Title
Malthus and Climate Change: Betting on a Stable Population

Authors
Kelly, David L.
Kolstad, Charles D.

Publication Date
1999-07-31

Malthus and Climate Change:
Betting on a Stable Population*

David L. Kelly
Department of Economics
University of Miami
Box 284126
Coral Gables, FL 33124

and

Charles D. Kolstad
Department of Economics
Bren School of Environmental Science and Management
and Environmental Studies Program
University of California
Santa Barbara, CA 93106

Current Version: July 31, 1999

*Research supported by US Department of Energy grant DE-FG03-94ER61944 to the Unversity of California Santa Barbara. We thank Robert Deacon, Alan Manne, seminar participants at the 1996 California Workshop on Environmental and Natural Resource Economics. Research assistance from Aran Ratcliffe is gratefully acknowledged.

Abstract

A standard assumption in integrated assessment models of climate change is that population and productivity are growing, but at a decreasing rate. We explore the significance of the assumption of population and productivity growth for greenhouse gas abatement. After all, there has been no long run slow down in the growth of productivity over the past few centuries, and the rate of population growth has actually been increasing for the past 19 centuries. Even if either of these growth rates were expected to slow, by how much is subject to great uncertainty. We show computationally that such continued growth greatly increases the severity of climate change. Indeed we find that climate change is a problem in large part "caused" by exogenous population and productivity growth. Rapid reductions in growth make climate change a small problem; smaller reductions in growth imply climate change is a very serious problem indeed. Analogously, reductions in the growth rate of population can be effective in controlling climate change.

1. Introduction

A standard assumption in computational models of climate change (integrated assessment models) is that population and productivity are growing, but at a decreasing rate. For example, a survey in Nordhaus and Yohe (1983) finds that most researchers assume a large slowdown in productivity growth over the next century. The mean estimate of the survey is that productivity growth will slow by more than half by 2025 (Nordhaus, 1994 assumes productivity growth slows by half every six decades). Similarly, the Intergovernmental Panel on Climate Change report (hereafter IPCC, 1990 and IPCC, 1992) estimates population growth will slow from 22 percent to 18 percent this decade, with zero growth by the year 2200, largely based on United Nations (1994) or World Bank (1991) predictions.

In this paper we explore the significance of the assumption of population and productivity growth for greenhouse gas abatement. After all, there has been no long run slow down in the growth of productivity over the past few centuries, and the rate of population growth has actually been increasing for the past 19 centuries. While most analysts expect a turn-around in the growth of population and productivity, exactly by how much and when is highly uncertain. We show computationally and theoretically that such continued growth greatly increases the severity of climate change. The conclusions are startling: there is almost a one-to-one correspondence between population and productivity growth assumptions, the degree of climate change, and hence the optimal response to climate change. Indeed we find that climate change is a problem in large part "caused" by exogenous population and productivity growth. By examining this question we also gain important insights about how assumptions in the growth of population and productivity drive the results that are common to almost all integrated assessment models.

This paper also extends the recent literature on the effect of population on externalities (see for example Harford, 1998) to the case of a stock pollutant in a production externality. Larger future populations increase the damage from current emissions, at least under the standard utilitarian paradigm. One can simultaneously observe the marginal damage from an additional unit of pollution, as well as the marginal damage from an additional person, with the mechanism being the increased damage from a larger population which suffers the pollution in future years.

In most integrated assessment models, emissions of greenhouse gases (GHGs) are proportional to the gross world product (GWP). Most models also assume the proportion declines over time, to reflect compositional changes in output, exogenous technical progress in abatement, or switching to fuels which release less GHGs. Models which make this assumption include Kolstad (1996) and Nordhaus (1994). Other models such as Manne, Mendelsohn, and Richels (1993) and Peck and Teisberg (1992) have an energy sector and model emissions as proportional to energy consumption. In the long run, energy usage rises with GWP, hence emissions and GWP rise together. Other models (primarily from the natural science literature) such as Lempert, Schlesinger, and Banks (1996) let emissions be exogenous. However, the exogenously specified emissions path is calibrated from projections which assume emissions are proportional to GWP. Hence nearly all integrated assessment models directly or indirectly make GHG emissions proportional to GWP.

The GWP in turn is modeled as a function of the different production inputs. These are capital, labor, and possibly an energy input. The world productivity variable and value share of each input are calculated using the Solow procedure, or growth accounting. According to growth accounting, productivity is simply the difference between the change in output and the change in production inputs at their average shares. Use of average shares in growth accounting in turn implicitly assumes that the economy is on the long run balanced growth path. By balanced growth, we mean that capital and labor (in productivity unit terms, hereafter adjusted labor) are growing at the same rate. This is equivalent to a constant capital to labor ratio. Thus output (per adjusted labor unit) will also grow at the same rate. The balanced growth assumption is probably reasonable for developed countries, although many researchers believe that output growth in developing countries generally has not reached the balanced growth rate.[1] There is a large literature which tries to explain long run growth in capital and output (Barro and Sala-i-Martin, 1995 provide an excellent summary).

[1] The reason for why developing countries sometimes exhibit slow growth is unclear. Usual reasons cited are differences in variables such as education and life expectancy (see for example, Summers and Heston, 1991) and differences in economic institutions and structure (see for example, Mauro, 1995 or North, 1996).

To see why the balanced growth result holds in an optimal growth model, suppose capital were growing at a slower rate than adjusted labor. Then over time the economy accumulates relatively more labor resources than capital resources. But then since labor has diminishing marginal returns, an extra unit of capital produces increasingly more output relative to another unit of adjusted labor. Eventually, the returns to investing in this capital relative to consumption become larger so that the economy invests at a greater rate. But then capital grows at a greater rate, moving back to the growth rate for adjusted labor. Hence balanced growth holds at the long run steady state.

Now add climate change to a model in which balanced growth holds in the long run and suppose for now there is no abatement. Since emissions are proportional to GWP, then emissions must also grow at the same rate as output, capital, and adjusted labor. However, since emissions cause damage from climate change, the return to capital is lower. Damage from climate change reduces future output for a given amount of capital. The return to consumption is unchanged by climate change. If the growth rate in adjusted labor is zero at the steady state, the growth rate in capital and emissions is also zero at the steady state and balanced growth holds, but with a lower capital to labor ratio. The capital to labor ratio will offset the damage from climate change with the loss in consumption from having less capital. Weitzman (1994) refers to the difference in capital between a model without an environmental externality and a model with an environmental externality as the "environmental drag." If investment in abatement is possible, the capital to output ratio will be closer to the optimal growth model capital to output ratio and there will be some steady state investment in abatement.

In this paper, we argue that the reduction in the return to capital caused by damage from climate change is very small. Hence the optimal solution found by climate change modelers is generally to grow at nearly the no-climate change rate. Since marginal abatement costs go to zero as abatement goes to zero, some amount of abatement is optimal.

Why is the damage from climate change small? This is basically because researchers assume that growth in adjusted labor stops long before climate change becomes a real problem. If adjusted labor stops growing, then capital stops growing, which implies output stops growing, but then so too does emissions, and therefore so does temperature change and damage from temperature change, all without cost.

Conversely, imagine a world in which adjusted labor units grew at a constant rate forever. Then capital grows at a constant rate, and so therefore does output and uncontrolled emissions. Because damages are convex, abatement increase approach 100 percent to keep net emissions finite. That is, emissions control approaches one and uncontrolled emissions goes to infinity, but net emissions remains finite. A corner solution of 100 percent emissions control differs greatly from emissions control when adjusted labor stops growing.

From these two thought experiments, one can see that assumptions about the growth in adjusted labor drive climate change results, as well as optimal policies. Indeed adjusted labor growth determines investment levels, emissions growth (or emission control), GHG levels, and temperatures. We find that if the growth rate in adjusted labor declines by 16 percent per decade instead of 31 percent per decade assumed in IPCC (1990), then the total steady state temperature increase rises from 5.83 degrees Celsius to an unprecedented 16 degrees.[2] Furthermore, control measures are severe: abatement nearly doubles, eventually rising to 26 percent of emissions, and the optimal capital level falls 36 percent below the balanced growth level. If the growth rate of adjusted labor declines by 31 percent per decade, then abatement rises to at most 14 percent of emissions and capital spending is at most 7 percent less than balanced growth.

Implications for near-term climate policy are significant, though less spectacular. Higher rates of population growth imply as much as 50 percent higher current marginal damages from carbon. If a carbon tax were used to implement the optimal level of emissions, the carbon tax would be set to the marginal damage. Though marginal damage from carbon is in the neighborhood of $5 per ton in 1995, the marginal damage of one more person is roughly two orders of magnitude higher, suggesting that attention to factors leading to population stabilization are in order for addressing climate change. Indeed, efficiency requires both a carbon tax and a tax on children.

[2]For reference, IPCC (1992) reports the current warming trend is about one half of one degree over the last 100 years. An increase of 3-4 degrees above modern temperatures last occurred an estimated 5000 years ago. Of course, models of climate change may not be appropriate if GHG levels more than double (in some cases considered here, GHG levels increase 6-fold). We assume that the structural model does not change if GHG levels rise about twice preindustrial levels.

While we use a model which is similar to standard and well-known models of climate change to facilitate comparisons, we believe the results apply generally to nearly all climate change models, because nearly all climate change models make similar assumptions about growth in population and productivity.

Another interpretation of our results is that efforts at GHG control might be appropriately supplemented by efforts to reduce population growth. While population control is sometimes mentioned as an important part of policies for other environmental problems, population control is rarely considered for climate change policy. This is especially surprising given the long run nature of climate change. Our analysis parallels Harford (1998) addresses child bearing choices and externalities in an economy with a consumption externality. Harford (1998) shows that both pollution taxes and child bearing taxes are needed to achieve the Parato optimum, with the optimal tax per child equal to the present value of the pollution taxes paid by the child.

In the next section we develop a simple theoretical framework from which we analyze how productivity and population growth affect decisions about investment and emissions. In the subsequent section, we give computational results for emissions and investment decisions under alternative specifications about the growth of adjusted labor. In section 4, we derive computationally the 1995 control rates under alternative specifications about the growth of adjusted labor. The final section is concluding remarks.

2. A Representative Model

Consider a simple model of climate change, the notation and structure of which is similar to Nordhaus (1994). Let time in years be discrete with $t = 0, 1, \ldots$. The population consists of L_t identical representative consumers (consumers are not differentiated over regions). Consumers value consumption of a composite commodity (C_t).

The consumer faces several resource constraints. Consumers use capital (K_t) and labor to produce the consumption good. Productivity is proportional to A_t (technology). Production is Cobb-Douglass:

$$Q_t = A_t K_t^{\gamma} L_t^{1-\gamma} \tag{2.1}$$

Here $\gamma \in (0,1)$ is the capital share. We let l_t denote the productivity weighted labor units (adjusted labor):

$$l_t = (A_t)^{\frac{1}{1-\gamma}} L_t \tag{2.2}$$

Therefore we can rewrite the production function as:

$$Q_t = K_t^{\gamma} l_t^{1-\gamma} \tag{2.3}$$

Emissions of GHGs (E_t) cause increases in atmospheric temperature above a base temperature (T_t), which cause environmental damage through reduced production. A fraction of the production can be spent on abatement (μ_t) which has a convex cost function and reduces emissions linearly. The abatement level is the primary concern of policy makers, especially in the current period. Net income (Y_t) is the fraction of production which is not spent on emission control.

$$Y_t \equiv \frac{\Omega(\mu_t)}{D(T_t)} Q_t \equiv \frac{1 - b_1 \mu_t^{b_2}}{1 + \theta_1 T_t^{\theta_2}} K_t^{\gamma} l_t^{1-\gamma} \tag{2.4}$$

In equation (2.4), $b_1, \theta_1 < 1$ and $b_2, \theta_2 > 1$ are positive constants.

The consumer faces the overall resource constraint that total income equals consumption plus net investment. Let δ_K be the depreciation rate, then:

$$C_t = Y_t - K_{t+1} + (1 - \delta_K) K_t \tag{2.5}$$

Uncontrolled emissions (emissions before abatement) in a given period depend largely on the specific mixture of fuels consumed. Some analysts suggest that in the future we will rely increasingly on energy sources that emit less GHGs per unit of released energy and that output will continue to become less energy intensive. We let this process be exogenous, represented by the variable σ_t, which corresponds to the emissions intensity of output. We assume:

$$\sigma_t = (1 - g_{\sigma,t}) \sigma_{t-1} \tag{2.6}$$

$$g_{\sigma,t} = (1 - \delta_\sigma)\, g_{\sigma,t-1} \tag{2.7}$$

Thus the emissions intensity of output improves at a declining rate. Note that improvements in the emissions intensity of output also implicitly represent sector shifts in output from goods which are GHG-intensive to produce to goods which are less GHG-intensive to produce.

The atmosphere absorbs fraction β_m of emissions. Let M_t denote the stock of GHGs above preindustrial levels and E_t the emissions, then:

$$E_t = \beta_m (1 - \mu_t)\, \sigma_t Q_t \tag{2.8}$$

$$M_{t+1} = E_t + (1 - \delta_M)\, M_t \tag{2.9}$$

Here δ_M is the "depreciation" of the stock of GHGs (primarily into the ocean). We let emissions be proportional to output and not net income Y_t. Hence, the purchase of abatement is not emission free, a factory must make the CO_2 scrubber. This simplifies theoretical analysis considerably because climate change does not cause a reduction in emissions and therefore future climate change through decreased net income.

We assume a two layer model for temperature change. Let surface temperature be the top layer (T_t) and deep ocean temperature (O_t) be the second layer, both measured in degrees Celsius above a base year. Let M_b denote the preindustrial level of GHGs, then the temperature model is:

$$T_{t+1} = (1 - \delta_T)\, T_t + r_1 \log\left(1 + \frac{M_t}{M_b}\right) + r_2 O_t \tag{2.10}$$

$$O_{t+1} = O_t + r_3 (T_t - O_t) \tag{2.11}$$

Here r_1 and $r_2, r_3, \delta_T \in (0, 1)$ are positive constants.

Preferences for the consumption good are specified by a twice differentiable and concave utility function. We assume utility has a constant relative risk aversion specification:

$$U\left[C_t\right] = \frac{C_t^{1-\eta} - 1}{1-\eta} \qquad \eta \geq 0 \tag{2.12}$$

Here the limiting case of $\eta = 1$ is logarithmic utility. The objective function W is the discounted sum of per-capita utilities over the infinite horizon. Let β denote the discount rate, then:

$$W = \sum_{t=0}^{\infty} \beta^t U\left[\frac{C_t}{L_t}\right] \tag{2.13}$$

Many integrated assessment models use a Ramsey aggregate objective function. The aggregate objective function weights utility at period t by the size of the population:

$$\hat{W} = \sum_{t=0}^{\infty} \beta^t L_t U\left[\frac{C_t}{L_t}\right] \tag{2.14}$$

The former objective function maximizes the utility of a representative consumer of the current generation while the latter maximizes the aggregate utility of all consumers including those yet to be born. One can show that both the aggregate and per-capita specifications yield the same stationary values, since the steady state is independent of the functional form of the utility function. Hence most of the results apply to either specification. However, the 1995 abatement decisions under aggregate utility are higher than under per-capita utility, because the social planner discounts the future less. When calculating 1995 abatement decisions, we consider both the aggregate and per-capita specifications.

We can rewrite the utility function (2.12) in terms of adjusted labor units.

$$U\left[\frac{C_t}{L_t}\right] = \frac{\left[\frac{C_t A_t^{\frac{1}{1-\gamma}}}{l_t}\right]^{1-\eta} - 1}{1-\eta} = A_t^{\frac{1-\eta}{1-\gamma}}\left(\frac{\left[\frac{C_t}{l_t}\right]^{1-\eta} - 1}{1-\eta}\right) + a_0 = U\left(c_t\right) + a_0 \tag{2.15}$$

Here:

$$a_0 = \frac{A_t^{\frac{1-\eta}{1-\gamma}} - 1}{1-\eta}\ ,\ c_t = \frac{C_t}{l_t} \tag{2.16}$$

Hence:

$$W=\sum_{t=0}^{\infty}\beta^{t}\left[A_{t}^{\frac{1-\eta}{1-\gamma}}U\left[c_{t}\right]+a_{0}\right] \tag{2.17}$$

$$\hat{W}=\sum_{t=0}^{\infty}\beta^{t}\left[A_{t}^{\frac{-\eta}{1-\gamma}}l_{t}U\left[c_{t}\right]+a_{0}\right] \tag{2.18}$$

Because a_0 is a constant, we may drop the a_0 terms from the objective function. Note that the only difference between the objective functions W and $\hat{W}$ is the term $A_t^{\frac{-\eta}{1-\gamma}} l_t$, which serves to alter the discount rate, consistent with our previous discussion. Hence the problem is to choose an investment level K_{t+1}, a consumption path C_t, and a level of GHG abatement μ_t which maximizes the objective function, subject to the environmental and economic constraints.

After solving out for consumption using the budget constraint we rewrite the optimization problem recursively using Bellman's equation. First we rewrite the exogenous variables σ and l as functions of the integer-valued state variable τ_t, which indexes the position of the exogenous variables. Let $k_t = \frac{K_t}{l_t}$ denote the capital to labor ratio and $g_{l,t}$ denote the growth rate in adjusted labor, which we assume is a decreasing function of time. Let $\nu : \Re^5 \to \Re$ denote the present discounted sum of utility given that the planner makes optimal decisions at each point in time (ie the value function). Then for problem W (problem $\hat{W}$ is analagous):

$$\nu\left(S\right)=\max_{k',\mu}\left\{A\left(\tau\right)^{\frac{1-\eta}{1-\gamma}}U\left[\frac{\Omega\left(\mu\right)}{D\left(T\right)}k^{\gamma}+\left(1-\delta_{k}\right)k-\left(1+g_{l}\left(\tau\right)\right)k'\right]+\beta\nu\left(S'\right)\right\} \tag{2.19}$$

Here primes denote next period's value and S is the vector of state variables:

$$S_t = \left[\begin{array}{ccccc} k_t & M_t & T_t & O_t & \tau_t \end{array}\right]'$$

The maximization is subject to:

$$\begin{bmatrix} k' \\ M' \\ T' \\ O' \\ \tau' \end{bmatrix} \equiv S' = G\left(S, \mu, k'\right) \equiv \begin{bmatrix} k' \\ \beta_m \sigma\left(\tau\right)\left(1-\mu\right) k^{\gamma} l\left(\tau\right) + \left(1-\delta_m\right) M \\ \left(1-\delta_T\right) T + r_1 \log\left(1+\frac{M}{M_b}\right) + r_2 O \\ O' = O + r_3\left(T-O\right) \\ \tau + 1 \end{bmatrix} \tag{2.20}$$

Two first order conditions, the three environmental constraints, and the exogenous behavior of the growth in adjusted labor and energy efficiency characterize the time series behavior of the model. In the appendix, we establish that the value function is increasing in the economic states k and τ, and decreasing in the environmental states M, T, and O. Differentiating the right hand side of the Bellman's equation (2.19) with respect to μ and k' (assuming an interior solution) gives the first order necessary conditions for the optimal abatement and investment:

$$A\left(\tau\right)^{\frac{1-\eta}{1-\gamma}} U'\left(c\right) \frac{\partial c}{\partial \mu} = \beta \frac{\partial \nu\left(G\left(S, \mu, k'\right)\right)}{\partial M'} \frac{\partial M'}{\partial \mu} \tag{2.21}$$

$$\left(1+g_l\left(\tau\right)\right) A\left(\tau\right)^{\frac{1-\eta}{1-\gamma}} U'\left(c\right) = \beta \frac{\partial \nu\left(G\left(S, \mu, k'\right)\right)}{\partial k'} \tag{2.22}$$

The left hand side of (2.21) is the marginal cost of abatement. Abatement reduces income which could be used for consumption. The right hand side is the marginal benefit of abatement: increased abatement reduces GHG concentrations in the following period, which in turn reduces the damage from climate change (the partial of ν with respect to M' is the marginal loss of value from an incremental increase in M'). Similarly, the left hand side of equation (2.22) represents the marginal utility of consumption, while the right hand side represents the marginal utility of investment. These must be equal for an optimal investment decision.

We can gain some insight as to how abatement changes with the growth rate in adjusted labor by examining the first order condition for optimal abatement (2.21) at the steady state.

At the steady state, the right hand side of (2.21), the marginal benefit, is (the details of the derivation are in the appendix):

$$\bar{MB} = \beta\phi_3\beta_m\bar{\sigma}\frac{\partial T'\left(\bar{M}\right)}{\partial M}\frac{\partial D\left(\bar{T}\right)}{\partial T}\frac{\Omega\left(\bar{\mu}\right)}{D\left(\bar{T}\right)^2}\bar{A}^{\frac{1-\eta}{1-\gamma}}U'\left(\bar{c}\right)\bar{Q}^2 \tag{2.23}$$

Here ϕ_3 is a positive constant and $\bar{c}$ is the stationary consumption level per unit of adjusted labor, and $\bar{\mu}$ is the stationary value of abatement. Since the stationary marginal benefit of abatement is convex in the stationary gross output $\bar{Q}$, the stationary marginal benefit of abatement is convex in adjusted labor. Hence the incentives to raise stationary control rates become stronger when the stationary adjusted labor is higher. The stationary marginal cost of abatement (the left hand side of 2.23) is linear in Q. Hence the marginal cost of abatement also rises with adjusted labor. The stationary first order condition reduces to:

$$\frac{\partial\Omega\left(\bar{\mu}\right)}{\partial\mu} = \phi_3\beta_m\bar{\sigma}\frac{\partial T'}{\partial M\left(\bar{M}\right)}\frac{\partial D}{\partial T\left(\bar{T}\right)}\bar{Y} \tag{2.24}$$

Hence since marginal costs are increasing in μ, as the stationary level of adjusted labor rises, the stationary abatement level rises.

Equation (2.24) suggests that there is a direct relationship between output (Y) and abatement. In the optimal growth model, growth in output is determined by growth in adjusted labor (balanced growth). To see how growth in output in the optimal growth model relates to growth in output in our model, we examine the marginal value of investment in (2.22), which is $\frac{\partial\nu(S)}{\partial k}$. After evaluating (2.19) at the optimal abatement and investment decisions, we differentiate both sides with respect to k and get:

$$\frac{\partial\nu\left(S\right)}{\partial k} = A\left(\tau\right)^{\frac{1-\eta}{1-\gamma}}U'\left(c\right)\frac{\partial C}{\partial k} + \beta\frac{\partial\nu\left(G\left(S,\mu,k'\right)\right)}{\partial M'}\frac{\partial M'}{\partial k} \tag{2.25}$$

The marginal utility of investment consists of two terms. The first term is the marginal benefit of investment: investment increases future capital stocks which provides increased income and therefore consumption. But there is a second term which is not present in the standard optimal growth model. The second term is the marginal damage from an increase in the capital stock. A marginal increase in the capital stock causes higher emissions next period, which will result in a higher stock of GHGs. Hence there is an increase in temperature and a loss of utility through reduced output.

Consumption has no effect on emissions. Climate change reduces the marginal utility of investment, hence the optimal decision is to consume more and invest less relative to the decision without climate change. We will argue, however, that given current assumptions about the growth in population and productivity this term is small and so investment is largely unchanged from the optimal growth model.

3. Alternative Assumptions about Adjusted Labor Growth

3.1. Decreasing Growth in Adjusted Labor

Next we analyze the deviation from balanced growth term and the marginal cost of abatement under different assumptions about the growth of labor and productivity. We assume adjusted labor grows at a continuously decreasing rate.[3]

$$l_t = (1 + g_{l,t})\, l_{t-1} \tag{3.1}$$

$$g_{l,t} = (1 - \delta_l)\, g_{l,t-1} \tag{3.2}$$

For $0 < \delta_l < 1$ we have a decreasing growth rate of adjusted labor. In the next subsection, we consider the limiting case of $\delta_l = 0$, which is constant growth of population and productivity. We will also look at the other limiting case of no growth, or $\delta_l = 1$.

Given that $0 < \delta_l < 1$, the model has a globally stable stationary state. In the stationary world, all variables are constant. The economy converges to the stationary state from below; ie. capital, labor, productivity, emissions, temperature, and GHG concentrations all increase until the stationary values are attained. As capital and the exogenous variables increase, the stock of GHGs increases. Increases in the stock of GHGs cause increases in the atmospheric temperature. This causes damage, which decreases the benefit of investment in equation

[3]This specification fits the assumptions of most climate change modelers reasonably well. See Energy Modeling Forum (1995).

(2.25). Hence the stationary state is an upper bound for the effect of climate change on capital investment.

To get an idea about the maximum values for investment and abatement, we numerically calculate the stationary investment levels with and without climate change. Table (1) gives the parameter values, which are close to those commonly used in integrated assessment models (most are from Nordhaus, 1994). The baseline value of $\delta_l = 0.31$ is calibrated to match average values used by climate change researchers. In the appendix we give the solution procedure for the stationary values of the controls and states, using the first order conditions, constraints, and stationary conditions. After solving for the stationary state, we find that climate change decreases the stationary state capital level by only 7 percent.

However, this is not so if the growth rate of adjusted labor fails to slow down as significantly (that is if δ_l falls below 0.31). Table (2) highlights values of stationary variables as the growth rate in adjusted labor is varied. The first column of Table (2) gives the decline in the growth rate of adjusted labor. The second column represents the percentage change in stationary capital levels from the model without climate change. The third column is the stationary abatement level. The fourth column is the stationary atmospheric temperature. The final column is the stationary value of adjusted labor. If adjusted labor slows down by 16 percent per decade instead of 31 percent, we see increased environmental drag: the stationary state capital level falls by 36 percent.

In other words, if population and productivity growth more closely follow historical growth rates and fail to slow down significantly, then climate change has very large effects. An interesting case is the special case of no growth in adjusted labor. Here temperature rises less than 1 degree (most of this due to already present inertia in the climate). Hence we can conclude that climate change is a problem in a sense "caused" by adjusted labor growth.

Just as the stationary state is the upper bound for capital and the deviation from the no-climate change growth path, the stationary state also represents the maximum climate change and the maximum abatement levels. Without a significant slowdown in adjusted labor, we see in Table (2) that emissions, climate change, and abatement all rise significantly. If adjusted labor growth slows by 16 percent per decade instead of the baseline 31 percent, the stationary abatement nearly doubles from 14 percent to 26 percent. The total climate change nearly triples from 5.83 degrees to 16 degrees.

In other words, climate change is a much more serious problem. In effect integrated assessment models rely on a significant slowdown in population and productivity to solve the climate change problem. Since population slows before climate change becomes a serious problem, only modest abatement is required.

Figures (1)-(2) illustrate the stationary results under various assumptions about the decline in the growth rate of adjusted labor. Here we consider a range of values for the growth rate of adjusted labor between a decline of 16 percent per decade and 31 percent per decade (well above the historically observed *negative* values for δ_l). Included in the simulations is the baseline value of 31 percent. Figure (1) shows the stationary capital levels of the model with and without climate change as a function of the decline in the growth rate of δ_l. Under current assumptions, there are only modest differences (there is little environmental drag). The difference magnifies exponentially as the decline rate shrinks. Figure (1) also shows GHG concentrations are convex in δ_l. Figure (2) gives the optimal stationary abatement levels as a function of δ_l. Temperature is also convex in δ_l.

3.2. Constant population and productivity growth

A common assumption in the economics literature is constant population growth. Constant population growth can be motivated by the difficulty in predicting future population growth. If future population growth rates are unpredictable, then todays growth rate is the best estimate of future growth rates. Furthermore, a common specification for the behavior of productivity is to assume that the growth rate of productivity is stochastic around a constant rate (for example see Jones, 1995). If we drop the stochastic part of productivity, we can model adjusted labor as growing at a constant rate.

Suppose then, adjusted labor grows at a constant rate, that is $\delta_l = 0$. Keep in mind that population growth may cease, but if technological change continues, adjusted labor keeps growing. Recall that the budget constraint in per-capita terms is:

$$c_t = \frac{1 - b_1 \mu_t^{b_2}}{1 + \theta_1 T_t^{\theta_2}} k_t^{\gamma} + (1 - \delta_K) k_t - (1 + g_l) k_{t+1} \tag{3.3}$$

Further:

$$E_t = \sigma_t (1 - \mu_t) k_t^{\gamma} l_t \tag{3.4}$$

$$M_{t+1} = \beta_m E_t + (1 - \delta_M) M_t \tag{3.5}$$

With constant growth of population and productivity the emissions level has an interior stationary value whereas the control variable does not. To see this, note that if emissions grows unbounded, then so to does the atmospheric temperature. This leads to zero consumption, which cannot be a maximum since the planner can always choose $\mu = 1$ at finite cost. Hence emissions and temperature are bounded from above. Emissions and temperature are bounded below by zero. Hence we substitute out for the abatement level and use emissions as the control variable. The resource constraint is then:

$$c_t = \frac{1 - b_1 \left(1 - \frac{E_t}{\sigma_t k_t^{\gamma} l_t}\right)^{b_2}}{1 + \theta_1 T_t^{\theta_2}} k_t^{\gamma} + (1 - \delta_K) k_t - (1 + g_l) k_{t+1} \tag{3.6}$$

Emissions, the emissions intensity of output, and the capital to labor ratio are all stationary, hence as adjusted labor approaches infinity, the cost function must approach b_1 or $\mu = 1$. One hundred percent abatement in the long run is quite different than the standard policy result of modest abatement in the long run.[4]

4. Implications for Current Period Abatement

The economy and climate converge to the stationary investment and abatement levels far into the future. Due to long lags especially in the response of the ocean, the climate does not converge to a stationary equilibrium for a few hundred years. Similarly, if adjusted labor continues to grow, the economy takes longer to reach stationary levels. One important policy

[4] The reader may find it unlikely that improvements in productivity generate strong effects on uncontrolled emissions, because (anecdotally) improvements in productivity come largely in non-GHG intensive sectors (eg services). However the emissions intensity improves over time, so most (83%) productivity advances are emission free. Thus growth in population and productivity drive results despite exogenous changes in fuel use and changes in sectoral composition.

issue is the optimal abatement level right now.

Tables (3) and (4) show the current period and future control rates obtained after solving the Bellman equation numerically for several values of δ_l. Kelly and Kolstad (1998) gives the details of our numerical solution method. Current abatement is lower under per-capita utility versus population utility. Hence we consider both types of utility functions in this section.

Table (3) indicates that if we assume adjusted labor growth slows by 16 percent per decade as opposed to 31 percent, there is a small increase in 1995 abatement. By the year 2055, abatement levels given adjusted labor grows at 16 percent are already 18 percent higher than abatement levels given adjusted labor grows at 31 percent. Hence assumptions about future adjusted labor growth significantly affect even current abatement decisions. Investment levels also quickly rise as the value of δ_l decreases.

The result that changes in the slowdown of adjusted labor affects current abatement decisions is perhaps surprising. For example, Manne and Richels (1995) (Figure 16) find near term abatement decisions are not sensitive to a one percent higher potential GDP growth rate from the year 2030 onward. On the other hand, the Nordhaus (1994) model overall is most sensitive to the growth rate variables in population and productivity (Table 6.2). However, the 1995 control rate is more sensitive to a few other variables (such as the discount rate) than to the decline rate in population and productivity. Such results are in fact consistent with our results, after some careful analysis. Changes in the decline rate affect near term growth rates in emissions (as observed in the booming 1990s). For example, Manne and Richels (1995) consider changes in the growth rates after 2020.

5. Carbon and Baby Taxes

In this section, we examine the relationship between the carbon tax and the damage caused by population growth. The idea is to examine the relative effectiveness of carbon taxes versus policies which restrict population growth. Our model has population growth being exogenous, yet we can still compute the marginal damage caused by an additional person.

Consider the competitive version of the above model, in which there exists L_t firms, each of which rents capital and labor from households at rates r_t and w_t respectively, in order to produce the consumption good. Because there is constant returns to scale, profits are exactly zero. The firms have access to the pollution abatement technology (μ_t) which firms are motivated to use because of a carbon tax (x_t in trillions of dollars per gigaton CO_2). The period-by-period optimization problem of firm i is then:

$$\max_{\mu_{it},K_{it},L_{it}} \left\{ \Pi_t = \frac{Q_{it}}{D\left(T_t\right)} - r_t K_{it} - w_t L_{it} - x_t E_{it} - \left(\Omega\left(\mu_{it}\right) - 1\right) \frac{Q_{it}}{D\left(T_t\right)} \right\} \tag{5.1}$$

The first order conditions reduce to:

$$r_t = \gamma \frac{\Omega\left(\mu_{it}\right)}{D\left(T_t\right)} A_t K_{it}^{\gamma-1} L_{it}^{1-\gamma} - x_t \left(1 - \mu_t\right) \sigma_t A_t K_{it}^{\gamma-1} L_{it}^{1-\gamma} \tag{5.2}$$

$$w_t = \left(1 - \gamma\right) \frac{\Omega\left(\mu_{it}\right)}{D\left(T_t\right)} A_t K_{it}^{\gamma} L_{it}^{-\gamma} - x_t \left(1 - \mu_t\right) \sigma_t A_t K_{it}^{\gamma} L_{it}^{-\gamma} \tag{5.3}$$

$$-\frac{\Omega_\mu\left(\mu_{it}\right)}{D\left(T_t\right)} = \beta_m \sigma_t x_t \tag{5.4}$$

General equilibrium requires that supply equals demand in the capital and labor markets, and that firms are identical. Hence $K_{it} = \frac{K_t}{L_t}$, $L_{it} = 1$, and $\mu_{it} = \mu_t$. Substituting these conditions into the first order conditions gives:

$$r_t = \gamma \frac{\Omega\left(\mu_t\right)}{D\left(T_t\right)} k_t^{\gamma-1} - x_t \left(1 - \mu_t\right) \sigma_t k_t^{\gamma-1} \tag{5.5}$$

$$w_t = \left(1 - \gamma\right) \frac{\Omega\left(\mu_t\right)}{D\left(T_t\right)} A_t^{\frac{1-2\gamma}{1-\gamma}} k_t^{\gamma} - x_t \left(1 - \mu_t\right) \sigma_t A_t^{\frac{1-2\gamma}{1-\gamma}} k_t^{\gamma} \tag{5.6}$$

$$-\frac{\Omega_\mu\left(\mu_t\right)}{D\left(T_t\right)} = \beta_m \sigma_t x_t \tag{5.7}$$

So the carbon tax creates incentives to use abatement technology and reduces incentives for labor and capital usage. Finally, suppose the government creates a balanced budget by returning tax revenues as lump-sum transfers to consumers. The government budget constraint sets $x_t E_t = TR_t$. Consumers thus make consumption and investment decisions as before, but using income from capital and labor rentals and transfers.

By comparing the first order conditions of the consumer and firm with the first order conditions associated with the social planner's problem (2.21) and (2.22), we see that the following carbon tax makes the first order conditions associated with the social planner's problem identical to the first order conditions associated with the competitive problem.

$$x_t = \frac{\frac{\beta \partial \nu}{\partial M_{t+1}}}{U'(c_t)} l_t \tag{5.8}$$

Thus the above tax induces the optimal allocation. The optimal carbon tax is the marginal loss of utility per ton of carbon converted to dollars. Note that the optimal carbon tax increases directly with adjusted labor, again indicating the sensitivity of the model to adjusted labor. Additionally, equation (5.7) implies that the optimal tax equals the marginal cost per unit of emissions reduction:

$$x_t = \frac{\Omega_\mu(\mu_t)}{\beta_m \sigma_t D(T_t)} \tag{5.9}$$

Given the optimal carbon tax, we can calculate the marginal damage per additional person, or the optimal baby tax (which is a proxy for population control policies). Consider the Bellman's equation from the social planning problem, modified to include an additional parameter which is the number of people in the base year L_0:

$$\nu(S; L_0) = \max_{k',\mu} \left\{ A(\tau)^{\frac{1-\eta}{1-\gamma}} U[c] + \beta \nu(S'; L_0) \right\} \tag{5.10}$$

Consider a small increase in population, but suppose the marginal person enters the world with enough capital to leave the capital to labor ratio unchanged (this allows us to focus directly on the externality). The marginal damage is the derivative of the value function with respect to an increase in the initial population.

$$\frac{\partial \nu}{\partial L_0}(S; L_0) = \beta \frac{\partial \nu}{\partial M'}(S'; L_0) \frac{\partial M'}{\partial L_0} + \beta \frac{\partial \nu}{\partial L_0}(S'; L_0) \tag{5.11}$$

Here we evaluate the controls at the optimum. Equation (5.11) implicitly defines the marginal damage from an extra person, which results from increased emissions now and into the infinite future, as well as more more descendents who also increase emissions in the future. Starting from an initial year of $t = 1995$ we can recursively solve for the marginal damage:

$$\frac{\partial \nu}{\partial L_0}\left(S_t; L_0\right) = \sum_{i=t}^{\infty} \beta^{i-t+1} \frac{\partial \nu}{\partial M_{i+1}}\left(S_{i+1}; L_0\right) \frac{\partial M_{i+1}}{\partial L_0} \tag{5.12}$$

From equation (2.9):

$$\frac{\partial M_{i+1}}{\partial L_0} = \frac{\partial E_i}{\partial L_0} + \left(1 - \delta_M\right) \frac{\partial M_i}{\partial L_0} \tag{5.13}$$

Thus $\frac{\partial M_{i+1}}{\partial L_0}$ is also obtained recursively as:

$$\frac{\partial M_{i+1}}{\partial L_0} = \sum_{j=0}^{i} \left(1 - \delta_M\right)^{i-j} \frac{\partial E_j}{\partial L_0} \tag{5.14}$$

We convert the damage from utility units to dollars per adjusted labor units by dividing by marginal utility and then to dollar units by multiplying by adjusted labor.

$$\text{MD} = \sum_{i=t}^{\infty} \beta^{i-t+1} \frac{\frac{\partial \nu}{\partial M_{i+1}}\left(S_{i+1}; L_0\right) \frac{\partial M_{i+1}}{\partial L_0}}{U'\left(c_t\right)} l_t \tag{5.15}$$

Finally, substituting in equation (5.9) gives:

$$\text{MD} = \sum_{i=t}^{\infty} \beta^{i-t+1} \frac{\Omega_\mu\left(\mu_i\right)}{\beta_m \sigma_i D\left(T_i\right)} \frac{\partial M_{i+1}}{\partial L_0} l_t \tag{5.16}$$

Or:

$$\text{MD} = \sum_{i=t}^{\infty} \beta^{i-t+1} x_i \frac{\partial M_{i+1}}{\partial L_0} \tag{5.17}$$

Hence:

$$\text{MD} = \sum_{i=t}^{\infty} \sum_{j=0}^{i} \beta^{i-t+1} \left(1 - \delta_M\right)^{i-j} x_i \frac{\partial E_j}{\partial L_0} \tag{5.18}$$

Apparently, the marginal damage per additional person is the sum of the marginal additions to the stock of GHGs created by the additional person and all descendents in each year, multiplied by the cost of the emissions, the discounted tax in each year. An extra person creates emissions into the infinite future, while production creates emissions only in the current period. Thus the population tax must consider emissions in the future, while the carbon tax considers only current emissions. This is similar to the recent results of Harford (1997) and Harford (1998) on population and stock externalities. The population tax is thus quite large relative to the carbon tax, about $270 per person versus $6.87 per ton in the base case maximizing population utility. Of course, the optimal carbon tax is paid each decade, while the population tax is paid only once per lifetime. The present discounted value of eight carbon tax payments (starting in 1995) corresponds to $35.10, still much less than the population tax. Table (5) gives the optimal taxes for both CO_2 and for population growth.

The population tax reflects the fact that there are two ways to damage the global environment: directly emitting GHGs and indirectly emitting GHGs by generating more people. Giving birth to one more person creates emissions from that additional person as well as emissions from the progeny of that additional person. This suggests that an additional tool to use in managing the greenhouse problem is managing population growth. Indeed, a carbon tax alone is not sufficient to induce optimal emissions, given the second implicit externality of population growth. Population growth can be managed in a variety of ways, including birth control programs, promotion of social security programs to reduce the reliance on children in old age, and development assistance to increase incomes and thus, presumably, birth rates. Table (5) suggests that if an additional birth can be avoided at a cost of $200 to $800, then it is desirable to do so. One caveat is appropriate here. Our model assumes a homogeneous population and the baby tax in Table (5) reflects average world incomes. High birth rates occur in lower income countries where per capita GHG emissions are lower. Thus the efficient baby tax in lower income countries is undoubtably lower than the averages in Table (5).

6. Conclusions

We have shown that the predictions of climate change models are very sensitive to assumptions about the growth in population and productivity. Furthermore, integrated assessment models generally assume that population and productivity will grow at an increasingly slower rate and one that is consistently less than has been observed historically. There are two possible justifications for slow growth in population.

Forecasters of population growth such as the World Bank (1991) often predict population growth will slow due to an income effect. As developing countries grow and increase income, incentives for large families decline, as has happened in the developed world. This implies that a possible policy response to climate change is to take steps to help developing countries reduce population growth through development aid, rather than specifically limiting or taxing greenhouse gas emissions. This would have numerous other benefits, such as helping to control other environmental problems brought about in part by a large population. This is supported by the high marginal damage brought about by introducing one more person into the population. However, several problems might arise. First, population growth in some regions might be the result of other factors unrelated to income. Second, the cause of heterogeneity in income growth around the world is unclear. As noted in the introduction, North (1996) and others have found the post war record of progress in economic development to be mixed at best. In any case, population might continue to increase in spite of a policy to help developing countries increase income.

But the most problematic part of the idea that higher per-capita income will slow population growth from the perspective of the integrated assessment modeler is that integrated assessment models assume productivity (and therefore per capita income) also stops growing. In other words the world economy must experience enough technological change to slow the growth of population, but not so much growth that emissions rise substantially.

Perhaps some modelers have (implicitly) an alternative idea: that population growth will slow or stop because, in a Malthusian way, the strain on environmental resources will become too great to support population growth beyond a certain point. One of these environmental resources is the condition of the climate. Thus causality plausibly can run in two directions for the relationship between population growth and climate change. Population growth affects the climate, as noted in this paper, but increased climate change can also affect the size of the population.

Regardless of the reasoning for the assumption, there are two possible solutions. The first is that we can reduce the drain on resources per-capita with climate change policies. The second is to try to slow the total population growth so that the total drain on resources is not so great. Climate change models implicitly pursue both the strategy of abatement and the strategy of limiting population growth. The social planner endogenously mitigates long term climate change by traditional abatement, but climate change is also reduced because population growth ends exogenously. The latter effect is very strong: a population of 10 billion puts a strain on the climate resource, but nothing like a population of 20 billion. A fruitful and more realistic direction for further research would be to take this into account. If abatement reduces per-capita emissions, then perhaps conditions will result in a larger population; the net effect would be a larger asymptotic population and little long run effect on climate change.

That technological growth will stop is a more difficult assumption to justify. However, it is very difficult to pin down how much technological change is in GHG-intensive industries in the long run. Anecdotally improvements in productivity come largely in areas of the economy which are not intensive in GHGs (ie service industries). However, the strong productivity-driven growth in the 1990s has produced a large increase in GHG emissions, due to higher consumption of gas and electricity by wealthier consumers. Further, because climate change is a global problem and because much GHG emissions are generated from electricity and gas consumption, developed countries cannot easily export GHG emissions. Finally, integrated assessment models are calibrated such that a high percentage of growth of productivity results in no GHG emissions, which is consistent with current and historical data. Still, in the very long run scarcity of fossil fuels and extensive regulation may result in greater improvements in emissions intensity than is observed in the past. A fruitful direction for further research is to include an endogenous model of technical change.

7. Appendix: Proofs of Main Results.

7.1. Proofs of theorems which sign the derivatives of the value function.

To sign the derivatives, we first write the value function recursively as:

$$\nu_t(S) = \max_{k',\mu} \{U[S,\mu,k'] + \beta\nu_{t-1}(G(S,\mu,k'))\} \tag{7.19}$$

The standard technique of proof by induction can be used to sign the derivatives.

PROPOSITION 1 *Suppose ν_0 is a decreasing function of M, T, and O. Then the derivative of the value function $\nu(S)$ with respect to M, T, and O is negative.*

We need only show that:

$$\frac{\partial\nu_{t-1}(S)}{\partial S_i} < 0 \qquad i = 2,3,4 \tag{7.20}$$

implies that:

$$\frac{\partial\nu_t(S)}{\partial S_i} < 0 \qquad i = 2,3,4 \tag{7.21}$$

To find the derivative of the value function with respect to the stock of GHGs, we evaluate equation (7.19) at the optimum $\mu = \mu^*$ and $k' = k'^*$. Taking the derivative of the resulting equation with respect to M gives:

$$\frac{\partial\nu_t(S)}{\partial M} = \beta(1-\delta_M)\frac{\partial\nu_{t-1}(G(S,\mu^*,k'^*))}{\partial M'} + \beta\frac{\partial T'}{\partial M}\frac{\partial\nu_{t-1}(G(S,\mu^*,k'^*))}{\partial T'} \tag{7.22}$$

Since the derivative of temperature change with respect to the stock of GHGs is positive, the above equation implies that if the derivative of ν_{t-1} with respect to M and T are less than zero, then the derivative of ν_t with respect to M is negative. Hence using the assumption of the proposition we have by induction:

$$\frac{\partial\nu(S)}{\partial M} < 0 \tag{7.23}$$

For the derivative of the value function with respect to temperature, we again evaluate equation (7.19) at the optimal controls and take the derivative with respect to T. We have:

$$\frac{\partial\nu_t(S)}{\partial T} = A^{\frac{1-\eta}{1-\gamma}}\frac{\partial U(C^*)}{\partial C}\frac{\partial C}{\partial T} +$$

$$\beta(1-\delta_T)\frac{\partial\nu_{t-1}(G(S,\mu^*,k'^*))}{\partial T'} + \beta r_3\frac{\partial\nu_{t-1}(G(S,\mu^*,k'^*))}{\partial O'} \tag{7.24}$$

Here C^* is the consumption when the controls are evaluated at the optimum. Note that derivative of consumption with respect to T is negative, r_3 is positive, and $0 < \delta_T < 1$. Hence the above equation implies that if the derivative of ν_{t-1} with respect to T and O are negative, then the derivative of ν_t with respect to T is negative. Hence using the assumption of the proposition we have by induction:

$$\frac{\partial \nu (S)}{\partial T} < 0 \tag{7.25}$$

The derivative of the value function with respect to O is:

$$\frac{\partial \nu_t (S)}{\partial O} = \beta r_2 \frac{\partial \nu_{t-1} (G (S, \mu^*, k'^*))}{\partial T'} + \beta (1 - r_3) \frac{\partial \nu_{t-1} (G (S, \mu^*, k'^*))}{\partial O'} \tag{7.26}$$

Here r_2 is positive and $0 < r_3 < 1$. Hence the above equation implies that if the derivative of ν_{t-1} with respect to T and O are negative, then the derivative of ν_t with respect to O is negative. Hence using the assumption of the proposition we have by induction:

$$\frac{\partial \nu (S)}{\partial O} < 0 \tag{7.27}$$

which completes the proof. □

Now that we have established the sign of the derivatives with respect to the environmental states, we can establish the sign of the derivatives with respect to the economic states. The proof requires a slightly more subtle argument, since there are terms with opposite signs in the derivative. Essentially, an increase in k or τ causes an increase in production, but also an increase in pollution. Therefore, the strategy is to increase the control enough so that pollution is unchanged, and then show that after paying the cost there is some production left over which can be consumed. We omit the proof of the sign of the derivative with respect to τ, since it follows analogously to the proof with respect to k.

PROPOSITION 2 *Suppose $b_2 > 1$ and $0 < b_1 < 1$. Then the partial derivative of the value function with respect to k is positive.*

Proof. Suppose the contrary, then for some k there exists an $\eta > 0$ such that:

$$\nu (S (k + \eta)) < \nu (S (k)) \tag{7.28}$$

Since all states are constant except for k, for ease of notation we drop the S and just use $\nu (k)$. Now at the optimum, we have:

$$\nu (k) = A^{\frac{1-\eta}{1-\gamma}} U (k, k_1'^*, \mu_1^*) + \beta \nu (G (k, k_1'^*, \mu_1^*)) \tag{7.29}$$

$$\nu (k + \eta) = A^{\frac{1-\eta}{1-\gamma}} U (k + \eta, k_2'^*, \mu_2^*) + \beta \nu (G (k + \eta, k_2'^*, \mu_2^*)) \tag{7.30}$$

Here μ_1^* is the optimal abatement when the capital level is k and μ_2^* is the optimal abatement when the capital level is $k + \eta$ and the same for the investment decision. By the definition of the maximum, we have:

$$\nu (k + \eta) \geq A^{\frac{1-\eta}{1-\gamma}} U (k + \eta, k_1'^*, \mu_2^*) + \beta \nu (G (k + \eta, k_1'^*, \mu_2^*)) \tag{7.31}$$

We now choose a level of abatement $\hat{\mu}$ so that $G(k+\eta, k_1'^*, \hat{\mu}) = G(k, k_1'^*, \mu_1^*)$. We equate the next period stock of GHGs to achieve the equality:

$$\begin{aligned} M_1' &= \beta_M \sigma(\tau)(1-\mu_1^*) k^\gamma l(\tau)^{1-\gamma} + (1-\delta_M) M = M_2' = \\ & \beta_M \sigma(\tau)(1-\hat{\mu})(k+\eta)^\gamma l(\tau)^{1-\gamma} + (1-\delta_M) M \end{aligned} \tag{7.32}$$

$$\Rightarrow \hat{\mu} = 1 - (1-\mu_1^*)\left(\frac{k}{k+\eta}\right)^\gamma \tag{7.33}$$

Now by definition of the maximum we have:

$$\nu(k+\eta) \geq A^{\frac{1-\eta}{1-\gamma}} U(k+\eta, k_1'^*, \hat{\mu}) + \beta\nu(G(k+\eta, k_1'^*, \hat{\mu})) \tag{7.34}$$

$$= A^{\frac{1-\eta}{1-\gamma}} U(k+\eta, k_1'^*, \hat{\mu}) + \beta\nu(G(k, k_1'^*, \mu_1^*)) \tag{7.35}$$

$$= A^{\frac{1-\eta}{1-\gamma}} \left[U(k+\eta, k_1'^*, \hat{\mu}) - U(k, k_1'^*, \mu_1^*)\right] + \nu(k) \tag{7.36}$$

Hence, if we can show that the difference in utilities is non-negative, we have a contradiction of (7.28). Since utility is strictly increasing, we have:

$$U(k+\eta, k_1'^*, \hat{\mu}) \geq U(k, k_1'^*, \mu_1^*)$$

if and only if:

$$\begin{aligned} &\frac{\Omega(\hat{\mu})}{D(T)}(k+\eta)^\gamma l(\tau)^{1-\gamma} + (1-\delta_K)(k+\eta) - k_1'^* \geq \\ &\frac{\Omega(\mu_1^*)}{D(T)} k^\gamma l(\tau)^{1-\gamma} + (1-\delta_K) k - k_1'^* \end{aligned} \tag{7.37}$$

Hence it is sufficient to show:

$$\Omega(\hat{\mu})(k+\eta)^\gamma \geq \Omega(\mu_1^*) k^\gamma \tag{7.38}$$

Plugging in for the cost function and $\hat{\mu}$ and rearranging gives:

$$1 \geq b_1\left(1-(1-\mu_1^*)\left(\frac{k}{k+\eta}\right)^\gamma\right)^{b_2} + \left(1-b_1\mu_1^{*b_2}\right)\left(\frac{k}{k+\eta}\right)^\gamma \tag{7.39}$$

Now it is easy to show that given the assumptions of the proposition, the derivative of the right hand side of the above equation with respect to μ is strictly positive. Hence the maximum value of the right hand side occurs at $\mu = 1$. Substituting in for $\mu = 1$ gives:

$$1 \geq b_1 + (1-b_1)\left(\frac{k}{k+\eta}\right)^\gamma \tag{7.40}$$

According to the assumptions of the proposition, the above equation holds since the ratio of capital stocks is less than one. Hence the difference in utilities is positive, which gives the contradiction. □

7.2. Derivation of stationary values.

We next derive the derivatives of the value function at the stationary point $S = S' = \bar{S}$. Clearly $\bar{\tau} = \infty$, with l and σ taking on their limiting values. Let ν_i denote the partial derivative of the value function with respect to state i, evaluated at the stationary point $\bar{S}$. The first order conditions at the stationary value of the states and controls is:

$$\bar{A}^{\frac{1-\eta}{1-\gamma}} U'\left(\bar{C}\right) \frac{\partial C}{\partial \mu} + \beta \frac{\partial M'}{\partial \mu} \nu_M = 0 \tag{7.41}$$

$$-\bar{A}^{\frac{1-\eta}{1-\gamma}} U'\left(\bar{C}\right) + \beta \nu_K = 0 \tag{7.42}$$

Here the last equation follows because the growth rate of adjusted labor is zero at the steady state.

The derivatives of the value function (at the optimal controls) with respect to the states evaluated at the stationary state is:

$$\nu_K = \bar{A}^{\frac{1-\eta}{1-\gamma}} U'\left(\bar{C}\right) \frac{\partial C}{\partial k} + \beta \frac{\partial M'}{\partial k} \nu_M \tag{7.43}$$

$$\nu_M = \beta \left(1 - \delta_M\right) \nu_M + \beta \frac{\partial T'}{\partial M} \nu_T \tag{7.44}$$

$$\nu_T = \bar{A}^{\frac{1-\eta}{1-\gamma}} U'\left(\bar{C}\right) \frac{\partial C}{\partial T} + \beta \left(1 - \delta_T\right) \nu_T + \beta r_3 \nu_O \tag{7.45}$$

$$\nu_O = \beta r_2 \nu_T + \beta \left(1 - r_3\right) \nu_O \tag{7.46}$$

Equations (7.43)-(7.46) form a linear system of four equations and four unknowns. The solution is then:

$$\nu_k = \bar{A}^{\frac{1-\eta}{1-\gamma}} U'\left(\bar{C}\right) \frac{\partial C}{\partial k} + \beta \phi_3 \bar{A}^{\frac{1-\eta}{1-\gamma}} U'\left(\bar{C}\right) \frac{\partial C}{\partial T} \frac{\partial M'}{\partial k} \frac{\partial T'}{\partial M} \tag{7.47}$$

$$\nu_M = \phi_3 \bar{A}^{\frac{1-\eta}{1-\gamma}} U'\left(\bar{C}\right) \frac{\partial T'}{\partial M} \frac{\partial C}{\partial T} \tag{7.48}$$

$$\nu_T = \phi_2 \bar{A}^{\frac{1-\eta}{1-\gamma}} U'\left(\bar{C}\right) \frac{\partial C}{\partial T} \tag{7.49}$$

$$\nu_O = \phi_1 \phi_2 \bar{A}^{\frac{1-\eta}{1-\gamma}} U'\left(\bar{C}\right) \frac{\partial C}{\partial T} \tag{7.50}$$

Where:

$$\phi_1 = \frac{\beta r_2}{1 - \beta\left(1 - r_3\right)} \tag{7.51}$$

$$\phi_2 = \frac{1}{1 - \beta\left(1 - \delta_T\right) - \beta r_3 \phi_1} \tag{7.52}$$

$$\phi_3 = \frac{\beta \phi_2}{1 - \beta\left(1 - \delta_M\right)} \tag{7.53}$$

Substituting the derivatives of the value function with respect to the states into the first order conditions gives:

$$\frac{\partial C}{\partial \mu} + \beta \phi_3 \frac{\partial M'}{\partial \mu} \frac{\partial T'}{\partial M} \frac{\partial C}{\partial T} = 0 \tag{7.54}$$

$$-1 + \beta \frac{\partial C}{\partial k} + \beta^2 \phi_3 \frac{\partial C}{\partial T} \frac{\partial M'}{\partial k} \frac{\partial T'}{\partial M} = 0 \tag{7.55}$$

To complete the model, we use the equations that govern the change in GHG levels, atmospheric temperature, and ocean temperature, evaluated at the stationary points:

$$\bar{M} = \frac{\beta_M \bar{\sigma}}{\delta_M}\left(1 - \bar{\mu}\right) \bar{k}^{\gamma} \bar{l}^{1-\gamma} \tag{7.56}$$

$$\bar{T} = \frac{r_1}{\delta_T - r_2} \log\left(1 + \frac{\bar{M}}{M_b}\right) \tag{7.57}$$

$$\bar{O} = \bar{T} \tag{7.58}$$

After substituting out for the ocean temperature, the first order conditions (7.54) and (7.55) and the constraints (7.56) and (7.57) form a non-linear system of 4 equations with unknowns $\bar{k}$, $\bar{M}$, $\bar{T}$, and $\bar{\mu}$. The numerical solution to the above system is analyzed in section 3.1.

Numerical Results and parameter values

Parameter	Value
η	1 (Logarithmic)
γ	0.25
b_1	0.0686
b_2	2.887
θ_1	0.01487
θ_2	2
β_m	6.4
δ_σ	0.11
δ_M	0.083
δ_K	0.6513
δ_T	0.4181
r_1	1.3368
r_2	0.0944
r_3	0.02
M_b	590
β	0.7441
$g_{l,1985}$	0.3653
l_{1985}	5.8626

Table 1: Parameter Values. Note that the decline rate δ_l is varied as discussed in the text.

δ_l	% Δ in $\bar{k}$	$\bar{\mu}$	$\bar{T}$	$\bar{l}$
1.00	-0.17	0.03	0.8	28.7
0.31	-6.88	0.14	5.83	100.42
0.28	-9.20	0.16	6.84	140.67
0.25	-12.69	0.18	8.20	215.70
0.22	-17.62	0.21	9.95	361.79
0.19	-25.35	0.23	12.52	742.34
0.16	-36.00	0.26	16.02	1895.37

Table 2: Stationary values and δ_l.

	W (per capita utility)				$\hat{W}$ (population utility)			
δ_l	1995	2025	2055	2095	1995	2025	2055	2095
1.000	0.0250	0.0264	0.0288	0.0295	0.0250	0.0264	0.0288	0.0295
0.310	0.0596	0.0801	0.0984	0.1138	0.0880	0.0974	0.1065	0.1154
0.250	0.0605	0.0820	0.1036	0.1256	0.0928	0.1074	0.1212	0.1361
0.225	0.0605	0.0809	0.1021	0.1242	0.0961	0.1104	0.1253	0.1448
0.190	0.0630	0.0839	0.1063	0.1331	0.1082	0.1238	0.1392	0.1609
0.160	0.0633	0.0874	0.1160	0.1548	0.1123	0.1281	0.1449	0.1716

Table 3: 1995-2095 GHG abatement (μ).

	W (per capita utility)				$\hat{W}$ (population utility)			
δ_l	1995	2025	2055	2095	1995	2025	2055	2095
1.000	19.43	19.90	19.92	19.92	19.43	19.90	19.92	19.92
0.310	32.31	74.20	128.30	200.13	41.86	83.39	132.55	193.36
0.250	32.66	75.11	140.55	243.92	48.93	107.96	189.48	132.55
0.225	32.83	75.54	141.22	244.18	50.98	115.72	211.63	375.31
0.190	34.70	81.30	158.80	299.07	57.95	127.95	233.40	437.38
0.160	35.01	85.53	189.78	429.72	58.10	129.51	249.08	522.24

Table 4: 1995-2095 investment levels. In trillions of dollars.

utility	δ_l	Baby tax ($ per person)	Carbon Tax ($ per ton)	1995 Emissions per person (Tons)
Per-Capita	0.31	$226.00	$3.30	0.7056
Per-Capita	0.16	$541.23	$3.70	0.6909
Population	0.31	$270.14	$6.87	0.7115
Population	0.16	$770.71	$10.90	0.7303

Table 5: Efficient tax rates on population and CO_2 and net emissions per person, 1995.

Graphs of Model Simulations and Historical and projected population

Figure 1: Optimal stationary capital levels and GHG concentrations as a function of δ_l.

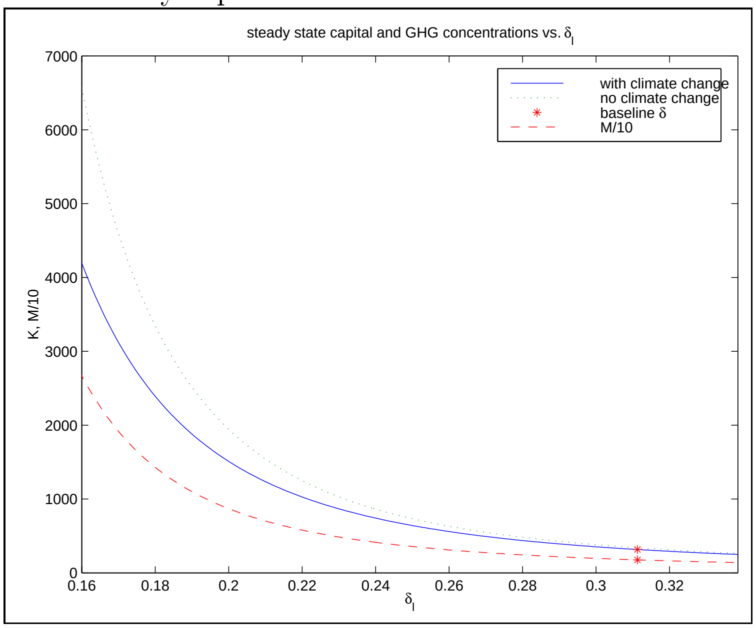


Figure 2: Optimal stationary state abatement and temperature as a function of δ_l.

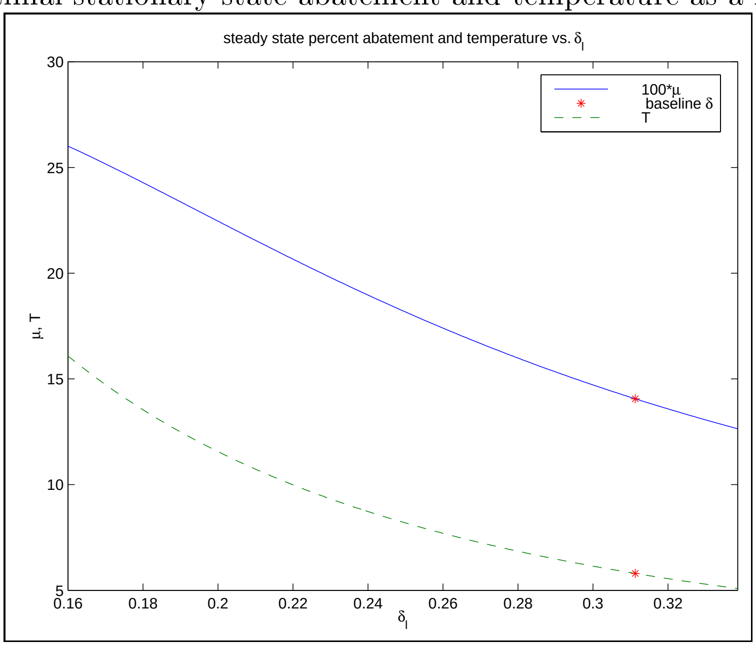
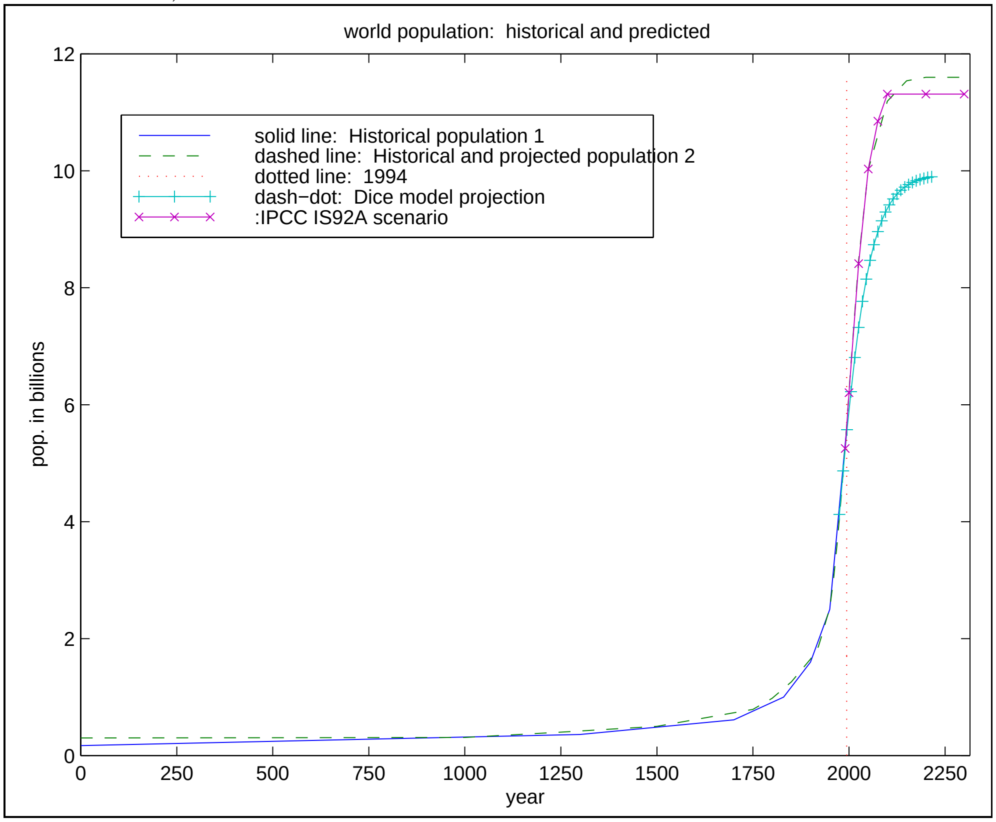


Figure 3: Historical and predicted population growth. Shown is the data set from the Smithsonian Institute, the United Nations data is similar.



Reference

Barro, R. J. and Sala-i-Martin, X. (1995). *Economic Growth.* McGraw-Hill, Inc., New York.

Energy Modeling Forum (1995). Second Round Study Design For EMF 14: Integrated Assessment of Climate Change. Pages 3-7. Stanford University.

Harford, J. D. (1997). Stock Pollution, Child-Bearing Externalities, and the Social Discount Rate. *Journal of Environmental Economics and Management*, *33*, 94–105.

Harford, J. D. (1998). The Ultimate Externality. *American Economic Review*, *88*, 260–265.

Houghton, J., Jenkins, G., and Ephraums, J. (Eds.). (1990). *Climate Change: The IPCC Scientific Assessment.* Cambridge University Press.

Houghton, J., Jenkins, G., and Ephraums, J. (Eds.). (1992). *Climate Change: The IPCC Scientific Assessment.* Cambridge University Press.

Jones, C. (1995). Time Series Tests of Endogenous Growth Models. *Quarterly Journal of Economics*, *110*, 495–525.

Kelly, D. L. and Kolstad, C. D. (1998). Bayesian Learning, Pollution, and Growth. *Journal of Economic Dynamics and Control*, *23*, 491–518.

Kolstad, C. (1996). Learning and Stock Effects in Environmental Regulation: The Case of Greenhouse Gas Emissions. *Journal of Environmental Economics and Management*, *31*, 1–18.

Lempert, R. J., Schlesinger, M. E., and Banks, S. C. (1996). When We Don't Know the Costs or Benefits: Adaptive Strategies for Abating Climate Change. *Climatic Change*, *33*, 235–274.

Manne, A., Mendelsohn, R., and Richels, R. (1993). MERGE: A Model for Evaluation Regional and Global Effects of GHG Reduction Policies. *Energy Policy*, *23*(1), 17–34.

Manne, A. and Richels, R. (1995). The Greenhouse Debate: Economic Efficiency, Burden Sharing, and Hedging Strategies. *Energy Journal*, *16*(4), 1–37.

Mauro, P. (1995). Corruption and Growth. *Quarterly Journal of Economics*, *110*, 681–712.

Nordhaus, W. and Yohe, G. (1983). Future Carbon Dioxide Emissions From Fossil Fuels. In *Changing Climate.* National Academy Press, Washington, DC.

Nordhaus, W. D. (1994). *Managing the Global Commons: The Economics of Climate Change*. MIT Press, Cambridge, MA.

North, D. (1990). *Institutions, Institutional Change, and Economic Performance*. Cambridge University Press, Cambridge, MA.

North, D. (1996). Some Fundamental Puzzles in Economic History/Development. Mimeo, Washington University, St. Louis.

Peck, S. and Teisberg, T. (1992). CETA: A Model for Carbon Emissions Trajectory Assessment. *Energy Journal*, *13*, 55–77.

Summers, R. and Heston, A. (1991). The Penn World Tables (Mark 5): An Expanded Set of International Comparisons, 1950-88. *Quarterly Journal of Economics*, *106*, 327–68.

United Nations (1994). *World Population Prospects: The 1994 Revision*. United Nations, New York.

Weitzman, M. L. (1994). On the "Environmental" Discount Rate. *Journal of Environmental Economics and Management*, *26*, 200–209.

World Bank (1991). *World Development Report*. Oxford University Press, New York.