}{3} \Big\{ \psi^{(13)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{1\perp}(u\bar{d})^\dagger_{A,0}(q\bar{q})^\dagger_{A,-1} - k^-_{1\perp}(u\bar{d})^\dagger_{A,0}(q\bar{q})^\dagger_{A,1}\right] \\
&+ \psi^{(14)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{1\perp}(u\bar{d})^\dagger_{S,0}(q\bar{q})^\dagger_{A,-1} + k^-_{1\perp}(u\bar{d})^\dagger_{S,0}(q\bar{q})^\dagger_{A,1}\right] \\
&+ \psi^{(15)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{1\perp}(u\bar{d})^\dagger_{A,-1}(q\bar{q})^\dagger_{A,0} - k^-_{1\perp}(u\bar{d})^\dagger_{A,1}(q\bar{q})^\dagger_{A,0}\right] \\
&+ \psi^{(16)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{1\perp}(u\bar{d})^\dagger_{A,-1}(q\bar{q})^\dagger_{S,0} + k^-_{1\perp}(u\bar{d})^\dagger_{A,1}(q\bar{q})^\dagger_{S,0}\right] \\
&+ \psi^{(17)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{2\perp}(u\bar{d})^\dagger_{A,0}(q\bar{q})^\dagger_{A,-1} - k^-_{2\perp}(u\bar{d})^\dagger_{A,0}(q\bar{q})^\dagger_{A,1}\right] \\
&+ \psi^{(18)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{2\perp}(u\bar{d})^\dagger_{S,0}(q\bar{q})^\dagger_{A,-1} + k^-_{2\perp}(u\bar{d})^\dagger_{S,0}(q\bar{q})^\dagger_{A,1}\right] \\
&+ \psi^{(19)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{2\perp}(u\bar{d})^\dagger_{A,-1}(q\bar{q})^\dagger_{A,0} - k^-_{2\perp}(u\bar{d})^\dagger_{A,1}(q\bar{q})^\dagger_{A,0}\right] \\
&+ \psi^{(20)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{2\perp}(u\bar{d})^\dagger_{A,-1}(q\bar{q})^\dagger_{S,0} + k^-_{2\perp}(u\bar{d})^\dagger_{A,1}(q\bar{q})^\dagger_{S,0}\right] \\
&+ \psi^{(21)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{3\perp}(u\bar{d})^\dagger_{A,0}(q\bar{q})^\dagger_{A,-1} - k^-_{3\perp}(u\bar{d})^\dagger_{A,0}(q\bar{q})^\dagger_{A,1}\right] \\
&+ \psi^{(22)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{3\perp}(u\bar{d})^\dagger_{S,0}(q\bar{q})^\dagger_{A,-1} + k^-_{3\perp}(u\bar{d})^\dagger_{S,0}(q\bar{q})^\dagger_{A,1}\right] \\
&+ \psi^{(23)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{3\perp}(u\bar{d})^\dagger_{A,-1}(q\bar{q})^\dagger_{A,0} - k^-_{3\perp}(u\bar{d})^\dagger_{A,1}(q\bar{q})^\dagger_{A,0}\right] \\
&+ \psi^{(24)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{3\perp}(u\bar{d})^\dagger_{A,-1}(q\bar{q})^\dagger_{S,0} + k^-_{3\perp}(u\bar{d})^\dagger_{A,1}(q\bar{q})^\dagger_{S,0}\right] \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi^{(25)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{1\perp}(u\bar{d})^\dagger_{A,0}(q\bar{q})^\dagger_{A,-1} + k^-_{1\perp}(u\bar{d})^\dagger_{A,0}(q\bar{q})^\dagger_{A,1}\right] \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi^{(26)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{1\perp}(u\bar{d})^\dagger_{S,0}(q\bar{q})^\dagger_{A,-1} - k^-_{1\perp}(u\bar{d})^\dagger_{S,0}(q\bar{q})^\dagger_{A,1}\right] \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi^{(27)}_{u\bar{d}q\bar{q}}(1,2,3,4) \\
&\times \left[k^+_{1\perp}(u\bar{d})^\dagger_{A,-1}(q\bar{q})^\dagger_{A,0} + k^-_{1\perp}(u\bar{d})^\dagger_{A,1}(q\bar{q})^\dagger_{A,0}\right] \\
&+ \mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi^{(28)}_{u\bar{d}q\bar{q}}(1,2,3,4)
\end{aligned}
$$

$$\times \left[k_{1\perp}^{+}(u\bar{d})^{\dagger}_{A,-1}(q\bar{q})^{\dagger}_{S,0} - k_{1\perp}^{-}(u\bar{d})^{\dagger}_{A,1}(q\bar{q})^{\dagger}_{S,0}\right]$$

$$+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi^{(29)}_{u\bar{d}q\bar{q}}(1,2,3,4)$$

$$\times \left[k_{2\perp}^{+}(u\bar{d})^{\dagger}_{A,0}(q\bar{q})^{\dagger}_{A,-1} + k_{2\perp}^{-}(u\bar{d})^{\dagger}_{A,0}(q\bar{q})^{\dagger}_{A,1}\right]$$

$$+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi^{(30)}_{u\bar{d}q\bar{q}}(1,2,3,4)$$

$$\times \left[k_{2\perp}^{+}(u\bar{d})^{\dagger}_{S,0}(q\bar{q})^{\dagger}_{A,-1} - k_{2\perp}^{-}(u\bar{d})^{\dagger}_{S,0}(q\bar{q})^{\dagger}_{A,1}\right]$$

$$+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi^{(31)}_{u\bar{d}q\bar{q}}(1,2,3,4)$$

$$\times \left[k_{2\perp}^{+}(u\bar{d})^{\dagger}_{A,-1}(q\bar{q})^{\dagger}_{A,0} + k_{2\perp}^{-}(u\bar{d})^{\dagger}_{A,1}(q\bar{q})^{\dagger}_{A,0}\right]$$

$$+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi^{(32)}_{u\bar{d}q\bar{q}}(1,2,3,4)$$

$$\times \left[k_{2\perp}^{+}(u\bar{d})^{\dagger}_{A,-1}(q\bar{q})^{\dagger}_{S,0} - k_{2\perp}^{-}(u\bar{d})^{\dagger}_{A,1}(q\bar{q})^{\dagger}_{S,0}\right]$$

$$+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi^{(33)}_{u\bar{d}q\bar{q}}(1,2,3,4)$$

$$\times \left[k_{3\perp}^{+}(u\bar{d})^{\dagger}_{A,0}(q\bar{q})^{\dagger}_{A,-1} + k_{3\perp}^{-}(u\bar{d})^{\dagger}_{A,0}(q\bar{q})^{\dagger}_{A,1}\right]$$

$$+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi^{(34)}_{u\bar{d}q\bar{q}}(1,2,3,4)$$

$$\times \left[k_{3\perp}^{+}(u\bar{d})^{\dagger}_{S,0}(q\bar{q})^{\dagger}_{A,-1} - k_{3\perp}^{-}(u\bar{d})^{\dagger}_{S,0}(q\bar{q})^{\dagger}_{A,1}\right]$$

$$+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi^{(35)}_{u\bar{d}q\bar{q}}(1,2,3,4)$$

$$\times \left[k_{3\perp}^{+}(u\bar{d})^{\dagger}_{A,-1}(q\bar{q})^{\dagger}_{A,0} + k_{3\perp}^{-}(u\bar{d})^{\dagger}_{A,1}(q\bar{q})^{\dagger}_{A,0}\right]$$

$$+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi^{(36)}_{u\bar{d}q\bar{q}}(1,2,3,4) \tag{29}$$

$$\times \left[k_{3\perp}^{+}(u\bar{d})^{\dagger}_{A,-1}(q\bar{q})^{\dagger}_{S,0} - k_{3\perp}^{-}(u\bar{d})^{\dagger}_{A,1}(q\bar{q})^{\dagger}_{S,0}\right]\Big\} |0\rangle \ .$$

For $|l_z| = 2$, the quark helicity must be $\lambda_{u\bar{d}} = \pm 1$ and $\lambda_{u\bar{u}} = \pm 1$. We have the following nine independent amplitudes:

$$|\pi^+\rangle^{|l_z|=2}_{u\bar{d}q\bar{q}} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4]$$

$$\times \frac{\delta_{ij}\delta_{kl}}{3}\Big\{\psi^{(37)}_{u\bar{d}q\bar{q}}(1,2,3,4)\left[k_{1\perp}^{+}k_{1\perp}^{+}(u\bar{d})^{\dagger}_{A,-1}(q\bar{q})^{\dagger}_{A,-1}\right.$$

$$\left. - k_{1\perp}^{-}k_{1\perp}^{-}(u\bar{d})^{\dagger}_{A,1}(q\bar{q})^{\dagger}_{A,1}\right]$$

$$+ \psi^{(38)}_{u\bar{d}q\bar{q}}(1,2,3,4)\left[k_{2\perp}^{+}k_{2\perp}^{+}(u\bar{d})^{\dagger}_{A,-1}(q\bar{q})^{\dagger}_{A,-1}\right.$$

$$\left. - k_{2\perp}^{-}k_{2\perp}^{-}(u\bar{d})^{\dagger}_{A,1}(q\bar{q})^{\dagger}_{A,1}\right]$$

$$+ \psi^{(39)}_{u\bar{d}q\bar{q}}(1,2,3,4)\left[k_{3\perp}^{+}k_{3\perp}^{+}(u\bar{d})^{\dagger}_{A,-1}(q\bar{q})^{\dagger}_{A,-1}\right.$$

$$\left. - k_{3\perp}^{-}k_{3\perp}^{-}(u\bar{d})^{\dagger}_{A,1}(q\bar{q})^{\dagger}_{A,1}\right]$$

$$+ \psi^{(40)}_{u\bar{d}q\bar{q}}(1,2,3,4)\left[k_{1\perp}^{+}k_{2\perp}^{+}(u\bar{d})^{\dagger}_{A,-1}(q\bar{q})^{\dagger}_{A,-1}\right.$$

$$\left. - k_{1\perp}^{-}k_{2\perp}^{-}(u\bar{d})^{\dagger}_{A,1}(q\bar{q})^{\dagger}_{A,1}\right]$$

$$+ \psi^{(41)}_{u\bar{d}q\bar{q}}(1,2,3,4)\left[k_{1\perp}^{+}k_{3\perp}^{+}(u\bar{d})^{\dagger}_{A,-1}(q\bar{q})^{\dagger}_{A,-1}\right.$$

$$\left. - k_{1\perp}^{-}k_{3\perp}^{-}(u\bar{d})^{\dagger}_{A,1}(q\bar{q})^{\dagger}_{A,1}\right] \tag{30}$$

$$+ \psi^{(42)}_{u\bar{d}q\bar{q}}(1,2,3,4)\left[k_{2\perp}^{+}k_{3\perp}^{+}(u\bar{d})^{\dagger}_{A,-1}(q\bar{q})^{\dagger}_{A,-1}\right.$$

$$\left. - k_{2\perp}^{-}k_{3\perp}^{-}(u\bar{d})^{\dagger}_{A,1}(q\bar{q})^{\dagger}_{A,1}\right]$$

$$+ \mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi^{(43)}_{u\bar{d}q\bar{q}}(1,2,3,4)\left[k_{1\perp}^{+}k_{1\perp}^{+}(u\bar{d})^{\dagger}_{A,-1}(q\bar{q})^{\dagger}_{A,-1}\right.$$

$$\left. + k_{1\perp}^{-}k_{1\perp}^{-}(u\bar{d})^{\dagger}_{A,1}(q\bar{q})^{\dagger}_{A,1}\right]$$

$$+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi^{(44)}_{u\bar{d}q\bar{q}}(1,2,3,4)\left[k_{2\perp}^{+}k_{2\perp}^{+}(u\bar{d})^{\dagger}_{A,-1}(q\bar{q})^{\dagger}_{A,-1}\right.$$

$$\left. + k_{2\perp}^{-}k_{2\perp}^{-}(u\bar{d})^{\dagger}_{A,1}(q\bar{q})^{\dagger}_{A,1}\right]$$

$$+ \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi^{(45)}_{u\bar{d}q\bar{q}}(1,2,3,4)\left[k_{3\perp}^{+}k_{3\perp}^{+}(u\bar{d})^{\dagger}_{A,-1}(q\bar{q})^{\dagger}_{A,-1}\right.$$

$$\left. + k_{3\perp}^{-}k_{3\perp}^{-}(u\bar{d})^{\dagger}_{A,1}(q\bar{q})^{\dagger}_{A,1}\right]\Big\} |0\rangle \ .$$

In summary, we have found 45 independent amplitudes. Similarly, we have another 45 amplitudes for the color structure $\frac{1}{3}\delta_{il}\delta_{jk}$. Together, we have 90 independent amplitudes for the $u\bar{d}q\bar{q}$ component.

The above formalism can also be used to construct the amplitudes for the $u\bar{d}s\bar{s}$ and $u\bar{d}c\bar{c}$ components in π^+. The total number of independent amplitudes is 90 in both cases. These amplitudes can be used to describe the intrinsic strange and/or charm contributions to the hadronic processes involving π, e.g., $J/\psi \to \rho\pi$ decays [27].

4 Wave-function amplitudes for the ρ^+ meson

The method in the last section can be straightforwardly used to construct the light-cone wave-function amplitudes for the ρ mesons. Strictly speaking, the ρ meson is not an eigenstate of the QCD hamiltonian; it appears as resonances in, for example, $\pi\pi$ scattering. However, we regard in the following discussion the ρ meson as if a bound state of quarks and gluons. The relevant studies of the distribution amplitudes for the ρ mesons can be found in [28, 29].

Because ρ is a vector meson, it has three helicity states, i.e., $\Lambda = 0, \pm 1$, corresponding to longitudinal ($\Lambda = 0$) and transverse ($\Lambda = \pm 1$) polarizations. The wave functions for the $\Lambda = 0$ state can be obtained, in principle, from those of the $\Lambda = \pm 1$ states by using angular momentum raising and lowering operators. In practice, however, these operators involve complicated quark-gluon interactions in light-cone quantization, and the constraint becomes a very complicated equation involving all higher Fock states. Since we are interested in the components of a few partons, we may regard the different helicity states as quasi-independent. Nonetheless, the $\Lambda = -1$ state can be obtained from the $\Lambda = +1$ state using a parity transformation.

The wave-function amplitudes for the helicity $\Lambda = 0$ state can be easily obtained from those in the last section, taking into account the difference on the $\hat{Y}$ transformation property between π and ρ, i.e.,

$$\hat{Y}|\rho^+, \Lambda=0\rangle = |\rho^+, \Lambda=0\rangle \ , \quad (31)$$

compared to (11). Hence the Fock expansion listed in the last section can be transformed to that of $|\rho^+, 0\rangle$, except that some signs must be changed. In particular, the total number of independent amplitudes will be the same.

For example, the two quark component for $|\rho^+, 0\rangle$ has two independent amplitudes:

$$|\rho^+, 0\rangle_{u\bar{d}}^{l_z=0} = \int \mathrm{d}[1]\mathrm{d}[2] \quad (32)$$
$$\times \psi_{u\bar{d}}^{(1)}(1,2)\frac{1}{\sqrt{3}}\left[u_{\uparrow i}^\dagger(1)\bar{d}_{\downarrow i}^\dagger(2) + u_{\downarrow i}^\dagger(1)\bar{d}_{\uparrow i}^\dagger(2)\right]|0\rangle$$

$$|\rho^+, 0\rangle_{u\bar{d}}^{|l_z|=1} = \int \mathrm{d}[1]\mathrm{d}[2] \quad (33)$$
$$\times \psi_{u\bar{d}}^{(2)}(1,2)\frac{1}{\sqrt{3}}\left[k_{1\perp}^- u_{\uparrow i}^\dagger(1)\bar{d}_{\uparrow i}^\dagger(2) - k_{1\perp}^+ u_{\downarrow i}^\dagger(1)\bar{d}_{\downarrow i}^\dagger(2)\right]|0\rangle \ .$$

Here we have used the same notation for the amplitudes, assuming that no confusion will arise. For the $u\bar{d}g$ component, we have

$$|\rho^+, 0\rangle_{u\bar{d}g}^{l_z=0} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]$$
$$\times \frac{T_{ij}^a}{2}\left\{\psi_{u\bar{d}g}^{(1)}(1,2,3)\right.$$
$$\times \left[(u\bar{d})_{A,1}^\dagger g_\downarrow^{a\dagger}(3) + (u\bar{d})_{A,-1}^\dagger g_\uparrow^{a\dagger}(3)\right]|0\rangle$$
$$+\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi_{u\bar{d}g}^{(2)}(1,2,3)$$
$$\left.\times \left[(u\bar{d})_{A,1}^\dagger g_\downarrow^{a\dagger}(3) - (u\bar{d})_{A,-1}^\dagger g_\uparrow^{a\dagger}(3)\right]\right\}|0\rangle \ , \quad (34)$$

$$|\rho^+, 0\rangle_{u\bar{d}g}^{|l_z|=1} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]$$
$$\times \frac{T_{ij}^a}{2}\left\{\psi_{u\bar{d}g}^{(3)}(1,2,3)\right.$$
$$\times \left[k_{1\perp}^+(u\bar{d})_{A,0}^\dagger g_\downarrow^{a\dagger}(3) + k_{1\perp}^-(u\bar{d})_{A,0}^\dagger g_\uparrow^{a\dagger}(3)\right]$$
$$+ \psi_{u\bar{d}g}^{(4)}(1,2,3)\left[k_{2\perp}^+(u\bar{d})_{A,0}^\dagger g_\downarrow^{a\dagger}(3) + k_{2\perp}^-(u\bar{d})_{A,0}^\dagger g_\uparrow^{a\dagger}(3)\right]$$
$$+ \psi_{u\bar{d}g}^{(5)}(1,2,3)\left[k_{1\perp}^+(u\bar{d})_{S,0}^\dagger g_\downarrow^{a\dagger}(3) - k_{1\perp}^-(u\bar{d})_{S,0}^\dagger g_\uparrow^{a\dagger}(3)\right]$$
$$+ \psi_{u\bar{d}g}^{(6)}(1,2,3)\left[k_{2\perp}^+(u\bar{d})_{S,0}^\dagger g_\downarrow^{a\dagger}(3)\right.$$
$$\left.\left. -k_{2\perp}^-(u\bar{d})_{S,0}^\dagger g_\uparrow^{a\dagger}(3)\right]\right\}|0\rangle \ , \quad (35)$$

$$|\rho^+, 0\rangle_{u\bar{d}g}^{|l_z|=2} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]$$
$$\times \frac{T_{ij}^a}{2}\left\{\psi_{u\bar{d}g}^{(7)}(1,2,3)\right.$$
$$\times \left[k_{1\perp}^+k_{1\perp}^+(u\bar{d})_{A,-1}^\dagger g_\downarrow^{a\dagger}(3) + k_{1\perp}^-k_{1\perp}^-(u\bar{d})_{A,1}^\dagger g_\uparrow^{a\dagger}(3)\right]$$
$$+ \psi_{u\bar{d}g}^{(8)}(1,2,3)$$
$$\times \left[k_{1\perp}^+k_{2\perp}^+(u\bar{d})_{A,-1}^\dagger g_\downarrow^{a\dagger}(3) + k_{1\perp}^-k_{2\perp}^-(u\bar{d})_{A,1}^\dagger g_\uparrow^{a\dagger}(3)\right]$$
$$+ \psi_{u\bar{d}g}^{(9)}(1,2,3)$$
$$\times \left[k_{2\perp}^+k_{2\perp}^+(u\bar{d})_{A,-1}^\dagger g_\downarrow^{a\dagger}(3)\right.$$
$$\left.\left. + k_{2\perp}^-k_{2\perp}^-(u\bar{d})_{A,1}^\dagger g_\uparrow^{a\dagger}(3)\right]\right\}|0\rangle \ . \quad (36)$$

We will not repeat the cases for four partons.

The helicity $\Lambda = 1$ ρ meson state, $|\rho^+, 1\rangle$, can be constructed similarly. For example, the $u\bar{d}$ component defined four independent amplitudes, corresponding to $l_z = 0,\ 1,\ 2$,

$$|\rho^+, 1\rangle_{u\bar{d}}^{l_z=0} = \int \mathrm{d}[1]\mathrm{d}[2] \quad (37)$$
$$\times \psi_{u\bar{d}}^{(1)}(1,2)\frac{1}{\sqrt{3}}\left[u_{\uparrow i}^\dagger(1)\bar{d}_{\uparrow i}^\dagger(2)\right]|0\rangle$$

$$|\rho^+, 1\rangle_{u\bar{d}}^{l_z=1} = \int \mathrm{d}[1]\mathrm{d}[2]$$
$$\times \left\{k_{1\perp}^+\psi_{u\bar{d}}^{(2)}(1,2)\frac{1}{\sqrt{3}}\left[u_{\uparrow i}^\dagger(1)\bar{d}_{\downarrow i}^\dagger(2) + u_{\downarrow i}^\dagger(1)\bar{d}_{\uparrow i}^\dagger(2)\right]\right. \quad (38)$$
$$\left. +k_{1\perp}^+\psi_{u\bar{d}}^{(3)}(1,2)\ \frac{1}{\sqrt{3}}\left[u_{\uparrow i}^\dagger(1)\bar{d}_{\downarrow i}^\dagger(2) - u_{\downarrow i}^\dagger(1)\bar{d}_{\uparrow i}^\dagger(2)\right]\right\}|0\rangle$$

$$|\rho^+, 1\rangle_{u\bar{d}}^{l_z=2} = \int \mathrm{d}[1]\mathrm{d}[2]$$
$$\times (k_{1\perp}^+)^2\psi_{u\bar{d}}^{(4)}(1,2)\frac{1}{\sqrt{3}}\left[u_{\downarrow i}^\dagger(1)\bar{d}_{\downarrow i}^\dagger(2)\right]|0\rangle \ . \quad (39)$$

Here again we use the same notation for the wave-function amplitudes although they can be very different from those in the $\Lambda = 0$ state. In fact, even the number of independent amplitudes for a given number of partons is different.

For the $u\bar{d}g$ component, the parton orbital angular momentum can be $l_z = -1, 0, 1, 2$, and 3. For $l_z = 0$, we find four independent amplitudes:

$$|\rho^+, 1\rangle_{u\bar{d}g}^{l_z=0} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]$$
$$\times \frac{T_{ij}^a}{2}\left\{\left(\psi_{u\bar{d}g}^{(1)}(1,2,3) + \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi_{u\bar{d}g}^{(2)}(1,2,3)\right)\right.$$
$$\times \left[(u\bar{d})_{A,0}^\dagger g_\uparrow^{a\dagger}(3)\right]$$
$$+ \left(\psi_{u\bar{d}g}^{(3)}(1,2,3) + \mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi_{u\bar{d}g}^{(4)}(1,2,3)\right)$$
$$\left.\times \left[(u\bar{d})_{S,0}^\dagger g_\uparrow^{a\dagger}(3)\right]\right\}|0\rangle \ . \quad (40)$$

For $l_z = -1$, we have two independent amplitudes:

$$|\rho^+, 1\rangle_{u\bar{d}g}^{l_z=-1} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]$$

$$\times \frac{T^a_{ij}}{2}\left\{\psi^{(5)}_{u\bar{d}g}(1,2,3)\left[k^-_{1\perp}(u\bar{d})^\dagger_{A,1}g^{a\dagger}_{\uparrow}(3)\right]\right.$$
$$\left.+\psi^{(6)}_{u\bar{d}g}(1,2,3)\left[k^-_{2\perp}(u\bar{d})^\dagger_{A,1}g^{a\dagger}_{\uparrow}(3)\right]\right\}|0\rangle\ . \quad (41)$$

For $l_z=1$, we have four independent amplitudes:

$$|\rho^+,1\rangle^{l_z=1}_{u\bar{d}g} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]$$
$$\times \frac{T^a_{ij}}{2}\left\{k^+_{1\perp}\psi^{(7)}_{u\bar{d}g}(1,2,3)\right.$$
$$\times\left[(u\bar{d})^\dagger_{A,1}g^{a\dagger}_{\downarrow}(3)+(u\bar{d})^\dagger_{A,-1}g^{a\dagger}_{\uparrow}(3)\right]$$
$$+k^+_{2\perp}\psi^{(8)}_{u\bar{d}g}(1,2,3)\left[(u\bar{d})^\dagger_{A,1}g^{a\dagger}_{\downarrow}(3)+(u\bar{d})^\dagger_{A,-1}g^{a\dagger}_{\uparrow}(3)\right]$$
$$+k^+_{1\perp}\psi^{(9)}_{u\bar{d}g}(1,2,3)\left[(u\bar{d})^\dagger_{A,1}g^{a\dagger}_{\downarrow}(3)-(u\bar{d})^\dagger_{A,-1}g^{a\dagger}_{\uparrow}(3)\right]$$
$$+k^+_{2\perp}\psi^{(10)}_{u\bar{d}g}(1,2,3)$$
$$\left.\times\left[(u\bar{d})^\dagger_{A,1}g^{a\dagger}_{\downarrow}(3)-(u\bar{d})^\dagger_{A,-1}g^{a\dagger}_{\uparrow}(3)\right]\right\}|0\rangle\ . \quad (42)$$

For $l_z=2$, we have six independent amplitudes:

$$|\rho^+,1\rangle^{l_z=2}_{u\bar{d}g} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]$$
$$\times \frac{T^a_{ij}}{2}\left\{k^+_{1\perp}k^+_{1\perp}\psi^{(11)}_{u\bar{d}g}(1,2,3)\left[(u\bar{d})^\dagger_{A,0}g^{a\dagger}_{\downarrow}(3)\right]\right.$$
$$+k^+_{1\perp}k^+_{2\perp}\psi^{(12)}_{u\bar{d}g}(1,2,3)\left[(u\bar{d})^\dagger_{A,0}g^{a\dagger}_{\downarrow}(3)\right]$$
$$+k^+_{2\perp}k^+_{2\perp}\psi^{(13)}_{u\bar{d}g}(1,2,3)\left[(u\bar{d})^\dagger_{A,0}g^{a\dagger}_{\downarrow}(3)\right]$$
$$+k^+_{1\perp}k^+_{1\perp}\psi^{(14)}_{u\bar{d}g}(1,2,3)\left[(u\bar{d})^\dagger_{S,0}g^{a\dagger}_{\downarrow}(3)\right]$$
$$+k^+_{1\perp}k^+_{2\perp}\psi^{(15)}_{u\bar{d}g}(1,2,3)\left[(u\bar{d})^\dagger_{S,0}g^{a\dagger}_{\downarrow}(3)\right] \quad (43)$$
$$\left.+k^+_{2\perp}k^+_{2\perp}\psi^{(16)}_{u\bar{d}g}(1,2,3)\left[(u\bar{d})^\dagger_{S,0}g^{a\dagger}_{\downarrow}(3)\right]\right\}|0\rangle\ .$$

For $l_z=3$, we have four independent amplitudes:

$$|\rho^+,1\rangle^{l_z=3}_{u\bar{d}g} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]$$
$$\times \frac{T^a_{ij}}{2}\left\{(k^+_{1\perp})^3\psi^{(17)}_{u\bar{d}g}(1,2,3)\left[(u\bar{d})^\dagger_{A,-1}g^{a\dagger}_{\downarrow}(3)\right]\right.$$
$$+(k^+_{2\perp})^3\psi^{(18)}_{u\bar{d}g}(1,2,3)\left[(u\bar{d})^\dagger_{A,-1}g^{a\dagger}_{\downarrow}(3)\right]$$
$$+(k^+_{1\perp})^2k^+_{2\perp}\psi^{(19)}_{u\bar{d}g}(1,2,3)\left[(u\bar{d})^\dagger_{A,-1}g^{a\dagger}_{\downarrow}(3)\right] \quad (44)$$
$$\left.+k^+_{1\perp}(k^+_{2\perp})^2\psi^{(20)}_{u\bar{d}g}(1,2,3)\left[(u\bar{d})^\dagger_{A,-1}g^{a\dagger}_{\downarrow}(3)\right]\right\}|0\rangle\ .$$

In total, we have 20 amplitudes, compared with the $\Lambda=0$ case where we have only nine ones. For simplicity, we will not consider those amplitudes with four partons.

5 Wave-function amplitudes for the nucleon

In this section, we enumerate the number of independent amplitudes for the nucleon, and more specifically for the proton. For the neutron, one just interchange the up and down quarks assuming isospin symmetry. Our expansion is also valid for the whole baryon octet, except the flavor structure need to be modified accordingly.

We consider only the state with positive helicity. The negative helicity state can be obtained simply from the modified parity transformation $\hat{Y}$. Three quark amplitudes have been studied extensively in [5]. The new result here includes three-quark plus one-gluon amplitudes. One can add an additional pair of sea quarks into the valence component, but the result is very complicated and we will not show it here.

5.1 The uud component

The quark distribution amplitudes describing the three-quark component of the proton have been studied extensively in the literature [4,14,28,30–32]. The wave-function amplitudes keeping full partons transverse-momentum dependence have been studied in [5]. From the approach advocated here, we immediately have for $l_z=0$ and $l_z=1$,

$$|P\uparrow\rangle^{l_z=0}_{uud} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]$$
$$\times\left(\psi^{(1)}_{uud}(1,2,3)+\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi^{(2)}_{uud}(1,2,3)\right) \quad (45)$$
$$\times\frac{\epsilon^{ijk}}{\sqrt{6}}u^\dagger_{i\uparrow}(1)\left(u^\dagger_{j\downarrow}(2)d^\dagger_{k\uparrow}(3)-d^\dagger_{j\downarrow}(2)u^\dagger_{k\uparrow}(3)\right)|0\rangle\ ,$$
$$|P\uparrow\rangle^{l_z=1}_{uud} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]$$
$$\times\left(k^+_{1\perp}\psi^{(3)}_{uud}(1,2,3)+k^+_{2\perp}\psi^{(4)}_{uud}(1,2,3)\right)$$
$$\times\frac{\epsilon^{ijk}}{\sqrt{6}}\left(u^\dagger_{i\uparrow}(1)u^\dagger_{j\downarrow}(2)d^\dagger_{k\downarrow}(3)\right.$$
$$\left.-d^\dagger_{i\uparrow}(1)u^\dagger_{j\downarrow}(2)u^\dagger_{k\downarrow}(3)\right)|0\rangle\ . \quad (46)$$

For $l_z=-1$, we have

$$|P\uparrow\rangle^{l_z=-1}_{uud} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]$$
$$\times k^-_{2\perp}\psi^{(5)}_{uud}(1,2,3)\frac{\epsilon^{ijk}}{\sqrt{6}}u^\dagger_{i\uparrow}(1)$$
$$\times\left(u^\dagger_{j\uparrow}(2)d^\dagger_{k\uparrow}(3)-d^\dagger_{j\uparrow}(2)u^\dagger_{k\uparrow}(3)\right)|0\rangle\ , \quad (47)$$

where we have used quark 2 and 3 antisymmetry. Likewise, we have

$$|P\uparrow\rangle^{l_z=2}_{uud} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]$$
$$\times k^+_{1\perp}k^+_{3\perp}\psi^{(6)}_{uud}(1,2,3) \quad (48)$$

$$\times\frac{\epsilon^{ijk}}{\sqrt{6}}u^\dagger_{i\downarrow}(1)\left(d^\dagger_{j\downarrow}(2)u^\dagger_{k\downarrow}(3)-u^\dagger_{j\downarrow}(2)d^\dagger_{k\downarrow}(3)\right)|0\rangle\ ,$$

where, in principle, there is an additional term $k^+_{2\perp}k^+_{3\perp}\psi^{(6')}_{uud}(1,2,3)$. However, after using 2 and 3 antisymmetry and 1 and 2 symmetry, it can shown that this term can be reduced to the term shown above.

5.2 The $uud+g$ component

Let us first consider the isospin symmetry. With the three quarks uud, one can construct two possible $I=1/2$ isospin combinations:

$$u(ud-du)\ ,\quad 2duu-udu-uud\ . \tag{49}$$

However, the second flavor structure can be reduced to the first one after some shuffling of the particle labels. Therefore, we shall only consider the first structure for the flavor wave function, taking into account all possible color and spin assignments for the three quarks and one gluon.

Since the gluon belongs to a color octet, the three quarks must couple to a color octet. For the three quarks, u_i, u_j, d_k, there are two possible ways to couple to a color octet:

$$3\times3\times3=(6+\bar{3})\times3=1+8+8+10\ .$$

We can have the first two quarks coupling to $\bar{3}$, and then couple them to the third quark to form a color octet. In this case, we have an overall color factor, $\epsilon^{ijl}T^a_{lk}$. Similarly, we can also have two other color factors, $\epsilon^{jkl}T^a_{li}$ and $\epsilon^{kil}T^a_{lj}$. However, the above three are not independent, because $\epsilon^{ijl}T^a_{lk}+\epsilon^{jkl}T^a_{li}+\epsilon^{kil}T^a_{lj}=0$. If we use the isospin structure $u_i(u_jd_k-d_ju_k)$, the best way to select the two independent color structures is to have the indices of jk to be antisymmetric or symmetric:

$$\epsilon^{jkl}T^a_{li}\ ,\quad \epsilon^{ijl}T^a_{lk}+\epsilon^{ikl}T^a_{lj}\ . \tag{50}$$

For the Fock component of $uudg$, the total quark helicity can be $\lambda_{uud}=-3/2,\ -1/2,\ 1/2,\ 3/2$, and the gluon helicity $\lambda_g=\pm1$. The parton orbital angular momentum projection can have the following values: $l_z=0,1,2,3,-1,-2$.

For $l_z=0$, the parton helicity can either be $\lambda_{uud}=3/2$ and $\lambda_g=-1$, or $\lambda_{uud}=-1/2$ and $\lambda_g=1$. For the first case, because the total quark helicity $\lambda_{uud}=3/2$, the three quarks are all in helicity-1/2 states, and we only have one spin structure, i.e., $u_{i\uparrow}(u_{j\uparrow}d_{k\uparrow}-d_{j\uparrow}u_{k\uparrow})$. Therefore, we can write down three independent amplitudes:

$$\begin{aligned}|P\uparrow\rangle^{l_z=0}_{uudg}&=\int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4]\\
&\times\Big(\psi^{(1)}_{uudg}(1,2,3,4)+\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi^{(2)}_{uudg}(1,2,3,4)\\
&\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi^{(3)}_{uudg}(1,2,3,4)\Big)\\
&\times\frac{\epsilon^{jkl}T^a_{li}}{2}\Big[u^\dagger_{i\uparrow}(1)\\
&\times\left(u^\dagger_{j\uparrow}(2)d^\dagger_{k\uparrow}(3)-d^\dagger_{j\uparrow}(2)u^\dagger_{k\uparrow}(3)\right)g^{a\dagger}_{\downarrow}(4)\Big]|0\rangle\ .\end{aligned}\tag{51}$$

Here, we have used the $2\leftrightarrow3$ symmetry to reduce the number of the independent amplitudes. For example, the $\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}$ term can be obtained from $\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}$ by 2 and 3 exchange, and hence the former is not independent. We have the following (anti-) symmetric properties for some amplitudes: $\psi^{(1)}_{uudg}(1,2,3,4)=-\psi^{(1)}_{uudg}(1,3,2,4)$ and $\psi^{(3)}_{uudg}(1,2,3,4)=\psi^{(3)}_{uudg}(1,3,2,4)$.

In the second case, we have the total quark helicity $\lambda_{uud}=-1/2$. There are three possible spin structures for the three quarks:

$$\begin{aligned}&u_{i\downarrow}(1)\left(u_{j\downarrow}(2)d_{k\uparrow}(3)-d_{j\downarrow}(2)u_{k\uparrow}(3)\right)\ ,\\
&u_{i\downarrow}(1)\left(u_{j\uparrow}(2)d_{k\downarrow}(3)-d_{j\uparrow}(2)u_{k\downarrow}(3)\right)\ ,\\
&u_{i\uparrow}(1)\left(u_{j\downarrow}(2)d_{k\downarrow}(3)-d_{j\downarrow}(2)u_{k\downarrow}(3)\right)\ .\end{aligned}\tag{52}$$

However, the associated color structures indicate that there is a (anti-) symmetric relation between the two indices j and k. Thus, the first two spin structures are equivalent to each other under 2 and 3 exchange. In the following, we will only keep one of the first two spin structures. The above observation also applies to the total quark helicity $\lambda_{uud}=1/2$ case. Taking this into account, we find seven independent amplitudes:

$$\begin{aligned}|P\uparrow\rangle^{l_z=0}_{uudg}&=\int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4]\\
&\times\Big\{\Big(\psi^{(4)}_{uudg}(1,2,3,4)+\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi^{(5)}_{uudg}(1,2,3,4)\\
&\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi^{(6)}_{uudg}(1,2,3,4)\\
&\quad\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi^{(7)}_{uudg}(1,2,3,4)\Big)\\
&\times\frac{\epsilon^{jkl}T^a_{li}}{2}\Big[u^\dagger_{i\downarrow}(1)\\
&\quad\times\left(u^\dagger_{j\uparrow}(2)d^\dagger_{k\downarrow}(3)-d^\dagger_{j\uparrow}(2)u^\dagger_{k\downarrow}(3)\right)g^{a\dagger}_{\uparrow}(4)\Big]\\
&+\Big(\psi^{(8)}_{uudg}(1,2,3,4)+\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi^{(9)}_{uudg}(1,2,3,4)\\
&\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi^{(10)}_{uudg}(1,2,3,4)\Big)\\
&\times\frac{\epsilon^{jkl}T^a_{li}}{2}\Big[u^\dagger_{i\uparrow}(1)\Big(u^\dagger_{j\downarrow}(2)d^\dagger_{k\downarrow}(3)\\
&\quad\quad-d^\dagger_{j\downarrow}(2)u^\dagger_{k\downarrow}(3)\Big)g^{a\dagger}_{\uparrow}(4)\Big]\Big\}|0\rangle\ .\end{aligned}\tag{53}$$

Again, the 2 and 3 symmetry in the above equation has been used to reduce the number of independent amplitudes. Moreover, it implies the relations $\psi^{(8)}_{uudg}(1,2,3,4)=-\psi^{(8)}_{uudg}(1,3,2,4)$ and $\psi^{(10)}_{uudg}(1,2,3,4)=\psi^{(10)}_{uudg}(1,3,2,4)$.

For $l_z=1$, the parton helicity can either be $\lambda_{uud}=1/2$ and $\lambda_g=-1$, or $\lambda_{uud}=-3/2$ and $\lambda_g=1$. In the first case, we define 10 independent amplitudes:

$$|P\uparrow\rangle^{l_z=1}_{uudg}=\int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4]$$

$$
\begin{aligned}
&\times \Big\{\Big(k_{1\perp}^{+}\Big(\psi_{uudg}^{(11)}(1,2,3,4)+\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi_{uudg}^{(12)}(1,2,3,4)\Big)\\
&+k_{2\perp}^{+}\Big(\psi_{uudg}^{(13)}(1,2,3,4)+\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi_{uudg}^{(14)}(1,2,3,4)\Big)\\
&+k_{3\perp}^{+}\Big(\psi_{uudg}^{(15)}(1,2,3,4)+\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi_{uudg}^{(16)}(1,2,3,4)\Big)\Big)\\
&\times\frac{\epsilon^{jkl}T_{li}^{a}}{2}\Big[u_{i\uparrow}^{\dagger}(1)\Big(u_{j\uparrow}^{\dagger}(2)d_{k\downarrow}^{\dagger}(3)-d_{j\uparrow}^{\dagger}(2)u_{k\downarrow}^{\dagger}(3)\Big)g_{\downarrow}^{a\dagger}(4)\Big]\\
&+\Big(k_{1\perp}^{+}\Big(\psi_{uudg}^{(17)}(1,2,3,4)+\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi_{uudg}^{(18)}(1,2,3,4)\Big)\\
&+k_{2\perp}^{+}\Big(\psi_{uudg}^{(19)}(1,2,3,4)+\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi_{uudg}^{(20)}(1,2,3,4)\Big)\Big)\\
&\times\frac{\epsilon^{jkl}T_{li}^{a}}{2}\\
&\times\Big[u_{i\downarrow}^{\dagger}(1)\Big(u_{j\uparrow}^{\dagger}(2)d_{k\uparrow}^{\dagger}(3)-d_{j\uparrow}^{\dagger}(2)u_{k\uparrow}^{\dagger}(3)\Big)g_{\downarrow}^{a\dagger}(4)\Big]\Big\}|0\rangle\ . \qquad (54)
\end{aligned}
$$

The symmetry between 2 and 3 leads to $\psi_{uudg}^{(17)}(1,2,3,4)=-\psi_{uudg}^{(17)}(1,3,2,4)$ and $\psi_{uudg}^{(18)}(1,2,3,4)=\psi_{uudg}^{(18)}(1,3,2,4)$. In the second case, we define four independent amplitudes:

$$
\begin{aligned}
|P\uparrow\rangle_{uudg}^{l_z=1}&=\int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4]\\
&\times\Big(k_{1\perp}^{+}\Big(\psi_{uudg}^{(21)}(1,2,3,4)+\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi_{uudg}^{(22)}(1,2,3,4)\Big)\\
&+k_{2\perp}^{+}\Big(\psi_{uudg}^{(23)}(1,2,3,4)+\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi_{uudg}^{(24)}(1,2,3,4)\Big)\Big)\\
&\times\frac{\epsilon^{jkl}T_{li}^{a}}{2}\Big[u_{i\downarrow}^{\dagger}(1)\Big(u_{j\downarrow}^{\dagger}(2)d_{k\downarrow}^{\dagger}(3)\\
&-d_{j\downarrow}^{\dagger}(2)u_{k\downarrow}^{\dagger}(3)\Big)g_{\uparrow}^{a\dagger}(4)\Big]|0\rangle\ , \qquad (55)
\end{aligned}
$$

where

$$\psi_{uudg}^{(21)}(1,2,3,4)=-\psi_{uudg}^{(21)}(1,3,2,4)$$

and

$$\psi_{uudg}^{(22)}(1,2,3,4)=\psi_{uudg}^{(22)}(1,3,2,4).$$

For $l_z=2$, the parton helicity must be $\lambda_{uud}=-1/2$ and $\lambda_g=-1$. We define 15 independent amplitudes:

$$
\begin{aligned}
|P\uparrow\rangle_{uudg}^{l_z=2}&=\int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4]\\
&\times\Big\{\Big((k_{1\perp}^{+})^2\Big(\psi_{uudg}^{(25)}(1,2,3,4)\\
&\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi_{uudg}^{(26)}(1,2,3,4)\Big)\\
&+(k_{2\perp}^{+})^2\Big(\psi_{uudg}^{(27)}(1,2,3,4)\\
&\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi_{uudg}^{(28)}(1,2,3,4)\Big)\\
&+(k_{3\perp}^{+})^2\Big(\psi_{uudg}^{(29)}(1,2,3,4)\\
&\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{2\beta}\psi_{uudg}^{(30)}(1,2,3,4)\Big)\\
&+k_{1\perp}^{+}k_{2\perp}^{+}\psi_{uudg}^{(31)}(1,2,3,4)+k_{1\perp}^{+}k_{3\perp}^{+}\psi_{uudg}^{(32)}(1,2,3,4)\\
&\quad+k_{2\perp}^{+}k_{3\perp}^{+}\psi_{uudg}^{(33)}(1,2,3,4)\Big)\\
&\times\frac{\epsilon^{jkl}T_{li}^{a}}{2}\Big[u_{i\downarrow}^{\dagger}(1)\Big(u_{j\uparrow}^{\dagger}(2)d_{k\downarrow}^{\dagger}(3)\\
&\quad-d_{j\uparrow}^{\dagger}(2)u_{k\downarrow}^{\dagger}(3)\Big)g_{\downarrow}^{a\dagger}(4)\Big]\\
&+\Big((k_{1\perp}^{+})^2\Big(\psi_{uudg}^{(34)}(1,2,3,4)\\
&\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi_{uudg}^{(35)}(1,2,3,4)\Big)\\
&+(k_{2\perp}^{+})^2\Big(\psi_{uudg}^{(36)}(1,2,3,4)\\
&\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi_{uudg}^{(37)}(1,2,3,4)\Big)\\
&+k_{1\perp}^{+}k_{2\perp}^{+}\psi_{uudg}^{(38)}(1,2,3,4)\\
&\quad+k_{2\perp}^{+}k_{3\perp}^{+}\psi_{uudg}^{(39)}(1,2,3,4)\Big)\\
&\times\frac{\epsilon^{jkl}T_{li}^{a}}{2}\Big[u_{i\uparrow}^{\dagger}(1)\Big(u_{j\downarrow}^{\dagger}(2)d_{k\downarrow}^{\dagger}(3)\\
&\quad-d_{j\downarrow}^{\dagger}(2)u_{k\downarrow}^{\dagger}(3)\Big)g_{\downarrow}^{a\dagger}(4)\Big]\Big\}|0\rangle\ , \qquad (56)
\end{aligned}
$$

where

$$\psi_{uudg}^{(34,39)}(1,2,3,4)=-\psi_{uudg}^{(34,39)}(1,3,2,4)$$

and

$$\psi_{uudg}^{(35)}(1,2,3,4)=\psi_{uudg}^{(35)}(1,3,2,4).$$

For $l_z=3$, the parton helicity must be $\lambda_{uud}=-3/2$ and $\lambda_g=-1$. We define eight independent amplitudes:

$$
\begin{aligned}
|P\uparrow\rangle_{uudg}^{l_z=3}&=\int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4]\\
&\times\Big((k_{1\perp}^{+})^3\Big(\psi_{uudg}^{(40)}(1,2,3,4)\\
&\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{2\alpha}k_{3\beta}\psi_{uudg}^{(41)}(1,2,3,4)\Big)\\
&+(k_{2\perp}^{+})^3\Big(\psi_{uudg}^{(42)}(1,2,3,4)\\
&\quad+\mathrm{i}\epsilon^{\alpha\beta}k_{1\alpha}k_{3\beta}\psi_{uudg}^{(43)}(1,2,3,4)\Big)\\
&+(k_{1\perp}^{+})^2k_{2\perp}^{+}\psi_{uudg}^{(44)}(1,2,3,4)\\
&\quad+(k_{2\perp}^{+})^2k_{1\perp}^{+}\psi_{uudg}^{(45)}(1,2,3,4)\\
&+(k_{2\perp}^{+})^2k_{3\perp}^{+}\psi_{uudg}^{(46)}(1,2,3,4)\\
&+\quad k_{1\perp}^{+}k_{2\perp}^{+}k_{3\perp}^{+}\psi_{uudg}^{(47)}(1,2,3,4)\Big)\\
&\times\frac{\epsilon^{jkl}T_{li}^{a}}{2}\Big[u_{i\downarrow}^{\dagger}(1)\Big(u_{j\downarrow}^{\dagger}(2)d_{k\downarrow}^{\dagger}(3)-d_{j\downarrow}^{\dagger}(2)u_{k\downarrow}^{\dagger}(3)\Big)\\
&\quad\times g_{\downarrow}^{a\dagger}(4)\Big]|0\rangle\ , \qquad (57)
\end{aligned}
$$

where

$$\psi_{uudg}^{(40,47)}(1,2,3,4)=-\psi_{uudg}^{(40,47)}(1,3,2,4)$$

and

$$\psi^{(41)}_{uudg}(1,2,3,4) = \psi^{(41)}_{uudg}(1,3,2,4).$$

For $l_z = -1$, the parton helicity must be $\lambda_{uud} = 1/2$ and $\lambda_g = 1$. In this case, we define 10 independent amplitudes:

$$\begin{aligned}
|P\uparrow\rangle^{l_z=-1}_{uudg} &= \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4] \\
&\times \Big\{ \Big(k^-_{1\perp} \Big(\psi^{(48)}_{uudg}(1,2,3,4) + \mathrm{i}\epsilon^{\alpha\beta} k_{2\alpha} k_{3\beta} \psi^{(49)}_{uudg}(1,2,3,4) \Big) \\
&+ k^-_{2\perp} \Big(\psi^{(50)}_{uudg}(1,2,3,4) + \mathrm{i}\epsilon^{\alpha\beta} k_{1\alpha} k_{3\beta} \psi^{(51)}_{uudg}(1,2,3,4) \Big) \\
&+ k^-_{3\perp} \Big(\psi^{(52)}_{uudg}(1,2,3,4) + \mathrm{i}\epsilon^{\alpha\beta} k_{1\alpha} k_{2\beta} \psi^{(53)}_{uudg}(1,2,3,4) \Big) \Big) \\
&\times \frac{\epsilon^{jkl} T^a_{li}}{2} \Big[u^\dagger_{i\uparrow}(1) \Big(u^\dagger_{j\uparrow}(2) d^\dagger_{k\downarrow}(3) \\
&\quad - d^\dagger_{j\uparrow}(2) u^\dagger_{k\downarrow}(3) \Big) g^{a\dagger}_\uparrow(4) \Big] \\
&+ \Big(k^-_{1\perp} \Big(\psi^{(54)}_{uudg}(1,2,3,4) + \mathrm{i}\epsilon^{\alpha\beta} k_{2\alpha} k_{3\beta} \psi^{(55)}_{uudg}(1,2,3,4) \Big) \\
&+ k^-_{2\perp} \Big(\psi^{(56)}_{uudg}(1,2,3,4) + \mathrm{i}\epsilon^{\alpha\beta} k_{1\alpha} k_{3\beta} \psi^{(57)}_{uudg}(1,2,3,4) \Big) \Big) \\
&\times \frac{\epsilon^{jkl} T^a_{li}}{2} \Big[u^\dagger_{i\downarrow}(1) \Big(u^\dagger_{j\uparrow}(2) d^\dagger_{k\uparrow}(3) - d^\dagger_{j\uparrow}(2) u^\dagger_{k\uparrow}(3) \Big) \\
&\quad \times g^{a\dagger}_\uparrow(4) \Big] \Big\} |0\rangle \ , \qquad (58)
\end{aligned}$$

where

$$\psi^{(54)}_{uudg}(1,2,3,4) = -\psi^{(54)}_{uudg}(1,3,2,4)$$

and

$$\psi^{(55)}_{uudg}(1,2,3,4) = \psi^{(55)}_{uudg}(1,3,2,4).$$

Finally, for $l_z = -2$, the parton helicity must be $\lambda_{uud} = 3/2$ and $\lambda_g = 1$. We find six independent amplitudes:

$$\begin{aligned}
|P\uparrow\rangle^{l_z=-2}_{uudg} &= \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4] \\
&\times \Big((k^-_{1\perp})^2 \Big(\psi^{(58)}_{uudg}(1,2,3,4) \\
&\quad + \mathrm{i}\epsilon^{\alpha\beta} k_{2\alpha} k_{3\beta} \psi^{(59)}_{uudg}(1,2,3,4) \Big) \\
&+ (k^-_{2\perp})^2 \Big(\psi^{(60)}_{uudg}(1,2,3,4) \\
&\quad + \mathrm{i}\epsilon^{\alpha\beta} k_{1\alpha} k_{3\beta} \psi^{(61)}_{uudg}(1,2,3,4) \Big) \\
&+ k^-_{1\perp} k^-_{2\perp} \psi^{(62)}_{uudg}(1,2,3,4) + k^-_{2\perp} k^-_{3\perp} \psi^{(63)}_{uudg}(1,2,3,4) \Big) \\
&\times \frac{\epsilon^{jkl} T^a_{li}}{2} \Big[u^\dagger_{i\uparrow}(1) \Big(u^\dagger_{j\uparrow}(2) d^\dagger_{k\uparrow}(3) - d^\dagger_{j\uparrow}(2) u^\dagger_{k\uparrow}(3) \Big) \\
&\quad \times g^{a\dagger}_\uparrow(4) \Big] |0\rangle \ , \qquad (59)
\end{aligned}$$

where

$$\psi^{(58,63)}_{uudg}(1,2,3,4) = -\psi^{(58,63)}_{uudg}(1,3,2,4)$$

and

$$\psi^{(59)}_{uudg}(1,2,3,4) = \psi^{(59)}_{uudg}(1,3,2,4).$$

The wave-function amplitudes for the other color structure, $(\epsilon^{ijl} T^a_{lk} + \epsilon^{ikl} T^a_{lj})/4$, can be defined similarly, except for the sign changes in the symmetric properties for some amplitudes. We have in total of $63 \times 2 = 126$ independent amplitudes for the $uudg$ Fock component in the proton.

Note that the above construction, where we have first considered the correct flavor structure for the three quarks, and then added all possible spin and color combinations, is not unique. One can also start with a general spin structure for the three quarks, and then consider the isospin constraints. For example, for the total quark helicity $\lambda_{uud} = 3/2$, the general spin structure will be

$$\phi(1,2,3) \epsilon^{ijl} T^a_{lk} u^\dagger_{i\uparrow}(1) u^\dagger_{j\uparrow}(2) d^\dagger_{k\uparrow}(3) |0\rangle \ , \qquad (60)$$

with the color coupling $\epsilon^{ijl} T^a_{lk}$. This color structure says that the indices i and j are antisymmetric, and the associated wave-function amplitude $\phi(1,2,3)$ has $1 \leftrightarrow 2$ symmetry. The isospin constraint indicates the following relations for the three-quark amplitude:

$$\begin{aligned}
&\phi(1,2,3) \\
&\times \epsilon^{ijl} T^a_{lk} \Big(u^\dagger_{i\uparrow}(1) u^\dagger_{j\uparrow}(2) d^\dagger_{k\uparrow}(3) + u^\dagger_{i\uparrow}(1) d^\dagger_{j\uparrow}(2) u^\dagger_{k\uparrow}(3) \\
&\quad + d^\dagger_{i\uparrow}(1) u^\dagger_{j\uparrow}(2) u^\dagger_{k\uparrow}(3) \Big) |0\rangle = 0 \ . \qquad (61)
\end{aligned}$$

Applying the above relation to (60), and taking into account the $1 \to 2$ symmetry for $\phi(1,2,3)$, one has for the component

$$\begin{aligned}
&\phi'(1,2,3) \qquad (62) \\
&\times \epsilon^{ijl} T^a_{lk} u^\dagger_{i\uparrow}(1) \Big(u^\dagger_{j\uparrow}(2) d^\dagger_{k\uparrow}(3) - d^\dagger_{j\uparrow}(2) u^\dagger_{k\uparrow}(3) \Big) |0\rangle \ ,
\end{aligned}$$

where the isospin wave function for the proton is explicit, and $\phi'(1,2,3) = \phi'(2,1,3)$. A similar analysis can be performed for the $\lambda_{uud} = \pm 1/2$ case. Working through all possibilities, we arrive at a different set of wave-function amplitudes, which are essentially equivalent to the above construction. As a check, for every orbital angular momentum projection l_z, we find the same number of independent amplitudes.

6 Wave-function amplitudes for the delta resonance

In this section, we extend the above classification of the wave-function amplitudes to the baryon decuplet, assuming again that these are bound states. We consider one specific example, the delta resonance, and other baryon decuplets can be obtained by changing the flavor structure. The distribution amplitudes for the Δ^{++} resonance have been studied in [4,33]. Δ^{++} has two independent helicity states, $\Lambda = 3/2$ and $1/2$, and the other two helicity states $\Lambda = -3/2,\ -1/2$ can be obtained by using the $\hat{Y}$ transformation (7).

Here we classify only the three-quark amplitudes, and the total quark helicity is $\lambda_{uuu} = 3/2,\ 1/2,\ -1/2,\ -3/2$. We first consider Δ^{++} with helicity $\Lambda = 3/2$, The quark orbital angular momentum projection then has the following values, respectively: $l_z = 0,\ 1,\ 2,\ 3$. According to the method in the previous sections, we find six independent wave-function amplitudes:

$$|\Delta, \Lambda = 3/2\rangle_{uuu}^{l_z=0} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3] \times \left(\psi_{uuu}^{(1)}(1,2,3) + \mathrm{i}\epsilon^{\alpha\beta} k_{1\alpha} k_{2\beta} \psi_{uuu}^{(2)}(1,2,3)\right) \times \frac{\epsilon^{ijk}}{\sqrt{6}} u_{i\uparrow}^\dagger(1) u_{j\uparrow}^\dagger(2) u_{k\uparrow}^\dagger(3)|0\rangle\ , \tag{63}$$

$$|\Delta, \Lambda = 3/2\rangle_{uuu}^{l_z=1} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3] k_{1\perp}^+ \psi_{uuu}^{(3)}(1,2,3) \times \frac{\epsilon^{ijk}}{\sqrt{6}} u_{i\uparrow}^\dagger(1) u_{j\uparrow}^\dagger(2) u_{k\downarrow}^\dagger(3)|0\rangle\ , \tag{64}$$

$$|\Delta, \Lambda = 3/2\rangle_{uuu}^{l_z=2} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3] \times \left(k_{1\perp}^+ k_{2\perp}^+ \psi_{uuu}^{(4)}(1,2,3) + k_{2\perp}^+ k_{3\perp}^+ \psi_{uuu}^{(5)}(1,2,3)\right) \times \frac{\epsilon^{ijk}}{\sqrt{6}} u_{i\uparrow}^\dagger(1) u_{j\downarrow}^\dagger(2) u_{k\downarrow}^\dagger(3)|0\rangle\ , \tag{65}$$

$$|\Delta, \Lambda = 3/2\rangle_{uuu}^{l_z=3} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3] \tag{66}$$

$$\times\ k_{1\perp}^+ k_{1\perp}^+ k_{2\perp}^+ \psi_{uuu}^{(6)}(1,2,3) \frac{\epsilon^{ijk}}{\sqrt{6}} u_{i\downarrow}^\dagger(1) u_{j\downarrow}^\dagger(2) u_{k\downarrow}^\dagger(3)|0\rangle\ ,$$

where we have used symmetry to reduce the number of independent amplitudes.

For the helicity $\Lambda = 1/2$ state of the Δ^{++} resonance, the classification is similar. The total quark helicity is the same as above, but the orbital angular projection l_z can be $l_z = 0,\ -1,\ 1,\ 2$. As a result, the three-quark Fock component for $\Delta^{++}(\Lambda = 1/2)$ has the following five independent wave-function amplitudes:

$$|\Delta, \Lambda = 1/2\rangle_{uuu}^{l_z=0} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3] \times \left(\psi_{uuu}^{(1)}(1,2,3) + \mathrm{i}\epsilon^{\alpha\beta} k_{1\alpha} k_{2\beta} \psi_{uuu}^{(2)}(1,2,3)\right) \times \frac{\epsilon^{ijk}}{\sqrt{6}} u_{i\uparrow}^\dagger(1) u_{j\uparrow}^\dagger(2) u_{k\downarrow}^\dagger(3)|0\rangle\ , \tag{67}$$

$$|\Delta, \Lambda = 1/2\rangle_{uuu}^{l_z=1} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3] \times\ k_{2\perp}^+ \psi_{uuu}^{(3)}(1,2,3) \frac{\epsilon^{ijk}}{\sqrt{6}} u_{i\uparrow}^\dagger(1) u_{j\downarrow}^\dagger(2) u_{k\downarrow}^\dagger(3)|0\rangle\ , \tag{68}$$

$$|\Delta, \Lambda = 1/2\rangle_{uuu}^{l_z=-1} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3] \times\ k_{2\perp}^- \psi_{uuu}^{(4)}(1,2,3) \frac{\epsilon^{ijk}}{\sqrt{6}} u_{i\uparrow}^\dagger(1) u_{j\uparrow}^\dagger(2) u_{k\uparrow}^\dagger(3)|0\rangle\ , \tag{69}$$

$$|\Delta, \Lambda = 1/2\rangle_{uuu}^{l_z=2} = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3] \tag{70}$$

$$\times\ k_{1\perp}^+ k_{2\perp}^+ \psi_{uuu}^{(5)}(1,2,3) \frac{\epsilon^{ijk}}{\sqrt{6}} u_{i\downarrow}^\dagger(1) u_{j\downarrow}^\dagger(2) u_{k\downarrow}^\dagger(3)|0\rangle\ .$$

We will not go beyond the three-quark Fock components, although these can be classified similarly.

Since the flavor structure for the baryon decuplet is completely symmetric, the light-cone expansion for the other states in the decuplet can be easily written down from the above results, apart from that the flavor structure need to be replaced accordingly. For example, the Δ^+ resonance has the symmetric flavor structure in the form of $(uud + udu + duu)/\sqrt{3}$. Substituting this into the above equations, the light-cone expansion of Δ^+ for the three-quark Fock component can be obtained. These amplitudes, together with those for the proton, are needed to calculate the proton–delta transition form factors in the asymptotic limit of QCD [34–36].

7 Asymptotic scaling of wave-function amplitudes

One of the important applications of the light-cone wave-function amplitudes is to calculate hard exclusive processes. The relative importance of a particular amplitude in a process can be determined from its scaling behavior when the parton transverse momenta become large. In [6], a generalized power counting rule for the light-cone amplitudes of any Fock components of a hadron state has been derived. Let $\psi_n(x_i, k_i, l_{zi})$ be a general amplitude describing a n partons Fock component of a hadron state with orbital angular momentum projection of $l_z = \sum_i l_{zi}$. The leading power behavior of the wave-function amplitude in the limit that all transverse momenta are uniformly large goes as [6]

$$\psi_n(x_i, k_{i\perp}, l_{zi}) \sim \frac{1}{(k_\perp^2)^{[n+|l_z|+\min(n'+|l'_z|)]/2-1}}\ , \tag{71}$$

which is determined by a mixing amplitude with smallest $n' + |l'_z|$. Since the wave-function amplitude has mass dimension of $-(n+|l_z|-1)$, the coefficient of the asymptotic form must have a soft mass dimension $\Lambda_{\mathrm{QCD}}^{\min(n'+|l'_z|)-1}$. We have the following selection rules for the amplitude mixings. First of all, because of angular momentum conservation, wave-function amplitudes belonging to different hadron helicity states do not mix. Second, because of the vector coupling in QCD, the quark helicity in a hard process does not change. Therefore, the pion amplitude $\psi_{u\bar{d}}^{(2)}$ does not mix with $\psi_{u\bar{d}}^{(1)}$, because the total quark helicity differs. An example of the non-trivial amplitude mixing is between the pion's two-quark–one-gluon and two-quark amplitudes.

The power counting rule (71) can be used to predict the scaling behaviors for all the light-cone wave-function amplitudes we have written down in the above for the mesons and baryons. We will not go into many details of these predictions; rather we consider some examples for the π^+ and proton. According to (71), the scaling behaviors for the two-parton light-cone amplitudes of π^+ are

$$\psi^{(1)}_{u\bar{d}}(1,2) \sim 1/k_\perp^2, \quad \psi^{(2)}_{u\bar{d}}(1,2) \sim 1/k_\perp^4 \ . \tag{72}$$

The $u\bar{d}g$ Fock amplitudes have the following scaling:

$$\psi^{(1,3,4,5,6)}_{u\bar{d}g}(1,2,3) \sim 1/k_\perp^4, \quad \psi^{(2,7,8,9)}_{u\bar{d}g}(1,2,3) \sim 1/k_\perp^6 \ , \tag{73}$$

where the mixings with the two-parton components give the dominant contribution at large $k_\perp$.

For the three-quark Fock component of the proton, we have the following scaling behaviors for the light-cone amplitudes:

$$\psi^{(1)}_{uud} \sim 1/k_\perp^4, \quad \psi^{(2,3,4,5)}_{uud} \sim 1/k_\perp^6, \quad \psi^{(6)}_{uud} \sim 1/k_\perp^8 \ . \tag{74}$$

Here, the scaling behaviors of $\psi^{(1,3,4,5,6)}_{uud}$ at large $k_\perp$ are determined by self-mixings, while that of $\psi^{(2)}_{uud}$ is determined by mixing with $\psi^{(1)}_{uud}$.

8 Summary and conclusion

Following [6], we studied in this paper how to classify the independent wave-function amplitudes for a hadron state. We discussed in detail how the spin, flavor (for quark) and color of the partons are systematically coupled. We have found these amplitudes for pion and proton up to and including four partons. We also worked out the leading light-cone wave amplitudes for the Δ resonance and the ρ meson.

A general power counting rule for the light-cone wave-function amplitude has been derived based on perturbative QCD [6]. Using this rule, we have predicted the asymptotic scaling behavior of a number of amplitudes for π^+ and the proton. This general power counting rule can be used as a constraint in modeling the light-cone wave-function amplitudes.

Many applications can be made based on the formalism presented here. One example is the generalized power counting rule for high-energy exclusive processes [6], including processes involving non-zero parton orbital angular momentum and hadron helicity flip. A number of processes have been briefly discussed in [6]. A more detailed discussion of the generalized counting rule will be presented elsewhere. In a different direction, one can also parameterize the light-cone wave-function amplitudes and fit them to many relevant experimental data, such as the elastic form factors, parton distributions, and generalized parton distributions.

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