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COUPLING TO FAST WAVES NEAR THE LOWER HYBRID FREQUENCY

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Calculations show that the fast wave near the lower hybrid frequency ($\omega_{ci} \ll \omega \ll \omega_{ce}$) may be launched efficiently into large tokamak plasmas. Coupling efficiency is calculated for waves, which may be useful for current drive experiments in the Princeton Large Torus, the Tokamak Fusion Test Reactor, and reactor grade plasmas.

INTRODUCTION

Significant advances have been made with lower hybrid (LH) waves for generating steady-state currents in tokamaks.¹ Work has proceeded swiftly from small linear devices^{2,3} generating <1 A to large tokamaks^{4,7} generating over 100 kA. The tokamak experiments appear to work well only for densities where the wave frequency is well above the LH frequency, $\omega^2 \geq 4\omega_{LH}^2 = 4\omega_{pi}^2/(1 + \omega_{pe}^2/\omega_{ce}^2)$. Additionally, the high-energy densities cause some uncertainty about the future role of parametric decay^{8,9} and turbulence.¹⁰

The fast wave near the LH frequency is unlikely, according to cold plasma theory, to produce significant ion heating. Hence, most earlier work concentrated on the slow wave. The waveguide arrays necessary for launching the fast wave are also more complicated than those needed for the slow wave. With strong attention given now to LH current drive in addition to ion heating, it is worthwhile to examine the feasibility of fast wave current drive (FWCD). Providing accessibility conditions are satisfied, the fast wave propagates through the LH resonance layer and therefore has favorable prospects for generating good current distribution with FWCD.

In this paper, we address the question of coupling efficiency from an external launching structure into

large tokamak plasmas. Portions of this question have been addressed by Golant,¹¹ Berger et al.,¹² and Theilhaber and Bers.¹³ We calculate transmission coefficients for fast waves with parameters useful for FWCD.

WAVE PHYSICS

The geometry for launching the fast wave from a grill antenna array is shown in Fig. 1. The waveguide grill (see Brambilla¹⁴) is oriented with the waveguide-induced vacuum electric field in the $\pm y$ direction when the confining magnetic field is in the z direction and plasma density increases in the x direction. An approximate dispersion relation for the fast wave is (when $\omega_{ci} \ll \omega \ll \omega_{pe} \ll \omega_{ce}$)

$$N_x^2 = \frac{\epsilon_{xy}^2 - (N_z^2 - 1)(N_z^2 + N_y^2 - 1)}{(N_z^2 - 1)}, \quad (1)$$

where

$$N = ck/\omega$$

$$\epsilon_{xy} \approx \omega_{pe}^2/\omega\omega_{ce}.$$

The wave is evanescent in the plasma until $N_x^2 > 0$.

The fast wave is evanescent until densities are reached satisfying (approximately)

$$\omega_{pe}^2 > \omega\omega_{ce}(N_z^2 - 1)^{1/2}(N_z^2 + N_y^2 - 1)^{1/2}, \quad (2)$$

and the wave can then propagate into the plasma until the approximate accessibility condition

$$N_z^2 \geq 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \quad (3)$$

would be violated. The requirements that the vacuum electric field be in the $\pm y$ direction and that Eq. (3) be satisfied means that the standard waveguide grill must be modified. Dielectric stuffing or the use of ridged waveguides are possible solutions to this problem.

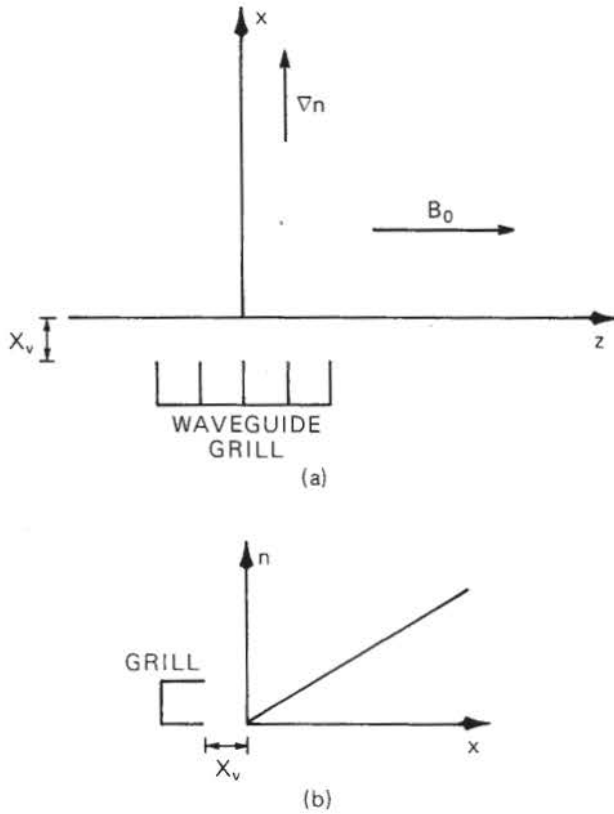


Fig. 1. (a) Grill plasma configuration for fast wave launching model, and (b) plasma density profile model for coupling calculation.

COUPLING MODEL

The evanescent region in the low-density plasma edge can lead to an imperfect coupling of energy from the waveguide into the fast wave in the plasma. Only if a substantial fraction of energy can be transmitted to the fast wave is FWCD feasible. The question of coupling from a waveguide to a plasma was addressed in a general formalism by Bers and Theilhaber,¹⁵ and the specific case of $N_y = 0$ was discussed by Golant¹¹ and Theilhaber and Bers.¹³ When $N_y \neq 0$, the boundary conditions are changed and there is some coupling with the density gradient. As shown later, the launching of waves with a finite N_y has a substantial advantage over the previous cases because of much improved coupling. We calculate the transmission coefficient of a single mode fast wave from the antenna into the plasma, while the technique of Brambilla¹⁴ should be used when a superposition of modes is studied.

A linear density rise is used, as shown in Fig. 1, with a vacuum gap between the waveguide, positioned at $x = -x_v$, and the plasma. The polarization of the launched wave is in the y direction with the waveguide

grill built and phased to control N_y and N_z . The energy in the reflected wave from the surface is assumed not to interfere with the antenna operation and not to reenter the plasma after further possible reflections. This assumption could be removed when a Brambilla-style superposition of modes is calculated.

The vacuum electric fields are evanescent because the fast wave accessibility requires launching waves with $N_z > 1$. In the plasma edge where the fast and slow waves couple weakly [we consider $N_z \geq 1.25$; see Eq. (25) in Ref. 11], the wave electric field can be found from

$$\begin{aligned} E_y'' + (\epsilon_{\perp} - N_z^2)E_y - iN_y E_x' + i\epsilon_{xy}E_x &= 0, \\ (\epsilon_{\perp} - N_y^2 - N_z^2)E_x - iN_y E_y' - i\epsilon_{xy}E_y &= 0, \end{aligned}$$

and

$$E_z \approx 0, \quad (4)$$

from whence the wave magnetic field is given by

$$\begin{aligned} H_y &= N_z E_x, \\ H_z &= -i \frac{\partial E_y}{\partial u} - N_y E_x, \end{aligned}$$

and

$$H_x = -N_z E_y, \quad (5)$$

where

$$\begin{aligned} \epsilon_{xy} &= \omega_{pe}^2 / \omega \omega_{ce} \\ \epsilon_{\perp} &= 1 + \omega_{pe}^2 / \omega_{ce}^2 - \omega_{pi}^2 / \omega^2 \approx 1 \\ \epsilon_{\parallel} &= 1 - \omega_{pe}^2 / \omega^2 - \omega_{pi}^2 / \omega^2 \end{aligned}$$

and the prime denotes d/du where $u = \omega x/c$. Equation (4) reduces to

$$E_y'' + \left[\frac{\epsilon_{xy}^2 - N_y \epsilon_{xy}'}{N_z^2 - 1} - (N_y^2 + N_z^2 - 1) \right] E_y = 0. \quad (6)$$

A wave with negative N_y reduces the evanescent distance, in effect increasing N_x^2 at a given position compared to the case of $N_y = 0$.

For the linear density profile, Eq. (6) reduces further to

$$E_y'' + (\alpha^2 u^2 - 2\alpha a) E_y = 0, \quad (7)$$

where

$$\begin{aligned} n &= n_0 u / u_L \\ \epsilon_{xy} &= \bar{\epsilon} u / u_L \\ u_L &= \text{density gradient scale length} \\ \alpha &= \frac{\bar{\epsilon}}{u_L (N_z^2 - 1)^{1/2}} \\ a &= \frac{1}{2\alpha} \left[\frac{\bar{\epsilon} N_y}{u_L (N_z^2 - 1)} + N_y^2 + N_z^2 - 1 \right]. \end{aligned}$$

Equation (7) has the parabolic cylinder function¹⁶ as the solution

$$E_y = AU \left[ia, \exp\left(-i \frac{\pi}{4}\right) \sqrt{2\alpha} u \right], \tag{8}$$

where A is the amplitude to be determined by the boundary conditions. Making tangential components of \mathbf{E} and \mathbf{H} continuous across the vacuum/plasma interface gives

$$A = E_0 \frac{2 \exp[-(N_y^2 + N_z^2 - 1)^{1/2} u_v]}{U(ia, 0)(1 - M)}, \tag{9}$$

with

$$M = \exp\left(-\frac{i\pi}{4}\right) \left(\frac{2\alpha}{N_y^2 + N_z^2 - 1}\right)^{1/2} \frac{U'(ia, 0)}{U(ia, 0)}.$$

The transmission coefficient is obtained by finding the ratio of the x component of the Poynting vector of the fast wave to the Poynting vector of the wave emitted from the waveguide grill:

$$T(N_y, N_z) = \frac{(N_z^2 - 1) \exp\left(-\frac{\pi a}{2}\right) \exp[-2(N_y^2 + N_z^2 - 1)^{1/2} u_v] \left|\Gamma\left(\frac{1}{4} + \frac{ia}{2}\right)\right|^2}{\pi N_z \sqrt{\alpha} \left[1 + \frac{(N_y^2 + N_z^2 - 1)^{1/2}}{2\pi \sqrt{\alpha}} \exp\left(\frac{\pi a}{2}\right) \left|\Gamma\left(\frac{1}{4} + \frac{ia}{2}\right)\right|^2 + \frac{\cosh \pi a}{8\pi^2 \alpha} (N_y^2 + N_z^2 - 1) \left|\Gamma\left(\frac{1}{4} + \frac{ia}{2}\right)\right|^4\right]}, \tag{10}$$

where $\Gamma(z)$ is the gamma function.

APPLICATION TO LARGE TOKAMAKS

For tokamaks the transmission coefficient is found in a manner similar to the above calculations. Complications arise since the density profile is no longer a linear function of x . Hence, the theory was modified and computer codes were developed. The density profile was taken to be parabolic from $r = 0$ to $r = a$ with an edge plasma with exponentially decreasing density going as $\exp[-(r - a)/d]$ from $r = a$ to $r = a + s$ and vacuum from $r = a + s$ to $r = a + s + x_v$ where the mouth of the waveguide grill is located.

Figure 2 shows the transmission coefficient for a Princeton Large Torus (PLT)-type plasma using $f = 800$ MHz, $a = 42$ cm, $d = s = 1$ cm, $x_v = 10^{-2}$ cm, and $n(a) = 5 \times 10^{12}$ cm⁻³. The transmission reaches a maximum for $N_z \approx 1.7$ and $N_y \approx -1$. The maximum current drive efficiency for LH waves on the PLT has occurred for the coupler with $N_z \approx 2$. It is thought that reducing N_z to ≤ 2 might increase the LH current drive. Hence, the fast wave has maximum coupling efficiency in a range where good FWCD may be expected on the PLT.

For the Tokamak Fusion Test Reactor (TFTR), calculations were done for two modes of operation. Both modes use $d = s = 1$ cm, $x_v = 10^{-2}$ cm, and $f = 800$ MHz. Figure 3 shows coupling to a low-density, low-field plasma with $n(a) = 1.6 \times 10^{12}$ cm⁻³, $\langle n \rangle = 2 \times 10^{13}$ cm⁻³, and $B = 27$ kG. Maximum coupling of 67% is achieved for a low $N_z \approx 1.4$,

while $N_z \geq 1.3$ is required for the plasma center to be accessible to the fast wave. Figure 4 shows coupling for $n(a) = 8.2 \times 10^{12}$ cm⁻³, $\langle n \rangle = 10^{14}$ cm⁻³, and $B = 50$ kG. The best coupling now occurs for $N_z = 1.7$, well above the accessibility limit of $N_z = 1.46$ at the center of the plasma. The coupling efficiency in

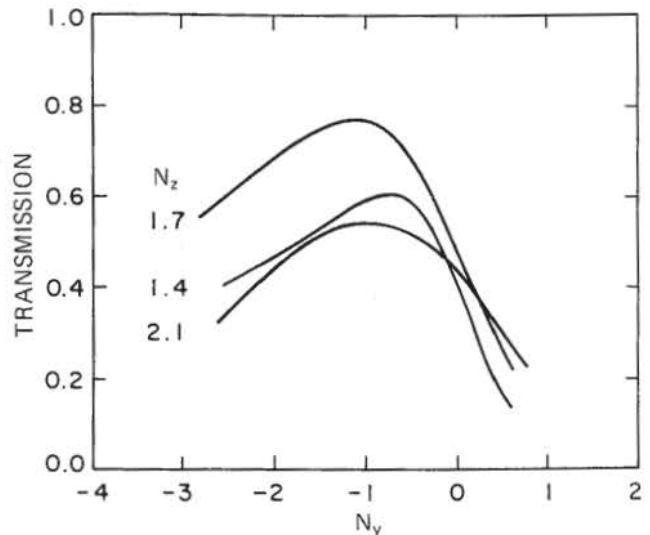


Fig. 2. Transmission coefficient versus N_y at various N_z values. The PLT plasma density profile $B_0 = 32$ kG and $f = 800$ MHz.

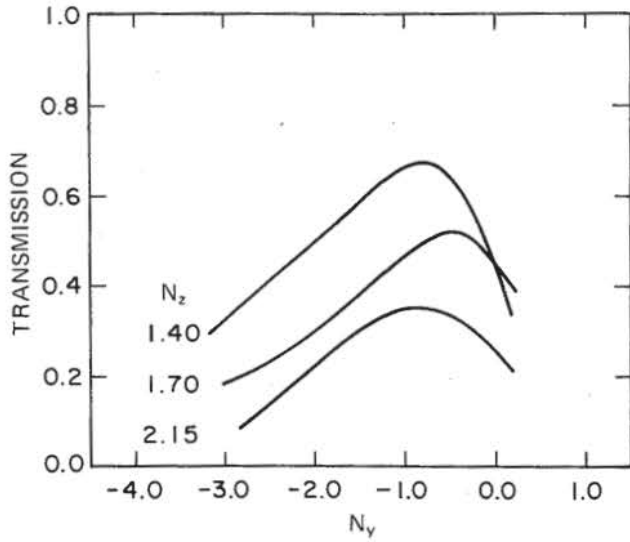


Fig. 3. Transmission coefficient for preliminary TFTR plasma density profile $B_0 = 27$ kG and $f = 800$ MHz.

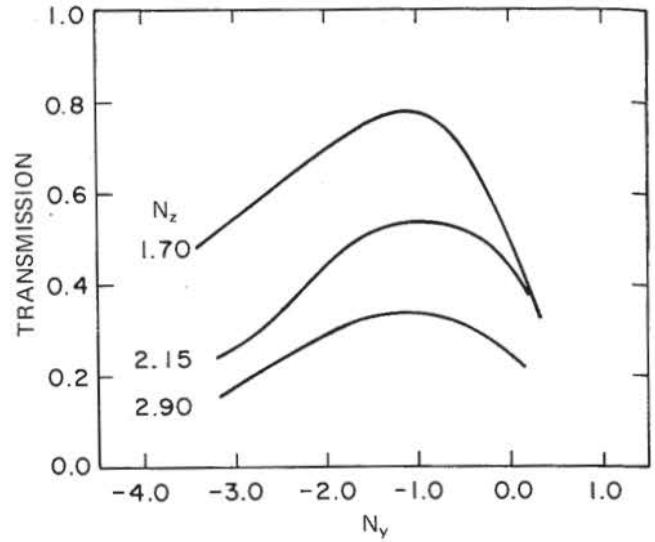


Fig. 4. Transmission coefficient for nominal TFTR plasma density profile $B_0 = 50$ kG and $f = 800$ MHz.

this case may be as high as ~80%. Hence, the fast wave couples better with a nominal operating parameters plasma in TFTR than the low-density, low-field preliminary parameters plasmas.

For a reactor plasma running at $B = 30$ kG, $f = 800$ MHz, and $n(a) = 3 \times 10^{13} \text{ cm}^{-3}$, with a toroidal belt limiter the wave launching structure might be placed in an edge plasma with $d = 1.4$ cm, $s = 2$ cm, and $x_v = 10^{-2}$ cm. Figure 5 shows that the transmission coefficient reaches a maximum with $N_z = 2.15$ and $N_y = -1.5$. A good coupling of 85% is achieved for this combination.

CONCLUSIONS

Waveguide arrays can be used to launch fast waves suitable for current drive in large tokamaks. By optimizing the wave spectrum, coupling coefficients as high as 85% can be achieved, with values in excess of 60% easily realizable in general for large tokamak plasmas. Accessibility criteria do not hinder the wave penetration into the plasma. For the experimental arrangements considered here, the evanescent layer is sufficiently thin that surface reflection should not be a substantial problem. For the optimal wave spectrum that couples the highest fraction of wave power into the plasma, the fast wave (which propagates directly through the LH layer if accessibility is satisfied) should allow good current profiles to be established or maintained in large tokamak plasmas.

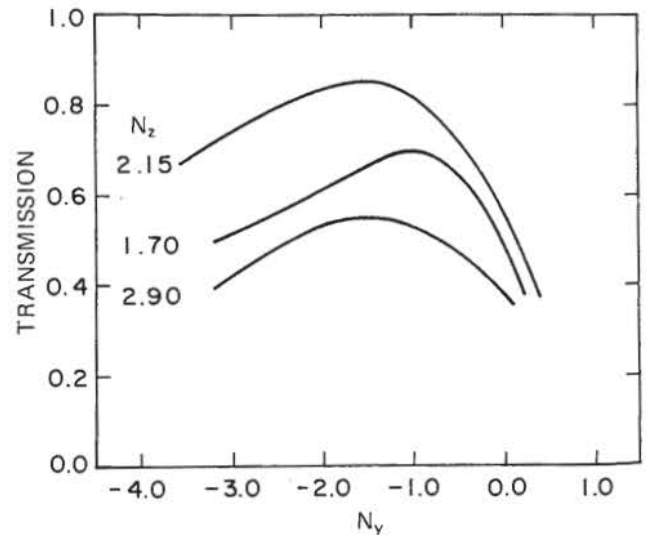


Fig. 5. Transmission coefficient for reactor grade plasma $B_0 = 30$ kG and $f = 800$ MHz.

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