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Author

Miller, C.W.

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Constance W. Miller

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A Numerical Model of Transient Two Phase Flow in a Geothermal Well

Constance W. Miller

Earth Sciences Division
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

ABSTRACT

The one-dimensional transient two phase flow in a geothermal well has been modelled with a finite difference approximation. The equations of mass, momentum and energy are solved using a partially implicit method. Terms that would place a severe time restriction on the calculation are solved implicity while other terms are solved explicitly for computational ease and efficiency. The wellbore model includes heat and mass transfer and is coupled to a simple reservoir model. It is used to investigate the transient behavior in a single or two phase well during well testing. Results show that when the reservoir has a relatively large value of kh, as exists in a geothermal field, the slope of the log (pressure) vs. log (time) curve is not necessarily a unit slope. The early time behavior of this curve is controlled by the interaction of the flow in the reservoir and that in the well, and can be used to determine near bore values of kh. Heat loss in the wellbore is shown to also affect the pressure vs. time plot.

INTRODUCTION

Well testing is one method of assessing reservoir properties. The behavior of the pressure vs. time curve is used to determine the value of kh and ϕch of the reservoir. Because the value of kh is generally much larger in a geothermal reservoir than an oil or gas field, the reservoir itself responds faster in the former case, and the transient behavior in the well itself does not die out before the reservoir starts to respond. Pressure transient curves derived in the petroleum literature assume that the changes in the well are relatively uniform. This situation is not necessarily true in a geothermal field. To fully analyze the well test results in a geothermal field, the transient nature of the flow in the well itself must be understood. While several numerical codes have been written to simulate two phase flow in the wellbore (Sugira, et al., 1979; Gould, 1974; Ryley, 1964; Juprasert and Sanyal, 1977), the ones reported in the geothermal field all assume steady state. The codes can be used to estimate the wellhead conditions after the well has been flowing for some time, but they are less useful in analyzing well test data. A steady state model naturally assumes that the mass into the well is equal to the mass out of the well, which is not true during the early testing of well or when the fluid temperature within the well is changing significantly. Also, it is not possible to use reservoir models which assume Darcy type flow to model the wellbore flow. The basic nature of the flow in the two cases is different when transients are important. For the fluid in the wellbore, the flow can be shown to be governed by a wave equation with damping, and in the reservoir the fluid flow is controlled by a diffusion-like equation.

A code to model one dimensional transient two phase flow in a well has been developed. It is coupled with a reservoir model of simple, one phase radial flow in a porous media. (Initially flashing only in the wellbore is being considered.) At early times, the flow in the reservoir is basically radial, so the model can be used to predict the drawdown pressure curve for single phase flow and when there is flashing in the wellbore. Some interesting results have been obtained using the model as will be illustrated at the end.

NUMERICAL MODEL

The basic problem is to solve the transient equations of mass, momementum, and energy for one-dimensional flow. For the initial development of the numerical method, two phase homogenous flow was modelled. The equations solved were:

$$\text{continuity,}\quad \frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho u) = 0 \qquad (1)$$

$$\text{momentum,}\quad \frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial x} + \frac{\partial P}{\partial x} + \rho g + \frac{1}{2}\frac{f\rho u^2}{D} = 0 \qquad (2)$$

$$\text{energy}\quad \frac{\partial \rho e}{\partial t} + \frac{\partial \rho u e}{\partial x} + P\frac{\partial u}{\partial x} + \frac{4\pi H(T_r - T_w)}{D} = 0 \qquad (3)$$

where ρ and e are the mass averaged values of density and energy respectively in the two phase region. The velocity of the gas and liquid are assumed to be equal, i.e. slip= 0, and furthermore it is assumed that thermodynamic equilibrium exists so that ρ can be written as a function of P and e. The extension of the model to include slip is straightforward (Miller, 1979a) assuming that the holdup and friction factors are known. Non-equilibrium flow can also be included but this case does require a second equation of mass. The frictional effects are expressed as the friction factor times 1/2 $\rho u^2/D$. Meaningful results can be obtained without going to a more elaborate description of the friction factor and slip.

The basic equations listed above were solved using a finite difference approximatin with a partially implicit method. Terms that would impose restrictive time steps if evaluated explicity are evaluated implicity while all other terms are evaluated explicitly for computational efficiency. In finite difference form, the equations become:

$$\frac{\rho_i^{\ell+1} - \rho_i^{\ell}}{\Delta t} + \frac{(\rho u)_{i+\frac{1}{2}}^{\ell+1} - (\rho u)_{i-\frac{1}{2}}^{\ell+1}}{\Delta x} = 0 \qquad (4)$$

$$\frac{(\rho u)_{i+\frac{1}{2}}^{\ell+1} - (\rho u)_{i+\frac{1}{2}}^{\ell}}{\Delta t} + \frac{[(\rho u^2)_{i+\frac{1}{2}} - (\rho u^2)_{i-\frac{1}{2}}]^{\ell}}{\Delta x} + \frac{P_{i+1}^{\ell+1} - P_i^{\ell+1}}{\Delta x} + \rho_{i+\frac{1}{2}}\, g - \frac{f(\rho u^2)_{i+\frac{1}{2}}^{\ell}}{2D} = 0 \qquad (5)$$

$$\frac{\rho_i(e_i^{\ell+1} - e_i^{\ell})}{\Delta t} + (\rho u)_i \frac{(e_i^{\ell} - e_{i-1}^{\ell})}{\Delta x} + P_i^{\ell+1}\frac{(u_{i+\frac{1}{2}} - u_{i-\frac{1}{2}})}{\Delta x} + \frac{4\pi H(T_r - T_w)}{D} = 0 \qquad (6)$$

Note that the velocity is not calculated at the same nodal point as the thermodynamic variables, i.e. at i+1/2 instead of at i.

The solution procedures involves combining the three equations above in addition to the equation of state, resulting in one equation for the new pressures, $P_i^{\ell+1}$. To facilitate this method, the equation of state is written in the form, $d\rho = (d\rho/dP)_e dP + (d\rho/de)_\rho de$ instead of $\rho = fn(P,e)$. The finite differenced form of this equation is

$$\rho_i^{\ell+1} - \rho_i^{\ell} = \left(\frac{\partial\rho}{\partial\rho}\right)_e (P_i^{\ell+1} - P_i^{\ell}) + \left(\frac{1}{\rho}\frac{\partial\rho}{\partial e}\right)\rho_i^{\ell}\,(e_i^{\ell+1} - e_i^{\ell}) \qquad (7)$$

Note that $\rho_i^{\ell}\,(e_i^{\ell+1} - e_i^{\ell})$ is given by Eq. 6, and the term $(1/\rho)\,(d\rho/de)$ varies linearly in the two phase region while $(d\rho/de)$ changes abruptly. Because the derivatives in Eq. 7 are evaluated explicitly, the new value of ρ calculated with Eq. 7 is compared with the value computed from $\rho = fn(P,e)$. If the difference between the two calculations is greater than a specified value, an iteration is necessary. In that case, the partial derivatives are averaged between the new and old values.

Equations 4-7 are combined. In the continuity equation, the expression for the new value of (ρu) is given by Eq. 5, and ρ is written in terms of pressure. The resulting expression for the new pressure is

$$\left[-2-a^2\left(\frac{\partial\rho}{\partial P}\right)_e\right] P_i^{\ell+1} + P_{i+1}^{\ell+1} + P_{i-1}^{\ell+1} = -a^2\left(\frac{\partial\rho}{\partial P}\right)_e P_i^{\ell} + a[\rho u_{i+\frac{1}{2}} - \rho u_{i-\frac{1}{2}}]^{\ell} - (\rho_{i+\frac{1}{2}} - \rho_{i-\frac{1}{2}})^{\ell} g\,\Delta x - \frac{f\Delta x}{2D}[\rho u^2_{i+\frac{1}{2}} - \rho u^2_{i-\frac{1}{2}}]^{\ell} + a^2\left(\frac{1}{\rho}\frac{\partial\rho}{\partial e}\right)\rho_i^{\ell}\,(e_i^{\ell+1} - e_i^{\ell}) - (\rho u^2_{i+\frac{1}{2}} - 2\rho u^2_{i-\frac{1}{2}} + \rho u^2_{i-3/2})^{\ell},$$

the difference $\rho_i^{\ell}\,(e_i^{\ell+1} - e_i^{\ell})$ given by Eq. 6. Equation 8 is a tri-diagonal matrix and the solution is straight forward if the boundary conditions are specified.

The boundary conditions considered were (1) specification of pressure and mass flow rate or velocity at either the wellhead or downhole, and (2) specification of the pressure at both wellhead and downhole. The pressure must be specified at one of the boundaries. The pressure at the second end is either known or is calculated from the momentum equation, i.e., given $\partial(\rho u)/\partial t$, the pressure at the boundary can be calculated using Eq. 5.

Once the new pressures are calculated, the new energy is determined with Eq. 6, the new density is given by Eq. 7, and the velocities are computed from either the continuity or momentum equation. If the mass flow rate is specified as a function of time, the velocity is calculated with the continuity equation. Given $(\rho u)_{i+1}^{\ell+1}$, the velocity at posiiton i is

$$u_i^{\ell+1} = [(\rho u)_{i+1}^{\ell+1} + \frac{\Delta x}{\Delta t}(\rho_i^{\ell+1} - \rho_i^{\ell})] \,/\, \rho_i^{\ell+1}$$

Because ρu is known at the wellhead, the velocities can be computed successively down the wellbore. If instead of knowing the mass or volume flowrate out of the well the pressure is given, the velocities are determined with the momentum equation.

The wellbore model was connected to a reservoir model that assumed single phase radial homogenous flow in the porous medium. The fluid was allowed to flow into the wellbore over a finite length. The intent was to mainly investigate the transient wellbore flow but the reservoir flow was included so that the drawdown pressure in the well would be consistent with the amount of fluid that flowed from the reservoir into the well. The reservoir flow equation,

$$\frac{\partial P}{\partial t} = \frac{k}{\mu c\phi}\,\frac{1}{r}\,\frac{\partial}{\partial r}\, r\, \frac{\partial P}{\partial r} \qquad (9)$$

was solved on a variable grid. The mass flow from the reservoir was matched with the mass flow into the well.

The temperature change around the wellbore was solved in a similar manner, i.e., the temperature change is calculated using the conduction equation which is similar in form to Eq. 9. A variable grid system was also used in this case.

Given the initial conditions in the well and in the reservoir, and the boundary conditions, the numerical model solves for the transient behavior in the wellbore. The basic nature of the interaction of the wellflow and reservoir flow can be understood.

EXAMPLE CALCULATIONS

The numerical model has been used to determine the early time behavior of the wellbore flow for both single phase and two phase flow. Examples of the calculations are given below. Figure 1 is a plot of the pressure changes that propagate down the wellbore after a stepwise change in flowrate at the wellhead. In this figure, the calculations were done for a liquid-filled well flowing under a positive head. The well is flowing steadily at one rate and then the flowrate is increased. At early times after the flowrate change, the increase in produced fluid is removed from wellbore storage instead of from the reservoir. A pressure drop propagates down the well. After a certain amount of time, depending on the compressibility of the fluid, the pressure pulse interacts with the formation/well boundary. In the particular case plotted, the reservoir has a large value of kh/μ, and it is capable of supplying more fluid for this pressure drop than the well could. The case results in a reverse pressure pulse which propagates back up the well, cancelling part of the initial pressure drop. The pressure pulse oscillates until it is finally damped out by the interaction with the boundaries.

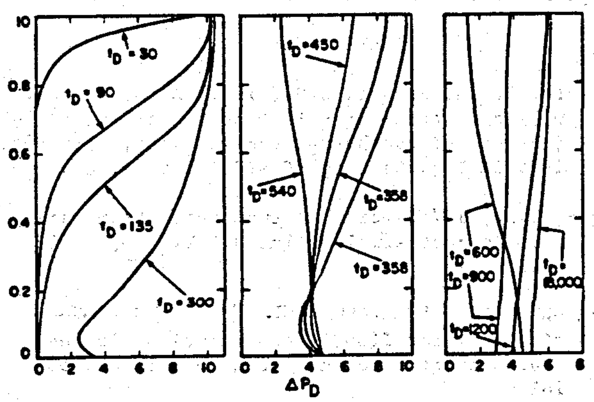

Figure 1. Pressure pulse propagating down the well for a stepwise flowrate change and for a single phase fluid.

Figure 2 shows the same calculations for a flashed system. Again, the fluid is slowly flowing and then the flowrate is suddenly increased. The pressure pulse propagates down the well. However in this case, there is a brine/two phase boundary. The dotted line in the figure gives the approximate location of the flash point. (Obviously as the flowrate is increased, the flash level drops.) When the pressure pulse reaches this boundary, it is partly reflected and partly transmitted. The reflected pulse propagates back toward the surface. In the single phase region, the propagation of the signal is much faster. The oscillations are mainly in the two phase region.

One can use the program to determine the pressure drawdown during the early time of a well test. It has been shown (Miller, 1979b) that the initial slope of a log log plot of pressure versus time in well testing is not necessarily unity as derived in the petroleum literature. As seen in the figures above there is a time delay until the downhole pressure registers the change made at the wellhead. Wellbore storage curves are derived assuming the

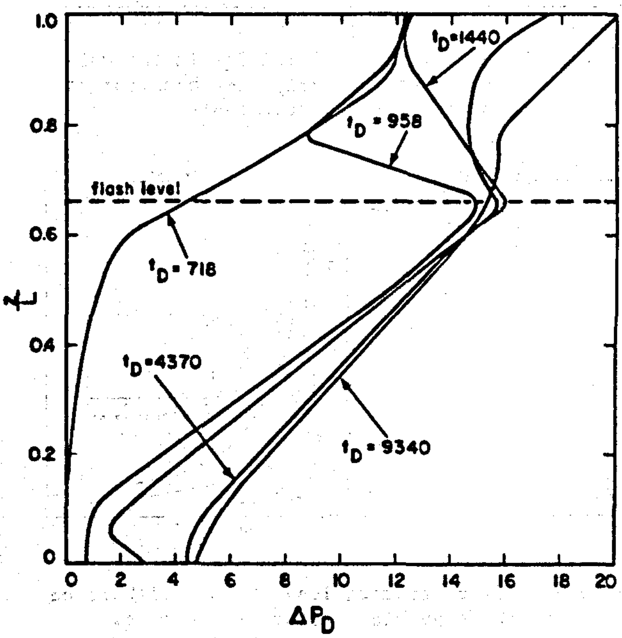

Figure 2. Pressure pulse propagating down the well for a stepwise flowrate change and for a flash level at about z/L = 0.65.

fluid in the well responds as a well mixed fluid. By being able to model the transient flow in the wellbore, it has been possible to calculate the expected drawdown in the well taking into account the non-uniformities in the well. The results show that another non-dimensional time t_{RW} must also be determined as well as the average wellbore storage coefficient C_D. The plot shows calculations for flashed and unflashed wells. The parameter t_{RW} of defined as

$$(\frac{\mu}{kh}) \frac{D^2}{8} \frac{1}{\rho} (\frac{\partial \rho}{\partial P})$$

As kh/μ decreases, t_{RW} increases and the early time behavior of the log P vs. log t approaches a one to one plot. As kh/μ increases, t_{RW} decreases and the slope of the log P vs. log t curve is steeper than unity.

The numerical model can also be used to determine the effect of heat loss to the rock surrounding the wellbore during a well test. The calculations shown are done for a well that has been flowing and is reasonably "warm." The assumed temperature profile is given by the insert in Figure 4.

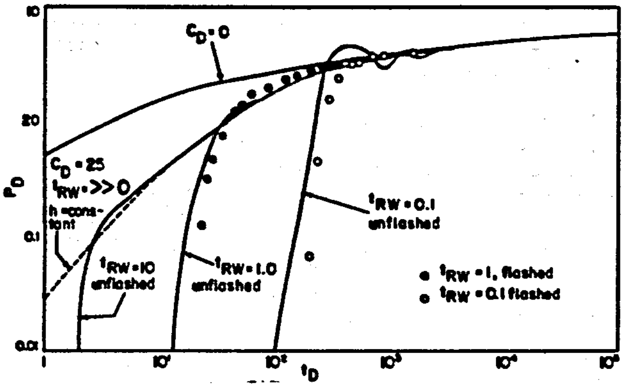

Figure 3. Effect of non-uniformities in the well on the expected early time behavior of downhole pressure transients.

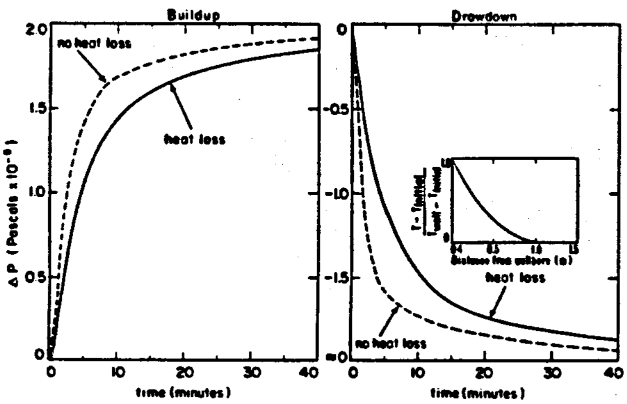

Figure 4. Effect of heat loss in the wellbore on the downhole pressure transients.

The well has been flowing steadily. Then the flow-rate is decreased. The figure compares the buildup curve and subsequent drawdown curve with and without heat transfer. It is seen by the calculations that even when the well has been flowing for several hours and the rock arround the bore has been heated, heat transfer during a well test is still important and must be considered. When the well test data is plotted, the slope of the P vs. log t curve in the pseudosteady region is significantly affected by the heat transfer. Also the time to reach the pseudosteady region is longer when heat transfer is important.

CONCLUSION

To be able to analyze well test data in a geothermal field, a transient wellbore model is necessary. The developed model is capable of handling two phase transient flow in a wellbore. The basic solution procedure is fundamentally different from the many steady state models reported because of the inclusion of the transient terms. The steady state solution is just a limiting case of the transient flow. Example calculations show that the non-uniformities in pressure in the wellbore can result in different early time behavior of the pressure vs. time curve than described by a lumped model. The unit slope on the log log scale is just a special case when kh/μ is small as in an oil or gas field. Also heat transfer alters the slope of the curve in the pseudosteady region so the slope of P vs. log t is no longer $q\mu/4\pi kh$.

NOMENCLATURE

a	$\Delta x/\Delta t$
c	reservoir compressibility
C_D	wellbore storage coefficient
D	well diameter
e	specific energy
f	friction factor
g	gravity
H	heat transfer coefficient
kh	permeability-thickness
P	pressure
P_D	non dimentional pressure
r	radial direction
t	time
t_D	non dimensional time $(4k/\phi\mu cD^2)t$
t_{RW}	ratio of reservoir response to well response
T_r	reservoir temperature
T_w	temperature of fluid in well
u	velocity
x	axial direction
z	height above reservoir
μ	absolute viscosity
ϕ	porosity
ρ	density

ACKNOWLEDGEMENTS

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