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Essays on Contract Theory and Industrial Organization

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Zhuoran Lu

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Abstract of the Dissertation

Essays on Contract Theory and Industrial Organization

by

Zhuoran Lu

Doctor of Philosophy in Economics

University of California, Los Angeles, 2018

Professor Simon Adrian Board, Co-Chair

Professor Moritz Meyer-ter-Vehn, Co-Chair

This dissertation consists of three essays on contract theory and industrial organization.

The first chapter studies a signaling model in which a strategic player determines the cost structure of signaling. A principal chooses a price schedule for a product, and an agent with a hidden type chooses how much to purchase as a signal to the market. When the market observes the price schedule, the principal charges monopoly prices, and the agent purchases less than the first-best. In contrast, when the market does not observe the price schedule, the principal charges lower prices, and the agent purchases more than in the observed case; those of the highest types purchase more than the first-best. In terms of payoffs, the principal gains lower profits, whereas the agent obtains higher utility than in the observed case. When the intensity of signaling activity is sufficiently high, the observed case yields higher social welfare than the unobserved case. The model can be applied to schools choosing tuition, retailers selling luxury goods and media companies selling advertising messages.

The second chapter studies nonlinear pricing for horizontally differentiated products that provide signaling values to consumers with private information, who choose how much to purchase as a signal to the receivers. I characterize the optimal symmetric price schedules under different market structures. Under monopoly, when the receivers observe the price schedule, the market is partially covered, and quantity is downward distorted if there is

little horizontal differentiation. As the degree of horizontal differentiation rises, the market coverage rises, and the downward distortion decreases. When the degree is sufficiently high, for a certain level of signaling intensity, the monopolistic allocation achieves the first-best; for higher signaling intensities, quantity is upward distorted at the low end. In contrast, when the receivers do not observe the price schedule, the market is always partially covered, and the allocation is more dispersed than that in the observed case. Specifically, higher types purchase more than in the observed case, with the highest types purchasing more than the first-best, whereas lower types purchase less than in the observed case, with more types excluded from the market. When the market structure changes from monopoly to duopoly, market competition results in a higher market coverage and larger quantities for both the observed and unobserved case.

The third chapter analyzes a principal-agent model to study how the architecture of peer monitoring affects the optimal sequence for teamwork. The agents work on a joint project, each responsible for an individual task. The principal determines the sequence of executing tasks as well as the rewards upon success of the project, the probability of which depends on each agent's effort and ability, with the objective of inducing full effort with minimum rewards. Agents may observe one another's effort based on an exogenous network and the endogenous sequence. We focus on networks composed of stars, and find a simple algorithm to characterize the optimal sequence of task assignment. The optimal sequence reflects the trade-off between the magnitude and the coverage of reward reduction in incentive design. In a single star, less capable periphery agents precede their center while more capable ones succeed their center. In complex networks consisting of multiple stars, periphery agents precede their center early in the sequence but succeed their center late in the sequence. When the number of peripheries differ across stars, a "V-shape" emerges: agents in large stars are allocated towards both ends of the sequence, while those in small ones towards the middle.

The dissertation of Zhuoran Lu is approved.

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2018

To my parents and my wife

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CHAPTER 1

Monopolistic Nonlinear Pricing for Signals

1.1 Introduction

In classic signaling models, the sender's preference depends only on his intrinsic type. This paper investigates situations in which the signaling cost also depends on the choice made by a third-party strategic player. For example, when a student obtains education to signal his ability, the university sets the tuition; when a consumer purchases a luxury good to signal his wealth, the retailer chooses the price; when a seller incurs advertising expenses to signal a product's quality, the media company determines the cost of advertising messages. A key observation is that since signaling cost is endogenous, how receivers interpret and respond to the sender's signal will depend on whether they observe the third party's choice.

In this paper, I derive the optimal price schedule for a principal selling a product to an agent who is endowed with a hidden type and chooses how much to purchase as a signal to the market. The equilibrium depends critically on whether the market observes the price schedule. When the market observes the price schedule, the principal internalizes the agent's signaling incentive when screening the agent, leading to a downward distortion in quantity. In contrast, when the market does not observe the price schedule, the agent is more sensitive to price changes, since the market will attribute a difference in quantity to agent preference heterogeneity. This provides the principal with an incentive to lower prices. In equilibrium, the agent chooses a higher quantity and obtains higher utility than in the observed case, whereas the principal gains lower profits than in the observed case.

This paper has meaningful implications for the price transparency of goods that provide signaling values to consumers. In the case of job market signaling, my model suggests that education is more costly and students are worse off when the tuition scheme (more precisely, the net prices for school) is observed by employers than otherwise. This implies that policies that improve the transparency of the net prices at colleges and universities, e.g., U.S. Code §1015a,[1] may *unintentionally* raise education expenses and harm students. This is because these policies allow schools to commit to high prices and not dilute the signaling value of a high-cost education by means of fee waivers or financial aid.

In addition, my model implies that a signaling good will yield higher profits if the seller can make the price publicly observed and commit to it. This echoes some real-world business practices. For example, luxury brands, such as Louis Vuitton, Tiffany and Hermes, enjoy a reputation of never or very rarely being on sale. This helps the sellers better commit to high prices, thereby reinforcing the signaling values of luxury goods. In the advertising industry, the high costs of each year's Super Bowl commercials are widely reported, thereby enhancing the signaling value of these costly commercials; in China, the TV station CCTV even broadcasts the auctions for some of its popular TV show commercials to accentuate their signaling values.

For the purpose of exposition, I present my model in conformity with the seminal work of Spence (1973) with productive education. I extend that model by adding a pre-signaling stage in which a school (*principal*) sets its tuition scheme and a worker (*agent*) chooses his education level to signal his privately known ability (*type*) to competing employers (*market*). In Section 1.3, as a reference point, I briefly revisit Spence's model by fixing tuition at zero. This is the case when schools are competitive and set the price equal to the marginal cost. In the least-cost separating equilibrium, all types except the lowest one choose more education than the first-best, as they attempt to separate themselves from lower types.

In Section 1.4, I introduce the school and study the case in which employers observe the

[1]Since 2011, American colleges and universities have been required to provide reasonable estimates of the net prices, including tuition, miscellaneous fees and personal expenses, that students will pay for school. See "U.S. Code §1015a - Transparency in college tuition for consumers" for details.

tuition scheme. In the school-optimal separating equilibrium (which is also the least-cost separating equilibrium), all types except the highest one choose less education than the first-best. This result is in contrast to that of Spence's model. The downward distortion is due to the schools screening. Having a cost advantage in education, a higher type can secure higher utility than a lower type by choosing the same education level. To incentivize truth-telling, the school must leave information rents to the worker, meaning that the marginal profit is less than the social surplus. Thus, the school under-supplies education.

While this mechanism is similar to screening models such as Mussa and Rosen (1978), my model also incorporates signaling, which can mitigate the downward distortion caused by screening. To illustrate, suppose that employers can observe the worker's ability, thereby eliminating signaling. When a higher type imitates a lower type, he not only incurs a lower total cost than the latter but also obtains a higher wage due to his higher ability. The second effect means that the worker can extract more information rents from the school; thus, the screening distortion is worse compared to when signaling is present.

In Section 1.5, I turn to the case in which employers do not observe the tuition scheme. In the school-optimal separating equilibrium (which is also the least-cost separating equilibrium), the school sets lower tuition rates and the worker chooses more education than when employers observe the tuition scheme. This difference is driven by a *signal jamming effect*. Because employers cannot observe the actual cost of education, they do not know whether a difference in education level is caused by a tuition change or worker cost heterogeneity. For example, suppose that the school lowers tuition so that the worker obtains more education than in the initial state. When employers observe the tuition scheme, they cut wages, since any education level now corresponds to a lower-ability worker. This dampens the worker's demand for additional education. In contrast, when employers do not observe the tuition scheme, they do not adjust wages despite that tuition changes. Consequently, the demand for education is more elastic, making the price cut relatively more profitable for the school. In equilibrium, employers correctly anticipate the schools incentive to cut tuition and offer lower wages, as education is inflated. This reduces the worker's willingness to pay, and thus, the school achieves lower profits when employers do not observe the tuition scheme.

Since the school is worse off when employers do not observe the tuition scheme, one may wonder why the school does not disclose tuition to employers. The reason is that the school cannot credibly announce the price absent intervention such as mandatory disclosure. Note that once employers believe the school's announcement, the latter would secretly cut prices to make a profitable deviation. Such an observation may explain the fact that while the listed tuition at American colleges and universities is rising, these schools offer students various and inclusive forms of financial aid.[2] The rationale is that employers cannot easily observe the details of such financial aid and thus do not know the actual cost of education. By raising the published tuition while simultaneously reducing the undisclosed net prices through stipends, schools persuade employers that their students are smarter than is actually the case, thereby allowing the schools to collect higher revenues from students.

Finally, in Section 1.6, I discuss the application and extension of the model and conclude my paper. All omitted proofs are provided in the Appendix.

1.1.1 Related Literature

This paper is most closely related to the literature on signaling. The paper contributes to the literature on signaling games by allowing a strategic player to affect signaling cost. In classic signaling models (e.g., Spence 1973, Riley 1985, Milgrom and Roberts 1986, Bagwell and Riordan 1991, Bagwell and Bernheim 1996), with an exogenous cost function, signaling activity gives rise to over-investment in costly actions. Spence (1974), Ireland (1994) and Andersson (1996) suggest taxing signaling activity to undo the signaling effect to restore the

[2]According to the reports by The College Board (www.collegeboard.org): "from 2007-08 through 2010-11, the percentage of institutional grant aid that helped to meet students financial need at private nonprofit four-year colleges and universities ranged from a low of 90% to a high of 93%" (*Trends in Student Aid 2011*, The College Board); "between 2008-09 and 2013-14, the $3,800 increase (in 2013 dollars) in average institutional grant aid for first-time full-time students at private bachelors institutions covered 95% of the $4,000 increase in tuition and fees" (*Trends in Student Aid 2016*, The College Board).

first-best. The associated tax scheme is thus the welfare-maximizing tax on signals. In my model, when the market observes the price schedule, I solve for the profit-maximizing tax on signals, which "over-taxes" signaling and causes a downward distortion in quantity.

The paper is also closely related to the literature on screening. Screening models, such as Mussa and Rosen (1978) and Maskin and Riley (1984), typically assume that buyers derive intrinsic utility from consuming the seller's product. My model differs in the sense that the product has further a signaling value, and a buyer's utility depends on the information that the product conveys. As such, my model contains both screening and signaling and clearly states the interaction between the two forces. Calzolari and Pavan (2006) study information disclosure in a sequential screening model. They show that the upstream principal leaves more information rents to the agent if she discloses information about the agents type to the downstream principal. Analogously, in my model, the principal leaves more rents to the agent than she would otherwise if the market can observe the agent's type, which is perfect information disclosure. The difference is that the market in my model is competitive; thus, unlike in their model, the disclosure of the agent's type creates no value for the market.

The model is closest to Rayo (2013). This paper also considers a principal who sells a signal to an agent with a hidden type, assuming that the principals mechanism is observed by the market. Whereas I assume additive separability in the market's action (e.g., wage) and the agent's type (e.g., ability), Rayo's adopts a multiplicative structure, and thus, the principals revenue depends on whether the allocation of signal is separating or pooling; this necessitates the use of novel screening techniques. The contribution of my paper is to study the case in which the market cannot observe the principal's mechanism, and comparing this to the observed case as well as a variety of other benchmarks. This enables me to assess how the transparency of pricing affects the degree of signaling and welfare.

The unobserved tuition case belongs to the class of signal jamming models proposed by Fudenberg and Tirole (1986). For example, in Holmström (1999), the labor market cannot distinguish the impact of the worker's ability from that of his effort on output. In response, the worker works harder to improve the market's perception of his ability. In comparison, in my model, the labor market cannot distinguish the impact of the worker's ability from that of tuition on education level. Thus, the school has an incentive to secretly cut tuition, thereby improving the market's perception and stimulating demand. In Chan, Li, and Suen (2007), a school has an incentive to inflate grades to improve the market's perception of its students. They show that grade inflation features strategic complements when the qualities of students are correlated across schools. In contrast to their model, my model incorporates screening in addition to signaling, as the school cannot observe its students' abilities.

Finally, my paper relates closely to the literature on intermediate price transparency. Inderst and Ottaviani (2012) shows how product providers compete through commissions paid to consumer advisers. Commissions bias advice; thus, an increase in a firms commission reduces consumers' willingness to pay if they observe the commission. Analogously, in my model, cheaper tuition reduces the signaling value of education, and thus, tuition cuts are less effective at stimulating demand than they would be otherwise when employers observe tuition. In Janssen and Shelegia (2015), a manufacturer chooses a wholesale price, retailers choose retail prices, and consumers search for the best deal. They argue that retailers are less sensitive to wholesale price changes when consumers do not observe the price than otherwise, as uninformed consumers are more likely to keep searching when the retail price rises. By contrast, in my model the worker is more sensitive to tuition changes when employers do not observe the tuition scheme than otherwise, as uninformed employers will have better (worse) beliefs over the worker's ability if they observe a higher (lower) education level.

1.2 The Model

Players and actions There is a single school (*principal*), a worker (*agent*) and multiple identical and competing firms, also referred to as *the labor market*. At the beginning of the game, the school chooses a tuition scheme $T(z):\mathbb{R}_+ \to \mathbb{R}_+$, where z stands for education level and $T(z)$ is the tuition at z. Subsequently, the worker decides how much education to purchase from the school based on the tuition scheme. For simplicity, I do not explicitly model firms' actions; rather, I directly assume that they offer the worker a wage equal to his expected productivity (see below).

The worker's productivity depends on his ability (*type*) θ and his education choice z. Specifically, θ is a random variable, which distributes over the interval $[\underline{\theta}, \bar{\theta}]$, according to a distribution function $F(\theta)$ with a positive density function $f(\theta)$. Denote by $Q(z, \theta)$ the productivity of a type-θ worker having education level z. I assume that $Q(z, \theta)$ is twice differentiable and increasing in both arguments. Formally, $Q_z(z, \theta)$, $Q_\theta(z, \theta) > 0$ if $z > 0$. I also assume that a worker with no education has zero productivity irrespective of his ability; that is, $Q(0, \theta) \equiv 0$. I consider this assumption realistic since many jobs require a minimal education level. For example, a lawyer candidate must graduate from a law school, and medical school education is prerequisite for being a licensed practitioner of medicine. In the Appendix, as a supplementary exercise, I present the analysis for the case in which education is unproductive.

Information The worker's education level is publicly observed. However, neither the school nor the labor market observes the worker's ability, but both know its distribution. In this paper, I mainly study two variants of the model: in the *observed* case, the tuition scheme is observed by the labor market; in the *unobserved* case, it is unobserved by the labor market. In each case, the labor market announces and commits to a wage schedule $W(z) : \mathbb{R}_+ \to \mathbb{R}_+$, based on the available information.

Payoffs I normalize the school's marginal costs of educating to zero and abstract from fixed costs. Suppose that the school chooses some tuition scheme T; then, let $z(\theta; T)$ be the education level chosen by a type-θ worker under T. Given the tuition scheme T and the wage schedule W, a type-θ worker who chooses education level z has utility:

$$U(z, \theta) = W(z) - T(z) - C(z, \theta),$$

where $C(z,\theta)$ is the worker's cost of effort for education. I assume that $C(z,\theta)$ is twice differentiable, increasing and strictly convex in z, and unbounded: $C_z(z,\theta) > 0$ if $z > 0$, and $C_{zz}(z,\theta) > k$ for some $k > 0$. Moreover, the standard *single-crossing property* holds: $C_{z\theta}(z,\theta) < 0$ if $z > 0$. This condition captures the feature that a higher-ability worker has lower marginal effort costs than a lower-ability worker. I also normalize $C(0,\theta)$ to 0 for all $\theta \in [\underline{\theta}, \bar{\theta}]$. This implies that, combined with $C_z(z,\theta) > 0$ and $C_{z\theta}(z,\theta) < 0$ if $z > 0$, $C_\theta(z,\theta) < 0$ if and only if $z > 0$. Finally, I assume that the worker can obtain a zero-utility outside option by acquiring no education and not entering the labor market.

First-best benchmark Define $S(z,\theta)$ as the social surplme function, i.e.,

$$S(z,\theta) = Q(z,\theta) - C(z,\theta).$$

Assume that $S(z,\theta)$ is strictly quasiconcave in z and has a unique maximizer $z^{fb}(\theta) \geq 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Then, the first-order condition implies that

$$S_z(z^{fb}(\theta),\theta) = Q_z(z^{fb}(\theta),\theta) - C_z(z^{fb}(\theta),\theta) = 0. \tag{1.1}$$

To ensure that $z^{fb}(\theta)$ is increasing, I assume further that $S_{z\theta}(z,\theta) > 0$ if $z > 0$. Then, the monotonicity holds according to Milgrom and Shannon (1994, Theorem 4). It is also readily confirmed that $S(z^{fb}(\theta),\theta)$ is increasing in θ.

Equilibrium I use *perfect Bayesian equilibrium* as the solution concept throughout this paper. In the observed case, an equilibrium consists of the school's tuition scheme T^o and conditional on any tuition scheme T, the worker's education function $z^o(\theta;T)$ and the labor market's wage schedule $W^o(z;T)$, such that

(i) For each T, the following holds: (a) given $W^o(z;T)$, $z^o(\theta;T)$ maximizes $U(z,\theta)$; (b) $W^o(z;T) = \mathbb{E}[Q(z,\theta)|z^o(\theta;T)]$ such that the labor market's posterior belief about the worker's ability, or simply *the market belief*, is updated using Bayes' rule.

(ii) Given $z^o(\theta;T)$, T^o maximizes the school's expected profit, i.e.,

$$T^o \in \arg\max_T \int_{\underline{\theta}}^{\bar{\theta}} T(z^o(\theta;T))dF(\theta).$$

In the unobserved case, the market's inference is independent of the actual tuition scheme but is conditional on a *conjectured* scheme; in equilibrium, the conjecture is correct. In this case, an equilibrium consists of a tuition scheme T^u and a wage schedule W^u (more precisely, $W^u(z;T^u)$), and conditional on any T, an education function $z^u(\theta;T)$, such that

(i) Given W^u, for each T, $z^u(\theta;T)$ maximizes $U(z,\theta)$; $W^u(z) = \mathbb{E}[Q(z,\theta)|z^u(\theta;T^u)]$ such that the market belief is updated using Bayes' rule.

(ii) Given $z^u(\theta;T)$, T^u maximizes the school's expected profit, i.e.,

$$T^u \in \arg\max_T \int_{\underline{\theta}}^{\bar{\theta}} T(z^u(\theta;T))dF(\theta).$$

Note that the equilibrium conditions have one important difference between the observed and unobserved case: in the unobserved case, the market belief needs to be correct only on the equilibrium path, whereas in the observed case, the market belief has to be correct following every tuition scheme that is chosen by the school.

Equilibrium selection For both the observed and unobserved case, while there possibly exist multiple equilibria, I focus on the *school-optimal separating equilibrium*, that is, the equilibrium that yields the highest payoff for the school, provided that on the equilibrium path, $z(\theta)$ is one-to-one if $z(\theta) > 0$.[3] To ensure that a separating equilibrium indeed exists, I impose an assumption on the cost function $C(z,\theta)$ and the distribution function $F(\theta)$.

[3] I do not impose any restriction on $z(\theta)$ off the equilibrium path.

Assumption 1.1. *$C_{z\theta\theta}(z,\theta) \geq 0$ and $F(\theta)$ has a non-decreasing hazard rate.*

The reason that I select the school-optimal separating equilibrium is because in Spence's model, the unique equilibrium that survives the D1 refinement (Banks and Sobel 1987) is the *least-cost separating equilibrium* (Riley 1979) in which $z(\theta)$ is one-to-one and the lowest type $\underline{\theta}$ chooses the first-best $z^{fb}(\underline{\theta})$.[4] In the school-optimal separating equilibrium, given the equilibrium tuition scheme, the continuation game constitutes the least-cost separating equilibrium, thereby allowing me to compare the associated equilibrium predictions with that of Spence's model. Moreover, in a discrete-type version of the model, a pooling equilibrium, in which all participating types choose an identical education level, does not exist in either case whenever the proportion of the highest type is sufficiently large.[5] Hence, I consider the school-optimal separating equilibrium a reasonable equilibrium to study.

Finally, given the equilibrium selection rule, one can easily conclude that the school has a higher equilibrium payoff in the observed case than in the unobserved case, as it can secure a weakly higher expected profit in the observed case by maintaining T^u.

1.2.1 Communication Mechanisms

Appealing to the revelation principle, I consider communication mechanisms between the school and worker in both the observed and unobserved case. It is without loss of generality to adjust the timing as follows. First, the school offers a contract $\langle z(\theta), T(z)\rangle$ to the worker. Then, the labor market publishes a wage schedule $W(z)$ based on the information available: in the observed case, it observes the contract; in the unobserved case, it does not. Finally, the

[4] Since $\underline{\theta}$ is the worst market belief, the lowest type does not fear further punishment from deviating to its full-information optimal education level. This initial condition leads to the separating equilibrium in which the worker obtains the least education.

[5] Suppose that, in either the observed or unobserved case, a pooling equilibrium exists such that all participating types choose the same education; then, the equilibrium wage is a constant which equals the average productivity, and the equilibrium tuition is a fixed fee and makes the lowest participating type just indifferent. But whenever the proportion of the highest type is sufficiently large, it is optimal for the school to serve only the highest type and exclude all lower types, leading to a contradiction.

worker reports his type to only the school. Reporting a type $\hat{\theta}$, the worker obtains education level $z(\hat{\theta})$, pays tuition $T(z(\hat{\theta}))$ and then receives wage $W(z(\hat{\theta}))$.

Worker's problem In both cases, given a contract $\langle z(\theta), T(z)\rangle$ and the associated wage schedule $W(z)$, a type-θ worker chooses a report $\hat{\theta}$ to maximize his utility

$$U(\hat{\theta}, \theta) = W(z(\hat{\theta})) - T(z(\hat{\theta})) - C(z(\hat{\theta}), \theta).$$

The mechanism $\{\langle z(\theta), T(z)\rangle, W(z)\}$ is *incentive compatible* (IC) if the worker is willing to truthfully report his type and is *individually rational* (IR) if the worker obtains a non-negative utility level. A type-θ worker's equilibrium payoff is represented by $U(\theta) := U(\theta, \theta)$.

School's problem In the observed case, the school chooses a contract to maximize its expected profit subject to incentive compatibility, individual rationality, and the market belief being correct. In the unobserved case, since the market's inference is independent of the school's choice, given the wage schedule, the school chooses a contract to maximize its expected profit subject to incentive compatibility and individual rationality.

Preliminaries In both cases, an allocation $\langle z(\theta), U(\theta)\rangle$ is *implementable* if it is incentive compatible and individually rational. Appealing to Mas-Colell, Whinston, and Green (1995, Proposition 23.D.2), I characterize all implementable allocations by the following lemma.

Lemma 1.1. *In both cases, an allocation $\langle z(\theta), U(\theta)\rangle$ is implementable if and only if*

(i) $z(\theta)$ is non-decreasing.

(ii) Define $\theta_0 \equiv \inf\{\theta | z(\theta) > 0\}$; then, for $\theta > \theta_0$,

$$U(\theta) = U(\theta_0) + \int_{\theta_0}^{\theta} -C_\theta(z(s), s) ds$$

subject to $U(\theta_0) \geq 0$.

By Lemma 1.1, I rewrite the school's problem for both cases. Note that incentive compatibility means that $T(z(\theta)) = W(z(\theta)) - C(z(\theta), \theta) - U(\theta)$ and that $U(\theta_0)$ is optimally set to 0. Substituting and integrating by parts, the school's problem can be stated as

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \left\{ W(z(\theta)) - C(z(\theta), \theta) + \frac{1 - F(\theta)}{f(\theta)} C_\theta(z(\theta), \theta) \right\} dF(\theta) \tag{1.2}$$

subject to $z(\theta)$ being non-decreasing.

In the observed case, correctness of the market belief means that $W(z) = \mathbb{E}[Q(z, \theta)|z(\theta)]$ for any implementable allocation $z(\theta)$ that the school chooses. Then, from the law of total expectation, program (1.2) is equivalent to

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \left\{ S(z(\theta), \theta) + \frac{1 - F(\theta)}{f(\theta)} C_\theta(z(\theta), \theta) \right\} dF(\theta) \tag{1.3}$$

subject to $z(\theta)$ being non-decreasing. Intuitively, because firms break even in expectation, the school maximizes the expected difference between social surplus and consumer surplus, as in Mussa and Rosen (1978) or Maskin and Riley (1984). It suffices to solve program (1.3) for the equilibrium characterization of the observed case. If the solution $z^o(\theta)$ is increasing over $[\theta_0, \bar{\theta}]$, then the school-optimal separating equilibrium is obtained.

In the unobserved case, without loss of generality, the school chooses an allocation $z(\theta)$, while simultaneously, the labor market chooses a wage schedule W. Then, the equilibrium conditions can be simplified as follows: (i) given W^u, $z^u(\theta)$ solves the school's problem in (1.2); (ii) $W^u(z) = \mathbb{E}[Q(z, \theta)|z^u(\theta)]$ such that the market belief is updated using Bayes' rule. In the case of multiple equilibria, I select the school-optimal separating equilibrium.

1.2.2 A Simple Example with Two Types

To develop simple intuitions for my general results, I establish a numerical example with binary types. I assume that $\theta \in \{\theta_L, \theta_H\}$ with $0 < \theta_L < \theta_H$ and that each type is realized with equal probability. I also assume that $Q(z, \theta) = \theta z$ and $C(z, \theta) = z^2/(2\theta)$. It is readily confirmed that Assumption 1.1 holds in this example.

As a first step, I study the observed case. Suppose that the contract $\{(z_L^o, T_L^o), (z_H^o, T_H^o)\}$ solves the school's problem and that the associated wage schedule is given by $\{W_L^o, W_H^o\}$. Then, incentive compatibility and individual rationality imply that

$$C(z_L^o, \theta_L) - C(z_L^o, \theta_H) \leq U(\theta_H) - U(\theta_L) \leq C(z_H^o, \theta_L) - C(z_H^o, \theta_H), \tag{IC}$$

$$U(\theta_i) = W_i^o - T_i^o - C(z_i^o, \theta_i) \geq 0,\ i = L, H. \tag{IR}$$

Note that $T_i^o = W_i^o - C(z_i^o, \theta_i) - U(\theta_i)$, $i = L, H$; thus, the school's expected profit equals

$$\Pi^o = 0.5\,(T_L^o + T_H^o) = 0.5\,[W_L^o + W_H^o - C(z_L^o, \theta_L) - C(z_H^o, \theta_H) - U(\theta_L) - U(\theta_H)]\,.$$

As is standard in the literature, both the downward IC and the low type's IR constraints are binding. Moreover, a correct market belief means that $W_L^o + W_H^o = \theta_L z_L^o + \theta_H z_H^o$ regardless of whether it is separating ($z_L^o \neq z_H^o$). Substituting these results and the model assumptions into Π^o, I write the school's problem as follows:

$$\max_{z_L, z_H}\ \theta_L z_L - \frac{z_L^2}{2\theta_L} - \frac{(\theta_H - \theta_L) z_L^2}{2\theta_L \theta_H} + \theta_H z_H - \frac{z_H^2}{2\theta_H}\ \ s.t.\ \ z_H \geq z_L.$$

Then, the first-order conditions imply that

$$z_L^o = \frac{\theta_H \theta_L^2}{2\theta_H - \theta_L} < \theta_L^2 = z_L^{fb},\ z_H^o = \theta_H^2 = z_H^{fb}.$$

Since $z_L^o < z_H^o$, we obtain the school-optimal separating equilibrium. In this equilibrium, the low type chooses less education than the first-best, while the high type chooses exactly the first-best. Intuitively, the high type benefits from his cost advantage over the low type, as indicated by the downward IC constraint, and thus, he extracts an *information rent* that is increasing in the low type's education level. This induces the school to under-supply education to the low type. Since there are only two types, there is no distortion of the high type's education level. From preliminary calculations, one can completely characterize the equilibrium outcome. In particular, the school's equilibrium payoff equals

$$\Pi^o = 0.5\,(T_L^o + T_H^o) = \frac{\theta_H \theta_L^3}{8\theta_H - 4\theta_L} + \frac{\theta_H^3}{4}.$$

Then, I consider the unobserved case. Suppose that the labor market believes naively that the school's contract is the same as that in the observed case and thus offers the same wage schedule. Will the school retain the same contract? It depends. To see why, consider an alternative contract $\{(z', T')\}$ such that $z' = z_H^o$ and $T' = W_H^o - C(z_H^o, \theta_L) < T_H^o$. That is, the school only offers the high education level from the observed case and reduces tuition to the level that also attracts the low type. Thus, the school's new expected profit equals

$$\Pi' = T' = \theta_H^3 - \frac{\theta_H^4}{2\theta_L}.$$

One can show that if θ_L and θ_H are close enough to one another, e.g., $\theta_L = 1$ and $\theta_H = 1.1$, then this deviation is indeed profitable ($\Pi' \approx 0.60$ while $\Pi^o \approx 0.56$). The idea is that with the wage schedule being fixed, the school secretly cuts its prices to gain market share; if the labor market could observe the tuition, such price cuts would undermine the signaling value of education and thus would not be profitable. If the gap between types is small enough, then the increase in quantity dominates the reduction in price, making the deviation profitable. In contrast, if the gap is relatively big, e.g., $\theta_L = 1$ and $\theta_H = 2$ (in this case, $\Pi' = 0$ while $\Pi^o = \frac{13}{6}$), and any off-equilibrium-path education is believed to be chosen by the low type, then the equilibrium of the observed case can also be sustained in the unobserved case. This is because the high education level is relatively high, and thus, the school finds it unprofitable to induce the low type to imitate the high type by secretly cutting the price.

Then, what is the equilibrium of the unobserved case if the aforementioned deviation is profitable? Here, I characterize the school-optimal separating equilibrium without proof. First, the offer to the low type (z_L^u, T_L^u) is the same as (z_L^o, T_L^o). Second, the high education level z_H^u satisfies the school's incentive compatibility constraint as follows:

$$\theta_L z_L^u - \frac{z_L^{u2}}{2\theta_L} - \frac{(\theta_H - \theta_L) z_L^{u2}}{2\theta_L\theta_H} = \theta_H z_H^u - \frac{z_H^{u\,2}}{2\theta_L} - \frac{(\theta_H - \theta_L) z_H^{u\,2}}{2\theta_L\theta_H} \;\; s.t. \;\; z_H^u \geq z_L^u. \tag{SIC}$$

This constraint indicates that the school weakly prefers truthfully revealing the worker's type to inducing the low type to imitate the high type. Third, W^u equals $Q(z, \theta_H)$ if $z = z_H^u$ and equals $Q(z, \theta_L)$ otherwise. Finally, T_H^u is derived by substituting z_H^u into $S(z, \theta_H) - U(\theta_H)$, where $U(\theta_H) = C(z_L^u, \theta_L) - C(z_L^u, \theta_H)$, as the downward IC constraint is binding.

This equilibrium outcome reveals the second heuristic result. In the unobserved case, the high type selects more education than in the observed case (note that the low type's situation does not change). One can derive this result from the SIC constraint. For example, if $\theta_L = 1$ and $\theta_H = 1.1$, then $z_H^u \approx 1.43$ while $z_H^o = z_H^{fb} = 1.21$. The underlying intuition is that in the unobserved case, the high education level must be excessively high such that the school finds it unprofitable to induce the low type to imitate the high type. Since in both cases the downward IC constraint is binding and the low type's education level is the same, the high type's utility does not change between the two cases. This implies that the high education level must be cheaper in the unobserved case, and thus, the school has a lower profit than in the observed case. For example, if $\theta_L = 1$ and $\theta_H = 1.1$, then $T_H^u \approx 0.61$ while $T_H^o = 0.63$. The reason is that the high type selects the efficient quantity in the observed case but selects more than the efficient quantity in the other, and thus, less social surplus is generated in the unobserved case.[6] To make the high type no worse off, the school must charge a lower price for the high education level. Intuitively, since education is inflated ($z_H^u > z_H^o$), the signaling value of the high education is diluted. This means that the worker has a lower willingness to pay, and thus, the school achieves lower profits in the unobserved case.

[6]This implies that social welfare is lower in the unobserved case in the two-type model. In the general model, however, the welfare comparison between the two cases is ambiguous, as I will show in Section 1.5.

1.3 Job Market Signaling without Tuition

I now return to the general model. As a reference point, I revisit Spence's signaling game in which tuition is fixed at zero. One could interpret such a benchmark as the case in which schools are competitive and thus choose tuition equal to the marginal cost. In this case, an equilibrium consists of an education function $z^s(\theta)$ and a wage schedule $W^s(z)$, such that (i) given $W^s(z)$, $z^s(\theta)$ maximizes $U(z,\theta)$; (ii) $W^s(z) = \mathbb{E}[Q(z,\theta)|z^s(\theta)]$ with the market belief updated using Bayes' rule. I focus on the least-cost separating equilibrium, in which $z^s(\theta)$ is one-to-one and the lowest type $\underline{\theta}$ chooses the first-best $z^{fb}(\underline{\theta})$. In the following, I apply the general results of Mailath (1987) to this specific setting to characterize the equilibrium.

Proposition 1.1. *The least-cost separating equilibrium exists, such that*

(i) $z^s(\underline{\theta}) = z^{fb}(\underline{\theta})$*;* $z^s(\theta)$ *satisfies the first-order condition*

$$Q_z(z^s(\theta),\theta) + Q_\theta(z^s(\theta),\theta)\cdot\theta^{s\prime}(z^s(\theta)) - C_z(z^s(\theta),\theta) = 0, \tag{1.4}$$

where $\theta^s(z)$ *is the inverse function of* $z^s(\theta)$*, being differentiable on* $[\underline{\theta},\bar{\theta}]$*.*

(ii) $z^s(\theta)$ *is increasing over* $[\underline{\theta},\bar{\theta}]$*, and thus,* $W^s(z^s(\theta)) = Q(z^s(\theta),\theta)$ *for all* $\theta \in [\underline{\theta},\bar{\theta}]$*.*

Note that the first two terms on the left-hand side (LHS) of (1.4) are the total derivative of $W^s(z)$. In particular, the second term is non-negative given the monotonicity of $z^s(\theta)$. Since $S(z,\theta)$ is strictly quasiconcave, comparing (1.4) with (1.1) implies that $z^s(\theta) \geq z^{fb}(\theta)$ for all $\theta \geq \underline{\theta}$, with equality holding at $\underline{\theta}$ only. This comparison is illustrated in Figure 1.1. I now summarize this result in the following corollary.

Corollary 1.1. *In Spence's signaling game, the worker chooses more education than the first-best. Specifically,* $z^s(\theta) \geq z^{fb}(\theta)$ *on* $[\underline{\theta},\bar{\theta}]$*, with strict inequality for* $\theta > \underline{\theta}$*.*

Corollary 1.1 indicates that the worker's signaling activity leads to over-education. The intuition is well-understood. Under complete information, the marginal benefit of education

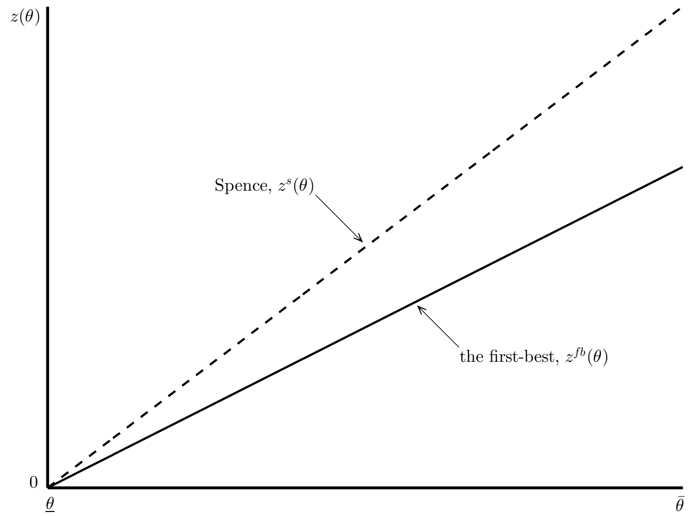

Figure 1.1: The Signaling Effect

This figure compares $z^s(\theta)$ with $z^{fb}(\theta)$ over $[\underline{\theta}, \bar{\theta}]$. This figure assumes that $Q(z,\theta) = \theta z + z$, $C(z,\theta) = z^2 + z - \theta z$, and $\theta \sim U[0,1]$. In this example, $z^{fb}(\theta) = \theta$ and $z^s(\theta) = \frac{3}{2}\theta$.

is its marginal contribution to human capital. In contrast, when ability is privately known, in addition to the human capital effect, there is a *signaling effect*; that is, a higher education level makes the labor market regard the worker as having higher ability. Thus, the marginal benefit of education is higher than under complete information. Since the marginal cost is the same, education is over-invested in when the workers ability is private information.

1.4 Labor Market Observes Tuition

Starting with this section, I take the school's strategic behavior into account. Here, I consider the case in which the labor market observes the tuition scheme. According to Section 1.2.1, it suffices to solve the school's problem in (1.3) for the equilibrium characterization. It is heuristic to interpret the integrand in (1.3) as the school's marginal profit in the observed case. Define

$$MP^o(z,\theta) := S(z,\theta) + \frac{1-F(\theta)}{f(\theta)} C_\theta(z,\theta).$$

As is standard in the literature, I solve the school's problem by relaxing the monotonicity constraint of $z(\theta)$ first and verity it ex post to justify the approach. This is equivalent to pointwise optimization for $MP^o(z,\theta)$. Inspired by Martimort and Stole (2009), I say that the school's marginal profit in the observed case is *regular* if $MP^o(z,\theta)$ is strictly quasiconcave in z and $MP^o_z(z,\theta)$ is increasing in θ. Given Assumption 1.1, regularity holds. Therefore, $MP^o(z,\theta)$ has a unique maximizer $z^*(\theta)$, which is increasing. Note that $z^*(\theta)$ might be negative for some region of θ; as such, I set $z(\theta)$ to 0 instead of $z^*(\theta)$. Thus, $MP^o(z(\theta),\theta)$ is non-decreasing in θ and is non-negative. The cutoff type θ^o_0 is thus either the maximal root of $MP^o(z(\theta),\theta)=0$ if it exists, or $\underline{\theta}$ otherwise. In summary, the optimal education allocation $z^o(\theta)$ is given by

$$z^o(\theta)=\begin{cases} z^*(\theta) & \text{if } \theta \geq \theta^o_0 \\ 0 & \text{otherwise.} \end{cases} \tag{1.5}$$

To complete the characterization of equilibrium, back out $z^o(\theta)$'s inverse function $\theta^o(z)$ over $[\theta^o_0,\bar{\theta}]$ given that $z^o(\theta)$ is increasing over $[\theta^o_0,\bar{\theta}]$. Plugging $\theta^o(z)$ into $Q(z,\theta)$ yields the equilibrium wage schedule $W^o(z)$ on $[z^o(\theta^o_0),z^o(\bar{\theta})]$. Finally, the equilibrium tuition scheme $T^o(z)$ on $[z^o(\theta^o_0),z^o(\bar{\theta})]$ is given by

$$T^o(z^o(\theta))=S(z^o(\theta),\theta)-U(\theta)=S(z^o(\theta),\theta)+\int_{\theta^o_0}^{\theta} C_\theta(z^o(s),s)ds. \tag{1.6}$$

For the off-path education levels, I assume without loss that the school sets exorbitantly high prices such that no type is willing to deviate to there in any case. Then, given $T^o(z)$, the school-optimal separating equilibrium is also the least-cost separating equilibrium in the sense that the cutoff type chooses his full-information optimal quantity under the total cost function $T^o(z)+C(z,\theta)$. Moreover, since $z^o(\theta)$ coincides with the unconstrained optimizer $z^*(\theta)$ on path, this equilibrium is further *the school-optimal equilibrium*. I now summarize the equilibrium outcome of the observed case in the next proposition.

Proposition 1.2. *In the observed case, the school-optimal separating equilibrium exists. On the equilibrium path, the education function* $z^o(\theta)$ *is given by (1.5); the tuition scheme* $T^o(z)$ *is given by (1.6); and the wage schedule* $W^o(z)$ *equals* $Q(z, \theta^o(z))$.

Note that $MP_z^o(z,\theta)$ is less than $S_z(z,\theta)$, holding weakly on the boundary. Consequently, regularity implies that $z^o(\theta) \leq z^{fb}(\theta)$ on $[\theta_0^o, \bar{\theta}]$, with equality holding at $\bar{\theta}$ only. If $\theta_0^o > \underline{\theta}$, then $z^o(\theta) = 0$ for all $\theta \in [\underline{\theta}, \theta_0^o)$. To summarize, we have the following corollary:

Corollary 1.2. *In the observed case, the worker chooses less education than the first-best. Specifically,* $z^o(\theta) \leq z^{fb}(\theta)$ *on* $[\underline{\theta}, \bar{\theta}]$*, with strict inequality on* $(\underline{\theta}, \bar{\theta})$.

Corollary 1.2 states that when the labor market observes the tuition scheme, education is under-supplied. This result stands in stark contrast to that of Spence's model. The altered equilibrium prediction results from the school's screening activity. Specifically, having a cost advantage in education, a higher-ability worker can secure higher utility than a lower-ability worker by choosing the same education as the latter. Therefore, to incentivize truth-telling, the school has to leave information rents to the worker. This means that the marginal profit of education is less than the social surplus generated; therefore, the school under-supplies education. In particular, an interval of types at the low end of the domain will be excluded from education if it is too costly to serve them.

Remark. *Suppose that education is a pure signal (i.e.,* $Q(z,\theta) \equiv Q(\theta)$*) as in Spence (1973); I show in the Appendix that the school-optimal separating equilibrium yields a virtually socially optimal outcome, such that* $z^o(\theta)$ *is arbitrarily close to* $z^{fb}(\theta) \equiv 0$*. Specifically, the school allocates increasing and infinitesimal education to different types, with the lowest type having no education. Thus, the school's profit is arbitrarily close to the first-best social welfare minus the lowest type's utility* $Q(\underline{\theta})$*. To interpret, since education is unproductive, the school provides little and different education in the form of different types of degrees to separate types; a worker without any degree is regarded as having the lowest ability.*

1.4.1 Screening vs Signaling

While the equilibrium prediction for the observed case is due to the mechanism of monopoly screening, my model also contains signaling. Note that given the tuition scheme $T^o(z)$, the subgame is indeed Spence's signaling game as if the worker had a cost function in the form of $T^o(z)+C(z,\theta)$. From the same argument as in Corollary 1.1, the education levels in the observed case are distorted, due to signaling, above the "efficient" level with respect to the total cost of education. This fact reveals that the equilibrium outcome of the observed case results from the interaction between screening and signaling.

Corollary 1.2 indicates that when both screening and signaling are present and exert the opposite effects—screening induces under-education, but signaling induces over-education—screening outweighs signaling. This is because as a Stackelberg leader, the school internalizes the worker's signaling incentive when screening his type. To see this, note that

$$T^{o\prime}(z) = W^{o\prime}(z) - C_z(z,\theta^o(z)) = \frac{d}{dz}\left[Q(z,\theta^o(z))\right] - C_z(z,\theta^o(z)).$$

Substituting this equation into the first-order condition of $MP^o(z,\theta)$, we have

$$T^{o\prime}(z) = Q_\theta(z,\theta^o(z)) \cdot \theta^{o\prime}(z) + \frac{1-F(\theta^o(z))}{f(\theta^o(z))}\left[-C_{z\theta}(z,\theta^o(z))\right]. \tag{1.7}$$

On the right-hand side (RHS) of (1.7), the first term captures the signaling effect, and the second term is the marginal information rent extracted by the worker. Note that signaling induces over-education, which reduces the school's profit in two ways: on one hand, it lowers total surplus; on the other hand, it provides the worker with more information rents. Thus, the optimal tuition scheme must undo these two effects, as indicated by (1.7). In contrast, if the school were a welfare-maximizing social planner, it would only undo the signaling effect by levying Pigovian taxes (Spence 1974). Denote by $T^{fb}(z)$ the welfare-maximizing tax on education. The marginal tax is equal to the signaling effect at the first-best, i.e.,

$$T^{fb'}(z) = Q_\theta(z, \theta^{fb}(z)) \cdot \theta^{fb'}(z), \tag{1.8}$$

where $\theta^{fb}(z)$ is the inverse function of $z^{fb}(\theta)$.[7] Since the second term on the RHS of (1.7) is positive, it follows from comparing (1.7) with (1.8) that the profit-maximizing tax on education "over-taxes" signaling activity and thus leads to under-education.

To see how signaling makes a difference, consider the situation in which the labor market also observes the worker's ability without changing any other element of the model. In this case, the wage equals the actual productivity, and signaling is eliminated. This means that the worker's intrinsic value for education is the social surplus $S(z,\theta)$. Since $S_\theta = Q_\theta - C_\theta > 0$, a higher type can be seen as a higher-value buyer of education. Thus, the school has the same monopoly screening problem as in Mussa and Rosen (1978). Specifically, the school chooses a contract $\langle z(\theta), T(z)\rangle$ to maximize its expected profit subject to the IC and IR constraints. Analogous to Lemma 1.1, an allocation $\langle z(\theta), U(\theta)\rangle$ is implementable if and only if (i) $z(\theta)$ is non-decreasing; (ii) $U(\theta_0) \geq 0$ and for $\theta > \theta_0$,

$$U(\theta) = U(\theta_0) + \int_{\theta_0}^{\theta} S_\theta(z(s), s) ds.$$

Thus, the school's problem in such Mussa and Rosen's screening game can be stated as

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \left\{ S(z(\theta), \theta) - \frac{1 - F(\theta)}{f(\theta)} S_\theta(z(\theta), \theta) \right\} dF(\theta)$$

subject to $z(\theta)$ being non-decreasing. Analogously, define the school's marginal profit as

$$MP^{mr}(z, \theta) := S(z, \theta) - \frac{1 - F(\theta)}{f(\theta)} S_\theta(z, \theta).$$

[7] Since the lowest type $\underline{\theta}$ chooses the first-best $z^{fb}(\underline{\theta})$ in equilibrium, he should be exempt from such tax; that is, $T^{fb}(z^{fb}(\underline{\theta})) = 0$. Then, directly integrating (1.8) yields the welfare-maximizing tax scheme $T^{fb}(z)$.

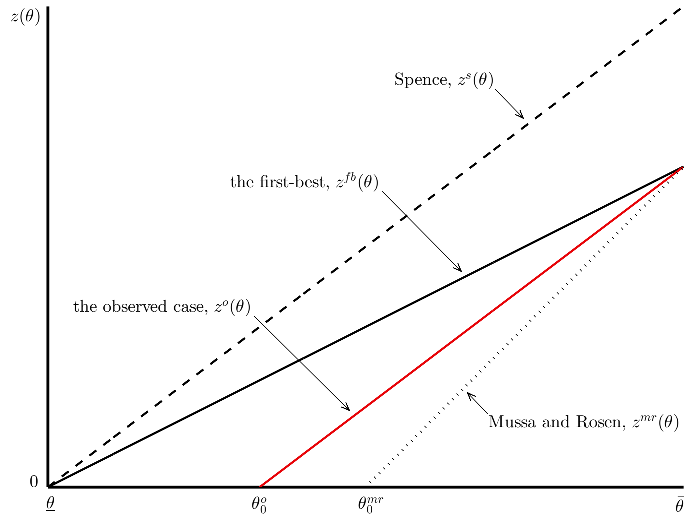

Figure 1.2: Screening vs Signaling

This figure compares $z^{mr}(\theta)$ with $z^o(\theta)$ over $[\underline{\theta}, \bar{\theta}]$ based on Figure 1.1. This figure assumes the same numerical example as Figure 1.1, such that $z^o(\theta) = \frac{3\theta-1}{2}$ and $z^{mr}(\theta) = 2\theta - 1$.

Similarly, I say that $MP^{mr}(z,\theta)$ is regular if it is strictly quasiconcave in z and $MP_z^{mr}(z,\theta)$ is increasing in θ.[8] Denote by $z^{mr}(\theta)$ and θ_0^{mr} the optimal allocation and the cutoff type in Mussa and Rosen's model, respectively. Suppose that $MP^{mr}(z,\theta)$ is regular, then one can characterize $z^{mr}(\theta)$ and θ_0^{mr} analogously to the observed case.

I am interested in how the allocation in Mussa and Rosen's model differs from that in the observed case. On the extensive margin, because $S_\theta > -C_\theta$, $MP^{mr}(z,\theta) \leq MP^o(z,\theta)$, with strict inequality for $\theta < \bar{\theta}$. Hence, if $\theta_0^o > \underline{\theta}$, then $\theta_0^{mr} > \theta_0^o$; that is, more types are excluded in Mussa and Rosen's model. On the intensive margin, if $Q_{z\theta} > 0$ on $[0, z^{fb}(\bar{\theta})]$,[9] then $z^{mr}(\theta) \leq z^o(\theta)$, with strict inequality on $[\theta_0^o, \bar{\theta})$, meaning that under-education is more serious in Mussa and Rosen's model. These findings are illustrated in Figure 1.2.

For welfare comparison, note that education is already under-supplied in the observed

[8]Given Assumption 1.1, $MP^{mr}(z,\theta)$ is regular if $Q_{z\theta\theta} \leq 0$.

[9]This condition is not restrictive; indeed, given that $Q_\theta(z,\theta) > 0$ and $Q(0,\theta) \equiv 0$, we have $Q_{z\theta}(z,\theta) > 0$ on $[0,\bar{z}]$ for some $\bar{z} > 0$. Given this condition, $MP_z^{mr}(z,\theta) < MP_z^o(z,\theta)$ on $[0, z^{fb}(\bar{\theta})]$ for $\theta < \bar{\theta}$.

case, yet the downward distortion is larger in Mussa and Rosen's model; thus, the observed case has higher social welfare. Moreover, since $MP^{mr}(z^{mr}(\theta),\theta) \leq MP^{o}(z^{o}(\theta),\theta)$ with strict inequality on $[\theta_0^o,\bar{\theta})$ and $\theta_0^{mr} \geq \theta_0^o$, it is readily confirmed that the school's expected profit is also higher in the observed case. In summary, we have the following proposition:

Proposition 1.3. *If both $MP^{o}(z,\theta)$ and $MP^{mr}(z,\theta)$ are regular, and $Q_{z\theta} > 0$ on $[0,z^{fb}(\bar{\theta})]$, then under-education is greater when signaling is eliminated. Specifically, $z^{mr}(\theta) \leq z^{o}(\theta)$, with strict inequality on $[\theta_0^o,\bar{\theta})$; if $\theta_0^o > \underline{\theta}$, then $\theta_0^{mr} > \theta_0^o > \underline{\theta}$. Consequently, social welfare and the school's expected profit are strictly higher when signaling is present than otherwise.*

Proposition 1.3 indicates that signaling can mitigate the downward distortion caused by screening. Intuitively, when the labor market observes the worker's ability, if a higher type imitates a lower type by choosing the same education, he not only has a lower total cost than the latter but also obtains a higher wage due to his higher productivity. In contrast, when the labor market does not observe the worker's ability, the higher type can no longer directly reap the benefit from higher productivity, and thus, he acquires more education to signal his ability. The signaling incentive reduces the worker's willingness to imitate lower types. Therefore, the school leaves lower information rents to the worker when signaling is present, as we have the following inequality:

$$\underbrace{\frac{1-F(\theta)}{f(\theta)}[-C_\theta(z,\theta)]}_{\text{information rents with signaling}} \leq \underbrace{\frac{1-F(\theta)}{f(\theta)}S_\theta(z,\theta)}_{\text{information rents without signaling}}$$

which holds with equality at the highest type $\bar{\theta}$ only. Consequently, signaling mitigates the screening distortion.

1.4.2 Signaling Intensity, Market Structure and Welfare

Knowing that signaling can mitigate the downward distortion due to screening, one may wonder how the mitigation corresponds to the intensity of signaling. Intuitively, the more intense signaling is, the more downward distortion is mitigated. Unfortunately, with general functional forms, such comparative statics is very complex; indeed, it is even hard to define the intensity of signaling. For tractability, I consider the numerical example below.

Example Assume that $Q(z,\theta)=\gamma\theta z+z$ with $\gamma>0$, $C(z,\theta)=z^2+z-\theta z$, and $\theta\sim U[0,1]$. From previous results, we have $z^{fb}(\theta)=\frac{(\gamma+1)\theta}{2}$, $z^s(\theta)=\frac{(2\gamma+1)\theta}{2}$ and $z^o(\theta)=\frac{(\gamma+2)\theta-1}{2}$. Define the intensity of signaling to be the ratio of the over-invested education in Spence's model, i.e., $z^s(\theta)-z^{fb}(\theta)$, to the first best education level $z^{fb}(\theta)$ for $\theta>0$. Substituting the education functions, we have

$$\frac{z^s(\theta)-z^{fb}(\theta)}{z^{fb}(\theta)}=\frac{\gamma}{\gamma+1}.$$

Clearly, the intensity of signaling is increasing in the parameter γ. To see the idea, note that the larger γ is, the stronger complementarity between the worker's ability and education is. In Spence's model, higher education induces the labor market to regard the worker as having higher ability; thus, if ability complements education to a larger extent, the marginal benefit of education will be higher, thereby enhancing signaling through education.

Then, I examine how the signaling intensity affects signaling mitigating the screening distortion. Similarly, I define the extent of the downward distortion in the observed case as the ratio of the under-supplied education, i.e., $z^{fb}(\theta)-z^o(\theta)$, to the first best education level $z^{fb}(\theta)$ for $\theta>0$. Substituting, we have

$$\frac{z^{fb}(\theta)-z^o(\theta)}{z^{fb}(\theta)}=\frac{1-\theta}{(\gamma+1)\theta}.$$

For any fixed $\theta\in(0,1)$, the extent of the downward distortion is decreasing in γ. This means that the more intense signaling is, the more screening distortion is mitigated.

Recall that in Spence's signaling game, signaling reduces social welfare, as it leads to over-education. In the observed case, by contrast, signaling raises social welfare because it mitigates the screening distortion. From the above analysis, one can infer that if signaling is sufficiently intense, then the welfare loss in Spence's model will exceed that in the observed case; thus, the observed case will yield higher social welfare. To be concrete, I formulate the difference in social welfare between Spence's model and the observed case:

$$\int_{\underline{\theta}}^{\bar{\theta}} [S(z^o(\theta),\theta) - S(z^s(\theta),\theta)]\, dF(\theta) = \frac{(\gamma^2+\gamma-1)(\gamma^2+3\gamma+1)}{12(\gamma+2)^2}.$$

It is clear that the observed case yields higher social welfare if and only if $\gamma > \frac{\sqrt{5}-1}{2}$.

This finding has welfare implications for the market structure of signals (which refer to education here). Note that when the market is served by perfectly competitive sellers of signals, the equilibrium outcome is predicted by Spence's model; when the market is served by a monopoly with a publicly observed price schedule, the equilibrium outcome is predicted by the observed case. Therefore, when the buyer's signaling incentive is sufficiently strong, a monopoly can yield higher social welfare than a perfectly competitive market. This implies that introducing competition among signal sellers is not necessarily socially beneficial, as doing so might aggravate over-investment in signaling activity.

Furthermore, since signaling exerts the opposite welfare effects between Spence's model and the observed case, an instrument that affects the intensity of signaling will also exert the opposite welfare effects between the two cases. Specifically, any instrument that attenuates signaling is socially beneficial in the Spencian world but harmful in the observed case. For example, students' grades substitute for their education levels in signaling; thus, grading is beneficial in the Spencian world but harmful in the observed case. If grades become less informative, e.g., due to grade inflation, then signaling through education will be enhanced, as students will attempt to separate themselves from others (Daley and Green 2014). This reveals that grade inflation is socially beneficial in the observed case by alleviating under-education, while it is harmful in the Spencian world because it aggravates over-education.

1.5 Labor Market Does Not Observe Tuition

In this section, I turn to the case in which the labor market does not observe the tuition scheme. Given some wage schedule $W(z)$, the school solves the problem in (1.2). Similar to the observed case, I define the school's marginal profit in the unobserved case as

$$MP^u(z,\theta) := W(z) - C(z,\theta) + \frac{1-F(\theta)}{f(\theta)} C_\theta(z,\theta).$$

It is heuristic to call the last two terms the school's *virtual cost*, and I define

$$G(z,\theta) := C(z,\theta) - \frac{1-F(\theta)}{f(\theta)} C_\theta(z,\theta).$$

In doing so, I establish an auxiliary game analogous to Spence's signaling game, in which the worker's cost function is given by $G(z,\theta)$ and utility function by $MP^u(z,\theta)$.

This analogy simplifies the equilibrium characterization of the unobserved case. If there exists an equilibrium with non-decreasing education levels for the auxiliary game, then one can construct an equilibrium for the unobserved case based on that. Specifically, assign the auxiliary game's equilibrium outcome to $\{z^u(\theta), W^u(z)\}$. I conclude that $z^u(\theta)$ solves the school's problem given $W^u(z)$, as it maximizes $MP^u(z,\theta)$ pointwise and is non-decreasing. Moreover, $W^u(z)$ is derived from the correct market belief over $z^u(\theta)$. Thus, $z^u(\theta)$ and $W^u(z)$ satisfy the equilibrium conditions of the unobserved case. As $z^u(\theta)$ has also determined the cutoff type θ_0^u, the tuition scheme $T^u(z)$ can be derived analogously to the observed case.[10] This closes the equilibrium characterization of the unobserved case.

In the following, I instead study the auxiliary game and focus on the school-optimal separating equilibrium. Given Assumption 1.1, we have $G_{z\theta}(z,\theta) < 0$ if $z > 0$, and thus, the single-crossing property holds. This condition means that it is less costly for the school to

[10] Unlike the observed case, it entails some loss of generality to assume that tuition is exorbitantly high for the off-path education, as the school cannot influence the market's belief over the tuition scheme. However, it is natural to smoothly extend $T^u(z)$ to $\mathbb{R}_+$; I show in the Appendix that dosing so is incentive compatible.

serve a higher-ability worker. The next proposition shows that the school-optimal separating equilibrium exists in the unobserved case.

Proposition 1.4. *In the unobserved case, the school-optimal separating equilibrium exists, such that*

(i) $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$*;* $z^u(\theta)$ *satisfies the first-order condition*

$$Q_z(z^u(\theta),\theta) + Q_\theta(z^u(\theta),\theta)\cdot\theta^{u\prime}(z^u(\theta)) - G_z(z^u(\theta),\theta) = 0, \tag{1.9}$$

where $\theta^u(z)$ *is the inverse function of* $z^u(\theta)$*, being differentiable on* $[\theta_0^u, \bar{\theta}]$.

(ii) $z^u(\theta)$ *is increasing over* $[\theta_0^u, \bar{\theta}]$*, and thus,* $W^u(z^u(\theta)) = Q(z^u(\theta),\theta)$ *for all* $\theta \in [\theta_0^u, \bar{\theta}]$.

Proposition 1.4 characterizes the equilibrium education function $z^u(\theta)$. It indicates that the cutoff type and his education level coincide for both the observed and unobserved case. In the Appendix, I show that if there is no exclusion in the observed case (i.e., $\theta_0^o = \underline{\theta}$), then the unobserved case has a unique separating equilibrium outcome, which is given above; otherwise (i.e., $\theta_0^o > \underline{\theta}$) there exists a continuum of separating equilibrium outcomes, and in each of them, $\theta_0^u \geq \theta_0^o$ and $z^u(\theta_0^u) \geq z^o(\theta_0^o)$. In addition, it is shown in the Appendix that the school-optimal separating equilibrium is also the least-cost separating equilibrium in the sense that the cutoff type chooses his full-information optimal education level under the total cost function $T^u(z) + C(z,\theta)$.

The next theorem presents the paper's main result. In contrast with the observed case, the worker chooses more education in the unobserved case. In particular, a worker who has a higher ability than the cutoff type chooses strictly more education in the unobserved case.

Theorem 1.1. *In contrast with the observed case, the worker chooses more education in the unobserved case. Specifically,* $z^u(\theta) \geq z^o(\theta)$ *on* $[\underline{\theta}, \bar{\theta}]$*, with strict inequality for* $\theta > \theta_0^u$.

Proof. Given Assumption 1.1, $MP^o(z,\theta)$ is strictly quasiconcave in z. Because $z^o(\theta)$ is the unique maximizer of $MP^o(z,\theta)$, it suffices to prove that $MP_z^o(z^u(\theta),\theta) \leq 0$, with strict inequality for $\theta > \theta_0^u$. This is given by the following:

$$\begin{aligned} MP_z^o(z^u(\theta),\theta) &= S_z(z^u(\theta),\theta) + \frac{1-F(\theta)}{f(\theta)} C_{z\theta}(z^u(\theta),\theta) \\ &= Q_z(z^u(\theta),\theta) - G_z(z^u(\theta),\theta) \\ &\leq Q_z(z^u(\theta),\theta) + Q_\theta(z^u(\theta),\theta)\cdot\theta^{u\prime}(z^u(\theta)) - G_z(z^u(\theta),\theta) \\ &= 0. \end{aligned}$$

The second equality is given by the definition of $G(z,\theta)$; the inequality results from the monotonicity of $z^u(\theta)$ on $[\theta_0^u,\bar{\theta}]$; the last equality is due to (1.9). Furthermore, for $\theta > \theta_0^u$, the second term in (1.9) is positive, and thus, the above inequality becomes strict. □

As I informally argued in Section 1.2 that the school has a lower equilibrium payoff in the unobserved case, this argument is formally proven by the corollary below.

Corollary 1.3. *In the unobserved case, the school's expected profit Π^u is strictly lower than its expected profit Π^o in the observed case.*

Proof. From Proposition 1.4, we have $MP^u(z^u(\theta),\theta) = MP^o(z^u(\theta),\theta)$. Since $z^o(\theta)$ is the unique maximizer of $MP^o(z,\theta)$ and $z^u(\theta) > z^o(\theta)$ for $\theta > \theta_0^u = \theta_0^o$, we have

$$\Pi^o - \Pi^u = \int_{\theta_0^o}^{\bar{\theta}} [MP^o(z^o(\theta),\theta) - MP^o(z^u(\theta),\theta)]dF(\theta) > 0.$$

Thus, the school is worse off in the unobserved case. □

The difference between the observed and unobserved case is driven by a signal jamming effect. The worker's signal is "jammed" in the unobserved case since the labor market does not observe the actual cost of education. Specifically, the labor market cannot distinguish the impact of a change in tuition from that of cost heterogeneity on the change in education. To illustrate, suppose that the school lowers tuition so that the worker chooses more education than in the initial state. When the labor market observes the tuition change, it cuts wages, as any education level now corresponds to a lower-ability worker. In contrast, when the labor market does not observe the tuition change, it does not adjust wages despite that tuition changes; thus, the worker is willing to pay more for additional education. Conversely, if the school raises tuition such that education decreases, then the labor market will *raise* wages in the observed case; thus, the worker's willingness to pay is lower in the unobserved case. This reveals that the worker is more sensitive to tuition changes in the unobserved case.

From the school's perspective, the demand is more elastic in the unobserved case. Note that the LHS of (1.9) represents the marginal profit of education in the unobserved case; the second term represents the signal jamming effect and is positive. In comparison, in the observed case, rearranging the first-order condition of $MP^o(z,\theta)$, we have

$$MP^o_z(z^o(\theta),\theta) = Q_z(z^o(\theta),\theta) - G_z(z^o(\theta),\theta).$$

Thus, the school's marginal profit is higher in the unobserved case than in the observed case. This provides the school with an incentive to "fool" the labor market with secret price cuts; that is, the school secretly supplies more education and persuades the labor market that the worker is more productive than is actually the case. In equilibrium, the labor market correctly anticipates the schools incentive and offers lower wages, as education is inflated. This reduces the worker's willingness to pay, and thus, the school achieves lower profits.

1.5.1 Implications for Tuition Transparency

I have shown that tuition cuts lead to smaller increases in demand in the observed case than in the unobserved case. This is because when the tuition cuts are publicly observed, the increase in demand is mitigated by the cheaper tuition reducing the signaling value of education. Hence, tuition cuts are less profitable in the observed case. Here, I show further that tuition is always more expensive in the observed case. Specifically, the tuition scheme in the unobserved case is uniformly lower than that in the observed case over the common

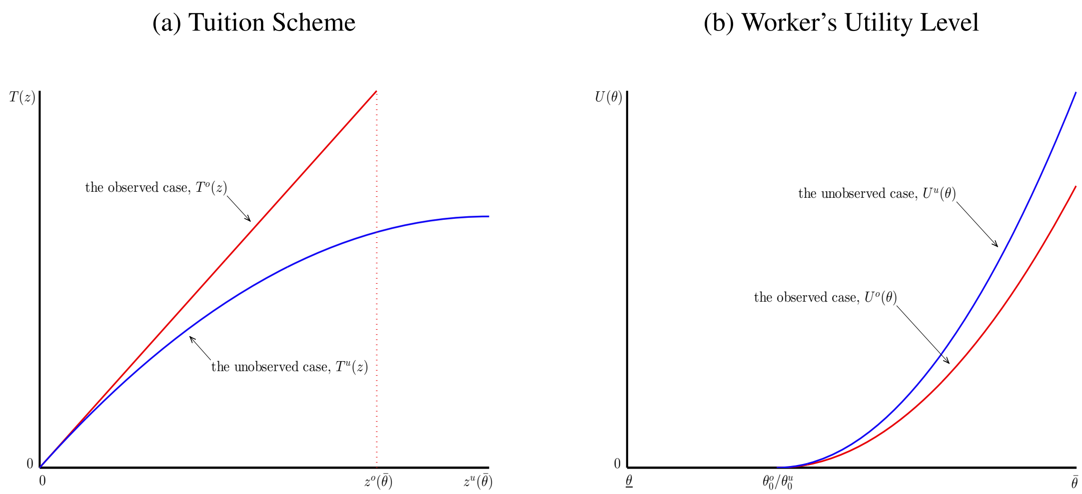

Figure 1.3: Implications for Tuition Transparency

This figure compares tuition rates and the worker's utility level between the observed and unobserved case. This figure considers the same numerical example as Figure 1.1, such that (a) $T^o(z) = \frac{z}{3}$ and $T^u(z) = -\frac{z^2}{4} + \frac{z}{3}$; (b) $U^o(\theta) = \frac{3}{4}(\theta - \frac{1}{3})^2$ and $U^u(\theta) = (\theta - \frac{1}{3})^2$.

domain of education. This is illustrated in Panel (a) of Figure 1.3.

Proposition 1.5. *$T^u(z) \leq T^o(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$, with strict inequality for $z > z^o(\theta_0^o)$.*

Proof. From the worker's first-order condition in both cases, we have

$$\frac{d}{dz}[W^o(z) - T^o(z)] = C_z(z, \theta^o(z)) \text{ and } \frac{d}{dz}[W^u(z) - T^u(z)] = C_z(z, \theta^u(z)).$$

From Theorem 1.1, $z^o(\theta) \leq z^u(\theta)$ on $[\theta_0^o, \bar{\theta}]$. Since both $z^o(\theta)$ and $z^u(\theta)$ are increasing, $\theta^o(z) \geq \theta^u(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. Thus, $C_z(z, \theta^o(z)) \leq C_z(z, \theta^u(z))$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. This implies that $W^o(z) - T^o(z) \leq W^u(z) - T^u(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. Since $W^o(z) = Q(z, \theta^o(z))$ and $W^u(z) = Q(z, \theta^u(z))$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$, $W^o(z) \geq W^u(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. Hence, it is readily confirmed that $T^o(z) \geq T^u(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. □

Furthermore, from the worker's first-order condition in the unobserved case, we have

$$T^{u\prime}(z) = W^{u\prime}(z) - C_z(z, \theta^u(z)).$$

Substituting this equation into (1.9), and noticing that $W^u(z) = Q(z, \theta^u(z))$, we obtain

$$T^{u\prime}(z) = \frac{1 - F(\theta^u(z))}{f(\theta^u(z))} \left[-C_{z\theta}(z, \theta^u(z)) \right]. \tag{1.10}$$

Equation (1.10) states that in the unobserved case, the marginal tuition equals the marginal information rent extracted by the worker. In contrast to the observed case, as indicated by the comparison between (1.10) and (1.7), the optimal tuition scheme in the unobserved case does not undo the signaling effect. The reason is that the loss in the social surplus caused by over-education will be compensated by the labor market overpaying the worker, as the labor market will overestimate the worker's ability if the school secretly cuts tuition. In addition, (1.10) states that the marginal tuition vanishes at the highest education level. This implies that the school offers quantity discounts (i.e., $T(z)/z$ is declining) for higher education levels in the unobserved case. This echoes the classic screening model of Maskin and Riley (1984), in which quantity discounts are also optimal at the right tail of the distribution.

In terms of the worker's payoff, note that in both cases, the market belief about tuition is correct in equilibrium; thus, given the equilibrium tuition scheme, the continuation game is indeed Spence's signaling game as if the worker's cost function was given by the total cost. Because the tuition scheme is uniformly lower in the unobserved case, the signaling costs are lower in this case. Consequently, the worker has a higher utility level in the unobserved case than in the observed case. This is illustrated in Panel (b) of Figure 1.3. Formally, we have the following proposition.

Proposition 1.6. *$U^u(\theta) \geq U^o(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, with strict inequality for $\theta > \theta_0^o$.*

Proof. For $\theta \in (\theta_0^o, \bar{\theta}]$, by Lemma 1.1 and Theorem 1.1, we have

$$U^u(\theta) - U^o(\theta) = \int_{\theta_0^o}^{\theta} [C_\theta(z^o(s), s) - C_\theta(z^u(s), s)] ds > 0.$$

The inequality is due to $C_{z\theta} < 0$ and $z^o(\theta) < z^u(\theta)$. For $\theta \in [\underline{\theta}, \theta_0^o]$, $U^u(\theta) = U^o(\theta) = 0$. $\square$

Propositions 1.5 and 1.6 imply that policies that improve the transparency of net prices at colleges and universities through mandatory disclosure may unintentionally induce more expensive education and harm students. These policies, such as U.S. Code §1015a, require schools to publicly disclose their net prices, which are often not previously observed by employers. Such an intervention allows schools to commit to high prices and not dilute the signaling value of a high-cost education by means of fee waivers, financial aid, and so forth. Hence, the net prices that students actually pay may be higher under such policies.

1.5.2 Welfare and Education Comparison

Now, I conduct a welfare analysis for the unobserved case. As a reference point, note that $z^o(\theta_0^o) < z^{fb}(\theta_0^o)$ and $z^o(\bar{\theta}) = z^{fb}(\bar{\theta})$. Because $z^u(\theta) \geq z^o(\theta)$, holding strictly for $\theta > \theta_0^o$, continuity implies that $z^u(\theta)$ intersects $z^{fb}(\theta)$ from below at least once. Moreover, under some mild conditions—the following Assumption 1.2, for example—I show that $z^u(\theta)$ is single-crossing $z^{fb}(\theta)$, i.e., there is a unique cutoff type such that all lower types obtain less education than the first-best while the others obtain more than the first-best (see Figure 1.4).

Assumption 1.2. *The function*

$$Q_\theta(z^{fb}(\theta),\theta)\cdot\theta^{fb'}(z)+\frac{1-F(\theta)}{f(\theta)}C_{z\theta}(z^{fb}(\theta),\theta) \tag{$*$}$$

is single-crossing with respect to θ.

Proposition 1.7. *Given Assumption 1.2, there exists a unique type* $\theta^w \in (\theta^*,\bar{\theta})$ *such that* $z^u(\theta) < z^{fb}(\theta)$ *on* $[\underline{\theta},\theta^w)$ *and* $z^u(\theta) > z^{fb}(\theta)$ *on* $(\theta^w,\bar{\theta}]$*, where* $\theta^* > \theta_0^u$ *is the root of* $(*)$.

In the unobserved case, there are two competing forces that pull the education function away from the first-best benchmark. On the one hand, the signal jamming effect provides the school with an incentive to supply more education. On the other hand, more education means more information rents to the worker. Since the cost of information rents ultimately vanishes as type approaches the top, the school unambiguously over-supplies education on

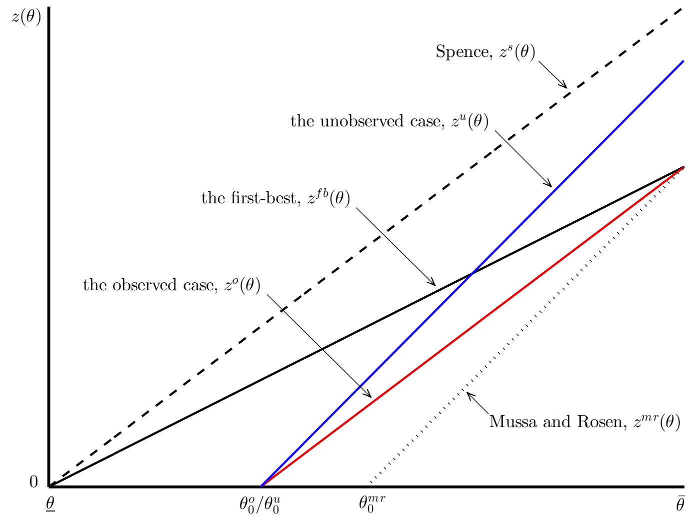

Figure 1.4: Equilibrium Education Functions

This figure illustrates all the equilibrium education functions that have been discussed in the paper. This figure considers the same numerical example as Figure 1.1, such that $z^u(\theta) = 2\theta - \frac{2}{3}$, and recall that $z^{fb}(\theta) = \theta$, $z^s(\theta) = \frac{3}{2}\theta$, $z^o(\theta) = \frac{3\theta-1}{2}$ and $z^{mr}(\theta) = 2\theta - 1$.

some upper interval of the spectrum. Assumption 1.2 ensures that the relative significance of the two forces alters only once, thus it rules out the possibility of multiple intersections between $z^u(\theta)$ and $z^{fb}(\theta)$. Proposition 1.7 reveals that under-education is slighter on a lower interval of the spectrum in the unobserved case than in the observed case; it also provides a lower bound for the length of this interval.

However, since over-education also occurs in the unobserved case, whether the observed or unobserved case yields higher social welfare remains ambiguous. Heuristically, if there is slight under-education in the observed case, then over-education will be a relatively more serious issue in the unobserved case; thus, the observed case will yield higher social welfare. Recall that in the observed case, the more intense signaling is, the slighter under-education there is. Thus, the more intense signaling is, the greater over-education in the unobserved case, as the school will find it more profitable to fool the labor market by secretly suppling more education. To illustrate, I revisit the example in Section 1.4.

Example Assume that $Q(z,\theta) = \gamma\theta z + z$ with $\gamma > 0$, $C(z,\theta) = z^2 + z - \theta z$, and $\theta \sim U[0,1]$. Applying Proposition 1.4, we have $z^u(\theta) = (\gamma+1)\theta - \frac{\gamma+1}{\gamma+2}$. Recall that $z^{fb}(\theta) = \frac{(\gamma+1)\theta}{2}$ and $z^o(\theta) = \frac{(\gamma+2)\theta-1}{2}$. It is readily confirmed that the welfare cutoff type $\theta^w = \frac{2}{\gamma+2}$, which is decreasing in γ. This implies that the over-education region is increasing in the intensity of signaling. Moreover, the difference in social welfare between the two cases is given by

$$\int_{\theta_0^o}^{\bar{\theta}} [S(z^o(\theta),\theta) - S(z^u(\theta),\theta)]\, dF(\theta) = \frac{\gamma(\gamma-1)(\gamma+1)^3}{12(\gamma+2)^3}.$$

Clearly, the RHS is positive if and only if $\gamma > 1$; that is, if signaling is sufficiently intense, then the observed case yields higher social welfare than the unobserved case.

My last proposition indicates that the education function in the unobserved case $z^u(\theta)$ is bounded above by that of Spence's signaling game $z^s(\theta)$. This is illustrated in Figure 1.4.

Proposition 1.8. *In the unobserved case, the worker chooses strictly less education than in Spence's signaling game, that is, $z^u(\theta) < z^s(\theta)$ on $[\underline{\theta}, \bar{\theta}]$.*

The intuition is clear, as the unobserved case is essentially Spence's signaling game with higher costs, meaning that it yields lower education levels. Proposition 1.8 implies that if signaling is sufficiently intense so that over-education is prevalent in the unobserved case, then social welfare is higher in the unobserved case than in Spence's model.[11]

In this paper, I performed a pairwise comparison between different education functions. I first showed that signaling alone leads to over-education, i.e., $z^s(\theta) > z^{fb}(\theta)$. Then, after accounting for the school's strategic behavior, the equilibrium education functions vary with the labor market's information. In Table 1.1, I summarize the correspondence between the equilibrium education function and information structure.

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[11] In the previous numerical example, if and only if γ is larger than some cutoff that is less than $\frac{\sqrt{5}-1}{2}$, the unobserved case yields higher social welfare than Spence's model. Thus, I have derived all the three cutoffs for a pairwise welfare comparison between Spence's model, the observed and unobserved case. These cutoffs partition the domain of γ into four divisions in which the three cases rank differently in terms of social welfare. It is clear that as the intensity of signaling rises (i.e., γ increases), the case that yields the highest social welfare will be, respectively, Spence's model, the unobserved case and the observed case.

Table 1.1: Education Functions under Different Information Structures

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="2">Labor Market Observes Tuition</td></tr>
<tr><td>No</td><td>Yes</td></tr>
<tr><td rowspan="2">Labor Market Observes Type</td><td>No</td><td>$z^u(\theta)$</td><td>$z^o(\theta)$</td></tr>
<tr><td>Yes</td><td colspan="2">$z^{mr}(\theta)$</td></tr>
</table>

As illustrated by Table 1.1, when the labor market observes the worker's ability and the school's tuition scheme, the model is Mussa and Rosen's screening game. A higher-ability worker benefits from his productivity and cost advantage over others. To incentivize truth-telling, the school leaves information rents to the worker and thus under-supplies education, that is, $z^{mr}(\theta) < z^{fb}(\theta)$. When the labor market observes only the tuition scheme, a higher-ability worker cannot benefit directly from his productivity advantage, and thus, signaling arises. Signaling mitigates the screening distortion since the school incurs lower information rents that stem from worker cost heterogeneity only. Thus, $z^{mr}(\theta) < z^o(\theta) < z^{fb}(\theta)$. Finally, when the labor market observes neither the tuition scheme nor the worker's ability, the worker becomes more sensitive to tuition changes, and thus, the demand for education is more elastic than in the observed case. This makes price cuts relatively more profitable for the school and induces it to supply more education; therefore, $z^o(\theta) < z^u(\theta)$.

Remark. *If education is a pure signal, all the results in this section are valid. Specifically, in the school-optimal separating equilibrium, the lowest type has no education and the others obtain increasing and positive amounts of education. Recall that in the observed case, the lowest type has zero education and the other types obtain infinitesimal education. Thus, the main result still holds. Moreover, in the unobserved case the school gains lower profits while the worker has a higher utility level than in the observed case. Finally, because education is unproductive and the unobserved case yields higher education levels, social welfare is higher in the observed case. See the Appendix for further details.*

1.6 Summary and Discussion

In this paper, I developed classic signaling models by allowing a third party to affect the signaling cost. I used this framework to analyze how a school with market power, e.g., a top business school, manages job market signaling by designing a tuition scheme. The equilibrium depends critically on whether employers observe the tuition scheme. In the observed case, the school internalizes the worker's signaling incentive when screening his type, causing under-education. In the unobserved case, the worker's signal is jammed and he is more sensitive to tuition changes. This leads to a more elastic demand for education and induces the school to lower tuition rates. In equilibrium, the worker chooses more education and obtains higher utility than in the observed case, whereas the school achieves lower profits than in the observed case.

My framework has policy implications for the transparency of tuition at colleges and universities. Mandatory disclosure policies, such as U.S. Code §1015a, make the net prices of education public information. On the one hand, this reduces the search costs of students, thereby stimulating the competition between schools and lowering prices; on the other hand, this also allows schools to commit to high prices and not dilute the signaling value of a high-cost education by means of fee waivers, financial aid and so forth. It is thus possible that such policies ultimately raise education costs and harm students. Hence, policymakers should not overlook the potential drawbacks of these mandatory disclosure policies.

My framework has welfare implications for the market structure of signals. When the market is served by perfectly competitive sellers, the equilibrium outcome is predicted by Spence's model; when the market is served by a monopoly with a publicly observed price schedule, the equilibrium outcome is predicted by the observed case. We show that when the buyer's signaling incentive is sufficiently strong, a monopoly can yield higher social welfare than a perfectly competitive market. This implies that introducing competition among the sellers of signals is not necessarily socially beneficial.

My framework also draws attention to the positive side of grade inflation at colleges and universities. Grade inflation, which is documented by a large body of empirical work (e.g., Johnson 2006, Rojstaczer and Healy 2010), induces students to obtain more education to signal their intrinsic and unobserved abilities. My model suggests that when schools have market power and under-supply education due to screening, grade inflation can mitigate the screening distortion by encouraging education, thereby raising social welfare.

1.6.1 Applications of the Model

In addition to job market signaling, my model can be applied to other vertical relationships in which signaling prevails, such as conspicuous consumption and advertising. In the case of conspicuous consumption, a retailer (*principal*) chooses a price schedule $T(z)$ for a luxury good, where z denotes the quality of the good. Then, as in Bagwell and Bernheim (1996), a consumer (*agent*) chooses the quality of the good he will purchase to signal his unobserved wealth (*type*) θ to the social contact (*market*); the social contact observes z and forms some belief about θ. In the spirit of the seminal work of Veblen (1899), the social contact rewards the consumer based on z. The reward scheme $W(z)$ is given by the social contacts expected benefit $\mathbb{E}[Q(z,\theta)]$ from the consumer; the function $Q(z,\theta)$ is increasing in both arguments, as the social contact obtains higher utility by making friends with richer people and sharing goods of higher quality with them. Moreover, the consumer derives intrinsic utility from the luxury good. The intrinsic utility is denoted by $V(z,\theta)$, which is increasing in the quality z. More important, the single-crossing condition holds: $V_{z\theta}(z,\theta) > 0$. This condition captures the feature that a wealthier individual has higher marginal utility from consuming a luxury good. For example, a buyer of a yacht can voyage more often if he is richer, as he is better able to afford the fuel costs and maintenance fees. In terms of payoffs, the retailers profit equals the revenue $T(z)$ minus the cost $C(z)$; the consumer's net utility equals the reward $W(z)$ plus the intrinsic utility $V(z,\theta)$ and minus the price $T(z)$. Then, given such a similar setup, one can easily replicate the analysis we have conducted in the education application.

Then, I turn to the application of advertising. In this case, a media company (*principal*) chooses a price schedule $T(z)$ for advertising messages, where z denotes advertising level. Then, as in Milgrom and Roberts (1986), a producer (*agent*) that has just developed a new product chooses its advertising level to signal the unobserved quality (*type*) θ of the product to consumers (*market*); consumers observe z and form some belief about θ. The producers revenue has two sources: the purchase in the introductory stage and the repeat purchase in the post-introductory stage. The introductory revenue $R^i(z,\hat{\theta})$ depends on the advertising level z and the expected quality $\hat{\theta}$, and is increasing in both arguments, as more advertising results in higher consumer awareness, and better consumer perception allows the producer to charge a higher price. In contrast, the post-introductory revenue $R^p(z,\theta)$ depends on the actual quality θ instead of $\hat{\theta}$, since the products quality is revealed after the introductory stage. $R^p(z,\theta)$ is increasing in z, as more introductory advertising results in a larger base of repeat purchase. More important, the single-crossing condition holds: $R^p_{z\theta}(z,\theta) > 0$. This is due to the complementarity between advertising and quality; that is, the marginal revenue of the introductory advertising is higher if the product is of higher quality, thereby allowing the producer to charge higher prices in the post-introductory stage. In terms of payoffs, the media company's profit equals the revenue $T(z)$ minus the cost $C(z)$; the producer's profit equals the sum of the introductory revenue $R^i(z,\hat{\theta})$ and post-introductory revenue $R^p(z,\theta)$ minus the price $T(z)$, with production costs normalized to zero. In particular, if $R^i(z,\hat{\theta})$ is linear in $\hat{\theta}$, then $R^i(z,\hat{\theta}) = \mathbb{E}[R^i(z,\theta)]$, and thus, the setup is similar to the education application and the analysis will be analogous.

A remark on the advertising model is as follows: the introductory price that is chosen by the producer may also be used as a signal of quality, as in Milgrom and Roberts (1986). In this regard, the producer faces a trade-off between signaling by price and signaling by advertising, depending on which channel is more effective; it is possible that both types of signal coexist in equilibrium. In turn, the possibility of multiple signals will also affect the media company's pricing strategy, making the analysis more complicated.

1.6.2 Extensions of the Model

The current paper's results still hold if we change non-linear tuition to linear tuition or change continuous types to discrete types. A somewhat special case is linear tuition with discrete types. Without loss of generality, suppose that there are only two types, low and high, and the school chooses a uniform tuition rate. In the observed case, the least-cost separating equilibrium exists, in which the high type obtains more education than the low type, and the latter is indifferent between revealing own type and imitating the former. In the unobserved case, however, such an equilibrium does not exist because the high education level is so high that the low type strictly prefers to reveal his type. The intuition is similar: the high education level must be relatively too high for the low type to imitate the high type, such that the school finds it unprofitable to cut the price and gain market share.

A new economic force arises in this case as the school is unable to price discriminate. That is, when the high type strictly prefers to separate himself from the low type, the school has an incentive to squeeze him by raising tuition. The reason is that the high type is less sensitive to tuition changes, as a decrease in education will cause him to be regarded as the low type even if this decrease is due to higher tuition. Therefore, the school faces a trade-off between squeezing the high type and maintaining the low type's market share. If the gap between the two types and the proportion of the high type are large enough, squeezing the high type is relatively more profitable, such that in equilibrium the low type is excluded from education and the high type is indifferent between choosing the equilibrium high education level and deviating downward optimally.

1.7 Appendix

1.7.1 Omitted Proofs

Proof of Proposition 1.1.

Proof. Let $U(\theta,\hat{\theta},z)$ be type-θ's payoff if he chooses education level z and is believed as type-$\hat{\theta}$. In particular, $U(\theta,\theta,z)$ equals $S(z,\theta)$, which is strictly quasiconcave in z and has a unique maximizer. Moreover, $U_2(\theta,\hat{\theta},z) = Q_\theta(z,\hat{\theta}) > 0$, $U_{13}(\theta,\hat{\theta},z) = -C_{z\theta}(z,\theta) > 0$, and $U_3(\theta,\hat{\theta},z)/U_2(\theta,\hat{\theta},z)$ is increasing in θ. Appealing to Mailath (1987, Theorem 3), I prove that a function $z(\theta)$ is incentive compatible if it satisfies (i) and (ii) of Proposition 1.1 and such a function uniquely exists given the initial condition. I assume that the market holds the worst belief off the equilibrium path, so that no type is willing to deviate to there. Thus, the least-cost separating equilibrium exists and is characterized by Proposition 1.1. □

Proof of Proposition 1.4.

Proof. I first prove that a separating equilibrium exists in the unobserved case. Fix some admissible initial point $(\theta_0^u, z^u(\theta_0^u))$. Given Assumption 1.1, $MP^o(z,\theta)$ is regular. Replacing $C(z,\theta)$, $U(z,\theta)$ and $S(z,\theta)$ by $G(z,\theta)$, $MP^u(z,\theta)$ and $MP^o(z,\theta)$, respectively, we obtain immediately the existence by replicating the proof of Proposition 1.1.

To find the school-optimal separating equilibrium, it suffices to pin down the initial point. I consider two cases. First, $\theta_0^o = \underline{\theta}$. Note that the lowest possible wage for any education level $z > 0$ is $Q(z,\underline{\theta})$. Thus, for every pair (z,θ) with $z > 0$, we have

$$MP^u(z,\theta) \geq Q(z,\underline{\theta}) - G(z,\theta) \geq Q(z,\underline{\theta}) - G(z,\underline{\theta}) = MP^o(z,\underline{\theta}).$$

The second inequality is due to $G_{z\theta} < 0$ if $z > 0$. Since $\theta_0^o = \underline{\theta}$, $MP^u(z^o(\underline{\theta}),\underline{\theta}) \geq 0$; that is, the marginal profit of the lowest type can be at least non-negative. Thus, $\theta_0^u = \underline{\theta}$. Note too that $MP^u(z^u(\theta),\theta) = MP^o(z^u(\theta),\theta)$, as types reveal in equilibrium. Then, it is optimal for the school to choose $z^u(\underline{\theta}) = z^o(\underline{\theta})$, because the labor market cannot punish this choice by holding a worse belief than $\underline{\theta}$ and $z^o(\underline{\theta})$ maximizes $MP^o(z,\underline{\theta})$ by definition. Thus, if $\theta_0^o = \underline{\theta}$, then the separating equilibrium outcome is unique such that $(\theta_0^u, z^u(\theta_0^u)) = (\underline{\theta}, z^o(\underline{\theta}))$.

Second, $\theta_0^o > \underline{\theta}$. In this case, $(\theta_0^u, z^u(\theta_0^u))$ and thus the equilibrium outcome is not unique. From Mailath (1987, Theorem 3), for every separating equilibrium, $z^u(\theta)$ satisfies (1.9) and is increasing. Analogously to the proof of Theorem 1.1, we have $z^u(\theta) \geq z^o(\theta)$ on $[\theta_0^u, \bar{\theta}]$ with strict inequality for $\theta > \theta_0^u$. This implies that $MP^u(z^u(\theta),\theta) \leq MP^o(z^o(\theta),\theta)$, as $z^o(\theta)$ is the unique maximizer of $MP^o(z,\theta)$, and $MP^u(z^u(\theta),\theta) = MP^o(z^u(\theta),\theta)$. By the definition of the cutoff type, we have that $\theta_0^u \geq \theta_0^o$ in every separating equilibrium of the unobserved case. Thus, we have determined the lower bound of $(\theta_0^u, z^u(\theta_0^u))$. In Section 1.7.2, I will show that the school-optimal separating equilibrium exists in this case such that $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$. In summary, in both cases, we have $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$. Therefore, Proposition 1.4 is proven. □

Proof of Proposition 1.7.

Proof. We only need to study the interval $(\theta_0^u, \bar{\theta})$. I have shown that $z^u(\theta)$ intersects $z^{fb}(\theta)$ from below at least once. Note that $z^u(\theta_0^u) = z^o(\theta_0^u) < z^{fb}(\theta_0^u)$ and $z^u(\bar{\theta}) > z^o(\bar{\theta}) = z^{fb}(\bar{\theta})$. If there are multiple intersections, then $z^u(\theta)$ intersects $z^{fb}(\theta)$ at least three times. Denote by $W^{fb}(z)$ the wage schedule in the first-best benchmark. Since both $W^u(z)$ and $W^{fb}(z)$ are increasing, it suffices to prove that $W^u(z)$ intersects $W^{fb}(z)$ only once. Suppose that $z^u(\theta)$ intersects $z^{fb}(\theta)$ at some θ^w, then $W^u(z)$ intersects $W^{fb}(z)$ at $z^{fb}(\theta^w)$. By differentiation,

$$W^{u\prime}(z^{fb}(\theta^w)) = C_z(z^{fb}(\theta^w),\theta^w) - \frac{1-F(\theta^w)}{f(\theta^w)} C_{z\theta}(z^{fb}(\theta^w),\theta^w),$$
$$W^{fb\prime}(z^{fb}(\theta^w)) = Q_z(z^{fb}(\theta^w),\theta^w) + Q_\theta(z^{fb}(\theta^w),\theta^w) \cdot \theta^{fb\prime}(z^{fb}(\theta^w)).$$

The first equation results from the first-order condition of $MP^u(z,\theta)$; the second is just the total derivative of $W^{fb}(z)$.

Then, from (1.1), we have that $W^{fb\prime}(z^{fb}(\theta^w)) - W^{u\prime}(z^{fb}(\theta^w))$ equals

$$Q_\theta(z^{fb}(\theta^w),\theta^w)\cdot\theta^{fb\prime}(z^{fb}(\theta^w)) + \frac{1-F(\theta^w)}{f(\theta^w)}C_{z\theta}(z^{fb}(\theta^w),\theta^w).$$

Given Assumption 1.2, the RHS can change its sign only once for different values of θ^w. Suppose that $z^u(\theta)$ intersects $z^{fb}(\theta)$ more than once, then the directions of the first three intersections are from below, from above, and from below; thereby, $W^u(z)$ intersects $W^{fb}(z)$ first from above, then from below, and then from above. This means that the LHS of the above equation will change its sign more than once, a contradiction. Thus, we have that $z^u(\theta)$ intersects $z^{fb}(\theta)$ only once and from below. Then, $W^{fb\prime}(z^{fb}(\theta^w)) - W^{u\prime}(z^{fb}(\theta^w)) > 0$, meaning that the RHS of the above equation is positive. By the definition of θ^*, we have

$$Q_\theta(z^{fb}(\theta^*),\theta^*)\cdot\theta^{fb\prime}(z^{fb}(\theta^*)) + \frac{1-F(\theta)}{f(\theta)}C_{z\theta}(z^{fb}(\theta^*),\theta^*) = 0.$$

It follows from Assumption 1.2 that $\theta^w > \theta^*$. Therefore, the proposition is proven. □

Proof of Proposition 1.8.

Proof. We only need to prove that $z^u(\theta) < z^s(\theta)$ on $[\theta_0^u, \bar{\theta}]$. From the first-order conditions, we can derive $W^u(z)$ and $W^s(z)$, respectively, by the initial value problems (IVP) below:

$$W^{u\prime}(z) = G_z(z,\theta^u(w,z)) \text{ and } W^{s\prime}(z) = C_z(z,\theta^s(w,z)),$$

with the initial points $(z^o(\theta_0^o), W^u(z^o(\theta_0^o)))$ and $(z^o(\theta_0^o), W^s(z^o(\theta_0^o)))$ for $W^u(z)$ and $W^s(z)$, respectively. It is easy to see that $G_z(z,w) \geq C_z(z,w)$ in any common domain of (z,w). From Corollary 1.2, we have $z^o(\theta) < z^s(\theta)$ on $[\underline{\theta},\bar{\theta}]$, and thus, $\theta_0^o > \theta^s(z^o(\theta_0^o))$. This implies that $W^u(z^o(\theta_0^o)) > W^s(z^o(\theta_0^o))$. Then, appealing to Hartman (1964, Corollary 4.2, page 27), we have $W^u(z) > W^s(z)$ in any common domain. This implies that $\theta^u(z) > \theta^s(z)$ in any common domain; therefore, $z^u(\theta) < z^s(\theta)$ on $[\theta_0^u, \bar{\theta}]$. The proposition is thus proven. □

1.7.2 Equilibrium Selection for the Unobserved Case

Here, I discuss equilibrium selection for the unobserved case. I present two lemmas. By Lemma 1.2, I characterize the school-optimal separating equilibrium given that $\theta_0^o > \underline{\theta}$; by Lemma 1.3, I show that the school-optimal separating equilibrium is also the least-cost separating equilibrium with respect to the total cost of education.

Lemma 1.2. *Given that $\theta_0^o > \underline{\theta}$, the school-optimal separating equilibrium exists in the unobserved case, such that $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$.*

Proof. As a first step, I show that the cutoff type's education level $z^u(\theta_0^u)$ is an increasing function of θ_0^u. From the proof of Proposition 1.4, we have $\theta_0^u \geq \theta_0^o > \underline{\theta}$. Thus,

$$MP^u(z^u(\theta_0^u), \theta_0^u) = MP^o(z^u(\theta_0^u), \theta_0^u) = 0.$$

Given Assumption 1.1, $MP^o(z, \theta)$ is regular; $z^o(\theta_0^u)$ is the unique maximizer of $MP^o(z, \theta_0^u)$. From the proof of Proposition 1.4, we have $z^u(\theta_0^u) \geq z^o(\theta_0^u)$ for each separating equilibrium. Then, regularity implies that $z^u(\theta_0^u)$ is the unique solution to the above equation given an admissible θ_0^u, and $z^u(\theta_0^u)$ is increasing. Thus, $z^u(\theta_0^u)$ is an increasing function of θ_0^u.

Second, I show that for any two admissible initial points $(\hat{\theta}_0^u, \hat{z}^u(\theta))$ and $(\tilde{\theta}_0^u, \tilde{z}^u(\theta))$, if $\hat{\theta}_0^u < \tilde{\theta}_0^u$, then $\hat{z}^u(\theta) < \tilde{z}^u(\theta)$ in any common domain. From Mailath (1987, Theorem 3), for every separating equilibrium, $z^u(\theta)$ satisfies (1.9). Rearranging (1.9), we have

$$z^{u\prime}(\theta) = \frac{Q_\theta(z^u(\theta), \theta)}{G_z(z^u(\theta), \theta) - Q_z(z^u(\theta), \theta)}.$$

From the first paragraph, if $\hat{\theta}_0^u < \tilde{\theta}_0^u$, then $\hat{z}^u(\hat{\theta}_0^u) < \tilde{z}^u(\tilde{\theta}_0^u)$. Appealing to Hartman (1964, Corollary 4.2, page 27), we have $\hat{z}^u(\theta) < \tilde{z}^u(\theta)$ in the common domain $[\tilde{\theta}_0^u, \bar{\theta}]$.

Third, I characterize $z^u(\theta)$ for all separating equilibria. To do so, I have to determine the domain of $z^u(\theta_0^u)$, which depends on the market belief off the equilibrium path. As have been shown, the lower bound of θ_0^u is θ_0^o, which is supportable if any off-path education is

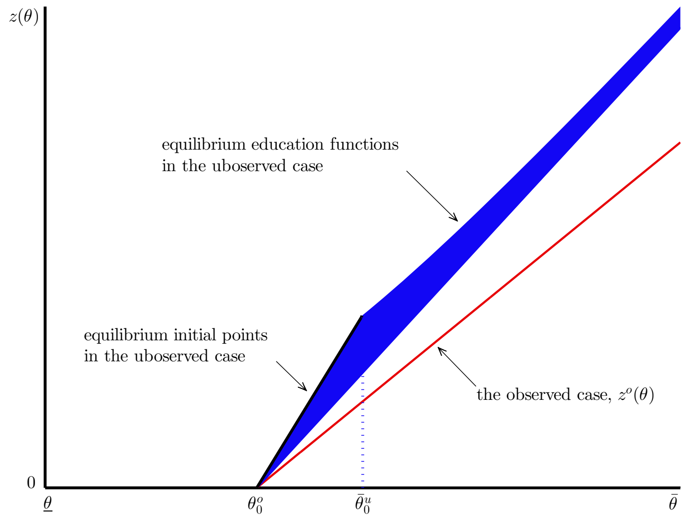

Figure 1.5: The Set of Separating Equilibria in the Unobserved Case

This figure illustrates the family of separating equilibria in the unobserved case given that $\theta_0^o > \underline{\theta}$. The blue area depicts the set of equilibrium education functions. This region is uniformly above the equilibrium education function in the observed case $z^o(\theta)$. The bold line is the set of equilibrium initial points with the cutoff type ranging from θ_0^o to $\bar{\theta}_0^u$. Each point uniquely determines an equilibrium education function $z^u(\theta)$ and thus an equilibrium outcome. This figure considers the same numerical example as Figure 1.1, such that the set of the initial points is $\{(\theta,z)|z(\theta) = 3\theta - 1; \frac{1}{3} \leq \theta \leq \frac{1}{2}\}$.

believed to be chosen by type θ_0^o. As the off-path belief gets gradually harsher, θ_0^u increases continuously, until the labor market holds the worst belief $\underline{\theta}$ off the equilibrium path. It is without loss of generality to confine the off-path education to $[0, z^u(\theta_0^u))$ when $z^u(\theta_0^u) > 0$. Denote by $\bar{\theta}_0^u$ the upper bound of θ_0^u, which is pinned down by

$$\max_{z<z^u(\bar{\theta}_0^u)} \{Q(z,\underline{\theta}) - G(z,\bar{\theta}_0^u)\} = MP^u(z^u(\bar{\theta}_0^u), \bar{\theta}_0^u) = 0.$$

That is, the school is indifferent between allocating type-$\bar{\theta}_0^u$ the optimal off-path education such that it is believed as type-$\underline{\theta}$ and maintaining the equilibrium allocation. Therefore, we have determined the domain of $z^u(\theta_0^u)$. Then, picking any $\theta_0^u \in [\theta_0^o, \bar{\theta}_0^u]$, we can uniquely pin down a $z^u(\theta)$. Figure 1.5 illustrates the education functions of all separating equilibria.

Finally, I prove that the initial point of the school-optimal separating equilibrium is $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$. Choose two equilibrium education functions, $\hat{z}^u(\theta)$ and $\tilde{z}^u(\theta)$, such that $\hat{z}^u(\theta) < \tilde{z}^u(\theta)$ on the common support $[\tilde{\theta}_0^u, \bar{\theta}]$. Since $z^u(\theta) \geq z^o(\theta)$ on $[\theta_0^u, \bar{\theta}]$ for every separating equilibrium, we have $\hat{z}^u(\theta) - z^o(\theta) < \tilde{z}^u(\theta) - z^o(\theta)$ on $[\tilde{\theta}_0^u, \bar{\theta}]$. Thus, regularity implies that $MP^u(\hat{z}^u(\theta), \theta) > MP^u(\tilde{z}^u(\theta), \theta)$ on $[\hat{\theta}_0^u, \bar{\theta}] \supset [\tilde{\theta}_0^u, \bar{\theta}]$. Then,

$$\Pi^u(\hat{\theta}_0^u) - \Pi^u(\tilde{\theta}_0^u) = \int_{\hat{\theta}_0^u}^{\bar{\theta}} MP^u(\hat{z}^u(\theta), \theta) dF(\theta) - \int_{\tilde{\theta}_0^u}^{\bar{\theta}} MP^u(\tilde{z}^u(\theta), \theta) dF(\theta) > 0.$$

The inequality is due to the fact that both the integrand and the integral domain of $\Pi^u(\hat{\theta}_0^u)$ are bigger than those of $\Pi^u(\tilde{\theta}_0^u)$. This result reveals that the lower the cutoff type, the higher the school's equilibrium payoff. Since $\theta_0^u \in [\theta_0^o, \bar{\theta}_0^u]$, the school-optimal separating equilibrium must be the one in which $\theta_0^u = \theta_0^o$, and thus, $z^u(\theta_0^u) = z^o(\theta_0^o)$. □

Lemma 1.3. *In the school-optimal separating equilibrium, the cutoff type θ_0^u chooses his full-information optimal education level under the total cost function $T^u(z) + C(z, \theta_0^u)$, i.e.,*

$$z^u(\theta_0^u) = \arg\max_z \; Q(z, \theta_0^u) - T^u(z) - C(z, \theta_0^u).$$

Proof. First, I characterize $T^u(z)$ on $\mathbb{R}_+$. On the equilibrium path, $T^u(z)$ is given by

$$T^u(z^u(\theta)) = S(z^u(\theta), \theta) - U^u(\theta) = S(z^u(\theta), \theta) + \int_{\theta_0^u}^{\theta} C_\theta(z^u(s), s) ds.$$

Then, I smoothly extend $T^u(z)$ to $\mathbb{R}_+$. First, from (1.10), we have $\lim_{z \to z^u(\bar{\theta})^-} T^{u\prime}(z) = 0$. It is thus natural to extend $T^u(z)$ horizontally upto $+\infty$. Second, if $z^u(\theta_0^u) > 0$, then I smoothly extend $T^u(z)$ to the left by extending the solution to the IVP that is defined by the differential equation in (1.10) and the initial condition that $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$, until $T^u(z)$ or z reaches 0, whichever is earliest. The rest part of $T^u(z)$ is fixed at 0. Thus, $T^u(z)$ is fully characterized on $\mathbb{R}_+$. To ensure that such $T^u(z)$ is incentive compatible, I simply assume that the labor market holds the worst belief $\underline{\theta}$ for any off-path education level, so that no type will deviate to the off-path.

Thus, given $T^u(z)$, it suffices to prove that the following first-order condition holds.

$$Q_z(z^u(\theta_0^u),\theta_0^u) - T^{u\prime}(z^u(\theta_0^u)) - C_z(z^u(\theta_0^u),\theta_0^u) = 0. \tag{1.11}$$

Note that $MP^o(z,\theta)$ is regular, $z^o(\theta_0^o)$ maximizes $MP^o(z,\theta_0^o)$ and $z^u(\theta_0^u) = z^o(\theta_0^o)$, thus

$$MP_z^o(z^u(\theta_0^u),\theta_0^u) = Q_z(z^u(\theta_0^u),\theta_0^u) - G_z(z^u(\theta_0^u),\theta_0^u) = 0.$$

Substituting $G_z(z^u(\theta_0^u),\theta_0^u)$ using (1.10) and the definition of $G(z,\theta)$, we obtain (1.11). Therefore, the lemma is proven. □

1.7.3 Unproductive Education

Here, I consider the case in which education is a pure signal. Without loss of generality, I assume that $Q(\theta) = \theta > 0$ on $[\underline{\theta},\bar{\theta}]$. Thus, the social surplus function $S(z,\theta)$ is decreasing in z, meaning that zero education is socially optimal, i.e., $z^{fb}(\theta) \equiv 0$.

I start with the observed case and focus on the school-optimal separating equilibrium. Unlike the productive education case, the worker's outside option is now endogenous, which depends on the off-path belief. Without loss of generality, I assume that the labor market holds the worst belief $\underline{\theta}$ for all positive off-path education levels. The equilibrium wage for zero education, $W(0)$, equals $\mathbb{E}[\theta|\theta \in [\underline{\theta},\theta_0)]$ with the market belief updated by Bayes' rule. Thus, the worker's outside option is endogenously given by $\mathbb{E}[\theta|\theta \in [\underline{\theta},\theta_0)]$. Analogously to Section 1.2.1, the school's problem can be stated as

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \left\{ S(z(\theta),\theta) + \frac{1-F(\theta)}{f(\theta)} C_\theta(z(\theta),\theta) - \mathbb{E}[\theta|\theta \in [\underline{\theta},\theta_0)] \right\} dF(\theta).$$

subject to $z(\theta)$ being increasing on $[\theta_0,\bar{\theta}]$. Substituting $Q(\theta)$ and $G(z,\theta)$ into the integral, we can succinctly write the school's value function as

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \{\theta - G(z(\theta),\theta) - \mathbb{E}[\theta|\theta \in [\underline{\theta},\theta_0)]\}\, dF(\theta).$$

Suppose that in equilibrium $\theta_0 > \underline{\theta}$, then by differentiating the value function with respect to θ_0 and rearranging, we have that the derivative equals

$$f(\theta_0) \cdot G(z(\theta_0),\theta_0) - \frac{f(\theta_0)}{F(\theta_0)} \left(\theta_0 - \mathbb{E}[\theta|\theta \in [\underline{\theta},\theta_0)]\right).$$

Note that $z(\theta_0)$ must be zero. Suppose not, then $z(\theta) > 0$ for all $\theta \in [\theta_0, \bar{\theta}]$, as $z(\theta)$ is increasing. But since the marginal profit $\theta - G(z,\theta)$ is decreasing in z, it is profitable to reduce all positive $z(\theta)$ by a fixed small amount, a contradiction. Hence, $G(z(\theta_0),\theta_0) = 0$. Note too that $\theta_0 > \mathbb{E}[\theta|\theta \in [\underline{\theta},\theta_0)]$. Hence, the above derivative is negative, meaning that the school can make a profitable deviation by lowering θ_0, a contradiction. Thus, $\theta_0 = \underline{\theta}$ in equilibrium. Then, the school's problem can be reduced to

$$\max_{z(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} [\theta - G(z(\theta),\theta)]\, dF(\theta) - \underline{\theta}.$$

subject to $z(\theta)$ being increasing on $[\underline{\theta}, \bar{\theta}]$. Note that the integrand is decreasing in z. Thus, the school has an incentive to allocate as little of education as possible to the worker. Since the school cannot charge the worker for zero education, the "optimal" allocation is increasing and infinitesimal education. Formally, $z^o(\theta)$ is increasing on $[\underline{\theta}, \bar{\theta}]$ with $z^o(\underline{\theta}) = 0$; for any $\varepsilon > 0$, we have $z^o(\theta) < \varepsilon$. Under this allocation, social welfare is arbitrarily close to the first-best level, and the school's payoff is equal to social welfare minus arbitrarily small information rents and a positive rent $\underline{\theta}$ for participation.

I now turn to the unobserved case. Similarly, I assume that the labor market holds the worst belief for all positive off-path education levels. Since the wage schedule is independent of the actual tuition scheme, the school's problem can be similarly stated as

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \{W(z) - G(z(\theta),\theta) - \mathbb{E}[\theta|\theta \in [\underline{\theta},\theta_0)]\} \, dF(\theta).$$

subject to $z(\theta)$ being increasing on $[\theta_0,\bar{\theta}]$. In equilibrium, $W(z(\theta)) = \theta$, thus, by the same argument as in the observed case, we have $\theta_0 = \underline{\theta}$. Consequently, the analysis in Section 1.5 is completely applicable to this case. Specifically, the school-optimal separating equilibrium exists in the unobserved case, such that $z^u(\underline{\theta}) = 0$, $z^u(\theta)$ is increasing over $[\underline{\theta},\bar{\theta}]$, and $z^u(\theta)$ satisfies the first-order condition

$$Q_\theta(z^u(\theta),\theta)\cdot\theta^{u\prime}(z^u(\theta)) - G_z(z^u(\theta),\theta) = 0.$$

Since $z^u(\theta)$ is positive on $\theta \in (\underline{\theta},\bar{\theta}]$, we have $z^u(\theta) \geq z^o(\theta)$ on $[\underline{\theta},\bar{\theta}]$, with strict inequality for $\theta > \underline{\theta}$; that is, the worker obtains more education in the unobserved case than in the observed case, and thus, Theorem 1.1 still holds. Since education is unproductive, social welfare is unambiguously higher in the observed case. Indeed, by the definition in Section 1.4, signaling intensity is infinity for unproductive education, as the equilibrium education levels are positive in Spence's model (Spence 1973), but the fist-best education level is zero; thus, the observed case yields higher social welfare. Since education levels are higher in the unobserved case, the school leaves more information rents to the worker, and thus, the worker is better off in the unobserved case. However, since social welfare is lower in the unobserved case, the school, which is the residual claimant, must be worse off than in the observed case. Thus, Corollary 1.3 also holds.

CHAPTER 2

Competitive Nonlinear Pricing for Signals

2.1 Introduction

Starting with the seminal work of Mussa and Rosen (1978) and Maskin and Riley (1984) on monopolistic nonlinear pricing, there is a large literature on nonlinear pricing in competitive settings. These models typically assume that buyers derive intrinsic value from consuming the products. Recently, Lu (2018) studies monopolistic nonlinear pricing for products that provide signaling values to consumers and assesses how the transparency of pricing affects the degree of signaling and welfare. In contrast, this paper studies nonlinear pricing for horizontally differentiated products that provide signaling values to consumers, and further investigates how (horizontal) competition affects sellers pricing strategies and the degree of signaling and welfare. The paper is also closely related to Rochet and Stole (2002) and Yang and Ye (2008) in the sense that the only substantial difference is that the products in this model have signaling value in addition to intrinsic value. Thus, the paper is complementary to the three recent papers, and establishes a close connection between each other.

In this paper, I derive the optimal symmetric price schedules, under different market structures, for horizontally differentiated products that provide signaling values to consumers with private information. The equilibrium depends critically on whether the signal receivers observe the sellers' price schedules, as well as on the market structure. I first consider the case in which a monopolist maximizes the joint profit of all products. When the receivers observe each product's price schedule, the (vertical) market is partially covered, and quantity is downward distorted if there is little horizontal differentiation. As consumers valuations for a product become more horizontally differentiated, the market coverage rises, and the downward distortion decreases. When the degree of horizontal differentiation is sufficiently high, for some intermediate level of signaling intensity, the monopolistic allocation can in fact achieve the first-best; for higher signaling intensities, quantity is upward distorted at the low end. In contrast, when the receivers do not observe any product's price schedule, the market is always partially covered, and the allocation is more dispersed than in the observed case. Specifically, an interval of higher types purchase more than in the observed case, with the highest types purchasing more than the first-best, whereas the rest types purchase less than in the observed case, with more types excluded from the market. When the market structure changes from monopoly to duopoly, in which each seller maximizes the profit of own product, market competition results in a higher market coverage and larger quantities for both the observed and unobserved case.

For the purpose of exposition, I present my model in terms of Spence's education model (Spence 1973) with productive education. In the model, two identical schools choose their own tuition scheme, and a worker chooses which school to attend and how much education to purchase to signal his privately known ability (*vertical type*) to competing employers. The worker's ability distributes uniformly over $[0, 1]$. Following Yang and Ye (2008), I model horizontal differentiation by assuming that the worker incurs transportation costs to attend school. The worker's distance to a school (*horizontal type*) distributes uniformly over $[0, \frac{1}{2}]$. As a benchmark, I consider the case in which there is no horizontal differentiation. Then, a symmetric Bertrand competition induces both schools to set price at the marginal cost, and the model returns to Spence's signaling game. In the least-cost separating equilibrium, all types except the lowest vertical type choose more education than the first-best, as they attempt to separate themselves from lower vertical types.

In Section 2.3, I consider the case in which a monopolist maximizes the joint profit of the two schools. I start with the observed case in which employers observe each school's tuition scheme. In the symmetric school-optimal separating equilibrium, when there is little horizontal differentiation, the vertical market is partially covered and has two segments: in the fully covered range, all horizontal types purchase education; in the partially covered range, only those close to either school purchase education. Moreover, all vertical types except the highest one purchase less education than the first-best. This result stands in contrast to that of the Bertrand-Spence benchmark. The downward distortion results from the interaction of three forces: market penetration, screening and signaling. Since a higher type can benefit from his cost advantage over lower types, the monopolist has to leave information rents to the worker to incentivize truth-telling. In the fully covered range, since the market share is maximized, the marginal profit of education is unambiguously lower than the social surplus, thus the monopolist under-supplies education. In the partially covered range, in contrast, the monopolist can benefit from rent provision to gain market share. However, when there is little horizontal differentiation, the screening effect is dominant, leading to a downward distortion. As the degree of horizontal differentiation rises, to maintain the market share in the partially covered range, the monopolist offers the worker a higher rent by both raising the market coverage and allocating more education to the worker.

When horizontal differentiation is sufficiently strong, the allocation depends critically on the intensity of signaling. As is pointed out by Lu (2018), in the monopoly observed case, signaling mitigates the screening distortion. This is because the worker's signaling incentive reduces his willingness to imitate lower types, and thus, the school leaves lower information rents to the worker than when signaling is absent. When signaling intensity is relatively low, screening outweighs signaling and market penetration, resulting in a downward distortion with a partially covered vertical market. When signaling intensity is at some intermediate level—when the workers productivity and cost heterogeneity are equally significant—the monopolistic allocation achieves the first-best for all types. That is, the effects of signaling and market penetration exactly offset that of screening, thereby restoring the social optimum. In contrast, full-efficiency can never occur when signaling is absent, because otherwise the monopolist had to offer the worker a rent equal to the social surplus, leading to zero profit. Again, this is because the worker extracts more information rents when signaling is absent.

For even higher signaling intensities, at the low end of the market where the monopolist has to charge very low prices to obtain market share, signaling outweighs screening, leading to over-education in this range.

Then, I turn to the unobserved case in which employers do not observe any school's tuition scheme. In the symmetric school-optimal separating equilibrium, the market coverage is lower, and education levels are more dispersed than in the observed case. Specifically, an interval of higher types choose more education than in the observed case, whereas the others choose less education than in the observed case. As in Lu (2018), this difference is driven by a *signal jamming effect*. Since employers cannot observe the actual cost of education, they will attribute a difference in education level to worker cost heterogeneity despite that tuition changes. Consequently, the worker's demand for education becomes more elastic than in the observed case. This provides the monopolist with an incentive to secretly supply more education. Suppose that, as in Lu (2018), there is no horizontal differentiation and thus the market contains only the fully covered range, then the vertical market is partially covered due to screening, and education levels are uniformly higher in the unobserved case than in the observed case. As the degree of horizontal differentiation rises, the partially covered range emerges, and the monopolist offers lower types more education to gain market share. However, due to incentive compatibility, doing so leaves higher types higher information rents. Since in the unobserved case those higher types already obtain higher rents than in the observed case, the monopolist finds it unprofitable to offer those lower types the same education levels as in the observed case. Therefore, at any positive degree of horizontal differentiation, opposite to higher types, an interval of lower types obtain less education in the unobserved case than in the observed case, meaning that the market coverage is lower in the unobserved case. The length of such an interval is increasing in the degree of horizontal differentiation and vanishes as the degree approaches zero.

In Section 2.4, I consider duopoly in which each school maximizes own profit given the others tuition scheme. Again, I start with the observed case. In contrast to monopoly, under duopoly, market competition results in a higher market coverage, higher education levels, and a higher equilibrium payoff to the worker. Intuitively, under duopoly, the two schools compete with each other in the fully covered range by providing the worker with more rents than under monopoly. This relaxes the incentive compatibility constraint for lower types. Specifically, each school fears less about allocating more education to lower types thereby providing higher types with more rents, as higher types will enjoy more rents anyway due to market competition. Therefore, the schools increase education supply for all participating types, and include some of those who are not served in the monopoly case.

In the unobserved case, based on numerical computation, I obtain qualitatively identical results as in the observed. However, the intuition is a bit subtler. Suppose that both schools retain the contract of the monopoly case, and thus, the labor market offers the same wage schedule. Then, given the other's tuition scheme, each school has an incentive to supply more education for two reasons. The first reason is the competition in rent provision between the two schools, as is suggested above. The second reason is that due to the signal jamming effect, each school has an incentive to secretly supply more education to "fool" the market thereby making a profitable deviation. Similarly, while higher types receive more education, so do lower types, as the incentive compatibility constraint relaxes. Thus, education levels are uniformly higher under duopoly than under monopoly; accordingly, the market coverage is higher under duopoly as well.

In Section 2.5, I conclude my paper. All omitted proofs are presented in the Appendix.

2.1.1 Related Literature

This paper is most closely related to three recent papers on nonlinear pricing: Rochet and Stole (2002), Yang and Ye (2008) and Lu (2018). Rochet and Stole (2002) studies both monopoly and duopoly nonlinear pricing in a Hotelling model. In this paper, horizontal types are interpreted as consumers outside options, thereby giving rise to random participation. Their analysis focuses on the case in which the vertical market is always fully covered. As such, they show that under monopoly, there is either bunching or efficient allocation at the bottom. Under duopoly, when the market is fully covered, both sellers offer an efficient cost-plus-fee tariff (Armstrong and Vickers 2001 has obtained a similar result).

Yang and Ye (2008) complements Rochet and Stole (2002)'s analysis by focusing on the case in which the lowest participating type is endogenously determined. By doing so, they investigate the effects of horizontal differentiation and competition on the market coverage and quality distortion. The paper shows that under monopoly, the vertical market is always partially covered and bunching never happens. Moreover, quantity is downward distorted with efficiency achieved only on the top. When the market structure changes from monopoly to duopoly, the market coverage rises, and quality distortion decreases.

In contrast to Rochet and Stole (2002) and Yang and Ye (2008), the products in my model possess signaling value. Signaling affects the equilibrium allocation by mitigating the screening distortion. In particular, when horizontal differentiation is sufficiently strong, for some certain level of signaling intensity, the monopolistic allocation can fully achieve the first-best; for higher signaling intensities, there is an upward distortion at the low end of the vertical market. These results cannot be obtained in the other two papers in which signaling is absent. Recently, Ye and Zhang (2017) studies monopolistic nonlinear pricing with consumer entry. Different from the mechanism in my paper, they show that consumer entry can mitigate the screening distortion too. Under certain conditions, the first-best can also be achieved by the monopolistic allocation.

Lu (2018) studies monopolistic nonlinear pricing for products that provide signaling values to consumers and examines the effects of the transparency of pricing on the degree of signaling and welfare. As in classic screening models, Lu (2018) makes two simplifying assumptions: the consumers possess one-dimensional private information, and make type-independent participation decisions. In contrast, the current paper studies nonlinear pricing for horizontally differentiated products with signaling value. Hence, the consumers have two-dimensional types and make type-dependent participation decisions. The results of Lu

(2018) can be regarded as the limit results of the current paper with respect to the degree of horizontal differentiation. Thus, there is no discontinuity in the results of Lu (2018) when we disturb the participation constraint somewhat.

There are several other papers that study nonlinear pricing for both horizontally and vertically differentiated products in competitive settings. For example, Gilbert and Matutes (1993), Stole (1995), Verboven (1999), Villas-Boas and Schmidt-Mohr (1999), Ellison (2005), and Armstrong and Vickers (2001). Like Rochet and Stole (2002), all these papers assume that the vertical market is always fully covered, thereby precluding the effects of horizontal competition on the market coverage.

2.2 The Model

Players and actions There are n schools, a worker and a competitive labor market. At the beginning of the game, each school i chooses a tuition scheme $T_i(z) : \mathbb{R}_+ \to \mathbb{R}_+$, where z stands for education level and $T_i(z)$ is the tuition at z. Then, observing all the tuition schemes, the worker chooses at most one school to attend, and upon attendance how much education to purchase from the school. The worker's education choice is thus characterized by which school he attends and how much education he chooses. Finally, the labor market offers the worker a wage equal to his expected productivity (see below).

The worker's productivity depends on his ability θ and education level z, irrespective of which school he attends.[1] The worker's ability θ is a random variable, which distributes uniformly over the unit interval: $\theta \sim U[0,1]$. Let $Q(z,\theta)$ be the productivity of a worker with ability θ and education level z. Specifically, I assume that

$$Q(z,\theta) = \gamma\theta z + z,$$

[1] That is, the education provided by each school is equally productive. Nonetheless, the wage offered by the labor market may still depend on which school the worker attends.

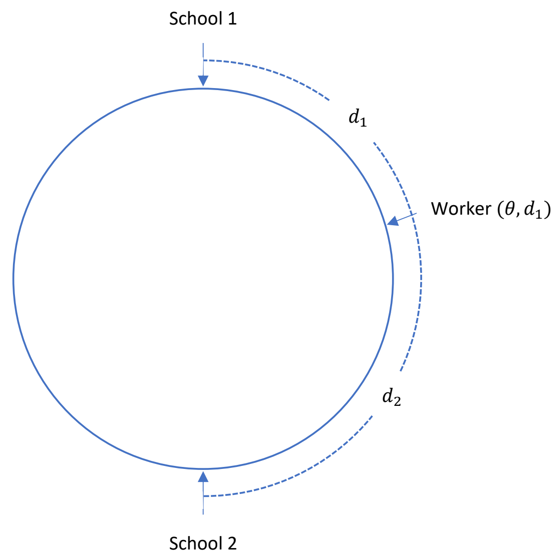

Figure 2.1: A Duopoly Education Market

where $\gamma > 0$ is a parameter. Thus, the productivity function is increasing in both arguments and is supermodular, meaning that both education and the worker's ability are productive, and complement each other. In addition, a worker with no education has zero productivity irrespective of his ability. This corresponds to that many jobs require a minimal education level. For example, a lawyer candidate must student from a law school, and medical school education is prerequisite for being a licensed practitioner of medicine.

The worker incurs a transportation cost if he attends a school. Specifically, the worker is located randomly and uniformly along a unit-length circle. The locations of all the schools split the circle evenly. Let d_i be the distance between the worker and school i. If the worker chooses to attend school i, then he incurs a transportation cost kd_i, where $k > 0$ is the unit transportation cost. Note that the worker's preference depends on his ability θ and his location that is summarized by $\{d_i\}$. Thus, the worker is characterized by a two-dimensional type $(\theta, \{d_i\})$, where the first preference parameter θ is called the worker's *vertical* type and the second parameter $\{d_i\}$ the worker's *horizontal* type, respectively, with both parameters independent of each other. Figure 2.1 illustrates the locations of two schools and the worker in a duopoly education market.

Information The worker's education choice is publicly observed. Whereas the distribution of the worker's type is common knowledge, neither the schools nor the labor market observes the worker's type. In this paper, for each market structure we consider, I study two variants of the model: in the *observed* case, all the tuition schemes are observed by the labor market; in the *unobserved* case, no tuition scheme is observed by the labor market. In each case, based on the available information, the labor market announces and commits to a wage schedule $W_i(z) : \mathbb{R}_+ \to \mathbb{R}_+$ for each school i's student.

Payoffs I normalize each school's production cost to zero. Thus, school i's per-customer profit equals the tuition revenue T_i. If a type-$(\theta, \{d_i\})$ worker attends school i and chooses education level z, then he obtains a *gross* utility given by

$$V_i(z,\theta) = W_i(z) - T_i(z) - C(z,\theta),$$

and accordingly a *net* utility given by

$$U_i(z,\theta,d_i) = V_i(z,\theta) - kd_i,$$

where $C(z,\theta)$ is the worker's cost of effort for education. Specifically, I assume that

$$C(z,\theta) = z^2 + (1-\theta)z.$$

Note that $C(z,\theta)$ is increasing and strictly convex in z, and that $C(0,\theta) = 0$ for any θ. More importantly, the standard *single-crossing property* holds: $C_{z\theta}(z,\theta) < 0$ if $z > 0$. This condition captures the feature that a higher-ability worker has lower marginal effort costs than a lower-ability worker. I also assume that the worker can obtain a zero-utility outside option by neither attending school nor entering the labor market.

First-best benchmark Define $S(z,\theta)$ as the social surplus function (which is net from the transportation cost). Then, we have

$$S(z,\theta) = Q(z,\theta) - C(z,\theta) = (\gamma+1)\theta z - z^2$$

It follows that the first-best education level is given by $z^{fb}(\theta) = \frac{(\gamma+1)\theta}{2}$. Substituting $z^{fb}(\theta)$ into $S(z,\theta)$, we have $S^{fb}(\theta) = \frac{(\gamma+1)^2\theta^2}{4}$.

Equilibrium Throughout the paper, I use *symmetric perfect Bayesian equilibrium* as the solution concept. Specifically, in the observed case, an equilibrium consists of each school i's tuition scheme T_i^o and conditional on any tuition scheme profile $\{T_i\}$, the worker's education choice $z_i^o(\theta;\{T_i\})$ and the labor market's wage schedule $W_i^o(z;\{T_i\})$ for each i, such that

(i) For each $\{T_i\}$: (a) given $W_i^o(z;\{T_i\})$, $z_i^o(\theta;\{T_i\})$ maximizes U_i; (b) $W_i^o(z;\{T_i\}) = \mathbb{E}[Q(z,\theta)|z_i^o(\theta;\{T_i\})]$ such that the labor market's posterior belief about the worker's ability, or simply *the market belief*, is updated using Bayes' rule.

(ii) Given $z_i^o(\theta;\{T_i\})$ and $\{T_{-i}^o\}$, T_i^o maximizes the school's expected profit, i.e.,

$$T_i^o \in \arg\max_{T_i} \int_0^1 T_i(z_i^o(\theta;\{T_i\}))d\theta$$

subject to that $T_j^o = T_i^o$ for any $j \neq i$.

In the unobserved case, the market belief is independent of the actual tuition schemes but is conditional on *conjectured* schemes. Since the solution concept focuses on symmetric equilibrium, I assume that the conjecture of tuition scheme is identical across schools, and in equilibrium, it is correct. Thus, in this case, an equilibrium consists of each school i's tuition scheme T_i^u and the associated wage schedule W^u (more precisely, $W^u(z;\{T_i^u\})$), and conditional on each profile $\{T_i\}$, an education function $z_i^u(\theta;\{T_i\})$ for each i, such that

(i) Given W^u, for each $\{T_i\}$, $z_i^u(\theta;\{T_i\})$ maximizes U_i; $W^u(z) = \mathbb{E}[Q(z,\theta)|z_i^u(\theta;\{T^u\})]$ such that the market belief is updated using Bayes' rule.

(ii) Given $z_i^u(\theta;\{T_i\})$ and $\{T_{-i}^u\}$, T_i^u maximizes the school's expected profit, i.e.,

$$T_i^u \in \arg\max_{T_i} \int_0^1 T_i(z_i^u(\theta;\{T_i\}))d\theta$$

subject to that $T_j^u = T_i^u$ for any $j \neq i$.

Note that the equilibrium conditions have one important difference between the observed and unobserved case: in the unobserved case, the market belief needs to be correct only on the equilibrium path, whereas in the observed case, the market belief has to be correct following every tuition scheme that is chosen by the school.

Note too that all schools are symmetric and the worker's location distributes uniformly along the circle. Since I consider symmetric equilibrium, following Yang and Ye (2008), I claim without argument that the analysis for a n-school oligopoly model can be translated into that of a duopoly model if we normalize k to $k' = 2k/n$.[2] Since I consider any $k > 0$, it is without loss of generality to focus on the duopoly model. Thus, in the subsequent, I focus on a duopoly education market as depicted in Figure 2.1. Hence, the worker's horizontal type can be simply characterized by d_i, $i = 1,2$.

Equilibrium selection Due to the flexibility of off-path belief, there possibly exist multiple equilibria even though we consider symmetric equilibrium. Following Lu (2018), for both the observed and unobserved case, I focus on the *school-optimal separating equilibrium*; that is, the equilibrium that yields the highest payoff for the schools, provided that on the equilibrium path, $z(\theta)$ is one-to-one if $z(\theta) > 0$.[3]

[2]See Section 5 of Yang and Ye (2008) for greater details.

[3]I do not impose any restriction on $z(\theta)$ off the equilibrium path.

2.2.1 Direct Mechanisms

It is well known that in common agency games, it is no longer without loss of generality to restrict attention to direct mechanisms by applying the revelation principle.[4] In this regard, following Rochet and Stole (2002), I restrict my attention to deterministic contracts.[5] Note that the worker's gross utility, upon purchasing from school i, depends only on his vertical type θ. Thus, it is without loss of generality to consider direct mechanisms such that the allocation depends only on the vertical type the worker reports to a school. For brevity, in the subsequent, I often interchange *vertical type* and *type*, provided there is no confusion.

Hence, for both the observed and unobserved case, it is without loss of generality to adjust the timing as follows. First, each school i offers a contract $\langle z_i(\theta), T_i(z)\rangle$ to the worker. Then, the labor market posts a wage schedule $W_i(z)$ for each school i's student based on the information available: in the observed case, it observes all the contracts; in the unobserved case, it does not observe any contract. Finally, the worker chooses at most one school to attend, and upon attendance he reports his type to only this school. If the worker chooses to attend school i and reports a type $\hat{\theta}$, then he obtains education level $z_i(\hat{\theta})$, pays tuition $T_i(z_i(\hat{\theta}))$ and then receives a wage $W_i(z_i(\hat{\theta}))$.

Worker's problem In both cases, given each school i's contract $\langle z_i(\theta), T_i(z)\rangle$ and the associated wage schedule $W_i(z)$, a type-θ worker chooses some school i to attend, and upon

[4]Martimort and Stole (2002) demonstrates an example in which the revelation principle may fail when competing principals deviate to more complicated mechanisms that incorporate off-path messages. The reason for such failure, as indicated by McAfee (1993), is that the mechanisms offered by other competing principals may also be the agent's private information when making his decision. This implies that such private information can potentially be used by competing principals in designing revelation mechanisms.

[5]see Rochet and Stole (2002) for a detailed discussion of restrictions due to focusing on deterministic contracts and excluding the possibility of random contracts. In contrast, Manelli and Vincent (2006) and Thanassoulis (2004) consider random contracts for indivisible goods in multi-dimensional screening games.

attendance a report $\hat{\theta}_i$ to the school to maximize his net utility:

$$U_i(\hat{\theta}_i, \theta, d_i) = W_i(z_i(\hat{\theta}_i)) - T(z_i(\hat{\theta}_i)) - C(z_i(\hat{\theta}_i), \theta) - kd_i.$$

The mechanism $\{\langle z_i(\theta), T_i(z)\rangle, W_i(z)\}$ is *incentive compatible* (IC) if the worker is willing to truthfully report his type and is *individually rational* (IR) if the worker obtains a non-negative utility level by attending school i. A type-θ worker's equilibrium payoff is given by $U(\theta, d_i) := \max_i U_i(\theta, \theta, d_i)$, and the corresponding gross utility by $V(\theta) := V_i(\theta, \theta)$.

School's problem In the observed case, given the other school's contract, each school chooses a contract to maximize its expected profit subject to the IC and IR constraints, and the correctness of the market belief. In the unobserved case, given the wage schedules and the other school's contract, each school chooses a contract to maximize its expected profit subject to only the IC and IR constraints.

2.2.2 Preliminary Analysis

For both the observed and unobserved case, an allocation $\langle z(\theta), U(\theta, d_i)\rangle$ is *implementable* if it is incentive compatible and individually rational. Since the worker's net utility is separable in z and d_i, an allocation $\langle z(\theta), U(\theta, d_i)\rangle$ is incentive compatible if and only if the allocation of education level and gross utility, $\langle z(\theta), V(\theta)\rangle$, is incentive compatible. By Mas-Colell, Whinston, and Green (1995, Proposition 23.D.2), we obtain the following lemma:

Lemma 2.1. *In both cases, an allocation $\langle z(\theta), V(\theta)\rangle$ is incentive compatible if*

(i) $z(\theta)$ *is non-decreasing.*

(ii) Define $\theta_0 \equiv \inf\{\theta | z(\theta) > 0\}$*; then, for* $\theta > \theta_0$*,*

$$V(\theta) = V(\theta_0) + \int_{\theta_0}^{\theta} -C_\theta(z(s), s)ds = V(\theta_0) + \int_{\theta_0}^{\theta} z(s)ds.$$

Note that if the lowest participating type θ_0 is an interior type, i.e., $\theta_0 \in (0,1)$, then by continuity, $V(\theta_0)$ is optimally set to 0.[6] Following Armstrong and Vickers (2001), I think each school as directly providing utility to the worker. Let $V_i(\theta)$ be school i's *rent provision* to a type-θ worker. According to Lemma 2.1, if school i's allocation $\langle z_i(\theta), V_i(\theta)\rangle$ is incentive compatible, then we must have

$$V_i'(\theta) = z_i(\theta),\ T_i(z_i(\theta)) = W_i(z_i(\theta)) - C(z_i(\theta), \theta) - V_i(\theta).$$

This means that any incentive compatible contract can be characterized by a rent provision schedule $V_i(\theta)$, and thus, individual rationality holds if and only if $V_i(\theta) - kd_i \geq 0$.

Given the rent provision schedules $\{V_i(\theta)\}$, $i = 1,2$, the worker decides whether to attend school, if so, which school to attend. If a type-(θ, d_i) worker chooses to attend school i, then we must have

$$V_i(\theta) - kd_i \geq \max\left\{0, V_{-i}(\theta) - k(\frac{1}{2} - d_i)\right\}.$$

This is equivalent to

$$d_i \leq \min\left\{\frac{V_i(\theta)}{k}, \frac{1}{4} + \frac{V_i(\theta) - V_{-i}(\theta)}{2k}\right\} := s_i(\theta).$$

Hence, school i's market share for each vertical type θ is given by $2s_i(\theta)$. Since $V_i(\theta)$ is increasing in θ, there is a cutoff type θ_1 above which the horizontal market is fully covered; that is, if the worker has a vertical type $\theta \in [\theta_1, 1]$, then he attends school irrespective of his horizontal type d_i. As such, I call the interval $[\theta_1, 1]$ *the competition range*, as in Yang and Ye (2008). In contrast, for $\theta \in [\theta_{0_i}, \theta_1)$, the horizontal market is partially covered; thus, I call the interval $[\theta_{0_i}, \theta_1)$ *the local monopoly range*. Note that θ_1 is endogenously given by

$$V_1(\theta_1) + V_2(\theta_1) = \frac{k}{2}.$$

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[6] $V(\theta_0)$ is not necessarily 0 if θ_0 is the lowest type $\underline{\theta}$. In general, if the lowest type can generate positive social surplus, then the school may leave a positive "rent" $V(\underline{\theta})$ to type $\underline{\theta}$, in order to gain the market share.

Then, I can represent the schools' expected payoffs with respect to the rent provision schedules. Given V_{-i}, school i's expected profit equals twice

$$\int_{\theta_{0_i}}^{1} \left[W_i(z_i(\theta)) - C(z_i(\theta),\theta) - V_i(\theta)\right] s_i(\theta) d\theta.$$

By decomposing the above integral into the local monopoly range and the competition range, we have that school i's expected profit is twice

$$\int_{\theta_{0_i}}^{\theta_1} \left[W_i(z_i(\theta)) - C(z_i(\theta),\theta) - V_i(\theta)\right] \frac{V_i(\theta)}{k} d\theta \\ + \int_{\theta_1}^{1} \left[W_i(z_i(\theta)) - C(z_i(\theta),\theta) - V_i(\theta)\right] \cdot \left[\frac{1}{4} + \frac{V_i(\theta) - V_{-i}(\theta)}{2k}\right] d\theta. \tag{2.1}$$

In the observed case, correctness of the market belief means that $W_i(z) = \mathbb{E}[Q(z,\theta)|z_i(\theta)]$ for any implementable allocation $z_i(\theta)$ that the school chooses. Then, from the law of total expectation, (2.1) can be rewritten as

$$\int_{\theta_{0_i}}^{\theta_1} \left[S(z_i(\theta),\theta) - V_i(\theta)\right] \frac{V_i(\theta)}{k} d\theta \\ + \int_{\theta_1}^{1} \left[S(z_i(\theta),\theta) - V_i(\theta)\right] \cdot \left[\frac{1}{4} + \frac{V_i(\theta) - V_{-i}(\theta)}{2k}\right] d\theta. \tag{2.2}$$

Thus, given V_{-i}, school i's problem is to choose a contract $\langle z_i(\theta), V_i(\theta)\rangle$ to maximize (2.2), subject to $z_i(\theta)$ being non-decreasing and that $V_i'(\theta) = z_i(\theta)$. If the solution to this program is identical for schools $i = 1,2$ with $z_i(\theta)$ being increasing over $[\theta_0, 1]$, then we obtain a symmetric school-optimal separating equilibrium.

In the unobserved case, given V_{-i} and W_i, each school i's problem is to choose a contract $\langle z_i(\theta), V_i(\theta)\rangle$ to maximize (2.1), subject to $z_i(\theta)$ being non-decreasing and that $V_i'(\theta) = z_i(\theta)$. Without loss of generality, assume that each school chooses a contract, while simultaneously, the labor market chooses a corresponding wage schedule. Then, the equilibrium conditions can be simplified as follows: for each school $i = 1,2$, (i) given V_{-i}^u and W_i^u,

$\langle z_i^u(\theta), V_i^u(\theta)\rangle$ solves school i's problem; (ii) $W_i^u(z) = \mathbb{E}[Q(z,\theta)|z_i^u(\theta)]$ such that the market belief is updated using Bayes' rule. In the case of multiple equilibria, I select a symmetric school-optimal separating equilibrium.

2.2.3 A Bertrand-Spence Benchmark

As a benchmark, I consider a duopoly education market in which the worker can attend any school at zero transportation cost, i.e., $k = 0$. In this case, a symmetric Bertrand competition induces both schools to set tuition at the marginal cost. The model is thus translated to a Spence's signaling game (Spence 1973). An equilibrium consists of an education function $z^s(\theta)$ and a wage schedule $W^s(z)$, such that (i) given $W^s(z)$, $z^s(\theta)$ maximizes $U(z,\theta)$; (ii) $W^s(z) = \mathbb{E}[Q(z,\theta)|z^s(\theta)]$ with the market belief updated using Bayes' rule. As in Lu (2018), I focus on the *least-cost separating equilibrium* such that $z^s(0) = z^{fb}(0)$. Applying Lu (2018, Proposition 3.1), we have that the least-cost separating equilibrium exists, such that

$$z^s(\theta) = \frac{(2\gamma+1)\theta}{2} \text{ on } [0,1],\ W^s(z) = \frac{2\gamma}{2\gamma+1}z^2 + z \text{ on } [0,\gamma+\frac{1}{2}].$$

It follows that $z^s(\theta) > z^{fb}(\theta)$ for all $\theta > 0$. The intuition is well understood. Specifically, since the worker's ability is private information, he attempts to separate himself from lower ability workers by acquiring more education, thereby leading to over-education. Given the analytical solution of $z^s(\theta)$, the signaling effect is explicitly given by

$$Q_\theta(z^s(\theta),\theta)\cdot\theta^s(z) = \frac{2\gamma z}{2\gamma+1} > 0.$$

The signaling effect reflects the feature that a higher education level makes the labor market regard the worker as having higher ability.

Furthermore, one can parameterize the intensity of signaling in this model. Let us define the intensity of signaling to be the ratio of the over-invested education in Spence's model, i.e., $z^s(\theta) - z^{fb}(\theta)$, to the first best education level $z^{fb}(\theta)$ for $\theta > 0$. Substituting, we have

$$\frac{z^s(\theta) - z^{fb}(\theta)}{z^{fb}(\theta)} = \frac{\gamma}{\gamma + 1}.$$

Clearly, the intensity of signaling is increasing in the parameter γ. To see the idea, note that the larger γ is, the stronger complementarity between the worker's ability and education is. Due to the signaling effect, a higher education level induces the labor market to regard the worker as having higher ability; thus, if ability complements education to a larger extent, the marginal benefit of education will be even higher, thereby enhancing signaling activity.

2.3 Monopoly

In this section, as a well-controlled benchmark, I consider a monopoly education market in which both schools are owned by a monopolist. The monopolist's objective is thus to maximize the joint profit of the two schools. Since the distribution of the worker's type is uniform and the schools' locations and technologies are symmetric, I assume that for both the observed and unobserved case, the two schools choose an identical contract in equilibrium, thereby resulting in symmetric market shares. I start the analysis with the observed case.

2.3.1 The Observed Case

Since both schools offer an identical contract in equilibrium, we can drop the subscripts to simplify (2.2). The monopolist's problem can be stated as

$$\max \underbrace{\int_{\theta_0}^{\theta_1} [S(z(\theta),\theta) - V(\theta)] \frac{V(\theta)}{k} d\theta}_{\text{Phase I: partially covered range}} + \underbrace{\int_{\theta_1}^{1} [S(z(\theta),\theta) - V(\theta)] \frac{1}{4} d\theta}_{\text{Phase II: fully covered range}}$$

$$s.t.\ V'(\theta) = z(\theta),\ z'(\theta) \geq 0,\ V(\theta_1) = \frac{k}{4}.$$

If further $\theta_0 \in (0,1]$, then $V(\theta_0) = 0$; otherwise, $V(\theta_0)$ should be chosen optimally.

As is standard in the literature, I solve the above program by relaxing the monotonicity constraint of $z(\theta)$ first and verity it ex post to justify the approach. The monopolist's problem is a two-phase optimal control problem: in Phase I, the horizontal market is partially covered; in Phase II, in contrast, the horizontal market is fully covered. Define the *Hamiltonian* of the two phases as follows:

$$H_1(z,V,\lambda,\theta) = [S(z,\theta) - V]\frac{V}{k} + \lambda z = [(\gamma+1)\theta z - z^2 - V]\frac{V}{k} + \lambda z,$$
$$H_2(z,V,\lambda,\theta) = [S(z,\theta) - V]\frac{1}{4} + \lambda z = [(\gamma+1)\theta z - z^2 - V]\frac{1}{4} + \lambda z,$$

where z is a control variable, V is a state variable and λ is the associated adjoint variable. From the Maximum Principle,[7] if $\langle z^*(\theta), V^*(\theta)\rangle$ solves the monopolist's problem, then for each phase $i = 1,2$, we must have

$$z^*(\theta) = \arg\max_z H_i(z, V^*(\theta), \lambda(\theta), \theta),$$
$$\dot{\lambda}(\theta) = -\frac{\partial}{\partial V} H_i(z^*(\theta), V^*(\theta), \lambda(\theta), \theta),$$

along with the transversality condition $\lambda(1) = 0$.

It follows that Phase I can be characterized by the following second order autonomous ordinary differential equation (ODE):

$$(\gamma+3)V - 2\ddot{V}V - \dot{V}^2 = 0. \tag{2.3}$$

To solve (2.3), I first consider the case in which the vertical market is partially covered; that is, the optimal lowest participating type $\theta_0^* \in (0,1]$, and thus, $V(\theta_0^*) = 0$. Given this

[7] See Seierstad and Sydsaeter (1986) for details.

boundary condition, it can be proven that the unique solution to (2.3) is given by[8]

$$V^*(\theta) = \frac{\gamma+3}{8}(\theta-\theta_0^*)^2,\ z^*(\theta) = \frac{\gamma+3}{4}(\theta-\theta_0^*).$$

Analogously, in Phase II, we have the ODE: $\ddot{V} = \dot{z} = \frac{\gamma+2}{2}$. Moreover, the transversality condition $\lambda(1) = 0$ implies that $z(1) = \frac{\gamma+1}{2}$. Thus, the solution to Phase II is given by

$$V^*(\theta) = \frac{(\gamma+2)\theta^2}{4} - \frac{\theta}{2} + \beta(\theta_1^*),\ z^*(\theta) = \frac{(\gamma+2)\theta}{2} - \frac{1}{2},$$

where $\beta(\theta_1^*)$ depends on the optimal switching type θ_1^* which remains to be determined. Since in both phases, $z(\theta)$ is increasing in θ, the monotonicity constraint is automatically satisfied, meaning that a symmetric school-optimal separating equilibrium exists.

To determine θ_1^* and thus θ_0^*, I impose the smooth pasting conditions: $V(\theta_1^{*-}) = V(\theta_1^{*+})$ and $z(\theta_1^{*-}) = z(\theta_1^{*+})$.[9] Combined with the condition $V(\theta_1^*) = \frac{k}{4}$, θ_1^* and θ_0^* are given by

$$\theta_0^* = \frac{1}{\gamma+2} - \frac{(\gamma+1)\sqrt{2(\gamma+3)k}}{2(\gamma+2)(\gamma+3)},\ \theta_1^* = \frac{1}{\gamma+2} + \frac{\sqrt{2(\gamma+3)k}}{2(\gamma+2)}. \tag{2.4}$$

It thus follows that for $\theta \in [\theta_1^*, 1]$, $V(\theta)$ is given by

$$V^*(\theta) = \frac{k}{4} + (\theta-\theta_1^*)\left[\frac{(\gamma+2)(\theta+\theta_1^*)}{4} - \frac{1}{2}\right].$$

Note that θ_0^* is an interior solution if and only if $k < \frac{2(\gamma+3)}{(\gamma+1)^2}$. We shall consider two cases. First, $\gamma \leq 1$. In this case, when $k \geq \frac{2(\gamma+3)}{(\gamma+1)^2}$, we have $\theta_1^* \geq 1$, meaning that Phase II is never entered. Then, θ_0^* is pinned down by the transversality condition $\lambda(1) = 0$, such that $\theta_0^* = \frac{1-\gamma}{\gamma+3} \geq 0$, with equality holding at $\gamma = 1$ only. Hence, if $\gamma < 1$, then θ_0^* is always

[8] Rochet and Stole (2002) shows that if a convex solution to (2.3) exists, then given specific boundary conditions, it is unique. See Rochet and Stole (2002, Appendix, p. 304-305) for details.

[9] The smooth pasting conditions are implied by the Weierstrass-Erdmann necessary condition.

an interior solution; in addition, Phase I (the partially covered range) always exists, whereas Phase II (the fully covered range) exists only if $k < \frac{2(\gamma+1)^2}{\gamma+3}$. Second, $\gamma > 1$. In this case, when $k = \frac{2(\gamma+3)}{(\gamma+1)^2}$, we have $\theta_1^* < 1$, meaning that Phase II always exists. Thus, if $k \geq \frac{2(\gamma+3)}{(\gamma+1)^2}$, then $\theta_0^* \leq 0$; that is, the vertical market is fully covered. As a result, $V(\theta)$ is free at the lowest type $\theta = 0$ and the boundary condition $V(0) = 0$ does not necessarily hold. Since such a case is more complicated, I postpone further analysis until I have summarized the results of the case in which the vertical market is partially covered.

Suppose that $\gamma \leq 1$, then the monopolist's optimal symmetric contract exists and has been characterized in the above analysis. Let $\langle z^{m_o}(\theta), V^{m_o}(\theta)\rangle$ be the equilibrium contract in the monopoly observed case, and $\theta_0^{m_o}$ and $\theta_1^{m_o}$ be the lowest participating type and switching type, respectively. Then, we have that $z^{m_o}(\theta)$ is increasing on $[\theta_0^{m_o}, 1]$; in particular, if $\gamma < 1$, then $\theta_0^{m_o} > 0$ always holds. Hence, the school-optimal separating equilibrium is obtained. Indeed, this equilibrium is the one that yields the highest expected profit for the monopolist among all equilibria. To summarize, we have the following proposition:

Proposition 2.1. *Suppose that $\gamma \leq 1$, then in the monopoly observed case, the symmetric school-optimal separating equilibrium exists. Specifically, for $k \in \left(0, \frac{2(\gamma+1)^2}{(\gamma+3)}\right)$,*

$$z^{m_o}(\theta) = \begin{cases} \frac{\gamma+3}{4}(\theta - \theta_0^{m_o}) & \textit{if } \theta_0^{m_o} \leq \theta < \theta_1^{m_o} \\ \frac{(\gamma+2)\theta}{2} - \frac{1}{2} & \textit{if } \theta_1^{m_o} \leq \theta \leq 1, \end{cases}$$

where $\theta_0^{m_o}$ and $\theta_1^{m_o}$ are given by θ_0^* and θ_1^* in (2.4), respectively. For $k \geq \frac{2(\gamma+1)^2}{(\gamma+3)}$,

$$z^{m_o}(\theta) = \frac{\gamma+3}{4}\left(\theta - \theta_0^{m_o}\right) \quad \textit{if } \theta_0^{m_o} \leq \theta \leq 1,$$

where $\theta_0^{m_o} = \frac{1-\gamma}{\gamma+3}$. If $\gamma < 1$, then for any $k > 0$, $\theta_0^{m_o} > 0$ and $z^{m_o}(\theta) \leq z^{fb}(\theta)$ on $[0,1]$ with equality holding at $\theta = 1$ only; if $\gamma = 1$, then for $k \geq \frac{2(\gamma+1)^2}{(\gamma+3)}$, $z^{m_o}(\theta) = z^{fb}(\theta)$ on $[0,1]$.

The monopolist's optimal contract has two noticeable features. First, when $\gamma < 1$, there is always under-education on both the extensive and intensive margin. Specifically, there is always a positive measure of vertical types who are excluded from education, i.e., $\theta_0^{m_o} > 0$. In addition, for all but the highest vertical type, education level is downward distorted, i.e., $z^{m_o}(\theta) < z^{fb}(\theta)$ on $[\theta_0^{m_o}, 1)$. This result stands in contrast to that of Spence's model in which there is always over-education. Second, perhaps it is more striking that when $\gamma = 1$, if the market contains only the partially covered range, then the monopoly optimal contract in fact achieves the first-best! In contrast, Lu (2018) studies monopolistic nonlinear pricing for signals with deterministic participation and find that the optimal contract can achieve the first-best only asymptotically.[10] Moreover, Rochet and Stole (2002) and Yang and Ye (2008) study monopolistic nonlinear pricing for non-signaling goods with random participation, and both papers find that the optimal contract always exhibits a downward distortion with efficiency achieved only on the boundary.[11]

The above features result from the interaction between three forces: market penetration, the monopolist's screening and the worker's signaling. To be specific, let us first consider the fully covered range. Note that the monopolist's market share is already maximized for each vertical type, thus it cannot benefit from supplying more rents to gain market share. Since a higher vertical type can benefit from his cost advantage over lower types, the monopolist has to leave information rents to the worker to incentivize truth-telling. This implies that the marginal profit of education is unambiguously less than the marginal social surplus in the fully covered range, and thus, the monopolist under-supplies education. In Lu (2018), the market contains only the fully covered range, and hence, there is always a downward distortion with efficiency achieved only on the top.

In contrast, in the partially covered range, the monopolist can benefit from rent provision to obtain market share. In this case, increasing education supply has two opposite effects.

[10] Using a numerical example similar to the current model, Lu (2018) shows that $z^{m_o}(\theta)/z^{fb}(\theta) \to 1$ as $\gamma \to \infty$. See Section 4.2 of Lu (2018) for details.

[11] See Rochet and Stole (2002, Proposition 4) and Yang and Ye (2008, Proposition 1).

On one hand, it reduces per-customer profit by providing the worker with more information rents. On the other hand, a larger rent also results in a larger market share. The optimal allocation rule thus must balance these two opposite effects. But the question is: why does under-education always occur in the partially covered range when $\gamma < 1$, whereas the social optimum can be fully achieved when $\gamma = 1$?

To answer this question, one should understand the effects of signaling on the optimal allocation. As is pointed out by Lu (2018), in the observed case under monopoly, signaling can mitigate the screening distortion. To see this, note that given the monopolist's tuition scheme, the subgame is indeed a Spence's signaling game as if the worker's cost function was given by $T(z) + C(z, \theta)$; thus, the signaling effect induces the worker to "over-invest" in education in terms of total cost. The signaling incentive reduces the worker's willingness to intimate lower types, thus the worker extracts lower information rents than when signaling is absent. To illustrate, suppose that the labor market can observe the worker's ability, thereby eliminating signaling. As a result, we return to Rochet and Stole (2002) or Yang and Ye (2008). In this case, the IC constraint is given by $V'(\theta) = S_\theta(z(\theta), \theta)$. In contrast, in the current environment, it is given by $V'(\theta) = C_\theta(z(\theta), \theta) < S_\theta(z(\theta), \theta)$. This reveals that the monopolist leaves lower information rents to the worker when signaling is present.

Recall that γ measures signaling intensity: the larger γ is, the more intense signaling is. Thus, if γ is relative big, then the screening distortion can be mitigated to a relatively great extent. Specifically, if the market has only the partially covered range, then it can be easily verified that the ratio $z^{m_o}(\theta)/z^{fb}(\theta)$ is increasing in γ for all θ, and arrives at 1 when $\gamma = 1$; that is, the effects of market penetration and signaling can exactly offset that of screening, thereby restoring the first-best allocation.

At first glance, it may be surprising that under asymmetric information, the monopolistic profit-maximizing pricing can be welfare-maximizing. To see the intuition, note that if the market contains only the partially covered range, then the marginal profit of each type θ is given by $[S(z(\theta), \theta) - V(\theta)]V(\theta)/k$. Suppose that the monopolist can observe the worker's vertical type, then the optimal contract $\langle z^*(\theta), V^*(\theta)\rangle$ is simply given by $z^*(\theta) = z^{fb}(\theta)$ and $V^*(\theta) = S(z^{fb}(\theta), \theta)/2$. This is because $V = S/2$ maximizes the marginal profit for any S and $z^*(\theta) = z^{fb}(\theta)$ maximizes $S(z, \theta)$. However, even without the ability to contract on θ, such an allocation can still be implemented without violating the IC constraint $V'(\theta) = z(\theta)$ when $\gamma = 1$. In contrast, in the contracting problems of Rochet and Stole (2002) and Yang and Ye (2008), the first-best can never be fully achieved. Suppose not, then $z^*(\theta) = z^{fb}(\theta)$ on $[0, 1]$. It follows from the envelope theorem and the associated IC constraint that

$$S^*(\theta) = S(z^{fb}(\theta), \theta) = \int_0^\theta S_\theta(z^{fb}(s), s)ds = V^*(\theta).$$

Thus, the marginal profit is always zero, which cannot be optimal. Again, this is due to that the monopolist supplies more information rents when signaling is absent.

It is worth noting that when $\gamma = 1$, the worker's productivity and cost heterogeneities are equally significant (i.e., $Q_{z\theta} = C_{z\theta}$). Since the lowest type is totally unproductive, at the social optimum, each type's social surplus is exactly twice his information rent, as is shown in the above. Thus, the first-best can be fully achieved if the market has only the partially covered range. Moreover, it can be easily verified that in equilibrium the signaling effect that is measured by $Q_\theta \cdot \theta'(z)$ equals the marginal tuition $T'(z)$. This implies that when the market contains only the partially covered range, the monopolistic optimal tuition scheme levies *Pigovian tax* on signaling, which undoes the signaling effect and thus restores the first-best. In contrast, in Lu (2018), the market has only the fully covered range; thus, the optimal tuition scheme "over-taxes" signaling and leads to a downward distortion.

Going forward, I turn to the case in which $\gamma > 1$. When $k < \frac{2(\gamma+3)}{(\gamma+1)^2}$, we have $\theta_0^* > 0$, i.e., the vertical market is partially covered. Thus, the equilibrium contract is characterized by Proposition 2.1. In contrast, when $k \geq \frac{2(\gamma+3)}{(\gamma+1)^2}$, we have $\theta_0^* \leq 0$, i.e., the vertical market is fully covered. In this case, the equilibrium contract in the fully covered range is also given by Proposition 2.1; thus, the boundary conditions $V(\theta_1^*) = \frac{k}{4}$ and $\dot{V}(\theta_1^*) = \frac{(\gamma+2)\theta_1^*-1}{2}$ remain. However, the initial state $V(0)$ is now free. This means that the adjoint variable λ satisfies:

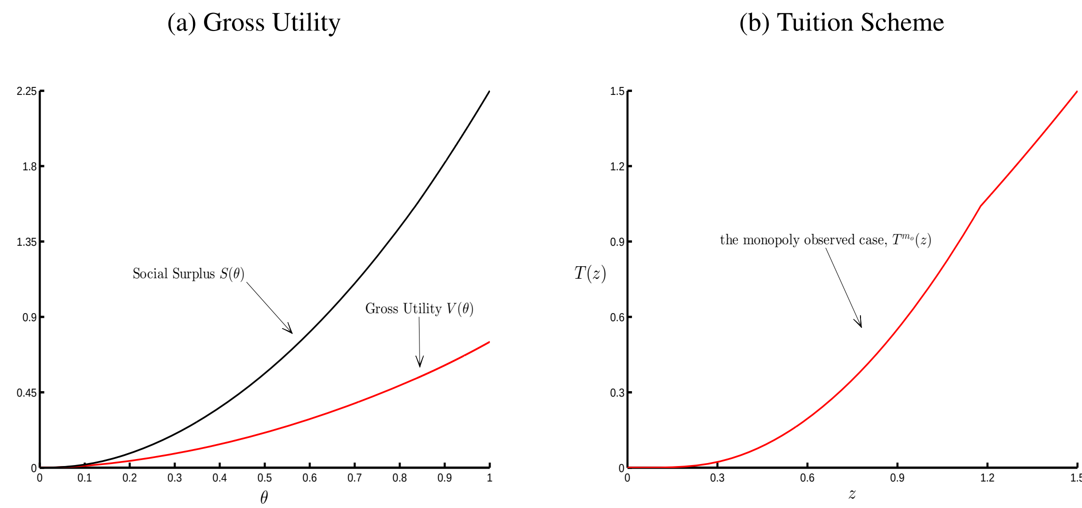

Figure 2.2: A Convex Solution in the Monopoly Observed Case

$\lambda(0) = 0$.[12] It follows that efficiency occurs at the bottom, i.e., $z^*(0) = z^{fb}(0) = 0$.[13] This yields an extra boundary condition: $\dot{V}(0) = 0$. In summary, when $k \geq \frac{2(\gamma+3)}{(\gamma+1)^2}$, the optimal contract in the partially covered range is given by the solution to the following problem:

$$(\gamma+3)V - 2\ddot{V}V - \dot{V}^2 = 0,$$

$$s.t.\ \dot{V}(0) = 0,\ V(\theta_1) = \frac{k}{4},\ \dot{V}(\theta_1) = \frac{(\gamma+2)\theta_1 - 1}{2}.$$

Note that this is not a standard boundary value problem (BVP), as the boundary conditions involve an endogenous endpoint θ_1. As far as I know, no existing BVP theorem can be applied directly to show the existence and uniqueness of the solution to this problem, not mention deriving an analytical solution. In this regard, I solve the problem using numerical methods.[14] In Figure 2.2, panel (a) depicts a convex solution $V(\theta)$ when $\gamma = 2$ and $k = 2$, and panel (b) depicts the associated tuition scheme $T^{m_0}(z)$.

[12]See Seierstad and Sydsaeter (1986, p. 185-186) for details.

[13]Rochet and Stole (2002) provides an intuitive discussion about efficiency at the bottom in its Appendix.

[14]The MATLAB code for all numerical calculations of this paper is available upon request.

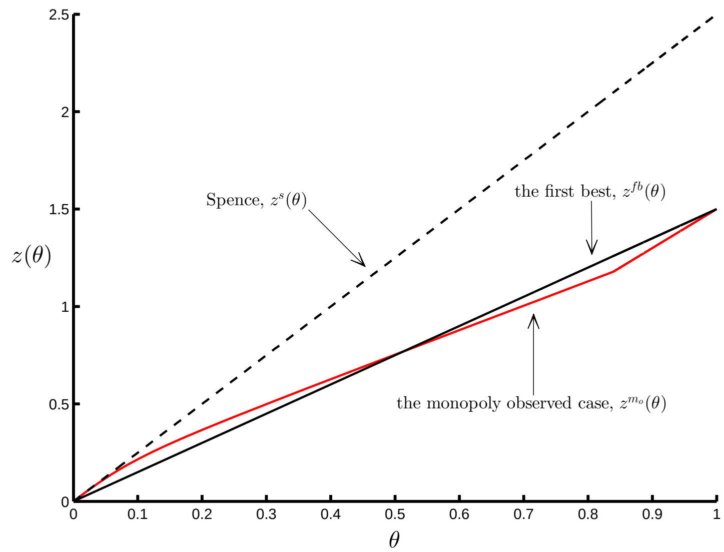

Figure 2.3: Over-Education at the Low End

The equilibrium outcome has a salient feature: that is, when the unit transportation cost k is sufficiently large, over-education occurs at the low end of the spectrum of θ. Moreover, if the market contains the fully covered range, then there exists a cutoff type such that all lower vertical types obtain more education than the first-best, whereas the others obtain less than the first best. This is illustrated in Figure 2.3 which assumes the same numerical example as in Figure 2.2. To summarize, we have the following proposition:

Proposition 2.2. *Suppose that $\gamma > 1$, then for sufficiently large k, if the market contains both the partially covered and fully covered range, then there exists a cutoff $\tilde{\theta} \in (0, \theta_1^{m_o})$, such that $z^{m_o}(\theta) > z^{fb}(\theta)$ on $(0, \tilde{\theta})$, whereas $z^{m_o}(\theta) < z^{fb}(\theta)$ on $(\tilde{\theta}, 1)$.*

As is depicted by Figure 2.3, $z^{m_o}(\theta)$ is single-crossing $z^{fb}(\theta)$ from above in the interior of the partially covered range. The intuition for over-education occurring at the low end is that when $\gamma > 1$, signaling is relatively intense; if the transportation cost is relatively high, then to gain market share, the monopolist charges low prices, especially at the low end of the vertical market. As illustrated in panel (b) of Figure 2.2, the tuition scheme is flat and close to 0 for low education levels. Thus, at the low end, signaling outweighs screening, leading to over-education. In addition, from Figure 2.3, $z^{m_o}(\theta)$ is bounded above by $z^s(\theta)$ which is the equilibrium education function in Spence's model. Intuitively, since tuition is fixed at zero, education is the least costly in Spence's signaling game, compared to other models; thus, the worker obtains the highest education level in Spence's model.

I am interested in the effects of the unit transportation cost k on allocation, in particular, on the (vertical) market coverage and quantity distortion. Similar to Yang and Ye (2008), k measures the degree of horizontal differentiation: a larger k means that the two schools are more horizontally differentiated. Following directly the previous analysis, Corollary 2.1 below shows that when the fully covered range exists and the vertical market is not fully covered, as horizontal differentiation increases, the monopolist raises the market coverage, offers more rents, and reduces the downward distortion in education level. When horizontal differentiation is eliminated, i.e., $k = 0$, the equilibrium outcome coincides with that of the observed case of Lu (2018). Formally, we have:

Corollary 2.1. *In the monopoly observed case, when k is such that $\theta_0^{m_o} > 0$, $\theta_1^{m_o} < 1$, as k increases: (i) $z^{m_o}(\theta)$ increases on $\theta \in (\theta_0^{m_o}, \theta_1^{m_o}]$ but remains the same on $(\theta_1^{m_o}, 1]$; (ii) $V^{m_o}(\theta)$ increases for $\theta \in (\theta_0^{m_o}, 1]$; (ii) the market coverage $[\theta_0^{m_o}, 1]$ enlarges, whereas the fully covered range $[\theta_1^{m_o}, 1]$ shrinks. If $k = 0$, then the equilibrium outcome coincides with that of the observed case of Lu (2018).*

Intuitively, as horizontal differentiation rises, to maintain market share in the partially covered range, the monopolist has to provide the worker with more rents, which, according to Lemma 2.1, can be achieved by either raising the market coverage, i.e., reducing θ_0, or supplying more education. The optimal allocation requires a balance between these two approaches. Corollary 2.1 shows that both methods will be employed in equilibrium when k increases. Corollary 2.1 also states that as k increases, the fully covered range shrinks. This is because the switching type θ_1's education level is pinned down by the IC constraint in the fully covered range, which does not directly depend on k; as education levels increase in the partially covered range, θ_1 must be higher accordingly. In other words, as the fixed fee (opportunity cost) of attending school kd_i increases, the marginal consumer θ_1 must have a higher valuation for education.

2.3.2 The Unobserved Case

Now, I turn to the unobserved case. Since I consider symmetric equilibrium, I assume that the labor market offers the same wage to both schools' student for a given education level, thereby allowing me to drop the subscript of the wage schedule. Then, given some wage schedule $W(z)$, the monopolist solves:

$$\max \underbrace{\int_{\theta_0}^{\theta_1} [W(z(\theta)) - C(z(\theta),\theta) - V(\theta)] \frac{V(\theta)}{k} d\theta}_{\text{Phase I: partially covered range}} + \underbrace{\int_{\theta_1}^{1} [W(z(\theta)) - C(z(\theta),\theta) - V(\theta)] \frac{1}{4} d\theta}_{\text{Phase II: fully covered range}}$$

$$s.t.\ V'(\theta) = z(\theta),\ z'(\theta) \geq 0,\ V(\theta_1) = \frac{k}{4}.$$

If further $\theta_0 \in (0,1]$, then we have $V(\theta_0) = 0$; otherwise, as in the observed case, we have to choose $V(\theta_0)$ optimally.

Similarly to the observed case, define the *Hamiltonian* of the two phases as follows:

$$H_1(z,V,\lambda,\theta) = [W(z) - C(z,\theta) - V]\frac{V}{k} + \lambda z,$$
$$H_2(z,V,\lambda,\theta) = [W(z) - C(z,\theta) - V]\frac{1}{4} + \lambda z,$$

The key difference from the observed case is that here $W(z)$ is endogenously determined by the equilibrium conditions. Suppose that the school-optimal separating equilibrium exists, and let $\langle z^*(\theta), V^*(\theta)\rangle$ solves the monopolist's problem, then from the Maximum Principle, we have the following first oder conditions (F.O.C.):

$$\frac{\partial}{\partial z} H_1(z^*(\theta), V^*(\theta), \lambda(\theta), \theta) = [W'(z^*(\theta)) - C_z(z^*(\theta),\theta)]\frac{V^*(\theta)}{k} + \lambda(\theta) = 0,$$
$$\frac{\partial}{\partial z} H_2(z^*(\theta), V^*(\theta), \lambda(\theta), \theta) = [W'(z^*(\theta)) - C_z(z^*(\theta),\theta)]\frac{1}{4} + \lambda(\theta) = 0,$$

along with the evolution rule for λ:

$$\dot{\lambda}(\theta) = -\frac{\partial}{\partial V} H_i(z^*(\theta), V^*(\theta), \lambda(\theta), \theta),\ i = 1,2,$$

and the transversality condition $\lambda(1) = 0$.

Moreover, in equilibrium, the market belief should be correct: $W(z^*(\theta)) = Q(z^*(\theta), \theta)$. Thus, $W'(z^*(\theta)) = Q_z(z^*(\theta), \theta) + Q_\theta(z^*(\theta), \theta) \cdot \theta^{*'}(z)$. Combining these conditions and substituting the model assumptions, we obtain an autonomous ODE for Phase I:

$$(2\gamma+3)V - \frac{\gamma V \dot{V} \dddot{V}}{\ddot{V}^2} - 2\ddot{V}V + \frac{\gamma \dot{V}^2}{\ddot{V}} - \dot{V}^2 = 0. \tag{2.5}$$

To solve (2.5), I first consider the case in which the vertical market is partially covered, i.e., $\theta_0^* \in (0,1]$, and thus, $V(\theta_0^*) = 0$. Given this boundary condition, it can be verified that the solution to (2.5) is given by

$$V^*(\theta) = \frac{4\gamma+3}{8}(\theta - \theta_0^*)^2,\ z^*(\theta) = \frac{4\gamma+3}{4}(\theta - \theta_0^*);$$

consequently, the wage schedule in Phase I is given by

$$W^*(z) = \frac{4\gamma}{4\gamma+3} z^2 + (\gamma\theta_0^* + 1)z,$$

where the lowest participating type θ_0^* remains to be determined.

Then, I consider Phase II. Because $\lambda'(\theta) = \frac{1}{4}$ and $\lambda(1) = 0$, $\lambda(\theta) = \frac{\theta-1}{4}$ in Phase II. Substituting $\lambda(\theta)$ and the condition $W(z) = Q(z, \theta(z))$ into the F.O.C. for z in Phase II, we obtain the following ODE:

$$W'(z) = 2(z - \frac{W}{\gamma z} + \frac{\gamma+1}{\gamma}).$$

The general solution to this ODE is given by

$$W(z) = \frac{\gamma}{\gamma+1}z^2 + \frac{2(\gamma+1)}{\gamma+2}z + c \cdot z^{\frac{\gamma}{2}},$$

where c is some parameter. To fully characterize $W(z)$, we need to pin down c. As is argued previously, the current model converges to Lu (2018) as $k \to 0$. Thus, I apply Lu (2018, Poposition 5.1) to the current model, assuming that $k = 0$, and conclude that $c = 0$. As such, we have fully characterized $W(z)$ for Phase II.

It thus follows that in Phase II, $V^*(\theta)$ and $z^*(\theta)$ are given by

$$V^*(\theta) = \frac{\gamma+1}{2}\theta^2 - \frac{\gamma+1}{\gamma+2}\theta + \beta(\theta_1^*),\ z^*(\theta) = (\gamma+1)(\theta - \frac{1}{\gamma+2})$$

where $\beta(\theta_1^*)$ depends on the optimal switching type θ_1^* that remains to be determined. Then, from smooth pasting and the condition $V(\theta_1^*) = \frac{k}{4}$, θ_1^* and θ_0^* are thus given by

$$\theta_0^* = \frac{1}{\gamma+2} - \frac{\sqrt{2(4\gamma+3)k}}{4(\gamma+1)(4\gamma+3)},\ \theta_1^* = \frac{1}{\gamma+2} + \frac{\sqrt{2(4\gamma+3)k}}{4(\gamma+1)}. \tag{2.6}$$

Substituting θ_1^*, we have that in Phase II, $V(\theta)$ is given by

$$V^*(\theta) = \frac{k}{4} + (\theta - \theta_1^*)\left[\frac{(\gamma+1)(\theta+\theta_1^*)}{4} - \frac{\gamma+1}{\gamma+2}\right].$$

In addition, from (2.6), if $\frac{8(\gamma+1)^4}{(4\gamma+3)(\gamma+2)^2} < k < \frac{8(\gamma+1)^2(\gamma+3)^2}{(4\gamma+3)(\gamma+2)^2}$, then $\theta_0^* > 0$ and $\theta_1^* > 1$; thus, Phase I exists, whereas Phase II is never entered. In this case, θ_0^* is pinned down by the transversality condition $\lambda(1) = 0$, such that $\theta_0^* = \frac{1}{2\gamma+3} > 0$. Thus, for any $k > 0$, we have $\theta_0^* > 0$, that is, the vertical market is always partially covered.

Since $z^*(\theta)$ is increasing in both Phase I and II, and the initial condition is optimally chosen, we obtain the school-optimal separating equilibrium. Let $\langle z^{m_u}(\theta), V^{m_u}(\theta)\rangle$ be the equilibrium contract in the unobserved case under monopoly, and $\theta_0^{m_u}$ and $\theta_1^{m_u}$ be the lowest participating type and switching type, respectively. I summarize the equilibrium education allocation of the unobserved case in the proposition below.

Proposition 2.3. *For any $\gamma > 0$, in the monopoly unobserved case, the symmetric school-optimal separating equilibrium exists. Specifically, for $k \in \left(0, \frac{8(\gamma+1)^4}{(4\gamma+3)(\gamma+2)^2}\right)$,*

$$z^{m_u}(\theta) = \begin{cases} \frac{4\gamma+3}{4}(\theta - \theta_0^{m_u}) & \text{if } \theta_0^{m_u} \le \theta < \theta_1^{m_u} \\ (\gamma+1)\theta - \frac{\gamma+1}{\gamma+2} & \text{if } \theta_1^{m_u} \le \theta \le 1, \end{cases}$$

where $\theta_0^{m_u}$ and $\theta_1^{m_u}$ are given by θ_0^* and θ_1^* in (2.6), respectively. For $k \ge \frac{8(\gamma+1)^4}{(4\gamma+3)(\gamma+2)^2}$,

$$z^{m_u}(\theta) = \frac{4\gamma+3}{4}(\theta - \theta_0^{m_u}), \text{ if } \theta_0^{m_u} \le \theta \le 1,$$

where $\theta_0^{m_u} = \frac{1}{2\gamma+3}$. It follows that there exists a cutoff $\tilde{\theta} \in (\theta_0^{m_u}, 1)$, such that $z^{m_u}(\theta) < z^{fb}(\theta)$ on $(\theta_0^{m_u}, \tilde{\theta})$, whereas $z^{m_u}(\theta) > z^{fb}(\theta)$ on $(\tilde{\theta}, 1)$.

As is immediately implied by Proposition 2.3, the degree of horizontal differentiation that is measured by k has similar effects on education supply, the worker's gross utility and the market coverage as in the observed case. Specifically, we have the following corollary:

Corollary 2.2. *In the monopoly unobserved case, when the market has the fully covered range, as k increases: (i) $z^{m_u}(\theta)$ increases for $\theta \in (\theta_0^{m_u}, \theta_1^{m_u}]$ but remains the same for $\theta \in (\theta_1^{m_u}, 1]$. (ii) $V^{m_u}(\theta)$ increases for $\theta \in (\theta_0^{m_u}, 1]$; (iii) the market coverage $[\theta_0^{m_u}, 1]$ enlarges, whereas the fully covered range $[\theta_1^{m_u}, 1]$ shrinks. If $k = 0$, then the equilibrium outcome coincides with that of the unobserved case of Lu (2018).*

I am interested in the difference in allocation between the observed and unobserved case. Proposition 2.3 shows that in contrast to the observed case, in the unobserved case, both under-education and over-education occur in equilibrium. Specifically, there exists a cutoff type such that all lower types obtain less education than the first-best, whereas the others obtain more than the first best. The next proposition shows further that in the unobserved case, the (vertical) market coverage is smaller than that of the observed case, whereas the fully covered range is larger in the unobserved case. Moreover, there exists a cutoff type in the partially covered range of the unobserved case, such that all lower types obtain less education in the unobserved case than in the observed case, whereas the others obtain more education in the unobserved case. Formally, we have:

Proposition 2.4. *For any $\gamma, k > 0$, $\theta_0^{m_u} > \theta_0^{m_o}$ and $\theta_1^{m_u} < \theta_1^{m_o}$. Furthermore, there exists a cutoff $\tilde{\theta} \in (\theta_0^{m_u}, \theta_1^{m_u})$, such that $z^{m_u}(\theta) < z^{m_o}(\theta)$ on $(\theta_0^{m_o}, \tilde{\theta})$, whereas $z^{m_u}(\theta) > z^{m_o}(\theta)$ on $(\tilde{\theta}, 1]$. The length of the interval $(\theta_0^{m_o}, \tilde{\theta})$ is increasing in k, and vanishes as $k \to 0$.*

Proposition 2.4 states that education levels are always higher in the unobserved case than in the observed case within the fully covered range of both cases. This result generalizes that of Lu (2018) in which the market contains only the fully covered range in both cases, and thus, the worker obtains more education in the unobserved case than in the observed case. As in Lu (2018), this result is driven by a *signal jamming effect*. Specifically, in the unobserved case, since the labor market cannot observe the actual cost of education, it does not know whether a difference in education level is caused by a tuition change or worker cost heterogeneity. Suppose that the monopolist lowers tuition so that the worker obtains more education than in the initial state. When the labor market observes the tuition scheme, it cuts wages, since any education level now corresponds to a lower-ability worker. This dampens the worker's demand for additional education. In contrast, when the labor market does not observe the tuition scheme, it does not adjust wages despite that tuition changes. Thus, the demand for education is more elastic in the unobserved case. This provides the monopolist with an incentive to secretly supply more education and persuades the labor market that the worker is more productive than is actually the case. Since in the observed case efficiency occurs at the top, over-education must occur at the high end in the unobserved case, as is predicted by Proposition 2.3. In equilibrium, the labor market correctly anticipates the monopolist's incentive and offers lower wages, as education is inflated.

(a) $\gamma = 1, k = 0.5$ (b) $\gamma = 1, k = 1$

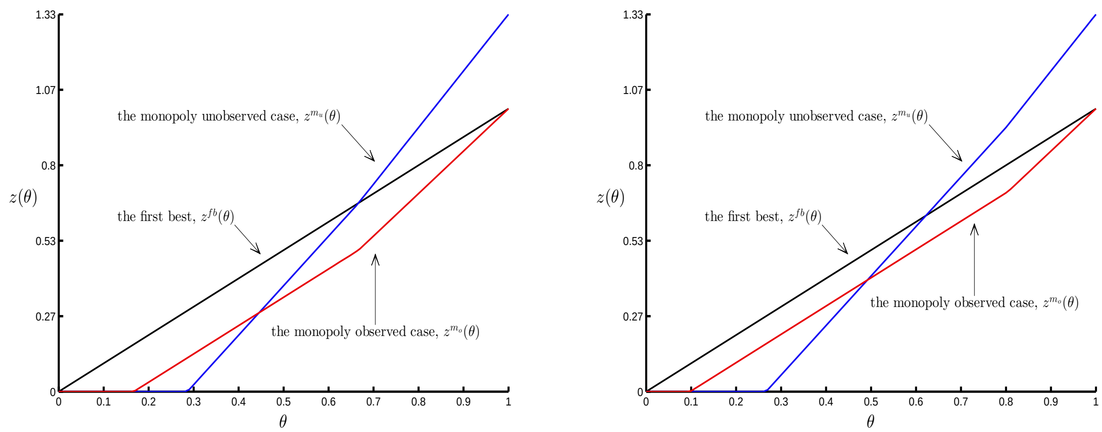

Figure 2.4: Education Comparison between the Observed and Unobserved Case

Furthermore, Proposition 2.4 reveals a significant distinction between Lu (2018) and the current model. That is, an interval of vertical types at the low end obtain more education in the observed case than in the unobserved case; the length of this interval is increasing in the degree of horizontal differentiation and vanishes as the degree approaches zero. Intuitively, when horizontal differentiation rises, to maintain the market share in the partially covered range, the monopolist offers the worker more rents by increasing both the market coverage and education levels. However, the increase in education supply is smaller in the unobserved case than in the observed case, especially at the low end of the market. This is because if the monopolist allocates the same education level to lower types as in the observed case, then the monopolist should allocate more education and leave more rents to higher types to remain incentive compatibility. But since those higher types already obtain higher education levels than in the observed case, supplying more education to them is not profitable. Thus, an interval of lower vertical types obtain less education in the unobserved case than in the observed case. As horizontal differentiation increases, this interval enlarges, meaning that the market coverage is larger in the observed case at any degree of horizontal differentiation. Propositions 2.3 and 2.4 imply that the education function is steeper in the unobserved case than in the observed. These features are illustrated in Figure 2.4.

Proposition 2.4 implies that an interval of lower vertical types obtain lower gross utility in the unobserved case than in the observed case, whereas the others obtain higher gross utility in the unobserved case, and the length of this interval is increasing in the degree of horizontal differentiation. Similarly, the tuition scheme in the unobserved case T^{m_u} is higher than that in the observed case T^{m_o} at the left tail of the common domain, such a region is also increasing in horizontal differentiation. These results differ from that of Lu (2018) in which tuition rates are uniformly lower and the worker is always better-off in the unobserved case than in the observed case. To summarize:

Proposition 2.5. *For any $\gamma, k > 0$, there exists a cutoff $\tilde{\theta} \in (\theta_0^{m_u}, 1)$, such that $V^{m_u}(\theta) < V^{m_o}(\theta)$ on $(\theta_0^{m_o}, \tilde{\theta})$, whereas $V^{m_u}(\theta) > V^{m_o}(\theta)$ on $(\tilde{\theta}, 1]$. The length of the interval $(\theta_0^{m_o}, \tilde{\theta})$ is increasing in k, and vanishes as $k \to 0$. Furthermore, there exists a cutoff $\tilde{z} \in (0,1)$, such that $T^{m_u}(z) > T^{m_o}(z)$ on $(0, \tilde{z})$, whereas $T^{m_u}(z) < T^{m_o}(z)$ on $(\tilde{z}, 1]$. The length of the interval $(0, \tilde{z})$ is increasing in k, and vanishes as $k \to 0$.*

From Corollaries 2.1 and 2.2, we have that the worker's gross utility is increasing in the degree of horizontal differentiation in both the observed and unobserved case. This means that if the worker is close to either school, i.e., $\min\{d_1, d_2\}$ is small enough, then his net utility is also increasing in the degree of horizontal differentiation. Intuitively, as horizontal differentiation increases, the worker's value for education becomes more dispersed, which corresponds to a clockwise rotation in demand (Johnson and Myatt 2006). Consequently, the monopolist lowers prices as the marginal consumer's willingness to pay is lower. This benefits the infra-marginal consumers who are close to either school. Proposition 2.5 thus implies that a low-ability worker who is close to either school benefits more from the rise in horizontal differentiation in the observed case than in the unobserved case. This is because, as is pointed out in the previous argument, the demand for eduction is more elastic in the unobserved case than in the observed case.

2.4 Duopoly

In this section, I consider a duopoly education market in which each school chooses its own contract to maximize its expected profit, given the other school's contract. The purpose of this section is to investigate the effects of market competition on education supply and the market coverage, compared to the monopoly benchmark, for the observed and unobserved case separately. For ease of comparison, I focus on the case in which the vertical market is partially covered in the monopoly case. As such, I assume throughout this section that $k < \bar{k} := \min\left\{\frac{2(\gamma+1)^2}{\gamma+3}, \frac{2(\gamma+3)}{(\gamma+1)^2}\right\}$. As in the monopoly case, I consider symmetric equilibrium. I start the analysis with the observed case.

2.4.1 The Observed Case

Suppose that a symmetric equilibrium exists, such that both schools choose an identical contract $\langle z^*(\theta), V^*(\theta)\rangle$. Thus, given the other school choosing $\langle z^*(\theta), V^*(\theta)\rangle$, school i's best response is to choose $\langle z_i(\theta), V_i(\theta)\rangle = \langle z^*(\theta), V^*(\theta)\rangle$. Given its expected profit in (2.2), school i's problem can be stated as

$$\max \underbrace{\int_{\theta_{0_i}}^{\theta_1} [S(z_i(\theta),\theta) - V_i(\theta)] \frac{V_i(\theta)}{k} d\theta}_{\text{Phase I: the local monopoly range}} + \underbrace{\int_{\theta_1}^{1} [S(z_i(\theta),\theta) - V_i(\theta)] \cdot \left[\frac{1}{4} + \frac{V_i(\theta) - V^*(\theta)}{2k}\right] d\theta}_{\text{Phase II: the competition range}}.$$

$$s.t.\ V_i'(\theta) = z(\theta),\ z_i'(\theta) \geq 0,\ V_i(\theta_1) + V^*(\theta_1) = \frac{k}{2}.$$

If further $\theta_{0_i} \in (0,1]$, then $V_i(\theta_{0_i}) = 0$; otherwise, $V_i(\theta_{0_i})$ is free. Analogously, let us define the Hamiltonian of the two phases and substitute $S(z,\theta)$, then we have:

$$H_1(z_i, V_i, \lambda, \theta) = [(\gamma+1)\theta z_i - z_i^2 - V_i]\frac{V_i}{k} + \lambda z_i,$$
$$H_2(z_i, V_i, \lambda, \theta) = [(\gamma+1)\theta z_i - z_i^2 - V_i] \cdot \left(\frac{1}{4} + \frac{V_i - V^*}{2k}\right) + \lambda z_i,$$

Note that Phase I is exactly the same as that in the monopoly case. If $\theta_{0_i} \in (0,1]$, then the solution to Phase I is also given by:

$$V^*(\theta) = \frac{\gamma+3}{8}(\theta-\theta_0^*)^2,\ z^*(\theta) = \frac{\gamma+3}{4}(\theta-\theta_0^*).$$

Then, I consider Phase II. By the Maximum Principle, we obtain the necessary conditions:

$$\begin{aligned} 0 &= \left[(\gamma+1)\theta z^*(\theta) - z^{*2}(\theta)\right]\cdot\frac{1}{4} + \lambda(\theta), \\ \dot{\lambda}(\theta) &= \frac{1}{4} - \frac{1}{2k}\left[(\gamma+1)\theta z^*(\theta) - z^{*2}(\theta) - V^*(\theta)\right], \end{aligned}$$

combined with the transversality condition $\lambda(1) = 0$. Eliminating $\lambda(\theta)$ from the above two equations, we obtain the following ODE:

$$\ddot{V}^* = \frac{\gamma+2}{2} - \frac{1}{k}[(\gamma+1)\theta\dot{V}^* - \dot{V}^{*2} - V^*].$$

In equilibrium, $V_i(\theta) = V^*(\theta)$, and thus, $V^*(\theta_1) = \frac{k}{4}$. From smooth pasting and the solution to Phase I, we have $\dot{V}^*(\theta_1) = z^*(\theta_1) = \frac{\sqrt{2(\gamma+3)k}}{4}$. In addition, $\lambda(1) = 0$ implies that $\dot{V}^*(1) = z^*(1) = \frac{\gamma+1}{2}$. Thus, the existence of equilibrium reduces to the existence of $\theta_1 \in (0,1]$ and the existence of a convex solution $V(\theta)$ over $[\theta_1, 1]$, satisfying:

$$\ddot{V} = \frac{\gamma+2}{2} - \frac{1}{k}[(\gamma+1)\theta\dot{V} - \dot{V}^2 - V] \tag{2.7}$$

$$s.t.\ V(\theta_1) = \frac{k}{4},\ \dot{V}(\theta_1) = \frac{\sqrt{2(\gamma+3)k}}{4},\ \dot{V}(1) = \frac{\gamma+1}{2}.$$

Note that (2.7) is not a standard BVP, as it involves an endogenous endpoint θ_1. As far as I know, no existing BVP theorem can be applied directly to show the existence and uniqueness of the solution to this problem. The oder-reduce techniques introduced by Rochet and Stole (2002) and Yang and Ye (2008) cannot be applied to (2.7) either. In this regard, I solve problem (2.7) using numerical methods. Let $\langle z^{d_o}(\theta), V^{d_o}(\theta)\rangle$ be the equilibrium

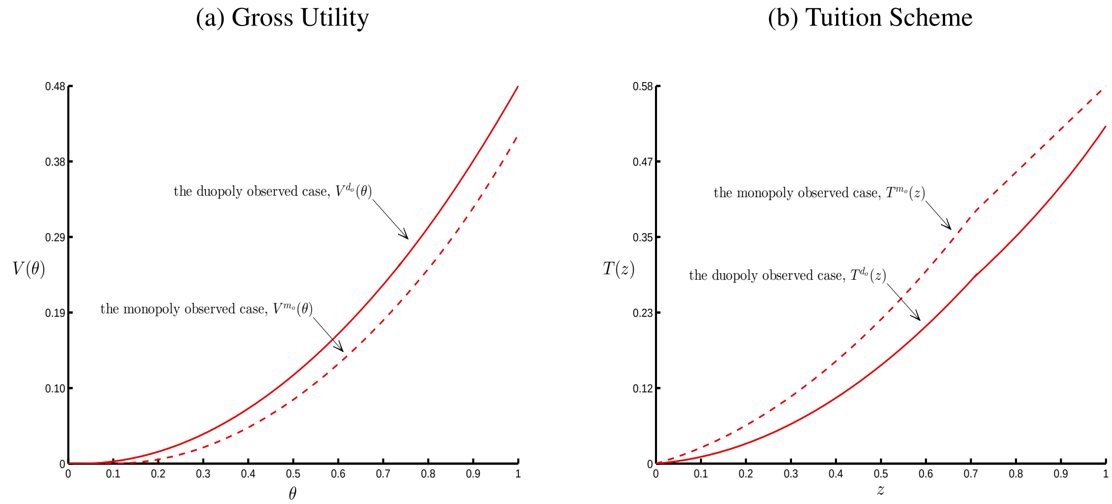

Figure 2.5: A Convex Solution in the Duopoly Observed Case

contract in the duopoly observed case, and $\theta_0^{d_o}$ and $\theta_1^{d_o}$ be the lowest participating type and switching type, respectively. In Figure 2.5, panel (a) depicts a convex solution $V^{d_o}(\theta)$ when $\gamma = 1$ and $k = 1$, along with the worker's gross utility under monopoly $V^{m_o}(\theta)$; panel (b) depicts the associated tuition scheme $T^{d_o}(z)$, along with that of the monopoly observed case $T^{m_o}(z)$. It turns out that under duopoly, tuition is lower and the worker obtains higher utility than under monopoly. I now summarize the equilibrium education allocation as follows:

Proposition 2.6. *Suppose that $k \in (0, \bar{k})$, then in the duopoly observed case, the symmetric school-optimal separating equilibrium exists, such that*

$$z^{d_o}(\theta) = \begin{cases} \frac{\gamma+3}{4}(\theta - \theta_0^{d_o}) & \text{if } \theta_0^{d_o} \leq \theta < \theta_1^{d_o} \\ \dot{V}^{d_o}(\theta) & \text{if } \theta_1^{d_o} \leq \theta \leq 1, \end{cases}$$

where $V^{d_o}(\theta)$ and $\theta_1^{d_o}$ are the solution to problem (2.7), and $\theta_0^{d_o} = \theta_1^{d_o} - \sqrt{\frac{2k}{\gamma+3}}$.

Proposition 2.6 implies that under duopoly, the equilibrium is discontinuous at $k = 0$. When $k = 0$, the equilibrium is a Bertrand-Spence equilibrium in which tuition is pushed down to 0 due to a symmetric Bertrand competition, and thus, the market is fully covered and the education selection is predicted by Spence's model. However, since social surplus is

(a) Education Supply (b) Market Coverage

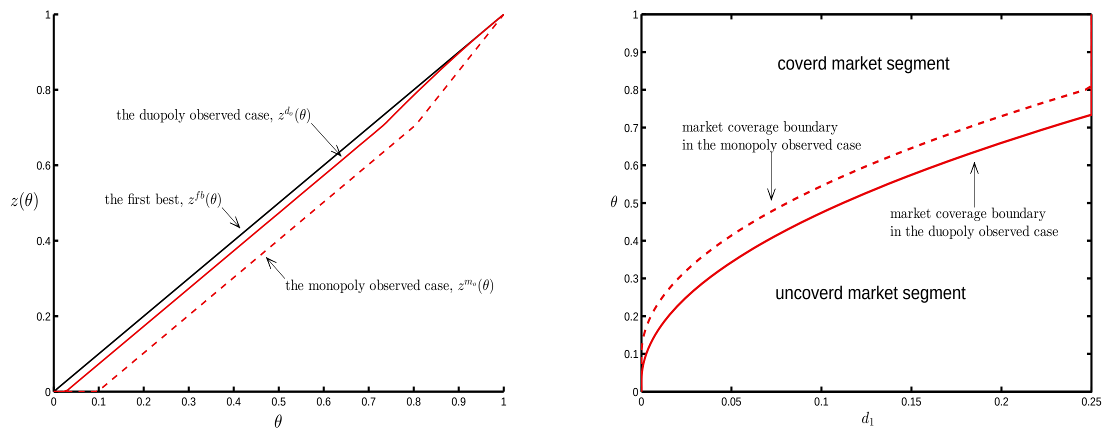

Figure 2.6: The Impacts of Market Competition in the Observed Case

close to 0 for sufficiently low types, so long as $k > 0$, each school becomes a local monopoly for those types. Thus, each school finds it profitable to exclude some very low types from education. Since the threshold is endogenously determined, it leads to distortion for infra-marginal types. In contrast, in Armstrong and Vickers (2001) and Rochet and Stole (2002), the lowest type can generate sufficiently high social surplus, thus, when the market is fully covered, both competing duopolists offer a cost-plus-fee tariff. This is because given that the competitor chooses such a pricing strategy, each duopolist finds it more profitable to make an efficient offer with a higher fixed fee than any inefficient offer.

Going forward, I investigate the impacts of market competition on education supply and the market coverage. The next proposition shows that in contrast to the monopoly case, both education supply and the market coverage are higher under duopoly. This is illustrated in Figure 2.6. A similar result has been obtained by Yang and Ye (2008).

Proposition 2.7. *Given $k \in (0, \bar{k})$, we have $\theta_0^{d_o} < \theta_0^{m_o}$ and $z^{d_o}(\theta) > z^{m_o}(\theta)$ for $\theta \in (\theta_0^{d_o}, 1)$. It follows that in contrast to the monopoly case, more worker types (in terms of both vertical and horizontal type) receive education, and each participating type obtains higher net utility.*

Intuitively, under duopoly, the two schools compete with each other in the fully covered range by providing the worker with more rent than in the monopoly case, thereby extending the fully covered range. Moreover, this relaxes the IC constraint. Specifically, each school fears less about allocating more education to lower types thereby providing higher types with more rent, as higher types will enjoy more rent anyway due to market competition. Hence, the schools increase education supply for all participating types, and include some of those who are not served in the monopoly case.

2.4.2 The Unobserved Case

Now, I turn to the unobserved case. Suppose that a symmetric equilibrium exists, in which both schools choose an identical contract $\langle z^*(\theta), V^*(\theta)\rangle$, and the labor market offers the same wage schedule $W^*(z)$ for both schools' student. Given the wage schedule $W^*(z)$ and that the other school chooses $\langle z^*(\theta), V^*(\theta)\rangle$, the school's problem can be stated as

$$\max \underbrace{\int_{\theta_{0_i}}^{\theta_1} [W^*(z_i(\theta)) - C(z_i(\theta),\theta) - V_i(\theta)] \frac{V_i(\theta)}{k} d\theta}_{\text{Phase I: the local monopoly range}}$$
$$+ \underbrace{\int_{\theta_1}^{1} [W^*(z_i(\theta)) - C(z_i(\theta),\theta) - V_i(\theta)] \cdot \left[\frac{1}{4} + \frac{V_i(\theta) - V^*(\theta)}{2k}\right] d\theta}_{\text{Phase II: the competition range}}.$$

$$s.t.\ V_i'(\theta) = z(\theta),\ z_i'(\theta) \geq 0,\ V_i(\theta_1) + V^*(\theta_1) = \frac{k}{2}.$$

If further $\theta_{0_i} \in (0,1]$, then $V_i(\theta_{0_i}) = 0$; otherwise, $V_i(\theta_{0_i})$ is free. Analogously, let us define the Hamiltonian of the two phases and substitute $S(z,\theta)$, then we have:

$$H_1(z_i, V_i, \lambda, \theta) = [W^*(z_i) - C(z_i,\theta) - V_i]\frac{V_i}{k} + \lambda z_i,$$
$$H_2(z_i, V_i, \lambda, \theta) = [W^*(z_i) - C(z_i,\theta) - V_i] \cdot \left(\frac{1}{4} + \frac{V_i - V^*}{2k}\right) + \lambda z_i,$$

As in the observed case, Phase I coincides with that in the monopoly case. If $\theta_{0_i} \in (0,1]$, then the solution to Phase I is also given by:

$$V^*(\theta) = \frac{4\gamma+3}{8}(\theta-\theta_0^*)^2,\; z^*(\theta) = \frac{4\gamma+3}{4}(\theta-\theta_0^*).$$

Then, I consider Phase II. By the Maximum Principle, we obtain the necessary conditions:

$$\begin{aligned} 0 &= \left[W^{*'}(z^*(\theta)) - 2z^*(\theta) - 1 + \theta\right]\cdot\frac{1}{4} + \lambda(\theta), \\ \dot{\lambda}(\theta) &= \frac{1}{4} - \frac{1}{2k}\left[W^*(z^*(\theta)) - z^{*2}(\theta) - (1-\theta)z^*(\theta) - V^*(\theta)\right], \end{aligned}$$

combined with the transversality condition $\lambda(1) = 0$. Moreover, the correctness of market belief means that $W^*(z) = Q(z, \theta^*(z))$. Then, substituting $W^*(z)$ and eliminating λ from the above two equations, we obtain the following ODE:

$$\dddot{V}^* = \frac{(\gamma+2)\ddot{V}^* + (\gamma-2)\ddot{V}^{*2}}{\gamma\dot{V}^*} + \frac{2}{\gamma k}\left[\frac{V^*\ddot{V}^*}{\dot{V}^*} - (\gamma+1)\theta\dot{V}^* + \dot{V}^*\ddot{V}^*\right].$$

In equilibrium, $V_i(\theta) = V^*(\theta)$, and thus, $V^*(\theta_1) = \frac{k}{4}$. From smooth pasting and the solution to Phase I, we have $\dot{V}^*(\theta_1) = z^*(\theta_1) = \frac{\sqrt{2(4\gamma+3)k}}{4}$. In addition, $\lambda(1) = 0$ combined with the F.O.C. for z implies that $W^{*'}(z(1)) - 2z(1) = 0$, meaning that $\frac{[\gamma-2\dot{V}^*(1)]\dot{V}^*(1)}{\ddot{V}^*(1)} = \gamma+1$. Thus, the existence of equilibrium reduces to the existence of $\theta_1 \in (0,1]$ and the existence of a convex solution $V(\theta)$ over $[\theta_1, 1]$, satisfying:

$$\dddot{V} = \frac{(\gamma+2)\ddot{V} + (\gamma-2)\ddot{V}^2}{\gamma\dot{V}} + \frac{2}{\gamma k}\left[\frac{V\ddot{V}}{\dot{V}} - (\gamma+1)\theta\dot{V} + \dot{V}\ddot{V}\right] \tag{2.8}$$

$$s.t.\; V(\theta_1) = \frac{k}{4},\; \dot{V}(\theta_1) = \frac{\sqrt{2(4\gamma+3)k}}{4},\; \frac{[\gamma-2\dot{V}(1)]\dot{V}(1)}{\ddot{V}(1)} = \gamma+1.$$

Clearly, there is no closed form solution to (2.8) in general. Thus, I solve (2.8) numerically. Let $\langle z^{d_u}(\theta), V^{d_u}(\theta)\rangle$ be the equilibrium contract in the duopoly unobserved case, and $\theta_0^{d_u}$ and $\theta_1^{d_u}$ be the lowest participating type and switching type, respectively. In Figure 2.7, panel (a) depicts a convex solution $V^{d_u}(\theta)$ when $\gamma = 1$ and $k = 1$, along with the worker's gross utility

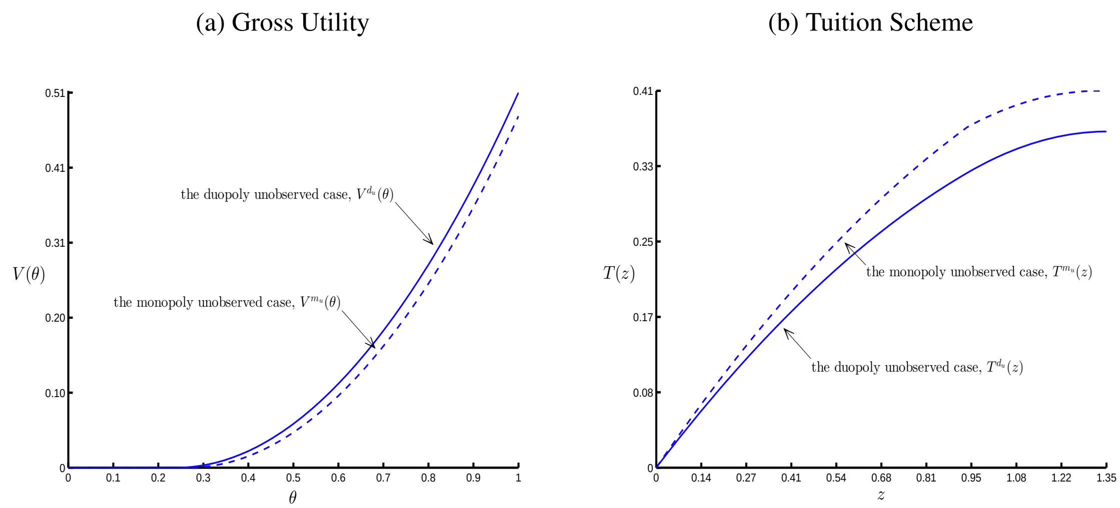

Figure 2.7: A Convex Solution in the Duopoly Unobserved Case

under monopoly $V^{m_u}(\theta)$; panel (b) depicts the associated tuition scheme $T^{d_u}(z)$, along with that of the monopoly unobserved case $T^{m_u}(z)$. Similar to the observed case, under duopoly, tuition is lower and the worker obtains higher utility than under monopoly. I now summarize the equilibrium education allocation as follows:

Proposition 2.8. *Suppose that $k \in (0, \bar{k})$, then in the duopoly unobserved case, the symmetric school-optimal separating equilibrium exists, such that*

$$z^{d_u}(\theta) = \begin{cases} \frac{4\gamma+3}{4}(\theta - \theta_0^{d_u}) & \text{if } \theta_0^{d_u} \leq \theta < \theta_1^{d_u} \\ \dot{V}^{d_u}(\theta) & \text{if } \theta_1^{d_u} \leq \theta \leq 1, \end{cases}$$

where $V^{d_u}(\theta)$ and $\theta_1^{d_u}$ are the solution to problem (2.8), and $\theta_0^{d_u} = \theta_1^{d_u} - \sqrt{\frac{2k}{4\gamma+3}}$.

As in the observed case, the equilibrium is discontinuous at $k = 0$ in the unobserved case. This is because when $k = 0$, the equilibrium is a Bertrand-Spence equilibrium as in the observed case. But so long as $k > 0$, both schools become a local monopoly for sufficiently low types. Consequently, both schools find it profitable to exclude some very low types and thus induce distortion for infra-marginal types.

In addition, I am interested in the impacts of market competition on education supply

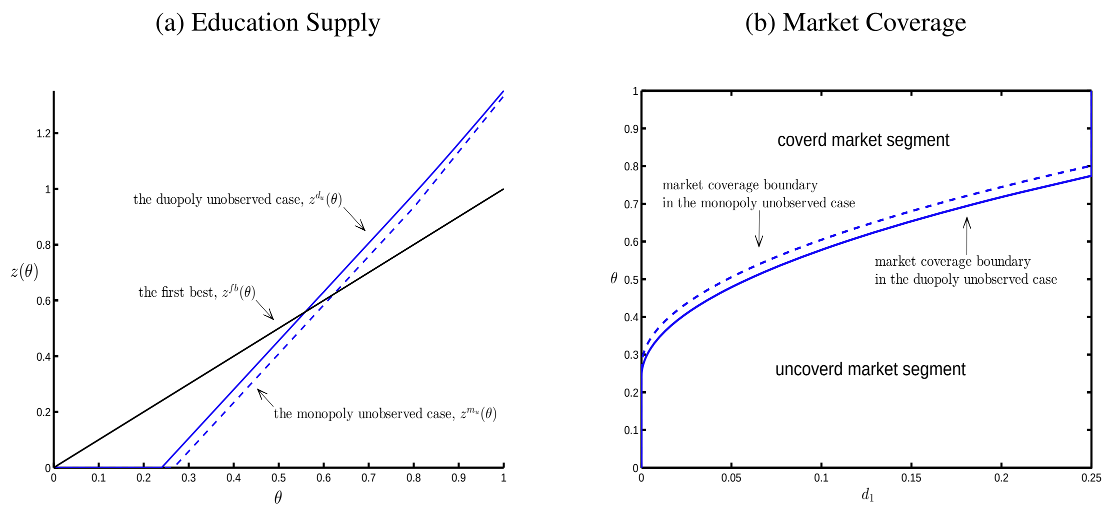

Figure 2.8: The Impacts of Market Competition in the Unobserved Case

and the market coverage in the unobserved case. Unfortunately, I cannot obtain a clear result rigorously. This is mainly due to that in the unobserved case under both monopoly and duopoly, the highest type's education level is not fixed at the first-best but is determined endogenously in equilibrium. Thus, the method in the proof of Proposition 2.7 cannot be applied, and I cannot derive any result from the corresponding ODEs either. Figure 2.8 illustrates a numerical example assuming that $\gamma = 1$ and $k = 1$. It turns out that education supply and the market coverage are indeed higher under duopoly than under monopoly.

The intuition of Figure 2.8 deserves some comments. Suppose that both schools retain the contract of the monopoly case, and thus, the labor market offers the same wage schedule. Then, given the other's contract, each school has an incentive to supply more education. The reason is twofold. First, as in the observed case, each school has an incentive to supply more education to steal the market share from the other in the competition range. Second, since the labor market does not observe the actual tuition scheme, supplying more education can induce the labor market to regard the worker as having higher ability, thereby increasing the worker's willingness to pay. Thus, both schools will rise education supply in the competition range. This in turn relaxes the IC constraint for lower types. Since the signal jamming effect also exists in the local monopoly range, each school will supply more education in this range, and will also include some of those who are not served under monopoly. A noticeable feature of Figure 2.8 is that the increase in education supply is relatively small at the high end of the market. A possible intuition is that both schools already allocate too much education, compared to that in the observed case, to these types under monopoly. Thus, the schools find it less profitable to allocate more education to those high types.

2.5 Conclusion

In this paper, I studied nonlinear pricing for horizontally differentiated products that provide signaling values to consumers, who choose how much to purchase as a signal to the receivers. I characterized the optimal symmetric price schedules under different market structures. The equilibrium depends critically on whether the receivers observe the sellers' price schedules, as well as on the market structure. Under monopoly, when the receivers observe the price schedule, the market is partially covered, and quantity is downward distorted if there is little horizontal differentiation. As the degree of horizontal differentiation rises, the market coverage rises, and the downward distortion decreases. When the degree is sufficiently high, for some intermediate level of signaling intensity, the monopolistic allocation achieves the first-best; for higher signaling intensities, quantity is upward distorted at the low end. In contrast, when the receivers do not observe any price schedule, the market is always partially covered, and the allocation is more dispersed than that in the observed case. When the market structure changes from monopoly to duopoly, market competition results in a higher market coverage and larger quantities for both the observed and unobserved case.

By studying the products that provide signaling values to consumers who possess private information, my model obtains qualitatively different welfare implications from standard competitive nonlinear pricing models. Moreover, this framework allows us to examine the interaction between horizontal competition and the transparency of pricing, and to assess the joint effects of these two forces on the equilibrium allocation and welfare.

2.6 Appendix

2.6.1 Omitted Proofs

Proof of Proposition 2.4.

Proof. Since for any $\gamma, k > 0$, $\theta_0^{m_u} > 0$, we only need to focus on the case in which both $\theta_0^{m_o}$ and $\theta_0^{m_u}$ are positive. I first prove that $\theta_0^{m_u} > \theta_0^{m_o}$. From (2.4) and (2.6), we have

$$\begin{aligned}\theta_0^{m_u} - \theta_0^{m_o} &= \frac{(\gamma+1)\sqrt{2(\gamma+3)k}}{2(\gamma+2)(\gamma+3)} - \frac{\sqrt{2(4\gamma+3)k}}{4(\gamma+1)(4\gamma+3)} \\ &= \frac{\gamma+1}{2(\gamma+2)}\sqrt{\frac{2k}{\gamma+3}}\left[1 - \frac{\gamma+2}{2(\gamma+1)^2}\sqrt{\frac{\gamma+3}{4\gamma+3}}\right]\end{aligned}$$

It can be easily verified that the value of the above bracket is positive for any $\gamma > 0$. Thus, we have $\theta_0^{m_u} > \theta_0^{m_o}$. Similarly, we have

$$\theta_1^{m_u} - \theta_1^{m_o} = \frac{\sqrt{2(4\gamma+3)k}}{4(\gamma+1)}\left[1 - \frac{2(\gamma+1)}{\gamma+2}\sqrt{\frac{\gamma+3}{4\gamma+3}}\right]$$

It can be easily verified that the value of the above bracket is negative for any $\gamma > 0$. Thus, we have $\theta_1^{m_u} < \theta_1^{m_o}$. This completes the proof of the first statement.

Then, I prove that there exists a cutoff $\tilde{\theta} \in (\theta_0^{m_u}, \theta_1^{m_u})$, such that $z^{m_u}(\theta) < z^{m_o}(\theta)$ on $(\theta_0^{m_o}, \tilde{\theta})$, but $z^{m_u}(\theta) > z^{m_o}(\theta)$ on $(\tilde{\theta}, 1]$. From Propositions 2.1 and 2.3, $z^{m_u}(1) > z^{m_o}(1)$. Since $z^{m_u}(\theta_0^{m_u}) = z^{m_o}(\theta_0^{m_o}) = 0$ and $\theta_0^{m_u} > \theta_0^{m_o}$, we have that $z^{m_u}(\theta)$ crosses $z^{m_o}(\theta)$ at least once. Let $\tilde{\theta}$ be one of the intersecting points. I will prove that $\tilde{\theta} \in (\theta_0^{m_u}, \theta_1^{m_u})$. Suppose not, then we have $\tilde{\theta} \in [\theta_1^{m_u}, 1)$. We shall consider two cases: (i) $\tilde{\theta} \in [\theta_1^{m_u}, \theta_1^{m_o})$; (ii) $\tilde{\theta} \in (\theta_1^{m_o}, 1)$. Suppose that (i) holds, then we have

$$(\gamma+1)\tilde{\theta} - \frac{\gamma+1}{\gamma+2} = \frac{\gamma+3}{4}(\tilde{\theta} - \theta_0^{m_o}).$$

It follows that

$$\tilde{\theta} - \theta_0^{m_o} = \frac{4\gamma+4}{3\gamma+1}\left(\frac{1}{\gamma+2} - 1\right) < 0,$$

leading to a contradiction. Then, I consider (ii). Suppose that (ii) holds, then we have

$$(\gamma+1)\tilde{\theta} - \frac{\gamma+1}{\gamma+2} = \frac{(\gamma+2)\tilde{\theta}}{2} - \frac{1}{2}.$$

It follows that $\tilde{\theta} = \frac{1}{\gamma+2} < \theta_1^{m_o}$, leading to a contradiction. Thus, we have $\tilde{\theta} \in (\theta_0^{m_u}, \theta_1^{m_u})$. It remains to show that such a $\tilde{\theta}$ is unique. To see this, note that $z^{m_u'}(\theta) > z^{m_o'}(\theta)$ on $(\theta_0^{m_u}, 1)$, as $\tilde{\theta} \in (\theta_0^{m_u}, \theta_1^{m_u})$. Since $z^{m_u}(\tilde{\theta}) = z^{m_o}(\tilde{\theta})$, $z^{m_u}(\theta)$ is single-crossing $z^{m_o}(\theta)$ from below at $\tilde{\theta}$. This completes the proof of this statement. Finally, note that both $\theta_0^{m_o}$ and $\theta_1^{m_u}$ converge to $\frac{1}{\gamma+2}$ as $k \to 0$. Thus, $(\theta_0^{m_o}, \theta_1^{m_u})$ vanishes as $k \to 0$. The proposition is thus proven. □

Proof of Proposition 2.5.

Proof. I first prove that there exists a cutoff $\tilde{\theta} \in (\theta_0^{m_u}, 1)$, such that $V^{m_u}(\theta) < V^{m_o}(\theta)$ on $(\theta_0^{m_o}, \tilde{\theta})$, but $V^{m_u}(\theta) > V^{m_o}(\theta)$ on $(\tilde{\theta}, 1]$. From Lemma 2.1, we have

$$V^{m_u}(1) = \frac{k}{4} + \int_{\theta_1^{m_u}}^{1} z^{m_u'}(\theta)d\theta > \frac{k}{4} + \int_{\theta_1^{m_o}}^{1} z^{m_o'}(\theta)d\theta = V^{m_o}(1).$$

The inequality is due to that $\theta_1^{m_u} < \theta_1^{m_o}$ and $z^{m_u'}(\theta) > z^{m_o'}(\theta)$ on $(\theta_0^{m_u}, 1)$ from the proof of Proposition 2.4. Since $V^{m_u}(\theta_0^{m_u}) = V^{m_o}(\theta_0^{m_o}) = 0$ and $\theta_0^{m_u} > \theta_0^{m_o}$, it follows from Lemma 2.1 and the single-crossing between $z^{m_u}(\theta)$ and $z^{m_o}(\theta)$ that $V^{m_u}(\theta)$ is single-crossing $V^{m_o}(\theta)$ from below at some $\tilde{\theta} \in (\theta_0^{m_u}, 1)$. This completes the proof of this statement.

Then, I prove that there exists a cutoff $\tilde{z} \in (0, z^{m_o}(1))$, such that $T^{m_u}(z) > T^{m_o}(z)$ on $(0, \tilde{z})$, but $T^{m_u}(z) > T^{m_o}(z)$ on $(\tilde{z}, z^{m_o}(1)]$. Let $\tilde{\theta}'$ be the cutoff such that $z^{m_u}(\theta) < z^{m_o}(\theta)$ on

$(\theta_0^{m_o}, \tilde{\theta}')$, but $z^{m_u}(\theta) > z^{m_o}(\theta)$ on $(\tilde{\theta}', 1]$. From the worker's F.O.C. in both cases, we have

$$W^{m_o'}(z) - T^{m_o'}(z) = C_z(z, \theta^{m_o}(z)) \ , W^{m_u'}(z) - T^{m_u'}(z) = C_z(z, \theta^{m_u}(z)).$$

Integrating both differential equations from 0 to $z^{m_o}(1)$, we have

$$\begin{aligned} W^{m_o}(z^{m_o}(1)) - T^{m_o}(z^{m_o}(1)) &= C(z^{m_o}(1), 1), \\ W^{m_u}(z^{m_o}(1)) - T^{m_u}(z^{m_o}(1)) &= C(z^{m_o}(1), \theta^{m_u}(z^{m_o}(1))). \end{aligned}$$

According to Proposition 2.4, $\theta^{m_u}(z^{m_o}(1)) < 1$. Since $W^{m_o}(z) = Q(z, \theta^{m_o}(z))$ and $W^{m_u}(z) = Q(z, \theta^{m_u}(z))$, $W^{m_o}(z^{m_o}(1)) > W^{m_u}(z^{m_o}(1))$. Also, since $C_{z\theta} < 0$, $C(z^{m_o}(1), \theta^{m_u}(z^{m_o}(1))) > C(z^{m_o}(1), 1)$. It thus follows that $T^{m_o}(z^{m_o}(1)) > T^{m_u}(z^{m_o}(1))$.

Moreover, since $z^{m_u}(\theta) < z^{m_o}(\theta)$ on $(\theta_0^{m_o}, \tilde{\theta}')$ and both $z^{m_u}(\theta)$ and $z^{m_o}(\theta)$ are increasing, $\theta^{m_u}(z) > \theta^{m_o}(z)$ on $(\theta_0^{m_o}, \tilde{\theta}')$. Thus, $C_z(z, \theta^{m_u}(z)) < C_z(z, \theta^{m_o}(z))$ on $(\theta_0^{m_o}, \tilde{\theta}')$. It follows that $W^{m_u}(z) - T^{m_u}(z) < W^{m_o}(z) - T^{m_o}(z)$ on $(0, z^{m_o}(\tilde{\theta}')]$. Since $W^{m_o}(z) = Q(z, \theta^{m_o}(z))$ and $W^{m_u}(z) = Q(z, \theta^{m_u}(z))$ on $(0, z^{m_o}(\tilde{\theta}')]$, $W^{m_u}(z) > W^{m_o}(z)$ on $(0, z^{m_o}(\tilde{\theta}')]$. Thus, it is readily confirmed that $T^{m_u}(z) > T^{m_o}(z)$ on $(0, z^{m_o}(\tilde{\theta}')]$. However, since $T^{m_o}(z^{m_o}(1)) > T^{m_u}(z^{m_o}(1))$, continuity implies that $T^{m_u}(z)$ must intersect $T^{m_o}(z)$ at some $\tilde{z} > z^{m_o}(\tilde{\theta}')$. It remains to prove that such $\tilde{z}$ is unique. To see this, note that for both the observed case

$$\begin{aligned} T^{m_o}(z) &= S(z, \theta^{m_o}(z)) - V^{m_o}(\theta^{m_o}(z)), \\ T^{m_u}(z) &= S(z, \theta^{m_u}(z)) - V^{m_u}(\theta^{m_u}(z)). \end{aligned}$$

Differentiating both equations with respect to z, we have

$$\begin{aligned} T^{m_o'}(z) &= S_z(z, \theta^{m_o}(z)) + S_\theta(z, \theta^{m_o}(z)) \cdot \theta^{m_o'}(z) - V^{m_o'}(\theta^{m_o}(z)) \cdot \theta^{m_o'}(z) \\ T^{m_u'}(z) &= S_z(z, \theta^{m_u}(z)) + S_\theta(z, \theta^{m_u}(z)) \cdot \theta^{m_u'}(z) - V^{m_u'}(\theta^{m_u}(z)) \cdot \theta^{m_u'}(z). \end{aligned}$$

Substituting $S(z,\theta)$ into the above equations and note that $V'(\theta) = z(\theta)$, we have

$$T^{m'_o}(z) = (\gamma+1)\theta^{m_o}(z) - 2z + \gamma z \cdot \theta^{m'_o}(z),$$
$$T^{m'_u}(z) = (\gamma+1)\theta^{m_u}(z) - 2z + \gamma z \cdot \theta^{m'_u}(z).$$

Since $\tilde{z} > z^{m_o}(\tilde{\theta}')$, for any $z \in (\tilde{z}, z^{m_o}(1))$, we have $\theta^{m_o}(z) > \theta^{m_u}(z)$ by the definition of $\tilde{\theta}'$. From the proof of Proposition 2.4, we have $z^{m'_u}(\theta) > z^{m'_o}(\theta)$ on $(\theta_0^{m_u}, 1)$. This implies that $\theta^{m'_o}(z) > \theta^{m'_u}(z)$ on $(0, z^{m_o}(1))$. Thus, we have $T^{m'_o}(z) > T^{m'_u}(z)$ on $(\tilde{z}, z^{m_o}(1))$. Since $T^{m_o}(\tilde{z}) = T^{m_u}(\tilde{z})$, such $\tilde{z}$ must be unique. The statement is thus proven. The rest part of the proposition follows immediately from Proposition 2.4. Thus, Proposition 2.5 is proven. □

Proof of Proposition 2.7.

Proof. I first prove that $\theta_0^{d_o} < \theta_0^{m_o}$. Suppose not, then $\theta_0^{d_o} \geq \theta_0^{m_o}$. Note that $\theta_1^{d_o} - \theta_0^{d_o} = \theta_1^{m_o} - \theta_0^{m_o} = \sqrt{\frac{2k}{\gamma+3}}$, thus $\theta_1^{d_o} \geq \theta_1^{m_o}$. From Propositions 2.1 and 2.6, we have

$$z^{d_o}(\theta_1^{d_o}) = z^{m_o}(\theta_1^{m_o}) = \frac{\sqrt{2(\gamma+3)k}}{4}.$$

Since $\theta_1^{d_o} \geq \theta_1^{m_o}$, for $\theta \in [\theta_1^{m_o}, \theta_1^{d_o}]$, $z^{m'_o}(\theta) = \frac{\gamma+2}{2}$. This implies that $z^{m_o}(\theta_1^{d_o}) > z^{d_o}(\theta_1^{d_o})$.

Moreover, from (2.7), we have that for $\theta \in [\theta_1^{d_o}, 1]$,

$$\begin{aligned} z^{d'_o}(\theta) &= \frac{\gamma+2}{2} - \frac{1}{k}[(\gamma+1)\theta z^{d_o}(\theta) - z^{d_o^2}(\theta) - V^{d_o}(\theta)] \\ &= \frac{\gamma+2}{2} - \frac{1}{k}[S(z^{d_o}(\theta),\theta) - V^{d_o}(\theta)] \end{aligned}$$

In equilibrium, each school must gain positive profit for each type in the fully covered range, i.e., $S(z^{d_o}(\theta),\theta) > V^{d_o}(\theta)$ for $\theta > \theta_1^{d_o}$. Thus, we have that for $\theta \in [\theta_1^{d_o}, 1]$,

$$z^{d'_o}(\theta) < z^{m'_o}(\theta) = \frac{\gamma+2}{2}.$$

Since $z^{m_o}(\theta_1^{d_o}) > z^{d_o}(\theta_1^{d_o})$, we have $z^{m_o}(1) > z^{d_o}(1)$. This contradicts the fact that $z^{m_o}(1) = z^{d_o}(1) = z^{fb}(1)$. Thus, we have $\theta_0^{d_o} < \theta_0^{m_o}$. This also implies that $\theta_1^{d_o} < \theta_1^{m_o}$.

Then, I prove that $z^{d_o}(\theta) > z^{m_o}(\theta)$ on $(\theta_0^{d_o}, 1)$. First, consider $\theta \in (\theta_0^{d_o}, \theta_1^{d_o}]$. Since on this interval $z^{m_o'}(\theta) = z^{d_o'}(\theta)$ and $\theta_0^{d_o} < \theta_0^{m_o} = \frac{\gamma+2}{2}$, we have $z^{d_o}(\theta) > z^{m_o}(\theta)$ for all $\theta \in (\theta_0^{d_o}, \theta_1^{d_o}]$. Second, consider $\theta \in (\theta_1^{d_o}, \theta_1^{m_o}]$. Since $z^{d_o}(\theta_1^{d_o}) = z^{m_o}(\theta_1^{m_o})$ and both $z^{d_o}(\theta)$ and $z^{m_o}(\theta)$ are increasing, we have $z^{d_o}(\theta) > z^{m_o}(\theta)$ for all $\theta \in (\theta_1^{d_o}, \theta_1^{m_o}]$. Finally, consider $\theta \in (\theta_1^{m_o}, 1]$. Due to the above analysis, we have $z^{d_o'}(\theta) < z^{m_o'}(\theta)$ on $(\theta_1^{m_o}, 1]$. Since $z^{m_o}(1) = z^{d_o}(1)$, $z^{d_o}(\theta) > z^{m_o}(\theta)$ for all $\theta \in (\theta_1^{m_o}, 1)$. Thus, we have $z^{d_o}(\theta) > z^{m_o}(\theta)$ on $(\theta_0^{d_o}, 1)$.

Since $\theta_0^{d_o} < \theta_0^{m_o}$ and $z^{d_o}(\theta) > z^{m_o}(\theta)$ on $(\theta_0^{d_o}, 1)$, from Lemma 2.1, $V^{d_o}(\theta) > V^{m_o}(\theta)$ on $(\theta_0^{d_o}, 1)$. It is thus readily confirmed that more types, w.r.t. both horizontal and vertical types, are served in the market under duopoly. Thus, the proposition is proven. □

CHAPTER 3

Optimal Sequence for Teamwork

3.1 Introduction

With the emergence of new business models and advanced communication technology, a large variety of workplace architectures have come into existence: typical examples include the "open space" or "war room" model adopted by Bloomberg, Google, Goldman Sachs, etc., and in contrast loose-fitting designs such as Virtual Locations promoted by Amazon. Such an architecture essentially determines how information *can* flow internally among peers. For instance, a worker's job attitudes or efforts can be observed by neighboring colleagues in an open office, but not by someone remote or partitioned. As indicated by empirical evidence, workers' productivity and willingness to work respond positively to observed efforts of peers (Ichino and Maggi 2000; Heywood and Jirjahn 2004; Gould and Winter 2009; Mas and Moretti 2009). In this context, a principal responsible for incentive design is essentially endowed with a monitoring structure among agents that helps reduce the cost of inducing full effort: the more peers an agent can affect via a change in decision (e.g., from working to shirking), which means a greater impact on the success of the whole team, the less incentive needed for her effort exertion. However, there is still considerable room for designing the optimal incentives, as in many situations the principal can dictate the order of tasks, and hence how information *will* flow, as well as the outcome-based rewards. This paper seeks to answer a naturally spurred question in this context: given a workplace architecture, what is the optimal sequence of assigning tasks, and what is the optimal associated reward scheme?

Winter (2010) has provided a thorough discussion on the optimal incentive design under an exogenous task assignment sequence. In contrast, our paper focus on endogenizing the sequence in the principal's problem, and makes two contributions. First, we extend Winter's model to include the design of sequence as the principal's available option in addition to the reward scheme. Since there is a one-to-one relation between every sequence and the optimal rewards that follow, the optimal sequence predicted by our model is essentially unique for a wide range of architectures. Second, we explicitly characterize the optimal sequence for several typical classes of architectures, including simple ones such as cliques and stars, and composite ones such as core-periphery networks. We find that the solution relies heavily on the shape of the architecture as well as heterogeneity among agents. In general, the optimal sequence allocates agents with more transparent actions to intermediate positions, and less capable/important agents are assigned tasks earlier than their more capable/important peers.

Our model considers a group of multiple agents working on a joint project, where each agent is responsible for an individual task. Externally, the agents face the standard moral hazard problem. Each agent needs to decide whether or not to exert effort, which cannot be observed by the principal, and the success probability of the whole project is determined by the joint effort profile. In most parts of our analysis, the agents' tasks are complementary. We allow agents to be heterogeneous, in the sense that some agent's effort may impose a greater influence on the success probability than others. Internally, the agents are connected to one another via an effort observation structure, which represents a workplace architecture in various applications. We model this structure as a network: a link ij between agent i and agent j implies that i observes j's effort if j makes his decision before i, and vice versa. In other words, feasibility of effort observation between two linked agents is bilateral in nature, while the actual flow of internal information is determined by the order of task execution.

The principal faces two problems of incentive design. First, she chooses and commits to a sequence of task assignment, in every period of which at least one agent is asked to make their decision on their corresponding task. Second, she offers a reward to each agent contingent on the final outcome of the project. The principal's objective is to minimize the total cost – that is, the sum of rewards upon success of the project – while inducing full effort from the agents.

Once the sequence is fixed, the network essentially produces a unique acyclic flow of internal information. We can then apply Winter (2010)'s result to characterize the optimal reward scheme. Also, a straightforward application of results from other prior works (e.g., Winter 2006) shows that the optimal sequence in a fully connected network, or a clique, is the reverse order in importance, i.e. the agent who is least influential to the project's success moves first, the second least influential agent moves second, and so forth. Our main analysis thus focuses on more complex and possibly asymmetric networks.

Our first novel result characterizes the optimal sequence in a star network. Star networks are representative of many important workplace relationships and social interactions, for instance, the center being the general contractor for a construction project and the peripheries being the subcontractors who work on different parts of the building job and communicate with the general contractor. We propose a simple algorithm to find the optimal sequence, which is unique subject to a number of trivial variations with identical reward schemes. In the sequence, the center takes up a position between two subgroups of peripheries, allowing the principal to offer less incentives to peripheries before the center at the cost of more incentives to those after the center. The optimum represents the balance between marginal benefit and marginal cost. In addition, more important agents move later in the sequence, so that every agent after the center is more important than any agent before the center. Intuitively, when an early mover's decision affects a group of more important peers, his shirking will trigger a greater reduction in the project's success probability. Hence, his implicit cost of shirking rises, which is always to the principal's advantage.

Not all workplace architectures or relationships can be approximated by simple structures such as cliques and stars. Instead, a complex architecture may be regarded as the composition of multiple simple ones, as in large projects that require the collaboration of several small teams. A typical class of such architectures is core-periphery networks, in which the centers of multiple stars are interlinked. We first consider a core-periphery structure for a vertical project, i.e., the order of executing tasks between stars is fixed while that within each star is decided by the principal. This architecture can represent a multi-phase project with vertical collaboration, e.g., the development of drugs that include preclinical, investigational and post-marketing phases. As above, we identify an algorithm that characterizes the essentially unique optimal sequence. In the sequence, one and only one of the stars plays a special role. Before it, all periphery agents of each star execute their tasks before their corresponding center, and obtain lower rewards when the project succeeds; after it, on the contrary, all periphery agents of each star execute their tasks after their corresponding center, and obtain higher rewards when the project succeeds. Such dichotomy results from monotonicity in the influence of an agent's action according to the position their star takes: the earlier their star is in the series, having the agent work his task before the center means more peers whom he can affect via internal information, and thus, a higher benefit from reducing his reward for the principal; at the same time, fewer agents would affect his action if he were placed after the center, implying a lower opportunity cost from reducing the rewards of those agents.

We further consider a core-periphery network for a horizontal project, that is, a set of inter-connected stars in which the order of executing tasks between stars is also decided by the principal. Projects that require horizontal collaboration of multiple departments, such as those for different components of an assembled final product like a cell phone or a motor vehicle, are typical examples of this architecture. We provide a partial characterization of the optimal sequence. On one hand, the sequence also features a "special" star before which all peripheries precede their center in task assignment and after which all peripheries follow their center. On the other hand, the number of peripheries in the optimal sequence must exhibit a "V-shape": before the "special" star, stars with more peripheries are assigned tasks first, while after the "special" star the pattern reverses. Placing a large star towards the end of the sequence allows the principal to raise the maximum size of reward reduction for each earlier agent, while placing one towards the beginning of the sequence exposes more agents to a larger-scale reward reduction.

The theoretical literature on incentive design for teamwork is extensive and growing. The trade-off an agent faced between working and shirking originates from the classical literature on moral hazard (Holmstrom 1982; Holmstrom and Milgrom 1991; Itoh 1991). Subsequent studies developed this literature to static contracting on teamwork with a number of variations, such as externalities (McAfee and McMillan 1991; Segal 1999; Babaioff et al. 2012), specialization versus multitasking (Balmaceda 2016), and loss-averse agents (Balmaceda 2018). Che and Yoo (2001), Segal (2003), Bernstein and Winter (2012) studied contracting problems in a dynamic context, with the main focus on how various forms of externalities affect the optimal contracts. A comprehensive study on the role of internal information in teamwork, with an exogenous sequence of task assignment, has been given by Winter (2004, 2006, 2010). This is the main strand of literature that our work contrasts to by endogenizing the sequence. Finally, experimental studies on behavior in team production have also indicated that an agent's contribution in teamwork is highly responsive to internal information (Carpenter et al. 2009; Steiger and Zultan 2014) and that unequal rewards tend to facilitate coordination and improve efficiency (Goerg, Kube, and Zultan 2010).

The rest of the paper is organized as follows. Section 3.2 describes the model. Section 3.3 presents the result for a fully connected network. Sections 3.4 and 3.5 present the results for a star network and a core-periphery network, respectively. Section 3.6 concludes. All omitted proofs are provided in the Appendix.

3.2 The Model

Players and actions A principal (*she*) owns a project that is collectively managed by a set N of n agents. Each agent (*he*) is responsible for a single task and chooses whether to exert effort or not. Formally, each agent's action space is given by $A = \{0, 1\}$, with $a = 1$ if the agent chooses to exert effort and $a = 0$ if he shirks. The cost of effort is $c > 0$ and constant over all agents. To save on notation, we normalize c to 1 without loss of generality. Hereafter, we interchange the terms *work* and *exert effort*.

Network The organizational structure, also referred to as the *network* of the agents, is characterized by an intrinsic and undirected graph g of unordered pairs (i, j) of agents who are directly linked. This network could origin from the workplace architecture, the authority structure, informal social networks and so forth. The structure of g is common knowledge.

Technology The organization's technology is a mapping from a profile of effort decisions to a probability of the project's success. For a subset $S \subseteq N$ of working agents, the probability of the project's success is $p(S)$. Throughout the paper, we assume that p is increasing in the sense that if $T \subset S$, then $p(T) < p(S)$. Moreover, we distinguish between the technology's properties of complementarity and substitutability. A technology p satisfies *complementarity* among agents if for every two sets of agents S and T with $T \subset S$ and every agent $i \notin S$, we have $p(S \cup \{i\}) - p(S) > p(T \cup \{i\}) - p(T)$; that is, i's effort is more effective if the set of other agents who exert effort enlarges. By contrast, we say that p satisfies *substitutability* among agents if $p(S \cup \{i\}) - p(S) \leq p(T \cup \{i\}) - p(T)$. In addition, we distinguish between different agents' importances to the project. We say that agent i is (weakly) more important than j if for every coalition $S \subseteq N$ with $i, j \in S$, we have $p(S \backslash \{i\}) \leq p(S \backslash \{j\})$; that is, i's shirking is more detrimental than j's to the probability of success. We assume that the set N is totally ordered in terms of importance.

Mechanism Before the agents execute their tasks, the principal has to choose a sequence of execution (permutation) π such that agent i is the π_i-th, with $\pi_i \in \{1, \ldots, n\}$, player to act. In addition, the principal has to design a reward scheme $v = (v_1, \ldots, v_n)$ such that agent i receives $v_i \geq 0$ if the project turns out to be successful, and receives zero payoff otherwise. A mechanism $\{\pi, v\}$ consists of a sequence of execution π and a reward scheme v for the agents. Throughout, we assume that the principal can commit to the mechanism.

Information The principal cannot monitor the agents' effort choices, but knows whether the project is a success or a failure after all effort decisions have been made.

The agents' *internal information* about their peers' effort choices is jointly determined by the graph g and the permutation π. Specifically, agent i observes agent j's action, or simply *i sees j*, before i acts if and only if i and j are directly linked in g (i.e., $(i,j) \in g$), and i acts after j (i.e., $\pi_i > \pi_j$).[1] For every pair (g,π), we define $N_i(g,\pi) := \{j|(i,j) \in g, \pi_i > \pi_j\}$ to be the set of agents whom agent i sees given the internal information determined by (g,π). To save on notation, we write N_i for the set $N_i(g,\pi)$ henceforth.

Principal's problem Consider the game that is defined by the set of agents N, the agents' action space A, the network g and a mechanism $\{\pi, v\}$. A strategy for agent i is a function $s_i : 2^{N_i} \to \{0,1\}$ which specifies the agent's effort choice as a function of his information on the effort decisions of other agents who are in N_i. For every strategy profile $s = (s_1, \ldots, s_n)$, agent i's expected utility is given by

$$u_i(s) = p(W(s))v_i - s_i,$$

where $W(s)$ is the set of agents who work given s.

A mechanism $\{\pi, v\}$ is *effort-inducing (EFI)* with respect to the network if there exists a *perfect Bayesian equilibrium (PBE)* s^* for the associated game such that all the agents exert effort (i.e., $W(s^*) = N$). The principal's problem is to design an EFI mechanism that yields minimal total payoffs to the agents among all EFI mechanisms. We call this mechanism an optimal EFI mechanism. In particular, for a fixed permutation π, a reward scheme $v^*(\pi)$ is optimal if $\{\pi, v^*(\pi)\}$ is an optimal EFI mechanism. The principal's objective is economically meaningful when she has a relatively high value for the project and each agent is sufficiently productive, such that exerting effort increases the probability of a success to a great extent. Alternatively, one can consider the mechanism that maximizes the principal's net profit (i.e., the project's value minus the agents' rewards). We find that doing so does not provide new insights, while complicates the analysis remarkably.

[1] If i and j act simultaneously, then neither of them can see the other.

Remark. *Our model makes a notable assumption for the information structure, that is, an agent can observe only the actions of those who are directly connected to him. This stands in contrast to Winter (2006)'s model in which the agents can see all their predecessors' actions. Clearly, if an agent could observe each preceding action, then the information he possesses depends only on the sequence of execution, making the network irrelevant. Consequently, one would derive the same results as in Winter's model. In order to examine the impacts of network topology on incentive design, we impose this assumption. Moreover, the assumption is reasonable when communication about others' efforts is costly between individuals. For example, it might be difficult to provide evidence showing that a worker shirked; in many companies, it is unprofessional to discuss colleagues' job performances, preventing workers from sharing such information with others.*

3.2.1 Preliminary Analysis

In this subsection, we characterize the optimal reward scheme $v^*(\pi)$ given a fixed sequence of execution π. Whereas this result has been established by Winter (2010), we keep this part in the paper for the sake of the completeness of analysis. We start the characterization with a complementary technology.

Let the technology p satisfies complementarity. Define $M_i(g,\pi)$, M_i for short, to be the set of agents satisfying that for each $j \in M_i$ there exists a sequence $\{k_r\}$ such that j sees k_1 sees k_2 sees $\dots k_r$ sees i. That is, everyone in M_i can ultimately learn i's action if an agent could share his information with those who see him. Proposition 3.1 characterizes the optimal reward scheme $v^*(\pi)$ with respect to an arbitrary permutation π.

Proposition 3.1. *Suppose that p satisfies complementarity. For any fixed permutation π, the optimal reward scheme $v^*(\pi)$ exists and pays agent i $v_i^* = [p(N) - p(N \backslash (\{i\} \cup M_i))]^{-1}$.*

The intuition of Proposition 3.1 is that when the agents execute their tasks sequentially, they are facing an implicit threat of shirking; that is, the exposure of a low effort might induce an agent who observes this behavior to shirk and consequently trigger a *domino effect* of shirking, making a success less likely. This implicit threat reduces the agent's incentive cost. Under complementarity and an optimal reward scheme, it is indeed sequentially rational for an agent to shirk once he sees someone shirking, making the implicit threat credible. In addition, Proposition 3.1 implies that if agent i's action becomes more transparent in the sense that the set M_i increases, then the principal should pay i less since i has a greater implicit threat now. In general, an agent's decision is more transparent when he acts earlier in the sequence and has more links connected to him.

We now turn to the case of substitutability. In contrast to the case of complementarity, under a substitutable technology, the internal information has no impact on incentives as if all the agents acted simultaneously. To implement full effort, the principal must provide the agents sufficient incentives when they believe that all their peers are working. However, due to the substitutability, such a reward scheme gives an agent an even stronger incentive to work when he sees someone shirking. This eliminates the implicit threat of shirking that is critical in the complementarity case, thereby preventing the principal from reducing the incentive costs. Formally, we have the following result:

Proposition 3.2. *Suppose that p satisfies substitutability. For every fixed permutation π, the optimal reward scheme $v^*(\pi)$ is identical and pays agent i $v_i^* = [p(N) - p(N \backslash \{i\})]^{-1}$.*

Proposition 3.2 indicates that peer information does not reduce incentive costs under a substitutable technology. This is because under an effort-inducing reward scheme the agents are incentivized to substitute own effort for those whom they see shirking. Thus, if an agent chooses to shirk, it does not affect the others' decisions. This means that an effort-inducing reward scheme has to provide an agent sufficient incentive when he believes that all his peers exert effort. It is tempting to note that such a reward scheme is also required by a full-effort Nash equilibrium of the game in which the agents choose their efforts simultaneously. Since the optimal reward scheme depends not on the information structure, every permutation is equivalent payoff-wise. Hence, in the subsequent sections, we assume that the technology satisfies complementarity. Since N is a finite set, Proposition 3.1 ensures that an optimal EFI mechanism exists; it thus remains to characterize such a mechanism.

3.3 Fully Connected Network

As a benchmark, in this section, we study a fully connected network in which all the agents are interconnected. This network topology yields the richest transparency in the sense that under any permutation, each agent can observe all preceding actions. As Proposition 3.1 characterized the optimal reward scheme for an arbitrary sequence of execution π, it remains to seek for the optimal permutation π^*. Instead of characterizing the optimal sequence for a fully connected network directly, we provide some more general results.[2] As a first step, we show that if two agents are linked in a network, then they cannot act simultaneously in the optimal sequence. This is summarized by Lemma 3.1 below:

Lemma 3.1. *For any two agents i and j such that $(i, j) \in g$, we have that $\pi_i^* \neq \pi_j^*$ in the optimal sequence π^*.*

Proof. Suppose not, then $\pi_i^* = \pi_j^*$. Consider a new permutation π' which differs from π^* only in that j acts in π' immediately after i and before all the agents who act after i in π^*; thus, $\pi_j' > \pi_j^*$ and $\pi_k' = \pi_k^*$ for any agent $k \neq j$. This implies that $N_j^* \subseteq N_j'$ and $M_j' = M_j^*$. Consider an agent $k \neq j$. Clearly, if $\pi_k^* > \pi_j^*$, then $M_k' = M_k^*$. If $\pi_k^* \leq \pi_j^*$, then we partition M_k^* into two parts: $M_{k\backslash j}^*$ and $M_k^* \backslash M_{k\backslash j}^*$, where $M_{k\backslash j}^*$ is the set of agents who will remain in M_k^* if all j's links are eliminated and the agents act in the order of π^*. Pick any agent $l \in M_k^*$. If $l \in M_{k\backslash j}^*$, then clearly he will remain in M_k' under π'. If $l \in M_k^* \backslash M_{k\backslash j}^*$, it must be that $l \in M_j^*$. Since $N_j^* \subseteq N_j'$ and $M_j' = M_j^*$, l will still remain in M_k', and thus, $M_k^* \subseteq M_k'$. In summary, for any agent $k \in N$, we have $M_k^* \subseteq M_k'$, meaning that $v_k^*(\pi') \leq v_k^*(\pi^*)$ due to Proposition 3.1. But since $(i, j) \in g$ and $\pi_i^* = \pi_j^*$, we have $M_i' \subset M_i^*$; thus, $v_i^*(\pi') < v_i^*(\pi^*)$. This means that the total payoffs to the agents are strictly lower under π' than under π^*, leading to a contradiction. Thus, the lemma is proven. □

[2] Thus, the results of this section will be more general than that of Winter (2006).

Lemma 3.1 states that under complementarity, it is always suboptimal to make two linked agents acting simultaneously. This is because doing so reduces the transparency of actions, thereby mitigating the implicit threat of shirking and raising incentive costs. Lemma 3.1 implies that in a fully connected network in which all the agents are interconnected, the optimal sequence is such that the agents act sequentially in the order $1,2,\ldots,n$, though the specific order of each agent remains unknown. To fully characterize the optimal sequence, we relabel the agents in the way that agent i is (weakly) less important than $i+1$, $i \leq n-1$, with agent n being the most important. In addition, we say that agents i and j are *neighbors* if $(i,j) \in g$. Proposition 3.3 below shows that if two agents are neighbors and share the same neighbors other than themselves, then the optimal sequence satisfies that if one agent acts immediately after the other, then the more important agent acts later. This implies that the optimal sequence for a fully connected network is the identity permutation.

Proposition 3.3. *For any two agents i and j such that $(i,j) \in g$, $\{k|(i,k) \in g\}\backslash\{j\} = \{k|(j,k) \in g\}\backslash\{i\}$ and i is more important than j, if in the optimal sequence π^*, $|\pi_i^* - \pi_j^*| = 1$, then $\pi_i^* = \pi_j^* + 1$. Thus, if g is a fully connected network and the agents are increasingly important, then the optimal EFI mechanism $\{\pi^*, v^*\}$ satisfies: (i) π^* is the identity permutation; (ii) agent i receives payoff $v_i^* = [p(N) - p(\{j|j<i\})]^{-1}$. In particular, if two agents are equally important, then switching their orders in π^* still results in an optimal sequence and does not change the total payoffs.*

Proof. We first prove that $\pi_i^* = \pi_j^* + 1$. Suppose not, then $\pi_j^* = \pi_i^* + 1$. Now switch i and j and call the new permutation π'. Since $\{k|(i,k) \in g\}\backslash\{j\} = \{k|(j,k) \in g\}\backslash\{i\}$ and $\pi_j^* > \pi_i^*$, we have $N_i' \cup \{i\} = N_j^* \cup \{j\}$, $N_j' = N_i^*$, $M_i' = M_j^*$ and $M_j' \cup \{j\} = M_i^* \cup \{i\}$. Consider an agent $k \neq i$, j. By Lemma 3.1, for any of i's neighbors l, we have either $\pi_l^* < \pi_i^*$ or $\pi_l^* > \pi_j^*$. Thus, there are two possibilities to consider.[3] First, i, $j \notin M_k^*$. Since $N_i' \cup \{i\} = N_j^* \cup \{j\}$ and $N_j' = N_i^*$, the switch between i and j will not affect M_k; thus, $M_k' = M_k^*$, meaning that

[3] The case that $j \in M_k^*$ but $i \notin M_k^*$ is impossible. This is because if $j \in M_k^*$ then $\pi_k^* < \pi_i^*$, meaning that $i \in M_k^*$ as $\{k|(i,k) \in g\}\backslash\{j\} = \{k|(j,k) \in g\}\backslash\{i\}$, leading to a contradiction.

$v_k^*(\pi') = v_k^*(\pi^*)$. Second, $i \in M_k^*$, then $j \in M_k^*$. Since $M_i' = M_j^*$ and $M_j' \cup \{j\} = M_i^* \cup \{i\}$, the switch will not affect M_k; thus, $M_k' = M_k^*$ and $v_k^*(\pi') = v_k^*(\pi^*)$. Note that $M_i' = M_j^*$ and i is more important than j, thus we have

$$p(N \backslash (\{j\} \cup M_j^*)) = p((N \backslash M_j^*) \backslash \{j\}) > p((N \backslash M_j^*) \backslash \{i\}) = p(N \backslash (\{i\} \cup M_i')).$$

It follows from Proposition 3.1 that $v_i^*(\pi') < v_j^*(\pi^*)$. Moreover, since $M_j' \cup \{j\} = M_i^* \cup \{i\}$, we have $v_j^*(\pi') = v_i^*(\pi^*)$. This implies that the total payoffs to the agents are strictly lower under π' than under π^*, leading to a contradiction.

The second part of the proposition is thus immediate. Note that if g is a fully connected network, then by Lemma 3.1, the agents act sequentially under π^*. If further the agents are increasingly important, then the above result means that the optimal sequence is the identity permutation, and thus, the optimal reward scheme is given by Proposition 3.1 accordingly. Finally, if two agents are equally important, then it is readily confirmed that switching these agents does not affect any agent's incentive cost. Therefore, the proposition is proven. □

Proposition 3.3 suggests that in a fully connected network, the principal should delay more important tasks towards the end of the production process if feasible. Intuitively, when an agent shirks under the optimal reward scheme, he triggers all his successors to shirk. If the agent and his successors together are more important to the project, then he faces a greater implicit threat of shirking and is thus easier to be incentivized. Analogously, on the equilibrium path, agent i makes his decision as if he worked on an independent project; if he chooses to exert effort, then the project yields a high output equal to $p(N)v_i^*$, with v_i^* fixed; otherwise, the project yields a low output equal to $p(\{j|j < i\})v_i^*$. By allocating more important agents into later stages, the principal essentially reduces the low output level without changing the high output; clearly, the agent will be more willing to exert effort.

Proposition 3.3 implies that agents who are allocated to later stages under the optimal mechanism are compensated more generously, even if all the agents are equally important *ex ante*. The idea is particularly transparent for fully connected networks. That is, on the equilibrium path, each agent could alternatively free ride on his predecessors' efforts instead of exerting effort himself, while the optimal reward just offsets his free riding incentive; an agent who involves in a later stage can free ride on more predecessors' efforts, and thus, his incentive cost is larger. Moreover, due to complementarity, the gap between two adjacent agents' rewards increases in their orders. This is because complementarity corresponds to an increasing return-to-scale technology; as more agents exerted efforts, the free riding behavior becomes more detrimental, and thus, the principal incurs increasingly more incentive costs to induce effort. These results are summarized by the following corollary:

Corollary 3.1. *Suppose that g is a fully connected network, then the optimal reward scheme v^* satisfies: v_i^* is increasing and strictly convex in i under the optimal sequence π^*.*

Proof. From Proposition 3.3, the gap between v_{i+1}^* and v_i^* is given by

$$v_{i+1}^* - v_i^* = \frac{p(\{j|j < i+1\}) - p(\{j|j < i\})}{[p(N) - p(\{j|j < i+1\})][p(N) - p(\{j|j < i\})]}.$$

The numerator of the right-hand side (RHS) is increasing in i due to the complementarity of p; the denominator is decreasing in i due to the monotonicity of p, and thus, $v_{i+1}^* - v_i^*$ is increasing in i. The monotonicity of v_i^* follows immediately from Proposition 3.3. □

Since a fully connected network yields the richest transparency, it imposes the greatest implicit threat of shirking on the agents. The corollary below states that the total payoffs to the agents are the least in fully connected networks among all network topologies. Thus, we obtain a sharp lower bound for the total payoffs an optimal EFI mechanism incurs.

Corollary 3.2. *A fully connected network yields minimal total payoffs to the agents.*

Proof. Let g_1 be a fully connected network and g_2 be an arbitrary network with the same amount of vertices as g_1. From Lemma 3.1, a permutation with some simultaneous moves is weakly suboptimal for g_2.[4] Without loss, assume that in the optimal sequence $\pi^*(g_2)$ of g_2 the agents act sequentially in the order $1, 2, \ldots, n$. Consider the permutation $\pi(g_1)$ of g_1 such that each agent has the same order in $\pi(g_1)$ as in $\pi^*(g_2)$. Clearly, for each agent i, M_i is weakly larger under $\pi(g_1)$. Then from Proposition 3.1, the optimal reward scheme pays i less under $\pi(g_1)$. Since $\pi(g_1)$ is not necessarily optimal, the optimal total payoffs must be (weakly) lower under g_1. Thus, the corollary is proven. □

Corollary 3.2 indicates that a fully connected network is the best network topology for the principal as it yields the richest internal information. Such a network can represent the emerging workplace architecture "war room" that is implemented by different organizations. The movement to such open-space environment allows workers to monitor their peers more easily, making the peer information about effort more transparent. Consequently, it enhances the implicit incentive of working that is imposed by this mutual observability. In our model, the network g is exogenously given. Suppose that the principal can improve the connection between agents (i.e., by adding links to g) at relatively low costs, then she might find it profitable to transform g into a fully connected network.

3.4 Star Network

Starting with this section, we study the optimal sequence for some typical network topologies that have not been studied by the literature. Here, we consider star networks. A star network satisfies that there exists some node i such that every link in the network involves node i; thus, agent i is termed as the *center* of the star, and the rest of the agents are termed as the *peripheries* of the star. The layout of a star network is depicted in Figure 3.1.

Star network structures are common in organizations. For example, in most scientific labs, when a project leader coordinates with his/her fellow researchers, the leader often

[4] Note that if agent i is not linked to anyone else, then his incentive cost is fixed at $[p(N) - p(N \backslash \{i\})]^{-1}$. Thus, whether there is another agent acting simultaneously does not affect i's incentive cost.

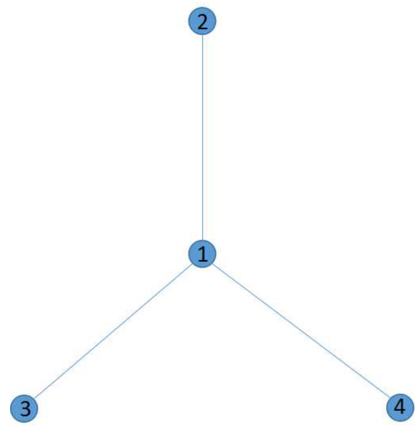

Figure 3.1: An Example of Star Network

works as the center of the team, while each fellow researcher works on an individual task and communicates the progress to the project leader. Such a team thus has a star network structure. Usually, the principal investigator (PI) of the lab can only observe the outcome of the entire project and chooses how to reward the team based on the outcome. In large-scale constructions, a general contractor who is in charge of the overall coordination of a project performs part of the building work and contracts subcontractors to perform specific and independent tasks. Such a team also can be represented by a star network, with the general contractor being the center and the subcontractors being the peripheries. In most cases, the owner/developer (principal) observes the quality of the entire building during the inspection phase and remunerates the contractors based on the quality.

To find the optimal sequence for a star network, it suffices to characterize the set of the center's successor(s), with the possibility of an empty set. For ease of exposition, we relabel the peripheries by importance from 1 to $n-1$, with a higher index referring to a more important agent. Provided there is no confusion, let the center be the n-th agent who is not necessarily the most important. As a useful general result, Lemma 3.2 below shows that if two agents share the same nonempty set of neighbors, then the optimal sequence satisfies that if the two agents have a neighbor who acts between them, then the more important one of the two agents acts in a later stage than the other.

Lemma 3.2. *For any two agents i and j such that* $\{k|(i,k) \in g\} = \{k|(j,k) \in g\} \neq \emptyset$ *and i is more important than j, if in the optimal sequence* π^**, there exists some* $k' \in \{k|(i,k) \in g\}$ *such that* $\pi_i^* \wedge \pi_j^* < \pi_{k'}^* < \pi_i^* \vee \pi_j^*$*, then* $\pi_i^* > \pi_j^*$*.*

Lemma 3.2 implies that if in the optimal sequence the center has a nonempty set of predecessors and successors, respectively, then the center's successors are uniformly more important than his predecessors. The intuition has been suggested previously; that is, if more important agents act in later stages, then shirking will induce agents with higher importance to shirk and is thus more detrimental to the probability of a success, leading to lower incentive costs. The relative importance between the center's predecessors and successors implies that the optimal sequence for a star network can be summarized by a sufficient statistic, that is, the number of the center's successor(s).[5] Let m be the number of the center's successor(s), with $0 \leq m \leq n-1$. Then, the center has $n-1-m$ predecessors; if each of them shirks, then the center and all his successors shirk correspondingly. Similarly, if the center shirks, then all his successors shirk too. However, the center's successors cannot trigger others' shirking, since their actions are unobservable to others. Thus, from Proposition 3.1, the total payoffs to the agents under an optimal reward scheme v^* is given by

$$v^*(m) = \underbrace{\sum_{i=1}^{n-1-m} \frac{1}{p(N) - p(\{j|j<n-m\} \backslash \{i\})}}_{\text{payoffs to the predecessors}} + \underbrace{\frac{1}{p(N) - p(\{j|j<n-m\})}}_{\text{payoff to the center}} + \underbrace{\sum_{n-m}^{n-1} \frac{1}{p(N) - p(N \backslash \{i\})}}_{\text{payoffs to the successors}}.$$

To find the optimizer m^*, we compare $v^*(m)$ with $v^*(m+1)$; the difference between the two items is the marginal effects of increasing the center's successors on the total rewards.

[5]This is because the relative orders between the center's predecessors or successors does not affect their incentive costs, as each individual's action is equally transparent for predecessors and successors, respectively.

By preliminary calculation, for any m with $0 \leq m \leq n-2$, we have

$$\begin{aligned}
v^*(m+1) - v^*(m) = & \sum_{i=1}^{n-2-m} \frac{1}{p(N) - p(\{j|j < n-m-1\} \backslash \{i\})} \\
& - \sum_{i=1}^{n-2-m} \frac{1}{p(N) - p(\{j|j < n-m\} \backslash \{i\})} \\
& - \frac{1}{p(N) - p(\{j|j < n-m\})} + \frac{1}{p(N) - p(N \backslash \{n-m-1\})}. \qquad (3.1)
\end{aligned}$$

The summation of the first three terms on the RHS of (3.1) is the net change in payoffs to the center and his predecessors. Since p is increasing, this value is negative, i.e., by increasing the center's successors, the total payoffs to the center and his predecessors decrease. The reason is twofold: first, increasing the center's successor reduces the number of the rest of the agents; more important, doing so makes the efforts of the center and his predecessors more transparent, thereby enhancing the implicit threat of shirking for these agents and reducing the incentive costs. In this regard, we call these terms together the marginal benefit (MB) of increasing the center's successors. Formally, we define

$$\begin{aligned}
MB(m) := & \sum_{i=1}^{n-2-m} \left[\frac{1}{p(N) - p(\{j|j < n-m\} \backslash \{i\})} - \frac{1}{p(N) - p(\{j|j < n-m-1\} \backslash \{i\})} \right] \\
& + \frac{1}{p(N) - p(\{j|j < n-m\})}.
\end{aligned}$$

In contrast, the last term on the RHS of (3.1) is positive, which is the extra reward to the new successor. Analogously, we call this term the marginal cost (MC) of increasing the center's successors. Formally, we define,

$$MC(m) := \frac{1}{p(N) - p(N \backslash \{n-m-1\})}.$$

Note that $MC(m)$ is non-decreasing in m. This is because each new successor of the center is (weakly) less important than the current ones, and thus, his incentive cost is higher. It follows that if $MB(m)$ is decreasing in m, then there exists a unique optimizer m^* (either an interior solution or a corner solution). Lemma 3.3 below shows that under complementarity, $MB(m)$ is indeed decreasing in m.

Lemma 3.3. *$MB(m)$ is decreasing in m.*

The idea is that on the equilibrium path, the center and his predecessor could alternatively free ride on the other predecessors' efforts. As the center obtains more successors, there are fewer agents whose efforts one can free ride on. Since complementarity corresponds to an increasing return-to-scale technology, free riding becomes less detrimental as the number of the center's successors rises. Furthermore, since the center's successors are uniformly more important than the predecessors as required by the optimal sequence, the shirking of the center and his predecessors will trigger on average less important agents to shirk as the center obtains more successors, meaning that the implicit threat of shirking is relatively weaker. In summary, on both the extensive and intensive margin, increasing the center's successors becomes less effective in reducing the incentive costs of the center and his predecessors as the number of the center's successors increases.

Lemma 3.3 ensures that the optimal sequence is essentially unique and can be succinctly characterized by an integer m^* which is the smallest m such that $MB(m) \leq MC(m)$. The next proposition shows that in the optimal sequence, the center never acts the first; if the center is sufficiently more important than all the peripheries, then he acts the last.

Proposition 3.4. *Suppose that g is a star network with $n \geq 3$ agents, then the optimal EFI mechanism $\{\pi^*, v^*\}$ satisfies: (i) the center has m^* successor(s) with $0 \leq m^* \leq n-2$ and each of them is more important than all the center's predecessors; (ii) the optimal reward scheme v^* is characterized by Proposition 3.1 accordingly. Moreover, if $[p(N) - p(N \backslash \{n-1\})] < \delta\, [p(N) - p(N \backslash \{n\})]$ for some small $\delta > 0$, then $m^* = 0$, where agent $n-1$ is the most important periphery agent and agent n is the center.*

Proof. The optimal sequence is given by Lemmas 3.2 and 3.3. Indeed, Lemma 3.3 implies that $m^* = \min\{m | MB(m) \leq MC(m)\}$. Then, the optimal reward scheme v^* is characterized by Proposition 3.1 accordingly. To see that m^* is bounded above by $n-2$, note that

$$MB(n-2) = \frac{1}{p(N) - p(\{1\})} < \frac{1}{p(N) - p(N \backslash \{1\})} = MC(n-2),$$

and thus, $m^* \leq n-2$. To prove the last statement of the proposition, note that

$$MC(0) = \frac{1}{p(N) - p(N \backslash \{n-1\})},$$

and that

$$\begin{aligned} MB(0) &= \sum_{i=1}^{n-2} \left[\frac{1}{p(N) - p(\{j|j<n\} \backslash \{i\})} - \frac{1}{p(N) - p(\{j|j<n-1\} \backslash \{i\})} \right] \\ &\quad + \frac{1}{p(N) - p(N \backslash \{n\})} \\ &< \sum_{i=1}^{n-2} \frac{1}{p(N) - p(\{j|j<n\} \backslash \{i\})} + \frac{1}{p(N) - p(N \backslash \{n\})} \\ &< \frac{n-1}{p(N) - p(N \backslash \{n\})}. \end{aligned}$$

Let $\delta = \frac{1}{n}$; thus, if $[p(N) - p(N \backslash \{n-1\})] < \delta \left[p(N) - p(N \backslash \{n\}) \right]$, then $MB(0) < MC(0)$. This implies that $m^* = 0$. The proposition is thus proven. □

The intuition of why the center never acts the first is straightforward. Note that for star networks, the first mover's incentive cost is constant, as his shirking always induces everyone to shirk. Rather than making the center the first mover, letting one periphery acts the first will lead to one more agent whose action can be observed by others, thereby improving transparency and reducing incentive costs. Hence, the optimal sequence requires that the center never acts the first.

Proposition 3.4 states that if the center is relatively more important than the peripheries, in the sense that the center's shirking is much more detrimental to the probability of success, then the center should act the last. Intuitively, if the center is relatively more important, then his predecessors' incentive costs are relatively low due to the large implicit threat of shirking.

In contrast, the center's successors do not have such implicit threat and thus have relative high incentive costs.[6] Consequently, increasing the center's successors is unprofitable.

Remark. *In particular, if all the agents are equally important, then it can be easily proven that* $1 \leq m^* \leq n-2$*; that is, the center always acts in an interior stage. This is because if the agents are equally important, then each agent i's incentive cost depends only on the cardinality of* M_i*, irrespective of his identity. By allocating the center into an interior stage, the mechanism allows the peripheral agents to learn their peers' actions through the center, as if the center acted as an internal communication device. This makes the agents' actions more transparent, thereby reducing the incentive costs.*

The previous analysis indicates that there exists a simple algorithm to find the optimal sequence for star networks. Specifically, one just needs to allocate the peripheries into the set of the center's successors one by one from the most important to the least, until the first time when $MB(m) \leq MC(m)$. In practice, this process is remarkably simpler than searching the optimal sequence for a general network topology. Moreover, the algorithm remains valid even if the relative order between the center and some peripheries is unadjustable. This can be achieved by applying the original algorithm to the remaining peripheries. See the Appendix for further details.

As a comparative-statics analysis, we study the impacts of the importance of individual task on the optimal sequence for star networks. Specifically, we examine how the number of the center's successors in the optimal sequence varies with the importance of individual task. For ease of exposition, in the following, we consider a numerical example and assume that the agents are equally important to the project.

Example Suppose that g is a star network with $n \geq 3$ agents, and that the project is a success if and only if all tasks are successful. Each task is successful with probability 1 if

[6] Indeed, for any $1 \leq m \leq n-2$, the incentive costs of the center and his predecessors' are bounded above by $[p(N) - p(N \backslash \{n\})]^{-1}$, whereas that of a center's successor is bounded below by $[p(N) - p(N \backslash \{n-1\})]^{-1}$. By assumption, the latter incentive cost is more than $1/\delta$ times of the former, for some small $\delta > 0$.

the agent works, and is successful with probability $\alpha \in (0,1)$ if the agent shirks. Hence, a lower probability α means that the failure of an individual task has critical implications on the entire project. Let w be the number of agents who work, then $p(w) = \alpha^{n-w}$ because all tasks are independent. Clearly, p is increasing and satisfies complementarity. Applying the previous results, we express $MB(m)$ and $MC(m)$ explicitly in the following:

$$MB(m,\alpha) = \frac{n-m-2}{1-\alpha^{m+2}} - \frac{n-m-2}{1-\alpha^{m+3}} + \frac{1}{1-\alpha^{m+1}},$$
$$MC(m,\alpha) = \frac{1}{1-\alpha}.$$

It follows that for fixed $\alpha \in (0,1)$, $MB(m)$ is decreasing in m, and that $MB(0) > MC(0)$ and $MB(n-2) < MC(n-2)$. Thus, the optimizer m^* exists and is an interior solution for any $\alpha \in (0,1)$. In addition, from basic mathematical analysis, we have that for fixed m, both $MB(\alpha)$ and $MC(\alpha)$ are increasing and strictly convex in α, and that $MB(\alpha)$ is single-crossing $MC(\alpha)$ from below in the domain $\in (0,1)$. This is illustrated in Figure 3.2. It thus follows that the optimizer $m^*(\alpha)$ is non-decreasing in α.[7] Formally, we have:

Corollary 3.3. *In the optimal sequence π^*, the number of the center's successors $m^*(\alpha)$ is non-decreasing in α for $\alpha \in (0,1)$.*

Corollary 3 implies that the more important each task is, the fewer successors the center has in the optimal sequence. Intuitively, if each task is important to the project's success, then each agent has a relatively strong incentive to work. Thus, the implicit threat of shirking is not crucial in providing incentive. This means that improving the transparency of actions by increasing the center's successors is not effective in reducing incentive costs. In contrast, if each individual task has little effect on the project's success, then each agent has a relatively strong incentive to shirk. In this case, the implicit threat of shirking plays an important role in providing incentive. Thus, the principal should make shirking behaviors more transparent by increasing the center's successors, thereby enhancing the implicit threat of shirking.

[7] Since m is an integer, $m^*(\alpha)$ is not necessarily increasing in α.

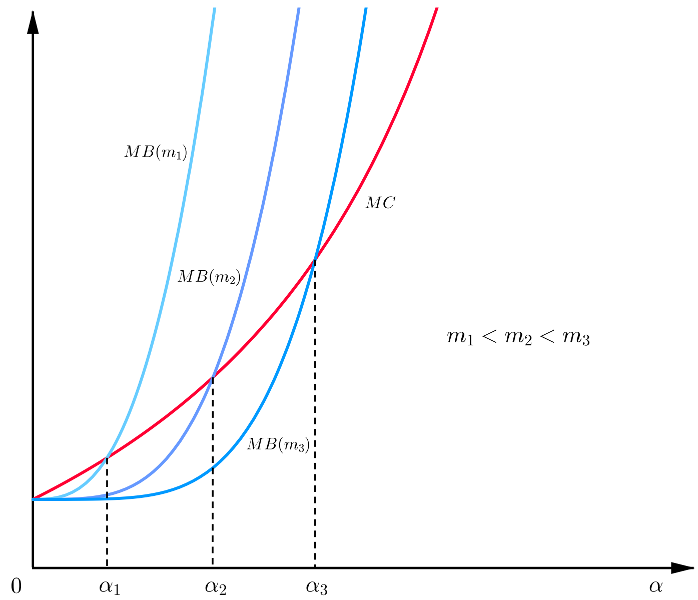

Figure 3.2: Marginal Benefit and Marginal Cost as a Function of Importance

3.5 Core-Periphery Network

In Sections 3.3 and 3.4, we studied two simple network topologies, fully connected networks and star networks. However, not all organizational structures can be approximated by such simple networks; rather, in many organizations, a more complex structure might emerge as a composition of multiple simple ones. For example, in large projects that require the collaboration of several teams, the organizational structure can be represented by a network composed of multiple stars. In this section, we study a typical class of such networks – core-periphery networks, in which the centers of multiple stars are interconnected. The layout of a core-periphery network is illustrated in Figure 3.3.

Based on the nature of production process, we consider two cases. In a *vertical* project, the relative order between different stars is fixed while that of the agents within each star is determined by the principal. This feature can represent a multi-phase project with vertical collaboration, such as the development of drugs that includes preclinical, investigational and post-marketing phases. The orders of different phases cannot be interchanged. In contrast, in a *horizontal* project, the principal can additionally determine the relative order between

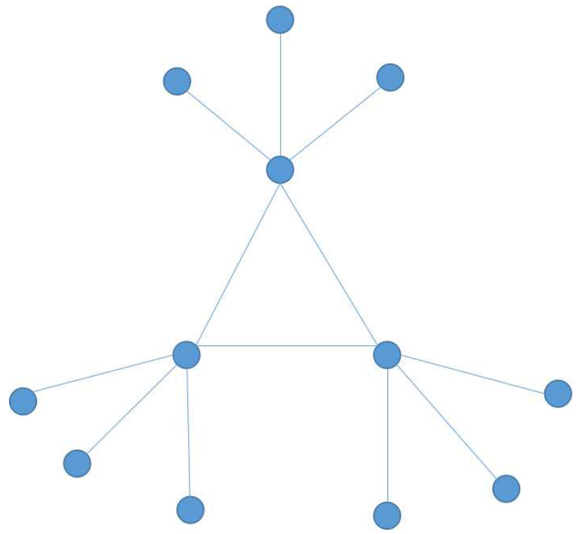

Figure 3.3: An Example of Core-Periphery Network

different stars. Projects that require horizontal collaboration of multiple departments such as those for different components of an assembled final product (e.g., a cell phone and a motor vehicle) typically have such feature. For tractability, we assume throughout this section that all the agents are equally important to the project. This implies that for any subset W of working agents, $p(W) = p(|W|)$, where $|W|$ is the cardinality of set W.

We first consider a vertical project. Let r be the number of stars in the network g. For ease of exposition, we assume that all stars act sequentially, and label the stars by their relative order such that each agent in star i acts before any agent in star $i+1$. Let s_i be the number of agents in star i; thus, we have $\sum_{i=1}^{r} s_i = n$. The principal's problem is to choose the sequence of execution within each star separately. Since the relative order between different stars is fixed, the internal sequence of each star jointly determines the sequence of execution for the entire project. Note that given the sequence, the core-periphery network yields the same transparency as the network in which the centers of the stars are linked in a chain according to their relative orders (see Figure 3.4). This is because for each agent i, M_i is identical between the two networks under the same sequence of execution. We call this new network *connected-stars*. In the subsequent, we will focus on the connected-stars network as it can be illustrated in an easier way.

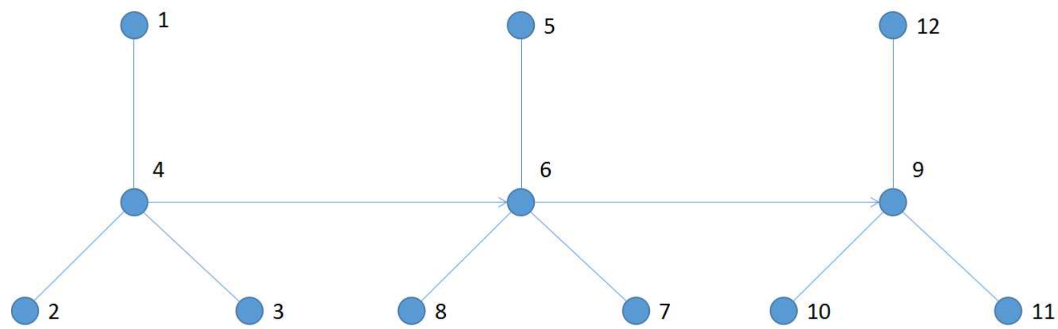

Figure 3.4: A Vertical Project Conducted in a Connected-Stars Network

In this sequence, the relative orders between different stars is fixed. Each agent acts before every agent of a star to his right. The label of each node denotes the order of the agent in the entire sequence. Given the sequence, such a network yields the same transparency as the core-periphery network in Figure 3.3.

To find the optimal sequence, we introduce, analogously to star networks, the marginal benefit and marginal cost of increasing the center's successors within a single star, namely, the marginal effects of allocating one more periphery to the set of the center's successors. To be succinct, we use the terms *marginal benefit* (MB) and *marginal cost* (MC). As explained before, the marginal benefit stems from the improvement of the transparency of preceding actions as well as the decrease in the number of the center's predecessors; the marginal cost is simply the extra payoff to the new successor. Since the agents are equally important, each agent i's incentive cost depends only on the cardinality of M_i. Thus, within any specific star, if a periphery acts after the center, then his incentive costs is fixed at $[p(n) - p(n-1)]^{-1}$. This implies that the marginal cost is constant across stars. The marginal benefit, in contrast, has an inter-star effect; that is, for any star except the first, increasing the center's successors within this star will improve the transparency of not only the center's action, but also all preceding actions that the center can learn on equilibrium path. To illustrate, consider the example in Figure 3.4. If alternatively agent 5 acts as agent 6's successor, then not only 6's action becomes more transparent, but also agent 1 to 4's actions are more transparent too, as these actions can be learned by agent 6 on the equilibrium path. Indeed, by moving agent 5 after 6, $|M_i|$ increases by 1, for $i = 1,2,3,4$ and 6, and thus, v_i^* becomes lower.

Analogously to star networks, let m_i be the number of peripheries who act after the center within star i, with $0 \leq m_i \leq s_i - 1$; thus, $MB(m_i)$ stands for the marginal benefit of allocating one more periphery of star i into the set of its center's successors; the marginal cost MC is fixed at $[p(n) - p(n-1)]^{-1}$. We first establish a useful result by Lemma 3.4 below. It indicates that if in the optimal sequence a center acts after some of his peripheries, then all preceding centers act the last within own star; if in contrast, a center acts before some of his peripheries, then all subsequent centers act the first within own star.

Lemma 3.4. *Suppose that the project is vertical, then the optimal sequence π^* satisfies: (i) if there exists a star i, with $i > 1$, such that $s_i - 1 - m_i^* > 0$, then for any star $j < i$, $m_j^* = 0$; (ii) if there exists a star i, with $i < r$, such that $m_i^* > 0$, then for any star $j > i$, $m_j^* = s_j - 1$.*

Proof. We first prove statement (i). Suppose that in the optimal sequence, there exist two stars i and j, with $i > j$, such that $s_i - 1 - m_i^* > 0$ and $m_j^* > 0$, then we must have $MB(m_i^*) \leq MC \leq MB(m_j^* - 1)$; otherwise, the principal can make a locally profitable deviation by raising m_i^* by 1 or reducing m_j^* by 1. On the other hand, we must also have $MB(m_i^*) > MB(m_j^* - 1)$. To see this, let K_1 be the set of agents such that for any agent $k_1 \in K_1$, if m_i^* increases then $|M_{k_1}|$ increases. Similarly, let K_2 be the set of agents such that for any agent $k_2 \in K_2$, if m_j^* increases (equivalently, $m_j^* - 1$ increases) then $|M_{k_2}|$ increases. Given the organizational structure, since $i > j$, we must have $K_2 \subset K_1$. This implies that $MB(m_i^*) > MB(m_j^* - 1)$, leading to a contradiction. The proof of statement (ii) is analogous. Therefore, the lemma is proven. □

The idea of Lemma 3.4 is straightforward. Since the stars' centers are ordered in a chain, the marginal benefit of increasing the center's successors within a star is always higher than that of a star in an earlier stage, as actions in later stages can impose the implicit threat of shirking on more preceding agents. Thus, if a center does not act the first within own star, then we have that the marginal benefit of increasing this center's successors is less than the marginal benefit. This implies that all preceding centers should act the last within own star, as the corresponding marginal benefits are even lower. Analogously, if a center does not act the last within own star, then when he has fewer successors, the marginal benefit is higher than the marginal cost, meaning that all subsequent centers should act the first within own stars, as the corresponding marginal benefit are even higher.

Note that in any sequence, each star can only have three possible internal sequences. We say that a star is a *type I* star if the center acts the last within own star, is a *type II* star if the center acts in an interior stage within own star, and is a *type III* star if the center acts the first within own star. Lemma 3.4 implies that in the optimal sequence, there can be at most one type II star, and all preceding stars (if any) should be type I and all subsequent stars (if any) should be type III. The other possible case is that there are several type I stars followed by type III stars, with the possibility that there are only type I stars or type III stars.

Consequently, we could establish a simple algorithm to find the optimal sequence for a core-periphery network under a vertical project. Specifically, we first assume that all stars are type I stars. Then, from the last star to the first, we allocate the peripheries one by one into the set of the center's successors. Note that the optimal sequence must emerge in some stage of this process. Thus, if the marginal benefit is non-increasing through this process, then the optimal sequence is obtained once $MB \leq MC$. The next proposition shows that such an algorithm is indeed valid.

Proposition 3.5. *Suppose that the project is vertical, then the optimal sequence π^* can be obtained through the following procedure: first, make each star a type I star; second, from the last star to the first, allocate the peripheries one by one into the set of the center's successors, until $MB(m_i^*) \leq MC$ for some star i in which m_i^* peripheries act after the center. In the optimal sequence, no center acts the first or the last in the entire sequence. The optimal reward scheme v^* is given by Proposition 3.1 accordingly.*

The algorithm characterized by Proposition 3.5 allows us to find the optimal sequence in a monotonic way, as for star networks. This remarkably simplifies the searching process. However, we have to point out that this simple algorithm relies on the assumption that the agents are equally important. If the agents are differently important, then the algorithm might not hold, as the marginal cost is not necessarily monotone across stars, whereas the algorithm for star networks is robust even if the agents differ in importance.

We now turn to a horizontal project. The only difference is that the principal is now able to choose the relative order between different stars, while an agent of a star in an earlier stage still acts before each agent of any subsequent star. Furthermore, Lemma 3.1 implies that in the optimal sequence all stars should be ordered sequentially, as simultaneous moves reduce the transparency. Thus, once the relative order between stars is determined, the rest of the analysis is identical to a vertical project. Although we are unable to fully characterize an algorithm to pinpoint the optimal sequence for a horizontal project, we find a useful property of the optimal sequence which can remarkably simplify the searching process. This is summarized by the proposition below.

Proposition 3.6. *Suppose that the project is horizontal, then the optimal sequence π^* satisfies: for any $1 \leq i \leq r-1$, (i) if both stars i and $i+1$ are type I stars, then $s_i \geq s_{i+1}$; (ii) if both stars i and $i+1$ are type III stars, then $s_i \leq s_{i+1}$.*

Proof. We first prove statement (i). Suppose not, then $s_{i+1} > s_i$. Thus, for any periphery j of star i and any periphery k of star $i+1$, we have $|M_j^*| = |M_k^*| + 1$. Now switch star i and $i+1$ with both stars remained as type I. Call this new permutation π'. Note that after the switch, $|M_j'| = |M_k^*|$ and $|M_k'| = |M_j^*|$, whereas $|M_l'| = |M_l^*|$ for any agent l who is not a periphery of either star i or $i+1$. From Proposition 3.1, the difference in total payoffs equals

$$
\begin{aligned}
v^*(\pi^*) - v^*(\pi') = & \left[\frac{s_i - 1}{p(n) - p(n-1-|M_j^*|)} + \frac{s_{i+1} - 1}{p(n) - p(n-1-|M_k^*|)} \right] \\
& - \left[\frac{s_i - 1}{p(n) - p(n-1-|M_j'|)} + \frac{s_{i+1} - 1}{p(n) - p(n-1-|M_k'|)} \right] \\
= & \frac{s_{i+1} - s_i}{p(n) - p(n-1-|M_k^*|)} - \frac{s_{i+1} - s_i}{p(n) - p(n-1-|M_j^*|)} \\
> & 0.
\end{aligned}
$$

This implies that π^* is not an optimal sequence, leading to a contradiction.

Then, we prove statement (ii). Suppose not, then $s_i > s_{i+1}$. Let agent j be the center of star i and agent k be the center of star $i+1$. Thus, we have $|M_j^*| = |M_k^*| + s_i$. Now switch star i and $i+1$ with both stars remained as type III. Call this new permutation π'. Note that after the switch, $|M_j'| = |M_k^*| - s_{i+1} + s_i$ and $|M_k'| = |M_j^*|$, whereas $|M_l'| = |M_l^*|$ for any agent l who is not the center of either star i or $i+1$. Thus, the difference in total payoffs equals

$$
\begin{aligned}
v^*(\pi^*) - v^*(\pi') = & \left[\frac{1}{p(n) - p(n-1-|M_j^*|)} + \frac{1}{p(n) - p(n-1-|M_k^*|)} \right] \\
& - \left[\frac{1}{p(n) - p(n-1-|M_j'|)} + \frac{1}{p(n) - p(n-1-|M_k'|)} \right] \\
= & \frac{1}{p(n) - p(n-1-|M_k^*|)} - \frac{1}{p(n) - p(n-1-|M_j'|)} \\
= & \frac{1}{p(n) - p(n-1-|M_k^*|)} - \frac{1}{p(n) - p(n-1-|M_k^*| - s_i + s_{i+1})} \\
> & 0.
\end{aligned}
$$

The inequality is due to that $s_i > s_{i+1}$. This implies that π^* is not an optimal sequence, leading to a contradiction. Thus, the proposition is proven. □

Proposition 3.6 indicates that in the optimal sequence, if multiple consecutive stars are all type I stars, then a star with more agents is allocated to an earlier stage; in contrast, if these stars are all type III stars, then a star with more agents is allocated to a later stage. Thus, if in the optimal sequence both type I and type III stars are present, including the case that these two types are connected by a single type II star, then the permutation is "V-shaped" in terms of the number of agents within each star. Specifically, starting from the very beginning, we first observe a series of type I stars with the number of agents within each star decreasing. Then, there may or may not be a single type II star which does not necessarily have fewer agents than previous stars. Finally, we observe a series of type III stars with the number of agents within each star increasing. In summary, Proposition 3.6 rules out many possible permutations, though the optimal one has not been obtained yet.

3.6 Conclusion

In this paper, we characterized the optimal effort-inducing mechanism for teamwork with network-based internal information for typical networks composed of stars. Our framework highlights the endogeneity of the task assignment sequence and provides a simple algorithm to derive the optimal sequence. An agent's position is tightly related to his importance to the project as well as his connectivity in the network. More important agents move later in the sequence and receive higher rewards, while better connected agents take up intermediate positions, reflecting a balance between incentives for early and later agents. The general question of how to fully characterize the optimal incentive scheme in an arbitrary network remains open; richer studies in this direction may shed more light on incentive design in many contemporary circumstances with complex channels of internal information.

3.7 Appendix

3.7.1 Omitted Proofs

Proof of Proposition 3.1.

Proof. We first prove that $\{\pi, v^*(\pi)\}$ is an EFI mechanism. Consider a strategy profile s^* such that $s_i^* = 1$ if and only if $a_j = 1$ for all $j \in N_i$ or N_i is empty; that is, an agent works unless he sees someone shirking. This strategy profile can be sustained by a PBE with the set of beliefs: if $a_j = 1$ for all $j \in N_i$ or $N_i = \emptyset$, then $a_k = 1$ for all $k \in N \backslash (N_i \cup \{i\} \cup M_i)$; that is, an agent, not seeing anyone shirking, believes that those whom he cannot see and who cannot see him through a sequence of agents will exert effort. To verify this statement, note that if agent i shirks then by induction every $j \in M_i$ shirks as well. In contrast, if i works then he believes that all the other agents work too unless he sees someone shirking. Suppose i is the first to act, then he believes that if he works then all the other agents also work, and if he shirks then he will induce each agent in M_i to shirk. Thus, i prefers working to shirking if and only if the difference in expected reward exceeds the effort cost, i.e.,

$$[p(N) - p(N \backslash (\{i\} \cup M_i))]v_i \geq 1 \tag{3.2}$$

Clearly, v_i^* satisfies (3.2). It follows by induction that for all $\pi_i \in \{2, \ldots, n\}$, i prefers to work on equilibrium path if and only if (3.2) holds, as he sees no one shirking. Off the path, if i sees a nonempty subset $S_i \subseteq N_i$ of agents shirking, then he knows that each $j \in S_i$ will induce everyone in M_j to shirk. Let $R_i := \bigcup_{j \in S_i} M_j \cup S_i$ be the set of agents whom i believes shirk. Thus, if i works then his expected utility equals $p(N \backslash R_i)v_i^* - 1$. In contrast, if i shirks then his expected utility equals $p((N \backslash R_i) \backslash (\{i\} \cup M_i))v_i^*$. We now provide a useful lemma.

Lemma 3.5. *Suppose p satisfies complementarity, then for any two nonempty sets of agents B, $C \subset N$, we have $p(N) - p(N \backslash B) > p(N \backslash C) - p((N \backslash C) \backslash B)$.*

Proof. If p satisfies complementarity, then for two nonempty sets T and S with $T \subset S$ and two agents i, $j \notin S$, we have

$$\begin{aligned} p(S \cup \{i\} \cup \{j\}) - p(S) &= p(S \cup \{i\} \cup \{j\}) - p(S \cup \{i\}) + p(S \cup \{i\}) - p(S) \\ &> p(T \cup \{i\} \cup \{j\}) - p(T \cup \{i\}) + p(T \cup \{i\}) - p(T) \\ &= p(T \cup \{i\} \cup \{j\}) - p(T). \end{aligned}$$

This implies by induction that for any nonempty set $Q \subset N$ with $Q \cap S = \emptyset$ we have

$$p(S \cup Q) - p(S) > p(T \cup Q) - p(T). \tag{3.3}$$

Then, let $T = (N \backslash C) \backslash B$, $S = (N \backslash B)$, and $Q = B$. It is readily confirmed that $T \subset S$ and $Q \cap S = \emptyset$; thus, the lemma is proven using (3.3). □

From Lemma 3.5, we conclude that $[p(N \backslash R_i) - p((N \backslash R_i) \backslash (\{i\} \cup M_i))]v_i^* < 1$. This means that i prefers to shirk whenever he sees someone shirking. Thus, s^* and the set of beliefs that are constructed above indeed constitute a PBE with full effort.

It remains to show that any alternative reward scheme v' with $v'_i < v^*_i$ cannot constitute a PBE with full effort. Suppose not, then the probability of success is $p(N)$ on the equilibrium path. If i shirks unilaterally, then he can at most trigger those in M_i to shirk, irrespective of the strategy profile. In other words, i's effort externality is confined to the coalition M_i. Since p is increasing, the difference in expected reward is less than the effort cost. Hence, i can make a profitable deviation by shirking, leading to a contradiction. Note that all these arguments go through for any fixed π, thus we have proven the proposition. □

Proof of Proposition 3.2.

Proof. As usual, we first prove that $\{\pi, v^*(\pi)\}$ is an EFI mechanism. Consider a strategy profile s^* with $s^*_i \equiv 1$, that is, an agent always exerts effort irrespective of his information set. This strategy profile can be sustained by a PBE with the set of beliefs that $a_j = 1$ for all $j \notin N_i$; that is, an agent believes that those whom he cannot see will exert effort. Note that if agent i sees no one shirking then he believes that all the other agents work. Hence, he prefers to work if and only if $[p(N) - p(N \backslash \{i\})]v_i \geq 1$, which holds for v^*_i. In contrast, if i sees a nonempty subset of agents $S_i \subseteq N_i$ who shirk, then his expected utility equals $p(N \backslash S_i)v^*_i - 1$ if he works; equals $p((N \backslash S_i) \backslash \{i\})v^*_i$ if he shirks. Then by substitutability, we have $p(N \backslash S_i) - p((N \backslash S_i) \backslash \{i\}) \geq p(N) - p(N \backslash \{i\})$. This implies that i still prefers to work. Hence, s^* and the set of beliefs constitute a PBE. Finally, we argue that there does not exist a reward scheme v' with $v'_i < v^*_i$ that admits a PBE with full effort. Suppose not, then i must prefer working to shirking if he encounters no shirking. Due to substitutability, if i shirks unilaterally then each $j \in M_i$ prefers to work, as argued above. This means that the difference in expected reward equals $p(N) - p(N \backslash \{i\})$. Since i is indifferent under v^*_i, he must prefer shirking under v', a contradiction. Hence, $v^*(\pi)$ is indeed optimal. □

Proof of Lemma 3.2.

Proof. Suppose not, then $\pi_i^* < \pi_{k'}^* < \pi_j^*$. Since (i,k'), $(j,k') \in g$, we have $j \in M_i^*$. Now switch i and j and call the new permutation π'. Since $\{k|(i,k) \in g\} = \{k|(j,k) \in g\}$, we have $(i,j) \notin g$, and thus, $N_i' = N_j^*$, $N_j' = N_i^*$, $M_i' = M_j^*$ and $M_j' \cup \{j\} = M_i^* \cup \{i\}$. Consider an agent $k \neq i$, j. There are three possibilities to consider. First, i, $j \notin M_k^*$. Since $N_i' = N_j^*$ and $N_j' = N_i^*$, the switch between i and j will not affect M_k, and thus, $M_k' = M_k^*$, meaning that $v_k^*(\pi') = v_k^*(\pi^*)$. Second, $i \in M_k^*$. It follows that $j \in M_k^*$ as $j \in M_i^*$. Since $M_i' = M_j^*$ and $M_j' \cup \{j\} = M_i^* \cup \{i\}$, the switch will not affect M_k; thus, $M_k' = M_k^*$ and $v_k^*(\pi') = v_k^*(\pi^*)$. Third, $j \in M_k^*$ but $i \notin M_k^*$. This means that $(i,k) \in g$, and by Lemma 3.1, that $\pi_i^* < \pi_k^* < \pi_j^*$. It follows that $M_k^* \backslash \{j\} = M_k' \backslash \{i\}$, and thus, we have

$$\begin{aligned}
p(N\backslash(\{k\} \cup M_k^*)) &= p(N\backslash(\{k\} \cup (M_k^* \backslash \{j\}) \cup \{j\})) \\
&= p((N\backslash(\{k\} \cup (M_k^* \backslash \{j\}))\backslash\{j\}) \\
&= p((N\backslash(\{k\} \cup (M_k' \backslash \{i\}))\backslash\{j\}) \\
&> p((N\backslash(\{k\} \cup (M_k' \backslash \{i\}))\backslash\{i\}) \\
&= p(N\backslash(\{k\} \cup (M_k' \backslash \{i\}) \cup \{i\})) = p(N\backslash(\{k\} \cup M_k')).
\end{aligned}$$

The inequality above is due to that i is more important than j. Then, from Proposition 3.1, we have $v_k^*(\pi') < v_k^*(\pi^*)$. Moreover, since $M_i' = M_j^*$, we have

$$p(N\backslash(\{j\} \cup M_j^*)) = p((N\backslash M_j^*)\backslash\{j\}) > p((N\backslash M_j^*)\backslash\{i\}) = p(N\backslash(\{i\} \cup M_i')).$$

It follows from Proposition 3.1 that $v_i^*(\pi') < v_j^*(\pi^*)$. Finally, since $M_j' \cup \{j\} = M_i^* \cup \{i\}$, we have $v_j^*(\pi') = v_i^*(\pi^*)$. This implies that the total payoffs to the agents are strictly lower under π' than under π^*, leading to a contradiction. Thus, the lemmas is proven. $\square$

Proof of Lemma 3.3.

Proof. Define $\Delta MB(m) \equiv MB(m+1) - MB(m)$. From basic calculation, we have

$$\begin{aligned}\Delta MB(m) = &\sum_{i=1}^{n-3-m}\left[\frac{1}{p(N)-p(\{j|j<n-m-1\}\backslash\{i\})} - \frac{1}{p(N)-p(\{j|j<n-m-2\}\backslash\{i\})}\right]\\ &- \sum_{i=1}^{n-3-m}\left[\frac{1}{p(N)-p(\{j|j<n-m\}\backslash\{i\})} - \frac{1}{p(N)-p(\{j|j<n-m-1\}\backslash\{i\})}\right]\\ &+ \left[\frac{1}{p(N)-p(\{j|j<n-m-1\})} - \frac{1}{p(N)-p(\{j|j<n-m\})}\right]\\ &+ \left[\frac{1}{p(N)-p(\{j|j<n-m-2\})} - \frac{1}{p(N)-p(\{j|j<n-m\}\backslash\{n-m-2\})}\right].\end{aligned}$$

Note that given a specific i, the term

$$\left[\frac{1}{p(N)-p(\{j|j<n-m\}\backslash\{i\})} - \frac{1}{p(N)-p(\{j|j<n-m-1\}\backslash\{i\})}\right]$$

is decreasing in m. This is due to the exactly same reasoning in the proof of Corollary 3.1. Thus, the difference between the above two summations is negative. In addition, the value of the third and forth bracket in the expression of $\Delta MB(m)$ are both negative, since p is increasing. Therefore, $\Delta MB(m)$ is negative, meaning that $MB(m)$ is decreasing in m. □

Proof of Proposition 3.5.

Proof. We first prove that the marginal benefit is decreasing within each star. Consider a star i, with $1 \leq i \leq r$. Let m_i be the number of peripheries who act after the center within star i, with $0 \leq m_i \leq s_i - 1$. Given the result of Lemma 3.4, it is without loss of generality to assume that for any star $j < i$, $m_j = 0$, and that for any star $j > i$, $m_j = s_j - 1$. Note that the marginal benefit of raising m_i has two independent components: first, improving the transparency of the actions within i and reducing the number of the center's predecessors; second, improving the transparency of all preceding actions that can be learned by the center of star i on the equilibrium path. Lemma 3 has shown that the first component is decreasing in m_i, thus it suffices to show that the second component is also decreasing in m_i. Denote the second component $MB_{j<i}(m_i)$. Since $m_j = 0$ for any star $j < i$, the total payoffs to the agents of these stars, $v^*_{j<i}(m_i)$, is given by

$$v^*_{j<i}(m_i) = \sum_{j=1}^{i-1} \left[\underbrace{\frac{s_j - 1}{p(n) - p(\sum_{k=1}^{j-1} s_k + s_j + s_i - 3 - m_i)}}_{\text{payoffs to the peripheries}} + \underbrace{\frac{1}{p(n) - p(\sum_{k=1}^{j-1} s_k + s_j + s_i - 2 - m_i)}}_{\text{payoff to the center}} \right].$$

By definition, $MB_{j<i}(m_i) = v^*_{j<i}(m_i+1) - v^*_{j<i}(m_i)$, with $0 \leq m_i \leq s_i - 2$. Because p satisfies complementarity, it can be easily shown that $MB_{j<i}(m_i)$ is indeed decreasing in m_i. Thus, the marginal benefit is decreasing within star i.

Then, we prove that the marginal benefit is decreasing over stars. This follows directly from the proof of Lemma 3.4. Specifically, a periphery of star i who acts after the center of i can impose an implicit threat of shirking on more agents than his counterparts in any star $j < i$. This implies that the marginal benefit is decreasing across stars. In summary, the marginal benefit is decreasing through the process characterized by Proposition 3.5.

To see that the center of star 1 does not act the first, we consider the marginal benefit when $m_1 = s_1 - 2$ and $m_j = s_j - 1$ for all star $j > 1$. However, this is equal to the marginal benefit of a single star when $m = n - 2$, and from Proposition 3.4, $MB(n-2) < MC$. Thus, the center of star 1 does not act the first. Similarly, to see that the center of star r does not act the last, we consider the marginal benefit when $m_i = 0$ for any star i. However, this equals the marginal benefit of a single star when $m = 0$. From Proposition 3.4, we have

$$MB(0) = \frac{n-2}{p(n) - p(n-2)} - \frac{n-2}{p(n) - p(n-3)} + \frac{1}{p(n) - p(n-1)} > \frac{1}{p(n) - p(n-1)} = MC.$$

Thus, the center of star r does not act the last. Therefore, the proposition is proven. □

3.7.2 Optimal Sequence for Partially Adjustable Star

Here, we demonstrate that the algorithm of searching the optimal sequence for star networks remains valid if the relative order between the center and some peripheries is unadjustable.

Suppose that due to technology constraint, a subset $K_1 \subset N$ of peripheries have to execute their tasks before the center, a subset $K_2 \subset N$ of peripheries have to execute their tasks after the center, and the remaining agents are perfectly flexible for ordering. Let t be the number of agents in the third group, and relabel the peripheries from 1 to $t-1$, with a higher index referring to a more important agent. Let agent t be the center. Suppose that among these $t-1$ peripheries, m act after the center, then from Lemma 3.2, they are more important than the other peripheries. Thus, the total payoffs to the agents is given by

$$\begin{aligned} v^*(m) = & \underbrace{\sum_{i=1}^{t-1-m} \frac{1}{p(N) - p(\{j|j<t-m\}\backslash\{i\} \cup K_1)}}_{\text{payoffs to the remaining predecessors}} \\ & + \underbrace{\frac{1}{p(N) - p(\{j|j<t-m\} \cup K_1)}}_{\text{payoff to the center}} + \underbrace{\sum_{t-m}^{t-1} \frac{1}{p(N) - p(N\backslash\{i\})}}_{\text{payoffs to the remaining successors}} \\ & + \underbrace{\sum_{k_1 \in K_1} \frac{1}{p(N) - p(\{j|j<t-m\} \cup K_1 \backslash \{k_1\})}}_{\text{payoff to the agents in } K_1} + \underbrace{\sum_{k_2 \in K_2} \frac{1}{p(N) - p(N\backslash\{k_2\})}}_{\text{payoffs to the agents in } K_2}. \end{aligned}$$

Analogously, the marginal benefit $MB(m;t)$ is given by

$$\begin{aligned} & \sum_{i=1}^{t-2-m} \left[\frac{1}{p(N) - p(\{j|j<t-m\}\backslash\{i\} \cup K_1)} - \frac{1}{p(N) - p(\{j|j<t-m-1\}\backslash\{i\} \cup K_1)} \right] \\ & + \sum_{k_1 \in K_1} \left[\frac{1}{p(N) - p(\{j|j<t-m\} \cup K_1 \backslash \{k_1\})} - \frac{1}{p(N) - p(\{j|j<t-m-1\} \cup K_1 \backslash \{k_1\})} \right] \\ & + \frac{1}{p(N) - p(\{j|j<t-m\})}, \end{aligned}$$

and the marginal cost $MC(m;t)$ is given by $[p(N) - p(N\backslash\{t-m-1\})]^{-1}$.

It can be shown analogously to Lemma 3.3 that $MB(m;t)$ is decreasing in m, as p satisfies complementarity. On the other hand, $MC(m;t)$ is increasing in m, as an agent with lower index is less important to the project. Thus, the optimizer m^* is either a corner solution or an interior solution such that $m^* = \min\{m|MB(m) \leq MC(m)\}$, meaning that the algorithm in Section 3.4 is still valid for searching the optimal sequence for star networks.

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