OPTIMAL OPERATION OF A MULTIPLE RESERVOIR SYSTEM

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ABSTRACT

This report presents a methodology to obtain optimal reservoir operation policies for a large-scale reservoir system. The model maximizes the system annual energy generation while satisfying constraints imposed on the operation of the reservoir network. The model incorporates the stochasticity of river flows and keeps future operating schedules up-to-date with the actual realization of those random variables. It yields medium-term (one-year ahead) optimal release policies that allow the planning of activities within the current water year, with the possibility of updating preplanned activities to account for uncertain events that affect the state of the system. The solution approach is a sequential dynamic decomposition algorithm that keeps computational requirements and dimensionality problems at low levels. The model is applied to a nine-reservoir portion of the California Central Valley Project and the results are compared with those from conventional operation methods currently in use, showing that the use of the model can improve the levels of energy production (about 30 percent increase) while the optimal release policies meet satisfactorily all other functions of the reservoir system. Sensitivity analysis is conducted to assess the optimality of the solutions and several alternative formulations of the model are developed and tested, the results showing the robustness of the optimal policies to the choice of model.

CHAPTER 1

INTRODUCTION

Optimal operation of reservoir systems is of fundamental importance for the adequate function of regional economies and environments as well as for the well being of the population served by such reservoir developments. Reservoirs provide a wide variety of indispensable services that affect every facet of modern society. Those services include the provision of water supply for human consumption, agricultural and industrial activities, hydropower generation, flood control prtection, ecological and environmental enhancement, navigation, and recreation. Large-scale reservoir developments have made possible the economic growth and flourishment of entire regions and nations. In parallel with the increasing expansion of population centers and economic activity, the demands exerted on the water stored by the reservoirs has been augmenting steadily. With the size of the systems almost at a maximum possible due to water availability limitations -- with the most suitable locations for development already harnessed -- and with tighter budgetary constraints, it has become mandatory to operate the reservoir systems in an efficient manner, so as to effectively and reliably provide their intended services to the public.

Efficient operation of reservoir systems, although desirable, is by no means a trivial task. First, there are multiple components (reservoirs, canals, rivers, pumping plants, etc.) that must be operated jointly. Second, there exist many, usually conflicting, interests and constraints that influence the management of the system. Third, there exist uncontrollable and uncertain elements that determine the reservoirs' operation such as streamflows. Fourth, conditions such as water demands, institutional regulations, and even infrastructural elements of the system, are continuously evolving due to the inherent dynamic nature of society and technology.

In the context of a much needed efficient management of reservoir systems, mathematical optimization models for reservoir operation become a natural ally, a valuable tool for improved planning of complex operational schedules for any reservoir system. Several aspects make optimization models prone to suitable use in reservoir management: the usually well defined structure and links between the physical components of the system, the quantitative nature of demands and constraints imposed on the system, and the advent of sophisticated computational equipment.

Modeling of reservoir management constitutes an elegant and classic example of the application of optimization theory to resource allocation. Reservoir management modeling, however, cannot be accomplished without problems of its own; among which predominantly figure its stochastic nature (streamflows, demands, etc.), the large size of the models (dimensionality), and the limitations of mathematical tools that force the modeler to compromise between accurate system modeling and complexity of the resulting optimization models.

This report is devoted to the development and application of a large-scale optimization model for the management of the northern portion of the California Central Valley Project (NCVP). The NCVP is a multireservoir, multipurpose system that constitutes the backbone of central and northern California water supply development. The product of this research is a model for computing monthly water release policies that would maximize the annual system hydropower generation while satisfying all constraints imposed on the operation of the NCVP. Emphasis is given to the following items throughout the development of the model:

(i) The optimization model must be able to represent the physical features of the system as closely as possible and include the pertinent links that exist between its different components. Those links are of hydraulic, hydrological, and operational nature. Links of operational nature refer to the effects that water releases from any reservoir have on the operation of any other reservoir.

(ii) The model must be tractable both numerically and in its implementability. The number and nature of the computations as well as the computer storage requirements have been kept to a practical manageable size, despite the fact that the model is aimed at large-scale problems. Also, the model has been developed to match the real operation conditions faced by the reservoir managers of the NCVP. The mathematical development of the model is kept at a general level (and subsequently tailored to the NCVP), so that the approach can be extended and applied to other systems with minor modifications.

(iii) The uncertainty of streamflows is handled in a simple way, and allows to keep the operating policy up to date with the actual evolution of the reservoir storages.

(iv) Along with the task of incorporating streamflow stochasticity and maintaining an adequate resemblance of the real operating scenario, conceptual rigor has not been sacrificed. Included in the analysis are convergence, type of optimality achieved (local or global), computational and computer storage burdens, existence of multiple optimal solutions, and a discussion of the advantages and disadvantages of the method relative to some popular competing models.

(v) Test of the model with a large-scale model (the NCVP) under different scenarios: below-average, average, and above-average streamflow conditions.

This study develops the optimization model for the monthly operation of the system, within a one-year horizon. That is the way in which many systems, including the NCVP, are managed: tentative or guiding operation policies are released at the beginning of every water year that represent the best judgement of the managing staff as to what must be done in the incoming water year. As the year progresses, at the beginning of each month, the policy to be followed the rest of the year is revised to accomodate the actual evolution of the system and correct the departures from what originally was expected to occur (due to streamflow variability, changes in the system's constraints, etc.).

To summarize, the objectives of this study are: (1) to develop a model to find optimal (monthly) operation policies for reservoir systems; (2) to apply the model to a large-scale reservoir system (NCVP), including streamflow stochasticity and actual operating constraints; and (3) to analyze the theoretical and computational features of the solution algorithm.

The remaining of the report is organized as follows. Chapter 2 defines the background terminology used in this work to set a common ground of understanding. Chapter 3 is a literature review of research done in large-scale reservoir system operation. Chapter 4 describes the solution algorithm that constitutes the backbone of the optimization model (the progressive optimality algorithm, POA). Chapter 5 contains the application of the POA to the NCVP. The objective function, constraints, and the structure of the mathematical model are presented. The streamflow forecasting model is also included in this chapter. Chapter 6 gives a discussion of results. Chapter 7 contains extensions of the model developed in Chapter 5. Those extensions include the use of system-dependent features and nonlinearities in the objection function and constraints. Those extensions are the product of the findings from the analysis of results in Chapter 6. Chapter 8 is a summary and final statement of important conclusions and further research needed. Appendix A contains physical data for the system as well as streamflow record. Finally, Appendix B shows listings of the computer programs.

CHAPTER 2

TERMINOLOGY

This chapter defines basic concepts that are employed in optimization models contained in this report.

2.1 Control or Decision Variable

The control variable in the reservoir operation problem is the volume of water released (through penstocks) and spilled during a month. The decision of the amount of water to release and spill is made at the beginning of each month.

2.2 State Variable

The state variable is the water stored in a reservoir at any time. The models use beginning- and end-of-month storages. For a multireservoir network, the state of the system is represented by a vector-valued state variable.

2.3 Constraints

Constraints are bounds or equalities that must be satisfied by state and decision variables or any function of those variables. They originate from technical, physical, institutional, or other considerations.

2.4 Feasible Region

A feasible region is a set of decision and state variables that satisfy all the system's constraints. If a solution (or a multiple set of solutions) exists, it must be contained within the feasible region.

2.5 Objective Function

An objective function is a scalar-valued function that maps the set of feasible decision variables into the real numbers, R^1. It measures the consequences or performance of a decision taken during any month and consequently the total performance achieved by the sequence of decisions made within a year.

2.6 Convex Set

If for any two points $\underline{x}_1$, $\underline{x}_2$ of a set K and a scalar $\alpha \in [0, 1]$ the linear combination $\underline{x} = \alpha\underline{x}_1 + (1 - \alpha)\underline{x}_2$ is also in K, then K is a convex set. Constrained minimization problems in which the objective function is convex and the set of constraints defines a concave set are termed convex programming problems. Linear programming (LP) and positive semidefinite linearly-constrained quadratic programming (PSD-QP) problems are examples of convex problems. For a maximization problem, the conditions to qualify as a convex problem are a concave objective function and a convex feasible set.

2.7 Optimization Model

The optimization model consists of the objective function and constraints of a problem and its solution algorithm.

2.8 Global and Local Optima

A feasible point $\underline{x}^*$ is a global optimum if $f(\underline{x}^*) > f(\underline{x})$ for all $\underline{x}$ contained in the feasible region, in which $f(\cdot)$ is the value of the objective function. A strong local optimum is a point $\hat{\underline{x}}$ such that $f(\hat{\underline{x}}) > f(\underline{x})$ for all $\underline{x}$ in the neighborhood of $\hat{\underline{x}}$ such that $\{\underline{x}; \| \hat{\underline{x}} - \underline{x} \|_2 \leq r\}$, in which r is a small number. Outside this neighborhood there may be points such that $f(\hat{\underline{x}}) \leq f(\underline{x})$, in which $\underline{x}$ is a feasible point.

2.9 Initial Policy

Many optimization techniques require an initial (starting) value for each of the variables of the optimization problem. In general, that initial value will not be a local or global optimum, but it must be feasible. In the reservoir operation problems considered in this report, the initial policy will be a sequence of reservoir storage vectors from the first month to the last month. Corresponding to that sequence of state variables is a sequence of decision variables (reservoir releases). Selection of initial policies will be discussed in Chapter 5.

2.10 Optimal Policy

A release policy consists of a sequence of release (penstock and spills) decisions, i.e., a sequence of vector-valued decision variables from the first month to the last month. Associated with a release policy is a state trajectory, a sequence of vector-valued state variables. The state trajectory is unique only if there is a one-to-one mapping between decision and state variable, in which case the system is invertible. A release policy is optimal if the objective function is maximized by such release policy.

2.11 Multiple Optimal Solutions

Some convex programming problems (e.g., LP and PSD-QP) have an infinite number of (feasible) solutions that are all optimal. That set of optimal solutions is also convex. An important consequence of the existence of multiple optimal solutions is that many alternative ways exist to achieve the same level of performance.

CHAPTER 3

REVIEW OF RESERVOIR OPERATION MODELS

This chapter reviews large-scale reservoir operation optimization models. The models are grouped into deterministic and stochastic models. That grouping is somewhat arbitrary because many models combine deterministic and stochastic features or deterministic, stochastic, simulation, and some system-dependent empirical techniques. Stochastic models will be characterized by the explicit inclusion of stochastic variability (usually, streamflow) in the objective function and/or the constraints.

In general, operation models of large-scale reservoir systems involve heavy computational and computer storage requirements. Some kind of sequential or decomposition scheme usually is adopted to succesfully implement the solution algorithms. A discussion of some of those algorithms is presented in Chapter 4. In short, Sections 3.1 and 3.2 contain reviews of proposed deterministic and stochastic reservoir operation models, respectively. For the sake of brevity, the reviews do not exhaust the numerous approaches that have been proposed. Finally, Section 3.3 briefly discusses major difficulties associated with reservoir operation models.

3.1 Determistic Models

Hall et al. (1969) were among the first investigators to propose a deterministic model for the monthly operation of a reservoir system. They used a version of Larson's (1968) state increment dynamic programming (SIDP) to solve for the monthly releases of a two-reservoir system. A historical low flow record was used to determine the annual firm energy. Roefs and Bodin (1970) proposed an analytical model that uses a combination of streamflow simulation, deterministic optimization, and multivariate analysis of the deterministic results. A brief discussion was included on the convexity of the model's objective function (the power formula). However, model's results were not provided.

Larson and Korsak (1970) and Korsak and Larson (1970) presented applications of dynamic programming successive approximations (DPSA) to a hypothetical four-reservoir system. They provided rigorous conditions for convergence and global optimality of the DPSA approach, conditions that have been unfortunately overlooked by most subsequent applications of DPSA by other investigators. Heidari et al. (1971) reported on the application of SIDP to Larson's simple four-reservoir system. They attributed the theoretical foundation of their study to Jacobson and Mayne (Jacobson, 1968a, 1968b, 1968c; Jacobson and Mayne, 1970), but we prefer to classify their method as a simplified version of Larson's SIDP, the so-called direct iterative SIDP (see Larson and Casti, 1982, for a recapitulation of SIDP). By using heuristic arguments, it was reported that convergence was achieved at a (unique) global optimum. However, subsequent research (e.g., Nopmongcol and Askew, 1976) showed multiple optimal solutions for the same problem.

Fults and Hancock (1972) used SIDP to obtain the daily operation policy of a two-reservoir system. The optimality of the results was checked by a heuristic approach in which different initial policies were input to the model, reportedly yielding the same value of the objective function. That approach, however, is not a proof of the convexity of the objective function, as concluded by Fults and Hancock (this topic is discussed further in Chapter 4). Liu and Tedrow (1973) presented a method that uses conventional dynamic programming (DP), a multivariable search technique, and simulation to develop rule curves. They simulated a two-reservoir system under extreme dry and wet conditions to develop a feasible operating range.

Hicks et al. (1974) developed a nonlinear programming model to obtain monthly operation policies for the Pacific Northwest hydroelectric system, consisting of 10 large reservoirs and 26 smaller run-of-the-river poundages. They used a modified conjugate gradient method to solve a problem with a nonlinear objective function and linear constraints. Assertion was made that only local optima are guaranteed. The optimization period used historical low flows. The optimization model yielded results on energy production that were about one percent greater than those obtained from a cut-and-dry approach. CPU execution time was reported to be considerable, 41 minutes per run.

Becker and Yeh (1974) used a linear programming-dynamic programming (LP-DP) approach for the monthly operation of a five-reservoir portion of the California Central Valley Project (CVP). The performance index consisted of minimization of potential energy losses of water backed up in the reservoirs. They argued that this type of criterion is justified by the high relative value of firm energy in comparison with that of firm water. Streamflows were assumed to be known for the one-year ahead planning horizon. By assuming that two of the reservoirs are kept at constant elevation throughout the year and a third reservoir is operated by a rule curve, the number of state variables was reduced to two, allowing an efficient implementation of the model. The model yielded a 35% energy production increase over contract levels. Fults et al. (1976) applied SIDP to a four-reservoir system in northern California, in which a highly nonlinear objective function represented power production. Two different initial policies gave different release policies, indicating the existence of multiple local optima. Because neither release policy yielded a global optimum, the authors concluded that SIDP does not apply to that reservoir system. However, it is not clear whether that was caused by the fact that the conditions for global optimality of SIDP were not met or by an inadequate state increment step, or both. The previously cited work of Fults and Hancock (1972) dealt with a subsystem of the four-reservoir system of Fults et al. (1976) in which SIDP reportedly converged to a global optimum.

Nopmongcol and Askew (1976) reported an application of what they termed multilevel incremental dynamic programming to Larson and Korsak's (1970) hypothetical four-reservoir example. It will be argued in Chapter 4 that this optimization scheme is Larson's DPSA. Nopmongcol and Askew argued that the approach of Heidari et al. (1971) is different from Larson and Korsak's (1970) SIDP and based their argument on the way that the time intervals were treated (fixed time steps). We prefer to think that the fixed step is a particular case of Larson's SIDP scheme and continue to classify the approach of Heidari et al. as merely SIDP. Further, Nopmongcol and Askew incorrectly argued that global optimality depends on their proposed multilevel scheme when, in fact, global optimality can only be achieved if Korsak and Larson's (1970) conditions are met, regardless of whether one or more levels are used in the solution.

Yeh et al. (1978) employed a combination of SIDP and DPSA (called IDPSA, after an abbreviation for SIDP-DPSA) to find optimal hourly operational criteria for the northern portion of the CVP system. They used a three-level approach in which monthly schedules (Becker and Yeh,

1974) were used as input to the daily model (Yeh et al., 1976) and the results of the daily model served as a basis to the hourly model. In essence, the system is decomposed into smaller subsystems and DPSA is used to advance the solution towards a local optimum. Each separate subsystem optimization is obtained by SIDP. The model also used an expanded approximate LP with a subsequent adjustment technique to develop an initial feasible policy, which is subsequently used to start the DPSA iterations. Results showed that the output from the approximate LP problem differed only by about 0.25 percent from the value of the objective function obtained after the IDPSA had converged to a local optimum. That is, the initial and final policies led to approximately the same solutions. It is argued that the tightly constrained feasible region did not leave much room for substantial improvements. No comparative results were given for different system decompositions and/or different initial policies. Although the necessary conditions for optimality (Larson and Korsak, 1970; Korsak and Larson, 1970) are difficult to evaluate in practice, the absence of different system decomposition and initial policy results does not allow an evaluation of the global optimality of the solution.

Murray and Yakowitz (1979) extended the differential dynamic programming (DDP) technique of Jacobson and Mayne (1970) by introducing linear constraints into DDP. The constrained DDP (denoted plainly as DDP) will be described in Chapter 4. Despite some attractive properties of DDP, results are not available on the application of the method to a real large-scale reservoir system.

Turgeon (1981) applied the principle of progressive optimality algorithm (POA) to maximize energy production in a system of four reservoirs in series. He considered deterministic inflows in an hourly operation model which incorporated time delays in the continuity equation. He erroneously asserted that convexity is not needed to obtain a global solution. Turgeon (1982a) also presented a modified version of SIDP (for a fixed time interval) in conjuction with DPSA (see Yeh et al., 1978) to the monthly operation of four reservoirs in series. The approach guarantees convergence to a global optimum for convex problems. However, Turgeon (1982a) did not provide a proof of the convexity of the problem under consideration. In particular, Turgeon's objective function is nondifferentiable on the decision variable and includes the energy head, which is usually a nonlinear function of reservoir storage and the latter is in turn related to the decision variable by the continuity equation. Under those circumstances, it is not possible to argue that a global optimum was obtained because convexity was not shown to hold. Indeed, since the solution procedure is DPSA, Korsak and Larson's (1970) conditions for global optimality must hold in order to obtain a global optimum.

Yeh and Becker (1982) presented a multiobjective technique for the operation of a portion of the CVP system. The technique requires the subjective criterion of a decision maker to choose among the multiple objectives served by the system. The constraint method of solution, in essence an iterative LP-DP approach, is used to solve the multiobjective programming problem. Due to the multiobjective nature of the problem and the effect that the decisions of a manager exert on the solution, it is difficult to establish the optimality of the approach unless extensive simulation runs are made.

Yazicigil et al. (1983) reported an application of LP to the daily operation of a four-reservoir system. The approach used concepts developed by the Corps of Engineers for water conservation such as balancing and zoning (Hydrologic Engineering Center, 1977). The concept of noninferior set was also used to establish tradeoff curves between different objectives. The optimality of the approach depends on the preset rules established by the Corps of Engineers.

Mariño and Mohammadi (1983b) extended the monthly operation model of Becker and Yeh (1974) to allow for maximization of both water releases and energy from the reservoir system. The proposed model is a combination of LP (used for month-by-month optimization) and DP (used for annual optimization). At every stage of the DP, a series of LP's are solved. Because the contract levels of water and energy are usually based on conservative estimates of natural inflows, the system is likely to be capable of providing more than those contract levels. To allow for the extra water releases and energy production, the values of the right-hand side of the contract constraints (in the LP model) are parametrically increased from the contract levels to the maximum possible levels in each month. To select the best beginning-of-month reservoir storage, a forward DP is used so that the water releases and energy produced during the month are maximized. The efficiency of the algorithm is improved through the use of parametric LP (reduces computation time) and an iterative solution procedure (reduces computation time and storage requirements). Those efficiency measures allowed the use of minicomputers (Mariño and Mohammadi, 1983c), which are more suitable for frequent updating purposes (because of their lower cost). The use of the model was illustrated for Shasta and Folsom reservoirs (CVP). In addition, Mohammadi and Mariño (1983a) reported on an efficient algorithm for the monthly operation of a multipurpose reservoir with a choice of objective functions. They considered water and energy maximization over the year, maximization of annual water and energy with flood control considerations, and maximization of water and energy for months with relatively high water and energy demands. The choice of objective functions gives the reservoir operator flexibility to select the objective that would best satisfy the needs of the demand area. Finally, Mohammadi and Mariño (1983b) presented a daily operation model for maximization of monthly water and energy output from a system of two parallel reservoirs. The daily model uses optimum monthly water and energy contract levels obtained from the monthly release policy. Those contract levels are adjusted to allow for differences between the daily and monthly forecast of inflow. The efficiency of the algorithm allowed the use of minicomputers. The model was applied to Shasta and Folsom reservoirs (CVP), with computation time requirements of less than two minutes.

3.2 Stochastic Models

It is emphasized that the division between stochastic and deterministic models is rather thin. Interest herein is centered on applications to large-scale reservoir systems. ReVelle et al. (1969) presented one of the first attempts to incorporate stochasticity into reservoir operation. The pivotal idea was to use a linear decision relationship between storage and release (linear decision rule, LDR) and to express the constraints of the system as probabilistic entities (chance constraints, CC) which are subsequently converted to deterministic equivalents. Many subsequent papers exploited the LDR approach (e.g., ReVelle and Kirby, 1970; Eastman and ReVelle, 1973; Gundelach and ReVelle, 1975), but most are aimed at determining the capacity of a single reservoir. The main difficulties with the CC-LDR approach arise from the fact that since storages and releases are random variables and almost any constraint on the operation of a system contains those variables, it is quite possible that some of those constraints will involve functions of storages and/or releases for which the distribution function is not known or cannot be derived and thus the deterministic equivalent cannot be developed. In addition, if storages and/or releases appear in the objective function of the model, then the expected value of returns is the relevant performance index that must be maximized. Usually, that leads to unsurmountable computational work for multistate problems. Furthermore, the choice of the reliability levels in the CC is rather arbitrary. The most serious drawback of LDR is that it forces the (random) state of the system (storage) to be equal to the sum of release and a constant, which is conceptually incorrect. Loucks et al. (1981) provided a list of references dealing with CC-LDR or some modification of the approach.

Takeuchi and Moreau (1974) described a combination of LP, stochastic DP, and simulation to obtain monthly operation policies that minimize short-term and long-term expected losses of a five-reservoir system. Drawbacks of the approach are the need for loss functions and the discretization of state variables. The approach also requires the knowledge or development (by using simulation) of joint conditional density functions of reservoir states given that any one reservoir is in a fixed state. The authors did not perform sensitivity analysis to establish the type of solution obtained. Perhaps, the most serious drawback of the technique is its large computational requirements.

Colorni and Fronza (1976) were the first to use the concept of reliability programming for the monthly operation of a single reservoir. They used a single reliability constraint to express the reliability of the system. Unlike the CC approach discussed earlier, the reliability level is part of the solution and thus is not specified beforehand.

Turgeon (1980) presented applications of system decomposition (DPSA) and aggregation/decomposition for the weekly operation of six reservoir-hydroplants in a stochastic environment. The decomposition approach works only for systems composed of parallel subsystems of reservoirs in series along rivers. In the decomposition technique, each subsystem is lumped into a composite equivalent and thus the (weekly) operation policies are developed for each composite subsystem as a whole rather than for each single reservoir. He claimed that DPSA yields nonoptimal results if a local feedback scheme is used but renders near optimal results when open-loop control is utilized. That is a rather surprising result because feedback or closed-loop controls usually perform better than its open-loop counterpart under stochastic disturbances; feedback control allows for self correction whereas the preplanned open-loop control does not (Bertsekas, 1976). The aggregation/decomposition scheme lumps all but one of the parallel subsystems into a composite unit and solves a two-state variable problem. The process is repeated for all complexes and then an adjustment is made to the solution of the aggregation phase to obtain a final solution. Results showed that the aggregation/decomposition method is superior to the decomposition technique for a six-reservoir system. CPU time was about 150 minutes per run by each method. It is ascertained that coarser state discretization would lead to smaller processing times but that could hamper the convergence to a global optimum (if it exists).

Simonovic and Mariño (1980) extended Colorni and Fronza's (1976) monthly operation model to allow for two reliability constraints, one for flood protection and the other for drought protection. The solution procedure consisted of an LP optimization and a two-dimensional Fibonaccian search technique. The search technique was used to select the reliability levels, and the LP was used to determine the releases for those reliabilities and thus evalute the objective function. Simonovic and Mariño (1981) also developed a methodology for the development of risk-loss functions in the reliability programming approach to a single reservoir. Flood and drought risk-loss functions were developed by using economic data. The reliability programming model was later applied to a three-reservoir system by Simonovic and Mariño (1982). Fixed reliability levels for each month in each reservoir were used to overcome computer storage requirements. Large memory requirements are needed for multireservoir systems; only local optimality can be guaranteed by the approach.

Mariño and Simonovic (1981) developed a two-step algorithm for the design of a multipurpose reservoir. The model was formulated as a CCLP which maximized downstream releases. The first step transforms the CC model into its deterministic equivalent through the use of an iterative convolution procedure. The second step finds the optimum size of the reservoir by solving the deterministic LP developed in step one. The model allows the use of random inflows and random demands together with other deterministic demands.

Mariño and Mohammadi (1983a) improved the work of Simonovic and Mariño (1980, 1981, 1982) by developing a new reliability programming approach for the monthly operation of a single multipurpose reservoir. The model uses CCLP and DP and differs from other reliability programming approaches with respect to the following three points. First, the development of risk-loss functions is not necessary. Development of risk-loss functions usually requires economic data that are often unreliable. Second, the reliability levels are not assumed to be constant throughout the year, and are different from month to month. This will eliminate the need for unnecessary extra releases in summer for high flood reliabilities, and extreme low releases during winter for high drought reliabilities. Third, the reliability of the hydroelectric energy production is included in the model. Most other existing stochastic models include energy production in the objective function and maximize the expected revenue of the produced energy.

Bras et al. (1983) devised a version of stochastic DP with a continuous updating of flow transition probabilities and system objectives. The innovative feature is the update of the transition probabilities of the flow states by considering the most recent information on the streamflow process. The method is used to find operating rules for a single reservoir. The updating stochastic DP (called "adaptive") was compared with the traditional steady-state method and a heuristic approach, and the objective function was varied to reflect different operating scenarios. For the realistic case in which spillage is considered, the heuristic approach yielded a higher firm energy than the traditional and the (adaptive) stochastic DP methods and there is indication that no advantage is derived from using the adaptive scheme. The type of optimality obtained is not addressed by the authors, but it obviously depends on the state and inflow discretization among other things (see, e.g., Knowles, 1981, for optimality conditions in control problems).

3.3 Discussion

The literature review in Sections 3.1 and 3.2 indicates that the methods that most satisfactorily handle the problems of dimensionality that arise in large-scale systems are mostly in the deterministic domain. Stochastic DP has been confined to one or two reservoirs; its worst handicap is that it requires discretization of state and inflows. For multistate systems, the storage requirements increase exponentially, severely limiting the applicability of stochastic DP. The reliability programming approach (e.g., Simonovic and Mariño, 1982) possesses significant computational requirements. For cases in which some finite increment is used to search for the solution of the reliability programming problem, the storage requirements increase exponentially in size when fine grids are used. In the deterministic framework, some methods overcome the dimensionality problem and for several there exist well established convergence and optimality criteria (Korsak and Larson, 1970; Murray, 1978). Unfortunately, those criteria exist under restrictive conditions (see Chapter 4).

Most researchers have not presented an analysis of the solutions obtained to reservoir operation problems (i.e., are the solutions unique, local, or global?). Although the theoretical postulates of optimality are difficult to check in many practical applications, testing of solutions (involving the effects of i) different initial policies, ii) alternate decomposition schemes, and iii) different grid sizes in some discretization schemes) usually is not reported. The existence of multiple solutions usually is not addressed. Perhaps, the key to a better understanding and successful implementation of the proposed reservoir operation models lies in the ability to take advantage of special system-dependent features that may lead to modeling simplifications (e.g., treating spillages as a function of storages, linearizing constraints, and considering storages of regulating reservoirs as constant) and in performing post-optimality analysis of the results.

Incorporation of the stochastic nature of streamflows in a suitable way into reservoir operation models needs further research. In this regard, seemingly contradictory findings have been reported but in effect those findings are compatible. For instance, by considering a time horizon that varied from a month to a year, Yeh et al. (1982) concluded that accurate streamflow forecasts lead to considerable benefits. In contrast, by using a time horizon of fifty-three years, Bras et al. (1983) found that no benefits were derived from inflow forecasts. Those apparently contradictory results bring to the surface the difference between short-term and long-term planning studies for reservoir operation. The major inference to be drawn is that for short-term time scales, especially for prediction of incoming flows during wet periods, it makes a great difference to have good inflow forecasts. For long-term studies, the dynamics of the system will center around average inflow values, making any type of forecasting scheme of marginal value.

The time framework of a study also raises an important question. When is an operation policy a real-time control scheme? Strictly speaking, real-time or on-line operation of a dynamic system requires two basic elements (Meier et al., 1971): an optimal state estimator and a control scheme that maximizes a given objective function. This issue has been exhaustively studied in the control literature (see, e.g., IEEE Transactions on Automatic Control, 1971) and frequently approached by using a Kalman filter for the state-space solution imbedded into a quadratic optimization scheme. Studies (Yeh et al., 1976, 1978; Bras et al., 1983) that have some ingredients of dynamic control (e.g., updated inflow forecasts) are essentially static operating modes. This is because the adaptive capability of a closed-loop control is not exploited. Yazicigil (1980) used a technique that has some attractive updating features. His optimization technique finds an optimal policy that is conditioned on today's forecasts and the actual state of the system. That policy is implemented for the current period (day) and is updated as soon as the applied control and stochastic inflow deviate the system from its expected state. That leads to a sequential solution of the optimization model at the beginning of each period. The research reported herein follows an analogous approach to a large-scale system to develop operation policies that are conditioned on updated flow forecasts and actual (observed) state values. Jamieson and Wilkinson (1972) outlined the principles of a sound real-time reservoir operation model. Unfortunately, the implementation of such a model has not been reported. In addition, Labadie et al. (1975) and Wenzel et al. (1976) presented attractive formulations for the real-time operation of flood-control reservoirs. Again, no results for on-line application have been reported.

In summary, some of the major difficulties associated with past research in reservoir operation modeling are: (i) the problem of dimensionality that practically has vanished stochastic DP from large-scale applications; (ii) the choice of adequate performance criteria and the assessment of the optimality of the results; and (iii) the incorporation of stochasticity (should stochasticity be handled in a a priori way (chance constraints), in a *a posteriori* way (reliability programming), in a static manner (stochastic DP), by simulation, or by some sequential updating scheme?).

CHAPTER 4

A REVIEW OF SOLUTION PROCEDURES

As indicated in the previous chapter, many optimization models have been proposed for the operation of reservoir systems. This chapter discusses some methods that have been successful, to some extent, in coping with large-scale systems: discrete dynamic programming (DP), the backbone of more sophisticated methods; state increment dynamic programming (SIDP); dynamic programming successive approximations (DPSA); differential dynamic programming (DDP); linear quadratic Gaussian method (LQG); nonlinear programming (NLP); and progressive optimality algorithm (POA). The main interest of the discussion is to highlight the relative advantages and disadvantages of the methods. Special attention is given to storage and computational requirements.

4.1 Formulation of the Problem

A general reference problem can be written as

$$\underset{\underline{u}_t,\ \forall t}{\text{maximize}}\ \underset{\underline{x}_t}{E}\left[\sum_{t=1}^{N} F(\underline{x}_t,\ \underline{u}_t(\underline{x}_t))\right] \tag{4.1}$$

subject to

$$\underline{x}_{t+1} = f_1(\underline{x}_t,\ \underline{u}_t,\ \underline{v}_t),\ \forall t \tag{4.2}$$

$$\underline{y}_{t+1} = f_2(\underline{x}_t,\ \underline{w}_t),\ \forall t \tag{4.3}$$

$$g(\underline{u}_t) \in \underline{U}_t,\ \forall t \tag{4.4}$$

The objective function, eq. (4.1), is to be maximized for some optimal control $\underline{u}_t$. The control $\underline{u}_t$ is deterministic and m-dimensional. The expectation E is with respect to a random n-dimensional state variable $\underline{x}_t$. The function F is generally nonlinear, time variant, and scalar valued. Notice that eq. (4.1) is written in the form of a closed-loop control, in which $\underline{u}_t$ is a function of a realization of $\underline{x}_t$. Equation (4.2) is the dynamics of the system, or the continuity equation in reservoir operation jargon. It describes the evolution of the system when a control $\underline{u}_t$ is applied to the system and a random disturbance $\underline{v}_t$ affects the state of the system. Equation (4.3) is the observation process, or that which can be observed about the state of the system because the measurement of state variables is not always complete (i.e., not all the components of the states may be observed) and is noise corrupted by $\underline{w}_t$. Equation (4.4) states that the control variable $\underline{u}_t$ must lie within a prescribed region $\underline{U}_t$ of the m-dimensional space R^m.

Notice that the general case described by eqs. (4.1)-(4.4) is in practice unsolvable. That is because the functions are arbitrarily complex and the noises $\underline{v}_t$ and $\underline{w}_t$ are, in theory, also arbitrarily distributed. Many simplifications of the general case are important because they may give reasonable approximations to the real world. In contrast to the stochastic closed-loop control is the open-loop formulation. Stochastic open-loop control differs from closed-loop control in that the former does not depend on the state of the system to decide on the actions to take. In general, that leads to suboptimal policies with respect to the closed-loop policies. In reservoir operation, deterministic open-loop control is the most frequent adopted method. This type of deterministic problem can be written as

$$\underset{\underline{u}_t,\ \forall t}{\text{maximize}} \sum_{t=1}^{N} F(\underline{x}_t, \underline{u}_t) \tag{4.5}$$

subject to

$$\underline{x}_{t+1} = f_3(\underline{x}_t, \underline{u}_t) \tag{4.6}$$

$$g_1(\underline{u}_t) \in \underline{U}_t \tag{4.7}$$

$$g_2(\underline{x}_t) \in \underline{X}_t \tag{4.8}$$

in which eq. (4.8) describes the feasible set for the state $\underline{x}_t$ and the meaning of the other equations can be inferred from the earlier general case.

The explicit incorporation of stochasticity makes the problem much more complex because i) information must be obtained about the statistical properties of the random variables, and ii) in the closed-loop control there exists a coupled problem of state estimation and control optimization. The structure of the general stochastic problem, eqs. (4.1)-(4.4), reveals that for given distributions of the state $\underline{x}_t$ and the noise $\underline{v}_t$, the distribution of $\underline{x}_{t+1}$ depends on the type of function f_1 and the control $\underline{u}_t$. Thus, from this perspective, the chance constraint approach of specifying probability ranges for the state variables may lead to improper distribution-function modeling. On the other hand, any deterministic problem of the form given by eqs. (4.5)-(4.8) could, in theory, be expressed as a gigantic nonlinear programming model with m x N variables in which m is the dimension of $\underline{u}_t$ and N is the number of time periods. Some of the reliability programming models discussed in Section 3.2 fall in this category. Other methods attempt to skip the large-size formulation and recur to sequential algorithms to achieve a solution (e.g., DP, SIDP, and DPSA), but not without introducing problems of their own. Some methods (e.g., some versions of stochastic DP and the POA) take a middle course of action by trying to capture the stochastic nature of a real-world system with some type of forecasting and continuous update of the states of the system. That is a compromise between the real stochastic problem and its deterministic counterpart.

Interestingly, the deterministic problem can be as difficult to solve as any other problem. For large-scale modeling, no matter which model is chosen, computational burden is a real hurdle. For any specific system, a careful analysis of available methods (or development of new methods) must be pursued before making a decision on which solution technique to adopt. The following discussion on some popular models points out some of the relative advantages and disadvantages of several optimization schemes.

4.2 Discrete Dynamic Programming (DP)

Deterministic DP is one of the first methods used to find optimal policy sequences and state trajectories of optimization problems of a sequential nature (Bellman, 1957; Bellman and Dreyfus, 1962). The fundamental basis of DP for solving problems stated as in eqs. (4.5)-(4.8) is the principle of optimality: "If an optimal trajectory is broken into two pieces, then the last piece is itself optimal." This simple statement holds the key for the sequential approach that can be used to solve many dynamic problems. It is also the basis for more advanced computational algorithms (e.g., SIDP, DPSA, stochastic DP, POA).

Dreyfus and Law (1977) and Larson (1982) discussed DP in detail. In essence, DP determines an optimal trajectory recursively, either in a backward or forward fashion. Assume that a backward recursion is used. The method constructs a sequence of decisions $\underline{u}_t$ for t = N, N-1,..., 1. The values that each state variable ($\underline{x}_t$) can take are a finite number of discrete values (d). Assume that d is the same for each component of $\underline{x}_t$. The optimal policy $\{\underline{u}_t\}$ is constructed by using the relation

$$\underset{\underline{u}_t}{\text{maximize}}\ F_t = V_t(\underline{x}_t, \underline{u}_t) + F_{t+1}[f_3(\underline{x}_t, \underline{u}_t)] \tag{4.9}$$

in which $F_{N+1} = C$ (a given constant), t = N, N - 1,..., 1, V_t is the current return obtained from taking a decision $\underline{u}_t$ when the state is $\underline{x}_t$, and $F_{t+1}(\cdot)$ is the optimal return (from period t + 1 on) derived from state $\underline{x}_{t+1}$ which in turn is equal to $f_3(\underline{x}_t, \underline{u}_t)$, i.e., the value obtained from the dynamics of the sytem [see eq. (4.6)] for $\underline{x}_t$ and $\underline{u}_t$. Thus, by concatenating the present stage optimal decision with a corresponding optimal trajectory for the subsequent stages, an optimal trajectory is obtained for periods t, t + 1,..., N (the initial and final states can be either fixed or free, depending on the nature of the problem). To select a policy in eq. (4.6), the constraints on state and control variables are checked to ensure that a feasible decision ($\underline{u}_t$) is being made. If a decision is infeasible, then that decision is discarded and others are examined.

From the preceding brief description, two advantages of DP are readily observed: i) functions such as F_t, V_t, f_3, and others can be of arbitrary form, namely nondifferentiable, time variant, nonlinear, etc., and ii) the method can handle constraints with complex mathematical structures. However, those advantages usually cannot be exploited when the number of state variables is greater than two. Chow et al. (1975) showed that the storage requirements to evaluate the recursive equation [eq. (4.9)] at each stage, or period, is

$$P = 2d^n \tag{4.10}$$

in which d is the number of feasible values that each component of the state vector can take and n is the dimension of the state vector $\underline{x}_t$. The memory required to store the current optimal values of the decision variables for each feasible value of the state variables at all stages (so that the optimal trajectory can be retrieved) is

$$T = Nmd^n \tag{4.11}$$

in which m is the dimension of the decision vector ($\underline{u}_t$) and N is the number of periods in which $\underline{u}_t$ must be determined (i.e., the number of optimization periods). Equations (4.10) and (4.11) show that storage requirements grow exponentially with n. That constitutes the most serious drawback for the application of DP, the so-called curse of dimensionality. The necessary conditions for optimality of DP follow from the classical calculus of variations (Bellman and Dreyfus, 1962) and the maximum principle (Pontriagyn et al., 1962). In practice, however, it is difficult to check and/or satisfy those conditions. Usually, the ability to detect a local optimum depends on the adequacy of the state discretization.

4.3 State Increment Dynamic Programming (SIDP)

Larson introduced the SIDP approach in 1968. Some investigators (Heidari et al., 1971; Chow et al., 1975) have used the term discrete differential dynamic programming (DDDP) to describe Larson's SIDP with a fixed time step while others (Hall et al., 1969; Turgeon, 1982a, 1982b) have termed the approach incremental dynamic programming (IDP). In essence, SIDP is an iterative technique in which the recursive equation of DP [eq. (4.9)] is used to search for an improved trajectory among the admissible discrete states in the neighborhood of a trial trajectory. Those admissible states near the state trajectory form a so-called corridor. In determining the optimum trajectory in the corridor, the constraints on state and decision variables must be satisfied. The following three steps describe the SIDP technique (Turgeon, 1982b):

1) Find an initial feasible trajectory and denote it by $\{\underline{x}_t^{(0)}\}$.

2) Solve the recursive equation of DP [eq. (4.9)] for $\underline{x}_t = \underline{x}_t^{(0)} - \underline{\alpha}$, $\underline{x}_t^{(0)}$, and $\underline{x}_t^{(0)} + \underline{\alpha}$ in which $\underline{\alpha}$ is a preselected vector of increments. The recursive DP equation can be either forward (Hall et al., 1969) or backward (Turgeon, 1982a) in time. The solution of eq. (4.9) for all t is denoted by $\{\underline{x}_t^{(1)}\}$.

3) If $|\underline{x}_t^{(1)} - \underline{x}_t^{(0)}| \leqq \underline{\varepsilon}$, $\underline{\varepsilon} \rightarrow 0$ (any other criterion could be used), $\forall t$, then stop. Otherwise, set $\{\underline{x}_t^{(0)}\} = \{\underline{x}_t^{(1)}\}$ and go to step 2.

Figure 4.1 shows some of the elements of SIDP for n = 1. Conceptually, SIDP does not add anything to DP except for the iterative search algorithm. Despite the iterative character of SIDP, it can be established that the storage requirements to evaluate eq. (4.9) at each stage are

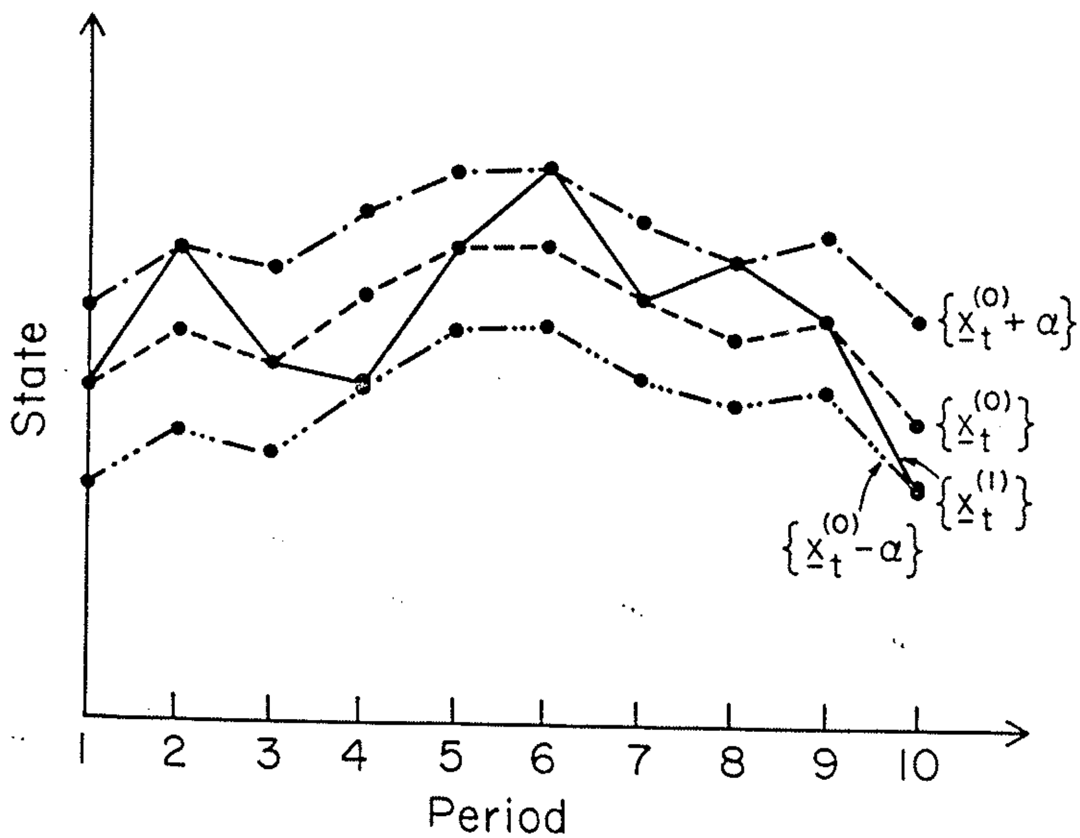


Fig. 4.1 Schematic representation of a trial trajectory $\{\underline{x}_t^{(0)}\}$, the corridor boundaries $\{\underline{x}_t^{(0)} \pm \alpha\}$, and a substitute trial trajectory $\{\underline{x}_t^{(1)}\}$.

$$P = 2(3)^n \tag{4.12}$$

and the corresponding memory requirements (T) are

$$T = Nm(3)^n \tag{4.13}$$

Thus, memory requirements vary exponentially with n. There may be storage difficulties when $n \geqq 3$, or even $n \geqq 2$. In addition, computations must be made K times, in which K is the number of iterations needed to converge to an optimal trajectory. Unfortunately, the convergence of SIDP is at best linear (Murray and Yakowitz, 1979) and K may become substantially large. Further, the selection of the increment $\underline{\alpha}$ is critical in this algorithm. A poorly chosen value of $\underline{\alpha}$ (which can vary from iteration to iteration and/or from period to period) may lead to nonoptimal solutions. Turgeon (1982a) developed a procedure to select $\underline{\alpha}$ so that convergence to a local optimum is guaranteed. He showed that for a convex objective function (and linear constraints) his modified SIDP leads to a global convergence.

4.4 Dynamic Programming Successive Approximations (DPSA)

In the DPSA approach, a problem of several control variables is decomposed into a number of subproblems containing only one control variable (Korsak and Larson, 1970; Larson and Korsak, 1970). The algorithm proceeds as follows:

1) Denote the ith component of the n-dimensional state variable by x_t^i. Select feasible control and trajectory sequences (initial policy) and denote them by $\{\underline{u}_t^{(0)}\}$ and $\{\underline{x}_t^{(0)}\}$, respectively. The state variables are discretized whereas the controls are not.

2) Select one component of the state vector, say x_t^i. All states $x^j(t)$, $j \neq i$, $t = 1, 2, \ldots, N$, are kept fixed at the values $\underline{x}_t^{(0)}$. Maximize the objective function over all x_t^i's, $t = 1, 2, \ldots, N$, by some method (e.g., one-dimensional DP or SIDP) in the usual way except that now there are n controls (decision variables) to vary at each time stage. The maximization is subject to the n-1 fixed states x_t^j, $j \neq i$, thereby leaving one degree of freedom in the controls to allow the variation of x_t^i. For this purpose, it is necessary to have a one-to-one mapping between the controls and the states expressed by the continuity equation (i.e., the system must be invertible).

3) After the optimal control and trajectories are developed by using the state component x_t^i, the method proceeds with step 2 for a different state component x_t^j until all the components ($i = 1, 2, \ldots, n$) have been treated once.

4) Repeat steps 2 and 3 until no further improvement can be made.

Bellman (1957) and Bellman and Dreyfus (1962) showed that decomposition schemes of the type of DPSA converge in a finite number of iterations. Korsak and Larson (1970) showed that there exist necessary and sufficient conditions for global optimality. Because of the popularity of DPSA in reservoir operation studies, it is worthwhile to summarize those optimality conditions. First, the law of motion or continuity equation [eq. (4.6)] must be linear, e.g.,

$$f(\underline{x}_t, \underline{u}_t) = \Phi_t \, \underline{x}_t + \Gamma_t \, \underline{u}_t, \; \forall t \tag{4.14}$$

in which Φ_t and Γ_t are suitably dimensioned matrices and the Γ_t's are nonsingular. Second, the objective function [see eq. (4.5)] is given by

$$\underset{\underline{x}_t,\ \underline{u}_t}{\text{maximize}} \sum_{t=1}^{N} \frac{1}{2}(\underline{x}_t^T\, Q_t\, \underline{x}_t + \underline{u}_t^T\, R_t\, \underline{u}_t) + A_t\, \underline{x}_t + B_t\, \underline{u}_t \tag{4.15}$$

Equation (4.15) can be expressed as a function of the $\underline{x}_t$'s only by substituting $\underline{u}_t$ by $\underline{u}_t = f^{-1}(\underline{x}_t,\ \underline{x}_{t+1})$, the inverse, unique mapping between control and state variables. Furthermore, all the matrices Q_t and R_t in eq. (4.15) must be positive definite (if the matrices are positive semidefinite, the optimality conditions are only necessary for a local optimum). Notice that eq. (4.15) must be a quadratic strictly convex function for the necessary and sufficient conditions to hold. Third, the objective function must be bounded for all feasible values of $\underline{x}_t$ and $\underline{u}_t$. Fourth, the decision and state variables must be continuous. The preceding four conditions are very restrictive. From a review of the literature on reservoir studies, it appears that no investigation fits those restrictions. That implies that whatever results are obtained when the necessary and sufficient conditions are violated, the only way to ascertain something about the optimality of those results is through well designed, heuristic approaches or sensitivity analyses.

In addition to the optimality issue, there are other difficulties that arise in the practical implementation of DPSA. One difficulty is the selection of the x_t^i's (i.e., how is the system going to be decomposed?). In some systems, the topology of the network gives a clue as to the selection of an adequate decomposition scheme, but this is not always the case. If DP is selected to solve the one-dimensional problems that arise from the DPSA decomposition, then the storage requirements (P) are

$$P = 2(n-1)d \qquad (4.16)$$

in which d is the number of discrete values of x_t^i, i = 1, 2,..., n. The factor (n-1) appears from the fact that while x_t^i is being maximized, the remaining x_t^j's ($j \neq i$) are being held fixed, and the total value of the objective function depends on the effect that changes on u_t^j ($j \neq i$) have on the objective function. The memory requirements (T) can be expressed by

$$T = ndN \qquad (4.17)$$

Equations (4.16) and (4.17) show that the curse of dimensionality is beaten by DPSA. However, the number of calculations of the program may become severely large because each iteration consists of a complete sweep of one-dimensional problems for every x_t^i (i = 1, 2,..., n). Furthermore, because the convergence rate (improvement from iteration to iteration) is slow, the number of iterations would be relatively large, especially if the initial feasible policy is not near the optimum.

As indicated in Section 3.2, several reservoir operation models have made use of a combination of SIDP and DPSA. Lower-dimensional problems that arise from the decomposition have been solved by techniques that require discretization of the state variables, thus hampering the convergence properties of the algorithm. It is evident that those lower-dimensional optimizations could be approached by some type of algorithm that does not require discretization (e.g., LP and quadratic programming).

4.5 Differential Dynamic Programming (DDP)

The DDP method was developed by Jacobson and Mayne (1970). The extension made by Murray and Yakowitz (1979) to include linear constraints into the problem is discussed herein. The DDP algorithm to solve linearly constrained problems can be summarized as follows:

1) Set the iteration counter k equal to zero. Construct a feasible control sequence $\{\underline{u}_t^{(k)}\}$ and a state trajectory $\{\underline{x}_t^{(k)}\}$. Set the time index t equal to N (the last stage).

2) Recalling the objective function as stated in eq. (4.5), obtain a second-order Taylor series expansion of $F(\underline{x}_N, \underline{u}_N)$ about $\underline{x}_N^{(k)}$ and $\underline{u}_N^{(k)}$ and denote it by $\hat{F}(\underline{x}_N, \underline{u}_N)$. Then the following quadratic programming problem is solved

$$\underset{\underline{u}_N}{\text{maximize}}\ \hat{F}(\underline{x}_N, \underline{u}_N) \tag{4.18}$$

subject to

$$\underline{u}_N \in \underline{U}_N \tag{4.19}$$

Notice that eq. (4.18) is maximized with respect to $\underline{u}_N$. The states $\underline{x}_N$ and $\underline{x}_{N+1}$ are treated as constant. Murray and Yakowitz (1979) solved eqs. (4.18) and (4.19) by using Fletcher's (1981) active set method. The solution is of the form

$$\underline{u}_N = \underline{\alpha}_N + \beta_N\ \underline{x}_N \tag{4.20}$$

Store the m-dimensional vector $\underline{\alpha}_N$ and the m x n matrix β_N. The approximation to the optimal value of F is denoted by

$$V_N = F(\underline{x}_N, \underline{u}_N) \tag{4.21}$$

in which $\underline{u}_N$ is defined by eq. (4.20)

3) Proceeding in a backward fashion, set t = t - 1 and define

$$Q(\underline{x}_t, \underline{u}_t) = F(\underline{x}_t, \underline{u}_t) + V_{t+1}[f(\underline{x}_t, \underline{u}_t)] \tag{4.22}$$

Perform a second-order Taylor series expansion of Q about $\underline{x}_t^{(k)}$ and $\underline{u}_t^{(k)}$ and denote it by $\hat{Q}(\underline{x}_t, \underline{u}_t)$. Solve the quadratic programming problem

$$\underset{\underline{u}_t}{\text{maximize}}\ \hat{Q}(\underline{x}_t, \underline{u}_t) \tag{4.23}$$

subject to

$$\underline{u}_t \in U_t \tag{4.24}$$

maintaining $\underline{x}_t$ as a constant. The solution of eqs. (4.23) and (4.24) is of the form

$$\underline{u}_t = \underline{\alpha}_t + \beta_t\ \underline{x}_t \tag{4.25}$$

Store $\underline{\alpha}_t$ and β_t. Approximate $Q(\underline{x}_t, \underline{u}_t)$ by $Q(\underline{x}_t, \underline{\alpha}_t + \beta_t\ \underline{x}_t)$.

4) Set $V_t = Q(\underline{x}_t, \underline{\alpha}_t + \beta_t\ \underline{x}_t)$ and t = t-1. Go to eq. (4.22) of step 3. Repeat the loop defined by steps 3 and 4 until a complete sweep

(from t = N to t = 1) is attained. At this point, a complete sequence of parameters $\{\underline{\alpha}_t, \beta_t\}$, for all t, is stored.

5) Construct a successor policy $\{\underline{u}_t^{(k+1)}\}$ by recursively calculating

$$\underline{u}_t^{(k+1)} = \underline{\alpha}_t + \beta_t \underline{x}_t \tag{4.26}$$

and

$$\underline{u}_{t+1}^{(k+1)} = f(\underline{x}_t^{(k+1)}, \underline{u}_t^{(k+1)}) \tag{4.27}$$

in which $\underline{x}_1$ is given. If the two consecutive control policies satisfy an adequate convergence criterion, then stop. Otherwise, set t = N and go to step 2. In practice, the forward run of step 5 usually requires a modification to avoid infeasible successor policies. This is achieved by solving a (forward) sequence of quadratic programming problems, as outlined in Murray (1978) and Murray and Yakowitz (1979).

It is evident from the preceding discussion that DDP is an involved method to implement. The main potential shortcoming of DDP in multi-reservoir problems is that it requires second-order differentiability for the objective function and first-order differentiability for the constraints. If that is not possible to achieve, then numerical differentiation must be used, which can erode the fast convergence properties credited to DDP. Although the active set method can be readily implemented for positive semidefinite matrices in the quadratic objective function, it is difficult to encounter that situation in reservoir operation problems (see Chapter 5). It turns out that the active set method for the indefinite case is relatively more complicated than for the semidefinite case and it guarantees local optimality at best (Fletcher, 1981). On the other hand, DDP has some good properties that apply to differentiable, positive definite cases. First, the memory and computational requirements grow proportionally with mn and m^3, respectively, in which m and n are the respective dimensions of $\underline{u}_t$ and $\underline{x}_t$. The computational requirements of DDP are given by $2am^3NK$ in which a is a constant independent of m and K is the number of complete iterations (from t = N to t = 1). The storage requirement is approximately given by

$$T = cNm(n+1) \tag{4.28}$$

in which c is a positive constant greater than one. Clearly, from eq. (4.28), there is no dimensionality problem. Second, there is no need to discretize the state and/or decision variables. Third, the convergence rate of the method is quadratic (Murray, 1978). Fourth, the continuity equation needs not be invertible. Fifth, the DDP converges to a global optimum, provided that the objective function is strictly convex.

4.6 Linear Quadratic Gaussian Method (LQG)

The previous solution procedures (DP, SIDP, DPSA, and DDP) were discussed within a deterministic context. This section reviews a classical solution approach (LQG) to stochastic optimal control problems. The LQG method assumes a linear continuity equation, a quadratic objective function, and Gaussian disturbances to the system under consideration. It is a true real-time (or on-line) feedback control approach that deals in a natural way with the coupled state estimation-control (closed loop) optimization. The theoretical background of the approach goes back to Kalman and Koepcke (1958) and further advances were made in the 1960's.

The discussion that follows is based on the works of Meier et al. (1971), Dorato and Levis (1971), and Willems (1978). The problem under consideration can be stated as

$$\underset{\underline{u}_t,\ \forall t}{\text{minimize}} \quad \underset{\underline{x}_t,\ \underline{z}_t}{E} \left[\sum_{t=1}^{N} \underline{x}_t^T Q_t \underline{x}_t + \underline{u}_t^T(\underline{z}_t) R_t \underline{u}_t(\underline{z}_t) \right] + \underline{x}_{N+1}^T S \underline{x}_{N+1} \tag{4.29}$$

subject to

$$\underline{x}_{t+1} = \phi_t \underline{x}_t + B_t \underline{u}_t + \underline{e}_t \tag{4.30}$$

$$\underline{z}_t = C_t \underline{x}_t + \underline{\theta}_t \tag{4.31}$$

In eq. (4.29), the expectation is taken with respect to the random variables $\underline{x}_t$ and $\underline{z}_t$; Q_t is a symmetric (n x n) positive semidefinite matrix; R_t and S are symmetric (m x m and n x n, respectively) positive definite matrices; $\underline{x}_t$ is the n x 1 state vector; $\underline{u}_t$ is the m x 1 control vector; and $\underline{z}_t$ is the r x 1 ($r \leqq n$) measurement process. In eq. (4.30), the continuity equation, ϕ_t and B_t are suitably dimensioned (known) matrices with ϕ_t nonsingular; $\underline{e}_t$ is a (normal) random noise with $E(\underline{e}_t) = 0$, $E(\underline{e}_t \underline{e}_\tau^T) = \ddagger_t \delta_{t\tau}$, and $E(\underline{e}_t \underline{\theta}_\tau^T) = 0$ ($\forall\tau$), $E(\underline{e}_t \underline{x}_0^T) = 0$ and $\underline{\theta}_t$ is a (normal) random noise with $E(\underline{\theta}_t) = 0$, $E(\underline{\theta}_t \underline{\theta}_\tau^T) = \Theta \delta_{t\tau}$ ($\forall\tau$) and $E(\underline{\theta}_t \underline{x}_0^T) - 0$; $\underline{x}_0$ is the initial state at time t = 0 with $E(\underline{x}_0) = \bar{\underline{x}}_0$ and

$E(\underline{x}_0 - \bar{\underline{x}}_0)(\underline{x}_0 - \bar{\underline{x}}_0)^T = \Sigma_0$, and $\delta_{t\tau}$ is the Kronecker delta. In eq. (4.31), the observation process, C_t is an n x n matrix and $\underline{\theta}_t$ is the measurement noise. Equation (4.31) expresses the fact that the controller may not be able to observe $\underline{x}_t$ exactly, but only can measure it with some error. The key to the solution of the LQG problem [eqs. (4.29)-(4.31)] relies upon the separation principle (Wonham, 1970; Joseph and Tou, 1961). Basically, the separation principle states that the solution of the stochastic problem can be divided into two parts: control (the selection of the optimum decision to input into the system) and estimation (the computation of the state conditional probability density function).

To give the solution of the problem, it is necessary to define the conditional mean ($\hat{\underline{x}}_{t|t}$) and the conditional variance ($\Sigma_{t|t}$) of the state:

$$\hat{\underline{x}}_{t|t} = \underset{\underline{x}_t}{E}(\underline{x}_t | \underline{z}_0, \underline{z}_1, \ldots, \underline{z}_t; \underline{u}_1\ \underline{u}_2, \ldots, \underline{u}_{t-1}, \bar{\underline{x}}_0, \Sigma_0) \quad (4.32)$$

$$\Sigma_{t|t} = \underset{\underline{x}_t}{E}(\underline{x}_t - \hat{\underline{x}}_{t|t})\ (\underline{x}_t - \hat{\underline{x}}_{t|t})^T \quad (4.33)$$

The estimation problem can be solved by using a Kalman filter (Kalman, 1960; Kalman and Bucy, 1961; Jazwinsky, 1970; Gelb, 1974). The estimator is

$$\hat{\underline{x}}_{t+1|t+1} = \phi_t\ \hat{\underline{x}}_{t|t} + B_t\ \underline{u}_t + K_{t+1}\ [\underline{z}_{t+1} - C_t(\phi_t\ \hat{\underline{x}}_{t|t} + B_t\ \underline{u}_t)] \quad (4.34)$$

with initial value $\hat{\underline{x}}_{0|0} = \bar{\underline{x}}_0$, in which K_{t+1} is the Kalman gain matrix given by

$$K_{t+1} = \Sigma_{t+1|t} C_{t+1}^T (C_{t+1} \Sigma_{t+1|t} C_{t+1}^T + \Theta_{t+1})^{-1} \tag{4.35}$$

where

$$\Sigma_{t+1|t} = (I - K_{t+1} C_{t+1}) (\phi_t \Sigma_{t|t-1} \phi_t^T + \Xi_t) \tag{4.36}$$

with initial value $\Sigma_{0|-1} = \Sigma_0$. Notice that the on-line estimation can proceed only in one direction: forward in time. This is accomplished by using eqs. (4.34)-(4.36) recursively.

The solution of the control part of the problem can be obtained by applying the principle of optimality to eq. (4.29). That is, a recursive (backward) equation is solved at each period t,

$$I_t(\hat{\underline{x}}_{t|t}) = \min_{\underline{u}_t} \{\hat{\underline{x}}_{t|t}^T Q_t \hat{x}_{t|t} + \underline{u}_t^T R_t \underline{u}_t$$

$$+ \underset{\hat{\underline{x}}_{t+1|t+1}}{E} [I_{t+1}(\hat{\underline{x}}_{t+1|t+1}) \mid \hat{\underline{x}}_{t|t}, \underline{u}_t]\} \tag{4.37}$$

starting with the terminal condition $I_{N+1}(\hat{\underline{x}}_{N+1|N+1}) = \underline{x}_{N+1}^T S \underline{x}_{N+1}$. It can be shown (Meier and Larson, 1971; Willems, 1978) that the optimal control $\underline{u}_t^*$ that minimizes eq. (4.37), for all t, is

$$\underline{u}_t^* = -G_t \hat{\underline{x}}_{t|t} \tag{4.38}$$

in which $\hat{\underline{x}}_{t|t}$ is obtained from eq. (4.34) and

$$G_t = (B_t^T P_{t+1} B_t + R_t)^{-1} B_t^T P_{t+1} \phi_t \tag{4.39}$$

The matrix P_{t+1} can be obtained recursively by using the relation

$$P_t = Q_t + \phi_t^T P_{t+1} \phi_t - \phi_t^T P_{t+1} B_t (B_t^T P_{t+1} B_t + R_t)^{-1} B_t^T P_{t+1} \phi_t \tag{4.40}$$

with $P_{t+1} = S$. The fundamental result is that the optimal control is given by a linear function of the optimal state estimator. It is evident that $\underline{u}_t^*$ is a closed-loop control, $\underline{u}_t^*(\underline{x}_t) = -G\, \hat{\underline{x}}_{t|t}$. The solution to the linear quadratic problem when the state is known exactly (i.e., there is no measurement error) is also linear and given by $\underline{u}_t^* = -G\, \underline{x}_t$, i.e., $\underline{x}_t$ is substituted for $\hat{\underline{x}}_{t|t}$. Moreover, in the linear quadratic problem, the disturbance $\underline{e}_t$ [see eq. (4.30)] needs not be Gaussian.

The preceding discussion reveals that some computations must be made on-line while a significant portion of the calculations can be made off-line (e.g., K_t, $\Sigma_{t+1|t}$, G_t, and P_t for all t), even at the predesign level. Assuming that $r = m = n$, the storage requirements of the method are proportional to Nn^2 in which N is the number of stages under consideration. The computational work is about $an^3(n+1)N$ in which a is a positive real number greater than one. Notice that for $a = 4$ and $n = 10$, the number of multiplications at each stage (period) is about 44,000. That number does not include other arithmetic and logic operations. Thus, the computational work seems to be the issue to be dealt with in this technique (see Mendel, 1971, and Samant and Sorenson, 1974, for exact storage and computational requirements under different circumstances).

4.7 Nonlinear Programming (NLP)

Nonlinear programming is a very important and diverse branch of optimization theory, but it is far better suited for static optimization (i.e., time-independent problems). It does not work for stochastic real-time control problems, of which reservoir operation is an example. In fact, applications of NLP to reservoir operation always convert the problem into a deterministic framework by using chance constraints, or by forecasting or simulating inflows or other random variables. The mathematical structure of the model is then written as if the problem were deterministic and some NLP algorithm is used to obtain a solution. The most adverse factor in this procedure is that the naturally dynamic problem, after its conversion into a gigantic NLP model, is expressed in terms of mN decision variables (m is the number of decision variables at each period and N is the number of optimization periods). Usually, that implies heavy computational work. For instance, if the resulting problem were a quadratic programming problem and Fletcher's (1981) active set method were used for its solution, the computational burden would be on the order of $(mN)^3$. If the same problem were attacked by a sequential scheme (e.g., the progressive optimality algorithm, POA, of Section 4.8), then the number of calculations would be on the order of m^3NK, in which K is the number of iterations needed to attain convergence. To illustrate, if $m = 5$, $N = 50$, and $K = 6$ (typical in the POA), then the NLP and POA approaches would result in 250^3 and 300×5^3 calculations, respectively.

4.8 Progressive Optimality Algorithm (POA)

The progressive optimality algorithm (POA) solves a multistage dynamic problem as a sequence of two-stage optimization problems. It is based on the principle of progressive optimality (PPO) (Howson and Sancho, 1975) which states that: "The optimal path has the property that each pair of decision sets is optimal in relation to its initial and terminal values." The PPO is derived from Bellman's (1957) principle of optimality.

This section presents an extension of the work of Howson and Sancho (1975) to the case in which there exists bounds on state and decision variables. Also discussed in this section are some powerful programming features of the POA that can be used to accelerate its convergence rate. Section 4.9 contains proofs of the convergence of the POA for bounded states and decisions. Specifically: (i) if a solution to a multistage dynamic problem exists, then the POA converges to that solution in a finite number of steps; and (ii) if the objective function is strictly concave and the constraints define a convex set (interest here is in maximization), then a global optimum is obtained. These proofs are extensions of those given by Howson and Sancho (1975) for the unconstrained (after substitution of the law of motion into the objective function) problem.

Before outlining the solution approach, several of the favorable features of the POA are summarized:

1) The decision and state variables need not be discretized.

2) The two-stage optimization problem (objective function and constraints) can be solved by any adequate algorithm, either of the DP or NLP families. Thus, the objective function and constraints can be of

arbitrary form, although for convergence to a global optimum the problem must be concave (or convex for the case of minimization).

3) The dynamics of the system need not be invertible for the application of the POA, although that implies the possible existence of multiple solutions.

4) Time delays in the continuity equation can be incorporated easily, as done by Turgeon (1981).

5) Convergence rates depend on the scheme utilized in the two-stage maximization. When a fast convergent algorithm like Fletcher's (1981) active set method is used, the POA convergence rate is better than linear and perhaps close to quadratic if a good initial policy is chosen; however, it has not been possible to establish quadratic convergence rigorously. Actual applications of the POA (Chapters 5 and 7) have shown convergence in less than nine iterations, five being the average.

6) Assuming that the active set method is used to solve the two-stage optimization problem, the computation effort to complete one iteration (from time $t = 1$ to $t = N$) is an^3N and $b(2m + n)^3N$ for invertible and noninvertible continuity equations, respectively, in which a and b are positive constants larger than one and independent of m and n, the respective dimensions of the decision and state variables.

7) Storage requirements are proportional to nN and $(2m + n)N$ for invertible and noninvertible cases, respectively. Those requirements account for the storage of current state trajectories. Parameter (transition matrices in the continuity equation) storage requirements vary greatly from one application to another, but for most applications (e.g., reservoirs without pumped storage) they are proportional to n^2 and $(2m + n)^2$ for invertible and noninvertible cases, respectively.

Consider the problem:

$$\text{maximize} \sum_{t=1}^{N} F(\underline{x}_t, \underline{u}_t) \tag{4.41}$$

subject to

$$\underline{x}_{t+1} = f(\underline{x}_t, \underline{u}_t), \ \forall t \tag{4.42}$$

$$\underline{x}_t \in \underline{X}_t, \ \forall t \tag{4.43}$$

$$\underline{u}_t \in \underline{U}_t, \ \forall t \tag{4.44}$$

$$\underline{x}_1, \underline{x}_{N+1} \ \text{ fixed} \tag{4.45}$$

in which $\underline{x}_t$ is an n-dimensional state vector, $\underline{u}_t$ is an m-dimensional decision vector, $\underline{X}_t$ and $\underline{U}_t$ are feasible sets, t denotes time (t = 1, 2,..., N + 1), and F and f are functions of arbitrary form (nonconcavity implies that local optimum is guaranteed only). For simplicity, assume that f in eq. (4.42) is invertible (this assumption can be relaxed as shown in Chapters 5 and 7).

According to the PPO, a solution to the problem given by eqs. (4.41)-(4.45) is accomplished by maximizing a sequence of overlapping two-stage problems:

$$\underset{\underline{u}_{t-1}, \underline{u}_t}{\text{maximize}} \ [F(\underline{x}_{t-1}, \underline{u}_{t-1}) + F(\underline{x}_t, \underline{u}_t)] \tag{4.46}$$

subject to

$$\underline{x}_{t+1} = f(\underline{x}_t, \underline{u}_t), \ \underline{x}_t = f(\underline{x}_{t-1}, \underline{u}_{t-1}) \tag{4.47}$$

$$\underline{u}_{t-1} \in \underline{U}_{t-1} \tag{4.48}$$

$$\underline{u}_t \in \underline{U}_t \tag{4.49}$$

$$\underline{x}_t \in \underline{X}_t \tag{4.50}$$

$$\underline{x}_{t-1} = \underline{x}^{(k)}_{t-1}, \ \underline{x}_{t+1} = \underline{x}^{(k)}_{t+1} \tag{4.51}$$

Notice that the maximization in eq. (4.46) is with respect to $\underline{u}_{t-1}$ and $\underline{u}_t$. As indicated in eqs. (4.48)-(4.50), $\underline{u}_{t-1}$, $\underline{u}_t$, and $\underline{x}_t$ must belong to feasible sets. Further, as specified in eq. (4.51), the beginning and ending states are fixed.

The problem [eqs. (4.46)-(4.51)] can be converted to a maximization problem written in terms of the state variables only by using the expressions

$$\underline{u}_{t-1} = f^{-1}(\underline{x}_{t-1}, \underline{x}_t) = g_{t-1}(\underline{x}_t) \tag{4.52}$$

$$\underline{u}_t = f^{-1}(\underline{x}_{t+1}, \underline{x}_t) = g_t(\underline{x}_t) \tag{4.53}$$

in which f^{-1} is the inverse continuity equation (eq. 4.47). By substituting eqs. (4.52) and (4.53) into eqs. (4.46), (4.48), and (4.49) one obtains

$$\underset{\underline{x}_t}{\text{maximize}} \; [F(\underline{x}_{t-1}, g_{t-1}(\underline{x}_t)) + F(\underline{x}_t, g_t(\underline{x}_t))] \tag{4.54}$$

subject to

$$g_{t-1}(\underline{x}_t) \in \underline{U}_{t-1} \tag{4.55}$$

$$g_t(\underline{x}_t) \in \underline{U}_t \tag{4.56}$$

$$\underline{x}_t \in \underline{X}_t \tag{4.57}$$

$$\underline{x}_{t-1} = \underline{x}_{t-1}^{(k)}, \; \underline{x}_{t+1} = \underline{x}_{t+1}^{(k)} \tag{4.58}$$

in which k denotes the iteration number.

The solution steps of the POA algorithm are:

1) The initial and final states $\underline{x}_1$ and $\underline{x}_{N+1}$ are fixed. Set the iteration counter k equal to one (k = 1). Find an initial feasible policy $\{\underline{u}_t^{(k)}\}$ and its corresponding state trajectory $\{\underline{x}_t^{(k)}\}$.

2) Solve the problem given by eqs. (4.54)-(4.58) by using any convenient method.

3) Denote the solution obtained in step 2 by $\underline{x}_t^*$. Set $\underline{x}_t^{(k+1)} = \underline{x}_t^*$. Increase the time index by one (i.e., set t equal to t + 1) and go to step 2. Repeat steps 2 and 3 until a complete iteration is performed (t = 1 through t = N). This is the end of the kth iteration.

4) Perform a convergence test, e.g., $\| \underline{u}_t^{(k)} - \underline{u}_t^{(k-1)} \|_2^2 \leq \varepsilon$, $\varepsilon \to 0$, for all t. If the test is satisfied, stop; otherwise, increase the iteration index, k = k +1, set t = 1, and go to step 2, or stop if k

has exceeded a specified limit. Notice that at the end of each iteration the new values $\{\underline{x}_t^{(k+1)}\}$ are used to derive the corresponding control sequence $\{\underline{u}_t^{(k+1)}\}$ by means of the continuity equation. Figure 4.2 shows a general block diagram for the POA.

Some remarks about the POA are necessary. First, it would appear that the iterations progress from $t = 1$ to $t = N$, but this is not necessarily so. The algorithm can be modified in such a way that successive local optimizations are made at some periods in which relatively higher improvements in the objective function are observed. Once the rate of improvement falls below some preset value (e.g., 10 percent improvement in the objective function with respect to the previous iteration), the algorithm would advance towards ending the iteration at $t = N$. Figures 4.3 and 4.4 show two different advancing schemes. This flexibility of the POA, to be able to take full advantage of localized conditions, accelerates the overall convergence rate at little expense in programming complexity. Second, time lags in the continuity equation are incorporated in a straightforward manner. Consider reservoir i which is directly connected to (downstream) reservoir j. Let d_{ij} be the travel time between reservoirs i and j; x_t^j and u_t^j the respective state and decision variables for reservoir j at time t; and $u_{t-d_{ij}}^i$ the decision for reservoir i at time $t - d_{ij}$. The continuity equation for reservoir j can be then written as

$$x_{t+1}^j = x_t^j - u_t^j + u_{t-d_{ij}}^i + y_t^j \tag{4.59}$$

in which y_t^j is the sum of diversions/accretions, net losses, and runoff into reservoir j. Equations similar to (4.59) must be written for every

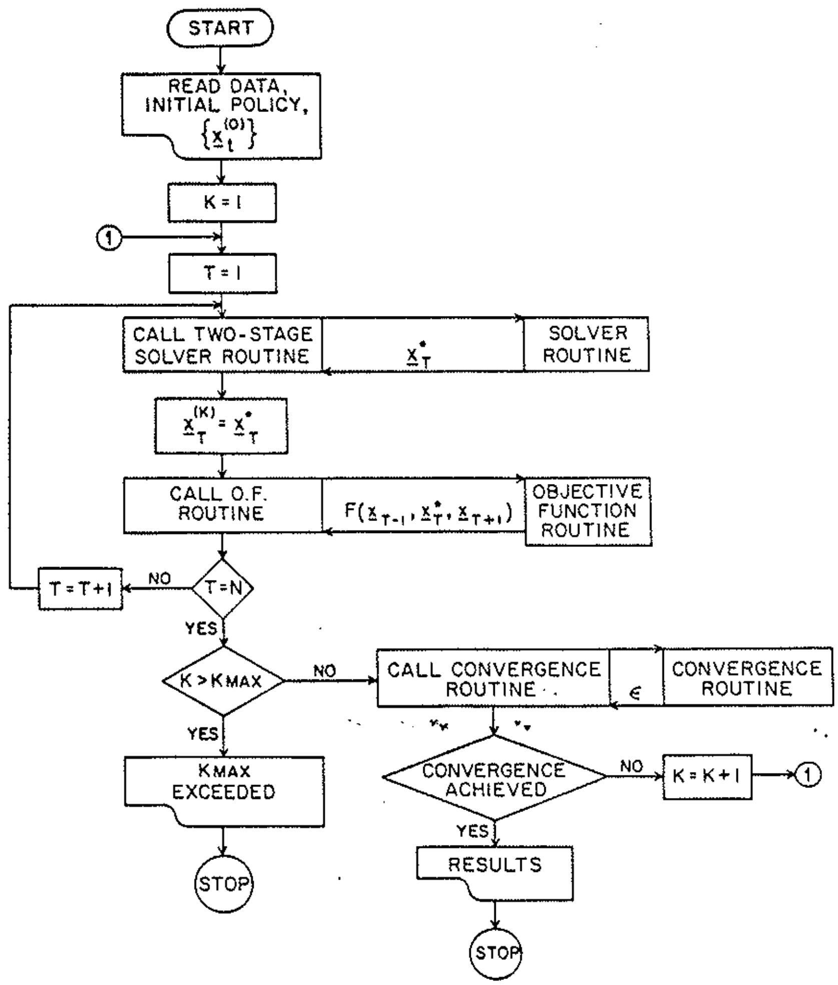


Fig. 4.2 Standard progressive optimality algorithm (POA) flow diagram.

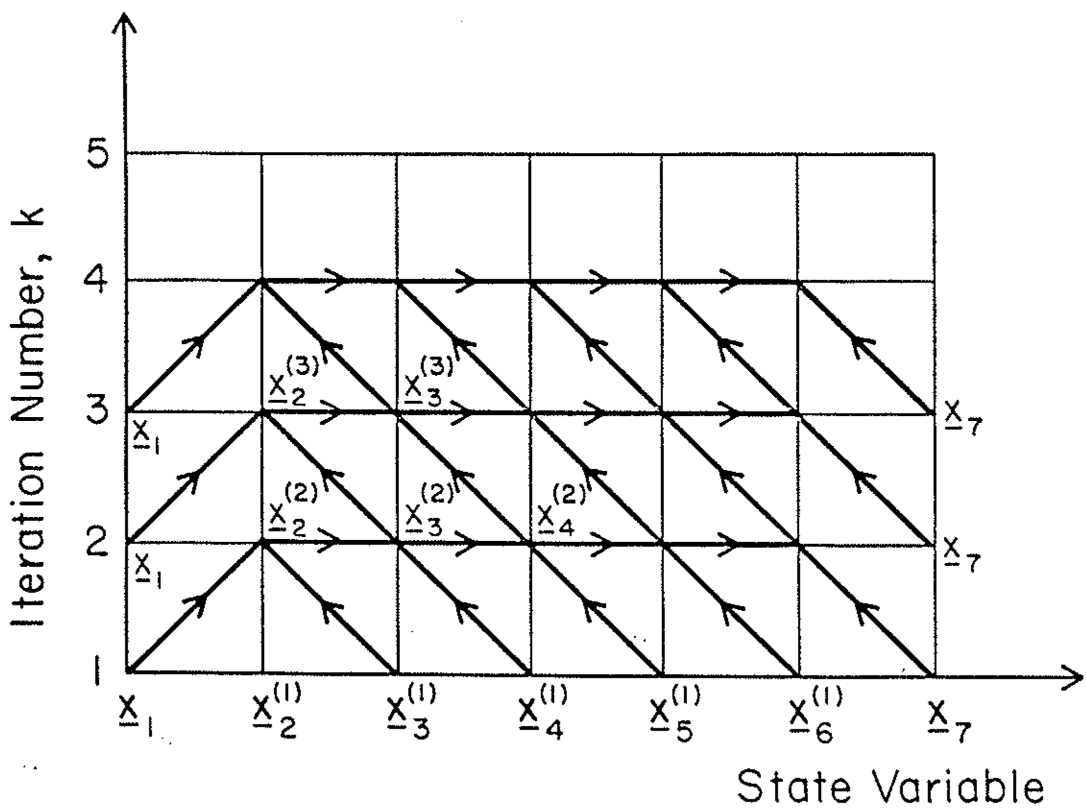


Fig. 4.3 Standard POA (To achieve state $\underline{x}_3^{(3)}$: (i) $\underline{x}_1$ and $\underline{x}_3^{(1)}$ yield $\underline{x}_2^{(2)}$; (ii) $\underline{x}_2^{(2)}$ and $\underline{x}_4^{(1)}$ yield $\underline{x}_3^{(2)}$; (iii) $\underline{x}_3^{(2)}$ and $\underline{x}_5^{(1)}$ yield $\underline{x}_4^{(2)}$; (iv) $\underline{x}_1$ and $\underline{x}_3^{(2)}$ yield $\underline{x}_2^{(3)}$; and (v) $\underline{x}_2^{(3)}$ and $\underline{x}_4^{(2)}$ yield $\underline{x}_3^{(3)}$).

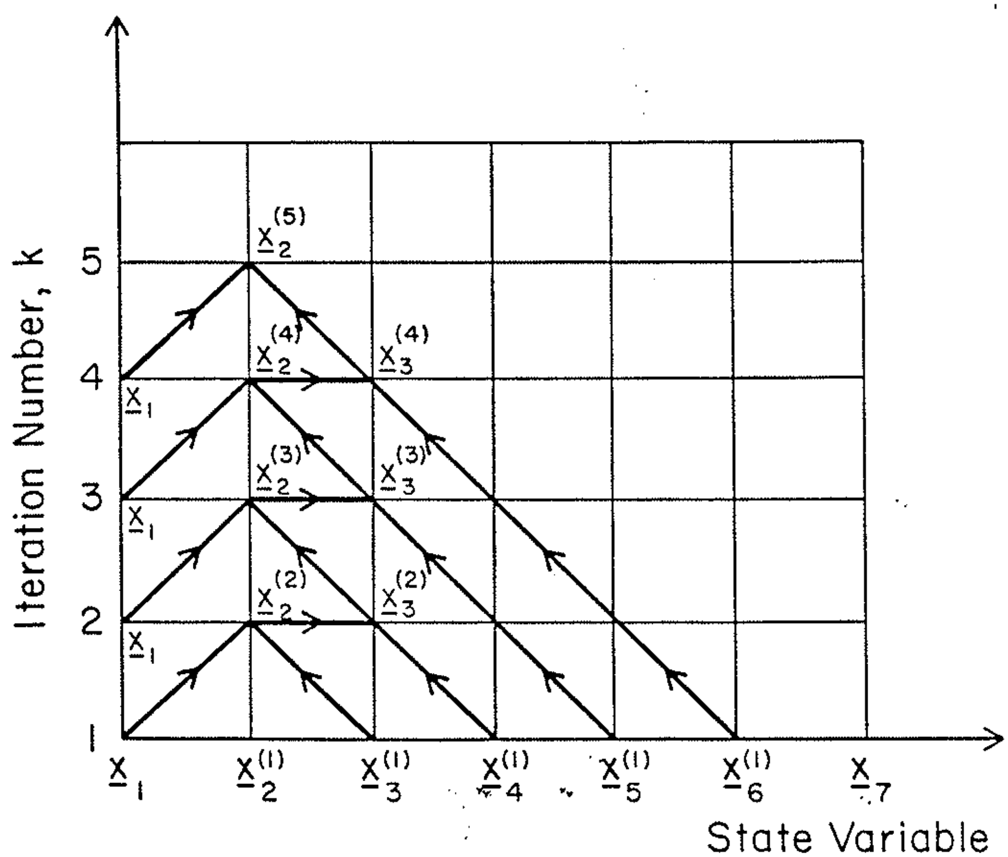


Fig. 4.4 Modified POA (To achieve state $\underline{x}_2^{(5)}$: (i) $\underline{x}_1$ and $\underline{x}_3^{(1)}$ yield $\underline{x}_2^{(2)}$; (ii) $\underline{x}_2^{(2)}$ and $\underline{x}_4^{(1)}$ yield $\underline{x}_3^{(2)}$; (iii) $\underline{x}_1$ and $\underline{x}_3^{(2)}$ yield $\underline{x}_2^{(3)}$; (iv) $\underline{x}_2^{(3)}$ and $\underline{x}_5^{(1)}$ yield $\underline{x}_3^{(3)}$; (v) $\underline{x}_1$ and $\underline{x}_3^{(3)}$ yield $\underline{x}_2^{(4)}$; (vi) $\underline{x}_2^{(4)}$ and $\underline{x}_6^{(1)}$ yield $\underline{x}_3^{(4)}$; and (vii) $\underline{x}_1$ and $\underline{x}_3^{(4)}$ yield $\underline{x}_2^{(5)}$). This scheme should be used when significant improvements (e.g., 10% improvement in the objective function with respect to the previous iteration) arise from the two-stage problem involving periods 1 and 2).

reservoir in the network and the POA must be programmed so that previous decisions ($\underline{u}_{t-d}$) are retrieved from storage to be used in the continuity equation, eq. (4.59).

4.9 Convergence Proofs of the POA for Bounded States and Decisions

It is assumed that the law of motion is invertible and that the objective function and constraints define a concave problem so that each two-stage maximization has a unique, global optimum.

Theorem 1. The POA produces monotonically increasing approximations to the optimal value which is the upper bound of the approximations. Proof: Let $\{\underline{c}_t\}$, t = 2, 3,..., N ($\underline{c}_1$, = $\underline{x}_1$ and $\underline{c}_{N+1} = \underline{x}_{N+1}$ are fixed) be an initial approximation to the optimal sequence $\{\bar{\underline{x}}_t\}$, t = 1, 2,..., N, N+1 with $\bar{\underline{x}}_1 = \underline{x}_1$ and $\bar{\underline{x}}_{N+1} = \underline{x}_{N+1}$ fixed. Define

$$I_N(\underline{x}_1) = \underset{\underline{x}_2,\ldots,\,\underline{x}_N}{\text{maximum}} \sum_{t=2}^{N+1} F(\underline{x}_{t-1}, \underline{x}_t) \tag{4.60}$$

Then,

$$I_N(\underline{x}_1) = \underset{\underline{x}_2,\ldots,\,\underline{x}_N}{\text{maximum}} \sum_{t=2}^{N+1} F(\underline{x}_{t-1}, \underline{x}_t)$$

$$= \underset{\underline{x}_2}{\text{maximum}} \left[F(\underline{x}_1, \underline{x}_2) + \underset{\underline{x}_3,\ldots,\,\underline{x}_N}{\text{maximum}} \sum_{t=3}^{N+1} F(\underline{x}_{t-1}, \underline{x}_t)\right]$$

$$= \underset{\underline{x}_2}{\text{maximum}} \left[F(\underline{x}_1, \underline{x}_2) + I_{N-1}(\underline{x}_2)\right], \; N \geqq 3$$

$$= F(\underline{x}_1, \bar{\underline{x}}_2) + F(\bar{\underline{x}}_2, \bar{\underline{x}}_3) + \ldots + F(\bar{\underline{x}}_N, \underline{x}_{N+1}) \tag{4.61}$$

from Bellman's principle of optimality. From the POA,

$$F(\underline{x}_1, \underline{c}_2) + F(\underline{c}_2, \underline{c}_3) \leq \max_{\underline{x}_2} [F(\underline{x}_1, \underline{x}_2) + F(\underline{x}_2, \underline{c}_3]$$

$$= F(\underline{x}_1, \bar{\underline{x}}_2^1) + F(\bar{\underline{x}}_2^1, \underline{c}_3) \quad (4.62)$$

in which $\bar{\underline{x}}_2^1$ is the optimal value obtained for $\underline{x}_2$, given $\underline{x}_1$ and $\underline{c}_3$, during the first iteration sweep. In a similar manner,

$$F(\bar{\underline{x}}_2^1, \underline{c}_3) + F(\underline{c}_3, \underline{c}_4) \leq \max_{\underline{x}_3} [F(\bar{\underline{x}}_2^1, \underline{x}_3) + F(\underline{x}_3, \underline{c}_4)]$$

$$= F(\bar{\underline{x}}_2^1, \bar{\underline{x}}_3^1) + F(\bar{\underline{x}}_3^1, \underline{c}_4) \quad (4.63)$$

in which $\bar{\underline{x}}_3^1$ is the optimal value of $\underline{x}_3$, given $\bar{\underline{x}}_2^1$ and $\underline{c}_4$, during the first iteration sweep. Similarly,

$$F(\bar{\underline{x}}_{j-1}^n, \bar{\underline{x}}_j^{n-1}) + F(\bar{\underline{x}}_j^{n-1}, \bar{\underline{x}}_{j+1}^{n-1}) \leq \max_{\underline{x}_j} [F(\bar{\underline{x}}_{j-1}^n, \underline{x}_j) + F(\underline{x}_j, \bar{\underline{x}}_{j+1}^{n-1})]$$

$$= F(\bar{\underline{x}}_{j-1}^n, \bar{\underline{x}}_j^n) + F(\bar{\underline{x}}_j^n, \bar{\underline{x}}_{j+1}^{n-1}) \quad (4.64)$$

in which $\bar{\underline{x}}_j^n$ is the optimal value of $\underline{x}_j$, given $\bar{\underline{x}}_{j-1}^n$ and $\bar{\underline{x}}_{j+1}^{n-1}$ (the nth and (n - 1)th iteration values for $\underline{x}_{j-1}$ and $\underline{x}_{j+1}$, respectively), during the nth iteration. Therefore, for the nth iteration,

$$F(\underline{x}_1, \underline{\bar{x}}_2^{n-1}) + F(\underline{\bar{x}}_2^{n-1}, \underline{\bar{x}}_3^{n-1}) + \ldots + F(\underline{\bar{x}}_N^{n-1}, \underline{x}_{N+1})$$

$$\leqq F(\underline{x}_1, \underline{\bar{x}}_2^{n}) + F(\underline{\bar{x}}_2^{n}, \underline{\bar{x}}_3^{n}) + \ldots + F(\underline{\bar{x}}_N^{n}, \underline{x}_{N+1}) \quad (4.65)$$

$$\leqq F(\underline{x}_1, \underline{\bar{x}}_2) + F(\underline{\bar{x}}_2, \underline{\bar{x}}_3) + \ldots + F(\underline{\bar{x}}_N, \underline{x}_{N+1}) \quad (4.66)$$

The last relation shows that indeed the iterations are monotonically increasing and bounded above by the optimal value.

Theorem 2. If the iterations converge in a finite number of steps, then the resulting $\{\underline{\bar{x}}_t^n\}$ is the optimal trajectory $\{\underline{\bar{x}}_t\}$. Proof: If convergence occurs in a finite number of iterations, then relation (4.65) becomes an equality that would hold for n + 1, n + 2, etc., because a steady state is reached. Denote the steady-state trajectory by $\{\underline{\hat{x}}_t\}$. Relation (4.65) can be then written as

$$F(\underline{x}_1, \underline{\bar{x}}_2^{n}) + F(\underline{\bar{x}}_2^{n}, \underline{\bar{x}}_3^{n}) + \ldots + F(\underline{\bar{x}}_N, \underline{x}_{N+1}) \equiv \hat{F}_N(\underline{x}_0)$$

$$\equiv F(\underline{x}_1, \underline{\hat{x}}_2) + F(\underline{\hat{x}}_2, \underline{\hat{x}}_3) + \ldots + F(\underline{\hat{x}}_N, \underline{x}_{N+1})$$

$$= \max_{\underline{x}_2} [F(\underline{x}_1, \underline{x}_2) + F(\underline{x}_2, \underline{\hat{x}}_3) + \ldots + F(\underline{\hat{x}}_{N-1}, \underline{\hat{x}}_N) + F(\underline{\hat{x}}_N, \underline{x}_{N+1})]$$

$$= \max_{\underline{x}_2} \{F(\underline{x}_1, \underline{x}_2) + \max_{\underline{x}_3} [F(\underline{x}_2, \underline{x}_3) + F(\underline{x}_3, \underline{\hat{x}}_4)]\}$$

$$+ F(\underline{\hat{x}}_4, \underline{\hat{x}}_5) + \ldots + F(\underline{\hat{x}}_N, x_{N+1})$$

$$= \max_{\underline{x}_2} \{F(\underline{x}_1, \underline{x}_2) + \max_{\underline{x}_3} [F(\underline{x}_2, \underline{x}_3) + \max_{\underline{x}_4} (F(\underline{x}_3, \underline{x}_4)$$

$$+ \max_{\underline{x}_5} (F(\underline{x}_4, \underline{x}_5) + \ldots))]\}$$

$$= \max_{\underline{x}_2} \{F(\underline{x}_1, \underline{x}_2) + \underset{\underline{x}_3, \underline{x}_4, \ldots, \underline{x}_N}{\text{maximum}} [F(\underline{x}_1, \underline{x}_2) + \ldots + F(\underline{x}_N, \underline{x}_{N+1})]\}$$

$$= \underset{\underline{x}_2, \underline{x}_3, \ldots, \underline{x}_N}{\text{maximum}} \sum_{t=2}^{N+1} F(\underline{x}_{t-1}, \underline{x}_t)$$

$$= I_N(\underline{x}_1) \text{ from eq. (4.60).}$$

Thus, $\{\hat{\underline{x}}_t\} = \{\bar{\underline{x}}_t\}$.

CHAPTER 5

OPTIMAL OPERATION POLICIES FOR THE NCVP

This chapter presents a development of monthly optimal release policies for the Northern Central Valley Project (NCVP). Special attention is given to the incorporation of actual characteristics and operational constraints of the NCVP system into the operation model. The model was developed by using feedback from the Central Valley Operations Office of the U.S. Bureau of Reclamation, Sacramento, California. The model uses a combination of deterministic dynamic optimization (progressive optimality algorithm, POA) and a continuously updating streamflow forecasting technique as the basis for determining monthly operation policies for any water year. The updating technique allows the revision of previous policies as the stochastic inputs deviate the system from expected values. Thus, available at all times are optimal policies that correspond to the actual realized states of the system and up to date with the most recent streamflow information.

This chapter is organized as follows. Section 5.1 gives a description of the NCVP system. Section 5.2 presents the streamflow forecasting technique used in this study. Section 5.3 develops the optimization model structure for the NCVP. Finally, Section 5.4 discusses the selection of an initial operating policy. Discussion of results is presented in Chapter 6.

5.1 Description of the NCVP System

The system under analysis is composed of the following reservoirs: Clair Engle, Lewiston, Whiskeytown, Shasta, Keswick, Natoma, Folsom, New Melones, and Tullock. Figure 5.1 shows a schematic representation

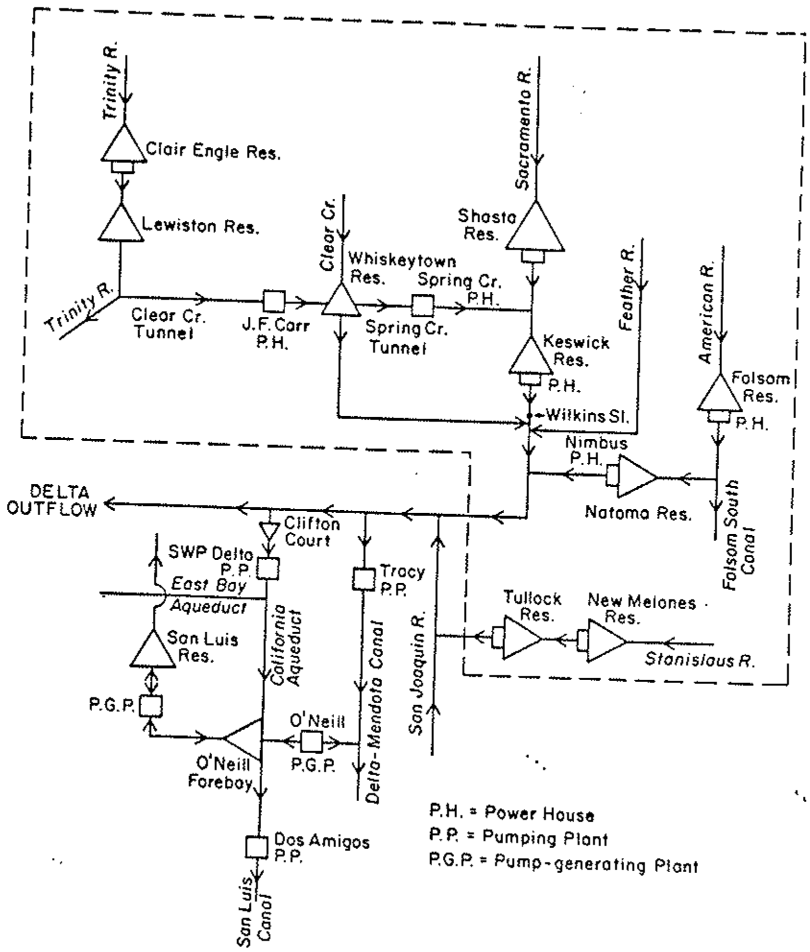


Fig. 5.1 Schematic representation of Central Valley Project.

of the CVP system. The portion of the system analyzed in this study is shown within the dashed lines.

The NCVP is managed jointly by the U.S. Bureau of Reclamation and the California Department of Water Resources. It stores flood and snowmelt waters and releases them at appropriate times to serve different functions. The main purposes of the NCVP are: provision of water for irrigation (I), municipal and industrial uses (MI), environmental control and enhancement (E), fish and wildlife requirements (F), river navigation (N), water quality control (WQ), flood regulation (FC), hydropower (HP), recreation (R), and control of ocean intrusion and erosion. Fults and Hancock (1972, 1974) and Madsen and Coleman (1974) presented a thorough geographical, institutional, and historical description of the CVP. The discussion herein will be centered on the nine reservoirs mentioned earlier and, specifically, on the joint operation of those reservoirs. From now on, the term system will refer to those nine reservoirs.

Table 5.1 shows basic data of the NCVP. Table 5.2 contains capacity data of the tunnels, canals, and penstocks that form part of the system. The reservoirs of the NCVP must operate jointly to perform the multiple functions enumerated in Table 5.1. The system release policy is subject to physical and technical constraints that arise from the capacity and technology of the facilities, as well as institutional and environmental regulations. It is evident from Table 5.1 that Shasta, Clair Engle, Folsom, and New Melones are the larger reservoirs within the system. Lewiston, Whiskeytown, Keswick, Natoma, and Tullock play an important role as regulating reservoirs. A regulating reservoir maintains an adequate flow magnitude downstream of a larger reservoir and a stable hydraulic head for a downstream power plant.

Table 5.1. Basic NCVP Data.

Reservoir	Managing Institution	First Year of Operation	Capacity (Kaf)	Installed Capacity* (Mw)	Functions Served
Shasta	USBR	1944	4552	559	I, FC, HP, MI, WQ, N, R, F
Clair Engle	USBR	1960	2448	128	I, FC, HP, MI, WQ, N, R, F
Lewiston	USBR	1962	14.7	–	Regulation, HP, F, R
Whiskeytown	USBR	1963	241	154 (J.F. Carr) 190 (Spring Cr.)	Regulation, I, HP, MI, R, F
Keswick	USBR	1948	23.8	90	Regulation, HP, R, F
Folsom	USBR	1955	1010	198	I, FC, HP, MI, WQ, R, F
Natoma	USBR	1955	8.8	15	Regulation, HP, R, F
New Melones	USBR	1978	2600	383	I, WQ, FC, F, HP, R
Tullock	Oakdale Ir. Dist.	1958	67	17	I, HP, Regulation

*As of July 1982.

Table 5.2. Capacities of Tunnels and Penstocks.

Facility	Capacity (cfs)
Folsom South Canal	3500
Clear Creek Tunnel	3600
Spring Creek Tunnel	4500
Shasta Penstocks	16500
Trinity Penstocks	3500
J. F. Carr Penstocks	3600
Spring Creek Penstocks	4500
Keswick Penstocks	14500
Nimbus Penstocks	5000
Folsom Penstocks	8000
New Melones Penstocks	8300
Tullock Penstocks	1800

1 cfs = 0.02832 m^3/s.

Winter flows in the Trinity River are stored for later release from Clair Engle Lake. Normally, water behind Trinity Dam (at Clair Engle Lake) is released through the Trinity Power Plant and regulated downstream in Lewiston Reservoir. The major portion of the water reaching Lewiston Dam is diverted to the Sacramento River watershed via the 11-mile long Clear Creek Tunnel. The remaining water is released to the Trinity River to support fishery. Water diverted through Clear Creek Tunnel exits through the Judge Francis Carr Power Plant in Whiskeytown Reservoir. It is possible to release water from Whiskeytown Dam to Clear Creek or make diversions through the Spring Creek Tunnel.

Sacramento River water is stored for later release from Shasta Dam. Ordinarily, water is released from Shasta Reservoir through the Shasta Power Plant and flows downstream to Keswick Reservoir. The inflow to Keswick includes releases from Shasta and Spring Creek Tunnel. Releases from Keswick may be made through the Keswick Power Plant and flow in excess of the power plant penstock capacity are spilled to the Sacramento River.

Folsom Dam stores American River water and releases it under normal operation through its power house. Excess flows are spilled to the American River. Part of those releases is diverted by the Folsom South Canal and the remainder goes into Lake Natoma, which acts as a regulating reservoir. As much water as possible is released through the Nimbus Power Plant at Lake Natoma, with any excess water spilled to the American River. The American and Sacramento Rivers converge near the city of Sacramento and flow to the Sacramento-San Joaquin Delta to serve several purposes.

New Melones Dam stores Stanislaus River flows to release them during the summer for agricultural use. It also releases water to maintain water quality standards in the San Joaquin River. Tullock Reservoir, upstream from New Melones, is primarily a regulating reservoir.

One of the main functions of the NCVP system is flood control protection. The flood control pool of the reservoirs is governed by regulations established by the U.S. Corps of Engineers. Those regulations stipulate that a certain amount of space be held empty in a reservoir during the period October 1-April 30. Table 5.3 summarizes the flood control regulations for Trinity Dam at Clair Engle Reservoir. Figures 5.2 through 5.5 show the flood control provisions for Shasta, Folsom, New Melones, and Tullock, respectively. There are no flood control regulations for the other reservoirs in the system (Lewiston, Whiskeytown, Keswick, Natoma, and Tullock) whose main function is flow regulation.

Nimbus and Keswick Power Plants are low-head installations whereas the other power plants are high-head facilities. Yeh et al. (1978) gave a set of performance curves for several of the generating units of the NCVP. A release of one acre-foot of water from Trinity Dam generates power at Trinity, J. F. Carr, Spring Creek, and Keswick Power Plants and, because of the system configuration, represents about five times as much power as one acre-foot of water released from Folsom Dam. A one-acre-foot release from Shasta is considered by the U.S. Bureau of Reclamation (USBR) to produce about 1.5 the power that can be obtained from one acre-foot of water released from Folsom Dam (Yeh et al., 1976).

Table 5.3. Flood Control Provisions for Trinity Dam at Clair Engle Reservoir.

Period (1)	Reservoir Storage and Water Surface Elevation (2)	Maximum Flood Control Releases (3)
Nov. 1-Feb. 28	Less than 1850 Kaf (elev. 2330.3 ft)	None required
Nov. 1-March 31	1850-2100 Kaf (elev. 2330.3-2347.6 ft)	3600 cfs
Nov. 1-March 31	2100-2450 Kaf (elev. 2347.6-2370.1 ft)	6000 cfs
Nov. 1-March 31	Greater than 2450 Kaf (elev. 2370.1 ft)	30000 cfs

According to existing U.S. Corps of Engineers regulations, the reservoir elevation should be maintained below 2347.6 ft during Nov. 1-March 31 each year.

If during the period indicated in column (1) the reservoir is within the storage levels of column (2), then release the flow in column (3).

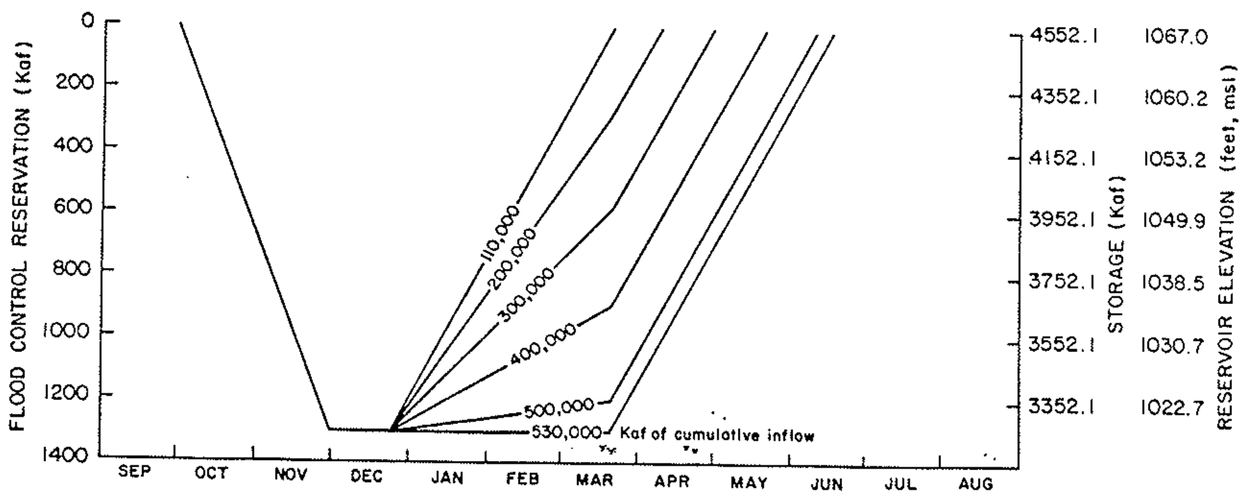


Fig. 5.2 Shasta flood control diagram. See notes on next page (Source: Central Valley Operations Office, U.S. Bureau of Reclamation, Sacramento, Ca.).

Notes

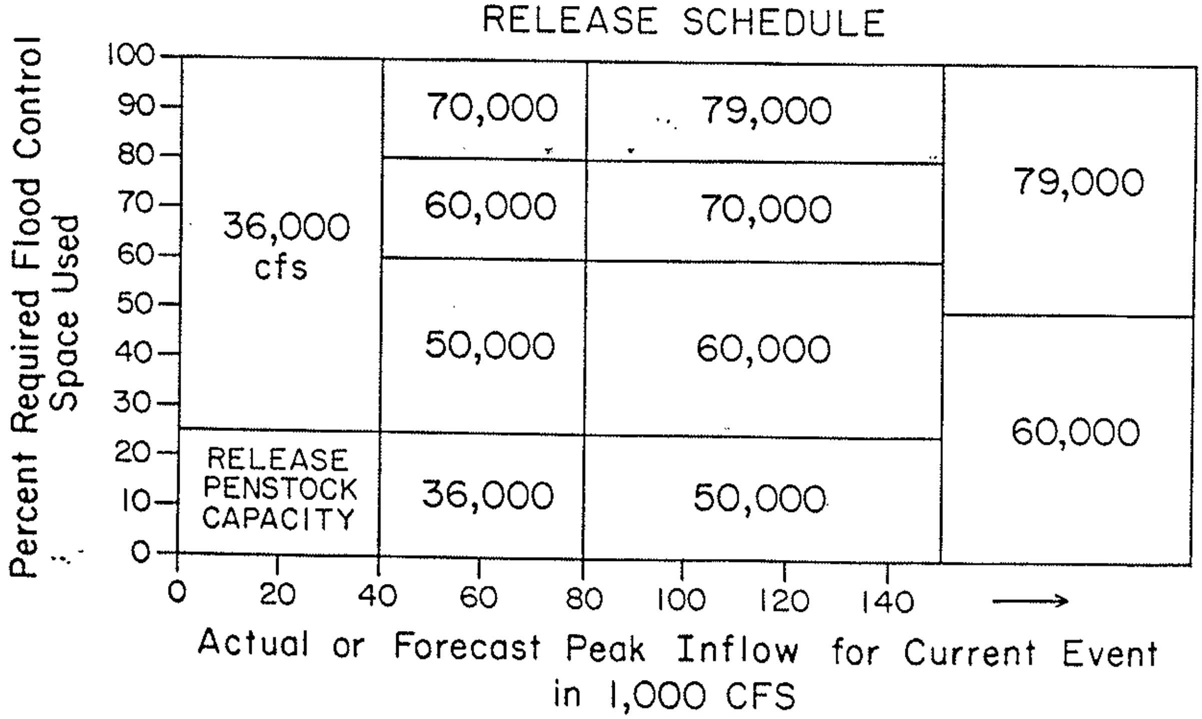
1. Rainflood parameters relate the accumulation of seasonal inflow to the required flood control space reservation on any given day. Parameter values are computed daily, from the accumulation of seasonal inflow by adding the current day's inflow in cubic feet per second (cfs) to 95% of the parameter value computed through the preceding day. The flood control diagram is initialized each flood season by assuming a parameter value of 100,000 cfs day on 1 October.

2. Except when releases are governed by the emergency spillway release diagram currently in force (File No. SA-26-92), water stored in the flood control reservation, defined hereon, shall be released as rapidly as possible, subject to the following conditions:

   a. That releases are made according to the Release Schedule hereon.
   b. That flows in Sacramento River below Keswick Dam do not exceed 79,000 cfs.
   c. That flows in Sacramento River at Bend Bridge gage do not exceed 100,000 cfs.
   d. That releases are not increased more than 15,000 cfs or decreased more than 4,000 cfs in any 2-hour period.

3. For example, if the percentage of the flood control space used is 90% and forecast inflow is 60,000 cfs, then release 70,000 cfs.

RELEASE SCHEDULE



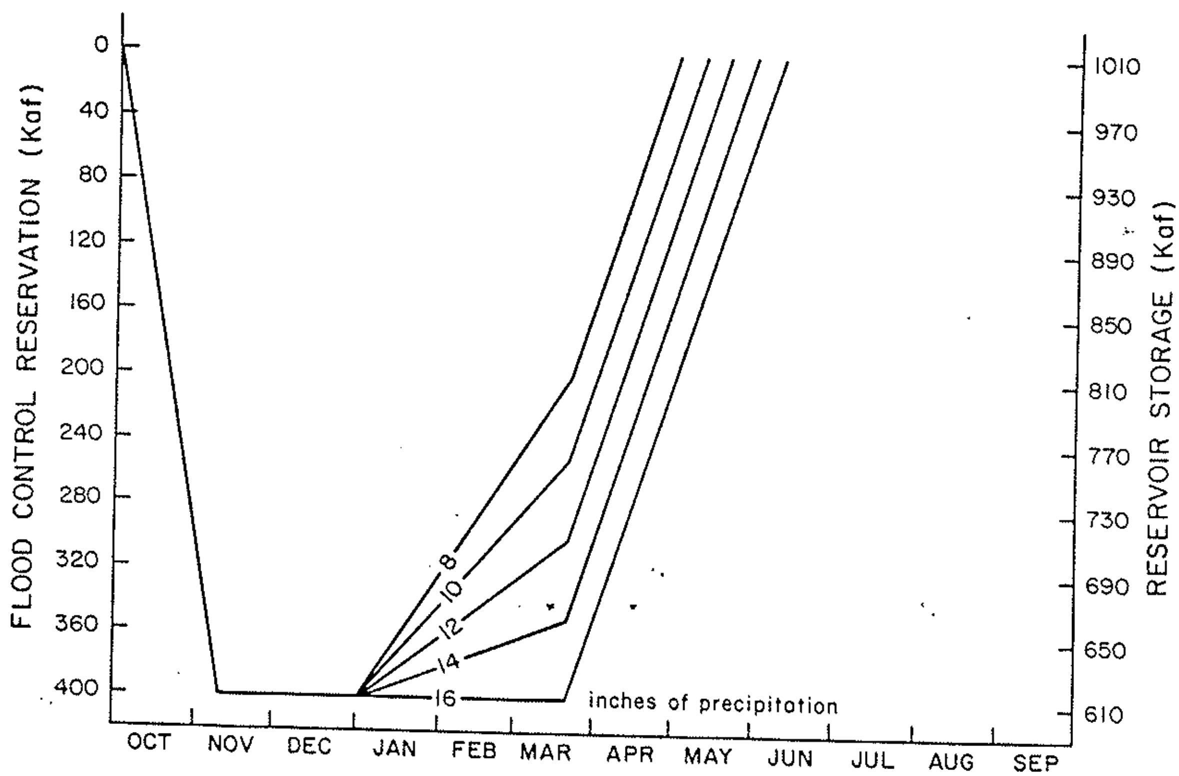


Fig. 5.3 Folsom flood control diagram. See notes on next page (Source: Central Valley Operations Office, U.S. Bureau of Reclamation, Sacramento, Ca.).

Notes

1. Rainfall parameters define the flood control space reservation on any given day and are computed daily from the weighted accumulation of seasonal basin mean precipitation by adding the current day's precipitation in inches to 97% of the parameter computed the preceding day. Sample computation of required space is shown below.

Sample Computation of Required Space

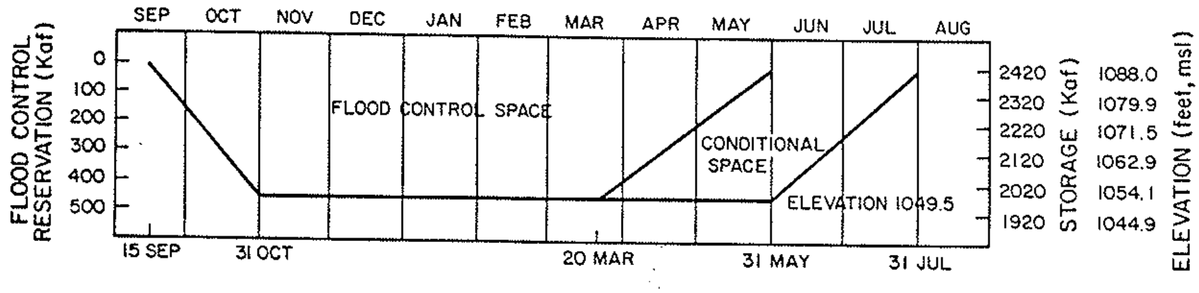
Date	Precip. (in.)	Previous day's weighted precip. x 0.97	Weighted precip. accumulation	Required flood control space (Kaf)
7 Nov	0.0	= 0.0	0.00	370,000
8 Nov	1.0	0.00 x 0.97 = 0.0	1.00	380,000
9 Nov	1.5	1.00 x 0.97 = 0.97	2.47	390,000
10 Nov	3.0	2.47 x 0.97 = 2.396	5.40	400,000
11 Nov	0.0	5.40 x 0.97 = 5.238	5.24	400,000
12 Nov	0.0	5.24 x 0.97 = 5.081	5.08	400,000
			7.00	400,000
30 Dec	2.0	7.00 x 0.97 = 6.790	8.79	400,000
31 Dec	1.0	8.79 x 0.97 = 8.526	9.53	400,000
1 Jan	1.0	9.53 x 0.97 = 9.244	10.24	398,200
2 Jan	0.0	10.24 x 0.97 = 9.932	9.93	396,200
3 Jan	0.0	9.93 x 0.97 = 9.634	9.63	394,000
4 Jan	0.0	9.63 x 0.97 = 9.341	9.34	391,700

2. Except when larger releases are required by the emergency spillway release diagram currently in force (File No. AM-1-26-585), water stored within the flood control reservation, defined hereon, shall be released as rapidly as possible subject to the following conditions:

   a. Outflows at the tailwater of Nimbus Dam in excess of power plant capacity may not exceed the lesser of 115,000 cfs or the maximum rate of inflow to Folsom Lake experienced during the current flood event.

   b. Between 6 November and 1 December, the maximum release may be limited to the Folsom power plant capacity if less than 40,000 acre feet of water is stored in the flood control space.

   c. Releases will not be increased more than 15,000 cfs or decreased more than 10,000 cfs during any 2-hour period.



Notes

1. Whenever water is stored in the Flood Control Space, it shall be released as rapidly as possible without causing flows in the Stanislaus River at Orange Blossom Bridge to exceed 8,000 cfs insofar as possible.

2. Whenever water is stored in the Conditional Flood Control Space, releases shall be made at a sufficient rate, based on anticipated snowmelt runoff, so that the pool elevation will not exceed 1088 ft subject to the limitations in paragraph 1 above.

3. Control of Stanislaus River flood flows requires coordinated operation with Tullock Reservoir.

4. Reservoir zoning is:

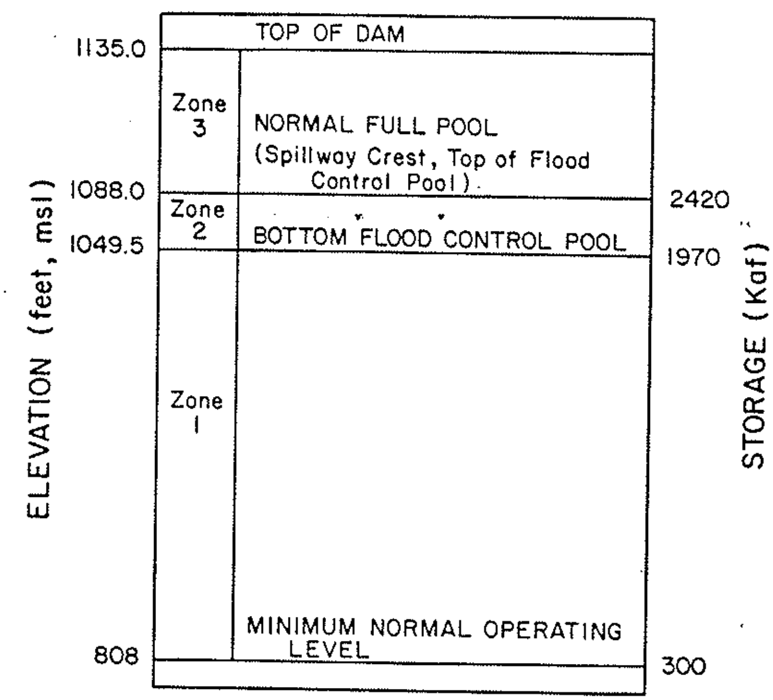


Fig. 5.4 New Melones flood control diagram. See notes on next page (Source: Central Valley Operations Office, U.S. Bureau of Reclamation, Sacramento, Ca.).

in which

Zone 1 Normal power and conservation operation: Flows in the Stanislaus River are not to exceed 3,500 cfs at Orange Blossom Bridge to minimize damages to adjacent low-lying agricultural lands.

Zone 2 Follow operation procedure in Flood Control Diagram on previous page. Notify Sacramento District personnel.

Zone 3 Initiate emergency operation and notify local authorities for possible evacuation of flood plain. The Flood Control and Irrigation Outlet will be operated to maintain the objective flow (8,000 cfs at Orange Blossom Bridge) as long as possible by gradually closing the outlets as pool rises above normal full pool elevation (1088 ft). For a receding pool, outlets will remain close until objective flow has been obtained. The outlets will then be opened to maintain a total flow of 8,000 cfs at Orange Blossom Bridge. When the water surface has receded to normal full pool elevation, resume flood control operation as in Zone 2. Notify Sacramento District personnel and request assistance if desired.

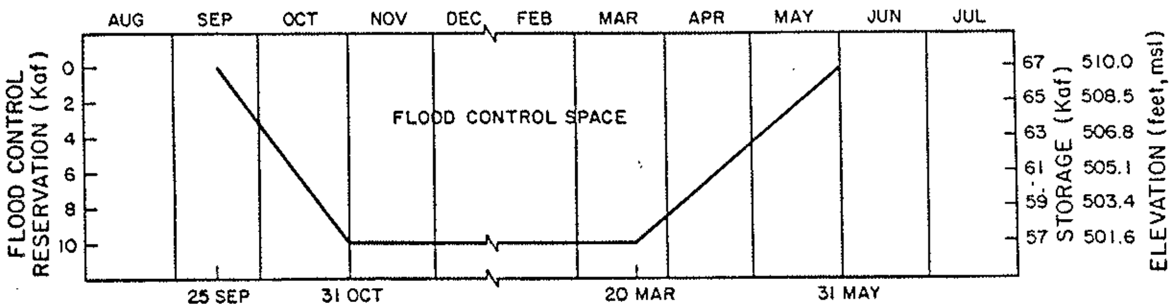


Fig. 5.5 Tullock flood control diagram (Source: Central Valley Operations Office, U.S. Bureau of Reclamation, Sacramento, Ca.).

Notes

1. Water stored in the flood control space will be released as rapidly as possible without causing flows in the Stanislaus River at Orange Blossom Bridge to exceed 8,000 cfs.

2. Flood control releases will not be changed more than 1,000 cfs per hour.

3. Elevations correspond to the Oakdale Irrigation District Stage-Storage Gage with a datum of 1.5 ft above the 1929 mean sea level.

The total power output of the system is delivered mostly to Pacific Gas and Electricity (PG & E), which uses it as peaking capacity to satisfy its power demand. The dependable capacity of the system (the power generation which under the "most adverse" flow conditions of record can be relied upon to its share of the power load) has been established at 860 megawatts (George Link, USBR, personal communication, 1982). The system managers notify PG & E 24 hr in advance of the next-day release volume and the utility returns a request for scheduling that release volume in such a way that it best matches PG & E's load curve. The dependable capacity of 860 megawatts (Mw) is used in this study to establish a lower bound on power generation. As will be shown in Chapter 6, this lower bound results in a redundant constraint due to the extremely low value of the dependable capacity of the system.

The longest-term operation activities of the CVP are planned for each water-year. On October 1, the USBR estimates future streamflows for the next 12 months. Based on that forecast, a tentative release policy is proposed for the 12-month period. Because actual streamflows deviate from their expected values and institutional and/or technical conditions may vary from month to month, streamflow forecasts are updated at the beginning of each month and the release policy is revised for the remaining months of the water year. The revised policy is based on actual (observed) storages, updated streamflow forecasts, and changes in circumstances (e.g., state and federal directives affect the operation of the system frequently). The proposed optimization model of the NCVP (Section 5.3 and Chapter 7) is developed to fit this recurrent revising scheme for release policies.

Data Relevant to the Constraints of the System

The optimization model of Sections 5.3 and 7.7 requires a quantitative statement of the constraints on the operation of the system. Figure 5.6 shows a schematic representation of the NCVP and the points at which accretions/diversions exist. Table 5.4 contains information on flow requirements. There exist regulations on maximum and minimum storages that arise from flood control provisions, recreation, aesthetic concerns, power plant performance, etc. Table 5.5 shows reservoir capacity allocation for the system under study. Tables 5.6 and 5.7 give maximum and minimum storages for the reservoirs, respectively. Those bounds on storage will be used as constraints in the optimization model of Section 5.3. The model will also be constrained by the maximum and minimum reservoir releases shown in Tables 5.8 and 5.9, respectively. Appendix A contains storage-area-elevation data for all reservoirs in the system.

Flood control constraints for Shasta and Folsom depend on cumulative rainfall and inflows to the reservoirs. Depending on those cumulative values, the flood control diagrams for Shasta and Folsom specify the flood control space to be provided (Figs. 5.2 and 5.3, respectively). In this study, the less stringent (upper) curve of the diagrams will be used to determine the necessary flood control storages. It will be shown in Chapter 6 that with an adequate streamflow forecasting technique (e.g., 15% deviation between actual and forecast flows), the current flood control provisions are very conservative.

Power generation will be considered in the objective function and constraints of the optimization model. The amount of power generated by the system depends on the effective hydraulic head at the intake of the

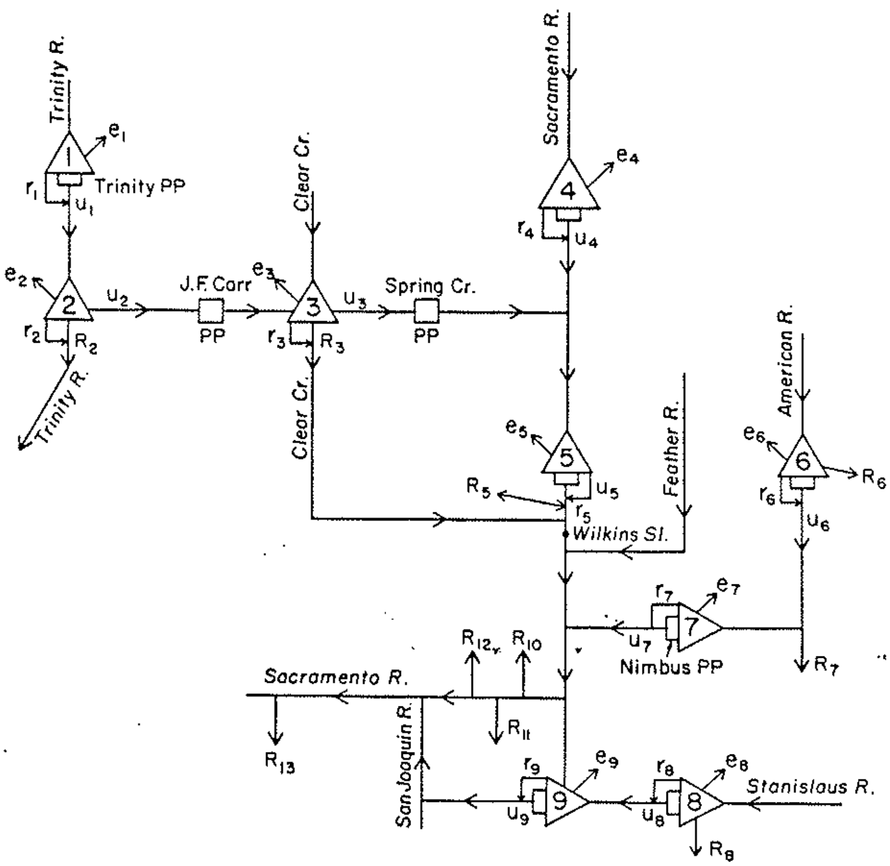


Fig. 5.6 Schematic representation of NCVP diversions, losses, releases, and spills.

Table 5.4. NCVP Flow Requirements (in cfs).

Requirement	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Function
Trinity River min. diversion (R_2)	300	300	300	300	300	300	300	300	300	300	300	300	F
Trinity River max. release ($r_2 + R_2$)	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	FC
Clear Creek min. diversion (R_3)	50	100	100	50	50	50	50	50	50	50	50	50	F
Accretion (+) or diversion (−), (R_5)	1000 (+)	2000 (+)	3000 (+)	4000 (+)	7000 (+)	4000 (+)		1000 (−)	3000 (−)	4000 (−)	3500 (−)	100 (−)	MI, I
Wilkins Slough min. flow	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	N
Folsom South Canal min. diversion (R_7)	50	50	50	50	50	50	50	50	50	50	50	50	MI, I
Folsom min. diversion (R_6)	30	30	30	30	30	30	30	30	30	30	30	30	MI
Shasta max. release ($u_4 + r_4$)	79000	79000	79000	79000	79000	79000	79000	79000	79000	79000	79000	79000	FC
Keswick max. release ($u_5 + r_5$)	49000	49000	49000	49000	49000	49000	49000	49000	49000	49000	49000	49000	FC
Keswick min. release ($u_5 + r_5$)	3250	3250	3250	3250	3250	3250	3250	3250	3250	3250	3250	3250	F
Lake Natoma min. release, dry year ($u_7 + r_7$)	250	250	250	250	250	250	250	250	250	250	250	250	F

Table 5.4. (continued)

Requirement	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Function
Lake Natoma min. release, wet year ($u_7 + r_7$)	500	500	500	500	500	500	500	500	500	500	500	500	F
Sacramento min. diversion (R_{10})	75	10	10	50	50	35	220	410	520	550	530	250	MI, I
Natoma desired release ($u_7 + r_7$)	1250	1250	1250	1250	1250	1250	1250	1250	1250	1250	1250	1250	F
New Melones min. diversion (R_8)	336	0	0	0	0	420	1429	1681	1849	1849	1765	1765	I
New Melones min. release ($u_8 + r_8$)	20	50	202	538	538	538	639	639	118	118	118	118	F
New Melones min. release, dry year ($u_8 + r_8$)	168	101	34	0	0	34	101	68	168	168	168	168	WQ
New Melones min. release, average year ($u_8 + r_8$)	136	68	0	0	0	0	101	55	136	136	136	136	WQ
New Melones min. release, wet year ($u_8 + r_8$)	68	0	0	0	0	0	0	0	101	101	101	101	WQ
Tracy accretion (+) or diversion (-) (R_{11})	1980 (-)	895 (-)	35 (-)	940 (+)	670 (+)	168 (+)	1025 (-)	2030 (-)	3100 (-)	4500 (-)	4240 (-)	2800 (-)	MI, I
Tracy pumping plant diversion (R_{12})	4200	4200	4200	4200	4200	4600	4600	3000	3000	6200	6200	4600	MI, I

Table 5.4. (continued)

Requirement	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Function
NCVP share of Delta requirement (R_{13})	1220–1530	1640–2050	2200–2800	2480–3100	1480–1850	2200–2750	2840–3550	5360–6700	4160–5200	3200–4000	1360–1700	720–900	WQ, E
Trinity min. penstock flow, total (u_1)	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	HP
Lewiston min. penstock flow, total (u_2)	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	HP
Whiskeytown min. penstock flow, total (u_3)	1300	1300	1300	1300	1300	1300	1300	1300	1300	1300	1300	1300	HP
Shasta min. penstock flow, total (u_4)	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	HP
Keswick min. penstock flow, total (u_5)	- - - - - - - - - - - - - - - - - Not available (see note 3) - - - - - - - - - - - - - - - - - -												
Folsom min. penstock flow, total (u_6)	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	HP
Natoma min. penstock flow, total (u_7)	- - - - - - - - - - - - - - - - - Not available (see note 3) - - - - - - - - - - - - - - - - - -												
New Melones min. penstock flow, total (u_8)	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	HP
Tullock min. penstock flow, total (u_9)	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	HP

Table 5.4. (continued)

Notes:

(1) All flows are in cfs.

(2) A desired release need not be met under drought conditions.

(3) Minimum penstock releases for Natoma and Keswick are not relevant because other constraints such as minimum flows for navigation and fish are higher.

(4) For the reach between Keswick and Natoma there is a stringent requirement: whatever flow exists as of October 15 (must be greater than 5000 cfs) at Wilkins Slough, it must be maintained at least at that level through December 31 to permit normal fish spawning.

(5) Data provided by the Central Valley Operations Office, USBR, Sacramento, CA.

Table 5.5. Reservoir Capacity Allocations (Storages in Kaf and elevations in ft).

Zone	Shasta	Folsom	Clair Engle	Keswick	Whiskey-town	Lewiston	Natoma	New Melones	Tullock
Dead Pool									
Top of pool elev.	737.75	205.5	1995.5	504.2	972.0	1843.3	102.4	543.0	439.0
Pool storage	115.8	0	10.0	0.10	0.008	0.04	1.8	0	14.1
Cumulative storage	115.8	0	10.0	0.10	0.008	0.04	1.8	0	
Inactive Pool									
Top of pool elev.	840.0	327.0	2145.0	574.0	1100.0	1898.0	118.5	808.0	†
Pool storage	471.3	90.0	302.6	16.2	27.4	11.7	4.2	300.0	†
Cumulative storage	587.1	90.0	312.6	16.3	27.4	11.74	6.0	300.0	†
Active Pool									
Top of pool elev.	1018.55	427.0	2370.0	587.0	1210.0	1902.0	125.0	1049.5	†
Pool storage	2664.9	520.0	2135.2	7.5	213.6	2.9	2.8	1970.0	†
Cumulative storage	3552.0	610.0	2448.0	23.8	241.0	14.7	8.8	2270.0	†
Joint Use Pool									
Top of pool elev.	1067.0	466.0	*	*	*	*	*	1088.0	509.3
Pool storage	1300.0	400.0	*	*	*	*	*	150.0	52.0
Cumulative storage	4552.0	1010.0	*	*	*	*	*	2420.0	66.1
Surcharge Pool									
Top of pool elev.	1070.7	475.4	2388.2	589.8	1220.5	**	126.5	1135.0	511.9
Pool storage	109.8	110.0	313.2	1.4	35.02	**	0.8	451.0	3.3
Cumulative storage	4661.8	1485.4	2761.2	25.2	276.0	**	9.6	2871.0	69.4

* Included in active pool; † not specified; ** no surcharge available.

Notes:
1. Active pool is used for all functions of the reservoirs, including flood control.
2. Joint use gives priority to flood control, but it is also used for other functions. The differentiation between joint use and active pool is somewhat arbitrary.
3. Source: Central Valley Operations Office, USBR, Sacramento, CA.

Table 5.6. Maximum Reservoir Storages (in Kaf).

Reservoir	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Reason
Clair Engle	2450	2100	2100	2100	2100	2100	2450	2450	2450	2450	2450	2450	FC
Lewiston	14.7	14.7	14.7	14.7	14.7	14.7	14.7	14.7	14.7	14.7	14.7	14.7	Reserv. capacity
Whiskeytown	241	241	241	241	241	241	241	241	241	241	241	241	Reserv. capacity
Shasta	4200	3600	3400	3600	3700	3800	3900	4200	4552	4552	4552	4552	FC
Keswick	23.8	23.8	23.8	23.8	23.8	23.8	23.8	23.8	23.8	23.8	23.8	23.8	Reserv. capacity
Folsom (wet year)	950	800	1000	1000	1000	1000	1000	1000	1010	1010	1010	1010	FC
Folsom (normal, dry year)	950	700	700	700	700	750	800	900	1010	1010	1010	1010	FC
Natoma	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	Reserv. capacity
New Melones	2220	2020	2020	2020	2020	2020	2120	2220	2420	2420	2420	2420	FC
Tullock	60	57	57	57	57	60	61	67	67	67	67	67	FC

FC indicates that a flood control provision or diagram was used to establish the maximum storage. Above this level, water will be spilled.

Source: Central Valley Operations Office, USBR, Sacramento, CA.

Table 5.7. Minimum Reservoir Storages (in Kaf).

Reservoir	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Reason
Clair Engle	300	300	300	300	300	300	300	300	300	300	300	300	Dead pool
Lewiston	6	6	6	6	6	6	6	6	6	6	6	6	HP
Whiskeytown	27.4	27.4	27.4	27.4	27.4	27.4	27.4	27.4	27.4	27.4	27.4	27.4	HP
Shasta	600	600	600	600	600	600	600	600	600	600	600	600	Dead pool
Keswick	3	3	3	3	3	3	3	3	3	3	3	3	Dead pool
Folsom	90	90	90	90	90	90	90	90	583	583	583	583	Oct-May: dead pool Jun-Sep: recreation
Natoma	2	2	2	2	2	2	2	2	2	2	2	2	Dead pool
New Melones	300	300	300	300	300	300	300	300	300	300	300	300	Dead pool
Tullock	12	12	12	12	12	12	12	12	12	12	12	12	Dead pool

Dead pool restriction implies that reservoir elevations are inadmissible for a variety of reasons: power generation, aesthetics, etc.

Source: Central Valley Operations Office, USBR, Sacramento, CA.

Table 5.8. Maximum Reservoir Releases, Excluding Spills, that are Limited to Penstock Capacity (in Kaf/month).

Reservoir	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
Clair Engle	209	209	209	209	209	209	209	209	209	209	209	209
Lewiston	214	214	214	214	214	214	214	214	214	214	214	214
Whiskeytown	268	268	268	268	268	268	268	268	268	268	268	268
Shasta	982	982	982	982	982	982	982	982	982	982	982	982
Keswick	863	863	863	863	863	863	863	863	863	863	863	863
Folsom	476	476	476	476	476	476	476	476	476	476	476	476
Natoma	300	300	300	300	300	300	300	300	300	300	300	300
New Melones	476	476	476	476	476	476	476	476	476	476	476	476
Tullock	110	110	110	110	110	110	110	110	110	110	110	110

Table 5.9. Minimum Reservoir Releases (in Kaf/month).

Reservoir	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
Clair Engle	89.3	89.3	89.3	89.3	89.3	89.3	89.3	89.3	89.3	89.3	89.3	89.3
Lewiston	59.5	59.5	59.5	59.5	59.5	59.5	59.5	59.5	59.5	59.5	59.5	59.5
Whiskeytown	77	77	77	77	77	77	77	77	77	77	77	77
Shasta	300	300	300	300	300	300	300	300	300	300	300	300
Keswick	238	194	194	194	194	194	298	357	476	536	506	303
Folsom (drought)	100	100	100	100	100	100	100	100	100	100	100	100
Folsom (normal, wet periods)	180	180	180	180	180	180	180	180	180	180	180	180
Natoma	75	75	75	75	75	75	75	75	75	75	75	75
New Melones (drought)	50	50	50	50	50	50	50	50	50	50	50	50
New Melones (normal, wet periods)	100	100	100	100	100	100	100	100	100	100	100	100
Tullock (drought)	50	50	50	50	50	50	50	50	50	50	50	50
Tullock (normal, wet periods)	60	60	60	60	60	60	60	60	60	60	60	60

These minimum releases are obtained from diverse requirements. Several of those requirements are redundant and the binding or critical requirement has been selected.

turbine, the flow through the penstocks, and the efficiency of the turbines. Curves that relate the rate of energy production (in megawatt-hour per kiloacre-foot, Mwh/Kaf) to reservoir storage (in Kaf) were developed from actual records of operation for the reservoirs since their power plants began operating. Those curves can be used to compute the energy generated in any period without having to include constraints on power production, which are nonlinear and usually create numerical and analytical difficulties. In addition, the rate of energy production becomes a linear function of the storage, and the head does not appear in the energy equation. The rate of energy production vs. reservoir storage curves were tested with actual operation data for the NCVP. In comparison with actual energy output, the error in the predicted energy generation was less than 2 percent. As shown in Chapter 7, it is straightforward to incorporate nonlinear rates of energy production.

Figures 5.7 through 5.13 show the energy production vs. reservoir storage curves for all reservoirs but Keswick and Nimbus Power Plant at Natoma Reservoir. The energy vs. storage relations for those two low-head installations were developed by using regression analysis from historical energy production records. The data on Figs. 5.7-5.13 were approximated by the following linear equations (for Keswick and Nimbus, the energy rate curves were approximated by linear relations that were tested against actual operation records):

Trinity (at Clair Engle Lake)

$$\xi_T = 221 + 0.0858\ \bar{x}_T \tag{5.1}$$

$r^2 = 94.6\%$

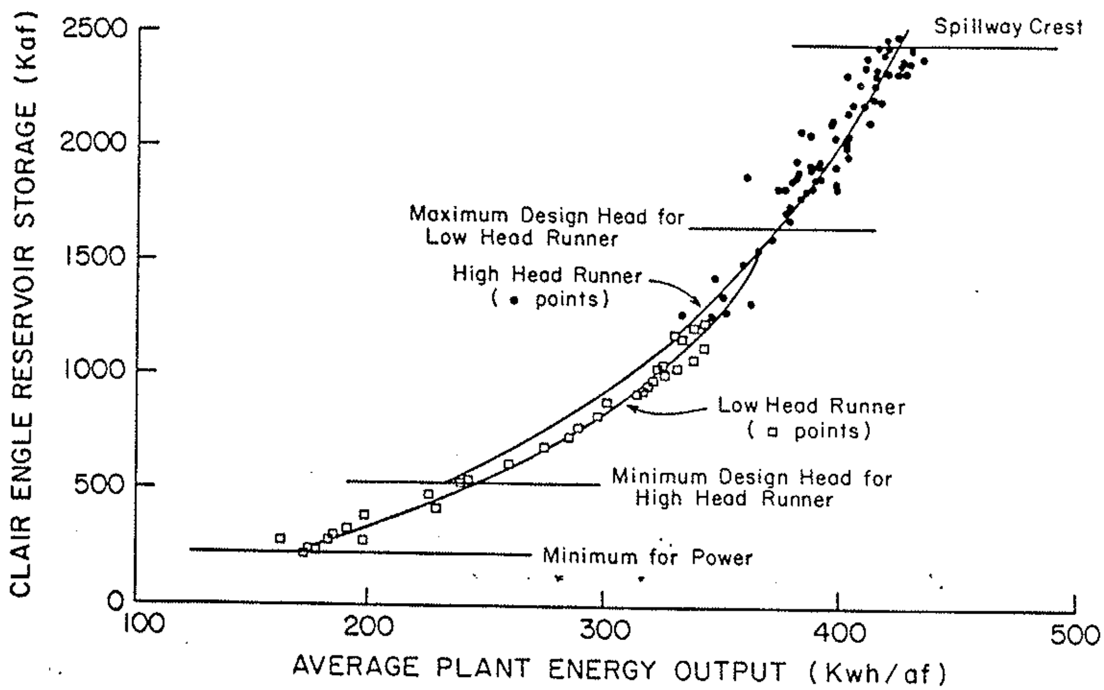


Fig. 5.7 Trinity power plant (at Clair Engle Lake) gross generation curve.

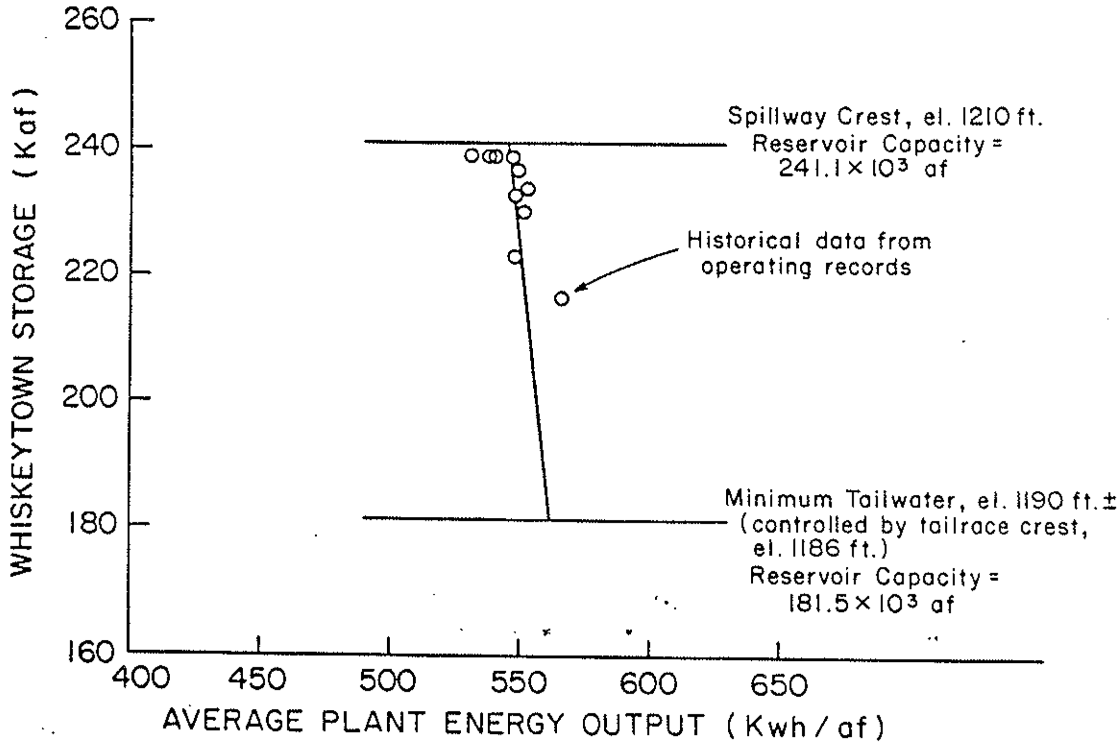


Fig. 5.8 Judge Francis Carr power plant gross generation curve.

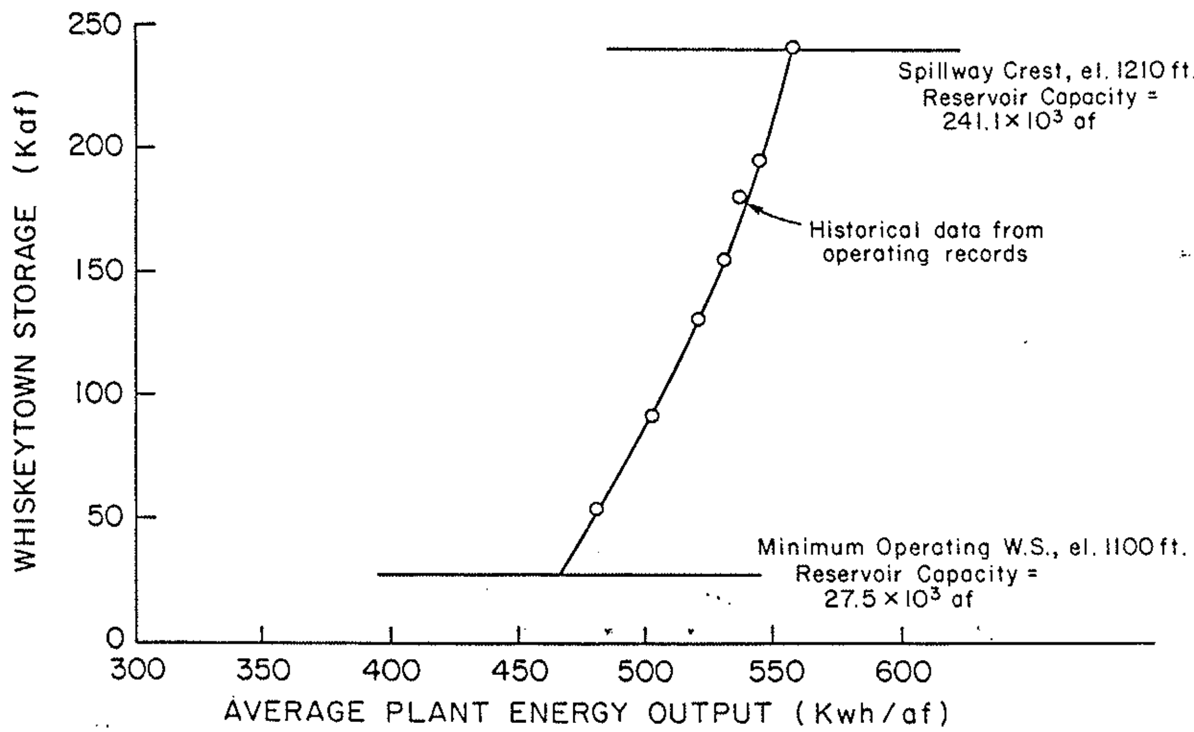


Fig. 5.9 Spring Creek power plant gross generation curve.

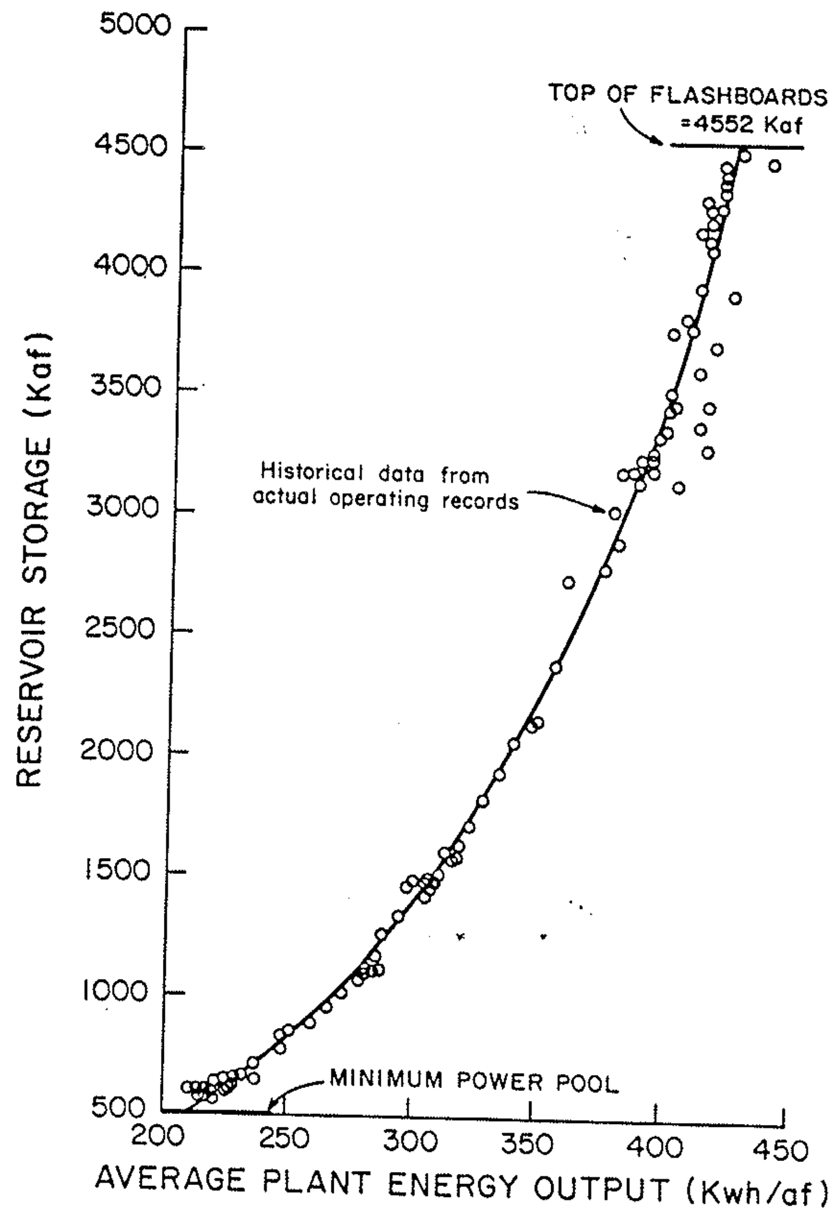


Fig. 5.10 Shasta power plant gross generation curve.

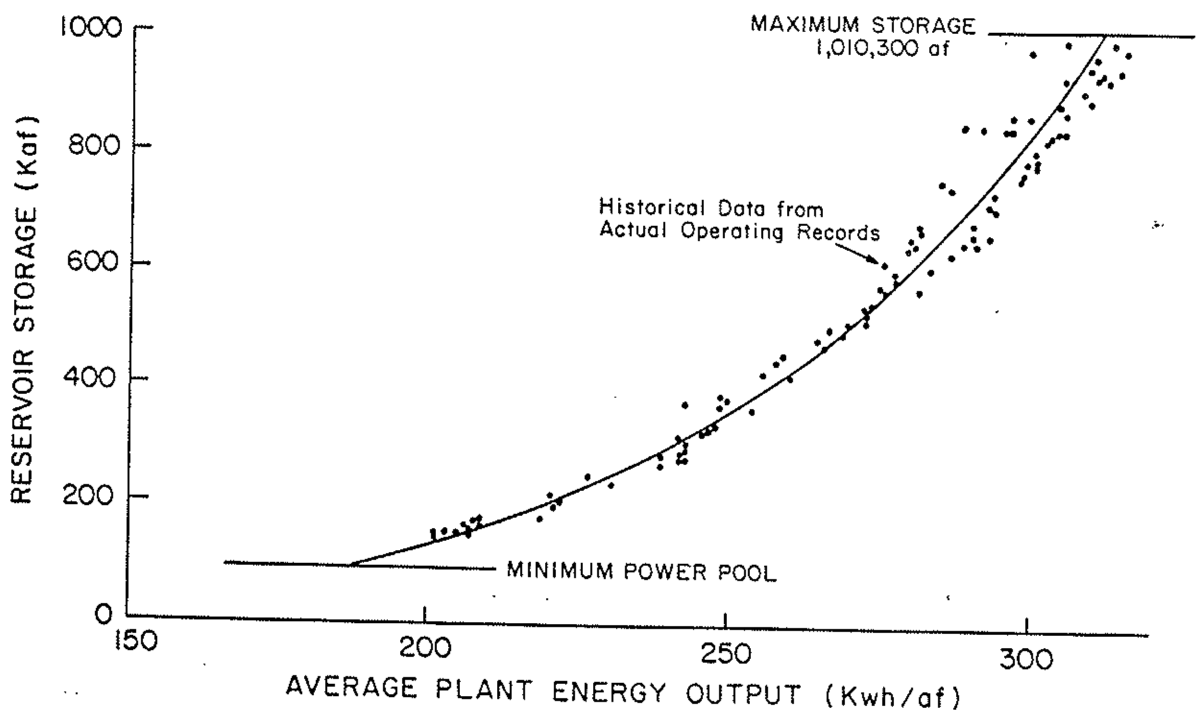


Fig. 5.11 Folsom power plant gross generation curve.

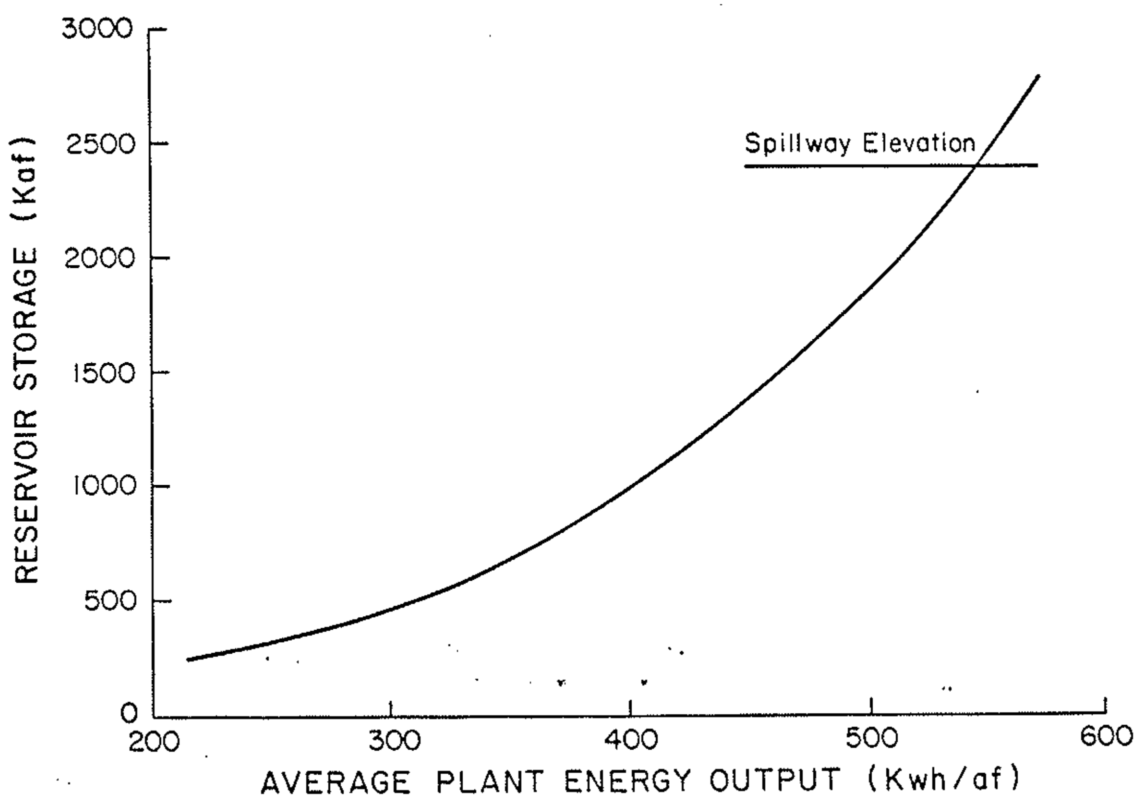


Fig. 5.12 New Melones power plant gross generation curve.

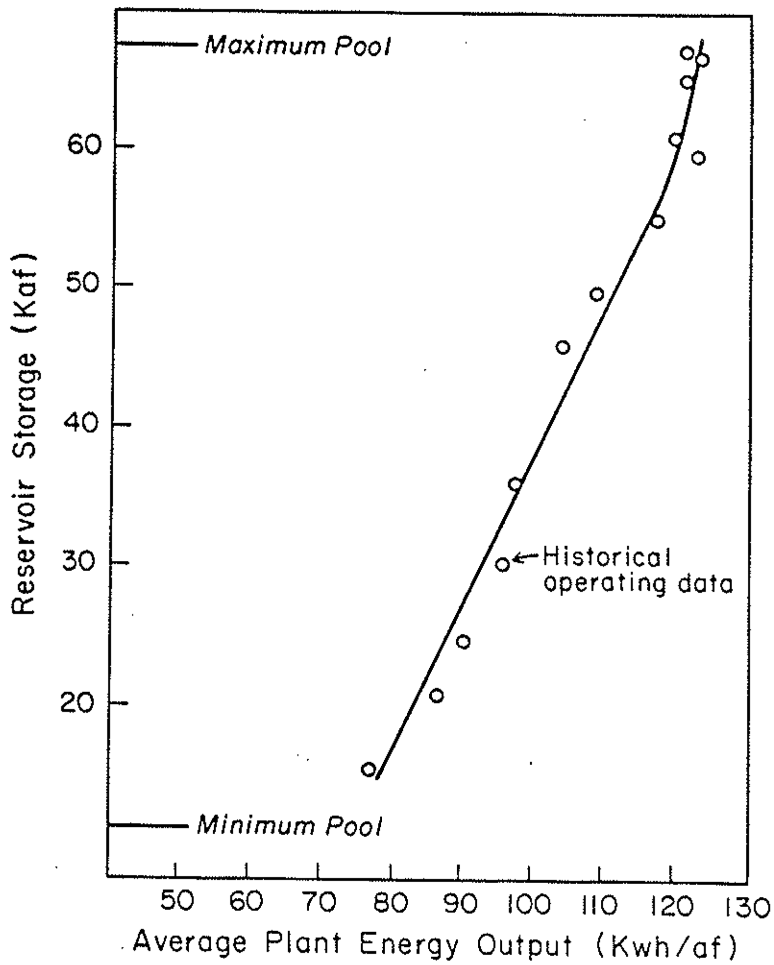


Fig. 5.13 Tullock power plant gross generation curve.

Judge Francis Carr

$$\xi_{JFC} = 575 + 0.9\ \bar{x}_L \tag{5.2}$$
$r^2 = 99.0\%$

Spring Creek

$$\xi_{SC} = 460 + 0.434\ \bar{x}_W \tag{5.3}$$
$r^2 = 98.0\%$

Shasta

$$\xi_S = 234 + 0.0462\ \bar{x}_S \tag{5.4}$$
$r^2 = 94.8\%$

Keswick

$$\xi_K = 80.3 + 0.6\ \bar{x}_K \tag{5.5}$$
$r^2 = 92.0\%$

Folsom

$$\xi_F = 201 + 0.120\ \bar{x}_F \tag{5.6}$$
$r^2 = 95.8\%$

Nimbus

$$\xi_N = 26.3 + 0.8\ \bar{x}_N \tag{5.7}$$
$r^2 = 91.0\%$

New Melones

$$\xi_{NM} = 268 + 0.123\ \bar{x}_{NM} \tag{5.8}$$
$r^2 = 98.0\%$

Tullock

$$\xi_{TU} = 64.9 + 0.931\ \bar{x}_{TU} \tag{5.9}$$
$$r^2 = 99.4\%$$

In eq. (5.1), ξ_T is the energy rate in Mwh/Kaf for Trinity Dam (at Clair Engle Lake), $\bar{x}_T$ is the average reservoir storage in Kaf during a specified period, and r^2 is the adjusted regression correlation coefficient. Other terms in eqs. (5.2)-(5.9) are defined similarly. The treatment of nonlinear energy production rates is fully addressed in Chapter 7.

Evaporation and direct rainfall input are considered for the larger reservoirs only, i.e., Clair Engle, Shasta, Folsom, and New Melones. Historical records of operation for Trinity and New Melones were used to derive monthly coefficients of net loss rates (evaporation minus direct rainfall input). For Shasta and Folsom, those coefficients were derived from data provided by Hall et al. (1969). Table 5.10 gives net loss rates data for the reservoirs. The total net loss in any month t (e_t) in Kaf is expressed by

$$e_t = c_t\ A_t \tag{5.10}$$

in which A_t is the average surface area of the reservoir in month t (kiloacre, Ka) and c_t is the net loss rate during month t (ft/month). It is possible to express eq. (5.10) as a function of average storage if an area-storage relation is available. Several linear functions that relate area (A) and storage (x) were obtained for the four reservoirs mentioned earlier:

Table 5.10. Reservoir Net Rate Losses, c_t (in ft/month).

Month	Clair Engle	Shasta	Folsom	New Melones
Jan	0.040	-0.020	-0.020	0.020
Feb	0.047	-0.001	-0.005	0.023
Mar	0.090	0	-0.001	0.050
Apr	0.215	0.002	0.001	0.008
May	0.340	0.005	0.003	0.120
Jun	0.484	0.006	0.004	0.200
Jul	0.715	0.008	0.006	0.260
Aug	0.635	0.007	0.005	0.150
Sep	0.450	0.005	0.004	0.060
Oct	0.170	0.003	0.001	0.030
Nov	0.070	0	0	0.001
Dec	0.022	-0.003	-0.002	0

Clair Engle

$$A_T = 3.33 + 0.0078\ \bar{x}_T \qquad (5.11)$$

$r^2 = 97.0\%$

Shasta

$$A_S = 3.99 + 0.0061\ \bar{x}_S \qquad (5.12)$$

$r^2 = 96.0\%$

Folsom

$$A_F = 2.67 + 0.0094\ \bar{x}_F \qquad (5.13)$$

$r^2 = 95.0\%$

New Melones

$$A_{NM} = 2.91 + 0.0088\ \bar{x}_{NM} \qquad (5.14)$$

$r^2 = 96.0\%$

in which area is in Ka and storage in Kaf. Equations (5.11)-(5.14) were developed from the area-storage data of Appendix A.

By substituting eqs. (5.11)-(5.14) into their respective equivalents to eq. (5.10), the net loss for each month t can be expressed as a function of average storage ($\bar{x}$) as follows:

Clair Engle

$$e_{tT} = 3.33\ c_{tT} + 0.0078\ c_{tT}\ \bar{x}_{tT} \qquad (5.15)$$

Shasta

$$e_{tS} = 3.99\ c_{tS} + 0.0061\ c_{tS}\ \bar{x}_{tS} \qquad (5.16)$$

Folsom

$$e_{tF} = 2.67\ c_{tF} + 0.0094\ c_{tF}\ \bar{x}_{tF} \tag{5.17}$$

New Melones

$$e_{tNM} = 2.91\ c_{tNM} + 0.0088\ c_{tNM}\ \bar{x}_{tNM} \tag{5.18}$$

Benefits Accruing from the Operation of the System

Due to the multiobjective nature of the NCVP operation, multiple benefits arise from the operation of the system. Flood control benefits arise from the reduced damage caused by floods that would otherwise occur without the project. As an example of the services provided to the public, Madsen and Coleman (1974) estimated that in 1970 the system averted flood damages for about $55 million (in 1970 dollars).

Irrigation benefits can be measured by the cost of providing an alternative source of supply. The criterion of alternative cost can also be applied to economic benefits accruing from hydropower, municipal and industrial use, and navigation. Jaquette (1978) estimated the cost of developing new reservoir water supply at $100 per acre-foot. The issue of benefits computation is more complicated with regard to water quality and fisheries. The economics of reservoir operation is a topic that needs further research.

For the purpose of this study, the performance of the system is measured by the total energy generated during a water year. As shown in Chapter 6, it is rational to use power revenue as a performance criterion because a large amount of power generation is usually associated with increased water deliveries for other purposes and with adequate flood control storages. It is acknowledged that the use of power generation as the only performance index is a limited criterion, although one that in the case of the NCVP operation model leads to an adequate mathematical structure and satisfactory operation policies as indicated by the results in Chapters 6 and 7. Chapter 6 and Section 7.8 contain a detailed discussion of power generation and its relation to releases and storages.

5.2 Streamflow Forecasting Technique

The worth of streamflow forecasting has been discussed by several investigators (Jettmar and Young, 1975; Klemes et al., 1981). It is generally agreed that the type of model used to forecast streamflows for long-term operation is not critical since most statistical models are good at predicting mean flows, which dominate long-term operational analysis. The accuracy of short-term forecasts is, however, crucial in short-term control of stochastic dynamic systems. Wenzel et al. (1976), Yeh et al. (1982), and others found that good streamflow prediction is important for short-term reservoir operation, especially in relation to flood control. Short-term forecast of river flows is of primary importance in the operation of the NCVP.

For this study, a new technique is developed to forecast river flows. Forecast flows are used as input to the monthly optimization model of reservoir operation. The parameters of the forecasting method (i.e., transition probabilities) are updated each month and future forecasts are correspondingly updated to include the last information available.

The conceptual basis of the forecasting method is to view the realization of monthly flows as replication of a multivariate seasonal autoregressive (AR) process. In this study, there are five rivers feeding the system: Sacramento River at Shasta, American River at Folsom, Trinity River at Clair Engle, Stanislaus River at New Melones, and Clear Creek at Whiskeytown. Thus, each month of the αth year, a five-component streamflow vector, $\underline{y}_{t\alpha}$, is observed; this constitutes one realization of the AR process. In the development that follows, $\underline{y}_{t\alpha}$ represents a five-dimensional vector of river flows during month t and water year α. For example, $\underline{y}_{2,30}$ is the river flow in November of the 30th year of flow data. The index t goes from t = 2 to τ = 12 and α varies from 1 to N, the number of years of the flow record. In the NCVP system, the first component of vector $\underline{y}_{t\alpha}$ corresponds to Shasta, the second to Folsom, the third to Clair Engle, the fourth to New Melones, and the fifth to Whiskeytown.

The expression for the first-order AR model is

$$\underline{y}_t = B_t \underline{y}_{t-1} + \underline{e}_t \tag{5.19}$$

in which $\underline{y}_t$ is a p-component vector with mean $E(\underline{y}_t) = \underline{0}$ (in this study, $\underline{y}_t$ is a five-dimensional flow vector whose mean has been subtracted); B_t is a p x p matrix (the "transition" matrix); and $\underline{e}_t$ is a sequence of independent random vectors with expected values $E(\underline{e}_t) = \underline{0}$ and covariance matrices $E(\underline{e}_t \underline{e}_t^T) = \Sigma_t$, and independent of $\underline{y}_{t-1}$, $\underline{y}_{t-2}, \ldots$ Let the covariance of $\underline{y}_t$ be $E(\underline{y}_t \underline{y}_t^T) = R_t$. It follows that

$$R_t = B_t R_{t-1} B_t^T + \Sigma_t \tag{5.20}$$

If the observations are made for t = 1, 2,..., τ and if $\underline{y}_1$ and the $\underline{e}_t$'s are normal, then the model for the observation period is specified by $R_1, B_2, \ldots, B_\tau, \Sigma_2, \ldots \Sigma_\tau$.

The identification criterion used to estimate the parameters R_t, B_t, and Σ_t is the maximization of the likelihood function of the process (Anderson, 1978) given by eq. (5.19). Let $\underline{y}_{t\alpha}$ be the p-component vector of measurements of the αth replicate at the tth time point ($\alpha = 1, 2, \ldots, N$ and $t = 1, 2, \ldots, \tau$). Assume that $\underline{e}_t$ and $\underline{y}_1$ are normally distributed as $N(0, \Sigma_t)$ and $N(0, R_1)$, respectively. The probability density function of the sequence $\underline{y}_{1\alpha}, \underline{y}_{2\alpha}, \ldots, \underline{y}_{\tau\alpha}$, $\alpha = 1, 2, \ldots, N$, is

$$\prod_{\alpha=1}^{N} \frac{1}{(2\pi)^{1/2\tau p} \, |R_1|^{1/2} \, \Pi_{t=2}^{\tau} \, |\Sigma_t|^{1/2}} \exp\left\{ -\frac{1}{2}\left[\underline{y}_{1\alpha}^T \, R_1^{-1} \, \underline{y}_{1\alpha} + \left(\sum_{t=2}^{\tau} \underline{y}_{t\alpha} - B_t \, \underline{y}_{t-1,\alpha}\right)^T \Sigma_t^{-1} \left(\sum_{t=2}^{\tau} (\underline{y}_{t\alpha} - B_t \, \underline{y}_{t-1,\alpha})\right)\right]\right\} \tag{5.21}$$

By taking the logarithm of eq. (5.21) and differentiating with respect to the elements of B_t, the maximum likelihood estimators (MLE) of B_2, $B_3, \ldots, B_\tau$ are

$$\hat{B}_t = C_t(1) \, C_{t-1}^{-1}(0), \quad t = 2, 3, \ldots, \tau \tag{5.22}$$

The MLE of R_1, $\Sigma_2, \ldots, \Sigma_\tau$ are respectively

$$\hat{R}_1 = C_1(0) \tag{5.23}$$

and

$$\hat{\Sigma}_t = C_t(0) - \hat{B}_t C_{t-1}(0) \, \hat{B}_t^T, \quad t = 2, 3, \ldots, \tau \tag{5.24}$$

In eqs. (5.22)-(5.24),

$$C_t(j) = \frac{1}{N} \sum_{\alpha=1}^{N} \underline{y}_{t\alpha} \, \underline{y}_{t-j,\alpha}^T \tag{5.25}$$

Clearly, $C_t(j)$ is the sample cross-correlation matrix between the observations at period t and those at period t-j. Notice that knowledge of $\hat{R}_1$, $\hat{\Sigma}_t$, and $\hat{B}_t$ allows the computation of $\hat{R}_2, \ldots, \hat{R}_\tau$ by means of eq. (5.20).

Application of the AR model to the five streams considered in the NCVP yielded the parameters $\hat{B}_t$ (estimate of the transition matrices) and $\hat{\Sigma}_t$ (estimate of the noise covariance matrices) shown in Table 5.11. Prediction of future inflows having the last period realization of $\underline{y}_t$ (i.e., $\underline{y}_{t-1}$) is accomplished by using the following expression recursively:

$$E(\underline{y}_{t+1} \mid \underline{y}_t) = \hat{B}_t \, \underline{y}_t \tag{5.26}$$

Inflow forecasts computed from eq. (5.26) converge to the historical means after eight periods. Thus, the best way to use eq. (5.26) is by updating the forecasts once a value of $\underline{y}_t$ is observed, i.e., by changing the base time $\underline{y}_t$ in eq. (5.26) and considering only the most recently updated forecasts for operation planning. Also, parameters $\hat{B}_t$, $\hat{\Sigma}_t$, and $\hat{R}_t$ can be modified as new realizations $\underline{y}_t$ become available so that those estimates can be kept up-to-date with the most recent information.

Tables 5.12, 5.13, and 5.14 show the results of the application of this technique to forecast flows at Shasta, Folsom, Clair Engle, New Melones, and Whiskeytown for a below-average inflow year (1975-1976), an average inflow year (1974-1975), and an above-average inflow year (1979-1980), respectively. The results represent the one-step ahead predictors (i.e., forecasts computed by updating the base $\underline{y}_t$ from month to month) and are within ±15% of the actual values. Those are the

Table 5.11. Transition and Noise Covariance Matrices for NCVP Monthly Inflows.

	$\hat{B}_t$						$\hat{\Sigma}_t$				
					November						
	(1)	(2)	(3)	(4)	(5)		(1)	(2)	(3)	(4)	(5)
(1)	-1.364	0.966	2.484	-1.203	15.987	(1)	43942	12562	12767	3170	3458
(2)	1.550	-1.627	-1.063	0.678	9.089	(2)	12562	14443	3609	4641	927
(3)	-0.617	0.454	1.532	-0.275	3.127	(3)	12767	3609	3828	889	1000
(4)	0.609	-0.632	-0.419	0.430	1.846	(4)	3170	4641	889	1646	254
(5)	-0.211	0.157	0.303	-0.064	1.659	(5)	3458	927	1000	254	301
					December						
(1)	2.819	-0.150	-4.239	0.796	-11.553	(1)	137207	89106	33530	19177	9779
(2)	0.522	1.235	-1.665	-0.797	-0.070	(2)	89106	71133	23417	15787	6033
(3)	0.136	0.185	0.069	-0.097	-0.685	(3)	33530	23417	10068	4825	2285
(4)	0.099	0.194	-0.358	0.621	-0.031	(4)	19177	15787	4825	3797	1333
(5)	0.204	0.049	-0.345	-0.130	-0.715	(5)	9779	6033	2285	1333	806
					January						
(1)	-0.328	-1.158	3.827	0.714	12.980	(1)	198138	72449	40653	18720	14298
(2)	-0.407	0.052	1.435	0.331	5.949	(2)	72449	51209	12519	14003	4195
(3)	-0.017	-0.306	0.810	0.244	2.373	(3)	40653	12519	9818	3080	3008
(4)	-0.105	0.080	0.299	0.000	1.267	(4)	18720	14003	3080	4223	981
(5)	-0.012	-0.141	0.343	0.111	1.091	(5)	14298	4195	3008	981	1280
					February						
(1)	-0.051	0.395	-1.736	-0.087	5.864	(1)	150781	44115	35487	8380	16084
(2)	-0.137	0.778	-0.408	-1.049	1.021	(2)	44115	20723	10885	4937	4381
(3)	-0.000	0.150	-0.098	-0.493	0.561	(3)	35487	10885	10243	2177	3872
(4)	-0.058	0.095	0.032	0.293	0.172	(4)	8380	4937	2177	1474	839
(5)	-0.014	-0.007	-0.149	0.087	0.816	(5)	16084	4381	3872	839	1869

Table 5.11. (continued)

	$\hat{B}_t$						$\hat{\Sigma}_t$				
March											
	(1)	(2)	(3)	(4)	(5)		(1)	(2)	(3)	(4)	(5)
(1)	-0.155	-0.268	0.023	0.589	4.632	(1)	73389	27668	16408	7346	7197
(2)	-0.133	0.568	-0.343	-0.378	1.967	(2)	27668	22765	6166	6261	1961
(3)	-0.003	-0.094	0.043	0.084	0.656	(3)	16408	6166	5139	1858	1643
(4)	0.001	-0.013	-0.156	0.499	0.481	(4)	7346	6261	1858	1997	498
(5)	-0.061	-0.005	-0.071	0.026	1.034	(5)	7197	1961	1643	498	946
April											
(1)	0.463	-0.158	-0.743	1.159	1.427	(1)	75661	30516	11745	7927	5157
(2)	-0.090	0.951	-0.659	-0.737	1.475	(2)	30516	20567	7318	6490	1822
(3)	-0.056	0.029	0.267	0.208	0.768	(3)	11745	7318	4690	2737	495
(4)	-0.074	0.192	-0.091	0.546	0.361	(4)	7927	6490	2737	2597	354
(5)	-0.011	-0.015	-0.023	0.120	0.584	(5)	5157	1822	495	354	467
May											
(1)	0.513	-0.195	-0.390	0.587	0.107	(1)	13074	8785	5884	3268	756
(2)	0.171	0.858	-0.641	0.313	0.368	(2)	8785	14556	5120	5989	355
(3)	0.037	-0.258	0.394	0.588	2.463	(3)	5884	5120	3958	2653	330
(4)	0.153	-0.071	-0.280	1.275	0.765	(4)	3268	5989	2653	4311	132
(5)	0.021	-0.033	-0.026	0.052	0.158	(5)	756	355	330	132	65
June											
(1)	0.640	-0.033	-0.017	-0.156	-1.800	(1)	2862	1552	1322	517	86
(2)	0.464	0.513	-0.449	-0.101	0.015	(2)	1552	4301	1084	1928	107
(3)	0.170	0.069	0.395	-0.233	0.270	(3)	1322	1084	1542	678	45
(4)	0.316	0.015	-0.317	0.462	0.835	(4)	517	1928	678	1759	30
(5)	0.009	-0.012	0.007	0.002	0.312	(5)	86	107	45	30	12

Table 5.11. (continued)

	$\hat{B}_t$						$\hat{\Sigma}_t$				
July											
	(1)	(2)	(3)	(4)	(5)		(1)	(2)	(3)	(4)	(5)
(1)	0.207	-0.055	0.167	0.122	0.768	(1)	367	130	81	101	7
(2)	-0.098	0.102	-0.078	0.369	4.095	(2)	130	1053	93	401	20
(3)	-0.005	-0.014	0.369	0.005	-0.342	(3)	81	93	157	59	3
(4)	0.034	-0.046	0.014	0.381	-0.457	(4)	101	401	59	292	6
(5)	-0.023	-0.005	-0.010	0.002	1.063	(5)	7	20	3	6	6
August											
(1)	0.660	-0.117	-0.130	0.105	1.143	(1)	151	118	32	12	8
(2)	-0.024	0.560	-0.283	-0.091	4.317	(2)	118	713	33	148	9
(3)	0.025	0.014	0.189	-0.027	-0.243	(3)	32	33	19	7	0
(4)	0.069	-0.021	-0.123	0.207	0.670	(4)	12	148	7	84	2
(5)	0.011	-0.007	-0.029	0.017	0.730	(5)	8	9	-0	2	2
September											
(1)	0.653	-0.122	0.087	0.785	1.777	(1)	195	27	27	-2	4
(2)	-0.038	0.750	-0.145	0.206	1.580	(2)	27	115	4	4	0
(3)	-0.070	-0.001	0.593	0.064	0.116	(3)	27	4	20	6	2
(4)	-0.036	0.063	-0.099	0.757	-0.127	(4)	-2	4	6	23	2
(5)	0.047	0.034	-0.183	0.053	0.190	(5)	4	0	2	2	5

(1) Shasta; (2) Folsom; (3) Clair Engle; (4) New Melones; (5) Spring Creek.

Table 5.12. Actual (A) and Forecast (F) Monthly Inflows (in Kaf) for Year with Below-Average Inflows, October 1975-September 1976.

Month	Shasta		Folsom		Clair Engle		New Melones		Whiskeytown	
	A	F	A	F	A	F	A	F	A	F
Oct	326	326	125	125	22	22	45	45	12	12
Nov	297	310	94	110	48	50	37	45	14	18
Dec	294	285	116	142	56	71	44	55	9	16
Jan	282	266	100	128	34	49	43	50	8	15
Feb	366	320	134	119	55	61	40	52	16	21
Mar	436	398	154	120	75	74	46	58	17	20
Apr	391	366	83	93	127	108	49	60	25	18
May	305	289	89	73	177	149	52	61	6	11
Jun	224	201	42	38	50	61	24	21	8	9
Jul	215	198	63	52	22	19	31	24	7	8
Aug	248	210	66	54	19	14	35	29	9	10
Sep	227	210	75	63	10	7	25	27	8	6

Table 5.13. Actual (A) and Forecast (F) Monthly Inflows (in Kaf) for Year with Average Inflows, October 1974-September 1975.

Month	Shasta		Folsom		Clair Engle		New Melones		Whiskeytown	
	A	F	A	F	A	F	A	F	A	F
Oct	270	270	113	113	6	6	37	37	13	13
Nov	299	305	120	125	17	18	37	39	7	13
Dec	359	372	116	138	38	42	34	46	16	25
Jan	344	385	158	179	46	55	43	51	15	22
Feb	850	749	270	315	110	93	86	73	57	44
Mar	1401	1199	423	412	260	210	141	129	140	107
Apr	824	982	341	385	169	188	104	118	59	71
May	793	815	506	510	393	341	261	273	34	36
Jun	464	505	348	365	270	266	308	317	23	24
Jul	287	310	139	146	66	71	79	63	12	10
Aug	256	269	120	121	19	18	40	35	10	11
Sep	258	244	133	118	13	16	35	29	8	10

Table 5.14. Actual (A) and Forecast (F) Monthly Inflows (in Kaf) for Year with Above-Average Inflows, October 1979-September 1980.

Month	Shasta		Folsom		Clair Engle		New Melones		Whiskeytown	
	A	F	A	F	A	F	A	F	A	F
Oct	273	273	89	89	38	38	39	39	9	10
Nov	338	430	100	85	80	91	35	40	16	18
Dec	347	580	145	120	85	100	51	55	18	18
Jan	1241	1058	987	685	236	190	278	285	47	40
Feb	1402	1218	712	880	306	279	263	270	110	100
Mar	886	958	475	580	156	150	163	155	61	55
Apr	568	618	403	462	197	210	169	170	32	28
May	426	505	407	418	201	212	280	223	14	14
Jun	271	305	237	250	105	110	244	265	12	11
Jul	236	240	180	192	48	49	128	139	8	8
Aug	205	224	109	115	9	10	41	48	8	9
Sep	222	220	129	131	8	8	42	43	9	8

values of inflows that will be used in the optimization model to update the operation policies every month. The implication of the month-to-month update is that any operation policy will be followed strictly during the incoming month only. Thereafter, the policy will be revised to account for changing conditions (flows, constraints, etc.).

If the $\underline{e}_t$'s were not assumed normal, then the estimators $\hat{R}_t$, $\hat{B}_t$, and $\hat{\Sigma}_t$ would still have the values reported earlier but the estimates should be interpreted as least squares estimators. Also, if the $\underline{e}_t$'s were not assumed normal, then the approach to forecast streamflows presented herein will not allow hypothesis testing. Among the hypotheses that can be tested with this approach are: i) stationarity (is $B_t = B$ for all t?); ii) independence between the vectors $\underline{y}_t$ (is $B_t = 0$ for all t?); iii) independence of subvectors (if $\underline{y}_t^T$ is partitioned as $(\underline{y}_t^1, \underline{y}_t^2)$, are the subvectors $\underline{y}_t^1$ and $\underline{y}_t^2$ independent? In other words, if $\underline{y}_t^1$ contains the inflows to Shasta, Whiskeytown, and Clair Engle and $\underline{y}_t^2$ contains the inflows to Folsom and New Melones, are the flows in $\underline{y}_t^1$ independent of those in $\underline{y}_t^2$?); iv) order of the model (is a first-order model adequate?); and v) whether a parameter matrix B_t remains the same or changes to another matrix B_t' during a set of years (a change in the matrix could be caused by the construction of a regulating reservoir).

5.3 Optimization Model for the NCVP

This section presents a mathematical formulation of the NCVP optimization model. The model uses a sequential optimization technique, the progressive optimality algorithm (POA), which was described earlier. The POA solves for an optimal release policy for the nine-reservoir system shown in Fig. 5.6. In essence, the way in which any operation schedule is used by the NCVP managers is as follows. At the beginning of a water year, the managers announce the next guiding release policy, which is updated each month after actual flows and water demands are known. The random nature of inflows is handled by making statistical forecasts of flows for the remaining months of the current water year. Those forecasts are updated monthly to account for the most recent actual realization of river flows. Updated flow forecasts as well as actual reservoir storages are input to the model and a revised policy is found for the remainder of the water year. The performance criterion, or objective function, of the model consists of maximization of the power generated throughout the year.

The following notation is used in this section:

$\underline{x}_t$ = 9-dimensional vector whose components x_t^i are the beginning of month storages at each reservoir; i denotes reservoir number, i = 1, 2,..., 9 (Fig. 5.6).

$\underline{u}_t$ = 9-dimensional control or decision vector that represents water released through penstocks; its components are u_t^i. Decisions are made at the beginning of month t.

$\underline{r}_t$ = 9-dimensional spillage vector; its components are r_t^i. Spillages will be treated as decision variables in different ways, according to alternative models developed in this section and in Chapter 7.

Figure 5.14 shows the relationship between the time index t and the vectors $\underline{x}_t$, $\underline{u}_t$, and $\underline{r}_t$.

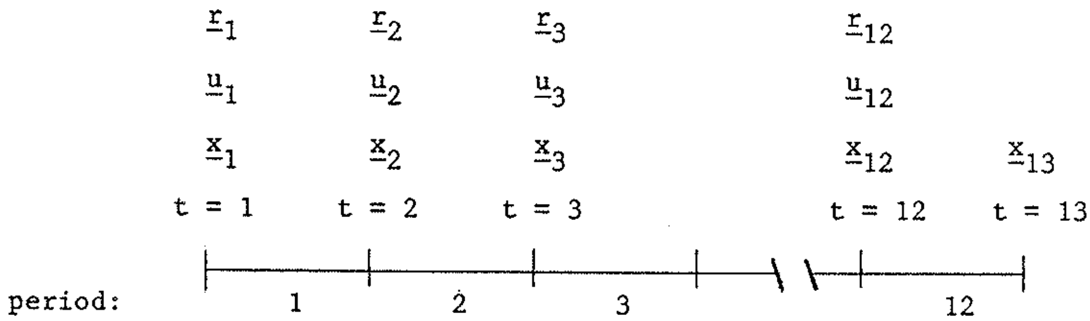


Figure 5.14. Relation between t and vectors $\underline{x}_t$, $\underline{u}_t$, and $\underline{r}_t$.

Spillage will occur when the maximum flood control level is exceeded. If the penstock discharge plus the release through other outlet structures reach the maximum allowable discharge, then the reservoir level is allowed to reach into the surcharge storage and the total release is maintained at a maximum until the reservoir is brought to an adequate level (as specified in the flood control diagram). Short-term control of large flood events, which occur occasionally, are best handled by on-line stochastic control methods where decisions must be taken in a time framework as short as a few minutes. Clearly, monthly operation schemes cannot capture those fast dynamic events in the best way. Nonetheless, monthly decision policies are valuable tools for planning purposes. In fact, it is through those monthly policies that many reservoir systems plan their future activities.

The first step in developing the optimization model for the operation of the NCVP system is to write the law of motion, or continuity equation, for each reservoir in the system (see Fig. 5.6):

Reservoir 1 (Clair Engle)

$$x_{t+1}^1 = x_t^1 - u_t^1 - r_t^1 + y_t^1 - e_t^1 \tag{5.27}$$

Reservoir 2 (Lewiston)

$$x_{t+1}^2 = x_t^2 - u_t^2 - r_t^2 + u_t^1 + r_t^1 - e_t^2 - R_t^2 \tag{5.28}$$

Reservoir 3 (Whiskeytown)

$$x_{t+1}^3 = x_t^3 - u_t^3 - r_t^3 + u_t^2 + y_t^3 - e_t^3 - R_t^3 \tag{5.29}$$

Reservoir 4 (Shasta)

$$x_{t+1}^4 = x_t^4 - u_t^4 - r_t^4 + y_t^4 - e_t^4 \tag{5.30}$$

Reservoir 5 (Keswick)

$$x_{t+1}^5 = x_t^5 - u_t^5 - r_t^5 + u_t^3 + u_t^4 + r_t^4 - e_t^5 \tag{5.31}$$

Reservoir 6 (Folsom)

$$x_{t+1}^6 = x_t^6 - u_t^6 - r_t^6 + y_t^6 - e_t^6 - R_t^6 \tag{5.32}$$

Reservoir 7 (Natoma)

$$x_{t+1}^7 = x_t^7 - u_t^7 - r_t^7 + u_t^6 + r_t^6 - e_t^7 - R_t^7 \tag{5.33}$$

Reservoir 8 (New Melones)

$$x_{t+1}^8 = x_t^8 - u_t^8 - r_t^8 + y_t^8 - e_t^8 - R_t^8 \tag{5.34}$$

Reservoir 9 (Tullock)

$$x_{t+1}^9 = x_t^9 - u_t^9 - r_t^9 + u_t^8 + r_t^8 - e_t^9 \tag{5.35}$$

in which y_t = river inflows during month t, e_t = net losses, and R_t = water demands. Equations (5.27)-(5.35) can be expressed in vector-matrix notation as

$$\underline{x}_{t+1} = I\ \underline{x}_t + \Gamma_1\ \underline{u}_t + \Gamma_2\ \underline{r}_t + \underline{z}_t \tag{5.36}$$

in which I denotes the identity matrix and

$$\Gamma_1 = \begin{bmatrix} -1 & & & & & & & & \\ 1 & -1 & & & & & & & \\ & 1 & -1 & & & & & & \\ & & & -1 & & & & & \\ & & 1 & 1 & -1 & & & & \\ & & & & & -1 & & & \\ & & & & & 1 & -1 & & \\ & & & & & & & -1 & \\ & & & & & & & 1 & -1 \end{bmatrix} \tag{5.37}$$

$$\Gamma_2 = \begin{bmatrix} -1 & & & & & & & & \\ 1 & -1 & & & & & & & \\ & & -1 & & & & & & \\ & & & -1 & & & & & \\ & & & 1 & -1 & & & & \\ & & & & & -1 & & & \\ & & & & & 1 & -1 & & \\ & & & & & & & -1 & \\ & & & & & & & 1 & -1 \end{bmatrix} \tag{5.38}$$

Notice that Γ_1 and Γ_2 are nonsingular lower triangular matrices that considerably simplify the numerical computations. The triangularity of Γ_1 and Γ_2 arises from a proper numbering of the network as done in Fig. 5.6. Equation (5.36) can be rewritten as

$$\Gamma_1 \underline{u}_t + \Gamma_2 \underline{r}_t = \underline{x}_{t+1} - \underline{x}_t - \underline{z}_t \tag{5.39}$$

Recall that e_t^i, the net losses for reservoir i during month t, is given by [see eqs. (5.15)-(5.18)]

$$e_t^i = d_t^i + c_t^i (x_{t+1}^i + x_t^i) \tag{5.40}$$

in which d_t^i and c_t^i are coefficients [see eqs (5.15)-(5.18)] and $(x_{t+1}^i + x_t^i)$ signifies the use of average storage. Substitution of eq. (5.40) into eq. (5.39) gives

$$\Gamma_1 \underline{u}_t + \Gamma_2 \underline{r}_t = A_{t+1} \underline{x}_{t+1} - B_t \underline{x}_t - \underline{v}_t \tag{5.41}$$

in which A_{t+1} is a diagonal matrix whose (diagonal) elements are $1 + c_t^i$, $i = 1, 2, \ldots, 9$; B_t is a diagonal matrix whose (diagonal) elements are $1 - c_t^i$; and $\underline{v}_t$ is the vector

$$\underline{v}_t = \begin{bmatrix} y_t^1 - d_t^1 \\ -R_t^2 - d_t^2 \\ y_t^3 - R_t^3 - d_t^3 \\ y_t^4 - d_t^4 \\ - d_t^5 \\ y_t^6 - R_t^6 - d_t^6 \\ -R_t^7 - d_t^7 \\ y_t^8 - R_t^8 - d_t^8 \\ - d_t^9 \end{bmatrix} \tag{5.42}$$

Because of the small values of the coefficient c_t^i [see eqs. (5.15)-(5.18)], the impact of net losses will be negligible. However, that may not be true in regions of different climatic conditions. Equation (5.41) can be also stated as

$$A_{t+1} \underline{x}_{t+1} = B_t \underline{x}_t + \Gamma_1 \underline{u}_t + \Gamma_2 \underline{r}_t + \underline{v}_t \tag{5.43}$$

or

$$\underline{x}_{t+1} = \phi_t \underline{x}_t + P_{1t} \underline{u}_t + P_{2t} \underline{r}_t + F_t \underline{v}_t \tag{5.44}$$

in which $\phi_t = A_{t+1}^{-1} B_t$; $P_{1t} = A_{t+1}^{-1} \Gamma_1$; $P_{2t} = A_{t+1}^{-1} \Gamma_2$; and $F_t = A_{t+1}^{-1}$. Equation (5.44) is the law of motion, or the equation of continuity, for the NCVP system (a linear equation). Vectors $\underline{u}_t$ and $\underline{r}_t$ represent deterministic control terms and $F_t \underline{v}_t$ is a stochastic disturbance term. Notice that $\underline{v}_t$ is a stochastic vector because it includes the random inflows y_t^i. In addition, it is evident from eq. (5.42) that if water requirements (R_t^i) were also considered random, then $\underline{v}_t$ would include only d_t^i as a nonrandom term. It is emphasized that in this study, the flows are forecast and then a deterministic problem is solved with the most recent forecast of flows used in the vector $\underline{v}_t$. Equations (5.41) and (5.44) play a central role in the development that follows.

The objective function of the optimization model consists of maximization of the energy generated during each year. The energy generated at reservoir i during month t (in Mwh) is

$$E_t^i = \xi_t^i u_t^i = [a^i + b^i (x_t^i + x_{t+1}^i)] u_t^i \tag{5.45}$$

in which ξ_t^i is the energy rate given by eqs. (5.1)-(5.9) and u_t^i is the penstock release from reservoir i during month t. For the whole system, the total energy generated during any month t can be expressed as

$$E_t = [\underline{a} + B(\underline{x}_t + \underline{x}_{t+1})]^T \underline{u}_t \tag{5.46}$$

in which

$\underline{a}^T$ = (221.0 575.0 460.0 234.0 80.3 201.0 26.3 268.0 64.9) and contains the constant terms a^i in eq. (5.45) which are explicitly given in eqs. (5.1)-(5.9).

$$B = \begin{bmatrix} 0.0429 & & & & & & & & \\ & 0.4500 & & & & & & & \\ & & 0.2170 & & & & 0 & & \\ & & & 0.0231 & & & & & \\ & & & & 0.3000 & & & & \\ & & & & & 0.0600 & & & \\ & & 0 & & & & 0.4000 & & \\ & & & & & & & 0.0615 & \\ & & & & & & & & 0.4660 \end{bmatrix}$$

in which the shown diagonal terms are the b^i's of eq. (5.45) and are equal to the factor that multiplies the average storage in eqs. (5.1)-(5.9) divided by two.

$\underline{u}_t$ = 9-dimensional penstock releases.

Recall that the POA maximizes a sequence of two-stage problems, i.e., maximize $E_{t-1} + E_t$ for t = 2, 3,..., 12 subject to a set of constraints. It follows from eq. (5.46) that

$$E_{t-1} + E_t = [\underline{a} + B(\underline{x}_{t-1} + \underline{x}_t)]^T \underline{u}_{t-1}$$

$$+ [\underline{a} + B(\underline{x}_t + \underline{x}_{t+1})]^T \underline{u}_t \qquad (5.47)$$

It is clear from eq. (5.41) that

$$\underline{u}_t = - \Gamma_1^{-1} \Gamma_2 \underline{r}_t + \Gamma_1^{-1} A_{t+1} \underline{x}_{t+1} - \Gamma_1^{-1} B_t \underline{x}_t - \Gamma_1^{-1} \underline{v}_t$$

$$= - \Gamma \underline{r}_t + C_{t+1} \underline{x}_{t+1} - D_t \underline{x}_t - F \underline{v}_t \qquad (5.48)$$

A similar expression can be developed for $\underline{u}_{t-1}$. Substitution of those expressions for $\underline{u}_t$ and $\underline{u}_{t-1}$ into eq. (5.47) yields

$$E_{t-1} + E_t = \underline{q}_t^T \underline{r}_t + \underline{p}_t^T \underline{r}_{t-1} + \underline{g}_t^T \underline{x}_t - \underline{x}_t^T B \Gamma \underline{r}_t$$

$$- \underline{x}_t^T B \Gamma \underline{r}_{t-1} + \underline{x}_t^T G_t \underline{x}_t + k_t \qquad (5.49)$$

in which

$$\underline{q}_t^T = - \underline{a}^T \Gamma - \underline{x}_{t+1}^T B \Gamma \qquad (5.50)$$

$$\underline{p}_t^T = - \underline{a}^T \Gamma - \underline{x}_{t-1}^T B \Gamma \tag{5.51}$$

$$\underline{g}_t^T = - \underline{a}^T D_t + \underline{x}_{t+1}^T C_{t+1}^T B - \underline{v}_t^T F^T B - \underline{x}_{t-1}^T B D_t + \underline{a}^T C_t$$

$$- \underline{x}_{t+1}^T D_{t-1}^T B - \underline{v}_{t-1}^T F^T B + \underline{x}_{t-1}^T B C_t \tag{5.52}$$

$$G_t = B C_t - B D_t \tag{5.53}$$

$$k_t = \underline{a}^T C_{t+1} \underline{x}_{t+1} - \underline{a}^T F \underline{v}_t + \underline{x}_{t+1}^T B C_{t+1} \underline{x}_{t+1}$$

$$- \underline{x}_{t+1}^T B F \underline{v}_t - \underline{a}^T D_{t-1} \underline{x}_{t-1} - \underline{a}^T F \underline{v}_{t-1}$$

$$- \underline{x}_{t-1}^T B D_{t-1} \underline{x}_{t-1} - \underline{x}_{t-1} B F \underline{v}_{t-1} \tag{5.54}$$

Notice that k_t is a constant because $\underline{x}_{t-1}$ and $\underline{x}_{t+1}$ are held fixed at every two-stage maximization. Thus, k_t plays no role in the two-stage solutions. Let

$$\underline{\theta} = \begin{bmatrix} \underline{x}_t \\ \underline{r}_t \\ \underline{r}_{t-1} \end{bmatrix} \tag{5.55}$$

that is, construct an augmented vector containing the variables for which the optimization is to be made. Equation (5.49) can be now expressed as

$$E_{t-1} + E_t = [\underline{g}_t^T, \underline{q}_t^T, \underline{p}_t^T]\ \underline{\theta} + \underline{\theta}_t^T H_t^1\ \underline{\theta}_t + k_t \tag{5.56}$$

in which

$$H_t^1 = \frac{1}{2}\begin{bmatrix} G_t + G_t^T & -B\Gamma & -B\Gamma \\ -\Gamma^T B & 0 & 0 \\ -\Gamma^T B & 0 & 0 \end{bmatrix} \tag{5.57}$$

where 0 is a 9 x 9 null matrix. By dropping the constant term k_t, eq. (5.56) can be rewritten as

$$E_{t-1} + E_t = \underline{\theta}_t^T H_t^1\ \underline{\theta}_t + \underline{s}_t^T\ \underline{\theta}_t \tag{5.58}$$

in which

$$\underline{s}_t^T = [\underline{g}_t^T, \underline{q}_t^T, \underline{p}_t^T] \tag{5.59}$$

Thus, it has been demonstrated that the two-stage objective function is quadratic. It is worthwhile to recall that the continuity equation was used to develop eq. (5.58) and thus it will not appear as a constraint in the constraint set associated with eq. (5.58).

The final step to complete the formulation of the two-stage optimization problem is to include the constraint set associated with eq. (5.58). First, recall that the original two-stage problem is

$$\underset{\underline{x}_t,\ \underline{r}_t,\ \underline{r}_{t-1}}{\text{maximize}} \qquad E_{t-1} + E_t \tag{5.60}$$

subject to

$$\underline{x}_t = f_{t-1}\ (\underline{x}_{t-1}, \underline{w}_{t-1}, \underline{v}_{t-1}), \qquad \underline{x}_{t+1} = f_t\ (\underline{x}_t, \underline{w}_t, \underline{v}_t) \tag{5.61}$$

$$\underline{w}_{t-1} \in \underline{W}_{t-1}, \qquad \underline{w}_t \in \underline{W}_t \tag{5.62}$$

$$\underline{u}_{t-1} \in \underline{U}_{t-1}, \qquad \underline{u}_t \in \underline{U}_t \tag{5.63}$$

$$\underline{r}_{t-1} \in \underline{R}_{t-1}, \qquad \underline{r}_t \in \underline{R}_t \tag{5.64}$$

$$\underline{x}_t \in \underline{X}_t \tag{5.65}$$

in which $\underline{x}_{t-1}$, $\underline{x}_{t+1}$ are fixed; $\underline{w}_{t-1} = \underline{u}_{t-1} + \underline{r}_{t-1}$; $\underline{w}_t = \underline{u}_t + \underline{r}_t$; $\underline{w}_t$ is total release (penstock plus spillage); and $\underline{W}_{t-1}$, $\underline{W}_t$, $\underline{U}_{t-1}$, $\underline{U}_t$, $\underline{R}_{t-1}$, $\underline{R}_t$, and $\underline{X}_t$ are feasible regions for the corresponding variables. Notice that eq. (5.60) is written as a function of $\underline{x}_t$, $\underline{r}_{t-1}$, and $\underline{r}_t$ in agreement with eq. (5.49). Equation (5.61), the continuity equation for months t-1 and t, is defined by eq. (5.44). It has been shown that eq. (5.60) can be written in terms of an augmented vector $\underline{\theta}_t$, as expressed by eq. (5.58). In developing eq. (5.58), use was made of eq. (5.44). Equation (5.62) can be expressed in terms of the components of $\underline{\theta}_t$ by making use of eq. (5.41), i.e., eq. (5.62) (constraints on total releases) implies

$$(-\Gamma + I)\underline{r}_t + C_{t+1}\ \underline{x}_{t+1} - D_t\ \underline{x}_t - F\ \underline{v}_t \in \underline{W}_t \tag{5.66}$$

for month t, or in terms of $\underline{\theta}_t$,

$$C_{t+1}\ \underline{x}_{t+1} + [-D_t \quad (-\Gamma + I) \quad 0]\underline{\theta}_t - F\ \underline{v}_t \in \underline{W}_t \tag{5.67}$$

where 0 is the null matrix. The Delta requirements impose another constraint on the total release $\underline{u}_t + \underline{r}_t$, namely $\underline{c}^T(\underline{u}_t + \underline{r}_t) \in De_t$, i.e.,

$$\underline{c}^T(-\Gamma + I)\underline{r}_t + \underline{c}^T\ C_{t+1}\ \underline{x}_{t+1} - \underline{c}^T\ D_t\ \underline{x}_t - \underline{c}^T\ F\ \underline{v}_t \in De_t \tag{5.68}$$

for month t, or in terms of $\underline{\theta}_t$,

$$\underline{c}^T\ C_{t+1}\ \underline{x}_{t+1} + \underline{c}^T[-D_t \quad (-\Gamma + I) \quad 0]\underline{\theta}_t - \underline{c}^T\ F\ \underline{v}_t \in De_t \tag{5.69}$$

where $\underline{c}^T = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1]$ and De_t is a set of feasible values for Delta water deliveries. Similarly, eq. (5.63) can be written in terms of the components of $\underline{\theta}_t$ by using eq. (5.48), i.e., eq. (5.63) (constraints on penstock releases) implies

$$-\Gamma\ \underline{r}_t + C_{t+1}\ \underline{x}_{t+1} - D_t\ \underline{x}_t - F\ \underline{v}_t \in \underline{U}_t \tag{5.70}$$

for month t, or in terms of $\underline{\theta}_t$,

$$[-D_t \quad -\Gamma \quad 0]\underline{x}_t + C_{t+1}\ \underline{x}_{t+1} - F\ \underline{v}_t \in \underline{U}_t \tag{5.71}$$

Clearly, eqs. (5.64) and (5.65) can be converted directly into constraints expressed in terms of $\underline{\theta}_t$, i.e., eq. (5.64) implies

$$[0 \quad I \quad 0]\underline{\theta}_t \in \underline{R}_t \tag{5.72}$$

for month t, and eq. (5.65) (constraints on storage values) implies

$$[I \quad 0 \quad 0]\underline{\theta}_t \in \underline{X}_t \tag{5.73}$$

for month t. The appropriate equations for month t-1 are obtained in a similar manner. All the constraints, eqs. (5.61)-(5.65) and those expressed by eqs. (5.66)-(5.73), are linear. Constraints on power generation, which are intrinsically nonlinear, have been avoided by the way in which energy generation is computed, as explained in Section 5.1. In any case, nonlinear power constraints can be linearized by using a Taylor expansion (this is addressed in Chapter 7, which deals with special features and extensions).

The original two-stage optimization problem, eqs. (5.60)-(5.65), has been transformed into a quadratic programming problem, namely

$$\underset{\underline{\theta}_t}{\text{maximize}} \quad \underline{\theta}_t^T H_t^1 \underline{\theta}_t + \underline{s}_t^T \underline{\theta}_t \tag{5.74}$$

subject to

$$A_t^1 \underline{\theta}_t \leq \underline{b}_t^1 \tag{5.75}$$

$$[I \quad 0 \quad 0]\underline{\theta}_{t-1}, \ [I \quad 0 \quad 0]\underline{\theta}_{t+1} \text{ fixed}$$

in which

$$A_t^1 = \begin{bmatrix} -D_t & -\Gamma+I & 0 \\ C_t & 0 & -\Gamma+I \\ -\underline{c}^T(-D_t & -\Gamma+I & 0) \\ -\underline{c}^T(C_t & 0 & -\Gamma+I) \\ -D_t & -\Gamma & 0 \\ D_t & \Gamma & 0 \\ C_t & 0 & -\Gamma \\ -C_t & 0 & \Gamma \\ 0 & I & 0 \\ 0 & 0 & I \\ I & 0 & 0 \\ -I & 0 & 0 \end{bmatrix} \qquad (5.76)$$

$$\underline{b}_t^1 = \begin{bmatrix} \underline{W}_t - C_{t+1}\,\underline{x}_{t+1} + F\,\underline{v}_t \\ \underline{W}_{t-1} + D_t\,\underline{x}_{t-1} + F\,\underline{v}_{t-1} \\ -De_t + \underline{c}^T C_{t+1}\,\underline{x}_{t+1} - \underline{c}^T F\,\underline{v}_t \\ -De_{t-1} - \underline{c}^T D_{t-1}\,\underline{x}_{t-1} - \underline{c}^T F\,\underline{v}_{t-1} \\ \underline{U}_{t,max} - C_{t+1}\,\underline{x}_{t+1} + F\,\underline{v}_t \\ -\underline{U}_{t,min} + C_{t+1}\,\underline{x}_{t+1} - F\,\underline{v}_t \\ \underline{U}_{t-1,max} + D_{t-1}\,\underline{x}_{t-1} + F\,\underline{v}_{t-1} \\ -\underline{U}_{t-1,min} - D_{t-1}\,\underline{x}_{t-1} - F\,\underline{v}_{t-1} \\ \underline{R}_t \\ \underline{R}_{t-1} \\ \underline{X}_{t,max} \\ -\underline{X}_{t,min} \end{bmatrix} \qquad (5.77)$$

Notice that eq. (5.75) lumps into one expression all the (linear) constraints on $\underline{\theta}_t$ that arise from eqs. (5.61)-(5.65), some of which were explicitly derived earlier [see eqs. (5.67), (5.69), (5.71), (5.72), and (5.73)].

Solution of eqs. (5.74) and (5.75) by the POA would yield the optimal sequences $\{\underline{\theta}_t^*\}$ and accordingly $\{\underline{x}_t^*\}$, $\{\underline{u}_t^*\}$, and $\{\underline{r}_t^*\}$. The vector $\underline{\theta}_t$ is of dimension 3 x 9 = 27. Thus, the computational effort involved in methods such as Fletcher's (1981) active set method would be proportional to a number between $a_1(27)^2$ and $a_2(27)^3$, in which a_1 and a_2 are positive real numbers independent of the dimension of $\underline{\theta}_t$ (Gill and

Murray, 1977). The aforementioned computational burden is not limiting in a fast modern computer but it is still unpleasantly large. Storage requirements would be proportional to $(27)^2$, a reasonably low number.

To reduce the computational work, without comprimising the validity of the model, two assumptions can be made and alternative simpler models associated with those assumptions can be developed. The first assumption deals with net losses. Examination of matrices A_{t+1} and B_t in eq. (5.43) shows that their (diagonal) coefficients $1 + c_t^i$ and $1 - c_t^i$, respectively, are close to one because the values of c_t^i are in all cases less than 0.0044. This can be verified by substituting the values of c_t^i (Table 5.10) into eqs. (5.15)-(5.18). Thus, for all practical purposes, A_{t+1} and B_t can be both set equal to the identity matrix. In effect, this assumption requires the equation of continuity, eq. (5.44), to have stationary parameters, i.e.,

$$\underline{x}_{t+1} = \underline{x}_t + \Gamma_1 \underline{u}_t + \Gamma_2 \underline{r}_t + \underline{z}_t$$

$$\Rightarrow \quad u_t = \Gamma_1^{-1} \underline{x}_{t+1} - \Gamma_1^{-1} \underline{x}_t - \Gamma_1^{-1} \underline{z}_t = G\,(\underline{x}_{t+1} - \underline{x}_t - \underline{z}_t) \qquad (5.78)$$

By using this assumption, the quadratic objective function of the model becomes linear, as will be shown later. The second assumption relates to the handling of spillages. Spillages can be assumed to be at a zero level whenever reservoir storage is below the maximum permissible flood control storage. Thus, $\underline{w}_t = \underline{u}_t + \underline{r}_t$ will be expressed simply as $\underline{w}_t = \underline{u}_t$. This assumption is valid for average (normal) and below-average (dry) inflow years, as will be shown by the numerical results obtained for the NCVP. For above-average (wet) inflow years, however, spillages will occur. This is because inflows will far exceed penstock releases and spillages must be made to avoid overtopping of the dam. Thus, under this assumption, spillages can be handled by maximizing with respect to $\underline{u}_t$. When $\underline{u}_t$ reaches its upper bound during high-inflow periods, the value of $\underline{u}_t$ is set equal to penstock capacity. Any excess release necessary to keep the flood control storage at adequate levels (following flood control provisions) is called spillage. Clearly, $\underline{r}_t$ is not handled explicitly as a decision variable; however, that has little effect on the solution. This is because $\underline{r}_t$ will be zero whenever storages are within permissible levels since no benefits arise from spilling water that can be routed through penstocks. For on-line control of large flood events, where decisions are taken only minutes apart and the output of the optimization model is used by automatic mechanisms that control penstock and spillway gates, it is preferable to use the full optimization model [eqs. (5.74) and (5.75)] that takes care of penstock, spillage, and all possible constraints automatically. Even in that situation, excessively high inflows may lead to an intervention by the operator to override some constraints imposed on the model. That can happen when storages reach dangerous levels. By treating spillage in the manner described earlier, the dimension of the decision vector $\underline{\theta}_t$ [eq. (5.74)] is reduced form 27 x 1 to 9 x 1.

Simplified Linear Model

The two assumptions just described lead to a reformulation of the mathematical structure of the two-stage problem. The two-stage energy production $E_{t-1} + E_t$ can be expressed as

$$E_{t-1} + E_t = [\underline{a} + B(\underline{x}_{t-1} + \underline{x}_t)]^T \underline{u}_{t-1} + [\underline{a} + B(\underline{x}_t + \underline{x}_{t+1})]^T \underline{u}_t$$

$$= [\underline{a} + B(\underline{x}_{t-1} + \underline{x}_t)]^T [G \underline{x}_t - G \underline{x}_{t-1} - G \underline{z}_t]$$

$$+ [\underline{a} + B(\underline{x}_t + \underline{x}_{t+1})]^T [G \underline{x}_{t+1} - G \underline{x}_t - G \underline{z}_t]$$

$$= \underline{h}_t^T \underline{x}_t + c_t \tag{5.79}$$

in which

$$\underline{h}_t^T = (\underline{x}_{t-1}^T - \underline{x}_{t+1}^T)[BG - (BG)^T] - (\underline{z}_{t-1}^T + \underline{z}_t^T)(BG)^T \tag{5.80}$$

$$c_t = [-\underline{a}^T G - \underline{z}_{t+1}^T (BG)^T] \underline{x}_{t-1} - \underline{x}_{t-1}^T (BG) \underline{x}_{t-1} - \underline{a}^T G \underline{z}_{t-1} + [\underline{a}^T G$$

$$- \underline{z}_t^T (BG)^T] \underline{x}_{t+1} + \underline{x}_{t+1}^T (BG) \underline{x}_{t+1} - \underline{a}^T G \underline{z}_t = \text{constant} \tag{5.81}$$

The development of eqs. (5.79)-(5.81) used the inverse of eq. (5.76) to express $\underline{u}_t$ and $\underline{u}_{t-1}$, namely

$$\underline{u}_t = \Gamma_1^{-1}(\underline{x}_{t+1} - \underline{x}_t - \underline{z}_t) = G(\underline{x}_{t+1} - \underline{x}_t - \underline{z}_t) \tag{5.82}$$

and

$$\underline{u}_{t-1} = \Gamma_1^{-1}(\underline{x}_t - \underline{x}_{t-1} - \underline{z}_{t-1}) = G(\underline{x}_t - \underline{x}_{t-1} - \underline{z}_{t-1}) \tag{5.83}$$

Interestingly, the objective function (eq. (5.75)] has become a linear function of the state variable ($\underline{x}_t$) only. The set of constraints associated with the two-stage maximization are:

Releases during month t greater than $\underline{U}_{t,min}$

$$\underline{u}_t \geq \underline{U}_{t,min} \Rightarrow G\underline{x}_t \leq -(\underline{U}_{t,min} + G\underline{z}_t - G\underline{x}_{t+1}) \tag{5.84}$$

Releases during month t-1 greater than $\underline{U}_{t-1,min}$

$$\underline{u}_{t-1} \geq \underline{U}_{t-1,min} \Rightarrow -G\underline{x}_t \leq -(\underline{U}_{t-1,min} + G\underline{x}_{t-1} + G\underline{z}_{t-1}) \tag{5.85}$$

Releases during month t less than $\underline{U}_{t,max}$

$$\underline{u}_t \leq \underline{U}_{t,max} \Rightarrow -G\underline{x}_t \leq (\underline{U}_{t,max} - G\underline{x}_{t+1} + G\underline{z}_t) \tag{5.86}$$

Releases during month t-1 less than $\underline{U}_{t-1,max}$

$$\underline{u}_{t-1} \leq \underline{U}_{t-1,max} \Rightarrow G\underline{x}_t \leq (\underline{U}_{t-1,max} + G\underline{x}_{t-1} + G\underline{z}_{t-1}) \tag{5.87}$$

Delta requirements during month t

$$\underline{c}^T\underline{u}_t \geq De_t \Rightarrow \underline{c}^T G\ \underline{x}_t \leq -(De_t - \underline{c}^T G\ \underline{x}_{t+1} + \underline{c}^T G\ \underline{z}_t) \tag{5.88}$$

where $\underline{c}^T = (0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 1)$, from Fig. 5.6.

Delta requirements during month t-1

$$\underline{c}^T \underline{u}_{t-1} \geq De_{t-1} \Rightarrow - \underline{c}^T G \, \underline{x}_t \leq - (De_{t-1} + \underline{c}^T G \, \underline{x}_{t-1} + \underline{c}^T G \, \underline{z}_{t-1}) \qquad (5.89)$$

Minimum storages for month t

$$\underline{x}_t \geq \underline{X}_{t,min} \qquad (5.90)$$

Maximum storages for month t

$$\underline{x}_t \leq \underline{X}_{t,max} \qquad (5.91)$$

Notice that the continuity equations for months t and t-1, respectively,

$$\underline{x}_{t+1} = \underline{x}_t + \Gamma_1 \, \underline{u}_t + \underline{z}_t \qquad (5.92)$$

and

$$\underline{x}_t = \underline{x}_{t-1} + \Gamma_1 \, \underline{u}_{t-1} + \underline{z}_{t-1} \qquad (5.93)$$

have been considered in eq. (5.79), i.e., the continuity equation was used to develop the objective function. Also, the rates of energy generation used in developing eq. (5.79) [see eqs. (5.1)-(5.9)] take into account the effects of head and flow in the generation of power so that power constraints are not present in the constraint set. In addition, minimum power delivery (set at 860 Mw dependable capacity in the NCVP) is extremely low and results in a redundant constraint. It is stressed that power constraints, when relevant, can always be linearized by a Taylor expansion. Linearization methods are treated in Chapter 7.

In summary, the full model [eqs. (5.74) and (5.75)] for the two-stage maximization has been simplified to the following linear programming model (dropping the constant term c_t):

$$\underset{\underline{x}_t}{\text{maximize}} \quad \underline{h}_t^T \, \underline{x}_t \tag{5.94}$$

subject to

$$A \, \underline{x}_t \leqq \underline{b}_t \tag{5.95}$$

$\underline{x}_{t+1}$, $\underline{x}_{t-1}$ fixed

in which

$$A = \begin{bmatrix} G \\ -G \\ -G \\ G \\ \underline{c}^T G \\ -\underline{c}^T G \\ -I \\ I \end{bmatrix} \tag{5.96}$$

$$\underline{b}_t = \begin{bmatrix} -\underline{U}_{t,min} - G\underline{z}_t + G\underline{x}_{t+1} \\ -\underline{U}_{t-1,min} - G\underline{x}_{t-1} - G\underline{z}_{t-1} \\ \underline{U}_{t,max} - G\underline{x}_{t+1} + G\underline{z}_t \\ \underline{U}_{t-1,max} + G\underline{x}_{t+1} + G\underline{z}_{t-1} \\ -De_t + \underline{c}^T G\underline{x}_{t+1} - \underline{c}^T G\underline{z}_t \\ -De_{t-1} - \underline{c}^T G\underline{x}_{t-1} - \underline{c}^T G\underline{z}_{t-1} \\ -\underline{X}_{t,min} \\ \underline{X}_{t,max} \end{bmatrix} \tag{5.97}$$

Notice that eqs. (5.94) and (5.95) are expressed in terms of a 9-dimensional unknown vector, $\underline{x}_t$. Thus, the two fundamental assumptions discussed previously (neglect net losses and treat spillage as excess over penstock capacity) have reduced the computational burden considerably. Equations (5.94) and (5.95) will be solved sequentially by using the POA. Once the optimal sequence $\{\underline{x}_t^*\}$ is known, the optimal release policies $\{\underline{u}_t^*\}$ can be readily obtained through the continuity equation, eq. (5.78).

Simplified Quadratic Model 1

Consider the case in which net losses are taken into account (i.e., the first assumption used in developing eqs. (5.94) and (5.95) is relaxed), but the spillage assumption is maintained. In this case, the two-stage maximization problem becomes a quadratic maximization problem with a 9-dimensional decision vector:

$$\underset{\underline{x}_t}{\text{maximize}}\ \underline{g}_t^T\,\underline{x}_t + \underline{x}_t^T\,G_t\,\underline{x}_t + k_t \tag{5.98}$$

subject to

$$A_t^2\,\underline{x}_t \leq \underline{b}_t^2 \tag{5.99}$$

$\underline{x}_{t+1}$, $\underline{x}_{t-1}$ fixed

in which

$$A_t^2 = \begin{bmatrix} D_t \\ -C_t \\ -D_t \\ C_t \\ \underline{c}^T\,D_t \\ -\underline{c}^T\,C_t \\ -I \\ I \end{bmatrix} \tag{5.100}$$

$$\underline{b}_t^2 = \begin{bmatrix} -\underline{U}_{t,min} + C_{t+1}\,\underline{x}_{t+1} - F\underline{v}_t \\ -\underline{U}_{t-1,min} - D_{t-1}\,\underline{x}_{t-1} - F\underline{v}_{t-1} \\ \underline{U}_{t,max} - C_{t+1}\,\underline{x}_{t+1} + F\underline{v}_t \\ \underline{U}_{t-1,max} + D_{t-1}\,\underline{x}_{t-1} + F\underline{v}_{t-1} \\ -De_t + \underline{c}^T C_{t+1}\,\underline{x}_{t+1} + \underline{c}^T F\underline{v}_t \\ -De_{t-1} - \underline{c}^T D_{t-1}\,\underline{x}_{t-1} - \underline{c}^T F\underline{v}_{t-1} \\ -\underline{X}_{t,min} \\ \underline{X}_{t,max} \end{bmatrix} \tag{5.101}$$

where k_t is defined by eq. (5.54), g_t and G_t are defined by eqs. (5.52) and (5.53), respectively, and other terms are defined by eqs. (5.84)-(5.91).

Simplified Quadratic Model 2

If net losses are not considered but spillages are included explicitly in the continuity equation, then the two-stage problem becomes

$$\underset{\underline{\theta}_t}{\text{maximize}}\ \underline{\theta}_t^T H\,\underline{\theta}_t + \hat{\underline{s}}_t\,\underline{\theta}_t + k_t' \tag{5.102}$$

subject to

$$A_3\,\dot{\underline{\theta}}_t \leq \underline{b}_t^3 \tag{5.103}$$

$$[I \quad 0 \quad 0]\,\underline{\theta}_{t-1},\ [I \quad 0 \quad 0]\,\underline{\theta}_{t+1}\ \text{fixed}$$

in which

$$\underline{\theta}_t^T = [\underline{x}_t^T,\ \underline{r}_t^T,\ \underline{r}_{t-1}^T] \tag{5.104}$$

$$H = \frac{1}{2}\begin{bmatrix} 0 & -B\Gamma & -B\Gamma \\ -\Gamma^T B & 0 & 0 \\ -\Gamma^T B & 0 & 0 \end{bmatrix} \tag{5.105}$$

$$\underline{\hat{s}}_t = [\underline{\hat{g}}_t,\ \underline{\hat{q}}_t,\ \underline{\hat{p}}_t] \tag{5.106}$$

$$\underline{\hat{g}}_t = \underline{x}_{t-1}^T\ (BG - G^T B) + \underline{x}_{t+1}^T\ (G^T B - BG) - (\underline{z}_t^T + \underline{z}_{t-1}^T)\ G^T B \tag{5.107}$$

$$\underline{\hat{q}}_t = -\ \underline{a}^T\ \Gamma - \underline{x}_{t+1}^T\ B\ \Gamma \tag{5.108}$$

$$\underline{\hat{p}}_t = -\ \underline{a}^T\ \Gamma - \underline{x}_{t-1}^T\ B\ \Gamma \tag{5.109}$$

$$\begin{aligned} k_t' = &-\ \underline{a}^T\ (-\Gamma_1^{-1}\ \underline{x}_{t+1} + \Gamma_1^{-1}\ \underline{x}_{t-1} + F\ \underline{v}_{t-1} + F\ \underline{v}_t) \\ &+ \underline{x}_{t+1}^T\ B\ \Gamma_1^{-1}\ \underline{x}_{t+1} - \underline{x}_{t-1}\ B\ \Gamma_1^{-1}\ \underline{x}_{t-1} - \underline{x}_{t+1}^T\ B\ F\ \underline{v}_t \\ &- \underline{x}_{t-1}\ B\ F\ \underline{v}_{t-1} \end{aligned} \tag{5.110}$$

$$= \text{constant}$$

$$A_3 = \begin{bmatrix} -G & -\Gamma+I & 0 \\ G & 0 & -\Gamma+I \\ -\underline{c}^T(-G & -\Gamma+I & 0) \\ -\underline{c}^T(G & 0 & -\Gamma+I) \\ -G & -\Gamma & 0 \\ G & \Gamma & 0 \\ G & 0 & -\Gamma \\ -G & 0 & \Gamma \\ 0 & I & 0 \\ 0 & 0 & I \\ I & 0 & 0 \\ -I & 0 & 0 \end{bmatrix} \qquad (5.111)$$

$$\underline{b}_t^3 = \begin{bmatrix} \underline{W}_t - G\,\underline{x}_{t+1} + F\,\underline{v}_t \\ \underline{W}_{t-1} + G\,\underline{x}_{t-1} + F\,\underline{v}_{t-1} \\ -De_t + \underline{c}^T G\,\underline{x}_{t+1} - \underline{c}^T F\,\underline{v}_t \\ -De_{t-1} - \underline{c}^T G\,\underline{x}_{t-1} - \underline{c}^T F\,\underline{v}_{t-1} \\ \underline{U}_{t,max} - G\,\underline{x}_{t+1} + F\,\underline{v}_t \\ -\underline{U}_{t,min} + G\,\underline{x}_{t+1} - F\,\underline{v}_t \\ \underline{U}_{t-1,max} + G\,\underline{x}_{t-1} + F\,\underline{v}_{t-1} \\ -\underline{U}_{t-1,min} - G\,\underline{x}_{t-1} - F\,\underline{v}_{t-1} \\ \underline{R}_t \\ \underline{R}_{t-1} \\ \underline{X}_{t,max} \\ -\underline{X}_{t,min} \end{bmatrix} \qquad (5.112)$$

and 0 is a 9 x 9 null matrix. Notice that a quadratic programming problem is obtained, although there is no quadratic term on $\underline{x}_t$ as is evident from the matrix H. Also, H is a constant matrix as opposed to the time-variant matrix H_t^1 of eq. (5.74). Because this development resembles the one that led to the model given by eqs. (5.74) and (5.75), the particular constraints of the constraint set (5.103) are obtained in forms analogous to those in eqs. (5.67), (5.71), (5.72), and (5.73). Table 5.15 summarizes the four alternative versions of the optimization model.

The alternative two-stage maximization problems developed in this section are not exclusive against each other; the different formulations

Table 5.15. Versions of the Optimization Model.

Optimization Model	Assumption		Equations
	Neglect Net Losses of Evaporation, Seepage, and Direct Rainfall to the Reservoir	Spillages treated as Excesses over Penstock Capacity	
Full Model	No	No	(5.74), (5.75)
Linear Model	Yes	Yes	(5.94), (5.95)
Quadratic Model 1	No	Yes	(5.98), (5.99)
Quadratic Model 2	Yes	No	(5.102), (5.103)

can be used for different months according to the validity of the assumptions built into each of the formulations. For example, because spillages are not likely to occur during the summer season, either the linear model or the quadratic model 1 can be used to perform the two-stage maximization. During flood periods, when it is advisable to include spillages as part of the decision variables, the two-stage problem can be solved by using either the full model or the quadratic model 2. From a numerical point of view, the linear model is the most attractive due to the well understood nature of linear programming. Chapter 6 presents some results obtained from the use of the simplified linear model developed in this section. Chapter 7 gives an application of a more general quadratic model. The quadratic case introduces a series of computational complexities that can be resolved by using the methods in Section 7.8.

5.4 Selection of Initial Operation Policy

Two considerations are usually important in developing an initial operation policy: (i) the computational effort should be relatively small as compared to the total computational work necessary to develop an optimal policy; and (ii) the initial policy should lie within the "radius of convergence" about a global optimum if it exists, and it should be "good" enough so as to keep the number of iterations required for convergence to a local optimum relatively low. For tightly constrained systems, the second condition is always met. In fact, for the problem given by eqs. (5.94) and (5.95), any feasible policy will always converge to a global optimum, as will be discussed in Chapter 6. The simplest, and usually the best, way to develop an initial operation policy is by a trial-and-error approach based on previous operational experience of the system under different scenarios. That approach was followed in this study, with the cooperation of the staff of the U.S. Bureau of Reclamation at Sacramento. There are, of course, other approaches to develop initial operation policies (see, e.g., Yeh et al., 1978). Figure 5.15 shows a flow chart of the NCVP optimization model.

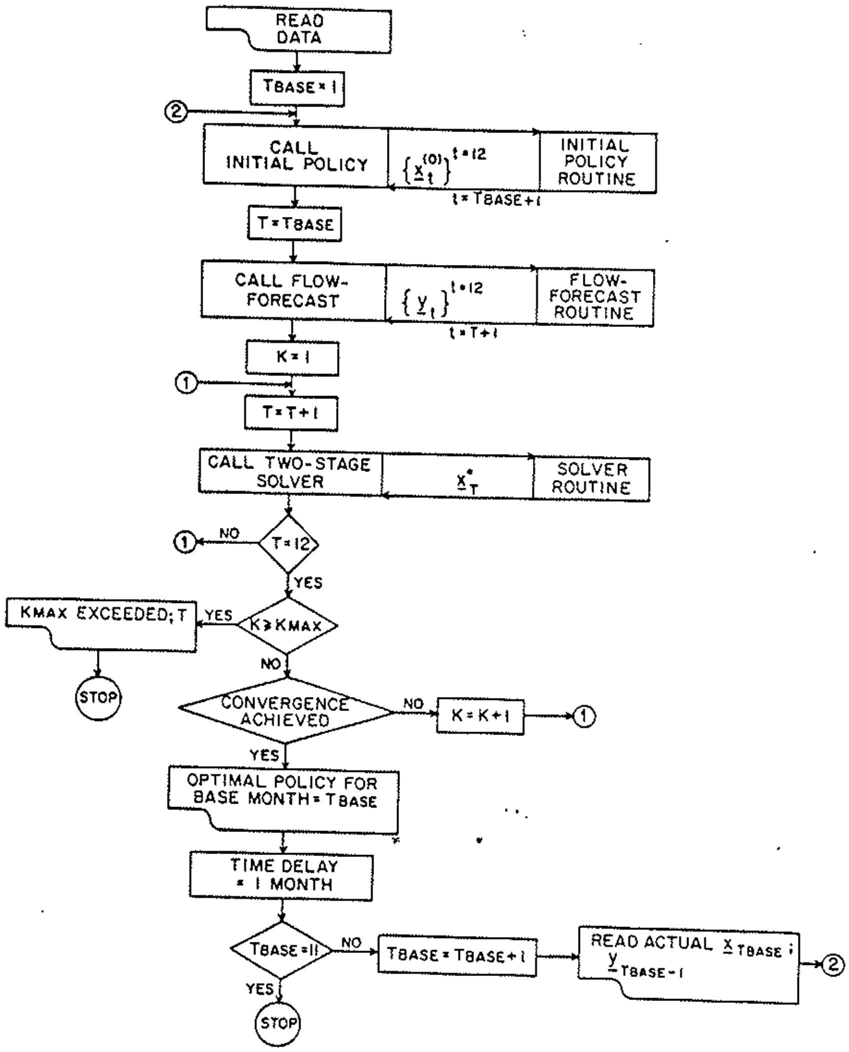


Fig. 5.15 NCVP monthly optimization flow chart.

CHAPTER 6

DISCUSSION OF RESULTS

This chapter discusses the application of the optimization model, developed in Chapter 5, to the NCVP system under three different scenarios: average (1974-75, 1979-80), below-average (1975-76), and above-average (1973-74) streamflow conditions. Section 6.1 gives sets of initial storage and release policies for the aforementioned streamflow conditions. Section 6.2 discusses the corresponding optimal release policies. On the basis of those results, Section 6.3 shows that the operation of the NCVP system can be analyzed by using a simplified optimization model. The optimal operation policies discussed in this chapter were computed by using the simplified linear model given by eqs. (5.94) and (5.95). Application of the models of full dimensionality (i.e., handling releases explicitly) and introduction of nonlinear energy rates and constraints are given in Chapter 7.

6.1 Initial Policies

Initial operating policies for the NCVP system were developed by using a trial-and-error procedure that considers some heuristic criteria used by NCVP managers to set up their release policies. In essence, desired reservoir storages at the end of the water year are selected and a feasible (initial) release policy that achieves those targets is chosen. As the system operation progresses through the year, actual flow conditions may lead to a revision of the ending storages selected initially. The overall philosophy is that reservoir storages must be kept high at the beginning of the dry season (usually, May) to meet increasing agricultural and Delta water requirements during the summer.

Also, the operation during the rainy season (November-March) is conservative in the sense that a substantial flood storage volume is allocated to store eventual large-runoff events. Clearly, there is a tradeoff between the desire to maintain the reservoir levels below some specified elevation during the rainy season and the desire to have as large a storage volume as possible at the beginning of the dry season. A general rule would be to maintain reservoir storages at the maximum permissible levels during the rainy season and to make large releases during the dry season. Interestingly, because most power installations in the NCVP are of the high-head type, a greater generation of power will not result from the largest releases but from some optimal reservoir elevation associated with moderate releases. The largest releases would drive reservoir levels below the range at which turbines can operate efficiently.

Two initial policies (policies I and II) were selected for storage and corresponding releases. This was done to determine if each initial policy yields the same optimal release policy. As will be discussed in Section 6.2, different initial policies generally yield different optimal release policies. However, all those optimal release policies give the same value for the objective function of the model (i.e., the same annual energy generation), thus indicating the existence of multiple optimal solutions. That is a common phenomenon in linear programming and convex (as opposed to strictly convex) quadratic problems.

Tables 6.1-6.14 show initial storage policies and corresponding initial release policies for the NCVP system. There are two initial policies (policies I and II) for water years with average (1974-75,

Table 6.1. Initial Storage Policy, 1974-75 (Policy I).

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1600	14.7	241	3000	23.8	600	8.8	1650	57
Nov	1506	14.7	241	2870	23.8	533	8.8	1567	57
Dec	1423	14.7	241	2769	23.8	473	8.8	1504	57
Jan	1361	14.7	241	2728	23.8	409	8.8	1438	57
Feb	1307	14.7	241	2672	23.8	387	8.8	1381	57
Mar	1317	14.7	241	2922	23.8	457	8.8	1367	57
Apr	1482	14.7	241	3723	23.8	580	8.8	1383	57
May	1501	14.7	241	3947	23.8	621	8.8	1302	57
Jun	1744	14.7	241	4040	23.8	827	8.8	1363	57
Jul	1814	14.7	241	3854	23.8	875	8.8	1461	57
Aug	1680	14.7	241	3491	23.8	764	8.8	1330	57
Sep	1499	14.7	241	3097	23.8	684	8.8	1165	57
Oct	1358	14.7	241	2755	23.8	617	8.8	995	57

Storages in Kaf. Streamflow conditions for water year 1974-75 were average (see inflow data in Appendix A).

Table 6.2. Initial Release Policy, 1974-75 (Policy I).

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	100	82	92	400	492	180	177	100	100
Nov	100	82	83	400	483	180	177	100	100
Dec	100	82	92	400	492	180	177	100	100
Jan	100	82	94	400	494	180	177	100	100
Feb	100	82	136	600	736	200	197	100	100
Mar	95	77	214	600	814	300	297	100	100
Apr	150	132	188	600	788	300	297	100	100
May	150	132	163	700	863	300	297	100	100
Jun	200	173	193	650	843	300	297	100	100
Jul	200	170	179	650	829	250	247	100	100
Aug	200	170	177	650	827	200	197	100	100
Sep	150	123	128	600	728	200	197	100	100

Releases in Kaf.

Table 6.3. Initial Storage Policy, 1974-75 (Policy II).

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1600	14.7	241	3000	23.8	600	8.8	1650	57
Nov	1470	14.7	241	2720	23.8	528	8.8	1567	57
Dec	1351	14.7	241	2469	23.8	463	8.8	1504	57
Jan	1253	14.7	241	2278	23.8	394	8.8	1438	57
Feb	1163	14.7	241	2072	23.8	367	8.8	1381	57
Mar	1137	14.7	241	2372	23.8	452	8.8	1367	57
Apr	1261	14.7	241	3223	23.8	690	8.8	1383	57
May	1294	14.7	241	3497	23.8	755	8.8	1302	57
Jun	1551	14.7	241	3740	23.8	961	8.8	1363	57
Jul	1685	14.7	241	3654	23.8	1009	8.8	1461	57
Aug	1615	14.7	241	3391	23.8	920	8.8	1330	57
Sep	1498	14.7	241	3097	23.8	764	8.8	1165	57
Oct	1358	14.7	241	2755	23.8	617	8.8	995	57

Storages in Kaf.

Table 6.4. Initial Release Policy, 1974-75 (Policy II).

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	136	118	128	550	678	185	182	100	100
Nov	136	118	119	550	699	185	182	100	100
Dec	136	118	128	550	678	185	182	100	100
Jan	136	118	130	550	680	185	182	100	100
Feb	136	118	172	550	722	185	182	100	100
Mar	136	118	255	550	805	185	182	100	100
Apr	136	118	174	550	724	276	273	100	100
May	136	118	149	550	699	300	297	100	100
Jun	136	109	129	550	688	300	297	100	100
Jul	136	106	117	550	677	228	225	100	100
Aug	136	106	113	550	675	276	273	100	100
Sep	149	109	111	600	723	280	277	100	100

Releases in Kaf.

Table 6.5. Initial Storage Policy, 1979-80 (Policy I).

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1632.0	14.7	241	3035	23.8	673.0	8.8	1600	60
Nov	1570.2	14.7	241	2708	23.8	472.0	8.8	1419	57
Dec	1550.6	14.7	241	2546	23.8	282.1	8.8	1254	57
Jan	1545.8	14.7	241	2293	23.8	127.0	8.8	1105	57
Feb	1692.2	14.7	241	2834	22.8	663.9	8.8	1183	57
Mar	1908.3	14.7	241	3536	22.8	925.5	8.8	1246	60
Apr	1974.7	14.7	241	3722	22.8	950.4	8.8	1184	61
May	2071.4	14.7	241	3790	22.8	903.0	8.8	1068	67
Jun	2172.7	14.7	241	3790	22.8	859.7	8.8	1048	67
Jul	2078.1	14.7	241	3561	22.8	896.7	8.8	982	67
Aug	1926.5	14.7	241	3297	22.8	876.5	8.8	800	67
Sep	1735.1	14.7	241	3002	22.8	785.6	8.8	536	67
Oct	1542.6	14.7	241	2622	22.8	624.8	8.8	323	57

Storages in Kaf. Streamflow conditions for water year 1979-80 were above average (see inflow data in Appendix A).

Table 6.6. Initial Release Policy, 1979-80 (Policy I).

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	100	82	84	600	684	290	287	200	103
Nov	100	82	85	500	585	290	187	200	100
Dec	90	72	79	600	679	300	297	200	100
Jan	90	72	83	700	784	610	290	200	100
Feb	90	72	96	700	856	610	290	200	97
Mar	90	72	117	700	817	610	290	200	99
Apr	100	82	105	500	605	610	290	200	94
May	100	82	98	426	524	610	290	200	100
Jun	200	173	177	500	677	200	197	200	100
Jul	200	170	175	500	675	200	197	200	100
Aug	200	170	174	500	674	200	197	200	100
Sep	200	173	175	600	775	290	287	150	60

Releases in Kaf. Folsom releases during Feb-May include a spillage of 160 Kaf. Excesses over penstock capacity at Tullock are spilled.

Table 6.7. Initial Storage Policy, 1979-80 (Policy II).

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1632.0	14.7	241	3035	23.8	673.0	8.8	1600	60
Nov	1520.2	10.7	150	2808	15.8	472.0	8.8	1469	50
Dec	1450.6	10.7	150	2446	15.8	282.1	8.8	1304	50
Jan	1385.8	10.7	150	2093	15.8	127.0	8.8	1205	40
Feb	1522.2	10.7	150	2634	15.8	663.9	8.8	1283	57
Mar	1728.3	10.7	150	3336	15.8	925.5	8.8	1346	47
Apr	1784.7	10.7	150	3522	15.8	950.4	8.8	1284	57
May	1831.4	10.7	150	3390	15.8	903.0	8.8	1168	57
Jun	1882.7	10.7	150	3216	15.8	859.7	8.8	1138	57
Jul	1888.1	14.7	146	3187	22.8	896.7	8.8	1052	67
Aug	1836.5	14.7	144	2983	22.8	876.5	8.8	860	67
Sep	1745.1	14.7	141	2888	22.8	785.6	8.8	586	67
Oct	1542.6	14.7	241	2622	22.8	624.8	8.8	323	57

Storages in Kaf.

Table 6.8. Initial Release Policy, 1979-80 (Policy II).

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	150	136	229	500	737	290	287	150	60
Nov	150	132	135	700	835	290	287	200	100
Dec	150	132	139	700	839	300	297	150	60
Jan	100	82	93	700	793	610	290	200	83
Feb	100	82	106	700	806	610	290	200	110
Mar	100	82	127	700	827	610	290	200	90
Apr	150	132	155	700	855	610	290	200	100
May	150	132	148	600	748	610	290	210	110
Jun	100	69	77	300	370	200	197	220	110
Jul	100	70	77	440	517	200	197	210	110
Aug	100	70	77	300	377	200	197	210	110
Sep	210	183	85	486	571	290	287	200	110

Releases in Kaf. Folsom releases during Feb-May include a spillage of 150 Kaf.
Excesses over penstock capacity at Tullock are spilled.

Table 6.9. Initial Storage Policy, 1975-76 (Policy I).

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1358	14.7	241	2755	23.8	617	8.8	995	57
Nov	1280	14.7	241	2781	23.8	642	8.8	970	57
Dec	1238	14.7	241	2778	23.8	636	8.8	957	57
Jan	1194	14.7	241	2772	23.8	652	8.8	951	57
Feb	1128	14.7	241	2754	23.8	652	8.8	944	57
Mar	1083	14.7	241	2820	23.8	686	8.8	934	57
Apr	1058	14.7	241	2956	23.8	740	8.8	905	57
May	1085	14.7	241	3047	23.8	723	8.8	819	57
Jun	1162	14.7	241	2852	23.8	712	8.8	721	57
Jul	1062	14.7	241	2576	23.8	654	8.8	621	57
Aug	934	14.7	241	2191	23.8	617	8.8	521	57
Sep	803	14.7	241	1939	23.8	583	8.8	421	57
Oct	713	14.7	241	1866	23.8	558	8.8	321	57

Storages in Kaf. Streamflow conditions for water year 1975-76 were below average (see inflow data in Appendix A).

Table 6.10. Initial Release Policy, 1975-76 (Policy I).

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	100	82	91	300	391	100	97	50	50
Nov	90	72	80	300	380	100	97	50	50
Dec	100	82	85	300	385	100	97	50	50
Jan	100	82	87	300	387	100	97	50	50
Feb	100	82	95	300	395	100	97	50	50
Mar	100	82	96	300	396	100	97	50	50
Apr	100	82	104	300	404	100	97	50	50
May	100	82	85	500	585	100	97	50	50
Jun	150	123	128	500	628	100	97	50	50
Jul	150	120	124	600	724	100	97	50	50
Aug	150	120	126	500	626	100	97	50	50
Sep	100	73	78	300	378	100	97	50	50

Releases in Kaf.

Table 6.11. Initial Storage Policy, 1975-76 (Policy II).

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1358	14.7	241	2755	23.8	617	8.8	995	57
Nov	1290	14.7	241	2781	23.8	642	8.8	970	57
Dec	1248	14.7	241	2778	23.8	636	8.8	957	57
Jan	1214	14.7	239	2772	23.8	652	8.8	951	57
Feb	1158	14.7	239	2754	23.8	652	8.8	944	57
Mar	1123	14.7	241	2820	23.8	686	8.8	934	57
Apr	1108	14.7	241	2956	23.8	740	8.8	905	57
May	1102	14.7	241	2947	23.8	723	8.8	819	57
Jun	1146	14.7	241	2852	23.8	712	8.8	721	57
Jul	1063	14.7	241	2576	23.8	654	8.8	621	57
Aug	952	14.7	241	2241	23.8	617	3.3	521	57
Sep	838	14.7	241	2039	23.8	583	8.8	421	57
Oct	713	14.7	241	1866	23.8	558	8.8	321	57

Storages in Kaf.

Table 6.12. Initial Release Policy, 1975-76 (Policy II).

Month	CLair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	90	72	81	300	381	100	97	50	50
Nov	90	72	80	300	380	100	97	50	50
Dec	90	72	77	300	377	100	97	50	50
Jan	90	72	77	300	377	100	97	50	50
Feb	90	72	83	300	383	100	97	50	50
Mar	90	72	86	300	386	100	97	50	50
Apr	133	115	137	400	537	100	97	50	50
May	133	115	118	400	558	100	97	50	50
Jun	133	106	121	500	621	100	97	50	50
Jul	133	103	107	550	657	100	102	50	50
Aug	133	103	109	450	559	100	92	50	50
Sep	135	108	113	400	513	100	97	50	50

Releases in Kaf.

Table 6.13. Initial Storage Policy, 1973-74 (Policy I).

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1500	14.7	241	2950	23.8	600	8.8	1400	60
Nov	1343	14.7	241	2593	23.8	389	8.8	1160	57
Dec	1546	14.7	241	3307	23.8	337	8.8	989	57
Jan	1587	14.7	241	3508	23.8	383	8.8	853	57
Feb	1912	14.7	241	4400	23.8	664	8.8	757	57
Mar	1824	14.7	241	4013	23.8	697	8.8	685	60
Apr	1924	14.7	241	4400	23.8	994	8.8	703	61
May	1972	14.7	241	4400	23.8	1010	8.8	713	67
Jun	2135	14.7	241	4084	23.8	1010	8.8	817	67
Jul	2195	14.7	241	3666	23.8	861	8.8	799	67
Aug	2060	14.7	241	3342	23.8	711	8.8	653	67
Sep	1873	14.7	241	3019	23.8	641	8.8	491	67
Oct	1673	14.7	241	2690	23.8	616	8.8	318	67

Storages in Kaf.

Table 6.14. Initial Release Policy, 1973-74 (Policy I).

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	200	182	191	669	860	300	297	250	253
Nov	209	191	296	863	1131	400	397	250	250
Dec	209	191	277	982	1250	400	397	250	250
Jan	209	191	351	1207	1475	400	397	250	250
Feb	209	191	246	1082	1659	250	247	150	147
Mar	209	191	331	1199	1659	400	397	150	149
Apr	209	191	281	1198	1466	574	571	110	103
May	209	191	226	982	1208	493	473	100	100
Jun	209	182	202	832	1034	476	473	110	100
Jul	209	179	189	650	839	325	322	100	100
Aug	209	179	188	600	788	200	197	100	100
Sep	209	182	189	600	789	150	147	100	107

Releases in Kaf. Releases that exceed penstock capacity include spillage.

1979-80) and below-average (1975-76) inflows. A single initial policy was considered for water year 1973-74, representing above-average inflows. Development of the initial policies indicated that for below-average streamflow conditions there is little room for optimizing the operation of the system, because prevailing low inflows barely meet the system's demands by releasing flows near their minimum permissible values. Thus, initial release policies I and II for 1975-76 turn out to be close to optimal, as will become evident in Section 6.2. For both average and above-average streamflow conditions, there is a larger feasible region and the gains from the optimization model can be significant. During the winter months of an extremely wet year such as 1973-74, the initial policy is nearly optimal because the reservoirs are at near capacity during those months and total releases are set equal to maximum permissible flows. In those circumstances, the optimization model allows the determination of the best feasible release policy that simultaneously minimizes the spillage and maximizes the power generation. Because the reservoirs are at a high stage after the winter, substantial improvements in energy generation can be obtained during the subsequent summer months.

Some of the initial policies were refined so as to make them near possible optimal releases whereas others were deliberately set to be poor (but feasible) initial estimates. This was done to estimate the number of iterations and CPU time needed by the POA to reach optimality. Inital policies I for average (1974-75, 1979-80) and below-average (1975-76) inflow years were carefully refined, attempting to be near their respective optimal policies. In those cases, convergence to the optimum was attained in six to eight iterations. In contrast, initial policies II for average-inflow years 1974-75 and 1979-80 were purposely developed to be far from good initial policies. That was accomplished by releasing heavily during wet (winter) months to maintain a year-round low head and a corresponding decrease in power generation. That is also nonoptimal from the standpoint of agricultural and Delta requirements, because those demands are low in the winter and thus larger than necessary flows will be of no use. This strategy forces summer releases to be at minimum permissible levels, when an additional acre-foot of water during this season has a greater marginal value than in the rainy season. Those deliberately-poor initial policies resulted in an increase in the number of iterations needed to attain convergence, ranging now from eight to ten iterations. Table 6.15 summarizes the required iterations and CPU times for the specified initial policies and inflow conditions. It is evident that the CPU time increases as the inflow conditions vary from below-average to average. That is because, for below-average flow conditions, the feasible region becomes so tight that there is little freedom to optimize any policy. Any feasible initial policy will be very close to an optimal release policy. As flow volumes increase to average-flow conditions, there is a corresponding increase in CPU time. Notice that policies II for average-flow years 1974-75 and 1979-80 (which were deliberately chosen to be inferior to their counterparts, policies I) also required more CPU time. For extremely wet conditions such as water year 1973-74, the feasible region becomes very tight during the winter and that implies a reduction in CPU time as shown in Table 6.15.

Table 6.15. Number of Iterations to Attain Convergence and CPU Time Requirements.

Inflow Condition	Policy	No. of Iterations to Attain Convergence	Burroughs B7800 CPU Time (min.)
Average (1974-75)	I	6	6.01
	II	9	8.94
Average (1979-80)	I	8	8.51
	II	10	10.32
Below Average (1975-76)	I	3	2.98
	II	3	2.79
Above Average (1973-74)	I	8	7.28

6.2 Optimal Operation Policies

Optimal state trajectories (i.e., end of month storages) and their corresponding release policies were obtained by applying the POA to the initial policies of Section 6.1. Tables 6.16 and 6.17 show the optimal strategies for average-flow conditions (1974-75) corresponding to the first initial policy (policy I, Tables 6.1 and 6.2). Table 6.17 also shows the energy produced by the optimal policy as well as the monthly water deliveries to the Delta. Tables 6.18 and 6.19 show similar information corresponding to the second initial policy (policy II, Tables 6.3 and 6.4). Clearly, for Clair Engle, Shasta, and New Melones reservoirs, the optimal state policies (end-of-month storages and releases) resulting from initial policy I are different from those resulting from initial policy II. For Folsom reservoir, the optimal end-of-month storages and release policies are the same for initial policies I and II. For the remaining five smaller reservoirs, the end-of-month storages are the same for initial policies I and II, but their release policies are different. The fact that optimal state trajectories (call them State I and State II) are equal does not imply that their corresponding optimal release policies (policy I and policy II, respectively) will be the same. Optimal release policies will be the same only if the optimal state trajectories are equal for all reservoirs in the system. Tables 6.17 and 6.19 also show that the value of the objective function of the model (total energy generated during the year) is the same for release policies I and II. This implies that there are multiple ways of achieving the optimum performance index. That is not surprising because the problem under analysis is in essence a large LP problem posed as a dynamic process and solved

Table 6.16. Optimal State Trajectory Corresponding to Initial Policy I, 1974-75.

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1600.0	14.7	241	3000.0	23.8	600	8.8	1650	57
Nov	1516.7	14.7	241	2970.0	23.8	533	8.8	1567	57
Dec	1439.7	14.7	241	2969.0	23.8	473	8.8	1504	57
Jan	1388.4	14.7	241	3028.0	23.8	409	8.8	1438	57
Feb	1225.4	14.7	241	3004.6	23.8	387	8.8	1381	57
Mar	1209.3	14.7	241	3153.7	23.8	477	8.8	1367	60
Apr	1380.0	14.7	241	3900.0	23.8	720	8.8	1383	61
May	1340.0	14.7	241	4200.0	23.8	762	8.8	1302	67
Jun	1524.0	14.7	241	4485.0	23.8	965	8.8	1363	67
Jul	1585.0	14.7	241	4288.0	23.8	1010	8.8	1461	67
Aug	1529.0	14.7	241	3813.0	23.8	969	8.8	1330	67
Sep	1448.0	14.7	241	3283.0	23.8	787	8.8	1165	67
Oct	1358.0	14.7	241	2755.0	23.8	617	8.8	995	57

Storages in Kaf.

Table 6.17. Optimal Energy Production, Release Policy, and Delta Releases Corresponding to Initial Policy I, 1974-75.

Month	Clair Engle		Lewiston		Whiskeytown		Shasta		Keswick	
	Energy	Release	Energy	Release	Energy	Release	Energy	Release	Energy	Release
Oct	31675.3	89.3	41940.8	71.3	45901.5	81.3	111572.1	300.0	36063.4	381.3
Nov	32696.0	94.0	44705.5	76.0	43473.7	77.0	111357.3	300.0	35656.7	377.0
Dec	30569.7	89.3	41940.8	71.3	45901.5	81.3	111759.2	300.0	36063.4	381.3
Jan	69624.6	209.0	112351.9	191.0	114612.6	203.0	137169.9	367.4	53948.4	570.4
Feb	41039.1	126.1	63587.7	108.1	91520.7	162.1	263718.3	700.9	81622.5	863.0
Mar	29654.8	89.3	41940.8	71.3	117604.9	208.3	259876.9	654.7	81622.5	863.0
Apr	70576.8	209.0	112351.9	191.0	139454.7	247.0	220661.6	524.0	72921.2	771.0
May	71867.9	209.0	112351.9	191.0	125339.9	222.0	220788.7	508.0	69043.4	730.0
Jun	74064.6	209.0	107057.9	182.0	114048.0	202.0	288629.8	661.0	81622.5	863.0
Jul	43260.1	122.0	54117.2	92.0	57024.0	101.0	320903.4	762.0	81622.5	863.0
Aug	34871.3	100.0	41176.1	70.0	43473.7	77.0	312763.2	786.0	81622.5	863.0
Sep	33796.4	99.0	42352.6	72.0	43473.7	77.0	293553.6	786.0	81622.5	863.0
Total	563696.5		815875.1		981829.0		2652754.2		793431.5	

Table 6.17. (continued)

Month	Folsom		Natoma		New Melones		Tullock		Delta
	Energy	Release	Energy	Release	Energy	Release	Energy	Release	Release
Oct	48416.4	180.0	5901.2	177.0	46584.6	100.0	11802.4	100.0	658.3
Nov	47044.8	180.0	5901.2	177.0	45686.7	100.0	11802.4	100.0	654.0
Dec	45705.6	180.0	5901.2	177.0	44893.3	100.0	11802.4	100.0	658.3
Jan	44776.8	180.0	5901.2	177.0	44136.9	100.0	11802.4	100.0	847.4
Feb	45511.2	180.0	5901.2	177.0	43700.2	100.0	11583.9	97.0	1137.0
Mar	49107.6	180.0	5901.2	177.0	43712.5	100.0	12007.3	99.0	1139.0
Apr	86686.1	299.0	9868.6	296.0	43312.8	100.0	11707.5	94.0	1161.0
May	92299.9	303.0	10002.0	300.0	43189.8	100.0	12734.4	100.0	1130.0
Jun	96808.5	303.0	10002.0	300.0	44167.6	100.0	12734.4	100.0	1263.0
Jul	57553.2	180.0	5901.2	177.0	43964.7	100.0	12734.4	100.0	1140.0
Aug	92520.7	302.0	9968.7	299.0	42144.2	100.0	12734.4	100.0	1262.0
Sep	86427.7	303.0	10002.0	300.0	40084.0	100.0	13495.2	110.0	1273.0
Total	792858.5		91151.7		525577.1		146941.2		12323.0

Energy in Mwh and releases in Kaf. Delta releases are the sum of Keswick, Natoma, and Tullock releases. Spillages at Lewiston and Whiskeytown to satisfy fish requirements are as given in Table 5.4. Total annual energy equals 7,364,114.8 Mwh.

Table 6.18. Optimal State Trajectory Corresponding to Initial Policy II, 1974-75.

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1600.0	14.7	241.0	3000.0	23.8	600.0	8.8	1650.0	57.0
Nov	1516.7	14.7	241.0	2970.0	23.8	533.0	8.8	1617.0	57.0
Dec	1330.6	14.7	241.0	2969.0	23.8	473.0	8.8	1574.0	57.0
Jan	1159.6	14.7	241.0	3028.0	23.8	409.0	8.8	1508.0	57.0
Feb	996.6	14.7	241.0	2974.3	23.8	387.0	8.8	1441.0	57.0
Mar	950.2	14.7	241.0	3153.7	23.8	477.0	8.8	1414.0	60.0
Apr	1120.9	14.7	241.0	3900.0	23.8	720.0	8.8	1419.0	61.0
May	1200.6	14.7	241.0	4200.0	23.8	762.0	8.8	1322.0	67.0
Jun	1457.0	14.7	241.0	4552.0	23.8	965.0	8.8	1373.0	67.0
Jul	1561.0	14.7	241.0	4312.0	23.8	1010.0	8.8	1461.0	67.0
Aug	1529.0	14.7	241.0	3813.0	23.8	969.0	8.8	1320.0	67.0
Sep	1448.0	14.7	241.0	3283.0	23.8	787.0	8.8	1145.0	67.0
Oct	1358.0	14.7	241.0	2755.0	23.8	617.0	8.8	995.0	57.0

Storages in Kaf.

Table 6.19. Optimal Energy Production, Release Policy, and Delta Releases Corresponding to Initial Policy II, 1974-75.

Month	Clair Engle		Lewiston		Whiskeytown		Shasta		Keswick	
	Energy	Release	Energy	Release	Energy	Release	Energy	Release	Energy	Release
Oct	31675.3	89.3	41940.8	71.3	45901.5	81.3	11572.1	300.0	36063.4	381.3
Nov	69693.6	203.1	108881.4	185.1	105070.9	186.1	111357.3	300.0	45975.3	486.1
Dec	68516.4	209.0	112351.9	191.0	113483.4	201.0	111759.2	300.0	47384.6	501.0
Jan	65521.7	209.0	112351.9	191.0	114612.6	203.0	148204.2	397.7	56814.2	600.7
Feb	47626.6	156.4	81411.0	138.4	108627.9	192.4	251848.4	670.6	81622.5	863.0
Mar	27669.6	89.3	41940.8	71.3	117604.9	208.3	259876.9	654.7	81622.5	863.0
Apr	28628.9	89.3	41940.8	71.3	71872.8	127.3	220661.6	524.0	61600.0	651.3
May	45762.5	136.6	69764.1	118.6	84463.3	149.6	192351.5	441.0	55858.9	590.6
Jun	58178.4	166.0	81764.0	139.0	89770.4	159.0	308885.9	704.0	81622.5	863.0
Jul	34649.0	98.0	39999.6	68.0	43473.7	77.0	331446.4	786.0	81622.5	863.0
Aug	34871.3	100.0	41176.1	70.0	43473.7	77.0	312763.2	786.0	81622.5	863.0
Sep	33796.4	99.0	42352.6	72.0	43473.7	77.0	293553.6	786.0	81622.5	863.0
Total	546589.6		815875.1		981829.0		2654280.3		793431.5	

Table 6.19. (continued)

Month	Folsom		Natoma		New Melones		Tullock		Delta
	Energy	Release	Energy	Release	Energy	Release	Energy	Release	Release
Oct	48416.4	180.0	5894.1	177.0	23446.0	50.0	5901.2	50.0	608.3
Nov	47044.8	180.0	5894.1	177.0	37139.7	80.0	9441.9	80.0	743.1
Dec	45705.6	180.0	5894.1	177.0	45754.3	100.0	11802.4	100.0	778.0
Jan	44776.8	180.0	5894.1	177.0	49430.0	110.0	12982.6	110.0	887.7
Feb	45511.2	180.0	5894.1	177.0	50124.8	113.0	13136.4	110.0	1150.0
Mar	49107.6	180.0	5894.1	177.0	49087.5	111.0	13341.5	110.0	1150.0
Apr	86686.1	299.0	9856.8	296.0	50642.3	116.0	13700.3	110.0	1057.3
May	92299.9	303.0	9990.0	300.0	47711.7	110.0	14007.8	110.0	1000.6
Jun	96808.5	303.0	9990.0	300.0	48652.0	110.0	14007.8	110.0	1273.0
Jul	57553.2	180.0	5894.1	177.0	48293.5	110.0	14007.8	110.0	1150.0
Aug	92520.7	302.0	9956.7	299.0	46155.7	110.0	14007.8	110.0	1272.0
Sep	86427.7	303.0	9990.0	300.0	31968.8	80.0	11041.6	90.0	1253.0
Total	792858.5		91042.2		528406.3		147379.2		12323.0

Energy in Mwh and releases in Kaf. Delta releases are the sum of Keswick, Natoma, and Tullock releases. Spillages at Lewiston and Whiskeytown to satisfy fish requirements are as given in Table 5.4. Total annual energy equals 7,351,691.7 Mwh.

sequentially by the POA. This is also common in convex quadratic problems.

It was found that an optimal release policy is achieved by releasing less water than the maximum possible penstock capacity. Because hydropower production depends on the storage level (the larger the head, the greater the energy production for a given discharge), an optimal release policy is a feasible tradeoff point between a high head and a small release and a low head and a large release. Such a tradeoff point is the optimal solution given by the POA. Because the power installations in the NCVP are of the high-head type, except Nimbus (at Natoma) and Keswick, the tradeoff point is shifted towards a relatively high head with a moderate discharge. Figure 6.1 shows a plot of energy production vs. release for Folsom reservoir. Ths plot was developed by assuming an initial reservoir storage of 800 Kaf and by releasing a constant amount of water until the storage reached 200 Kaf. Similar energy vs. release relations can be developed for the other reservoirs. If, in addition to hydropower, other benefits of water to downstream users would have been included in the energy vs. release relation, then it is reasonable to expect larger releases. Rows of Tables 6.17 and 6.19 corresponding to Clair Engle reservoir illustrate that releases are relatively moderate as compared to the total penstock capacity of the reservoir given in Table 5.2.

It is evident from the results in Tables 6.17 and 6.19 that all releases are passed through the penstocks, and spillages do not occur. That presumes, of course, that all the power plants are in operation 100 percent of the time (an optimistic presumption). In practice, idle power installations can be included in the model by varying penstock

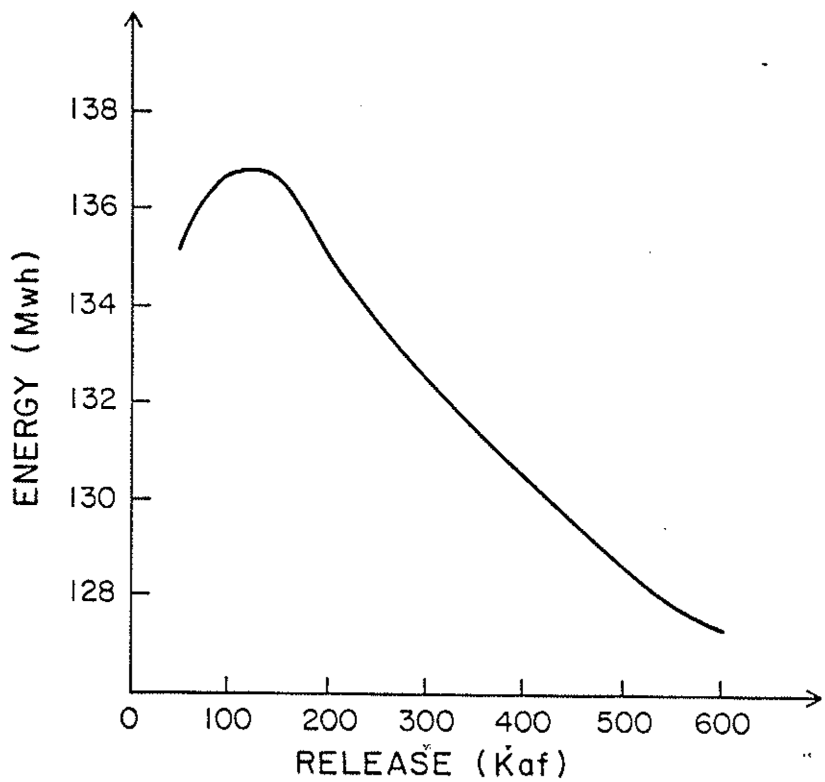


Fig. 6.1 Typical energy vs. release curve (developed by using Folsom reservoir data).

capacity in the constraint set, as required by operational conditions. The point is that if conditions were ideal, Tables 6.16-6.19 would provide two alternative optimal ways to operate the system. That establishes an upper bound in the performance of the system (as measured by power generation) towards which any adopted optimal policy would be aimed.

Tables 6.20-6.23 show optimal strategies for average-flow year 1979-80. Although the solutions are different for initial policies I and II, they yield the same total energy for the year. Inflow during 1979-80 is greater than in average-flow year 1974-75 and consequently, total energy production is also greater in 1979-80 (compare Tables 6.21 and 6.23 with Tables 6.17 and 6.19). Results in Tables 6.21 and 6.23 also show that spillages occured at Lake Natoma and Tullock when annual inflows increased from 1206 Kaf in 1974-75 to 1734 Kaf in 1979-80. Figure 6.2 shows a graphical relation of total annual inflow and total annual energy for the system.

For water year 1975-76 with below-average streamflows, initial policies I and II are near optimal policies. Because of tight feasible-region conditions, the benefits from running the optimization model are marginal. Initial policies I and II yield the same optimal state and release policies. Tables 6.24 and 6.25 show the optimal end-of-month storages and releases resulting from initial policy I. The gain in energy production associated with the optimal policies relative to the energy production associated with both initial policies I and II is about 1 percent.

From the results in Tables 6.16-6.25 it is evident that the optimal state trajectories for the smaller reservoirs are to keep them full all

Table 6.20. Optimal State Trajectory Corresponding to Initial Policy I, 1979-80.

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1632.0	14.7	241.0	3035.0	23.8	673.0	8.8	1600.0	60.0
Nov	1577.2	14.7	241.0	3008.0	23.8	567.6	8.8	1462.0	57.0
Dec	1547.8	14.7	241.0	3046.0	23.8	364.7	8.8	1337.0	57.0
Jan	1543.7	14.7	178.1	2729.0	23.8	206.6	8.8	1185.0	57.0
Feb	1690.8	14.7	183.4	3184.0	23.8	733.5	8.8	1253.0	57.0
Mar	1907.6	14.7	201.7	3800.0	23.8	985.1	8.8	1303.0	60.0
Apr	1974.7	14.7	241.0	3900.0	23.8	1000.0	8.8	1230.0	61.0
May	2080.4	14.7	241.0	4168.0	23.8	1010.0	8.8	1098.0	67.0
Jun	2072.7	14.7	241.0	4294.0	23.8	1010.0	8.8	1068.0	67.0
Jul	1969.1	14.7	241.0	4134.0	23.8	1010.0	8.8	992.0	67.0
Aug	1808.5	14.7	241.0	3691.0	23.8	992.5	8.8	800.0	67.0
Sep	1637.1	14.7	241.0	3187.0	23.8	798.6	8.8	536.0	67.0
Oct	1542.6	14.7	241.0	2622.0	22.8	624.8	8.8	323.0	57.0

Storages in Kaf.

Table 6.21. Optimal Energy Production, Release Policy, and Delta Releases Corresponding to Initial Policy I, 1979-80.

Month	Clair Engle		Lewiston		Whiskeytown		Shasta		Keswick	
	Energy	Release	Energy	Release	Energy	Release	Energy	Release	Energy	Release
Oct	33356.7	93.0	44117.3	75.0	43473.7	77.0	112078.0	300.0	35664.2	377.0
Nov	38985.9	109.8	53999.5	91.8	53523.5	94.8	112154.2	300.0	37348.1	394.8
Dec	31578.7	89.3	41940.8	71.3	77793.4	141.2	243955.3	664.0	76171.9	805.2
Jan	32126.6	89.3	41940.8	71.3	41460.3	77.0	291284.0	786.0	81639.8	863.0
Feb	33520.7	89.3	41940.8	71.3	41854.6	77.0	310729.7	786.0	81639.8	863.0
Mar	34608.3	89.3	41940.8	71.3	42817.1	77.0	323729.8	786.0	81639.8	863.0
Apr	35941.7	91.0	41940.8	73.0	54201.0	96.0	126111.2	300.0	37461.6	396.0
May	83426.1	209.0	112351.8	191.0	116871.0	207.0	128841.7	300.0	47962.2	507.0
Jun	82428.2	209.0	107057.9	182.0	105014.5	186.0	184764.0	431.0	58368.2	617.0
Jul	80059.3	209.0	105293.2	179.0	103885.3	184.0	281620.3	679.0	81639.8	863.0
Aug	66386.9	180.0	88234.5	150.0	86947.5	154.0	278553.2	709.0	81639.8	863.0
Sep	36455.7	102.0	44117.3	75.0	43473.7	77.0	289027.5	785.0	81639.8	863.0
Total	588874.9		764875.5		811315.6		2682848.9		782815.0	

Table 6.21. (continued)

Month	Folsom		Natoma		New Melones		Tullock		Delta
	Energy	Release	Energy	Release	Energy	Release	Energy	Release	Release
Oct	53544.8	194.0	6373.6	191.4	71641.1	157.0	7165.3	60.0	628.4
Nov	77852.2	303.0	9990.0	300.0	70422.2	160.0	7081.4	60.0	754.8
Dec	71289.2	303.0	9990.0	300.0	85889.9	203.0	12156.5	103.0	1208.2
Jan	118406.8	460.0	9990.0	300.0	87766.8	210.0	12982.6	110.0	1273.0
Feb	139893.4	460.0	9990.0	300.0	90566.3	213.0	13136.4	110.0	1273.0
Mar	147248.8	460.0	9990.0	300.0	89417.5	211.0	13341.5	110.0	1273.0
Apr	126260.2	392.6	7745.6	232.6	88813.2	216.0	13700.3	110.0	738.6
May	131038.7	406.7	8215.1	246.7	84253.9	210.0	14007.8	110.0	863.7
Jun	76361.4	237.0	7792.2	234.0	82884.9	210.0	14007.8	110.0	961.0
Jul	63362.9	197.3	6470.2	194.3	79423.7	210.0	14007.8	110.0	1167.3
Aug	93465.2	303.0	9990.0	300.0	70032.8	200.0	12734.4	100.0	1263.0
Sep	86780.4	303.0	9990.0	300.0	48124.3	150.0	7361.0	60.0	1223.0
Total	1185503.9		106526.7		949236.5		141683.0		12627.0

Energy in Mwh and releases in Kaf. Delta releases are the sum of Keswick, Natoma, and Tullock releases. In addition to the above penstock releases, spillages are 160 Kaf at Lake Natoma during Jan-Apr and 100 Kaf at Tullock during Oct-Sep. Total annual energy equals 8,013,680.0 Mwh.

Table 6.22. Optimal State Trajectory Corresponding to Initial Policy II, 1979-80.

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1632.0	14.7	241.0	3035.0	23.8	673.0	8.8	1600.0	60.0
Nov	1461.2	14.7	241.0	3008.0	23.8	567.6	8.8	1462.0	57.0
Dec	1332.6	14.7	241.0	3007.6	23.8	364.7	8.8	1337.0	57.0
Jan	1232.3	14.7	241.0	2666.1	23.8	206.6	8.8	1228.0	57.0
Feb	1379.4	14.7	183.4	3184.0	23.8	733.5	8.8	1313.0	57.0
Mar	1596.2	14.7	201.7	3800.0	23.8	985.1	8.8	1363.0	60.0
Apr	1663.3	14.7	241.0	3900.0	23.8	1000.0	8.8	1290.0	61.0
May	1770.7	14.7	241.0	4168.0	23.8	1010.0	8.8	1158.0	67.0
Jun	1882.7	14.7	241.0	4294.0	23.8	1010.0	8.8	1128.0	67.0
Jul	1789.1	14.7	241.0	4265.0	23.8	1010.0	8.8	1052.0	67.0
Aug	1731.5	14.7	241.0	3768.0	23.8	992.5	8.8	860.0	67.0
Sep	1637.1	14.7	241.0	3187.0	23.8	798.6	8.8	586.0	67.0
Oct	1542.6	14.7	241.0	2622.0	22.8	624.8	8.8	323.0	57.0

Storages in Kaf.

Table 6.23. Optimal Energy Production, Release Policy, and Delta Releases Corresponding to Initial Policy II, 1979-80.

Month	Clair Engle		Lewiston		Whiskeytown		Shasta		Keswick	
	Energy	Release	Energy	Release	Energy	Release	Energy	Release	Energy	Release
Oct	73922.9	209.0	112351.9	191.0	108966.6	193.0	112078.0	300.0	46637.8	493.0
Nov	71238.5	209.0	112351.9	191.0	109531.2	194.0	126209.8	338.4	50365.0	532.4
Dec	61406.8	185.5	98528.5	167.5	98521.7	174.5	251345.5	688.5	81639.8	863.0
Jan	29740.6	89.3	41940.8	71.3	77238.1	139.9	266923.2	723.1	81639.8	863.0
Feb	31134.7	89.3	41940.8	71.3	41854.6	77.0	310729.7	786.0	81639.8	863.0
Mar	32222.3	89.3	41940.8	71.3	42817.1	77.0	323729.8	786.0	81639.8	863.0
Apr	32890.9	89.3	41940.8	71.3	53241.2	94.3	126111.2	300.0	37300.8	394.3
May	33731.4	89.3	41940.8	71.3	49289.1	87.3	128841.7	300.0	36638.6	387.3
Jun	75325.5	199.0	101175.6	172.0	99368.5	176.0	129513.9	300.0	45029.6	476.0
Jul	39435.6	106.0	44705.5	76.0	45732.1	81.0	307539.2	733.0	77004.4	814.0
Aug	37647.8	103.0	42940.8	73.0	43473.7	77.0	310203.2	786.0	81639.8	863.0
Sep	36455.7	102.0	44117.3	75.0	43473.7	77.0	289027.5	785.0	81639.8	863.0
Total	555152.9		765875.5		813507.7		2682252.6		782815.0	

Table 6.23. (continued)

Month	Folsom		Natoma		New Melones		Tullock		Delta
	Energy	Release	Energy	Release	Energy	Release	Energy	Release	Release
Oct	53544.8	194.4	6373.6	191.4	71641.1	157.0	7165.3	60.0	744.4
Nov	77852.2	303.0	9990.0	300.0	70422.2	160.0	7081.4	60.0	892.4
Dec	71289.2	303.0	9990.0	300.0	68119.6	160.0	7081.4	60.0	1223.0
Jan	118406.8	460.0	9990.0	300.0	81884.4	193.0	10976.2	93.0	1256.0
Feb	139893.4	460.0	9990.0	300.0	92138.3	213.0	13136.4	110.0	1273.0
Mar	147248.8	460.0	9990.0	300.0	90974.7	211.0	13341.5	110.0	1273.0
Apr	126260.2	392.6	7745.6	232.6	90407.2	216.0	13700.3	110.0	736.4
May	131038.7	406.7	8215.1	246.7	85803.7	210.0	14007.8	110.0	744.0
Jun	76361.4	237.0	7792.2	234.0	84434.7	210.0	14007.8	110.0	820.0
Jul	63362.9	197.3	6470.2	194.3	80973.5	210.0	14007.8	110.0	1118.3
Aug	93465.2	303.0	9990.0	300.0	74955.1	210.0	14007.8	110.0	1273.0
Sep	86780.4	303.0	9990.0	300.0	64780.7	200.0	13495.2	110.0	1273.0
Total	1185503.9		106526.7		956535.1		142009.2		12626.5

Energy in Mwh and releases in Kaf. Delta releases are the sum of Keswick, Natoma, and Tullock releases. In addition to the above penstock releases, spillages are 160 Kaf at Lake Natoma during Jan-Apr and 100 Kaf at Tullock during Oct-Sep. Total annual energy equals 7,990,178.6 Mwh.

INFLOW TO RESERVOIRS

YEAR	NEW MELONES	SHASTA	FOLSOM	CLAIR ENGLE	WHISKEYTOWN	TOTALS INFLOW	TOTALS ENERGY
1973-74	1498	10 796	4408	2672	771	20146	11.14
1974-75	1206	6405	2786	1408	394	12198	7.35
1975-76	470	3611	1142	695	139	6057	4.53
1979-80	1734	6415	3972	1471	344	13936	8.01

INFLOWS IN 10^3 ACRE FEET
ENERGY IN 10^6 MWH

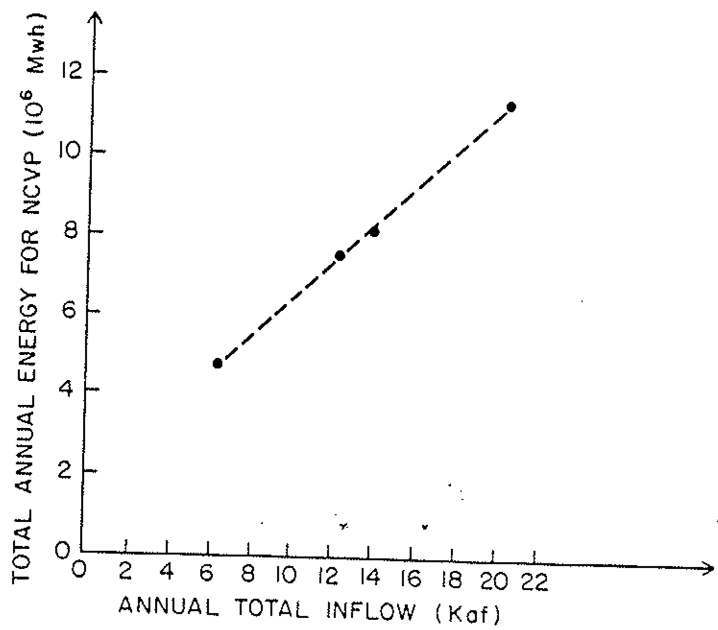


Fig. 6.2 Total annual energy vs. total annual inflow for the NCVP.

Table 6.24. Optimal State Trajectory Corresponding to Initial Policies I and II, 1975-76.

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1358.0	14.7	241.0	2755.0	23.8	617.0	8.8	995.0	57.0
Nov	1290.7	14.7	241.0	2781.0	23.8	642.0	8.8	970.0	57.0
Dec	1249.4	14.7	241.0	2778.0	23.8	636.0	8.8	957.0	57.0
Jan	1213.4	14.7	241.0	2772.0	23.8	652.0	8.8	951.0	57.0
Feb	1157.4	14.7	241.0	2754.0	23.8	652.0	8.8	944.0	57.0
Mar	1123.1	14.7	241.0	2820.0	23.8	686.0	8.8	934.0	57.0
Apr	1108.8	14.7	241.0	2956.0	23.8	740.0	8.8	905.0	57.0
May	1146.5	14.7	241.0	3047.0	23.8	723.0	8.8	819.0	57.0
Jun	1164.5	14.7	241.0	3052.0	23.8	712.0	8.8	721.0	57.0
Jul	1005.5	14.7	241.0	2952.0	23.8	654.0	8.8	621.0	57.0
Aug	884.0	14.7	241.0	2622.5	23.8	617.0	8.8	521.0	57.0
Sep	802.0	14.7	241.0	2425.0	23.8	583.0	8.8	421.0	57.0
Oct	713.0	14.7	241.0	1866.0	23.8	558.0	8.8	321.0	57.0

Storages in Kaf.

Table 6.25. Optimal Energy Production, Release Policy, and Delta Releases Corresponding to Initial Policies I and II, 1975-76.

Month	Clair Engle		Lewiston		Whiskeytown		Shasta		Keswick	
	Energy	Release	Energy	Release	Energy	Release	Energy	Release	Energy	Release
Oct	29882.4	89.3	41940.8	71.3	45336.9	80.3	108564.5	300.0	35976.4	380.3
Nov	29466.3	89.3	41940.8	71.3	44772.3	79.3	108723.9	300.0	35881.8	379.3
Dec	30052.2	92.0	43529.0	74.0	43473.7	77.0	108661.5	300.0	35664.2	377.0
Jan	29043.7	90.0	42352.6	72.0	43473.7	77.0	108495.2	300.0	35664.2	377.0
Feb	28471.8	89.3	41940.8	71.3	47595.3	84.3	108827.8	300.0	36354.8	384.3
Mar	28285.6	89.3	41940.8	71.3	48159.9	85.3	110227.7	300.0	36449.4	385.3
Apr	28375.3	89.3	41940.8	71.3	52676.6	93.3	111800.8	300.0	37206.2	393.3
May	50902.6	159.0	82940.4	141.0	81301.5	144.0	112466.1	300.0	42002.4	444.0
Jun	65645.4	209.0	107057.8	182.0	105579.1	187.0	120752.3	324.0	48340.6	511.0
Jul	43345.5	143.5	66764.1	113.5	66339.8	117.5	197528.8	544.5	62625.2	662.0
Aug	29626.3	101.0	41764.3	71.0	43473.7	77.0	156191.1	445.5	49428.5	522.5
Sep	28313.4	99.0	42352.6	72.0	43473.7	77.0	261834.0	786.0	81639.8	863.0
Total	421410.5		636464.8		665656.3		1614073.6		537233.5	

Table 6.25. (continued)

Month	Folsom		Natoma		New Melones		Tullock		Delta
	Energy	Release	Energy	Release	Energy	Release	Energy	Release	Release
Oct	27654.0	100.0	3230.1	97.0	19442.4	50.0	5901.2	50.0	527.3
Nov	27768.0	100.0	3230.1	97.0	19325.5	50.0	5901.2	50.0	526.3
Dec	27828.0	100.0	3230.1	97.0	19267.1	50.0	5901.2	50.0	524.0
Jan	27924.0	100.0	3230.1	97.0	19227.1	50.0	5901.2	50.0	524.0
Feb	28128.0	100.0	3230.1	97.0	19174.8	50.0	5901.2	50.0	531.3
Mar	28656.0	100.0	3230.1	97.0	19054.9	50.0	5901.2	50.0	532.3
Apr	28878.0	100.0	3230.1	97.0	18701.3	50.0	5901.2	50.0	540.3
May	28710.0	100.0	3230.1	97.0	18135.5	50.0	5901.2	50.0	591.3
Jun	28296.0	100.0	3230.1	97.0	17526.7	50.0	5901.2	50.0	658.0
Jul	27726.0	100.0	3230.1	97.0	16911.6	50.0	5901.2	50.0	809.0
Aug	27300.0	100.0	3230.1	97.0	16296.7	50.0	5901.2	50.0	669.5
Sep	26946.0	100.0	3230.1	97.0	15681.7	50.0	5901.2	50.0	1010.0
Total	335814.0		38761.2		218745.3		70814.4		7443.3

Energy in Mwh and releases in Kaf. Delta releases are the sum of Keswick, Natoma, and Tullock releases. Spillages at Lewiston and Whiskeytown to satisfy fish requirements are as given in Table 5.4. Total annual energy equals 4,538,973.6 Mwh.

year. That stems from the ratio of the capacities of the major reservoirs to their corresponding downstream regulating reservoirs. The largest capacity ratio of the system is 241/2448 = 10%, corresponding to Clair Engle and Whiskeytown reservoirs. When a capacity ratio becomes less than the largest capacity ratio of the system, all the state variables corresponding to downstream, smaller, regulating reservoirs can be treated as constant and equal to the maximum capacity of the regulating reservoirs. Those nodes in the network can thus be treated as transmission points only. In that manner, the optimization model would be considerably simplified, as shown in Chapter 7. The number of state variables in the NCVP system would be reduced from nine to four: Clair Engle, Shasta, Folsom, and New Melones. Care must be taken in reformulating the model in terms of the reduced number of state variables because the constraints that hold for the operation of the smaller reservoirs must still be satisfied. For example, if constraints representing penstock and spillage capacities are not observed, releases from Shasta reservoir could wash away Keswick reservoir.

Tables 6.26 and 6.27 show the optimal strategies for above-average flow year 1973-74. Notice that substantial spillage occurs in the optimal release policies. Also, the higher storage levels and greater releases that occur in this year result in an increased total energy production, as shown in Fig. 6.2. The energy vs. volume of inflow curve, a fairly straight curve, is applicable to the range of inflow volumes depicted in Fig. 6.2. For total annual inflow volumes smaller than 6 Kaf or greater than 22 Kaf, the performance characteristics of the plant installations would deviate the curve from a straight line.

Table 6.26. Optimal State Trajectory, 1973-74.

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1500.0	14.7	241.0	2950.0	23.8	600.0	8.8	1400.0	60.0
Nov	1334.0	14.7	241.0	2602.0	23.8	442.0	8.8	1203.0	57.0
Dec	1546.0	14.7	241.0	3307.0	23.8	390.0	8.8	1039.0	57.0
Jan	1587.0	14.7	241.0	3508.0	23.8	436.0	8.8	893.0	57.0
Feb	1912.0	14.7	241.0	4400.0	23.8	717.0	8.8	787.0	57.0
Mar	1824.0	14.7	241.0	4013.0	23.8	697.0	8.8	712.0	60.0
Apr	1924.0	14.7	241.0	4400.0	23.8	994.0	8.8	729.0	61.0
May	1972.0	14.7	241.0	4400.0	23.8	1010.0	8.8	733.0	67.0
Jun	2135.0	14.7	241.0	4257.0	23.8	1010.0	8.8	827.0	67.0
Jul	2195.0	14.7	241.0	3839.0	23.8	1010.0	8.8	809.0	67.0
Aug	2060.0	14.7	241.0	3491.0	23.8	967.0	8.8	653.0	67.0
Sep	1873.0	14.7	241.0	3093.0	23.8	794.0	8.8	491.0	67.0
Oct	1673.0	14.7	241.0	2690.0	23.8	616.0	8.8	318.0	60.0

Storages in Kaf.

Table 6.27. Optimal Energy Production, Release Policy, and Delta Releases, 1973-74.

Month	Clair Engle		Lewiston		Whiskeytown		Shasta		Keswick	
	Energy	Release	Energy	Release	Energy	Release	Energy	Release	Energy	Release
Oct	71598.9	209.0	112351.9	191.0	112918.8	200.0	239085.8	660.0	81356.0	860.0
Nov	68910.4	200.0	107057.9	182.0	146229.8	259.0	323074.2	872.0	81639.8	863.0
Dec	74279.8	209.0	112351.9	191.0	151311.2	268.0	384380.8	982.0	81639.8	863.0
Jan	77561.4	209.0	112351.9	191.0	151311.2	268.0	409174.7	982.0	81639.8	863.0
Feb	79686.3	209.0	112351.9	191.0	138890.1	246.0	420630.2	982.0	81639.8	863.0
Mar	79793.9	209.0	112351.9	191.0	151311.2	268.0	420630.2	982.0	81639.8	863.0
Apr	81120.9	209.0	112351.9	191.0	151311.2	268.0	429409.0	982.0	81639.8	863.0
May	83012.8	209.0	112351.9	191.0	127598.2	226.0	351087.2	809.0	65274.0	690.0
Jun	85012.2	209.0	107057.8	182.0	114048.0	202.0	350286.6	832.0	81639.8	863.0
Jul	84339.8	209.0	105293.2	179.0	106708.3	189.0	271839.7	674.0	81639.8	863.0
Aug	81452.7	209.0	105293.2	179.0	106143.7	188.0	260611.0	675.0	81639.8	863.0
Sep	77982.8	209.0	107057.9	182.0	106708.3	189.0	247753.8	674.0	81639.8	863.0
Total	944751.9		1318223.4		1564490.0		4107963.1		963028.0	

Table 6.27. (continued)

Month	Folsom		Natoma		New Melones		Tullock		Delta
	Energy	Release	Energy	Release	Energy	Release	Energy	Release	Release
Oct	65089.4	247.0	8125.2	244.0	88613.5	207.0	7165.3	60.0	1164.0
Nov	100368.0	400.0	9990.0	300.0	98629.6	243.0	10976.2	93.0	1256.0
Dec	100224.0	400.0	9990.0	300.0	100572.7	260.0	12982.6	110.0	1273.0
Jan	108072.0	400.0	9990.0	300.0	96543.2	260.0	12982.6	110.0	1273.0
Feb	86609.5	303.0	9990.0	300.0	55108.8	153.0	13136.4	110.0	1273.0
Mar	120984.0	400.0	9990.0	300.0	53849.8	151.0	13341.5	110.0	1273.0
Apr	152910.2	476.0	9990.0	300.0	41517.9	116.0	13700.3	110.0	1273.0
May	153367.2	476.0	9990.0	300.0	40033.4	110.0	14007.8	110.0	1100.0
Jun	105359.4	327.0	5028.3	151.0	40547.5	110.0	14007.8	110.0	1124.0
Jul	69677.2	218.0	6426.9	193.0	39370.4	110.0	14007.8	110.0	1166.0
Aug	92918.0	303.0	9990.0	300.0	33835.6	100.0	12734.4	100.0	1263.0
Sep	86536.8	303.0	9990.0	300.0	31775.4	100.0	13276.8	107.0	1270.0
Total	1242115.7		109480.4		720397.9		152319.7		14708.0

Energy in Mwh and releases in Kaf. Delta releases are the sum of Keswick, Natoma, and Tullock releases. In addition to the above penstock releases, spillages are: i) Whiskeytown: 28 Kaf (Nov), 9 Kaf (Dec), 83 Kaf (Jan), 63 Kaf (Mar), 13 Kaf (Apr); ii) Shasta: 225 Kaf (Jan), 100 Kaf (Feb), 409 Kaf (Mar), 216 Kaf (Apr); iii) Keswick: 268 Kaf (Nov), 387 Kaf (Dec, Jan, Mar, Apr), 365 Kaf (Feb), 345 Kaf (May), 171 Kaf (Jun); iv) Folsom: 98 Kaf (Apr), 17 Kaf (May); v) Natoma: 97 Kaf (Nov, Dec, Jan, Mar), 271 Kaf (Apr), 173 Kaf (May, Jun), 22 Kaf (Jul); and vi) Tullock: 150 Kaf (Oct, Nov, Dec, Jan), 40 Kaf (Feb, Mar). Total annual energy equals 11,122,770.1 Mwh.

Table 6.28 summarizes the energy production levels obtained for water year 1979-80. The ratio of actual energy (E_a) and maximized energy (E_m) varies from 29 percent at New Melones Power Plant to 72 percent at Shasta Power Plant. Those ratios should be interpreted as an indication of the opportunity that exists to improve energy generation rather than as a true measure of the suboptimality of actual policies. That is because actual operation of a power plant considers idle time due to maintenance and breakdowns, which according to some studies (e.g., Tudor Engineering Co., 1980) varies from 10 to 40 percent of the total 8760 hours per year. It was not possible to model power installation failures due to (i) the randomness of their occurrence and cause, (ii) the time that the breakdowns last, and (iii) the varying effect of power plant halts in the integrated energy network. Poor and limited records on past failures add to the difficulty of including failures into the optimization model. During idle time, releases are merely spillage and energy generation does not occur. For example, at New Melones there were three months of complete halt in the operation of the power plant and, in addition, legal battles kept the reservoir from being filled completely, which also affected the actual power production adversely. Recall that the optimized results were obtained on the assumption that the plants were on-line 100 percent of the time, which is the ideal situation. Thus, uncertain factors that affect the operation of the power plants (e.g., failures, repairs, maintenance, and even institutional constraints) make practically impossible to assess the true state of subutilization of the system. By assuming that the E_a/E_m ratio is a reasonable estimate of the annual availability factor (the

Table 6.28. Actual and Maximized Energy Production (in 10^3 Mwh) for Water Year 1979-80.

Month	Trinity Power Plant at Clair Engle Lake		Judge Francis Carr Power Plant		Spring Creek Power Plant		Shasta Power Plant		Keswick Power Plant		Folsom Power Plant	
	Actual	Max.	Actual	Max.	Actual	Max.	Actual	Max.	Actual	Max.	Actual	Max.
Oct	30.4	73.9	36.8	112.4	42.3	109.0	76.5	112.1	19.0	46.6	37.7	53.5
Nov	9.6	71.2	4.8	112.4	19.5	109.5	89.6	126.2	20.7	50.3	41.3	77.9
Dec	17.1	61.4	15.4	98.5	19.2	98.5	111.1	251.3	23.2	81.6	37.2	71.3
Jan	6.2	29.7	*	41.9	23.9	77.2	237.4	266.9	47.6	81.6	107.0	118.4
Feb	17.7	31.1	3.4	41.9	55.3	41.8	209.9	310.7	39.2	81.6	84.3	139.9
Mar	73.4	32.2	78.2	41.9	99.5	42.8	228.3	323.7	49.9	81.6	130.0	147.2
Apr	44.9	32.9	53.5	41.9	54.7	53.2	112.9	126.1	29.0	37.3	99.8	126.3
May	21.2	33.7	21.2	41.9	18.6	49.3	164.4	128.8	33.7	36.6	82.0	131.0
Jun	51.5	75.3	55.2	101.2	59.6	99.4	212.9	129.5	47.3	45.0	66.6	76.4
Jul	54.3	39.4	57.2	44.7	57.2	45.7	237.9	307.5	52.2	77.0	84.4	63.4
Aug	75.7	37.6	87.2	42.9	85.9	43.5	165.7	310.2	43.6	81.6	35.6	93.5
Sep	71.1	36.5	84.7	44.2	88.6	43.5	86.8	289.0	29.3	81.6	40.5	86.8
Total	473.10	817.5	497.6	765.9	624.3	813.7	1933.4	2682.2	434.7	782.8	846.4	1185.5
E_a/E_m	0.58		0.65		0.77		0.72		0.56		0.71	

Table 6.28. (continued)

Month	Nimbus Power Plant at Lake Natoma		New Melones Power Plant	
	Actual	Max.	Actual	Max.
Oct	4.6	6.4	*	71.6
Nov	4.7	10.0	*	70.4
Dec	4.7	10.0	*	68.1
Jan	8.7	10.0	19.6	81.9
Feb	6.7	10.0	23.6	92.1
Mar	10.6	10.0	47.8	90.9
Apr	9.7	7.7	55.8	90.4
May	9.1	8.2	39.5	85.8
Jun	7.6	7.8	23.3	84.4
Jul	9.7	6.5	38.5	80.9
Aug	4.3	10.0	22.1	74.9
Sep	4.6	10.0	9.0	64.8
Total	85.0	106.5	279.2	956.5
E_a/E_m	0.80		0.29	

*Power plant not in operation. E_a = actual energy production; E_m = maximized energy proratio of the number of hours a power plant is available for actual operation to the 8760 hours in a year), an overall ratio $E_a/E_m = 5.2/8.077 = 0.64$ is obtained. Considering that a typical annual availability factor is about 0.85 (Tudor Engineering Co., 1980), a value of $E_a/E_m = 0.64$ would imply a potential increase of up to 33 percent in the annual energy production of the system. For the actual annual energy produced during average-flow year 1979-80 (5.2×10^6 Mwh from Table 6.28) that would mean potentially a maximum additional 1.9×10^6 Mwh.

Benefits of the optimization model can also be measured in terms of increased water deliveries to downstream users. For example, the Delta requires a delivery of 3850 Kaf of water per year. Optimal release policies in Table 6.21 indicate a total annual release of 14773 Kaf, more than three times the required amount. For May-August, when most agricultural activities take place, additional water could be supplied for leaching and crop-growing purposes. The Delta requirements for May-August are about 2698 Kaf (see Table 5.4). For the same period, optimal releases indicate that 4813 Kaf were delivered in 1979-80. This suggests the possibility of a conjunctive use of surface water and groundwater reservoirs. Also, with increased deliveries, cultivated areas could be expanded or better leaching of salts might be achieved, resulting in an expanded economic output. Fish spawning, water quality, and navigation would also benefit from increased water deliveries.

Further insight into the differences between actual operation policies and those resulting from the optimization model can be gained from Figs. 6.3 and 6.4. Figure 6.3 shows actual and optimal state trajectories (for policies I and II) for Shasta reservoir. It is evident that substantially smaller storages are maintained from November

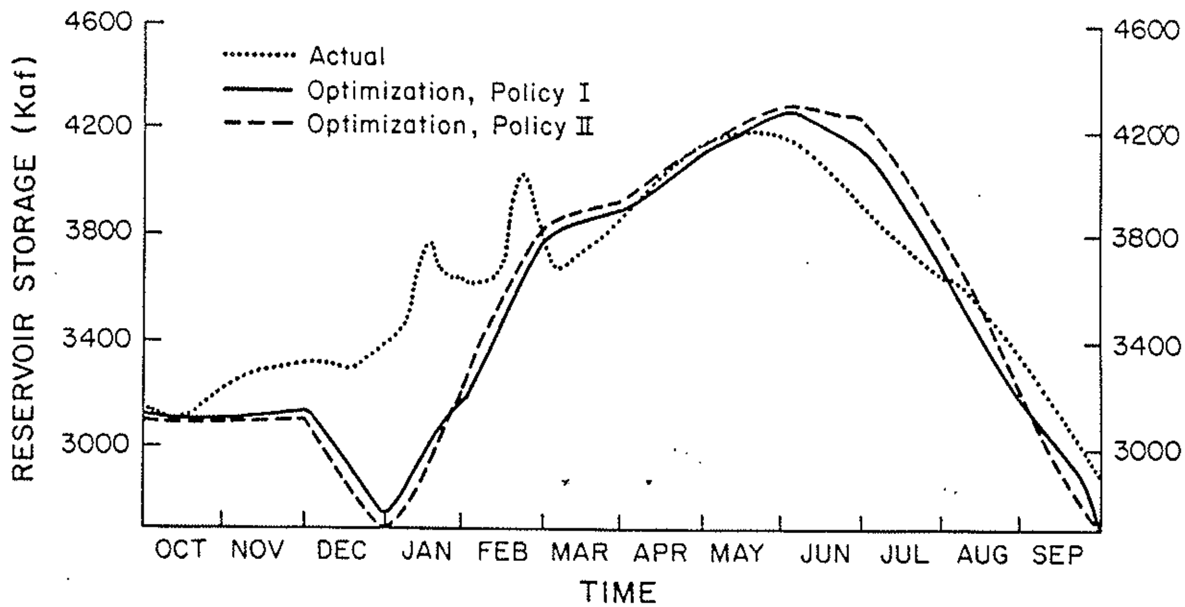


Fig. 6.3 Operation of Shasta reservoir (water year 1979-80).

to February in the optimal policies. That is accomplished by releasing large volumes of water through the penstocks, resulting in greater available flood control storages than in the actual operation. Thus, the level of energy generation during November-February is higher with the optimal trajectory because the releases are routed through the penstocks at a larger magnitude relative to the actual operation. Also, when the high inflows of January-April occur, the actual operation follows the flood control regulations by spilling large volumes of water because the empty volume in the flood control pool is not as large as that attained with the optimal state trajectories. In March-June, the optimal state trajectories maintain higher storage elevations than in the actual operation. That also results in increased energy production because releases during March-June are set at penstock capacity (see Tables 6.21 and 6.23), with higher storage levels resulting in increased rates of energy generation. Also in March-June, releases from the optimal state policies are at near penstock capacity because the inflows during those months maintain the reservoir filled to near capacity and releases must be kept high to avoid overtopping. The lower storages during March-June in the actual operation are due to water spillages that drive the reservoir level to lower stages. Those spillages reflect the conservativeness of the actual operation policy. Because they bypass the power plant, those spillages do not generate energy. In contrast, the reliance of the optimal trajectories on greater penstock outflows and smaller spillages reflects (i) the foreknowledge of future inflows (within a certain range of error) that arises from streamflow forecast and (ii) the knowledge that for a given release, the higher the storage level the higher the energy generation rate. During July-September, there is a steady drawdown of the reservoir storage level in the actual and optimal policies, reflecting increased demands for water and energy during the summer. Because the optimal state trajectories start at a higher elevation in July and end at a slightly lower level in September than the actual policy, the rate of water release during this period is higher for the optimal policies. That results in a greater generation of energy for the optimal policies in the summer as is evident in Table 6.28. Optimal policies I and II follow a similar pattern throughout the year and, as shown in Tables 6.21 and 6.23, result in the same total energy production. The actual state trajectory shows high peaks in January and February that are due to short-term floods that raise the reservoir level for a few days. Those floods are partly spilled and do not contribute to energy generation at the reservoir power plant. Those flood events are not captured with the monthly optimization model. The model considers flood flows through a larger total monthly inflow. That results in an overestimation of energy production in the optimization model because flood inflow volumes are distributed through the month, allowing to be handled as larger penstock outflows rather than spillages.

Figure 6.4 shows the actual and optimal state trajectories for Folsom reservoir. The operational features are similar to those discussed earlier for Shasta reservoir (this is true also for the other two major reservoirs, Clair Engle and New Melones). The optimal policy relies on large penstock releases in October-December, drawing the storage level low and creating a large empty volume for high inflows that occur from January to April. During January and February, the empty space created by the optimal releases of previous months is filled by runoff. Penstock releases are kept at a maximum and some spillages occur to avoid overtopping (see Tables 6.21 and 6.23). In March-June, the reservoir is kept at near full capacity and releases through the penstocks are at a maximum. That results in a head-release combination that yields the largest energy production rates. In contrast, the actual policy maintains a steadier reservoir elevation through the year, with large spillages used to accomodate the large winter floods (peaks shown in Fig. 6.4 for January, February, and March). The main explanation for the significant difference between the actual and optimal trajectories during February-June is the conservative approach of the NCVP management with regard to flood control. In essence, the managers of the system preserve a substantial empty space to "prepare for the worst." The optimization model does not take into account those intangible considerations. Instead, for specified inflow forecasts, constraints, and power function, the model chooses a state trajectory that maximizes energy production. It must be stressed that the NCVP management considers flood control as the major function of the reservoir system and that there are strong institutional pressures to avoid any possible adverse outcome from unexpected floods. Thus, a comparison of actual and model policies cannot be made solely on the basis of a single parameter, namely energy production. Rather, the performance of the operation model (as measured by power output) must be interpreted as a possibility for improvement in a real-world context that involves many factors not considered or reflected in the optimization model.

CHAPTER 7

SYSTEM-DEPENDENT FEATURES: AN EXTENSION OF THE OPTIMIZATION MODEL

This chapter develops alternative optimization models that take advantage of system-dependent features. The models are simpler than those presented in Chapter 5 because they have fewer decision variables. In addition, the applicability of the full optimization model is generalized to the case of nonlinear objective function and/or constraints. The resulting model has the minimum possible number of decision variables, thus being computationally efficient. Section 7.1 presents a full model that considers the simplifications introduced by the existence of regulating reservoirs. Sections 7.2-7.4 develop linear and quadratic models that constitute alternative simplifications of the full model of Section 7.1. Sections 7.5 and 7.6 discuss the necessary modifications to the model when the objective function and constraints, respectively, are nonlinear. Section 7.7 gives a complete description of a general model that handles nonlinearities in the objective function and/or constraints and treats spillages explicitly as decision variables. The general model is of minimum dimensionality. Finally, Section 7.8 shows the results of applying the general model of Section 7.7 to the NCVP.

7.1 Modeling Regulating Reservoirs

This section develops a simplified full model in which regulating reservoirs are treated in a special manner. Section 6.2 showed that Lewiston, Whiskeytown, Keswick, Natoma, and Tullock reservoirs can be treated as regulating units. Because those reservoirs can be maintained full throughout the year, they act as transmission nodes in the system network. That implies that as many state variables as there are regulating reservoirs in the system can be eliminated from the optimization model, greatly simplifying the mathematical structure of the model.

In general, for a large reservoir-small reservoir subsystem in series, the adequacy of treating a small reservoir as a regulating unit depends on the capacity ratio of the subsystem (see Section 6.2). The largest capacity ratio in the NCVP corresponds to the Clair Engle-Whiskeytown subsystem, with a value of 0.10. The development that follows shows that the optimization model presented in Chapter 5 can be modified to obtain a simpler but completely equivalent model. Although the development is based on the NCVP system configuration, the principles hold for any other reservoir network. The notation introduced in Chapter 5 will also be used in this chapter.

Reservoir storages in the regulating reservoirs are set equal to the reservoirs' maximum capacities (or any other feasible volume as long as that volume is fixed). That implies $x_t^2 = 14.7$, $x_t^3 = 241$, $x_t^5 = 23.8$, $x_t^7 = 8.8$, and $x_t^9 = 57$ Kaf for all $t = 1, 2, \ldots, 13$. The upper indices 2, 3, 5, 7, and 9 refer to the number of the regulating reservoirs, which are Lewiston, Whiskeytown, Keswick, Natoma, and Tullock, respectively. Recall that the equation of continuity, eq. (5.48), is

$$\underline{u}_t = - \Gamma_1^{-1} \Gamma_2 \underline{r}_t + \Gamma_1^{-1} A_{t+1} \underline{x}_{t+1} - \Gamma_1^{-1} B_t \underline{x}_t - \Gamma_1^{-1} \underline{v}_t \qquad (5.48)$$

To fix the storages at the regulating reservoirs as constant, a reordering of the elements in the vectors of eq. (5.48) must be made. Vector $\underline{u}_t$ is partitioned as follows:

$$\underline{u}_t = \begin{bmatrix} \underline{u}_t^{(1)} \\ \underline{u}_t^{(2)} \end{bmatrix} \tag{7.1}$$

in which the subvectors $\underline{u}_t^{(1)}$ and $\underline{u}_t^{(2)}$ contain the releases from the nonregulating and regulating reservoirs, respectively. Similar partitions can be made for vectors $\underline{r}_t$, $\underline{v}_t$, $\underline{x}_t$, and $\underline{x}_{t+1}$. For vectors $\underline{x}_t$ and $\underline{x}_{t+1}$, the subvectors $\underline{x}_t^{(2)}$ and $\underline{x}_{t+1}^{(2)}$ contain fixed storages, which are the capacities of the regulating reservoirs. It is convenient to use elementary matrices (Householder, 1975) for the automatic implementation of the reordering. For the NCVP, the elementary matrix S,

$$S = \begin{bmatrix} 1 & & & & & & & & \\ & & & 1 & & & & & \\ & & & & & 1 & & & \\ & & & & & & & 1 & \\ & 1 & & & & & & & \\ & & 1 & & & & & & \\ & & & & 1 & & & & \\ & & & & & & 1 & & \\ & & & & & & & & 1 \end{bmatrix} \tag{7.2}$$

reorders the vector $\underline{u}_t$ in the form

$$(S\underline{u}_t)^T = (u_t^1 \;\; u_t^4 \;\; u_t^6 \;\; u_t^8, \quad u_t^2 \;\; u_t^3 \;\; u_t^5 \;\; u_t^7 \;\; u_t^9) \tag{7.3}$$

in which

$$\underline{u}_t^{(1)} = (u_t^1 \;\; u_t^4 \;\; u_t^6 \;\; u_t^8) \tag{7.4}$$

$$\underline{u}_t^{(2)} = (u_t^2 \quad u_t^3 \quad u_t^5 \quad u_t^7 \quad u_t^9) \tag{7.5}$$

Notice that $\underline{u}_t^{(1)}$ contains the releases from the nonregulating reservoirs: Clair Engle (u_t^1), Shasta (u_t^4), Folsom (u_t^6), and New Melones (u_t^8). A similar reordering can be obtained by premultiplying vectors $\underline{x}_t$, $\underline{x}_{t+1}$, $\underline{r}_t$, and $\underline{v}_t$ by S. Vector reordering also implies a rearrangement of the matices in eq. (5.48),

$$\Gamma_1 = \begin{bmatrix} -1 &&&&&&&& \\ & -1 &&&&&&& \\ && -1 &&&&&& \\ &&& -1 &&&&& \\ 1 &&&& -1 &&&& \\ &&&& 1 & -1 &&& \\ & 1 &&&& 1 & -1 && \\ && 1 &&&&& -1 & \\ &&& 1 &&&&& -1 \end{bmatrix} \tag{7.6}$$

$$\Gamma_2 = \begin{bmatrix} -1 &&&&&&&& \\ & -1 &&&&&&& \\ && -1 &&&&&& \\ &&& -1 &&&&& \\ 1 &&&& -1 &&&& \\ &&&&& -1 &&& \\ & 1 &&&&& -1 && \\ && 1 &&&&& -1 & \\ &&& 1 &&&&& -1 \end{bmatrix} \tag{7.7}$$

$$A_{t+1} = \begin{bmatrix} 1+c_t^1 & & & & & & & & \\ & 1+c_t^4 & & & & & & & \\ & & 1+c_t^6 & & & & & & \\ & & & 1+c_t^8 & & & & & \\ & & & & 1 & & & & \\ & & & & & 1 & & & \\ & & & & & & 1 & & \\ & & & & & & & 1 & \\ & & & & & & & & 1 \end{bmatrix} \tag{7.8}$$

$$B_t = \begin{bmatrix} 1-c_t^1 & & & & & & & & \\ & 1-c_t^4 & & & & & & & \\ & & 1-c_t^6 & & & & & & \\ & & & 1-c_t^8 & & & & & \\ & & & & 1 & & & & \\ & & & & & 1 & & & \\ & & & & & & 1 & & \\ & & & & & & & 1 & \\ & & & & & & & & 1 \end{bmatrix} \tag{7.9}$$

In eqs. (7.8) and (7.9), the ones along the diagonal indicate that losses are considered only for the nonregulating reservoirs. Coefficients c_t^i are as defined in eq. (5.40). The reordered equation of continuity can be written as

$$\begin{bmatrix} \Gamma^1_{11} & \Gamma^1_{12} \\ \Gamma^1_{21} & \Gamma^1_{22} \end{bmatrix} \begin{bmatrix} \underline{u}^{(1)}_t \\ \underline{u}^{(2)}_t \end{bmatrix} = - \begin{bmatrix} \Gamma^2_{11} & \Gamma^2_{12} \\ \Gamma^2_{21} & \Gamma^2_{22} \end{bmatrix} \begin{bmatrix} \underline{r}^{(1)}_t \\ \underline{r}^{(2)}_t \end{bmatrix} + \begin{bmatrix} A^{t+1}_{11} & A^{t+1}_{12} \\ A^{t+1}_{21} & A^{t+1}_{22} \end{bmatrix} \begin{bmatrix} \underline{x}^{(1)}_{t+1} \\ \underline{x}^{(2)}_{t+1} \end{bmatrix} - \begin{bmatrix} B^t_{11} & B^t_{12} \\ B^t_{21} & B^t_{22} \end{bmatrix} \begin{bmatrix} \underline{x}^{(1)}_t \\ \underline{x}^{(2)}_t \end{bmatrix} - \begin{bmatrix} \underline{v}^{(1)}_t \\ \underline{v}^{(2)}_t \end{bmatrix} \tag{7.10}$$

in which Γ^1_{12}, Γ^2_{12}, A^{t+1}_{12}, and B^t_{12} are null matrices, and the partitions of the matrices are uniquely defined by the partitions of the respective vectors. It is emphasized that $\underline{x}^{(2)}_{t+1} = \underline{x}^{(2)}_t$ for all periods t. To simplify the notation, both $\underline{x}^{(2)}_{t+1}$ and $\underline{x}^{(2)}_t$ will be denoted by $\underline{k}$ from now on.

The energy equation also needs to be modified. That is,

$$E_t = \left\{ \begin{bmatrix} \underline{a}_1 \\ \underline{a}_2 \end{bmatrix} + \begin{bmatrix} B_{11} & 0 \\ 0 & B_{22} \end{bmatrix} \left[\begin{bmatrix} \underline{x}^{(1)}_t \\ \underline{k} \end{bmatrix} + \begin{bmatrix} \underline{x}^{(1)}_{t+1} \\ \underline{k} \end{bmatrix} \right] \right\} \begin{bmatrix} \underline{u}^{(1)}_t \\ \underline{u}^{(2)}_t \end{bmatrix} \tag{7.11}$$

in which

$$\underline{a}^T_1 = (a^1 \quad a^4 \quad a^6 \quad a^8) \tag{7.12}$$

$$\underline{a}^T_2 = (a^2 \quad a^3 \quad a^5 \quad a^7 \quad a^9) \tag{7.13}$$

$$B = \begin{bmatrix} B_{11} & 0 \\ 0 & B_{12} \end{bmatrix} = \begin{bmatrix} b_1 & & & & & & & & \\ & b_4 & & & & & & 0 & \\ & & b_6 & & & & & & \\ & & & b_8 & & & & & \\ & & & & b_2 & & & & \\ & & & & & b_3 & & & \\ & 0 & & & & & b_5 & & \\ & & & & & & & b_7 & \\ & & & & & & & & b_9 \end{bmatrix} \tag{7.14}$$

The coefficients in eqs. (7.12)-(7.14) are given by the values specified right after eq. (5.46). For period t-1, E_{t-1} is similarly expressed by setting t = t-1 in eq. (7.11). In the POA, the objective function is $E_{t-1} + E_t$ for each two-stage maximization. To develop an expression for $E_{t-1} + E_t$, it is necessary to substitute the continuity equation [eq. (7.10)] into the energy equation for periods t [eq. (7.11)] and t-1 [eq. (7.11) with index t replaced by t-1]. Solving for $\underline{u}_t$ in eq. (7.10) gives

$$\underline{u}_t = - \begin{bmatrix} \Gamma^1_{11} & \Gamma^1_{12} \\ \Gamma^1_{21} & \Gamma^1_{22} \end{bmatrix}^{-1} \Gamma_2 \, \underline{r}_t + \begin{bmatrix} \Gamma^1_{11} & \Gamma^1_{12} \\ \Gamma^1_{21} & \Gamma^1_{22} \end{bmatrix}^{-1} A_{t+1} \, \underline{x}_{t+1}$$

$$- \begin{bmatrix} \Gamma^1_{11} & \Gamma^1_{12} \\ \Gamma^1_{21} & \Gamma^1_{22} \end{bmatrix}^{-1} B_t \, \underline{x}_t - \begin{bmatrix} \Gamma^1_{11} & \Gamma^1_{12} \\ \Gamma^1_{21} & \Gamma^1_{22} \end{bmatrix}^{-1} \underline{v}_t$$

$$= \begin{bmatrix} H_{11}^{t+1} & 0 \\ H_{21}^{t+1} & H_{22}^{t+1} \end{bmatrix} \begin{bmatrix} \underline{x}_{t+1}^{(1)} \\ \underline{k} \end{bmatrix} - \begin{bmatrix} M_{11}^{t} & 0 \\ M_{21}^{t} & M_{22}^{t} \end{bmatrix} \begin{bmatrix} \underline{x}_{t}^{(1)} \\ \underline{k} \end{bmatrix}$$

$$- \begin{bmatrix} N_{11} & 0 \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} \underline{r}_{t}^{(1)} \\ \underline{r}_{t}^{(2)} \end{bmatrix} - \begin{bmatrix} \underline{w}_{t}^{(1)} \\ \underline{w}_{t}^{(2)} \end{bmatrix} \tag{7.15}$$

Substitution of eq. (7.15) into E_t [eq. (7.11)] and doing a similar substitution of $\underline{u}_{t-1}$ into E_{t-1} yields

$$E_{t-1} + E_t = k_t^2 + [\underline{p}_t^{(1)T} \quad \underline{p}_t^{(2)T}] \begin{bmatrix} \underline{r}_{t-1}^{(1)} \\ \underline{r}_{t-1}^{(2)} \end{bmatrix} + [\underline{q}_t^{(1)T} \quad \underline{q}_t^{(2)T}] \begin{bmatrix} \underline{r}_{t}^{(1)} \\ \underline{r}_{t}^{(2)} \end{bmatrix} + \underline{g}_t^T \underline{x}_t$$

$$+ \underline{x}_t^T G_t \underline{x}_t - \underline{x}_t^T B \underline{r}_{t-1} - \underline{x}_t^T B \underline{r}_t$$

$$= k_t^2 + \underline{p}_t^T \underline{r}_{t-1} + \underline{q}_t^T \underline{r}_t + \underline{g}_t^T \underline{x}_t + \underline{x}_t^T G_t \underline{x}_t - \underline{x}_t^T B \underline{r}_{t-1}$$

$$- \underline{x}_t^T B \underline{r}_t \tag{7.16}$$

in which

$$k_t^2 = (\underline{a}_1^T + \underline{x}_{t+1}^T B_{11}) (H_{11}^{t+1} \underline{x}_{t+1} - \underline{w}_t^{(1)}) + (\underline{a}_1^T + \underline{x}_{t-1}^T B_{11})$$

$$(- M_{11}^{t-1} \underline{x}_{t-1} - \underline{w}_{t-1}^{(1)}) + (\underline{a}_2^T + \underline{k}^T B_{22}) (H_{21}^{t+1} \underline{x}_{t+1} + H_{22}^{t+1} \underline{k}$$

$$- M_{22}^t \underline{k} - \underline{w}_t^{(2)} + H_{22}^t \underline{k} - M_{21}^{t-1} \underline{x}_{t-1} - M_{22}^{t-1} \underline{k} - \underline{w}_{t-1}^{(2)}) \quad (7.17)$$

$$\underline{p}_t^{(1)T} = - [(\underline{a}_1^T + \underline{x}_{t-1}^T B_{11}) N_{11} + (\underline{a}_2^T + \underline{k}^T B_{22}) N_{21}] \quad (7.18)$$

$$\underline{p}_t^{(2)T} = [(\underline{a}_2^T + \underline{k}^T B_{22}) N_{22}] \quad (7.19)$$

$$\underline{q}_t^{(1)T} = - [(\underline{a}_1^T + \underline{x}_{t+1}^T B_{11}) N_{11} + (\underline{a}_2^T + \underline{k}^T B_{22}) N_{21}] \quad (7.20)$$

$$\underline{q}_t^{(2)T} = [(\underline{a}_2^T + \underline{k}^T B_{22}) N_{22}] \quad (7.21)$$

$$\underline{g}_t^T = - (a_1^T + \underline{x}_{t+1}^T B_{11}) M_{11}^t + (\underline{a}_1^T + \underline{x}_{t-1}^T B_{11}) H_{11}^t$$

$$+ (\underline{a}_2^T + \underline{k}^T B_{22}) (H_{21}^t - M_{21}^t) + (H_{11}^{t+1} \underline{x}_{t+1} - \underline{w}_t^{(1)})^T B_{11}$$

$$- (M_{11}^{t-1} \underline{x}_{t-1} + \underline{w}_{t-1}^{(1)})^T B_{11} \quad (7.22)$$

$$G_t = - B_{11} M_{11}^t + B_{11} H_{11}^t \quad (7.23)$$

$$B = -\,(B_{11}N_{11} \quad 0) \tag{7.24}$$

In eqs. (7.16)-(7.24), the superscript (1) on subvectors $\underline{x}_{t-1}$, $\underline{x}_t$, and $\underline{x}_{t+1}$ has been dropped because those subvectors represent the storage subvectors for the nonregulating reservoirs.

By defining an augmented vector $\underline{\theta}_t$,

$$\underline{\theta}_t = \begin{bmatrix} \underline{x}_t \\ \underline{r}_t \\ \underline{r}_{t-1} \end{bmatrix} \tag{7.25}$$

eq. (7.16) can be rewritten as

$$E_{t-1} + E_t = [\underline{g}_t^T \quad \underline{q}_t^T \quad \underline{p}_t^T]\ \underline{\theta}_t + \underline{\theta}_t^T\, H_t^2\, \underline{\theta}_t + k_t^2 \tag{7.26}$$

in which

$$H_t^2 = \frac{1}{2}\begin{bmatrix} G_t + G_t^T & B & B \\ B^T & 0 & 0 \\ B^T & 0 & 0 \end{bmatrix} \tag{7.27}$$

where matrices $(G_t + G_t^T)$, B, and 0 have dimensions of 4 x 4, 4 x 9, and 9 x 9, respectively. By dropping the constant term k_t^2 and by defining $\hat{\underline{s}}_t^T = [\underline{g}_t^T \quad \underline{q}_t^T \quad \underline{p}_t^T]$, eq. (7.26) can be expressed as

$$E_{t-1} + E_t = \underline{\theta}_t^T\, H_t^2\, \underline{\theta}_t + \hat{\underline{s}}_t^T\, \underline{\theta}_t \tag{7.28}$$

Equation (7.28) is the objective function of the two-stage maximization, a quadratic function on $\underline{\theta}_t$. The dimension of the unknown vector $\underline{\theta}_t$ is 24 x 1, which is smaller than the analogous decision vector of eq. (5.56).

The maximization of eq. (7.28) is carried out subject to the following constraints:

Constraints on total releases (penstock plus spillage) for months t and t-1. For month t,

$$\underline{u}_t + \underline{r}_t \in \underline{W}_t \quad \Rightarrow \quad H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - M_t \begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix} + (I - N)\, \underline{r}_t - \underline{w}_t \in \underline{W}_t \quad (7.29)$$

which can be expressed as

$$H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} + [-M_1^t \quad I\text{-}N \quad 0]\, \underline{\theta}_t - M_2^t \underline{k} - \underline{w}_t \in \underline{W}_t \quad (7.30)$$

in which $M_t = [M_1^t \quad M_2^t]$. Similarly, for month t-1,

$$-M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} + [H_1^t \quad 0 \quad I\text{-}N]\, \underline{\theta}_t + H_2^t\, \underline{k} - \underline{w}_{t-1} \in \underline{W}_{t-1} \quad (7.31)$$

in which $H_t = [H_1^t \quad H_2^t]$.

Constraints on penstock releases for months t and t-1. For month t,

$$H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - M_1^t\, \underline{x}_t - M_2^t\, \underline{k} - N\, \underline{r}_t - \underline{w}_t \in \underline{U}_t \quad (7.32)$$

or in terms of $\underline{\theta}_t$,

$$H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} + [-M_1^t \quad -N \quad 0] \ \underline{\theta}_t - M_2^t \ \underline{k} - \underline{w}_t \in \underline{U}_t \tag{7.33}$$

Similarly, for month t-1,

$$H_1^t \ \underline{x}_t + H_2^t \ \underline{k} - M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} - N \ \underline{r}_{t-1} - \underline{w}_{t-1} \in \underline{U}_{t-1} \tag{7.34}$$

or, in terms of $\underline{\theta}_t$

$$[H_1^t \quad 0 \quad -N] \ \underline{\theta}_t + H_2^t \ \underline{k} - M \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} - \underline{w}_t \in \underline{U}_{t-1} \tag{7.35}$$

Constraints on Delta water deliveries for months t and t-1. For month t,

$$\underline{c}^T \ (\underline{u}_t + \underline{r}_t) \in De_t$$

$$\Rightarrow \ \underline{c}^T \ H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - \underline{c}^T \ (M_1^t \ \underline{x}_t + M_2^t \ \underline{k}) + \underline{c}^T \ (I - N) \ \underline{r}_t$$

$$- \underline{c}^T \ \underline{w}_t \in De_t \tag{7.36}$$

or, in terms of $\underline{\theta}_t$,

$$\underline{c}^T \ H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} + \underline{c}^T \ [-M_1^t \quad I-N \quad 0] \ \underline{\theta}_t - \underline{c}^T \ M_2^t \ \underline{k} - \underline{c}^T \ \underline{w}_t \in De_t \tag{7.37}$$

in which $\underline{c}^T = [0 \ \ 0 \ \ 0 \ \ 0 \ \ 1 \ \ 0 \ \ 1 \ \ 0 \ \ 1]$. Similarly, for month t-1,

$$\underline{c}^T (\underline{u}_{t-1} + \underline{r}_{t-1}) \in De_{t-1}$$

$$\Rightarrow \quad - \underline{c}^T M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} + \underline{c}^T H_1^t \underline{x}_t + \underline{c}^T H_2^t \underline{k} + \underline{c}^T (I-N) \underline{r}_{t-1}$$

$$- \underline{c}^T \underline{w}_{t-1} \in De_{t-1} \qquad (7.38)$$

or, in terms of $\underline{\theta}_t$,

$$\underline{c}^T [H_1^t \quad 0 \quad I-N] \underline{\theta}_t + \underline{c}^T H_2^t \underline{k} - \underline{c}^T M_t \begin{bmatrix} \underline{x}_{t-1}^{(1)} \\ \underline{k} \end{bmatrix} - \underline{c}^T \underline{w}_{t-1} \in De_{t-1} \qquad (7.39)$$

Constraints on spillage releases for months t and t-1. For month t,

$$[0 \quad I \quad 0] \underline{\theta}_t \in \underline{R}_t \qquad (7.40)$$

For month t-1,

$$[0 \quad 0 \quad I] \underline{\theta}_{t-1} \in \underline{R}_{t-1} \qquad (7.41)$$

Constraints on storage volumes for month t,

$$[I \quad 0 \quad 0] \underline{x}_t \in \underline{X}_t \qquad (7.42)$$

in which $\underline{x}_{t-1}$ and $\underline{x}_{t+1}$ are fixed. In eqs. (7.29)-(7.42), $\underline{W}_t$, $\underline{W}_{t-1}$, $\underline{U}_t$, $\underline{U}_{t-1}$, De_t, De_{t-1}, $\underline{R}_t$, $\underline{R}_{t-1}$, and $\underline{X}_t$ are feasible regions.

In summary, when the storages at the regulating reservoirs are constant, the (full) optimization model is (dropping the constant term k_t^2)

$$\text{maximize } \underline{\theta}_t^T H_t^2 \underline{\theta}_t + \underline{\hat{s}}_t^T \underline{\theta}_t \tag{7.43}$$

subject to

$$\hat{A}_t^2 \underline{\theta}_t \leq \underline{\hat{b}}_t^2 \tag{7.44}$$

$$[I \quad 0 \quad 0]\, \underline{\theta}_{t-1}, \qquad [I \quad 0 \quad 0]\, \underline{\theta}_{t+1} \text{ fixed}$$

in which

$$\hat{A}_t^2 = \begin{bmatrix} -M_1^t & I-N & 0 \\ H_1^t & 0 & I-N \\ -M_1^t & -N & 0 \\ M_1^t & N & 0 \\ H_1^t & 0 & -N \\ -H_1^t & 0 & N \\ -\underline{c}^T(-M_1^t & I-N & 0) \\ -\underline{c}^T(H_1^t & 0 & I-N) \\ 0 & I & 0 \\ 0 & 0 & I \\ I & 0 & 0 \\ -I & 0 & 0 \end{bmatrix} \tag{7.45}$$

$$\hat{\underline{b}}_t^2 = \begin{bmatrix} \underline{W}_t - H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} + M_2^t \, \underline{k} + \underline{w}_t \\ \underline{W}_{t-1} + M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} - H_2^t \, \underline{k} + \underline{w}_{t-1} \\ \underline{U}_{t,max} - H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} + M_2^t \, \underline{k} + \underline{w}_t \\ -\underline{U}_{t,min} + H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - M_2^t \, \underline{k} + \underline{w}_t \\ \underline{U}_{t-1,max} - H_2^t \, \underline{k} + M \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} + \underline{w}_{t-1} \\ -\underline{U}_{t-1,min} + H_2^t \, \underline{k} - M \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} - \underline{w}_{t-1} \\ -De_t + \underline{c}^T H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - \underline{c}^T M_2^t \, \underline{k} - \underline{c}^T \, \underline{w}_t \\ -De_{t-1} - \underline{c}^T M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} + \underline{c}^T H_2^t \, \underline{k} - \underline{c}^T \, \underline{w}_{t-1} \\ \underline{R}_t \\ \underline{R}_{t-1} \\ \underline{X}_{t,min} \\ -\underline{X}_{t,max} \end{bmatrix} \qquad (7.46)$$

The POA solves a sequence of problems of the form given by eqs. (7.43) and (7.44) to obtain optimal operating policies for the NCVP. The model given by eqs. (7.43) and (7.44) can be expressed in several simplified forms by using a procedure analogous to that discussed in Section 5.3.

7.2 Simplified Linear Model

A simplified version of eqs. (7.43) and (7.44) is developed in which (i) net reservoir losses are neglected and (ii) spillages are treated as excesses over penstock capacity. As will be shown, the simplified model has only four unknown variables, reducing the (full) optimization model to a simple sequence of LP problems.

If net reservoir losses are neglected and spillages are treated as excesses over penstock capacity, then the equation of continuity can be expressed as

$$\begin{bmatrix} \Gamma^1_{11} & \Gamma^1_{12} \\ \Gamma^1_{21} & \Gamma^1_{22} \end{bmatrix} \begin{bmatrix} \underline{u}_t^{(1)} \\ \underline{u}_t^{(2)} \end{bmatrix} = - \begin{bmatrix} \Gamma^2_{11} & \Gamma^2_{12} \\ \Gamma^2_{21} & \Gamma^2_{22} \end{bmatrix} \begin{bmatrix} \underline{r}_t^{(1)} \\ \underline{r}_t^{(2)} \end{bmatrix} + \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - \begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix} - \begin{bmatrix} \underline{v}_t^{(1)} \\ \underline{v}_t^{(2)} \end{bmatrix} \tag{7.47}$$

Equation (7.47) is obtained from eq. (7.10) by setting matrices A_{t+1} and B_t equal to identity matrices. It follows from eq. (7.47) that

$$\begin{bmatrix} \underline{u}_t^{(1)} \\ \underline{u}_t^{(2)} \end{bmatrix} = \begin{bmatrix} H_{11} & 0 \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - \begin{bmatrix} H_{11} & 0 \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix}$$

$$- \begin{bmatrix} N_{11} & 0 \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} \underline{r}_t^{(1)} \\ \underline{r}_t^{(2)} \end{bmatrix} - \begin{bmatrix} \underline{w}_t^{(1)} \\ \underline{w}_t^{(2)} \end{bmatrix}$$

$$= H \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - H \begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix} - N\, \underline{r}_t - \underline{w}_t \qquad (7.48)$$

where the matrices H and N are time-independent. Substitution of eq. (7.48), without the spillage terms because of assumption (ii), into eq. (7.11) yields the linear objective function

$$E_{t-1} + E_t = \underline{\hat{h}}_t^T\, \underline{x}_t + \hat{c}_t \qquad (7.49)$$

in which

$$\hat{c}_t = (\underline{a}_1^T + \underline{x}_{t+1}^T\, B_{11})\,(H_{11}\, \underline{x}_{t+1} - \underline{w}_t^{(1)}) - (\underline{a}_1^T + \underline{x}_{t-1}^T\, B_{11})$$

$$\cdot\, (H_{11}\, \underline{x}_{t-1} + \underline{w}_{t-1}^{(1)}) + (\underline{a}_2^T + \underline{k}^T\, B_{22})$$

$$\cdot\, [H_{21}\, (\underline{x}_{t+1} - \underline{x}_{t-1}) - \underline{w}_{t-1}^{(2)} - \underline{w}_t^{(2)}] \qquad (7.50)$$

$$\underline{\hat{h}}_t^T = (\underline{x}_{t-1}^T - \underline{x}_{t+1}^T) B_{11} H_{11} + [(H_{11} \underline{x}_{t+1} - \underline{w}_t^{(1)})^T$$

$$- (H_{11} \underline{x}_{t-1} + \underline{w}_{t-1}^{(1)})^T] B_{11} \qquad (7.51)$$

Associated with objective function is the following set of constraints: Minimum releases for months t and t-1. For month t,

$$\underline{u}_t \geqq \underline{U}_{t,min} \Rightarrow H \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - H_1 \underline{x}_t - H_2 \underline{k} - \underline{w}_t \geqq \underline{U}_{t,min} \qquad (7.52)$$

or equivalently,

$$H_1 \underline{x}_t \leqq H \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - H_2 \underline{k} - \underline{w}_t - \underline{U}_{t,min} \qquad (7.53)$$

in which $H = (H_1, H_2)$. For month t-1,

$$- H_1 \underline{x}_t \leqq - H \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} + H_2 \underline{k} - \underline{w}_{t-1} - \underline{U}_{t-1,min} \qquad (7.54)$$

Maximum releases for months t and t-1. For month t,

$$\underline{u}_t \leqq \underline{U}_{t,max} \Rightarrow - H_1 \underline{x}_t \leqq - H \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} + H_2 \underline{k} + \underline{w}_t + \underline{U}_{t,max} \qquad (7.55)$$

For month t-1,

$$H_1 \underline{x}_t \leqq H \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} - H_2 \underline{k} + \underline{w}_{t-1} + \underline{U}_{t-1,max} \tag{7.56}$$

Delta water requirements for months t and t-1. For month t,

$$\underline{c}^T \underline{u}_t \geqq De_t \Rightarrow \underline{c}^T H_1 \underline{x}_t \leqq \underline{c}^T H \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - \underline{c}^T H_2 \underline{k} - \underline{c}^T \underline{w}_t - De_t \tag{7.57}$$

For month t-1,

$$- \underline{c}^T H_1 \underline{x}_t \leqq - \underline{c}^T H \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} + \underline{c}^T H_2 \underline{k} - \underline{c}^T \underline{w}_{t-1} - De_{t-1} \tag{7.58}$$

where $\underline{c}^T = (0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1)$.

Minimum storages for month t,

$$\underline{x}_t \geqq \underline{X}_{t,min} \Rightarrow - \underline{x}_t \leqq - X_{t,min} \tag{7.59}$$

Maximum storages for month t,

$$\underline{x}_t \leqq \underline{X}_{t,max} \tag{7.60}$$

with $\underline{x}_{t-1}$, $\underline{x}_{t+1}$ fixed. In eqs. (7.52)-(7.60), $\underline{U}_{t,min}$, $\underline{U}_{t,max}$, De_t, $\underline{X}_{t,min}$, and $\underline{X}_{t,max}$ are feasible sets. In summary, the two-stage maximization model is (dropping the constant term $\hat{c}_t$)

$$\text{maximize } \underline{\hat{h}}_t^T \underline{x}_t \tag{7.61}$$

subject to

$$\hat{A}\ \underline{x}_t \leqq \hat{\underline{b}}_t \tag{7.62}$$

$\underline{x}_{t-1}$, $\underline{x}_{t+1}$ fixed

in which

$$\hat{A} = \begin{bmatrix} H_1 \\ -H_1 \\ -H_1 \\ H_1 \\ \underline{c}^T H_1 \\ -\underline{c}^T H_1 \\ I \\ -I \end{bmatrix} \tag{7.63}$$

$$\hat{\underline{b}}_t = \begin{bmatrix} -\underline{U}_{t,min} + H \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - H_2 \underline{k} - \underline{w}_t \\ -\underline{U}_{t-1,min} - H \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} + H_2 \underline{k} - \underline{w}_{t-1} \\ \underline{U}_{t,max} - H \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} + H_2 \underline{k} + \underline{w}_t \\ \underline{U}_{t-1,max} + H \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} - H_2 \underline{k} + \underline{w}_{t-1} \\ -De_t + \underline{c}^T H \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - \underline{c}^T H_2 \underline{k} - \underline{c}^T \underline{w}_t \\ -De_{t-1} - \underline{c}^T H \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} + \underline{c}^T H_2 \underline{k} - \underline{c}^T \underline{w}_{t-1} \\ \underline{X}_{t,max} \\ -\underline{X}_{t,min} \end{bmatrix} \quad (7.64)$$

Equation (7.62) is a compact form of eqs. (7.52)-(7.60). Because the unknown vector $\underline{x}_t$ is a four-dimensional vector, solution of eqs. (7.61) and (7.62) by the POA is computationally efficient.

7.3 Simplified Quadratic Model 1

This section develops a simplified optimization model in which net reservoir losses are taken into account but spillages are treated as excesses over penstock capacity. The model is suitable for climatic conditions in which evaporation becomes an important component of the mass balance at the reservoir and/or geological features cause substantial seepage. The objective function for the two-stage maximization becomes

$$E_{t-1} + E_t = k_t^2 + \underline{g}_t^T \underline{x}_t + \underline{x}_t^T G_t \underline{x}_t \tag{7.65}$$

in which k_t^2, $\underline{g}_t^T$, and G_t are given by eqs. (7.17), (7.22), and (7.23), respectively. The set of constraints on the decision vector $\underline{x}_t$ of eq. (7.65) is given by the following expressions:

Constraints on penstock releases for months t and t-1, respectively,

$$H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - M_1^t \underline{x}_t - M_2^t \underline{k} - \underline{w}_t \in \underline{U}_t \tag{7.66}$$

$$-M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} + H_1^t \underline{x}_t + H_2^t \underline{k} - \underline{w}_{t-1} \in \underline{U}_{t-1} \tag{7.67}$$

Constraints on Delta water deliveries for months t and t-1, respectively,

$$\underline{c}^T H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - \underline{c}^T M_1^t \underline{x}_t - \underline{c}^T M_2^t \underline{k} - \underline{c}^T \underline{w}_t \in De_t \tag{7.68}$$

$$-\underline{c}^T M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} + \underline{c}^T H_1^t \underline{x}_t + \underline{c}^T H_2^t \underline{k} - \underline{c}^T \underline{w}_{t-1} \in De_{t-1} \tag{7.69}$$

Constraints on storage volumes for month t,

$$\underline{x}_t \in \underline{X}_t \tag{7.70}$$

Because spillages are not part of the decision vector in the objective function [eq. (7.65)], constraints on spillages are not explicitly shown in the set (7.66)-(7.70). However, the computer program must check those spillages to ensure that they are within a feasible range when they occur. In compact form, the model specified by eqs. (7.65)-(7.70) can be expressed as (dropping the constant term k_t^2)

$$\text{maximize } \underline{g}_t^T \underline{x}_t + \underline{x}_t^T G_t \underline{x}_t \tag{7.71}$$

subject to

$$A_t^3 \underline{x}_t \leq \hat{\underline{b}}_t^3 \tag{7.72}$$

in which

$$A_t^3 = \begin{bmatrix} M_1^t \\ -H_1^t \\ -M_1^t \\ H_1^t \\ \underline{c}^T M_1^t \\ -\underline{c}^T H_1^t \\ I \\ -I \end{bmatrix} \tag{7.73}$$

$$\underline{b}_t^3 = \begin{bmatrix} -\underline{U}_{t,min} - \underline{w}_t - M_2^t \underline{k} + H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} \\ -\underline{U}_{t-1,min} - \underline{w}_t + H_2^t \underline{k} - M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} \\ \underline{U}_{t,max} + \underline{w}_t + M_2^t \underline{k} - H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} \\ \underline{U}_{t-1,max} + \underline{w}_{t-1} - H_2^t \underline{k} + M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} \\ -De_t - \underline{c}^T \underline{w}_t - \underline{c}^T M_2^t \underline{k} + \underline{c}^T H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} \\ -De_{t-1} - \underline{c}^T \underline{w}_{t-1} + \underline{c}^T H_2^t \underline{k} - \underline{c}^T M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} \\ \underline{X}_{t,max} \\ -\underline{X}_{t,min} \end{bmatrix} \tag{7.74}$$

Notice that the model expressed by eqs. (7.71) and (7.72) has only four unknown variables. Equations (7.71) and (7.72) define a quadratic linearly-constrained problem. Because the problem has only inequality constraints, it is a linear complementarity problem (Fletcher, 1981), which can be efficiently solved by simplex-like methods such as Dantzig-Wolfe and Lemke algorithms. Care must be taken to ensure that G_t is positive definite before applying these methods. If G_t is indefinite, other methods (e.g., the method used in Section 7.8) are more suitable. The linear complementarity structure applies to the quadratic problems developed in Section 5.3, the quadratic problem (7.43)-(7.44), and the quadratic problem to be developed next.

7.4 Simplified Quadratic Model 2

If net reservoir losses are neglected but spillages are included as part of the decision variables, then the objective function of the model can be expressed as

$$\text{maximize } E_{t-1} + E_t = [\hat{\underline{h}}_t^T \quad \hat{\underline{q}}_t^T \quad \hat{\underline{p}}_t^T]\underline{\theta}_t + \underline{\theta}_t^T \hat{H} \underline{\theta}_t + \hat{c}_t \tag{7.75}$$

in which $\underline{\theta}_t$ is given by eq. (7.25), $\hat{c}_t$ is given by eq. (7.50), $\hat{\underline{h}}_t^T$ is given by eq. (7.51), and

$$\hat{\underline{q}}_t^T = [-(\underline{a}_2^T + \underline{k}^T B_{22})N_{21} - (\underline{a}_1^T + \underline{x}_{t+1} B_{11})N_{11}, \\ -(\underline{a}_2^T + \underline{k}^T B_{22})N_{22}] \tag{7.76}$$

$$\hat{\underline{p}}_t^T = [-(\underline{a}_2^T + \underline{k}^T B_{22})N_{21} - (\underline{a}_1^T + \underline{x}_{t-1} B_{11})N_{11}, \\ -(\underline{a}_2^T + \underline{k}^T B_{22})N_{22}] \tag{7.77}$$

$$\hat{H} = \frac{1}{2}\begin{bmatrix} 0 & -B_{11}N_{11} & -B_{11}N_{11} \\ -N_{11}^T B_{11} & 0 & 0 \\ -N_{11}^T B_{11} & 0 & 0 \end{bmatrix} \tag{7.78}$$

where the null matrix in the upper left corner of eq. (7.78) is of dimension 4 x 4 and the remaining null matrices are of dimension 9 x 9.

The set of constraints can be expressed as

$$A_t^* \underline{\theta}_t \leqq \underline{b}_t^* \tag{7.79}$$

$[I \quad 0 \quad 0]\underline{x}_{t-1}$, $[I \quad 0 \quad 0]\underline{x}_{t+1}$ fixed

in which

$$A_t^* = \begin{bmatrix} -H_1 & I-N & 0 \\ H_1 & 0 & I-N \\ -H_1 & -N & 0 \\ H_1 & N & 0 \\ H_1 & 0 & -N \\ -H_1 & 0 & N \\ -\underline{c}^T(-H_1 & I-N & 0) \\ -\underline{c}^T(H_1 & 0 & I-N) \\ 0 & I & 0 \\ 0 & 0 & I \\ I & 0 & 0 \\ -I & 0 & 0 \end{bmatrix} \tag{7.80}$$

$$\underline{b}_t^* = \begin{bmatrix} \underline{W}_t + \underline{w}_t + H_2\,\underline{k} - H\begin{bmatrix}\underline{x}_{t+1}\\ \underline{k}\end{bmatrix} \\ \underline{W}_{t-1} + \underline{w}_{t-1} - H_2\,\underline{k} + H\begin{bmatrix}\underline{x}_{t-1}\\ \underline{k}\end{bmatrix} \\ \underline{U}_{t,max} + \underline{w}_t + H_2\,\underline{k} - H\begin{bmatrix}\underline{x}_{t+1}\\ \underline{k}\end{bmatrix} \\ -\underline{U}_{t,min} - \underline{w}_t - H_2\,\underline{k} + H\begin{bmatrix}\underline{x}_{t+1}\\ \underline{k}\end{bmatrix} \\ \underline{U}_{t-1,max} + \underline{w}_{t-1} - H_2\,\underline{k} + H\begin{bmatrix}\underline{x}_{t-1}\\ \underline{k}\end{bmatrix} \\ -\underline{U}_{t-1,min} - \underline{w}_{t-1} + H_2\,\underline{k} - H\begin{bmatrix}\underline{x}_{t-1}\\ \underline{k}\end{bmatrix} \\ -De_t - \underline{c}^T\,\underline{w}_t - \underline{c}^T H_2\,\underline{k} + \underline{c}^T H\begin{bmatrix}\underline{x}_{t+1}\\ \underline{k}\end{bmatrix} \\ -De_{t-1} - \underline{c}^T\,\underline{w}_{t-1} + \underline{c}^T H_2\,\underline{k} - \underline{c}^T H\begin{bmatrix}\underline{x}_{t-1}\\ \underline{k}\end{bmatrix} \\ \underline{R}_t \\ \underline{R}_{t-1} \\ \underline{X}_{t,max} \\ -\underline{X}_{t,min} \end{bmatrix} \tag{7.81}$$

where the components of A_t^*, which arise from the different constraints, are ordered as in matrix $\hat{A}_t^2$ in eq. (7.45). The matrices in eq. (7.80) are obtained from eq. (7.48), i.e., eq. (7.48) can be rewritten as

$$\begin{bmatrix} \underline{u}_t^{(1)} \\ \underline{u}_t^{(2)} \end{bmatrix} = [H_1 \quad H_2] \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - [H_1 \quad H_2] \begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix} - \begin{bmatrix} N_{11} & 0 \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} \underline{r}_t^{(1)} \\ \underline{r}_t^{(2)} \end{bmatrix}$$

$$- \begin{bmatrix} \underline{w}_t^{(1)} \\ \underline{w}_t^{(2)} \end{bmatrix} = H \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - H \begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix} - N \underline{r}_t - \underline{w}_t \qquad (7.82)$$

Solutions to problems (7.43)-(7.44), (7.61)-(7.62), (7.71)-(7.72), and (7.75)-(7.79), are exactly the same as those of their corresponding analogs (5.74)-(5.75), (5.94)-(5.95), (5.98)-(5.99), and (5.102)-(5.103). The models in Sections 7.1-7.4 are more advantageous than those in Section 5.3 because they have fewer decision variables and hence there is a reduction in the computational burden.

7.5 Nonlinearity in the Energy Rate Equation

Development of the optimization models in Chapter 5 and Sections 7.1-7.4 assumed that the energy generation rate (ξ_t) is a linear function of reservoir storage (x_t). That assumption is reasonable for the power plants in the NCVP system, in which yearly reservoir levels commonly vary from about 30 to 80 percent of the reservoir capacity, as can be observed in Figs. 5.7 - 5.13. For other reservoir systems in which the energy rate curves depart considerably from a linear trace, the problem can be approached as explained in this section.

Assume that the energy rate for reservoir i (ξ_t^i) is a quadratic function of the average reservoir storage,

$$\xi_t^i = a^i + b^i(x_t^i + x_{t+1}^i/2) + c^i(x_t^i + x_{t+1}^i/2)^2 \tag{7.83}$$

in which a^i, b^i, and c^i are constant coefficients. In order to use the earlier optimization models, it is necessary to linearize the expression for ξ_t^i about a value $x_{t,0}^i$ (which is a scalar variable). That is, $x_{t,0}^i$ is the best guess of the optimum value of the storage for reservoir i at the beginning of month t. It is reasonable to set $x_{t,0}^i$ equal to the initial guess made to develop the initial policy. A first-order Taylor series approximation about $x_{t,0}^i$ in eq. (7.83) yields

$$\xi_t^i \cong \hat{a}_t^i + \hat{b}_t^i\, x_t^i \tag{7.84}$$

in which

$$\hat{a}_t^i = a^i + \frac{b^i}{2}(x_{t,0}^i + x_{t+1}^i) + \frac{c^i}{4}(x_{t,0}^i + x_{t+1}^i)^2 - \left[\frac{b^i}{2} + \frac{c^i}{2}(x_{t,0}^i + x_{t+1}^i)\right]x_{t,0}^i \tag{7.85}$$

$$\hat{b}_t^i = \frac{b^i}{2} + \frac{c^i}{2}(x_{t,0}^i + x_{t+1}^i) \tag{7.86}$$

The linearized energy rate equation [eq. (7.84)] would then be used to form the objective function of the optimization model, which is constructed in the usual manner (see, e.g., Section 7.1).

Suppose that the two-stage maximization problem has been set up and eq. (7.84) has been used to form the objective function. From earlier developments in Chapter 5 and Sections 7.1-7.4, the two-stage maximization problem is

$$\text{maximize } \underline{\theta}_t^T H_t \underline{\theta}_t + \underline{q}_t^T \underline{\theta}_t \tag{7.87}$$

subject to

$$A_t \underline{\theta}_t \leqq \underline{b}_t \tag{7.88}$$

$$[I \quad 0 \quad 0]\underline{\theta}_{t-1}, \quad [I \quad 0 \quad 0]\underline{\theta}_t \text{ fixed}$$

The problem represented by eqs. (7.87) and (7.88) could be solved by any convenient quadratic or dynamic programming algorithm (see Section 7.8). However, after a solution is obtained, it is not possible to advance to the next two-stage maximization because the vector $\underline{\theta}_t^{1T} = (\underline{x}_{t,1} \;\; \underline{r}_{t,1} \;\; \underline{r}_{t-1,1})$ that solves eqs. (7.87) and (7.88) depends on the assumed vector $\underline{x}_{t,0}$ (the vector of values used to perform the expansion of the energy values for all the reservoirs) used earlier to linearize the rates of energy generation ξ_t^i's. A second solution to the two-stage problem is then carried out by linearizing the energy rate equation [eq. (7.83)] about $\underline{x}_{t,1}$, in which $\underline{x}_{t,1}$ is the leading subvector of $\underline{\theta}_t^1$. The new solution, $\underline{\theta}_t^{2T} = (\underline{x}_{t,2} \;\; \underline{r}_{t,2} \;\; \underline{r}_{t-1,2})$ would then be used to perform another linearization of the energy rate equation about $\underline{x}_{t,2}$ and the two-stage problem is solved again. The procedure is repeated until the subvector $\underline{x}_{t,j}$ used to linearize the energy rate equation in the jth iteration is approximately equal to the solution subvector $\underline{x}_{t,j+1}$ obtained in the jth solution of the two-stage problem. Upon convergence of $\underline{x}_{t,j}$ (the convergence criterion is chosen by the user), one can proceed to the solution of the two-stage problem corresponding to month t+1 in the usual way of advance as done by the POA.

7.6 Nonlinearities in the Constraints

Nonlinearities in the constraints of reservoir operation models typically arise with respect to power generation and spillage releases. Nonlinear constraints do not invalidate any of the earlier developments in this chapter or in Chapter 5. In fact, the impact of nonlinear constraints would be only in the formulation of the two-stage maximization model, which now becomes (dropping constant terms in the objective function)

$$\text{maximize } \underline{\theta}_t^T \hat{H}_t \underline{\theta}_t + \hat{\underline{q}}_t^T \underline{\theta}_t \tag{7.89}$$

subject to

$$\hat{A}_t^1 \underline{\theta}_t \leqq \hat{\underline{b}}_t^1 \tag{7.90}$$

$$f_i(\underline{\theta}_t) \in R_i, \quad i = 1, \ldots, r \tag{7.91}$$

$$[I \quad 0 \quad 0]\underline{\theta}_{t-1}, \quad [I \quad 0 \quad 0]\underline{\theta}_t \text{ fixed}$$

Equations (7.90) and (7.91) represent sets of linear and nonlinear constraints, respectively. There are r nonlinear constraints in which R_i denotes the feasible sets. Equation (7.89) can be developed by linearizing the energy rate equation, as discussed in the previous section. If the nonlinear constraints are linearized about values of the decision vector $\underline{\theta}_t$, then the presence of nonlinear constraints in the optimization model would lead to an iterative solution of each two-stage maximization problem. The presence of a nonlinear energy rate function and/or nonlinear constraints converts the two-stage maximization problems into nonlinear problems, as opposed to linear or quadratic linearly-constrained problems. In principle, those nonlinear problems could be solved directly by nonlinear methods (Fletcher, 1981; McCormick, 1983; Wismer and Chattergy, 1983). However, due to the large-scale nature of those problems, it is more convenient to recourse to linearization of the objective function and/or constraints and solve a larger number of simple problems, within the framework of the models developed earlier.

Because spillage is the only source of nonlinearity in the constraints of the optimization models considered in this report, a procedure to handle nonlinearities in the spillage releases will be discussed. The procedure is also applicable to other types of nonlinear constraints. Typically, the flow over a spillway can be expressed as

$$r_t^i = \tilde{c}^i \, (h_t^i - d^i)^{\eta^i} \, \delta^i \tag{7.92}$$

in which $\delta^i = 0$ if $h_t^i < d^i$, $\delta^i = 1$ if $h_t^i \geqq d^i$, the index i denotes the ith reservoir, coefficients $\tilde{c}^i$, d^i, and η^i are constant, and h_t^i is the reservoir water-surface elevation during month t. The introduction of δ^i is necessary to prevent the computation of spillage whenever the reservoir level h_t^i is below the spillway crest d^i. Equation (7.92) is used for the sake of argument; other functions can be handled similarly, as will be explained later. A relation expressing h_t^i as a function of storage (x_t^i) can be developed from reservoir storage-elevation data (see Appendix A). Let that relation be

$$h_t^i = g^i(x_t^i) \tag{7.93}$$

in which g^i is any function that represents the fit between h_t^i and x_t^i. Substitution of eq. (7.93) into eq. (7.92) gives

$$r_t^i = \tilde{c}^i[g^i(x_t^i) - d^i]^{\eta^i}\delta^i \tag{7.94}$$

Linearization of r_t^i in eq. (7.94) about $x_{t,0}^i$ (as in Section 7.5) yields

$$r_t^i \cong \hat{c}_t^i + \hat{d}_t^i\, x_t^i \tag{7.95}$$

in which

$$\hat{c}_t^i = \Big\{\tilde{c}^i[g^i(x_{t,0}^i) - d^i]^{\eta^i} - x_{t,0}^i\, \eta^i\tilde{c}^i[g^i(x_{t,0}^i)$$

$$- d^i]^{\eta^i-1}\left[\frac{dg}{dx_t^i}\right]_{x_{t,0}^i} \Big\}\, \delta^i \tag{7.96}$$

$$\hat{d}_t^i = \Big\{\eta^1\tilde{c}^i[g^i(x_{t,0}^i) - d^i]^{\eta^i-1} \left[\frac{dg}{dx_t^i}\right]_{x_{t,0}^i} \Big\}\, \delta^i \tag{7.97}$$

For a system of reservoirs, constraints on spillages during month t can be expressed as

$$\underline{r}_t = \underline{c}_t + D_t\, \underline{x}_t \leqq \underline{R}_t \tag{7.98}$$

in which $\underline{c}_t^T = [\hat{c}_t^1 \quad \hat{c}_t^2 \quad \ldots \quad \hat{c}_t^n]$, n is the number of reservoirs in the system, D_t is a diagonal matrix whose (diagonal) elements are the $\hat{d}_t^i$'s, $\underline{x}_t$ is the reservoir storage vector, and $\underline{R}_t$ is the vector of feasible values. Similarly, for month t-1,

$$\underline{r}_{t-1} = \underline{c}_{t-1} + D_{t-1}\, \underline{x}_t \leqq \underline{R}_{t-1} \tag{7.99}$$

Thus, the problem defined by eqs. (7.89)-(7.91) can now be stated in terms of a set of linear constraints, i.e.,

$$\text{maximize } \underline{\theta}_t^T \hat{H}_t\, \underline{\theta}_t + \hat{\underline{q}}_t^T\, \underline{\theta}_t \tag{7.100}$$

subject to

$$\begin{bmatrix} \hat{A}_t^1 & \\ D_t & 0 \\ D_{t-1} & 0 \end{bmatrix} \underline{\theta}_t \leqq \begin{bmatrix} \hat{\underline{b}}_t^1 \\ \underline{R}_t - \underline{c}_t \\ \underline{R}_{t-1} - \underline{c}_{t-1} \end{bmatrix} \tag{7.101}$$

$[I \quad 0 \quad 0]\underline{\theta}_{t-1}, \quad [I \quad 0 \quad 0]\underline{\theta}_t$ fixed

in which the null matrices in eq. (7.101) are of dimension n x 2n.

The solution of the two-stage problem represented by eqs. (7.100) and (7.101) is obtained in an iterative manner. The procedure is as follows:

1) Set the iteration (counter) index k equal to zero. Expand the energy rate equation and/or spillage constraints about $\underline{x}_{t,k}$ [this has been accomplished already by developing eqs. (7.100) and (7.101)].

2) Solve the two-stage problem given by eqs. (7.100) and (7.101).

3) If the subvector $\underline{x}_{t,k+1}$ (which is part of the solution $\underline{\theta}_{t,k+1}$ in Step 2) is approximately equal to $\underline{x}_{t,k}$, proceed with the POA algorithm for the next (period t+1) two-stage problem. Otherwise linearize the nonlinear components of the problem about $\underline{x}_{t,k}$, increase the counter k by one, and go to Step 2.

Expressing spillages as a function of average storage results in an underestimation of the volume of spilled water when there is a rapid increase in the reservoir's water elevation during any month (from a level below the spillway crest to another level above the spillway crest). Similarly, an underestimation of the volume of spilled water results when there is a sharp decline in the monthly water level of the reservoir from a point above the spillway crest to another point below the spillway crest. This underestimation of the volume of spilled water could cause significant unbalances in the equation of continuity. That can happen during periods of high inflows into the reservoir. Because the duration of those high-inflow events is relatively short as compared to one month, the natural way to overcome the underestimation of spillage is to shorten the duration of the stages in the POA (e.g., from one month to fifteen days for periods of high inflows only). The POA algorithm does not require any modification to handle variable-duration stages, except for the way in which the input data are prepared. Finally, for reservoirs with large surface areas (as in Shasta, Folsom, Clair Engle, and New Melones), high-inflow volumes result in moderate increases in water level elevations, thus reducing the likelihood of inaccurate spillage computation.

The next section illustrates the application of the concepts in Sections 7.5 and 7.6 to the NCVP system.

7.7 A Full Model of Minimum Dimensionality

This section demonstrates that when the energy rates and spillages are modeled as nonlinear expressions, the model represented by eqs. (7.100) and (7.101) can be expressed as a quadratic linearly-constrained model whose dimensionality (number of unknown variables) is equal to the number of nonregulating reservoirs. The resulting quadratic problem is the most general form of the reservoir operation model that can be obtained because it accounts for nonlinearities in the objective function and/or constraints, and optimizes both penstock and spillage releases simultaneously and explicitly. The generalization of the model results in a problem of minimum dimensionality (four dimensions in the case of the NCVP).

Spillages for months t and t-1, respectively, can be expressed as

$$\begin{bmatrix} \underline{r}_t^{(1)} \\ \underline{r}_t^{(2)} \end{bmatrix} = \begin{bmatrix} \underline{c}_t \\ \underline{K} \end{bmatrix} + \begin{bmatrix} D_t & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix} \tag{7.102}$$

$$\begin{bmatrix} \underline{r}_{t-1}^{(1)} \\ \underline{r}_{t-1}^{(2)} \end{bmatrix} = \begin{bmatrix} \underline{c}_{t-1} \\ \underline{K} \end{bmatrix} + \begin{bmatrix} D_{t-1} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} \tag{7.103}$$

in which

$$\underline{K} = \begin{bmatrix} \tilde{c}^2(h_t^2 - d^2)\eta^2\delta^2 \\ \tilde{c}^3(h_t^3 - d^3)\eta^3\delta^3 \\ \tilde{c}^5(h_t^5 - d^5)\eta^5\delta^5 \\ \tilde{c}^7(h_t^7 - d^7)\eta^7\delta^7 \\ \tilde{c}^9(h_t^9 - d^9)\eta^9\delta^9 \end{bmatrix} \qquad (7.104)$$

where coefficients $\tilde{c}^i$, d^i, η^i are as defined in eq. (7.92) and $\underline{c}_t$, D_t, $\underline{c}_{t-1}$, and D_{t-1} are as defined in eqs. (7.98) and (7.99), respectively. Partitions $[(\cdot)^{(1)} \quad (\cdot)^2]$ refer to the nonregulating and regulating reservoir subvectors, respectively, and $\underline{k}$ is the subvector of (fixed) regulating reservoir storages whose components are x_t^2, x_t^3, x_t^5, x_t^7, and x_t^9. Those values were defined in Section 7.1.

The continuity equation for months t and t-1, respectively, can be expressed as

$$\underline{u}_t = H_{t+1}\begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - \begin{bmatrix} M_{11}^t & 0 \\ M_{21}^t & M_{22}^t \end{bmatrix}\begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix} - N\left[\begin{bmatrix} \underline{c}_t \\ \underline{K} \end{bmatrix} + \begin{bmatrix} D_t & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix}\right] - \underline{w}_t$$

$$= H_{t+1}\begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - \begin{bmatrix} \hat{M}_{11}^t & 0 \\ \hat{M}_{21}^t & \hat{M}_{22}^t \end{bmatrix}\begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix} - \begin{bmatrix} \hat{\underline{w}}_t^{(1)} \\ \hat{\underline{w}}_t^{(2)} \end{bmatrix} \qquad (7.105)$$

$$\underline{u}_{t-1} = \begin{bmatrix} H_{11}^{t} & 0 \\ H_{21}^{t} & H_{22}^{t} \end{bmatrix} \begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix} - \begin{bmatrix} M_{11}^{t-1} & 0 \\ M_{21}^{t-1} & M_{22}^{t-1} \end{bmatrix} \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix}$$

$$- N \begin{bmatrix} \underline{c}_{t-1} \\ \underline{K} \end{bmatrix} - \begin{bmatrix} N_{11}D_{t-1} & 0 \\ N_{21}D_{t-1} & 0 \end{bmatrix} \begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix} - \underline{w}_{t-1}$$

$$= \begin{bmatrix} \hat{H}_{11}^{t} & 0 \\ \hat{H}_{21}^{t} & \hat{H}_{22}^{t} \end{bmatrix} \begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix} - \begin{bmatrix} M_{11}^{t-1} & 0 \\ M_{21}^{t-1} & M_{22}^{t-1} \end{bmatrix} \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} - \begin{bmatrix} \hat{\underline{w}}_{t-1}^{(1)} \\ \hat{\underline{w}}_{t-1}^{(2)} \end{bmatrix} \qquad (7.106)$$

in which

$$\begin{bmatrix} \hat{M}_{11}^{t} & 0 \\ \hat{M}_{21}^{t} & \hat{M}_{22}^{t} \end{bmatrix} = \begin{bmatrix} M_{11}^{t}+N_{11}D_t & 0 \\ M_{21}^{t}+N_{21}D_t & M_{22}^{t} \end{bmatrix} \qquad (7.107)$$

$$\begin{bmatrix} \hat{\underline{w}}_{t}^{(1)} \\ \hat{\underline{w}}_{t}^{(2)} \end{bmatrix} = \begin{bmatrix} \underline{w}_{t}^{(1)} \\ \underline{w}_{t}^{(2)} \end{bmatrix} + N \begin{bmatrix} \underline{c}_t \\ \underline{K} \end{bmatrix} \qquad (7.108)$$

$$\begin{bmatrix} \hat{H}_{11}^{t} & 0 \\ \hat{H}_{21}^{t} & \hat{H}_{22}^{t} \end{bmatrix} = \begin{bmatrix} H_{11}^{t}-N_{11}D_{t-1} & 0 \\ H_{21}^{t}-N_{21}D_{t-1} & H_{22}^{t} \end{bmatrix} \qquad (7.109)$$

$$\begin{bmatrix} \hat{\underline{w}}_{t-1}^{(1)} \\ \hat{\underline{w}}_{t-1}^{(2)} \end{bmatrix} = \begin{bmatrix} \underline{w}_{t-1}^{(1)} \\ \underline{w}_{t-1}^{(2)} \end{bmatrix} + N \begin{bmatrix} \underline{c}_{t-1} \\ \underline{K} \end{bmatrix} \qquad (7.110)$$

and H_{t+1}, M_t, N_t, and $\underline{w}_t$ are as defined in eq. (7.15).

The rates of energy generation for months t and t-1, respectively, are

$$\xi_t = \begin{bmatrix} \underline{\hat{a}}_t^{(1)} \\ \underline{\hat{a}}^{(2)} \end{bmatrix} + \begin{bmatrix} \hat{B}_{11}^t & 0 \\ 0 & \hat{B}_{22} \end{bmatrix} \begin{bmatrix} \underline{x}_t \\ \underline{\hat{k}} \end{bmatrix} \tag{7.111}$$

$$\xi_{t-1} = \begin{bmatrix} \underline{\hat{a}}_{t-1}^{(1)} \\ \underline{\hat{a}}^{(2)} \end{bmatrix} + \begin{bmatrix} \hat{B}_{11}^{t-1} & 0 \\ 0 & \hat{B}_{22} \end{bmatrix} \begin{bmatrix} \underline{x}_t \\ \underline{\hat{k}} \end{bmatrix} \tag{7.112}$$

in which

$$\underline{\hat{a}}_t^{(1)T} = (\hat{a}_t^1 \quad \hat{a}_t^4 \quad \hat{a}_t^6 \quad \hat{a}_t^8) \tag{7.113}$$

$$\underline{\hat{a}}_{t-1}^{(1)T} = (\hat{a}_{t-1}^1 \quad \hat{a}_{t-1}^4 \quad \hat{a}_{t-1}^6 \quad \hat{a}_{t-1}^8) \tag{7.114}$$

$$\underline{\hat{a}}^{(2)T} = (a^2 \quad a^3 \quad a^5 \quad a^7 \quad a^9) \tag{7.115}$$

$$\hat{B}_{11}^t = \begin{bmatrix} \hat{b}_t^1 & & & 0 \\ & \hat{b}_t^4 & & \\ & & \hat{b}_t^6 & \\ 0 & & & \hat{b}_t^8 \end{bmatrix} \tag{7.116}$$

$$\hat{B}_{11}^{t-1} = \begin{bmatrix} \hat{b}_{t-1}^1 & & & 0 \\ & \hat{b}_{t-1}^4 & & \\ & & \hat{b}_{t-1}^6 & \\ 0 & & & \hat{b}_{t-1}^8 \end{bmatrix} \tag{7.117}$$

$$\hat{B}_{22} = \begin{bmatrix} b^2 + c^2x_t^2 & & & & \\ & b^3 + c^3x_t^3 & & 0 & \\ & & b^5 + c^5x_t^5 & & \\ 0 & & & b^7 + c^7x_t^7 & \\ & & & & b^9 + c^9x_t^9 \end{bmatrix} \tag{7.118}$$

$$\hat{k} = \begin{bmatrix} x_t^2 \\ x_t^3 \\ x_t^5 \\ x_t^7 \\ x_t^9 \end{bmatrix} \tag{7.119}$$

where the $\underline{\hat{a}}_t^i$'s and $\underline{\hat{a}}_{t-1}^i$'s in eqs. (7.113) and (7.114), respectively, are defined by

$$\underline{\hat{a}}_t^{(i)} = a^i + \frac{b^i}{2}(x_{t,0}^i + x_{t+1}^i) + \frac{c^i}{4}(x_{t,0}^i + x_{t+1}^i)^2$$

$$- \left[\frac{b^i}{2} + \frac{c^i}{2}(x_{t,0}^i + x_{t+1}^i)\right]x_{t,0}^i \tag{7.120}$$

$$\underline{\hat{a}}_{t-1}^{(i)} = a^i + \frac{b^i}{2}(x_{t,0}^i + x_{t-1}^i) + \frac{c^i}{4}(x_{t,0}^i + x_{t-1}^i)^2$$

$$- \left[\frac{b^i}{2} + \frac{c^i}{2}(x_{t,0}^i + x_{t-1}^i)\right]x_{t,0}^i \tag{7.121}$$

The $\hat{b}_t^i$'s and $\hat{b}_{t-1}^i$'s of eqs. (7.116) and (7.117), respectively, are defined by

$$\hat{b}_t^i = \frac{b^i}{2} + \frac{c^i}{2}(x_{t,0}^i + x_{t+1}^i) \tag{7.122}$$

$$\hat{b}_{t-1}^i = \frac{b^i}{2} + \frac{c^i}{2}(x_{t,0}^i + x_{t-1}^i) \tag{7.123}$$

The a^i's, b^i's, and c^i's of eqs. (7.115), (7.118), and (7.120)-(7.123) are defined in eq. (7.83). The x_t^i's in eqs. (7.118) and (7.119) are the constant values of storage at the regulating reservoirs, whose values were defined in Section 7.1.

From eqs. (7.105), (7.106), (7.111), and (7.112), the energy equation for the two-stage problem becomes

$$E_t + E_{t-1} = \underline{\xi}_t^T \underline{u}_t + \underline{\xi}_{t-1}^T \underline{u}_{t-1} = k_t^* + \underline{q}_t^{*T} \underline{x}_t + \underline{x}_t^T H_t^* \underline{x}_t \tag{7.124}$$

in which

$$\begin{aligned} k_t^* = {} & [\underline{\hat{a}}^{(2)T} + \underline{\hat{k}}^T \hat{B}_{22}] [H_{21}^{t+1} \underline{x}_{t+1} + H_{22}^{t+1} \underline{k} - \hat{M}_{22}^t \underline{k} - \underline{\hat{w}}_t^{(2)} \\ & + \hat{H}_{22}^t \underline{k} - M_{21}^{t-1} \underline{x}_{t-1} - M_{22}^{t-1} \underline{k} - \underline{w}_{t-1}^{(2)}] + [\underline{\hat{a}}_t^{(1)T} \\ & \cdot (H_{11}^{t+1} \underline{x}_t - \underline{\hat{w}}_t^{(1)}) - \underline{\hat{a}}_{t-1}^{(1)T} (M_{11}^{t-1} \underline{x}_{t-1} + \underline{\hat{w}}_{t-1}^{(1)})] \end{aligned} \tag{7.125}$$

$$\underline{q}_t^{*T} = - \underline{\hat{a}}_t^{(1)T} \hat{M}_{11}^t - (\underline{\hat{a}}^{(2)T} + \underline{\hat{k}}^T \hat{B}_{22}) (\hat{M}_{21}^t + \hat{H}_{21}^t)$$

$$+ \underline{x}_{t+1}^T (\hat{B}_{11}^t H_{11}^{t+1})^T - \underline{\hat{w}}_t^{(1)T} \hat{B}_{11}^t + \underline{\hat{a}}_{t-1}^{(1)T} \hat{H}_{11}^t$$

$$- \underline{x}_{t-1}^T (\hat{B}_{11}^{t-1} M_{11}^{t-1})^T - \underline{\hat{w}}_{t-1}^{(1)T} \hat{B}_{11}^{t-1} \tag{7.126}$$

$$H_t^* = \hat{B}_{11}^{t-1} \hat{H}_{11}^t - \hat{B}_{11}^t \hat{M}_{11}^t \tag{7.127}$$

The unknown vector $\underline{x}_t$ in eq. (7.124) is of dimension 4 x 1. The constraints associated with eq. (7.124) are:

Constraints on total releases for months t and t-1. For month t,

$$\underline{u}_t + \underline{r}_t \leq \underline{W}_t$$

$$\Rightarrow \hat{\hat{M}}_1^t \underline{x}_t \leq \underline{W}_t + \hat{M}_2^t \underline{k} - H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} + \underline{\hat{\hat{w}}}_t \tag{7.128}$$

in which

$$\hat{\hat{M}}_t = [\hat{\hat{M}}_1^t \quad \hat{\hat{M}}_2^t] = \begin{bmatrix} D_t & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \hat{M}_{11}^t & 0 \\ \hat{M}_{21}^t & \hat{M}_{22}^t \end{bmatrix} \tag{7.129}$$

$$\underline{\hat{\hat{w}}}_t = \underline{\hat{w}}_t - \begin{bmatrix} \underline{c}_t \\ \underline{K} \end{bmatrix} \tag{7.130}$$

where matrix D_t is defined by eq. (7.98), $\hat{M}_t$ by eq. (7.107), $\underline{\hat{w}}_t$ by eq. (7.108), and $(\underline{c}_t^T \quad \underline{K}^T)$ by eqs. (7.102) and (7.104), respectively. For month t-1,

$$\hat{\hat{H}}_1^t \underline{x}_t \leqq \underline{W}_{t-1} - \hat{H}_2^t \underline{k} + M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} + \hat{\hat{\underline{w}}}_{t-1} \tag{7.131}$$

in which

$$\hat{\hat{H}}_t = [\hat{\hat{H}}_1^t \quad \hat{\hat{H}}_2^t] = \begin{bmatrix} \hat{H}_{11}^t & 0 \\ \hat{H}_{21}^t & \hat{H}_{22}^t \end{bmatrix} + \begin{bmatrix} D_{t-1} & 0 \\ 0 & 0 \end{bmatrix} \tag{7.132}$$

$$\hat{\hat{\underline{w}}}_{t-1} = \hat{\underline{w}}_{t-1} - \begin{bmatrix} \underline{c}_{t-1} \\ \underline{K} \end{bmatrix} \tag{7.133}$$

where D_{t-1} is defined by eq. (7.103), $\hat{H}_t$ by eq. (7.106), and $\hat{\underline{w}}_{t-1}$ by eq. (7.110).

Constraints on maximum penstock releases for months t and t-1. For month t,

$$\underline{u}_t \leqq \underline{U}_{t,max}$$

$$\Rightarrow \; -M_1^t \underline{x}_t \leqq \underline{U}_{t,max} + \hat{M}_2^t \underline{k} - H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} + \hat{\underline{w}}_t \tag{7.134}$$

in which $\hat{M}_1^t = [\hat{M}_1^t \quad \hat{M}_2^t]$. For month t-1,

$$\underline{u}_{t-1} \leqq \underline{U}_{t-1,max}$$

$$\Rightarrow \; \hat{H}_1^t \underline{x}_t \leqq \underline{U}_{t-1,max} - \hat{H}_2^t \underline{k} + M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} + \hat{\underline{w}}_{t-1} \tag{7.135}$$

Constraints on minimum penstock releases for month t and t-1. For month t,

$$\underline{u}_t \geqq \underline{U}_{t,min}$$

$$\Rightarrow \hat{M}_1^t \underline{x}_t \leqq \underline{U}_{t,min} - \hat{M}_2^t \underline{k} + H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - \hat{\underline{w}}_t \tag{7.136}$$

For month t-1,

$$\underline{u}_{t-1} \geqq \underline{U}_{t,min}$$

$$\Rightarrow -\hat{H}_1^t \underline{x}_t \leqq -\underline{U}_{t-1,min} + \hat{H}_2^t \underline{k} - M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} - \hat{\underline{w}}_{t-1} \tag{7.137}$$

Constraints on Delta requirements for months t and t-1. For month t,

$$\underline{c}^T(\underline{u}_t + \underline{r}_t) \geqq De_t$$

$$\Rightarrow -\underline{c}^T \hat{\hat{M}}_1^t \underline{x}_t \leqq -De_t - \underline{c}^T \hat{M}_2^t \underline{k} + \underline{c}^T H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - \underline{c}^T \hat{\hat{\underline{w}}}_t \tag{7.138}$$

For month t-1,

$$\underline{c}^T(\underline{u}_{t-1} + \underline{r}_{t-1}) \geqq De_{t-1}$$

$$\Rightarrow -\underline{c}^T \hat{\hat{H}}_1^t \underline{x}_t \leqq -De_{t-1} + \underline{c}^T \hat{H}_2^t \underline{k} - \underline{c}^T M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} - \underline{c}^T \hat{\hat{\underline{w}}}_{t-1} \tag{7.139}$$

in which $\underline{c}^T = (0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1)$.

Constraints on spillages for months t and t-1. For month t,

$$\underline{r}_t \leq \underline{R}_t \Rightarrow \begin{bmatrix} \underline{c}_t \\ \underline{K} \end{bmatrix} + \begin{bmatrix} D_t & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix} \leq \underline{R}_t \Rightarrow \begin{bmatrix} D_t \\ 0 \end{bmatrix} \underline{x}_t \leq \underline{R}_t - \begin{bmatrix} \underline{c}_t \\ \underline{K} \end{bmatrix} \qquad (7.140)$$

For month t-1,

$$\underline{r}_{t-1} \leq \underline{R}_{t-1} \Rightarrow \begin{bmatrix} \underline{c}_{t-1} \\ \underline{K} \end{bmatrix} + \begin{bmatrix} D_{t-1} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{x}_t \\ \underline{k} \end{bmatrix} \leq \underline{R}_{t-1}$$

$$\Rightarrow \begin{bmatrix} D_{t-1} \\ 0 \end{bmatrix} \underline{x}_t \leq \underline{R}_{t-1} - \begin{bmatrix} \underline{c}_{t-1} \\ \underline{K} \end{bmatrix} \qquad (7.141)$$

Constraints on maximum and minimum storage releases for month t,

$$\underline{x}_t \leq \underline{X}_{t,max} \qquad (7.142)$$

$$-\underline{x}_t \leq -\underline{X}_{t,min} \qquad (7.143)$$

In compact form, the two-stage maximization model can be expressed as (dropping the constant term k_t^*)

$$\underset{\underline{x}_t}{\text{maximize}} \quad \underline{q}_t^{*T} \underline{x}_t + \underline{x}_t^T H_t^* \underline{x}_t \qquad (7.144)$$

subject to

$$A_t^{**} \underline{x}_t \leq \underline{b}_t^{**} \qquad (7.145)$$

$\underline{x}_{t-1}, \underline{x}_{t+1}$ fixed

in which

$$A_t^{**} = \begin{bmatrix} \hat{\hat{M}}_1^t \\ \hat{\hat{H}}_1^t \\ -\hat{M}_1^t \\ \hat{H}_1^t \\ \hat{M}_1^t \\ -\hat{H}_1^t \\ -\underline{c}^T \hat{\hat{M}}_1^T \\ -\underline{c}^T \hat{\hat{H}}_1^T \\ D_t \\ 0 \\ D_{t-1} \\ 0 \\ I \\ -I \end{bmatrix} \qquad (7.146)$$

$$
\underline{b}_t^{**} = \begin{bmatrix}
\underline{W}_t + \hat{M}_2^t \, \underline{k} - H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} + \underline{\hat{\hat{w}}}_t \\
\underline{W}_{t-1} + \hat{H}_2^t + M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} + \underline{\hat{\hat{w}}}_{t-1} \\
\underline{U}_{t,max} + \hat{M}_2^t \, \underline{k} - H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} + \underline{\hat{w}}_t \\
\underline{U}_{t-1,max} - \hat{H}_2^t \, \underline{k} + M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} + \underline{\hat{w}}_{t-1} \\
-\underline{U}_{t,min} - \hat{M}_2^t \, \underline{k} + H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - \underline{\hat{w}}_t \\
-\underline{U}_{t-1,min} + \hat{H}_2^t \, \underline{k} - M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} - \underline{\hat{w}}_{t-1} \\
-De_t - \underline{c}^T \hat{M}_2^t \, \underline{k} + \underline{c}^T H_{t+1} \begin{bmatrix} \underline{x}_{t+1} \\ \underline{k} \end{bmatrix} - \underline{c}^T \underline{\hat{\hat{w}}}_t \\
-De_{t-1} + \underline{c}^T \hat{H}_2^t \, \underline{k} - \underline{c}^T M_t \begin{bmatrix} \underline{x}_{t-1} \\ \underline{k} \end{bmatrix} - \underline{c}^T \underline{\hat{\hat{w}}}_t \\
\underline{R}_t - \begin{bmatrix} \underline{c}_t \\ \underline{K} \end{bmatrix} \\
\underline{R}_{t-1} - \begin{bmatrix} \underline{c}_{t-1} \\ \underline{K} \end{bmatrix} \\
\underline{X}_{t,max} \\
-\underline{X}_{t,min}
\end{bmatrix} \tag{7.147}
$$

For the NCVP system, the dimensionality of the problem given by eqs. (7.144) and (7.145) is four (i.e., there are four unknown variables). For other systems, the dimensionality will be equal to the number of nonregulating reservoirs. Because of the simple structure of the problem (quadratic objective function and linear inequality constraints), reliable algorithms can be used to obtain a solution (Gill and Murray, 1974, 1977).

When the problem represented by eqs. (7.144) and (7.145) is solved, the solution $\underline{x}^*_{t,1}$ is compared with $\underline{x}_{t,0}$ (the vector used to perform the first linearization of the objective function and/or constraints). If an adequate convergence criterion is satisfied, then advance to the next two-stage maximization corresponding to period t+1. If convergence is not attained, a new linearization is performed about $\underline{x}^*_{t,1}$. The resulting problem is solved and the convergence is checked to decide whether to advance to the next two-step maximization or to keep solving the current (period t) two-stage problem until convergence is obtained.

The model [eqs. (7.144) and (7.145)] should be used whenever spillages can be expressed as function of storages because that leads to a "full model" of very low dimensionality. If spillages cannot be expressed as function of storages (either those functions are not available or it is not proper to model spillages as function of storages only), then the "full models" of Sections 5.3 and 7.1 are alternative ways of modeling the reservoir system, at the expense of an increase in the dimensionality of the problem.

The different two-stage maximization formulations developed in Sections 7.1-7.4 and in this section can be used for different periods interchangeably. The linear model of Section 7.2 may prove to be the simplest model to use in the summer season. The full model presented in this section is a likely choice to be used during flooding events.

7.8 Application of General Nonlinear Model

The full model of minimum dimensionality developed in Section 7.7 [eqs. (7.144) and (7.145), which includes nonlinear energy generation rates and spillages, is applied to the NCVP. Water year 1979-80 is selected for the application with the purpose of comparing the results with those shown in Figs. 6.3 and 6.4, obtained from the linear model of Section 5.3 and from actual operation records.

Recall that the model obtained in Section 7.7 is

$$\underset{\underline{x}_t}{\text{maximize}}\ \underline{q}_t^{*T}\ \underline{x}_t + \underline{x}_t^T\ H_t^*\ \underline{x}_t \tag{7.144}$$

subject to

$$A_t^{**}\ \underline{x}_t \leq \underline{b}_t^{**} \tag{7.145}$$

$$\underline{x}_{t-1},\ \underline{x}_{t+1}\quad \text{fixed}$$

in which the decision vector $\underline{x}_t$ is a four-dimensional vector that contains the storages of Clair Engle, Shasta, Folsom, and New Melones stacked in that order. The constraint set was linearized and put into the form of eq. (7.145). A linearization procedure was also used to state eq. (7.144) as a quadratic function (see details in Sections 7.5-7.7). Thus, the problem stated by eqs. (7.144)-(7.145) must be solved iteratively as explained in Section 7.7. Before applying the generalized model, several tasks must be performed: (i) development of nonlinear (quadratic) energy rates; (ii) development of spillage equations; (iii) development of initial policies; and (iv) selection of a suitable algorithm to solve the quadratic linearly-constrained problem posed by eqs. (7.144)-(7.145).

Energy Generation Rates

The motivation for developing quadratic functions for the energy rate curves shown in Figs. 5.7-5.13 is to obtain a better fit throughout the entire operation range as compared to the linear functions in eqs. (5.1)-(5.9). Three of the energy rate curves remain linear, namely those for Judge Francis Carr Power Plant (Fig. 5.3), Keswick, and Nimbus, and no quadratic functions need to be developed for those power plants. By using the data displayed in Figs. 5.7-5.13, the following energy rates are obtained:

Trinity (at Clair Engle Lake)

$$\xi_T = 133.0 + 0.228\ \bar{x}_T - 0.468 \times 10^{-4}\ \bar{x}_T^2 \qquad (7.148)$$

$r^2 = 99.3\%$

Judge Francis Carr

$$\xi_{JFC} = 606.3 - 0.254\ x_w \qquad (7.149)$$

$r^2 = 99.8\%$

Spring Creek

$$\xi_{SC} = 445.0 + 0.738\ x_W - 1.10 \times 10^{-3}\ x_w^2 \qquad (7.150)$$

$r^2 = 99.8\%$

Shasta

$$\xi_S = 169.0 + 0.107\ \bar{x}_S - 0.115 \times 10^{-4}\ \bar{x}_S^2 \qquad (7.151)$$

$r^2 = 99.6\%$

Keswick

$$\xi_K = 80.3 + 0.6\ x_K \tag{7.152}$$
$$r^2 = 95.8\%$$

Folsom

$$\xi_F = 171.0 + 0.265\ \bar{x}_F - 0.130 \times 10^{-3}\ \bar{x}_F^2 \tag{7.153}$$
$$r^2 = 98.7\%$$

Nimbus

$$\xi_N = 26.3 + 0.80\ x_N \tag{7.154}$$
$$r^2 = 91.0\%$$

New Melones

$$\xi_{NM} = 169.0 + 0.275\ \bar{x}_{NM} - 0.479 \times 10^{-4}\ \bar{x}_{NM}^2 \tag{7.155}$$
$$r^2 = 98.6\%$$

Tullock

$$\xi_{TU} = 63.4 + 1.020\ x_{TU} - 1.37 \times 10^{-3}\ x_{TU} \tag{7.156}$$
$$r^2 = 99.9\%$$

In eq. (7.148), ξ_T is the energy rate in MWh/Kaf for Trinity Power Plant (at Clair Engle Lake), $\bar{x}_T$ is the average reservoir storage in Kaf during a specified period, and r^2 is the adjusted regression correlation coefficient. Other terms in eqs. (7.149)-(7.156) are defined similarly. In eq. (7.149), the energy rate depends on the storage of the downstream Whiskeytown reservoir. This can be explained by realizing that the storage at Lewiston is fixed and the energy gradient line from the intake of Clear Creek tunnel to its discharging point (at Whiskeytown) is determined by the reservoir elevation at Whiskeytown. Due to the larger size of Whiskeytown as compared to Lewiston (241 and 14.7 Kaf, respectively), it is likely that (slight) changes in elevations would occur at Whiskeytown and those changes would determine the differential head at J. F. Carr Power Plant and consequently its energy production rate. In fact, that is the case and it explains the negative slope in eq. (7.149), which is consistent with Fig. 5.8. Since it was found earlier (Chapter 6) that Whiskeytown acts as a regulating reservoir, then for all practical purposes the storage at Whiskeytown (x_W) can be assumed fixed and equal to the average storage ($\bar{x}_W$). That is the reason for using x_W, rather than $\bar{x}_W$, in eqs. (7.149) and (7.150). Also, due to the regulating nature of Keswick, Lake Natoma (where Nimbus Power Plant is located), and Tullock, the (fixed) storages equal the average storages and thus the overbar has been omitted in eqs. (7.152), (7.154), and (7.156). In addition, it is evident that eqs. (7.149), (7.152), and (7.154) are linear functions (actual operation records yielded linear relations), as can be appreciated for J. F. Carr Power Plant in Fig. 5.3; for Keswick and Nimbus, the development of the linear curves was discussed briefly in Section 5.1. It is clear that the approach developed in Sections 7.5-7.7 can handle a combination of linear and nonlinear energy rate functions. The (fixed) storage values for x_W, x_K, x_N, and x_{TU} are given by x_t^3, x_t^5, x_t^7, and x_t^9, respectively, in Section 7.1. Notice that the storage at Lewiston has no role in the expression for energy rates, yet it must be included in the continuity equation. That leads to minor changes in eqs. (7.144)-(7.145) which will be considered later in this subsection.

To set up the system energy production rate in matrix form, an expression of the form of eq. (7.84) is necessary for each power plant. By using eqs. (7.148)-(7.156), it is straightforward to develop an expression of the form of eq. (7.84) for any of the power plants. For example, for i = 1 (i.e., at Trinity),

$$\xi_t^1 \cong \hat{a}_t^1 + \hat{b}_t^1 x_t^1 \tag{7.157}$$

in which

$$\hat{a}_t^1 = 133.0 + \frac{0.228}{2}(x_{t,0}^1 + x_{t+1}^1) - \frac{0.468 \times 10^{-4}}{4}(x_{t,0}^1 + x_{t+1}^1)^2$$
$$- \left[\frac{0.228}{2} - \frac{0.468 \times 10^{-4}}{2}(x_{t,0}^1 + x_{t+1}^1)\right] x_{t,0}^1 \tag{7.158}$$

$$\hat{b}_t^1 = \frac{0.228}{2} - \frac{0.468 \times 10^{-4}}{2}(x_{t,0}^1 + x_{t+1}^1) \tag{7.159}$$

The numerical values appearing in eqs. (7.158) and (7.159) were obtained from eq. (7.148) and were substituted appropriately by an inspection of the terms defined in eqs. (7.85) and (7.86). Equation (7.84) for the case of a linear energy rate [say, eq. (7.149), corresponding to J. F. Carr Power Plant] is given by

$$\xi_t^2 \doteq a^2 + b^2 x_t^3 \tag{7.160}$$

in which $a^2 = 606.3$ and $b^2 = -0.254$. When the energy rate is quadratic and the reservoir storage is fixed (e.g., at Tullock), eq. (7.84) takes the form

$$\xi_t^9 = a^9 + \hat{b}^9\ x_t^9 \tag{7.161}$$

in which $a^9 = 63.4$ and $\hat{b}^9$ is set equal to $b^9 + c^9\ x_t^9$, i.e.,

$$b^9 = 1.020 - 1.37 \times 10^{-3}\ x_t^9 \tag{7.162}$$

The corresponding form of eq. (7.84) for any other power plant can be developed along the same lines as done in eqs. (7.157)-(7.162).

The discussion on energy rates is completed by providing the vector-matrix expression of the energy rates for months t and t - 1 [similar to eqs. (7.111) and (7.112), respectively] that are needed in the two-stage problem. From the development in Section 7.7 that led to eq. (7.111) and from earlier discussion in this subsection, it follows that for month t

$$\underline{\xi}_t = \begin{bmatrix} \underline{\hat{a}}_t^{*(1)} \\ \underline{\hat{a}}^{*(2)} \end{bmatrix} + \begin{bmatrix} \hat{B}_{11}^{*t} & 0 \\ 0 & \hat{B}_{22}^{*} \end{bmatrix} \begin{bmatrix} \underline{x}_t \\ \underline{\hat{k}}^{*} \end{bmatrix} \tag{7.163}$$

in which

$$\underline{\hat{a}}_t^{*(1)T} = \underline{a}_t^{(1)T} = (\underline{\hat{a}}_t^1\ \ \underline{\hat{a}}_t^4\ \ \underline{\hat{a}}_t^6\ \ \underline{\hat{a}}_t^8) \tag{7.164}$$

$$\underline{\hat{a}}^{*(2)T} = (a^2\ \ a^3\ \ a^5\ \ a^7\ \ a^9) = (606.3,\ 445.0,\ 80.3,\ 26.3,\ 63.4) \tag{7.165}$$

$$\hat{B}_{11}^{*t} = \hat{B}_{11}^{t} = \text{diag}\ (\hat{b}_t^1\ \ \hat{b}_t^4\ \ \hat{b}_t^6\ \ \hat{b}_t^8) \tag{7.166}$$

$$\hat{B}^{*}_{22} = \text{diag}\ (-0.254,\ 0.738 - 1.10 \times 10^{-3}\ x^{3}_{t},\ 0.60,\ 0.80,\ 1.020$$

$$- 1.37 \times 10^{-3}\ x^{9}_{t}) \qquad (7.167)$$

$$\hat{\underline{k}}^{*T} = (x^{3}_{t}\ \ x^{3}_{t}\ \ x^{5}_{t}\ \ x^{9}_{t}) \qquad (7.168)$$

In eq. (7.164), the components $\hat{a}^{i}_{t}$ are given by eq. (7.120). In eq. (7.166), the symbol diag $(\cdot, \cdot, \ldots, \cdot)$ denotes a diagonal matrix with elements $(\cdot, \cdot, \ldots, \cdot)$, which in this case are given by eq. (7.122). For month t-1, instead of eqs. (7.164) and (7.166), use eqs. (7.121) and (7.123), respectively; all other terms are the same as for eq. (7.163). It should be evident the differences between eq. (7.111) and (7.163). Such differences arise from several facts: (i) x^{2}_{t} plays no role in defining the energy rates in this section; (ii) the energy rates are of both linear and quadratic types herein, whereas in Section 7.7 the energy rates were assumed to be quadratic; and (iii) linearization of energy equations is limited to a few reservoirs (Clair Engle, Shasta, Folsom, and New Melones) herein, whereas in Section 7.7 linearization was performed for all the power plants. Finally, for month t-1, $\underline{\xi}_{t-1}$ can be analogously developed by shifting the time index backwards by 1.

Spillage Discharge Equations

Recall form eq. (7.92) that for the ith reservoir, the spillage discharge as a function of reservoir elevation is

$$r^{i}_{t} = \tilde{c}^{i}\ (h^{i}_{t} - d^{i}_{t})^{\eta^{i}}\ \delta^{i} \qquad (7.92)$$

with all terms already defined in eq. (7.92). To develop the spillway discharge equations, use was made of the spillway discharge tables in Appendix A. Those tables contain discrete values of discharge for various storage elevations and opening of gates. The spillway structures in some cases (e.g., Shasta) are not simple overflow structures and thus the exponent η departs from its theoretical value of $\eta = 1.5$. In other cases, as in Nimbus, the discharge is the result of a combined underflow from radial gates and the action of a spillway structure (gated spillways). For purely underflow gates, the theoretical exponent η is 0.5, but due to the more complicated design of the actual structures, η takes values either smaller or greater than 0.5. The reader is referred to Appendix A for more details on spillway discharge.

The exponential interpolation of the spillway discharge tables yielded the following equations (flows are in cfs and elevations are in ft above mean sea level):

Trinity (at Clair Engle reservoir)

$$Q = 781\ (h_t^1 - 2370)^{1.29} \qquad (7.169)$$

$r^2 = 98.4\%$

Lewiston

$$Q = 412\ (h_t^2 - 1871)^{0.626} \qquad (7.170)$$

$r^2 = 99.8\%$

Equation (7.170) was developed for one of two gates with an opening of 2 ft.

Whiskeytown

$$Q = 992\ (h_t^3 - 1208)^{1.52} \tag{7.171}$$

$r^2 = 98.7\%$

Shasta

$$Q = 314\ (h_t^4 - 1039)^{1.56} \tag{7.172}$$

$r^2 = 99.9\%$

Equation (7.172) represents the discharge for one of three drum gates operating at a setting in which the high elevation point of the gate is at an elevation of 1039 ft.

Keswick

$$Q = 720\ (h_t^5 - 547)^{0.436} \tag{7.173}$$

$r^2 = 99.2\%$

Equation (7.173) gives the discharge for one of four gates operating at an opening of 2 ft.

Folsom

$$Q = 242\ (h_t^6 - 420)^{0.466} \tag{7.174}$$

$r^2 = 99.9\%$

Equation (7.174) represents the discharge for one of eight gates operating at a gate opening of 1 ft.

Nimbus

$$Q = 437\ (h_t^7 - 110)^{0.317} \tag{7.175}$$

$r^2 = 99.9\%$

Equation (7.175) gives the discharge for one of 18 gates operating at a normal opening of 1 ft.

New Melones

$$Q = 420\ (h_t^8 - 1088)^{1.55} \tag{7.176}$$

$r^2 = 99.6\%$

Tullock

$$Q = 750\ (h_t^9 - 495)^{0.478} \tag{7.177}$$

$r^2 = 95.0\%$

Equation (7.177) was obtained from data provided by the Oakdale Irrigation District. Equations (7.169)-(7.177) need to be (i) converted from cfs to acre-ft/month before they can be used in the development that follows and (ii) expressed in terms of storage because the optimization is expressed in terms of storage rather than elevation, as was expressed in eq. (7.94). The spillage equations allow a better modeling of spilled water if those expressions depend on forecast inflows, i.e., depending on the magnitude of the forecast inflow, the spillway discharge equation will be based on a different gate opening. This implies that in the programming of the spillway discharges, an equation should be developed for each different gate setting for every reservoir that has a gated spillway (all reservoirs except Trinity, Whiskeytown, and

New Melones). That was done in this study, and thus eqs. (7.169)-(7.177) are an example of one of the several sets of equations that were used in the solution of the problem.

From the elevation-storage data of Appendix A, shapes of elevation vs. storage curves were analyzed to determine appropriate interpolation functions. The interval of interest is for the range of elevations above the spillway crest, otherwise the spillage would be zero, which means that only the shape of the elevation vs. storage at high stages is of concern. Fortunately, from the perspective of numerical simplicity, the plots were nearly straight lines for all but low elevations. Figure 7.1 depicts an illustration in which the elevation-storage curve for Clair Engle shows high nonlinearity for elevations below 2010 ft and an almost perfect linear curve everywhere else. Similar behavior was determined to exist in the other major reservoirs (Shasta, Folsom, and New Melones) for which the elevation-storage curves are needed. A similar pattern holds for the smaller reservoirs, but for those the interest is centered at a single elevation because the storage is held constant and there is no need for elevation-storage curves. The following linear functions were developed for the four major reservoirs:

Clair Engle

$$h^1 = 2142 + 0.0971\ x^1 \qquad (7.178)$$

$r^2 = 97.2\%$

Shasta

$$h^4 = 871 + 0.0444\ x^4 \qquad (7.179)$$

$r^2 = 99.3\%$

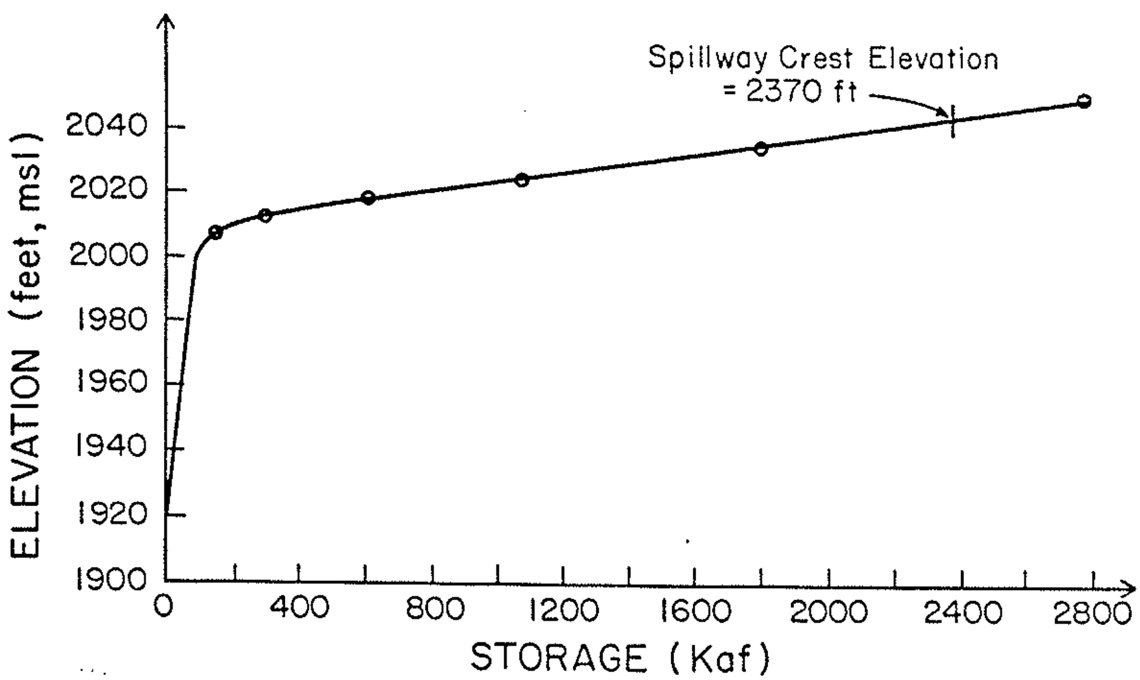


Fig. 7.1 Elevation vs. storage (Clair Engle reservoir).

Folsom

$$h^6 = 364 + 0.101\ x^6 \tag{7.180}$$

$$r^2 = 99.5\%$$

New Melones

$$h^8 = 860 + 0.0945\ x^8 \tag{7.181}$$

$$r^2 = 99.5\%$$

To complete the information regarding elevation vs. storage, elevations in ft above mean sea level corresponding to constant storages at Lewiston, Whiskeytown, Keswick, Natoma, and Tullock ($x_t^2 = 14.7$, $x_t^3 = 241$, $x_t^5 = 23.8$, $x_t^7 = 8.8$, and $x_t^9 = 57.0$ Kaf, respectively) are 1901.1, 1210.0, 587.4, 125.1, and 501.6 ft, respectively (from elevation-storage data in Appendix A). Upon substitution of eqs. (7.178)-(7.181) into eqs. (7.169), (7.172), (7.174), and (7.176), respectively, and after subsequent linearization, the following expressions are obtained:

Clair Engle

$$r_t^1 \cong \hat{c}_t^1 + \hat{d}_t^1\ x_t^1 \tag{7.182}$$

in which

$$\hat{c}_t^1 = \left\{ \tilde{c}^1 \left[g^1\!\left(\frac{x_{t,0}^1 + x_{t+1}^1}{2} \right) - d^1 \right]^{\eta^1} - x_{t,0}^1\ \eta^1\ \tilde{c}^1 \left[g^1\!\left(\frac{x_{t,0}^1 + x_{t+1}^1}{2} \right) - d^1 \right]^{\eta^1 - 1} \left[\frac{dg}{dx_t^1} \right]_{x_{t,0}^1} \right\} \delta^1$$

$$= \left\{ 781 \left[\left(2142 + 0.0971 \left(\frac{x_{t,0}^{1} + x_{t+1}^{1}}{2} \right) \right) - 2370 \right]^{1.29} - x_{t,0}^{1} \ (1.29)(781) \right.$$

$$\left. \cdot \left[\left(2142 + 0.0971 \left(\frac{x_{t,0}^{1} + x_{t+1}^{1}}{2} \right) \right) - 2370 \right]^{0.29} (0.0971) \right\} \delta^{1} \quad (7.183)$$

$$\hat{d}_t^1 = \left\{ \eta^1 \tilde{c}^1 \left[g^1 \left(\frac{x_{t,0}^{1} + x_{t+1}^{1}}{2} \right) - d^1 \right]^{\eta^1 - 1} \left[\frac{dg}{dx_t^1} \right]_{x_{t,0}^1} \right\} \delta^1$$

$$= \left\{ (1.29)(781) \left[\left(2142 + 0.0971 \left(\frac{x_{t,0}^{1} + x_{t+1}^{1}}{2} \right) \right) \right. \right.$$

$$\left. \left. - 2370 \right]^{0.29} (0.0971) \right\} \delta^{1} \quad (7.184)$$

Shasta

$$r_t^4 \cong \hat{c}_t^4 + \hat{d}_t^4 \ x_t^4 \quad (7.185)$$

in which

$$\hat{c}_t^4 = \left\{ \tilde{c}^4 \left[g^4 \left(\frac{x_{t,0}^{4} + x_{t+1}^{4}}{2} \right) - d^4 \right]^{\eta^4} - x_{t,0}^4 \ \eta^4 \ \tilde{c}^4 \left[g^4 \left(\frac{x_{t,0}^{4} + x_{t+1}^{4}}{2} \right) \right. \right.$$

$$\left. \left. - d^4 \right]^{\eta^4 - 1} \left[\frac{dg}{dx_t^4} \right]_{x_{t,0}^4} \right\} \delta^4$$

$$= \left\{ 314 \left[\left(871 + 0.0444 \left(\frac{x_{t,0}^{4} + x_{t+1}^{4}}{2} \right) \right) - 1039 \right]^{1.56} - x_{t,0}^{4} \ (1.56)(314) \right.$$

$$\left. \cdot \left[871 + 0.0444 \left(\frac{x_{t,0}^{4} + x_{t+1}^{4}}{2} \right) \right) - 1039 \right]^{0.56} (0.0444) \right\} \delta^{4} \quad (7.186)$$

$$\hat{d}_t^4 = \left\{ \eta^4 \tilde{c}^4 \left[g^4 \left(\frac{x_{t,0}^4 + x_{t+1}^4}{2} \right) - d^4 \right]^{\eta^4 - 1} \left[\frac{dg}{dx_t^4} \right]_{x_{t,0}^4} \right\} \delta^4$$

$$= \left\{ (1.56)(314) \left[\left(871 + 0.0444 \left(\frac{x_{t,0}^4 + x_{t+1}^4}{2} \right) \right) - 1039 \right]^{0.56} (0.0444) \right\} \delta^4 \qquad (7.187)$$

Folsom

$$r_t^6 \cong \hat{c}_t^6 + \hat{d}_t^6 \, x_t^6 \qquad (7.188)$$

in which

$$\hat{c}_t^6 = \left\{ \tilde{c}^6 \left[g^6 \left(\frac{x_{t,0}^6 + x_{t+1}^6}{2} \right) - d^6 \right]^{\eta^6} - x_{t,0}^6 \, \eta^6 \, \tilde{c}^6 \left[g^6 \left(\frac{x_{t,0}^6 + x_{t+1}^6}{2} \right) - d^6 \right]^{\eta^6 - 1} \left[\frac{dg}{dx_t^6} \right]_{x_{t,0}^6} \right\} \delta^6$$

$$= \left\{ (242) \left[\left(364 + 0.101 \left(\frac{x_{t,0}^6 + x_{t+1}^6}{2} \right) \right) - 420 \right]^{0.466} - x_{t,0}^6 \, (0.466)(242) \cdot \left[\left(364 + 0.101 \left(\frac{x_{t,0}^6 + x_{t+1}^6}{2} \right) \right) - 420 \right]^{-0.534} (0.101) \right\} \delta^6 \qquad (7.189)$$

$$\hat{d}_t^6 = \left\{ \eta^6 \tilde{c}^6 \left[g^6 \left(\frac{x_{t,0}^6 + x_{t+1}^6}{2} \right) - d^6 \right]^{\eta^6 - 1} \left[\frac{dg}{dx_t^6} \right]_{x_{t,0}^6} \right\} \delta^6$$

$$= \left\{(0.466)(242)\left[\left(364 + 0.101\left(\frac{x^6_{t,0} + x^6_{t+1}}{2}\right)\right) - 420\right]^{-0.534}(0.101)\right\}\delta^6 \tag{7.190}$$

New Melones

$$r^8_t \cong \hat{c}^8_t + \hat{d}^8_t\, x^8_t \tag{7.191}$$

in which

$$\hat{c}^8_t = \left\{\tilde{c}^8\left[g^8\left(\frac{x^8_{t,0} + x^8_{t+1}}{2}\right) - d^8\right]^{\eta^8} - x^8_{t,0}\,\eta^8\,\tilde{c}^8\left[g^8\left(\frac{x^8_{t,0} + x^8_{t+1}}{2}\right) - d^8\right]^{\eta^8-1}\left[\frac{dg}{dx^8_t}\right]_{x^8_{t,0}}\right\}\delta^8$$

$$= \left\{(420)\left[\left(860 + 0.0945\left(\frac{x^8_{t,0} + x^8_{t+1}}{2}\right)\right) - 1088\right]^{1.55} - x^8_{t,0}\,(1.55)(420) \cdot \left[860 + 0.0945\left(\frac{x^8_{t,0} + x^8_{t+1}}{2}\right)\right) - 1088\right]^{1.55}(0.0945)\right\}\delta^8 \tag{7.192}$$

$$\hat{d}^8_t = \left\{\eta^8\tilde{c}^8\left[g^8\left(\frac{x^8_{t,0} + x^8_{t+1}}{2}\right) - d^8\right]^{\eta^8-1}\left[\frac{dg}{dx^8_t}\right]_{x^8_{t,0}}\right\}\delta^8$$

$$= \left\{(1.55)(420)\left[\left(860 + 0.0945\left(\frac{x^8_{t,0} + x^8_{t+1}}{2}\right)\right) - 1088\right]^{0.55}(0.0945)\right\}\delta^8 \tag{7.193}$$

In eqs. (7.183) and (7.184), $\delta^1 = 1$ if the reservoir elevation is above the spillway crest and $\delta^1 = 0$ otherwise; and $x^1_{t,0}$ is the value of storage about which the linearization is made. Similar definitions apply to eqs. (7.186)-(7.187), (7.189)-(7.190), and (7.192)-(7.193). The spillway discharge functions for the regulating reservoirs [eqs. (7.170), (7.171), (7.173), (7.175), and (7.177)] need not be linearized, and together with eqs. (7.182), (7.185), (7.188), and (7.191) can be used to form the (linear) spillage constraints in matrix form as indicated by eqs. (7.102) and (7.103), without any modification whatsoever.

After development of the energy and spillage equations, the two-stage problem [eqs. (7.144) and (7.145)] must be modified to fit the particular features of the actual system. It turns out that the only modification is in the objective function, as was explained after eq. (7.163). The corollary is that the constraint equation, eq. (7.145), whose terms were given in eqs. (7.146) and (7.147), remains unchanged. The objective function, eq. (7.144), takes the following form:

$$E_t + E_{t-1} = \underline{\xi}_t^T \underline{u}_t + \underline{\xi}_{t-1}^T \underline{u}_{t-1}$$

$$= k_t^{**} + \underline{q}_t^{**T} \underline{x}_t + \underline{x}_t^T H_t^{**} \underline{x}_t \qquad (7.194)$$

in which

$$k_t^{**} = [\underline{\hat{a}}^{*(2)T} + \underline{\hat{k}}^{*T}\ \hat{B}_{22}^{*}]\ [H_{21}^{t+1}\ \underline{x}_{t+1} + H_{22}^{t+1}\ \underline{k} - \hat{M}_{22}^{t}\ \underline{k} - \underline{\hat{w}}_t^{(2)}$$

$$+ \hat{H}_{22}^{t}\ \underline{k} - M_{21}^{t-1}\ \underline{x}_{t-1} - M_{22}^{t-1}\ \underline{k} - \underline{w}_{t-1}^{(2)}] + [\underline{\hat{a}}_t^{*(1)T}\ (H_{11}^{t+1}\ \underline{x}_t$$

$$- \underline{\hat{w}}_t^{(1)}) - \underline{\hat{a}}_{t-1}^{*(1)T}\ (M_{11}^{t-1}\ \underline{x}_{t-1} + \underline{\hat{w}}_{t-1}^{(1)})] \tag{7.195}$$

$$\underline{g}^{**T} = - \underline{\hat{a}}_t^{*(1)T}\ \hat{M}_{11}^{t} - (\underline{\hat{a}}^{*(2)T} + \underline{\hat{k}}^{*T}\ \hat{B}_{22}^{*})\ (\hat{M}_{21}^{t} + \hat{H}_{21}^{t})$$

$$+ \underline{x}_{t+1}^{T}\ (\hat{B}_{11}^{*t}\ H_{11}^{t+1})^{T} - \underline{\hat{w}}_t^{(1)T}\ \hat{B}_{11}^{*(1)t} + \underline{\hat{a}}_{t-1}^{*(1)T}\ \hat{H}_{11}^{t}$$

$$- \underline{x}_{t-1}^{T}\ (\hat{B}_{11}^{*t-1}\ M_{11}^{t-1})^{T} - \underline{w}_{t-1}^{(1)T}\ \hat{B}_{11}^{*t-1} \tag{7.196}$$

$$H_t^{**} = \hat{B}_{11}^{*t-1}\ \hat{H}_{11}^{t} - \hat{B}_{11}^{*t}\ \hat{M}_{11}^{t} \tag{7.197}$$

where $\underline{\hat{a}}_t^{*(1)}$, $\underline{\hat{a}}^{*(2)}$, $\hat{B}_{11}^{*t}$, $\hat{B}_{22}^{*}$, and $\underline{\hat{k}}^{*}$ were defined after eq. (7.163) and all other terms were defined in Section 7.7. The final form of the two-stage problem for the NCVP system is (dropping the constant term k_t^{**})

$$\underset{\underline{x}_t}{\text{maximize}}\ \underline{g}_t^{**T}\ \underline{x}_t + \underline{x}_t^{T}\ H_t^{**}\ \underline{x}_t \tag{7.198}$$

subject to

$$A_t^{**} \underline{x}_t \leq \underline{b}_t^{**} \tag{7.199}$$

$\underline{x}_{t-1}$, $\underline{x}_{t+1}$ fixed

where the decision (unknown) vector $\underline{x}_t$ is of dimension four.

Initial Policies

To initiate the POA, a feasible initial policy is required. Tables 7.1-7.4 show two different initial policies for water year 1979-80. Because the purpose is to compare the results obtained with the linear and nonlinear models of Chapter 5 and Section 7.7, respectively, the initial ($\underline{x}_1$) and ending ($\underline{x}_{13}$) storages are equal to those used in Chapter 6. It can be seen in Tables 7.1-7.4 that the initial release policies I and II are different; however, unlike the initial policies of Chapter 6, they could not be made significantly distinct because once the spillages are written as a function storage one loses the freedom to manipulate them as decision variables. The motivation behind the use of different initial policies is to test whether they lead to the same optimal release policy or not. The development of the initial policies was based on previous operation experience of the NCVP and a trial-and-error approach to ensure feasibility.

Solution Algorithm

The master algorithm is the POA, which was described in Section 4.8. The logic of the POA advances the approximate release policies $\{\underline{u}_t^{(k)}\}$, k = 0, 1,..., K, to an optimal policy $\{\underline{u}_t^{(*)}\}$ which satisfies user-specified convergence tests. However, the POA assumes that at every stage, the following two-stage (quadratic) problem is solved:

Table 7.1 Initial Storage Policy, 1979-80 (Policy I)

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1632.0	14.7	241	3035	23.8	673	8.8	1600	60
Nov	1570.2	14.7	241	2708	23.8	472	8.8	1419	57
Dec	1550.6	14.7	241	2546	23.8	372	8.8	1254	57
Jan	1545.8	14.7	241	2293	23.8	267	8.8	1105	57
Feb	1692.2	14.7	241	2834	23.8	714	8.8	1183	60
Mar	1908.3	14.7	241	3536	23.8	793	8.8	1246	61
Apr	1974.7	14.7	241	3722	23.8	807	8.8	1184	67
May	2071.4	14.7	241	3790	23.8	805	8.8	1068	67
Jun	2172.7	14.7	241	3790	23.8	862	8.8	1048	67
Jul	2078.1	14.7	241	3561	23.8	874	8.8	982	67
Aug	1926.5	14.7	241	3297	23.8	864	8.8	800	67
Sep	1735.1	14.7	241	3002	23.8	783	8.8	536	67
Oct	1542.6	14.7	241	2622	22.8	625	8.8	323	57

Storages in Kaf

Table 7.2. Initial Release Policy, 1979-80 (Policy I)

Month	Clair Engle		Lewiston		Whiskeytown		Shasta		Keswick	
	Spill	Penstock	Spill	Penstock	Spill	Penstock	Spill	Penstock	Spill	Penstock
Oct		100	26	74	3	77		600	56	621
Nov		100	26	74	6	84		500	56	528
Dec		100	26	74	6	86		600	56	630
Jan		100	26	74	6	115		700	56	759
Feb		100	26	74	6	178		700	56	822
Mar		95	26	69	6	124		700	56	768
Apr		150	26	124	6	150		500	56	594
May		150	26	124	6	132	3	423	56	502
Jun		200	26	174	6	180		500	56	624
Jul		200	26	174	6	176		500	56	520
Aug		200	26	174	6	176		500	56	520
Sep		150	26	124	6	127		600	56	571

Table 7.2. (Continued)

Month	Folsom		Natoma		New Melones		Tullock	
	Spill	Penstock	Spill	Penstock	Spill	Penstock	Spill	Penstock
Oct		290	20	267		200	93	110
Nov		200	20	267		200	90	110
Dec		250	15	232		200	90	110
Jan	240	300	237	300		200	90	110
Feb	333	300	330	300		200	87	113
Mar	161	300	158	300		200	89	111
Apr	155	250	152	250		200	84	116
May	150	200	147	200		200	90	110
Jun	25	200	22	200		200	90	110
Jul	10	180	7	180		200	90	110
Aug	10	180	7	180		200	90	110
Sep	10	277	7	277		150	90	70

Releases in Kaf

Table 7.3 Initial Storage Policy, 1979-80 (Policy II)

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1632.0	14.7	241	3035	23.8	673	8.8	1600	57
Nov	1520.2	14.7	241	2808	23.8	512	8.8	1469	57
Dec	1450.6	14.7	241	2446	23.8	362	8.8	1304	57
Jan	1385.8	14.7	241	2093	23.8	257	8.8	1205	57
Feb	1522.2	14.7	241	2634	23.8	700	8.8	1283	57
Mar	1728.3	14.7	241	3336	23.8	950	8.8	1346	57
Apr	1784.7	14.7	241	3522	23.8	1000	8.8	1284	57
May	1831.4	14.7	241	3390	23.8	950	8.8	1168	57
Jun	1882.7	14.7	241	3216	23.8	1000	8.8	1138	57
Jul	1888.1	14.7	241	3187	23.8	887	8.8	1052	57
Aug	1836.5	14.7	241	2983	23.8	867	8.8	860	57
Sep	1745.1	14.7	241	2888	23.8	776	8.8	586	57
Oct	1542.6	14.7	241	2622	23.8	625	8.8	323	57

Storages in Kaf

Table 7.4. Initial Release Policy, 1979-80 (Policy II)

Month	Clair Engle		Lewiston		Whiskeytown		Shasta		Keswick	
	Spill	Penstock	Spill	Penstock	Spill	Penstock	Spill	Penstock	Spill	Penstock
Oct		150	18	132	6	135		500	40	595
Nov		150	18	132	6	142		700	40	802
Dec		150	18	132	6	144		700	40	804
Jan		100	18	82	6	123		700	40	783
Feb		100	18	82	6	186		700	40	846
Mar		100	18	82	6	137		700	40	797
Apr		100	18	82	6	108		700	40	768
May		150	18	132	6	140		600	40	700
Jun		100	18	82	6	88		300	30	358
Jul		100	18	82	6	84		440	30	494
Aug		100	18	82	6	84		300	30	354
Sep		210	18	192	6	195		486	30	651

Table 7.4. (Continued)

Month	Folsom		Natoma		New Melones		Tullock	
	Spill	Penstock	Spill	Penstock	Spill	Penstock	Spill	Penstock
Oct		250		247		150	50	100
Nov		250		247		200	100	100
Dec		250		247		150	50	100
Jan	244	300	241	300		200	100	100
Feb	162	300	159	300		200	100	100
Mar	125	300	122	300		200	100	100
Apr	153	300	100	250		200	100	100
May	157	200	104	250		210	110	100
Jun	100	250	147	200		220	120	100
Jul		200	47	150		210	110	100
Aug		200	47	150		210	110	100
Sep	30	250	47	150		200	100	100

Releases in Kaf

$$\text{maximize } k_t^{**} + \underline{q}_t^{**T}\, \underline{x}_t + \underline{x}_t^T\, H_t^{**}\, \underline{x}_t \tag{7.200}$$

subject to

$$A_t^{**}\, \underline{x}_t \leq \underline{b}_t^{**} \tag{7.201}$$

$\underline{x}_{t-1}$, $\underline{x}_{t+1}$ fixed

The simple quadratic structure of the problem given by eqs. (7.200) and (7.201) is misleading because, in general, the problem has an indefinite Hessian matrix (i.e., a matrix with positive and negative eigenvalues). The matrix H_t^{**} is reconstructed at every step of the POA iteration and it is not possible to ensure a priori whether the matrix is indefinite or not. Thus, it would be inappropriate to apply simple algorithms (e.g., Lemke's and principal pivoting) that are suitable to solve the linear complementarity problem, which has a structure similar to eqs. (7.200)-(7.201), because those algorithms are successful for positive definite cases only. On the other hand, it is advisable to take advantage of the quadratic linearly-constrained structure of eqs. (7.200)-(7.201), rather than to recourse to the more complicated nonlinear programming methods, which are computationally more inefficient than quadratic programming (QP) algorithms. Moreover, nonlinear methods do not achieve a solution in a finite number of steps as do well designed QP methods, and usually suffer from serious ill-conditioning (see Fletcher, 1981, for an assessment of nonlinear programming). It appears that the best method available to solve eqs. (7.200)-(7.201) is the active set method as modified by Gill and Murray (1974, 1977) to handle the general case of indefinite Hessian matrix. This study adopts the method presented by Gill and Murray (1977). The reader is referred to that reference for a thorough description of the method and its implementation. Also, Fletcher (1981) gives an excellent exposition of the role of active set methods to solve QP problems. In essence, at any iteration, the active set method defines a set of constraints that are active or binding, and solves an equality-constrained QP problem to obtain the longest feasible step correction along a direction of negative curvature ($\underline{\delta}^{(k)}$). At the new iterate point, $\underline{x}^{(k+1)} = \underline{x}^{(k)} + \underline{\delta}^{(k)}$, an analysis of Lagrange multipliers tests whether convergence has been achieved, if not, the information about $\underline{\delta}^{(k)}$ and the values of the Lagrange multipliers determine a new active set of constraints to initiate another iteration. In general terms, the adopted QP algorithm consists of the following steps: (i) determine an initial feasible point $\underline{x}^{(1)}$ and its corresponding set of active constraints $A^{(1)}$ (from Section 8.4 of Fletcher, 1981); (ii) solve an equality-constrained QP problem to determine the correction $\underline{\delta}^{(1)}$ so that $\underline{x}^{(2)} = \underline{x}^{(1)} + \underline{\delta}^{(1)}$ (Gill and Murray, 1977); and (iii) test for convergence: if all the Lagrange multipliers are $\leqq 0$, stop, otherwise define the new active set, increase the iteration superscript by one, and go to step (ii) for the next iteration (Gill and Murray, 1977). This active set algorithm finds a solution to eqs. (7.200)-(7.201) in a finite number of steps, is stable numerically, and uses previous information on any new iteration so as to keep the computational burden as low as possible. The implementation of the POA with this method to solve the two-stage problems was carried over to solve the full model described in Section 7.7. The results are given in the next subsection. Appendix B includes the computer program used in the solution of the full model; the subroutine used to solve the quadratic two-stage subproblems is part of this program.

Discussion of Results

The optimal state, release trajectories, and energy production for initial policies I and II are shown in Tables 7.5-7.7 and 7.8-7.10, respectively. Optimal policies I and II are clearly different except for the subsystem New Melones-Tullock where initial policies I and II yielded the same optimal release and state sequences. It is evident that the regulating reservoirs (Lewiston, Whiskeytown, Keswick, Natoma, and Tullock) show constant reservoir elevations as specified in the development of the model in Section 7.7. Both solutions I and II yielded the same volume of Delta releases as specified in Tables 7.7 and 7.8 (annual total Delta release = 14697 Kaf). The total annual energy production (the value of the objective function) is almost the same for policies I and II, 7.764×10^6 and 7.772×10^6 MWh, respectively. For all practical purposes, it can be claimed that the two alternative optimal policies produce a comparable performance as measured by energy production. Table 7.11 summarizes the results obtained from the linear model of Section 5.3 (discussed in Chapter 6) and the full model of Section 7.7. The linear model results in larger Delta releases (14773 Kaf) than those obtained with the nonlinear model (14697 Kaf, for both policies I and II) and also in larger annual energy production ($8.077\ 10^6$ MWh as compared to 7.764×10^6 and 7.772×10^6 MWh for the two optimal policies of the full model). The larger Delta releases is an intuitive result because the linear model treats spillages as decision variables whereas the nonlinear model considers them as functions of storage and the degree of freedom available with respect

Table 7.5 Optimal State Trajectory, 1979-80 (Policy I)

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1632	14.7	241	3035	23.8	673	8.8	1600	57
Nov	1571	14.7	241	3008	23.8	572	8.8	1462	57
Dec	1549	14.7	241	3046	23.8	372	8.8	1338	57
Jan	1545	14.7	241	2729	23.8	217	8.8	1190	57
Feb	1692	14.7	241	3184	23.8	728	8.8	1250	57
Mar	1909	14.7	241	3800	23.8	964	8.8	1300	57
Apr	1974	14.7	241	3800	23.8	939	8.8	1233	57
May	2079	14.7	241	3896	23.8	992	8.8	1100	57
Jun	2071	14.7	241	3771	23.8	999	8.8	1067	57
Jul	1967	14.7	241	3592	23.8	936	8.8	990	57
Aug	1806	14.7	241	3148	23.8	872	8.8	800	57
Sep	1635	14.7	241	2753	23.8	786	8.8	536	57
Oct	1543	14.7	241	2622	23.8	624	8.8	323	57

Storages in Kaf

Table 7.6. Optimal Release Policy, 1979-80 (Policy I)

Month	Clair Engle		Lewiston		Whiskeytown		Shasta		Keswick	
	Spill	Penstock	Spill	Penstock	Spill	Penstock	Spill	Penstock	Spill	Penstock
Oct		99	26	73	5	77		300	50	327
Nov		102	26	76	5	87		300	50	337
Dec		89	26	63	5	76		664	50	690
Jan		89	26	63	5	105		786	50	841
Feb		89	26	63	5	168		786	110	844
Mar		91	26	65	5	121	11	875	220	787
Apr		92	26	66	5	93	78	394	50	515
May		209	26	183	5	192	56	495	50	693
Jun		209	26	183	5	190	14	436	50	590
Jul		209	26	183	5	186		680	50	816
Aug		180	26	154	5	157		600	50	707
Sep		100	26	74	5	78		353	50	381

Table 7.6. (Continued)

Month	Folsom Spill	Folsom Penstock	Natoma Spill	Natoma Penstock	New Melones Spill	New Melones Penstock	Tullock Spill	Tullock Penstock
Oct	20	170	19	168		157	55	102
Nov		300	19	278		159	55	104
Dec		300	19	278		199	110	89
Jan		476	184	289		218	110	108
Feb	70	406	184	289		213	110	103
Mar	156	344	184	313		205	110	95
Apr	45	305	62	285		217	110	107
May	45	355	124	273		213	110	103
Jun	23	277	33	264		211	110	101
Jul	22	222	19	222		210	110	100
Aug	18	177	19	173		200	110	90
Sep	14	277	19	269		150	55	95

Releases in Kaf

Table 7.7
Optimal Energy Production, Penstock Release, and
Delta Releases Corresponding to Initial Policy I, 1979-80.

Month	Clair Engle		Lewiston		Whiskeytown		Shasta		Keswick	
	Energy	Release	Energy	Release	Energy	Release	Energy	Release	Energy	Release
Oct	37433	99	39791	73	43041	77	116194	300	30928	327
Nov	39353	102	43062	79	49748	89	116255	300	32063	339
Dec	33257	89	34340	63	42482	76	254542	664	65260	690
Jan	33762	89	34340	63	58692	105	302473	786	79542	841
Feb	34864	89	34340	63	93907	168	316296	786	79826	844
Mar	36332	91	35431	65	67635	121	358348	875	74435	787
Apr	37066	92	35976	66	51984	93	162038	394	48709	515
May	84571	209	99751	183	107322	192	203834	495	65544	693
Jun	84151	209	99751	183	106204	190	178312	436	55802	590
Jul	82909	209	99751	183	103968	186	273052	680	77177	816
Aug	69643	180	83943	154	87758	157	232812	600	66868	707
Sep	38475	100	41427	76	44718	80	131846	353	36035	381
Total	611816		681903		857459		2646002		712189	

Table 7.7. (Continued)

Month	Folsom Energy	Folsom Release	Natoma Energy	Natoma Release	New Melones Energy	New Melones Release	Tullock Energy	Tullock Release	Delta Release
Oct	48550	170	5601	168	75007	157	11943	102	721
Nov	80135	300	9269	278	73158	159	12177	104	843
Dec	71330	300	9269	278	87574	199	10421	89	1236
Jan	127182	476	9635	289	94439	218	12646	108	1582
Feb	122672	406	9635	289	94094	213	12060	103	1640
Mar	104773	343	10402	313	90293	205	11123	95	1709
Apr	93230	305	9502	285	92140	217	12529	107	1129
May	108621	355	9102	273	87485	213	12060	103	1353
Jun	84679	277	8802	264	84647	211	11826	101	1148
Jul	67559	222	7401	222	79119	210	11709	100	1317
Aug	53338	177	5768	173	66265	200	10538	90	1149
Sep	83472	277	8968	269	41742	150	11123	95	869
Total	1045538		103354		965963		140155		14696

Energy in MWh and Releases in Kaf.

Table 7.8 Optimal State Trajectory, 1979-80 (Policy II)

Month	Clair Engle	Lewiston	Whiskeytown	Shasta	Keswick	Folsom	Natoma	New Melones	Tullock
Oct	1632	14.7	241	3035	23.8	673	8.8	1600	57
Nov	1480	14.7	241	3008	23.8	540	8.8	1462	57
Dec	1370	14.7	241	3008	23.8	350	8.8	1338	57
Jan	1285	14.7	241	2667	23.8	200	8.8	1190	57
Feb	1421	14.7	241	3180	23.8	711	8.8	1250	57
Mar	1627	14.7	241	3770	23.8	947	8.8	1300	57
Apr	1683	14.7	241	3780	23.8	962	8.8	1233	57
May	1785	14.7	241	3900	23.8	905	8.8	1100	57
Jun	1816	14.7	241	3950	23.8	992	8.8	1067	57
Jul	1811	14.7	241	3721	23.8	939	8.8	990	57
Aug	1734	14.7	241	3257	23.8	819	8.8	800	57
Sep	1633	14.7	241	2860	23.8	748	8.8	536	57
Oct	1543	14.7	241	2622	23.8	624	8.8	323	57

Storages in Kaf

Table 7.9. Optimal Release Policy, 1979-80 (Policy II)

Month	Clair Engle		Lewiston		Whiskeytown		Shasta		Keswick	
	Spill	Penstock	Spill	Penstock	Spill	Penstock	Spill	Penstock	Spill	Penstock
Oct		190	26	164	5	168		300	25	443
Nov		190	26	164	5	175		338	50	463
Dec		170	26	144	5	157		688	50	795
Jan		100	26	74	5	116		728	50	794
Feb		100	26	74	5	179		812	220	771
Mar		100	26	74	5	130		876	220	786
Apr		95	26	69	5	96	78	370	50	494
May		170	26	144	5	153	57	319	50	479
Jun		110	26	84	5	91	35	465	50	541
Jul		125	26	99	5	102		700	50	752
Aug		109	26	84	5	87		602	25	664
Sep		99	26	73	5	77		460	25	512

Table 7.9. (Continued)

Month	Folsom Spill	Folsom Penstock	Natoma Spill	Natoma Penstock	New Melones Spill	New Melones Penstock	Tullock Spill	Tullock Penstock
Oct	18	204	19	200		157	55	102
Nov		290	19	268		159	55	104
Dec		295	19	273		199	110	89
Jan		476	184	289		218	110	108
Feb	52	424	184	289		213	110	103
Mar	150	310	184	273		205	110	95
Apr	79	381	62	395		217	110	107
May	45	275	125	192		213	110	103
Jun	22	268	30	257		211	110	101
Jul	22	278	19	278		210	110	100
Aug	19	161	19	158		200	110	90
Sep	15	238	19	231		150	55	95

Releases in Kaf

Table 7.10. Optimal Energy Production, Penstock Release, and Delta Releases Corresponding to Initial Policy II, 1979-80.

Month	Clair Engle		Lewiston		Whiskeytown		Shasta		Keswick	
	Energy	Release	Energy	Release	Energy	Release	Energy	Release	Energy	Release
Oct	71147	190	98821	164	93907	168	116194	300	41899	443
Nov	68945	190	98821	164	97820	175	130740	338	43791	463
Dec	60043	170	86770	144	87758	157	253854	688	75191	795
Jan	35581	100	44590	74	64840	116	279207	728	75097	794
Feb	37178	100	44590	74	100055	179	326388	812	72921	771
Mar	38215	100	44590	74	72666	130	358322	876	74340	786
Apr	36825	95	41577	69	53661	96	151813	370	46723	494
May	66606	170	86770	144	85522	153	131367	319	45304	479
Jun	43182	110	50616	84	50866	91	190603	465	51168	541
Jul	48762	125	59654	99	57015	102	280986	700	71124	752
Aug	41878	109	50616	84	48071	86	233229	602	62707	663
Sep	37328	99	43987	73	43041	77	172908	460	48425	512
Total	585690		751402		855222		2625611		708690	

Table 7.10. (Continued)

Month	Folsom Energy	Folsom Release	Natoma Energy	Natoma Release	New Melones Energy	New Melones Release	Tullock Energy	Tullock Release	Delta Release
Oct	57916	204	6668	200	75007	157	11943	102	844
Nov	76323	290	8935	268	73158	159	12177	104	959
Dec	69403	295	9102	273	87574	199	10421	89	1336
Jan	126014	476	9635	289	94439	218	12646	108	1535
Feb	127770	424	9635	289	94094	213	12060	103	1677
Mar	94706	310	9101	273	90293	205	11123	95	1668
Apr	116240	381	13169	395	92140	217	12529	107	1218
May	83984	275	6401	192	87485	213	12060	103	1059
Jun	81920	268	8568	257	84647	211	11826	101	1089
Jul	84371	278	9269	278	79119	210	11709	100	1309
Aug	48111	161	5268	158	66265	200	10538	90	1066
Sep	69404	238	7702	231	41741	150	11123	95	937
Total	1035802		103453		965962		140155		14697

Energy in MWh and Releases in Kaf.

to setting spillage values (as was done in the linear case) is lost. Clearly, the nonrecoverable spillages occur at Lewiston and Whiskeytown only because spillages at other reservoirs return to the rivers that discharge in the Delta. The increased energy achieved with the linear model has a more subtle explanation. First, Figures 7.2 and 7.3 show the state trajectories at Shasta and Folsom for the different models. It is evident that nonlinear policies I and II follow a pattern similar to the linear policy but, overall, maintain a lower storage elevation. That is explained by the fact that when spillages are functions of storage, there is a penalty for achieving higher levels because the spilled (non-energy producing) water increases exponentially with the differential of reservoir elevation minus spillway crest elevation. Second, it can be expected that penstock releases will increase (in the nonlinear model) to keep reservoir levels from reaching such wasteful high levels. Because energy production is linear in the penstock release (recall that $E_t = \underset{\sim}{\xi}_t^T \underline{u}_t$), it would follow that the nonlinear model is more likely to generate more energy than the linear model. The resolution of the contradiction established by this argument and the observed results (which indicate more energy from the linear model) lies in the fact that energy production is a quadratic function on storages, through the energy rates, and that offsets the effect of the higher penstock release, for in the linear case the storages are higher. In the more realistic nonlinear model, the tradeoff between higher elevations and smaller releases is more complex than in the linear case, discussed in Chapter 6. It can be observed in Table 7.11 that values of E_a/E_m (actual over maximized annual energy ratios) are higher for the nonlinear model than for the linear model. The overall E_a/E_m ratio for

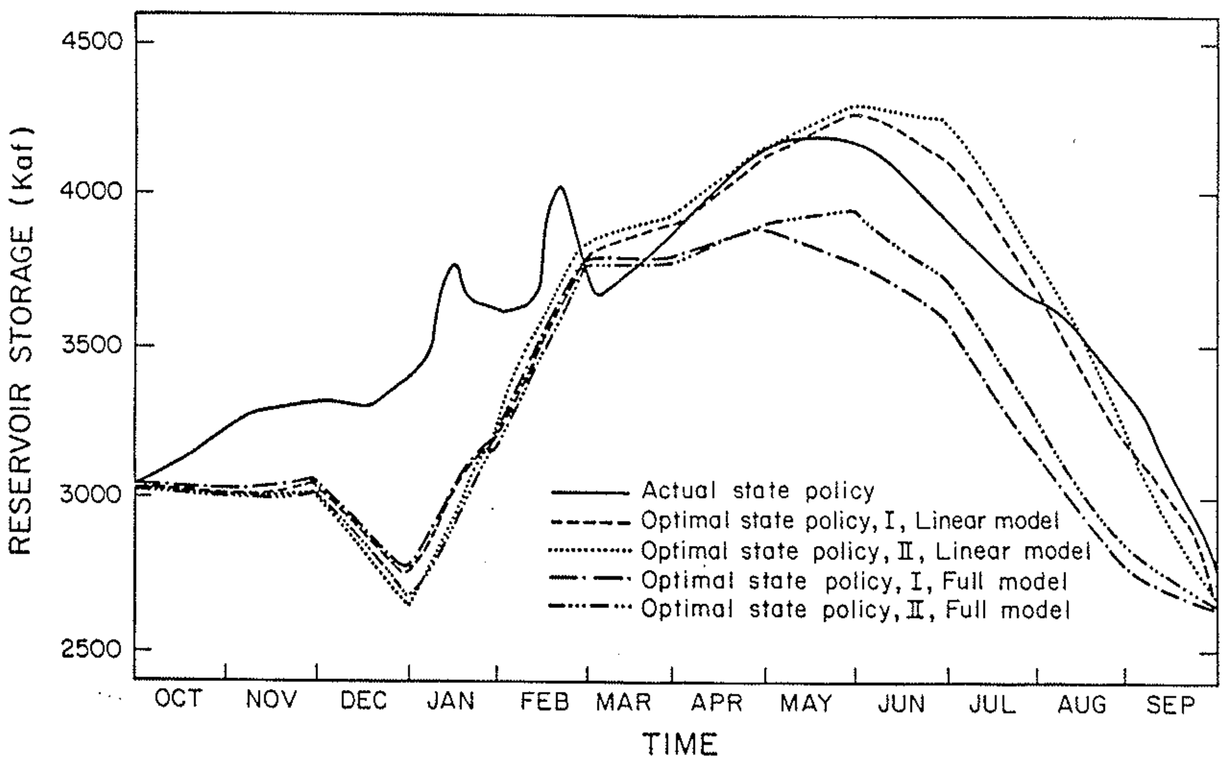


Fig. 7.2 Operation of Shasta reservoir (water year 1979-80).

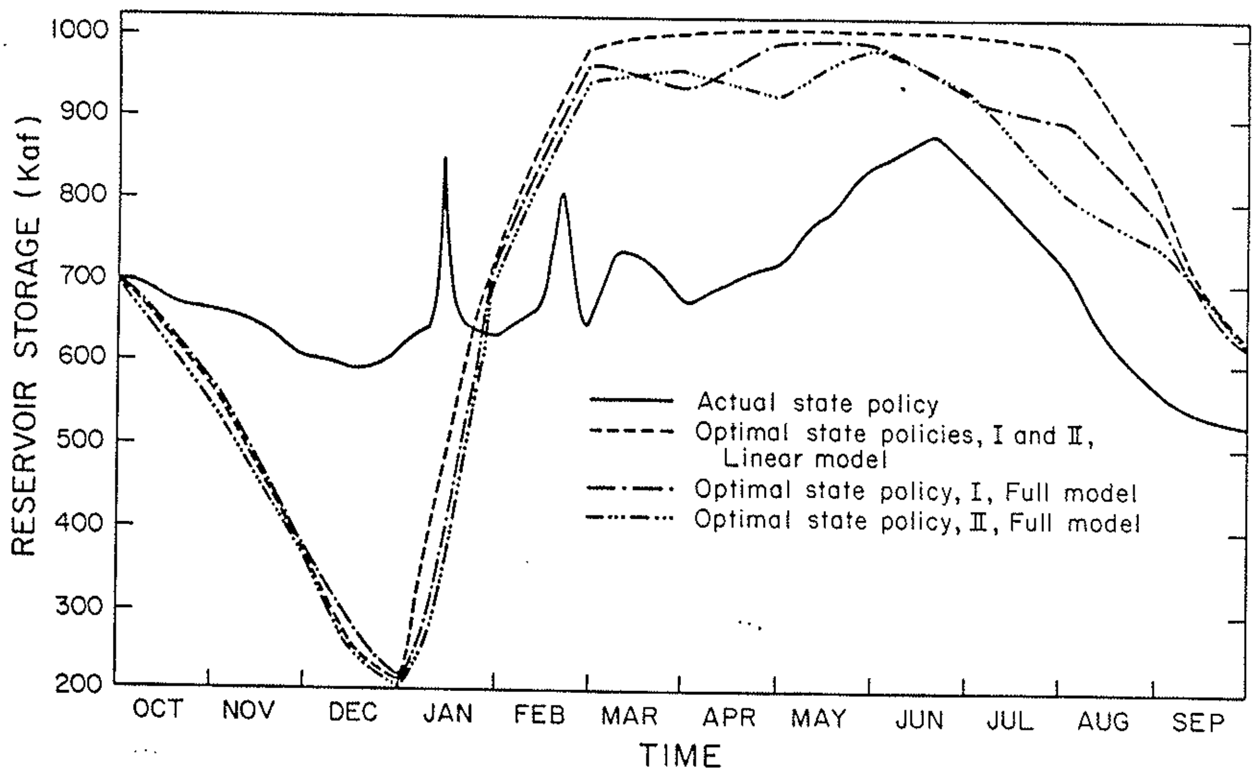


Fig. 7.3 Operation of Folsom reservoir (water year 1979-80).

Table 7.11. Actual and Maximized Energy Production (in 10^3 Mwh) for 1979-80

	Trinity Power Plant at Clair Engle				Judge Francis Carr Power Plant				Spring Creek Power Plant			
Month	Actual	Linear	Full I	Full II	Actual	Linear	Full I	Full II	Actual	Linear	Full I	Full II
Oct	30.4	73.9	37.4	71.1	36.8	112.4	39.8	98.8	42.3	111.2	43.0	93.9
Nov	9.6	71.2	39.4	68.9	4.8	112.4	43.1	98.8	19.5	115.2	49.7	97.8
Dec	17.1	61.4	33.3	60.0	15.4	98.5	34.3	86.8	19.2	101.3	42.5	87.8
Jan	6.2	29.7	33.8	35.6	*	41.9	34.3	44.6	23.9	95.5	58.7	64.8
Feb	17.7	31.1	34.9	37.2	3.4	41.9	34.3	44.6	55.3	87.0	93.9	100.1
Mar	73.4	32.2	36.3	38.2	78.2	41.9	35.4	44.6	99.5	50.0	67.6	72.7
Apr	44.9	32.9	37.0	36.8	53.5	41.9	36.0	41.6	54.7	56.6	52.0	53.7
May	21.2	33.7	84.6	66.6	21.2	41.9	99.8	86.8	18.6	46.5	107.3	85.5
Jun	51.5	75.3	84.2	43.1	55.2	101.2	99.8	50.6	59.6	102.2	106.2	50.9
Jul	54.3	39.4	82.9	48.8	57.2	44.7	99.8	59.7	57.2	45.7	104.0	57.2
Aug	75.7	37.6	69.6	41.9	87.2	42.9	84.0	50.6	85.9	44.0	87.8	48.1
Sep	71.1	36.5	38.5	37.3	84.7	44.2	41.4	44.0	88.6	45.7	44.7	43.0
Total	473.1	555.1	611.8	585.7	497.6	765.9	681.9	751.4	623.3	901.0	857.5	855.2
E_a/E_m		0.58	0.77	0.81		0.65	0.73	0.66		0.69	0.73	0.73

Table 7.11. (Continued)

Month	Shasta Power Plant				Keswick Power Plant				Folsom Power Plant			
	Actual	Linear	Full I	Full II	Actual	Linear	Full I	Full II	Actual	Linear	Full I	Full II
Oct	76.5	112.1	116.2	116.2	19.0	47.0	30.9	41.9	37.7	53.5	48.6	57.9
Nov	89.6	126.2	116.2	130.7	20.7	51.3	32.1	43.8	41.3	77.9	80.1	76.3
Dec	111.1	251.3	254.5	253.9	23.2	81.6	65.3	75.2	37.2	71.3	71.3	69.0
Jan	237.4	266.9	302.5	279.2	47.6	81.6	79.5	75.1	107.0	118.4	127.2	126.0
Feb	209.9	310.7	316.3	326.4	39.2	81.6	79.8	72.9	84.3	139.9	122.7	127.8
Mar	228.3	323.7	358.3	358.3	49.9	81.6	74.4	74.3	130.0	147.2	104.8	94.7
Apr	112.9	126.1	162.0	151.8	29.0	37.3	48.7	46.7	99.8	126.3	93.2	116.2
May	164.4	128.8	203.8	131.4	33.7	36.2	65.5	45.3	82.0	131.0	108.6	84.0
Jun	212.9	129.5	178.3	190.6	47.3	45.5	55.8	51.2	66.6	76.4	84.7	81.9
Jul	237.9	307.5	273.1	281.0	52.2	77.0	77.2	71.1	84.4	63.4	65.6	84.4
Aug	165.7	310.2	232.8	232.2	43.6	81.6	66.9	62.7	35.6	93.5	53.3	48.1
Sep	86.8	289.0	131.8	172.9	29.3	81.6	36.0	48.4	40.5	86.8	83.5	69.4
Total	1933.4	2682.2	2646.0	2625.6	434.7	784.5	712.2	708.7	846.4	1185.5	1045.5	1035.8
E_a/E_m		0.72	0.73	0.74		0.55	0.61	0.61		0.71	0.81	0.82

Table 7.11. (Continued)

	Nimbus Power Plant at Lake Natoma				New Melones Power Plant			
Month	Actual	Linear	Full I	Full II	Actual	Linear	Full I	Full II
Oct	4.6	6.4	5.6	6.7	*	71.6	75.0	75.0
Nov	4.7	10.0	9.3	8.9	*	70.4	73.2	73.2
Dec	4.7	10.0	9.3	9.1	*	68.1	87.6	87.6
Jan	8.7	10.0	9.6	9.6	19.6	81.9	94.4	94.4
Feb	6.7	10.0	9.6	9.6	23.6	92.1	94.1	94.1
Mar	10.6	10.0	10.4	9.1	47.8	90.9	90.3	90.3
Apr	9.7	7.7	9.5	13.2	55.8	90.4	92.1	92.1
May	9.1	8.2	9.1	6.4	39.5	85.8	87.5	87.5
Jun	7.6	7.8	8.8	8.6	23.3	84.4	84.6	84.6
Jul	9.7	6.5	7.4	9.3	38.5	80.9	79.1	79.1
Aug	4.3	10.0	5.8	5.3	22.1	74.9	66.3	66.3
Sep	4.6	10.0	9.0	7.7	9.0	64.8	41.7	41.7
Total	84.9	106.5	103.4	103.5	279.2	956.5	966.0	966.0
E_a/E_m		0.80	0.82	0.82		0.29	0.29	0.29

* Power plant not in operation. E_a = actual energy production; E_m = maximized energy production.

policy I of the nonlinear model is 5.2/7.764 = 0.67, slightly larger than the 0.64 obtained in Chapter 6. If it is assumed that the typical availability factor is 0.85 (see discussion on Chapter 6), then a potential increase of up to 27% over energy actually produced could be achieved by using release policies from the nonlinear model. A 27% increase will be about 1.4×10^6 MWh per year with average inflow conditions. Besides these observations on energy generation, the similar form of the state trajectory for the linear and nonlinear models suggest that the overall policy of the latter can be explained with the arguments used in Chapter 6 to justify the optimal trajectory in the former. Namely, high inflow forecasts result in a drawdown of reservoirs in December, mainly by routing large volumes of water through penstocks. Reservoir elevations are relatively steady throughout the winter, maintained at the best elevation so that the tradeoff between elevation and discharge is optimal in the sense that for given conditions, the total energy would be maximized. The larger volumes of water released during the summer (4967 Kaf in May-August) than the requirements at the Delta (2698 Kaf in May-August) obtained with the nonlinear model (see policy I, Table 7.7) point again to the feasibility of an extension of agricultural activities as discussed in Chapter 6. Finally, at the expense of a moderate increase in the difficulty of the nonlinear model, both in its formulation and solution, it appears that the nonlinear model should be preferred over the linear model due to the closer representation to the actual system that it commands. However, the results of this section show the robustness of optimal policies to the choice of model: the nonlinear and linear models produced relatively close results despite the significant different assumptions, mathematical structure, and numerical solution inherent to each model.

CHAPTER 8

SUMMARY AND CONCLUSIONS

Alternative optimization models have been developed to obtain reservoir operation policies. Two different models have been used to find optimal release policies for the NCVP system. Several conclusions can be drawn from this study:

1) It is possible to increase the annual energy production of the system for below-average, average, and above-average inflow conditions. For a sample year of average inflow conditions, an upper bound to the possible increase of annual energy production was found to be of the order of 1.4 - 1.9 x 10^6 Mwh, from both the linear and nonlinear models, respectively.

2) Delta and agricultural water deliveries can be increased by adopting the optimal release policies. For a year of average inflow conditions, the water released from the system exceeded the agricultural demands by a factor of 1.6 (for both models). That suggests the possibility of increasing irrigated areas, providing better leaching of agricultural fields, and improving conjunctive management of surface water and groundwater reservoirs.

3) Much of the improvement achieved by the optimal operation policies developed in this study relative to the actual implemented operation schedules is due to: (i) an accurate river inflow forecasting technique; (ii) a highly conservative set of flood-control provisions currently enforced in the operation of the NCVP; and (iii) an integrative analysis, intrinsic in the optimization model, that allows to represent all the links and constraints that act simultaneously and

interactively within the system. Clearly, this integral conceptualization cannot be achieved by a heuristic approach based solely on experience.

4) It is difficult to establish a direct comparison between the optimal and actual implemented operation policies. This is due to the following reasons: (i) there is a significant amount of idle time in the power plants of the system that is caused by breakdowns and maintenance. Those operation halts occur randomly during any season of the year. In addition, those idle periods are difficult to consider in any optimization model because it is not known when they will occur, how long they will last, and what repercussion they will have in the integrated energy network; (ii) there are legal and institutional regulations that are highly variable which affect the directives of the NCVP management staff; and (iii) the managers of the system consider many intangible effects that cannot be properly considered with a mathematical model. This is especially true for establishing flood-control regulations, where a conservative attitude prevails with regard to flood management.

5) The improved performance reported by the use of the optimization models should be viewed as an upper bound to the possible gains that could be derived from the use of mathematical models. The better the forecasting of river inflows, the greater the annual availability factor of the power plants. Also, the more knowledgeable the system managers become with reservoir optimization models, the closer the performance of the system will be to the upper bounds obtained under the conditions assumed by the models. Clearly, the use of mathematical models and the better understanding that emanates from their use should result in a feedback to the models, with their probable reformulation

and modification that would bring closer the unpleasant difficulties of any real-world system and the sophistications inherent to any mathematical model. With regard to the two different models, linear and nonlinear, the optimal policies proved to be robust to the choice of model, i.e., the results obtained from both models were relatively close. However, the nonlinear model gives a better representation of the physical features of the system. Both models have equal dimensionality, although the solution of the two-stage problem in the nonlinear formulation is more complicated. The implementation of the nonlinear model is worthwhile because that model can be expected to provide more reliable results under varying conditions than the linear model.

The experience gained in this investigation has made evident several areas related to reservoir operation that deserve further study:

1) Reliable forecasts of river inflows. Conceptual hydrologic and statistical methods that would allow to predict accurate streamflows to the system are perhaps the most needed element in reservoir operation planning. Development of techniques for river-flow forecasting is a major task due to the size of the basins and the difficulties in modeling the hydrologic elements that interact to determine the volume of runoff feeding the reservoirs. For short-term events, the use of satellite information and raingage networks offers a possibility to improve flood management. Statistical forecasting techniques for real-time prediction of floods also may prove useful in developing flood-control strategies.

2) Interrelationships that exist between the functions served by the reservoir system and the economic environment in which the system is imbedded. The state of the economy determines to a great extent the

demands of water for agricultural activities and the energy requirements for industrial uses. Price schedules of water releases for agricultural uses also determine to some extent the quantities of water requested by the agricultural sector.

3) Conjunctive use of surface water and groundwater reservoirs. Improved management of surface water reservoirs leads to a greater recharge potential at groundwater basins. Thus, excess releases over contractual levels could be recharged to groundwater reservoirs rather than discharged into the ocean.

4) Implementation and installation of software that would automatize the operation of reservoir systems. This is a possible way to perform better control decisions for routing flood events by integrating inflow forecasts, control decision making, and execution of control policies into a unique, coordinated operation.

There has been substantial research published in the past years dealing with reservoir management. The stage is set to translate the best of this academic effort into working, operational techniques that are applied on a day-to-day basis by system managers. There must be a feedback from researchers to system operators, and vice versa, so that confidence and experience is gained in the use of optimization models. The real challenge in reservoir management is to make the best academic findings and methods operational. The time has come for researchers and practitioners to share their knowledge and experience to achieve more efficiently the goal that both pursue: a better use of the natural and societal resources available.

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