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Author
Liu, Shuo

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Essays on Over-the-Counter Financial Markets

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by

Shuo Liu

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ABSTRACT OF THE DISSERTATION

Essays on Over-the-Counter Financial Markets

by

Shuo Liu

Doctor of Philosophy in Economics

University of California, Los Angeles, 2020

Professor Pierre-Olivier Weill, Chair

This dissertation consists of three chapters that study dealer's endogenous search effort in over-the-counter (OTC) financial markets and its effect on asset's liquidity risk in U.S. corporate bond markets. In Chapter 1, I study dealer's search intensity using a transaction-level data set on U.S. corporate bonds. The main target of this chapter is to test whether dealer's search intensity is endogenously determined by their idiosyncratic states and how search intensity affects market efficiency. Existing literatures commonly do not consider dealer's continuous adjustment of search intensity in search-and-match models and there is no paper using transaction-level data to estimate the dealer-level state-dependent search intensity. In this paper, I propose a search-and-match model with dealers' endogeneous and state-dependent search intensity and estimate it using the TRACE data for the U.S. corporate bond market. I find that: [1] if we rank all dealers by their private valuations for holding the bond, the dealer of the middle-level private valuation will choose the highest level of search intensity, and she works as the "dealer of dealers" to reallocate bond positions from the low-type dealers to the high-type dealers; [2] the estimated model gives us a quantitative evaluation of the inefficiency due to the decentralized market structure. At the average level

across all sub-markets in our sample, the model estimates that dealers' search cost is 0.75% of bond's face value, and there is on average 8.64% of bond positions being misallocated, comparing with a counterfactual frictionless market. In conclusion, the decentralized market structure generates 8.96% welfare loss relative to the frictionless one.

In Chapter 2, I study the correlation between corporate bond's misallocation among dealers and liquidity risk. This chapter bridges the literature on search-and-match models and the literature on explaining the non-default component of corporate bond's credit spread variations. In this paper, I propose a measure of bond's misallocation among dealers. This measure is based on a structural search-and-match model, and is defined as the cross-sectional covariance of dealers' idiosyncratic private valuations for holding the bond and their actual inventory positions in the bond. Using the TRACE data for the U.S. corporate bond market, I construct a panel data which contains yearly series of empirical estimates of bond's misallocation and liquidity risk, and verify that: at the bond level, a higher magnitude of misallocation among the dealers is associated with a higher magnitude of liquidity risk. This finding gives a preliminary market microstructural evidence supporting that: the distribution of market maker's states correlates with the magnitude of asset's liquidity risk.

In Chapter 3, I theoretically study the social optimal policy function of dealer's meeting technology in over-the-counter markets. This chapter contributes to the existing literature by considering the dealer-level state-dependent meeting technology in a random search model and obtaining explicit-form solutions of the social optimal policy functions. In the model, I allow the agents (dealers) to freely adjust their meeting technologies based on two types of idiosyncratic states: asset position and liquidity need. I find that in the social optimal policy functions, there is no intermediation in the sense that no dealer will choose to search simultaneously on both the buy side and sell side of the market. This result applies for a general form of search-cost function.

The dissertation of Shuo Liu is approved.

Lee Ohanian

Zhipeng Liao

Bernard Herskovic

Matthew Saki Bigio Luks

Pierre-Olivier Weill, Committee Chair

University of California, Los Angeles

2020

To my wife Yang Yu and my parents

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VITA

Education	University of California, Los Angeles	Los Angeles, USA
	C.Phil., Economics	2016
	M.A., Economics	2016
	Columbia University	New York City, NY, USA
	M.S., Operations Research	2013
	Nankai University	Tianjin, China
	B.Ec., Financial Engineering (with distinction)	2012
Experience	Federal Reserve Bank of St. Louis	St. Louis, MO, USA
	Dissertation Intern	2019
	International Monetary Fund	Washington D.C., USA
	Ph.D. Research Intern	2018
Award	UCLA Dissertation Year Fellowship, 2019	
	UCLA Graduate Student Fellowship, 2014	
	UCLA Graduate Student Research Assistantship, 2017	
	Honor Pass in Macroeconomics Field Paper, 2016	
	Honor Pass in Econometrics Field Paper, 2016	

CHAPTER 1

Dealers' Search Intensity in U.S. Corporate Bond Markets

1.1 Introduction

In the U.S. corporate bonds markets, dealers manage bond inventories to provide liquidity to customers. Inventory management is facilitated by a decentralized over-the-counter (OTC) interdealer market subject to search frictions: dealers need to locate other dealer-counterparties with whom to trade. To overcome search frictions, dealers need to decide how much time and how many resource to spend in building connections with other dealers or to hire how many traders to staff their trading desks. Empirical papers on trading structures of decentralized financial markets show that dealers exhibit persistently heterogeneous trading frequencies[1]. Does this market structure emerge from dealers' heterogeneous search efforts? How does dealers' choice of search efforts affect market efficiency? Examing these questions will help provide a framework to study how search frictions affect the welfare of market participants through affecting dealers' trading activities and asset liquidity.

In this paper, we propose and estimate a search-based model for the U.S. corporate bond market, extending Hugonnier, Lester, and Weill (2018). There are two aspects of

[1] The recent empirical studies on OTC markets commonly use a conceptual framework inherited from the analysis of static networks to document a "core-periphery" trading structure within the interdealer market. Such market structure is documented by Di Maggio, Kerman, and Song (2017) for the U.S. corporate bond market, Li and Schürhoff (2014) for the U.S. treasury bond market, Hollifield, Neklyudov, and Spatt (2017) for the U.S. securitizations market, and Bech and Atalay (2010) for the fed funds market.

contribution: on the theory side, the innovation is to consider dealers' endogeneous and state-dependent choice of search intensity based on dealers' idiosyncratic states (holding position and private valuation[2] for each bond). Dealers' endogeneous search intensity drives heterogeneous frequency of trade and ultimately determines the impact of search costs on equilibrium liquidity yield spread[3]; on the empirical side, we offer a structural estimation of dealers' search intensities and search model parameters by using the academic version of TRACE data. This dataset includes the information on the identities of the dealer-counterparties in each transaction. Using the estimated search intensities, we validate the theoretical predictions on dealers' heterogeneous roles in the intermediation process. Using the estimated search model, we further quantify the over-the-counter inefficiencies in terms of welfare per capita and bond misallocation in U.S corporate bond markets.

This model has the following features that distinguish it from the other search-based models which also explain dealers' heterogeneous frequency of trade:

First, dealers' heterogeneous search intensities can be identified from transaction-level data. The identification depends on the "separation" of the dealer sector from the customer sector and the assumed matching technology in the model: [1] the separation of the dealer sector from the customer sector allows us to identify the relative level of search intensities across the dealers separately on the buy and sell sides of the market. If moving across dealers on the same side, all the dealers have the same probability of meeting and trading with a customer. This implies that, on either side of the market, dealers' number of completed transactions with customers are proportional to their search intensities. This allows us to identify the relative trend of searching activity across the dealers on the same side; [2] the matching technology allows us to use the realized transactions between two specific dealers,

[2]In the spirit of Duffie, Gârleanu, and Pedersen (2005), market participants have idiosyncratic private valuation type (preference) for the target asset which is modelled as a "consol". By holding the asset, market participants obtain flow utility, the level of which equals the level of their valuation for the asset.

[3]Recent empirical analyses show that the interdealer search frictions drive the large unexplained common factor in bond-level yield spread changes (see Friewald and Nagler (2018), Bao, O'Hara, and Zhou (2016)).

dealers with the maximum and the minimum private valuations, to identify the ratio of the dealer-sector's aggregate buying-search intensity over its aggregate selling-search intensity, because the matching technology assumes for each dealer, the probability of contacting (being contacted by) a trading counterparty is proportional to the counterparty's (the dealer's) search intensity[4]. This ratio is further equal to the ratio of the conditional[5] probability of trading with a customer on the buy side over that on the sell side, for *every* dealer. Then using this identified ratio, we further exclude the effect of "conditional probability of trading with customers" from the difference in realized dealer-customer transactions between the buy and sell sides, and we identify the remaining part as being from the difference between each dealer's buying- and selling- search intensities[6]. Then we finally identify each dealer's total searching activity by summing over her buying- and selling- search intensities.

Second, the model characterizes the shape of the distribution of search intensity among dealers, and connects it with dealers' heterogeneous roles in the intermediation of bonds. My model generates the following two predictions that can both be empirically verified: **[1]** dealers' total search intensity is a hump-shaped function of dealers' private valuation. Within each cross section, dealers of intermediate private valuations choose higher total search intensities and dealers of extreme (either low or high) private valuations choose relatively lower total search intensities. Moreover, the lower total search intensities of the low(high)-type dealers are mainly driven by lower selling(buying) intensities. This prediction implies that the intermediate-type dealers behave as the intermediary and they trade actively on both sides of the market to intermediate the bond from low-type dealers to high-type ones. Empirically verifying this prediction makes my model complement the results of the other

[4]This matching technology is discussed and used by Mortensen (1982), Shimer and Smith (2001), and Üslü (2019)

[5]Here by "conditional", we mean conditional on searching on the buy or sell side of the market, a dealer has a realized trade with a customer instead of another dealer.

[6]Another key assumption in my model is that dealers on either side of the market follow a unique policy function to decide on their search intensity, and dealers change their search intensities whenever they switch from one side of the market to the other.

search-based models; and [**2**]: dealers play heterogeneous roles in the intermediation process by specializating in transactions of different directions. Low-type dealers spend more resources searching on the buy side and specialize in buying the bond from customers and selling to other dealers; high-type dealers spend more resources searching on the sell side and specialize in selling the bond to customers and buying from other dealers. Intermediate-type dealers on average invest in equal amounts of average buying and selling intensities, and contribute most to intermediating the bond from low-type dealers and/or customers to high-type ones.

Finally, state-dependent search intensity allows the model to be used as a framework to study how search frictions affect the welfare of market participants through driving dealers' trading activities. The estimates of model parameters indicate nontrivial market inefficiencies compared with frictionless markets in terms of welfare per capita and bond misallocation.[7] In this paper, we define each market as a combination of bond j and quarter q, and conduct counterfactual analysis as in Gavazza (2016) for each $Market(j, q)$. The main findings include: [**1**] search frictions generate on average an 8.96% welfare loss across all markets, compared to corresponding frictionless markets. For each market, we calculate welfare as the difference between the total utility flow and the total search costs spent by all the dealers. For each counterfactual frictionless market, welfare is equal to the total utility flow but with no bond misallocation; [**2**] for each bond, there is on average 8.64% of total shares being mis-allocated, in the sense of being held by customers and/or dealers with private valuations lower than the marginal investor in a frictionless market; and [**3**] The levels of these two dimensions of inefficiencies exhibit high variations across bonds and over time.

[7]Bond misallocation means the proportion of amount of bond that is being held by agents (either dealers or customers) with valuation types lower than that of the marginal agent.

Related literature

The model with state-dependent search intensity contributes to the theoretical literature initiated by Duffie, Gârleanu, and Pedersen (2005) that uses a search-and-match model to study asset price and liquidity in over-the-counter markets. My model studies fully decentralized market structure by setting a random search environment, which is similar to one strand of the literature developed by Duffie, Gârleanu, and Pedersen (2007), Vayanos and Wang (2007), Vayanos and Weill (2008), Weill (2008), Afonso (2011), Gavazza (2011), Praz (2014), Trejos and Wright (2016), Afonso and Lagos (2015), Atkeson, Eisfeldt, and Weill (2015). Another strand of literature focuses on *semi-decentralized* market structure in which dealers trade in a frictionless centralized interdealer market which allows them to immediately offload inventories through trading with other dealers, as in Weill (2007), Lagos and Rocheteau (2009), Feldhütter (2011), Pagnotta and Philippon (2018a), and Lester, Rocheteau, and Weill (2015).

My model is most related to Hugonnier, Lester, and Weill (2018) in the setting of dealers' heterogeneous valuation types and the incorporation of both dealer and customer sectors. The main difference in my model is that we consider dealers' explicit choice of state-dependent search intensity based on their idiosyncratic states. In Hugonnier, Lester, and Weill (2018), dealers are endowed with homogeneous search intensities.

My model is different from Shen, Wei, and Yan (2018) who is the first to consider the search intensity decision. They discuss the endogenous entry and exit of investors into an over-the-counter market based on investors' idiosyncratic trading needs and a common search cost, which focuses more on the extensive margin of choosing whether to search or not. Once entering the market, investors will adopt the same level of search intensity. My paper instead considers dealers' intensive margin of choosing how fast to search within the market, based on dealers' idiosyncratic trading needs and bond positions. The empirical identification of dealers' heterogeneous search intensities shows that the intensive margin of choosing the search speed is significant within the dealer sector.

There is a contemporaneous strand of literature that also considers heterogeneous search intensity as the main mechanism of endogeneous intermediation under a random search environment: Neklyudov (2012) considers exogeneously heterogeneous search intensity among dealers and two discrete valuation types; Üslü (2019) introduces *ex-ante* heterogeneity in meeting rates into a fully decentralized market model with unrestricted asset holdings; Farboodi, Jarosch, and Shimer (2017b) consider *ex-ante* choice of (distribution of) search intensity at the initial time, after which each agent maintains a fixed level of search intensity even though their private valuations may change, but my model allows dealers to change their search intensities as long as their state variables change.[8]

Moreover, my model relates to papers with alternative, other than search intensity, mechnisms of endogeneous intermediation, including Farboodi (2014) on bank heterogeneous risk exposure, Neklyudov and Sambalaibat (2015) on dealers' serving clients with different liquidity needs, Wang (2016) on the trade-off between trade competition and inventory efficiency, Farboodi, Jarosch, and Menzio (2017a) on dealers' heterogeneous bargaining power, and Bethune, Sultanum, and Trachter (2018) on private information and heterogeneous screening ability, among others.

This paper fills the gap in empirical analysis on heterogeneous search intensity/frequency of trade in the search-based literature. In current papers, heterogeneity in dealers' frequency of trade is mostly motivated by the documented core-periphery structure based on the network approach, as in Li and Schürhoff (2014) for the U.S. treasury bond market, Di Maggio, Kerman, and Song (2017) for the U.S. corporate bond market, Hollifield, Neklyudov, and Spatt (2017) for the U.S. securitizations market, and Bech and Atalay (2010) for the fed funds market. By using transaction-level data on corporate bonds, this paper quantifies this

[8]There exist other related papers that consider other mechnisms, other than heterogeneous search intensity, to generate heterogeneous frequency of trade among market participants. For example, in Farboodi, Jarosch, and Menzio (2017a), dealers' frequency of trade is driven by heterogeneous bargaining power instead of their search intensity.

interdealer core-periphery structure by a search-based approach. From the search perspective, the core dealers are the ones choosing higher total search intensity over both sides of the market and the periphery ones choose relatively lower total search intensity.

Finally, my paper empirically identifies dealers' search intensity in an over-the-counter financial market, based on which dealers' search cost and financial asset misallocation are quantified. In terms of estimation, my paper is most related to Gavazza (2016), who estimates a search-and-bargaining model of a decentralized market by using transaction data on business aircraft, and quantifies the effects of trading frictions and the existence of dealers on asset price, allocation and social welfare. Hendershott, Li, Livdan, and Schürhoff (2017) also do structural estimation for a one-to-many search-and-match model with endogeneous network size and transaction prices, and quantify the effects of client-dealer relations on execution quality in the OTC market for corporate bonds. Other papers that structually estimate search models focus mostly on labor markets, including Eckstein and Wolpin (1990), and Eckstein and Van den Berg (2007), among others.

1.2 Model

The model is an extension of Hugonnier, Lester, and Weill (2018), but with state dependent dealer search intensity.

1.2.1 Environment

Market participants and preferences Market participants include a continuum of customers with measure normalized to 1 and a continuum of dealers with measure $m \leq 1$. Dealers and customers trade a long-lived indivisible bond in fixed supply $s < 1 + m$, and each participant's holding a is assumed to be either zero or one.[9] Market participants are

[9] This $\{0, 1\}$ assumption for bond holding and the indivisibility of bonds determine that the trading volume in each transaction equals one.

all risk neutral and discount future utility flow at rate r. By holding one unit of bond, each participant obtains a utility flow per unit time, which is equal to her idiosyncratic private valuation type.[10]

Customers' private valuation type takes two possible values, either low or high, denoted by $y \in \{y_\ell, y_h\}$ with $y_\ell < y_h$. Each customer draws a new private valuation with intensity α. Private valuation processes are independent across customers and independent of everything else. Customers' new private valuation y' follows a discrete distribution with $P(y' = y_c) = \pi_c$, $c = \ell, h$. In a stationary equilibrium, π_c is equal to the measure of customers with type c.

Dealers' private valuation type $\delta \in [\delta_\ell, \delta_h]$ follows an arbitrary continuous distribution with pdf $f(\delta)$. As in Hugonnier, Lester, and Weill (2018), we assume dealers' private valuations are stable over time.[11]

Search, matching, and trade All market participants randomly search and trade in the market. Each dealer chooses the optimal search intensity $\lambda_a^*(\delta)$ as a function of her current asset position $a \in \{0, 1\}$ and private valuation type $\delta \in [\delta_\ell, \delta_h]$. The flow cost of choosing $\lambda_a^*(\delta)$ is given by $c \times \lambda_a^*(\delta)^2$ with $c > 0$. The value of c captures the market level of search friction. Customers have constant search intensity $\rho > 0$. We assume dealers search to meet and trade with both other dealers and customers, while customers search to meet and trade only with dealers.

We adopt the matching technology discussed by Mortensen (1982), Shimer and Smith (2001), and Üslü (2019). The intensity with which a dealer with search intensity λ contacts or is contacted by another dealer with search intensity λ' equals $m(\lambda, \lambda') = 2 \times \frac{m}{1+m} \times \lambda \frac{\lambda'}{\Lambda}$, where $\frac{m}{1+m}$ is the probability of meeting a dealer conditional on a meeting and Λ is the aggregate level of all dealers' search intensities. Therefore the intensity of meeting a specific

[10]The private valuation can be determined by dealers' idiosyncratic liquidity needs, financing costs, hedging needs, and etc. Within each cross section, dealers can be ranked by their private valuation types.

[11]In the data, dealers' trading behavior (total trading volume, fractions of trading volume with different directions, and centrality, etc) is much more stable than customers'.

trading counterparty is not only proportional to the corresponding physical measure but also proportional to the counterparty's search intensity. Similarly, the intensity with which a dealer with search intensity λ contacts or is contacted by a customer equals $m(\lambda, \rho) = \lambda \times \left(\frac{1}{1+m} + \frac{\rho}{m\Lambda}\right)$.

Once two participants meet, trade only happens when there exist positive gains from trade, and transaction price is determined by Nash bargaining. We assume a dealer's bargaining power with other dealers is equal to $\frac{1}{2}$. Dealers' bargaining power with customers is equal to θ s.t. $0 < \theta < 1$.

1.2.2 Model solutions and stationary equilibrium

Within each group of dealers of the same private valuation type $\delta \in [\delta_\ell, \delta_h]$, there exist dealer-owners and dealer-non-owners. We denote the density of dealer-owners of type δ by $\phi_1(\delta)$ and that of dealer-non-owners of the same type by $\phi_0(\delta)$. Dealer-owners hold one unit of bond and search to sell the bond to other dealers or customers. Once a sale is completed, they become dealer-non-owners and search to buy one unit of bond from other dealers or customers. There are four groups of customers: high- and low-type owners and non-owners. We denote the corresponding measures by μ_{h1}, μ_{h0}, $\mu_{\ell 1}$, $\mu_{\ell 0}$.

A dealer/customer's willingness to pay for the bond is determined by her reservation value, which is equal to the difference between the values of holding and not-holding the bond. Therefore, if $V_a(\delta)$ is the value of a dealer with type $\delta \in [\delta_\ell, \delta_h]$ and holding position $a \in \{0, 1\}$, then the reservation value is $\triangle V(\delta) = V_1(\delta) - V_0(\delta)$. Similarly, the value and reservation value of a customer with type $y \in \{y_\ell, y_h\}$ and holding position $a \in \{0, 1\}$ are denoted by $W_a(y)$ and $\triangle W(y) = W_1(y) - W_0(y)$.

1.2.2.1 Dealers' reservation value

As is standard, $V_a(\delta)$, with $a \in \{0,1\}$ and $\delta \in [\delta_\ell, \delta_h]$, satisfies the HJB equation:

$$
\begin{aligned}
rV_a(\delta) = \max_{\lambda} \quad & \Big\{-c\lambda^2 + a\delta \\
& + \sum_{c\in\{\ell,h\}} \lambda \left(\frac{1}{1+m} + \frac{\rho}{m\Lambda}\right) \mu_{c,1-a}\theta\left((2a-1)(\triangle W(y_c) - \triangle V(\delta))\right)^+ \\
& + \int_{\delta_\ell}^{\delta_h} 2\lambda \frac{m}{1+m} \frac{\lambda^*_{1-a}(\delta')}{\Lambda} \phi_{1-a}(\delta') \frac{\left((2a-1)(\triangle V(\delta') - \triangle V(\delta))\right)^+}{2} d\delta' \Big\} \qquad (1.1)
\end{aligned}
$$

where $x^+ = \max\{0, x\}$, $\lambda^*_0(\delta)$ is the optimal search intensity of a dealer non-owner with type δ, $\lambda^*_1(\delta)$ is the optimal search intensity of a dealer owner with type δ, and Λ is the aggregate level of all dealers' search intensities $\Lambda = \int_{\delta_\ell}^{\delta_h} \lambda^*_0(\delta)\phi_0(\delta)d\delta + \int_{\delta_\ell}^{\delta_h} \lambda^*_1(\delta)\phi_1(\delta)d\delta$.

According to (1.1), by choosing search intensity λ, a dealer of type δ who holds $a = 1$ unit of bond pays flow cost $c\lambda^2$ and enjoys the utility flow δ until one of following two events occur: first, with intensity $2\lambda\frac{m}{1+m}$ the dealer owner contacts or is contacted by a dealer non-owner of higher private valuation type and receives half of the trade surplus; second, with intensity $\lambda\left(\frac{1}{1+m} + \frac{\rho}{m\Lambda}\right)$ the dealer owner contacts or is contacted by a customer non-owner with type y_h and receives θ of the trade surplus. Similar interpretations work for dealer non-owners with holding position $a = 0$ and not enjoying any utility flow.

Given distributions, reservation values, and all other dealers' optimal search intensities, by FOCs of the HJB equation (1.1), the optimal search intensities $\lambda^*_1(\delta)$ and $\lambda^*_0(\delta)$ satisfy the following conditions:

$$
\begin{aligned}
2c\lambda^*_1(\delta) = & \left(\frac{\rho}{m\Lambda} + \frac{1}{1+m}\right) \mu_{h0}\theta(\triangle W(y_h) - \triangle V(\delta)) \\
& + \frac{m}{1+m} \int_{\delta}^{\delta_h} \frac{\lambda^*_0(\delta')\phi_0(\delta')}{\Lambda} (\triangle V(\delta') - \triangle V(\delta)) d\delta' \qquad (1.2)
\end{aligned}
$$

$$
\begin{aligned}
2c\lambda_0^*(\delta) &= \left(\frac{\rho}{m\Lambda} + \frac{1}{1+m}\right)\mu_{\ell 1}\theta(\triangle V(\delta) - \triangle W(y_\ell)) \\
&\quad + \frac{m}{1+m}\int_{\delta_\ell}^{\delta} \frac{\lambda_1^*(\delta')\phi_1(\delta')}{\Lambda}(\triangle V(\delta) - \triangle V(\delta'))d\delta' \qquad (1.3)
\end{aligned}
$$

$\forall \delta \in [\delta_\ell, \delta_h]$. Then the HJB equation for the reservation value function $\triangle V(\delta)$ is:

$$
\begin{aligned}
r\triangle V(\delta) &= -c\lambda_1^{*2}(\delta) + c\lambda_0^{*2}(\delta) + \delta + 2\lambda_1^*(\delta)\frac{m}{1+m}\int_{\delta}^{\delta_h} \frac{\lambda_0^*(\delta')}{\Lambda}\phi_0(\delta')\frac{\triangle V(\delta') - \triangle V(\delta)}{2}d\delta' \\
&+ \lambda_1^*(\delta)\left(\frac{1}{1+m} + \frac{\rho}{m\Lambda}\right)\mu_{h0}\theta(\triangle W(y_h) - \triangle V(\delta)) \\
&- 2\lambda_0^*(\delta)\frac{m}{1+m}\int_{\delta_\ell}^{\delta} \frac{\lambda_1^*(\delta')}{\Lambda}\phi_1(\delta')\frac{\triangle V(\delta) - \triangle V(\delta')}{2}d\delta' \\
&- \lambda_0^*(\delta)\left(\frac{1}{1+m} + \frac{\rho}{m\Lambda}\right)\mu_{\ell 1}\theta(\triangle V(\delta) - \triangle W(y_\ell)) \qquad (1.4)
\end{aligned}
$$

The equations (1.2)-(1.4) presume the monotonicity of reservation value function $\triangle V(\delta)$ and that dealers always want to buy from low-type customers and sell to high-type customers.[12]

1.2.2.2 Customers' reservation value

The reservation value of a customer with private valuation type $y \in \{y_\ell, y_h\}$ satisfies the following HJB equation by similar steps:

$$
r\triangle W(y) = y + \sum_{j\in\{\ell,h\}} \alpha\pi_j\left(\triangle W(y_j) - \triangle W(y)\right)^+ \qquad (1.5)
$$

$$
\begin{aligned}
&+ \left(\frac{\rho}{m\Lambda} + \frac{1}{1+m}\right)(1-\theta)\int_{\delta_\ell}^{\delta_h} \lambda_0^*(\delta)\phi_0(\delta)(\triangle V(\delta) - \triangle W(y))^+ d\delta \\
&- \left(\frac{\rho}{m\Lambda} + \frac{1}{1+m}\right)(1-\theta)\int_{\delta_\ell}^{\delta_h} \lambda_1^*(\delta)\phi_1(\delta)(\triangle W(y) - \triangle V(\delta))^+ d\delta
\end{aligned}
$$

[12]The presumption of monotonicity of $\triangle V(\delta)$ is a guess and will be verified in the proof of Proposition 1. The presumptions that dealers always want to buy from (sell to) low-type (high-type) customers require a parametric restriction, as in Hugonnier, Lester, and Weill (2018). We will verify numerically that these restrictions hold in the numerical examples.

The difference in a customer's reservation value from that of a dealer is: with intensity α, a customer switches her private valuation type. Again, equation (1.5) presumes that dealers always want to buy from low-type customers and sell to high-type customers.

1.2.2.3 Distribution of dealers and customers

The densities of dealer owner $\phi_1(\delta)$ satisfy the following inflow-outflow equations in equilibrium:

$$\frac{2m}{1+m}\phi_1(\delta)\lambda_1^*(\delta)\int_{\delta}^{\delta_h}\frac{\lambda_0^*(\delta')}{\Lambda}\phi_0(\delta')d\delta' + \left(\frac{\rho}{m\Lambda}+\frac{1}{1+m}\right)\phi_1(\delta)\lambda_1^*(\delta)\mu_{h0} \tag{1.6}$$

$$= \frac{2m}{1+m}\phi_0(\delta)\lambda_0^*(\delta)\int_{\delta_\ell}^{\delta}\frac{\lambda_1^*(\delta')}{\Lambda}\phi_1(\delta')d\delta' + \left(\frac{\rho}{m\Lambda}+\frac{1}{1+m}\right)\phi_0(\delta)\lambda_0^*(\delta)\mu_{\ell 1}$$

$\forall\delta \in [\delta_\ell, \delta_h]$. In (1.6), the left-hand side is the outflow due to trading with dealer non-owners with higher types and high-type customer non-owners. The right-hand side is the inflow due to trading with dealer owners with lower types and low-type customer owners. Given the condition $\phi_1(\delta) + \phi_0(\delta) = f(\delta)$, $\forall\delta \in [\delta_\ell, \delta_h]$, the inflow-outflow equation of $\phi_0(\delta)$ is redundant.

The measures of high-type customer non-owner μ_{h0} and low-type customer owner $\mu_{\ell 1}$ satisfy the following inflow-outflow equations:

$$\alpha\mu_{\ell 0}\pi_h = \mu_{h0}\left(\frac{\rho}{m\Lambda}+\frac{1}{1+m}\right)\int_{\delta_\ell}^{\delta_h}\lambda_1^*(\delta)\phi_1(\delta)d\delta + \alpha\mu_{h0}\pi_\ell \tag{1.7}$$

$$\alpha\mu_{h1}\pi_\ell = \mu_{\ell 1}\left(\frac{\rho}{m\Lambda}+\frac{1}{1+m}\right)\int_{\delta_\ell}^{\delta_h}\lambda_0^*(\delta)\phi_0(\delta)d\delta + \alpha\mu_{\ell 1}\pi_h \tag{1.8}$$

In both (1.7) and (1.8), the left-hand side represents the inflow due to type switch and the right-hand side represent the outflow due to switching type and trading with dealers. Given the measures of high-type customer π_h and low-type customer π_ℓ, the inflow-outflow equations of μ_{h1} and $\mu_{\ell 0}$ are also redundant.

Then we define stationary equilibrium as follows:

Definition 1.2.1: *A stationary equilibrium contains* $\triangle V(\delta)$, $\phi_0(\delta)$, $\phi_1(\delta)$, $\lambda_0^*(\delta)$, $\lambda_1^*(\delta)$ *and* $\triangle W(y_\ell)$, $\triangle W(y_h)$, $\mu_{\ell 0}$, $\mu_{\ell 1}$, μ_{h0}, μ_{h1}, *such that*

1. *Given distributions* $\phi_0(\delta)$, $\phi_1(\delta)$, $\mu_{\ell 0}$, $\mu_{\ell 1}$, μ_{h0}, μ_{h1}, *and* $f(\delta), \delta \in [\delta_\ell, \delta_h]$*:*

   – $\triangle V(\delta), \lambda_0^*(\delta), \lambda_1^*(\delta)$ *solve dealers' HJB equation* (1.4) *and first-order conditions for search intensities* (1.2)-(1.3)*;*

   – $\triangle W(y_\ell)$, $\triangle W(y_h)$ *solve customers' HJB equation* (1.5)*.*

2. *Given* $\lambda_0^*(\delta)$, $\lambda_1^*(\delta)$, ρ, *the endogeneous distributions* $\phi_0(\delta)$, $\phi_1(\delta)$, $\mu_{\ell 0}$, $\mu_{\ell 1}$, μ_{h0}, μ_{h1} *satisfy:*

   – $\phi_0(\delta) + \phi_1(\delta) = f(\delta), \forall \delta \in [\delta_\ell, \delta_h]$ *where* $\int_{\delta_\ell}^{\delta_h} f(\delta) d\delta = m$*;*

   – $\mu_{\ell 1} + \mu_{\ell 0} = \pi_\ell$, $\mu_{h1} + \mu_{h0} = \pi_h$ *where* $\pi_\ell + \pi_h = 1$*;*

   – *the inflow-outflow equations* (1.6)-(1.8)*.*

3. *Market clears:*

   – $\int_{\delta_\ell}^{\delta_h} \phi_1(\delta) d\delta + \mu_{\ell 1} + \mu_{h1} = s$

For the existence of such a stationary equilibrium, we consider a continuous and compact mapping based on a system of equations. This system of equations includes dealers' and customers' HJB equations, the first-order conditions for search intensities, the evolution equation of the asset-owner density function $\phi_1(\delta)$, the evolution equation of the high-type customer-nonowner density μ_{h0}, and the evolution equation of the low-type customer-owner $\mu_{\ell 1}$.[13] A proof of existence will not be included in this paper, and a similar proof based on Schauder's fixed-point theorem can be referred in Liu (2018).[14]

[13] All the evolution equations are based on the inflow-outflow equations that at each time the net change in the density of a specific group of agents is obtained by subtracting the outflow from the inflow of that group.

[14] In the numerical algorithm, we obtain the other equilibrium components ϕ_0, μ_{h1}, and $\mu_{\ell 0}$ by equilibrium conditions $\phi_0(\delta) + \phi_1(\delta) = f(\delta)$, $\mu_{\ell 1} + \mu_{\ell 0} = \pi_\ell$, and $\mu_{h1} + \mu_{h0} = \pi_h$. The market clear condition is used for checking whether the model solution converges to a fixed point.

1.2.3 Model predictions

Compared with models with either constant or exogeneous search intensity, this model allows us to study how search intensity varies as a function of private valuation types and also varies between owners and non-owners within each type, since search intensity is dealers' state-dependent choice. The distribution of search intensity determines that of trading volume across dealers, which further implies the role played by dealers in the intermediation process.

1.2.3.1 Dealers' heterogeneous search intensities

We define the total search intensity for a dealer of type $\delta \in [\delta_\ell, \delta_h]$ as follows:

$$\bar{\lambda}(\delta) = \phi_1(\delta) \times \lambda_1^*(\delta) + \phi_0(\delta) \times \lambda_0^*(\delta)$$

where $\phi_1(\delta) \times \lambda_1^*(\delta)$ is interpreated as the selling intensity of a dealer with type δ, and $\phi_0(\delta) \times \lambda_0^*(\delta)$ is the buying intensity of a dealer with type δ. Total search intensity is empirically relevant such that it can be regarded as a measure of a dealer's search behavior over both buy and sell sides at medium frequency. Alternatively, it measures a dealer's instantaneous search behavior with many traders. Formally, we can imagine a dealer is a continuum coalition of traders with identical type but idiosyncratic trading histories. The size of the coalition of type-δ traders is equal to $f(\delta)$, where a fraction $\phi_1(\delta)$ of the traders own the bond and a fraction $\phi_0(\delta)$ of the traders do not own the bond.

As for the distribution of total search intensity $\bar{\lambda}(\delta)$ among the cross section of dealers, we have the following proposition:

Proposition 1: *In any stationary equilibrium with $\triangle W(y_\ell) < \triangle V(\delta) < \triangle W(y_h)$, $\forall \delta \in [\delta_\ell, \delta_h]$:*

1. *$\lambda_1^*(\delta)$ is strictly decreasing in δ, $\lambda_0^*(\delta)$ is strictly increasing in δ;*

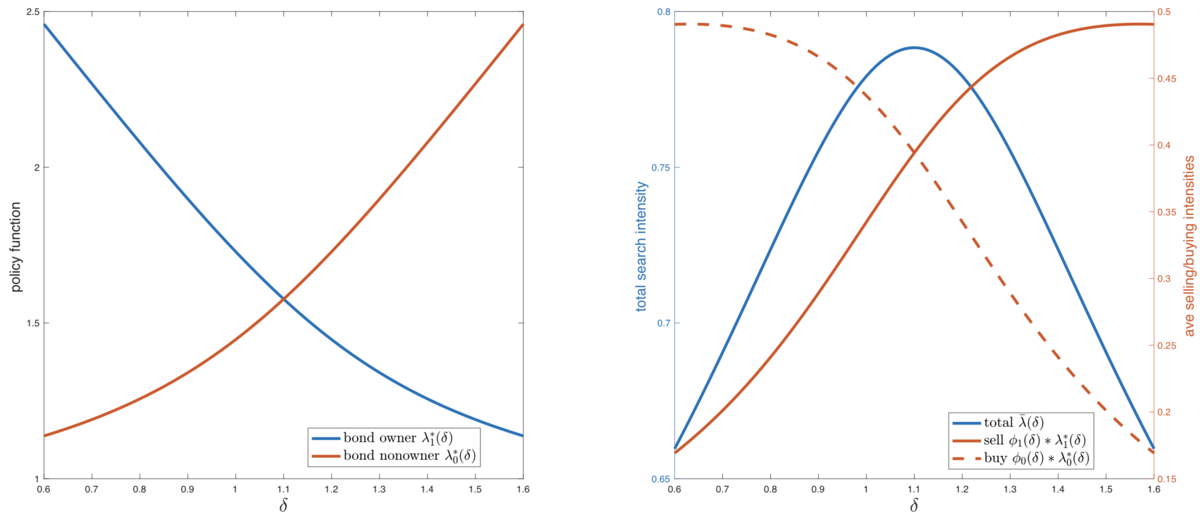

Figure 1.1: Policy functions and total search intensity

($s = \pi_h = 0.5$, $y_\ell = 0.5$, $y_h = 1.7$, $\delta_\ell = 0.6$, $\delta_h = 1.6$, $\alpha = \rho = m = \theta = 0.5$, $c = r = 0.05$)

2. *If the distribution of private valuation $f(\delta)$ is a uniform distribution on $[\delta_\ell, \delta_h]$, there exists a symmetric equilibrium s.t. $\phi_1(\delta) = \phi_0(\delta_\ell + \delta_h - \delta)$ and $\lambda_1^*(\delta) = \lambda_0^*(\delta_\ell + \delta_h - \delta)$, $\forall \delta \in [\delta_\ell, \delta_h]$. In this symmetric equilibrium, $\exists c^* > 0$ s.t. for any $c < c^*$, $\bar{\lambda}(\delta)$ is hump-shaped and attains its maximum at intermediate type $\frac{\delta_\ell + \delta_h}{2}$.*

All proofs are in the Appendix 1.A.1.1. The condition $c < c^*$ implies that the hump-shaped property applies for the not very high level of search friction. When $c > c^*$, this property may fail. Specifically, for a very high level of c, the function of $\bar{\lambda}(\delta)$ may switch to be u-shaped.

For a single dealer owner holding one unit of bond, the lower the dealer's private valuation, the more willing she is to search fast (on the sell side) to sell the bond, due to the higher marginal gains of searching. Similarly, for a single dealer non-owner, the higher the dealer's private valuation, the higher the gains from searching (on the buy side) to buy the bond.

Dealers' total search intensity is hump-shaped with private valuation $\delta \in [\delta_\ell, \delta_h]$, which is driven by a composition effect in a market with a low enough level of search friction. Dealers with extreme valuations (either very high or very low) choose lower total search intensities than the dealer with middle valuation. To understand this finding, consider a dealer with a very high valuation. When this dealer is on the buy side, she chooses very high search intensity because she values the bond more than most other dealers (on the buy side), she has incentive to search quickly to acquire the bond from the dealers on the sell side. Once she acquires the bond, she switches to the sell side and chooses a very low search intensity, since there are very few dealers on the sell side with private valuations that are higher than hers. As a result, in stationary equilibrium, this high-valuation dealer buys very quickly and, is more likely to be on the sell side of the market, with a low search intensity. The key that this high-valuation dealer is able to buy quickly and spends more time on the sell side is the low level of search friction which enables her to quickly acquires the bond. So in terms of total value with densities $\phi_0(\delta)$ and $\phi_1(\delta)$ being as weights, her total search intensity is at a low level. Similar result works for the low-valuation dealer, she sells very quickly and is more likely to be on the buy side, also with a low search intensity, which makes her total search intensity at a low level. By contrast, the dealer with middle valuation has equal weights to be on the buy and the sell sides of the market, with relatively high search intensity on both sides. So considering both sides of the market, she searches more actively than other dealers.

Another way to interprete the hump-shaped property of total search intensity $\bar{\lambda}(\delta)$ is it depends on the gap between the absolute changes in the buying and selling intensities, for per unit change in private valuation type within the cross section of dealers. For example, as type varies from low to high in the lower range, for per unit increase, the decrease in probability of trade happening conditional on a meet for dealer owners $|\frac{d}{d\delta}\left(\int_{\delta}^{\delta_h}\frac{\lambda_0^*(\delta')}{\Lambda}\phi_0(\delta')d\delta'\right)| = \frac{\lambda_0^*(\delta)\phi_0(\delta)}{\Lambda}$ is always larger than the increase in that for dealer non-owners $|\frac{d}{d\delta}\left(\int_{\delta_\ell}^{\delta}\frac{\lambda_1^*(\delta')}{\Lambda}\phi_1(\delta')d\delta'\right)| = \frac{\lambda_1^*(\delta)\phi_1(\delta)}{\Lambda}$, because the buying intensity is always above the selling intensity in the neighbourhood of each $\delta \in [\delta_\ell, \bar{\delta}]$. This drives the amount by which the selling intensity $\lambda_1^*(\delta)\phi_1(\delta)$ curve increases to be larger than that by which the buying intensity $\lambda_0^*(\delta)\phi_0(\delta)$ curve decreases, for per unit increase in δ, to maintain that the total number of selling transactions equals that of buying transactions. This further determines that $\bar{\lambda}(\delta)$ increases with δ in the lower range. Similiar explanations apply for the decreasing of $\bar{\lambda}(\delta)$ in the higher range.

1.2.3.2 Dealers' heterogeneous roles in the intermediation process

In this section, we further characterize the implication of endogeneous and state-dependent search intensity on dealers' heterogeneous roles in the intermediation process.

For each dealer of private valuation type δ, we calculate volumes of four types of transactions: sell-to-customer $V_{S2C}(\delta)$, buy-from-customer $V_{BfC}(\delta)$, sell-to-dealer $V_{S2D}(\delta)$, and buy-from-dealer $V_{BfD}(\delta)$. Figure 1.2 compares the levels of these four types of transactions among dealers. Figure 1.3 shows how the proportion of each type in dealers' *total* trading volume varies with private valuation types. Both examples are under the symmetry restrictions. We also construct the gross, intermediation and net trading volumes, similarly defined in Atkeson, Eisfeldt, and Weill (2015), Hugonnier, Lester, and Weill (2018), and Üslü (2019), separately for the interdealer and dealer-customer markets. Equations of all types of volumes are in Appendix 1.A.3.

The distribution of endogeneous search intensity and the distribution of various types of trading moments jointly imply that: [1] lower-type dealers, on average, invest in higher buying intensity and contribute most to buying the bond from (low-type) customer owners and selling to higher-type dealer non-owners. Therefore, low-type dealers are net buyers in the dealer-customer market and net sellers in the interdealer market; [2] intermediate-type dealers, on average, invest in equal amounts[15] of average buying and selling intensities and contribute most to intermediating the bond from lower-type participants to higher-type ones in both the interdealer and dealer-customer markets. In the interdealer market, the intermediate-type dealers behave as the dealer of dealers and tend to lie in the middle of intermediation chains defined in Hugonnier, Lester, and Weill (2018); Meanwhile in the dealer-customer market, these dealers also directly buy and sell to customers at equal

[15] The result that it is the intermediate-type dealers that invest in equal average buying and selling intensities is based on the symmetry restriction on stationary equilibrium.

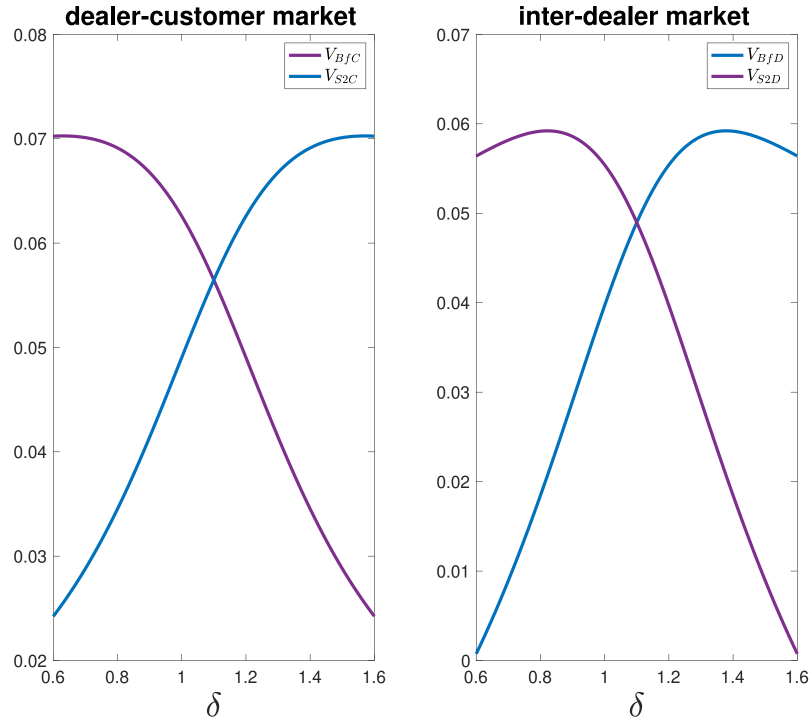

Figure 1.2: Model implied levels of different transaction types
($s = \pi_h = 0.5$, $y_\ell = 0.5$, $y_h = 1.7$, $\delta_\ell = 0.6$, $\delta_h = 1.6$, $\alpha = \rho = m = \theta = 0.5$, $c = r = 0.05$)

amounts[16]; [3] higher-type dealers, which are closer to the high-type customer buyers, on average invest in higher selling intensity and contribute most to selling the bond to (high-type) customers and buying from lower-type dealer owners. Therefore, they are net sellers in the dealer-customer market and net buyers in the interdealer market.

1.3 Identification

Testing the model's predictions creates two key challenges: the first is to identify dealers' private valuations, the second is to measure dealers' search intensities. In this section, by using bond transaction-level data with assigned dealer identities, we construct a measure of

[16]Interpreting by intermediation chains, the intermediate-type dealers also contribute most to constructing chains with only one dealer (themselves) to connect customer sellers and buyers.

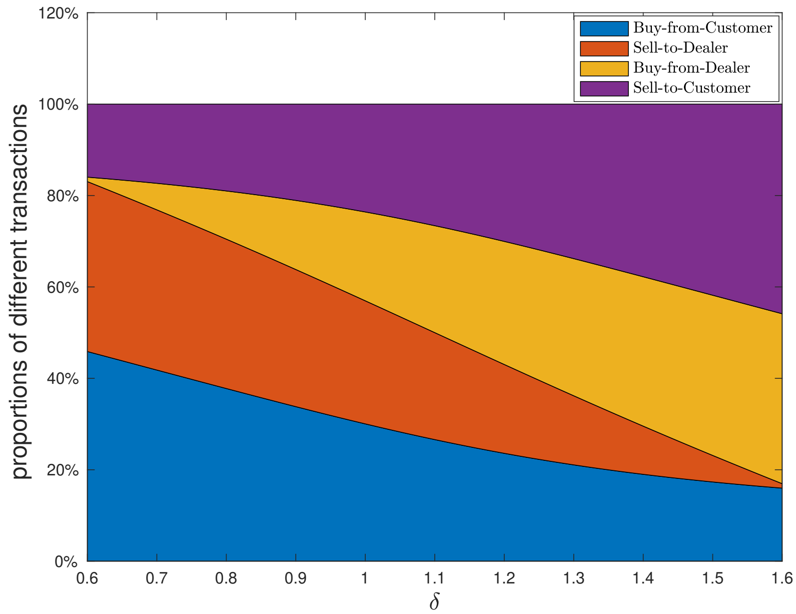

Figure 1.3: Model implied proportions of different transaction types
($s = \pi_h = 0.5$, $y_\ell = 0.5$, $y_h = 1.7$, $\delta_\ell = 0.6$, $\delta_h = 1.6$, $\alpha = \rho = m = \theta = 0.5$, $c = r = 0.05$)

dealers' private valuation based on the Nash bargaining assumption and separately identify dealers' buying- and selling intensities using a group of transaction-related moments.

1.3.1 Data description

We use the Academic Corporate Bond TRACE Data set provided by the Financial Industry Regulatory Authority (FINRA). This data set contains dealers' reports to the Trade Reporting and Compliance Engine (TRACE) which disclose information on all transactions in corporate bonds. One advantage of the data is we can observe identities of the dealers in all transactions. This allows us to track how the bonds are transacted between the dealers, so that we can characterize how actively each dealer trades with the other dealers and/or the outside bond investors.[17]

[17] In the analysis, we define all registered members of FINRA as dealers and all non-registered outside trading counterparties as customers. The main registered firm members of FINRA include broker-dealer firms,

We filtered the data following the procedure in Dick-Nielsen (2014), and we recover the trading counterparties in locked-in and give-up trades[18].We merge the cleaned data with the Mergent Fixed Income Securities Database (FISD) and Wharton Research Data Services (WRDS) Bonds Return Database to obtain bond fundamental characteristics and credit ratings. We construct a monthly panel containing both dealer-wise and bond-wise variables[19].

Following the academic literature using the same data set, we further filtered the data by excluding some "unusual bonds" and some specific types of transactions: [1] We exclude bonds with optional characteristics, such as variable coupon, convertiable, exchangable, and puttable, etc, and we also exclude asset-backed securities and private placed instruments; [2] To faciliate measuring each dealer's search intensity for each single bond, we further drop the inactively traded bonds, defined as those traded in fewer than 25 months throught the whole sample period; [3] Finally, we exclude the "on-the-run" transactions which happened within three months since bonds' offering dates, to only consider secondary market transactions.

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funding portals, and capital acquisition brokers, etc, which are all dealer-like firms. The ID numbers assigned by FINRA to registered members are all virtual IDs. In the data, non-registered trading counterparties are assigned with the ID of "C" by FINRA.

[18]By the user guide of FINRA, a "Give Up" trade report is reported by one FINRA member on behalf of another FINRA member who is the real one to buy or sell the bonds and thus has a reporting responsibility. For such reports, we call the FINRA members, who asked other members to submit reports for them, the true trading counterparties; Locked-in report is a trade report representing both sides of a transaction. FINRA members such as Alternative Trading Systems (ATSs), Electronic Communications Networks (ECNs), and clearing firms have the ability to match buy and sell orders, and therefore to report on behalf of multiple parties using a single trade report submitted to FINRA and indicate that the trade is locked-in. Similarly, we call the FINRA members who submit the buy or sell orders, instead of those clearing platforms, as the true trading counterparties. In the error filters, for these two types of trades, we use the IDs of the true trading counterparties as dealers' IDs and we adjust the reported prices accordingly to account for the agency fees charged by reporting firms and clearing platforms (ATSs, ECNs, and clearing firms).

[19]The raw data is high-frequency data that records the time of each transaction in seconds. In empirical literature using TRACE data to analyze U.S. corporate bond market liquidity, it is common practice to process the data to monthly frequency as corporate bonds are relatively illiquid compared with stock markets, see Bao, Pan, and Wang (2011), Crotty (2013), Friewald and Nagler (2016), and Friewald and Nagler (2018), etc. Specifically, An (2019) documents that dealers' average inventory duration in the U.S. corporate bond market is around three weeks by using the same data, which implies that the average frequency dealers adjust their inventories is around one month.

The final sample ranges from Jan 2005 to Sep 2015, and contains 10760 bonds traded by 3050 dealers. The total outstanding amount of all bonds in our sample is $5.37 trillion. The average bond rating is BBB by the S&P rating categories. Among these bonds, around 84% are investment grade and the remaining ones are high-yield or non-rated.[20] The Panel A in Table 1.1 reports additional bond fundamentals.

The summary statistics in the Panel B of Table 1.1 suggests that we can possibly ignore the different values of transaction size since the standard deviation of trading volume is much lower than its average level. Then we can assume all transactions have the same size as the average level, and use the number of realized transactions to calculate the transaction-related moments to estimate the model.

1.3.2 Identifying dealers' private valuation

In the model with a continuum of dealers, for a dealer with type $\delta \in [\delta_\ell, \delta_h]$, her transaction price with another dealer with type $\delta' \in [\delta_\ell, \delta_h]$ is:

$$P(\delta, \delta') = \frac{\triangle V(\delta) + \triangle V(\delta')}{2}$$

On the sell side of a dealer with type δ, since $\triangle V(\delta') > \triangle V(\delta)$, so the lowest selling price is exactly equal to $\triangle V(\delta)$ for continuum of dealers. Vice versa, on the buy side of a dealer with type δ, since $\triangle V(\delta') < \triangle V(\delta)$, the highest buying price is exactly equal to $\triangle V(\delta)$. Again based on monotonicity of $\triangle V(\delta)$, in data, we construct the following consistent estimator[21]

[20] By the S&P rating categories, investment grade are S&P BBB or higher; and high-yield(junk) are below or equal to S&P BBB-.

[21] In finite samples, on the buy side of each dealer, the maximum purchasing price is a downward biased estimate for the dealer's marginal valuation; on the sell side, the minimum selling price is an upward biased estimate for the dealer's marginal valuation. Taking the average of the sample maximum purchasing price and the sample minimum selling price will make the bias cancel out. In small samples with dealers' unbalanced buy and sell trades, the levels of the upward bias and the downward bias may not be equal. Then to make the bias cancel out completely, the weights assigned on the two extreme prices can be adjusted according to the realized number of buy and sell trades.

Table 1.1: Descriptive Statistics on the Final Sample of TRACE Data (Jan 2005 - Sep 2015)

Panel A: bond fundamental characteristics (10760 bonds)

	Mean	Std. dev.	Q5	Q50	Q95
Offering amount ($million)	458.97	577.99	5.74	300.00	1500.00
Coupon(%)	5.72	1.88	2.50	5.65	9.00
Maturity (years)	11.29	7.61	3.28	9.99	30.03
Amount outstanding($million)	499.35	615.95	6.88	350.00	1750.00
Credit rating	8.53 (BBB)	3.94	3.00 (AA)	8.00 (BBB+)	16.00 (B-)
Age (years)	3.70	2.55	0.48	3.17	8.72
Month turnover (%)	6.92	11.42	0.39	3.57	23.76

Note: [1] For variables "Offering amount ($million)," "Coupon(%)," and "Maturity (years)," we calculate summary statistics based on bond-wise observations; for variables "Amount outstanding($million)," "Credit rating," "Age (years)," and "Month turnover (%)," we calculate summary statistics based on bond-month observations as these variables change over time; [2] Month turnover is calculated using bonds' monthly total trading volumes (par amounts) and dividing by bonds' average amount outstanding for that month.

Panel B: dealer trading activity (3050 dealers)

	All	Sale to customer	Buy from customer	Interdealer
Num of trades (million)	57.62	20.88	15.43	21.31
Total par value($trillion)	27.80	10.57	10.52	6.70
Average par value ($million)	0.48	0.51	0.68	0.31
Average vol (thousand)	482.41	506.25	681.86	314.59
Std. vol (thousand, all bonds)	4.47	5.47	4.46	3.22
Std. vol (thousand, within bond)	1.58	1.62	1.89	0.87

Note: [1] Total par value ($ trillion) is calculated by summing up the par values of all transactions. Average par ($million) is calculated through dividing "Total par value ($trillion)" by "Num trades." [2] Trading volume ("trade vol") is in unit of share of bonds. "Std. vol (thousand, all bonds)" is the standard deviation of all trading volumes (unit: share) by pooling all dealers transactions for all bonds in corresponding markets (customer-dealer or interdealer market). "Std. vol (thousand, within bonds)" is the average standard deviation of trading volumes within each bond. "Std. vol (thousand, within bonds)" measures whether volume per trade has a large dispersion among the cross section of dealers within each bond.

as a proxy for dealers' private valuation type δ:

$$\hat{\delta}^j_{i,t} = \frac{max\{Buy^j_{i,n^{j,B}_{i,t}}\} + min\{Sell^j_{i,n^{j,S}_{i,t}}\}}{2}$$

where $\{Buy^j_{i,n^{j,B}_{i,t}}\}$ ($\{Sell^j_{i,n^{j,S}_{i,t}}\}$) is the collection of all buying (selling) prices by dealer i for bond j within month t and $n^{j,B}_{i,t}$ ($n^{j,S}_{i,t}$) is the correspondinng number of total buying (selling) transactions (including both dealer-customer and interdealer transactions) within month t. [22]

1.3.3 Identifying dealers' search intensity

We identify dealers' heterogeneous search intensities separately on the buy- and sell-side of the market using the following transaction-related moments,[23] where variables with a hat are obtained directly from the data:

1. expected number of selling transactions for each dealer of type $\delta \in [\delta_\ell, \delta_h]$:

$$\widehat{Trade}_S(\delta) = \phi_1(\delta)\lambda_1^*(\delta)\left[\underbrace{\left(\frac{1}{1+m} + \frac{\rho}{m\Lambda}\right)\mu_{h0}}_{\text{trading with customers}} + \underbrace{\frac{2m}{1+m}\int_\delta^{\delta_h}\frac{\lambda_0^*(\delta')}{\Lambda}\phi_0(\delta')d\delta'}_{\text{trading with higher-type dealer non-owners}}\right] \tag{1.9}$$

2. expected number of buying transactions for each dealer of type $\delta \in [\delta_\ell, \delta_h]$:

$$\widehat{Trade}_B(\delta) = \phi_0(\delta)\lambda_0^*(\delta)\left[\underbrace{\left(\frac{1}{1+m} + \frac{\rho}{m\Lambda}\right)\mu_{\ell 1}}_{\text{trading with customers}} + \underbrace{\frac{2m}{1+m}\int_{\delta_\ell}^{\delta}\frac{\lambda_1^*(\delta')}{\Lambda}\phi_1(\delta')d\delta'}_{\text{trading with lower-type dealer owners}}\right] \tag{1.10}$$

[22] In quantitative analysis, we define each market by one bond j and one quarter q. Each dealer i's private valuation for bond j in quarter q is calculated as the weighted average of all monthly private valuations $\hat{\delta}^j_{i,t}$ in quarter q weighted by dealer i's monthly total trading volume in bond j.

[23] We calculate the moments at the bond and month/quarter level.

3. for each selling transaction made by a dealer of type $\delta \in [\delta_\ell, \delta_h]$, the probability that the dealer δ's trading counterparty is another dealer rather than a customer:

$$\widehat{Pr}\left[SellToDealers|Sell\right](\delta) = \frac{\frac{2m}{1+m}\int_{\delta}^{\delta_h} \frac{\lambda_0^*(\delta')}{\Lambda}\phi_0(\delta')d\delta'}{\left(\frac{1}{1+m} + \frac{\rho}{m\Lambda}\right)\mu_{h0} + \frac{2m}{1+m}\int_{\delta}^{\delta_h} \frac{\lambda_0^*(\delta')}{\Lambda}\phi_0(\delta')d\delta'} \tag{1.11}$$

4. for each buying transaction made by a dealer of type $\delta \in [\delta_\ell, \delta_h]$, the probability that the dealer δ's trading counterparty is another dealer rather than a customer:

$$\widehat{Pr}\left[BuyFromDealers|Buy\right](\delta) = \frac{\frac{2m}{1+m}\int_{\delta_\ell}^{\delta} \frac{\lambda_1^*(\delta')}{\Lambda}\phi_1(\delta')d\delta'}{\left(\frac{1}{1+m} + \frac{\rho}{m\Lambda}\right)\mu_{\ell 1} + \frac{2m}{1+m}\int_{\delta_\ell}^{\delta} \frac{\lambda_1^*(\delta')}{\Lambda}\phi_1(\delta')d\delta'} \tag{1.12}$$

where $\widehat{Trade}$ and $\widehat{Pr}$ are the number of transactions and probability of trading with dealers conditional on a trade happening, for each dealer on either the buy- or sell-side of the market.

We show how to identify the total-selling-intensity function $\phi_1(\delta)\lambda_1^*(\delta)$ and total-buying-intensity function $\phi_0(\delta)\lambda_0^*(\delta)$ both up to a constant. For notational simplicity, we define the following measures based on data moments $\widehat{Trade}$ and $\widehat{Pr}$ for each dealer with type $\delta \in [\delta_\ell, \delta_h]$:

$$\widehat{f_1}(\delta) = \left(1 - \widehat{Pr}\left[SellToDealers|Sell\right](\delta)\right) \times \widehat{Trade}_S(\delta) \tag{1.13}$$

$$\widehat{f_2}(\delta) = \left(1 - \widehat{Pr}\left[BuyFromDealers|Buy\right](\delta)\right) \times \widehat{Trade}_B(\delta)$$

$$\widehat{f_3}(\delta) = \widehat{Pr}\left[SellToDealers|Sell\right](\delta) \times \widehat{Trade}_S(\delta)$$

$$\widehat{f_4}(\delta) = \widehat{Pr}\left[BuyFromDealers|Buy\right](\delta) \times \widehat{Trade}_B(\delta) \tag{1.14}$$

where $\widehat{f_1}(\delta)$ is the number of selling-to-customer transactions for a dealer with type δ and $\widehat{f_2}(\delta)$ is the corresponding number of buying-from-customer transactions.

The following Proposition 2 gives the identification results of the selling- and buying intensities up to a same constant relating to the measure of all dealers m. Proof of Proposition

2 is in Appendix 1.A.1.2.

Proposition 2: *Assuming that the selling intensity of the minimum-type dealer-owner equals the buying intensity of the maximum-type dealer non-owner:*

$$\phi_1(\delta_\ell)\lambda_1^*(\delta_\ell) = \phi_0(\delta_h)\lambda_0^*(\delta_h)$$

the following functions of private valuation type and the ratio of aggregate buying intensity versus aggregate selling intensity $\frac{\Lambda_0}{\Lambda_1}$ *are identified by:*

$$\frac{2m}{1+m}\phi_1(\delta)\lambda_1^*(\delta) = \frac{\widehat{f}_3(\delta_\ell) + \widehat{f}_4(\delta_h)}{\widehat{f}_1(\delta_\ell)} \times \widehat{f}_1(\delta)$$

$$\frac{2m}{1+m}\phi_0(\delta)\lambda_0^*(\delta) = \frac{\widehat{f}_3(\delta_\ell) + \widehat{f}_4(\delta_h)}{\widehat{f}_2(\delta_h)} \times \widehat{f}_2(\delta)$$

$$\frac{\Lambda_0}{\Lambda_1} = \frac{\widehat{f}_3(\delta_\ell)}{\widehat{f}_4(\delta_h)}$$

where $\widehat{f}_1(\delta) - \widehat{f}_4(\delta)$ *are defined by* (1.13)-(1.14).

The key to the identification results is: the intensities of trading with customers *conditional* on the choice of selling/buying intensities are constant across dealers of different private valuation types, on either the sell- or buy side of the market. Therefore $\widehat{f}_1(\delta)$ ($\widehat{f}_2(\delta)$) is equal to $\phi_1(\delta)\lambda_1^*(\delta)$ ($\phi_0(\delta)\lambda_0^*(\delta)$) multiplied by a constant that equals the intensity of selling to (buying from) customers. However, the value of the intensity of trading with customers on the sell side is different from that on the buy side. To identify $\phi_1(\delta)\lambda_1^*(\delta)$ and $\phi_0(\delta)\lambda_0^*(\delta)$ up to a same constant, we focus on different types of transactions only for dealers of the minimum and maximum private valuation types. The reason we specifically focus on these two extreme private valuation types, δ_ℓ and δ_h, is that there always exists a positive trading surplus between the δ_ℓ-type dealer-owner (δ_h-type dealer-nonowner) and all other dealer nonowners (owners), which reduces the intensity of trading with dealers to be only proportional to the probability of meeting a dealer $\frac{2m}{1+m}$ and aggregate buying intensity Λ_0 (aggregate selling intensity Λ_1) for the δ_ℓ-type dealer-owner (δ_h-type dealer-nonowner). Additionally, the assumption in Proposition 2 helps to identify the ratio $\frac{\Lambda_0}{\Lambda_1}$ by only using interdealer transactions for δ_ℓ- and δ_h-type dealers. Therefore we can further disentangle the probability of meeting a dealer $\frac{2m}{1+m}$ from either Λ_0 or Λ_1. The final step is to compare the number of dealer-customer transactions with the number of interdealer transactions separately for δ_ℓ- and δ_h-type dealers, which allows us to replace the intensities of trading with customers (different between the two sides) with the same constant $\frac{2m}{1+m}$.

1.3.4 Identifying other parameters

System of equations to identify parameters except for c and θ With identified search intensity functions by the group of transaction-related moments above, we use the following system of equations to identify the model parameters except for c and θ:

$$\frac{\mu_{h0}}{\mu_{\ell 1}} = \frac{\Lambda_0}{\Lambda_1} \tag{1.15}$$

$$\alpha\mu_{\ell 0}\pi_h = \alpha\mu_{h0}\pi_\ell + \mu_{h0}\left(\frac{1}{1+m} + \frac{\rho}{m\Lambda}\right)\int_{\delta_\ell}^{\delta_h} \lambda_1^*(\delta)\phi_1(\delta)d\delta \tag{1.16}$$

$$\alpha\mu_{h1}\pi_\ell = \alpha\mu_{\ell 1}\pi_h + \mu_{\ell 1}\left(\frac{1}{1+m} + \frac{\rho}{m\Lambda}\right)\int_{\delta_\ell}^{\delta_h} \lambda_0^*(\delta)\phi_0(\delta)d\delta \tag{1.17}$$

$$\pi_h = \mu_{h0} + \mu_{h1}$$

$$\pi_\ell = \mu_{\ell 0} + \mu_{\ell 1}$$

$$\pi_h + \pi_\ell = 1$$

$$s = \mu_{h1} + \mu_{\ell 1} + m_1 \tag{1.18}$$

$$\left[\frac{1}{1+m} + \frac{\rho}{m\Lambda}\right] \int_{\delta_\ell}^{\delta_h} \phi_0(\delta)\lambda_0^*(\delta) d\delta = ContactC2D \tag{1.19}$$

$$\left(1 + \frac{(1+m)\rho}{m\Lambda}\right) \frac{\mu_{h0}}{m} = \frac{2\widehat{f}_1(\delta_\ell)}{\widehat{f}_3(\delta_\ell) + \widehat{f}_4(\delta_h)} \tag{1.20}$$

$$\left(1 + \frac{(1+m)\rho}{m\Lambda}\right) \frac{\mu_{\ell 1}}{m} = \frac{2\widehat{f}_2(\delta_h)}{\widehat{f}_3(\delta_\ell) + \widehat{f}_4(\delta_h)} \tag{1.21}$$

where (1.15)-(1.18) are equilibrium conditions, $ContactC2D$ in (1.19) is the intensity with which customers meet dealer-buyers similarly defined in Hugonnier, Lester, and Weill (2018), (1.20)-(1.21) are based on the identification results of Proposition 2, and $\widehat{f}_1(\delta) - \widehat{f}_4(\delta)$ are defined by (1.13)-(1.14). Details of the identification are in Appendix 1.A.2.1.

Calibration The identification of parameters from the system of equations (1.15)-(1.21) depends on calibrating bond supply per capita s, and also targeting on intensity $ContactC2D$ and the fraction of shares of a bond that held by dealers $\frac{m_1}{s}$.

Our calibration works as follows: [1] we calibrate the bond supply per capita s through dividing the amount outstanding variable from the FISD database by the average trading volume across all transactions which is a measure of average trade size, then by the number of customers N. For the value of N, we follow the approach in Hugonnier, Lester, and Weill (2018) assuming half of the household population from the U.S. Census is directly or indirectly investing in financial market in general,[24] and then applying the average ratio of

[24] This assumption in Hugonnier, Lester, and Weill (2018) is also motivated by data from the Survey of Consumer Finance (SCF) and Bricker and et al (2017).

shares of corporate bonds in household holdings of liquidity assets relative to that of municipal bonds throughout 2002 to 2015 by the factbook of the SIFMA. Our calibration of the number of customers per bond is then around $N = 35896$; [2] the intensity with which customers meet dealer-buyers $\left[\frac{1}{1+m^j} + \frac{\rho^j}{m^j \overline{\Lambda}^j}\right] \int_{\delta_\ell^j}^{\delta_h^j} \phi_0(\delta)\lambda_0^*(\delta)d\delta$ is derived from the average trading delay for customers to contact dealers through voice-based OTC trading in corporate bonds, which is calibrated by an approach similar to that of Pagnotta and Philippon (2018a). Since we mainly report estimated model parameters at bond and quarter levels, and also corporate bond market is relatively more liquid than the municipal bond market, we calibrate the average trading delay as one business day for a customer to meet a dealer-buyer. Therefore, $ContactC2D$ equals 60 per quarter; and [3] we calibrate the (average) fraction of shares of a bond held by dealers based on the data on security broker-dealers' holding positions of corporate bonds from Flow of Funds. The average fraction over the sample period 2005-2015 is around 2.82%. Details of calculation are in Appendix 1.A.2.2.

The identification of the measure of dealers per capita m depends on the former identifications of aggregate buying/selling intensities, the calibration of the fraction held by dealers and also the following assumption: [25]

Assumption 1: *Dealer-owners' average selling intensity is equal to dealer-non-owners' average buying intensity:*

$$\int_{\delta_\ell}^{\delta_h} \lambda_1^*(\delta') \frac{\phi_1(\delta')}{m_1} d\delta' = \int_{\delta_\ell}^{\delta_h} \lambda_0^*(\delta') \frac{\phi_0(\delta')}{m_0} d\delta'$$

Therefore m can be identified as $m_1 \times \frac{\widehat{f}_3(\delta_\ell)+\widehat{f}_4(\delta_h)}{\widehat{f}_4(\delta_h)}$.

[25] We also alternatively identify m by using a calibrated number of customers N and the number of dealers that ever provide liquidity within each month, quarter or throughout the whole sample period, since TRACE data allows us to identify dealer counterparties for each completed transaction. The main concern of this identification approach other than relying on Assumption 1 is that the physical size of dealers will highly depend on choosing the length of unit period since the corporate bond market is illiquid relative to equity markets and dealers may not complete any transactions if the length of the unit period is too short.

Price-related moments for estimating c and θ Based on identified search intensities and model parameters above, we use the following group of price-related conditions to estimate search cost coefficient c and dealers' bargaining power to customers θ:

1. average interdealer transaction price:

$$m_1: \quad E(P_{DD}) = \frac{\int_{\delta_\ell}^{\delta_h} \lambda_1^*(\delta)\phi_1(\delta) \int_{\delta}^{\delta_h} \frac{\lambda_0^*(\delta')\phi_0(\delta')}{\Lambda} \frac{(\triangle V(\delta')+\triangle V(\delta))}{2} d\delta' d\delta}{\int_{\delta_\ell}^{\delta_h} \lambda_1^*(\delta)\phi_1(\delta) \int_{\delta}^{\delta_h} \frac{\lambda_0^*(\delta')\phi_0(\delta')}{\Lambda} d\delta' d\delta} \tag{1.22}$$

2. average price of transactions that customers sell to dealers within bond j:

$$m_2: \quad E(P_{CD}) = \int_{\delta_\ell}^{\delta_h} \frac{\lambda_0^*(\delta)\phi_0(\delta)}{\Lambda_0} [(1-\theta)\triangle V(\delta) + \theta \triangle W(y_\ell)] d\delta$$

3. average price of transactions that dealers sell to customers within bond j:

$$m_3: \quad E(P_{DC}) = \int_{\delta_\ell}^{\delta_h} \frac{\lambda_1^*(\delta)\phi_1(\delta)}{\Lambda_1} [(1-\theta)\triangle V(\delta) + \theta \triangle W(y_h)] d\delta \tag{1.23}$$

1.4 Quantitative analysis

1.4.1 Estimation procedure

The estimation contains two main steps. In the first step, we construct B-spline nonparametric estimators[26] of unknown functions $\widehat{f}_1(\delta)$-$\widehat{f}_4(\delta)$ and obtain fitted values. Then we plug in fitted unknown functions back to the group of moment conditions (1.9)-(1.10) and use the generalized method of moments (GMM) to estimate the two following constant terms: $\left(1+\frac{(1+m)\rho}{m\Lambda}\right)\frac{\mu_{h0}}{m}$ and $\left(1+\frac{(1+m)\rho}{m\Lambda}\right)\frac{\mu_{\ell 1}}{m}$, subject to constraints (1.15)-(1.19). In the second step, we follow Hansen (1982) and Gavazza (2016) to use the two-step simulated method

[26]Expressions of estimators are in Appendix 1.A.2.3.

of moments (SMM) approach to estimate the unknown parameters $\psi = \begin{bmatrix} c & \theta \end{bmatrix}^T$ by plugging estimated parameters in the first step into moment conditions (1.22)-(1.23). By similar notations, the two-step estimator takes the form

$$\hat{\psi} = \underset{\psi \in \Psi}{argmin} \frac{(m(\psi) - m_s)'}{m_s} \Omega(\tilde{\psi}) \frac{(m(\psi) - m_s)}{m_s}$$

where $m(\psi) = \begin{bmatrix} m_1(\psi) & m_2(\psi) & m_3(\psi) \end{bmatrix}^T$ is the vector of price-related moments that computed from the model stationary equilibrium solutions which are evaluated at the parameter vector ψ; $m_s = \begin{bmatrix} m_{1,s} & m_{2,s} & m_{3,s} \end{bmatrix}^T$ is the vector of sample moments; Ψ is the parameter space. We firstly use identity matrix as the weight matrix to calculate the preliminary consistent estimate $\tilde{\psi}$ of ψ, then we use the consistent estimate of the inverse of asymptotic variance-covariance matrix $\Omega(\tilde{\psi})$ as the weight matrix in the second step. We minimize the percentage deviation of model-implied moments from sample moments.

Estimates We define each market by one bond j and one quarter q and denote it as $Market(j, q)$. We further restrict that there are at least 25 observations within each market, and each observation is defined by one dealer i in $Market(j, q)$ who trades on both sides of the market. This restriction further shrinks our sample [27] used for estimation to include 6301 bonds and 47634 markets. For each dealer i's state variables in $Market(j, q)$, we construct the dealer's private valuation for bond j by calculating the volume-weighted average of dealer i's monthly private valuations $\hat{\delta}_{i,t}^{j}$ within quarter q.

For each $Market(j, q)$, we estimate the dealers' search intensity functions and model

[27] The reason we choose one quarter as the time period for each market is that for each market, we would like to have a relatively large size of cross section of observations, which allows us to obtain more accurate estimates. The median size of cross section (number of dealers) across all markets is 38 dealers for quarterly data, compared with 12 dealers for monthly data. For robustness check, we re-do all quantitative analysis for markets defined by monthly data, i.e., each market is defined as a bond j and month t, and the results are qualitatively same, except that there is quite a proportion of dealers only trading on one side (only buy or only sell) within one month, which could generate negative estimated search intensities for the direction with no transaction.

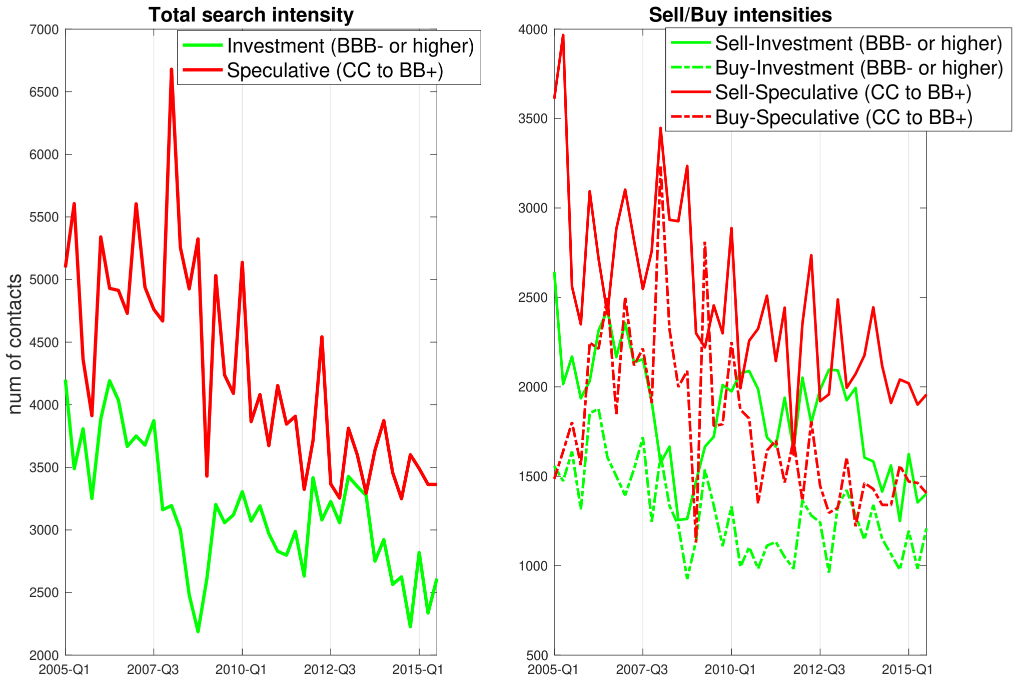

Figure 1.4: Average estimated search intensities across bond grades

parameters using the moments and equilibrium conditions in the previous section. Figure 1.4 shows quarterly average search intensities separately for investment grade and speculative grade bonds by S&P ratings. Search intensities are generally more volatile for lower-rated bonds and manifest a decreasing trend after the financial crisis. Moreover, selling intensities are on average higher than buying intensities, which is consistent with the fact that the estimated measure of high-type customers is lower than that of low-type customers. This requires the whole dealer sector to search more actively on the sell side.

Most estimates of parameters exhibit large variation across markets and are right-skewed. Specifically, the estimate of the measure of high-type customer π_h has a very close distribution to that of bond supply per capita s, which indicates that the marginal investors in most frictionless markets have private valuation types equal or close to that of the high-type customer y_h. This is shown in Table 1.10 and Figure 1.5. The calibration of bond supply per

Table 1.2: Estimated and Calibrated Parameters for 47634 markets (6301 bonds)

	Estimates			
Parameter	Description	Mean	Median	Std. dev.
ρ	customer search intensity (per quarter)	3.23	1.92	3.53
α	customer intensity of switching type (per quarter)	1.46	0.89	1.63
m	measure of dealers	0.006	0.005	0.004
π_h	measure of high-type customers	0.08	0.06	0.06
π_ℓ	measure of low-type customers	0.92	0.94	0.06
c	coefficient of search cost function $c \times \lambda^2$	0.83	0.85	0.17
θ	dealers' bargaining power to customers	0.73	0.65	0.27
	Calibration			
s	bond supply (per capita)	0.09	0.06	0.07

Note: "Mean" and "Median" are calculated over all markets, with each market defined by one bond and one quarter.

customer s is calculated by firstly dividing each bond's amount outstanding by the bond's average trading volume (among all market participants) within each $Market(j, q)$, and then dividing the result further by the calibrated number of customers per bond $N = 35896$, as explained in Section 1.3.4. The estimated model parameters and corresponding model implied components (both on a quarterly basis) in Table 1.2 and Table 1.3 will be used for welfare analysis in Section 1.4.3. Table **??** in Appendix 1.A.2.4 compares the fitted values of model-implied moments with empirical moments calculated from data.

Table 1.3: Model-implied endogeneous components for 47634 markets (6301 bonds)

Measures	Description	Mean	Median	Std. dev.
μ_{h0}	measure of high-type customer-non-owner	0.0078	0.0044	0.0095
$\mu_{\ell 1}$	measure of low-type customer-owner	0.0029	0.0021	0.0024
Λ_1	aggregate dealer-sector selling intensity	15.95	7.00	23.97
Λ_0	aggregate dealer-sector buying intensity	21.78	14.05	22.10
Λ	aggregate dealer-sector total search intensity	38.48	22.83	43.75
m_1	measure of all dealer-owners	0.0025	0.0017	0.0020
m_0	measure of all dealer-non-owners	0.0034	0.0034	0.0043

Note: "Mean" and "Median" are calculated over all markets, with each market defined by one bond and one quarter.

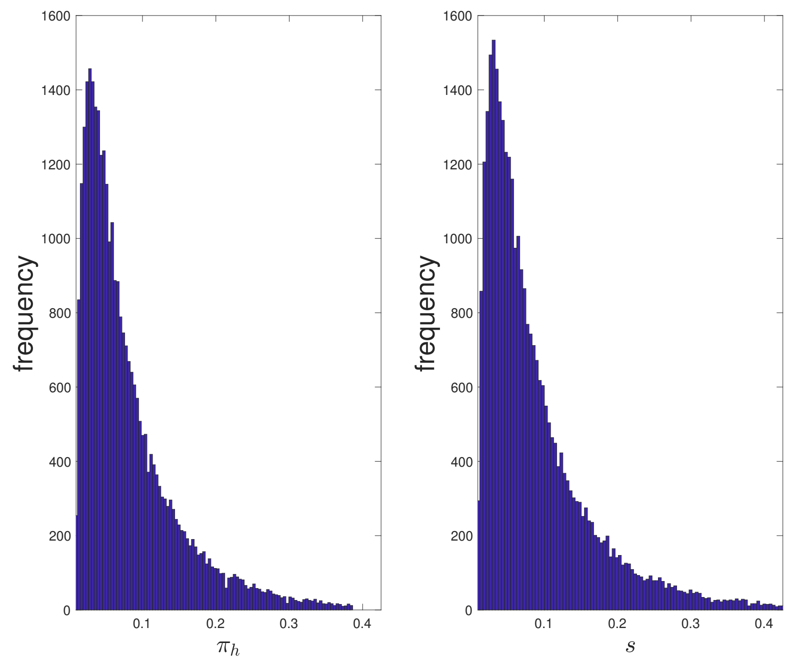

Figure 1.5: Summary of estimate of π_h and bond supply per capita s

1.4.2 Results about search intensity and trading roles

Distribution of search intensity among dealers We give examples of two sub-markets to intuitively show how (identified) search intensities are distributed among dealers, as in Figure 1.6. Market-1 has the maximum number of dealers among all sub-markets, and Market-2 has the median number of dealers. In both markets, the distributions of search intensities are "hump-shaped". This is consistent with the theoretical predictions of the model with endogeneous search efforts. Moreover, the distribution of search intensity deviates from that of dealers' private valuations. This implies that it is more likely dealers choose heterogeneous search intensities.

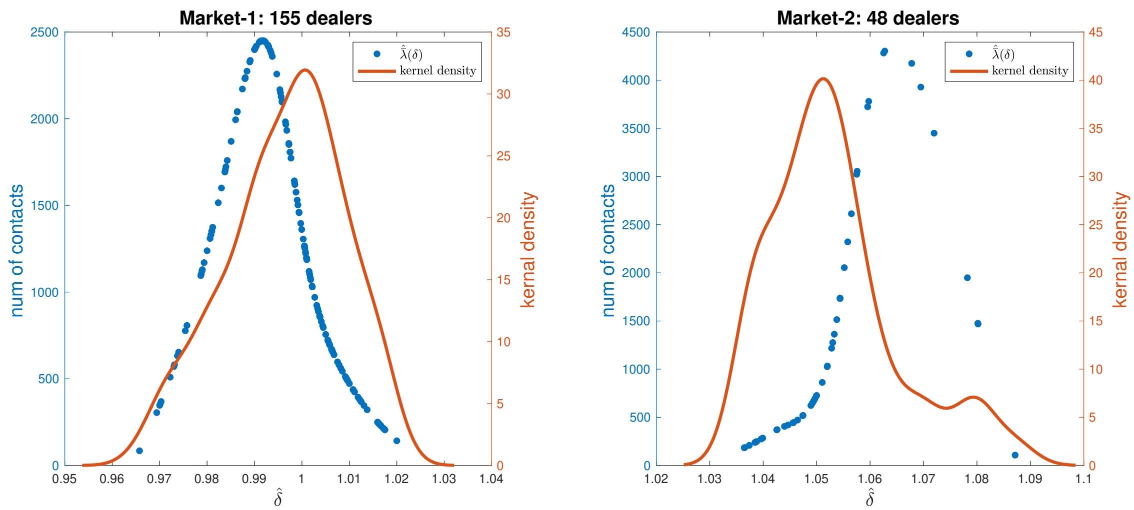

Figure 1.6: Examples of two markets

(Market-1: bond 013817AP6, 2010-Q3, BBB-, terms to maturity 8.6 years, 750k shares; Market-2: bond 803111AM5, 2010-Q3, BBB, terms to maturity 22.2 years, 500k shares.)

Using data on all the markets, we fit search intensities as a quadratic function of dealers' scaled private valuation $\hat{\delta}^j_{S,i,q}$, which is computed through dividing quarterly private valuation $\hat{\delta}^j_{i,q}$ by cross-dealer mean level $\bar{\hat{\delta}}^j_q$ for each $Market(j,q)$ [28]. The quardratic fitting has the

[28]The scaled private valuation is expressed in percentage of the cross-dealer mean level, thus being a measure of the distance of dealers' private valuation to the cross-dealer mean level. The reason we divide

following form:

$$\widehat{\overline{\lambda}}^{j}_{i,q} = \beta_0 + \beta_1 \times \hat{\delta}^{j}_{S,i,q} + \beta_2 \times (\hat{\delta}^{j}_{S,i,q})^2 + \Gamma_1 X^{j}_{q} + \Gamma_2 Y_{i,q} + \tau_i + \phi_j + \eta_y + \epsilon^{j}_{i,q} \tag{1.24}$$

where the vector X^j_q includes as bond-related controls bond j's credit rating, bond j's HHI (Herfindahl index) calculated by using market shares of all dealers to measure whether transactions are concentrated to a specific group of dealers, bond j's previous-three-month turnover, amount outstanding, time to maturity and coupon rate; the vector $Y_{i,q}$ includes as dealer-related controls dealer i's quarterly eigenvector centrality[29] in the interdealer network, dealer i's "HHI for bonds" calculated by using her trade shares of all bonds, and dealer i's "HHI for trade types" calculated by using her trade shares of different trading directions (customer-to-dealer, dealer-to-customer or dealer-to-dealer). These two HHI indices are to measure whether a dealer specializes in a specific bond or trading direction; fixed effects by dealer τ_i, bond ϕ_j and year η_y are also controlled. We also include selling intensity $\widehat{\overline{\lambda}}^{S,j}_{i,q}$ and buying intensity $\widehat{\overline{\lambda}}^{B,j}_{i,q}$ as dependent variables. In Table 1.4, we mainly report estimates of β_1 and β_2. Regression results of (1.24) verify that, within each $Market(j,q)$, total search intensity $\widehat{\overline{\lambda}}^{j}_{i,q}$ is hump-shaped over dealers' private valuation. Specifically, the composition effect implied by the model is consistently verified by the fact that the increasing total search intensity in the lower range of private valuation is driven by faster increase in selling intensity $\widehat{\overline{\lambda}}^{S,j}_{i,q}$ than decrease in buying intensity $\widehat{\overline{\lambda}}^{B,j}_{i,q}$, which is shown in Figure 1.7; similarly, the decreasing total search intensity in the higher range of private valuation is driven by

the raw private valuations by cross-dealer mean level is to control for unobserved factors that drive bonds to be traded at a discount or premium.

[29] "Eigenvector centrality" is one measure of vertices' network centralities. By using all the interdealer transactions, we construct an interdealer network in which we regard each dealer as one "vertice" and each transaction record as a link connecting two vertices. The advantage of using eigenvector centrality is it incorporates not only direct but also indirect trading counterparties for each dealer and thus more accurately measures each dealer's importance in the network by assigning scores to them. The higher the value of eigenvector centrality, the more central and important the dealer is in the interdealer network. We calculate daily values of eigenvector centrality on a rolling basis. Specifically, for each day, we use all the previous-90-day transactions of each dealer to calculate her eigenvector centrality for the current day. Then we calculate the quarterly average by using daily values.

Table 1.4: Distribution of search intensity among dealers (quadratic form)

$Dep^j_{i,q}$	$\widehat{\overline{\lambda}}^{j}_{i,q}$	$\widehat{\overline{\lambda}}^{S,j}_{i,q}$	$\widehat{\overline{\lambda}}^{B,j}_{i,q}$
$\hat{\delta}^j_{S,i,q}$ (%)	1788.54***	968.84***	466.04***
	(36.92)	(35.77)	(28.12)
$(\hat{\delta}^j_{S,i,q})^2$	-8.93***	-4.54***	-2.57***
	(-36.89)	(-33.65)	(-30.97)
# of obs	1,500,047	1,500,047	1,500,047
$Adj\ R^2$	0.1547	0.1241	0.1689
Dealer×Bond×Year FE	YES	YES	YES

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are clustered in dealer#bond#year.

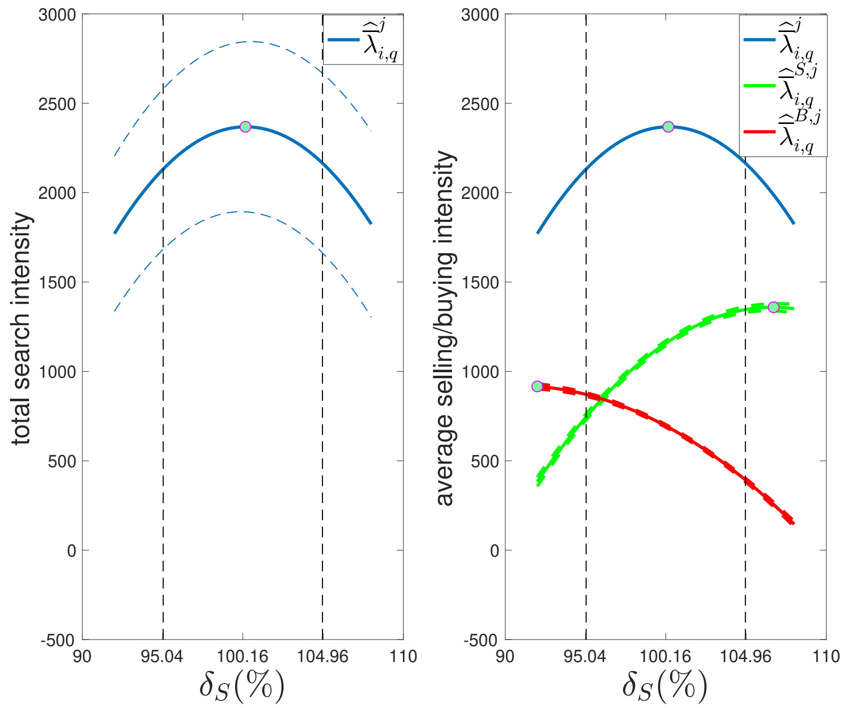

Figure 1.7: Distribution of search intensity among dealers (quadratic form)

faster decrease in buying intensity $\widehat{\overline{\lambda}}^{B,j}_{i,q}$ than increase in selling intensity $\widehat{\overline{\lambda}}^{S,j}_{i,q}$.

Dealers' heterogeneous trading roles We verify the model predictions on dealers' heterogeneous roles in the intermediation process by replacing the dependent variables in (1.24) with the following empirical moments[30]: number of sell-to-customer transactions $V^j_{S2C,i,q}$, number of buy-from-customer transactions $V^j_{BfC,i,q}$, number of sell-to-dealer transactions $V^j_{S2D,i,q}$, and number of buy-from-dealer transactions $V^j_{BfD,i,q}$. As in the theoretical part, we also construct the gross number of transactions and proportion of intermediation transactions, separately for the interdealer and dealer-customer markets. Regression results in Appendix 1.A.3 are also consistent with model predictions.

Table 1.5: Distribution of transactions of different directions

$Dep^j_{i,q}$	$V^j_{S2C,i,q}$	$V^j_{BfC,i,q}$	$V^j_{S2D,i,q}$	$V^j_{BfD,i,q}$
$\hat{\delta}^j_{S,i,q}$ (%)	2.29***	1.02***	2.08***	1.99***
	(28.56)	(21.68)	(21.76)	(28.09)
$(\hat{\delta}^j_{S,i,q})^2$	-0.0111***	-0.0054***	-0.0111***	-0.0094***
	(-27.41)	(-23.06)	(-23.37)	(-26.52)
# of obs	1,500,090	1,500,090	1,500,090	1,500,090
Adj R^2	0.1731	0.2278	0.2193	0.2249
Dealer×Bond×Year FE	YES	YES	YES	YES

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are clustered in dealer#bond#year.

Regression results in Table 1.5 and Figure 1.8 verify that as private valuation ranges from low to high, dealers switch from "buying from customers and selling to dealers" to "buying from dealers and selling to customers." Dealers with private valuations closer to the mean level, by composition effect, on aggregate trade more actively than other dealers in both the dealer-customer and interdealer markets, which is further shown by the curve of the gross number of transactions in Appendix 1.A.3. Moreover, those dealers trade closer

[30] Here we mainly show the results for dependent variables as the *number* of transactions of different directions, which is consistent with the low standard deviation of trading volume in Panel B of Table 1.1, and also consistent with the measures of search intensities which are also identified using the number of transactions. In the Appendix, we show the results for dependent variables as the *volume* of transactions of different directions for the robustness check.

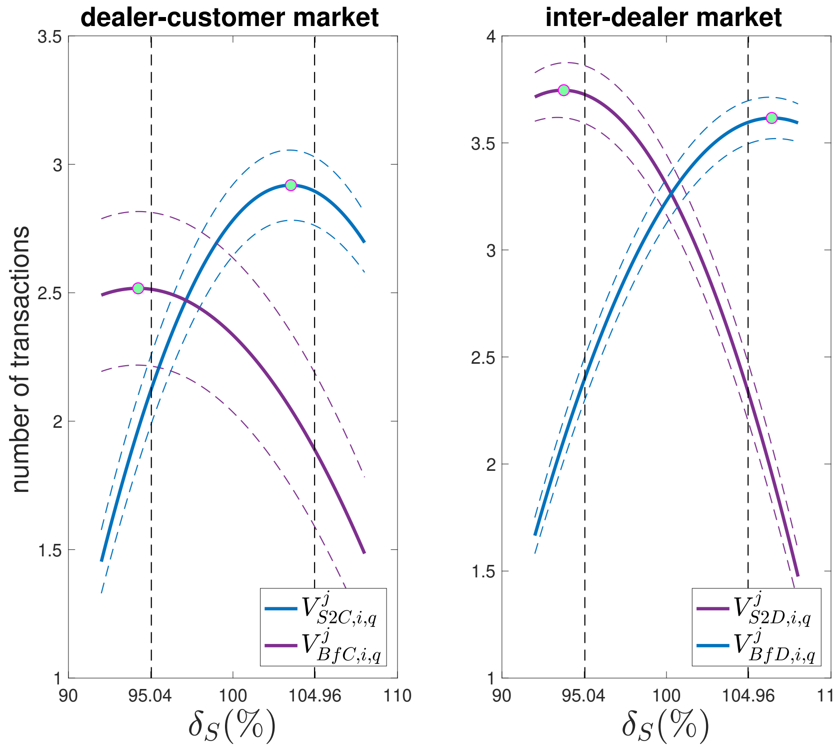

Figure 1.8: Distribution of transactions of different directions (quadratic form)

amounts in the buy- and sell side of the market to intermediate shares of bonds from low-type customers/dealers to high-type ones. We further characterize how dealers' specializing in transactions of different directions correlates with the signed distance[31] of dealers' private valuations relative to the mean level across all dealers. Figure 1.9 shows how the proportions of different types of transactions vary with the signed distance. We also characterize the relationship separately for each subperiod in Appendix 1.A.5.2.

Figure 1.9 indicates that: [1] for dealers of each level of private valuation, the aggregate proportion of selling transactions (either to customer or to other dealers) is close to that

[31] The signed distance is defined as $\frac{\hat{\delta}^j_{i,q} - \hat{\bar{\delta}}^j_q}{|\hat{\delta}^j_{h,q} - \hat{\delta}^j_{l,q}|}$, i.e., the normalized distance in (1.50) without absolute value on the numerator. The signed distance measures not only how far each dealer's private valuation is to the corresponding cross-dealer mean level, but also indicates whether the value is below or above the mean level. The difference in the scaled private valuation is that it also controls for the dispersion of all dealers' private valuations for the same bond and same month.

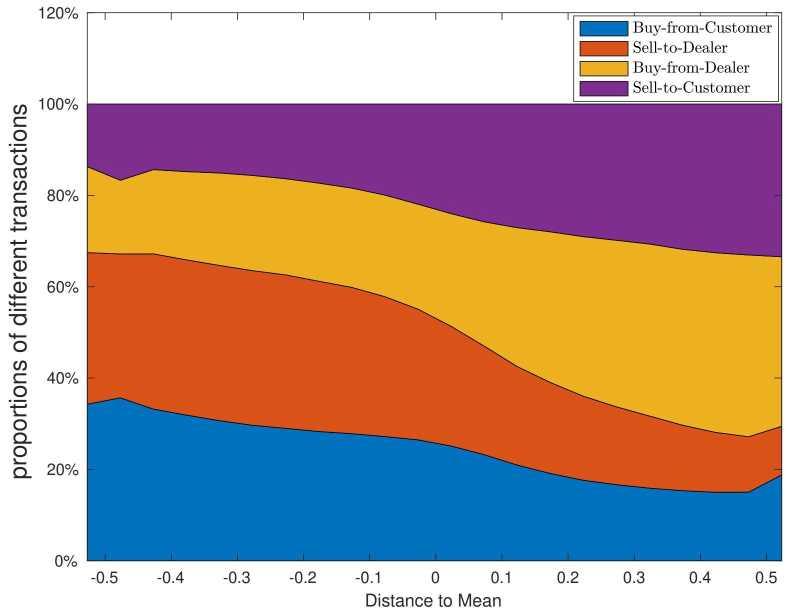

Figure 1.9: Dealer's private valuation and proportions of transactions in different directions (values are averages taken over all bonds and all quarters)

of buying transactions; [2] as private valuation ranges from low to high, on the buy side, dealers switch from "buying *mainly* from customers" to "buying *mainly* from other dealers." Similarly on the sell side, dealers switch from "selling *mainly* to other dealers" to "selling *mainly* to customers"; [3] dealers in the lower range of private valuations take the main roles to buy from low-type customers and sell to higher-type dealers, and similarly, dealers in the higher range of private valuations take the main roles to buy from lower-type dealers and sell to high-type customers; [4] dealers with private valuations close to the mean level trade equally on either of the four types.

1.4.3 Market inefficiencies compared to a frictionless market

In terms of market inefficiency, we conduct a counterfactual analysis to evaluate how search frictions between customers and/or dealers affect bond prices and misallocation, based on estimated model parameters in Section 1.4.1. Here search frictions refers to frictions to contact/locate potential counterparties caused by the decentralized structures in both the dealer-customer market and the interdealer market. The counterfactual scenarios would be Walrasian markets of the same model parameters but with centralized exchanges to which both customers and dealers have frictionless access.

Walrasian price Consider the corresponding Walrasian (frictionless) market in which there is a central exchange where customers and dealers can buy or sell the target bond immediately at equilibrium price P, which is unique within each stationary equilibrium (or $Market(j,q)$ for bond j and quarter q).

As is standard, the Walrasian price $P = \frac{u^*}{r}$ where u^* is the private valuation (utility flow) of the marginal investor which is defined as the asset owner with the lowest private valuation type in a frictionless market:

$$u^* = \begin{cases} y_h & \text{if } s <= \pi_h; \\ \{u \in [\delta_\ell, \delta_h] : \pi_h + \int_{u^*}^{\delta_h} f(\delta) d\delta = s\} & \text{if } \pi_h < s < \pi_h + m; \\ y_\ell & \text{if } s >= \pi_h + m. \end{cases}$$

For most markets, the marginal investor in corresponding frictionless markets are high-type customers or high-type dealers.[32] Based on estimates of parameters in Section 1.4.1, there are 10.2% of markets with $s \leq \pi_h$ and 89.8% with $\pi_h < s < \pi_h + m$. Therefore, in the remaining section, we only consider the two cases of $s \leq \pi_h$ and $\pi_h < s < \pi_h + m$. In the latter case, we denote the marginal-investor private valuation type as δ^*. The derivation

[32] In the TRACE data for the U.S. corporate bond market, since we are not able (or allowed by FINRA) to uncover the true identities of registered members (or dealers), some of the registered members may be actually customers based on their trading behavior, for example, they may mostly trade on one side of the market, but under regulation of FINRA. Since in our sample, we exclude the dealers that only trade on one side of the market, this may lead to an under-estimation of the measure of high-type customers π_h, and thus the proportion of markets with the marginal investor as high-type customers in corresponding frictionless markets.

and estimation of Walrasian price P are in Appendix 1.A.4.

We report the Walrasian prices and average OTC transaction prices $\overline{P}$ (and all other measures of interest) in Table 1.6, separately for markets with the frictionless-market marginal investor as high-type customers ($s \leq \pi_h$) and markets with the marginal investor as dealers ($s > \pi_h$). In the first group of markets, the average Walrasian price is higher than the realized average transaction price, which is consistent with similar counterfactual analysis on over-the-counter markets as in Gavazza (2016). However, in the second group of markets, the average Walrasian price is lower than the realized transaction price. The possible reason is that,[33] for most markets, total search costs on the sell side are higher than those on the buy side, which requires the average transaction price to be higher to compensate for the dealers on the sell side. Additionally, since the private valuation of the marginal investor is lower than that of the high-type customers, for dealers with private valuations in between, there still exist positive gains from intermediation, which makes it possible for the transaction price to be higher than the Walrasian price.

Bond misallocation In this paper, bond misallocation is defined as the proportion of bond amount outstanding being held by low-type customers and/or dealers with private valuation types lower than that of the marginal investor in a corresponding frictionless market.

The ratio of bond misallocation $Rmis$ is correspondingly defined as below:

$$Rmis = \begin{cases} \frac{s-\mu_{h1}}{s} \text{ or } \frac{\mu_{\ell 1}+m_1}{s} & \text{if } s \leq \pi_h; \\ \frac{\mu_{\ell 1}+\int_{\delta_\ell}^{\delta^*} \phi_1(\delta)d\delta}{s} & \text{if } s > \pi_h. \end{cases}$$

The average misallocation ratio over all markets is 8.64% with standard deviation as 2.93%. Moreover, there is a significant different misallocation ratio between markets with the marginal investor as high-type customers (i.e., $s \leq \pi_h$) and markets with the marginal

[33] This is indicated by formula of Walrasian price (1.49) in Appendix 1.A.4.

investor as dealers (i.e., $s > \pi_h$). In the former case, the average misallocation ratio is 17.11%, which is more than twice that of the latter case, 7.96%. Therefore, the effect of search frictions on bond misallocation is larger for markets with the marginal investor as high-type customers. For a robustness check, we also calculated the value of $\frac{\mu_{h0}+\int_{\delta^*}^{\delta_h}\phi_0(\delta)d\delta}{s}$ for the latter case, which is very close to that of $\frac{\mu_{\ell 1}+\int_{\delta_\ell}^{\delta^*}\phi_1(\delta)d\delta}{s}$.[34]

Total flow utility Total flow utility is defined as the summation of all bond-owners' utility flows in stationary equilibrium:

$$Tot\_utility = \int_{\delta_\ell}^{\delta_h} \phi_1(\delta)\delta d\delta + \mu_{h1}y_h + \mu_{\ell 1}y_\ell$$

Total flow utility measures the total benefits of all market participants by holding the bond and positively contributes to the total welfare of each market. In Table 1.6, total flow utility is in percentage of bond face value and further scaled by the number of customers per bond. The gap between Walrasian markets with $s \leq \pi_h$ and $s > \pi_h$ is mainly driven by the gap in the supply of bond per capita s.

Dealers' search costs The average search cost per contact for the whole dealer sector is calculated by dividing total search cost by the aggregate level of all dealers' search intensities Λ.

$$Ave\_SearchCost = c \times \int_{\delta_\ell}^{\delta_h} \left(\frac{\lambda_1^{*2}(\delta)\phi_1(\delta) + \lambda_0^{*2}(\delta)\phi_0(\delta)}{\Lambda} \right) d\delta$$

where $\Lambda = \int_{\delta_\ell}^{\delta_h}(\lambda_1^*(\delta)\phi_1(\delta) + \lambda_0^*(\delta)\phi_0(\delta))d\delta$. In Table 1.6, the total search cost is also scaled by the number of customers per bond.

In Table 1.6, total search costs are larger for OTC markets with $s \leq \pi_h$, which is due to limited supply of bonds, higher bond misallocation, and thus higher gains from interme-

[34] The mean of the difference is 0.02% with standard deviation as 0.4%. To estimate $\phi_1(\delta)$, we assume $\phi_1(\delta)$ is proportionate to dealers' standardized inventory position, subject to $\phi_1(\delta_\ell) = 0$ and $\int_{\delta_\ell}^{\delta_h}\phi_1(\delta)d\delta = m_1$.

diation that motivate more dealers to spend on searching. Over all markets in our sample, the mean level of search cost per contact is 0.75% of face value, with standard deviation as 1.48%. With conjecture that search cost per contact is compensated by bond price (yield), using the approximated relationship[35] between bond price and yield, we calculate that the search cost per contact 0.75% corresponds to approximately 22.7 basis points of bond yield.

Finally, we calculate the welfare (per customer) as the gap between total flow utility and total search costs. Compared with Walrasian markets, OTC markets exhibit a close level of total flow utility (per capita) but nontrivial total search costs (per capita), which on average reduces the welfare by about 8.96% relative to Walrasian markets.

1.5 Conclusion

In this paper, we propose a search-based model for the U.S. corporate bond market with dealers' endogeneous and state-dependent search intensity. The model generates the following implications that can be empirically verified: [1] endogeneous intermediation: dealers with intermediate private valuation type choose higher search intensities than others and intermediate shares of bonds from low-type to high-type dealers. Low-type dealers mainly trade on the buy side to buy bonds from customer-sellers. High-type dealers mainly trade on the sell side to sell bonds to customer-buyers; [2] over-the-counter efficiencies: the estimated model indicates nontrivial market inefficiency that, taking the average over all markets in our sample, dealers' search cost per contact is 0.75% of the bond's face value, which generates an 8.96% welfare loss relative to corresponding frictionless markets, and there is on average 8.64% of bond misallocation. Moreover, the level of market inefficiency exhibits large variation across different bonds and over time.

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[35]The approximated formula is: $ApproxYTM = \frac{C+\frac{F-P}{n}}{\frac{F+P}{2}}$ where C is bond coupon/interest payment, F is face value, P is transaction price, and n is years to maturity. See `https://financeformulas.net/Yield_to_Maturity.html`.

Table 1.6: Comparison with corresponding frictionless markets for 47634 markets (6301 bonds)

	Markets with $s \leq \pi_h$	
	OTC market (mean/std.dev of all markets)	Walrasian market (mean/std.dev of all markets)
$\frac{\mu_{\ell 1}+m_1}{s}$ (%)	17.11%	0
	(6.85%)	
Tot flow utility (% of face value)	4.73%	4.77%
	(4.03%)	(4.04%)
Tot search costs (% of face value)	0.99%	0
	(2.76%)	
Welfare (per customer)	3.74%	4.77%
	(3.71%)	(4.04%)
$\overline{P}$	99.58%	102.34%
(average transaction price)	(8.06%)	(6.95%)
	Markets with $s > \pi_h$	
	OTC market (mean/std.dev of all markets)	Walrasian market (mean/std.dev of all markets)
$\frac{\mu_{\ell 1}+\int_{\delta_\ell}^{\delta^*} \phi_1(\delta)d\delta}{s}$ (%)	7.96%	0
	(2.13%)	
Tot flow utility (% of face value)	10.24%	10.27%
	(8.91%)	(8.96%)
Tot search costs (% of face value)	0.49%	0
	(1.56%)	
Welfare (per customer)	9.75%	10.27%
	(7.68%)	(8.96%)
$\overline{P}$ (% of face value)	103.02%	101.88%
(average transaction price)	(7.08%)	(9.64%)

Appendix 1.A Appendix of Chapter 1

1.A.1 Proof of propositions

1.A.1.1 Proof of proposition 1

In this proof, we assume stationary equilibrium exists.[36]

[1] $\lambda_1^{*\prime}(\delta) < 0$ and $\lambda_0^{*\prime}(\delta) > 0$:

By (1.2)-(1.4), we obtain:

$$r\triangle V(\delta) = \delta + c\lambda_1^{*2}(\delta) - c\lambda_0^{*2}(\delta) \tag{1.25}$$

$$\lambda_1^{*\prime}(\delta) = (-\triangle V'(\delta))\frac{1}{2c}\left[\left(\frac{\rho}{m\Lambda} + \frac{1}{1+m}\right)\mu_{h0}\theta + \frac{m}{1+m}\int_{\delta}^{\delta_h}\frac{\lambda_0^*(\delta')\phi_0(\delta')}{\Lambda}d\delta'\right] \tag{1.26}$$

$$\lambda_0^{*\prime}(\delta) = \triangle V'(\delta)\frac{1}{2c}\left[\left(\frac{\rho}{m\Lambda} + \frac{1}{1+m}\right)\mu_{\ell 1}\theta + \frac{m}{1+m}\int_{\delta_\ell}^{\delta}\frac{\lambda_1^*(\delta')\phi_1(\delta')}{\Lambda}d\delta'\right] \tag{1.27}$$

(1.25)-(1.27) $\implies$

$$\begin{aligned} r\triangle V'(\delta) &= 1 + 2c \times \lambda_1^*(\delta)\lambda_1^{*\prime}(\delta) - 2c \times \lambda_0^*(\delta)\lambda_0^{*\prime}(\delta) \qquad (1.28)\\ &= 1 + (-\triangle V'(\delta))\left(\lambda_1^*(\delta)\left[\left(\frac{\rho}{m\Lambda} + \frac{1}{1+m}\right)\mu_{h0}\theta + \frac{m}{1+m}\int_{\delta}^{\delta_h}\frac{\lambda_0^*(\delta')\phi_0(\delta')}{\Lambda}d\delta'\right]\right.\\ &\qquad \left. + \lambda_0^*(\delta)\left[\left(\frac{\rho}{m\Lambda} + \frac{1}{1+m}\right)\mu_{\ell 1}\theta + \frac{m}{1+m}\int_{\delta_\ell}^{\delta}\frac{\lambda_1^*(\delta')\phi_1(\delta')}{\Lambda}d\delta'\right]\right) \end{aligned}$$

[36] The proof of existance of stationary equilibrium is similar as in Liu (2018) and Hugonnier, Lester, and Weill (2018).

(1.28) $\implies$

$$\begin{aligned}&\triangle V'(\delta)\\&=\frac{1}{r+\lambda_1^*(\delta)X_1(\delta)+\lambda_0^*(\delta)X_0(\delta)}\\&>0\end{aligned} \tag{1.29}$$

where

$$\begin{aligned}X_1(\delta)&=\left(\frac{\rho}{m\Lambda}+\frac{1}{1+m}\right)\mu_{h0}\theta+\frac{m}{1+m}\int_{\delta}^{\delta_h}\frac{\lambda_0^*(\delta')\phi_0(\delta')}{\Lambda}d\delta'\\X_0(\delta)&=\left(\frac{\rho}{m\Lambda}+\frac{1}{1+m}\right)\mu_{\ell 1}\theta+\frac{m}{1+m}\int_{\delta_\ell}^{\delta}\frac{\lambda_1^*(\delta')\phi_1(\delta')}{\Lambda}d\delta'\end{aligned}$$

(1.26)-(1.27) and (1.29) $\implies$

$$\lambda_1^{*\prime}(\delta)<0 \text{ and } \lambda_0^{*\prime}(\delta)>0$$

[**2**] If symmetric restrictions apply and the distribution $f(\delta)$ is uniform distribution, then $\exists c^*>0$, s.t. for any $c<c^*$:

$$\bar{\lambda}'(\delta)>0,\quad \forall\delta\in[\delta_\ell,\frac{\delta_\ell+\delta_h}{2}] \text{ and } \bar{\lambda}'(\delta)<0,\quad \forall\delta\in[\frac{\delta_\ell+\delta_h}{2},\delta_h]$$

Proof: When symmetric restrictions apply and $f(\delta)\equiv\overline{U}$, such that $\overline{U}=\frac{1}{\delta_h-\delta_\ell}$, search intensity policy functions and density functions trivally satisfy the following conditions:

$$\lambda_1^*(\frac{\delta_\ell+\delta_h}{2})=\lambda_0^*(\frac{\delta_\ell+\delta_h}{2}) \tag{1.30}$$

$$\lambda_1^*(\delta)>\lambda_0^*(\delta) \text{ and } \phi_1(\delta)<\phi_0(\delta),\qquad \forall\delta\in[\delta_\ell,\frac{\delta_\ell+\delta_h}{2}) \tag{1.31}$$

$$\lambda_1^*(\delta) < \lambda_0^*(\delta) \text{ and } \phi_1(\delta) > \phi_0(\delta), \qquad \forall \delta \in (\frac{\delta_\ell + \delta_h}{2}, \delta_h]$$

$$\phi_1'(\delta) > 0 \text{ and } \phi_0'(\delta) < 0, \qquad \forall \delta \in [\delta_\ell, \delta_h]$$

$$\lambda_1^{*\prime}(\delta) = -\lambda_0^{*\prime}(\delta_h + \delta_\ell - \delta) \text{ and } \phi_1'(\delta) = -\phi_0'(\delta_h + \delta_\ell - \delta), \qquad \forall \delta \in [\delta_\ell, \delta_h]$$

$$\mu_{h0} = \mu_{\ell 1} \text{ and } \Lambda_0 = \Lambda_1 \tag{1.32}$$

where $\mu_{h0} = \mu_{\ell 1}$ is obtained by inflow-outflow equations (1.7)-(1.8) and also $\Lambda_0 = \Lambda_1$. Then by definition,

$$\begin{aligned}\bar{\lambda}'(\delta) &= \phi_1'(\delta)\lambda_1^*(\delta) + \phi_1(\delta)\lambda_1^{*\prime}(\delta) + \phi_0'(\delta)\lambda_0^*(\delta) + \phi_0(\delta)\lambda_0^{*\prime}(\delta) \\ &= \underbrace{\phi_1'(\delta)(\lambda_1^*(\delta) - \lambda_0^*(\delta))}_{*} \\ &+ \frac{1}{2c}\triangle V'(\delta)\left(\underbrace{(\phi_0(\delta)\mu_{\ell 1} - \phi_1(\delta)\mu_{h0})\left(\frac{\rho}{m\Lambda} + \frac{1}{1+m}\right)\theta}_{**} + (\phi_0(\delta)a(\delta) - \phi_1(\delta)b(\delta))\frac{m}{1+m}\right)\end{aligned} \tag{1.33}$$

where

$$a(\delta) = \int_{\delta_\ell}^{\delta} \frac{\lambda_1^*(\delta')\phi_1(\delta')}{\Lambda} d\delta'$$

$$b(\delta) = \int_{\delta}^{\delta_h} \frac{\lambda_0^*(\delta')\phi_0(\delta')}{\Lambda} d\delta'$$

By (1.30)-(1.32), both terms $*$ and $**$ in (1.33) are positive for $\forall \delta \in [\delta_\ell, \frac{\delta_\ell + \delta_h}{2})$. To characterize the sign of $\phi_0(\delta)a(\delta) - \phi_1(\delta)b(\delta)$ in the range of $[\delta_\ell, \frac{\delta_\ell + \delta_h}{2})$, we use the inflow-outflow equation (1.6) for $\phi_1(\delta)$:

$$\frac{2m}{1+m}\phi_1(\delta)\lambda_1^*(\delta)\int_\delta^{\delta_h}\frac{\lambda_0^*(\delta')}{\Lambda}\phi_0(\delta')d\delta'+\left(\frac{\rho}{m\Lambda}+\frac{1}{1+m}\right)\phi_1(\delta)\lambda_1^*(\delta)\mu_{h0}$$
$$=\frac{2m}{1+m}\phi_0(\delta)\lambda_0^*(\delta)\int_{\delta_\ell}^{\delta}\frac{\lambda_1^*(\delta')}{\Lambda}\phi_1(\delta')d\delta'+\left(\frac{\rho}{m\Lambda}+\frac{1}{1+m}\right)\phi_0(\delta)\lambda_0^*(\delta)\mu_{\ell 1}$$

$\Longrightarrow$

$$\begin{aligned}\frac{\phi_0(\delta)}{\phi_1(\delta)}&=\frac{\lambda_1^*(\delta)}{\lambda_0^*(\delta)}\frac{\frac{2m}{1+m}\int_\delta^{\delta_h}\frac{\lambda_0^*(\delta')}{\Lambda}\phi_0(\delta')d\delta'+\left(\frac{\rho}{m\Lambda}+\frac{1}{1+m}\right)\mu_{h0}}{\frac{2m}{1+m}\int_{\delta_\ell}^{\delta}\frac{\lambda_1^*(\delta')}{\Lambda}\phi_1(\delta')d\delta'+\left(\frac{\rho}{m\Lambda}+\frac{1}{1+m}\right)\mu_{\ell 1}}\\&=\frac{\lambda_1^*(\delta)}{\lambda_0^*(\delta)}\frac{\frac{2m}{1+m}b(\delta)+\left(\frac{\rho}{m\Lambda}+\frac{1}{1+m}\right)\mu_{h0}}{\frac{2m}{1+m}a(\delta)+\left(\frac{\rho}{m\Lambda}+\frac{1}{1+m}\right)\mu_{\ell 1}}\end{aligned}\tag{1.34}$$

By inflow-outflow equations of measures of high-type customer-non-owner and low-type customer-owner (1.7)-(1.8),

$$\mu_{h0}=\frac{\alpha\mu_{\ell 0}\pi_h}{\left(\frac{\rho}{m\Lambda}+\frac{1}{1+m}\right)\Lambda_1+\alpha\pi_\ell}$$

$$\mu_{\ell 1}=\frac{\alpha\mu_{h1}\pi_\ell}{\left(\frac{\rho}{m\Lambda}+\frac{1}{1+m}\right)\Lambda_0+\alpha\pi_h}$$

Since $\mu_{\ell 0}<\pi_\ell\leq 1$, $\mu_{h1}<\pi_h\leq 1$, $\Lambda_0=\Lambda_1=\frac{\Lambda}{2}$, also $\Lambda_1\to\infty$ and $\Lambda_0\to\infty$ as $c\to 0$, we have:

$$\lim_{c\to 0}\mu_{h0}=\lim_{c\to 0}\mu_{\ell 1}=0\tag{1.35}$$

Then by (1.31) and (1.34)-(1.35), we have:

$$\begin{aligned}\lim_{c\to 0}\frac{\phi_0(\delta)}{\phi_1(\delta)}&=\frac{\lambda_1^*(\delta)}{\lambda_0^*(\delta)}\frac{\frac{2m}{1+m}b(\delta)}{\frac{2m}{1+m}a(\delta)}\\&>\frac{b(\delta)}{a(\delta)},\qquad\forall\delta\in[\delta_\ell,\frac{\delta_\ell+\delta_h}{2})\end{aligned}$$

$\Longrightarrow$

$$\underset{c\to 0}{lim}\left(\phi_0(\delta)a(\delta)-\phi_1(\delta)b(\delta)\right)>0, \qquad \forall\delta\in[\delta_\ell,\frac{\delta_\ell+\delta_h}{2})$$

Then in (1.33), we have for $\forall\delta\in[\delta_\ell,\frac{\delta_\ell+\delta_h}{2})$:

$$\underset{c\to 0}{lim}\bar{\lambda}'(\delta)>\frac{1}{2c}\triangle V'(\delta)\frac{m}{1+m}\underset{c\to 0}{lim}\left(\phi_0(\delta)a(\delta)-\phi_1(\delta)b(\delta)\right)>0$$

by $\triangle V'(\delta)>0$, and both terms $*$ and $**$ in (1.33) are positive for $\forall\delta\in[\delta_\ell,\frac{\delta_\ell+\delta_h}{2})$. Finally, by symmetry conditions,

$$\bar{\lambda}'(\delta)=-\bar{\lambda}'(\delta_h+\delta_\ell-\delta) \qquad \forall\delta\in[\delta_\ell,\delta_h]$$

$\Longrightarrow \forall\delta\in(\frac{\delta_\ell+\delta_h}{2},\delta_h]$:

$$\underset{c\to 0}{lim}\bar{\lambda}'(\delta)<0$$

$\square$

1.A.1.2 Proof of proposition 2

We use P to denote the *intensity* of trading at different directions *conditional* on the choice of selling/buying intensity for each individual dealer, and use Pr to denote the conditional probability that trading counterparty is a dealer or customer *conditional* on that transaction of specific direction happens. Notations with hat refer to identified data moments. Specifically, for each dealer with type $\delta\in[\delta_\ell,\delta_h]$, we denote:

$$P(S2D|\delta)=\frac{2m}{1+m}\int_{\delta}^{\delta_h}\frac{\lambda_0^*(\delta')}{\Lambda}\phi_0(\delta')d\delta'$$

$$P(BfD|\delta)=\frac{2m}{1+m}\int_{\delta_\ell}^{\delta}\frac{\lambda_1^*(\delta')}{\Lambda}\phi_1(\delta')d\delta'$$

$$P(S2C) = \left(\frac{1}{1+m} + \frac{\rho}{m\Lambda}\right)\mu_{h0}$$

$$P(BfC) = \left(\frac{1}{1+m} + \frac{\rho}{m\Lambda}\right)\mu_{\ell 1}$$

In (1.9)-(1.10), replace $P(S2D|\delta)$ and $P(BfD|\delta)$ by (1.11)-(1.12), the following two functions are identified:

$$\widehat{f}_1(\delta) = \Big(1 - \widehat{Pr}\left[SellToDealers|Sell\right](\delta)\Big) \times \widehat{Trade}_S(\delta) = \phi_1(\delta)\lambda_1^*(\delta) \times P(S2C) \quad (1.36)$$

$$\widehat{f}_2(\delta) = \Big(1 - \widehat{Pr}\left[BuyFromDealers|Buy\right](\delta)\Big) \times \widehat{Trade}_B(\delta) = \phi_0(\delta)\lambda_0^*(\delta) \times P(BfC) \quad (1.37)$$

In (1.9)-(1.10), replace $P(S2C)$ and $P(BfC)$ with $P(S2D|\delta)$ and $P(BfD|\delta)$ by (1.11)-(1.12), the following two functions are identified:

$$\widehat{f}_3(\delta) = \widehat{Pr}\left[SellToDealers|Sell\right](\delta) \times \widehat{Trade}_S(\delta) = \frac{2m}{1+m}\phi_1(\delta)\lambda_1^*(\delta)\int_{\delta}^{\delta_h}\frac{\lambda_0^*(\delta')}{\Lambda}\phi_0(\delta')d\delta' \quad (1.38)$$

$$\widehat{f}_4(\delta) = \widehat{Pr}\left[BuyFromDealers|Buy\right](\delta) \times \widehat{Trade}_B(\delta) = \frac{2m}{1+m}\phi_0(\delta)\lambda_0^*(\delta)\int_{\delta_\ell}^{\delta}\frac{\lambda_1^*(\delta')}{\Lambda}\phi_1(\delta')d\delta' \quad (1.39)$$

Plug in δ_ℓ in (1.38) and plug in δ_h in (1.39), obtain:

$$\widehat{f}_3(\delta_\ell) = \frac{2m}{1+m}\phi_1(\delta_\ell)\lambda_1^*(\delta_\ell)\frac{\Lambda_0}{\Lambda} \quad (1.40)$$

$$\widehat{f}_4(\delta_h) = \frac{2m}{1+m}\phi_0(\delta_h)\lambda_0^*(\delta_h)\frac{\Lambda_1}{\Lambda} \quad (1.41)$$

by assumption $\phi_1(\delta_\ell)\lambda_1^*(\delta_\ell) = \phi_0(\delta_h)\lambda_0^*(\delta_h)$, we obtain:

$$\frac{\widehat{f}_3(\delta_\ell)}{\widehat{f}_4(\delta_h)} = \frac{\Lambda_0}{\Lambda_1} \quad , \quad \frac{\frac{\widehat{f}_3(\delta_\ell)}{\widehat{f}_4(\delta_h)}}{1+\frac{\widehat{f}_3(\delta_\ell)}{\widehat{f}_4(\delta_h)}} = \frac{\Lambda_0}{\Lambda} \quad , \quad \frac{1}{1+\frac{\widehat{f}_3(\delta_\ell)}{\widehat{f}_4(\delta_h)}} = \frac{\Lambda_1}{\Lambda} \tag{1.42}$$

then plug (1.42) into (1.40)-(1.41), we obtain:

$$\widehat{f}_3(\delta_\ell) + \widehat{f}_4(\delta_h) = \frac{2m}{1+m}\phi_1(\delta_\ell)\lambda_1^*(\delta_\ell) = \frac{2m}{1+m}\phi_0(\delta_h)\lambda_0^*(\delta_h) \tag{1.43}$$

Plug in δ_ℓ in (1.36) and plug in δ_h in (1.37), obtain:

$$\widehat{f}_1(\delta_\ell) = \phi_1(\delta_\ell)\lambda_1^*(\delta_\ell) \times P(S2C)$$

$$\widehat{f}_2(\delta_h) = \phi_0(\delta_h)\lambda_0^*(\delta_h) \times P(BfC) \tag{1.44}$$

Since $\frac{2m}{1+m}$, $P(S2C)$ and $P(BfC)$ are all constants (within each market), by (1.43)-(1.44), we obtain:

$$\frac{\widehat{f}_1(\delta_\ell)}{\widehat{f}_3(\delta_\ell) + \widehat{f}_4(\delta_h)} = \frac{P(S2C)}{\frac{2m}{1+m}} \tag{1.45}$$

$$\frac{\widehat{f}_2(\delta_h)}{\widehat{f}_3(\delta_\ell) + \widehat{f}_4(\delta_h)} = \frac{P(BfC)}{\frac{2m}{1+m}} \tag{1.46}$$

The (1.45)-(1.46) allow us to replace trading intensities $P(S2C)$ and $P(BfC)$ in (1.36)-(1.37), obtain:

$$\widehat{f}_1(\delta) = \phi_1(\delta)\lambda_1^*(\delta) \times \frac{\widehat{f}_1(\delta_\ell)}{\widehat{f}_3(\delta_\ell) + \widehat{f}_4(\delta_h)} \times \frac{2m}{1+m}$$

$$\widehat{f}_2(\delta) = \phi_0(\delta)\lambda_0^*(\delta) \times \frac{\widehat{f}_2(\delta_h)}{\widehat{f}_3(\delta_\ell) + \widehat{f}_4(\delta_h)} \times \frac{2m}{1+m}$$

then $\frac{2m}{1+m}\phi_1(\delta)\lambda_1^*(\delta)$ and $\frac{2m}{1+m}\phi_0(\delta)\lambda_0^*(\delta)$ can be identified as:

$$\frac{2m}{1+m}\phi_1(\delta)\lambda_1^*(\delta) = \frac{\widehat{f}_3(\delta_\ell) + \widehat{f}_4(\delta_h)}{\widehat{f}_1(\delta_\ell)} \times \widehat{f}_1(\delta)$$

$$\frac{2m}{1+m}\phi_0(\delta)\lambda_0^*(\delta) = \frac{\widehat{f}_3(\delta_\ell) + \widehat{f}_4(\delta_h)}{\widehat{f}_2(\delta_h)} \times \widehat{f}_2(\delta)$$

and then $\frac{2m\Lambda_1}{1+m}$ and $\frac{2m\Lambda_0}{1+m}$ can also be identified by calculating the full integral over $[\delta_\ell, \delta_h]$. □

1.A.2 Identification and estimation

1.A.2.1 Identy parameters except for c and θ

Based on the second group of restrictions as (1.15)-(1.19), rewrite (1.45) as:

$$\frac{\widehat{f}_1(\delta_\ell)}{\widehat{f}_3(\delta_\ell) + \widehat{f}_4(\delta_h)} = \frac{P(S2C)}{\frac{2m}{1+m}} = \frac{1+m}{2m}\left(\frac{1}{1+m} + \frac{\rho}{m\overline{\Lambda}}\right)\mu_{h0} \tag{1.47}$$

By (1.47), (1.15), (1.19) and identification of $\frac{2m\Lambda_0}{1+m}$, we identify:

$$\mu_{h0} = \frac{\frac{\widehat{f}_1(\delta_\ell)}{\widehat{f}_3(\delta_\ell)+\widehat{f}_4(\delta_h)}}{ContactC2D} \times \frac{2m\Lambda_0}{1+m} \tag{1.48}$$

then plug (1.48) in (1.15), we identify:

$$\mu_{\ell 1} = \frac{\frac{\frac{\widehat{f}_1(\delta_\ell)}{\widehat{f}_3(\delta_\ell)+\widehat{f}_4(\delta_h)}}{ContactC2D} \times \frac{2m\Lambda_0}{1+m}}{\frac{\Lambda_0}{\Lambda_1}}$$

After identifying μ_{h0} and $\mu_{\ell 1}$, by (1.18), we identify:

$$\mu_{h1} = s - m_1 - \mu_{\ell 1} = s - m_1 - \frac{\frac{\frac{\widehat{f}_1(\delta_\ell)}{\widehat{f}_3(\delta_\ell)+\widehat{f}_4(\delta_h)}}{ContactC2D} \times \frac{2m\Lambda_0}{1+m}}{\frac{\Lambda_0}{\Lambda_1}}$$

then π_h, π_l and $\mu_{\ell 0}$ are identified as:

$$\begin{aligned}\pi_h = \mu_{h0} + \mu_{h1} &= \frac{\frac{\widehat{f}_1(\delta_\ell)}{\widehat{f}_3(\delta_\ell)+\widehat{f}_4(\delta_h)}}{ContactC2D} \times \frac{2m\Lambda_0}{1+m} + s - m_1 - \mu_{\ell 1} \\ &= s - m_1 + \frac{\frac{\widehat{f}_1(\delta_\ell)}{\widehat{f}_3(\delta_\ell)+\widehat{f}_4(\delta_h)}}{ContactC2D} \times \frac{2m\Lambda_0}{1+m} \times \frac{\Lambda_0 - \Lambda_1}{\Lambda_0}\end{aligned}$$

$$\pi_l = 1 - s + m_1 + \frac{\frac{\widehat{f}_1(\delta_\ell)}{\widehat{f}_3(\delta_\ell)+\widehat{f}_4(\delta_h)}}{ContactC2D} \times \frac{2m\Lambda_0}{1+m} \times \frac{\Lambda_1 - \Lambda_0}{\Lambda_0}$$

$$\begin{aligned}\mu_{\ell 0} = \pi_l - \mu_{\ell 1} &= 1 - s + m_1 + \frac{\frac{\widehat{f}_1(\delta_\ell)}{\widehat{f}_3(\delta_\ell)+\widehat{f}_4(\delta_h)}}{ContactC2D} \times \frac{2m\Lambda_0}{1+m} \times \frac{\Lambda_1 - \Lambda_0}{\Lambda_0} - \frac{\frac{\frac{\widehat{f}_1(\delta_\ell)}{\widehat{f}_3(\delta_\ell)+\widehat{f}_4(\delta_h)}}{ContactC2D} \times \frac{2m\Lambda_0}{1+m}}{\frac{\Lambda_0}{\Lambda_1}} \\ &= 1 - s + m_1 - \frac{\frac{\widehat{f}_1(\delta_\ell)}{\widehat{f}_3(\delta_\ell)+\widehat{f}_4(\delta_h)}}{ContactC2D} \times \frac{2m\Lambda_0}{1+m}\end{aligned}$$

By assumption $\int_{\delta_\ell}^{\delta_h} \lambda_1^*(\delta')\frac{\phi_1(\delta')}{m_1}d\delta' = \int_{\delta_\ell}^{\delta_h} \lambda_0^*(\delta')\frac{\phi_0(\delta')}{m_0}d\delta'$, m_0 is trivally identified as:

$$m_0 = m_1 \times \frac{\Lambda_0}{\Lambda_1}$$

and also

$$m = m_0 + m_1 = m_1 \times \frac{\Lambda}{\Lambda_1}$$

Given identification of $\frac{2m}{1+m}\phi_1(\delta)\lambda_1^*(\delta)$, $\frac{2m}{1+m}\phi_0(\delta)\lambda_0^*(\delta)$, $\frac{2m\Lambda_0}{1+m}$ and $\frac{2m\Lambda_1}{1+m}$: Λ_0, Λ_1, Λ and two functions $\phi_1(\delta)\lambda_1^*(\delta)$, $\phi_0(\delta)\lambda_0^*(\delta)$ are then also identified.

Plug identified m, Λ and μ_{h0} back into (1.47), we can identify ρ:

$$\rho = \left(\frac{ContactC2D}{\Lambda_0} - \frac{1}{1 + m_1 \times \frac{\Lambda}{\Lambda_1}} \right) \times m_1 \times \frac{\Lambda}{\Lambda_1} \times \Lambda$$

the condition (1.47) comes from (1.45), and ρ is overidentified by conditions (1.45) and (1.46), so we can estimate ρ separately in each of the two conditions and then take average.

Finally, we turn to conditions (1.16) and (1.17) (these two conditions are same and can be reduced to one condition) to identify α.

1.A.2.2 Calibration of average fraction of positions held by broker-dealer sector

Table 1.7: Holding positions on corporate and foreign bonds ($billion) by different sectors

Year	Ratio of Broker-dealer (%)	Broker-dealer (asset+liability)	Total assets
2005	4.59	378.1	8236.1
2006	4.57	424.3	9275.2
2007	4.20	447.6	10653.5
2008	2.17	220.9	10167.1
2009	2.36	247.3	10477.4
2010	3.06	319.2	10441.2
2011	1.87	196.3	10502.5
2012	2.09	230.2	10995.8
2013	2.17	241.3	11134.7
2014	2.06	239.4	11600
2015	1.91	223.4	11722.2
Average	2.82	288	10473.3

Sources: Flow of Funds L.213, Federal Reserve Board.

1.A.2.3 B-spline nonparametric estimator of unknown functions

The B-spline nonparametric estimator of unknown functions $\widehat{f}_i(\delta)$, $i = 1, 2, 3, 4.$ in (1.36)-(1.39) have the following forms:

$$\widehat{f}_i(\delta) = \sum_{k=1}^{5} \beta_{k,i}^{j} B_k^j(\delta)$$

where $B_k^j(\delta), k = 1, 2, 3, 4, 5$ are B-spline basis functions of dealers' type δ for bond j, for a natural cubic spline with degree of freedom equals 5 (4 intercept knots).

1.A.2.4 Model fits

For model fitting results, please refer to **?** and **?**.

1.A.3 Dealers' gross, net and intermediation trading volumes

1.A.3.1 Model prediction

In both the dealer-customer market DC and the interdealer market DD, for each dealer of private valuation type δ, $G_M(\delta)$ denotes the gross trading volume over both sides of the interdealer market, $N_M(\delta)$ denotes the net trading volume which equals to the absolute level of difference in the amount of bond between buying and selling transactions by dealer δ with other dealers, and $I_M(\delta)$ denotes the intermediation volume which equals gross trading volume minus net trading volume and it represents the magnitude of intermediation service that dealer δ provides to all other dealers in market $M \in \{DC, DD\}$.

$$G_{DD}(\delta) = \frac{2m}{1+m}\phi_1(\delta)\lambda_1^*(\delta)\int_{\delta}^{\delta_h} \frac{\lambda_0^*(\delta')}{\Lambda}\phi_0(\delta')d\delta' + \frac{2m}{1+m}\phi_0(\delta)\lambda_0^*(\delta)\int_{\delta_\ell}^{\delta} \frac{\lambda_1^*(\delta')}{\Lambda}\phi_1(\delta')d\delta'$$

$$N_{DD}(\delta) = \left|\frac{2m}{1+m}\phi_1(\delta)\lambda_1^*(\delta)\int_{\delta}^{\delta_h} \frac{\lambda_0^*(\delta')}{\Lambda}\phi_0(\delta')d\delta' - \frac{2m}{1+m}\phi_0(\delta)\lambda_0^*(\delta)\int_{\delta_\ell}^{\delta} \frac{\lambda_1^*(\delta')}{\Lambda}\phi_1(\delta')d\delta'\right|$$

Table 1.8: Model Fit (Part A) for 47634 markets (6301 bonds)

Theoretical Moment (1)	Empirical Value (2)	Fitted Value (3)
$\int_{\delta_\ell^j}^{\delta_h^j} \frac{\phi_1^j(\delta)\lambda_1^{j*}(\delta)}{m^j} \left[\left(\frac{1}{1+m^j} + \frac{\rho^j}{m^j\overline{\Lambda}^j}\right) \mu_{h0}^j\right] d\delta$	2.401 (1.255)	2.364 (4.643)
$\int_{\delta_\ell^j}^{\delta_h^j} \frac{\phi_0^j(\delta)\lambda_0^{j*}(\delta)}{m^j} \left[\left(\frac{1}{1+m^j} + \frac{\rho^j}{m^j\overline{\Lambda}^j}\right) \mu_{\ell 1}^j\right] d\delta$	1.914 (0.985)	1.877 (3.443)
$\frac{\int_{\delta_\ell^j}^{\delta_h^j} \lambda_1^{j*}(\delta)\phi_1^j(\delta) \int_{\delta}^{\delta_h^j} \frac{\lambda_0^{j*}(\delta')\phi_0^j(\delta')}{\Lambda^j} \frac{(\triangle V^j(\delta')+\triangle V^j(\delta))}{2} d\delta' d\delta}{\int_{\delta_\ell^j}^{\delta_h^j} \lambda_1^{j*}(\delta)\phi_1^j(\delta) \int_{\delta}^{\delta_h^j} \frac{\lambda_0^{j*}(\delta')\phi_0^j(\delta')}{\Lambda^j} d\delta' d\delta}$	96.986 (10.121)	93.264 (15.291)
$\int_{\delta_\ell^j}^{\delta_h^j} \frac{\lambda_0^{j*}(\delta)\phi_0^j(\delta)}{\Lambda_0^j}[(1-\theta^j)\triangle V^j(\delta) + \theta^j \triangle W^j(y_\ell^j)]d\delta$	96.475 (10.087)	84.014 (14.891)
$\int_{\delta_\ell^j}^{\delta_h^j} \frac{\lambda_1^{j*}(\delta)\phi_1^j(\delta)}{\Lambda_1^j}[(1-\theta^j)\triangle V^j(\delta) + \theta^j \triangle W^j(y_h^j)]d\delta$	97.689 (9.521)	94.054 (14.899)

Note: The expectation operator is over all dealers within each bond j. For both Empirical Value and Fitted Value, the mean level and standard deviation across all markets are reported.

$$
\begin{aligned}
I_{DD}(\delta) &= G_{DD}(\delta) - N_{DD}(\delta) \\
&= \frac{4m}{1+m} \times \min\left\{\phi_1(\delta)\lambda_1^*(\delta) \int_{\delta}^{\delta_h} \frac{\lambda_0^*(\delta')}{\Lambda}\phi_0(\delta')d\delta', \phi_0(\delta)\lambda_0^*(\delta) \int_{\delta_\ell}^{\delta} \frac{\lambda_1^*(\delta')}{\Lambda}\phi_1(\delta')d\delta'\right\}
\end{aligned}
$$

Both gross and intermediation trading volumes manifest the ability/incentive of dealers to re-

Table 1.9: Model Fit (Part B) Mapping between Theoretical and Empirical Moments

Theoretical Moment (1)	Empirical Moment (2)
$\int_{\delta_\ell^j}^{\delta_h^j} \frac{\phi_1^j(\delta)\lambda_1^{j*}(\delta)}{m^j}\left[\left(\frac{1}{1+m^j}+\frac{\rho^j}{m^j\overline{\Lambda}^j}\right)\mu_{h0}^j\right]d\delta$	$E\left(Trade_S^j \times Pr^j\left[SellToDealers\mid Sell\right]\right)$
$\int_{\delta_\ell^j}^{\delta_h^j} \frac{\phi_0^j(\delta)\lambda_0^{j*}(\delta)}{m^j}\left[\left(\frac{1}{1+m^j}+\frac{\rho^j}{m^j\overline{\Lambda}^j}\right)\mu_{\ell 1}^j\right]d\delta$	$E\left(Trade_B^j \times Pr^j\left[BuyFromDealers\mid Buy\right]\right)$
$\dfrac{\int_{\delta_\ell^j}^{\delta_h^j}\lambda_1^{j*}(\delta)\phi_1^j(\delta)\int_{\delta}^{\delta_h^j}\frac{\lambda_0^{j*}(\delta')\phi_0^j(\delta')}{\Lambda^j}\frac{(\triangle V^j(\delta')+\triangle V^j(\delta))}{2}d\delta' d\delta}{\int_{\delta_\ell^j}^{\delta_h^j}\lambda_1^{j*}(\delta)\phi_1^j(\delta)\int_{\delta}^{\delta_h^j}\frac{\lambda_0^{j*}(\delta')\phi_0^j(\delta')}{\Lambda^j}d\delta' d\delta}$	$E(P_{DD}^j)$
$\int_{\delta_\ell^j}^{\delta_h^j}\frac{\lambda_0^{j*}(\delta)\phi_0^j(\delta)}{\Lambda_0^j}[(1-\theta^j)\triangle V^j(\delta)+\theta^j\triangle W^j(y_\ell^j)]d\delta$	$E(P_{CD}^j)$
$\int_{\delta_\ell^j}^{\delta_h^j}\frac{\lambda_1^{j*}(\delta)\phi_1^j(\delta)}{\Lambda_1^j}[(1-\theta^j)\triangle V^j(\delta)+\theta^j\triangle W^j(y_h^j)]d\delta$	$E(P_{DC}^j)$

Note: The expectation operator is over all dealers within each bond j. For both Empirical Value and Fitted Value, the mean level and standard deviation across all markets are reported.

allocate the bond within the interdealer market. Figure 1.10 shows that dealers of intermediate private valuation type search and trade most actively on both sides of the market, and thus provide the highest level of intermediation service compared with other dealers.

Table 1.10: Summary of estimate of π_h and bond supply per capita s

Variable	Mean	Std dev	Min	Q25	Q50	Q75	Max
π_h	0.0814	0.0660	0.0095	0.0347	0.0603	0.1061	0.3845
s	0.0882	0.0725	0.0101	0.0369	0.0650	0.1152	0.4251

Note: π_h is the measure of high-type customers; s is the bond supply (per capita).

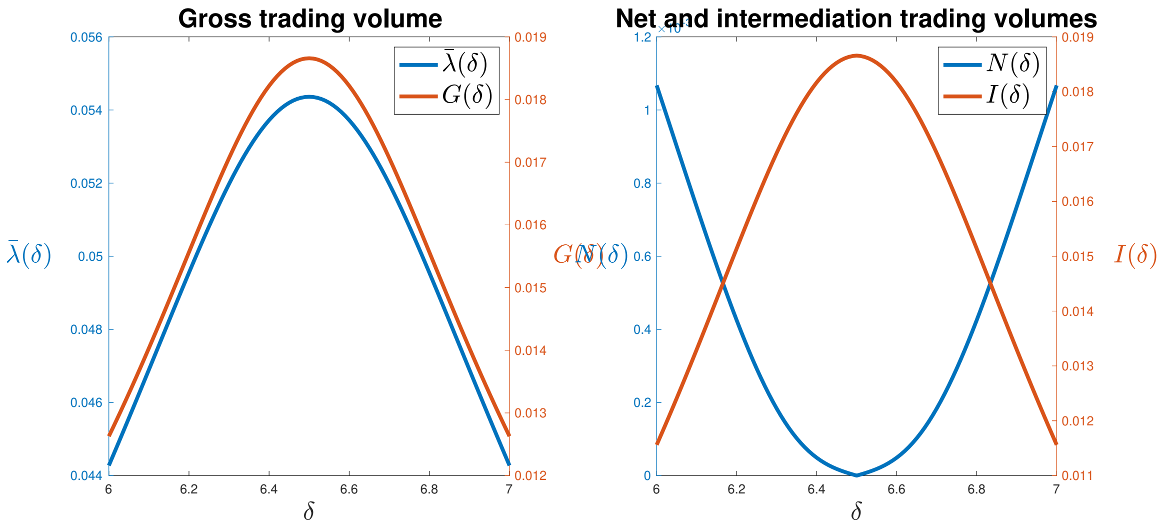

Figure 1.10: Distribution of dealers' trading moments within the interdealer market

1.A.3.2 Empirical validation

Table 1.11: Distribution of trading moments among dealers (quadratic form)

$Dep^j_{i,t}$	$Gross\_Vol^j_{i,t}$	$Inter\_Vol^j_{i,t}(\%)$	$Net\_Vol^j_{i,t}(\%)$	$Std\_Inv^j_{i,t}$
$\hat{\delta}^j_{S,i,t}$ (%)	23906.38***	0.5849***	-0.5849***	0.0056***
	(3.73)	(3.53)	(-3.53)	(2.31)
$(\hat{\delta}^j_{S,i,t})^2$	-116.1845***	-0.0029***	0.0029***	-4.51e-05***
	(-3.76)	(-3.66)	(3.66)	(-3.34)
# of obs	11,606,655	11,606,655	11,606,655	5,964,679
Adj R^2	0.0779	0.4031	0.4031	0.1073
Dealer×Bond×Year FE	YES	YES	YES	YES

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are clustered in dealer#bond#year.

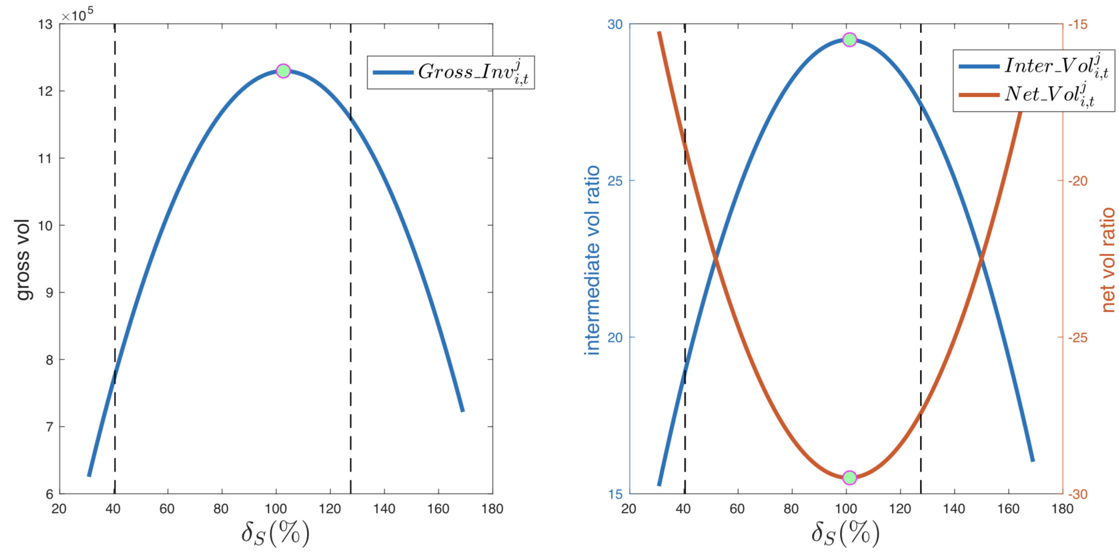

Figure 1.11: Distribution of trading moments among dealers (quadratic form)

Regression results in Table 1.11 verifie that the slopes of gross volume, intermediation volume and volatility of inventory positions are consistent with that of total search intensity. The implication for endogeneous intermediation is: dealers with higher total search intensities trade actively on both sides of the market to intermediate bonds from low-type dealers to high-type ones, through maintaining more volatile inventory positions and lower net trading volume.

1.A.4 Market inefficiency

As in the earlier version of Hugonnier, Lester, and Weill (2018), the objectives of both customers and dealers are to choose an asset-holding process $a_t \in \{0, 1\}$, subject to their utility-type process, to maximize the following objective function:

$$
\begin{aligned}
E_{0,u}\left[\int_0^\infty u_t a_t e^{-rt} dt - \int_0^\infty P e^{-rt} da_t\right] &= E_{0,u}\left[\int_0^\infty u_t a_t e^{-rt} dt - P e^{-rt} a_t\Big|_0^\infty + \int_0^\infty P a_t e^{-rt}(-r) dt\right] \\
&= E_{0,u}\left[\int_0^\infty u_t a_t e^{-rt} dt + P a_0 + \int_0^\infty P a_t e^{-rt}(-r) dt\right] \\
&= P a_0 + E_{0,u}\left[\int_0^\infty a_t e^{-rt}(u_t - rP) dt\right]
\end{aligned}
$$

where u_t denotes customers' or dealers' utility-type process, $y_t \in \{y_\ell, y_h\}$ or $\delta_t \in [\delta_\ell, \delta_h]$, the expectation operator $E_{0,u}$ is conditional on initial time and initial utiltiy type u, a_0 is initial holding position, and $da_t \in \{1, -1\}$.

The optimal asset-holding process for both customers and dealers are:

$$a_t = \begin{cases} 1 & \text{if } u_t > rP; \\ 1 \text{ or } 0 & \text{if } u_t = rP; \\ 0 & \text{if } u_t < rP. \end{cases}$$

By market clear condition, there exists $\exists! u^* \in [\delta_\ell, \delta_h] \cup \{y_\ell, y_h\}$ s.t. $P = \frac{u^*}{r}$ and u^* has the expression:

$$u^* = \begin{cases} y_h & \text{if } s <= \pi_h; \\ inf\{u \in [\delta_\ell, \delta_h] : \pi_h + m - \int_{\delta_\ell}^{u} f(\delta) d\delta \le s\} & \text{if } \pi_h < s < \pi_h + m; \\ y_\ell & \text{if } s >= \pi_h + m. \end{cases}$$

Based on estimation results, we calculate the corresponding Walrasian prices based on reservation values of the marginal investors in OTC markets which solve (1.2)-(1.5):

$$\text{For } \pi_h < s < \pi_h + m: \qquad P = \frac{\delta^*}{r} = \triangle V(\delta^*) - \frac{c\lambda_1^{*2}(\delta^*) - c\lambda_0^{*2}(\delta^*)}{r} \tag{1.49}$$

$$\text{For } s \le \pi_h: \qquad P = \frac{y_h}{r} = \triangle W(y_h) \left(1 + \frac{\alpha\pi_\ell + \left(\frac{\rho}{m\Lambda} + \frac{1}{1+m}\right)(1-\theta)\Lambda_1}{r} \right)$$

$$- \frac{\alpha\pi_\ell \triangle W(y_\ell) + \int_{\delta_\ell}^{\delta_h} \left(\frac{\rho}{m\Lambda} + \frac{1}{1+m}\right)(1-\theta)\lambda_1^*(\delta)\phi_1(\delta)\triangle V(\delta) d\delta}{r}$$

1.A.5 Robustness check

1.A.5.1 Distribution of search intensity among dealers by monthly data

Fit search intensities as quardratic function of private valuation The quardratic fitting using monthly data has the following form:

$$\widehat{\overline{\lambda}}_{i,t}^{j} = \beta_0 + \beta_1 \times \hat{\delta}_{S,i,t}^{j} + \beta_2 \times (\hat{\delta}_{S,i,t}^{j})^2 + \Gamma_1 X_t^j + \Gamma_2 Y_{i,t} + \tau_i + \phi_j + \eta_y + \epsilon_{i,t}^{j}$$

where the controls are similarly defined except for monthly basis. In Table 1.12, the results verify that total search intensity is still a hump-shaped function of dealers' (scaled) private valuation. In the lower range of private valuation, the upwards slope of total search intensity is driven by the increase in average selling intensity; and in the higher range, the downwards slope of total search intensity is driven by the decrease in average buying intensity.

Use measure of distance to mean-level private valuation as control For each dealer i, we calculate $\frac{|\hat{\delta}_{i,t}^{j} - \hat{\overline{\delta}}_t^j|}{|\delta_{h,t}^{\hat{j}} - \delta_{l,t}^{\hat{j}}|}$ as the measure of distance of dealer i's private valuation type $\hat{\delta}_{i,t}^{j}$ to the cross-dealer mean level $\hat{\overline{\delta}}_t^j$ among the cross section of dealers within each bond j, which is further normalized by the difference between the maximum and minimum private valuations. To verify the model prediction about the shape of total search intensity among each cross section of dealers, we run the following regression:

$$\widehat{\overline{\lambda}}_{i,t}^{j} = \beta_0 + \beta_1 \times \frac{|\hat{\delta}_{i,t}^{j} - \hat{\overline{\delta}}_t^j|}{|\delta_{h,t}^{\hat{j}} - \delta_{l,t}^{\hat{j}}|} + \Gamma_1 X_t^j + \Gamma_2 Y_{i,t} + \tau_i + \phi_j + \eta_t + \epsilon_{i,t}^{j} \quad (1.50)$$

where all the other controls are same as (1.24).

Regression results are in Table 1.13. where we also include $Trade_{i,t}^{j}$, $\widehat{\overline{\lambda}}_{i,t}^{S,j}$ and $\widehat{\overline{\lambda}}_{i,t}^{B,j}$ as dependent variables. The results indicate that average selling intensity always increases with private valuation on both sides of the cross-dealer mean level, and average buying intensity increases on the left side of the mean level and decreases on the right side. By composition effect, total search intensity

Table 1.12: Distribution of search intensity among dealers (quadratic form)

$Dep_{i,t}^{j}$	$\widehat{\overline{\lambda}}_{i,t}^{j}$	$\widehat{\overline{\lambda}}_{i,t}^{S,j}$	$\widehat{\overline{\lambda}}_{i,t}^{B,j}$
$\hat{\delta}_{S,i,t}^{j}$ (%)	8.2698***	5.4695***	3.3484***
	(6.30)	(8.97)	(3.58)
$(\hat{\delta}_{S,i,t}^{j})^2$	-0.0423***	-0.0172***	-0.0279***
	(-5.91)	(-6.33)	(-5.23)
$HHI_{i,t}^{bond}$ (thousands)	1.9031	3.0577***	-1.2108**
	(1.64)	(2.78)	(-2.46)
$HHI_{i,t}^{type}$ (thousands)	-6.9322***	-5.8018***	-0.6129
	(-4.27)	(-3.81)	(-0.83)
$HHI_{t}^{j,concen}$ (thousands)	-17.097***	-9.7139***	-7.6099***
	(-19.44)	(-12.42)	(-17.36)
$EV_{i,t}$	110.0916***	46.4680**	64.4953***
	(4.85)	(2.13)	(9.55)
$Rating_{t}^{j}$	2.0028***	3.1608***	-1.1348***
	(5.05)	(8.12)	(1.11)
$Pre3Mturnover_{t}^{j}$ (%)	0.2300***	0.1108***	0.1211***
	(6.31)	(5.89)	(5.6)
$amtout_{t}^{j}$ (million) (%)	-0.022***	0.0019	-0.0248***
	(-9.96)	(1.56)	(-13.81)
TTM_{t}^{j} (days)	1.1006***	0.9427***	0.2178*
	(3.57)	(3.25)	(1.84)
$Coupon^{j}$ (%)	-0.7678**	-2.9136	-10.8165
	(-2.28)	(-0.56)	(-1.54)
# of obs	11,606,655	11,434,333	11,406,360
$Adj\ R^2$	0.0593	0.0493	0.1241
Dealer×Bond×Year FE	YES	YES	YES

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are clustered in dealer#bond#year.

increases on the left side of the mean level and decreases on the right side, which is mainly driven by decrease in buying intensity.

Table 1.13: Distribution of search intensity among dealers

$Dep^j_{i,t}$	$\widehat{\bar{\lambda}}^j_{i,t}$	$Trade^j_{i,t}$	$\widehat{\bar{\lambda}}^{S,j}_{i,t}$	$\widehat{\bar{\lambda}}^{B,j}_{i,t}$
$\frac{\lvert\hat{\delta}^j_{i,t}-\hat{\bar{\delta}}^j_t\rvert}{\lvert\delta^{\hat{j}}_{h,t}-\delta^{\hat{j}}_{l,t}\rvert}$	-136.2285***	-6.1430***	-85.3014***	-54.3122***
	(-24.77)	(-384.71)	(-13.58)	(-21.63)
$\frac{\lvert\hat{\delta}^j_{i,t}-\hat{\bar{\delta}}^j_t\rvert}{\lvert\delta^{\hat{j}}_{h,t}-\delta^{\hat{j}}_{l,t}\rvert} \times \mathbb{1}(\hat{\delta}^j_{i,t} > \hat{\bar{\delta}}^j_t)$			46.9200***	-40.2532***
			(7.27)	(-16.51)
$HHI^{bond}_{i,t}$ (thousand)	1.9887 *	2.82e-04	3.1318***	-1.1766***
	(1.71)	(0.05)	(2.86)	(-2.39)
$HHI^{type}_{i,t}$ (thousand)	-4.9329***	-0.0344***	-5.155***	0.2775
	(-3.04)	(-7.04)	(-3.39)	(0.37)
$HHI^{j,concen}_t$ (thousand)	-15.9301***	-0.446***	-9.1246***	-7.0549***
	(-18.12)	(-182.23)	(-11.68)	(-16.14)
$EV_{i,t}$	107.2886***	2.7191**	9.2142***	63.1459***
	(4.73)	(54.23)	(1.57)	(9.36)
$Rating^j_t$	1.5266***	0.0056***	2.2030***	-0.6837***
	(4.34)	(4.13)	(6.44)	(-6.45)
$Pre3Mturnover^j_t$ (%)	0.0022***	7.31e-05***	0.0010***	0.0013***
	(6.25)	(7.18)	(5.62)	(5.54)
$amtout^j_t$ (million)	-0.024***	-3.22e-04**	0.0028**	-0.0281***
	(-11.25)	(-2.52)	(2.40)	(-15.75)
TTM^j_t (days)	1.0714***	0.0067***	0.9386***	0.1920***
	(3.47)	(7.89)	(3.23)	(1.62)
$Coupon^j$ (%)	-0.6868**	-0.0459**	9.2142	-21.8105***
	(-2.19)	(-2.22)	(1.57)	(-3.08)
# of obs	11,606,655	11,606,655	11,434,333	11,406,360
$Adj\ R^2$	0.0593	0.1168	0.1241	0.1241
Dealer×Bond×Year FE	YES	YES	YES	YES

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are clustered in dealer#bond#year.

1.A.5.2 Proportions of different types of transactions in subperiods

We look at the relationship between the distribution of transactions of different types with distance of dealers' private valuations to cross-dealer mean level within each subperiod. Similar as Bessembinder, Jacobsen, Maxwell, and Venkataraman (2016), we divide the whole sample period into five

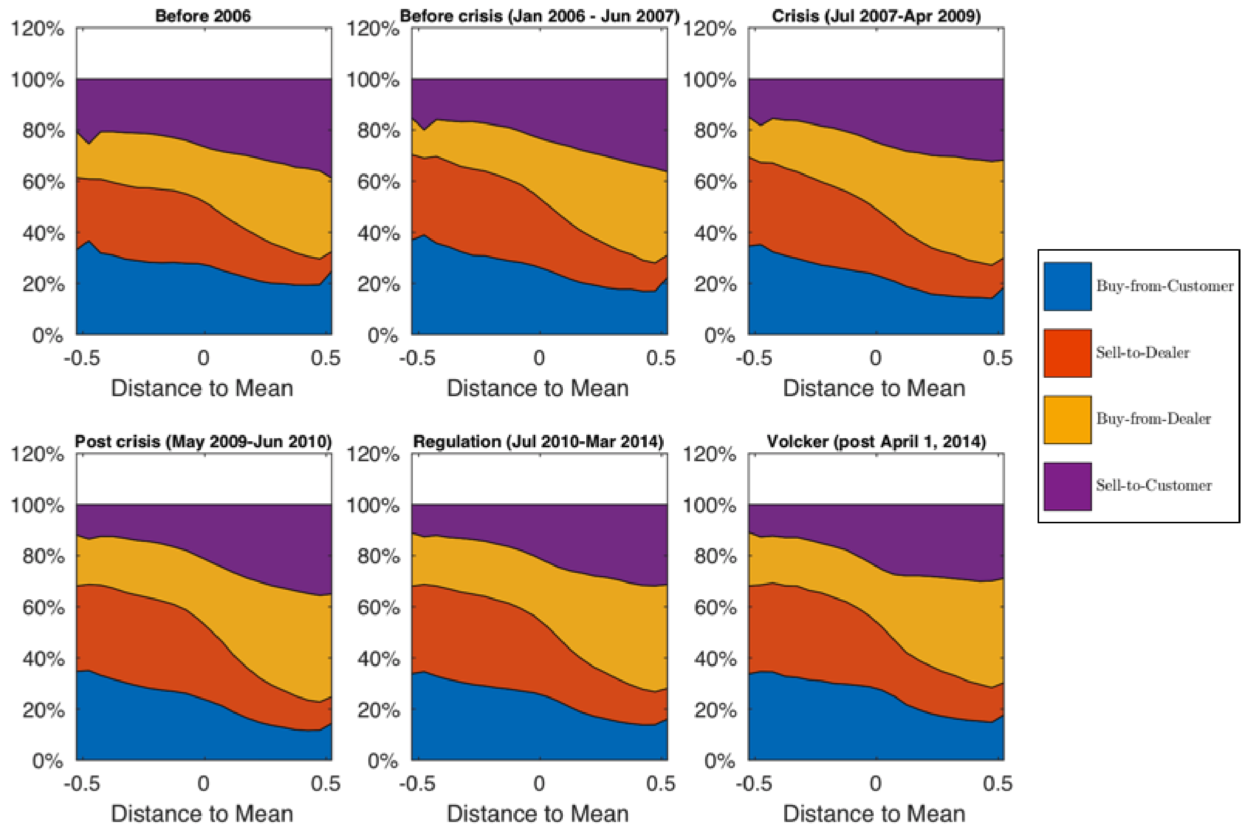

Figure 1.12: Dealer's private valuation and proportions of transactions in different directions (by subperiod)

subperiods: Pre-crisis (Jan 2006-Jun 2007), Crisis (Jul 2007-Apr 2009), Post-crisis (May 2009-Jun 2010), Regulation (Jul 2010-Mar 2014), Volcker (post April 1, 2014).

CHAPTER 2

Bond Misallocation and Liquidity Risk

2.1 Introduction

U.S. corporate bonds trade in decentralized over-the-counter (OTC) markets, in which dealers provide liquidity to customer investors. Empirical studies starting from Collin-Dufresn, Goldstein, and Martin (2001) document that there is a common non-default component in the variations of all corporate bonds' yield spreads over time. This component can not be captured by bond fundamentals, firm-level fundamentals or macroeconomic variables. Later studies show that this common component is closely related to a market-level liquidity factor. Motivated by the theoretical rationalization on that OTC market frictions drive the liquidity-related part of transaction price in decentralized markets, Friewald and Nagler (2018) empirically show that OTC market frictions, namely systemic inventory, search and bargaining frictions, jointly explain a large proportion of the common component. However, to my best knowledge, there have not been papers talking about whether those common frictions drive different bonds' yield spreads by different magnitudes, and which market microstructural factors can explain this heterogeneity.

In this paper, we construct a measure of "bond's misallocation among dealers" and we find that this measure is closely correlated with bonds' heterogeneous yield spread loadings on the common OTC search friction. The measure of bond's misallocation is based on a search-and-match model with dealers endogenously choosing search intensities based on their idiosyncratic states. Specifically, we define this measure as the cross-sectional covariance of dealers' private valuations for holding the bond and their actual inventory positions in the bond. So it can further be regarded as a summary statistic on the joint distribution of dealers' idiosyncratic states. At a lower level of the cross-sectional covariance, there will be more dealers of lower-type private valuations holding bond positions, comparing with a counterfactual frictionless market. In this case, we regard the bond positions as being more misallocated among the dealers, because in a frictionless market all bond positions are held by dealers of the highest private valuations. Correspondingly, a higher level of the cross-sectional covariance implies a lower level of bond's misallocation. This measure is motivated by the fact that in U.S. corporate bond markets, transactions happen bilaterally and the reallocations of bond positions rely on dealers' market-making and searching efforts. The common OTC search friction, together with the distribution of dealers' idiosyncratic states, drives dealers' market-making and searching decisions over time. The latter will further drive the distribution of realized transaction prices and thus the average yield spread variations over time.

Firstly, we use the TRACE data for the U.S. corporate bond market to test whether bonds have significantly heterogeneous factor loadings of yield spread on the common OTC search friction. Since the search friction is a market-level liquidity factor, in this paper, we also call the factor loading as "bond's liquidity risk attributed to search frictions". Specifically, it measures how much a bond's yield spread changes in response to one unit change in the common OTC search friction. Similar as Friewald and Nagler (2018), we use the length of intermediation chain as a measure of the OTC search friction. By theoretical rationalization in Hugonnier, Lester, and Weill (2018), the expected length of intermediation chain decreases with the level of search friction. Then we follow the procedures in the literature to estimate bonds' heterogeneous liquidity risk attributed to search frictions in a reduced-form multi-factor model. Our estimation results are consistent with Friewald and Nagler (2018) and we further show that there is a high variation in the magnitude of liquidity risk across different bonds. The standard deviation of the liquidity risk is more than three times of the mean level.

Secondly, we estimate the series of dealers' idiosyncratic states, namely dealers' private valuations for holding each bond and their inventory positions in each bond, following the procedures in Liu (2020) and Hansch, Naik, and Viswanathan (1998). With the estimated series, we construct a panel data which contains yearly series of empirical estimates of bond's misallocation and liquidity risk. By estimating a panel data model, we verify that: at the bond level, a higher magnitude of misallocation among the dealers is associated with a higher magnitude of liquidity risk. This finding gives a preliminary market microstructural evidence which supports that: in decentralized financial markets, the distribution of market maker's idiosyncratic states correlates with the magnitude of the asset's liquidity risk.

Finally, we give a simple numerical explanation on the verified correlation between bond's misallocation and liquidity risk, by solving the search-and-match model with dealers' endogeneous search efforts. The numerical solutions show that in a stationary equilibrium where the cross-sectional covariance of dealers' private valuation and inventory position is at a lower level, the higher level of bond misallocation motivates more dealers to choose a higher search intensity to buy or sell to adjust their holding positions. As a result, the average level of all dealers' search intensities is also high. With a qudratic-form search cost, the average level of marginal search cost in the dealer sector is monotonically increasing with the dealers' average search intensity. Since this average marginal cost will be compensated by bond's average yield spread, in equilibrium the bond has its average yield-spread being more exposed to shocks to OTC search frictions.

Related literature

This paper firstly contributes to the empirical literature initiated by Collin-Dufresn, Goldstein, and Martin (2001) that uncovers fundamental factors to explain U.S. corporate bonds' yield spread variations over time. In this literature, Collin-Dufresn, Goldstein, and Martin (2001) establish that there is an unexplained single common factor in corporate bonds' yield spreads after controlling for commonly used explanatory variables; Longstaff, Mithal, and Neis (2005) measure the size of the default and non-default components in corporate spreads, and show that the non-default component is related to bond-specific as well as macroeconomic measures of liquidity. Latter papers add other liquidity factors to improve the explanation, see Bao, Pan, and Wang (2011), De Jong and Driessen (2012), Bongaerts, De Jong, and Driessen (2017), Crotty (2013), Friewald and Nagler (2016), and He, Khorrami, and Song (2019), among others. Specifically, Friewald and Nagler (2018) attribute the unexplained part of the non-default component to over-the-counter (OTC) market frictions. In this paper, we specifically focus on bond's yield spread loading on search frictions. Using similar measures, we further document that there is a high variation in the magnitude of yield spread loading on OTC search frictions across different bonds. And we further construct a measure of bond's misallocation and correlates it with the magnitude of the yield spread loading.

The search-and-match model in this paper also belongs a theoretical literature initiated by Duffie, Gârleanu, and Pedersen (2005) that uses a search-and-match model to study asset price and liquidity in over-the-counter markets. My model studies fully decentralized market structure by setting a random search environment, which is similar to one strand of the literature developed by Duffie, Gârleanu, and Pedersen (2007), Vayanos and Wang (2007), Vayanos and Weill (2008), Weill (2008), Afonso (2011), Gavazza (2011), Praz (2014), Trejos and Wright (2016), Afonso and Lagos (2015), Atkeson, Eisfeldt, and Weill (2015). My model is most related to Hugonnier, Lester, and Weill (2018) in the setting of dealers' heterogeneous private valuation types and the incorporation of both dealer and customer sectors. The main difference in my model is that we consider dealers' explicit choice of state-dependent search intensity based on their idiosyncratic states. In Hugonnier, Lester, and Weill (2018), dealers are endowed with homogeneous search intensities. Based on my model, we construct the cross-sectional covariance of dealers' private valuations and bond holding positions as the measure of bond's misallocation among the dealers. Papers in this literature which also consider endogenous and/or heterogeneous search intensity include Shen, Wei, and Yan (2018), Neklyudov (2012), Üslü (2019), and Farboodi, Jarosch, and Shimer (2017b), etc.

This paper connects the empirical literature on explaining corporate bond's yield spread variations and the theoretical literature on studying OTC market structure using the structural search-and-match framework. Most papers in the theoretical literature focus mainly on how searching and trading activities determine the transaction price and volume between each pair of two trading counterparties. This paper instead focuses on giving a more structual explanation on bond-level yield spread patterns, rather than only bilateral-based terms of trade. Also this paper considers dealer-level market-making and searching behavior as a channel that connects the change in OTC search friction and the bond-level yield spread variations. The empirical verification also motivates future theoretical research.

2.2 Data description

We use the Academic Corporate Bond TRACE Data set provided by the Financial Industry Regulatory Authority (FINRA). This data set contains dealers' reports to the Trade Reporting and Compliance Engine (TRACE) which disclose information on all transactions in corporate bonds. One advantage of the data is we can observe identities of the dealers in all transactions. This allows us to track how the bonds are transacted between the dealers, so that we can construct intermediation chains, and also construct the measure of bond misallocation within the dealer sector. [1] We filtered the data following the procedure in Dick-Nielsen (2014), and we recover the trading counterparties in locked-in and give-up trades[2].We merge

[1]In the analysis, we define all registered members of FINRA as dealers and all non-registered outside trading counterparties as customers. Main registered firm of FINRA include broker-dealer firms, crowdfunding portals, and capital acquisition brokers, etc, which are all dealer-like firms. The ID numbers assigned by FINRA to registered members are all virtual IDs. In the data, non-registered trading counterparties are assigned with the ID of "C" by FINRA.

[2]By the user guide of FINRA, a "Give Up" trade report is reported by one FINRA member on behalf of another FINRA member who is the real one to buy or sell the bonds and thus has a reporting responsibility. For such reports, we call the FINRA members, who asked other members to submit reports for them, the true

the cleaned data with the Mergent Fixed Income Securities Database (FISD) and Wharton Research Data Services (WRDS) Bonds Return Database to obtain bond fundamental characteristics and credit ratings. We construct a monthly panel containing both dealer-wise and bond-wise variables[3].

Following the academic literature using the same data set, we further filtered the data by excluding some "unusual bonds" and some specific types of transactions: [1] We exclude bonds with optional characteristics, such as variable coupon, convertiable, exchangable, and puttable, etc, and we also exclude asset-backed securities and private placed instruments; [2] To estimate bonds' factor loadings on OTC search frictions, we further drop the inactively traded bonds, defined as those traded in fewer than 25 months throught the whole sample period; [3] Finally, we exclude the "on-the-run" transactions which happened within three months since bonds' offering dates, to only consider secondary market transactions.

The final sample ranges from Jan 2005 to Sep 2015, and contains 10760 bonds traded by 3050 dealers. The total outstanding amount of all bonds in our sample is \$5.37 trillion. The average bond rating is BBB by the S&P rating categories. Among these bonds, around 84% are investment grade and the remaining ones are high-yield or non-rated.[4] Bonds on

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trading counterparties; Locked-in report is a trade report representing both sides of a transaction. FINRA members such as Alternative Trading Systems (ATSs), Electronic Communications Networks (ECNs), and clearing firms have the ability to match buy and sell orders, and therefore to report on behalf of multiple parties using a single trade report submitted to FINRA and indicate that the trade is locked-in. Similarly, we call the FINRA members who submit the buy or sell orders, instead of those clearing platforms, as the true trading counterparties. In the error filters, for these two types of trades, we use the IDs of the true trading counterparties as dealers' IDs and we adjust the reported prices accordingly to account for the agency fees charged by reporting firms and clearing platforms (ATSs, ECNs, and clearing firms).

[3]The raw data is high-frequency data that records the time of each transaction in seconds. In empirical literature using TRACE data to analyze U.S. corporate bond market liquidity, it is common practice to process the data to monthly frequency as corporate bonds are relatively illiquid compared with stock markets, see Bao, Pan, and Wang (2011), Crotty (2013), Friewald and Nagler (2016), and Friewald and Nagler (2018), etc. Specifically, An (2019) documents that dealers' average inventory duration in the U.S. corporate bond market is around three weeks by using the same data, which implies that the average frequency dealers adjust their inventories is around one month.

[4]By the S&P rating categories, investment grade are S&P BBB or higher; and high-yield(junk) are below or equal to S&P BBB-.

average have time to maturity as 7.6 years. There are 57,623,804 transactions with total par amount as $27.8 trillion. The average trade size is $482.41 thousand with standard deviation as $4.47 thousand.

2.3 Liquidity risk attributed to search frictions

In this paper, we specifically focus on bonds' liquidity risk attributed to over-the-counter (OTC) search frictions (henceforth "liquidity risk" for short). Friewald and Nagler (2018) show that changes in OTC market frictions can explain a large portion of variations in bond yield spreads, by fitting a multi-factor model using the same data set. The OTC market frictions they consider include search frictions, inventory frictions, and bargaining frictions, etc. Specifically, we follow the similar procedure to use the weighted average length of intermediation chain as a measure of OTC search friction[5], and we regard the factor loading of bond yield spread[6] on the average chain length as a measure of the bond liquidity risk attributed to OTC search frictions. We will show that the magnitude of this measure of liquidity risk varies across different groups of bonds.

2.3.1 Length of intermediation chain

Intermediation chains were firstly constructed in Li and Schürhoff (2014) and Hollifield, Neklyudov, and Spatt (2017) to track how municipal bonds and securitization instruments are reallocated from a customer-seller to a customer-buyer through a series of dealers in the interdealer market. The length of an intermediation chain is defined as the number of dealers,

[5]The weighted average length of intermediation chains is equal to the average number of dealers being involved in the intermediation process. Details about this measure are discussed in Appendix 2.A.2.2.

[6]Yield spread is defined as the difference between corporate bond yield and the treasury yield whose term equals the corporate bond duration. Similar as in Crotty (2013), Friewald and Nagler (2018), etc, we calculate treasury yields of different terms through linearly interpolating between points on the treasury curve.

through which the assets passed during the reallocation process. By Hugonnier, Lester, and Weill (2018), the expected length of intermediation chain decreases with the level of search frictions in the interdealer market. Specifically, in a more frictional interdealer market, it is more difficult for dealers to meet and trade with each other, so that there will be fewer dealers being involved in each reallocation of assets between customers, then the average length of intermediation chain will be shorter.[7]

We calculate the average length of intermediation chain across all bonds for each month, using volumes of reallocation as weights. Figure 2.1 shows that the average chain length is relatively higher before the 2008 great financial crisis (GFS) when search frictions are relatively low in corporate bond secondary market. Then it decreases by as large as 6% during the crisis period when secondary market liquidity nearly dried up. Although the average chain length recovers slightly in the post-crisis period.[8], after Dodd-Frank act was signed into law in July, 2010, it further decreases by nearly 8% till the third quarter of 2015. This is consistent with the effects of Dodd-Frank act on restricting both dealers' proprietary tradings and dealers' liquidity provision to customers.

To verify that the average chain length is negatively correlated with the level of search frictions, we also plot the ratio of pre-arranged transactions among all transactions for each month. This ratio tends to be higher when market is more frictional so that dealers are less willing to commit their capital to liquidity provision, but more willing to pre-arrange trades between buyers and sellers. In Figure 2.1, the ratio of pre-arranged trades is negatively correlated with the average length of intermediation chain.

[7] As market-level search frictions increase, although intermediation chains will on average be shorter, it does not necessarily mean the reallocations of assets between customers take shorter time.

[8] Similar as Bessembinder, Jacobsen, Maxwell, and Venkataraman (2016), we divide the whole sample period into five subperiods: Pre-crisis (Jan 2006-Jun 2007), Crisis (Jul 2007-Apr 2009), Post-crisis (May 2009-Jun 2010), Regulation (Jul 2010-Mar 2014), Volcker (post April 1, 2014).

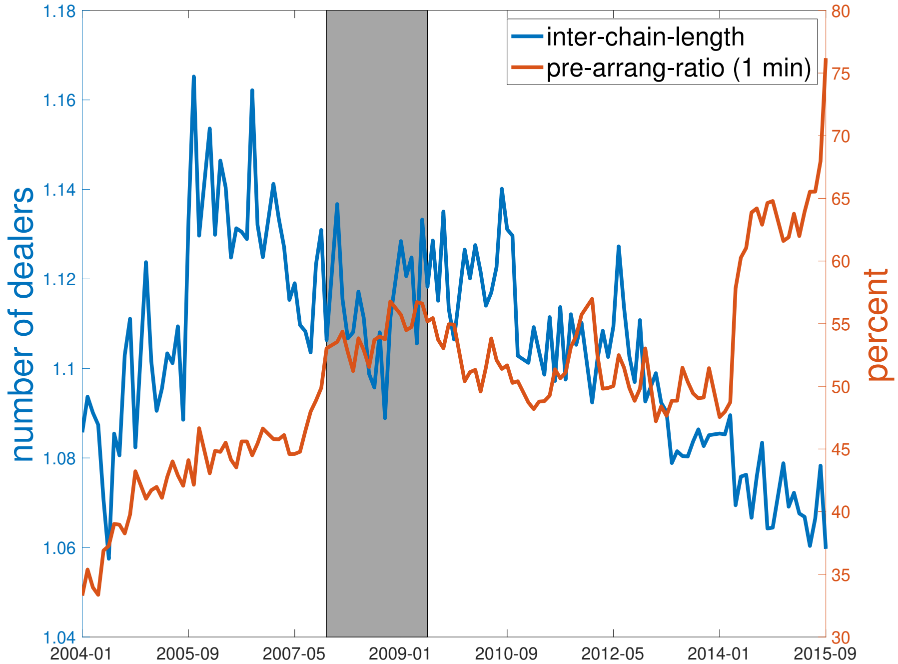

Figure 2.1: The value-weighted average length of intermediation chain (Jun 2004 - Sep 2015)

2.3.2 Bond liquidity risk

We estimate bonds' heterogeneous yield spread loadings on OTC search frictions using monthly panel data, and we use this factor loading as a measure of bond's liquidity risk attributed to search frictions. We calculate yield spread as the gap between bond yield and the same-maturity treasury yield. Then we regress the change in yield spread on multiple regressors, including the regressors about the change in other OTC market frictions in Friewald and Nagler (2018), regressors about the change in market fundamental factors (e.g. equity pricing factors, market volatility, etc) in Fama and French (1993), Carhart (1997), Crotty (2013), and bond fundamentals. The yield spread loading on OTC search frictions is the estimate of bond-wise coefficient on the regressor "average length of intermediation chain across all bonds". This coefficient measures how sensitively the non-default component of credit spread responds to the change in OTC search frictions. The model is as follows:

$$\triangle(YieldSpread)_{j,t} = \beta^j_{SysSearch}\triangle SystemChainLength_t + \beta^j_{SysNetConcen}\triangle SysNetConcen_t$$

$$+\beta^j_{MKT}R_{MKT,t} + \beta^j_{SMB}R_{SMB,t} + \beta^j_{HML}R_{HML,t} + \beta^j_{UMD}R_{UMD,t}$$

$$+\boldsymbol{\gamma}^j_1\triangle I_t + \boldsymbol{\gamma}^j_2\triangle B_t + \boldsymbol{\gamma}^j_3\triangle X^{(j)}_t + \epsilon_{j,t}$$

where $\triangle SystemChainLength_t$ is the change in the average length of intermediation chain, which is a proxy for shocks to OTC search frictions. Therefore, $\beta^j_{SysSearch}$ is the defined bond j's liquidity risk, and our main focus is to discuss how market structural factors (specifically, bond's misallocation among dealers) determine the magnitude of $\beta^j_{SysSearch}$. In Appendix 2.A.2.4, we show that the factor loading $\beta^j_{SysSearch}$ is significantly priced in bonds' yield spreads.

The multi-factor model includes other controls as follows: [1] change in interdealer network concentration $\triangle SysNetConcen_t$, which is measured by the summation of all dealers' average degree centralities[9] in month t; [2] returns on factor-portfolios $R_{MKT,t}$, $R_{SMB,t}$, $R_{HML,t}$ and $R_{UMD,t}$, namely market portfolio (S&P 500 portfolio), small-minus-big(SMB) portfolio, high-minus-low(HML) portfolio and up-minus-down(UMD) momentum-factor portfolio; [3] change in OTC inventory-related frictions $\triangle I_t = (\triangle inv_{t-1}; \triangle amtout_t; \triangle prearrange_t)$, in which $\triangle inv_{t-1}$ is the one-month-lagged change in all dealers' inventories in all bonds, $\triangle amtout_t$ is the change in all bonds' amount outstanding, $\triangle prearrange_t$ is the change in pre-arranged ratio of all transactions; [4] change in OTC bargaining frictions $\triangle B_t = (\triangle blocktrade_t; \triangle HHIdealer_t)$, in which $\triangle blocktrade_t$ is the change in ratio of block trades

[9] Degree centrality is another measure of vertices' centralities in a network. Unlike eigenvector centrality, degree centrality only takes into account all direct links directed from or to each vertice. For a network with n vertices, the theoretical maximum value of the summation of all vertices' degree centralities is $n(n-1)$. Therefore, summation of all dealers' degree centralities in the interdealer network is a better measure of the concentration of the network. The closer the summation is to $n(n-1)$, where n is the number of dealers, the less concentrated the interdealer network is.

and $\triangle HHIdealer_t$ is the change in average value of all bonds' HHI indices[10]; [4] all the other bond-wise and market-aggregate controls

$\triangle X_t = (\triangle(YieldSpread)_{j,t-1}, \triangle RF_t; (\triangle RF_t)^2; \triangle SLOPE_t; \triangle turnover_t^j; Rating_t^j; TTM_t^j)$ in Collin-Dufresn, Goldstein, and Martin (2001) and Friewald and Nagler (2018), in which $\triangle(YieldSpread)_{j,t-1}$ is the lagged term of change in yield spread, $\triangle RF_t$ is the change in 10-year treasury rate, $(\triangle RF_t)^2$ is the square value to capture potential non-linear effect, $\triangle SLOPE_t$ is the change in the slope of yield curve, $\triangle turnover_t^j$ is the change in bond j's current-month turnover rate, $Rating_t^j$ is bond j's credit rating in month t and TTM_t^j is bond j's time to maturity in month t.

The mean value of $\beta^j_{SysSearch}$ across all bonds is significantly negative, as shown in Table 2.1. This indicates that, when intermediation chains are longer (in other words, OTC search frictions decrease), bond's yield spread will decrease. The signs of other reported average coefficients in Table 2.1 are consistent with those in Friewald and Nagler (2018). The full regression results are in 2.4 in Appendix 2.A.2.1.

However, the average magnitude of $\beta^j_{SysSearch}$ is significantly heterogeneous among different groups of bonds. We divide the whole sample of bonds into different groups based on bonds' credit rating and time to maturity. Table 2.2 shows that the factor loading $\beta^j_{SysSearch}$ has higher absolute value for high-yield bonds and/or bonds with longer time to maturity. The higher the absolute value of $\beta^j_{SysSearch}$ is, the more sensitively bond j's yield spread responds to shocks to OTC search frictions. Our next focus is to construct a new market microstructural variable, "bond's misallocation among dealers", and use it to explain why different bonds have different magnitudes of liquidity risk attributed to OTC search frictions.

[10] Block trades are defined as trades with trading volume being larger than $\$1,000,000$. Each bond's HHI index is calculated by using all dealers' market shares in that bond. Both variables are proxy for systemic bargaining frictions in the U.S. corporate bond market: the higher the ratio of block trades is, the more bargaining power the corporate bond customers (investors) have, and the higher the average value of all bonds' HHI indices is, the more concentrated are bonds' transactions to a subset of dealers, therefore, the lower bargaining power of the customers (investors) have

Table 2.1: Bond-level liquidity risk

$\triangle(YieldSpread)_{j,t}$ (%)	(1)	(2)	(3)
$\triangle SystemChainLength_t$	-2.32***	-1.67***	-1.55***
	(-32.80)	(-21.38)	(-21.38)
$\triangle SysNetConcen_t$ (thousand)	-9.83e-03***	-4.77e-03***	-4.43e-03***
	(-48.16)	(-22.30)	(-20.10)
$\triangle inv_{t-1}$ ($trillion)	7.55***	5.55***	5.74***
	(24.07)	(16.39)	(17.51)
$\triangle prearrange_t$ (%)	0.26***	1.28***	1.08***
	(3.43)	(15.87)	(13.43)
$\triangle blocktrade_t$ (%)	-66.67***	-29.29***	-28.65***
	(-50.94)	(-22.37)	(-22.15)
$\triangle amtout_t$ ($trillion)	-0.32***	-0.47***	-0.38***
	(-6.17)	(-8.12)	(-6.54)
$\triangle HHIdealer_t$ (thousand)	-1.04***	-0.67***	-0.67***
	(-46.70)	(-28.99)	(-30.82)
$Mean\ Adj\ R^2$	0.18	0.35	0.37
$\#ofBonds$	11176	11176	9595
$\#ofObs$	515514	515514	479146
market aggregates and FFC 4 factors	NO	YES	YES
bond liquidity and fundamentals	NO	NO	YES

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. In Panel A, we exclude bonds with total number of observations smaller or equal to 19 for model (1)-(2) and smaller or equal to 25 for model (4). The reported estimated coefficients are average values taken across all bonds. The corresponding t-statistics are calculated by dividing each reported (average) coefficient value by the standard deviation of the estimates and scaling by the square root of the number of bonds.

2.4 Correlation between bond misallocation and liquidity risk

In this section, we construct a measure of bond's misallocation among dealers, and show that this measure is closely correlated with the magnitude of bond's liquidity risk. The main takeaway is: a bond which is more misallocated among the dealers has its yield spread being more exposed to shocks to OTC search frictions, because the dealers are more willing to reallocate the bond between themselves and a larger portion of the bond price will compensate the dealers for paying the search costs.

Table 2.2: Group-level liquidity risk

$\triangle(YieldSpread)_{j,t}$ (%)	(1)	(2)	(3)
$\triangle SystemChainLength_t$	-0.38***	-0.32***	-0.15*
	(-8.99)	(-6.29)	(-1.92)
$\triangle SystemChainLength_t \times \mathbb{1}\,(HY\_bonds)$		-0.19*	
		(-2.34)	
$\triangle SystemChainLength_t \times \mathbb{1}\,(TTM\_2nd)$			-0.57***
			(-5.20)
$\triangle SystemChainLength_t \times \mathbb{1}\,(TTM\_3rd)$			-0.43***
			(-3.98)
$Adj\ R^2$	0.1307	0.1307	0.1308
$\#ofBonds$	11703	11703	11703
$\#ofObs$	523586	523586	523586
market aggregates and FFC 4 factors	YES	YES	YES
bond liquidity and fundamentals	YES	YES	YES
OTC inventory and bargaining friction	YES	YES	YES

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. In Panel A, we exclude bonds with total number of observations smaller or equal to 19 for model (1)-(2) and smaller or equal to 25 for model (4). The reported estimated coefficients are average values taken across all bonds. The corresponding t-statistics are calculated by dividing each reported (average) coefficient value by the standard deviation of the estimates and scaling by the square root of the number of bonds. In Panel B: TTM_1st: 13 days∼3 years, TTM_2nd: 3∼6 years, TTM_3rd: > 6 years.

2.4.1 Bond misallocation among dealers

For each bond, we define its misallocation among dealers as the cross-sectional covariance of dealers' idiosyncratic private valuations[11] for holding the bond and their actual inventory positions in the bond. In the dealer sector, if there are more (less) low(high)-private-valuation dealers holding the bond, the level of this covariance will be lower, then we regard the bond as being "more misallocated" among the dealers.

[11] In the spirit of Duffie, Gârleanu, and Pedersen (2005), dealers' idiosyncratic private valuations can be understood as their idiosyncratic preferences in holding the bond, which can be determined by their idiosyncratic liquidity needs, financing costs, and hedging needs, etc. Within each bond, dealers can be ranked by their private valuation types. For example, a dealer who has a higher liquidity need or financing cost than other dealers will manifest a lower private valuation for holding the bond than others.

Theoretical counterpart of bond misallocation The measure of bond misallocation is based on a structural search model in Liu (2020). In this section, we give a review on the model environment and a simple numerical example to show that a higher (lower) level of the cross-sectional covariance of dealers' private valuations and bond inventories implies a lower (higher) magnitude of bond misallocation among dealers.

The model environment is: there are two sectors of agents in the market, a continuum of customers with physical measure normalized to 1 and a continuum of dealers with physical measure as $m \leq 1$. Dealers and customers search and trade a single bond with fixed supply s. Each participant's bond position a is assumed to be either zero or one.[12] Each participant has a private valuation for the bond: customers' private valuation takes two possible values, either low or high, denoted by $y \in \{y_\ell, y_h\}$ with $y_\ell < y_h$, and follows a discrete distribution $P(y' = y_c) = \pi_c$, $c = \ell, h$; dealers' private valuations $\delta \in [\delta_\ell, \delta_h]$ lie in between customers' low type and high type, and follow a continuous distribution $f_D(\delta)$. The two types of customers cannot directly trade with each other, so the bond needs to be intermediated through the dealer sector. One position of the bond can be sold from a low-type customer to a dealer, and transacted between several dealers, and then finally sold to a high-type customer. Each customer periodically receives an idiosyncratic shock with Poisson intensity α, which makes her valuation switch between the high type and low type. This shock generates the fundamental trading needs in the market. Dealers do not receive such valuation shock, so their relative valuations remain fixed over time. Dealers endogeneously choose their search efforts $\lambda(a, \delta)$ based on their idiosyncratic states (a, δ), and pay a search cost $c \times \lambda^2(a, \delta)$ at each time, where $c > 0$ is a proxy for the market-level OTC search frictions.

The model generates a stationary equilibrium which includes a density function $\phi_1(\delta)$. The value of $\phi_1(\delta)$ on each $\delta \in [\delta_l, \delta_h]$ is the probability that a dealer with private valuation δ holds one position of the bond, i.e. this dealer is a dealer-owner in stationary

[12]This $\{0, 1\}$ assumption for bond holding and the indivisibility of bonds determine that the trading volume in each transaction equals one.

equilibrium. Correspondingly, the probability that this dealer with private valuation δ is a dealer-nonowner is denoted as $\phi_0(\delta) = f_D(\delta) - \phi_1(\delta)$. As a result, the shape of the density function $\phi_1(\delta)$ determines how the bond is allocated among dealers, and it uniquely maps to the level of the cross-sectional covariance $Cov(\delta, a)$ of dealers' private valuations and bond positions. The covariance has the following form:

$$Cov(\delta, a) = \sum_{a\in\{0,1\}} \int_{\delta_\ell}^{\delta_h} (a - \overline{a}_d)(\delta - \overline{\delta}_d)\frac{\phi_a(\delta)}{m} d\delta = \int_{\delta_\ell}^{\delta_h} (\delta - \overline{\delta}_d)\frac{\phi_1(\delta)}{m} d\delta \tag{2.1}$$

where $\overline{\delta}_d$ is the average value of dealer's private valuation and has an expression as $\sum_{a\in\{0,1\}} \int_{\delta_\ell}^{\delta_h} \delta\phi_a(\delta)d\delta = \int_{\delta_\ell}^{\delta_h} \delta f_D(\delta)d\delta$. Based on the final expression $\int_{\delta_\ell}^{\delta_h}(\delta - \overline{\delta}_d)\frac{\phi_1(\delta)}{m}d\delta$ in (2.1), we can regard the cross-sectional covariance $Cov(\delta, a)$ as a weighted average of $(\delta-\overline{\delta}_d)$, with the values of the density function $\phi_1(\delta)$ as weights.

The value of $Cov(\delta, a)$ is negatively correlated with the level of the bond's misallocation among dealers. For example, if there is a larger proportion of the bond positions being held by low-private-valuation dealers, in the term $\int_{\delta_\ell}^{\delta_h}(\delta-\overline{\delta}_d)\frac{\phi_1(\delta)}{m}d\delta$ larger weights will be imposed on lower δ, which leads to a lower value of $Cov(\delta, a)$.

A numerical example of this model is shown in Figure 2.2. The area below the density function $\phi_1(\delta)$ is equal to the total amount of bond positions being held by dealers. Suppose in Walrasian (frictionless) market, the minimum private valuation among all the dealer-owners is the middle level $\frac{\delta_l+\delta_h}{2}$, then we call the dealer of this level of private valuation as the "marginal investor". Since in Walrasian market all of the bond positions are held by dealers of the highest private valuations, we assume there is *no* bond misallocation among dealers in this case. Then in any over-the-counter (OTC) market with search frictions, we regard any bond positions which are held by dealers with private valuations lower than this middle level $\frac{\delta_l+\delta_h}{2}$ as "being misallocated". In the right graph of Figure 2.2, there are two OTC markets with the same level of search frictions but different levels of bond misallocations. Market-1 has relatively lower amount of bond positions being misallocated than market-

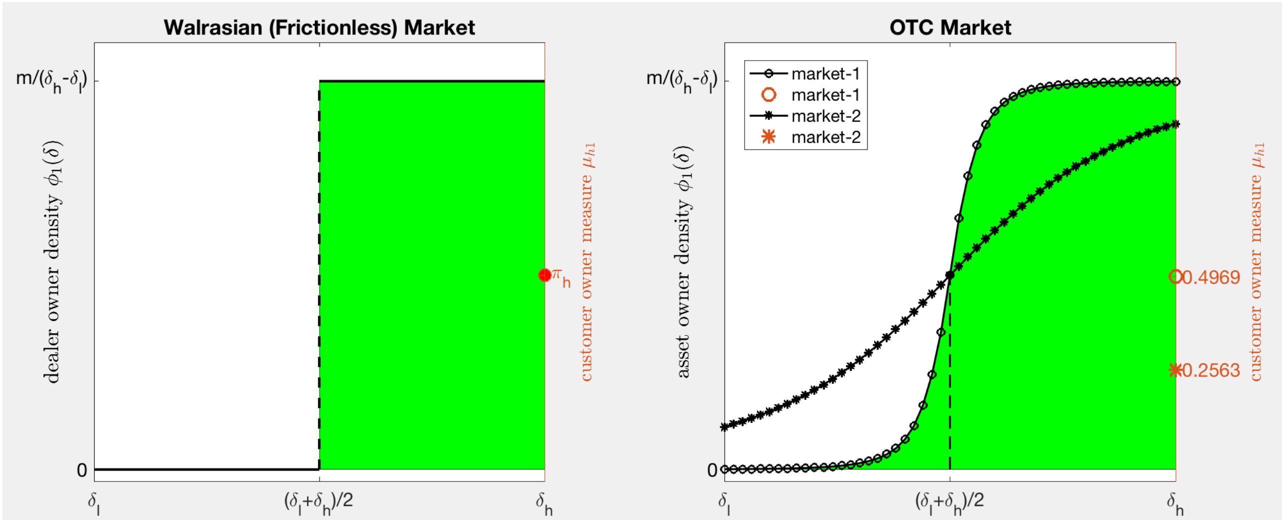

Figure 2.2: Numerical example of dealer owner density function and bond misallocation

(Walrasian market: $Cov(\delta, a) = 0.130$, $\mu_{h1} = \pi_h = 0.5$, $\mu_{l1} = 0$. market-1: $Cov(\delta, a) = 0.103$, $\mu_{h1} = 0.4969 < \pi_h$, $\mu_{l1} = 0.0031$, $c = 0.07$. market-2: $Cov(\delta, a) = 0.092$, $\mu_{h1} = 0.2563 < \pi_h$, $\mu_{l1} = 0.2437$, $c = 0.07$.)

2. Correspondingly, the cross-sectional covariance $Cov(\delta, a)$ is higher in market-1 than in market-2.

Data estimate of bond misallocation We follow the procedure in Liu (2020) to estimate the monthly series of dealers' idiosyncratic private valuations using realized transaction prices. Detailed explanations on the estimator is in Appendix 2.A.1. For each bond-month pair, each dealer's private valuation is the simple average of the dealer's maximum buying price and minimum selling price for that bond-month pair. The estimator of a dealer's private valuation type δ is as follows[13]:

$$\hat{\delta}^j_{i,t} = \frac{max\{Buy^j_{i,n^{j,B}_{i,t}}\} + min\{Sell^j_{i,n^{j,S}_{i,t}}\}}{2}$$

where $\{Buy^j_{i,n^{j,B}_{i,t}}\}$ ($\{Sell^j_{i,n^{j,S}_{i,t}}\}$) is the collection of all buying (selling) prices by dealer i for bond j in month t, and $n^{j,B}_{i,t}$ ($n^{j,S}_{i,t}$) is the correspondinng number of total buying (selling) transactions (including both dealer-customer and interdealer transactions) in month t.

We follow the procedure in Hansch, Naik, and Viswanathan (1998) to estimate the monthly series of dealers' inventory positions. We use $Q^j_{i,t}$ to denote the (unobservable) dealer i's inventory position in bond j and month t, s.t. $0 \leq t \leq T$, where T is the last month of our sample. We use $q^j_{i,t}$ to denote the corresponding observable signed net trading volume, which is positive (negative) when the dealer i increases (shrinks) her inventory position of bond j in month t. With unobservable initial inventory $Q^j_{i,0}$, $Q^j_{i,t}$ satisfies:

$$Q^j_{i,t} = Q^j_{i,0} + \sum_{s=1}^{t} q^j_{i,s}$$

Then we construct the standardized inventory for each dealer i, bond j and month t:

$$I^j_{i,t} = \frac{Q^j_{i,t} - \bar{Q}^j_i}{\sigma^j_i}$$

where $\bar{Q}^j_{i,t} = \frac{\sum_{s=0}^{T} Q^j_{i,s}}{T+1}$ and $\sigma^j_i = \sqrt{\frac{\sum_{s=0}^{T}(Q^j_{i,s} - \bar{Q}^j_{i,t})^2}{T}}$ are the sample mean and standard deviation of the monthly series.[14]

[13]In finite samples, on the buy side of each dealer, the maximum buying price is a downward biased estimate for the dealer's marginal valuation; on the sell side, the minimum selling price is an upward biased estimate for the dealer's marginal valuation. Taking the average of the sample maximum buying price and the sample minimum selling price will make the bias cancel out. In small samples with dealers' unbalanced buy and sell trades, the levels of the upward bias and the downward bias may not be equal. Then to make the bias cancel out completely, the weights assigned on the two extreme prices can be adjusted according to the realized number of buy and sell trades.

[14]For a robustness check, we also follow Friewald and Nagler (2016) to calculate $\bar{Q}^j_{i,t}$ and $\sigma^j_{i,t}$ only using

The standardized inventory $I^j_{i,t}$ essentially measures by how much the current inventory $Q^j_{i,t}$ deviates from the unobserved target level $\bar{Q}^j_{i,t}$, and the deviation is scaled by the volatility of the series within each pair of dealer i and bond j. By similar derivation in Hansch, Naik, and Viswanathan (1998), this standardization [1] excludes the effect of unobserved initial inventory position $Q^j_{i,0}$ after issuance[15], and writes standardized inventory as a linear combination of a series of signed net trading volumes $\{q^j_{i,s}\}$; and [2] controls for differences in risk aversion to guarantee the comparability of inventories across dealers (see Friewald and Nagler (2016)).

With the estimated monthly series $\{\hat{\delta}^j_{i,t}\}$ and $\{I^j_{i,t}\}$, we calculate the cross-sectional covariance for each year by the following two steps: firstly, for each pair of dealer i and bond j in year y, we separately calculate the dealer's yearly weighted average of private valuation $\hat{\delta}^j_{i,y}$ and yearly weighted average of inventory position $I^j_{i,y}$, using the dealer's monthly trading volumes in year y as weights; secondly, for bond j and year y, we pool all dealers' yearly private valuations $\{\hat{\delta}^j_{i,y}\}_{i\in D_y}$ and inventory positions $\{I^j_{i,y}\}_{i\in D_y}$ together, and calculate the cross-sectional covariance as follows:

$$\widehat{Cov}(\hat{\delta}^j_{i,y}, I^j_{i,y}) = \frac{1}{N^y_d} \sum_{i\in D_y} \left(\hat{\delta}^j_{i,y} - \overline{\hat{\delta}}^j_y\right) * \left(I^j_{i,y} - \overline{I}^j_y\right)$$

where D_y is the collection of all the dealers who completed at least one transaction in bond j on both the buy and sell sides of the market in year y, and N^y_d is the number of dealers in group D_y; $\overline{\hat{\delta}}^j_y$ and $\overline{I}^j_y$ are the simple cross-dealer means of private valuation and inventory position in year y.

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series of signed trading volumes within the fixed rolling time window $[t, t-R]$. We obtain similar results for our quantitative analysis.

[15]We calculate the series of standardized inventory $\{I^j_{i,t}\}$ before dropping bond transactions during a 3-month on-the-run period following issuance.

2.4.2 Correlation between bond misallocation and bond liquidity risk

In this section, we show that bonds with a lower level of cross-sectional covariance of dealers' private valuation and inventory position (i.e. a higher magnitude of misallocation) will have its yield spread more exposed to shocks to OTC search frictions (i.e. a higher magnitude of liquidity risk). This finding gives a preliminary market microstructural evidence which shows that: the distribution of market maker's states correlates with the magnitude of corporate bond's liquidity risk.

To verify this correlation, we construct a yearly panel data on corporate bonds' factor loadings on OTC search frictions $\beta^j_{SysSearch,y}$ and within-bond average cross-sectional covariance $\overline{\widehat{Cov}}_y(\hat{\delta}^j_{i,ey}, I^j_{i,ey})$. Specifically, $\beta^j_{SysSearch,y}$ is estimated for bond j which has transactions completed in year y, using bond j's all transactions within the time window $[1, y]$. Correspondingly, $\overline{\widehat{Cov}}_y(\hat{\delta}^j_{i,ey}, I^j_{i,ey})$ is constructed as a weighted average of bond j's yearly cross-sectional covariance throughout all years $ey \in [1, y]$. Therefore, to construct each point $(\beta^j_{SysSearch,y}, \overline{\widehat{Cov}}_y(\hat{\delta}^j_{i,ey}, I^j_{i,ey}))$ in the yearly panel data, we make use of all the cumulative information until year y on bond transactions, market microstructure, bond fundamentals, and market aggregates, etc.

We estimate the following reduced-form model to verify the correlation between bond's misallocation $\overline{\widehat{Cov}}_y(\hat{\delta}^j_{i,ey}, I^j_{i,ey})$ and liquidity risk $\beta^j_{SysSearch,y}$:

$$\beta^j_{SysSearch,y} = \alpha_0 + \alpha_1 * \overline{\widehat{Cov}}_y(\hat{\delta}^j_{i,ey}, I^j_{i,ey}) + \boldsymbol{\alpha}_2 F^j_y + \eta_y + \epsilon^j_y$$

where the vector F^j_y includes the weighted averages of bond fundamentals, proportions of interdealer and dealer-customer transactions, liquidity measures, etc, and the year fixed effect η_y controls the time window of cumulative information used to construct the data points.

The regression results in Table 2.3 indicate that, at the bond level, a higher magnitude of misallocation among the dealers (a lower level of $\overline{\widehat{Cov}}_y(\hat{\delta}^j_{i,ey}, I^j_{i,ey})$) is associated with a higher magnitude of liquidity risk (a higher absolute magnitude of $\beta^j_{SysSearch,y}$).

Table 2.3: Correlation of bond misallocation and liquidity risk

$\beta^j_{SysSearch,y}$ (< 0)	(1)	(2)	(3)	(4)
$\widehat{Cov}_y(\hat{\delta}^j_{i,ey}, I^j_{i,ey})$ ($1,000 \times \%$)	0.25***	0.25***	0.25***	0.22***
	(6.00)	(6.00)	(5.99)	(5.14)
$turnover^j_y$ (%)		0.06	0.09	0.06
		(0.78)	(1.07)	(0.50)
$Num\_DD^j_y$ (thousand)			-0.64***	-0.47**
			(-4.23)	(-3.07)
$Num\_DC^j_y$ (thousand)			0.26**	0.20*
			(2.92)	(2.11)
$Amtout^j_y$ (\$trillion)				-17.81
				(-0.24)
TTM^j_y (thousand days)				0.06***
				(5.47)
$Rating^j_y$				0.06***
				(4.56)
Adj R^2	0.02	0.02	0.03	0.03
F statistics	52.11	47.82	43.12	38.26
# of Bonds	4754	4754	4754	4754
# of Obs	22359	22359	22359	22359
Year FE	YES	YES	YES	YES

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. All the following variables are weighted averages within the time window $[1, y]$: $turnover^j_y$ is turnover rate which is the ratio of total trading volume to total outstanding amount; $Num\_DD^j_y$ and $Num\_DC^j_y$ are numbers of interdealer- and dealer-customer transactions; bond fundamentals include outstanding amount $Amtout^j_y$, time to maturity TTM^j_y, and credit rating $Rating^j_y$.

Finally, in Table 2.7 of Appendix 2.A.2.4, we show that the bond-level liquidity risk attributed to OTC search frictions is on average compensated by 8 bps yield spread across all bonds. We extend the yearly panel data by adding the cumulative weighted average yield spread for each point in the data. As a result, the value of compensated yield spread is also at a weighted aveage level on a cumulative basis. It also varies across different bonds with a maximum value as high as 66 bps. An increase in liquidity risk of one standard deviation is associated with around 18 bps.

2.5 Numerical explanation by search-and-match model

In this section, we apply the numerical solutions of the model in Section 2.4.1 under different sets of parameters to give an explanation for the correlation between bond's misallocation among dealers and liquidity risk. The numerical solutions imply that dealers' endogeneous and state-dependent search intensity works as an important channel which connects bond's misallocation and liquidity risk.

The mechnism is: in equilibrium where the covariance of dealers' private valuation and inventory position is at a lower level, there are more dealers holding bond positions that are less aligned with their private valuation types. Specifically, there is a larger proportion of dealers who hold higher(lower)-than-average inventory positions but have lower(higher)-than-average private valuation types. This motivates more dealers in the market to choose a higher level of search effort to buy or sell to adjust their holding positions. We denote the average level of search effort in the dealer sector as $\frac{\Lambda}{m}$, where $\frac{\Lambda}{m} = \int_{\delta_\ell}^{\delta_h} \lambda(1,\delta)\frac{\phi_1(\delta)}{m}d\delta + \int_{\delta_\ell}^{\delta_h} \lambda(0,\delta)\frac{\phi_0(\delta)}{m}d\delta$. With a qudratic-form search cost $c \times \left(\frac{\Lambda}{m}\right)^2$, the average level of marginal search cost in the dealer sector can be approximated by $2c \times \frac{\Lambda}{m}$. Since this average marginal cost will be compensated by bond's average price (yield), those bonds with a higher level of dealers' average search effort $\frac{\Lambda}{m}$ will have their average transaction price (yield) more exposed to shocks to OTC search frictions c.[16]

In Figure 2.3, we draw the numerical solutions of the stationary equilibria in six markets with different levels of bond misallocation $Cov(\delta, a)$. We focus on how bond's liquidity risk attributed to OTC search friction c varies across different markets *at the each level of* c. Specifically, we vary the Poisson intensity α at which customers' private valuation types

[16]There also exists a second-order effect of change in search friction c on the aggregate search intensity Λ, but the magnitude of this effect is dominated by the first-order effect when equilibrium aggregate search intensity is at a high level.

switch between low and high values, to generate the varying level of bond misallocation $Cov(\delta, a)$ across the markets.[17] In this figure, bond's average transaction price $\overline{P}$ is defined as the weighted average price across all transactions, and bond's price sensitivity to OTC search frictions is then defined as the corresponding derivative $\frac{\partial \overline{P}}{\partial c}$. This derivative has negative values since a higher level of search friction implies a lower level of average transaction price to compensate dealers and customers with a higher yield. Since bond's price fully determines its yield spread under fixed risk-free rate, $\frac{\partial \overline{P}}{\partial c}$ can also be regarded as a theoretical counterpart of the bond liquidity risk $\beta_{SysSearch}$ as estimated in data. In subgraph-D of Figure 2.3, we further construct the derivative of bond's yield with respect to search frictions c, which approximates the *negative* of the factor loading $\beta_{SysSearch}$.[18]

The numerical solutions verify that: at each fixed level of search frictions c, when we move from the market with the highest magnitude of bond misallocation ($\alpha = 0.75$) to the market with the lowest magnitude of bond misallocation ($\alpha = 0.25$), the absolute magnitude of bond's liquidity risk attributed OTC search frictions $\frac{\partial \overline{YTM}}{\partial c}$ will increase across markets. Specifically, in the market with the highest bond misallocation (($\alpha = 0.75$)), as OTC search friction c increaes by one unit, the bond's yield spread will increase by the largest value among all the markets.

[17] Intuitively, for a bond with a higher α, customers receive i.i.d. shocks on their private valuation types at a higher intensity which drives customers to more frequently search to trade with randomly selected dealers. This increases the likelihood that a low-type dealer-nonowner or a high-type dealer-owner meets and trades with customers, because there always exists a positive trading surplus between a high-type customer-nonowner (low-type customer-owner) and any dealer-owners (dealer-nonowners). Therefore there will be a larger proportion of dealers holding inventory positions which are not well-aligned with their private valuation types and the value of $Cov(\delta, a)$ will be lower.

[18] Bond's yield is approximated by the formula $ApproxYTM = \frac{C + \frac{F-P}{n}}{\frac{F+P}{2}}$ where we choose time to maturity $n = 5$, face value $F = 100$, and coupon rate $C = 0$.

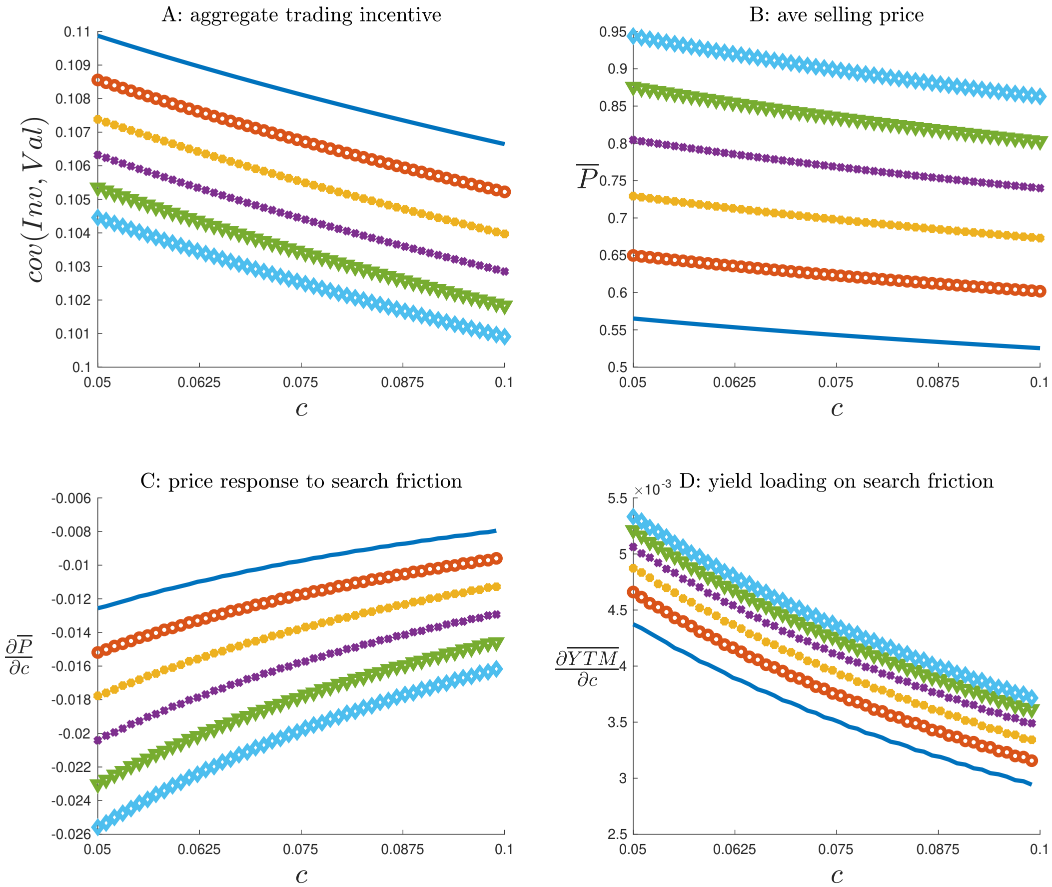

Figure 2.3: Aggregate trading incentive and bond price sensitivity to search friction ($s = \pi_h = 0.5$, $y_\ell = 0.5$, $y_h = 1.7$, $\delta_\ell = 0.6$, $\delta_h = 1.6$, $\rho = m = \theta = 0.5$, $r = 0.05$, $c \in [0.05, 0.1]$ and $\alpha \in [0.25, 0.75]$)

2.6 Conclusion

In this paper, we propose a measure of corporate bond's misallocation among dealers and document that this measure is closely correlated with corporate bond's liquidity risk attributed to OTC search frictions. This measure of bond's misallocation is based on a structural search-and-match model with dealers' endogeneous search efforts, and it is defined as the cross-sectional covariance of dealers' private valuations for holding the bond and their actual inventory positions in the bond. Using the TRACE data for the U.S. corporate bond market, we construct a panel data which contains yearly series of empirical estimates of bond's misallocation and liquidity risk, and we verify that: at the bond level, a higher magnitude of misallocation among the dealers (or a lower level of the cross-sectional covariance of dealers' private valuations and inventory positions) is associated with a higher magnitude of liquidity risk. This finding gives a preliminary market microstructural evidence which shows that: the distribution of market maker's states correlates with the magnitude of corporate bond's liquidity risk. The numerical solutions of the search-and-match model gives a preliminary explanation on how the bond's misallocation affects bond's liquidity risk attributed to OTC search frictions, through driving dealers' investment in search efforts.

Appendix 2.A Appendix of Chapter 2

2.A.1 Estimate of dealers' private valuation

In the search-and-match model, we denote dealer-owners' value function as $V_1(\delta)$ and dealer-nonowners' value function as $V_0(\delta)$, for $\delta \in [\delta_l, \delta_h]$. Then we define dealers' reservation value function for the bond is $\triangle V(\delta) = V_1(\delta) - V_0(\delta)$, for $\delta \in [\delta_l, \delta_h]$, which measures how much compensation each dealer requires for giving up holding one position of the bond. In the bilateral search environment, when two dealers (suppose one holds one position of the bond and the other does not hold any position) with different private valuations meet, trading only happens when the dealer-owner's private valuation is lower than that of the dealer-nonowner. The realized transaction price is determined by a symmetric Nash bargaining process. Specifically, for a dealer with a type $\delta \in [\delta_\ell, \delta_h]$, her transaction price with another dealer with a type $\delta' \in [\delta_\ell, \delta_h]$ is:

$$P(\delta, \delta') = \frac{\triangle V(\delta) + \triangle V(\delta')}{2}$$

where whether $P(\delta, \delta')$ is a selling or buying price depends on whether the dealer δ "holds the bond and search on her sell side" or "does not hold the bond and search on her buy side".

For transactions happening on the sell side of the dealer δ, since $\triangle V(\delta') > \triangle V(\delta)$(or the transaction would not happen), if it is possible for dealer δ to meet a continuum of other dealers, the lowest selling price is exactly equal to $\triangle V(\delta)$. Vice versa, on the buy side of the dealer δ, since $\triangle V(\delta') < \triangle V(\delta)$, the highest buying price is exactly equal to $\triangle V(\delta)$. Again based on monotonicity of $\triangle V(\delta)$, in data, we construct the following consistent estimator[19]

[19] In finite samples, on the buy side of each dealer, the maximum buying price is a downward biased estimate for the dealer's marginal valuation; on the sell side, the minimum selling price is an upward biased estimate for the dealer's marginal valuation. Taking the average of the sample maximum buying price and the sample minimum selling price will make the bias cancel out. In small samples with dealers' unbalanced

as a proxy for dealers' private valuation type δ:

$$\hat{\delta}^j_{i,t} = \frac{max\{Buy^j_{i,n^{j,B}_{i,t}}\} + min\{Sell^j_{i,n^{j,S}_{i,t}}\}}{2}$$

where $\{Buy^j_{i,n^{j,B}_{i,t}}\}$ ($\{Sell^j_{i,n^{j,S}_{i,t}}\}$) is the collection of all buying (selling) prices by dealer i for bond j within month t and $n^{j,B}_{i,t}$ ($n^{j,S}_{i,t}$) is the correspondinng number of total buying (selling) transactions (including both dealer-customer and interdealer transactions) within month t. [20]

2.A.2 Bond liquidity risk attributed to search frictions

2.A.2.1 Factors driving yield spread change

The full regression results are in Table 2.4.

2.A.2.2 Intermediation chain

The matching algorithm to construct intermediation chains is an extension of the algorithms in Hollifield, Neklyudov, and Spatt (2017) and Li and Schürhoff (2014). Similarly, the intermediation chains start from customer-sell-to-dealer trades and end at dealer-sell-to-customer trades. We also use the first-in-first-out(FIFO) matching algorithm to look for the next trades for each incomplete chain. The main difference is, we only allow the split matching in the first round of the loop. After the first round, we track a fixed par amount of a bond until finding the final customer buyer.

Each intermediation chain starts from a trade that a customer C_s sells some amount of a bond to a dealer D_1. We then look for the next trade completed by dealer D_1 selling to a customer

buy and sell trades, the levels of the upward bias and the downward bias may not be equal. Then to make the bias cancel out completely, the weights assigned on the two extreme prices can be adjusted according to the realized number of buy and sell trades.

[20] In quantitative analysis, we define each market by one bond j and one quarter q. Each dealer i's private valuation for bond j in quarter q is calculated as the weighted average of all monthly private valuations $\hat{\delta}^j_{i,t}$ in quarter q weighted by dealer i's monthly total trading volume in bond j.

or another dealer within a calendar time window from -1 day to +30 days around the initial C_s-sells-to-D_1 trade. The initial trade is then followed by a trade that the dealer D_1 sells the same amount (of the same bond) either to a customer C_e or to another dealer D_2. In the first case of selling-to-C_e, the current intermediation chain ends and it is recorded as a CDC chain, that is, there is one dealer on the chain; In the second case of selling-to-D_1, the current intermediation chain is not ended and is temporarily recorded as an incomplete chain CDD. We continue looking for trades completed by dealer D_2 selling to a customer or another dealer within the same calendar time window. This process will continue until finding a dealer-sell-to-customer trade of the same bond in same par amount.

We only consider "split matching" in the first round of loop in the sense that, given the initial C_s-sell-to-D_1 trade, we look for a trade with D_1 as the seller of the same bond and with the shortest time gap to the initial trade. Suppose the initial trade has par amount Q_1 and the next closest trade is "dealer D_1 sells Q_2 of the same bond to a dealer D_2". Then if $Q_1 > Q_2$, that is, the initial trade has larger par amount than the second trade, we split Q_1 into two pieces Q_2 and $Q_1 - Q_2$, and we record a new incomplete chain CDD with par amount Q_2 and put the remaining par amount $Q_1 - Q_2$ (sold by C_s to D_1) back to the pile of initial customer-to-dealer trades to be used to initiate new intermediation chains; If $Q_1 < Q_2$, similarly, we split Q_2 into two pieces Q_1 and $Q_2 - Q_1$, and we record a new incomplete chain CDD with par amount Q_1 and put the remaining par amount $Q_2 - Q_1$ (sold by D_1 to D_2) back to the pile of candidate interdealer trades that will be used to generate more intermediation chains. After the first round of the loop, for all incomplete chains CDD, we restrict that all matched trades on the same intermediation chain after the first round need to have exacty the same par amounts. Same as Li and Schürhoff (2014), we allow for up to 7 dealers on an intermediation chain. Figure 2.4 shows the "split matching" in the first round.

The matching algorithm matches a total of 6.7 million of complete intermediation chains. Table 2.5 reports the average trading information of intermediation chains of each length. The average trading size is generally lower for longer chains, which implies that it is more difficult for a larger amount of bond to be reallocated from the initial customer seller to the final customer buyer through too many dealers, since dealers may tend to split the large amount into smaller pieces when they trade with each other in the interdealer market. The total markup increases with the

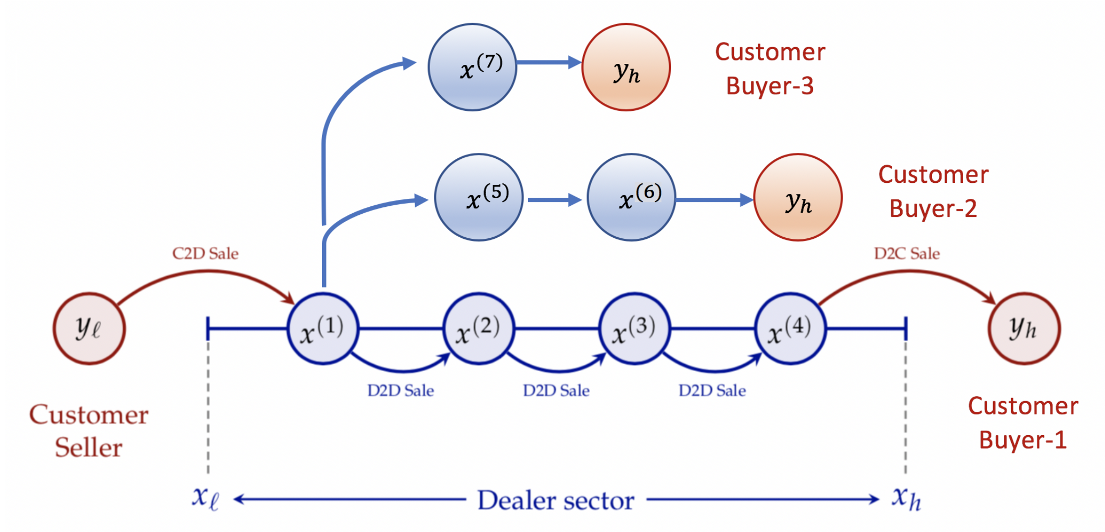

Figure 2.4: Split matching in constructing intermediation chains

chain length, because dealers on average buy at lower prices and sell at higher prices to gain the intermediation profit. The total time gap also increases with the chain length, which is consistent with our expectation that in an interdealer market with the level of search frictions fixed, it takes a longer time for dealers to implement more trades with each other to form longer chains.

Table 2.6 reports the average bond information of intermediation chains for each length, which implies that dealers' search dynamics are heterogeneous across different bonds. This also motivates the extension of my preliminary model to consider the case of multiple assets.

2.A.2.3 Heterogeneous bond-level liquidity risk

Figure 2.5 shows that for individual bonds, although the mean and median of $\beta^j_{SysSearch}$ are both negative, there exist quite a portion of bonds with positive $\beta^j_{SysSearch}$. Moreover, within the bonds of negative $\beta^j_{SysSearch}$, the absolute level of $\beta^j_{SysSearch}$ is heterogeneous across individual bonds. The more negative $\beta^j_{SysSearch}$ is, the more sensitively that bond j's yield responds to innovation in OTC search frictions. In Section 2.A.2.4, we verify that the factor loading $\beta^j_{SysSearch}$ is significantly priced in corporate bond yield spread, in the sense that bonds with more negative $\beta^j_{SysSearch}$ will on average exhibit a higher level of yield spread.

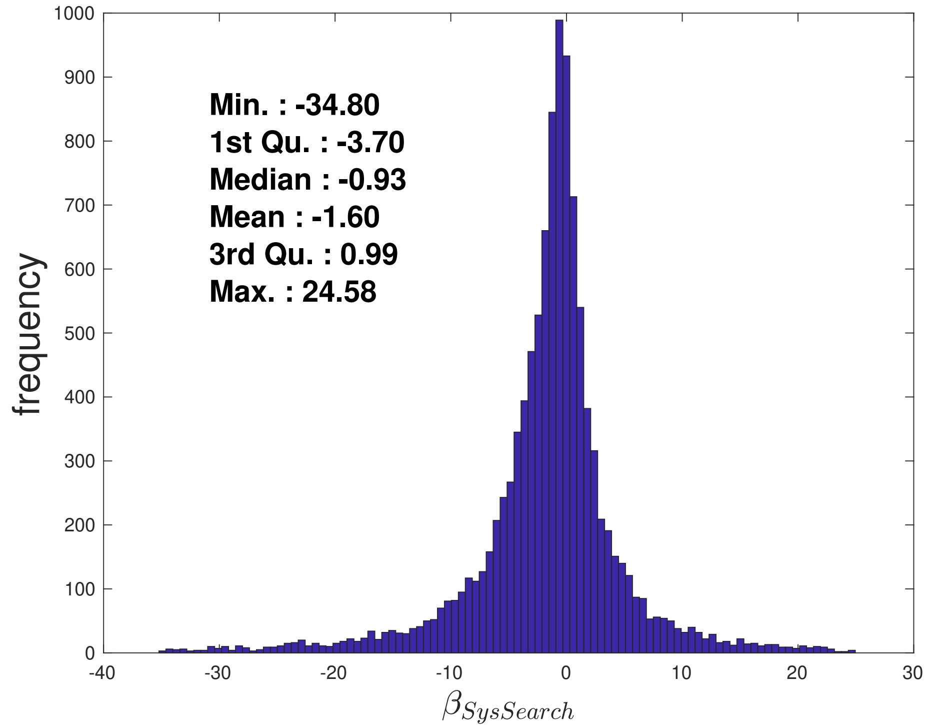

Figure 2.5: Distribution of yield loading on systemic search friction (bond-level yield spread loadings for 11176 bonds)

2.A.2.4 Bond liquidity risk and level of yield spread

In this section, we test whether corporate bond's liquidity risk is priced in cumulative weighted average yield spread. Again, we estimate a reduced-form panel data model using the yearly panel data. We extend the data by adding a cumulative weighted average yield spread $YieldSpread_{j,y}$ for each point in the data. The model is:

$$YieldSpread_{j,y} = \lambda_{SysSearch} * \beta^j_{SysSearch,y} + \lambda_{SysNetConcen} * \beta^j_{SysNetConcen,y}$$

$$+\lambda_{prearrange} * \gamma^j_{1,prearrange,y} + \lambda_{inv} * \gamma^j_{1,inv,y}$$

$$+\lambda_{blocktrade} * \gamma^j_{2,blocktrade,y} + \lambda_{HHIdealer} * \gamma^j_{2,HHIdealer,y} + \overline{BF}^j_y + \eta_y + \epsilon^j_y$$

where $\overline{BF}^j$ is a collection of bond-specific factors that are also important determinants of bond's yield spread, including bonds' liquidities measured by Amihud[21], trade concentration (among dealers), credit rating, bond-specific search frictions[22] and number of trades in segmented markets (interdealer market and dealer-customer market). All the points in the data are calculated by the time window $[1, y]$.

Table 2.7 shows that, nearly all of bond's exposures to OTC market frictions are consistently compensated by bond's yield spread. Specifically, since the estimated bond's liquidity risk $\beta^j_{SysSearch,y}$ is on average negative, the estimation results establish that a higher magnitude of liquidity risk (more negative $\beta^j_{SysSearch,y}$) implies a higher yield spread level. The regression results are robust when adding a collection of bond-specific factors or using truncated sample in which the max and min values of $\beta^j_{SysSearch,y}$ are both within three standard deviations from the mean level.

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[21] $Amihud^j_y$ is a liquidity measure proposed by Amihud (2002), which is calculated as the average absolute value of daily return divided by daily par dollar volume. Specifically, $Amihud^j_t = \frac{1}{d_{j,t}} \sum_{\tau=1}^{d_{j,t}} \frac{|r_{j,\tau}|}{Volume_{j,\tau}}$, where $d_{j,t}$ is the number of days with observed returns in month t for bond j, $r_{j,\tau}$ is the return for bond j on day τ, and $Volume_{j,t}$ is the par dollar volume traded on day τ.

[22] Bond-specific search frictions refer to the average time interval between consecutive trades on each intermediation chain, excluding the head and tail trades. The reason we exclude the head and tail segments of intermediation chains is that these trades are more likely to be pre-arranged or more likely imply directed search of investors instead of the random search we focus on.

Table 2.4: Bond yield loadings on multiple factors

$\triangle(YieldSpread)_{j,t}$ (%)	(1)	(2)	(3)
$\triangle SystemChainLength_t$	-2.32***	-1.67***	-1.55***
	(-32.80)	(-21.38)	(-21.38)
$\triangle SysNetConcen_t$ (thousand)	-9.83e-03***	-4.77e-03***	-4.43e-03***
	(-48.16)	(-22.30)	(-20.10)
$\triangle inv_{t-1}$ ($trillion)	7.55***	5.55***	5.74***
	(24.07)	(16.39)	(17.51)
$\triangle prearrange_t$ (%)	0.26***	1.28***	1.08***
	(3.43)	(15.87)	(13.43)
$\triangle blocktrade_t$ (%)	-66.67***	-29.29***	-28.65***
	(-50.94)	(-22.37)	(-22.15)
$\triangle amtout_t$ ($trillion)	-0.32***	-0.47***	-0.38***
	(-6.17)	(-8.12)	(-6.54)
$\triangle HHIdealer_t$ (thousand)	-1.04***	-0.67***	-0.67***
	(-46.70)	(-28.99)	(-30.82)
$\triangle RF_t$		-2.72***	-2.68***
		(-41.53)	(-22.15)
$(\triangle RF_t)^2$		-8.61***	-1.13***
		(-9.66)	(-14.96)
$R_{MKT,t}$		6.35***	6.31***
		(60.26)	(61.05)
$\triangle SLOPE_t$		0.24***	0.27***
		(19.93)	(22.67)
$R_{SMB,t}$		0.53***	0.44***
		(5.53)	(4.70)
$R_{HML,t}$		0.22**	0.08
		(2.05)	(0.75)
$R_{UMD,t}$		-2.58***	-2.60***
		(-34.07)	(-34.74)
$\triangle turnover_t^j$			-1.45e-03 ***
			(-3.10)
$Rating_t^j$			6.41e-03 ***
			(3.41)
TTM_t^j			1.20e-05 *
			(1.96)
$Mean\ Adj\ R^2$	0.18	0.35	0.37
$\#ofBonds$	11176	11176	9595
$\#ofObs$	515514	515514	479146

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Reported estimated coefficients are average values taken across all bonds. Similar to Friewald and Nagler (2018), the t-statistics are calculated by dividing each reported (average) coefficient value by the standard deviation of the estimates and scaling by the square root of the number of bonds.

Table 2.5: Chain Length and Trade Information (Jan 2005 - Sep 2015)

	Num (thousands)	Vol($1,000)	Markup(%)	Total time(mins)	Pre-arranged(%)
CDC	3982.47	1092.33	0.999	10591.89	21.26
C(2)DC	1180.52	181.57	1.317	15192.10	2.37
C(3)DC	1028.50	155.09	2.102	16253.38	1.73
C(4)DC	351.85	55.57	2.334	19404.53	0.52
C(5)DC	104.86	112.42	2.112	25066.72	0.07
C(6)DC	32.57	64.68	2.374	34231.25	0.03
C(7)DC	12.69	125.46	2.272	40545.61	0.03

Note: C(i)DC means there are i dealers on the chain; Vol($1,000) is the average trading volume per chain calculated for each length throughout the whole sample period; Markup(%) is the average total markup per chain calculated for each length throughout the whole sample period. For each chain, the total markup is calculated by using the last dealer-sell-to-customer price on the chain minus the initial customer-sell-to-dealer price, then dividing the difference by the initial customer-sell-to-dealer price; Total time(mins) is the average total time gap per chain calculated for each length throughout the whole sample period. For each chain, the total time gap (in minutes) is the length of time between the time point at which the last dealer-sell-to-customer trade happens and the time point at which the initial customer-sell-to-dealer trade happens; We record an intermediation chain as being pre-arranged if its total time is shorter than 1 minute.

Table 2.6: Chain Length and Bond Information (Jan 2005 - Sep 2015)

	Investment-grade(%)	Amount out($million)	Maturity(years)	TTM/TTO
CDC	68.21	881.92	10.85	22.54
C(2)DC	81.53	1169.34	10.54	4.89
C(3)DC	71.81	961.74	10.84	5.58
C(4)DC	68.99	964.9	11.30	3.46
C(5)DC	61.42	1042.67	11.24	4.72
C(6)DC	54.63	1370.50	11.27	4.04
C(7)DC	50.42	1490.65	11.20	5.26

Note: The higher the value of "Credit rating" is, the lower the credit rating of the bonds under an S&P rating scheme; Investment-grade(%) is the proportion of bonds that are investment grade ones with S&P credit ratings as BBB- or higher; Amount out($million) is the bonds' amount outstandings; Maturity(years) is the bonds' whole maturities; TTM/TTO is a calculated ratio of time to maturity versus time to offering, which is used to measure whether a bond is relatively young or not.

Table 2.7: Level of yield spread and factor loadings on systemic OTC market frictions

$YieldSpread_{j,y}$ (%)	(1)	(2)	(3)	(4)
$\beta^j_{SysSearch,y}$	-0.02***	-0.01***	-0.08***	-0.05***
	(-12.92)	(-10.81)	(-18.83)	(-13.53)
$\beta^j_{SysNetConcen,y}$	0.40	0.10	1.23*	2.08***
	(1.55)	(0.52)	(2.21)	(4.74)
$\gamma^j_{1,prearrange,y}$	226.8***	114.7***	738.8***	385.1***
	(23.56)	(15.76)	(41.13)	(26.82)
$\gamma^j_{1,inv,y}$	4.78e-03***	2.36e-03***	12.48e-03***	6.82e-03***
	(24.04)	(15.72)	(34.08)	(23.43)
$\gamma^j_{2,blocktrade,y}$	-24.73***	-10.81***	-58.19***	-28.91***
	(-40.18)	(-22.98)	(-54.17)	(-33.15)
$\gamma^j_{2,HHIdealer,y}$	-0.03***	-0.02***	-0.13***	-0.08***
	(-9.44)	(-8.91)	(-22.81)	(-19.16)
$Amihud^j_y$		212.4***		399.2***
		(6.35)		(7.44)
$Rating^j_y$		0.47***		0.37***
		(173.79)		(130.01)
$HHIdealer^j_y$ (1,000)		-0.10***		-0.11***
		(-12.16)		(-14.23)
$ChainTimeGap^j_y$ (mins)		9.91e-05***		9.50e-05***
		(15.81)		(15.57)
$Num\_DD^j_y$ (1,000)		0.42***		0.42***
		(10.61)		(11.30)
$Num\_DC^j_y$ (1,000)		-0.24***		-0.17***
		(-11.15)		(-7.93)
$Adj\ R^2$	0.08	0.48	0.20	0.51
Year FE	YES	YES	YES	YES
# of Obs	41332	41332	28932	28932
# of Bonds	11176	11176	8803	8803

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Regression (1) and (2) use all the bonds (obs). Regression (3) and (4) use the truncated sample which is obtained by dropping the bonds with $\beta^j_{SysSearch}$ ranked within the top and bottom 15% of the whole range of all the bonds, to eliminate the possible effect from extreme values. The reason we choose 15% as the cutoff is, by doing this, in the truncated sample, the max and min values are both within three standard deviations away from the mean.

CHAPTER 3

Agent's Social Optimal Meeting Technology in Over-the-Counter Markets

3.1 Introduction

Over-the-counter (OTC) market played an important role in the 2008 financial crisis. Nearly all of the securities and derivatives involved in the financial turmoil that began with a 2007 breakdown in the U.S. mortgage market were traded in OTC markets. [1] There have been some common stylized facts in OTC markets documented by a series of papers, one of which is the stable core-periphery interdealer network. For example, Li and Schürhoff (2014) documents the structure of dealer network in the municipal bonds market and concludes that the dealership exhibits a stable core-periphery structure based on measures such as the number of trading connections and the order flow between dealers; Hollifield, Neklyudov, and Spatt (2017) also documents the core-periphery network structure of the market for the 144a and registered instruments; Bech and Atalay (2010) uses federal fund loans data to analyze the topology of the daily networks and documents the similar pattern.

Existence of the core-periphery interdealer network can be attributed to dealers' heterogeneity in meeting technologies as in Farboodi, Jarosch, and Shimer (2017b). Dealers choosing more advanced meeting technology behave more active and have larger centrality in the interdealer network. Dealers choosing less advanced one behave less active and

[1] Randall Dodd, Markets: Exchange or Over-the-Counter, International Monetary Fund. `https://www.imf.org/external/pubs/ft/fandd/basics/markets.htm`

lie closer to the periphery of the network. In Neklyudov (2012), meeting technology is interpreted as trading frequency which is a result of costly investment in customer-relations capital (also highly correlated with interdealer activeness) and also legal support and extent of in-house expertise.

In this paper, we construct a search-and-bargain model with dealers being free[2] to choose and change their meeting technology (or the search intensity in the model) based on their own asset position and liquidity need, which is new to the current literature, to explain the formation of core-periphery interdealer network in different market environments. Then the model is applied to evaluate the effectiveness of policy responses targeting at different groups of dealers in response to an unexpected aggregate liquidity shock. Then we further discuss whether intermediation service is necessary in social optimal solution and its policy implication. In our model, the trading motive between two randomly matched counterparties comes from the difference in their current holding positions and private valuations for the target asset, which determines the current flow utility received from holding the asset.[3] Our model is closest to Hugonnier, Lester, and Weill (2018) and Farboodi, Jarosch, and Shimer (2017b). Hugonnier, Lester, and Weill (2018) contributes to the literature by firstly analyzing the microstructure and trading patterns in OTC market through the heterogeneity in trader's private valuation on the target asset. And they maintain the assumption of homogeneous search intensity among all traders. Farboodi, Jarosch, and Shimer (2017b) contributes to firstly discussing the formation and welfare consequences of endogenous heterogeneity in trader's search intensity (also interpreted as meeting technology) more from a social planner perspective. Based on their model setup, the meeting technology is invariant

[2]Here the "being free" means dealers are allowed to have time varying meeting technology or the adjustment cost of meeting technology can be regarded as zero, but dealers are still subject to investment cost of meeting technology. This mainly contrasts our model with that in Farboodi, Jarosch, and Shimer (2017b) in which they assume agents' meeting technologies, once initially chosen, will be time invariant, which is equally like assuming infinite adjustment cost.

[3]This setting of trading motive is consistent with a long and fast growing literature following Duffie, Gârleanu, and Pedersen (2005), which will be discussed more in Section 1.1.

once it is determined for each individual trader. While our model discusses the endogenous heterogeneity in trading frequency more from a competitive equilibrium perspective: agents choose their current search intensity based on their current utility type[4] and asset position, and we allow agents to adjust their search intensities once their utility types shift up or down, or their asset positions change through trading with others. In other words, there exists a one-to-one mapping between the two-dimensional state variable "utility type and asset position" and "search intensity" in our model.

In this paper, we mainly focus on the stationary equilibria in the interdealer market where the distribution of dealers' utility type is convex and symmetric with respect to the intermediate-level utility type. Such equilibria can give us equilibrium components which are more interesting and consistent with the economic intuition. We characterize the stationary equilibria and show that asset owner's optimal meeting technology is monotonically decreasing with respect to his valuation on the asset and asset nonowner's optimal technology is monotonically increasing with respect to his valuation, which is consistent with the general intuition that, for a nonowner (owner) with extremely high (low) valuation on the asset, he has very strong incentive to search inside the market to correct his misaligned asset position through trading with his potential counterparties. Then we characterize the weighted average optimal meeting technology for each group of agents with a certain level of utility type. We find that, in less-frictional market environments, which is mainly characterized by a lower searching cost and a lower Poisson intensity of idiosyncratic liquidity shock, the intermediate-utility-type agents will behave most active thus becoming the core-dealers; while in more-frictional market environments, where there is a higher searching cost and a higher intensity of idiosyncratic liquidity shock, the extreme-utility-type agents will behave relatively more active (or even most active) thus playing the role of core-dealers and the

[4]Since this paper mainly focuses on the interdealer market, we use "agents", "investors", "intermediators", "market makers" and "dealers" interchangably, but they all refer to the dealers in the OTC market in this paper. Also, in this paper, we use "search intensity", "trading frequency" and "meeting technology" interchangably, and we use "utility type", "liquidity needs" and "valuation on the asset" interchangably.

intermediate-utility-type agents will instead behave like periphery-dealers. Then we characterize each agent's contribution to different measures of market liquidity, e.g. expected instantaneous gross and intermediation trading volume, total intermediation profit, intermediation profit per trade, and etc. We find that, the magnitude of gross trading volume is consistent with agent's activeness but for the intermediation profit, which is also proxy for dealers' bid-ask spread, intermediate-utility-type agents always contribute the highest level of aggregate bid-ask spread and the lowest level of bid-ask spread per trade.

Besides the implication for the formation of core-periphery interdealer network, we discuss the effects of different rescue policies in response to a certain form of aggregate liquidity shock, where we assume a certain proportion of both asset owners and nonowners of higher-than-intermediate utility types will have their types shifted down by a certain amount, thus suddenly changing the distribution of dealers' utility type. We simply define the form of rescue policy to be that, policy targetting at a certain group of agents will maintain those agents' liquidity needs as their pre-shock levels right after the aggregate liquidity shock occurs. In reality, this policy is implemented through directly injecting liquidity into the dealers. We conclude that, in all the market environments, policy that targets on dealers of higher-than-intermediate utility types dominates the other ones in terms of recovering the whole market's liquidity level. Since such group of agents will choose different meeting technologies in different market environments and it is easier for regulatory institutions to identify dealers by their trading frequency and trading volume per unit of time, then our model gives the policy implication that in less-frictional market, it will be better to firstly inject liquidity into those less active (more periphery) dealers while in more frictional market, it will be better to firstly save those more active dealers.

Finally, we discuss the policy function of social optimal meeting technologies. We find that, it is always optimal for asset owners with higher-than-intermediate utility types to remain silent (not search to sell) and correspondingly, asset nonowners with lower-than-intermediate utility types to remain silent (not search to buy). In other words, there is no intermediation in our social optimal solution, since there does not exist any single agent being assigned with positive meeting technologies in both asset-owner and asset-nonowner densities. Moreover, agents with extremely mis-aligned asset positions will be assigned with higher level of meeting technologies compared with competitive equilibrium solution. These results, which are counterintuitive to the results in current literature, possibly come from the setting of our social welfare objective function. If we define the asset owners with higher-than-intermediate utility types and the asset nonowners with lower-than-intermediate utility types as well-aligned agents, then the social level of well-alignement will be the unique part that positively contributes to the social welfare. And the social level of investment cost in meeting technologies will be the other part that negatively contributes to the social welfare. Then it is intuitive that, for well-aligned agents, it is optimal to make them remain silent to save the investment cost and maintain their current asset holdings, unless they become mis-aligned ones due to idiosyncratic liquidity shocks; for extremely-misaligned agents, it is optimal to make them more actively search to trade to reduce the social level of misalignement.[5] Based on these key results, we can further solve out the explicit solution to the social welfare problem and it coincides exactly with the numerical ones searched out by MatLab. Specifically, in the case of linear cost function, we can obtain the one-dimention policy measure that social planner only needs to identify a marginal utility type for asset owners which is smaller than the intermediate utility type, and assign all asset owners lower than this marginal type with the maximum meeting technology; correspondingly, identify the symmetric marginal utility type for asset nonowners which is higher than the intermediate utility type, and assign all asset nonowners higher than this marginal type with the same maximum meeting technology.

[5]The conclusion that there is no intermediation in the social optimal solution is robust to several cost functions. We discuss this in appendix.

Related literature

There has been a fast-growing literature on documenting and modeling the stylized facts in the OTC asset markets. Besides the core-periphery interdealer network, Li and Schürhoff (2014) also documents the positive correlation between dealers' centrality and spreads they earn in municipal bond market, which is also termed as "centrality premium". The correlation of centrality with other statistics such as inventory, trading cost and difference in bargaining power, etc are also discussed. While in securities market for 144a and registered instruments, Hollifield, Neklyudov, and Spatt (2017) documents the negative correlation between dealers' centrality and spread, which is termed as "centrality discount". Afonso and Schoar (2013) researches the interbank lending market and documents that most banks inside the market form long-term stable lending relationships, which will affect how liquidity shocks are transmitted across the whole market, e.g. banks connected with concentrated lenders will be less affected by the shocks. Moreover, they discover other facts in interbank lending market, such as banks which borrow from a more concentrated and stable set of lenders tend to have smaller sizes, the observed concentration of relationships more likely reflects the need for liquidity hedging among all the market participants, etc. Siriwardane (2015) uses credit default swap (CDS) data to document that the market is dominated by only a handful of market makers (or net sellers) and such concentration of sellers increase the fragility of the market.

For model setup, Duffie, Gârleanu, and Pedersen (2005) firstly constructs a search-and-bargain model with investors of only two utility types and explicit market markers in an OTC market for a "consol". And the interdealer market structure is simplified to be a perfect competitive one which generates a unique interdealer market price. There are papers focusing only on pure dealer markets, e.g., Gârleanu (2009), Lagos, Rocheteau, and Weill (2011), Feldhütter (2011), Pagnotta and Philippon (2018b) and Lester, Rocheteau, and Weill (2015). Specifically, Lagos and Rocheteau (2009) develops a model of liquidity without restricting on asset positions of investors. For papers that model the whole decentralized market and endogenously generate dealers and customers from random searching and bilateral bargaining process, several versions of models are constructed. Duffie, Gârleanu, and Pedersen (2007) considers markets for both asset paying riskless dividend and asset paying risky dividend. Asset position support is restricted to be $\{0, 1\}$ and there are only two utility types (high and low). Weill (2008) extends by constructing a multi-asset model and maintain the restriction on asset positions of investors to be $\{0, 1\}$. Afonso and Lagos (2015) focuses on the market for federal funds and assumes the loan sizes (asset positions) are elements of a countable set. Other related papers are e.g. Vayanos and Wang (2007), Vayanos and Weill (2008), Afonso (2011), Gavazza (2011), Gavazza (2016) and Trejos and Wright (2016). Most of these papers restrict two utility types of the investors, which may potentially prevent the framework from characterizing the stylized core-periphery interdealer network documented in the empirical papers above and analyzing its policy implication. Hugonnier, Lester, and Weill (2018) contributes by allowing arbitrary i.i.d distribution of preference shock to investors and endogenously generate intermediation chains and core-periphery trading networks, which is consistent with the empirical findings. But they maintain the homogeneous search intensity in their model setup. Üslü (2019) constructs model combining unrestricted asset positions and exogenous heterogeneity in search intensities among investors, and he also contributes by using Fourier transformation to generate moments of stationary distributions of variables of interest. Besides Üslü (2019), there are other but not too many papers explicitly assuming heterogeneity in search intensity, e.g. Neklyudov (2012), Farboodi, Jarosch, and Shimer (2017b). The latter, as discussed above, evaluates the distribution of meeting technologies and further allows all agents to endogenously choose their meeting technologies to analyze how distribution of search intensity is generated. They also model investors with only two utility types thus there is no one-to-one mapping between utility type and meeting technology.

Another strand of literature mainly uses explicit network approach to model the formation of links and process of bargaining between traders in the OTC markets, instead of using search-and-bargain model to endogenously generate network characteristics. Related work includes Babus and Kondor (2018), Malamud and Rostek (2017), Alvarez and Barlevy (2015), Farboodi (2014), Gofman (2014), Chang and Zhang (2018). And there are also some papers (including some papers listed above) combing search and network characteristics, including Hugonnier, Lester, and Weill (2018), Farboodi, Jarosch, and Shimer (2017b), Neklyudov (2012), and Shen, Wei, and Yan (2015). Specifically, Atkeson, Eisfeldt, and Weill (2015) develops hybrid model to analyze entry and exit equilibrium conditions in the OTC market for credit default swap. With traders with homogeneous search intensity, they conclude that banks with intermediate risk exposure per trader (essentially like intermediate utility type) and large sizes endogenously enter the OTC market behaving like market maker to gain intermediation profit.

Besides strands of literatures above, there are also some other papers departing from search-and-bargain and network methods and research the OTC market from other perspectives. For example, Acharya and Bisin (2009), Duffie and Lubke (2010).

3.2 Model

We consider the OTC interdealer market for asset in the form of "consol" which pays one unit of dividend per unit of time. This asset is in fixed supply $s = \frac{1}{2}$.[6] There exists a continuum of agents (dealers) $[0, 1]$ who have heterogeneous utility type $\delta \in [0, 1]$ following arbitrary distribution $F_\delta(\delta)$ (with PDF as $f_\delta(\delta)$)[7]. Utility type δ can be interpreted as the current flow utility that agents can receive from holding one unit of such asset. There

[6]In this paper, we will mainly focus on the symmetric equilibria which are more tractable. Similar to Neklyudov(2015), since the switching rates between any two possible utility types are constantly equal to α, the fixed asset supply $s = \frac{1}{2}$ asssumption ensures that the mass of asset owners will be equal to that of asset owners in the steady state dynamic equilibria.

[7]We firstly consider symmetric case that $f_\delta(\delta)$ is convex and symmetric with respect to $\delta = \frac{1}{2}$. In numerical example, we specifically consider the uniform case that $f_\delta(\delta) = 1 \ \forall \delta \in [0, 1]$

is an idiosyncratic Poisson utility type shock arriving at intensity α. Based on their own characteristics (including current utility type, wealth level and asset holding), agents are free to choose their search intensity $\lambda \in [0, \bar{\lambda}]$, where $\bar{\lambda}$ is the upper bound of level of meeting technology that agents can choose[8]. Agents spend $C(\lambda) = c_1\lambda^2$ as flow cost to invest in and maintain their current search intensity λ.

In later analysis, we use the coefficient of flow cost c_1 as a measure of the magnitude of friction on the OTC market. The key difference in microstructure between OTC market and frictionless exchange (Walrasian) market is how fast/advanced meeting technology agents can choose to trade with each other, which can also be equally attributed to how large the c_1 is. Also, the Poisson intensity of idiosyncratic liquidity shock α will be used as a measure of the number of the misaligned agents in the market. Intuitively, higher α makes it easier for agents of high(low) utility types shift to low(high) utility types with their asset position unchanged before trading with others through searching and bargaining. We will talk more about this in Corollary 2 below.

We assume agents have CARA instantaneous utility as $u(c) = -e^{-\gamma c}$ with risk aversion coefficient γ. Agent's wealth is denoted by W and asset holding[9] is restricted to belong to $\{0, 1\}$. Other parameters are risk free interest rate r and agent's discount rate β.

3.2.1 HJB equation for reservation value

Let $U(W, \delta, a)$ be the value function of an agent with wealth level as W, utility type δ and asset holding a, where $a \in \{0, 1\}$. Then as in Duffie, Gârleanu, and Pedersen (2007), the

[8] $\bar{\lambda}$ can be ex-post proved to be finite.

[9] It can be proved expost that in the current model setup, once two counterparties are matched, since their reservation value is strictly increasing in utility type δ, they will either sell or buy as much as possible to obtain gains from trade, thus the asset holding can always be normalized to be either 0 or 1. And we implicitly assume here that short selling is allowed.

agent's problem is:

$$U(W,\delta,a) = \sup_{c,\lambda} E_t[-\int_t^{\infty} e^{-\beta(s-t)} e^{-\gamma c_s} ds | W_t = W, \delta_t = \delta, a_t = a]$$

s.t.

$$dW_t = (rW_t - c_t + a_t\delta_t - C(\lambda_t))dt - P[(W,\delta_t,a_t),(W',\delta_t',a_t')]da_t$$

$$\lim_{T\to\infty} e^{-\beta(T-t)} E_t[e^{-r\gamma W_T}] = 0$$

$$C(\lambda_t) = c_1\lambda_t^2 \quad (c_1 > 0)^{10}$$

where $P[(W,\delta_t,a_t),(W',\delta_t',a_t')]$ is a bilaterally bargained price from symmetric (each agent has same bargaining power) Nash bargaining game between two randomly matched counterparties with state variables as (W,δ_t,a_t) and (W',δ_t',a_t'). da_t is bilateral trading quantity and $da_t \in \{-1,1\}$.

Using notations $U(W,\delta,1) = U_1(W,\delta)$ and $U(W,\delta,0) = U_0(W,\delta)$ as value functions for asset owners and asset nonowners, as in Duffie, Gârleanu, and Pedersen (2007), we can guess and verify the form of $U_1(W,\delta)$ and $U_0(W,\delta)$, and we can get HJB equation for simplified value functions without state variable W as follows. It is important to note that, in this paper, we focus on the rational expectation competitive equilibrium in the sense that, all asset owners and asset nonowners adopt the common policy rule $\lambda_1^*(\delta)$ and $\lambda_0^*(\delta)$ to choose their optimal meeting technology. And the matching technology is constant return to scale.

$$\begin{aligned} rV_1(\delta) = \max_{\lambda_1(\delta)}\{\delta - C(\lambda_1(\delta)) + \alpha \int_0^1 (V_1(\delta') - V_1(\delta))dF_\delta(\delta') \\ + \lambda_1(\delta)\int_0^{\bar{\lambda}}\int_0^1 \frac{\lambda'}{\Lambda_0} \max\{\triangle V(\delta') - \triangle V(\delta), 0\}\Phi_0(d\delta',d\lambda')\} \end{aligned} \quad (3.1)$$

[10] It can be proved expost that if $C(\lambda)$ is linear form, then by F.O.C of λ, optimal $\lambda(\delta)$ will be either 0 or $\bar{\lambda}$. And for the simpliest version of model, we assume $C(\lambda)$ is convex with $C(0) = 0$ and $C'(0) = 0$, i.e. there is no fixed flow entry cost.

$$rV_0(\delta) = \max_{\lambda_0(\delta)} \{-C(\lambda_0(\delta)) + \alpha \int_0^1 (V_0(\delta') - V_0(\delta)) dF_\delta(\delta')$$
$$+ \lambda_0(\delta) \int_0^{\bar{\lambda}} \int_0^1 \frac{\lambda'}{\Lambda_1} max\{\triangle V(\delta) - \triangle V(\delta'), 0\} \Phi_1(d\delta', d\lambda')\} \tag{3.2}$$

s.t.

$$\triangle V(\delta) = V_1(\delta) - V_0(\delta)$$

$$\Lambda_1 = 2 \int_0^{\bar{\lambda}} \int_0^1 \lambda' \Phi_1(d\delta', d\lambda')$$

$$\Lambda_0 = 2 \int_0^{\bar{\lambda}} \int_0^1 \lambda' \Phi_0(d\delta', d\lambda')$$

then we get,

$$\lambda_1^*(\delta) = \frac{\int_0^{\bar{\lambda}} \int_\delta^1 \frac{\lambda'}{\Lambda_0} (\triangle V(\delta') - \triangle V(\delta)) \Phi_0(d\delta', d\lambda')}{2c_1} \tag{3.3}$$

$$\lambda_0^*(\delta) = \frac{\int_0^{\bar{\lambda}} \int_0^\delta \frac{\lambda'}{\Lambda_1} (\triangle V(\delta) - \triangle V(\delta')) \Phi_1(d\delta', d\lambda')}{2c_1} \tag{3.4}$$

where $\lambda_1^*(\delta)$ and $\lambda_0^*(\delta)$ are optimal (instantaneous) search intensity chosen by asset owner and asset nonowner of utility type (liquidity need) δ. $\triangle V(\delta)$ is the reservation value for agent of utility type δ. $\Phi_0(\delta', \lambda')$ is cumulative joint measure of utility type and (optimally) chosen search intensity for asset nonowners. $\Phi_1(\delta', \lambda')$ is cumulative joint measure of utility type and (optimally) chosen search intensity for asset owners. Λ_1 is the weighted average search intensity chosen by asset owners. Λ_0 is the weighted average search intensity chosen by asset nonowners.

Proposition 1 Given the distribution of utility type $F_\delta(\delta)$ with symmetric PDF $f_\delta(\delta)$ and the cumulative joint measures $\Phi_0(\delta', \lambda')$ and $\Phi_1(\delta', \lambda')$: the optimal meeting technology chosen by asset owners $\lambda_1^*(\delta)$ is a decreasing function of utility type δ; the optimal meeting technology chosen by asset nonowners $\lambda_0^*(\delta)$ is an increasing function of utility type δ; the reservation value $\triangle V(\delta)$ is a strictly increasing and positive function of utility type δ. Proof is in Appendix 3.A.1.

For an asset owner, if his utility type is relatively low (or he has higher liquidity need so he has relatively lower valuation for holding the asset), he will have strong incentive to sell his current asset as quickly as possible given his expectation on the joint distribution of asset nonowner's utility type and meeting technology, which means he will (at least temporarily) choose relatively more advanced meeting technology to increase his trading frequency. If asset owner's utility type is relatively high, which means he has lower liquidity need and he is more willing to hold the asset, then such asset owner will behave less active in the market since he has a relatively well-aligned asset position. Similar interpretation works for the optimal meeting technology chosen by asset nonowners. Asset nonowners of relatively high utility types will invest in more advanced technology to eagerly search for potential asset sellers in the market. And asset nonowners of relatively low utility types will have weak incentive to increase their trading frequencies, thus remaining relatively less active.

The increasing property of reservation value function $\triangle V(\delta)$ guarantees that once a lower-type owner and a higher-type nonowner are randomly matched, there will always be gains from trade, thus the bilateral trading quantity will never be zero. And the probability that one agent being matched with another one of the same utility type will approximately be zero.

Based on Proposition 1 and by assuming all agents in the market adopt the same policy rules $\lambda_1^*(\delta)$ and $\lambda_0^*(\delta)$, we can further simplify the HJB equations (3.1) and (3.2), and get the HJB equation for reservation value function as follows:

$$r\triangle V(\delta) = \delta + C(\lambda_0^*(\delta)) - C(\lambda_1^*(\delta)) + \alpha \int_0^1 (\triangle V(\delta') - \triangle V(\delta)) dF_\delta(\delta') \tag{3.5}$$

$$+\lambda_1^*(\delta) \int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0} (\triangle V(\delta') - \triangle V(\delta)) \phi_0(\delta') d\delta' - \lambda_0^*(\delta) \int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1} (\triangle V(\delta) - \triangle V(\delta')) \phi_1(\delta') d\delta'$$

s.t.

$$\lambda_1^*(\delta) = \frac{\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}(\triangle V(\delta') - \triangle V(\delta))\phi_0(\delta')d\delta'}{2c_1} \tag{3.6}$$

$$\lambda_0^*(\delta) = \frac{\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}(\triangle V(\delta) - \triangle V(\delta'))\phi_1(\delta')d\delta'}{2c_1} \tag{3.7}$$

$$\Lambda_0 = 2\int_0^1 \lambda_0^*(\delta')\phi_0(\delta')d\delta'$$

$$\Lambda_1 = 2\int_0^1 \lambda_1^*(\delta')\phi_1(\delta')d\delta' \tag{3.8}$$

$$\phi_0(\delta) = \int_0^{\bar{\lambda}} \Phi_0(d\delta, d\lambda')$$

$$\phi_1(\delta) = \int_0^{\bar{\lambda}} \Phi_1(d\delta, d\lambda')$$

Individual agent's expectation on the joint distribution of asset position (either 0 or 1), utility type δ and adopted search intensity λ can be simplified to the joint densities of asset position and utility type which are denoted by $\phi_0(\delta)$ and $\phi_1(\delta)$, since optimal meeting technology is monotonic with respect to utility type by Proposition 1.

3.2.2 Joint densities of utility type and asset holding

To further discuss the stationary equilibrium in next section, we need to characterize the law of motion for densities of asset owners $\phi_1(\delta)$ and nonowners $\phi_0(\delta)$ of each utility type δ. Let $\hat{f}_\delta(\delta)$ be the distribution of new utility type in response to idiosyncratic liquidity shock and $f_\delta(\delta) = \phi_0(\delta) + \phi_1(\delta)$ be the current distribution of utility type for the whole population. For simplicity, we only consider the case $\hat{f}_\delta(\delta) = f_\delta(\delta) = 1$ in stationary equilibrium[11], we have:

[11] In Section 5, we will consider one form of aggregate liquidity shock with the refinancing channel defined similar as in Duffie et al (2006). The refinancing channel means $\hat{f}_\delta(\delta) \neq f_\delta(\delta) = \phi_0(\delta) + \phi_1(\delta)$, in which the distribution of utiltiy type $f_\delta(\delta)$ can gradually recover to the pre-shock scenario due to the function of $\hat{f}_\delta(\delta)$.

$$\dot{\phi_1}(\delta) = -\alpha\phi_1(\delta) + \frac{\alpha}{2}\hat{f}_\delta(\delta) - 2\phi_1(\delta)\lambda_1^*(\delta)\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\phi_0(\delta')d\delta'$$
$$+ 2\phi_0(\delta)\lambda_0^*(\delta)\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta' = 0 \tag{3.9}$$

$$\dot{\phi_0}(\delta) = -\alpha\phi_0(\delta) + \frac{\alpha}{2}\hat{f}_\delta(\delta) - 2\phi_0(\delta)\lambda_0^*(\delta)\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta'$$
$$+ 2\phi_1(\delta)\lambda_1^*(\delta)\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\phi_0(\delta')d\delta' = 0 \tag{3.10}$$

In both equation (3.9) and (3.10), the first term is the outflow from asset owners (nonowners) of utility type δ due to idiosyncratic liquidity shock. The second term is the inflow due to idiosyncratic liquidity shock. For example in equation (3.9), it is the inflow of asset owners, originally with other utility types, having their types shifted exactly to δ due to liquidity shock. The third term is the outflow due to the implemented bilateral trades based on random searching and bargaining. For example in equation (3.9), asset owners of type δ are matched with asset nonowners of higher utility types, then this subgroup of asset owners will sell their assets to their counterparties and become asset nonowners. The fourth term is correspondingly the inflow due to implemented bilateral trades.

Moreover, $\phi_0(\delta)$ and $\phi_1(\delta)$ at each time point should also satisfy the following conditions[12]:

$$\phi_0(\delta) + \phi_1(\delta) = f_\delta(\delta) \tag{3.11}$$

$$\int_0^1 \phi_1(\delta)d\delta = \int_0^1 \phi_0(\delta)d\delta = \frac{1}{2} \tag{3.12}$$

Equation (3.11) is based on the definition of pdf $f_\delta(\delta)$ and joint densities $\phi_0(\delta)$ and $\phi_1(\delta)$, which shows that each group of agents of the same utility type contains both asset owners

[12]Here we ignore the time subscripts for simplicity since we will discuss the stationary equilibrium in next section.

and nonowners. Equation (3.12) is the market clear condition for the asset in fixed supply $s = \frac{1}{2}$.

3.3 Stationary equilibrium characterization

Definition 1 A stationary equilibrium contains a reservation value function $\triangle V(\delta)$, joint densities of asset position and utility type $\phi_0(\delta)$ and $\phi_1(\delta)$, and optimal meeting technology functions of asset owners and nonowners $\lambda_1^*(\delta)$ and $\lambda_0^*(\delta)$ such that:

1. joint measures $\phi_0(\delta)$ and $\phi_1(\delta)$ satisfy (3.9)-(3.12) for $\forall \delta \in [0, 1]$;

2. reservation value function $\triangle V(\delta)$ satisfies (3.5) subject to (3.6)-(3.8), given stationary distribution $F_\delta(\delta)$ and joint densities $\phi_0(\delta)$ and $\phi_1(\delta)$;

3. optimal meeting technologies satisfy (3.6)-(3.7), given stationary joint densities $\phi_0(\delta)$ and $\phi_1(\delta)$, distribution $F_\delta(\delta)$ and optimal reservation value function $\triangle V(\delta)$.

Proposition 2 There exists stationary equilibrium given uniform distribution of utility type $f_\delta(\delta) \equiv 1$, $\forall \delta \in [0, 1]$ for any $r > 0$, $\alpha > 0$ and $c_1 > 0$. Proof is in Appendix 3.A.2.

Based on (3.9)(3.10), the distribution of utility type seems to be totally exogeneous, since $\dot{f}(\delta) = \dot{\phi}_1(\delta) + \dot{\phi}_0(\delta) = 0$, $\forall \delta \in [0, 1]$. In other words, our model does not exclude the possibility of multiple equilibria characterized by different distributions of utility type. To discuss stationary equilibria, we implicitly assume the distribution of idiosyncratic liquidity shock is the same as the steady-state distribution of utility type in the market.[13] In Section 3.5, when there comes an aggregate liquidity shock, we abandon the above assumption and still maintain the distribution of idiosyncratic liquidity shock same as before, specifically we focus on the case $\hat{f}_\delta(\delta) = 1$ $\forall \delta \in [0, 1]$, which departs from the immediate post-shock distribution of utility type.

[13] It can be ex-post shown that the stationary equilibrium distribution of utility type will eventually converge to that of new utility types from idiosyncratic shock.

3.3.1 The frictionless benchmark

To understand the effect of OTC market search friction on market efficiency and welfare, we firstly characterize the frictionless benchmark, i.e. the Walrasian market. Upon receiving idiosyncratic liquidity shock at intensity α, every agent can adjust his asset position immediately to accommodate his new utility type at the unique price p on the market at each time point.

Assume one agent's current asset position is $a \in \{0, 1\}$ and his immediately adjusted new asset position is $a' \in \{0, 1\}$, $V_a^f(\delta)$ is the value function of agent with asset position a and utility type δ, then we have the HJB equation for frictionless market as:

$$rV_a^f(\delta) = \delta * a + \alpha \int_0^1 \underset{a'}{max} \left[V_{a'}^f(\delta') - V_a^f(\delta) - p(a' - a) \right] dF_\delta(\delta') \tag{3.13}$$

By first order condition of a', we get:

$$a' = \begin{cases} 1 & \text{if } \triangle V^f(\delta') > p; \\ 1 \text{ or } 0 & \text{if } \triangle V^f(\delta') = p; \\ 0 & \text{if } \triangle V^f(\delta') < p. \end{cases}$$

Given fixed asset price p, since $\triangle V^f(\delta)$ is strictly increasing on δ by (3.13), $\exists! \delta^* \in [0, 1]$ s.t. $\triangle V^f(\delta^*) = p$. Since p is market clearing price, as in Hugonnier et al (2016), we have the following market clear condition:

$$\delta^* = inf\{\delta \in [0, 1] : 1 - F_\delta(\delta) \leq \frac{1}{2}\}$$

which means for all agents with utility $\delta > \delta^*$, they will hold the asset and for all agents with utility $\delta < \delta^*$, they will not hold the asset. In other words, $\phi_1^f(\delta) = f(\delta)$, $\forall \delta \in [\delta^*, 1]$ and $\phi_0^f(\delta) = f(\delta)$, $\forall \delta \in [0, \delta^*]$. In the case of fixed asset supply $s = \frac{1}{2}$, $\delta^* = \frac{1}{2}$.

By (3.13), we have the reservation value function for frictionless market as:

$$r\triangle V^f(\delta) = \delta + \alpha(p - \triangle V^f(\delta))$$

Then for $\delta^* = \frac{1}{2}$:

$$\triangle V^f(\delta^*) = \frac{\delta^*}{r} = \frac{1}{2r} = p$$

Since upon receiving the idiosyncratic liquidity shock, every agent can immediately adjust her asset position. Let $a^f(\delta)$ be the instantaneously adjusted asset position for $\delta \in [0,1]$, at each time point, the expected instantaneous total trading volume in the market can be expressed as:

$$\begin{aligned}
TV^f &= \alpha \int_0^1 \int_0^1 |a^f(\delta') - a^f(\delta)| dF_\delta(\delta') dF_\delta(\delta) \\
&= 2\alpha \int_0^{\delta^*} \int_{\delta^*}^1 |a^f(\delta') - a^f(\delta)| dF_\delta(\delta') dF_\delta(\delta) \\
&= 2\alpha(1 - F_\delta(\delta^*))F_\delta(\delta^*) \\
&= 2\alpha s(1 - s) \\
&= \frac{\alpha}{2}
\end{aligned}$$

Intuitively, in frictionless market, all the tradings happen due to the idiosyncratic shock and all the tradings are completed between agents and the Walrasian auctioner. As in Gârleanu(2009) and Üslu(2015), if we sum over all agents' continuation utilities, we can obtain the measure of the social welfare with $f_\delta(\delta) \equiv 1, \forall \delta \in [0,1]$:

$$
\begin{aligned}
W^f &= \int_0^{+\infty} e^{-rt} \left(\int_0^1 \delta \phi_1^f(\delta) d\delta \right) dt \\
&= \frac{1}{r} \int_{\delta^*}^1 \delta f_\delta(\delta) d\delta \\
&= \frac{E[\delta; \delta > \delta^*]}{r} \\
&= \frac{3}{8r}
\end{aligned}
$$

In the following sections, we will calculate the instantaneous trading volume and maximized social welfare in OTC market with search friction, and compare them with the frictionless benchmark above. Also, the agents with mis-aligned asset positions contain both asset owners with $\delta \in [0, \delta^*]$ and asset nonowners with $\delta \in [\delta^*, 1]$.

3.3.2 Equilibrium with symmetric $f_\delta(\delta)$

We firstly consider the case of symmetric and convex distribution of utility type, that is, $f_\delta(\delta)$ is symmetric with respect to $\delta = \frac{1}{2}$ and decreasing in $\delta \in [0, \frac{1}{2}]$ and increasing in $\delta \in [\frac{1}{2}, 1]$. The reason we consider such exogenous distribution is, when $f_\delta(\delta)$ is convex, we can obtain the equilibrium solutions where $\phi_1(\delta)(\phi_0(\delta))$ is monotonically increasing(decreasing). Such equilibria are more interesting as being consistent with the intuition that within the group of agents with lower(higher) liquidity needs, there should exist a larger proportion of asset owners(nonowners). Moreover, we are more interested in the financial stability of the symmetric stationary equilibria characterized by Definition 2 under an aggregate liquidity shock to the whole market (details in Section 3.5).

Definition 2 For symmetric $f_\delta(\delta)$(with respect to $\delta = \frac{1}{2}$), the symmetric stationary equilibrium is defined as follows:

$$\phi_0(\delta) = \phi_1(1-\delta) \qquad \forall \delta \in [0,1] \tag{3.14}$$

$$\lambda_0^*(\delta) = \lambda_1^*(1-\delta) \qquad \forall \delta \in [0,1] \tag{3.15}$$

and all the components also satisfy Definition 1.

By (3.14)(3.15), reservation value $\triangle V(\delta)$ satisfies:

$$\triangle V(0) + \triangle V(1) = \triangle V(\delta) + \triangle V(1-\delta) \quad \forall \delta \in [0,1]$$

and

$$\frac{d^2 \triangle V(\delta)}{d\delta^2} > 0, \forall \delta \in [0, \frac{1}{2}); \quad \frac{d^2 \triangle V(\delta)}{d\delta^2} < 0, \forall \delta \in (\frac{1}{2}, 1]; \quad \frac{d^2 \triangle V(\frac{1}{2})}{d\delta^2} = 0.$$

Proof is in Appendix 3.A.3.

3.3.2.1 Joint densities $\phi_1(\delta)$ and $\phi_0(\delta)$ under symmetric $f_\delta(\delta)$

Proposition 3 For equilibrium with symmetric (either convex or concave) distribution of utility type $f_\delta(\delta)$, if the following condition is satisfied, we will obtain $\phi_0'(\delta) < 0 < \phi_1'(\delta)$, $\forall \delta \in [0,1]$: For $f_\delta(\delta)$, $\nexists\ \delta^* \in [0,1]$[14] s.t.

$$\left(\frac{\alpha}{2} + 2\lambda_0^*(\delta)a(\delta^*)\right) f_\delta'(\delta^*) + \frac{1}{c_1}\frac{d\triangle V(\delta^*)}{d\delta}\left(a(\delta^*)^2\phi_0(\delta^*) + b(\delta^*)^2\phi_1(\delta^*)\right) \tag{3.16}$$

$$+2\lambda_1^*(\delta^*)\lambda_0^*(\delta^*)\phi_1(\delta^*)\phi_0(\delta^*)\left(\frac{1}{\Lambda_0} + \frac{1}{\Lambda_1}\right) = 0$$

where $\lambda_0^*(\delta)$, $\lambda_1^*(\delta)$, Λ_0, Λ_1 and $\triangle V(\delta)$ follow (3.5)-(3.8). And notations $a(\delta)$ and $b(\delta)$ follow the Appendix 3.A.1. Proof is in Appendix 3.A.4.

When $f_\delta(\delta)$ is convex, the intuition behind condition (3.16) is, if we want to guarantee

[14] Actually, we just need the condition to be satisfied over $\delta \in [0, \frac{1}{2}]$

$\phi_1'(\delta) > 0$ on $\delta \in [0, \frac{1}{2}]$ (equally $\phi_0'(\delta) < 0$ on $\delta \in [\frac{1}{2}, 1]$), we require $f_\delta(\delta)$ not to drop too quickly within $\delta \in [0, \frac{1}{2}]$. If $f_\delta(\delta)$ drops too quickly, although higher-utility-type group tends to have a larger proportion of agents as asset owners, the density of the asset owners may still shrink. Similarly, when $f_\delta(\delta)$ is concave, if we want to guarantee $\phi_1'(\delta) > 0$ on $\delta \in [\frac{1}{2}, 1]$ (equally $\phi_0'(\delta) < 0$ on $\delta \in [0, \frac{1}{2}]$), we require $f_\delta(\delta)$ not to drop too quickly within $\delta \in [\frac{1}{2}, 1]$.

Specifically, if $f_\delta(\delta) \equiv 1\ \forall \delta \in [0, 1]$, then the condition (3.16) in Proposition 3 is automatically satisfied since the first term in (3.16) is zero and the sum of the following two terms is always strictly positive by Proposition 1. For the following part of the paper, unless otherwise specified, we automatically assume $f_\delta(\delta) \equiv 1\ \forall \delta \in [0, 1]$. By symmetry, $\phi_0(\frac{1}{2}) = \phi_1(\frac{1}{2}) = \frac{1}{2}$ and $|\phi_0'(\frac{1}{2})| = |\phi_1'(\frac{1}{2})|$. The reason that we focus on such equilibria is that, such equilibria are interesting and more consistent with the intuition that high-utility-type agents tend to have larger density to be an asset owner. The next proposition gives some comparative statics for stationary measures $\phi_1(\delta)$ and $\phi_0(\delta)$.

Proposition 4 For symmetric equilibrium with uniform distribution of utility type $f_\delta(\delta) \equiv 1$ $\forall \delta \in [0, 1]$: as c_1 and/or α increases, if $\lambda_1^*(0)$ decreases[15], then $\phi_1(\delta)(\phi_0(\delta))$ increases(decreases) for each $\delta \in [0, \frac{1}{2})$ and decreases(increases) for each $\delta \in (\frac{1}{2}, 1]$, and the magnitude of change shrinks as δ converges to $\frac{1}{2}$. Proof is in Appendix 3.A.5.

The intuition behind Proposition 4 is: when α increases, every agent's utility type becomes more unstable, then in each unit time, there will be more agents with mis-aligned asset positions when the magnitude of market friction c_1 does not change. Then for each specific

[15] The reason we need the condition "$\lambda_1^*(0)$ decreases" is that: by increasing the cost coefficient c_1, for asset owner with utility type $\delta = 0$, there will be two counteractive effects that there will be more asset nonowners with utility type higher than zero which potentially increases the benefit from searching but it will also be more expensive for asset owner of type zero to search. "$\lambda_1^*(0)$ decreases" will guarantee that the latter effect dominates. And this will determine the shape of the asset-owner density function since as search is discouraged, there will be more mis-aligned agents in the market; by increasing the parameter α, although the first effect above will encourage the asset owner of type zero to search but there will be more competitors of the same type also with mis-aligned asset positions, which potentially discourages the search at the same time, so it is also reasonable to assume that "$\lambda_1^*(0)$ decreases".

$\delta \in [0, \frac{1}{2})$ ($\delta \in (\frac{1}{2}, 1]$), now there will be a larger proportion of asset owners(nonowners), although the majority group is still asset nonowner; when the magnitude of market friction c_1 increases, it is more expensive to search inside the market, which discourages potential intermediation activities. Then there will also be a larger proportion of misaligned-asset-position agents for each $\delta \in [0, 1]$[16]. In a nutshell, higher Poisson intensity of idiosyncratic liquidity shock and/or more expensive searching will raise the level of market friction. Figure 3.1 gives a numerical example of the asset-owner density $\phi_1(\delta)$ where $f_\delta(\delta) \equiv 1$, $\forall \delta \in [0, 1]$ and we vary the values of α and c_1 at the same time, leaving the risk free rate r fixed. We let α change from 0.005 to 0.75 and c_1 change from 1 to 2. We can see, as the α and/or c_1 shrinks, the shape of asset-owner density will be closer to a centralized (Walrasian) case.

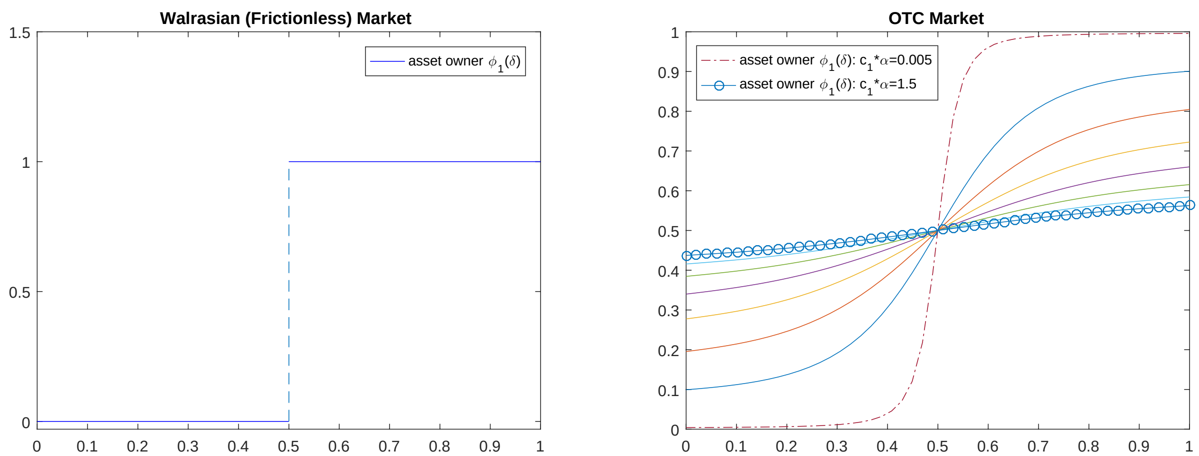

Figure 3.1: Equilibrium joint densities in Walrasian market and OTC markets

[16]In our model with fixed asset supply $s = \frac{1}{2}$, if the market is frictionless (i.e. Walrasian market), $\phi_0(\delta) = f_\delta(\delta) = 1$ ($\phi_1(\delta) = 0$) for all $\delta \in [0, \frac{1}{2})$ and $\phi_1(\delta) = f_\delta(\delta) = 1$ ($\phi_0(\delta) = 0$) for all $\delta \in [\frac{1}{2}, 1]$, that is, asset is allocated to the agents who currently values it most. In OTC market, we will call all the asset owners with utility type $\delta \in [0, \frac{1}{2})$ and all the asset nonowners with utility type $\delta \in [\frac{1}{2}, 1]$ as the mismatched-asset-position agent.

3.3.2.2 Weighted average meeting technology $\bar{\lambda}(\delta)$

Before defining the weighted average meeting technology $\bar{\lambda}(\delta)$, we firstly characterize the properties of marginal investor $\delta^* = s = \frac{1}{2}$ defined in Section 3.3.1 inside the OTC market.

Corollary 1 In symmetric equilibria, the marginal investor $\delta^* = s = \frac{1}{2}$ satisfies:

$$\lambda_1^*(\delta^*) = \lambda_0^*(\delta^*)$$

and

$$\triangle V(\delta^*) = \frac{\delta^* + \alpha E_\delta(\triangle V(\delta))}{\alpha + r} = \frac{\delta^*}{r} = p$$

where p is the unique market clearing price in Walrasian market benchmark.

Intuitively, agent of utility type δ^* is indifferent to becoming either asset owner or asset nonowner. In his reservation value function, this agent weights more his current and future utility types relative to his current asset position. Then his main incentive to enter the market is to provide intermediation service, that is, to purchase at lower prices and sell at higher prices, instead of adjusting his own asset position to become well-aligned agent. We will call this agent as pure intermediator. Intuitively, the pure intermediator's investment in meeting technology should be most elastic with respect to the market environment since this agent has no "'inelastic' hedging purpose" to be either a net buyer or a net seller.

Given equilibrium components $\phi_0(\delta)$ and $\phi_1(\delta)$, we can define the proportions of asset owners and nonowners within each group of agents of utility type $\delta \in [0, 1]$ as follows:

$$S_0(\delta) = \frac{\phi_0(\delta)}{f_\delta(\delta)}$$

$$S_1(\delta) = \frac{\phi_1(\delta)}{f_\delta(\delta)}$$

Then we can further define the weighted average meeting technology $\bar{\lambda}(\delta)$:

$$\bar{\lambda}(\delta) = S_1(\delta)\lambda_1^*(\delta) + S_0(\delta)\lambda_0^*(\delta) = \frac{\phi_1(\delta)}{f_\delta(\delta)}\lambda_1^*(\delta) + \frac{\phi_0(\delta)}{f_\delta(\delta)}\lambda_0^*(\delta)$$

where $\bar{\lambda}(\delta)$ can also be understood as people's expectation on the optimal meeting technology chosen by an individual agent (dealer) of utility type δ.

Specifically, when $f_\delta(\delta) \equiv 1 \ \forall \delta \in [0,1]$, the proportions of asset owners and nonowners are just equal to densities $\phi_1(\delta)$ and $\phi_0(\delta)$. If condition (3.16) in Proposition 3 is satisfied, we have $\phi_0'(\delta) < 0 < \phi_1'(\delta)$, which is consistent with the intuition that generally in a normal/less frictional OTC market, agents of lower liquidity needs more likely become asset owners and agents of higher liquidity needs more likely sell their assets to hold more cash thus becoming asset nonowners. Next we need to characterize the shape of $\bar{\lambda}(\delta)$ to figure out which group of agents will more likely choose more advanced meeting technology, thus behaving more active (closer to the core inside the interdealer network) and which group of agents will more likely behave less active. And how will the distribution of optimal meeting technology change with different market environments, which can jointly be determined by c_1 and α.

Proposition 5 For symmetric equilibrium with $f_\delta(\delta) \equiv 1 \ \forall \delta \in [0,1]$, the weighted average search intensity function $\bar{\lambda}(\delta)$ maintains the following properties in the range of reasonable parameter values[17]:

1. $\bar{\lambda}'(\frac{1}{2}) = 0$, $\bar{\lambda}'(0) < 0$, $\bar{\lambda}'(1) > 0$;

2. For each $\alpha > 0$ ($c_1 > 0$), $\exists c_1^*(\alpha) > 0$ ($\exists \alpha^*(c_1) > 0$), s.t. if $c_1 > c_1^*(\alpha)$ ($\alpha > \alpha^*(c_1)$):

   - $\bar{\lambda}'(\delta) < 0 \ \forall \delta \in [0, \frac{1}{2})$;

   - $\bar{\lambda}(0) > \bar{\lambda}(\frac{1}{2})$;

[17] Here "reasonable" values mainly refer to $r > 0$, $c_1 > 0$ and $\alpha > 0$ in Proposition 2, which guarantees the existence of stationary equilibrium.

- $\bar{\lambda}''(\frac{1}{2}) > 0$;

3. For each $\alpha > 0$ ($c_1 > 0$), $\exists c_1^{**}(\alpha) > 0$ ($\exists \alpha^{**}(c_1) > 0$), s.t. if $c_1 < c_1^{**}(\alpha)$ ($\alpha < \alpha^{**}(c_1)$):

    - $\exists \hat{\delta} \in (0, \frac{1}{2})$ s.t. $\bar{\lambda}'(\hat{\delta}) > 0$;
    - $\bar{\lambda}(0) < \bar{\lambda}(\frac{1}{2})$;
    - $\bar{\lambda}''(\frac{1}{2}) < 0$;

Proof is in Appendix 3.A.6.

By Proposition 5, we know $\delta = \frac{1}{2}$ is always a stationary point. Since it always applies that $\bar{\lambda}'(0) < 0$, if $\delta = \frac{1}{2}$ is a local maximum point in relatively smaller c_1, by Mean Value Theorem, there must exist a utility type $\delta' \in (0, \frac{1}{2})$ (symmetrically $1 - \delta' \in (\frac{1}{2}, 1)$) which is a local minimum point. For specific α, when c_1 changes from being small ($< c_1^{**}(\alpha)$) to being large ($> c_1^{*}(\alpha)$), this local minimum point will intuitively shifts from being close to $\delta = 0$ to being close to $\delta = \frac{1}{2}$ for the left part of $\bar{\lambda}(\delta)$. Similar idea works for given specific c_1 and α changes from being small ($< \alpha^{**}(c_1)$) to being large ($> \alpha^{*}(c_1)$). While this local minimum point is difficult to be technically identified in a general model setup and the main topic of our paper is to discuss how dealers switch their roles to behave either active or inactive in different market environments, then in the following sections we will ignore this local minimum point and mainly compare the expected optimal meeting technology between the intermediate utility type agent ($\delta = \frac{1}{2}$) and the extreme utility type agents ($\delta = 0$ and 1) in symmetric stationary equilibria in different market environments, to discuss which agent will behave as the core dealer and which agent will behave as the periphery one.

3.3.3 Quantitative example

In Figure 3.2-3.4, we give three groups of stationary equilibrium solutions. The value of c_1 is used as a measure of market friction and the value of α is used as a measure of market mis-alignment. By Proposition 5, as c_1 increases, extreme-value agents, who initially invest

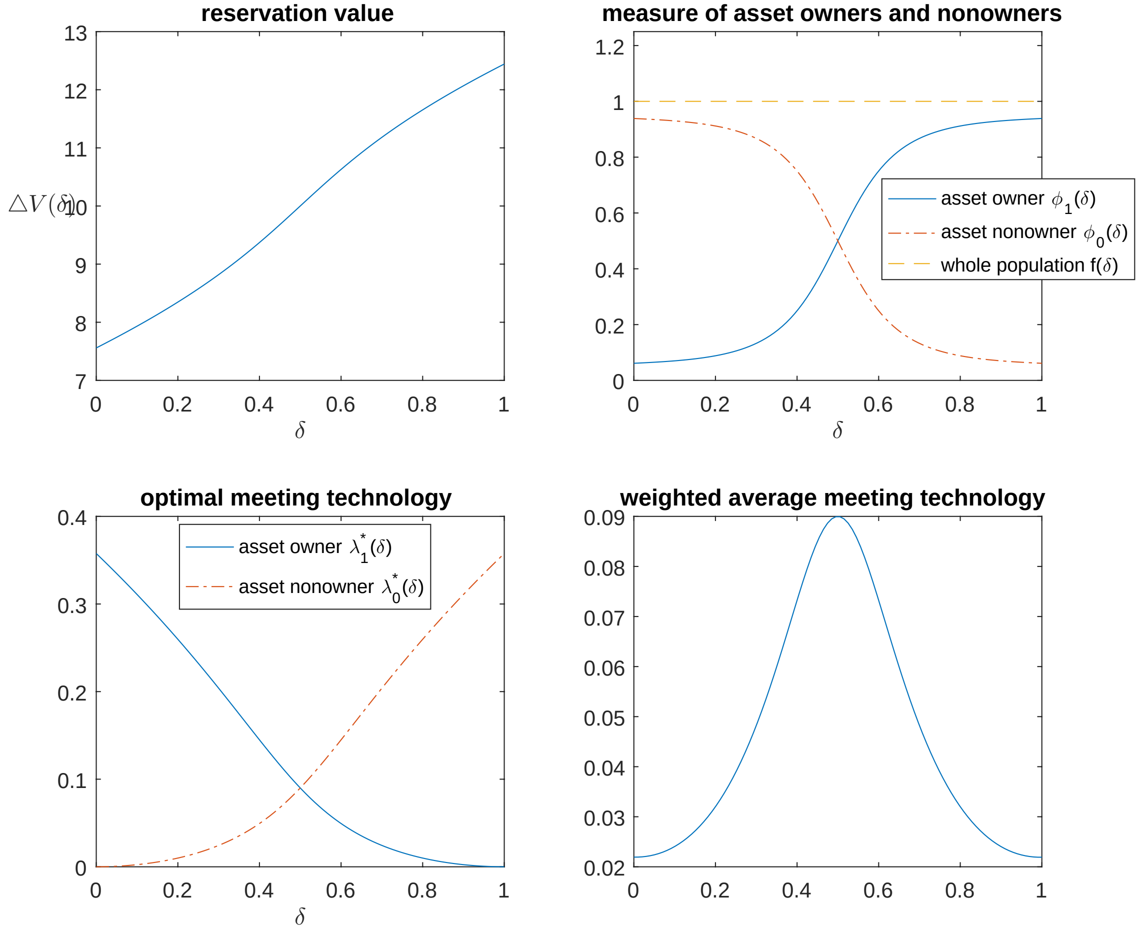

Figure 3.2: Stationary equilibrium solutions with $c_1 = 2$, $\alpha = 0.05$

in less advanced weighted average meeting technology than intermediate-value agents, will behave more active than the intermediate ones in more frictional market. By the graph of measures of asset owners and nonowners, as α increases, there are more mis-aligned agents in the market.

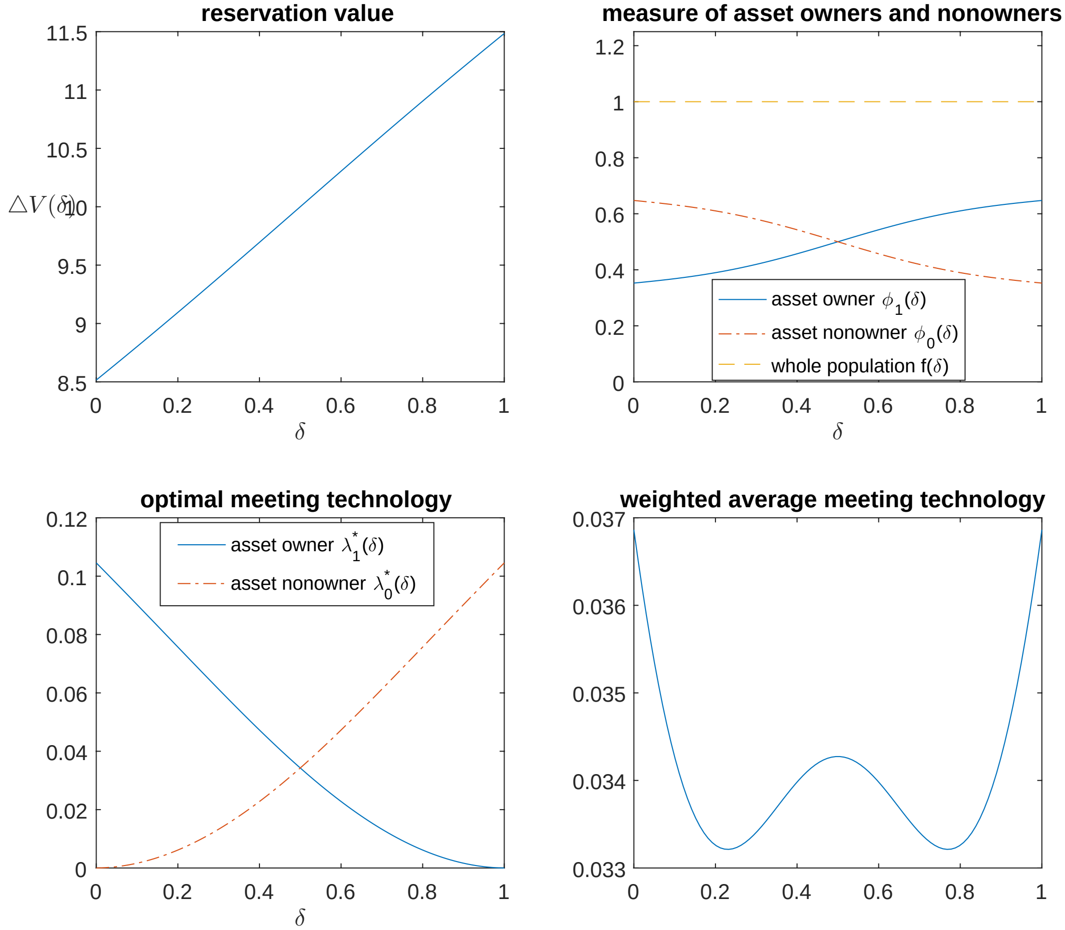

Figure 3.3: Stationary equilibrium solutions with $c_1 = 5$, $\alpha = 0.25$

3.4 Core-periphery interdealer network

Our analytical results have implications for the formation and evolution of the core-periphery interdealer network, which is documented to commonly exist in several OTC markets, including municipal bond market, securities market for 144a and registered instruments, federal funds market, and etc. The centrality of dealers in the interdealer network is mainly measured by the number of completed trades (larger than a certain scale) per unit time, which also represents the activeness of dealers. If one dealer behaves more active in the OTC

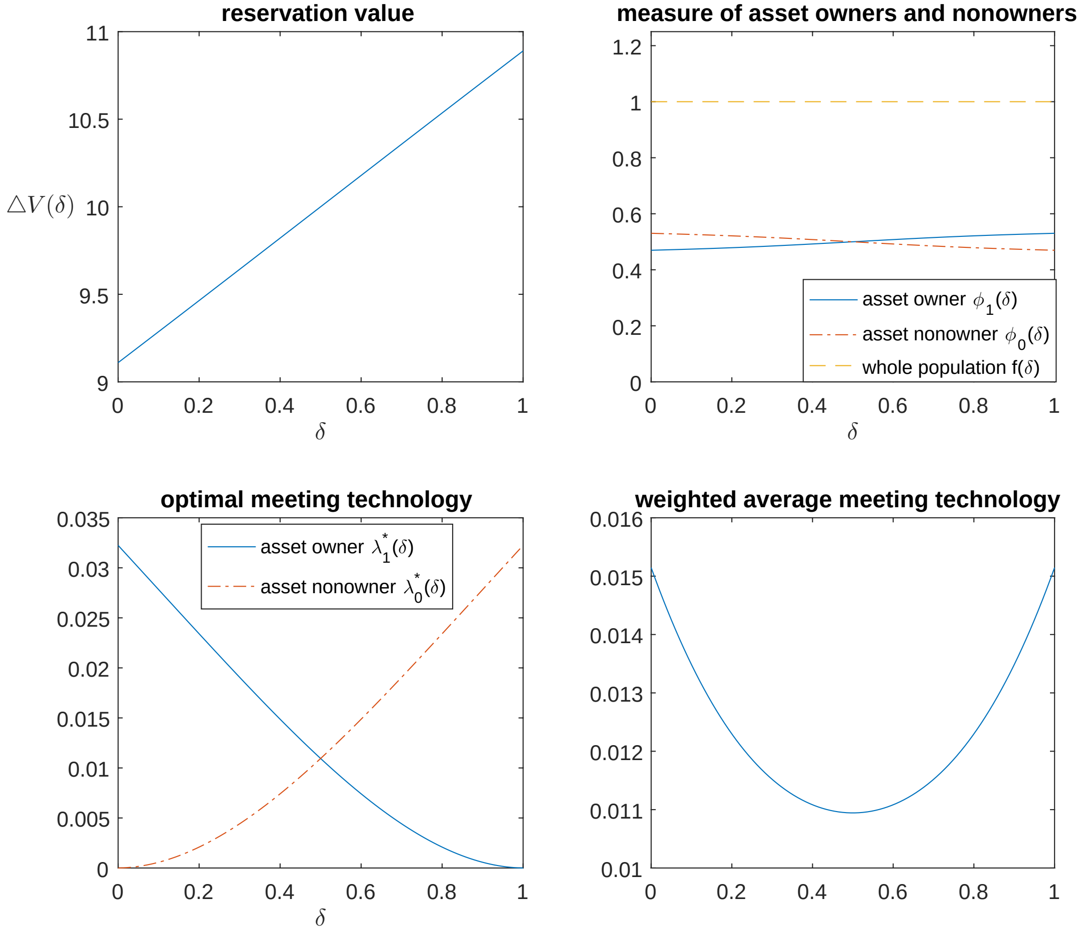

Figure 3.4: Stationary equilibrium solutions with $c_1 = 10$, $\alpha = 0.5$

market, i.e. searches more frequently for potential counterparties to trade with, he will have larger centrality and lie closer to the core of the network.

3.4.1 Trading frequency

We use the weighted average search intensity $\bar{\lambda}(\delta)$ as a measure of the trading frequency or centrality of the dealer of utility type δ. In unit time, the number of effective trades completed by an individual agent also equals his (expected instantaneous) gross trading volume (denoted as $G(\delta)$ below), since the trading quantity between any two matched agents has been normalized to be either +1 or −1. As a result, both $\bar{\lambda}(\delta)$ and $G(\delta)$ can be used as a measure of dealers' centrality. The question is, which dealers will averagely choose higher $\bar{\lambda}(\delta)$ and which dealers will averagely choose the lower one? It turns out that, the market environment determines the shape of $\bar{\lambda}(\delta)$ thus the distribution of optimal meeting technology. Referring to our baseline model, we can use the product $c_1 * \alpha$ as a measure of the market environment. As interpreted in the model setup, higher α means more misaligned agents in the market and higher c_1 means more expensive meeting technology investment, thus more frictional market.

For simplicity, we only focus on the extreme-value agent with $\delta = 0$ (or $\delta = 1$) and intermediate-value agent with $\delta = \frac{1}{2}$, although $\delta = 0$ (or $\delta = 1$) agent will never be the least active agent inside the market due to $\bar{\lambda}'(0) < 0$ by Proposition 5. Another reason is, we want to emphasize the motive of these two groups of agents to "switch" their positions in the interdealer network.

The former analytical result implies the key for the fact that "the less active agent in less frictional market becomes (relatively) more active in more frictional market" is, the former less active agent's trading motivation is more robust to the market environment. For agents with $\delta = 0$ ($\delta = 1$), their more robust weighted average meeting technology is mainly driven by the investment of asset owners (nonowenrs) in the group. The latter are highly motivated to search to hedge their highly mis-aligned asset positions instead of just waiting for their utility type to shift up or down. While for agent with $\delta = \frac{1}{2}$, since they are indifferent between holding the asset or not by Corrolary 1, their motivation to hedge their mis-aligned asset positions is the least among all agents. Then their trading motivation is mainly to gain intermediation profit through buying low and selling high at the cost of searching, which will be more affected by the level of market friction c_1. They can also be regarded as the most pure intermediator.

3.4.2 Measures of liquidity

3.4.2.1 Trading volumes

To evaluate the effect of core-periphery interdealer network on the liquidity level of the whole market, we use agent's trading volumes and profit gained from providing intermediation service[18] as measures of market liquidity. The intermediation service refers to the activity that dealers buy certain amount of asset from someone and sell the same amount of asset to the others inside the interdealer market. Correspondingly the trading volumes include expected instantaneous gross trading volume $G(\delta)$, net trading volume $N(\delta)$ and intermediation trading volume $I(\delta)$ of agents of utility type $\delta \in [0,1]$, which are defined as follows:

$$G(\delta) = 2\phi_1(\delta)\lambda_1^*(\delta)\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\phi_0(\delta')d\delta' + 2\phi_0(\delta)\lambda_0^*(\delta)\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta'$$

$$N(\delta) = \left| 2\phi_1(\delta)\lambda_1^*(\delta)\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\phi_0(\delta')d\delta' - 2\phi_0(\delta)\lambda_0^*(\delta)\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta' \right|$$

$$I(\delta) = G(\delta) - N(\delta) = 4*min\left\{\phi_1(\delta)\lambda_1^*(\delta)\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\phi_0(\delta')d\delta', \phi_0(\delta)\lambda_0^*(\delta)\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta'\right\}$$

where intermediation volume equals gross volume subtracts net volume and it represents the magnitude of intermediation service that each agent provides to the whole market. Both gross and intermediation trading volumes are manifestation of the ability of agents to reallocate assets among investors.

Similar to the formation of core-periphery interdealer network, the levels of friction and mis-alignment in the OTC market also determines which group of agents will make the main contribution to the market liquidity. Here we give an quantitative example to compare gross,

[18] The "intermediation profit" can be regarded as proxy for bid-ask spread gained by each agent, which comes from buying certain amount of asset at lower price and selling the same amount of asset at higher price.

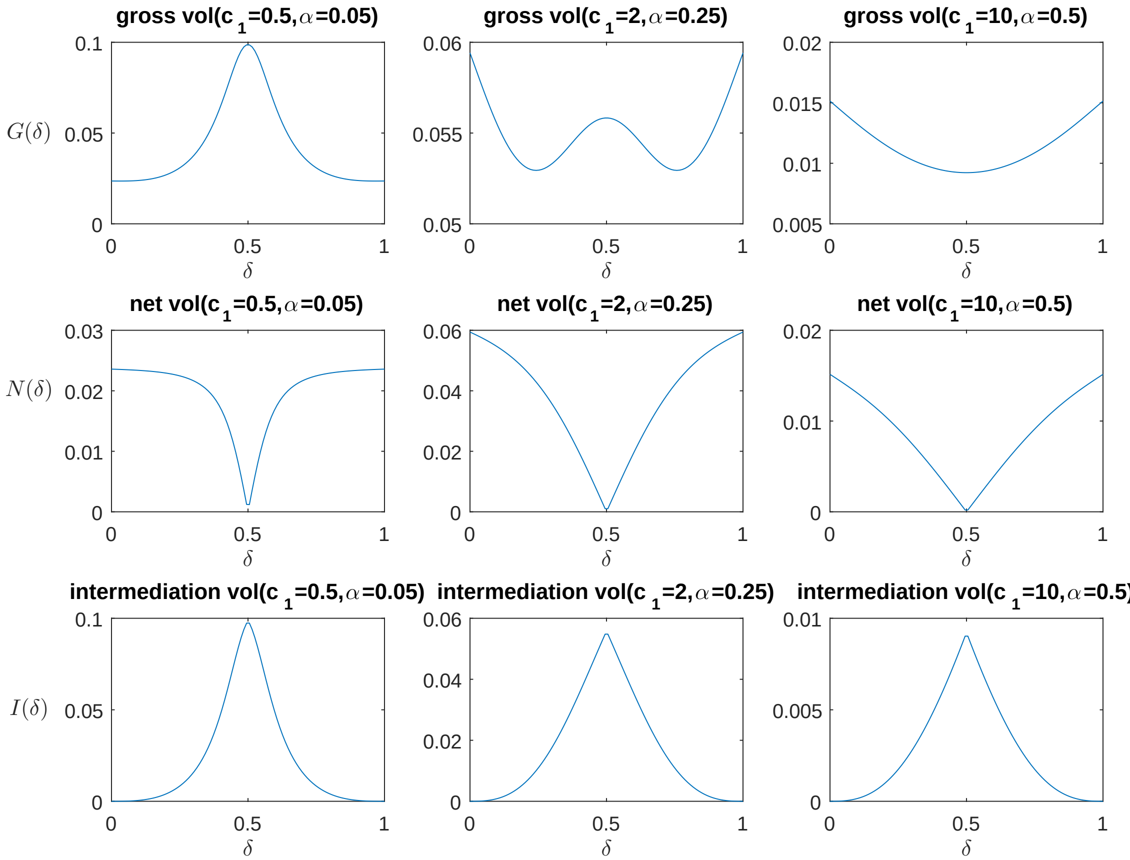

Figure 3.5: Trading volumes in different markets

net and intermediation trading volumes in three markets with different levels of friction.

Figure 3.5 verifies the former conclusion that intermediate-type agents behave as the most pure intermediator inside the market since they maintain the highest intermediation trading volume in all three markets with different levels of friction. Also, in less frictional market ($c_1 = 0.5, \alpha = 0.05$), the intermediate-type agents behave most active in the sense that they contribute the highest gross trading volume. While as market becomes more and more frictional, they behave relatively less and less active than the extreme-type agents. Moreover, based on the magnitudes of trading volumes: as agent's utility type becomes more and more extreme (closer to $\delta = 1$ (or $\delta = 1$)), most part of his gross trading volume comes from hedging his own mis-aligned asset position; as agent's utility type becomes more and more intermediate (closer to $\delta = \frac{1}{2}$), most part of his gross trading volume comes from providing intermediation service to the whole market.

3.4.2.2 Centrality profit per trade

To measure the main transaction cost in an illiquid market, former literatures mainly use intermediation profit per trade, which corresponds to dealers' bid-ask spread per trade in data. Since customers, as a whole, need to pay dealers bid-ask spreads to reallocate assets among themselves, the magnitude of bid-ask spread per trade can measure how easy it is for customers to buy from/sell to the dealers on average. Thus it can be used as one measure of market liquidity/illiquidity.

Intermediation profit per trade for each utility type $IP_p(\delta)$ is defined as:

$$IP_p(\delta) = \bar{P}_s(\delta) - \bar{P}_b(\delta) \quad \forall \delta \in (0,1)$$

where

$$\bar{P}_s(\delta) = \frac{\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0} \frac{\triangle V(\delta) + \triangle V(\delta')}{2} \phi_0(\delta') d\delta'}{\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0} \phi_0(\delta') d\delta'}$$

and

$$\bar{P}_b(\delta) = \frac{\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1} \frac{\triangle V(\delta) + \triangle V(\delta')}{2} \phi_1(\delta') d\delta'}{\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1} \phi_1(\delta') d\delta'}$$

Intuitively, $\bar{P}_s(\delta)$ and $\bar{P}_b(\delta)$ are agent δ's average selling price and average buying price[19].

In former literatures, Li and Schürhoff (2014) documents the positive correlation between centrality and bid-ask spread per trade (i.e. centrality premium) in municipal bond market while Hollifield, Neklyudov, and Spatt (2017) documents the negative correlation

[19]Since agents with $\delta = 0$ ($\delta = 1$) either remain silent or search to sell (buy), they do not provide intermediation service to the whole interdealer market. So we ignore these two utility types when discussing intermediation profit per trade.

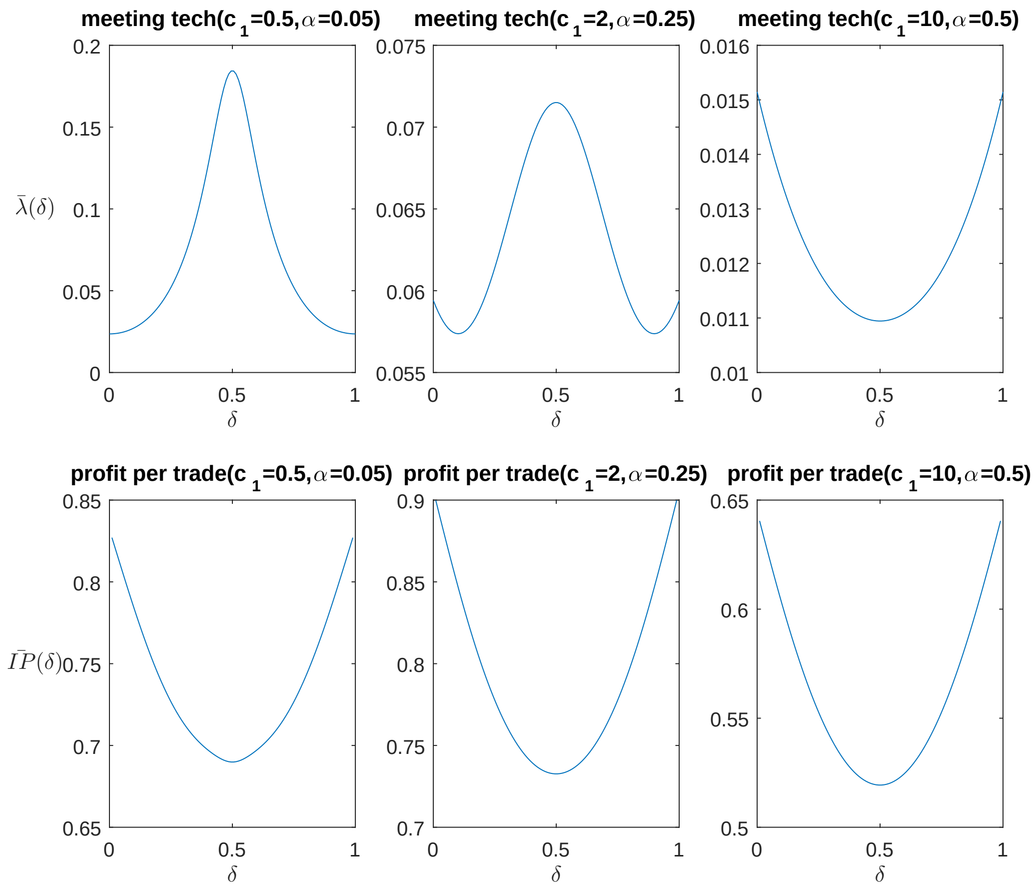

Figure 3.6: Correlation between centrality and intermediation profit per trade

(i.e.centrality discount) in asset-backed-securities market. Üslü (2019) attributes the correlation between centrality and bid-ask spread per trade to the level of market friction and the sign of correlation is also consistent with our model. Here we also give a quantitative example to analyze the sign of such correlation in three markets with different levels of friction and mis-alignment. In this example, we use weighted average meeting technology as a measure of dealers' centrality. Figure 3.6 shows that the intermediation profit per trade is always minimized at $\delta = \frac{1}{2}$ due to our assumption of heterogeneous valuation among all the dealers and the Nash bargaining process. Since there are different patterns of investment in meeting technologies across different markets, "centrality premium" will appear in more frictional market and “centrality discount” will appear in less frictional market. In other words, if the assumptions and the mechanism in our model are correct, we may conclude that: the core dealers in ABS market should on average have the liquidity needs closest to the social average level, and the core dealers in municipal bond market should on average have either the highest or the lowest liquidity needs among all the dealers.

3.5 Aggregate liquidity shock

We consider the aggregate liquidity shock in similar form as in Duffie, Gârleanu, and Pedersen (2007). In their paper, upon each aggregate liquidity shock, a randomly chosen fraction of agents will suffer a sudden jump of their current utility types from high state to low state. The aggregate liquidity shock is expected to occur following a Poisson process, which is newly added into the HJB equations of value functions. Yet in our model, since the distribution of utility type $f_\delta(\delta)$ has continuous support $[0,1]$, we consider the aggregate liquidity shock in a new form that, for each agent whose utility type is $\delta \in [\frac{1}{2}, 1]$, his utility type shifts to $\delta - \frac{1}{2}$, i.e. $\frac{1}{2}$ lower than his current type, with probability π. The shifts of utility types are independent among all the agents in $\delta \in [\frac{1}{2}, 1]$, thus we can apply the Law of Large Numbers. Figure 3.7 gives an example of aggregate liquidity shock with $\pi = 0.5$.

Additionally we maintain the self-refinancing channel as in Duffie, Gârleanu, and Pedersen (2007) so that, for each agent, the distribution of new utility type in response to idiosyncratic liquidity shock is assumed to be always uniform on $[0,1]$ and the distribution of utility type can recover to the pre-shock scenario through this channel. Since this aggregate shock is not a permanent one, it is more reasonable to assume that agents will expect a Poisson arrival of such aggregate liquidity shock in the future, thus generating the “permanent price effect”[20].

[20]In our paper, the “permanent price effect” refers to the effect of expectation on future aggregate liquidity shock on the social (purchase/sale) price level. If agents expect that there is a high probability that there will be an aggregate liquidity shock in next period, the social valuation on the asset will decrease thus making

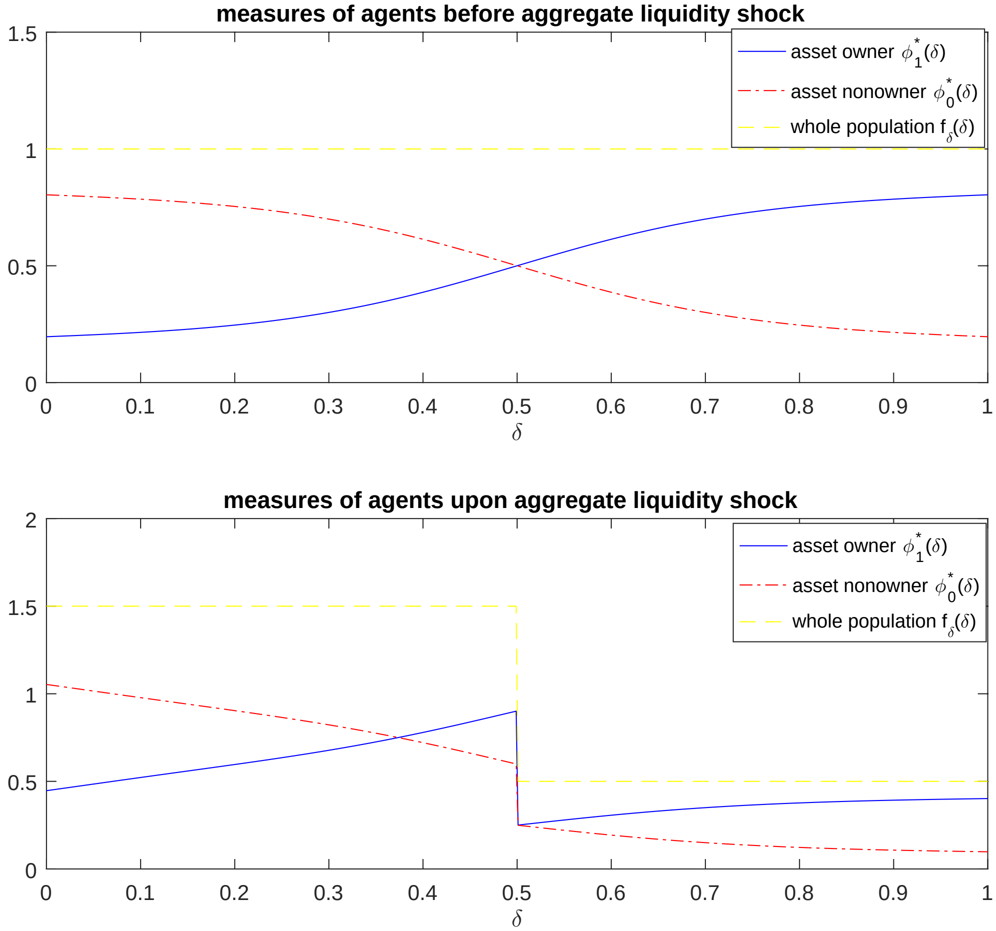

Figure 3.7: Measures of agents before and after the aggregate liquidity shock

Assuming t is the length of time after the most recent aggregate liquidity shock, we obtain the new HJB equations for agents indirectly affected $\delta \in [0, \frac{1}{2})$ and agents directly affected $\delta \in [\frac{1}{2}, 1]$:

---

the social (purchase/sale) price decrease.

For $\forall\delta \in [0, \frac{1}{2}]$,

$$\begin{aligned}\triangle\dot{V}(\delta,t) = r\triangle V(\delta,t) - \delta + c_1\lambda_1^{*2}(\delta,t) - c_1\lambda_0^{*2}(\delta,t) - \alpha\int_0^1 (\triangle V(\delta',t) - \triangle V(\delta,t))d\delta' \\ - \lambda_1^*(\delta,t)\int_0^1 \frac{\lambda_0^*(\delta',t)}{\Lambda_{0,t}}(\triangle V(\delta',t) - \triangle V(\delta,t))\phi_0(\delta',t)d\delta' \\ + \lambda_0^*(\delta,t)\int_0^\delta \frac{\lambda_1^*(\delta',t)}{\Lambda_{1,t}}(\triangle V(\delta,t) - \triangle V(\delta',t))\phi_1(\delta',t)d\delta' \\ - \eta(\triangle V(\delta,0) - \triangle V(\delta,t))\end{aligned}$$

For $\forall\delta \in [\frac{1}{2}, 1]$,

$$\begin{aligned}\triangle\dot{V}(\delta,t) = r\triangle V(\delta,t) - \delta + c_1\lambda_1^{*2}(\delta,t) - c_1\lambda_0^{*2}(\delta,t) - \alpha\int_0^1 (\triangle V(\delta',t) - \triangle V(\delta,t))d\delta' \\ - \lambda_1^*(\delta,t)\int_0^1 \frac{\lambda_0^*(\delta',t)}{\Lambda_{0,t}}(\triangle V(\delta',t) - \triangle V(\delta,t))\phi_0(\delta',t)d\delta' \\ + \lambda_0^*(\delta,t)\int_0^\delta \frac{\lambda_1^*(\delta',t)}{\Lambda_{1,t}}(\triangle V(\delta,t) - \triangle V(\delta',t))\phi_1(\delta',t)d\delta' \\ - \eta\left[\pi(\triangle V(\delta-0.5,0) - \triangle V(\delta,t)) + (1-\pi)(\triangle V(\delta,0) - \triangle V(\delta,t))\right]\end{aligned}$$

where η is the expected Poisson intensity of future aggregate liquidity shock.

The evolution equation and market clear condition for densities $\phi_1(\delta,t)$ and $\phi_0(\delta,t)$ after the aggregate liquidity shock are as follows:
For $\forall t > 0$ and $\forall\delta \in [0,1]$,

$$\dot{\phi}_1(\delta,t) = -\alpha\phi_1(\delta,t) + \frac{\alpha}{2}\hat{f}_\delta(\delta) - 2\phi_1(\delta,t)\lambda_1^*(\delta,t)\int_\delta^1 \frac{\lambda_0^*(\delta',t)}{\Lambda_{0,t}}\phi_0(\delta',t)d\delta'$$

$$+2\phi_0(\delta,t)\lambda_0^*(\delta,t)\int_0^\delta \frac{\lambda_1^*(\delta',t)}{\Lambda_{1,t}}\phi_1(\delta',t)d\delta'$$

$$\dot{\phi}_0(\delta,t) = -\alpha\phi_0(\delta,t) + \frac{\alpha}{2}\hat{f}_\delta(\delta) + 2\phi_1(\delta,t)\lambda_1^*(\delta,t)\int_\delta^1 \frac{\lambda_0^*(\delta',t)}{\Lambda_{0,t}}\phi_0(\delta',t)d\delta'$$

$$-2\phi_0(\delta,t)\lambda_0^*(\delta,t)\int_0^\delta \frac{\lambda_1^*(\delta',t)}{\Lambda_{1,t}}\phi_1(\delta',t)d\delta'$$

where $\hat{f}_\delta(\delta) \equiv 1 \; \forall\delta \in [0,1]$ and

$$\phi_0(\delta,t)+\phi_1(\delta,t)=f_\delta(\delta,t)$$

$$\int_0^1 \phi_0(\delta,t)d\delta = \int_0^1 \phi_1(\delta,t)d\delta = \frac{1}{2}$$

Figure 3.8 and 3.9 show the trends of market average purchase and sale prices and different measures of market liquidity before and after the aggregate liquidity shock.

We can see both the average purchase and sale prices will drop hugely right after the unexpected aggregate liquidity shock and then recover up. Due to agent's expectation on future aggregate liquidity shock, the new stationary equilibrium price levels will be permanently shifted down relative to the original ones. For different measures of market liquidity, there are immediate increases in market gross trading volume and intermediation trading volume, and immediate decrease in average bid-ask spread per trade right after the aggregate liquidity shock. Then as time goes by, the former two measures will go down until reaching a new lower-level stationary equilibrium. The reason for the immediate increases may be, right after the start of crisis, agents with their utility types suddenly shifted down will have strong incentives to sell the asset thus making all the agents to reallocate the fixed supply of asset among themselves. This will hugely increase the total trading volume inside the market. Then both trading volumes will go down since those agents will reach better-aligned asset positions. Eventually, due to expectation on future aggregate liquidity shock, the market liquidity measured by both trading volumes will also be permanently shifted down in the new stationary equilibrium, where agents become more conservative to search to trade with others. For average intermediation profit per trade, the reason for the immediate decrease is the decline in the average valuation among all dealers in the market. Although it mainly measures the average trading cost of customers, it does not necessarily mean the immediate

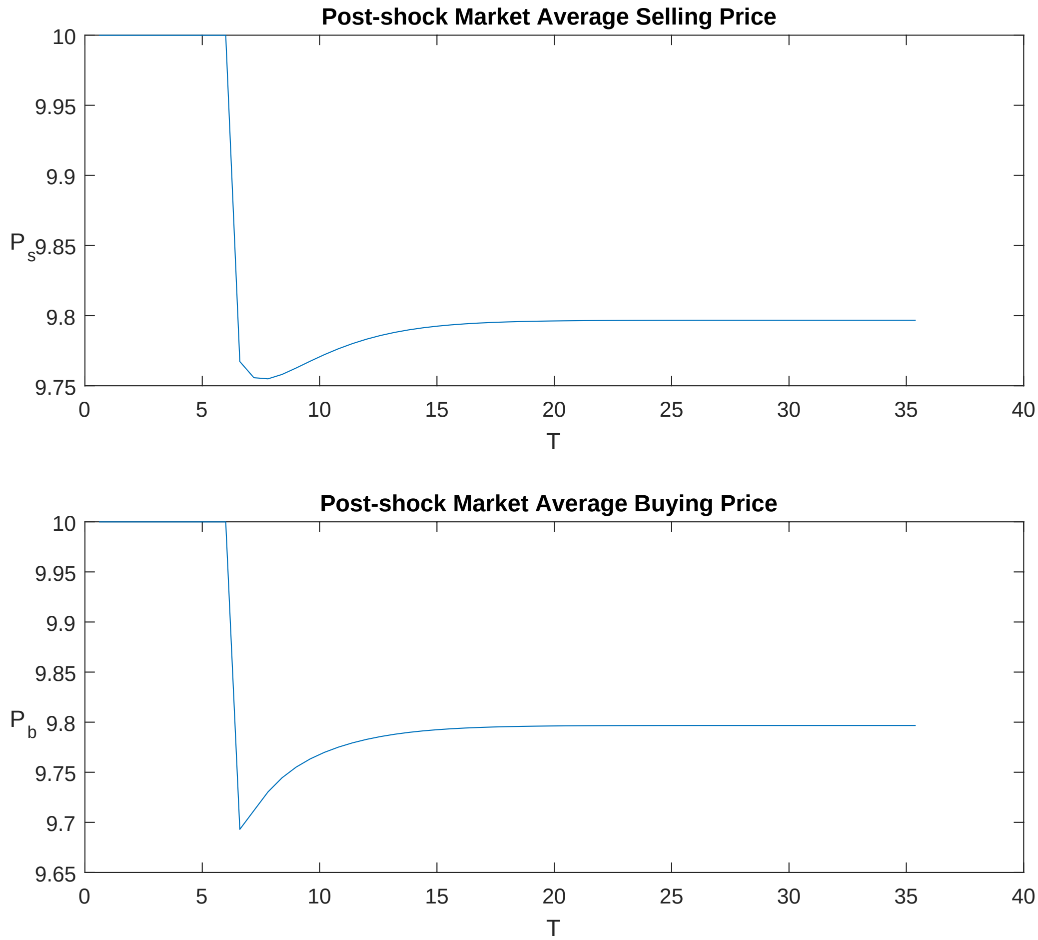

Figure 3.8: Market average prices with $\alpha = 0.25$, $c_1 = 1$ and $\eta = 0.1$

increase in market liquidity since dealers may trade off between lower average intermediation profit per trade and higher trading delay, which is beyond the discussion of this paper.

Policy choice targeting at different groups of dealers

We define the rescue policies as the actions taken by the monetary authority, to make the directly affected dealers' liquidity needs recover to their pre-shock levels, through, for example, directly injecting liquidity into the targeted dealers. Due to limited resources, rescue policies usually target on specific group of dealers with priority, that is, firstly inject

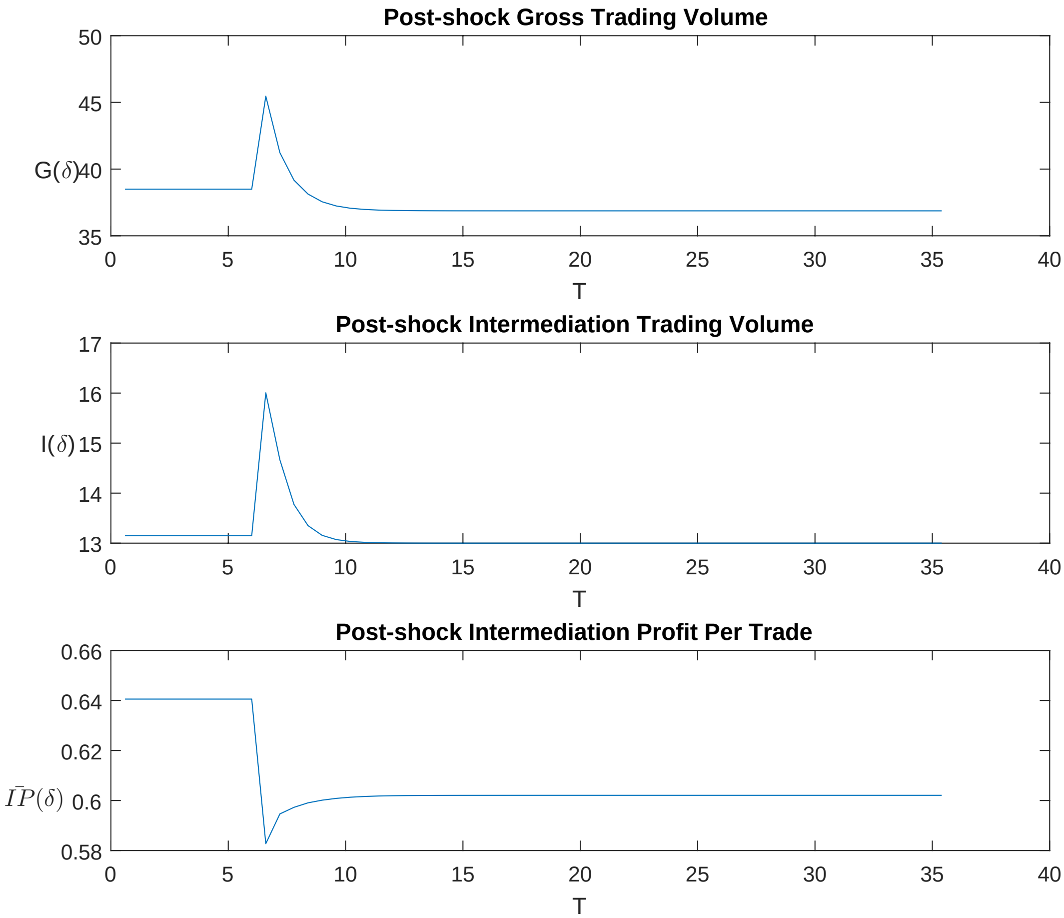

Figure 3.9: Measures of market liquidity with $\alpha = 0.25$, $c_1 = 1$ and $\eta = 0.1$

liquidity into the specific group of dealers.

We consider two policy choices separately targeting at both asset owners and nonowners with $\delta \in [\frac{1}{2}, \frac{3}{4}]$ (Policy 1) and both asset owners and nonowners with $\delta \in [\frac{3}{4}, 1]$ (Policy 2). By the changes in different measures of market liquidity under these two policies, we can determine which group of dealers are more important in the sense that maintaining their pre-shock utility types more helps maintaining the market liquidity.

Figure 3.10 shows the effects of different policies ("no policy", "Policy 1" and "Policy 2") in response to unexpected aggregate liquidity shock, under different levels of market

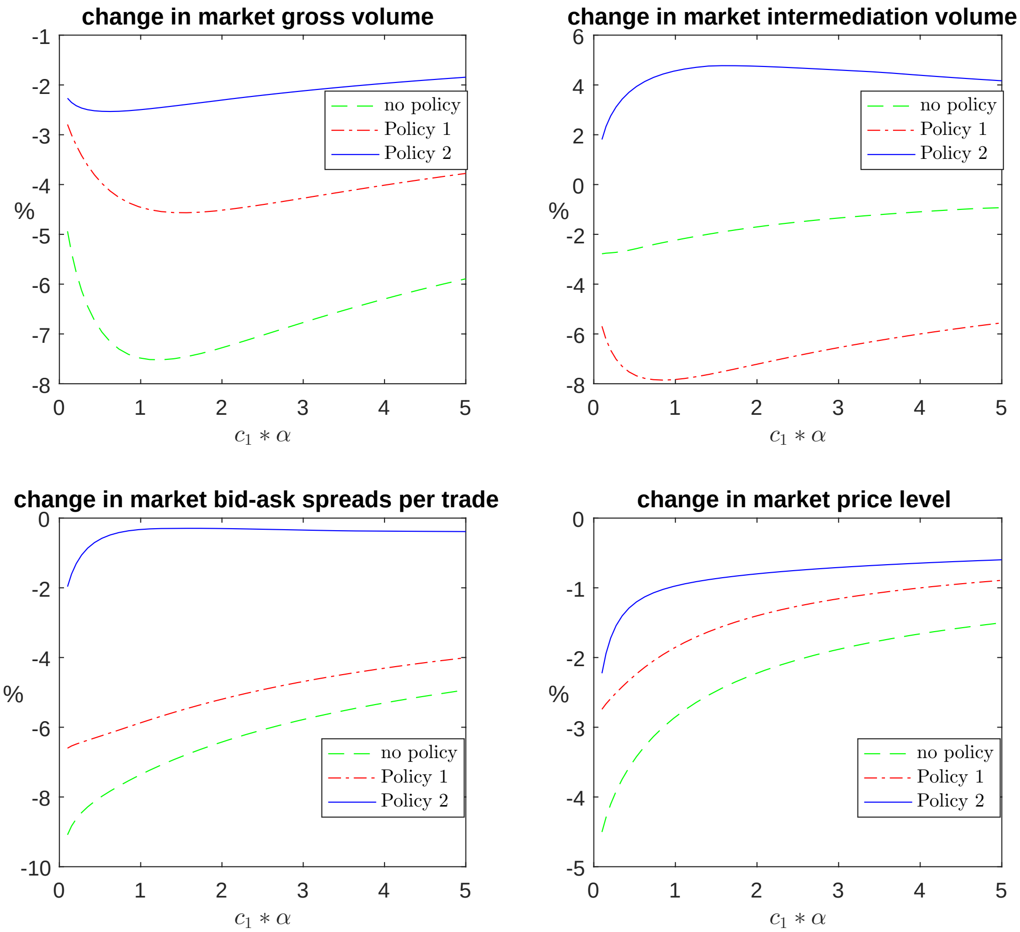

Figure 3.10: Net changes in new stationary equilibrium

(net changes are expressed as a percentage of the corresponding values in the old equilibrium)

friction. The effect is measured by changes in different measures of market liquidity in the new stationary equilibrium as a percentage of the initial stationary equilibrium. Figure 3.11 instead focuses on the dynamic process and shows the effects on the cumulative change in market liquidity before achieving the new stationary equilibrium.

By Figure 3.10 and 3.11, Policy 2 uniformly dominates Policy 1 and "no policy response" across different measures of market liquidity and different levels of market friction. Intuitively, higher-type agents contribute more to maintaining the market liquidity. While the

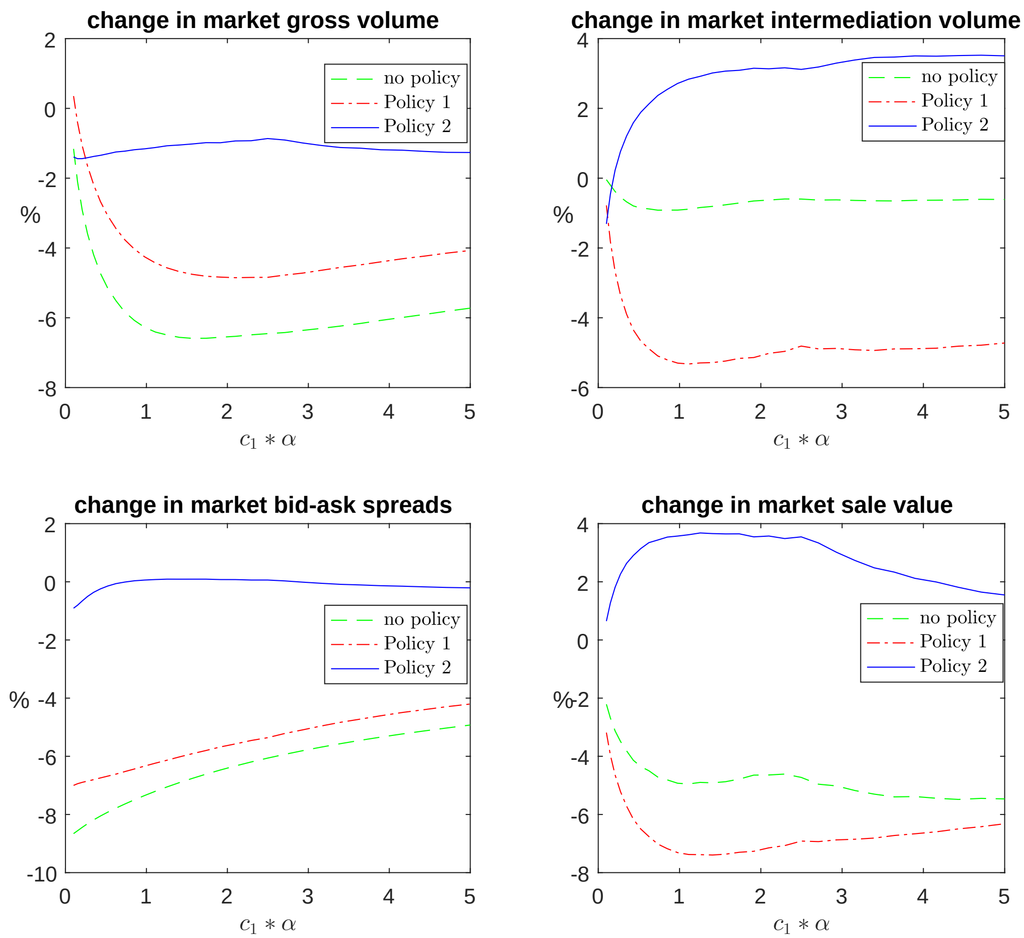

Figure 3.11: Cumulative changes before achieving new stationary equilibrium

(cumulative changes are expressed as a percentage of the corresponding values in the old equilibrium)

key implication of the model is, such higher-type agents may become core dealers in more frictional market and become periphery dealers in less frictional market. Then we conclude that the core dealers may not always be the most important ones that should be given priority to receive liquidity after the aggregate liquidity shock. To better maintain the level of market liquidity, policy makers need to firstly identify the market environment (either more frictional or less frictional), and attach more importance to the core dealers in market with higher level of friction and attach more importance to periphery dealers in market with lower level of friction.

3.6 Efficiency analysis

3.6.1 Social optimal choice of meeting technologies

As in the case of frictionless market, we use the sum of discounted instantaneous utility flows to measure the positive part of social welfare. The difference is, in frictional market, all agents are burdened with instantaneous investment costs of meeting technologies. In this section, we assume the investment cost is in quadratic form.

$$
\begin{aligned}
W &= \int_0^{+\infty} e^{-rt} \int_0^1 \delta\phi_1(\delta) d\delta dt - \int_0^{+\infty} e^{-rt} \int_0^1 c_1 \lambda_1^{*2}(\delta)\phi_1(\delta) d\delta dt \\
&\quad - \int_0^{+\infty} e^{-rt} \int_0^1 c_1 \lambda_0^{*2}(\delta)\phi_0(\delta) d\delta dt \\
&= \frac{1}{r}\left(\int_0^1 \delta\phi_1(\delta) d\delta - \int_0^1 c_1 \lambda_1^{*2}(\delta)\phi_1(\delta) d\delta - \int_0^1 c_1 \lambda_0^{*2}(\delta)\phi_0(\delta) d\delta\right)
\end{aligned}
$$

If we specifically focus on symmetric equilibria, the last two terms are equal, then social welfare is simplified to be:

$$
W = \frac{1}{r}\left(\int_0^1 \delta\phi_1(\delta) d\delta - 2\int_0^1 c_1 \lambda_1^{*2}(\delta)\phi_1(\delta) d\delta\right)
$$

Then we discuss in fixed market environment (characterized by fixed r, c_1 and α) and given the uniform distribution of utility type $f_\delta(\delta) \equiv 1 \ \forall \delta \in [0,1]$, what is the social optimal assignment of meeting technology to asset owners and nonowners of different utility types and how it is different from the competitive equilibrium one that optimally chosen by the agents. For simplicity, we only focus on symmetric assignment between asset owners and nonowners and we regard the density of asset owners $\phi_1^S(\delta)$ as the second control variable (function) that connected with $\lambda_1^{S*}(\delta)$ through the equilibrium constraint (3.17) below.[21]

Define the following normed linear spaces: $\Lambda_{S1} = \{\lambda_1^S(\delta) : \lambda_1^S(\delta) \in C^1[0,1]; \lambda_1^S(\delta) \geq 0 \text{ and } \lambda_1^{S'}(\delta) \leq 0, \forall\delta \in [0,1]\}$,[22] $\Phi_{S1} = \{\phi_1^S(\delta) : \phi_1^S(\delta) \in C^1[0,1]; 0 \leq \phi_1^S(\delta) \leq 1 \text{ and } \phi_1^{S'}(\delta) \geq 0, \forall\delta \in [0,1]; \int_0^1 \phi_1^S(\delta)d\delta = \frac{1}{2}\}$, all with the norm $\|f\| = \underset{0\leq\delta\leq1}{max}|f(\delta)|$. The simplified social planner problem $[SP]$ is:

$$\underset{\lambda_1^S(\delta)\in\Lambda_{S1},\phi_1^S(\delta)\in\Phi_{S1}}{max} \quad W = \int_0^1 (\delta - 2c_1\lambda_1^{S^2}(\delta))\phi_1^S(\delta)d\delta$$

s.t.

$$\phi_1^S(\delta) = \frac{1}{1 + \frac{\frac{\alpha}{2}+2\lambda_1^S(\delta)\int_0^{1-\delta}\frac{\lambda_1^S(\delta')}{\Lambda_1}\phi_1^S(\delta')d\delta'}{\frac{\alpha}{2}+2\lambda_1^S(1-\delta)\int_0^{\delta}\frac{\lambda_1^S(\delta')}{\Lambda_1}\phi_1^S(\delta')d\delta'}} \qquad \forall\delta \in [0,1] \tag{3.17}$$

and

$$\Lambda_1 = 2\int_0^1 \lambda_1^S(\delta')\phi_1^S(\delta')d\delta'$$

the constraint (3.17) is by the symmetry of $\lambda_1^S(\delta)$ and $\lambda_0^S(\delta)$ with respect to $\delta = \frac{1}{2}$, i.e. $\lambda_0^S(\delta) = \lambda_1^S(1-\delta), \forall\delta \in [0,1]$.

The key to explicitly solve the above social planner problem is to firstly obtain Proposition 6 that guarantees $\lambda_1^{S^*}(\delta) \equiv 0$ on the higher half of utility space $[\frac{1}{2}, 1]$.

Proposition 6 If $\lambda_1^{S*}(\delta)$ and $\phi_1^{S*}(\delta)$ solve the social planner problem $[SP]$, $\lambda_1^{S*}(\delta) \equiv 0$ for $\forall\delta \in [\frac{1}{2}, 1]$. Proof is in Appendix 3.A.7.

[21] The original social planner problem has $\lambda_1^{S*}(\delta)$ as its unique control variable. Given any $\lambda_1^{S*}(\delta)$, by the evolution equation of densities at stationary equilibrium, we can obtain one corresponding density function $\phi_1^S(\delta)$ through fixed point convergence.

[22] It is intuitive that the social optimal meeting technology of asset owners $\lambda_1^S(\delta)$ is a decreasing function. Suppose the social optimal function $\lambda_1^{S^*}(\delta)$ has two points $\delta_1 < \delta_2$ with $\lambda_1^{S^*}(\delta_1) < \lambda_1^{S^*}(\delta_2)$, then we can switch the meeting technologies of these two agents without increasing the total investment cost. Then the agent with lower utility type will be assigned with higher meeting technology thus having more oppotunities to sell his asset. Since lower-type asset owners are more likely to be mis-aligned agents, the above switching help improve the alignment of the whole market. Or for simplicity, we can just guess and verify later that the social optimal $\lambda_1^{S^*}(\delta)$ is a decreasing function on $[0,1]$.

The intuition behind Proposition 6 is, it is optimal to make only mis-aligned agents to actively search in the market to trade with others if the searching is not free and is in quadratic form. The unique positive part $\int_0^1 \delta\phi_1^S(\delta)d\delta$ in social objective function is maximized at $\phi_1^S(\delta) \equiv f_\delta(\delta) \equiv 1$ on $[\frac{1}{2}, 1]$ and zero elsewhere, which is also the frictionless case in Walrasian market. Then the level of social objective function is consistent with the magnitude of alignment of the whole market (or negatively correlated with the mis-alignment of the whole market). For already-well-aligned agents, it is optimal to make them silent and only assign positive meeting technologies to those mis-aligned agents.

By Proposition 6, we can explicitly obtain the expression of the social optimal meeting technology (details are in Appendix 3.A.9):

$$\lambda_1^{S*}(\delta) = \begin{cases} \frac{-2c_1\alpha^2+\sqrt{4c_1^2\alpha^4+4c_1\alpha^2(\frac{1}{2}-\delta)}}{2c_1\alpha} & \delta \in [0, \frac{1}{2}); \\ 0 & \delta \in [\frac{1}{2}, 1]. \end{cases} \tag{3.18}$$

We also give a quantitative example with $r = 0.05, \alpha = 0.1, c_1 = 1$ to compare the social optimal and competitive equilibrium meeting technologies and densities for both asset owners and nonowners. The weighted average meeting technology for asset owners (or nonowners) is $\Lambda^C = 0.0766$ in competitive equilibrium and $\Lambda^S = 0.0376$ in social optimal solution, which means the latter assignment costs less and is more efficient. The social welfare is $W^C = 6.5764$ in competitive equilibrium, which is lower than that of social optimal solution $W^S = 6.7820$.

Figure 3.12 compares the symmetric social optimal and competitive equilibrium meeting technologies. The solution $\lambda_1^{S*}(\delta)$ numerically searched out by MatLab exactly follows the explicit solution (3.18). Intuitively, more searching resources (meeting technologies) are assigned to extremely mis-aligned agents, for example, $\lambda_1^{S*}(0) > \lambda_1^*(0)$. Figure 3.13 compares the densities generated by competitive equilibrium and social optimal assignment of meeting technologies. We can see, the densities generated by social optimal assignment are closer to the Walrasian case which is the most efficient one.

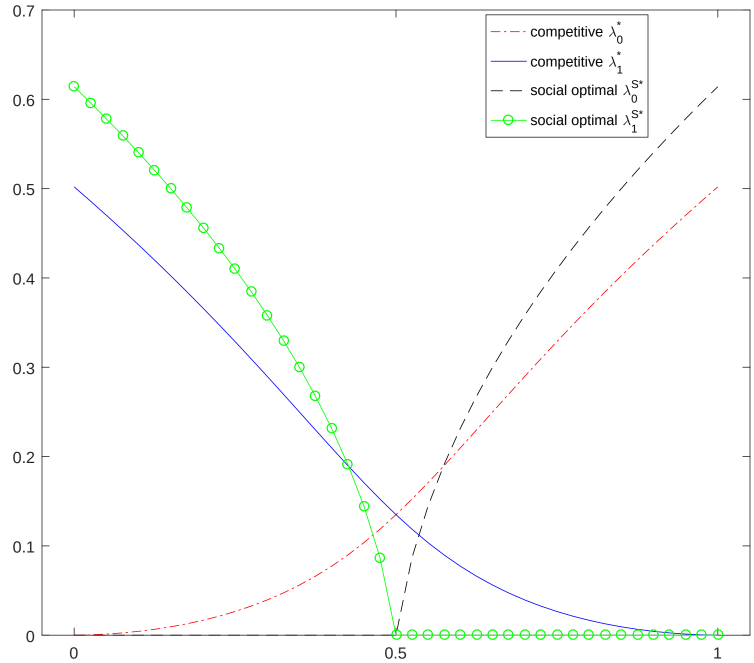

Figure 3.12: Competitive equilibrium and social optimal meeting technologies

Then Figure 3.14 shows that in social optimal solution, for each utility type, the expected instantaneous gross trading volume is equal to the net trading volume, which intuitively shows that the intermediation trading volume is constantly zero across all utility types. As a result, in social optimal solution, the profit per trade is essentially the expected revenue(cost) per trade for asset owners(non-owners) with utility type lower(higher) than $\frac{1}{2}$, the magnitudes of which are much larger than that of intermediation profit in competitive equilibrium solution (the right y axis in sub-figure "profit per trade($c_1 = 1, \alpha = 0.1$)). The social optimal gross trading volume for the whole market is 1.54 compared with that of the competitive equilibrium solution which is 2.33. This possibly implies that individual traders do not internalize the social externality into their decisions and there exist large part(in this case, approximately 34%) of inefficient tradings in the sense that they do not contribute to the well-alignment of target asset in the market.

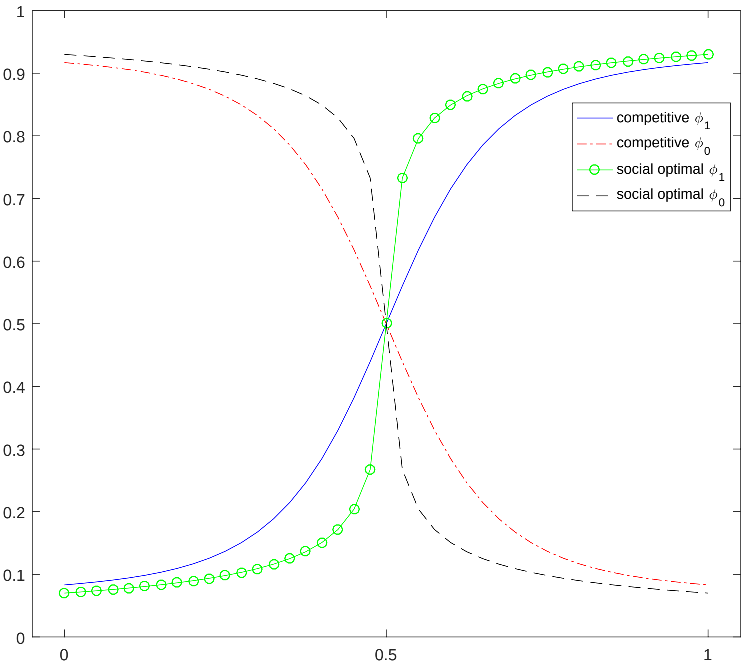

Figure 3.13: Competitive equilibrium and social optimal densities

3.6.2 Robustness of the optimality of "no intermediation"

Section 3.6.1 essentially concludes that in social optimal solution, there is "no intermediation" in interdealer market, since no agent will be assigned positive meeting technologies both when being asset owner and when being asset nonowner. To check the robustness, we give Proposition 7 below to show which forms of searching cost function will guarantee the optimality of "no intermediation".

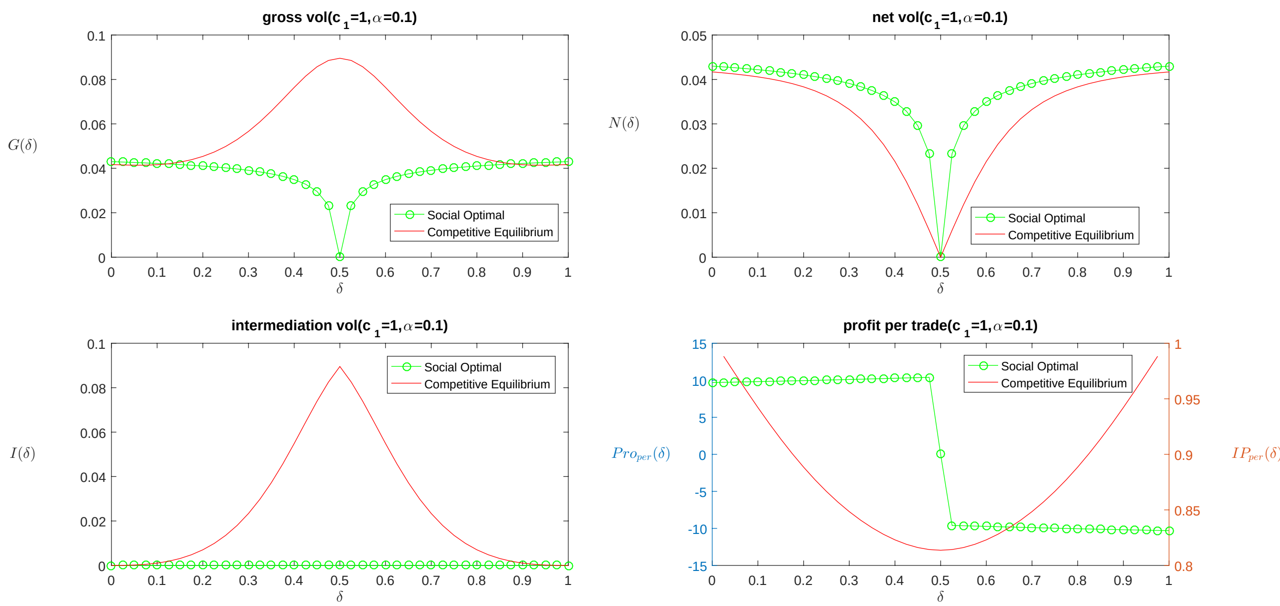

Figure 3.14: Liquidity measures between social optimal and competitive equilibrium solutions

Proposition 7 For any cost function $C(\lambda)$ that satisfies the following condition[23][24]

$$C'(\lambda) \begin{cases} \geq 0 & \delta = 0; \\ > 0 & \forall \delta \in (0, \lambda^{ub}]. \end{cases} \tag{3.19}$$

the social optimal meeting technology $\lambda_1^{S*}(\delta) \equiv 0$ for $\forall \delta \in [\frac{1}{2}, 1]$. Proof is in Appendix 3.A.8.

Condition (3.19) applies for most cost functions including quadratic form $C(\lambda) = c_1\lambda^2$, linear form $C(\lambda) = c_1\lambda$, concave form $C(\lambda) = c_1\lambda^p \quad p \in (0, 1)$ and etc, where the coefficient $c_1 > 0$. Then we conclude that the optimality of "no intermediation" is robust to the cost function form.

The key to this result is our assumption that agents are allowed to and also willing

[23] In condition (3.19), $C'(0) \geq 0$ includes the case that $C'(0) = +\infty$

[24] λ^{ub} is the upper bound of meeting technology for either asset owners or nonowners. If there is no upper bound, it is equal that $\lambda^{ub} = +\infty$.

to shift to new meeting technologies in response to idiosyncratic liquidity shock through a uniform policy rule. Compared with Farboodi, Jarosch, and Shimer (2017b), in their paper, each agent is endowed with/chooses a certain level of meeting technology, which, once chosen, cannot be changed forever. This can equally be regarded as the case that the adjustment cost of meeting technology is infinity. So there is no one-to-one mapping from agent's utility type to their meeting technology. Meanwhile, with some other reasonable assumptions, the main trading incentive comes from the difference in meeting technologies between every two matched agents, and the agents with more advanced meeting technologies will automatically play the role of intermediator. While in our paper, agents are free to adjust their meeting technologies without any adjustment cost, then agents of each specific utility type will either purely search to sell or purely search to buy at every time point. So the meaning of intermediation in our paper is a little bit different from that in Farboodi, Jarosch, and Shimer (2017b).

Next we will show, for the general cost function $C(\lambda)$ that satisfies the condition in Proposition 7, how to obtain the explicit expression of $\lambda_1^{S*}(\delta)$. By Proposition 7 and substituting $\lambda_1^{S*}(\delta) \equiv 0, \forall \delta \in [\frac{1}{2}, 1]$ into the equilibrium constraint to obtain the expression of $\phi_1^S(\delta)$, we can get the reduced-form social planner problem $[RP]$:

$$\begin{aligned}
\underset{\lambda_1^S(\delta)\in\Lambda_{S1}}{max} \quad W^*(\lambda_1^S(\delta)) &= \int_0^{\frac{1}{2}} (\delta - 2C(\lambda_1^S(\delta)))\frac{1}{1+\frac{\frac{\alpha}{2}+\lambda_1^S(\delta)}{\frac{\alpha}{2}}}d\delta + \int_{\frac{1}{2}}^1 \delta\frac{1}{1+\frac{\frac{\alpha}{2}}{\frac{\alpha}{2}+\lambda_1^S(1-\delta)}}d\delta \\
&= \int_0^{\frac{1}{2}} \left(\frac{-\alpha C(\lambda_1^S(\delta)) + \frac{\alpha}{2} + (1-\delta)\lambda_1^S(\delta)}{\alpha + \lambda_1^S(\delta)}\right) d\delta \\
&= \int_0^{\frac{1}{2}} f^C(\lambda_1^S(\delta), \delta) d\delta
\end{aligned}$$

s.t.

$$K_{\frac{1}{2}} = \int_0^{\frac{1}{2}} \frac{\frac{\alpha}{2}\lambda_1^S(\delta)}{\alpha + \lambda_1^S(\delta)} d\delta \leq \Lambda \tag{3.20}$$

where in (3.20), Λ is the restricted maximum meeting technology of the whole market.[25]

By Hamiltonian approach,

$$
\begin{aligned}
L(\delta, \lambda_1^S(\delta)) &= H(\delta, K_\delta) + \mu(\Lambda - K_{\frac{1}{2}}) \\
&= \frac{-\alpha C(\lambda_1^S(\delta)) + \frac{\alpha}{2} + (1-\delta)\lambda_1^S(\delta)}{\alpha + \lambda_1^S(\delta)} + m_\delta \frac{\frac{\alpha}{2}\lambda_1^S(\delta)}{\alpha + \lambda_1^S(\delta)} + \mu(\Lambda - K_{\frac{1}{2}})
\end{aligned}
$$

where

$$
K_\delta = \int_0^\delta \frac{\frac{\alpha}{2}\lambda_1^S(\delta')}{\alpha + \lambda_1^S(\delta')} d\delta' \quad \text{and} \quad \dot{K}_\delta = \frac{\frac{\alpha}{2}\lambda_1^S(\delta)}{\alpha + \lambda_1^S(\delta)}
$$

The necessary conditions for $\lambda_1^{S^*}(\delta) : [0, \frac{1}{2}) \rightarrow R^+$ to be optimal are:

$$
\dot{m}_\delta = -\frac{\partial H(\delta, K_\delta)}{\partial K_\delta} = 0
$$

$$
\bar{m} = m_{\frac{1}{2}} = \begin{cases} 0 & \text{if } \mu = 0; \\ -\frac{\partial W^*(\lambda_1^S(\delta))}{\partial K_{\frac{1}{2}}} & \text{if } \mu > 0. \end{cases}
$$

For simplicity, we consider the case $\Lambda = \lambda^{ub}$, i.e. budget constraint is always not binding, then $\mu = 0$ and $\bar{m} = 0$. Then we have:

$$
\begin{aligned}
\frac{\partial L(\delta, \lambda_1^S(\delta))}{\partial \lambda_1^S(\delta)} &= \frac{\partial}{\partial \lambda_1^S(\delta)} \left(\frac{-\alpha C(\lambda_1^S(\delta)) + \frac{\alpha}{2} + (1-\delta)\lambda_1^S(\delta)}{\alpha + \lambda_1^S(\delta)} \right) \\
&= \frac{\frac{\alpha}{2} - \alpha\delta + \alpha C(\lambda_1^S(\delta)) - \alpha(\alpha + \lambda_1^S(\delta))C'(\lambda_1^S(\delta))}{(\alpha + \lambda_1^S(\delta))^2}
\end{aligned}
$$

[25] For simplicity, usually we consider the case that Λ is equal to the upper bound of $\lambda_1(\delta)$, λ^{ub}, i.e. budget constraint (3.20) is always not binding, then the corresponding Lagrangian multipliers satisfy $\mu = 0$ and $\bar{m} = 0$.

and

$$\frac{\partial^2 L(\delta,\lambda_1^S(\delta))}{\partial {\lambda_1^S}^2(\delta)} = \frac{-\alpha C''(\lambda_1^S(\delta))(\alpha+\lambda_1^S(\delta))^2}{(\alpha+\lambda_1^S(\delta))^3} - \frac{(\alpha-2\alpha\delta+2\alpha C(\lambda_1^S(\delta))-2\alpha(\alpha+\lambda_1^S(\delta))C'(\lambda_1^S(\delta)))}{(\alpha+\lambda_1^S(\delta))^3}$$

Then for each $\delta \in [0, \frac{1}{2})$, the optimal ${\lambda_1^S}^*(\delta)$ needs to satisfy one of the following conditions:

1. $\frac{\partial L(\delta,\lambda_1^S(\delta))}{\partial \lambda_1^S(\delta)}|_{\lambda_1^S(\delta)={\lambda_1^S}^*(\delta)} = 0$ and $\frac{\partial^2 L(\delta,\lambda_1^S(\delta))}{\partial {\lambda_1^S}^2(\delta)}|_{\lambda_1^S(\delta)={\lambda_1^S}^*(\delta)} \leq 0$, then $0 < {\lambda_1^S}^*(\delta) < \lambda^{ub}$;
2. $\frac{\partial L(\delta,\lambda_1^S(\delta))}{\partial \lambda_1^S(\delta)} > 0$ for $\forall \lambda_1(\delta) \in [0, \lambda^{ub}]$, then ${\lambda_1^S}^*(\delta) = \lambda^{ub}$;
3. $\frac{\partial L(\delta,\lambda_1^S(\delta))}{\partial \lambda_1^S(\delta)} < 0$ for $\forall \lambda_1(\delta) \in [0, \lambda^{ub}]$, then ${\lambda_1^S}^*(\delta) = 0$;
4. For every $\delta \in [0, \frac{1}{2})$, $\nexists \lambda_1^S(\delta) \in [0, \lambda^{ub}]$ s.t. $\frac{\partial L(\delta,\lambda_1^S(\delta))}{\partial \lambda_1^S(\delta)} = 0$ and $\frac{\partial^2 L(\delta,\lambda_1^S(\delta))}{\partial {\lambda_1^S}^2(\delta)} \leq 0$, and $W(\lambda_1^S(\delta) \equiv 0) > W(\lambda_1^S(\delta) \equiv \lambda^{ub})$, then ${\lambda_1^S}^*(\delta) \equiv 0 \ \forall \delta \in [0, \frac{1}{2})$;
5. For every $\delta \in [0, \frac{1}{2})$, $\nexists \lambda_1^S(\delta) \in [0, \lambda^{ub}]$ s.t. $\frac{\partial L(\delta,\lambda_1^S(\delta))}{\partial \lambda_1^S(\delta)} = 0$ and $\frac{\partial^2 L(\delta,\lambda_1^S(\delta))}{\partial {\lambda_1^S}^2(\delta)} \leq 0$, and $W(\lambda_1^S(\delta) \equiv 0) < W(\lambda_1^S(\delta) \equiv \lambda^{ub})$, then ${\lambda_1^S}^*(\delta) \equiv \lambda^{ub} \ \forall \delta \in [0, \frac{1}{2})$.

In Appendix 3.A.9, we specifically solve the explicit expressions of $\lambda_1^{S*}(\delta)$ with cost functions of quadratic forms $C(\lambda) = c_1\lambda^2$ and $C(\lambda) = c_2\lambda^2 + c_3\lambda(c_2 < 0, c_3 > 0)$, linear form $C(\lambda) = c_1\lambda$ and concave form $C(\lambda) = c_1\lambda^p \quad p \in (0, 1)$, where in all cases $c_1 > 0$.

3.6.3 Unidimentional policy measure with linear investment cost

In this section, we specifically focus on the case of linear cost function $C(\lambda) = c_1\lambda$, which generates the unidimentional policy measure. In this case, social planner only needs to identify the marginal asset-owner utility type, and the policy will be to assign all the asset owners with utility types lower than the marginal one with the most advanced meeting technology λ^{ub} and make all the asset owners with utility types larger than the marginal one silent in the market. Symmetrically, we can obtain the marginal asset-nonowner utility type accordingly and assign all asset nonowners with utility types larger than this marginal type with the most advanced meeting technology and make those lower than this marginal type silent.

Proposition 8 In social planner problem, when $C(\lambda) = c_1\lambda$ $(c_1 > 0)$, the social optimal meeting technology of asset owner $\lambda_1^{S*}(\delta)$ satisfies:

1. If $c_1\alpha < \frac{1}{2}$,

$$\lambda_1^{S*}(\delta) = \begin{cases} \lambda^{ub} & \text{if } \delta \leq \delta_1^*; \\ 0 & \text{if } \delta > \delta_1^*. \end{cases}$$

   where $\delta_1^* = \frac{1}{2} - c_1\alpha$ is the marginal asset-owner utility type;

2. If $c_1\alpha \geq \frac{1}{2}$,

$$\lambda_1^{S*}(\delta) \equiv 0 \quad \forall \delta \in [0, 1]$$

Details are in Appendix 3.A.9.

Finally we give a quantitative example to compare the competitive equilibrium and the social optimal solutions. In the example, we set $r = 0.05, c_1 = 0.05, \alpha = 0.35, \lambda^{ub} = 1.4$ and obtain the competitive equilibrium welfare is 6.2031 and the social optimal welfare is 6.8953. Figure 3.15 compares the meeting technologies. It is straight forward to see that there is no intermediation in the social optimal solution, since the optimal meeting technologies do not overlap at the upper bound.

3.7 Conclusion

This paper develops a search-and-bargain model to evaluate the policy responses in different OTC market environments in response to a certain form of aggregate liquidity shock, which affects dealers' valuation on the target asset. In the model setup, dealers are free to choose and change their optimal meeting technology based on their characteristics: asset position

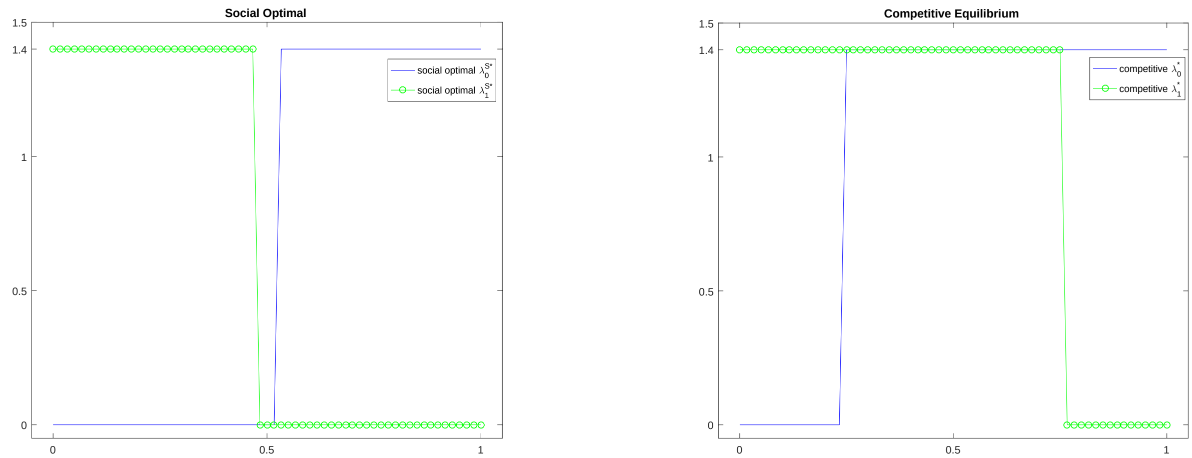

Figure 3.15: Social optimal and competitive equilibrium meeting technologies for linear cost ($\lambda^{ub} = 1.4$)

and liquidity need, which is new to the current literature. The model can generate the core-periphery interdealer network which is one of the common stylized facts of OTC markets that documented in former papers. We find that, dealers of intermediate utility types become the core dealers in less frictional market where meeting technology is relatively cheap and frequency of idiosyncratic liquidity shock is relatively low; while in the opposite more frictional market environment, dealers with extreme utility types will become the core ones and dealers with intermediate utility types will behave much less active than before. And these different relationships between dealers' liquidity needs and optimal meeting technologies have different potential implications for policy choice in response to the aggregate liquidity shock. In more frictional market, monetary authority with limited funding should firstly inject liquidity into the core dealers; while in less frictional market, monetary authority should firstly inject liquidity into the periphery ones. We also conclude that in the social optimal assignment of meeting technologies among all dealers, there is no intermediation in the sense that no dealer will be assigned positive meeting technologies both when being asset owner and when being asset nonowner. Specifically, we discuss the case of linear cost function, which generates the unidimentional policy measure.

In this paper, we implicitly assume that there is perfect information in the market since every agent has rational expectation on the distribution of utility types. As a result, the main searching motive in our model is to trade with others to either gain intermediation profit or hedge mis-aligned asset position. While the two most significant characteristics of OTC market are accessibility/searching friction and imperfect information. The future research direction may be to incorporate private information into the model and thus generates alternative searching motive to learn (e.g. the quality of target asset or the matched counterparty's expected valuation) from trading.

Appendix 3.A Appendix of Chapter 3

3.A.1 Proposition 1

We use guess and verify approach to prove the monotonicity of reservation value function $\triangle V(\delta)$. Suppose $\triangle V(\delta)$ is strictly increasing, then the subtraction "(3.1) minus (3.2)" will reduce to (3.5):

$$r\triangle V(\delta) = \delta + C(\lambda_0^*(\delta)) - C(\lambda_1^*(\delta)) + \alpha \int_0^1 (\triangle V(\delta') - \triangle V(\delta)) dF_\delta(\delta')$$

$$+\lambda_1^*(\delta)\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}(\triangle V(\delta') - \triangle V(\delta))\phi_0(\delta')d\delta' - \lambda_0^*(\delta)\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}(\triangle V(\delta) - \triangle V(\delta'))\phi_1(\delta')d\delta'$$

and expressions of individual and aggregate levels of optimal meeting technologies will reduce to (3.6)-(3.8). By (3.5)-(3.8), we obtain that for $\forall\delta$

$$\begin{aligned}&(\triangle V(\delta))^2\left(\frac{a(\delta)^2 - b(\delta)^2}{4c_1}\right) + \triangle V(\delta)\left(r + \alpha + \frac{B(\delta)b(\delta) - A(\delta)a(\delta)}{2c_1}\right)\\ &- \delta - \alpha E[\triangle V] - \frac{B(\delta)^2 - A(\delta)^2}{4c_1} = 0 \end{aligned} \tag{3.21}$$

with the notations as:

$$\begin{aligned} A(\delta) &= \int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}\triangle V(\delta')\phi_1(\delta')d\delta' \\ B(\delta) &= \int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\triangle V(\delta')\phi_0(\delta')d\delta' \\ a(\delta) &= \int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta' \\ b(\delta) &= \int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\phi_0(\delta')d\delta' \end{aligned} \tag{3.22}$$

$$E[\triangle V] = \int_0^1 \triangle V(\delta')f_\delta(\delta')d\delta' \tag{3.23}$$

Then denote LHS of equation (3.21) as F, by Implicit Function Theorem, we verify that

$$\frac{d\triangle V(\delta)}{d\delta} = -\frac{\partial F/\partial \delta}{\partial F/\partial \triangle V(\delta)} = \frac{1}{r+\alpha+\lambda_1^*(\delta)b(\delta)+\lambda_0^*(\delta)a(\delta)} > 0 \quad \forall \delta \in [0,1]$$

By first order conditions (3.3)(3.4), it is trival that

$$\lambda_1^*(1) = \lambda_0^*(0) = 0 \tag{3.24}$$

and by plugging the first order conditions into the HJB equations, the optimal meeting technology functions $\lambda_1^*(\delta)$ and $\lambda_0^*(\delta)$ satisfy

$$(\alpha+r)V_1(\delta) = \delta + c_1\lambda_1^{*2}(\delta) + \alpha E[V_1(\delta)] \tag{3.25}$$

$$(\alpha+r)V_0(\delta) = c_1\lambda_0^{*2}(\delta) + \alpha E[V_0(\delta)] \tag{3.26}$$

where the expectation $E[\cdot]$ is using the symmetric PDF $f_\delta(\delta)$,
(3.25)-(3.26) $\Longrightarrow$

$$(\alpha+r)\triangle V(\delta) = \delta + c_1\lambda_1^{*2}(\delta) - c_1\lambda_0^{*2}(\delta) + \alpha E[\triangle V(\delta)] \tag{3.27}$$

apply $E[\cdot]$ on both sides $\Longrightarrow$

$$(\alpha+r)E[\triangle V(\delta)] = E[\delta] + c_1\left(\int_0^1 \lambda_1^{*2}(\delta)f_\delta(\delta)d\delta - \int_0^1 \lambda_0^{*2}(\delta)f_\delta(\delta)d\delta\right) + \alpha E[\triangle V(\delta)] \tag{3.28}$$

Later by Corrollary 1, we will prove that if distribution of utility type ($f_\delta(\delta)$) is symmetric with respect to $\delta = \frac{1}{2}$, then the equilibrium optimal meeting technology functions $\lambda_1^*(\delta)$ and $\lambda_0^*(\delta)$ are symmetric to each other with respect to $\delta = \frac{1}{2}$, i.e. $\lambda_0^*(\delta) = \lambda_1^*(1-\delta)$ for $\forall\delta \in [0,1]$.

Here we just take this conclusion as given and then we can get:

$$\int_0^1 \lambda_1^{*2}(\delta) f_\delta(\delta) d\delta = \int_0^1 \lambda_0^{*2}(\delta) f_\delta(\delta) d\delta$$

Together with (3.28), we obtain:

$$E[\triangle V(\delta)] = \frac{E(\delta)}{r} > 0$$

Then by (3.24)(3.27),

$$(\alpha + r)\triangle V(0) = c_1 \lambda_1^{*2}(0) + \alpha E[\triangle V(\delta)] > 0$$

Together with $\frac{d\triangle V(\delta)}{d\delta} > 0 \quad \forall \delta \in [0,1]$, we obtain

$$\triangle V(\delta) > 0 \quad \forall \delta \in [0,1]$$

By (3.6)(3.7), we obtain:

$$\frac{d\lambda_1^*(\delta)}{d\delta} = \frac{-\frac{d\triangle V(\delta)}{d\delta} \int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0} \phi_0(\delta') d\delta'}{2c_1} < 0 \quad \forall \delta \in [0,1]$$

$$\frac{d\lambda_0^*(\delta)}{d\delta} = \frac{\frac{d\triangle V(\delta)}{d\delta} \int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1} \phi_1(\delta') d\delta'}{2c_1} > 0 \quad \forall \delta \in [0,1]$$

3.A.2 Proposition 2

Based on properties of the competitive equilibrium components $\triangle V(\delta)$, $\lambda_1^*(\delta)$ and $\phi_1(\delta)$, under $f_\delta(\delta) \equiv 1$, define the following normed linear spaces: $\triangle V_S = \{\triangle V(\delta) : \triangle V(\delta) \in C^1[0,1]; \triangle V(\delta) \geq 0 \text{ and } \triangle V'(\delta) > 0, \forall \delta \in [0,1]; E(\triangle V(\delta)) = \int_0^1 \triangle V(\delta) d\delta = \frac{1}{2r}\}$, $\Lambda_{S1} = \{\lambda_1^*(\delta) : \lambda_1^*(\delta) \in C^1[0,1]; \lambda_1^*(\delta) \geq 0 \text{ and } \lambda_1^{*'}(\delta) < 0, \forall \delta \in [0,1]\}$, $\Phi_{S1} = \{\phi_1(\delta) : \phi_1(\delta) \in C^1[0,1]; 0 \leq \phi_1(\delta) \leq 1 \text{ and } \phi_1'(\delta) > 0, \forall \delta \in [0,1]; \int_0^1 \phi_1(\delta) d\delta = \frac{1}{2}\}$, all with the norm $\|f\| =$

$\max\limits_{0\leq\delta\leq 1}|f(\delta)|$.

Vector of stationary equilibrium components $(\triangle V(\delta) \quad \lambda_1^*(\delta) \quad \lambda_0^*(\delta) \quad \phi_1(\delta) \quad \phi_0(\delta))^T$, by symmetry between $\lambda_1^*(\delta)$ and $\lambda_0^*(\delta)$ and symmetry between $\phi_1(\delta)$ and $\phi_0(\delta)$, is a fixed point of the following transformation $T : \triangle V_S \times \Lambda_{S1} \times \Phi_{S1} \longrightarrow \triangle V_S \times \Lambda_{S1} \times \Phi_{S1}$:[26]

$$T\begin{bmatrix} \triangle V(\delta) \\ \lambda_1^*(\delta) \\ \phi_1(\delta) \end{bmatrix} = \begin{bmatrix} T_1(\triangle V(\delta)) \\ T_2(\lambda_1^*(\delta)) \\ T_3(\phi_1(\delta)) \end{bmatrix}$$

where

$$\begin{aligned} T_1(\triangle V(\delta)) =& \frac{\delta + c_1\lambda_0^{*2}(\delta) - c_1\lambda_1^{*2}(\delta) + \alpha\int_0^1 \triangle V(\delta')dF_\delta(\delta')}{r+\alpha+\lambda_1^*(\delta)\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\phi_0(\delta')d\delta' + \lambda_0^*(\delta)\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta'} \\ &+ \frac{\lambda_1^*(\delta)\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\triangle V(\delta')\phi_0(\delta')d\delta' + \lambda_0^*(\delta)\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}\triangle V(\delta')\phi_1(\delta')d\delta'}{r+\alpha+\lambda_1^*(\delta)\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\phi_0(\delta')d\delta' + \lambda_0^*(\delta)\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta'} \\ =& \frac{1}{\alpha+r}\left(\delta + c_1\lambda_1^{*2}(\delta) - c_1\lambda_1^{*2}(1-\delta) + \frac{\alpha}{2r}\right) \end{aligned}$$

$$\begin{aligned} T_2(\lambda_1^*(\delta)) &= \frac{1}{2c_1}\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\triangle V(\delta')\phi_0(\delta')d\delta' \\ &= \frac{1}{2c_1}\int_0^{1-\delta} \frac{\lambda_1^*(\delta')}{\Lambda_1}(\triangle V(1-\delta') - \triangle V(\delta))\phi_1(\delta')d\delta' \\ &\text{s.t. } \Lambda_1 = 2\int_0^1 \lambda_1^*(\delta')\phi_1(\delta')d\delta' \end{aligned}$$

[26]For each "component mapping" ($T_1 - T_3$), we assume all the other equilibrium components are given and may/may not be the corresponding "fixed points".

$T_3(\phi_1(\delta)) =$

$$\frac{\alpha \int_0^1 \lambda_1^*(\delta')\phi_1(\delta')d\delta' + 2\lambda_1^*(1-\delta)\int_0^\delta \lambda_1^*(\delta')\phi_1(\delta')d\delta'}{2\alpha \int_0^1 \lambda_1^*(\delta')\phi_1(\delta')d\delta' + 2\lambda_1^*(1-\delta)\int_0^\delta \lambda_1^*(\delta')\phi_1(\delta')d\delta' + 2\lambda_1^*(\delta)\int_0^{1-\delta} \lambda_1^*(\delta')\phi_1(\delta')d\delta'}$$

Use $(\overline{\triangle V}(\delta), \overline{\lambda_1^*}(\delta), \overline{\phi_1}(\delta))$ as notations of fixed points of $\triangle V(\delta)$, $\lambda_1^*(\delta)$ and $\phi_1(\delta)$.

(1) Given fixed point $\overline{\lambda_1^*}(\delta)$, by transformation T_1,
$\overline{\triangle V}(\delta) = \frac{1}{\alpha+r}\left(\delta + c_1\overline{\lambda_1^*}^2(\delta) - c_1\overline{\lambda_1^*}^2(1-\delta) + \frac{\alpha}{2r}\right)$ is a fixed point of $\triangle V(\delta)$.

(2) Given fixed points $\overline{\triangle V}(\delta)$ and $\overline{\phi_1}(\delta)$, plug them into transformation T_2 which is trivally continuous, we can prove this T_2 works on normed linear space Λ_{S1} which is nonempty (trivally), convex and compact.

Convexity

For $\forall \lambda_1^{1*}(\delta), \lambda_1^{2*}(\delta) \in \Lambda_{S1}$ and $\forall \lambda \in (0,1)$, define the new function $\hat{\lambda}(\delta) = \lambda * \lambda_1^{1*}(\delta) + (1-\lambda) * \lambda_1^{2*}(\delta)$, it is trival that $\hat{\lambda}(\delta) \in C^1[0,1]$, $\hat{\lambda}(\delta) \geq 0$ and $\hat{\lambda}'(\delta) = \lambda * \lambda_1^{1*\prime}(\delta) + (1-\lambda) * \lambda_1^{2*\prime}(\delta) < 0$. So $\hat{\lambda}(\delta) \in \Lambda_{S1}$ for $\forall \lambda \in (0,1)$.

Boundedness

For $\forall \lambda_1^*(\delta) \in \Lambda_{S1}$,

$$\begin{aligned}
T_2(\lambda_1^*(\delta)) &= \frac{1}{2c_1}\int_0^{1-\delta} \frac{\lambda_1^*(\delta')}{\Lambda_1}(\overline{\triangle V}(1-\delta') - \overline{\triangle V}(\delta))\overline{\phi_1}(\delta')d\delta' \\
&\leq (\overline{\triangle V}(1) - \overline{\triangle V}(0))\frac{1}{2c_1}\int_0^{1-\delta} \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta' \\
&\leq (\overline{\triangle V}(1) - \overline{\triangle V}(0))\frac{1}{2c_1}\int_0^{1} \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta' \\
&= \frac{(\overline{\triangle V}(1) - \overline{\triangle V}(0))}{2c_1}
\end{aligned}$$

By Proposition 1, $\forall \triangle V(\delta) \in \triangle V_S$ is strictly increasing on $[0,1]$. We have,

$$0 < \triangle V(1) - \triangle V(0) = \frac{1 - 2c_1 {\lambda_1^*}^2(0)}{\alpha + r} < \frac{1}{\alpha + r} \tag{3.29}$$

then

$$T_2(\lambda_1^*(\delta)) < \frac{1}{2c_1(\alpha + r)}$$

Equicontinuity

Firstly we need to prove the boundedness of $\frac{d\triangle V(\delta)}{d\delta}$ for $\forall \triangle V(\delta) \in \triangle V_S$.

$$\frac{d\triangle V(\delta)}{d\delta} = \frac{1}{r + \alpha + \lambda_1^*(\delta)b(\delta) + \lambda_0^*(\delta)a(\delta)}$$

where $0 \leq a(\delta) = \int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta' \leq \frac{1}{2}$ and $0 \leq b(\delta) = \int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\phi_0(\delta')d\delta' \leq \frac{1}{2}$.
Also by (3.29), $0 < \lambda_1^*(0) = \underset{\delta \in [0,1]}{max} \lambda_1^*(\delta) < \frac{1}{\sqrt{2c_1}}$, then we get

$$\frac{1}{r + \alpha + \frac{1}{\sqrt{2c_1}}} < \frac{d\triangle V(\delta)}{d\delta} < \frac{1}{r + \alpha} = B_{dV}$$

Then for $\forall \lambda_1^*(\delta) \in \Lambda_{S1}$ and $\forall \delta \in [0,1]$: given $\forall \epsilon > 0$, we can always choose small enough $\hat{\triangle} = \frac{2c_1\epsilon}{B_{dV}} > 0$, such that, by (3.6)-(3.8),

$$|\lambda_1^*(\delta + \hat{\triangle}) - \lambda_1^*(\delta)| = \left| \frac{-\hat{\triangle}}{2c_1} \frac{d\overline{\triangle V}(\delta)}{d\delta} \int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\phi_0(\delta')d\delta' + o(\hat{\triangle}) \right| \leq 2 * \frac{1}{2c_1} \frac{2c_1\epsilon}{B_{dV}} B_{dV} \frac{1}{2} = \epsilon.$$

Since $\hat{\triangle}$ does not relate to specific value of δ, then any sequence of functions in normed linear space Λ_{S1} is uniform equicontinuous on $[0,1]$. Based on **boundedness** and **equicontinuity** above, and refer to Arzelä-Ascoli theorem, we prove the continuous transformation T_2, under given fixed points $\overline{\triangle V}(\delta)$ and $\overline{\phi_1}(\delta)$, maps Λ_{S1} to Λ_{S1}, where the normed linear space Λ_{S1} is nonempty, convex and compact. By Schauder's fixed point theorem, given fixed points $\overline{\triangle V}(\delta)$ and $\overline{\phi_1}(\delta)$, there exists fixed point $\overline{\lambda_1^*}(\delta)$ of $\lambda_1^*(\delta)$.

(3) Given fixed points $\overline{\lambda_1^*}(\delta)$ and $\overline{\triangle V}(\delta)$, plug $\overline{\lambda_1^*}(\delta)$ into transformation T_3 which is trivally continuous, we can prove this T_3 works on normed linear space Φ_{S1} which is nonempty (trivally), convex and compact.

Convexity

For $\forall \phi_1^1(\delta), \phi_1^2(\delta) \in \Phi_{S1}$ and $\forall \lambda \in (0,1)$, define the new function $\hat{\phi}(\delta) = \lambda * \phi_1^1(\delta) + (1-\lambda) * \phi_1^2(\delta)$, it is trival that $\hat{\phi}(\delta) \in C^1[0,1]$, $0 \leq \hat{\phi}(\delta) \leq \lambda + 1 - \lambda = 1$ and $\hat{\phi}'(\delta) = \lambda * \phi_1^{1\prime}(\delta) + (1-\lambda) * \phi_1^{2\prime}(\delta) > 0$. So $\hat{\phi}(\delta) \in \Phi_{S1}$ for $\forall \lambda \in (0,1)$.

Boundedness

By definition of normed linear space Φ_{S1}, it is trival that Φ_{S1} is bounded.

Equicontinuity

We already proved the boundedness of $\frac{d\overline{\triangle V}(\delta)}{d\delta}$ and thus the boundedness of $\overline{\lambda_1^*}'(\delta) = \frac{d\overline{\lambda_1^*}(\delta)}{d\delta} = \frac{-1}{2c_1}\frac{d\overline{\triangle V}(\delta)}{d\delta}\int_0^{1-\delta}\frac{\overline{\lambda_1^*}(\delta')}{\Lambda_1}\phi_1(\delta')d\delta'$. Next we need to prove the boundedness of $\frac{d\phi_1(\delta)}{d\delta}$.

$$
\begin{aligned}
&\frac{d\phi_1(\delta)}{d\delta} \\
&= \frac{d}{d\delta}\left[\frac{\alpha\int_0^1 \lambda_1^*(\delta')\phi_1(\delta')d\delta' + 2\lambda_1^*(1-\delta)\int_0^\delta \lambda_1^*(\delta')\phi_1(\delta')d\delta'}{2\alpha\int_0^1 \lambda_1^*(\delta')\phi_1(\delta')d\delta' + 2\lambda_1^*(1-\delta)\int_0^\delta \lambda_1^*(\delta')\phi_1(\delta')d\delta' + 2\lambda_1^*(\delta)\int_0^{1-\delta}\lambda_1^*(\delta')\phi_1(\delta')d\delta'}\right] \\
&= \frac{\left[\frac{\lambda_1^*(1-\delta)\lambda_1^*(\delta)\phi_1(\delta)}{\Lambda_1} - \lambda_1^{*\prime}(1-\delta)a(\delta)\right]\left[\frac{\alpha}{2} + 2\lambda_1^*(\delta)b(\delta)\right]}{\left(\alpha + 2\lambda_1^*(1-\delta)a(\delta) + 2\lambda_1^*(\delta)b(\delta)\right)^2} \\
&- \frac{\left[\lambda_1^{*\prime}(\delta)b(\delta) - \frac{\lambda_1^*(1-\delta)\lambda_1^*(\delta)\phi_1(1-\delta)}{\Lambda_1}\right]\left[\frac{\alpha}{2} + 2\lambda_1^*(1-\delta)a(\delta)\right]}{\left(\alpha + 2\lambda_1^*(1-\delta)a(\delta) + 2\lambda_1^*(\delta)b(\delta)\right)^2}
\end{aligned}
$$

We already proved the boundedness of $\lambda_1^*(\delta)$, $\lambda_1^{*\prime}(\delta)$, $a(\delta)$, $b(\delta)$, and $\exists \hat{\epsilon} > 0$ s.t. $\hat{\epsilon} \leq \Lambda_1 \leq 2\lambda_1^*(0)$, then if we plug in the given fixed points $\overline{\lambda_1^*}(\delta)$ and $\overline{\triangle V}(\delta)$ into the above equation, we will obtain the boundedness of $\frac{d\phi_1(\delta)}{d\delta}$, denote $\underset{\delta\in[0,1]}{max}|\frac{d\phi_1(\delta)}{d\delta}| = B_{d\phi_1}$.

Then for $\forall \phi_1(\delta) \in \Phi_{S1}$ and $\forall \delta \in [0,1]$: given $\forall \epsilon > 0$, we can always choose small enough

$\hat{\triangle} = \frac{\epsilon}{2B_{d\phi_1}} > 0$, such that,

$$|\phi_1(\delta + \hat{\triangle}) - \phi_1(\delta)| = \left| \frac{d\phi_1(\delta)}{d\delta} \hat{\triangle} + o(|\hat{\triangle}|) \right| \le 2 * \hat{\triangle} * \left| \frac{d\phi_1(\delta)}{d\delta} \right| \le 2 * \frac{\epsilon}{2B_{d\phi_1}} * B_{d\phi_1} = \epsilon.$$

Since $\hat{\triangle}$ does not relate to specific value of δ, then any sequence of functions in normed linear space Φ_{S1} is uniform equicontinuous on $[0, 1]$. Based on **boundedness** and **equicontinuity** above, and refer to Arzelä-Ascoli theorem, we prove the continuous transformation T_3, under given fixed points $\overline{\lambda_1^*}(\delta)$ and $\overline{\triangle V}(\delta)$, maps Φ_{S1} to Φ_{S1}, where the normed linear space Φ_{S1} is nonempty, convex and compact. By Schauder's fixed point theorem, given fixed points $\overline{\lambda_1^*}(\delta)$ and $\overline{\triangle V}(\delta)$, there exists fixed point $\overline{\phi_1}(\delta)$ of $\phi_1(\delta)$.

By (1)-(3) above, we prove that there exists fixed points $\overline{\triangle V}(\delta)$, $\overline{\lambda_1^*}(\delta)$, $\overline{\phi_1}(\delta)$ for the transformation $T : \triangle V_S \times \Lambda_{S1} \times \Phi_{S1} \longrightarrow \triangle V_S \times \Lambda_{S1} \times \Phi_{S1}$ defined above, given any parameters $r > 0$, $\alpha > 0$ and $c_1 > 0$.

3.A.3 Definition 2

By

$$\phi_0(\delta) = \phi_1(1 - \delta) \qquad \forall \delta \in [0, 1]$$

$$\lambda_0^*(\delta) = \lambda_1^*(1 - \delta) \qquad \forall \delta \in [0, 1]$$

we can obtain the following equalities:

$$\Lambda_1 = \int_0^1 \lambda_1^*(\delta')\phi_1(\delta')d\delta' = \int_1^0 \lambda_1^*(1-t)\phi_1(1-t)d(1-t) = \int_0^1 \lambda_0^*(t)\phi_0(t)dt = \Lambda_0$$

$$a(\delta) = \int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta' = \int_1^{1-\delta} \frac{\lambda_1^*(1-t)}{\Lambda_0}\phi_1(1-t)d(1-t)$$
$$= \int_{1-\delta}^1 \frac{\lambda_0^*(t)}{\Lambda_0}\phi_0(t)dt = b(1-\delta) \quad \text{(by (3.22)(3.23))}$$

$\Longrightarrow$

$$\frac{d\triangle V(\delta)}{d\delta}$$
$$= \frac{1}{r+\alpha+\lambda_1^*(\delta)b(\delta)+\lambda_0^*(\delta)a(\delta)} = \frac{1}{r+\alpha+\lambda_0^*(1-\delta)a(1-\delta)+\lambda_1^*(1-\delta)b(1-\delta)}$$
$$= \frac{d\triangle V(1-\delta)}{d(1-\delta)}$$

$\Longrightarrow$

$$\triangle V(\delta) - \triangle V(0) = \int_0^\delta \frac{d\triangle V(t)}{dt}dt = \int_0^\delta \frac{d\triangle V(1-t)}{d(1-t)}dt = \triangle V(1) - \triangle V(1-\delta)$$

$\Longrightarrow$

$$\triangle V(0) + \triangle V(1) = \triangle V(\delta) + \triangle V(1-\delta) \quad \forall \delta \in [0,1]$$

Also, with same notations in Appendix 3.A.1, in any stationary equilibrium,

$$\frac{d^2\triangle V(\delta)}{d\delta^2} = \frac{-1}{(r+\alpha+\lambda_1^*(\delta)b(\delta)+\lambda_0^*(\delta)a(\delta))^2}(\lambda_1^*(\delta)b(\delta)+\lambda_0^*(\delta)a(\delta))'$$

where

$$(\lambda_1^*(\delta)b(\delta)+\lambda_0^*(\delta)a(\delta))'$$
$$= \frac{1}{2c_1}\frac{d\triangle V(\delta)}{d\delta}\left[\left(\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta'\right)^2 - \left(\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\phi_0(\delta')d\delta'\right)^2\right]$$
$$+ \frac{\lambda_0^*(\delta)\lambda_1^*(\delta)}{\Lambda_0}(\phi_1(\delta)-\phi_0(\delta))$$

Specifically, in symmetric stationary equilibrium characterized in Definition 2,

$$\left(\int_0^{\delta} \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta'\right)^2 - \left(\int_{\delta}^{1} \frac{\lambda_0^*(\delta')}{\Lambda_0}\phi_0(\delta')d\delta'\right)^2 \begin{cases} < 0 & \forall\delta \in [0, \frac{1}{2}); \\ = 0 & \delta = \frac{1}{2}; \\ > 0 & \forall\delta \in (\frac{1}{2}, 1]. \end{cases}$$

$$\phi_1(\delta) - \phi_0(\delta) \begin{cases} < 0 & \forall\delta \in [0, \frac{1}{2}); \\ = 0 & \delta = \frac{1}{2}; \\ > 0 & \forall\delta \in (\frac{1}{2}, 1]. \end{cases}$$

$\Longrightarrow$

$$\frac{d^2 \triangle V(\delta)}{d\delta^2} \begin{cases} > 0 & \forall\delta \in [0, \frac{1}{2}); \\ = 0 & \delta = \frac{1}{2}; \\ < 0 & \forall\delta \in (\frac{1}{2}, 1]. \end{cases}$$

3.A.4 Proposition 3

We firstly consider symmetric and convex $f_\delta(\delta)$:

$$f'_\delta(\delta) \begin{cases} < 0 & \forall\delta \in [0, \frac{1}{2}); \\ = 0 & \delta = \frac{1}{2}; \\ > 0 & \forall\delta \in (\frac{1}{2}, 1]. \end{cases}$$

and

$$f_\delta(\delta) = f_\delta(1-\delta), \quad \forall\delta \in [0, 1]$$

$$f_\delta(\delta) = \phi_1(\delta) + \phi_0(\delta), \quad \forall\delta \in [0, 1]$$

By equilibrium condition (3.9) with $\hat{f}_\delta(\delta) = f_\delta(\delta)$,

we obtain (using the notations $A(\delta), B(\delta), a(\delta), b(\delta)$ in Appendix 3.A.1):

$$\begin{aligned}
&\frac{d\dot{\phi}_1(\delta)}{d\delta} \\
&= 0 \\
&= -\alpha\phi_1'(\delta) + \frac{\alpha}{2} f_\delta'(\delta) - 2\lambda_1^{*'}(\delta)\phi_1(\delta)b(\delta) - 2\lambda_1^*(\delta)\phi_1'(\delta)b(\delta) + 2\frac{\lambda_1^*(\delta)\phi_1(\delta)\lambda_0^*(\delta)\phi_0(\delta)}{\Lambda_0} \\
&+ 2\lambda_0^{*'}(\delta)\phi_0(\delta)a(\delta) + 2\lambda_0^*(\delta)(f_\delta'(\delta) - \phi_1'(\delta))a(\delta) + 2\frac{\lambda_1^*(\delta)\phi_1(\delta)\lambda_0^*(\delta)\phi_0(\delta)}{\Lambda_1}, \\
&\quad \forall\delta \in [0, 1] \qquad (3.30)
\end{aligned}$$

since the sum of all the terms not including $f_\delta'(\delta)$ or $\phi_1'(\delta)$ is positive, then

$$-\left(\alpha + 2\lambda_1^*(\delta)b(\delta) + 2\lambda_0^*(\delta)a(\delta)\right)\phi_1'(\delta) + \left(\frac{\alpha}{2} + 2\lambda_0^*(\delta)a(\delta)\right) f_\delta'(\delta) < 0, \quad \forall\delta \in [0, 1]$$

By the definition and sign of $f_\delta'(\delta)$, we obtain

$$\phi_0'(\delta) < 0 \qquad \forall\delta \in [0, \frac{1}{2})$$

$$\phi_1'(\delta) > 0 \qquad \forall\delta \in (\frac{1}{2}, 1]$$

$$\left(\frac{\alpha}{2} + 2\lambda_0^*(\delta)a(\delta)\right)\phi_0'(\delta) < \left(\frac{\alpha}{2} + 2\lambda_1^*(\delta)b(\delta)\right)\phi_1'(\delta) \qquad \forall\delta \in [0, 1]$$

and since $f_\delta'(\frac{1}{2}) = 0$

$$\phi_1'(\frac{1}{2}) = -\phi_0'(\frac{1}{2}) > 0$$

Suppose $\exists\delta_1^* \in [0, \frac{1}{2})$, s.t. $\phi_1'(\delta_1^*) < 0$ and $\nexists\delta^* \in [0, 1]$ s.t.

$$\left(\frac{\alpha}{2} + 2\lambda_0^*(\delta^*)a(\delta^*)\right) f_\delta'(\delta^*) + \frac{1}{c_1}\frac{d\triangle V(\delta^*)}{d\delta}\left(a(\delta^*)^2\phi_0(\delta^*) + b(\delta^*)^2\phi_1(\delta^*)\right) \qquad (3.31)$$

$$+2\lambda_1^*(\delta^*)\lambda_0^*(\delta^*)\phi_1(\delta^*)\phi_0(\delta^*)\left(\frac{1}{\Lambda_0}+\frac{1}{\Lambda_1}\right)=0$$

Suppose all equilibrium components are smooth, by Mean Value Theorem, $\exists\delta_2^* \in (\delta_1^*, \frac{1}{2})$ s.t. $\phi_1'(\delta_2^*) = 0$. By (3.30), we obtain

$$\left(\frac{\alpha}{2}+2\lambda_0^*(\delta_2^*)a(\delta_2^*)\right)f_\delta'(\delta_2^*)+\frac{1}{c_1}\frac{d\triangle V(\delta_2^*)}{d\delta}\left(a(\delta_2^*)^2\phi_0(\delta_2^*)+b(\delta_2^*)^2\phi_1(\delta_2^*)\right)$$

$$+2\lambda_1^*(\delta_2^*)\lambda_0^*(\delta_2^*)\phi_1(\delta_2^*)\phi_0(\delta_2^*)\left(\frac{1}{\Lambda_0}+\frac{1}{\Lambda_1}\right)=0$$

which contradicts with condition (3.31). Then we conclude $\nexists\delta_1^* \in [0, \frac{1}{2})$, s.t. $\phi_1'(\delta_1^*) < 0$. So if condition (3.31) is satisfied,

$$\phi_1'(\delta) > 0 \quad \forall\delta \in [0,1]$$

Similar idea works for the sign of $\phi_0'(\delta)$ on $\delta \in (\frac{1}{2}, 1]$. Then we conclude as long as condition (3.31) applies,

$$\phi_0'(\delta) < 0 < \phi_1'(\delta) \quad \forall\delta \in [0,1]$$

And the same conclusion applies when $f_\delta(\delta)$ is symmetric but concave.

3.A.5 Proposition 4

We use the Implicit Function Theorem to show the effects of α and c_1 on all the competitive equilibrium components $\triangle V(\delta)$, $\lambda_1^*(\delta)$ and $\phi_1(\delta)$ on $\delta \in [0,1]$. Since we only focus on symmetric equilibrium defined in **Definition 2**, we have the other two components as $\lambda_0^*(\delta) = \lambda_1^*(1-\delta)$ and $\phi_0(\delta) = \phi_1(1-\delta)$ for $\forall\delta \in [0,1]$.

We write the three competitive equilibrium conditions collectively as follows:

$$H(\triangle V(\delta), \lambda_1^*(\delta), \phi_1(\delta); \alpha, c_1) = \begin{bmatrix} H_1(\triangle V(\delta), \lambda_1^*(\delta), \phi_1(\delta); \alpha, c_1) \\ H_2(\triangle V(\delta), \lambda_1^*(\delta), \phi_1(\delta); \alpha, c_1) \\ H_3(\triangle V(\delta), \lambda_1^*(\delta), \phi_1(\delta); \alpha, c_1) \end{bmatrix} \equiv 0_{3\times 1}$$

where

$$\begin{aligned}
&H_1(\triangle V(\delta), \lambda_1^*(\delta), \phi_1(\delta); \alpha, c_1)\\
&= 2\alpha\phi_1(\delta)\int_0^1 \lambda_1^*(\delta)\phi_1(\delta)d\delta + 2\phi_1(\delta)\lambda_1^*(\delta)\int_0^{1-\delta} \lambda_1^*(\delta')\phi_1(\delta')d\delta'\\
&+ 2\phi_1(\delta)\lambda_1^*(1-\delta)\int_0^{\delta} \lambda_1^*(\delta')\phi_1(\delta')d\delta' - \alpha\int_0^1 \lambda_1^*(\delta)\phi_1(\delta)d\delta - 2\lambda_1^*(1-\delta)\int_0^{\delta} \lambda_1^*(\delta')\phi_1(\delta')d\delta'\\
&\equiv 0
\end{aligned}$$

$$\begin{aligned}
&H_2(\triangle V(\delta), \lambda_1^*(\delta), \phi_1(\delta); \alpha, c_1)\\
&= (\alpha + r)\triangle V(\delta) - \delta - c_1\lambda_1^{*2}(\delta) + c_1\lambda_1^{*2}(1-\delta) - \alpha E[\triangle V(\delta)]\\
&= (\alpha + r)\triangle V(\delta) - \delta - c_1\lambda_1^{*2}(\delta) + c_1\lambda_1^{*2}(1-\delta) - \frac{\alpha}{2r}\\
&\equiv 0
\end{aligned}$$

$$\begin{aligned}
&H_3(\triangle V(\delta), \lambda_1^*(\delta), \phi_1(\delta); \alpha, c_1)\\
&= 2c_1\lambda_1^*(\delta) - (\triangle V(0) + \triangle V(1) - \triangle V(\delta))\int_0^{1-\delta} \frac{\lambda_1^*(\delta')\phi_1(\delta')}{\Lambda_1}d\delta'\\
&\quad + \int_0^{1-\delta} \frac{\lambda_1^*(\delta')\phi_1(\delta')\triangle V(\delta')}{\Lambda_1}d\delta'\\
&= 2c_1\lambda_1^*(\delta) - (\triangle V(0) + \triangle V(1) - \triangle V(\delta))\int_0^{1-\delta} F(\delta')d\delta' + \int_0^{1-\delta} F(\delta')\triangle V(\delta')d\delta'\\
&\equiv 0
\end{aligned}$$

In the last but one equality, we use the notation $F(\delta) = \frac{\lambda_1^*(\delta)\phi_1(\delta)}{\Lambda_1}$ for simplicity.

By Implicit Function Theorem, we have the following general relation:

For any $i = 1, 2, 3$, any $\delta \in [0,1]$ and any incrementals [27] $h_{\triangle V}(\delta), h_{\lambda_1^*}(\delta), h_{\phi_1}(\delta)$,

$$\frac{\partial H_i}{\partial \triangle V(\delta)} h_{\triangle V}(\delta) + \frac{\partial H_i}{\partial \lambda_1^*(\delta)} h_{\lambda_1^*}(\delta) + \frac{\partial H_i}{\partial \phi_1(\delta)} h_{\phi_1}(\delta) + \frac{\partial H_i}{\partial c_1} \triangle c_1 \equiv 0 \tag{3.32}$$

$$\frac{\partial H_i}{\partial \triangle V(\delta)} h_{\triangle V}(\delta) + \frac{\partial H_i}{\partial \lambda_1^*(\delta)} h_{\lambda_1^*}(\delta) + \frac{\partial H_i}{\partial \phi_1(\delta)} h_{\phi_1}(\delta) + \frac{\partial H_i}{\partial \alpha} \triangle \alpha \equiv 0 \tag{3.33}$$

Specifically for $i = 1$, we have:

$$\frac{\partial H_1}{\partial \triangle V(\delta)} h_{\triangle V}(\delta) \equiv 0$$

$$\begin{aligned}
&\frac{\partial H_1}{\partial \phi_1(\delta)} h_{\phi_1}(\delta) + \frac{\partial H_1}{\partial \lambda_1^*(\delta)} h_{\lambda_1^*}(\delta) \\
&= \lim_{m \to 0} \Bigg\{ \frac{H_1(\lambda_1^*(\delta), \phi_1(\delta) + m h_{\phi_1}(\delta)) - H_1(\lambda_1^*(\delta), \phi_1(\delta))}{m} \\
&+ \frac{H_1(\lambda_1^*(\delta) + m h_{\lambda_1^*}(\delta), \phi_1(\delta)) - H_1(\lambda_1^*(\delta), \phi_1(\delta))}{m} \Bigg\} \\
&= 2\alpha\phi_1(\delta) \int_0^1 \lambda_1^*(\delta) h_{\phi_1}(\delta) d\delta + 2\alpha h_{\phi_1}(\delta) \int_0^1 \lambda_1^*(\delta)\phi_1(\delta) d\delta + 2\phi_1(\delta)\lambda_1^*(\delta) \int_0^{1-\delta} \lambda_1^*(\delta') h_{\phi_1}(\delta') d\delta' \\
&+ 2h_{\phi_1}(\delta)\lambda_1^*(\delta) \int_0^{1-\delta} \lambda_1^*(\delta')\phi_1(\delta') d\delta' + 2\phi_1(\delta)\lambda_1^*(1-\delta) \int_0^{\delta} \lambda_1^*(\delta') h_{\phi_1}(\delta') d\delta' \\
&+ 2h_{\phi_1}(\delta)\lambda_1^*(1-\delta) \int_0^{\delta} \lambda_1^*(\delta')\phi_1(\delta') d\delta' - \alpha \int_0^1 \lambda_1^*(\delta) h_{\phi_1}(\delta) d\delta - 2\lambda_1^*(1-\delta) \int_0^{\delta} \lambda_1^*(\delta') h_{\phi_1}(\delta') d\delta' \\
&+ 2\alpha\phi_1(\delta) \int_0^1 h_{\lambda_1^*}(\delta)\phi_1(\delta) d\delta + 2\phi_1(\delta) h_{\lambda_1^*}(\delta) \int_0^{1-\delta} \lambda_1^*(\delta')\phi_1(\delta') d\delta' \\
&+ 2\phi_1(\delta)\lambda_1^*(\delta) \int_0^{1-\delta} h_{\lambda_1^*}(\delta')\phi_1(\delta') d\delta' + 2\phi_1(\delta)\lambda_1^*(1-\delta) \int_0^{\delta} h_{\lambda_1^*}(\delta')\phi_1(\delta') d\delta' \\
&+ 2\phi_1(\delta) h_{\lambda_1^*}(1-\delta) \int_0^{\delta} \lambda_1^*(\delta')\phi_1(\delta') d\delta' - \alpha \int_0^1 h_{\lambda_1^*}(\delta)\phi_1(\delta) d\delta \\
&- 2\lambda_1^*(1-\delta) \int_0^{\delta} h_{\lambda_1^*}(\delta')\phi_1(\delta') d\delta' - 2h_{\lambda_1^*}(1-\delta) \int_0^{\delta} \lambda_1^*(\delta')\phi_1(\delta') d\delta' \\
&\equiv 0 \quad \text{on } \delta \in [0,1]
\end{aligned}$$

[27] We will define the incrementals more formally in Section 3.A.7.

$$\frac{\partial H_1}{\partial c_1}(\delta)\triangle c_1 \equiv 0 \quad \text{and} \quad \frac{\partial H_1}{\partial \alpha}(\delta)\triangle\alpha = \left(2\phi_1(\delta)\int_0^1 \lambda_1^*(\delta)\phi_1(\delta)d\delta - \int_0^1 \lambda_1^*(\delta)\phi_1(\delta)d\delta\right)\triangle\alpha \tag{3.34}$$

Since by (3.34),

$$\frac{\partial H_1}{\partial \alpha}(\delta)\triangle\alpha + \frac{\partial H_1}{\partial \alpha}(1-\delta)\triangle\alpha = 0$$

and by (3.32)(3.33),

$$\begin{aligned}
&\frac{\partial H}{\partial \phi_1(\delta)}h_{\phi_1}(\delta) + \frac{\partial H}{\partial \lambda_1^*(\delta)}h_{\lambda_1^*}(\delta) + \frac{\partial H}{\partial \phi_1(\delta)}h_{\phi_1}(1-\delta) + \frac{\partial H}{\partial \lambda_1^*(\delta)}h_{\lambda_1^*}(1-\delta) \\
&= 2(h_{\phi_1}(\delta) + h_{\phi_1}(1-\delta))\left(\alpha\int_0^1 \lambda_1^*(\delta')\phi_1(\delta')d\delta' + \lambda_1^*(\delta)\int_0^{1-\delta} \lambda_1^*(\delta')\phi_1(\delta')d\delta'\right. \\
&\left. +\lambda_1^*(1-\delta)\int_0^{\delta} \lambda_1^*(\delta')\phi_1(\delta')d\delta'\right) \\
&\equiv 0
\end{aligned} \tag{3.35}$$

we obtain that for either changing α or changing c_1:

$$h_{\phi_1}(\delta) + h_{\phi_1}(1-\delta) \equiv 0, \quad \forall\delta \in [0,1] \tag{3.36}$$

Specifically for $i = 2$:

$$\begin{aligned}
&(\alpha + r)h_{\triangle V}(\delta) + 2c_1\left(\lambda_1^*(1-\delta)h_{\lambda_1^*}(1-\delta) - \lambda_1^*(\delta)h_{\lambda_1^*}(\delta)\right) + \triangle c_1\left(\lambda_1^{*2}(1-\delta) - \lambda_1^{*2}(\delta)\right) \\
&\equiv 0, \quad \forall\delta \in [0,1]
\end{aligned} \tag{3.37}$$

and

$$\begin{aligned}
&(\alpha + r)h_{\triangle V}(\delta) + 2c_1\left(\lambda_1^*(1-\delta)h_{\lambda_1^*}(1-\delta) - \lambda_1^*(\delta)h_{\lambda_1^*}(\delta)\right) + \triangle\alpha\left(\triangle V(\delta) - \frac{1}{2r}\right) \\
&\equiv 0, \quad \forall\delta \in [0,1]
\end{aligned} \tag{3.38}$$

By (3.37)(3.38), we can also plug in $1-\delta$ without changing the equalities. Then we obtain that for either changing α or changing c_1:

$$h_{\triangle V}(\delta)+h_{\triangle V}(1-\delta)\equiv 0, \quad \forall\delta\in[0,1] \tag{3.39}$$

(3.36)(3.39) further give us:

$$h'_{\phi_1}(\delta)=h'_{\phi_1}(1-\delta), \quad \forall\delta\in[0,1] \tag{3.40}$$

$$h'_{\triangle V}(\delta)=h'_{\triangle V}(1-\delta), \quad \forall\delta\in[0,1]$$

$$h_{\triangle V}(\frac{1}{2})=h_{\phi_1}(\frac{1}{2})=0 \tag{3.41}$$

Then as long as we can identify the sign of $h'_{\phi_1}(\delta)$ (or $h'_{\triangle V}(\delta)$) for any $\delta\in[0,1]$, then it will be sufficient to characterize the change in the shape of asset-owner density $h_{\phi_1}(\delta)$ on the whole interval. Here we specifically focus on the utility type $\delta=0$, by condition H_1:

$$\phi_1(0)=\frac{\frac{\alpha}{2}}{\alpha+\lambda_1^*(0)} \tag{3.42}$$

By condition that if c_1 increases, $h_{\lambda_1^*}(0)<0$, then it is trival by (3.42) that $h_{\phi_1}(0)>0$;
By condition that if α increases, $h_{\lambda_1^*}(0)<0$, then by (3.42):

$$(\phi_1(0)-\frac{1}{2})\triangle\alpha+(\alpha+\lambda_1^*(0))h_{\phi_1}(0)+\phi_1(0)h_{\lambda_1^*}(0)=0 \tag{3.43}$$

since the first and third terms in (3.43) are both negative, then

$$h_{\phi_1}(0)>0 \tag{3.44}$$

If (3.44) applies when c_1 and/or α increases, then by (3.40)(3.41), it is trival to prove by contradiction that $h'_{\phi_1}(\delta)<0, \quad \forall\delta\in[0,1]$. $\square$

3.A.6 Proposition 5

3.A.6.1

For symmetric equilibrium with $f_\delta(\delta) \equiv 1 \; \forall \delta \in [0,1]$,

$$\bar{\lambda}(\delta) = \phi_1(\delta)\lambda_1^*(\delta) + \phi_0(\delta)\lambda_0^*(\delta)$$

Using the notations $A(\delta), B(\delta), a(\delta), b(\delta)$ in Appendix 3.A.1,

$$\lambda_1^{*\prime}(\delta) = -\frac{1}{2c_1}\frac{d\triangle V(\delta)}{d\delta}b(\delta)$$

$$\lambda_0^{*\prime}(\delta) = \frac{1}{2c_1}\frac{d\triangle V(\delta)}{d\delta}a(\delta)$$

$$\phi_0'(\delta) = \frac{2\left[(\lambda_1^*(\delta)b(\delta))'(\frac{\alpha}{2} + 2\lambda_0^*(\delta)a(\delta)) - (\lambda_0^*(\delta)a(\delta))'(\frac{\alpha}{2} + 2\lambda_1^*(\delta)b(\delta))\right]}{(\alpha + 2\lambda_0^*(\delta)a(\delta) + 2\lambda_1^*(\delta)b(\delta))^2}$$

$$\phi_1'(\delta) = -\phi_0'(\delta) = \frac{2\left[(\lambda_0^*(\delta)a(\delta))'(\frac{\alpha}{2} + 2\lambda_1^*(\delta)b(\delta)) - (\lambda_1^*(\delta)b(\delta))'(\frac{\alpha}{2} + 2\lambda_0^*(\delta)a(\delta)))\right]}{(\alpha + 2\lambda_0^*(\delta)a(\delta) + 2\lambda_1^*(\delta)b(\delta))^2}$$

$\Longrightarrow$

$$\lambda_1^{*\prime}(0) = -\frac{1}{4c_1}\frac{1}{r + \alpha + \frac{\lambda_1^*(0)}{2}}$$

$$\lambda_0^{*\prime}(0) = 0$$

$$\begin{aligned}\phi_0'(0) &= \frac{\alpha(\lambda_1^*(\delta)b(\delta))'|_{\delta=0} - (\lambda_0^*(\delta)a(\delta))'|_{\delta=0}(\alpha + 2\lambda_1^*(0))}{(\alpha + \lambda_1^*(0))^2} \\ &= \frac{-\frac{\alpha}{8c_1}\frac{1}{r+\alpha+\frac{\lambda_1^*(0)}{2}}}{(\alpha + \lambda_1^*(0))^2}\end{aligned}$$

$\Longrightarrow$

$$\begin{aligned}\bar{\lambda}'(0) &= \phi_1'(0)\lambda_1^*(0) + \phi_1(0)\lambda_1^{*\prime}(0) \\ &= \frac{-\frac{\alpha}{8c_1}\frac{\lambda_1^*(0)}{r+\alpha+\frac{\lambda_1^*(0)}{2}}}{(\alpha+\lambda_1^*(0))^2} - \frac{1}{4c_1}\frac{\phi_1(0)}{r+\alpha+\frac{\lambda_1^*(0)}{2}} \\ &< 0\end{aligned}$$

By symmetry,

$$\begin{aligned}&\bar{\lambda}'(1-\delta) \\ &= \phi_1'(1-\delta)\lambda_1^*(1-\delta) + \phi_1(1-\delta)\lambda_1^{*\prime}(1-\delta) + \phi_0'(1-\delta)\lambda_0^*(1-\delta) + \phi_0(1-\delta)\lambda_0^{*\prime}(1-\delta) \\ &= -\phi_0'(\delta)\lambda_0^*(\delta) - \phi_0(\delta)\lambda_0^{*\prime}(\delta) - \phi_1'(\delta)\lambda_1^*(\delta) - \phi_1(\delta)\lambda_1^{*\prime}(\delta) \\ &= -\bar{\lambda}'(\delta)\end{aligned}$$

then

$$\bar{\lambda}'(1) = -\bar{\lambda}'(0) > 0$$

and

$$\bar{\lambda}'(\frac{1}{2}) = -\bar{\lambda}'(1-\frac{1}{2}) = -\bar{\lambda}'(\frac{1}{2})$$

$\Longrightarrow$

$$\bar{\lambda}'(\frac{1}{2}) = 0$$

3.A.6.2

Lemma A.6.2 (1) As $c_1 \to +\infty$: $\lambda_1^*(\delta) \to 0$ and $\lambda_0^*(\delta) \to 0$ for $\forall \delta \in (0,1)$; (2) As $c_1 \to 0$: given $\forall \hat{\delta} \in (0,1)$ and $\forall M > 0$, $\lambda_1^*(\hat{\delta}) > M$ and $\lambda_0^*(\hat{\delta}) > M$.
Proof:

By boundedness of $\triangle V(\delta)$ (3.29), we have for $\forall \delta \in (0,1)$

$$0 < \int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}(\triangle V(\delta') - \triangle V(\delta))\phi_0(\delta')d\delta' < (\triangle V(1) - \triangle V(0)) \int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}\phi_0(\delta')d\delta' < \frac{1}{2(\alpha + r)}$$

$$0 < \int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}(\triangle V(\delta) - \triangle V(\delta'))\phi_1(\delta')d\delta' < (\triangle V(1) - \triangle V(0)) \int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}\phi_1(\delta')d\delta' < \frac{1}{2(\alpha + r)}$$

then it is trival that for any fixed α, as $c_1 \to +\infty$,

$$\lambda_1^*(\delta) = \frac{\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}(\triangle V(\delta') - \triangle V(\delta))\phi_0(\delta')d\delta'}{2c_1} \to 0$$

$$\lambda_0^*(\delta) = \frac{\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}(\triangle V(\delta) - \triangle V(\delta'))\phi_1(\delta')d\delta'}{2c_1} \to 0$$

By symmetry,

$$\lambda_1^*(\delta) = \frac{\int_0^{1-\delta} \frac{\lambda_1^*(\delta')}{\Lambda_1}(\triangle V(1-\delta') - \triangle V(\delta))\phi_1(\delta')d\delta'}{2c_1} \to 0$$

and also by Section 3.A.2, $\lambda_1^*(\delta) \in \Lambda_{S1}$, $\phi_1(\delta) \in \Phi_{S1}$ and $\triangle V(\delta) \in \triangle V_S$ where Λ_{S1}, Φ_{S1} and $\triangle V_S$ are compact sets. Then for each fixed $\hat{\delta} \in (0,1)$, by Extreme Value Theorem, $\exists(\lambda_{1S}^*(\delta), \phi_{1S}(\delta), \triangle V_S(\delta))$ s.t.

$$\begin{aligned}
&(\lambda_{1S}^*(\delta), \phi_{1S}(\delta), \triangle V_S(\delta)) \\
&= \underset{(\lambda_1^*(\delta), \phi_1(\delta), \triangle V(\delta)) \in \Lambda_{S1} \times \Phi_{S1} \times \triangle V_S}{argmax} \left\{ \int_0^{1-\delta} \frac{\lambda_1^*(\delta')}{\Lambda_1}(\triangle V(\delta) - \triangle V(1-\delta'))\phi_1(\delta')d\delta' \right\}
\end{aligned}$$

and

$$\begin{aligned}
&M^* \\
&= \underset{(\lambda_1^*(\delta), \phi_1(\delta), \triangle V(\delta)) \in \Lambda_{S1} \times \Phi_{S1} \times \triangle V_S}{max} \left\{ \int_0^{1-\delta} \frac{\lambda_1^*(\delta')}{\Lambda_1}(\triangle V(\delta) - \triangle V(1-\delta'))\phi_1(\delta')d\delta' \right\}
\end{aligned}$$

then for any other large constant $M > 0$, we can always find $c_1^*(\hat{\delta}) = -\frac{M^*}{2M}$ s.t.

$$\lambda_1^*(\hat{\delta}) = -\frac{\int_0^{1-\hat{\delta}} \frac{\lambda_1^*(\delta')}{\Lambda_1}(\triangle V(\delta) - \triangle V(1-\delta'))\phi_1(\delta')d\delta'}{2c_1} > M \quad \forall c_1 < c_1^*(\hat{\delta})$$

i.e. for each fixed $\hat{\delta} \in (0, 1)$,

$$\lambda_1^*(\hat{\delta}) \to +\infty \quad \text{as } c_1 \to 0$$

$\square$

$$\begin{aligned}\bar{\lambda}'(\delta) &= \phi_1'(\delta)\lambda_1^*(\delta) + \phi_1(\delta)\lambda_1^{*\prime}(\delta) + \phi_0'(\delta)\lambda_0^*(\delta) + \phi_0(\delta)\lambda_0^{*\prime}(\delta) \\ &= \phi_1'(\delta)(\lambda_1^*(\delta) - \lambda_0^*(\delta)) + \frac{1}{2c_1}\frac{d\triangle V(\delta)}{d\delta}(\phi_0(\delta)a(\delta) - \phi_1(\delta)b(\delta))\end{aligned}$$

where

$$\begin{aligned}\phi_1'(\delta) =& \frac{(\alpha + 4\lambda_1^*(\delta)b(\delta))\left(\frac{a^2(\delta)}{2c_1}\frac{d\triangle V(\delta)}{d\delta} + \frac{\lambda_0^*(\delta)\lambda_1^*(\delta)\phi_1(\delta)}{\Lambda}\right)}{(\alpha + 2\lambda_0^*(\delta)a(\delta) + 2\lambda_1^*(\delta)b(\delta))^2} \\ &+ \frac{(\alpha + 4\lambda_0^*(\delta)a(\delta))\left(\frac{b^2(\delta)}{2c_1}\frac{d\triangle V(\delta)}{d\delta} + \frac{\lambda_0^*(\delta)\lambda_1^*(\delta)\phi_0(\delta)}{\Lambda}\right)}{(\alpha + 2\lambda_0^*(\delta)a(\delta) + 2\lambda_1^*(\delta)b(\delta))^2}\end{aligned}$$

$$\Lambda = \Lambda_1 = \Lambda_0$$

(1) For each α, by Lemma A.6.2,

$$\begin{aligned}&\bar{\lambda}'(\delta) \\ &= \frac{1}{2c_1}\left\{2c_1\phi_1'(\delta)(\lambda_1^*(\delta) - \lambda_0^*(\delta)) + \frac{d\triangle V(\delta)}{d\delta}(\phi_0(\delta)a(\delta) - \phi_1(\delta)b(\delta))\right\}\end{aligned}$$

where

$$\begin{aligned}
&\underset{c_1\to+\infty}{lim} 2c_1\phi_1'(\delta) \\
&= \underset{c_1\to+\infty}{lim} \left\{ \frac{(\alpha+4\lambda_1^*(\delta)b(\delta))\left(a^2(\delta)\frac{d\triangle V(\delta)}{d\delta}+\frac{\lambda_0^*(\delta)2c_1\lambda_1^*(\delta)\phi_1(\delta)}{\Lambda}\right)}{(\alpha+2\lambda_0^*(\delta)a(\delta)+2\lambda_1^*(\delta)b(\delta))^2} \right. \\
&\left. +\frac{(\alpha+4\lambda_0^*(\delta)a(\delta))\left(b^2(\delta)\frac{d\triangle V(\delta)}{d\delta}+\frac{\lambda_0^*(\delta)2c_1\lambda_1^*(\delta)\phi_0(\delta)}{\Lambda}\right)}{(\alpha+2\lambda_0^*(\delta)a(\delta)+2\lambda_1^*(\delta)b(\delta))^2} \right\} \\
&= \frac{0}{\alpha^2} \\
&= 0 \quad \forall\delta\in(0,\frac{1}{2})
\end{aligned} \tag{3.45}$$

$$\underset{c_1\to+\infty}{lim}\frac{\phi_0(\delta)}{\phi_1(\delta)} = \underset{c_1\to+\infty}{lim}\frac{\frac{\alpha}{2}+2\lambda_1^*(\delta)b(\delta)}{\frac{\alpha}{2}+2\lambda_0^*(\delta)a(\delta)} = 1 < \frac{b(\delta)}{a(\delta)} \quad \forall\delta\in(0,\frac{1}{2}) \tag{3.46}$$

and notations $a(\delta)$ and $b(\delta)$ follow Section 3.A.1.

Then (3.45)(3.46) and "$\bar{\lambda}'(0)<0$" $\Longrightarrow$

$$\underset{c_1\to+\infty}{lim}\bar{\lambda}'(\delta)<0 \quad \forall\delta\in[0,\frac{1}{2}) \tag{3.47}$$

We also assume that

$$\lambda_1^*(\delta_1;c_1)=\Omega\left(\lambda_1^*(\delta_2;c_1)\right)(c_1\to+\infty) \quad \forall\delta_1,\delta_2\in[0,\frac{1}{2}] \tag{3.48}$$

$$\int_0^{1-\delta}\lambda_1^*(\delta';c_1)(\triangle V(\delta)-\triangle V(1-\delta'))\phi_1(\delta')d\delta'=\Omega(\Lambda_1(c_1))(c_1\to+\infty) \quad \forall\delta\in(0,1) \tag{3.49}$$

which are the negation of $\lambda_1^*(\delta_1;c_1)=o\left(\lambda_1^*(\delta_2;c_1)\right)(c_1\to+\infty)$ and $c_1\lambda_1^*(\delta;c_1)=o(1)(c_1\to+\infty)$.

Then

$$\bar{\lambda}(0)=\frac{\alpha\lambda_1^*(0)}{\alpha+\lambda_1^*(0)}$$

$$\bar{\lambda}(\frac{1}{2}) = \lambda_1^*(\frac{1}{2})$$

then by (3.48) and Lemma A.6.2,

$$\lim_{c_1\to+\infty} \frac{\bar{\lambda}(0)}{\bar{\lambda}(\frac{1}{2})} = \lim_{c_1\to+\infty} \frac{\alpha\frac{\lambda_1^*(0)}{\lambda_1^*(\frac{1}{2})}}{\alpha+\lambda_1^*(0)} = \frac{\lambda_1^*(0)}{\lambda_1^*(\frac{1}{2})} > 1 \tag{3.50}$$

Then we calculate that,

$$\bar{\lambda}''(\frac{1}{2}) = \frac{\frac{d\triangle V(\frac{1}{2})}{d\delta}}{2c_1^2\Lambda_1(\alpha+4\lambda_1^*(\frac{1}{2})b(\frac{1}{2}))} \times \underbrace{\left\{-4\frac{d\triangle V(\frac{1}{2})}{d\delta}b(\frac{1}{2})\left(\int_0^{\frac{1}{2}}\lambda_1^*(\delta')\phi_1(\delta')d\delta'\right) + \frac{c_1\lambda_1^*(\frac{1}{2})\alpha}{2} - 2\lambda_1^*(\frac{1}{2})c_1\lambda_0^*(\frac{1}{2})b(\frac{1}{2})\right\}}_{*} \tag{3.51}$$

so the sign of $\bar{\lambda}''(\frac{1}{2})$ depends on the sign of the $*$ term in (3.51).

As by Lemma A.6.2 and (3.49)

$$\lim_{c_1\to+\infty}\left\{-4\frac{d\triangle V(\frac{1}{2})}{d\delta}b(\frac{1}{2})\left(\int_0^{\frac{1}{2}}\lambda_1^*(\delta')\phi_1(\delta')d\delta'\right) + \frac{c_1\lambda_1^*(\frac{1}{2})\alpha}{2} - 2\lambda_1^*(\frac{1}{2})c_1\lambda_0^*(\frac{1}{2})b(\frac{1}{2})\right\} = \lim_{c_1\to+\infty}\frac{c_1\lambda_1^*(\frac{1}{2})\alpha}{2} > 0$$

then

$$\lim_{c_1\to+\infty}\bar{\lambda}''(\frac{1}{2}) > 0 \tag{3.52}$$

Then by (3.47)(3.50)(3.52), we conclude that for each fixed α, $\exists c_1^1(\alpha), c_1^2(\alpha), c_1^3(\alpha)$ s.t.

$$\bar{\lambda}'(\delta) < 0 \quad \forall\delta\in[0,\frac{1}{2}) \quad \text{for } \forall c_1 > c_1^1(\alpha)$$

$$\bar{\lambda}(0) > \bar{\lambda}(\frac{1}{2}) \quad \text{for } \forall c_1 > c_1^2(\alpha)$$

$$\bar{\lambda}''(\frac{1}{2}) > 0 \quad \text{for } \forall c_1 > c_1^3(\alpha)$$

Then

$$c_1^*(\alpha) = max\{c_1^1(\alpha), c_1^2(\alpha), c_1^3(\alpha)\}$$

(2) For each c_1, since the following components are bounded:

$$0 < \frac{d\triangle V(\delta)}{d\delta} < \frac{1}{r+\alpha} \quad \forall \delta \in [0,1]$$

$$0 \leq \lambda_1^*(\delta) < \frac{1}{2c_1(\alpha+r)} \quad \forall \delta \in [0,1]$$

$$0 \leq a(\delta) \leq \frac{1}{2} \quad \forall \delta \in [0,1]$$

$$0 \leq b(\delta) \leq \frac{1}{2} \quad \forall \delta \in [0,1]$$

$\Longrightarrow$

$$\begin{aligned}
\lim_{\alpha\to+\infty} \bar{\lambda}'(\delta) &= \lim_{\alpha\to+\infty} \left\{ \phi_1'(\delta)(\lambda_1^*(\delta) - \lambda_0^*(\delta)) + \frac{1}{2c_1}\frac{d\triangle V(\delta)}{d\delta}(\phi_0(\delta)a(\delta) - \phi_1(\delta)b(\delta)) \right\} \\
&= \lim_{\alpha\to+\infty} \frac{\frac{a^2(\delta)}{2c_1}\frac{d\triangle V(\delta)}{d\delta} + \frac{\lambda_0^*(\delta)\lambda_1^*(\delta)\phi_1(\delta)}{\Lambda} + \frac{b^2(\delta)}{2c_1}\frac{d\triangle V(\delta)}{d\delta} + \frac{\lambda_0^*(\delta)\lambda_1^*(\delta)\phi_0(\delta)}{\Lambda}}{\alpha} + \\
&\quad \underbrace{\lim_{\alpha\to+\infty} \frac{1}{2c_1}\frac{d\triangle V(\delta)}{d\delta}(\phi_0(\delta)a(\delta) - \phi_1(\delta)b(\delta))}_{**} \\
&= \underbrace{\lim_{\alpha\to+\infty} \frac{1}{2c_1}\frac{d\triangle V(\delta)}{d\delta}(\phi_0(\delta)a(\delta) - \phi_1(\delta)b(\delta))}_{**}
\end{aligned}$$

since

$$\lim_{\alpha\to+\infty} \frac{\phi_0(\delta)}{\phi_1(\delta)} = \lim_{\alpha\to+\infty} \frac{\frac{\alpha}{2} + 2\lambda_1^*(\delta)b(\delta)}{\frac{\alpha}{2} + 2\lambda_0^*(\delta)a(\delta)} = 1 < \frac{b(\delta)}{a(\delta)} \quad \forall \delta \in (0, \frac{1}{2})$$

so

$$\lim_{\alpha\to+\infty} \bar{\lambda}'(\delta) = \underbrace{\lim_{\alpha\to+\infty} \frac{1}{2c_1}\frac{d\triangle V(\delta)}{d\delta}(\phi_0(\delta)a(\delta) - \phi_1(\delta)b(\delta))}_{**} < 0 \quad \forall \delta \in (0, \frac{1}{2}) \tag{3.53}$$

And it is trival that

$$\lim_{\alpha\to+\infty}\frac{\bar{\lambda}(0)}{\bar{\lambda}(\frac{1}{2})}=\lim_{\alpha\to+\infty}\frac{\alpha\frac{\lambda_1^*(0)}{\lambda_1^*(\frac{1}{2})}}{\alpha+\lambda_1^*(0)}=\frac{\lambda_1^*(0)}{\lambda_1^*(\frac{1}{2})}>1 \tag{3.54}$$

$$\lim_{\alpha\to+\infty}\bar{\lambda}''(\frac{1}{2})=\lim_{\alpha\to+\infty}\frac{\lambda_1^*(\frac{1}{2})\frac{d\triangle V(\frac{1}{2})}{d\delta}}{4c_1\Lambda_1}>0 \tag{3.55}$$

Then by (3.53)(3.54)(3.55), we conclude that for each fixed c_1, $\exists\alpha^1(c_1),\alpha^2(c_1),\alpha^3(c_1)$ s.t.

$$\bar{\lambda}'(\delta)<0 \quad \forall\delta\in[0,\frac{1}{2}) \quad \text{for } \forall\alpha>\alpha^1(c_1)$$

$$\bar{\lambda}(0)>\bar{\lambda}(\frac{1}{2}) \quad \text{for } \forall\alpha>\alpha^2(c_1)$$

$$\bar{\lambda}''(\frac{1}{2})>0 \quad \text{for } \forall\alpha>\alpha^3(c_1)$$

Then

$$\alpha^*(c_1)=max\{\alpha^1(c_1),\alpha^2(c_1),\alpha^3(c_1)\}$$

3.A.6.3

$$\bar{\lambda}'(\delta)=\underbrace{\phi_1'(\delta)(\lambda_1^*(\delta)-\lambda_0^*(\delta))}_{3*}+\underbrace{\frac{1}{2c_1}\frac{d\triangle V(\delta)}{d\delta}}_{4*}(\phi_0(\delta)a(\delta)-\phi_1(\delta)b(\delta))$$

The terms 3∗ and 4∗ are always positive, so we only focus on the sign of $\phi_0(\delta)a(\delta)-\phi_1(\delta)b(\delta)$.

(1) For each α, by Lemma A.6.2, $\exists\hat{\delta}\in(0,\frac{1}{2})$ s.t.

$$\lim_{c_1\to0}\frac{\phi_0(\delta)}{\phi_1(\delta)}=\lim_{c_1\to0}\frac{2\lambda_1^*(\delta)b(\delta)}{2\lambda_0^*(\delta)a(\delta)}>\frac{b(\delta)}{a(\delta)} \quad \forall\hat{\delta}<\delta<\frac{1}{2}$$

where the inequality “>” is by $\lambda_1^*(\delta)>\lambda_0^*(\delta)$ for $\forall\delta\in(0,\frac{1}{2})$ and $0<a(\hat{\delta})<a(\delta)$.

Then $\exists\hat{\delta}\in(0,\frac{1}{2})$ s.t.

$$\lim_{c_1\to0}\bar{\lambda}'(\delta)>0 \quad \forall\hat{\delta}<\delta<\frac{1}{2} \tag{3.56}$$

And we also assume that

$$\lambda_1^*(\delta_1; c_1) = \Omega\left(\lambda_1^*(\delta_2; c_1)\right)(c_1 \to 0) \quad \forall \delta_1, \delta_2 \in [0, \frac{1}{2}]$$

which is the negation of $\lambda_1^*(\delta_1; c_1) = o\left(\lambda_1^*(\delta_2; c_1)\right)(c_1 \to 0)$.

Then

$$\underset{c_1 \to 0}{lim} \frac{\bar{\lambda}(0)}{\bar{\lambda}(\frac{1}{2})} = \underset{c_1 \to 0}{lim} \frac{\alpha \frac{\lambda_1^*(0)}{\lambda_1^*(\frac{1}{2})}}{\alpha + \lambda_1^*(0)} = 0 < 1 \tag{3.57}$$

Since

$$\begin{aligned} 0 &< c_1 \lambda_1^*(\frac{1}{2}) \\ &= \frac{1}{2} \int_{\frac{1}{2}}^{1} \frac{\lambda_0^*(\delta')}{\Lambda_0} (\triangle V(\delta') - \triangle V(\frac{1}{2})) \phi_0(\delta') d\delta' \\ &< (\triangle V(1) - \triangle V(0)) \frac{1}{2} \int_{\frac{1}{2}}^{1} \frac{\lambda_0^*(\delta')}{\Lambda_0} \phi_0(\delta') d\delta' \\ &< \frac{1}{4(\alpha + r)} \end{aligned}$$

and by Lemma A.6.2

$$\underset{c_1 \to 0}{lim} \lambda_0^*(\frac{1}{2}) = +\infty$$

then the dominant term in term "$*$" of (3.51) is "$-2\lambda_1^*(\frac{1}{2}) c_1 \lambda_0^*(\frac{1}{2}) b(\frac{1}{2})$".

So we have

$$\underset{c_1 \to 0}{lim} \bar{\lambda}''(\frac{1}{2}) = \underset{c_1 \to 0}{lim} \frac{\frac{d\triangle V(\frac{1}{2})}{d\delta}}{2c_1^2 \Lambda_1 (\alpha + 4\lambda_1^*(\frac{1}{2}) b(\frac{1}{2}))} \left\{ -2\lambda_1^*(\frac{1}{2}) c_1 \lambda_0^*(\frac{1}{2}) b(\frac{1}{2}) \right\} < 0 \tag{3.58}$$

Then by (3.56)(3.57)(3.58), we conclude that for each fixed α, $\exists c_1^4(\alpha), c_1^5(\alpha), c_1^6(\alpha)$ s.t. $\exists \hat{\delta} \in (0, \frac{1}{2})$ s.t.

$$\bar{\lambda}'(\delta) > 0 \quad \forall \hat{\delta} < \delta < \frac{1}{2} \quad \text{for } \forall c_1 < c_1^4(\alpha)$$

$$\bar{\lambda}(0) < \bar{\lambda}(\frac{1}{2}) \quad \text{for } \forall c_1 < c_1^5(\alpha)$$

$$\bar{\lambda}''(\frac{1}{2}) < 0 \quad \text{for } \forall c_1 < c_1^6(\alpha)$$

Then

$$c_1^{**}(\alpha) = max\{c_1^4(\alpha), c_1^5(\alpha), c_1^6(\alpha)\}$$

(2) For each c_1, similar to the case of "fixed α", to discuss the sign of $\bar{\lambda}'(\delta)$, we only focus on the sign of $\phi_0(\delta)a(\delta) - \phi_1(\delta)b(\delta)$.
$\exists\hat{\delta} \in (0, \frac{1}{2})$ s.t.

$$\lim_{\alpha\to 0}\frac{\phi_0(\delta)}{\phi_1(\delta)} = \lim_{\alpha\to 0}\frac{2\lambda_1^*(\delta)b(\delta)}{2\lambda_0^*(\delta)a(\delta)} > \frac{b(\delta)}{a(\delta)} \quad \forall\hat{\delta} < \delta < \frac{1}{2}$$

where the inequality ">" is by $\lambda_1^*(\delta) > \lambda_0^*(\delta)$ for $\forall\delta \in (0, \frac{1}{2})$ and $0 < a(\hat{\delta}) < a(\delta)$.
Then $\exists\hat{\delta} \in (0, \frac{1}{2})$ s.t.

$$\lim_{\alpha\to 0}\bar{\lambda}'(\delta) > 0 \quad \forall\hat{\delta} < \delta < \frac{1}{2} \tag{3.59}$$

To compare $\bar{\lambda}(0)$ and $\bar{\lambda}(\frac{1}{2})$, similarly

$$\lim_{\alpha\to 0}\frac{\bar{\lambda}(0)}{\bar{\lambda}(\frac{1}{2})} = \lim_{\alpha\to 0}\frac{\alpha\frac{\lambda_1^*(0)}{\lambda_1^*(\frac{1}{2})}}{\alpha + \lambda_1^*(0)} = 0 < 1 \tag{3.60}$$

Also, as $\alpha \to 0$, the term "$\frac{c_1\lambda_1^*(\frac{1}{2})\alpha}{2} \to 0$" in "$*$" term of (3.51), so it is trival that

$$\lim_{\alpha\to 0}\bar{\lambda}''(\frac{1}{2}) < 0 \tag{3.61}$$

Then by (3.59)(3.60)(3.61), we conclude that for each fixed c_1, $\exists\alpha^4(c_1), \alpha^5(c_1), \alpha^6(c_1)$ s.t.
$\exists\hat{\delta} \in (0, \frac{1}{2})$ s.t.

$$\bar{\lambda}'(\delta) > 0 \quad \forall\hat{\delta} < \delta < \frac{1}{2} \quad \text{for } \forall\alpha < \alpha^4(c_1)$$

$$\bar{\lambda}(0) < \bar{\lambda}(\frac{1}{2}) \quad \text{for } \forall\alpha < \alpha^5(c_1)$$

$$\bar{\lambda}''(\frac{1}{2}) < 0 \quad \text{for } \forall\alpha < \alpha^6(c_1)$$

Then

$$\alpha^{**}(c_1) = max\{\alpha^4(c_1), \alpha^5(c_1), \alpha^6(c_1)\}$$

□

3.A.7 Proposition 6

In section 3.A.7.2, we give three lemmas, the conclusions of which will be used in the main proof in section 3.A.7.1.

3.A.7.1 Main proof of Proposition 6

Define the following normed linear spaces: $\Lambda_{S1} = \{\lambda_1^S(\delta) : \lambda_1^S(\delta) \in C^1[0,1]; \lambda_1^S(\delta) \geq 0 \text{ and } \lambda_1^{S'}(\delta) \leq 0, \forall \delta \in [0,1]\}$, $\Phi_{S1} = \{\phi_1^S(\delta) : \phi_1^S(\delta) \in C^1[0,1]; 0 \leq \phi_1^S(\delta) \leq 1 \text{ and } \phi_1^{S'}(\delta) \geq 0, \forall \delta \in [0,1]; \int_0^1 \phi_1^S(\delta)d\delta = \frac{1}{2}\}$, all with the norm $\|f\| = \underset{0\leq\delta\leq1}{max}|f(\delta)|$. We can further transform the original social welfare problem to a new one with two control variables $\lambda_1^S(\delta) \in \Lambda_{S1}$ and $\phi_1^S(\delta) \in \Phi_{S1}$ and transfer the original equilibrium constraint as follows:

New Problem:

$$[P] \qquad \underset{\lambda_1^S(\delta)\in\Lambda_{S1}, \phi_1^S(\delta)\in\Phi_{S1}}{max} \quad W = \int_0^1 (\delta - 2c_1 {\lambda_1^S}^2(\delta))\phi_1^S(\delta)d\delta$$

s.t.

$$\begin{aligned}
&H(\lambda_1^S(\delta), \phi_1^S(\delta)) \\
&= 2\alpha\phi_1^S(\delta)\int_0^1 \lambda_1^S(\delta)\phi_1^S(\delta)d\delta + 2\phi_1^S(\delta)\lambda_1^S(\delta)\int_0^{1-\delta} \lambda_1^S(\delta')\phi_1^S(\delta')d\delta' \\
&+ 2\phi_1^S(\delta)\lambda_1^S(1-\delta)\int_0^{\delta} \lambda_1^S(\delta')\phi_1^S(\delta')d\delta' - \alpha\int_0^1 \lambda_1^S(\delta)\phi_1^S(\delta)d\delta - 2\lambda_1^S(1-\delta)\int_0^{\delta} \lambda_1^S(\delta')\phi_1^S(\delta')d\delta' \\
&\equiv 0
\end{aligned}$$

If there exists subset $\triangle^+ \subset [\frac{1}{2}, 1]$ s.t. $\lambda_1^{S*}(\delta) > 0, \forall \delta \in \triangle^+$,[28] and under uniform distribution of δ on $[0,1]$, the measure of subset $\triangle^+$ is $\int_0^1 \mathbb{1}_{\{\delta \in \triangle^+\}}(\delta) d\delta = m^+$, we choose $\hat{\epsilon} > 0$ and $\delta_{\hat{\epsilon}^2} \in (\frac{1}{2}, 1)$ such that the Lebesgue measure of the new subset $D = \triangle^+ \cap [\frac{1}{2}, \delta_{\hat{\epsilon}^2}]$ satisfies $\mu[D] = \mu[\triangle^+ \cap [\frac{1}{2}, \delta_{\hat{\epsilon}^2}]] = \hat{\epsilon}^2 < m^+$.

Based on the $\hat{\epsilon}$ and new subset D chosen above, we can construct a new solution point $\left(\lambda_1^{NS*}(\delta), \phi_1^{NS*}(\delta)\right)$ as follows:

$$\lambda_1^{NS*}(\delta) = \lambda_1^{S*}(\delta) + h_{\lambda_1^{S*}}(\delta)$$

where

$$h_{\lambda_1^{S*}}(\delta) = \begin{cases} -\hat{\epsilon}\lambda_1^{S*}(\delta), & \forall \delta \in D \\ 0, & \forall \delta \in [0,1] n D \end{cases}$$

and

$$\phi_1^{NS*}(\delta) = \phi_1^{S*}(\delta) + h_{\phi_1^S}(\delta)$$

where the incremental $h_{\phi_1^S}(\delta)$ is obtained from the following equation given the incremental

[28] $\triangle^+$ may be a union of several disjoint subintervals of $[0,1]$.

$h_{\lambda_1^{S*}}(\delta)$ chosen above:

$$
\begin{aligned}
&\frac{\partial H}{\partial \phi_1^S(\delta)} h_{\phi_1^S}(\delta) + \frac{\partial H}{\partial \lambda_1^{S*}(\delta)} h_{\lambda_1^{S*}}(\delta) \\
&= \lim_{m\to 0} \left\{ \frac{H(\lambda_1^{S*}(\delta), \phi_1^S(\delta) + m h_{\phi_1^S}(\delta)) - H(\lambda_1^{S*}(\delta), \phi_1^S(\delta))}{m} \right. \\
&\left. + \frac{H(\lambda_1^{S*}(\delta) + m h_{\lambda_1^{S*}}(\delta), \phi_1^S(\delta)) - H(\lambda_1^{S*}(\delta), \phi_1^S(\delta))}{m} \right\} \\
&= 2\alpha\phi_1^S(\delta) \int_0^1 \lambda_1^{S*}(\delta) h_{\phi_1^S}(\delta) d\delta + 2\alpha h_{\phi_1^S}(\delta) \int_0^1 \lambda_1^{S*}(\delta)\phi_1^S(\delta) d\delta \\
&+ 2\phi_1^S(\delta)\lambda_1^{S*}(\delta) \int_0^{1-\delta} \lambda_1^{S*}(\delta') h_{\phi_1^S}(\delta') d\delta' + 2h_{\phi_1^S}(\delta)\lambda_1^{S*}(\delta) \int_0^{1-\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta') d\delta' \\
&+ 2\phi_1^S(\delta)\lambda_1^{S*}(1-\delta) \int_0^{\delta} \lambda_1^{S*}(\delta') h_{\phi_1^S}(\delta') d\delta' + 2h_{\phi_1^S}(\delta)\lambda_1^{S*}(1-\delta) \int_0^{\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta') d\delta' \\
&- \alpha \int_0^1 \lambda_1^{S*}(\delta) h_{\phi_1^S}(\delta) d\delta - 2\lambda_1^{S*}(1-\delta) \int_0^{\delta} \lambda_1^{S*}(\delta') h_{\phi_1^S}(\delta') d\delta' \\
&+ 2\alpha\phi_1^S(\delta) \int_0^1 h_{\lambda_1^{S*}}(\delta)\phi_1^S(\delta) d\delta + 2\phi_1^S(\delta) h_{\lambda_1^{S*}}(\delta) \int_0^{1-\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta') d\delta' \\
&+ 2\phi_1^S(\delta)\lambda_1^{S*}(\delta) \int_0^{1-\delta} h_{\lambda_1^{S*}}(\delta')\phi_1^S(\delta') d\delta' + 2\phi_1^S(\delta)\lambda_1^{S*}(1-\delta) \int_0^{\delta} h_{\lambda_1^{S*}}(\delta')\phi_1^S(\delta') d\delta' \\
&+ 2\phi_1^S(\delta) h_{\lambda_1^{S*}}(1-\delta) \int_0^{\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta') d\delta' - \alpha \int_0^1 h_{\lambda_1^{S*}}(\delta)\phi_1^S(\delta) d\delta \\
&- 2\lambda_1^{S*}(1-\delta) \int_0^{\delta} h_{\lambda_1^{S*}}(\delta')\phi_1^S(\delta') d\delta' - 2h_{\lambda_1^{S*}}(1-\delta) \int_0^{\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta') d\delta' \\
&\equiv 0 \quad \text{on } \delta \in [0,1]
\end{aligned}
$$

Then we construct another subset $B \subset [0, \frac{1}{2}]$ which is symmetric with the subset D chosen above, i.e. for $\forall\delta \in B$, $1-\delta \in D$ and for $\forall\delta \in D$, $1-\delta \in B$. We will show the new solution point $\left(\lambda_1^{NS*}(\delta), \phi_1^{NS*}(\delta)\right)$ dominates the old one $\left(\lambda_1^{S*}(\delta), \phi_1^{S*}(\delta)\right)$ in the sense that the new point generates higher value of social welfare without violating the constraint. In the proof, we need to use the conclusions of Lemma 1, Lemma 2 and Lemma 3, the proof of which will be given after the main proof.

Given the chosen $h_{\lambda_1^{S*}}(\delta)$ above and the obtained $h_{\phi_1^S}(\delta)$ from $\frac{\partial H}{\partial \phi_1^S(\delta)} h_{\phi_1^S}(\delta) + \frac{\partial H}{\partial \lambda_1^{S*}(\delta)} h_{\lambda_1^{S*}}(\delta) \equiv 0, \forall \delta \in [0,1]$ accordingly, the marginal change in the value of objective function (the social welfare) taking the chosen $\hat{\epsilon}$ to zero is:

$$
\begin{aligned}
& \lim_{\hat{\epsilon}\to 0} \frac{1}{\hat{\epsilon}} \left(\frac{\partial W}{\partial \phi_1^S(\delta)} h_{\phi_1^S}(\delta) + \frac{\partial W}{\partial \lambda_1^{S*}(\delta)} h_{\lambda_1^{S*}}(\delta) \right) \\
&= \lim_{\hat{\epsilon}\to 0} \frac{1}{\hat{\epsilon}} \int_0^1 (\delta - 2c_1 {\lambda_1^{S*}}^2(\delta)) h_{\phi_1^S}(\delta) d\delta + 4c_1 \int_D {\lambda_1^{S*}}^2(\delta') \phi_1^S(\delta') d\delta' \\
&= \lim_{\hat{\epsilon}\to 0} \frac{1}{\hat{\epsilon}} \int_{B\cup D} (\delta - 2c_1 {\lambda_1^{S*}}^2(\delta)) h_{\phi_1^S}(\delta) d\delta + 4c_1 \int_D {\lambda_1^{S*}}^2(\delta') \phi_1^S(\delta') d\delta' \quad \text{(by Lemma 1)}
\end{aligned} \tag{3.62}
$$

Also by Lemma 1 and Lemma 2:

$$
\begin{aligned}
& \lim_{\hat{\epsilon}\to 0} \int_{B\cup D} (\delta - 2c_1 {\lambda_1^{S*}}^2(\delta)) h_{\phi_1^S}(\delta) d\delta \\
&= \lim_{\hat{\epsilon}\to 0} \left(\int_B (\delta - 2c_1 {\lambda_1^{S*}}^2(\delta)) h_{\phi_1^S}(\delta) d\delta + \int_D (\delta - 2c_1 {\lambda_1^{S*}}^2(\delta)) h_{\phi_1^S}(\delta) d\delta \right) \\
&= \lim_{\hat{\epsilon}\to 0} \left(- \int_B (\delta - 2c_1 {\lambda_1^{S*}}^2(\delta)) h_{\phi_1^S}(1-\delta) d\delta + \int_D (\delta - 2c_1 {\lambda_1^{S*}}^2(\delta)) h_{\phi_1^S}(\delta) d\delta \right) \\
&= \lim_{\hat{\epsilon}\to 0} \left(- \int_D (1 - \delta - 2c_1 {\lambda_1^{S*}}^2(1-\delta)) h_{\phi_1^S}(\delta) d\delta + \int_D (\delta - 2c_1 {\lambda_1^{S*}}^2(\delta)) h_{\phi_1^S}(\delta) d\delta \right) \\
&= \lim_{\hat{\epsilon}\to 0} \left(\int_D (2\delta - 1 + 2c_1 {\lambda_1^{S*}}^2(1-\delta) - 2c_1 {\lambda_1^{S*}}^2(\delta)) h_{\phi_1^S}(\delta) d\delta \right) \\
&> 0 \quad (D \subset [\frac{1}{2}, 1] \text{ and } {\lambda_1^{S*}}'(\delta) < 0)
\end{aligned} \tag{3.63}
$$

(3.62)(3.63) lead to:

$$
\lim_{\hat{\epsilon}\to 0} \frac{1}{\hat{\epsilon}} \left(\frac{\partial W}{\partial \phi_1^S(\delta)} h_{\phi_1^S}(\delta) + \frac{\partial W}{\partial \lambda_1^{S*}(\delta)} h_{\lambda_1^{S*}}(\delta) \right) > 0
$$

Then by Lemma 3, any point $\left(\lambda_1^S(\delta), \phi_1^S(\delta)\right)$ with $\lambda_1^S(\delta) > 0, \forall \delta \in \triangle^+$ where $\triangle^+ \subset [\frac{1}{2}, 1]$ cannot be a local extremum. $\square$

3.A.7.2 Three Lemmas

Lemma 1 The incremental $h_{\phi_1^S}(\delta)$ satisfies

$$h_{\phi_1^S}(\delta) = \begin{cases} O(\hat{\epsilon}), & \forall\delta \in B \cup D \\ o(\hat{\epsilon}), & \forall\delta \in [0,1]n(B \cup D) \end{cases}$$

and

$$\underset{\hat{\epsilon}\to 0}{lim} h_{\phi_1^S}(\delta) \begin{cases} > 0, & \forall\delta \in D \\ < 0, & \forall\delta \in B \end{cases}$$

Proof:

We use guess and verify approach. We guess $h_{\phi_1^S}(\delta) = O(\hat{\epsilon}), \forall\delta \in B \cup D$, and $h_{\phi_1^S}(\delta) = o(\hat{\epsilon}), \forall\delta \in [0,1]n(B \cup D)$. Divide both sides of $\frac{\partial H}{\partial\phi_1^S(\delta)} h_{\phi_1^S}(\delta) + \frac{\partial H}{\partial\lambda_1^{S*}(\delta)} h_{\lambda_1^{S*}}(\delta) \equiv 0$ by $\hat{\epsilon}$ and take $\hat{\epsilon}$ to zero, we get:

$$\begin{aligned}
&\underset{\hat{\epsilon}\to 0}{lim} \Bigg\{ 2\alpha\phi_1^S(\delta)\int_0^1 \lambda_1^{S*}(\delta)\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}}d\delta + 2\alpha\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}}\int_0^1 \lambda_1^{S*}(\delta)\phi_1^S(\delta)d\delta \\
&+ 2\phi_1^S(\delta)\lambda_1^{S*}(\delta)\int_0^{1-\delta} \lambda_1^{S*}(\delta')\frac{h_{\phi_1^S}(\delta')}{\hat{\epsilon}}d\delta' + 2\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}}\lambda_1^{S*}(\delta)\int_0^{1-\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta' \\
&+ 2\phi_1^S(\delta)\lambda_1^{S*}(1-\delta)\int_0^{\delta} \lambda_1^{S*}(\delta')\frac{h_{\phi_1^S}(\delta')}{\hat{\epsilon}}d\delta' + 2\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}}\lambda_1^{S*}(1-\delta)\int_0^{\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta' \\
&- \alpha\int_0^1 \lambda_1^{S*}(\delta)\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}}d\delta - 2\lambda_1^{S*}(1-\delta)\int_0^{\delta} \lambda_1^{S*}(\delta')\frac{h_{\phi_1^S}(\delta')}{\hat{\epsilon}}d\delta' \\
&+ 2\alpha\phi_1^S(\delta)\int_0^1 \frac{h_{\lambda_1^{S*}}(\delta)}{\hat{\epsilon}}\phi_1^S(\delta)d\delta + 2\phi_1^S(\delta)\frac{h_{\lambda_1^{S*}}(\delta)}{\hat{\epsilon}}\int_0^{1-\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta' \\
&+ 2\phi_1^S(\delta)\lambda_1^{S*}(\delta)\int_0^{1-\delta} \frac{h_{\lambda_1^{S*}}(\delta')}{\hat{\epsilon}}\phi_1^S(\delta')d\delta' + 2\phi_1^S(\delta)\lambda_1^{S*}(1-\delta)\int_0^{\delta} \frac{h_{\lambda_1^{S*}}(\delta')}{\hat{\epsilon}}\phi_1^S(\delta')d\delta' \\
&+ 2\phi_1^S(\delta)\frac{h_{\lambda_1^{S*}}(1-\delta)}{\hat{\epsilon}}\int_0^{\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta' - \alpha\int_0^1 \frac{h_{\lambda_1^{S*}}(\delta')}{\hat{\epsilon}}\phi_1^S(\delta)d\delta \\
&-2\lambda_1^{S*}(1-\delta)\int_0^{\delta} \frac{h_{\lambda_1^{S*}}(\delta')}{\hat{\epsilon}}\phi_1^S(\delta')d\delta' - 2\frac{h_{\lambda_1^{S*}}(1-\delta)}{\hat{\epsilon}}\int_0^{\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta' \Bigg\} \\
&\equiv 0 \quad \text{on } \delta \in [0,1]
\end{aligned} \tag{3.64}$$

(1) For $\forall \delta \in [0,1] n (B \cup D)$, the value of left hand side (LHS) of (3.64) satisfies:

$$
\begin{aligned}
& LHS_1 \\
&= \lim_{\hat{\epsilon}\to 0} \Bigg\{ 2\alpha \phi_1^S(\delta) \int_{B\cup D} \lambda_1^{S*}(\delta) \frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}} d\delta + 2\alpha \frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}} \int_0^1 \lambda_1^{S*}(\delta)\phi_1^S(\delta) d\delta \\
&+ 2\phi_1^S(\delta)\lambda_1^{S*}(\delta) \int_0^{1-\delta} \lambda_1^{S*}(\delta') \frac{h_{\phi_1^S}(\delta')}{\hat{\epsilon}} d\delta' \\
&+ 2\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}} \lambda_1^{S*}(\delta) \int_0^{1-\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta') d\delta' + 2\phi_1^S(\delta)\lambda_1^{S*}(1-\delta) \int_0^{\delta} \lambda_1^{S*}(\delta') \frac{h_{\phi_1^S}(\delta')}{\hat{\epsilon}} d\delta' \\
&+ 2\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}} \lambda_1^{S*}(1-\delta) \int_0^{\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta') d\delta' - \alpha \int_{B\cup D} \lambda_1^{S*}(\delta) \frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}} d\delta \\
&- 2\lambda_1^{S*}(1-\delta) \int_0^{\delta} \lambda_1^{S*}(\delta') \frac{h_{\phi_1^S}(\delta')}{\hat{\epsilon}} d\delta' \\
&- 2\alpha\phi_1^S(\delta) \int_D \lambda_1^{S*}(\delta)\phi_1^S(\delta) d\delta + 2\phi_1^S(\delta)\lambda_1^{S*}(\delta) \int_0^{1-\delta} \frac{h_{\lambda_1^{S*}}(\delta')}{\hat{\epsilon}} \phi_1^S(\delta') d\delta' \\
&+ 2\phi_1^S(\delta)\lambda_1^{S*}(1-\delta) \int_0^{\delta} \frac{h_{\lambda_1^{S*}}(\delta')}{\hat{\epsilon}} \phi_1^S(\delta') d\delta' + \alpha \int_D \lambda_1^{S*}(\delta)\phi_1^S(\delta) d\delta \\
& -2\lambda_1^{S*}(1-\delta) \int_0^{\delta} \frac{h_{\lambda_1^{S*}}(\delta')}{\hat{\epsilon}} \phi_1^S(\delta') d\delta' \Bigg\} \\
&= \lim_{\hat{\epsilon}\to 0} \Bigg\{ \alpha(2\phi_1^S(\delta)-1) \int_{B\cup D} \lambda_1^{S*}(\delta) \frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}} d\delta \\
&+ \underbrace{2\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}} \left(\alpha \int_0^1 \lambda_1^{S*}(\delta)\phi_1^S(\delta) d\delta + \lambda_1^{S*}(\delta) \int_0^{1-\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta') d\delta' \right)}_{\text{(*-1)}} \\
&+ \underbrace{2\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}} \left(\lambda_1^{S*}(1-\delta) \int_0^{\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta') d\delta' \right)}_{\text{(*-2)}} \\
&+ 2\phi_1^S(\delta)\lambda_1^{S*}(\delta) \int_0^{1-\delta} \lambda_1^{S*}(\delta') \frac{h_{\phi_1^S}(\delta')}{\hat{\epsilon}} d\delta' - 2\phi_1^S(1-\delta)\lambda_1^{S*}(1-\delta) \int_0^{\delta} \lambda_1^{S*}(\delta') \frac{h_{\phi_1^S}(\delta')}{\hat{\epsilon}} d\delta' \\
&- \alpha(2\phi_1^S(\delta)-1) \int_D \lambda_1^{S*}(\delta)\phi_1^S(\delta) d\delta \\
&+ 2\phi_1^S(\delta)\lambda_1^{S*}(\delta) \int_0^{1-\delta} \frac{h_{\lambda_1^{S*}}(\delta')}{\hat{\epsilon}} \phi_1^S(\delta') d\delta' - 2\phi_1^S(1-\delta)\lambda_1^{S*}(1-\delta) \int_0^{\delta} \frac{h_{\lambda_1^{S*}}(\delta')}{\hat{\epsilon}} \phi_1^S(\delta') d\delta' \\
&\equiv 0 \quad \text{on } \delta \in [0,1] n (B \cup D)
\end{aligned}
\tag{3.65}
$$

Denote the maximum of $\left|\lambda_1^{S*}(\delta)h_{\phi_1^S}(\delta)\right|$ over $B\cup D$ as A_1, the maximum of $\left|\lambda_1^{S*}(\delta)\phi_1^S(\delta)\right|$ over D as A_2, the maximum of $\left|h_{\lambda_1^{S*}}(\delta)\phi_1^S(\delta)\right|$ over D as A_3, then except for the $(*-1)+(*-2)$ term in equation (3.65), all the other terms are $o(\hat{\epsilon})$ terms:

$$\begin{aligned}&\lim_{\hat{\epsilon}\to 0}\left|\alpha(2\phi_1^S(\delta)-1)\int_{B\cup D}\lambda_1^{S*}(\delta)\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}}d\delta\right|\\ &\leq \lim_{\hat{\epsilon}\to 0}2\left|\alpha(2\phi_1^S(\delta)-1)\right|A_1\hat{\epsilon}^2\frac{1}{\hat{\epsilon}}=\lim_{\hat{\epsilon}\to 0}2\left|\alpha(2\phi_1^S(\delta)-1)\right|A_1\hat{\epsilon}=0\end{aligned}$$

$$\begin{aligned}&\lim_{\hat{\epsilon}\to 0}\left|2\phi_1^S(\delta)\lambda_1^{S*}(\delta)\int_0^{1-\delta}\lambda_1^{S*}(\delta')\frac{h_{\phi_1^S}(\delta')}{\hat{\epsilon}}d\delta'-2\phi_1^S(1-\delta)\lambda_1^{S*}(1-\delta)\int_0^{\delta}\lambda_1^{S*}(\delta')\frac{h_{\phi_1^S}(\delta')}{\hat{\epsilon}}d\delta'\right|\\ &\leq \lim_{\hat{\epsilon}\to 0}\left(\left|2\phi_1^S(\delta)\lambda_1^{S*}(\delta)\right|+\left|2\phi_1^S(1-\delta)\lambda_1^{S*}(1-\delta)\right|\right)A_1\hat{\epsilon}^2\frac{1}{\hat{\epsilon}}\\ &=\lim_{\hat{\epsilon}\to 0}\left(\left|2\phi_1^S(\delta)\lambda_1^{S*}(\delta)\right|+\left|2\phi_1^S(1-\delta)\lambda_1^{S*}(1-\delta)\right|\right)A_1\hat{\epsilon}\\ &=0\end{aligned}$$

$$\lim_{\hat{\epsilon}\to 0}\left|-\alpha(2\phi_1^S(\delta)-1)\int_D\lambda_1^{S*}(\delta)\phi_1^S(\delta)d\delta\right|\leq \lim_{\hat{\epsilon}\to 0}\left|\alpha(2\phi_1^S(\delta)-1)\right|A_2\hat{\epsilon}^2=0$$

$$\begin{aligned}&\lim_{\hat{\epsilon}\to 0}\left|2\phi_1^S(\delta)\lambda_1^{S*}(\delta)\int_0^{1-\delta}\frac{h_{\lambda_1^{S*}}(\delta')}{\hat{\epsilon}}\phi_1^S(\delta')d\delta'-2\phi_1^S(1-\delta)\lambda_1^{S*}(1-\delta)\int_0^{\delta}\frac{h_{\lambda_1^{S*}}(\delta')}{\hat{\epsilon}}\phi_1^S(\delta')d\delta'\right|\\ &\leq \lim_{\hat{\epsilon}\to 0}\left(\left|2\phi_1^S(\delta)\lambda_1^{S*}(\delta)\right|+\left|2\phi_1^S(1-\delta)\lambda_1^{S*}(1-\delta)\right|\right)A_3\hat{\epsilon}^2\frac{1}{\hat{\epsilon}}\\ &=\lim_{\hat{\epsilon}\to 0}\left(\left|2\phi_1^S(\delta)\lambda_1^{S*}(\delta)\right|+\left|2\phi_1^S(1-\delta)\lambda_1^{S*}(1-\delta)\right|\right)A_3\hat{\epsilon}\\ &=0\end{aligned}$$

Then to make equation (3.65) still apply, we conclude that

$$\lim_{\hat{\epsilon}\to 0}\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}}=0,\quad \forall\delta\in[0,1]n(B\cup D)$$

(2) For $\forall\delta \in D$, the value of left hand side (LHS) of (3.64) equals to the summation of LHS value in case **(1)** (LHS_1) and another extra term with incremental $h_{\lambda_1^{S*}}(\delta)$ outside the integrals:

$$
\begin{aligned}
& LHS_2 \\
& = LHS_1 + \lim_{\hat{\epsilon}\to 0} 2\phi_1^S(\delta)\frac{h_{\lambda_1^{S*}}(\delta)}{\hat{\epsilon}} \int_0^{\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta' \\
& = \lim_{\hat{\epsilon}\to 0} 2\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}} \left(\alpha \int_0^1 \lambda_1^{S*}(\delta)\phi_1^S(\delta)d\delta + \lambda_1^{S*}(\delta)\int_0^{1-\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta' \right. \\
& \qquad \left. + \lambda_1^{S*}(1-\delta)\int_0^{\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta'\right) \\
& - \lim_{\hat{\epsilon}\to 0} 2\phi_1^S(\delta)\frac{\hat{\epsilon}\lambda_1^{S*}(\delta)}{\hat{\epsilon}}\int_0^{1-\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta' + o(\hat{\epsilon}) \\
& = \lim_{\hat{\epsilon}\to 0} 2\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}} \left(\alpha \int_0^1 \lambda_1^{S*}(\delta)\phi_1^S(\delta)d\delta + \lambda_1^{S*}(\delta)\int_0^{1-\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta' \right. \\
& \qquad \left. + \lambda_1^{S*}(1-\delta)\int_0^{\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta'\right) \\
& - \lim_{\hat{\epsilon}\to 0} 2\phi_1^S(\delta)\lambda_1^{S*}(\delta)\int_0^{1-\delta} \lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta' + o(\hat{\epsilon}) \\
& \equiv 0 \quad \text{on } \delta \in D
\end{aligned}
$$

Then we conclude that

$$\lim_{\hat{\epsilon}\to 0} h_{\phi_1^S}(\delta) = O(\hat{\epsilon}) \text{ and } \lim_{\hat{\epsilon}\to 0} h_{\phi_1^S}(\delta) > 0, \quad \forall\delta \in D$$

(3) For $\forall\delta \in B$, the value of left hand side (LHS) of (3.64) equals to the summation of LHS value in case **(1)** (LHS_1) and some extra terms with incremental $h_{\lambda_1^{S*}}(1-\delta)$ outside the integrals:

$$
\begin{aligned}
&LHS_3 \\
&= LHS_1 - \underset{\hat{\epsilon}\to 0}{lim} 2\phi_1^S(1-\delta)h_{\lambda_1^{S*}}(1-\delta)\int_0^{\delta}\lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta' \\
&= \underset{\hat{\epsilon}\to 0}{lim} 2\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}}\left(\alpha\int_0^1\lambda_1^{S*}(\delta)\phi_1^S(\delta)d\delta + \lambda_1^{S*}(\delta)\int_0^{1-\delta}\lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta'\right. \\
&\left. +\lambda_1^{S*}(1-\delta)\int_0^{\delta}\lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta'\right) - \underset{\hat{\epsilon}\to 0}{lim} 2\phi_1^S(1-\delta)\frac{-\hat{\epsilon}\lambda_1^{S*}(1-\delta)}{\hat{\epsilon}}\int_0^{\delta}\lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta' + o(\hat{\epsilon}) \\
&= \underset{\hat{\epsilon}\to 0}{lim} 2\frac{h_{\phi_1^S}(\delta)}{\hat{\epsilon}}\left(\alpha\int_0^1\lambda_1^{S*}(\delta)\phi_1^S(\delta)d\delta + \lambda_1^{S*}(\delta)\int_0^{1-\delta}\lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta'\right. \\
&\left. +\lambda_1^{S*}(1-\delta)\int_0^{\delta}\lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta'\right) + \underset{\hat{\epsilon}\to 0}{lim} 2\phi_1^S(1-\delta)\lambda_1^{S*}(1-\delta)\int_0^{\delta}\lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta' + o(\hat{\epsilon}) \\
&\equiv 0 \quad \text{on } \delta\in B
\end{aligned}
$$

Then we conclude that

$$\underset{\hat{\epsilon}\to 0}{lim} h_{\phi_1^S}(\delta) = O(\hat{\epsilon}) \text{ and } \underset{\hat{\epsilon}\to 0}{lim} h_{\phi_1^S}(\delta) < 0, \quad \forall\delta\in B$$

□

Lemma 2 The incremental $h_{\phi_1^S}(\delta)$ satisfies

$$h_{\phi_1^S}(\delta) + h_{\phi_1^S}(1-\delta) = 0, \ \forall\delta\in[0,1]$$

given any form of incremental $h_{\lambda_1^{S*}}(\delta)$.

Proof:

By $\frac{\partial H}{\partial \phi_1^S(\delta)} h_{\phi_1^S}(\delta) + \frac{\partial H}{\partial \lambda_1^{S*}(\delta)} h_{\lambda_1^{S*}}(\delta) \equiv 0, \forall \delta \in [0,1]$, we use:

$$\frac{\partial H}{\partial \phi_1^S(\delta)} h_{\phi_1^S}(\delta) + \frac{\partial H}{\partial \lambda_1^{S*}(\delta)} h_{\lambda_1^{S*}}(\delta) + \frac{\partial H}{\partial \phi_1^S(\delta)} h_{\phi_1^S}(1-\delta) + \frac{\partial H}{\partial \lambda_1^{S*}(\delta)} h_{\lambda_1^{S*}}(1-\delta) = 0, \forall \delta \in [0,1]$$

then we can trivally get[29]:

$$h_{\phi_1^S}(\delta) + h_{\phi_1^S}(1-\delta) = 0 \quad \forall \delta \in [0,1]$$

for any $h_{\lambda_1^{S*}}(\delta)$. $\square$

Lemma 3 Let f achieve a local extremum subject to $H(x) = \theta$ at the point x_0 and assume that f and H are continuously Fréchet differentiable in an open set containing x_0 and that x_0 is a regular point of H. Then $f'(x_0)h = 0$ for all h satisfying $H'(x_0)h = \theta$. (This lemma is from "Optimization by Vector Space Methods" by David G.Luenberger, page 242.)

3.A.8 Proposition 7

By proof of Proposition 6, the cost function $C(\lambda) = c_1\lambda^2$ only appear in conditions (3.62)(3.63). Then if condition (3.19) applies:

$$C'(\lambda) \begin{cases} \geq 0 & \delta = 0; \\ > 0 & \forall \delta \in (0, \lambda^{ub}]. \end{cases}$$

[29] The result is similar as in (3.35).

then (3.62) becomes:

$$
\begin{aligned}
&\underset{\hat{\epsilon}\to 0}{lim}\frac{1}{\hat{\epsilon}}\left(\frac{\partial W}{\partial \phi_1^S(\delta)}h_{\phi_1^S}(\delta)+\frac{\partial W}{\partial \lambda_1^{S*}(\delta)}h_{\lambda_1^{S*}}(\delta)\right)\\
&=\underset{\hat{\epsilon}\to 0}{lim}\frac{1}{\hat{\epsilon}}\int_0^1(\delta-2C(\lambda_1^{S*}(\delta)))h_{\phi_1^S}(\delta)d\delta+2\int_D C'(\lambda_1^{S*}(\delta'))\lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta'\\
&=\underset{\hat{\epsilon}\to 0}{lim}\frac{1}{\hat{\epsilon}}\int_{B\cup D}(\delta-2C(\lambda_1^{S*}(\delta)))h_{\phi_1^S}(\delta)d\delta+2\int_D C'(\lambda_1^{S*}(\delta'))\lambda_1^{S*}(\delta')\phi_1^S(\delta')d\delta'\\
&\text{(by Lemma 1 in Section 3.A.7.2)}
\end{aligned}
\tag{3.66}
$$

where the second term is still positive since $D \subset [\frac{1}{2},1]$ and $C'(\lambda) > 0\ \forall\delta \in (0, \lambda^{ub}]$.

Also by Lemma 1 and Lemma 2 in Section 3.A.7.2:

$$
\begin{aligned}
&\underset{\hat{\epsilon}\to 0}{lim}\int_{B\cup D}(\delta-2C(\lambda_1^{S*}(\delta)))h_{\phi_1^S}(\delta)d\delta\\
&=\underset{\hat{\epsilon}\to 0}{lim}\left(\int_B(\delta-2C(\lambda_1^{S*}(\delta)))h_{\phi_1^S}(\delta)d\delta+\int_D(\delta-2C(\lambda_1^{S*}(\delta)))h_{\phi_1^S}(\delta)d\delta\right)\\
&=\underset{\hat{\epsilon}\to 0}{lim}\left(-\int_B(\delta-2C(\lambda_1^{S*}(\delta)))h_{\phi_1^S}(1-\delta)d\delta+\int_D(\delta-2C(\lambda_1^{S*}(\delta)))h_{\phi_1^S}(\delta)d\delta\right)\\
&=\underset{\hat{\epsilon}\to 0}{lim}\left(-\int_D(1-\delta-2C(\lambda_1^{S*}(1-\delta)))h_{\phi_1^S}(\delta)d\delta+\int_D(\delta-2C(\lambda_1^{S*}(\delta)))h_{\phi_1^S}(\delta)d\delta\right)\\
&=\underset{\hat{\epsilon}\to 0}{lim}\left(\int_D(2\delta-1+2C(\lambda_1^{S*}(1-\delta))-2C(\lambda_1^{S*}(\delta)))h_{\phi_1^S}(\delta)d\delta\right)\\
&>0\quad (C'(\lambda)\geq 0, D\subset[\frac{1}{2},1] \text{ and } \lambda_1^{S*\prime}(\delta)<0)
\end{aligned}
\tag{3.67}
$$

(3.66)(3.67) still lead to:

$$
\underset{\hat{\epsilon}\to 0}{lim}\frac{1}{\hat{\epsilon}}\left(\frac{\partial W}{\partial \phi_1^S(\delta)}h_{\phi_1^S}(\delta)+\frac{\partial W}{\partial \lambda_1^{S*}(\delta)}h_{\lambda_1^{S*}}(\delta)\right)>0
$$

Then we can still get the contradiction, then we conclude that any $\lambda_1^{S*}(\delta)$ that satisfies there exists subset $\triangle^+ \subset [\frac{1}{2},1]$ where $\lambda_1^{S*}(\delta) > 0, \forall\delta \in \triangle^+$ cannot be the social optimal solution.

3.A.9 Solution to the social planner problem with different cost functions

3.A.9.1 Convex cost function $C(\lambda) = c_1\lambda^2$

Social Optimal Solution

$$\frac{\partial L}{\partial \lambda_1(\delta)} = \frac{\frac{\alpha}{2} - \alpha\delta + \alpha c_1 \lambda_1^2(\delta) - \alpha(\alpha + \lambda_1(\delta))2c_1\lambda_1(\delta)}{(\alpha + \lambda_1(\delta))^2} = 0$$

and

$$\frac{\partial^2 L}{\partial \lambda_1^2(\delta)} = \frac{-2c_1\alpha^3 - \alpha(1-2\delta)}{(\alpha + \lambda_1(\delta))^4} < 0$$

Then solutions is:

$$\lambda_1^{S*}(\delta) = \frac{-2c_1\alpha^2 + \sqrt{4c_1^2\alpha^4 + 4c_1\alpha^2(\frac{1}{2} - \delta)}}{2c_1\alpha} \quad \forall\delta \in [0, \frac{1}{2}), \qquad \lambda_1^{S*}(\delta) \equiv 0 \quad \forall\delta \in [\frac{1}{2}, 1]$$

Competitive Equilibrium Solution

$$\lambda_1^*(\delta) = \frac{\int_\delta^1 \frac{\lambda_0^*(\delta')}{\Lambda_0}(\triangle V(\delta') - \triangle V(\delta))\phi_0(\delta')d\delta'}{2c_1}$$

$$\lambda_0^*(\delta) = \frac{\int_0^\delta \frac{\lambda_1^*(\delta')}{\Lambda_1}(\triangle V(\delta) - \triangle V(\delta'))\phi_1(\delta')d\delta'}{2c_1}$$

3.A.9.2 Linear cost function $C(\lambda) = c_1\lambda$

Social Optimal Solution

$$\frac{\partial L}{\partial \lambda_1(\delta)} = \frac{\frac{\alpha}{2} - \alpha\delta + \alpha C(\lambda_1(\delta)) - \alpha(\alpha + \lambda_1(\delta))C'(\lambda_1(\delta))}{(\alpha + \lambda_1(\delta))^2} = \frac{\frac{\alpha}{2} - c_1\alpha^2 - \alpha\delta}{(\alpha + \lambda_1(\delta))^2}$$

Then if $c_1\alpha < \frac{1}{2}$, the solution is:

$$\lambda_1^{S*}(\delta) = \begin{cases} \lambda^{ub} & \text{if } \delta \leq \frac{1}{2} - c_1\alpha; \\ 0 & \text{if } \delta > \frac{1}{2} - c_1\alpha. \end{cases}$$

If $c_1\alpha \geq \frac{1}{2}$,

$$\lambda_1^{S*}(\delta) \equiv 0 \quad \forall \delta \in [0, 1]$$

Competitive Equilibrium Solution

For competitive equilibrium solutions, given parameters c_1, α, r, $\exists \delta^*(c_1, \alpha, r)$, s.t. $\lambda_1^*(\delta) = \lambda^{ub}$ for $\forall \delta \in [0, \delta^*(c_1, \alpha, r)]$ and $\lambda_1^*(\delta) = 0$ for $\forall \delta \in (\delta^*(c_1, \alpha, r), 1]$; by symmetry, $\lambda_0^*(\delta) = \lambda^{ub}$ for $\forall \delta \in [1 - \delta^*(c_1, \alpha, r), 1]$ and $\lambda_0^*(\delta) = 0$ for $\forall \delta \in [0, 1 - \delta^*(c_1, \alpha, r))$. For simplicity to compare with social optimal solution, we give numerical case such that $1 - \delta^*(c_1, \alpha, r) < \frac{1}{2} < \delta^*(c_1, \alpha, r)$, i.e. there exists intermediation behavior in CE equilibrium.

3.A.9.3 Social optimal solution for concave cost function $C(\lambda) = c_1\lambda^p$, $p \in (0, 1)$

$$\begin{aligned} \frac{\partial L}{\partial \lambda_1(\delta)} &= \frac{\frac{\alpha}{2} - \alpha\delta + \alpha C(\lambda_1(\delta)) - \alpha(\alpha + \lambda_1(\delta))C'(\lambda_1(\delta))}{(\alpha + \lambda_1(\delta))^2} \\ &= \frac{\alpha(\frac{1}{2} - \delta + (1 - p)c_1{\lambda_1}^p(\delta) - \alpha c_1 p {\lambda_1}^{p-1}(\delta))}{(\alpha + \lambda_1(\delta))^2} \end{aligned}$$

- Case 1: $\lambda_1^{S*}(\delta)$ that satisfies the following equation is a **stationary point**:

$$\frac{1}{2} - \delta + (1 - p)c_1{\lambda_1}^p(\delta) - \alpha c_1 p {\lambda_1}^{p-1}(\delta) = 0 \tag{3.68}$$

Since

$$\begin{aligned}
\frac{\partial^2 L}{\partial \lambda_1^2(\delta)} &= \frac{\alpha\left(\lambda_1^{p-1}(\delta)\alpha c_1 p(p^2-3p+4)+\lambda_1^{p-2}(\delta)\alpha^2 c_1 p(1-p)^2\right)}{(\alpha+\lambda_1(\delta))^3} \\
&+ \frac{\alpha\left(\lambda_1^{p}(\delta)c_1(1-p)(p-2)+(2\delta-1)\right)}{(\alpha+\lambda_1(\delta))^3} \\
&= \frac{\alpha\left((\alpha c_1\lambda_1^{p-1}(\delta)-(\frac{1}{2}-\delta))p+\alpha c_1\lambda_1^{p-1}(\delta)(1-p)^2+(\frac{1}{2}-\delta)\frac{\alpha(1-p)^2}{\lambda_1(\delta)}\right)}{(\alpha+\lambda_1(\delta))^3} \\
&> 0 \quad \text{by (3.68)}
\end{aligned}$$

Then the stationary point is local min point.

- Case 2: Since $0 < p < 1$, then $\lambda_1(\delta) \equiv 0$ is a local max point, since $\frac{\partial L}{\partial \lambda_1(\delta)}|_{\lambda_1(\delta)=0} < 0$ $\forall \delta \in [0, \frac{1}{2})$, then the social welfare trivally $W^* = 5$ for $r = 0.05$.
- Case 3: $\lambda_1(\delta) \equiv \lambda^{ub}$ is a local max point if $\frac{1}{2} - \delta + (1-p)c_1(\lambda^{ub})^p - \alpha c_1 p(\lambda^{ub})^{p-1} > 0$ for $\forall \delta \in [0, \frac{1}{2})$.

The social optimal solution for concave cost function is either $\lambda_1^{S*}(\delta) = \lambda^{ub}$ or $\lambda_1^{S*}(\delta) = 0$ on $[0, \frac{1}{2})$ depending on parameters c_1, α, r, p. (Also need to verify expost that the generated $\triangle V^S(\delta)$ satisfies $\frac{d\triangle V(\delta)}{d\delta} > 0$ for $\forall \delta \in [0, 1]$.)

Numerical Example for Case 2: $\lambda^{ub} = 0.3, c_1 = 2, \alpha = 0.75, p = 0.5$ ($\lambda_1(\delta) \equiv 0$ is a local max point but $\lambda_1(\delta) \equiv \lambda^{ub}$ is not local max point)
Numerical Example for Case 3: $\lambda^{ub} = 1, c_1 = 1, \alpha = 0.05, p = 0.5$ (Both $\lambda_1(\delta) \equiv 0$ and $\lambda_1(\delta) \equiv \lambda^{ub}$ are local max points, but the marginal loss from deviating from $\lambda_1(\delta) \equiv \lambda^{ub}$ is large in this case)

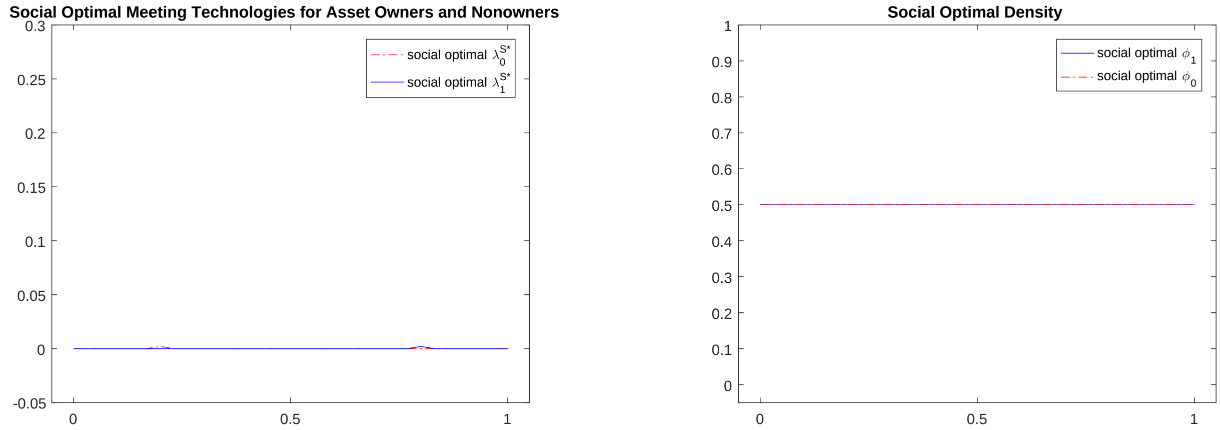

Figure 3.16: Case 2: Social optimal meeting technologies and densities for concave cost function $C(\lambda) = c_1\lambda^p$

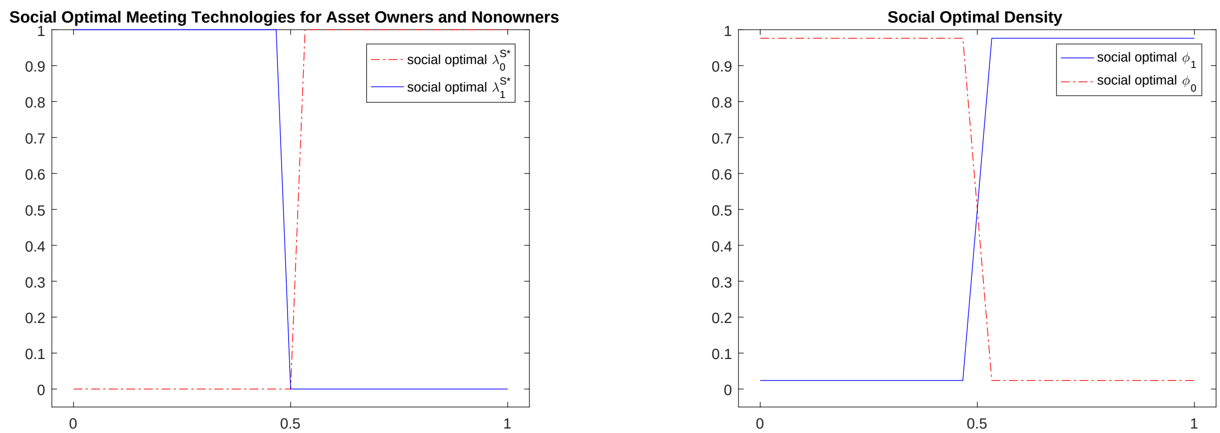

Figure 3.17: Case 3: Social optimal meeting technologies and densities for concave cost function $C(\lambda) = c_1\lambda^p$

3.A.9.4 Social optimal solution for $C(\lambda) = c_1\lambda^2 + c_2\lambda$ $(c_1 < 0, c_2 > 0)$

Finally, we give a numerical example for $C(\lambda) = c_1\lambda^2 + c_2\lambda$ $(c_1 < 0, c_2 > 0$ to double check the sufficient condition for $\lambda_1^{S*}(\delta) \equiv 0$ on $[\frac{1}{2}, 1]$.

$$C'(\lambda) \geq 0 \quad \forall \lambda \in [0, \lambda^{ub}]$$

$\Longrightarrow$

$$c_2 > -2c_1\lambda^{ub}$$

- Case 1: The analytical stationary point satisfies:

$$\frac{\partial L}{\partial \lambda_1(\delta)} = \frac{\frac{\alpha}{2} - \alpha\delta - \alpha^2 c_2 - 2\alpha^2 c_1 \lambda_1(\delta) - \alpha c_1 \lambda_1^2(\delta)}{(\alpha + \lambda_1(\delta))^2} = 0$$

  $\Longrightarrow$

$$\lambda_1{}^*(\delta) = \frac{-2\alpha c_1 + \sqrt{4\alpha^2 c_1^2 - 4c_1(\alpha c_2 + \delta - \frac{1}{2})}}{2c_1},$$
$$\forall \delta \in [0, \frac{1}{2}] \quad (\text{require } \alpha^2 c_1 - \alpha c_2 + \frac{1}{2} \leq 0)$$

  and

$$\frac{\partial^2 L}{\partial \lambda_1^2(\delta)} = \frac{2\alpha^2 c_2 - 2\alpha^3 c_1 + \alpha(2\delta - 1)}{(\alpha + \lambda_1(\delta))^4} \geq \frac{\alpha + \alpha(2\delta - 1)}{(\alpha + \lambda_1(\delta))^4} \geq 0 \quad (\text{by } \alpha^2 c_1 - \alpha c_2 + \frac{1}{2} \leq 0)$$

  so the stationary point is a local min point.

- Case 2: If $\alpha c_2 \geq \frac{1}{2}$, then $\lambda_1{}^*(\delta) \equiv 0 \ \forall \delta \in [0, \frac{1}{2}]$ is local maximum point.

- Case 3: If $\alpha c_2 + 2\alpha c_1 \lambda^{ub} + c_1(\lambda^{ub})^2 \leq 0$, then $\lambda_1{}^*(\delta) \equiv \lambda^{ub} \ \forall \delta \in [0, \frac{1}{2}]$ is local maximum point.

Numerical Example for Case 2: $\lambda^{ub} = 2, c_1 = -0.5, c_2 = 10, \alpha = 0.5$.

Numerical Example for Case 3: $\lambda^{ub} = 1.5, c_1 = -0.5, c_2 = 2, \alpha = 0.05$.

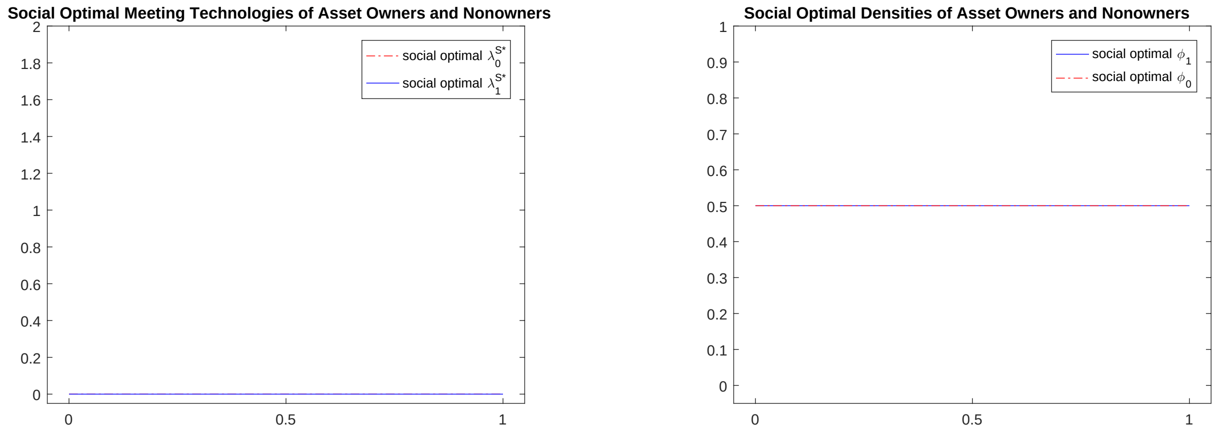

Figure 3.18: Case 2: Social optimal meeting technologies and densities for convex cost function $C(\lambda) = c_1\lambda^2 + c_2\lambda$

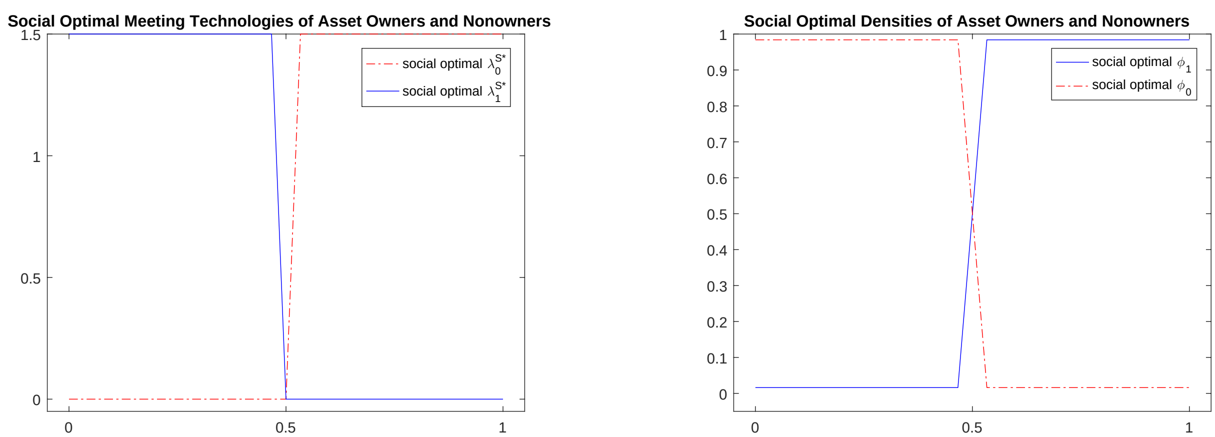

Figure 3.19: Case 3: Social optimal meeting technologies and densities for convex cost function $C(\lambda) = c_1\lambda^2 + c_2\lambda$

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