Title

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¿From Abstract to Concrete?
Evidence for designing learning platforms that adapt to user's proficiencies.

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Abstract

Digital-tablets distribute cognition through visual, auditory and haptic interactivity. We designed a tutor-game that explored how narratives ((S)trong/(W)eak) and gestures ((I)conic/(D)eictic) could be combined to situate embodied learning. Students played seven levels of a fractions game designed to teach them how to create and compare fractions. One hundred thirty-one students (N=131, age $\bar{x}$=8.78 yrs, 52.6% Female) were randomly assigned to one of four groups (SI, SD, WI, WD) in a 2x2 factorial experiment. Students completed pre/post direct and transfer assessments and tutor-game log data was mined to explore characteristics of students learning. Results revealed a significant interaction between narrative and gesture moderated by student proficiency. In effect, students new to fractions performed better in an abstract environment using deictic (pointing) gestures. However, as students' proficiencies improved, they learned better using iconically enactive gestures in strong narrative with setting, characters and a plot. This has important implications for designing adaptive learning platforms and curricula for teaching fractions.

Keywords: embodied, situated, grounded cognition; narrative, gestures; design-based research; DBR; data-mining; adaptive learning.

Introduction

Tutor-games provide learners with dynamic experiences that channel their visual (sight), aural (sound) and haptic (touch) perceptions into their cognitions (Baddeley, 1986; Ricker, AuBuschon & Cowan, 2010). As virtual portals, digital tablets allow educators to situate learning in various contexts that scaffold the processes that connect concepts (Barab et al., 2007; Brown, Collins & Duguid, 1989; Saxe, 1988; Lave, 1988; Schwartz & Bransford, 1999). The touch-based gestural interface of digital tablets accesses the haptic channel as a means for *embodying* concepts (Varela, Thompson, & Rosch, 1990; Barsalou, 1999; Glenberg & Kaschak, 2002; Lakoff & Johnson, 1980). The multi-modal ecology of digital tablets allows researchers to scaffold experiences that afford (Gibson, 1979) students freedom to explore with feedback that guides their learning (Dewey, 1938/1963).

Theoretical Background

Developing Narrative. Developing an effective narrative invests the audience in the continuity of the characters, locations, objects, actions and themes and invests them into the plot's trajectory (Graesser, Singer & Trabasso, 1994). These details (microstructure) are the access points to a larger interactive narrative (macrostructure) that situates the concepts (van Dijk & Kintsch, 1983). Thus, designers must create assets that engage players in problem spaces through the processes that foster correct mental model constructions (Johnson-Laird, 1980). Black and Bower (1980) found that the structure of stories, with actors, settings, problems and solutions, aided in participants inference making and recall. In effect, the coherence of narrative schemas helps participants chunk details into mental models (Black, Turner & Bower, 1979) and ideally, the audiences' investment in the narrative can motivate player's explorations of the processes for creating and comparing fractions in a problem space conducive for discovery (Brown, Collins & Duguid, 1989).

Developing Gestural Mechanics. Goldin-Meadow, Cook and Mitchell (2009) demonstrated that a pairing gesture (i.e., two fingers to identify two numbers as a pairing) facilitated elementary students strategies for arithmetic problems and demonstrates how gestures as abstractions are still rooted in relation to the body. In the cognitive science literature, gestures have been typically defined as spontaneous co-articulations with speech (Kendon, 1972; McNeill, 1992), but in the digital age, the physicality of gestures has been co-opted into gestural mechanics as an interface with touch and motion based digital technologies. Educators can leverage the mechanics of gestures as communications of concepts and strategies by simulating perceptual states to activate learners' understandings (Goldin-Meadow, 1999).

Exploratory studies (Swart et al., 2014) revealed types of gestures learners used when explaining fractions. Echoing Hostetter's and Alibali's (2008) *Gestures as Simulated Action*, students majoritively used either *iconic gestures (I)* (metaphorical, enactive, symbolic) that enact their understandings or *deictic (D) gestures* (pointing) that identify them (**Fig. 1**).

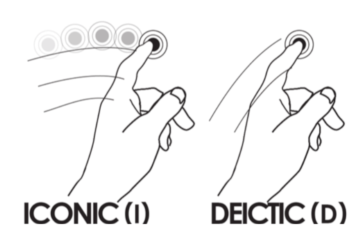

Fig. 1: *Iconic* & *Deictic* Gestures

The Tutor-Game: Mobile Movement Mathematics (M3).

The human ability to think mathematically manifests from the endowments of our perceptual systems. It includes our abilities to estimate the magnitudes of spaces and durations of time as well as enumerate objects by differentiating the intensities of stimuli in our surroundings (Dehaene, 1997). These experiences ground the embodied metaphors of mathematical thinking (Lakoff & Núñez, 2001; see **Fig 2**) and we recognize that fractions originate in the processes of fracturing wholes into parts. Thus, we chose to use *object* fracturing as the metaphor for developing a situatively embodied curriculum.

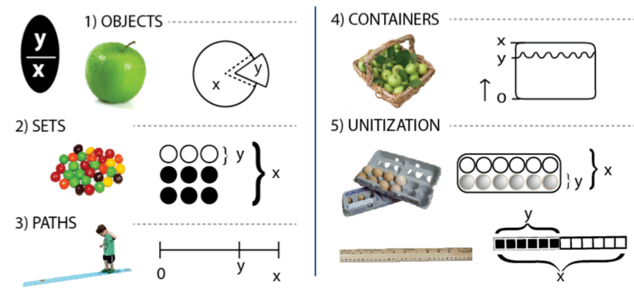

Fig. 2: Embodied Experiences of Mathematical Fractions

The tutor-game consisted of 7 levels of 5 fractions that were situated in either a *strong (S)* or *weak (W) narrative.* The strong narrative had a setting, characters and plot based on the PBS series *Cyberchase,* and was compared to a weak, non-descript environment without narrative elements (see Fig. 3). We characterized it as "weak" in lieu of "no" narrative to account for researchers inability to control for any internal narratives that students might devise.

Fig. 3. *Strong Narrative* (L) & Weak Narrative (R)

To play, students used either iconic or deictic gestures in a 2-part tutor-game: [Part 1] Players estimated, denominated, numerated and re-estimated using the fractivator (a hybrid of a rectangular area model and a number line (Siegler & Opfer, 2003); [Part 2] Players determined equivalency between fractions by ordering them, magnifying their height and delineating each onto a vertical number line (**Fig. 4**).

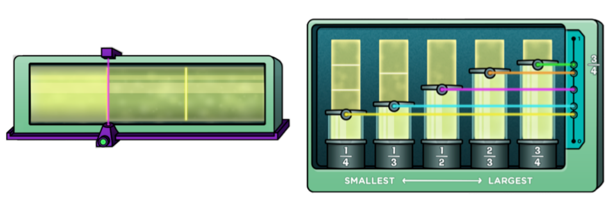

Fig. 4. Part 1 (Obj. Fracturing) & Part 2: Obj. Equivalency

The Experiment

In order to isolate for the impact of gesture (I vs. D) and narrative (S vs. W) on learning, we devised, designed and developed 4 versions of the digital tablet tutor-game (M3) that resulted in the following experimental conditions: *SI*, *SD*, *WI*, and *WD,* and all other factors (curriculum, assets, instructions, feedback, and scaffolding) were held constant.

Under the *gesture hypothesis* (embodied), iconic gestures with richer perceptual affordances (Black, Segal, Vitale and Fadjo, 2012) should help learners embody mathematical concepts better than deictic gestures. We predicted that *iconic* gestures, by grounding concepts in real-world actions, connect internal processes of our cognition and affect better than *deictic* gestures.

For the *narrative hypothesis* (situated), contextualizing problem spaces (via setting, characters and plot) helps learners engage in the construction of their own conceptual models. By situating learning, we predicted that the *strong* (S) *narrative* will produce higher levels of engagement and motivation and higher levels of learning compared to a *weak* (W) *narrative*.

The third hypothesis arises from the interplay of design and how independent factors will interact. The *interaction hypothesis* suggests that combinations of narrative types (S vs. W) and gesture types (I vs. D) will create learning environments that vary in their efficiency for the learner. In favor of the situated and embodied condition, we predicted that the SI condition would perform better than SD and or WI conditions, while the WD condition would perform better than SD and or WI.

The fourth hypothesis stems from our classroom observations of students' play and the prospect for differential efficiencies between SI and WD. The *proficiency hypothesis* suggests that learners' existing proficiencies at fractions will moderate how they play and learn. In favor of the situated and embodied condition, we predicted that students with lower proficiencies would benefit more from the situated embodied experience of the SI condition while students with higher proficiencies would benefit from the abstractions of the WI condition.

Methods

Participants. One hundred thirty-one participants from grades 3 (N=131; $\bar{x}_{age}$=8.78 years [1.36], 52.6% female) at afterschool programs in New York City obtained parental consent to participate in the program.

Procedure. Researchers formally tested a total of 131 students in specially designated classrooms where researchers and monitors proctored over the sessions, administered assessments, collected observational and video record the sessions. In a 2x2 randomized factorial, students were assigned to play one of four game-based environments (**S**trong-**I**conic (SI, n_{si} = 35), **S**trong-**D**eictic (SD, n_{sd} = 27), **W**eak-**I**conic (WI, n_{wi} = 34), **W**eak-**D**iectic (WD, n_{wd} = 35)). Each student completed 3 one-hour sessions that in total included pre-tests, game play, post-tests and exit-

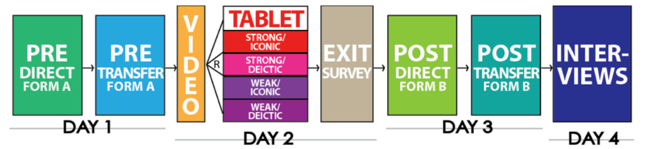

Fig. 5. 2x2 Randomize Factorial w Repeated Measures

surveys (*see* **Fig. 5**). Students' sessions were run in separate groups of 10 (5/condition) with a total of 2 sessions per day (total of 20 students/day; 5 per condition) for 3 days each week, extended over multiple weeks and some students participated in an optional 4th-day clinical interview.

Materials

Assessments.

Direct Pre/Post Test: Parallel Forms A & B of fraction problems directly from the game curriculum. Representations of fractions were similar to static versions of what students saw in the game, including estimation, denomination, numeration and determining equivalency between fractions (40 items).

General Pre/Post Test: Parallel Forms C & D of general fraction assessment that included problems using objects, collections of objects, number lines, numerical fractions, arithmetic, and word problems. Questions included items asking students to estimate, denominate, numerate and determine equivalency between fractions (43 items).

M3: Digital Tablet Tutor-Game.

Log Data: The backend of the game was designed to deliver user log data (i.e., telemetry data) to helps researchers create profiles of students' learning by tracking players' *time*, *accuracy/error*, *attempts* and *strategies* during tutor-game play.

Equipment.

iPad Air & Sony MDR-ZX100 Headphones: A class set of 10 each; *Flip Video UltraHD Camcorder:* 2 camcorders w/ Tripods for Video.

Results

Formal Assessments

Direct Assessments. ANOVA revealed a significant interaction between gesture and narrative on Direct Assessment Total Difference scores (post - pre), $F_{(1,126)}$ = 7.324, p < .008, d = .482, (1- β) = .766 (Figure 59). The significance of this interaction supports the both the narrative and gesture hypotheses that each can impact learning. Since the interaction is significant, the main effects of gesture or narrative are unclear. However, **Fig. 6** clearly depicts the interaction and illustrates how students in the SI and WD groups show significantly higher rates of learning across amongst all the M3 groups.

T-tests for independence revealed differences between conditions for Direct Assessment Total Difference scores, with students in the SI group ($\bar{x}_{pre}$ = .208, SD = 0.143) scoring higher than students in the SD group ($\bar{x}_D$ = .143, SD

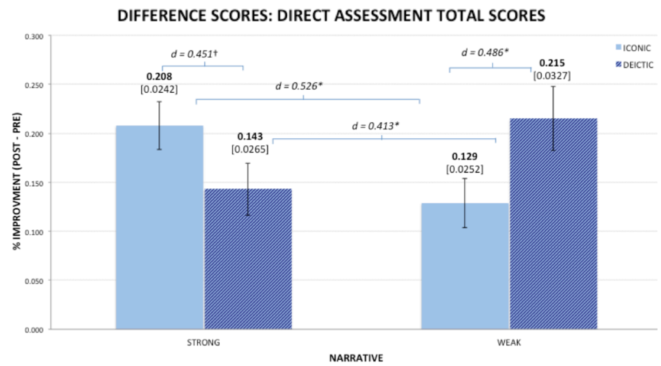

Fig. 6. ANOVA revealed a significant interaction between *gesture* and *narrative* on *Direct Assess total scores.*

= 0.138), $t_{(60)}$ = 1.79, p < .079, d = .451 and significantly higher than students in the WI group ($\bar{x}_D$ = .129, SD = 0.147), $t_{(67)}$ = 2.25, p < .028, d = .526) while the WD group ($\bar{x}_D$ = .215, SD = 0.194) scored higher than SD, $t_{(60)}$ = 1.79, p < .107 and significantly higher than WI, $t_{(67)}$ = 2.069, p < .041, d =.486.

Preliminarily, this suggests that the strong narrative combined with iconic gestures as well as the deictic gestures combined with weak narrative both provide a learning experience significantly more efficient than either the strong-deictic or weak-iconic pairings.

Transfer Assessment. ANOVA revealed no significant main effects of gesture or interaction between gesture and narrative for Transfer Assessment Numeration Difference scores $F_{(1,128)}$ = 1.70, p < .195, d = .229, (1- β) = .254. Though t-tests for independence of the difference scores (post – pre) were not significant between groups, the pattern

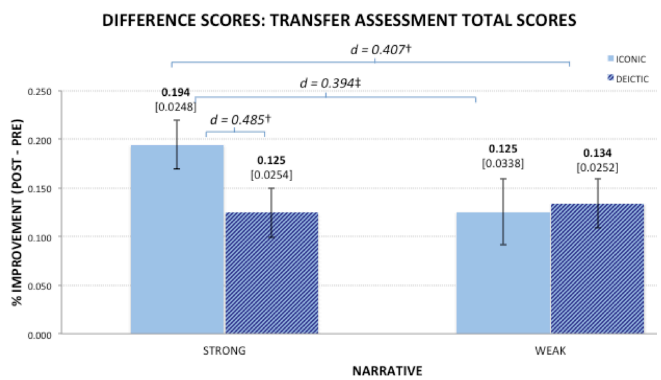

Fig. 7. ANOVA revealed a significant interaction between *gesture* and *narrative* on *Direct Assess total score.s*

of results in **Fig. 7** show that students in the *SI* group ($\bar{x}_{pre}$ = .147, SD = .192) scored higher than students in *SD* ($\bar{x}_D$ = .084, SD = 0.180), $t_{(60)}$ = 1.296, *p < .20, d = .330*, higher than students in *WI* ($\bar{x}_D$ = 0.07, SD = 0.246), $t_{(67)}$ = 1.272, *p < .305, d = .394*, and higher than students in the *WD* condition ($\bar{x}_D$ = .061, SD = 0.194), $t_{(68)}$ = 1.857, *p < .068, d = .443*.

A one-way contrast showed that the *SI* group performed significantly better than the other three groups t (127) = 1.763, SE = .122, p < .080. Unlike the direct assessment interaction, results from the transfer assessment suggested that the situated and embodied condition (SI) contributed to better transfer. Simply, enacting the processes of fracturing objects while situated in a narratively contextualized problem space seems to contribute to better transfer.

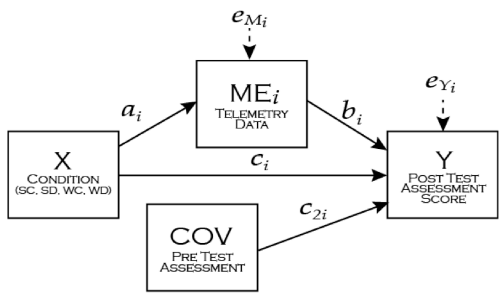

Fig. 8. HLR model regressing PreTest, Telemetry Data and Condition on PostTest scores. Direct Effect of X on Y; Indirect Effect of X on Y *via MEi* = $(a_i)(b_i)$; COV on Y = c_i

Tutor-game Log Data

Mediation with a Covariate Models. The next series of analyses looked principally at how condition and tutor-game play account for the variance in students' post-test scores while controlling for pre-test scores. **Fig. 8** depicts the conceptual path model used for the stepwise construction of the Hierarchical Linear Regressions (HLR) predicting the variance in the assessment scores.

The path model depicts how the variance in dependent variable (Y, post-test assessment score) is accounted for by the independent variable (X, condition – SI, SD, WI, WD), while controlling for a covariate (COV, pre-test assessment score) and mediated by students' tutor-game play (ME, telemetry data).

Direct Assessment Total Post-Test. The first HLR regresses condition, pre-test scores and tutor-game play on direct assessment total scores. The complete meditational covariate model significantly predicted the outcome of students *Direct Assessment Post-Test* scores $R = .645$, $F_{(7, 4577)} = 543.80$, $p < .001$. With the covariance of pre-test controlled, tutor-game play predicted a significant amount of the variance in *Direct Post-Test Assessment* scores (B = .623, SEB = 0.012, β = .607, $p < .001$, 95% CI [.599, .646]).

Transfer Assessment: Total Score. The complete model significantly predicted the outcome of students *Transfer Assessment Total Post-Test* scores $R = .632$, $F_{(8, 4576)} =$ 379.80, $p < .001$. With the covariance of pre-test controlled, tutor-game play predicted a significant amount of the variance in direct post-test assessment scores R = .626, $F_{(3,4580)} = 35.47$, p < .001..

Moderated Mediation Models. With solid evidence that both the SI and WD conditions were efficient environments for learning, it was important to clarify the nature of the interaction between narrative and gesture and determine if the situated embodied approach (SI) was better for low proficiency students (i.e., early learning is situationally embodied) or those with higher proficiencies. The second path model determines if students' initial proficiencies (MO_i, pre-test score) moderated how students played (ME_i, telemetry data) and improved on formal assessments (Y_i).

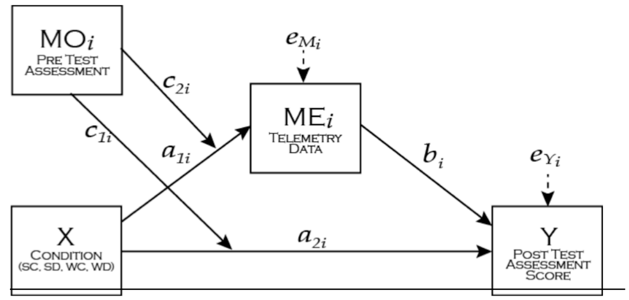

Fig. 9. HLR model of PreTest, Telemetry Data and Condition on PostTest scores. Direct Effect of X on Y; Indirect Effect of X on Y *via* $ME_i = (a_i)(b_i)$; MO_i on X → Y = c_i and ME_i → Y

In **Fig. 10**, we can see that there are two distinct slopes for the *SI* ($R^2 = .474$) and *WD* ($R^2 = .183$) conditions, indicating two distinct trajectories of improvement from pretest (x-axis) to post-test (y-axis) scores. The dashed red boxes indicate the median split between *low* and *high initial proficiencies*. Visual inspection suggests that the WD group shows better learning when their initial proficiencies are lower while the SI group seems to show better learning when their initial proficiencies are higher.

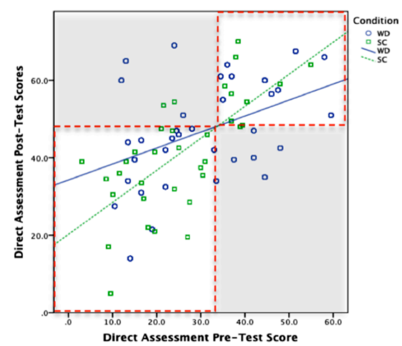

Fig. 10. Scatterplot of Pre-Test (X axis) and Post-Test (Y Axis) scores by groups (SI; WD).

The moderated meditational model of the proficiency hypothesis confirmed that student performances in the game on formal assessments were significantly moderated by their existing proficiencies with fractions. **Fig. 11 (top)** shows the moderated mediation of direct assessment scores by condition and proficiency $R = .630$, MSE = 122.36, $F_{(5, 2444)} = 353.72$, $p < .0001$. Students with lowest proficiencies (10th percentile (x_{pre} =11.50; B = -9.32, SE_B = .832, $t_{(2443)} = -11.20$, $p < .0001$, 95% CI [-10.95, -7.68]), benefitted the most if they were in the WD condition (β < 0) condition compared to the SI (β > 0), but as proficiency improved, students began to benefit more in the SI condition (90th percentile (x_{pre}=46.00; B = 5.29, SE_B = .645, $t_{(2443)} = 8.21$, $p < .0001$, 95% CI [4.03, 6.56]). We see a similar transition for low to high proficiencies from WD to SI for the transfer assessment (see **Fig. 11**, **bottom**). In this case, the transition from the WD to the SI condition takes place at lower initial proficiencies for transfer of learning.

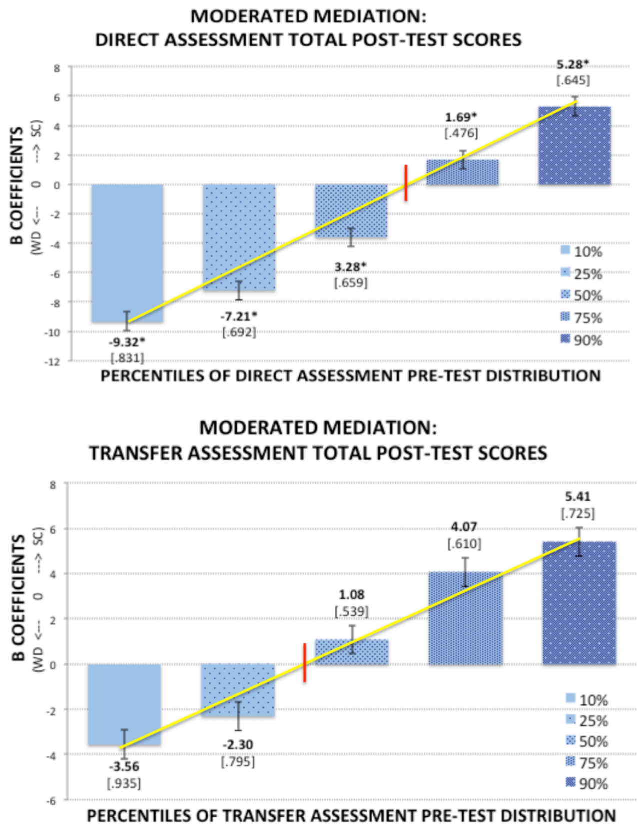

Fig. 11a & 11b. Moderated Mediation of formative assessment scores by the interaction between condition and existing fractions proficiency. Scores on the pre-test are stratified by percentiles along the x-axis (10th, 25th, 50th, 75th, 90th %), and values on the y-axis are the weights of the B coefficients for changes in *Direct Assessment Post-Test scores.* Coefficient values below the zero line on the y-axis indicate that the WD improved more on post-test at that percentile and coefficient values above the zero line indicate that students in the SI group improved more.

Discussion

The Gesture, Narrative & Interactions Hypotheses. The significant interaction between gesture and narrative on the direct assessment of the M3 curriculum shows that types of gestures may be conceptualized differently depending on the contexts in which they are embedded. It calls into question our original theoretical assumptions that situating cognition through narrative and embodying procedural learning through iconic gestures would produce better learning.

The HLRs on students direct and transfer assessment total scores showed that students tutor-game play, including their accuracy *denominating, numerating and estimating* significantly predicted learning, supporting the position that the act of splitting objects is central to learning fractions (Steffe, 2004; Norton & Wilkins, 2009). Improvement on transfer assessment seems to suggest that the procedural and conceptual knowledge that players are developing is robust enough that the curriculum prepared them for future learning (Schwartz & Bransford, 1998) of near transfer representations and new domains for fraction.

The Efficiency Principle. Although our initial hypotheses predicted the superlative performances by the SI conditions for both assessments, the significant interaction between gesture and narrative suggests that both the SI and WD conditions are both efficient platforms for learning. Schwartz, Bransford and Sears (2005) note that efficiency often means rapid retrieval with accurate appropriation and application of knowledge and skills for understanding, solving and explaining a problem. Though the situated embodied SI environment provided a perceptually rich experience (Black et al., 2012) that promoted better transfer, students using deictic gestures in the weak narrative (i.e., without seductive details, Harp & Mayer, 1998; Adams et al., 2012) also showed significantly better learning. Might the minimal and abstracted environment of the WD condition make procedures and concepts easily salient?

The Proficiency Principle. Students with low initial proficiencies benefitted more from playing in the WD version of the game, while students with higher initial proficiencies benefitted more in the SI environment. This finding was contrary to our hypothesis and the principle of *concreteness fading* (i.e., start concrete and fade to abstract; Fyfe, McNeil, Son & Goldstone, 2014). Still to be determined is how these results fit with *The Expertise Reversal Effect* (i.e., experts require reduced guidance; Sweller, Ayres, Kalyuga, & Chandler, 2003). Does the presence of the strong narrative make instruction and guidance invasive (i.e., reduced)? Nonetheless, the current results support findings from a study by Kaminski, Sloutsky and Heckler (2006; 2008) that found that students learned division with remainders better using abstract symbols rather than concrete real world depictions.

Significance

The current research demonstrated that combinations of different narratives and gestures produced differential learning. Ribbons and Malliet (2010) advocate for *simulational realism* in gaming. They argue that there must be balance between the rules that govern gaming experiences (e.g., gestures) and their relevance to the situated environment (e.g., the interactive narrative). This research suggests that when educators are designing pedagogy and curricula for mathematical fractions, students should begin working with abstractions and as their proficiency improves the learning platform should adapt to concrete experiences.

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