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Effective action for gauge theories and quark confinement*

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The effective action for spinor fields coupled to Abelian or non-Abelian gauge bosons is examined as a function of a composite or elementary field. Various authors have proposed a classical Lagrangian that may lead to quark confinement, and the question is asked whether such a Lagrangian could be an effective one generated by quantum correction to a more conventional one. It is shown that such a mechanism will not be operative for Abelian theories and speculations are made for non-Abelian ones.

I. INTRODUCTION

One of the dilemmas of the quark model of hadrons based on fractionally charged quarks is the absence of these quarks as real physical states. This has lead to various schemes for confining quarks, either permanently or up to a very high energy threshold. Some of these rely on the "color" version of this model, in which in addition to the usual SU(N) symmetry there is an SU(3) symmetry of color,[1] and the confinement is accomplished if one can ensure that only color singlets exist as physical objects.

It has been made plausible that such a confinement can be achieved in a strong-coupling limit[2] of a theory of a gauge field, most probably non-Abelian, coupled to quarks. A particular version of this idea, put forward by Kogut and Susskind[3] and by 't Hooft,[4] is based on a particular Lagrangian that appears *ad hoc* and is not renormalizable. The above authors note these facts and suggest that the Lagrangian is only an effective one. In this article we shall investigate whether quantum corrections to a more prosaic renormalizable theory could generate the modifications necessary for permanent quark binding.

At this stage it will be useful to review the ideas of Refs. 3 and 4. In addition to an Abelian or non-Abelian vector field $A_\mu(x)$ we postulate a neutral scalar field $\phi(x)$ and a Lagrangian

$$L = -\tfrac{1}{4}Z_3(\phi)F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - j_\mu A^\mu ,$$

(1)

with $j_\mu(x)$ the color current of the particles to be confined. We have not written explicitly the kinetic Lagrangian of the quarks. $Z_3(\phi)$ is some function of the field ϕ chosen to vanish at the minimum of the potential $V(\phi)$. $Z_3(\phi)$ may be viewed as a field-dependent dielectric constant.[3] In the static limit the Hamiltonian becomes

$$H = \frac{D^2}{2Z_3(\phi)} + \tfrac{1}{2}(\nabla\phi)^2 + V(\phi) ,$$

(2)

where $D_i = Z_3(\phi)F_{0i}$ with $\nabla \cdot D = j_0$. In the presence of charges the value of D is constrained by Gauss's theorem, and V may not attain its minimum as that would make the first term of H infinite. It is the competition between this term and $V(\phi)$ that leads to confinement. In Ref. 4 it is shown that in the case $Z_3(\phi) \sim [V(\phi) - V(\phi_{\min})]^\alpha$, with $\alpha > \tfrac{1}{3}$, the favored solution forces the displacement field D into a tube between sources of opposite charge, leading to a potential that increases as the sources separate. In the non-Abelian case the confinement is plausible if the color charges of the vector fields are integral while those of the quarks are fractional.

As mentioned earlier the above Lagrangian does not look very natural and is not renormalizable. We shall study the effective action generated by loop corrections[5] to a renormalizable quark–vector-boson Lagrangian to see when a form similar to Eq. (1) can arise. Within the above class we study both the Abelian and non-Abelian versions. As has been suspected, quark binding will probably be due to singular infrared behavior. We are able to show that the Abelian version *does not* lead, at least for small coupling constants, to binding of the form discussed above. For non-Abelian theories nothing rigorous can be said but we shall make some speculations indicating that such a mechanism may arise.

One other attractive feature of non-Abelian theories is asymptotic freedom.[6] Scalar mesons with their inherent four-meson couplings are often contrary to this freedom. Thus instead of introducing an elementary scalar field we shall consider a composite field, $\Sigma(x) = \overline{\Psi}(x)\Psi(x)$, and obtain an effective action involving it. With minor modifications we may discuss all our results in terms of an elementary scalar field. Whatever these scalar fields are, we do not want to induce any mechanism where a mass is imparted to the vector bosons, as this would destroy the long-range nature of the forces necessary to bind the quarks.

In Sec. II we set up the effective action formalism

and discuss the technical points involved in the introduction of a composite field.

In Sec. III we explicitly obtain this action in a one- and two-loop approximation, while in Sec. IV these approximations are improved through the application of the renormalization group. In Sec. V the results and speculations are summarized.

In the Appendix we develop the technique for calculating the effective action beyond just the effective potential.

II. EFFECTIVE ACTION AND COMPOSITE FIELDS

The renormalizable Lagrangians of the theories we shall consider will be of the form

$$L = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\slashed{\partial} - e\slashed{A})\psi + \frac{1}{2\xi}(\partial_\mu A^\mu)^2 . \tag{3}$$

$\overline{\psi}$ is a multicomponent spinor field, A_μ is a matrix representing the vector bosons, $A_\mu = A^\alpha_\mu t_\alpha$ and $F_{\mu\nu}$ in the gauge-invariant field strength matrix, $F_{\mu\nu} = F^\alpha_{\mu\nu}T_\alpha$; T is the adjoint representation of the color group under consideration, while t_α is the representation for the quarks. Except for discussions involving the renormalization group we do not consider vector-boson loops; thus at this stage we do not differentiate Abelian and non-Abelian theories. Similarly we do not have to consider Faddeev-Popov ghosts. The $(1/\xi)(\partial \cdot A)^2$ in Eq. (3) is a gauge-fixing term. We shall perform all our computations in the Lorentz gauge.[7]

The generating functional for the Green's functions is

$$Z[\eta,\overline{\eta},J_\mu,K] = \int [d\psi][d\overline{\psi}][dA_\mu]\exp\left\{\frac{i}{\hbar}\int d^4x[L(\psi,\overline{\psi},A_\mu) + \overline{\eta}\psi + \overline{\psi}\eta + J_\mu A^\mu + K\overline{\psi}_\alpha\psi_\alpha]\right\} . \tag{4}$$

The integrations are functional,[8,9] and in addition to the usual sources for the fields under consideration we introduce a source, $K(x)$, for the composite field $\overline{\psi}_\alpha(x)\psi(x)_\alpha$. The generator of the connected Green's functions is

$$W[\eta,\overline{\eta},J_\mu,K] = \frac{\hbar}{i}\ln Z[\eta,\overline{\eta},J_\mu,K] , \tag{5}$$

while that for the one-particle-irreducible Green's functions is obtained via a Legendre transform[8]

$$\frac{\delta W}{\delta\overline{\eta}} = \psi, \quad W\frac{\overleftarrow{\delta}}{\delta\eta} = \overline{\psi}, \quad \frac{\delta W}{\delta J_\mu} = A^\mu, \quad \frac{\delta W}{\delta K} = \Sigma + \overline{\psi}\psi ,$$

$$\tag{6a}$$

and

$$\Gamma[\psi,\overline{\psi},A_\mu,\Sigma] = W - \overline{\eta}\psi - \overline{\psi}\eta - J_\mu A^\mu - K(\Sigma + \overline{\psi}\psi) . \tag{6b}$$

Equations (6a) are used to express η, $\overline{\eta}$, J, and K in terms of ψ, $\overline{\psi}$, A, and Σ. Γ is the effective action we wish to discuss. Γ is a functional of the classical fields, and following Coleman and Weinberg[5] we will expand Γ in powers of momentum:

$$\Gamma[\psi,\overline{\psi},A_\mu,\Sigma] = \int d^4x[-V(\Sigma) + \overline{\psi}(i\slashed{\partial} - e\slashed{A})\psi Z_2(\Sigma)$$

$$-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}Z_3(\Sigma) + \cdots] . \tag{7}$$

V, Z_2, and Z_3 are functions, not functionals of their arguments. As we do not wish to break

Lorentz invariance we do not expect the Fermi fields to develop nonzero vacuum expectation values, and thus we consider the above functions evaluated at $\psi = 0$. Gauge invariance prevents any explicit dependence of V and Z_i's on A_μ.

$V(\Sigma)$ is the energy of the lowest state in which the expectation value of $\overline{\psi}(x)\psi(x)$, appropriately renormalized, equals Σ. In the true vacuum Σ attains the value given by minimizing V. If $Z_3(\Sigma)$ vanishes at that point, a confining mechanism as discussed in the introduction will be present.

Technically, inverting Eq. (6a) for K is fairly complicated, especially because of renormalization. Cornwall, Jackiw, and Tomboulis[10] have given an elegant and thorough discussion of the situation where the composite operator is nonlocal, e.g., $\overline{\psi}(x)\psi(y)$. One can extend their results to the local case; however, we shall develop a scheme for analyzing Eq. (4) directly.

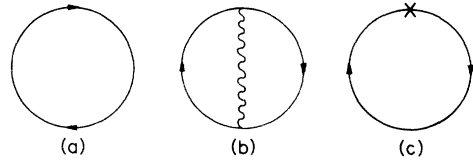We introduce two auxiliary fields $\sigma(x)$ and $\phi(x)$ and introduce the following generating functional:

FIG. 1. Diagrams contributing to the potential $V(\phi)$. The solid lines denote the propagator $(i\slashed{\partial} - \phi)^{-1}$ and the wavy line denotes the propagator $g_{\mu\nu}/\partial^2$. (c) indicates the one-loop renormalization due to divergences in the diagrams of Fig. 3.

$$\tilde{Z}[\eta,\bar{\eta},J,K,I]= \int [d\psi][d\bar{\psi}][dA_\mu][d\sigma][d\phi]\exp\left\{\frac{i}{\hbar}\int d^4x[L+\bar{\eta}\psi+\bar{\psi}\eta+J_\mu A^\mu+K\sigma+\phi(\sigma-\bar{\psi}\psi)+I\phi]\right\}.$$ (8)

Clearly, for $I=0$ the above generating functional reduces to that of Eq. (4),

$$\tilde{Z}[\eta,\bar{\eta},J,K,0]=Z[\eta,\bar{\eta},J,K] .$$ (9)

Following the standard procedure we introduce

$$\tilde{W}[\eta,\bar{\eta},J,K,I]=\frac{\hbar}{i}\ln\tilde{Z}[\eta,\bar{\eta},J,K,I]$$ (10)

and the effective action by

$$\frac{\delta\tilde{W}}{\delta\bar{\eta}}=\psi,\quad \tilde{W}\frac{\overleftarrow{\delta}}{\delta\eta}=\bar{\psi},\quad \frac{\delta\tilde{W}}{\delta J_\mu}=A^\mu,\quad \frac{\delta\tilde{W}}{\delta K}=\bar{\Sigma}+\bar{\psi}\psi,\quad \frac{\delta\tilde{W}}{\delta I}=\phi ,$$ (11a)

$$\tilde{\Gamma}=\tilde{W}-\bar{\eta}\psi-\bar{\psi}\eta-J\cdot A-K(\Sigma+\bar{\psi}\psi)-I\phi .$$ (11b)

The effective action of Eq. (6) may be obtained from (11) by first solving

$$\frac{\delta\tilde{\Gamma}}{\delta\phi}=0$$ (12)

for ϕ, obtaining $\phi=\phi_c[\psi,\bar{\psi},A,\Sigma]$, and then substituting it into $\tilde{\Gamma}$:

$$\Gamma(\psi\bar{\psi},A,\Sigma)=\tilde{\Gamma}(\psi\bar{\psi},A,\Sigma,\phi_c) .$$ (13)

In the above $\bar{\Sigma}$ and Σ will coincide at the critical value of ϕ as $I=0$. Likewise one may note that $\tilde{W}=W-IK$, from which it may be inferred that $\tilde{\Gamma}$ has the separation

$$\tilde{\Gamma}[\psi,\bar{\psi},A,\bar{\Sigma},\phi]=\int\bar{\Sigma}\phi\,dx+\tilde{\Gamma}'[\psi,\bar{\psi},A,\phi]$$ (14)

and Eq. (12) yields

$$\bar{\Sigma}+\frac{\delta\tilde{\Gamma}'}{\delta\phi}\bigg|_{\phi=\phi_c}=0 .$$ (15)

The potential will likewise have the form

$$V(\bar{\Sigma})=-\phi\bar{\Sigma}-U(\phi) .$$ (16)

Applying Eq. (15) we find that the minimum of V occurs at Σ corresponding to $\phi=0$. Similarly Z_3 will be a function of ϕ only and we shall look for situations where

$$Z_3(\phi=0)=0 .$$ (17)
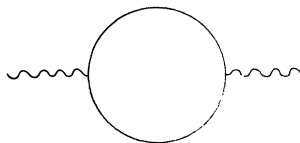
In passing we should mention an alternate scheme

for introducing a composite local field due to Gross and Neveu[11] and to Coleman, Jackiw, and Politzer.[12] In place of Eq. (8) the term $-\frac{1}{2}(\sigma-\bar{\psi}\psi-K)^2$ is added to the Lagrangian and the functional integration over σ is performed. We note that the $K\bar{\psi}\psi$ term is canceled and a source for the σ field is introduced in its place. We found the previous scheme more direct as it did not introduce an additional $(\bar{\psi}\psi)^2$ coupling and σ propagation.

III. LOOP EXPANSION

The approximate method we shall use for evaluating the effective action is the WKB or loop expansion. The details of the method for obtaining the potential were developed by Jackiw[13] and by Iliopoulos, Itzykson, and Martin.[14] A technique for evaluating the coefficients of higher-order derivatives was presented in Ref. 14; however, it does not appear generalizable to Lagrangians of more than one field. A more general method was alluded to in Ref. 13. As a functional evaluation of these terms has not been previously presented, we shall discuss its details in the Appendix.

Since the one-loop contribution to $Z_3(\phi)$ is of order e^2, we shall evaluate the potential $U(\phi)$ up to two loops.[15] As mentioned in the Introduction, we do not expect ψ to acquire a nonvanishing vacuum expectation value, and thus we set $\psi=0$. Likewise, gauge invariance requires $A_\mu=0$. The diagrams contributing to $U(\phi)$ are shown in Fig. 1. The one for $Z_3(\phi)$ is in Fig. 2. Figure 1(c) represents the inclusion of wave-function and $\phi\bar{\psi}\psi$ vertex renormalizations indicated in Fig. 3. Before presenting the results we will discuss the renormalization of the effective action. The coefficients of ϕ^2 and ϕ^4 in $U(\phi)$ are infinite. As we do not wish to ϕ field to propagate, we set

$$\frac{\partial^2 V}{\partial\phi^2}\bigg|_{\phi=0}=0 .$$ (18)

(Had we evaluated the coefficient of $\partial_\mu\phi\partial^\mu\phi$ in the

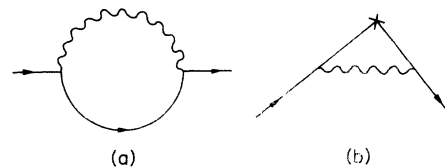FIG. 2. Contribution to $Z_3(\phi)$. Propagators are as in Fig. 1.

(a)   (b)

FIG. 3. Diagrams contributing to the wave-function and vertex corrections in the one-loop approximation. Propagators are the usual ones, namely $(i\not{p})^{-1}$ and $g_{\mu\nu}/\partial^2$. The cross denotes the insertion of a $\phi\bar{\psi}\psi$ vertex.

effective action, it too would have been set equal to zero at some renormalization point $\phi = M$.) The coefficient of ϕ^4 is determined by requiring

$$\frac{\partial^4 V}{\partial \phi^4}\bigg|_{\phi = M} = \lambda \, . \tag{19}$$

λ is at the moment an arbitrary parameter. The only other infinity that occurs in evaluating Γ is in the term $Z_2(\phi)\bar{\psi}\psi$; setting $\partial Z_2/\partial \phi|_{\phi = M} = 1$ removes this infinity.

Had we been dealing with an elementary field ϕ, the above renormalization conditions would have had a conventional interpretation in terms of wave-function and coupling-constant renormalizations. For composite fields the intepretation is different. The coefficient of $\phi\bar{\psi}\psi$ determines the magnitude of the $\bar{\psi}\psi$ content of Σ, while Eq. (19) sets the over-all magnitude Σ. The other conditions indicate that we are dealing with a composite field.

To the order indicated above we find [compare Eq. (16)]

$$U(\phi) = \frac{S\phi^4}{16\pi^2}\left[\ln\left(\frac{\phi^2}{M^2}\right) - \frac{25}{6}\right] - \frac{6e^2 NT(R)\phi^4}{(16\pi^2)^2}\left\{\ln^2\left(\frac{\phi^2}{M^2}\right) - \frac{35}{3} - 7\left[\ln\left(\frac{\phi^2}{M^2}\right) - \frac{25}{6}\right]\right\} - \frac{\lambda\phi^4}{24} \tag{20}$$

and

$$Z_3(\phi) = 1 - T(R)\frac{e^2}{12\pi^2}\ln\left(\frac{\phi^2}{M^2}\right) \, . \tag{21}$$

In the above S is the dimension of the spinor representation, N is the dimension of the adjoint representation, and $\text{Tr}(t^\alpha t^\beta) = \delta^{\alpha\beta}T(R)$.

IV. APPLICATION OF THE RENORMALIZATION GROUP

The approximate forms for $U(\phi)$ of Eq. (20) or alternately $V(\Sigma)$ of Eq. (16) and of $Z_3(\phi)$ of Eq. (21) may be improved through the use of the renormalization group. The potential $V(\Sigma)$ satisfies

$$\left[M\frac{\partial}{\partial M} + \beta(e)\frac{\partial}{\partial e} + \beta(e,\lambda)\frac{\partial}{\partial \lambda} - \gamma_\Sigma(e)\Sigma\frac{\partial}{\partial \Sigma}\right]V(\Sigma) = 0 \tag{22}$$

when $\beta(e)$ is the usual coefficient governing the dependence of the coupling constant on the renormalization point. $\beta(e,\lambda)$ serves the same function for λ. $\beta(e)$ has been evaluated previously[6]:

$$\beta(e) = -\frac{e^3}{16\pi^2}\frac{11}{3}C_2(G) + \frac{e^3}{16\pi^2}\frac{4}{3}T(R) \, . \tag{23}$$

$C_2(G)$ is the Casimir operator of the non-Abelian group in question; in the Abelian case $C_2(G) = 0$. $\gamma_\Sigma(e)$ is the anomalous dimension of the composite field and it may be evaluated directly from the divergent piece of Fig. 3(b):

$$\gamma_\Sigma(e) = -\frac{8C_2(G)e^2}{16\pi^2} \, . \tag{24}$$

Equivalently one may show that $U(\phi)$ satisfies a similar equation:

$$\left[M\frac{\partial}{\partial M} + \beta(e)\frac{\partial}{\partial e} + \beta(e,\lambda)\frac{\partial}{\partial \lambda} - \gamma_\phi(e)\phi\frac{\partial}{\partial \phi}\right]U(\phi) = 0 \, . \tag{25}$$

Applying this equation to $\partial^4 U/\partial \phi^4$ and setting $\phi = M$ (see Ref. 5) we obtain

$$\beta(e,\lambda) = -\frac{3}{\pi^2}(1 + e^2) + \frac{2\lambda e^2}{\pi^2} \, . \tag{26}$$

With the exception of the result for $\gamma_\phi(e)$, the above are presented for completeness and unlike the equation for $Z_3(\phi)$ will not be used subsequently.

$$\left[M\frac{\partial}{\partial M} + \beta(e)\frac{\partial}{\partial e} - 2\gamma_V(e) - \gamma_\phi(e)\phi\frac{\partial}{\partial \phi}\right]Z_3(\phi) = 0 \, , \tag{27}$$

where $\gamma_V(e)$ is the anomalous dimension of the vector field and has likewise been presented previously,[6]

$$\gamma_V(e) = -\frac{e^2}{16\pi^2}\frac{13}{3}C_2(G) + \frac{e^2}{16\pi^2}\frac{8}{3}T(R) \, . \tag{28}$$

Equation (27) may be solved readily; let $t = \ln\phi/M$, and remembering $Z_3(t = 0) = 1$ we obtain

$$Z_3(\phi) = \exp\left[-2\int_0^t dt'\bar{\gamma}_V(e'(t'))dt'\right] \, , \tag{29}$$

where

$$\bar{\gamma}_V(e') = \frac{\gamma_V(e')}{1 + \gamma_\phi(e')} \tag{30}$$

and $e'(t)$ is obtained from the solution of

$$\frac{\partial e'}{\partial t} = \bar{\beta}(e') = \frac{\beta(e')}{1 + \gamma_\phi(e')} \, , \quad e'(0) = 0 \, . \tag{31}$$

These results are applied to the confinement problem in the next section.

V. CONFINEMENT?

We recall that the minimum of $V(\Sigma)$ occurs at Σ corresponding to $\phi = 0$ or to $t = -\infty$.

A. Abelian theory

As Abelian field theories are infrared-stable, we expect the loop expansion to yield reliable result for $t \to -\infty$. We find

$$[e'(t)]^2 \sim e^2/(1 - 2ae^2 t),$$ (32)

$$\bar{\gamma}_v(e) \sim be^2,$$

with a and b positive. The above yield

$$Z_3(\phi) \to (1 - 2ae^2 t)^{b/a}.$$ (33)

$Z_3(\phi)$ does *not* approach zero as ϕ approaches zero. This dielectric mechanism is not present in Abelian theories for coupling constants in the region of attraction of the origin.

B. Non-Abelian theories

Three possibilities occur. The first is that $\bar{\beta}(e)$ has an infrared-stable zero at some finite value. It is unlikely that confinement would occur in this case. In the case where $\bar{\beta}(e)$ has no further zeros $e'(t)$ could have a singularity at a finite value of t, in which case the points $\phi = 0$ and $\phi = M$ are not simply related to each other and this argument sheds no light on the confinement problem.

The third and perhaps most attractive possibility is that $e'(t)$ grows indefinitely as $t \to -\infty$, and that $\bar{\gamma}_v(e')$ approaches a negative constant as $e \to \infty$,

$$\bar{\gamma}_v(e') \to -d.$$ (34)

In this situation

$$Z_3(\phi) \to \text{const} \times \left(\frac{\phi}{M}\right)^{2d}$$ (35)

and the sought-for confinement will occur for sufficiently large d. [This restriction would come from studying the solutions of Eq. (2).] Taking

seriously the results we have obtained for $\bar{\beta}(e)$ and $\bar{\gamma}_v(e)$ (not for β and γ alone) leads to this third possibility.

ACKNOWLEDGMENT

APPENDIX: HIGHER-ORDER DERIVATIVES IN THE EFFECTIVE ACTION

We shall present the technical details for the functional evaluation of the coefficients of terms involving derivatives of fields in the expansion of the effective action. An elegant method was presented in Ref. 14; unfortunately it appears useful only for a one-component theory. A more general technique was suggested in Ref. 13; however, the details were not presented. The results are simple and could have been guessed. In order to avoid notational difficulties we shall treat Bose fields exclusively. The result will be given in detail for a one-loop calculation; the extension is obvious.

We consider a theory with a bare Lagrangian

$$L = \tfrac{1}{2}\partial_\mu \phi_\alpha \partial^\mu \phi_\alpha - V^0(\phi_\alpha),$$ (A1)

where ϕ_α is a multicomponent field. Following the standard procedure for performing the functional integrations by a steepest-descent method[13,14] we obtain up to the one-loop level

$$\Gamma[\phi] = \Gamma_0[\phi] + \Gamma_1[\phi],$$ (A2)

with

$$\Gamma_0[\phi] = \int d^4x [\tfrac{1}{2}\partial_\mu \phi \partial^\mu \phi - V^{(0)}(\phi)],$$ (A3a)

$$\Gamma_1[\phi] = -i \ln \left\{ \int [d\psi_\alpha] \exp\left[i \int d^4x\, \psi_\alpha(x)[-\tfrac{1}{2}\partial^2 \delta_{\alpha\beta} - \tfrac{1}{2}V^{(0)}_{\alpha\beta}(\phi)]\psi_\beta \right] \right\}$$

$$+ i \ln \left\{ \int [d\psi_\alpha] \exp\left[i \int d^4x\, \psi_\alpha(x)(-\tfrac{1}{2}\partial^2 \delta_{\alpha\beta} - V^{(0)}_{\alpha\beta}(0))\psi_\beta \right] \right\}.$$ (A3b)

$V^{(0)}_{\alpha\beta}(\phi) = \partial^2 V^{(0)}/\partial \phi_\alpha \partial \phi_\beta$. In the situation where $\phi_\alpha(x)$ in the above is taken to be a constant, $\bar{\phi}_\alpha$, the functional integration may be performed explicitly and the resultant $\Gamma[\bar{\phi}]$ yields the one-loop correction to $V(\bar{\phi})$.

For nonconstant ϕ_α we expand the integrands in (A3b) in powers of $F_{\alpha\beta}(\phi(x), \bar{\phi}) = V^{(0)}_{\alpha\beta}(\phi(x)) - V^{(0)}_{\alpha\beta}(\bar{\phi})$ up to second order:

$$\Gamma_1[\phi] = -i \ln \left\{ \int [d^4\alpha] \exp\left[i \int d^4x\, \psi_\alpha(-\tfrac{1}{2}\partial^2 \delta_{\alpha\beta} - \tfrac{1}{2}V_{\alpha\beta}(\bar{\phi}))\psi_\beta \right] \right.$$

$$\times \left[1 - \frac{i}{2} \int d^4x\, \psi_\alpha(x)F_{\alpha\beta}(x)\psi_\beta(x) - \tfrac{1}{8} \int d^4x\, d^4y\, \psi_\alpha(x)F_{\alpha\beta}(x)\psi_\beta(x)\psi_\gamma(y)F_{\gamma\delta}(y)\psi_\delta(y) \right] \right\}.$$ (A4)

The functional integrations are performed and

$$\Gamma_1(\phi) = - \int d^4x \, V_1(\overline{\phi})$$

$$- i \ln \left\{ 1 + \tfrac{1}{2} \int d^4x \, \text{Tr}(\Delta(0, \overline{\phi}) F(x)) \right.$$

$$\left. + \tfrac{1}{8} \left[\int d^4x \, \text{Tr}(\Delta(0; \overline{\phi}) F(x)) \right]^2 + \tfrac{1}{4} \int d^4x \, d^4y \, \text{Tr}(\Delta(x - y; \overline{\phi}) F(x) \Delta(y - x, \overline{\phi}) F(y)) \right\} \tag{A5}$$

where

$$\Delta_{\alpha\beta}(x; \overline{\phi}) = \int \frac{d^4k}{(2\pi)^4} \, [\delta_{\alpha\beta} k^2 - V_{\alpha\beta}(\overline{\phi})]^{-1} e^{ik \cdot x} \, . \tag{A6}$$

Expanding the logarithm, one finds

$$\Gamma_1[\phi] = - \int d^4x \, V_1(\overline{\phi}) - \frac{i}{2} \int d^4x \, \text{Tr}(\Delta(0, \overline{\phi}) F(x)) - \frac{i}{4} \int d^4x \, d^4y \, \text{Tr}(\Delta(x - y; \overline{\phi}) F(x) \Delta(y - x; \overline{\phi}) F(y)) \, . \tag{A7}$$

Expanding $F(x)F(y)$ around $y = x$ we get

$$F(x)F(y) = F(x)F(x) + (y - x)^\mu F(x) \partial_\mu F(x) + \tfrac{1}{2}(y - x)^\mu (y - x)^\nu \partial_\mu \partial_\nu F(x) \, . \tag{A8}$$

Combining the above with (A7) and changing variables from y to $z = y - x$, we obtain

$$\Gamma_1(\phi) = - \int d^4x \, V_1(\overline{\phi}) - i \int d^4x \, \text{Tr}(\Delta(0, \overline{\phi}) F(x))$$

$$- \frac{i}{4} \int d^4x \, d^4z \, \text{Tr}[\Delta(z; \overline{\phi}) F(x) \Delta(-z; \overline{\phi}) F(x) + z^\mu \Delta(z; \overline{\phi}) F(x) \Delta(-z; \overline{\phi}) \partial_\mu F(x)$$

$$+ \tfrac{1}{2} z^\mu z^\nu \Delta(z; \overline{\phi}) \partial_\mu F(x) \Delta(-z; \overline{\phi}) \partial_\nu F(x)] \, . \tag{A9}$$

Comparing this with the momentum expansion of the effective action

$$\Gamma[\phi] = - \int d^4x \, V[\phi] - \tfrac{1}{2} \int \partial_\mu \phi Z^{\mu\nu}(\phi) \partial_\nu \phi \, d^4x$$

$$\approx - \int d^4x \, V[\overline{\phi}] - \int d^4x \frac{\partial V}{\partial \phi}(\phi - \overline{\phi}) - \tfrac{1}{2} \int d^4x \frac{\partial^2 V}{\partial \phi_\alpha \partial \phi_\beta}(\phi - \overline{\phi})_\alpha (\phi - \overline{\phi})_\beta - \tfrac{1}{2} \int d^4x \, \partial_\mu \phi \, Z^{\mu\nu}(\overline{\phi}) \partial_\nu \phi \tag{A10}$$

and noting that $\partial_\mu \phi$ is of the order of $\phi - \overline{\phi}$ or of F, we find the result of interest:

$$- \tfrac{1}{2} \partial_\mu \phi \, Z^{\mu\nu}(\overline{\phi}) \partial_\nu \phi = \frac{i}{8} \int d^4z \, z^\mu z^\nu \, \text{Tr}(\Delta(z; \phi) \partial_\mu V(\phi) \Delta(-z; \overline{\phi}) \partial_\nu V(\phi)) \, . \tag{A11}$$

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