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The Problem of Social Control and Coordination of Complex Systems in Sociology

A LOOK AT THE COMMUNITY CLEAVAGE PROBLEM

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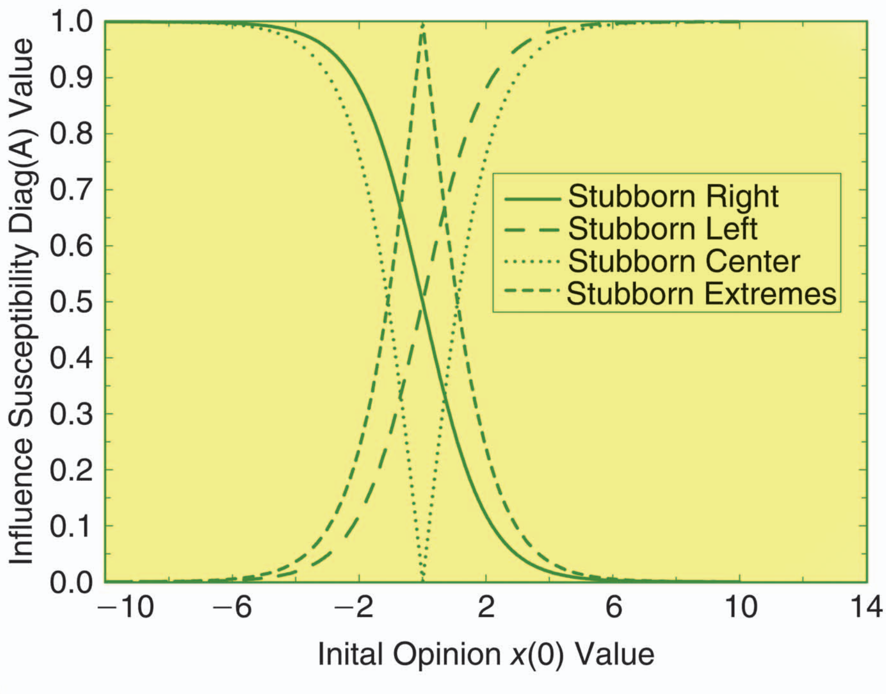

The coordination and control of social systems is the foundational problem of sociology. The discipline was established in Europe in the aftermath of the American and French Revolutions. With the dismantling of the hierarchical controls of European aristocratic systems, the examination of alternative mechanisms of coordination and control became a preoccupation. The Industrial Revolution (c. 1760–1840), which was occurring at the same time, reinforced this preoccupation by decoupling the power of purse and the power of

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positions of authority. While the hierarchies of church and state retained coercive power, the exercise of such power was increasingly contested. The moral compass of society, its general welfare, and its capacity to adapt to changing circumstances appeared to be aggregated properties of the unconstrained opinions and behaviors of a large collectivity of individuals. The definition of the problem of coordination and control, which emerged in Europe in the new discipline of sociology, still resonates and guides current sociological work. The problem definition [1], broadly stated, is this: if value is placed on nonhierarchical mechanisms of social control, then what mechanisms and structures (consistent with this value) allow a coordination and control of social systems?

Emile Durkheim's 1893 publication, The *Division of Labor in Society*, has been the most influential general paradigm for subsequent work on the problem [2]. His argument is as follows. There cannot now exist a society of individuals with a "common conscience" on the basis of which individuals' behaviors are regulated by their compliance to a shared set of standards. The coordination of a society without a "common conscience" must depend on decentralized mechanisms of social control and social institutions that together somehow eventuate in what was referred to as an "organic solidarity" that serves to promote the general welfare. Durkheim was particularly attracted to the idea that a revitalization of the medieval guilds might serve as a decentralized structural basis of coordination and control. Interestingly, he took the medieval guild institutions of universities as an example of social institutions that serve to promote the general welfare of the scientific community. By bringing scientists from different fields together, the larger scientific community is coordinated by the interactions of scientists in different fields within each university and the interactions of scientists in the same field located in different universities. Without universities, the scientific community would be substantially more fragmented into noncommunicating classes based on their particular interests.

The key point is that the classic problem of social control and coordination in sociology is defined in terms of a self-regulating system of individuals and their voluntary adjustments of behaviors. It seeks solutions in terms of the assemblage of institutions that are established or modified in response to enduring social conflicts and that may serve to resolve them. Furthermore, especially in the sociology that has developed in the United States, the effects of these social control institutions depend on their alterations of the typology of the interpersonal networks that join individuals and the extents to which individuals are susceptible to informal control. Emphasis is placed on endogenous mechanisms of interpersonal influence that unfold in connected structures, in which individuals' opinions and behaviors affect the opinions and behaviors of other individuals. These mechanisms, the network structures in which these mechanisms unfold, and the social institutions that broadly shape these structures are all viewed as theoretically central to an attack on the problem of social control and coordination. Efficiency is not emphasized in this literature. Instead, network structures with path redundancies that are robust to disturbances (edge failures and node deaths) are more theoretically important than the short paths that define a small world. Acceptable solutions to social problems are ones that are based on the uncoerced consent of individuals.

The sociological work on these problems is largely qualitative and presents an opportunity for substantial advancements in mathematical modeling. Strands of research in sociology that have a mathematical basis and bear on these problems include work on the social networks involved in collective action, social movements, and opinion-behavior dynamics. However, these strands of research are unfortunately far less developed than they might be if a larger community of mathematically oriented investigators was involved. This article introduces one instance of a well-defined problem that falls squarely into the general problem.

COMMUNITY CLEAVAGE PROBLEM

Pronounced community cleavages of opinion on issues is central to the problem of social control and coordination. The sociological perspective on enduring cleavages is that they are based on institutionalized conditions that constrain the capacity of interpersonal influence networks to generate points of agreement on some of the dimensions of the conflict. When the differences cannot be reconciled, the sociological perspective turns to a second-order problem regarding the capacity of interpersonal influence networks to secure an agreement on the legitimacy of rules that the collectivity will employ to decide on courses of action. Since no social-choice rule is objectively fair, the subjective grounds of perceived legitimacy are crucial. What is perceived as fair and unfair is subject to interpersonal influences. In other words, we can agree to disagree and agree on procedures to settle on a course of action. But if we cannot agree on such procedures, and if their implementation is vigorously contested, then we are in a deep hole.

The analysis of community cleavages begins with understanding the structural conditions under which a process of endogenous interpersonal influences on opinions, unfolding in a connected influence network, fails to generate consensus. A thorough understanding of these conditions is a necessary precursor to rational calculations on suitable and feasible adaptations of institutions that alter influence network typology and opinion distributions. Understanding the conditions under which interpersonal influence mechanisms *fail* to generate consensus turns out to be a hard problem, especially in the class of aperiodic irreducible influence networks in which these mechanisms may unfold.

Abelson's [3] investigation of various models of opinion change showed that a formal explanation of emergent consensus on specific issues is easily obtained but that a formal explanation of emergent differentiated opinion clusters is difficult: "Since universal ultimate agreement is an ubiquitous outcome of a very broad class of mathematical models, we are naturally led to inquire what on earth one must assume in order to generate the bimodal outcomes of community cleavage studies" [3, p. 153]. The response to the problem was a nonlinear simulation model in which the influence network structure is modified during the influence process as a function of individuals' evolving opinion differences. With the recent influx of investigators into the field of social networks from the natural and engineering sciences, this community cleavage problem has attracted attention as an interesting problem that might be successfully addressed. The new approaches that have been proposed are similar to those in [3]. For example, a fruitful and sustained line of inquiry has developed in a class of bounded-confidence models [4]–[6], but in contrast to the prior work [3], these recent models also provide analytical results.

A significant common property of all approaches to the problem is a relaxation of the assumption of a time-invariant influence structure and the adoption of some assumption that links structural changes and opinion differences. Because opinion differences are being modified by an interpersonal influence mechanism, an analysis of such systems is nontrivial. A basic question is whether a simpler linear time-invariant (LTI) state-space model might suffice to address Abelson's problem. This article demonstrates that a nonlinear model is not required and a linear model is adequate to address the problem. A static structural basis may exist for community cleavages and, by implication, a different static basis may ameliorate them. Thus, the analysis is simplified. Furthermore, analysis is enriched in the case of discrete-time LTI models, which may draw on the highly developed body of general theorems on graphs and matrices. In many of the proposed nonlinear models, persistent disagreement is based on a threshold cessation of influence between agents with sufficiently different opinions; in the linear model of this article, cessation is based on agents with memories who are taking into account their initial opinions.

The remainder of the article is organized into three sections. A model of opinion dynamics with stubborn agents is described that includes the seminal DeGroot model [7] as a special case. This model is then applied to the investigation of conditions under which different forms of opinion polarization are generated and ameliorated. The analysis of this section concentrates on the bimodal form of polarization. Finally, the general problem of social control and coordination, described previously, is revisited with a set of specific problems that control theorists may be especially well positioned to address.

INTERPERSONAL INFLUENCES WITH STUBBORN AGENTS

The discrete-time convex combination mechanism of opinion dynamics described in this section is a generalization of the French–Harary–DeGroot [8], [9], [7] line of work on such mechanisms. DeGroot's model, which includes the models in [8] and [9] as special cases, is

$$\mathbf{x}(k+1) = \mathbf{W}\mathbf{x}(k), \qquad k = 0, 1, \ldots, \tag{1}$$

where $\mathbf{x}(0) \in \mathbb{R}^{n \times 1}$ is the initial opinions of n individuals on an issue, and $\mathbf{W}$ is a row-stochastic matrix. This specification is consistent with the mass of observations in experimental social psychology obtained on small groups of individuals, which indicate that the postdiscussion opinions of group members are usually constrained to the group's range of prediscussion opinions defined by the maximum and minimum values of their prediscussion opinions [10]. The opinions may be positive or negative. In social psychology, signed opinions are referred to as attitudes and are generally defined as a positive or negative cognitive orientation of some intensity toward a particular object (such as an issue, event, person, or behavior). Equation (1) is seminal in work on opinion dynamics and also of interest in control theory [11] and economics [12]. A prediction of the model is that if the $\lim_{k \to \infty} \mathbf{W}^k$ exists and if $\mathbf{x}(0)$ is a vector of all different initial opinions, then the opinions of an (i, j) unordered pair of individuals will converge to an exact interpersonal agreement, $x_i(\infty) = x_j(\infty)$, when i and j directly or indirectly influence each other's opinions via the paths of the influence network corresponding to $\mathbf{W}$, and this agreement will occur regardless of i and j's levels of resistance to interpersonal influence, $0 \leq \{w_{ii}, w_{jj}\} < 1$, and the lengths of the shortest path(s) of influence from i to j and vice versa. If the limit exists and $\mathbf{W}$ is irreducible, that is, a group in which all n individuals mutually influence one another directly or indirectly, then all of its n individuals will converge to an exact consensus $x_1(\infty) = x_2(\infty) = \cdots = x_n(\infty)$. Because sociologists are attentive to the investigation of social conflicts, this property of the model is viewed as an important theoretical limitation [13]. For example, [14, p. 182] argued that "any serious theory of agreements and decisions must at the same time be a theory of disagreements and the conditions under which [agreements and] decisions cannot be reached." Sociologists are attentive to Durkheim's postulate that the existence of enduring interpersonal disagreements is a characteristic feature of social systems. The existence of equilibrium disagreements is restricted to reducible influence systems in (1).

The generalization of (1) that allows stubborn agents [15]–[17] is situated in a growing literature on stubborn agents in opinion dynamics. Some of these investigations are based on the generalization [18]–[20], [10], and others have proposed alternative models [4], [21]–[24]. The model (1) has been a sustained focus of attention in the social science on opinion dynamics since 1956 to the present, it is currently an increasingly prominent model in the control theory community, and its generalization has been intensively assessed with empirical findings [10], [25]. It has not been demonstrated that the dynamics of the generalization are consistent with a convergence to a bimodal opinion distribution in aperiodic irreducible time-invariant influence systems.

The generalization of (1) described in this section includes (1) as a special case and allows enduring opinion disagreements in the same $\{\mathbf{W}, \mathbf{x}(0)\}$ conditions that (1) predicts must generate agreements. DeGroot noted that the vector $\mathbf{x}(0)$ may be generalized to a $\mathbf{X}(0) \in \mathbb{R}^{n \times m}$ matrix, and in sociology this is sometimes useful. Initial opinions $\mathbf{X}(0)$ may consist of m different issues that are simultaneously being influenced and result in discrete combinations of opinions and associated behaviors; sociologists refer to such discrete combinations as social positions. Or it may be a single issue on which individuals' positions are defined by m coordinates. For example, it may be a row-stochastic matrix of weights accorded by individuals to m mutually exclusive actions; if these relative preferences are influenced, then all the $\mathbf{X}(k)$ will be row-stochastic in the iterates of the influence process.

Let $\mathbf{X}(0) \in \mathbb{R}^{n \times m}$ and $\mathbf{V}(k) \in \mathbb{R}^{n \times n}$. Then each row $i = 1, \ldots, n$ of $\mathbf{X}(k) = \mathbf{V}(k)\mathbf{X}(0)$ are the m coordinates of the opinion of individual i in the convex hull of $\mathbf{X}(0)$ for all $\mathbf{V}(k)$ in the domain of nonnegative matrices with rows sums constrained to one. Row-stochastic $\mathbf{V}(k)$ are generated by the generalization [16], [10] of (1)

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{AWX}(k) + (\mathbf{I}-\mathbf{A})\mathbf{X}(0), \\ &= \mathbf{V}(k)\mathbf{X}(0), \end{aligned} \tag{2}$$

$k = 0, 1, 2, \ldots$, where $\mathbf{W}$ is a nonnegative matrix with row sums constrained to one, $\mathbf{A}$ is a diagonal matrix constrained to $\mathbf{0} \le \mathbf{A} \le \mathbf{I}$, that is, $0 \le a_{ii} \le 1$ for all i, and

$$\mathbf{V}(k) = (\mathbf{AW})^k + \left[\sum_{i=0}^{k-1} (\mathbf{AW})^i\right](\mathbf{I}-\mathbf{A}), \qquad k > 0, \tag{3}$$

corresponds to an evolving matrix polynomial of walks in a network structure. All $\mathbf{V}(k)$ are row stochastic. The iterates of $\mathbf{V}(k)$ may be expressed as $\mathbf{V}(k) = \mathbf{AWV}(k-1) + (\mathbf{I}-\mathbf{A})$, $\mathbf{V}(0) = \mathbf{I}$, $k = 1, 2, \ldots$. The first iterate $\mathbf{V}(1) = \mathbf{AW} + \mathbf{I} - \mathbf{A}$ is row stochastic. All $\mathbf{WV}(k-1)$ are row stochastic, and all $\mathbf{V}(k) = \mathbf{AWV}(k-1) + (\mathbf{I}-\mathbf{A})$, $k > 1$, are row stochastic.

Conditions of Convergence

Let $\mathcal{G}$ be the directed graph of $\mathbf{AW}$ defined on edges $i \xrightarrow{a_{ii}w_{ij} > 0} j$ that correspond to the subset of positive elements of $\mathbf{AW}$, and let $\mathcal{G}$ contain at least one $i \xrightarrow{a_{ii}w_{ij} > 0} j, i \ne j$ edge, so that $\mathbf{A} \ne \mathbf{0}$ and $\mathbf{W} \ne \mathbf{I}$. Let $\mathbf{AW}$ be an aperiodic matrix. It is aperiodic if in $\mathcal{G}$ there is no integer greater than one that divides the length of every cycle of the graph. The matrix $\mathbf{AW}$ is either stochastic ($a_{ii} = 1$ for all i), strictly substochastic ($a_{ii} < 1$ for all i), or not-strictly substochastic ($a_{ii} < 1$ for at least one but not all i). Let $\rho(\mathbf{AW})$ denote spectral radius of $\mathbf{AW}$.

With the above definitions, which include restrictions on the typology of $\mathcal{G}$, the sequence $\{\mathbf{V}(k); k = 0, 1, \ldots\}$ of (3) converges if and only if the $\lim_{k\to\infty} (\mathbf{AW})^k$ exists. The following standard convergence conditions apply [26]:

- If $\mathbf{AW}$ is stochastic, then $\rho(\mathbf{AW}) = 1$, the sequence $\{(\mathbf{AW})^k; k = 0, 1, \ldots\}$ converges, and $\{\mathbf{V}(k); k = 0, 1, \ldots\}$ converges if and only if the eigenvalues of the spectrum of $\mathbf{AW}$ with $|\lambda| = \rho(\mathbf{AW}) = 1$ are all $\lambda = 1$.
- If $\mathbf{AW}$ is a strictly substochastic, then $\rho(\mathbf{AW}) < 1$, the sequence $\{(\mathbf{AW})^k; k = 0, 1, \ldots\}$ converges to $\lim_{k\to\infty} (\mathbf{AW})^k = \mathbf{0}$, and $\{\mathbf{V}(k); k = 0, 1, \ldots\}$ converges to $\mathbf{V} = (\mathbf{I}-\mathbf{AW})^{-1}(\mathbf{I}-\mathbf{A}) = \left[\sum_{k=0}^{\infty} (\mathbf{AW})^k\right](\mathbf{I}-\mathbf{A})$.
- If $\mathbf{AW}$ is not strictly substochastic, then $\rho(\mathbf{AW}) \le 1$, the sequence $\{(\mathbf{AW})^k; k = 0, 1, \ldots\}$ converges, and $\{\mathbf{V}(k); k = 0, 1, \ldots\}$ converges a) if $\lambda = 1$ is not included in the spectrum or b) if the eigenvalues of the spectrum of $\mathbf{AW}$ with $|\lambda| = \rho(\mathbf{AW}) = 1$ are all $\lambda = 1$.

In general, for any $\mathbf{W}$, $n \ge 2$ row-stochastic typology, if $0 < a_{ii} < 1$ for all i, then $\{\mathbf{V}(k); k = 0, 1, \ldots\}$ converges to $\mathbf{V} = (\mathbf{I}-\mathbf{AW})^{-1}(\mathbf{I}-\mathbf{A})$.

The above results are insensitive to an affine transformation of initial opinions,

$$\alpha + \beta\mathbf{X}(\infty) = \mathbf{V}[\alpha + \beta\mathbf{X}(0)]. \tag{4}$$

The scalars $\{\alpha, \beta\}$ pass through the system without altering $\mathbf{V}$, which is strictly a function of $\mathbf{AW}$.

Conditions of Consensus

The model simplifies to (1) in the special case of $\mathbf{A} = \mathbf{I}$ [7] and, more generally, in the special case of a binary $\mathbf{A}$ in which all a_{ii} values are either zero or one. But it allows emergent patterns of interpersonal disagreement in aperiodic irreducible influence networks. A simple example is

$$\mathbf{AW} = \begin{bmatrix} 0.40 & 0 \\ 0 & 0.60 \end{bmatrix} \begin{bmatrix} 0.25 & 0.75 \\ 0.50 & 0.50 \end{bmatrix},$$

for which $\lim_{k\to\infty} (\mathbf{AW})^k = \mathbf{0}$, and $\mathbf{V} = (\mathbf{I}-\mathbf{AW})^{-1}(\mathbf{I}-\mathbf{A})$ is

$$\mathbf{V} = \begin{bmatrix} 0.7\bar{7} & 0.2\bar{2} \\ 0.3\bar{3} & 0.6\bar{6} \end{bmatrix}.$$

Here, because the rows of $\mathbf{V}$ are not identical, a consensus is not generated for two individuals with different initial opinions. However, for two such initially disagreeing individuals with the above $\mathbf{W}$, if their $\mathbf{A}$ is allowed to approach $\mathbf{I}$, then their equilibrium opinions will approach an exact consensus. An exact equilibrium consensus is a simple object, that is, one position defined by m coordinates in which all n individuals are located.

In general, the existence of $\mathbf{V}$ with *identical rows* is a necessary and sufficient condition for an exact equilibrium consensus that does not depend on the initial opinions, that is, such a $\mathbf{V}$ will map the elements of any arbitrary $\mathbf{X}(0)$ onto one point. The existence of $\mathbf{V}$ with identical rows is consistent with only two forms of $\mathbf{A}$. Without loss of generality, consider the $m = 1$ case. If an exact equilibrium consensus is formed, that is,

$$x_*(\infty) \equiv x_1(\infty) = x_2(\infty) = \cdots = x_n(\infty),$$

then from the equilibrium scalar equation of the influence system

$$x_i(\infty) = a_{ii} \sum_{j=1}^{n} w_{ij} x_j(\infty) + (1 - a_{ii}) x_i(0), \tag{5}$$

it follows that

$$(1 - a_{ii})[x_*(\infty) - x_i(0)] = 0. \tag{6}$$

Hence, if $x_*(\infty) \neq x_i(0)$, then $a_{ii} = 1$ and, equivalently, if $a_{ii} < 1$, then $x_*(\infty) = x_i(0)$. If $x_*(\infty) - x_i(0) = 0$, then the value of a_{ii} remains undetermined (it may be any value), and if $a_{ii} = 1$, the amount of opinion change $x_*(\infty) - x_i(0)$ remains undetermined. For an exact equilibrium consensus that does not depend on initial opinion, there cannot be more than *one* individual with $x_*(\infty) = x_i(0)$, because with two or more such individuals, consensus is *restricted* to a constrained $\mathbf{x}(0)$ in which there are shared initial opinions. There cannot be more than one individual with $a_{ii} < 1$ because the initial opinions of two or more individuals with $a_{ii} < 1$ would have to be identical for an exact consensus. Hence, for the group as a whole, an exact equilibrium consensus that does not depend on individuals' initial opinions is only consistent with either $\mathbf{A} = \mathbf{I}$ or an $\mathbf{A}$ with one individual i for whom $a_{ii} < 1$ and $n - 1$ individuals $j \neq i$ with $a_{ii} = 1$. Finally, only aperiodic $\mathbf{AW}$s with a particular form of network structure are consistent with the existence of $\mathbf{V}$ with identical rows. Any directed graph corresponding to $\mathbf{AW}$ may be partitioned into maximal strong components whose members mutually influence one another, directly or indirectly. By definition, a trivial component has one member. The maximum size of a component is n, and when such a component exists, the entire graph is said to be strongly connected. The existence of $\mathbf{V}$ with identical rows is only consistent with an aperiodic $\mathbf{AW}$ that contains a unique strong component of s individuals (containing at most one individual with $a_{ii} < 1$) and all other $n - s$ individuals for whom $a_{jj} = 1$. In the special case of a network with one person for whom $a_{ii} = 0$, the unique strong component is trivially that person ($s = 1$).

Social Structure and Influence Centralities

All outcomes of the influence system are determined by three constructs that define the issue-specific social structure of the system: $\mathbf{X}(0)$, $\mathbf{A}$, and $\mathbf{W}$. The social structure is *assembled* by the individuals of the system, that is, their initial opinions, extents of attachment to their initial opinions, and weights accorded to other individuals. The derived $\mathbf{V} = [v_{ij}]$ matrix of the system describes the total (direct and indirect) influences of each j's initial opinion in determining i's equilibrium opinion on the issue. The relative influence centralities of the individuals, defined as

$$\boldsymbol{c} = \frac{1}{n} \mathbf{V}^T \mathbf{e}, \tag{7}$$

where $\mathbf{e}$ is a vector of ones, are their relative mean total influences, $\mathbf{c}^T \mathbf{e} = 1$. This influence centrality measure is equivalent to eigenvector centrality in the special case of $\mathbf{A} = \mathbf{I}$ and to PageRank centrality in the special case of $\mathbf{A} = \alpha \mathbf{I}$, $0 < \alpha < 1$ [27]–[29].

Derivations

The equilibrium equation of the model

$$\mathbf{X}(\infty) = \mathbf{AWX}(\infty) + (\mathbf{I} - \mathbf{A})\mathbf{X}(0) \tag{8}$$

presents several additional useful equations. The equation may be re-expressed as

$$\mathbf{X}(\infty) - \mathbf{X}(0) = \mathbf{A}[\mathbf{WX}(\infty) - \mathbf{X}(0)] \tag{9}$$

to describe the opinion change of each individual. It follows that individuals' extents of attachments to their initial opinion, that is, $a_{11}, \ldots, a_{nn}$ for each of the m columns of $\mathbf{X}(0), h = 1, \ldots, m$, satisfy

$$0 \leq a_{iih} = \frac{x_{ih}(\infty) - x_{ih}(0)}{\sum_{j=1}^{n} w_{ij} x_{jh}(\infty) - x_{ih}(0)} \leq 1, \quad i = 1, \ldots, n; \; h = 1, \ldots, m, \tag{10}$$

under the homogeneity constraint $a_{ii1} = a_{ii2} = \ldots = a_{iim}$ for all i. It follows that the initial opinions of the n individuals may be derived when $(\mathbf{I} - \mathbf{A})$ is nonsingular

$$\mathbf{X}(0) = (\mathbf{I} - \mathbf{A})^{-1}(\mathbf{I} - \mathbf{AW})\mathbf{X}(\infty). \tag{11}$$

An implication of this last equation is that, for a given $\mathbf{W}$ and given $\mathbf{X}(\infty)$, an arbitrary nonsingular $(\mathbf{I} - \mathbf{A})$ is associated with a unique $\mathbf{X}(0)$. The social structure $\{\mathbf{X}(0), \mathbf{A}, \mathbf{W}\}$ that is defined by this unique $\mathbf{X}(0)$, given $\mathbf{W}$ and the arbitrary nonsingular $(\mathbf{I} - \mathbf{A})$ is one member of an infinite set of social structures consistent with $\mathbf{W}$ and $\mathbf{X}(\infty)$.

To illustrate an employment of (11), let $n = 6$; let all w_{ij} of $\mathbf{W}$ be $w_{ij} = 1/6$; let the diagonal values of $\mathbf{A}$ be arbitrarily set to 0.58, 0.14, 0.42, 0.83, 0.53, 0.01, respectively, for $i = 1, ..., 6$; and let $x_i(\infty)$ be −10 for $i = 1, ..., 3$, and 10 for $i = 4, \ldots, 6$. The solved values for $\mathbf{x}(0)$ are −23.81, −11.63, −17.24, 58.82, 21.28, 10.10, respectively, for $i = 1, \ldots, 6$, With these values, $(\mathbf{I} - \mathbf{AW})^{-1}(\mathbf{I} - \mathbf{A})\mathbf{x}(0)$ returns the prespecified $\mathbf{x}(\infty)$. In these results, the modulus of the solved values for $\mathbf{x}(0)$ monotonically increase with the values of a_{ii}. A different set of diagonal values for $\mathbf{A}$, consistent with a nonsingular $(\mathbf{I} - \mathbf{A})$, are associated with a different $\mathbf{x}(0)$. For the same $\mathbf{W}$, these values will also generate the prespecified $\mathbf{x}(\infty)$. The illustrated solution shows that the model is consistent with the emergence of a bimodal distribution of opinions. Solutions obtained with (11) are of interest to sociologists when particular types of $\mathbf{X}(\infty)$ are associated with particular types of

$\{\mathbf{X}(0), \mathbf{A}, \mathbf{W}\}$ social structures. The results to be presented suggest that the polarization of opinions is associated with a particular type of social structure in which the a_{ii} values are a function of $x_i(0)$ values.

The model, and these derivations, describe one parsimonious framework for analyzing social control and coordination. In this framework, social control and coordination may be defined, abstractly, as an interpersonal influence system that transforms the m coordinates of n individuals' initial positions $\mathbf{X}(0)$ into particular $\mathbf{X}(\infty)$ positions within the convex hull of $\mathbf{X}(0)$. The designation of particular types of $\mathbf{X}(\infty)$ as desirable (for example, optimal) or undesirable (for example, suboptimal) is open to different definitions.

Validation and Extension

The model has been evaluated with data collected on small groups in laboratory [10], [30], [17] and field settings [20], [25]. The laboratory and field experiments collected subjects' pre- and postdiscussion opinions on specific issues, their postdiscussion subjective assessments of their openness or closure to influence on the issue (that is, a measure of the a_{ii} construct for each subject), and each subject's distribution of this value among the $n-1$ other members of the group. Based on the collected measures $\{\mathbf{x}(0), \mathbf{A}, \mathbf{W}\}$, the correspondence of predicted and observed postdiscussion opinions is investigated, and this correspondence in hundreds of trials on numerous groups provides substantial empirical support for the model. See the previously cited references for details of the experimental procedures and findings. Illustrations of the model's applications to the literature in experimental social psychology on small groups are presented in [10]. Applications to larger systems are reported in [16] and [20]. An attractive special case of the model, which assumes $a_{ii} = 1 - w_{ii}$ for all i, has been intensively investigated [10]. Relaxations of the model's time-invariant assumptions are entertained in [10] and are ongoing. It is axiomatic that this model will be imperfect at some level; for example, it is obvious that interpersonal influences do not occur in the simultaneous way posited by the model and that there are disorderly sequences of interpersonal influences in a group. An analysis of the model indicates that it may not be misleading when asynchronous influences are allowed [19]. The model's application to an analysis of the costs of disagreement relative to a social optimum have been advanced [18]. Other applications of the model in the domain of control theory include [31]–[34].

The model has been extended to cover behaviors [35], [10]. These behaviors are binary actions with probabilities that depend on the opinions. This extension is nontrivial. An analysis of opinion dynamics on a specific issue may be weakly or strongly associated with individuals' issue-related behaviors, in particular periods of time, and depending on the issue. The specification of conditions under which this linkage is weak or strong has been an ongoing prominent focus of research in social psychology, where it is referred to as the attitude–behavior linkage problem. For a review of this literature, which also attends to the linkage of individuals' evolving displayed opinions on specific issues and their issue-related behaviors, see [35] and its references. It may be noted that displayed opinions are behaviors in a "words-are-deeds" framework and that an analysis of opinion dynamics does not require, but in some cases may be enhanced by, the analysis of other issue-related behaviors.

POLARIZING FUNCTIONS

The application of the above model of opinion dynamics to Abelson's community cleavage problem requires a demonstration that equilibrium bimodal distributions of opinions are reliably generated in special cases of social structures defined by the three constructs of the model: $\mathbf{X}(0), \mathbf{A},$ and $\mathbf{W}.$ The analysis is restricted to social structures defined by a $n \times 1$ vector of all different initial opinions and aperiodic irreducible $\mathbf{AW}$. The assumption of all different initial opinions eliminates patterns of initial agreements as a factor of system outcomes. The restriction to aperiodic irreducible $\mathbf{AW}$ is consistent with Abelson's problem definition because it is not difficult to generate bimodal opinion distributions in reducible $\mathbf{AW}$. Furthermore, this restriction guarantees the existence $\mathbf{X}(\infty)$. The mathematical definition of bimodal (two modes) may be relaxed to include distributions with two local maxima. But these definitions do not capture the phenomenon of bimodal polarization and its magnitude, that is, a central interval that is sparsely populated and left and right intervals that are densely populated. The greater the span of the sparse central interval, the more pronounced the polarization with the important caveat that small quantified differences of opinion may be perceived by individuals as subjectively important depending on the issue. With signed opinions, the behaviors of individuals (for example, their votes) are expected to be consistent with their positive or negative inclinations regardless of the strength of these inclinations.

The bimodal community polarization involved in Abelson's problem is one of several forms of polarization that may be taken as presenting social control or coordination problems. It is useful to briefly situate the phenomenon of bimodal polarization in the broader literature of experimental social psychology that has defined group polarization as a shift (in specific directions) of the distribution of opinions among individuals who are discussing an issue. Work on group polarization in social psychology was triggered by the finding of risky shifts of opinion under the condition of small group discussion [36]. The work initially concentrated on group discussions of choice dilemmas: the minimum level of confidence required to accept a risky option with a high payoff than a less risky option with a low payoff. Group discussions of such issues appeared to generate a shift toward the adoption of the more risky option. Upon intense investigation, this provocative finding on risky shifts was discarded as a general proposition and replaced with a group polarization hypothesis defined as follows: group discussion will reinforce an initial positive (risky) or negative (cautious) inclination of the group, if such an initial inclination exists. This hypothesis, again upon intensive investigation, also proved to be insecure. Work on the problem eventually withered among experimental social psychologists as the cognitive revolution in social psychology took hold and displaced the investigation of social groups as a central matter. For a sociological treatment of the problem and a review of this literature, see [30]. The empirical evidence is mixed on whether group polarization, as defined above, can be accepted as a general proposition. Currently, the only generally accepted proposition is that interpersonal influences ubiquitously generate a slight or substantial shift of mean opinion on an issue toward either the maximum or minimum value of the initial opinion distribution of a group. Unlike Abelson, the above work on group polarization did not attempt to explain polarization in terms of a group dynamics model; instead, the attempt (which failed) was to explain polarization as a main effect of group discussion. For example, the hypothesis that group discussion, per se, generates risky shifts has been rejected.

With the model in (2), a shift in the distribution of opinions may be substantial in an aperiodic irreducible **AW** when the individuals located in one tail of an initial bell-shaped opinion distribution are stubborn (with low a_{ii} values) and all other individuals are accommodative (with high a_{ii} values). The existence of stubborn individuals in both tails of an initial opinion distribution, in combination with an accommodative central mass, sets up cross pressures that may dampen a marked shift of the central mass of opinion toward one tail of the initial opinion distribution. Less obvious are the implications of stubborn individuals in the central mass of an initial opinion distribution and accommodative individuals in its tails. This pattern is a prominent theoretical proposition in both the disciplines of social psychology and sociology. In social psychology, it appears as a proposition in the seminal work on social comparison theory [37]. In sociology, it ubiquitously appears in studies of social norms, where social norms are indicated by the mean values of opinion and behavior distributions. Individuals with opinions near the mean values of bell-shaped opinion distributions are viewed as compliant to the norm and stubborn to opinion change. Individuals with opinions that are distant from the mean values of such distributions are viewed as individuals without strong normative social support for their deviant opinions, who are more open to opinion change. This last special case of a social structure appears to be involved in the emergence of bimodal opinion distributions. It is a social structure based on an aperiodic irreducible **AW**, a bell-shaped initial opinion distribution $\mathbf{x}(0) \in \mathbb{R}^{n \times 1}$, and a matrix **A** in which stubborn attachment to initial opinions decreases with the distance of an initial opinion from the mean initial opinion.

Note that in all of the above special cases of emergent forms of polarization, the particular form of polarization depends on an association of individuals' initial opinions and their extents of attachment to their initial opinions. For a given time-invariant aperiodic irreducible **AW**, various forms of polarization may be explained by functions $a_{ii} = f(x_i(0))$, $i = 1, \ldots, n$ that map individuals' initial opinions onto their attachments to their initial opinions. Figure 1 displays four such functions. These four functions are defined below and their implications are illustrated.

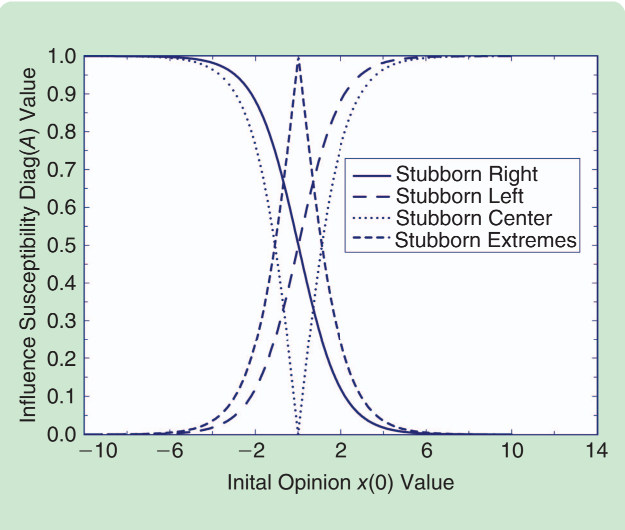

FIGURE 1 Four functions that map initial opinion $x(0)$ values onto influence susceptibility diag(A) values.

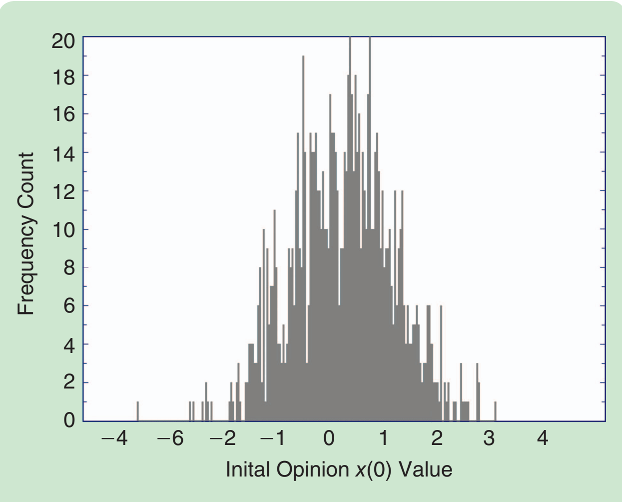

FIGURE 2 An initial opinion distribution that is a random sample ($n = 1000$) drawn from the standard normal distribution $N(\mu, \sigma^2)$, with $\mu = 0$ and $\sigma = 1$.

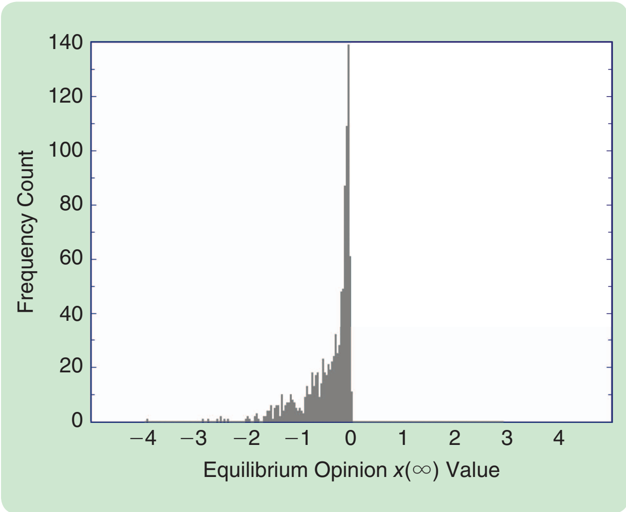

FIGURE 3 An equilibrium opinion distribution obtained from (2) with the $\mathbf{x}(0)$ of Figure 2, the polarization function (12), and $\mathbf{W}$ constructed as follows. The edges of an aperiodic irreducible Gilbert random graph $G(n,p)$ with $p=0.20$ are assigned values, randomly drawn from the uniform distribution, and then normalized to obtain a row-stochastic matrix $\mathbf{W}$.

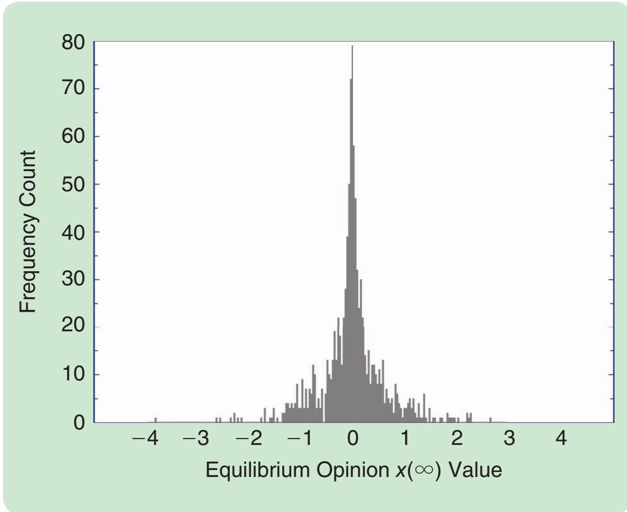

FIGURE 5 An equilibrium opinion distribution obtained from (2) with the $\mathbf{x}(0)$ of Figure 2, the $\mathbf{W}$ of Figure 3, and the polarization function (14).

Figure 2 displays a frequency distribution of initial opinions $\mathbf{x}(0)$ that is a random sample $(n=1000)$ drawn from the standard normal distribution $N(\mu,\sigma^2)$, with $\mu=0$ and $\sigma=1$. The $\mathbf{x}(0)$ values of Figure 2 will be employed in each of the models described below. A random $\mathbf{W}$ is formed as follows: the edges of an aperiodic irreducible Gilbert random graph $G(n,p)$ with $p=0.20$ are assigned values, randomly drawn from the uniform distribution, and then normalized to obtain a row-stochastic matrix $\mathbf{W}$. This $\mathbf{W}$ is likely to have at least one positive value on its diagonal. The same $\mathbf{W}$ will be employed in each of the models described below. The models differ only in the polarizing function that maps the initial opinion $\mathbf{x}(0)$ values onto the influence susceptibility values of the diagonal of $\mathbf{A}$.

Figure 3 forms $\mathbf{A}$ as follows for all i,

$$a_{ii}=\frac{\exp[x_i(0)-\bar{x}(0)]}{1+\exp[x_i(0)-\bar{x}(0)]}, \tag{12}$$

where $\bar{x}(0)$ denotes the value of the group's mean initial opinion. The distribution polarizes to the left.

Figure 4 forms $\mathbf{A}$ as follows for all i,

$$a_{ii}=1-\frac{\exp[x_i(0)-\bar{x}(0)]}{1+\exp[x_i(0)-\bar{x}(0)]}. \tag{13}$$

The distribution polarizes to the right.

Figure 5 forms $\mathbf{A}$ as follows for all i

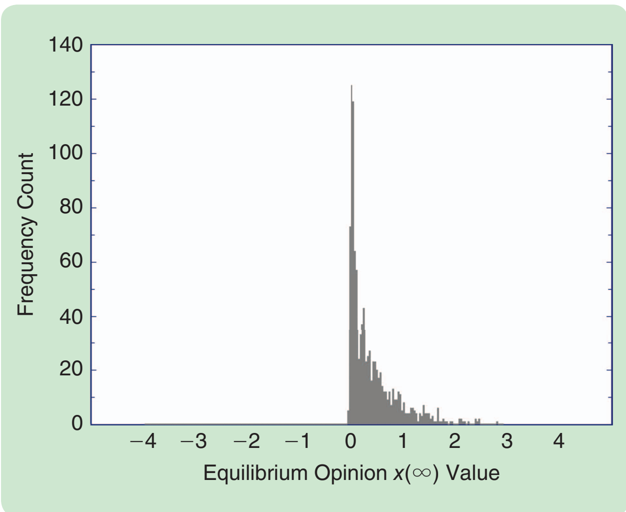

FIGURE 4 An equilibrium opinion distribution obtained from (2) with the $\mathbf{x}(0)$ of Figure 2, the $\mathbf{W}$ of Figure 3, and the polarization function (13).

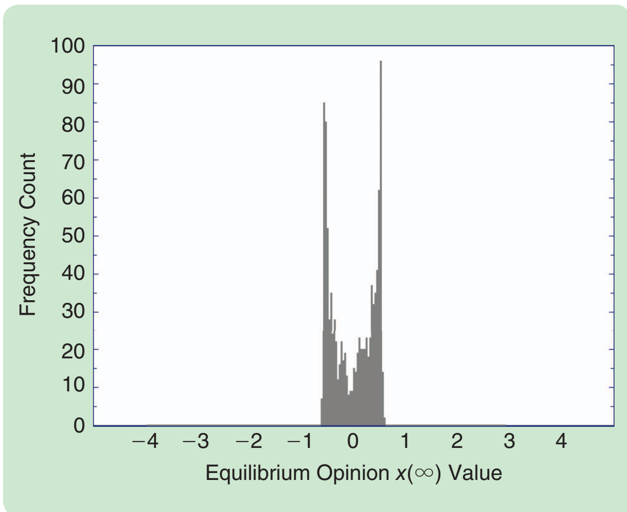

FIGURE 6 An equilibrium opinion distribution obtained from (2) with the $\mathbf{x}(0)$ of Figure 2, the $\mathbf{W}$ of Figure 3, and the polarization function (15).

$$a_{ii} = 1 - 2\left|\left(\frac{\exp[x_i(0) - \bar{x}(0)]}{1 + \exp[x_i(0) - \bar{x}(0)]} - \frac{1}{2}\right)\right|. \quad (14)$$

With intransigence in both the left and right tails, the opinion distribution does not polarize.

Figure 6 forms $\mathbf{A}$ as follows for all i,

$$a_{ii} = 2\left|\left(\frac{\exp[x_i(0) - \bar{x}(0)]}{1 + \exp[x_i(0) - \bar{x}(0)]} - \frac{1}{2}\right)\right|. \quad (15)$$

A bimodal distribution is formed.

In sum, group polarization in the social psychological literature on opinion dynamics is defined broadly as a movement of the distribution of opinions in specific directions. The movement illustrated in Figure 6 suggests that bimodal polarization arises when individuals' subjective certainty in the truth of their initial opinions decreases with the distance of their initial opinions from the group's mean initial opinion. Although the postulate of such a function is credible, there are no lines of empirical work in social psychology that point to one best specification of it. More broadly, there are no lines of work in social psychology that reliably indicate when different polarizing functions will exist. The present observations may serve to trigger advancements on these matters and mathematical work on alternative specifications of the polarizing functions that are entertained here.

SOCIAL CONTROL IN SELF-REGULATING SOCIAL SYSTEMS

In this final section, a set of open social control and coordination problems are posed. The mathematical definition of a problem is more precise than the definition of a problem in other fields, for example, the definitions of open problems in neuroscience. The presented list of problems are ones that may be solved by mathematical models that attend to the explanation of observable natural phenomena and provide researchable predictions.

The problems are posed in terms of the social structures of interpersonal influence systems. Because sociologists are concerned with natural social processes and social structures, the most useful advancements on these problems are ones that postulate *one* mechanism of interpersonal influence with constructs that allow an analysis of all these problems. In the model described in this article, social structures are defined by three issue-specific constructs: individuals' initial opinions $\mathbf{X}(0)$, their extents of attachment to these initial opinions $\mathbf{A}$, and their influence network $\mathbf{W}$. These social structures are assembled by the n individuals of the system, and the system's dynamics are generated by the repetitive responses of individuals to the displayed opinions of others on an issue. Problems of social control and coordination arise when the opinions and associated behaviors of individuals diminish the capacity of a society to adapt to changing circumstances. In the classical literature of sociology, acceptable solutions to these problems are ones based on individual consent, that is, solutions should not depend on coercion or designed regimes of mental and behavioral conditioning of individuals' initial opinion responses to issues or on such conditioning of their allocations of weights to others. Directive authority also is based on consent [38].

> Authority is the character of a communication (order) in a formal organization by virtue of which it is accepted by a contributor to or "member" of the organization as governing the action he contributes; that is, as governing or determining what he does or is not to do so far as the organization is concerned …. If a directive communication is accepted by one to whom it is addressed, its authority for him is confirmed or established. Disobedience of such a communication is a denial of its authority for him. Therefore, under this definition the decision as to whether an order has authority or not lies with the persons to whom it is addressed, and does not reside in "persons of authority" or those who issue these orders.

Similarly, consent is taken as the only legitimate basis of particular voting schemes, laws, and other formal procedures that allow collective decisions among disagreeing individuals and that are enforceable constraints on collective behavior. Thus, the changing circumstances to which a self-regulating society must adapt include perceived injustices and other grievances voiced by the individuals of a society.

Within the above framework, the key unsettled problem is the absence of a validated understanding of systems of interpersonal influences on individuals' opinions and behaviors. The particular model of such systems, described and employed in this article, is situated in a mathematical field of work that has grown explosively with the influx of investigators into it from the natural and engineering sciences. This influx has contributed elegant formulations. The number of proposed models greatly exceeds the number of efforts to empirically evaluate the assumptions upon which the models are based. The intimate dance between theory and empirical data that been instrumental in the advancement of science currently does not exist. Because researchers who are skilled in conducting experiments on human subjects are not necessarily expert in the mathematics of dynamic models, theorists might motivate such a dance were they to contribute specific researchable predictions from their models.

A second problem, suggested by the present model, is the development of a better understanding of nodal regulation of system outcomes. This problem may be defined as follows. Social structure is not imposed upon individuals; it is assembled by individuals. Each individual i of the system presents an initial position on an issue $x_i(0)$ that may be a position with one or more coordinates, an attachment to this initial position a_{ii}, and a set of weights $w_{i1}, \ldots, w_{in}$. The system's matrix constructs, temporal dynamics, and outcomes are determined by the assemblage of these individual-generated values. If the assembled $\mathbf{W}$ is a large-scale aperiodic irreducible matrix, then a massive restructuring of $\mathbf{W}$ may be required to markedly alter the system's trajectory. On the other hand, my exploratory unpublished simulations on the model described in this article point to the conjecture that a system's trajectory may be quickly altered (regardless of the particular network typology of a large-scale aperiodic irreducible $\mathbf{W}$) with changes of the a_{ii} values or with changes of the functions that associate individuals' initial opinions and their attachments to their initial opinions. Is this conjecture correct, is it useful, and what are its limitations? Perhaps control theorists have developed theories that are applicable to this question. Are there natural process analogs of existing designed processes that may be proposed and evaluated that alter these functions? Are there efficient algorithms for solving the general form of (10) to obtain $\mathbf{A}$, and under what conditions of $\mathbf{W}$, $\mathbf{X}(0)$, and $\mathbf{X}(\infty)$ are such solutions available?

A third problem, defined by the mathematical theory of social choice, concerns the analysis of combining individual opinions to reach a collective decision. Arrow's impossibility theorem [39] is obviously moot when interpersonal influences generate consensus. In broad strokes, this famous theorem states that an unambiguous preference ordering of alternatives cannot be determined under a set of assumptions on fair voting procedures. Whether or not consensus is formed, interpersonal influences may modify the rank ordering of individuals' preferences that are implicit in a row-stochastic $\mathbf{X}(0)$ with $m > 2$ columns corresponding to alternative courses of action. When individuals allocate weights to m alternatives, a strict or partial rank ordering of the alternatives is implicit based on these weights. The analysis of interpersonal influences on such $\mathbf{X}(0)$, and its implicit preference rankings, is an open problem that has a direct bearing on social choice theory.

Fourth, the theory of social choice [39] is broadly concerned with the analysis of procedures that allow collective decisions among disagreeing individuals. These procedures are formulated, modified, and enforced by particular individuals who occupy positions of authority. In a self-regulating system in which consent is a fundamental value, occupancy of positions of authority (and authority itself) is contingent on the consent of individuals whose behaviors are being regulated. In a self-regulating system based on consent, interpersonal influences on individuals' appraisals of other individuals should be intimately related to the tenure of individuals who occupy positions with regulative authority. In such systems, a deliberative body (usually a small group of individuals) that makes collective decisions in a particular issue domain may be defined as a *strictly* legitimate body if every person who is subject to a particular regulation has a strong positive appraisal of at least one member of the deliberative body. Weaker definitions of legitimacy may be defined. Currently, the dynamics of interpersonal appraisals is poorly understood; see the related literature on structural balance theory and its classic special case, which posits that $n \times n$ matrices of signed interpersonal appraisals must either consist of all positive appraisals or two subsets of individuals with all positive within-subset appraisals and all negative between-subset appraisals [40], [41]. In general, interpersonal influence systems may alter an appraisal matrix $\mathbf{X}(0)$ with $m \geq 1$ columns corresponding to particular individuals.

A fifth problem, defined by the theory of collective action [42], concerns the coordination of individuals' opinions and associated behaviors to achieve accepted specified collective goals or levels of welfare. The existing mathematical theory on collective action points to social dilemmas and free-rider problems under the assumption of rational choice. Applications of opinion dynamics models, which elaborate the existing mathematical analysis of the theory collective action under relaxed assumptions of rationality, is an unsettled open problem. A relaxation of rational choice assumptions is justified, when these assumptions are inconsistent with interpersonal influences on individuals' opinions. The literature on free riders, that is, individuals who do not contribute to but do benefit from a collective action, has not attended to the effects of interpersonal influences on individuals' choices of the distribution of their available resources (time, labor, funds) across $m > 1$ collective actions. Such distributions may be defined as a row-stochastic $\mathbf{X}(k)$ in which each column corresponds to a particular collective action and each row i corresponds to the relative importance of each such action for i under the assumption that all i support at least one of the m actions. If an i accords no weight to particular activity, then i is a free rider with respect that action when i derives benefits from it. But free riders are not a problem for a society when n individuals with different preferences for actions provide (as an aggregate) sufficient support for *each* of the m actions.

A sixth open problem is also a coordination problem. Sociologists define a social position as a set of individuals with identical or similar cognitive or behavioral orientations toward a set of objects, where these orientations include individuals' relations of various types with other individuals. The influence network of the group is one such relation. Field studies of social groups find two or more social positions in most groups. The organization of social relations within and between two or more social positions is referred to as the role structure of the group; see the landmark article on role structures [43]. The implicit undeveloped thesis of this work is that role structures have implications for the coordination of behaviors in groups with members who are differentiated by their social positions. The number of edges of the social relations within social positions and between particular pairs of social positions may be few or numerous. These role structures are taken to be stable with respect to their aggregate features, that is, their number of social positions and their densities of edges within positions and between the occupants of pairs of positions. A theoretical framework for addressing the implications of different role structures is lacking. The Granovetter hypothesis [44] is that a differentiated group may be coordinated on the basis of bridges (single edges) that join the differentiated social positions. But a reliability theory analysis of such coordination is in its infancy [45], [16], [20]. Specifically, with node deaths and edge failures, the homogeneity of opinions and behaviors that define a social position may be a fragile homogeneity, and the linkages among social positions may be a fragile structural basis of collective coordination.

A seventh open problem is defined as follows. Sociologists view social institutions as central in the analysis of social control and coordination. The nested (national, regional, and more local) array of a society's social institutions in the domains of education, economy, military, government, and judiciary affect the social structure of the society's interpersonal influence system. As indicated in the introduction of the article, the Durkheim hypothesis is that problems of social control and coordination may be ameliorated by restructuring social institutions in response to perceived problems. The assumption of this institution-oriented approach is that institutionalized responses to perceived problems may alter conditions of the social structure of interpersonal influence systems. In this article, the social structure is defined by three issue-specific matrix constructs. An open problem is whether a multilevel modeling approach can be developed in which individuals' scalar values in these influence system matrix constructs are treated as dependent on nested sets of conditions exogenous to the opinion dynamics system. Multilevel statistical models [46] are now widely employed in sociology, but their application to investigations of opinion dynamics is rare [25]. The assumptions entailed in multilevel statistical models are nontrivial. A correct methodology less restrictive in its assumptions, which would embed the constructs of an opinion dynamics system as functions of variables exogenous to the system, would be useful.

An eighth open problem is that the sociological literature on the coordination and control of social systems does not include a developing line of work on natural feedback mechanisms that would fit the mathematical description given by dynamical state-space models. There is, of course, a substantial literature on reactions (for example, social movements and political party mobilization) to perceived social problems (for example, specific injustices, inequalities, laws, and regulations). But there is nothing in the literature that comes close to the development of a theory of control with designed feedbacks that are suitable to social systems. It is not evident if there do exist natural feedback functions that alter opinion dynamics. The proposal of credible natural feedbacks that might be empirically evaluated would be useful.

The general thrust and novelty of the approach to these problems is its theoretical focus on models of opinion dynamics in interpersonal influence networks. The term "opinion dynamics" is convenient but unfortunate. Interpersonal influences alter individuals' cognitive orientations to objects, which may be particular issues, events, individuals, social categories of individuals, institutions, and so forth. The human brain responds to all displayed information that is available to it and the mounting evidence is that it does so automatically. The results of the cognitive revolution in social psychology indicate the importance of heuristic mechanisms in explanations of individuals' responses. In particular, see the work on rapid heuristic individual responses [47], [48] and especially work on the automaticity of individuals' attitudes and behaviors [49]. The postulate of a convex combination mechanism, with which heterogeneous opinions are automatically synthesized by individuals, may be taken as consistent with this literature. The seminal empirical work [50] on these responses indicates that individuals locate all displayed objects in a cognitive space defined by three dimensions (bad-good, passive-active, and weak-strong). The findings indicate that these dimensions of cognitive response are cross-cultural [51]. Models of opinion dynamics are, from this perspective, dealing with an extraordinarily fundamental matter, about which we currently do not have a secure understanding, but one that will be better understood as more investigators with mathematical expertise turn their attention to it.

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