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Authors

Sarnecka, Barbara W.
Carey, Susan

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The Development of Human Conceptual Representations: A Case Study

Susan Carey and Barbara W. Sarnecka

Harvard University

Susan Carey and Barbara W. Sarnecka, Department of Psychology, Harvard University.

Correspondence concerning this article should be addressed to Susan Carey, Laboratory for Developmental Studies, 33 Kirkland Street, Harvard University, Cambridge, MA, 02138.

Electronic correspondence should be addressed to scarey@wjh.harvard.edu

Abstract

This chapter discusses a uniquely human learning mechanism—bootstrapping-- whereby external symbolic systems (especially language) enable the creation of new, internal representational resources. The process is described in general terms and is also illustrated with a specific case: The acquisition of concepts for positive integers (1, 2, 3, etc.). First, the core knowledge systems that have numerical content are described, and it is shown that none of these systems alone is capable of representing the positive integers. Next, children's typical pattern of acquisition of the positive integers is described. Finally, bootstrapping is argued as the best explanation for the attested pattern of acquisition, and several types of evidence for this argument are reviewed.

The Problem

Human adults can think thoughts formulated over hundreds of thousands of concepts, concepts like *dog, banana, water, quark, George W. Bush, sidewalk, fourteen, pi*.... Non-human animals represent only the tiniest fraction of the human repertoire. The problem, then, is accounting for the difference. Where do human concepts come from?

Some Terminology

Concepts are mental representations with conceptual content, as opposed to perceptual or sensory content. Mental representations are characterized by their extensions (the entities in the world they pick out) and by their computational role (the inferences they support, the rules of combination that yield new representations, and so on).

Overview of the Argument

Logically, accounting for the adult human conceptual repertoire has two components: A characterization of innate representational resources and a characterization of the mechanisms that underlie developmental change. Here we are concerned only with learning mechanisms, although maturationally driven processes undoubtedly also play a role in early development.

With respect to innate conceptual representations, we endorse the core knowledge hypothesis. With respect to the mechanisms that underlie developmental change, we focus on bootstrapping processes—processes whereby the new representational resources created are more than the sum of their parts. They transcend, in some qualitative way, the representational resources that were their input. Thus, we take on Fodor's challenge to cognitive science—accounting for the construction of concepts more powerful than those the infant begins with.

Core Knowledge

Along with Baillargeon (see this volume), Carey and Spelke (1994), Leslie (1994), and many others, we endorse the existence of core knowledge. That is, we agree with the empirical claim that there are systems of representations with the following properties: 1) the real-world entities in the domains of core knowledge systems are identified by innate input analyzers, 2) acquisition of knowledge of these entities is supported, at least partially, by innate, domain specific, learning mechanisms, 3) core knowledge systems are often evolutionarily ancient, and 4) core knowledge systems operate throughout the whole life span, even in adulthood.

The ethological literature provides many examples of core knowledge systems in non-human animals. Lorenzian imprinting, for example, is an innately given learning mechanism that enables geese and many other birds to identify conspecifics. Mark Johnson and colleagues showed that identifying conspecifics is so important that evolution has provided two separate mechanisms for it. In addition to Lorenzian imprinting, chickens are also endowed with an innate schematic representation of what a chicken looks like. The two distinct entity identification systems (one based on movement, one based on a static schema of a bird-shaped entity) support the chick's learning to identify its mother (or the entity it should stay close to). The two input analyzers have distinct neural substrates, with distinct critical periods, and a complex interaction during development (Johnson & Morton, 1991).

Another famous example of a non-human core knowledge system is the learning mechanism through which Indigo buntings learn to identify the North Star, and then use it for celestial navigation for the rest of their lives. In a series of elegant studies culminating in planetarium experiments with arbitrary arrangements of the stars, Emlen (1975; Emlen, Wiltschko, Demong, Wiltschko, & Berian, 1976) showed that nestlings analyze the rotation of

the night sky, extracting the center of rotation as north. The representation that is the output of this domain-specific learning process then guides direction of flight when the bird's hormones indicate that it's time to fly north (in the spring) or south (in the fall).

We note these examples from ethology for two reasons. First, they illustrate what is meant by core knowledge. Second, they show that there is nothing theoretically problematic about the core-knowledge hypothesis. It would be astounding if human beings were the only animals without core knowledge systems. Indeed, there is good empirical evidence for human core knowledge of objects, causality, number, and intentional agency (Carey and Spelke, 1994; Gergely, Nadasdy, Csibra, & Biro, 1995; Johnson, 2003; Leslie, 1994; Woodward, 1998; Woodward, Sommerville, & Guajardo, 2001, see also Baillargeon, this volume).

Fodor's Challenge to Cognitive Science

Although we endorse the core-knowledge hypothesis, we also hold that human beings have the capacity to create representational resources that transcend core knowledge. That is, we believe that human beings can create systems of representation that allow for concepts not expressible by the core systems. This is precisely what Fodor (1975) famously argued was impossible. Fodor's argument was simple. All known learning mechanisms, he claimed, are forms of hypothesis testing, and one can't test a hypothesis if one cannot represent it. Thus, such mechanisms cannot be responsible for the creation of new representational resources.

In the famous Chomsky/Piaget debate (Piatelli-Palmarini, 1980), Piaget offered an equally simple counter-argument to Fodor's. All that is needed is a counter-example—if new representational capacities *do* arise in the course of development (either on individual or historical time scales) then this must be *possible*. Piaget's examples were from the history of mathematics. Early in the cultural history of mathematics, the concept of number included only

the positive integers; later development expanded the concept to include zero, negative integers, rational and irrational numbers, and so on. Before the construction of the rationals, thoughts involving $14/15$ could not be entertained. Similarly, before the construction of the irrationals, propositions about $\sqrt{2}$ or π could not be represented.

Thus, Fodor's challenge first requires that we describe successive conceptual systems, spelling out what is qualitatively new in the second. But we can also address the explanatory part of Fodor's challenge. It is not the case that no known learning mechanisms create new representational resources. Learning in some kinds of connectionist architectures yields emergent representations (see Shultz, this volume), and the learning mechanisms that discover Bayes net causal representations (see Gopnik, this volume) also are capable of positing previously unrepresented variables. Additionally, bootstrapping processes of many different kinds have been described (Block, 1986; Nersessian, 1992; Quine, 1960). Here we characterize a type of bootstrapping process that we believe to be uniquely human-- one that underlies the creation of new representational resources.

The Case Study

We fill out the picture sketched above through a particular case study: The acquisition of positive-integer concepts (i.e., *one*, *two*, *three*, and so forth.). We describe two systems of core knowledge with numerical content: 1) analog magnitude representations of number --Dehaene's "number sense," (Dehaene, 1997) and 2) a system of parallel individuation of small sets of individuals. When the individuals are objects, this system of representation encompasses the object-indexing and short-term memory mechanisms of mid-level vision--Pylyshyn's FINSTs (Pylyshyn, 1994; Pylyshyn & Storm, 1998); or Triesman's object files (Treisman, 1998). (See Feigenson, Dehaene & Spelke, 2004, for a much more complete account of these two core

systems.) We then show that although these systems of representation have numerical content, they do not have the power to represent the positive integers, and we characterize the bootstrapping process through which, at around age 3-1/2, children create a new representational system with the power to do so.

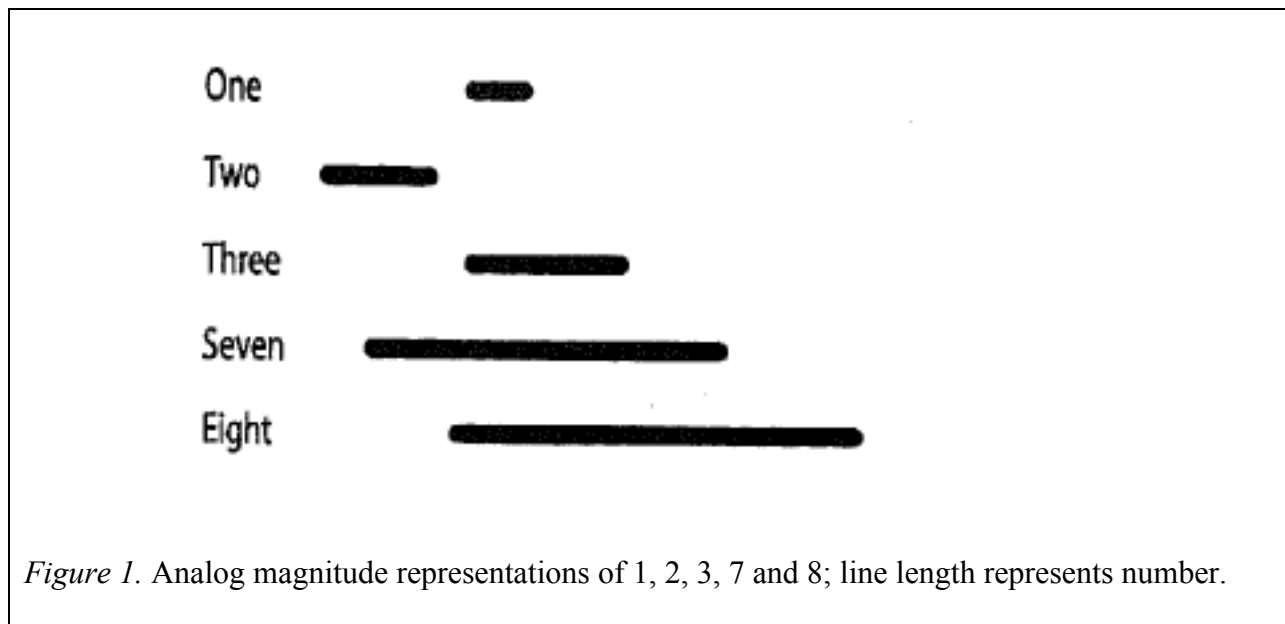
A system of representation can express the positive integers if it represents the cardinal values of sets, and also represents the successor relation among adjacent cardinal values. The numeral lists of natural language provide such a system, as long as numerals are deployed in a counting routine that assigns one numeral to each individual in a set, and in which the last numeral reached in a count corresponds to the cardinal value of the set. In other words, counting routines must respect the "cardinal principal" described by Gelman and Gallistel (1978). In this paper, we use "integer" or "positive integer" to refer to such representations. But it is important to note that representations can have numerical content and still fall short of being representations of the integers. Any representations of discrete quantity that are sensitive to numerical identity of individuals (sameness in the sense of *the same individual*), and over which computations with numerical content are defined (e.g., numerical comparison, addition, subtraction), have numerical *content*-- even if these representations capture cardinal values only roughly or even not at all. In this paper, we characterize representational systems in terms of both their format and the computations they enter into, thus specifying the type of numerical content they have.

Core System 1: Analog Magnitude Representations

Core System 1 is described by Stanislas Dehaene in his delightful book *The Number Sense* (Dehaene, 1997). Analog magnitudes are representations of the approximate cardinal values of large sets of individuals (at least up through several hundred). The representational tokens are neural magnitudes monotonically related to the number of individuals in a set (linearly

on some formulations; logarithmically on others). Because the symbols themselves get bigger as the represented entity gets bigger, they are called *analog* magnitudes. Figure 1 provides a sample analog magnitude representation of number, where the symbol is a line, and length is the dimension varying with set size. Mental analog magnitude number representations support numerical computations of many types, including comparison, addition and subtraction (Barth, Kanwisher, & Spelke, 2003; Barth, La Mont, Lipton, Dehaene, Kanwisher, & Spelke, in press; Dehaene, 1997).

Number is not the only dimension of experience represented by analog magnitudes—other examples include brightness, loudness, and temporal duration. In each case, as the physical magnitudes get bigger, it becomes increasingly harder to discriminate values that are the same absolute distance apart. You can see in Figure 1 that is harder to tell the symbols for seven and eight apart than it is to tell the symbols for two and three apart. In other words, the discriminability of any two values is a function of their ratio, as described by Weber's law.



To demonstrate Weber's law for yourself, tap out as fast as you can without counting (you can prevent yourself from counting by thinking the word "the" with each tap) the following

numbers of taps: 4, 24, 7, and 27. If you did this several times, you'd find your mean numbers of taps to be 4, 24, 7, and 27, with the range of variation very tight around 4 (usually 4, occasionally 3 or 5) and very great around 27 (from 14 to 40 taps, for example). Although the absolute difference separating 4 and 7 is the same as that separating 24 and 27, the distributions of taps around 4 and 7 will overlap much less than the distributions around 24 and 27. This happens because your ability to discriminate values depends not on their absolute numerical difference, but on their ratio, as characterized by Weber's law. This discriminability function is one of the psychophysical signatures of analog magnitude representations.

Space precludes our reviewing the elegant evidence for analog magnitude representations of number in non-human animals and human infants, but let us give just one example. Fei Xu and Elizabeth Spelke showed infants arrays of dots, one array at a time. Total array size, total volume of dots, density of dots, and so on were controlled in these studies, such that the only possible basis for the infants' discrimination was numeric. Seven-month-old infants were habituated either to arrays of eight or sixteen. After habituation they were presented with new displays, alternating between arrays of the same number of dots to which they had been habituated and arrays of the other number. Xu and Spelke found that the infants recovered interest to the new number, and also found evidence for Weber's law. The infants could discriminate eight from sixteen and sixteen from thirty-two (both ratios of 1:2), but not eight from twelve or sixteen from twenty-four (both ratios of 2:3) (Xu & Spelke, 2000).

Lipton and Spelke (2003) reported parallel findings when the individuals were streams of discrete sounds; the Weber fraction threshold of success at 7 months was 1:2, just as for dots in the Xu and Spelke studies. In addition, by 9 months of age, the ratio at which infants succeed is

2:3—in both modalities. Finally, Wood and Spelke (in press) extended these findings to streams of puppet jumps.

In sum, infants and animals (Dehaene, 1997; Gallistel, 1989) form numerical analog magnitude representations of fairly large sets of quite different types of individuals, but these representations are only approximate. Analog magnitude representations of number fall short of representing the positive integers. In this system one cannot represent *exactly* fifteen, or fifteen *as opposed to* fourteen. Nonetheless, analog magnitude representations clearly have numerical content: They refer to cardinal values of sets discrete individuals, and number-relevant computations are defined over them.

Core System 2: Parallel Individuation of Small Sets

A second system of representation with numerical content works very differently. Infants and nonhuman primates have the capacity to form mental representations of individuals and to create mental models of ongoing events in which each individual is represented by a single representational token. Figure 2 shows how, in this system, sets of one, two, or three crackers might be represented. In this figure, the format of representation of each cracker is iconic. However, the representations could certainly have other formats. What is important is that there is one representational token for each individual cracker in the set. Number is implicitly represented; the representational tokens in the model stand in one-to-one correspondence with the objects in the world.

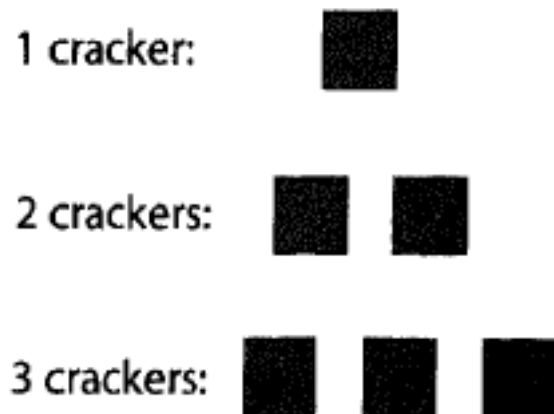


Figure 2. Representation of sets of 1, 2, and 3 crackers. One symbol for each individual, no summary symbol for cardinal value of sets.

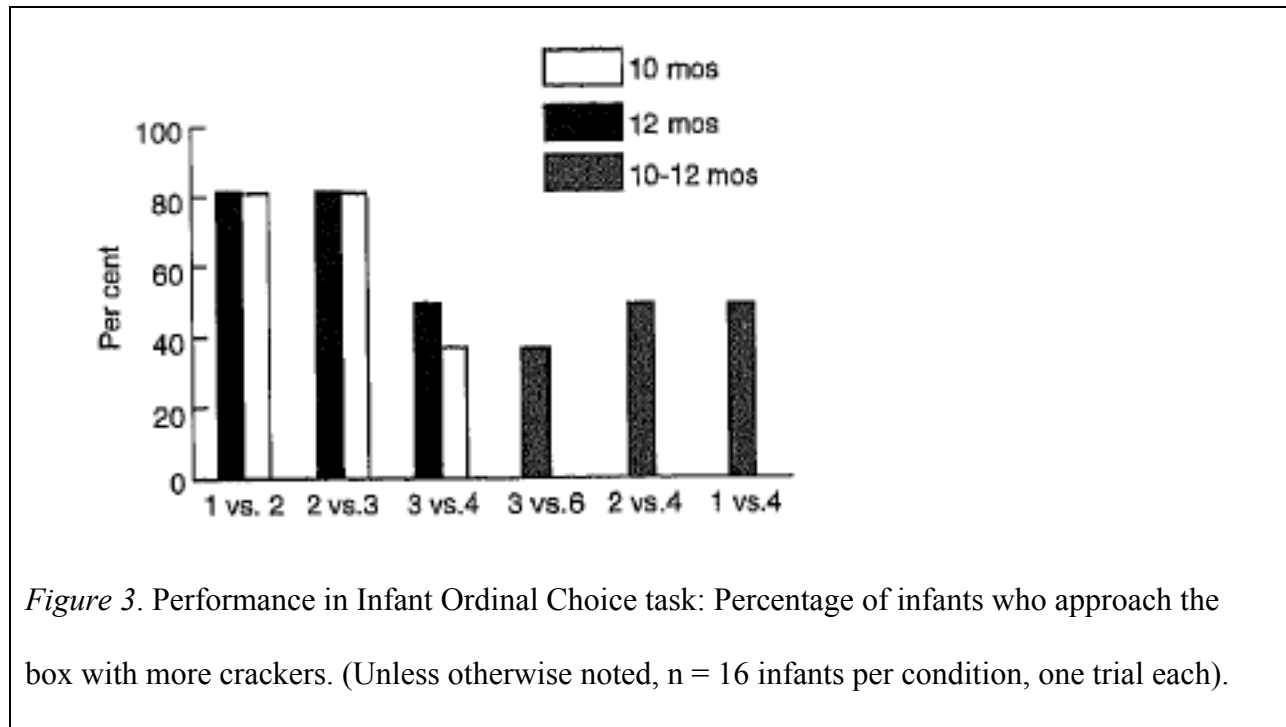
To get a feel for the evidence that infants do indeed deploy such models, consider the following experiment. Ten- to fourteen-month-old infants are shown a box they can reach into, but cannot see into. If you show an infant three objects being placed (one at a time or all at once) into this box, and then allow her to reach in to retrieve them one at a time, she shows by her pattern of reaching that she expects to find exactly three objects there. That is, after she has reached in and retrieved one object, if you have surreptitiously removed the other two, she searches persistently for more. Similarly, if she has retrieved two objects, but you have absconded with the third, she searches for the remaining one. So far, this is just another demonstration that infants represent number. However, an exploration of the limits on infants' performance of this task implicates a different system of representation from the analog magnitude system sketched above. In the box-reach task, performance breaks down at four objects. If the infant sees four objects being placed into the box and is allowed to retrieve two of them, or even just one of them, she does not keep reaching for the remaining objects. Remember

that in the analog magnitude system of representation, success at numerical comparison is a function of the ratios of the numbers being compared, and that infants' system of representation can handle sets of objects at least as big as thirty-two. But in this reaching task, infants succeed at ratios of 2:1 and 3:2, but fail at 4:2 and even 4:1 (Feigenson & Carey, 2003, in press). This result makes sense in terms of the limits on parallel attention and short-term memory that characterize mid-level, object-based attention (see Scholl, 2001 for review).

A second paradigm provides convergent evidence of the breakdown of performance at set sizes above three (Feigenson, Carey, & Hauser, 2002; Hauser, Carey, & Hauser, 2000). Ten- and 12-month-olds watch as crackers are placed, one at a time, into each of two tall opaque boxes. For example, three crackers are placed into the leftmost box and two crackers into the rightmost. Infants are then allowed to crawl toward one of the boxes. As Figure 3 shows, when the choices are 1 vs. 2 or 2 vs. 3 crackers, infants overwhelmingly approach the box with more crackers. But when the choices are 3 vs. 6, 2 vs. 4 or even 1 vs. 4, performance falls to chance. The comparison of 2 vs. 3 and 1 vs. 4 is particularly informative. Both events involve a total of 5 cracker placements, so they take equally long overall. In terms of Weber ratios, 1 vs. 4 is clearly easier to discriminate than 2 vs. 3. But children succeed on the latter and fail at the former. Again, as soon as either set exceeds 3 items, children fail.

To recap: When representing small sets, infants reveal the set-size limit characteristic of parallel individuation rather than the Weber ratio signature of analogue magnitude representations. This is the best evidence that the parallel individuation system depicted in Figure 2 underlies performance on the box-reach task and cracker-choice task (and indeed, performance on most infant tasks involving small sets, including Wynn's infant addition and subtraction studies and simple habituation studies; see Feigenson, Dehaene et al., 2004 for a review). See

Carey and Xu (2001) for other evidence supporting the identification of infant representations of small sets of objects with the object-files of mid-level, attentional parallel individuation.)



Unlike the system of analog magnitude number representations, the core knowledge system implicated in these studies is not dedicated to representing the cardinal values of sets. Rather, it represents each individual in the set separately. *Individual* is a deeply numerical concept; infants make use of a wide variety of information in establishing whether a given individual is the same one or a different one from another (*numerical identity*). The computations carried out over these representations include summing the total spatial extent of the individuals in the set (e.g., computing total volume, surface area, contour) and comparing sets on the basis of these magnitudes (Clearfield & Mix, 1999; Feigenson, Carey, & Hauser, 2002; Feigenson, Carey, & Spelke, 2002). But models of small sets of individuals can also be compared on the basis of 1-1 correspondence, establishing numerical equality or inequality,

numerical more or less (Feigenson, in press; Feigenson & Carey, 2003). Thus, although not a dedicated number representation system, Core System 2 has rich numerical content.

Detour: Infants Seem to Lack the Singular/Plural Distinction

In both the cracker-choice and box-reach paradigms, infants failed at 1 vs. 4 comparisons. These are striking failures. If a 12-month old infant sees 1 cracker placed in a box and 4 crackers placed in another box, the infant chooses at random between the boxes. If a 12- or 14-month-old infant sees 4 objects placed in a box into which the infant can reach but cannot see, she is satisfied after retrieving only one of them. We offered these data to support the conclusion that when tracking individuals, infants do not represent numbers beyond 3. But to succeed at these 1 vs. 4 tasks, infants need not represent *exactly 4* or even *approximately 4*; they need only represent the set of 4 as a plurality, and hence as more than 1. In other words, all they need is a singular/plural distinction. But they appear to have none.

We found these data so puzzling that we extended the box-reach task to older infants. Infants were either shown 3 or 4 balls, first displayed on top of the box and then placed inside it. All balls but 1 were surreptitiously removed, so when the infants reached in, they found only 1 ball. At all ages tested, when 3 balls had been placed in the box, infants persistently searched for further balls after retrieving just 1. But at 12, 14, 18 and 20 months of age, infants who had seen 4 balls placed into the box stopped searching after retrieving 1. Their behavior on these trials was the same as when they saw just 1 ball placed in the box and had retrieved it. It is not until 22 months of age that infants succeed, as a group, on this non-verbal singular/plural task (Barner, Thalwitz, Wood, & Carey, 2005).

Perhaps it should not be so surprising that non-verbal infants do not represent the singular/plural distinction. Neither of the core systems with numerical content includes a

computationally relevant break between single individuals on the one hand, and sets of more than one individual on the other. The analog magnitude system distinguishes among all sets whose ratio exceeds the Weber threshold of discriminability. It does not treat 8 and 16 as equivalent to each other, in contrast to 1. Similarly, the system of parallel individuation tracks each individual in sets of 1, 2, or 3, and does not lump 2 and 3 together as principally different from 1.

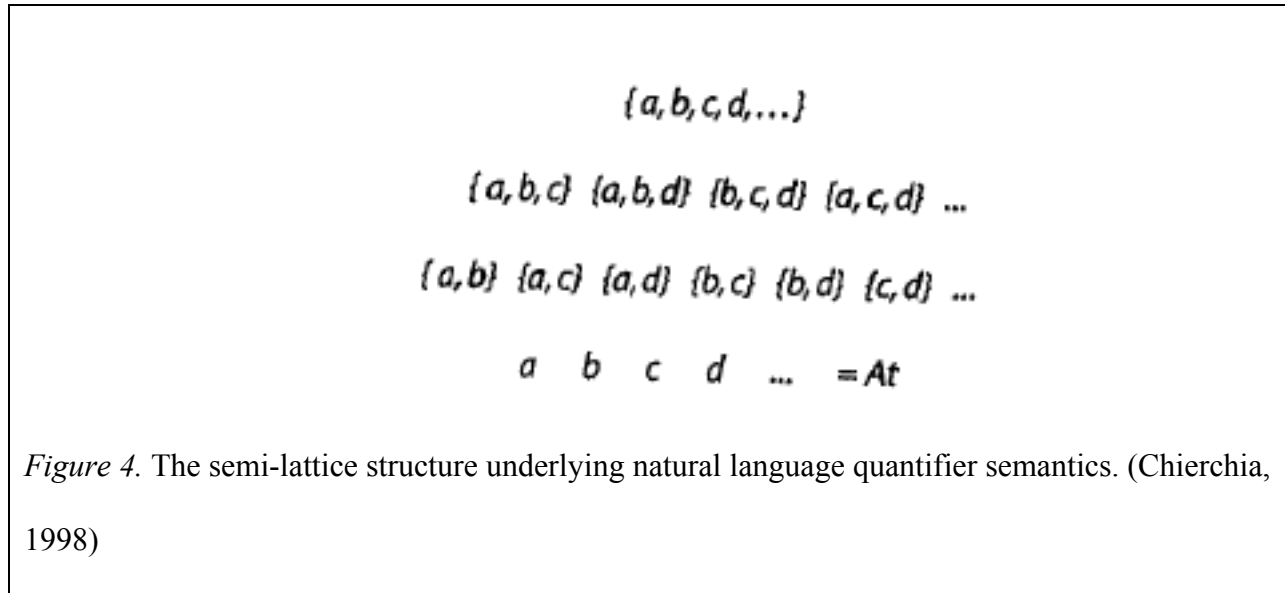
Interestingly, 20-month-old English and French learners also have not yet learned the linguistic markers of the singular/plural distinction, whereas 24-month-old French and English speaking children have (Koudier, Halberda, & Feigenson, 2004; Koudier, Halberda, Wood, & Carey, 2004; Wood, Koudier, & Carey, 2004). Ongoing work in our laboratory, and that of Sid Koudier in Paris, shows that the age when children begin to mark the singular/plural distinction in French and in English is 22 months—the very same age at which infants begin to succeed on the non-verbal 1-4 comparison task.

These data hint at another source of representational resources relevant to the acquisition of number concepts—the semantic distinctions that underlie number marking in natural language syntax and in natural language quantifier systems.

A Third Developmental Source of Number Representations: Natural Language Semantics

Linguists such as Chierchia (1998) and Link (1983) have provided unified treatments of the linguistically universal quantificational resources that underlie the singular/plural distinction, the count/mass distinction, quantification in classifier languages, and quantifiers themselves (“none,” “some,” “all,” “each,” etc.). These treatments rest on the semi-lattice of *sets* that can be constructed from a universe of *individuals* (the atoms, *At*, of Figure 4.) These linguistically universal devices require an explicit conceptual distinction between individuals and sets; having

such a distinction available for hypothesis testing might help the child construct a representation of integers.



Notice that in the two core systems with numerical content, there are no representational tokens with “individual” or “set” as their content, although there are attentional mechanisms that pick out sets of individuals for which one token for each individual is created (parallel individuation) or for which a summary representation of approximate numerosity is computed (analog magnitudes). The third system of representation, explicit natural language quantification, is also on-line before age 2, at least in languages with singular/plural distinctions.

Transcending Core Knowledge

We have described three systems of mental representations with numerical content that are available to children before age 2. The analog magnitude system and the system of parallel individuation of small sets meet the criteria for core knowledge: They are evolutionarily ancient, supported by domain-specific input analyzers, have content that goes beyond sensori-motor primitives, support domain-specific learning, and continue to articulate our mental models of the world throughout the life span. On some views of language acquisition, which we share,

acquisition of cross-linguistically universal quantificational devices is also likely to be supported by innate domain-specific constraints within a language-acquisition device, but for present purposes, all that matters is that these quantificational devices are available at the outset of the learning process that constructs the positive integers. Importantly, none of the three systems alone has the power to represent the positive integers.

The system of parallel individuation has no symbols for integers, no summary symbols for the cardinal values of sets. And it has a set-size limit of three or four. A person cannot think a thought formulated over the concept 7 using this system of representation alone. The analog magnitude system comes closer to a representation of the positive integers. At least it contains representational tokens for the approximate cardinal values of sets. However, it too cannot represent exactly 7, or exactly 15, or exactly 32... Furthermore, because quantities are compared via their ratios, analog magnitude representations of number obscure the successor relation. The difference between 2 and 3 is not experienced as is the difference between 8 and 9 (indeed, the latter cannot be discriminated at all). Finally, the number-marking systems of natural language distinguish singular (1), dual (2), sometimes triple or paucal (3 or few), and have quantifiers like “many” or “some” that pick out relative numerical magnitudes. However, barring numeral lists themselves, natural language quantification includes no representations of exact cardinal values above 3.

Interim Conclusion

We must be very careful when we interpret a researcher’s claim that prelinguistic infants represent number. This is true, but which numbers can they represent? Not rational or real numbers, and not even the positive integers. When we specify the representational systems available to infants, characterizing the format and content of the symbols therein and the

computations they support, we can be precise about what numerical content they include. This cashes out what we mean when we say prelinguistic infants represent number.

We have met the descriptive part of Fodor's challenge. Children in cultures where counting is salient create a representation of the positive integers by age 3-1/2 to 4 (Fuson, 1988, 1992; Wynn, 1992). We have characterized how representations of positive integers transcend (qualitatively) the three systems of representation that are universally available to human beings and that are evident in children under 2 years of age.

The claim that integer representations transcend core knowledge has implications for development, both over historical time and in the individual child. Whereas the representations given by core knowledge systems should be easily and universally accessible, representations of positive integers should be relatively difficult to create. And indeed, whether we consider cultures evolving number concepts in historical time, or individual children acquiring the number concepts available in their culture, the process is a slow one.

Gordon (2004) has described an isolated Amazonian culture—the Piraha—in which the adults demonstrate (on non-verbal tasks) normal analog magnitude and parallel individuation representations. The Piraha language also has quantifiers (including “one,” “two” and “many.”) However, the particular history of this people is such that they have not developed or imported, as most of the world's cultures now have, a numeral list. And Gordon shows that Piraha adults fail at a wide variety of non-verbal tasks requiring the mental representation of exact numbers above 3 or 4, although they do perfectly well on tasks tapping analog magnitude representations of sets above 4 (see Pica, Lerner, Izard & Dehaene, 2004, for convergent evidence from the Mundurucu, another isolated Amazonian culture). The Piraha case demonstrates the importance of cultural input (i.e., a culturally transmitted system of external symbols, like numeral words) in

individuals' development of positive-integer concepts. Similarly, Hurford (1987) reviews ethnographic studies of partial systems of number representation that historically precede (in any given language) the construction of a full, base-system, numeral-list representation of the positive integers.

Turning to individual development, many psychologists (e.g., Fuson, 1992; Schaeffer, Eggleston & Scott, 1974; Siegler & Robinson, 1982; Wynn, 1992) have noted how difficult it is for toddlers to come to understand the numerical meaning of counting. Of course, this difficulty is to be expected on the view that core knowledge lacks the resources to represent the positive integers.

Wynn's Difficulty-of-Learning Argument

If concepts for the exact numbers 1, 2, 3, 4, and 5 were available from infancy, then identifying the words to label those concepts should be a relatively simple matter. Consider the challenge that you, the reader, would face if you suddenly found yourself in a foreign country where you didn't speak the language. You know *what numerals are*—you just wouldn't know the local words for them. But if you saw people counting (at a market, for example) you would be able to figure out the numeral words quite easily. Let's say you heard and saw people counting, using the sequence *ash, naz, gim, batul, thropp*. Then, you noticed people asking for *gim* this or *gim* that and getting three things in return. You would conclude that *gim* meant 3. And once you knew that, it would take you no time at all to match 1, 2, 4, and 5 with *ash, naz, batul*, and *thropp*.

If representations of the positive integers were given by any one core knowledge system, (e.g., via a preverbal, mental list of numeral-like symbols, Gelman et al., 1978) we should expect children to learn the numerals of their native language the same way that adults learn the

numerals of a foreign language. That is to say, once one numeral is learned, the other numerals should follow almost immediately. But children's numeral learning is actually quite a lengthy process (see, e.g., Fluck & Henderson, 1996; Fuson, 1992; Siegler & Robinson, 1982; Wynn, 1990, 1992). As Karen Wynn has argued, the pattern of numeral learning that children actually show suggests that they are doing something much more difficult than simply labeling pre-existing number concepts. They seem actually to be *constructing* new number representations as they go along.

By about age 3, most children can recite the numeral sequence (count out loud) up to "ten" (Fuson, 1988; Miller, Smith, Zhu, & Zhang, 1995). They can also 'count' sets of 5 or 6 objects—that is, they recite the number sequence while pointing to each object in turn (Fuson, 1988; Sarnecka & Gelman, 2004; Wynn, 1990, 1992). However, children this age don't understand that the last numeral used in counting reveals the number of items in the whole set; that is, they have not yet mastered Gelman and Gallistel's (1978) "cardinality principle."

Several observations show that children count without understanding how counting represents number. For example, if you ask a two- or three-year-old to count (or watch you count) a row of 5 apples ("one, two, three, four, five") and then ask her "So, how many apples are there?" she will probably respond by recounting. It is virtually impossible to induce a young child to produce a cardinal response after a count, but if you succeed, she responds by guessing ("Um... two! No, wait-- three!")

Clearly, three-year-olds do not connect counting and numeral meanings in the same way that adults do. But it's not that young preschoolers know *nothing* about numerals. They do use numerals as quantifiers, although without regard to the actual set sizes each numeral denotes. (The second author's two-year-old son once demanded more M&M's by saying "I like some

plenty! I like some *too much!* I like some *lot!* I like some *eight!*") So the question is: what exactly *do* preschoolers know about numerals? To answer that question, Wynn and others came up with a variety of ways to probe numeral knowledge. Three such tasks are described below.

The Give-A-Number task. (Schaeffer et al., 1974; Wynn, 1990, 1992) In this task, children are given a pile of small objects (e.g., 15 toy fish) and are asked to give a certain number of them to a puppet. For example, the child might be asked "Can you give *two* fish to the monkey?" (See Figure 5, Part A.) The dependent measure is how many items the child actually gives.

The Point-to-X task. (Wynn, 1992). In this task the child is shown a pair of pictures (e.g., a picture of 2 fish next to a picture of 3 fish). The child is asked to choose one. For example, "Point to the picture of *two* fish." (See Figure 5, Part B.) The dependent measure is which picture the child chooses.

The What's-on-This-Card task. (Gelman, 1993; LeCorre, 2003) In this task, the child is shown a picture of 1 or more objects and asked simply "What's on this card?" Children typically respond with just the object name at first (e.g., "fish"). So feedback is given on the first trial, modeling a numeral response ("That's right! It's *two fish!*"). On subsequent trials, no feedback is given, but most children give numeral responses, as modeled.) The dependent measure is what numeral the child says. (See Figure 5, Part C.)

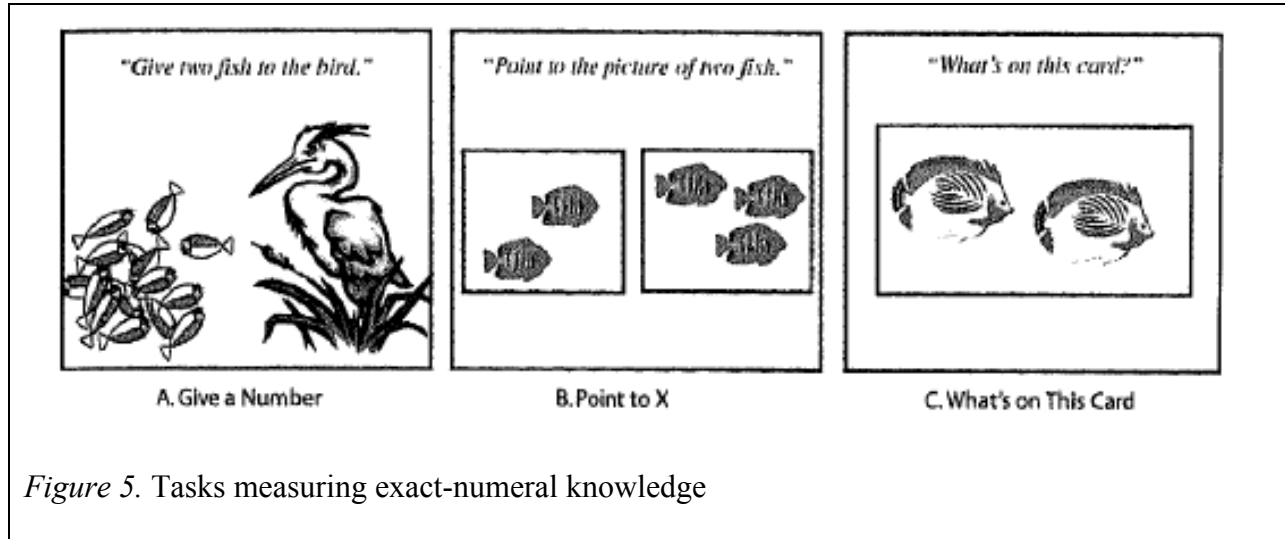


Figure 5. Tasks measuring exact-numeral knowledge

The Pattern of Numeral Learning

Data from the tasks above have uncovered a systematic pattern to numeral learning—a pattern that has now been replicated in many studies, and with child speakers of Japanese, Mandarin, and Russian as well as English (LeCorre, Li, & Jia, 2003; Li, LeCorre, Shui, Jia, & Carey, 2003; Sarnecka, Kamenskaya, Ogura, Yamana, & Yudovina, 2004).

The pattern is as follows. The child begins as a no-numeral-knower, meaning that she makes no distinctions among numerals. On the Give-N task, she gives either 1 item for every request or a handful for every request. On the Point-to-N task, she points at random; on the What's-on-this-Card task, she usually produces no numerals, answering with nouns alone. The non-numeral-knower level is followed by a series of intermediate levels—the “one”-knower, “two”-knower, and “three”-knower levels; children at any of these levels are called “subset-knowers,” for they know the exact meaning of only a subset of the numerals in their count list. We illustrate the pattern of responses of subset-knowers by describing a “two”-knower. On the Given-N-task, a “two”-knower gives the correct number asked for if it is “one” or “two,” but for all other numerals grabs a handful that is more than 2. On the Point-to-N task, a “two”-knower is correct for any pair of items that includes a set of 1 or 2 (i.e., presented with a set of 2 and 5,

chooses correctly when asked for “two” or “five”, but is random on every pair involving sets greater than 2 (e.g., chooses randomly when asked for the set of “three” between cards with 3 and 6). On the What’s-on-this-Card task, a “two”-knower says “one” for cards with 1, “two” for cards with 2, and then uses larger numerals randomly (often “three” for all sets) for cards containing 3 through 10 items. One of the striking findings in this body of literature is the within-child consistency on these three tasks—diagnosis as a “two”-knower on the basis of Give-a-Number predicts knower-level on the other two tasks, in spite of the widely different performance demands of the tasks (LeCorre, Brannon, Van de Walle, & Carey, 2004a, b; Wynn, 1992). These studies also show that subset-knowers use counting differently from children who understand how counting represents number. They do not spontaneously count to solve the Give-a-Number or What’s-on-this-Card tasks, and when they have given a wrong number on Give-a-Number and are asked to fix it, they usually leave the set unchanged or fix it in the wrong direction.

Between 3-1/2 and 4-1/2, a qualitative change occurs: the child becomes a *high-numeral-knower*. They solve all three tasks in Figure 5 for all numerals in their count list, using counting to do so for large numbers. If counting to check reveals they have made a mistake on Give-a-Number, they adjust the set correctly, without recounting. They have figured out how counting represents number; they have induced the cardinality principle.

Interim Conclusions

The data on subset-knowers supports our conclusions from the description of the two core systems of numerical content. The pattern of slow, piece-by-piece numeral learning and the qualitative differences between subset-knowers and high-numeral-knowers are evidence for discontinuity in number-concept development. If integer representations were part of core

knowledge, then numeral learning would be a simple mapping problem, as it is for adults learning the numerals of a foreign language. Instead, the data reviewed above indicate that children learn the numeral list, and as soon as they are “one”-knowers, they understand that numerals encode something having to do with number (e.g., they use numerals other than “one” as plural markers, as quantifiers meaning “some.”) They then take months to figure out what “two” means. Why should this be so, given that children can certainly *perceive* the difference between 1, 2, and 3 items even before they learn to talk? We submit that numeral learning takes so long because learning numerals requires children to construct *summary* representations of exact numbers—single representational tokens with content different from any in a core knowledge system.

We turn now to the second part of Fodor’s challenge to cognitive science—sketching a learning process that might underlie the creation of a representational resource with more power than its input.

A Bootstrapping Proposal

To explain how children construct the positive integers, we appeal to “bootstrapping.” Many psychologists, historians, and philosophers of science have used the metaphor of bootstrapping to describe learning processes whose endpoints transcend in some qualitative way their starting points. The choice of metaphor may seem puzzling, because pulling oneself up by one’s own bootstrap is clearly impossible, whereas the learning we seek to explain actually occurs, and thus is certainly possible. We keep the term because of its historical credentials and because it seeks to explain cases of learning that many have argued *are* impossible.

We use the term “bootstrapping” in its original sense from the literature in the philosophy of science, not as it has come to be used in the language acquisition literature. “Semantic

bootstrapping” and “syntactic bootstrapping” are processes hypothesized to help the child construct a mapping between innately specified syntactic and semantic categories. As we use the term, bootstrapping is a process that underlies the construction of new representational resources. In the bootstrapping process we propose, a set of symbols (and the relations among them) are first learned in terms of each other. At this point, the symbols have meaning in relation to the other symbols in the set, but are not yet interpreted (or at most are only partially interpreted) in terms of antecedent mental representations. Once the symbols and the relations among them are learned, they serve as placeholders, waiting to be filled in with richer and richer meanings. These richer meanings are connections between the placeholder symbols and representations in other parts of the mind (for example, the outputs of core knowledge systems). These connections are gradually made through analogical reasoning, inductive leaps, and inference to best explanation.

The resulting system has greater power than core knowledge systems for two reasons. First, part of a symbol's meaning comes from its place in the set – its relation to the other symbols with which it was learned. The symbols thus have content they could not have if each were learned on its own, defined completely in terms of meanings from the core systems. Second, bootstrapping allows representations from previously distinct representational systems to be combined and integrated. This allows for the formation of representations that could not be produced by any one of the original systems alone. Thus, bootstrapping is not just a way of acquiring knowledge, it is a way of acquiring *new representational resources*. To use a computer analogy, bootstrapping is to some other kinds of learning as software installation is to data entry. It doesn't just add more information, it enables new kinds of thinking.

Bootstrapping Exact-Number Concepts

How does bootstrapping explain the pattern of numeral learning described above? First, let's consider what children have to learn:

- The numeral list (i.e., the words and their order)
- The meaning of each numeral ("three" = 3, "seven" = 7, etc.)
- How the numeral list represents exact numbers (for any word "X" on the list whose cardinal meaning N is known, the next word on the list has a cardinal meaning $N+1$)

Next, let's consider some of the knowledge systems that children have available at the outset of learning:

- The parallel individuation system (tracks up to 3 individuals at the same time)
- Analog magnitude representations of number (yields large, approximate numbers)
- The quantificational semantics of natural language (providing the distinctions of individual vs. set, discrete vs. continuous quantification, singular vs. plural, etc.)
- The capacity for explicit symbols—the capacity to create lexical entries and to search for meanings.
- The capacity to represent serial order, including ordered lists of lexical items (e.g., to learn nonsense strings like "eenie, meenie, minie, mo.")

Now let's consider how a bootstrapping process might occur. Our proposal draws on many others (e.g., Bloom, 2000; Bloom & Wynn, 1997; Fuson, 1992; Hurford, 1987; Klahr & Wallace, 1976; Spelke & Tsivkin, 2001).

Numerals in a list. In the earliest phase of the learning process, two things happen independently. On the one hand, children learn a meaningless string of numeral words ("one, two, three...") just as they learn other meaningless strings ("eenie, meenie, minie, mo..."). At

this point, the counting routine may be associated with particular situations (e.g., with pointing, with reading particular books that involve counting, etc.), but the only meaning the numerals have are the relations that hold between them within the list (e.g. "five" is just the word after "four" and before "six"). This list—meaningless except for its internal order-- is the placeholder structure.

Numerals in sentences. Separately, children encounter the numerals as quantifiers—appearing in sentences without the other members of the list ("You can take *two* cookies. No, *two* is plenty... I said *two*! Hey! Come back here!"). Eventually, children learn the exact meanings of some of them. (First "one," then "two," then "three," in the pattern described above.) From the very start, children's knowledge of grammar constrains the hypotheses they entertain about what the numerals might mean. Imagine yourself at age two, hearing the sentence "We have two cats" (or its equivalent in your native language). Suppose you understand this whole sentence (words, syntax, morphology) except for the word "two." The rest of the sentence gives you lots of hints about what "two" might mean. First of all, you can tell that "two" is an adjectival form. And you already know that adjectival forms pick out properties, not individuals. So you know that "two" refers to some property (let's call it *two-ness*). Thereafter, your question is not "What is a *two*?" but rather, "What is *two-ness*?" and "What sorts of things have *two-ness*?" Next, you can tell by how it interacts with other words that "two" is a quantifier (rather than an adjective). For example, in English, only quantifiers can appear in the partative construction: "A lot of sunny weather," "five hundred of our closest friends," etc. Other languages also have special constructions in which only quantifiers can appear. For example, in Japanese, only quantifiers can appear in classifier constructions (Downing, 1996). Once you recognize the constructions that are typical of quantifiers in your language, you can infer that the

numerals are quantifiers—words that explicitly refer to quantity-- because they appear in these constructions. (Bloom & Wynn, 1997).

After children figure out that numerals are quantifiers, they begin to learn exact meanings for some of them just as they would learn the meanings of other quantifiers. "One" is learned like the singular determiner "a(n)" – as an explicit marker of sets containing 1 individual (indeed, in many languages, the numeral “one” and the indefinite article “a(n)” are the same lexical item.) If the structure indicated in Figure 4 is available to support the meanings of quantifiers, it is the source of explicit concepts of *individual* and *set*. The plural morpheme "-s" is learned as an explicit marker of sets containing more than 1 individual. At this point, the other numerals ("two," "three," "four," etc.) are treated simply as quantifiers that mark sets of more than 1 individual.

Next, "two" is learned just as dual markers are learned in languages that have singular/dual/plural morphology—that is, as a marker for pairs of individuals, for sets in the first level above the atoms in Figure 4. Higher numerals are seen as quantifiers referring to sets of more than 2. Finally “three” is learned just as trial markers are in the rare languages that have singular/dual/trial/plural morphology—that is, as a marker of triplets of individuals. Higher numerals are seen as quantifiers referring to sets of more than 3. The child can integrate these representations with outputs of the parallel individuation system, which are also representations of individuals (one individual-file open), pairs (two individual files open), and triplets (three individual files open).

The contribution of language. The question arises: If children do not have the concepts *exactly 1*, *exactly 2* and *exactly 3* beforehand, then how do they learn the singular/plural distinction, the dual marker, and the trial marker? This is the very crux of our argument. We

believe that the concepts of *set* and - *individual* actually come neither from the parallel individuation system nor from the analog magnitude system, for although attentional mechanisms pick out sets and individuals as input to both systems, neither has representations with the content *set*. Rather, these concepts become available in the course of language acquisition; they are part of LAD (the language acquisition device). It is *language* that requires the notion of a set, and language that draws a singular/plural distinction—that is, a special distinction between sets of 1 (individuals) and all other sets.

Similarly, we argue that although the parallel individuation system is what allows children to keep track of 1, 2, or 3 objects a time, it is language that induces children to create a *summary* representation with the content "2." In other words, it is language that spurs the creation of an internal symbol whose meaning is *that which is common to all situations where a pair of individuals are being tracked at the same time*. Associating linguistic markers with unique states of the parallel individuation system is only possible for up to three objects, because the parallel individuation system can only keep track of up to three individuals at once. This is why the piecemeal learning described above must end upon learning the exact meaning of “three” (or “four,” if parallel individual extends to 4 items in the preschool years, as it does for adults).

The leap from low- to high-numeral knowledge. Once these two senses (quantifier meaning and list position) of "one," "two," and "three" have been learned, the child is in a position to notice that the first three syllables in the counting list ("one, two, three") are the same as the singular, dual, and trial markers "one," "two," and "three." Having noticed this, the child draws an analogy based on two very different “follows” relations—the relation of words in the count list ("one" is followed by "two", which is followed by "three") and the relation of sets

denoted by the singular, dual, and trial markers (a single plus an individual makes a pair; a pair plus an individual makes a triple). This numerical meaning derives from the system of parallel individuation (remember, the infant represents the numerical relations between models containing 1, 2, and 3 individuals). So, moving forward a word in the count list can be likened to adding 1 individual to a set.

This idea (one word forward equals one more individual) captures the successor principle. On this basis, the child can assign meanings to numerals in the list beyond "three." The new insight also gives meaning to the object-counting routine—object counting, after all, is the very act of coordinating steps forward in the list with the addition of individuals to a set. So the child can finally understand what counting has to do with the question of how many—the logic of the cardinality principle becomes clear.

Thus, the child learns the integer meanings of “two” and “five” in different ways. (The *list* meanings of ‘two’ and ‘five’ would be something like *follows ‘one,’ precedes ‘three’* and *‘follows ‘four,’ precedes ‘six,’* respectively. These are both learned the same way—by memorizing the ordered list.) Returning to the integer meanings, “two” (i.e., *dual*) uses notions inherent in natural language (such as *individual* and *set*) plus the system of parallel individuation. The meaning of “five” (i.e., “*four*” plus 1) is given by the successor principle—a rule the child has induced by integrating the quantifier meanings and list meanings of the words “one,” “two,” and “three.”

Evidence for the Bootstrapping Proposal

Several types of evidence support this bootstrapping proposal. First, many studies have now shown that children initially learn the numeral list as a meaningless string (Fuson, 1992; LeCorre, Brannon, Van de Walle, & Carey, 2004a, b; Schaeffer et al., 1974). Second, the

morphosyntactic cues that constrain numeral meanings are part of the speech children hear and use (Bloom, 2000; Bloom & Wynn, 1997). Third, as discussed above, the partial meanings children assign to large numerals before inducing the successor principle are quantifier-like meanings. For example, one-knowers interpret numerals higher than “one” to mean *plural*, two-knowers interpret numerals higher than ‘two’ to mean *some, larger than 2*, etc. Fourth, children learning languages that do not obligatorily mark singular/plural (e.g. Mandarin and Japanese) learn the meaning of “one” many months later than English- and Russian-speaking children (Li et al., 2003; Sarnecka et al., 2004). This is true even though the English and Russian speakers are *not* better at counting objects, and even though the word “one” is used just as often in Japanese as in Russian. (Sarnecka et al., 2004)

Our bootstrapping proposal contains one conspicuous omission: We assign no role to the analog magnitude system. We realize that this omission may be viewed with alarm. First and foremost, magnitudes are a critical component of numeral meaning for adults, and they may be essential to the representation of numerical order (Dehaene, 1997; Lemer, Dahan, Spelke & Cohen, 2003). Second, the analog magnitude system can represent larger numerosities than can the parallel individuation system. So it could allow children to assign approximate numerical meanings to numerals beyond “three” before inducing the cardinal principle. It's not hard to imagine a bootstrapping proposal that would build on magnitudes rather than (or in addition to) the system of parallel individuation.

Despite the plausibility of these arguments, research using several different tasks has shown that it is not until *after* children work out how the numeral list represents number that they learn which analog magnitudes correspond to which words in the list (LeCorre, 2003). Thus, magnitudes (at least analog magnitude representations of sets greater than 4) do not play a role in

the initial construction of exact-number concepts. Rather, the bootstrapping process seems to rely exclusively on information yielded by the parallel individuation system and natural language.

Conclusions

We offer the bootstrapping process in answer to Fodor's explanatory challenge. Of course, new representational capacities cannot come from *nowhere*; on this we agree with Fodor. Many distinct representational systems with numerical content are drawn upon. The new representational power derives from two sources—the capacity to interrelate symbols directly, in terms of each other, and the capacity to combine representations from distinct systems of core knowledge by creating mappings between them. Through this process, aspects of representations implicit in one system and explicit in another, such as the notion of “set” needed to create a summary representation of cardinal values of pairs, enrich understanding of both systems.

Our sketch is, by necessity, only a caricature of the bootstrapping process underlying the construction of the count list. In other work, we have filled in more of the details (e.g., LeCorre & Carey, 2004; Sarnecka & Gelman, 2004; Sarnecka et al., 2004) but many of the relevant details simply are not yet known. For example, in the bootstrapping process as we've laid it out, only the serial order of the memorized count is hypothesized to function as the essential placeholder, but it is likely that the initially meaningless counting routine (learned like the gestures in “patty-cake, patty-cake...”) may also play a role in working out the numerical meaning of the numeral list. Or, for another example, the account leaves open whether it is *necessary* that the numerals be learned in order, “one” first, “two” second, and “three” next, or whether this is merely a matter of input frequency. It is interesting that language typologies find a similar ordering of quantifier systems; a language with a dual marker also has singular/plural markers, but not vice versa; a language with a trial marker also always has a dual marker, but not

vice versa (Corbett, 2000). We are currently engaged in a series of training studies that address these and other questions, and that test details of the bootstrapping proposal.

Language has two roles to play in this bootstrapping story. Unique to this case, the role of numerals as quantifiers helps the child begin to construct meanings for them. And like all bootstrapping of the sort we describe here (see Block, 1986, for a general characterization of this process from the point of view of conceptual role semantics), the numeral list is learned directly, with serial order its only meaning-relevant property, and serves as a placeholder to scaffold the construction of a representation of natural number. In all cases of conceptual-role bootstrapping, a set of explicit, external symbols (words or mathematical symbols) is learned directly, with their meanings initially characterized only, or mainly, in terms of the conceptual roles given by their interrelations. These symbols serve as placeholders, and model manipulation mechanisms (analogical mapping, limiting case analyses, and so on) serve to fill in those placeholders by relating these symbols to antecedent mental representations. Bootstrapping mechanisms of this sort have been posited to underlie the creation of new conceptual resources in the history of science and in the construction of intuitive theories in childhood. See, for some examples, Nersessian's (1992) analysis of Maxwell's construction of electromagnetic theory, Smith and colleagues' (Smith, Snir & Grosslight, 1992) analysis of conceptual change within elementary school aged children's theories of matter, and Carey's (1999) analysis of the construction of a vitalist biology in early childhood.

Because of its dependence on explicit, external, symbols, conceptual role bootstrapping is a uniquely human learning mechanism. It differs, in this regard, from other learning mechanisms sketched in these pages (e.g., connectionist learning algorithms). It also makes salient another aspect of human learning absent from the most of the other chapters in this book—human

learning is not exhausted by mechanisms in which the individual confronts statistical data in the world (as in algorithms for constructing Bayes-net representations of causal structure, or again, as in connectionist learning algorithms). Undoubtedly, human beings, like other animals, make use of a huge variety of learning algorithms, but only humans culturally create new representational resources and only human children make use of language to build these anew for themselves. The construction of the integer list is offered here as an example of this uniquely human process.

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Figure Captions

Figure 1. Analog magnitude representations of 1, 2, 3, 7 and 8; line length represents number.

Figure 2. Representation of sets of 1, 2, and 3 crackers. One symbol for each individual, no summary symbol for cardinal value of sets.

Figure 3. Performance in Infant Ordinal Choice task: Percentage of infants who approach the box with more crackers. (Unless otherwise noted, $n = 16$ infants per condition, one trial each).

Figure 4. The semi-lattice structure underlying natural language quantifier semantics. (Chierchia, 1998)

Figure 5. Tasks measuring exact-numeral knowledge