UCLA
Recent Work

Title
What Drives Equity Market Non-participation?

Permalink
https://escholarship.org/uc/item/9k0217bm

Author
Hsu, Jason C.

Publication Date
2003-11-01

What Drives Equity Market Non-participation?

Jason C. Hsu[†]

UCLA Anderson School of Management

This Draft: November 2003

Abstract:

This paper produces endogenous equity market non-participation in an economy with uninsurable labor income risk and heterogeneous skill levels. Prudence and impatience generate stationary household wealth levels that depend on income. Skill, and therefore labor income, heterogeneity leads to wealth heterogeneity, with high skill households accumulating high wealth and low skill households accumulating low wealth. A HARA class utility with subsistence consumption requirement generates decreasing RRA with respect to household wealth. Consequently, low skill households also have significantly higher local RRA. In addition low skill households have less human capital and therefore have lower diversification demand for stocks. Low wealth, high RRA and low diversification demand predicts that low skill households do not hold stocks in the face of a moderate ownership cost. In addition, the model predicts a humped lifecycle wealth accumulation pattern and a humped lifecycle stock allocation pattern. I also find that stockholders exhibit a greater aggregate willingness to supply risky capital during the expansion phase of a business cycle, despite the lower conditional equity premium.

1

I. Introduction:

Using the 1984 Panel Study of Income Dynamics (PSID) data, Mankiw and Zeldes (1991) find that only 23.2% of U.S. households hold more than $1000 in stocks directly. Blume and Zeldes (1993) and Haliassos and Bertaut (1995) find similar patterns of equity market non-participation using the Survey of Consumer Finance (SCF) data from 1992 and prior. Using the U.S. Consumer Expenditure Survey (CEX) database coving 1980 to 1996, Brav, Constantinides and Geczy (2002), and Vissing-Jorgensen (2002b) find that less than 30% of the U.S. households hold beneficial stock ownerships in material dollar amounts.[1] Attanasio, Banks and Tanner (2002) report the similar pattern of non-participation using the U.K. Family Expenditure Survey covering 1978 to 1995. Even during the heydays of Internet stock trading, Bertaut and Starr-McCluer (2000) find that less than 50% of households own stocks.

The high equity market non-participation rate documented is puzzling given the attractive premiums offered by stocks. Insights into this question would also help us understand the equity premium puzzle. Recent papers have found that equity market non-participation helps explain the smoothness in aggregate consumption growth (empirical: Attanasio, Banks and Tanner (2002), Brav, Constantinides and Geczy (2002), Vissing-Jorgense (2002); theoretical: Basak and Cuocco (1998), Constantinides, Donaldson and Mehra (2002) and Storesletten, Telmer and Yaron (2001)). It is then doubly important to understand the nature of non-participation and the asset-pricing implications associated with a high equity market non-participation rate. Is the perceived cost associated with equity investment the main driver of non-participation? If so, what are the impacts on asset prices from the recent decreases in trading costs and increases in financial savvy-ness in the population? Or perhaps, households simply are heterogeneous in their preferences, and non-participating households are just more risk averse than the stock-

---

[1] *Beneficial* stock ownership is defined as ownership where the household is directly exposed to stock price fluctuations. Claims to defined benefit pensions, which may be invested in the stock market, would not be considered *beneficial* ownership because pension benefits are guaranteed and are not linked to the performance of the stock market. However, claims to defined contribution plans, such as self-directed 401K plans would be considered as *beneficial*. In the U.S. more than 70% of all pension assets belong to defined benefit plans, suggesting that most pension participants should not be identified as equity market participants.

holding households. In this case, the measure of risk aversion implied by aggregate data might overstate the risk aversion that is appropriate for studying the equity market.

Standard portfolio choice models have had tremendous difficulties delivering a high rate of equity market non-participation in the population. This has led the current literature to focus on exploring high fixed-cost of stock ownership and departures from the standard expected utility framework (see Haliassos and Bertaut (1995) for an excellent discussion on this). Hong, Kubik and Stein (2001) believe that information cost, which is high for households not endowed with social networks, which facilitate the acquisition of financial knowledge and information, is responsible for non-participation. Ang, Bekaert and Liu (2002), appealing to an alternative utility function specification, suggest that disappointment aversion, which can be quite high for some households, account for non-participation. Cao, Wang and Zhang (2003), using a Knightian uncertainty approach, argue that some households are extremely uncertain about the right model for understanding equity returns and therefore use worst case scenarios to analyze investments; this leads to non-participation.

The more recent literature on portfolio choice finds that incorporating labor income deepens the non-participation puzzle![2] Households endowed with (risky) labor income demand greater stock allocations in their portfolios than households without labor income to diversify their human capital. Labor income, therefore, makes the high rate of equity market non-participation all the more puzzling. Models, based on alternative preferences or non-expected utility paradigms, have not addressed this issue at all.

In this paper, I propose a model, which delivers a high rate of equity market non-participation in the economy, without appealing to large market frictions, alternative preferences or non-expected utility framework. I also refrain from assuming preference heterogeneity (which is needed in Ang, Bekaert and Liu (2002) to deliver non-

---

[2] See Deaton and Lucas (1997) for an exhaustive exercise in portfolio allocation analysis with risky labor income, which predicts that households, endowed with (risky) labor income, would demand leveraged stock portfolios and there would be full equity market participation even with fairly high transaction costs associated with owning stocks; these theoretical predictions are clearly at odds with reported micro data on personal finances. For a perspective on how the portfolio allocation puzzle is related to the equity premium puzzle, see Deaton and Lucas (1996).

participation) or cognitive heterogeneity (which is needed in Cao, Wang and Zhang (2003) to deliver non-participation). In my model, a high degree of wealth heterogeneity, which arises endogenously, coupled with moderate fixed-cost, leads to severe non-participation.

Heterogeneity in household wealth is not addressed or linked to portfolio choice in the aforementioned theory papers on non-participation. This seems unsatisfactory since empirical studies have documented increasing probability of stock ownership with wealth. Endogenous wealth heterogeneity plays a key role in my economy and drives many of the interesting results.

Two key ingredients drive the non-participation rate in my economy. The main innovation comes from the introduction of a two-sector labor market. A fraction of the population is endowed with high skill and employed at high wage rate, while the rest of the population is endowed with low skill and employed at low wage rate. The second innovation comes from the assumption of subsistence consumption needs. This is implemented through a HARA class utility function with decreasing relative risk aversion. I show that prudence (driven by *volatile* labor income) and impatience (driven by *high* subjective discount rate) lead households to develop stationary lifecycle *wealth targets* or targeted levels of buffer stock savings. Below their wealth targets, prudence dominates and households accumulate wealth to build up buffer stocks to hedge against unemployment spills. Above their wealth targets, impatience dominates and households decumulate to avoid deferring too much current consumption.

The wealth targets, which are key in understanding the household's portfolio choice behavior, depend, in part, on the level of the household's permanent labor income as well as the volatility of that labor income stream. High skill, and therefore high income, households endogenously choose to accumulate high wealth, while low skill (low income) households choose to accumulate low wealth; this result, while intuitive is not trivial; if labor incomes were constant and guaranteed for life, households would choose to consume their entire income each period and accumulate no wealth. Since low skill households rationally choose to also become low net worth, they also make the conscious choice to be more relatively risk averse than high net worth households. This is the

mechanism through which the model generates heterogeneous relative risk aversion in the population without assuming heterogeneity in the preference parameters.

The greater relative risk aversions for the low income households translate to lower stock allocations for these households. The lower demand for stocks is further depressed by low income households' small human capital portfolio. In my model, equity returns, which are only weakly correlated with the labor income fluctuations, diversify human capital risk. As a result, low income households, who have less human capital, have lower diversification demand for stocks.

The combined effect of low net worth, high local RRA and low diversification demand for stocks means that low skill households allocate trivial dollar investments in stocks. In the face of moderate stock ownership cost, these households rationally choose to not participate in the equity market.

In this model, equity market non-participants are predominantly the low skill households. However, lifecycle considerations, which lead to a humped lifecycle wealth accumulation pattern and a humped lifecycle stock allocation pattern for households, further predict that a large fraction of young and retired high skill households would also not invest in stocks.

Households in this economy are assumed finitely-lived, which allows the characterization of lifecycle behaviors. In particular, the specification of the retirement income process has important implications for households' wealth accumulation policy. I show that the wealth target is not constant over the lifecycle of a household. In my (partial equilibrium) economy, working (not retired) households desire wealth targets that are about two to three times their annual after-tax income to hedge against *transient* labor income shocks and to smooth the large decrease in retirement income.[3] Once retired, households desire a zero wealth target since they received guaranteed pension income and therefore

---

[3] This is consistent with the low wealth-income ratio observed in micro data. See Kimball (1990a, 1990b) and Carroll (1997) for more discussion on the low wealth-income ratio puzzle.

have no precautionary savings motive.[4] Without precautionary savings motive, impatience dominates prudence and retirees decumulate their wealth to zero. This lifecycle *wealth target* pattern predicts a humped lifecycle *wealth accumulation* pattern. A young family enters the workforce with little wealth. It must *accumulate* toward its desired wealth target over time. Upon reaching its desired wealth target, wealth accumulation stops and the household wealth *plateaus* at this level for as long as a household is exposed to unemployment risk and the impending fall in income on retirement. Upon retirement, precautionary savings motive disappears immediately and this household begins to *decumulate* toward zero wealth; the decumulation is not instantaneous, as this household desires to smooth intertemporal consumption. These sequence of events combine to produce a humped wealth accumulation pattern over a household's lifecycle.

The humped lifecycle wealth accumulation pattern predicts a humped lifecycle stock allocation pattern for stockholders. As I mentioned before, a young household starts with very little wealth. Subsistence consumption need combined with a *borrowing constraint* make this household very relatively risk averse and shy away from stocks. As this young household accumulates toward its wealth target, it becomes less relatively risk-averse and its stock allocation increases. However as human capital is converted into income with age, a household's diversification demand for stocks decreases. Finally, stock allocation falls to zero in retirement after the retiree decumulates to zero wealth. These sequence of events combine to produce a humped stock allocation over a household's lifecycle.

Qualitatively, the model produces what it claims—significant equity market non-participation in the economy. I calibrate the model to examine its quantitative merit and to demonstrate the quantitative importance of the different model parameters. In particular, the income processes for the two skill groups are carefully calibrated to reflect higher unemployment rates and greater unemployment durations during the recession phase of a business cycle.

---

[4] The subjective discount rate in this economy is set higher than the expected risky return such that a household would not accumulate assets if it is endowed with a constant income stream.

In a calibrated economy, I find that high skill households would not invest in stocks at wealth levels as high as $100,000, which traditional portfolio choice models have not been able to produce. In addition, I find that stockholders (who are composed entirely of the high net worth households) allocate a greater fraction of their portfolio to stocks during expansions despite the lower expansion equity premium, and they allocate a smaller fraction during recessions despite the higher recession equity premium.[5] This is because during recessions, households are faced with greater uncertainty in their labor incomes; they are more likely to become unemployed, and when unemployed they are more likely to remain unemployed. The greater aggregate willingness to supply risky capital during expansions is consistent with the empirical observation that the equity premium is counter-cyclical.

The rest of the paper is organized as follows. Section II formulates the model and calibrates it to observed U.S. data. Section III characterizes the household's optimal portfolio choice problem. Section IV examines the household's lifecycle wealth targets and portfolio allocations and discusses why non-participations arise for the low skill households. Section V performs comparative statics on key model parameters. Section VI offers some directions on future research and concludes.

II. The Model

The model is introduced jointly with the calibration exercise to facilitate exposition. Some features of the calibrated model are included to provide a realistic environment for studying portfolio choice but are not essential for driving non-participation. Some modeling features, which ex ante seem important but turn out not to matter, are also included as the negative results are interesting nonetheless. Whenever exposition would allow, I highlight model features, which are included primarily for realism and/or for comparison with traditional models.

---

[5] The equity premium in this model is assumed counter-cyclical. The conditional expansion equity premium is assumed to be lower than the recession equity premium, which is consistent with historical return data. The return process is carefully spelled out in Section II.

In this model, a household is assumed finitely-lived and is characterized by its age cohort (a = young, middle-aged or retired), liquid net worth (w), skill level (s = high or low) and employment status (ε = employed or unemployed). Prior to retirement, a household receives a stochastic labor income (i); the income process is exogenous in the sense that unemployment and retirement are not voluntary. A high skill household receives a high income and vice versa for a low skill household. The economy can take on one of two states, an expansion or a recession state (M=X or R). In a recession, the unemployment rate and duration of unemployment are higher for both skill types. In addition, a high skill household has a non-zero probability of job displacement, where it finds a low skill job after unemployment. Households have available to them a risky and a risk free investment. The *real* risk free rate is exogenously fixed at 1%. The conditional distributions of the risky returns are specified as lognormal with a constant volatility of 18% and a recession equity premium of 3.6% and expansion equity premium of 3.4%. Finally, both high skill and low skill wage fluctuations are correlated with the equity market returns. Households observe the state of the economy (M) as well as their own private characteristics (a, s, ε, i, w). Investment and consumption decisions are formed based on these variables.

Household period utility function

One of the main goals of the paper is to remain within the expected utility framework while producing a high rate of equity market non-participation in the economy. Households have a Stone-Geary (HARA) period utility function with decreasing relative risk aversion:

$$U_t = E_t\left[\sum_{\tau=t}^{\infty} \beta^{(\tau-t)} 1_{\{a_\tau \neq d\}} u_\tau\right], \tag{1}$$

where:

$$
\begin{aligned}
u_\tau &= \frac{\left(C_\tau - \Lambda_{a_\tau}\right)^{1-\eta}}{1-\eta} \quad \text{for } C_\tau > \Lambda_{a_\tau} , \\
&= -\infty \quad \text{for } C_\tau \leq \Lambda_{a_\tau} ,
\end{aligned} \tag{2}
$$

where Λ_{a_τ} is the minimum period consumption that must be satisfied to ensure survival and where the indicator function $1_{\{a_\tau \neq d\}}$ sets the period utility to zero when the household

8

is deceased.[6] This specification differs from the standard CRRA (which is a limiting case when Λ_{a_τ} goes to 0) assumption in only one aspect—the introduction of a subsistence consumption level Λ_{a_τ}, which leads to decrease relative risk aversion with respect to consumption (and indirectly wealth).[7] Cohn, Lewellen, Lease and Schlarbaum (1975), Friend and Blume (1975) and Morion and Suarez (1983) find empirical support for decreasing RRA. A subsistence consumption requirement is also consistent with the notion that the basic goods and services are not perfectly scalable.

The subsistence consumption assumption is crucial in delivering endogenous non-participation and other interesting predictions. However, there is some leeway in specifying the subsistence levels. I tie my hands by calibrating the subsistence consumption levels for the three age cohorts (young, middle-aged and retired) are calibrated to equal a fraction (ϕ) of the 1995 U.S. household poverty thresholds published by the Census Bureau.[8] The household (annual) income poverty thresholds are \$10,259, \$15,569 and \$9219 respectively for the median 2 person household, 4 person household and 65 year or older household. I point out here that results are not qualitatively different if I assume a similar subsistence consumption level for all three age cohorts.

For the benchmark case, I set $\phi = 30\%$, the power coefficient $\eta = 3$ and the subjective discount factor $\beta = 0.88\%$. These variables are chosen to be reasonable while generating the right household wealth levels and wealth dispersion in the economy. I discuss later in Section V the selection of these parameters. The subjective discount rate assumed here is higher than what is usually assumed for representative agent models with no borrowing constraint. However, a high subjective discount rate is both empirically supported and theoretically desirable. Friedman (1952,1963), Griliches, Maddala, Lucas and Wallace (1962), Landsberger (1971) and Mohabbat and Simos (1977) find empirical supports from micro data that household annual subjective discount rates are significantly

---

[6] Since the agent cannot commit suicide, it does not matter what the utility at death is. In our particular setup, the utility, while dead, is actually higher than the utility of being alive! Thereby confirming the suspicion that all rational agents go to heaven.

[7] Note that this utility function is not the habit persistent utility function, since Λ_a is a constant and does not depend on past individual or aggregate consumptions.

[8] The age cohort structure is described later in great details.

higher than 10% (in fact some suggest discount rates in excess of 20%). Theoretically, the buffer-stock saving models, which have shown successes in explaining the high correlation between current income shocks and consumption growth, require high subjective discount rates relative to expected returns on investments.

The high subjective discount rate delivers the following model feature. Households would choose to consume their entire income if their income stream is constant for life. Household would therefore only save for precautionary purpose to insure against income fluctuations.

Note that the local curvature of the utility function or the relative risk aversion is not simply equal to η (as it is in the CRRA case) but also depends on the distance between the current consumption and the subsistence level:

$$\gamma_t = \eta \frac{C_t}{C_t - \Lambda_{a_t}}. \tag{3}$$

The inverse of the elasticity of intertemporal rate of substitution is, however, equal to η and does not depend on the current consumption or the subsistence level. Equation (3) allows me to deliver endogenous relative risk aversion heterogeneity without assuming exogenous preference heterogeneity.

Borrowing and short-selling constraints

In addition, I impose a borrowing constraint. Households are prevented from borrowing to consume. This assumption can be motivated in two ways. First, we know that, in reality, low net worth and unemployed households (the only households that would need to borrow to finance consumption) are likely to receive un-collateralized financing only at very high interest rates due to moral hazard and credit risk. Second, if I assume a stochastic collection process, where the lender could demand full loan repayment prior to the unemployed borrower finding new employment, the household's utility would fall to negative infinity; this would endogenously guarantee no borrowing.[9]

---

[9] See Zeldes (1989) for more discussion on this point.

Households are further restricted from financing consumption by short-selling stocks. This is a realistic assumption as investors, in general, cannot short sell stocks without sufficient hard assets to post to margin as collaterals.

State variables and their evolutionary dynamics

I introduce the different state variables and their dynamics formally in the following section. Households are assumed to observe these state variables with perfect precision and understand their evolutionary dynamics. In addition, I calibrate these dynamics to U.S. data. The calibration should help the reader assess the quantitative merit of the model in addition to its qualitative features. Specifically, I produce quantitative predictions on the portfolio compositions of high income vs. low income households, on non-participation rate as well as the wealth cutoff for equity participation, and on household wealth targets. In Section V, I consider the impacts on household portfolio choice when the parameters of the state variable dynamics deviate from the baseline case.

The macroeconomic variable:

1. State of the economy

I model fluctuations in the state of the economy to incorporate business cycle risk in the model. Business cycle risk impacts portfolio choices in important ways and is a convenient modeling mechanism for specifying the correlation between labor income fluctuations and risky asset returns. The unemployment rates, unemployment durations, wages as well as expected return on the equity market, which are specified later in this section, depend on the current state of the macroeconomy. For simplicity, the economy is assumed to take on only two possible states:

$$\boldsymbol{M} = X \text{ (if expansion) or } R \text{ (if recession)}, \tag{4}$$

where $\boldsymbol{M}$ follows a two state Markov process with the following transition probability matrix:

$$\Pi_{M \to M'} = \begin{bmatrix} \pi_{XX} & \pi_{XR} \\ \pi_{RX} & \pi_{RR} \end{bmatrix}, \tag{5}$$

11

and where the transition probabilities π_{XR}, π_{XX}, π_{RX} and π_{RR} are calibrated from the expected expansion duration (τ_X) and recession duration (τ_R) of the U.S. business cycle:

$$\pi_{XX} = 1 - \frac{1}{\tau_X}, \ \pi_{XR} = 1 - \pi_{XX}, \ \pi_{RR} = 1 - \frac{1}{\tau_R} \text{ and } \pi_{RX} = 1 - \pi_{RR}. \tag{6}$$

In addition the unconditional probabilities can be computed as:

$$\pi_X = \frac{\tau_X}{\tau_X + \tau_R} \text{ and } \pi_R = \frac{\tau_R}{\tau_X + \tau_R}. \tag{7}$$

Using NBER business cycle statistics, τ_X and τ_R are set to equal 12.7 quarters and 5.7 quarters respectively, implying $\pi_{XX} = 0.9213$, $\pi_{XR} = 0.0787$, $\pi_{RX} = 0.1754$, $\pi_{RR} = 0.8246$, $\pi_X = 0.6902$ and $\pi_R = 0.3098$.

The household characteristics:

2. Household skill level

The household skill level is the only source of exogenous heterogeneity in my economy. This heterogeneity then endogenously leads to all other heterogeneities in the economy. I assume only two skill levels (high vs. low) for the ease of exposition and computation. The intuition of the model still carries through if a wider spectrum of skill levels is considered.

For calibration purposes, I assume that 20% of the households are high skill while 80% are low skill. The high skill income level is matched to the median household income level for the top income quintile; the low skill income level is then matched to the median household income level for the bottom 80%. The assumption of only two skill levels can be justified, to a large degree, by the dichotomy in the U.S. household income levels. The top income quintile households earn on average 100% more than the next quintile (U.S. Census Bureau). The bottom four quintiles, by comparison, demonstrate a much smaller heterogeneity in their incomes. The stark dichotomy in earnings is likely due to do the dichotomy in education attainment. Households with professional degrees (such as MBAs) earn three times the earnings of the median households and about twice the

earnings of households with only an undergraduate education (Digest of Education Statistics 2002).

Formally:

$$s = h \text{ if high skill or } l \text{ if low skill,} \tag{8}$$

where s follows the following conditional Markov transition probability matrix:

$$\Pi^{\varepsilon}_{s \to s'} = \begin{bmatrix} \pi^{\varepsilon}_{hh} & \pi^{\varepsilon}_{hl} \\ \pi^{\varepsilon}_{lh} & \pi^{\varepsilon}_{ll} \end{bmatrix}, \qquad \varepsilon = \{e, u\}, \tag{9}$$

where 2 conditional transition probability matrices $\Pi^{\varepsilon}_{s \to s'}$ are defined over employment status (ε) and set such that a high skill household faces a small but non-zero probability of job displacement (transiting to a low skill state) after a period of unemployment. This is the additional human capital risk faced by high income households. An employed high skill household, however, faces no displacement probability. The rationale for this modeling choice is that human capital depreciates during unemployment. After an extended period of unemployment, the depreciation can be so severe that a high skill worker becomes unfit to be reemployed back into his old job. A low skill household, in my model, does not face job displacement risk. It follows directly then:

$$\Pi^{e}_{s \to s'} = \begin{bmatrix} \pi^{e}_{hh} & \pi^{e}_{hl} \\ \pi^{e}_{lh} & \pi^{e}_{ll} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \Pi^{u}_{s \to s'} = \begin{bmatrix} \pi^{u}_{hh} & \pi^{u}_{hl} \\ \pi^{u}_{lh} & \pi^{u}_{ll} \end{bmatrix} = \begin{bmatrix} 1 - \pi^{u}_{hl} & \pi^{u}_{hl} \\ 0 & 1 \end{bmatrix}. \tag{10}$$

Carroll (1992) estimates that there is a small ($<1\%$ per annum) unconditional probability that household labor income falls drastically; I interpret these rare events as corresponding to job displacement in my model and assume an unconditional probability of job displacement of 0.20% per quarter. Since low skill and employed workers do not experience displacement, the conditional probability of job displacement for an unemployed high skill can then be computed as:

$$\left(\pi_X \pi^{X,h}_u + \pi_R \pi^{R,h}_u \right) \pi^{u}_{hl} = 0.2\% \Rightarrow \pi^{u}_{hl} = 4.0\% . \tag{11}$$

13

Job displacement risk, while interesting in its own right, is not a main theme in this paper. The model does not depend on it to generate the qualitative features of non-participation, humped lifecycle wealth accumulation and stock allocation, and cyclical willingness to supply risky capital. Job displacement risk is only useful to quantitatively produce a median wealth level for the high skill households that is consistent with the observed median household wealth for the top income quintile families. I revisit this point again in Section V.

3. Employment status:

Unemployment spills are the primary driver of a household's savings behavior in my economy. The higher the unemployment rate and the longer the unemployment duration, the greater is the demand for precautionary savings.

Formally:

$$\varepsilon = u \text{ if unemployed or } e \text{ if employed,} \tag{12}$$

where ε follows the following conditional Markov transition probability matrix:

$$\Pi_{\varepsilon \to \varepsilon'}^{M,s} = \begin{bmatrix} \pi_{ee}^{M,s} & \pi_{eu}^{M,s} \\ \pi_{ue}^{M,s} & \pi_{uu}^{M,s} \end{bmatrix}, \quad M = \{X, R\} \text{ and } s = \{h, l\}, \tag{13}$$

where 4 conditional employment transition probability matrices $\Pi_{\varepsilon \to \varepsilon'}^{M,s}$ are defined for the permutations of macroeconomic states ($M = \{R, X\}$) and household skills ($s = \{h, l\}$). The transition probabilities are calibrated to match the U.S. unemployment rates $(\pi_u^{X,h}, \pi_u^{X,l}, \pi_u^{R,h}, \pi_u^{R,l})$ over the business cycle as well as the duration of unemployment for each labor skill sector $(\tau_u^{X,h}, \tau_u^{X,l}, \tau_u^{R,h}, \tau_u^{R,l})$. To complete the calibration exercise, I first compute the duration of employment:

$$\pi_u^{M,s} = \frac{\tau_u^{M,s}}{\tau_u^{M,s} + \tau_e^{M,s}} \Rightarrow \tau_e^{M,s} = \frac{\tau_u^{M,s}}{\pi_u^{M,s}} - \tau_u^{M,s}, \tag{14}$$

the transition probabilities can then be computed as:

$$\pi_{ee}^{M,s} = 1 - \frac{1}{\tau_e^{s,X}}, \ \pi_{eu}^{M,s} = 1 - \pi_{ee}^{M,s}, \ \pi_{uu}^{M,s} = 1 - \frac{1}{\tau_u^{s,X}} \ \text{and} \ \pi_{ue}^{M,s} = 1 - \pi_{uu}^{M,s}. \tag{15}$$

The calibration exercise here is complicated by the fact that quality unemployment statistics categorized by income levels are not available. Anecdotal evidences combined with unemployment data compiled by the BLS suggest that unemployment rates and unemployment durations are lower for the high skill laborers than for low skill laborers; at the same time unemployment rate fluctuation is larger for the high skill group. I calibrate the transition probability matrix to reflect these observations, while matching closely to the aggregate unemployment rates and durations reported in Imrohoroglu (1989).[10] The high skill households are assumed to have an unemployment rate $\pi_u^{X,h} = 2\%$ during expansions and $\pi_u^{R,h} = 12\%$ during recessions. The low skill households are assumed to have an unemployment rate $\pi_u^{X,l} = 4.5\%$ during expansions and $\pi_u^{R,l} = 12\%$ during recessions. Since 20% of the households are assumed high skill while the remaining 80% are low skill, the cohort unemployment rates then produce aggregate expansion and recession unemployment rates of 4% and 12%, which match Imrohoroglu's parameters exactly. In addition, unemployment durations are assumed identical for both skill groups at $\tau_u^{X,h} = \tau_u^{X,l} = 1.1$ quarters and $\tau_u^{R,h} = \tau_u^{R,l} = 1.4$ quarters respectively during expansions and recessions. The transition probabilities are then:

$$\Pi_{\varepsilon \to \varepsilon'}^{X,h} = \begin{bmatrix} 0.9814 & 0.0186 \\ 0.9091 & 0.0909 \end{bmatrix}, \ \Pi_{\varepsilon \to \varepsilon'}^{R,h} = \begin{bmatrix} 0.9029 & 0.0971 \\ 0.7143 & 0.2857 \end{bmatrix}, \tag{16}$$

and

$$\Pi_{\varepsilon \to \varepsilon'}^{X,l} = \begin{bmatrix} 0.9571 & 0.0429 \\ 0.9091 & 0.0909 \end{bmatrix}, \ \Pi_{\varepsilon \to \varepsilon'}^{R,l} = \begin{bmatrix} 0.9029 & 0.0971 \\ 0.7143 & 0.2857 \end{bmatrix}. \tag{17}$$

4.a. Household wage process

---

[10] Imrohoroglu (1989) presents a carefully calibrated real business cycle model, which studies the costs of macroeconomic fluctuations.

The U.S. Census Bureau reports a median household before-tax income of $23,830 per annum (in 1995 dollars) for the bottom 80% income households and a median income of $109,411 for the top 20% income households. I assume an after-tax annual wage income of $20,000 (effective tax rate of 16%) and $80,000 (effective tax rate of 26.9%) respectively for employed households.[11] I use the 1995 household income numbers because real income has grown at an unprecedented rate in the late 90's, while real income growth historically has been barely positive. In any case, the absolute levels of the high skill and low skill wage incomes are not important. Only the relative difference between the incomes as well as the level of the income relative to the required subsistence consumption are important.

In addition to the increase in the unemployment rate during recessions, downturns in the economy also impact wage negatively. For the low skill households, negative wage shocks can be reductions in overtime hours or reductions in real wages (as nominal wage increases fall short of inflation). For the high skill households they can be reductions in bonuses. One might expect the wage fluctuations for low skill households to be fairly modest; this is born out by the data. The labor income risk for low skill households, results almost entirely from unemployment. However, one might expect the wage fluctuations for high skill households to be more volatile and correlated with the stock market. This is because firm performance-linked compensations (whether in the form of vested stock options or profit sharing plans) are usually a significant portion of the total compensation awarded to high-ranking employees.[12]

The wage processes for the two skill groups are defined as follows:

---

[11] These numbers are derived using the H&R Block Tax Cut software with assumptions on dependents and other deductions in conjunction with a California state income tax assumption, as well as Medicare and social security tax withholdings.

[12] Principal-agent theory predicts that employees who need to exercise a substantial amount of discretion or decision-making in their jobs should have their compensation linked to firm performance to ensure good effort. Total compensation data from the BLS supports this claim as the average compensation of high income workers exhibit greater covariance and correlation with the equity market return.

$$
i_h = \begin{cases} i_{Xh} & \text{if } M = X \\ i_{Rh} & \text{if } M = R \end{cases} \text{ and } i_l = \begin{cases} i_{Xl} & \text{if } M = X \\ i_{Rl} & \text{if } M = R \end{cases}
\tag{18}
$$

where the wage fluctuations are calibrated to reflect pay reductions during recessions and to capture higher wage volatilities in the high skill sector. Note that in my model, wage does not fluctuate within an expansion or a recession phase; wage only fluctuates when the macroeconomic state transits from one phase to another.

The wage volatilities for the two skill groups are needed to calibrate the income process described in equation (18). I assume a quarterly wage volatility of 6.5% for the low skill households and 11.5% for the high skill households, which correspond to an aggregate volatility of 7.5% per quarter, which is also used by Deaton (1991), Koo (1995) and Heaton and Lucas (1997) in their calibration of wage dynamics.

Since the income processes are two-state Markov chains, I have:

$$
\mathrm{d}i_{Xl}^2 \pi_X + \mathrm{d}i_{Rl}^2 \pi_R = 0.065^2 \text{ and } \mathrm{d}i_{Xl}\pi_X + \mathrm{d}i_{Rl}\pi_R = 0 ,
\tag{19}
$$

and

$$
\mathrm{d}i_{Xh}^2 \pi_X + \mathrm{d}i_{Rh}^2 \pi_R = 0.115^2 \text{ and } \mathrm{d}i_{Xh}\pi_X + \mathrm{d}i_{Rh}\pi_R = 0 ,
\tag{20}
$$

where $\mathrm{d}i_{Xs} = \left(i_{Xs} - E[i_s]\right)\big/E[i_s]$ is the deviation of the period labor income from the expected income. Solving we have $\mathrm{d}i_{Xl} = 4.4\%$, $\mathrm{d}i_{Rl} = -9.7\%$, $\mathrm{d}i_{Xh} = 7.4\%$ and $\mathrm{d}i_{Rh} = -16.5\%$.

I examine in Section V the qualitative and quantitative impacts of the wage rate volatility assumed here. While wage fluctuations are realistic assumptions, they turn out not to matter qualitatively, as labor income risk is dominated by unemployment spills.

4.b. Retirement Income:

Retired households receive retirement benefits:

$$
b_s = \begin{cases} b_h & \text{if high skill at retirement} \\ b_l & \text{if low skill at retirement} \end{cases} ,
\tag{21}
$$

where, in the baseline case, b_h and b_l are assumed equal to 50% of the average income for the high skill and the low skill respectively plus the average social security income, which in 1995 dollars is equal to \$7628 pretax or roughly \$6000 after-tax at the marginal rate.

The retirement income assumption here is different from the zero pension income assumed in most lifecycle models. Storesletten, Telmer and Yaron (2001) among others endow retirees with no income to accentuate the risk of stockholding for retired households, which leads to non-participation for retirees. However, I achieve non-participation for retirees by the exact opposite route—by endowing retired households with guaranteed pension incomes. Essentially, retirees decumulate their wealth to zero because they no longer have the need to save for unemployment; this trivially produce equity market non-participation for retirees. I explore this point further in Section IV and V.

5. Age cohort

Households are characterized by the age cohorts to which they belong. There are three non-trivial age cohorts and a trivial age cohort. Specifically, a household can be:

$$\boldsymbol{a} = y \text{ (if young)}, \ m \text{ (if middle-aged)}, \ o \text{ (if retired)}, \ d \text{ (if dead)} \qquad (22)$$

where $\boldsymbol{a}$ follows the following four state Markov transition probability matrix:

$$\Pi_{a \to a'} = \begin{bmatrix} \pi_{yy} & \pi_{ym} & \pi_{yo} & \pi_{yd} \\ \pi_{my} & \pi_{mm} & \pi_{mo} & \pi_{md} \\ \pi_{oy} & \pi_{om} & \pi_{oo} & \pi_{od} \\ \pi_{dy} & \pi_{dm} & \pi_{do} & \pi_{dd} \end{bmatrix} = \begin{bmatrix} \pi_{yy} & \pi_{ym} & 0 & 0 \\ 0 & \pi_{mm} & \pi_{mo} & 0 \\ 0 & 0 & \pi_{oo} & \pi_{od} \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (23)$$

Unlike the standard lifecycle models, where age transition is deterministic, age transition in this model is stochastic. The transition probabilities are defined to match an expected duration of 20 years for staying in each age cohort ($\tau_y = \tau_m = \tau_o = 80$ quarters):

$$\pi_{aa} = 1 - \frac{1}{\tau_a} \Rightarrow \pi_{yy} = \pi_{mm} = \pi_{oo} = 0.9875 . \qquad (24)$$

18

Young households (these are 2 person households just entering the work force) expect to remain *young* for 20 years before they start to consume and invest like *middle-aged* households. Similarly, *middle-aged* households (4 person households with children) expect to continue to work for another 20 years before retiring. The stochastic age transition also means that death is stochastic.[13] This modeling feature allows me to examine Modigliani's conjecture that households often leave bequests (even when bequest motive is not present) because they wish to hedge against *living too long*.

It is worthwhile mentioning that my specification of the cohort transition process is distinctively different from the existing literature on overlapping generations, which assume deterministic death. In standard lifecycle models, death is known with perfect certainty for agents, which unrealistically reduces precautionary savings motive in people's old age. In addition, the current specification allows the model to be solved using a recursive technique commonly used for solving infinitely-lived agent economies; this recursive relationship is shown in the following section when the value function is presented. In contrast, standard computable lifecycle models are solved using backward induction, which is computationally more cumbersome.

6. Household liquid wealth follows the following dynamics:

$$w_{t+1} = w_t \left(1 - c_t\right)\left(1 + R_f + \theta_t \left(R_{t,t+1} - R_f\right)\right) - 1_{\{\theta_t > 0\}} \mu + 1_{\{\varepsilon_t = e\}} i_{Ms} + 1_{\{a_t = o\}} b_s , \qquad (25)$$

where c_t and θ_t are, respectively, the fraction of total wealth consumed and invested in stocks at time t. μ is the fixed cost of stock ownership and is set at \$400 per year in my baseline case.[14] $R_{t,t+1}$ and R_f are the returns from the risky and the risk free investment

---

[13] Note that young and middle-aged households do not face death directly. They face death only indirectly as there is some positive probability that a middle-age household could transit to the retiree cohort and then become deceased in two model periods (2 quarters).

[14] While there is much debate on the true cost of stock ownership (cost associated with obtaining a working knowledge of portfolio theory in addition to trading fees and account maintenance fees), former SEC Chairman Arthur Levitt provides the best anecdote on just how un-widespread investment knowledge is—"*While 63% [of all survey respondents] know the difference between a*

respectively and are described in more details later. $1_{\{\theta_t>0\}}$, $1_{\{\varepsilon_t=e\}}$ and $1_{\{a_t=o\}}$ are indicator functions. Note that a household can either receive a working wage i_{Ms}, a pension check b_s or nothing if it is unemployed.

I now describe the investment opportunities available to the households.

1. *Risk free investment*

Households can invest in risk free deposits earning 1% real return per annum. The real short-term default free interest rate has been about 1% per annum in the U.S. Weil (1989) finds this low interest rate puzzling. He argues that households would need to have negative subjective time discount rate to demand bank deposits yielding 1%. I show that households with a very high subjective discount rate would still have positive demands for the risk free investment at 1%. Because low skill households do not invest in stocks (this is shown later when the wealth accumulation policy and the portfolio policy functions are solved), their buffer-stock savings are stored entirely in the risk free asset.

2. *Equity investment*

Households can invest in a risky asset (the S&P500 index fund essentially), which offers a premium of 3.4% per annum over the risk free rate during expansions and 3.6% per annum during recessions. The volatility of the risky asset returns is constant at 18% per annum, and the risky returns are conditionally log-normal and have a 15% correlation with the fluctuations in the macroeconomy. While the historical average excess stock return measures about 7%, the literature on equity premium contends that a large fraction of the historical return is due to luck. Recent empirical works by Claus and Thomas (2001), Fama and French (2002) and Donaldson, Kamstra and Kramer (2003) find support that the true ex ante equity premium is only around 3.5%. Furthermore, the literature on time-varying risk premium concludes that the equity premium (and the Sharp Ratio) is counter-cyclical.

---

halfback and a quarterback, only 12% know the difference between a load and no-load mutual fund."

The economy, with an exogenously specified risk free rate and stock return process, allows only a partial equilibrium analysis. Therefore I refrain from speculating on the level of the conditional and unconditional risk premia or the factors driving them. I focus instead on the wealth accumulation pattern, portfolio characteristics and consumption behaviors, which support the specified return processes. The merit of the model is then assessed based on the ability of these predicted wealth, portfolio and consumption characteristics to match with the observed data.

3. *Fixed cost of stock ownership*

I assume a fixed cost of stock ownership μ, which is set to $100 per quarter in the baseline case. Other levels of fixed cost are also considered in Section V. The fixed cost of stock ownership include dollar equivalent cost for preparing and filing the IRS Schedule D form for capital gains and losses (which is estimated by the IRS to take 3 hours and 41 minutes), in addition to loads (front or back-end) as well as other related administrative fees for purchasing and holding mutual funds. On top of that brokerage firms also charge commissions and account maintenance fees. Industry sources have also informed me that a relationship with less than $100,000 are usually not considered profitable for brokerage firms unless the account trades actively or invests on margin; fees are usually designed to discourage these low net worth accounts. Entry costs, such as the cost of gaining financial knowledge is likely to deter participation also. However, I do not consider that in this paper.

While fixed cost of stock ownership is a reality, it is also important for modeling purpose. For HARA utility functions, households are locally (absolute) risk neutral and would always be willing to hold a tiny fraction of their portfolio in stocks (see Rabin (2000)). Therefore a fixed cost or transaction cost would be required to generate nonparticipation. In this paper, an important contribution is that the assumed fixed cost needs not be extremely large to preclude stock investment in a large fraction of the population.[15] I show later that even with a negligible fixed cost of $10 per quarter, a fraction of households in my economy would hold no stocks.

---

[15] Heaton and Lucas (1997) show that even with a 4% transaction cost on equity, households in their calibrated economy would still hold portfolios that are 100% in stocks.

III.   Solving the Optimal Portfolio Choice Problem

The household's portfolio choice problem is now completely specified. I solve for the optimal portfolio policy numerically as an analytical solution is not available. I use a recursive dynamic programming technique.[16] The state variables are $\boldsymbol{M}$, $\boldsymbol{\varepsilon}$, $\boldsymbol{s}$, $\boldsymbol{a}$, and $\boldsymbol{w}$, and the choice variables are $\boldsymbol{c}$ and $\boldsymbol{\theta}$. At this point, it is convenient to introduced a new 32 state Markov transition probability matrix $\Pi_{\{M\varepsilon sa\}\to\{M'\varepsilon's'a'\}}$ describing the dynamics of the four joint state variables $\{\boldsymbol{M}, \boldsymbol{\varepsilon}, \boldsymbol{s}, \boldsymbol{a}\}$. The entries in this 4-dimensional probability matrix are constructed from the transition probabilities defined in (5), (13), (9) and (23):

$$\pi_{\{M\varepsilon sa\}\to\{M'\varepsilon's'a'\}} = \pi_{MM'}\pi_{\varepsilon\varepsilon'}^{M,s}\pi_{ss'}^{\varepsilon}\pi_{aa'} \ . \tag{26}$$

The value function is then:

$$V\left(M_t,\varepsilon_t,s_t,a_t,w_t\right) = \max_{c_t,\theta_t}\left\{\frac{\left(c_t w_t - \Lambda_{a_t}\right)^{1-\gamma}}{1-\gamma} + \beta E_t\left[V\left(M_{t+1},\varepsilon_{t+1},s_{t+1},a_{t+1},w_{t+1}\right)\right]\right\},\tag{27}$$

where the conditional expectation is defined as:

$$E_t\left[V\left(M_{t+1},\varepsilon_{t+1},s_{t+1},a_{t+1},w_{t+1}\right)\right]$$
$$= \sum_{M_{t+1}}^{\{X,R\}}\sum_{\varepsilon_{t+1}}^{\{e,u\}}\sum_{s_{t+1}}^{\{h,l\}}\sum_{a_{t+1}}^{\{y,m,o,d\}} \pi_{\{M_t\varepsilon_t s_t a_t\}\to\{M_{t+1}\varepsilon_{t+1}s_{t+1}a_{t+1}\}} \cdot V\left(M_{t+1},\varepsilon_{t+1},s_{t+1},a_{t+1},w_{t+1}\right) , \tag{28}$$

where M_t, ε_t, s_t, a_t and w_t follow the dynamics described in (4), (12), (8), (22) and (25). Recall also that we restrict households from borrowing to consume or shorting stocks:

$$c_t < 100\% \text{ (no borrowing to consume)}, \tag{29}$$

and

$$0 \le \theta_t \text{ (no shorting).} \tag{30}$$

---

[16] A detailed document, complete with the Matlab codes, can be requested from the author.

Equation (27) defines a recursive relationship. Contraction mapping then guarantees the existence of a unique fixed point solution to the optimal portfolio choice problem. It is worth noting that unlike other models with lifecycle consideration, this solution uses a recursive method instead of backward induction, which is substantially more cumbersome.

IV.   Lifecycle Wealth Targets and Portfolio Allocation

I describe the optimal lifecycle wealth targets and portfolio allocations for high skill vs. low skill households in this section.

Lifecycle wealth targets

I first introduce results on households' lifecycle wealth targets and the associated wealth accumulation pattern instead of results on equity market non-participation because non-participation is a direct consequence of the low skill households choosing to accumulate low wealth. I define explicitly the lifecycle wealth targets (or the lifecycle buffer-stock saving targets) for each skill group. The *employed* high skill and low skill household's zero accumulation points $\{W^{h,y}(M),\ W^{h,m}(M),\ W^{h,o}(M)\}$ and $\{W^{l,y}(M),\ W^{l,m}(M),\ W^{l,o}(M)\}$ are defined as the levels of wealth at which the investment and consumption decisions make the expected next period wealth growth zero. I do not compute zero accumulation points for unemployed households because they are not meaningful (and also undefined). While unemployed, households decumulate at every level of wealth.

I can solve for these macro state dependent zero accumulation wealth levels implicitly by rearranging equation (25) and setting $E_t\left[\frac{w_{t+1}}{w_t}\right]=1$. Using the unconditional probabilities π_M, the household's *unconditional* or *steady state* lifecycle wealth targets are defined as:

$$\bar{W}^{s,a} = \sum_M^{\{X,R\}} \pi_M W^{s,a}(M). \tag{31}$$

Figure 2 illustrates graphically the determination of these steady state wealth targets. The wealth targets, so defined, are wealth levels that households maintain (on average across the business cycle) when they are employed. During unemployment spills, house-

23

holds decumulate away from that target to smooth consumption, but accumulate back toward it once they find employments.

What determine the levels of a household's lifecycle wealth target? In general, households might build up wealth for three reasons—1. to hedge against unemployment spills, 2. to hedge against living too long, 3. to prevent falling consumption in retirement and 4. to fuel future consumption growth. However, given the assumption of the guaranteed pension income and the high subjective discount rate, 2 and 4 are completely dominated by 1 and 3. I present below a (log-linearized) approximate equality relating portfolio returns to preference parameters as well as consumption growth (ΔC) moments:[17]

$$E[r] \approx \delta + \eta E[\Delta C] - \frac{1}{2}\eta^2 \left(\frac{C}{C - \Lambda_a}\right)^2 \mathrm{var}(\Delta C) + \eta \left(\frac{C}{C - \Lambda_a}\right) \mathrm{cov}(\Delta C, r), \qquad (32)$$

where the subjective discount rate $\delta \approx 1 - \beta$. *Impatience*, which is increasing in δ and the inverse of the intertemporal rate of substitution η, makes households detest future consumption growth at the expense of current consumption. Ignoring the second order expansion terms in equation (32), we see that when the expected portfolio return is low, households with high δ would not accumulate to fuel positive consumption growth. *Prudence*, which is increasing in η and relative risk aversion $\eta \frac{C}{C - \Lambda_a}$, counteracts impatience to make households defer current consumption even when the expected portfolio return is low. From the second term in the expansion, we see that if consumption growth is expected to be negative (which occurs when households retire without adequate savings), the aversion to falling consumption makes households accumulate wealth. From the third term in the expansion, we see that if consumption growth is expected to be volatile (which occurs when households do not have adequate savings to self-insure against unemployment), the aversion to consumption growth variability makes households accumulate wealth.

---

[17] This approximation comes from the following two approximations:

$$E[r - r_f] \approx \eta \left(\frac{C}{C - \Lambda_a}\right) \mathrm{cov}(\Delta C, r) \text{ and } r_f \approx \delta + \eta E[\Delta C] - \frac{1}{2}\eta^2 \left(\frac{C}{C - \Lambda_a}\right)^2 \mathrm{var}(\Delta C).$$

See Cochrane (1997) for a review on the derivation of the two approximations.

Prudence and impatience interact to produce household wealth processes that are stationary. While employed, a household accumulates additional buffer-stock to hedge against unemployment spill when its wealth level is below its desired wealth target and decumulates to avoid deferring too much current consumption when its wealth is above its desired wealth target. While unemployed, a household would decumulate to smooth its consumption profile. I present the lifecycle wealth targets for a high skill vs. a low skill household in the Table 1 below.

Table 1. Lifecycle Wealth Targets for High Skill vs. Low Skill Households (in $1000)

	Young	Middle-aged	Retired
High Skill	318	294	0
Low Skill	47	52	0

Note that a high skill household accumulates a significantly greater wealth than a low skill household. This result is not trivial, much as it may seem obvious. If labor income were riskless, equation (32) suggests (at an equity premium of 3.5% and subjective discount rate $\delta \approx 12\%$) that both the low and the high skill households would accumulate no wealth and would instead consume their entire labor income every period. Since labor income is risky, households must build up buffer stocks to smooth consumption during unemployment. A high skill household, with high income and therefore high average per period consumption, needs a larger buffer stock to maintain its lifestyle during an unemployment spill than would a low skill household.

The difference in portfolio behaviors between a high skill household and a low skill household additionally drives a wedge between their wealth targets. I show in the next section that *working* (non-retired) low skill households do not hold stocks. Working high skill household, however, invest in stocks heavily; high skill households choose to store part of their buffer-stocks in stocks because the benefit outweighs the fixed cost and the associated risk. Substantial stock allocations lead to additional volatility in consumption

growth, which accentuates the precautionary savings need, which drives the wealth target higher.

Note that the desired wealth targets are similar for the young and the middle-aged households belonging to the same skill group. This is because the risk of unemployment is identical for the young and the middle-aged. The minor difference in the wealth targets for the young and the middle-aged is attributed to the difference in the portfolio behaviors and the required level of subsistence consumption for the two age cohorts.

Lifecycle wealth accumulation

While the lifecycle wealth targets are not humped, the predicted lifecycle wealth accumulation pattern is humped. This is because young families start out with little or no wealth. Prudence drive them to accumulate toward their desired wealth targets as they earn and age. Retirees, on the other hand, have guaranteed pension income until dealth and are driven by impatience to decumulate to zero wealth. Figure 2 shows the speed of wealth accumulation for the different skill groups and age cohorts. Table 2 summarizes the expected accumulation time from zero to 80% of the desired wealth target for young households and the expected decumulation time to $20K for retired households.

Table 2. Expected Accumulation and Decumulation Time

	Young Household (staring with $0)	Retired Household(decumulating to $20K)
High Skill	accumulating to $240K	(starting with $300K)
	73 quarters	51 quarters
Low Skill	accumulating to $52K	(starting with $50K)
	12 quarters	11 quarters

Note that the speed of accumulation and decumulation falls as the household wealth nears the wealth target. Accumulation to the desired wealth target cannot occur instantaneously for obvious reasons. Households' savings are constrained by their incomes; in addition, households must consume each period to survive. The decumulation to zero for

the retirees also does not occur instantaneously. This is because households, while impatient to consume, dislike sharp reductions in consumptions even more.

In my model households do accumulate wealth to prepare for retirement in addition to hedge against unemployment. Upon retirement, precautionary savings motive disappears because pension incomes are guaranteed until death. However, because pension incomes are only a fraction of the previous labor incomes, retirees decmulate their wealth to smooth the fall in their consumptions. Similar to Modigliani's conjecture households in this model would often leave significant amount of assets at death even though no bequest motive is assumed. This is because they die before depleting their assets.

Equity market non-participation

I plot the portfolio allocations of high skill vs. low skill households for the different age cohorts in Figures 3, 4 and 5. Note that there are equity market participation cutoff wealth levels, below which households do not hold stocks. The equity market participation cutoff levels are summarized in Table 3 below.

Table 3. Equity Market Participation Cutoff Wealth Level
for High Skill vs. Low Skill Households (in $1000)

	Young	Middle-aged	Retired
High Skill	115	123	29
Low Skill	70	78	39

For the working low skill households, the cutoff wealth levels average $74,000 and for the high skill households, the cutoff wealth levels average around $119,000. Since low skill households have wealth targets that are lower than their participation cutoff wealth levels, they do not participate in the equity market in general. High skill households, on the other hand have wealth targets that are significantly above their participation cutoff wealth level; therefore, a large fraction of high skill households (those that have accumulated wealth) are participants in the stock market. Note that not all high skill households invest in stocks. Specifically, young high skill households that have not accumu-

27

lated more than $115,000 would not invest in stocks. Retired high skill households that have decumulated their wealth below $29,000 also do not invest in stocks.

The non-participation at lower wealth levels for both skill groups is driven by two factors: 1. high relative risk aversions at low wealth levels (see Figure 1) and 2. a fixed cost of stock ownership. The first factor is of primary importance and is a feature not found in other non-participation models. Shown in Figure 6 are the portfolio allocations when a $40 per year fixed cost of stock ownership is assumed. We see that even with an insignificant ownership cost, at low wealth levels (below $40,000), both the high skill and the low skill households choose to hold no stocks because their relative risk aversions, driven by subsistence consumption need, are extremely high.

In addition, I find that non-participation can occur for households with liquid wealth as high as $100,000. This is one feature that the traditional models on non-participation have been unable to generate—non-participation at high liquid wealth level, which has led Haliassos and Bertaut (1995) and Cao, Wang and Zhang (2003) to argue in favor of non-expected utility paradigm. In my model, high skill households that earn on average $80,000 after-tax, are still sufficiently conservative at a wealth level of $100,000; $100,000 is simply insufficient to buffer against unemployment spills and job displacement risk. The conditional consumption growth volatility at wealth levels near 100,000 is still sufficiently high to make high skill households shy away from equity investment.

Most models that have included risky labor income to study portfolio choices have focused on accentuating equity holding risk by making labor income fluctuations and labor income volatilities more correlated with the equity market. If labor income shocks are very large and persistent and its volatility is counter-cyclical, then households shy away from holding stocks. However, these models fail quantitatively because labor income shocks, in reality, are no where near as volatile and persistent as what is needed to drive equity market non-participation. In this paper, extremely volatile labor income would actually lead to increased likelihood of equity market participation. Low skill households faced with very large and persistent labor shocks would accumulate very large buffer-stock portfolios. In which case, a modest fixed cost of stock ownership would not discourage stock investments. On the other hand, reducing labor income volatility to near

zero would actually generate equity market non-participation trivially. Households, endowed with a near constant per period income would accumulate almost no wealth in this model and therefore would hold no stocks as long as there is a fixed cost.

Lifecycle portfolio allocations

1. Portfolios for Young vs. Middle-aged vs. Retiree

Examining the lifecycle stock allocation pattern in Figures 3, 4 and 5, we can see that young households are always more willing to commit a greater fraction of their portfolios, at every level of wealth, to stocks than middle-aged households. This is because stock investment diversifies human capital risk, and young households have more human capital. However, once households enter retirement, their stock allocation becomes extremely aggressive. This is because their retirement income is guaranteed for as long as they live, which means given the same investment portfolio, their conditional consumption growth volatility is lower than young or middle-aged households. Retired households are therefore more capable at bearing equity market risk. This feature is generally missing in most lifecycle models where retirees are assumed to receive no pension benefits; the "no pension income" modeling choice is usually made to prevent retirees from holding too much equity investment.[18]

In my economy, retirees have riskless pension income. Surprisingly a significant fraction of retirees in my economy also choose to hold no stock investments, even though they have an advantage in bearing equity risk. This is because the retirees have a wealth target of *zero*! With no pension income risk, impatience dominates prudence (in fact there is no precautionary savings motive anymore), and retirees decumulate their wealth toward zero. However, the decumulation, as I mention before, does not occur instantly, but rather over an extended period of time. This results in a dichotomy in the portfolio allocation of the retired households. A recently retired high skill household would still have a sizeable portfolio that is heavily invested in the stock market. As this retired household continues to decumulate, its wealth eventually falls below the participation threshold; at which point, this household holds a portfolio that is 0% invested in stocks.

---

[18] See Storesletten, Telmer and Yaron (2001) and Constantinides, Donaldson and Mehra (2002). In their economies, retirees do not hold stocks. This serves to accentuate equity market risks in the hands of the middle-aged workers, which then leads to a high equity premium.

This is consistent with reported portfolio behaviors. Specifically, the NYSE reports that only 12.6% of all stockholding households are age 65 or above, while 37.3% of stockholding households are between 45~65, suggesting that nearly 2/3 of all middle-aged stockholders become non-stockholders after retirement.

2. Portfolio for High Skill vs. Low Skill

In addition, I observe that working low skill households have a more aggressive stock allocation policy at wealth levels below $100,000. This is because their unemployment rate and unemployment duration fluctuations are less severe and the covariance between their labor income fluctuations and the equity returns is lower. However, this effect is soon dominated by two factors. First, since consumption is influenced largely by the level of the labor income, high skill households have a greater average consumption than low skill households at every level of wealth (see Figure 1). Thus low skill households have a greater relative risk aversion at every level of wealth (see Figure 1). Second, since high skill households have more human capital, therefore they have greater demand for equity, which diversifies labor income risk.

3. Portfolio for Employed vs. Unemployed

Comparing the portfolios for unemployed and employed households, I find that unemployed households for both skill groups have more conservative stock allocations for almost all levels of wealth. [19] This is because unemployment is persistent, which makes the conditional next period consumption growth volatility larger during unemployment than during employment. In addition, since high skill households face the additional risk of job displacement, the conditional next period consumption growth volatility during unemployment is further accentuated for this group.

4. Portfolio for Expansions vs. Recessions

Comparing portfolio strategies during expansions and recessions, I find that for almost all wealth levels, households in both skill groups have more aggressive stock allocations during expansions than during recessions. Households are more conservative (meaning

---

[19] At lower wealth levels, unemployed high skill households have more aggressive allocations than their employed counterpart due to hedge demand for displacement risk; this demand is very quickly dominated by the risk of extended unemployment spill.

they allocate less to stocks and also consume less) during recessions because the probability of becoming unemployed is heightened and the unemployment duration is lengthened substantially. More strikingly, households allocate less to stocks during recessions despite the higher conditional equity premium (and Sharpe Ratio) offered. The attractiveness of the higher equity premium during recessions does not dominate portfolio allocation until household wealth exceeds well over $2,000,000.

Since working high skill households, who are the stockholders in this economy, have lifecycle wealth targets around $300,000, their portfolio strategies are characterized by greater stock allocation during recessions. This indicates that the aggregate willingness to supply risky capital is greater during expansions and lower during recessions, which is consistent with the counter-cyclical equity premium assumed in this economy.

V.   Comments on Key Model Parameters

The subjective discount rate

The subjective discount rate assumed in this economy is roughly 12% per annum, which is high compared to the values that are usually used. As is mentioned before, high subjective discount rates are also found in buffer stock savings models (see Carroll (1997) for a review) and are empirically well supported. In this subsection, I illustrate how equity market non-participation is related to the choice of the subjective discount rate.

In Figures 7, I plot the lifecycle wealth targets and non-participation wealth level cutoffs for the two skill groups. Observe that at $\beta > 0.9$, working low skill households (which have accumulated to their wealth targets), would begin to participate in the equity market at a $400 per annum fixed cost of stock ownership. Observe from Figure 7 that the β for the U.S. economy could be estimated using this model by examining the median U.S. household wealth. The median wealth for the bottom 80% income households in 1995 dollars is $37,639 and, for the top 20% households is $273,819 (1998 Survey of Consumer Finance).[20] At $\beta = 0.88$, the low skill average household wealth target is between

---

[20] Note that I would want to select wealth targets that are higher than the observed median incomes because a significant fraction of households in the economy would be accumulating toward their wealth targets and their actual wealth would be less than their targeted wealth.

$45K to $55K and the high skill average household wealth target is between $290K~310K. Since many households in this economy would have wealth much lower than their wealth target, particularly the young households, a $\beta = 0.88$ turns out to be appropriate for matching to the U.S. household wealth data.

The literature on the risk free rate puzzle has claimed that a $\beta > 1$ is needed to generate a net demand for a low yielding risk free asset.[21] However, at $\beta = 0.88$, this economy has a net demand for the risk free asset yielding 1%. This result is largely driven by the low skill households, who do not invest in stocks, but instead store their entire buffer-stocks in the risk free vehicle.

The fixed cost of stock ownership

The fixed cost of stock ownership is crucial in delivering non-participation in the equity market. Without assuming a fixed cost, there would be 100% equity market participation. Examining Figure 6, where portfolio allocations are plotted for economies with a fixed cost of $400 per annum, $200 per annum and $40 per annum. We can see that equity market non-participation cutoff wealth levels are decreasing in the assumed fixed cost. However, at wealth levels below $40K, the demand for stocks is negligible even when there is almost no fixed cost associated with owning stocks. This suggests that my economy can have significant non-participation with a much lower fixed cost assumption.

Labor income volatility

I examine how labor income volatility impacts equity market non-participation. I set wage fluctuations to zero and for both skill groups and examine the household lifecycle wealth targets and the implications on non-participation. I perform the identical exercise for wage fluctuations that are twice the baseline case (2 * 7.5% = 15% per quarter). The results are summarized in Table 4 and 5.

Table 4. Lifecycle Wealth Targets (in $1000) for Wage Fluctuation = 0% vs. 15%

---

[21] In the risk free rate puzzle literature, a positive net supply of risk free assets is usually assumed since models do not explicitly model the government, which is the single largest net borrower.

	Young	Middle-aged	Retired
Wage Volatility = 0%			
High Skill	313	285	0
Low Skill	42	47	0
Wage Volatility = 15% per quarter			
High Skill	364	330	0
Low Skill	53	63	0

Table 5. Equity Market Participation Cutoff Wealth Level (in $1000)
for Wage Fluctuation = 0% vs. 15%

	Young	Middle-aged	Retired
Wage Volatility = 0%			
High Skill	71	91	29
Low Skill	66	71	39
Wage Volatility = 15% per quarter			
High Skill	150	143	29
Low Skill	73	82	39

Note that the labor income risk is reduced only moderately when we remove wage fluctuations; as is mentioned before unemployment spills dominate labor income risk. Without wage fluctuations, we see reduced household wealth targets across the board. However, the non-participation cutoff wealth levels also fall to reflect the reduced covariance between labor income and stock returns. With increased wage fluctuations, the exact opposite occurs. Household wealth targets as well as non-participation cutoff wealth

targets increase across the board. Qualitatively, wage fluctuation volatility play almost no part in driving the non-participation result.

Job displacement risk

I show here that job displacement risk for the high skill households is quantitatively necessary to produce the right wealth targets for the high skill households, although it is not qualitatively necessary for delivering equity market non-participation since it does not impact the low skill households. Table 6 shows the wealth targets and non-participation cutoffs when job displacement probability is zero.

Table 6. Lifecycle Wealth Targets
and Equity Market Participation Cutoff Wealth Levels (in $1000)
when Displacement Probability = 0%.

	Young	Middle-aged	Retired
Wealth Target			
High Skill	142	159	0
Low Skill	47	52	0
Non-participation Cutoff			
High Skill	111	119	29
Low Skill	70	78	39

Without job displacement risk, high skill households do not have very high wealth targets and therefore do not accumulate very high wealth. Because unemployment rate and unemployment duration during expansions are lower for the high skill households than for the low skill households, high skill households in fact desire a lower wealth-to-income ratio than the low skill households, which is counterfactual. Job displacement shock,

which is permanent and a very real concern for high income households, accentuates labor income risk sufficiently to induce households to build up a much larger wealth than they otherwise would.

Retirement benefits

I consider a different retirement benefit scenario here to illustrate households' desire to save for retirement. Table 7 shows the household wealth targets and non-participation cutoffs when retirees of both skill levels receive only social security benefits with no additional pension income. The average social security income in 1995 dollars is equal to $7628 pretax, which I assume to equal $7000 taxed.

Table 7. Lifecycle Wealth Targets
and Equity Market Participation Cutoff Wealth Levels (in $1000)
when Pension Income = $7K per Annum for Both Skill Groups.

	Young	Middle-aged	Retired
Wealth Target			
High Skill	625	943	0
Low Skill	70	109	0
Non-participation Cutoff			
High Skill	71	89	29
Low Skill	65	66	39

Note that the wealth targets for both skill groups increase significantly. In this economy, despite impatience to delay current consumption to fuel future consumption growth, households are nonetheless averse to sharp drops in their consumption. House-

holds therefore also accumulate wealth to smooth falling consumption in retirement, in addition to hedging against unemployment. The lower the retirement pension income is relative to the working income, the higher is the wealth target for the middle-aged households.

Surprisingly, low pension income for the low skill households lead to equity market participation for the working low skill households! This is because low skill households must accumulate a very high wealth to smooth consumption in retirement, when pension income is very low. With a large investment portfolio, the fixed cost is comparably less significant and stock investment makes sense for a low skill household.

Power coefficient η

Finally, I examine the effect of increasing the power coefficient η, which increases the RRA for households at every level of wealth. Standard intuition would suggest that non-participation would be more severe. This is however not true in this model. Table 8 shows the wealth targets and equity market participation cutoff wealth levels when the power coefficient η is increased from 3 to 4.

Table 8. Lifecycle Wealth Targets

and Equity Market Participation Cutoff Wealth Levels (in $1000)

when $\eta = 4$

	Young	Middle-aged	Retired
Wealth Target			
High Skill	447	540	0
Low Skill	78	88	0
Non-participation Cutoff			
High Skill	148	165	29
Low Skill	72	81	39

The increased η, which accentuates the precautionary savings motive, makes low skill households accumulate greater wealth. Again the larger investment portfolio allows low skill households to take advantage of stock investments in the presence of fixed cost of stock ownership, leading to the counter-intuitive result that higher risk aversion coefficient generates greater equity market participation!

VI.   Limitations, Future Research and Conclusion

Limitations

In this model, households would only accumulate wealth when faced with risky labor income. While this may be descriptive of the economy in general, it does not seem to explain why some individuals with no apparent labor income risk, such as tenured professors, would accumulate high wealth. It is likely that the right model for understanding wealth accumulation for the high net worth individuals would involve heterogeneity in agent's subjective discount rates.

Future Research

While the paper aims to develop a realistic portfolio choice model, which I believe it does with good success, leisure and housing choices are conspicuously missing. Real estate

represents the largest asset owned by households. Whether this asset should be treated as a durable consumer item or investment is a very difficult question and should deliver interesting portfolio behaviors.[22] In addition, voluntary unemployment and retirement are not allowed in the model; households in this paper are simply assume to not value leisure and prefer to work whenever work is available. It is suspected that allowing households to voluntary withdraw from the labor force or to work beyond retirement age would allow for more aggressive portfolio choices.[23]

Attanasio, Banks and Tanner (2002), Brav, Constantinides and Geczy (2002) and Vissing-Jorgensen (2002) find empirical evidence that non-participation resolves the equity premium puzzle. Basak and Cuoco (1998) and Contantinides, Donaldson and Mehra (2002) assume non-participation in their general equilibrium model to deliver high equity premium. This suggests that extending this paper to a general equilibrium setting could help address important asset pricing questions, not the least of which includes the equity premium puzzle.[24]

Conclusion

This paper delivers high non-participation rate in the economy by assuming a two-sector labor market, where households endowed with high skill earn high income and households with low skill earn low income. Unlike recent papers on non-participation, this paper remains within the expected utility framework and assumes a HARA class utility. In addition, households are endowed with homogeneous preferences and information; in contrast, many recent papers on non-participation depend crucially on heterogeneous information or preferences to drive non-participation.

Impatience (driven by high subjective discount rate) and prudence (driven by volatile labor income) interact to generate stationary lifecycle wealth targets for households, with high skill households accumulating high wealth and low skill households accumulating

---

[22] Piazzesi, Schneider and Tuzel (2003) offer an innovative way of examining the housing choice issue and the asset pricing implications that arise.

[23] See Liu and Neis (2002) for an economy with voluntary retirement.

[24] Hsu (2003) extends the current framework in a general equilibrium framework to examine the equity premium puzzle.

low wealth in equilibrium. Subsistence consumption requirement implies that households have decreasing RRAs. Consequently, low skill households also have significantly higher local RRA. In addition low skill households have less human capital and therefore have lower diversification demand for stocks. Low wealth, high RRA and low diversification demand predicts that low skill households do not hold stocks in the face of a moderate ownership cost.

The model generates testable predictions, which can be used to assess its merit. The calibrated labor income process, which has counter-cyclical volatility, generates portfolio behaviors, which suggest that there is a greater aggregate willingness to supply risky capital during expansions. This is consistent with the empirical finding of a counter-cyclical equity premium. The model also predicts humped lifecycle wealth accumulation as well as humped lifecycle stock allocation; both are consistent with empirical observations. In addition, the model offers some surprising predictions. Specifically, all else being equal, households with lower pension income accumulated greater wealth prior to retirement, which increase their probability of owning stocks. Furthermore, households who are endowed with higher power coefficient (risk aversion coefficient) are also more likely to accumulate greater wealth, which again increase their probability of owning stocks.

Reference:

Aiyagari, S. R., (1994), Uninsured Idiosyncratic Risk and Aggregate Saving, *Quarterly Journal of Economics*, Volume 109, Issues 3, 659-684.

Ameriks, J. and S. P. Zeldes, (2001), How Do Household Portfolio Shares Vary With Age?, working paper, Columbia University.

Ang, A., G. Bekaert and J. Liu, (2002), Why Stocks May Disappoint, working paper, Columbia University.

Attanasio, O., J. Banks and S. Tanner, (2002), Asset Holding and Consumption Volatility, *The Journal of Political Economy*, Volume 110, Issue 4, 771-792.

Attanasio, O. and S. J. Davis, (1996), Relative Wage Movements and the Distribut6ion of Consumption, *The Journal of Political Economy*, Volume 104, Issue 6, 1227-1262.

Barsky, R. B., M. S. Kimball, F. T. Juster and M. D. Shapiro, (1997), Preference parameters and behavioral heterogeneity: An experimental approach in the health and retirement survey, *Quarterly Journal of Economics*, Volume 112, Issue 2, 537-579.

Basak, S., (1999), On the fluctuations in consumption and market returns in the presence of labor and human capital: An equilibrium analysis, *Journal of Economic Dynamics and Control*, Volume 23, 1029-1064.

Basak, S. and D. Cuoco, (1998), An Equilibrium Model with Restricted Stock Market Participation, *The Review of Financial Studies*, Volume 11, Issue 2, 309-341.

Bodie, Z. and D. B. Crane, (1997), Personal Investing: Advice, theory and evidence, *Financial Analysts Journal*, Volume 53, No. 6, 13-23.

Bodie, Z., R. C. Merton, and W. F. Samuelson, (1992), Labor supply flexibility and portfolio choice in a life cycle model, *Journal of Economic Dynamics and Control*, Volume 16, 427-449.

Blume M. E. and S. P. Zeldes, (1994), Household stockownership patterns and aggregate asset pricing theories, working paper, University of Pennsylvannia.

Brav, A., G. M. Constantinides and C. C. Geczy, (2002), Asset Pricing with Heterogeneous Consumers and Limited Participation: Empirical Evidenc, *The Journal of Political Economy*, Volume 110, Issue 4, 793-824.

Campbell J. Y., (1998), Asset Prices, Consumption, and the Business Cycle, NBER working paper No. 6485.

_____, (2000), Asset Pricing at the Millennium, *The Journal of Finance*, Volume LV, No. 4, 1515-1567.

Cao, H. H., T. Wang and H. H. Zhang (2003), Model Uncertainty, Limited Participation and Asset Prices, working paper, University of North Carolina, Chapel Hill.

Carroll, C. D., (1992), The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence, *Brooking Papers on Economic Activity*, Issue 2, 61-156.

_____, (1997), Buffer-Stock Saving and The Life Cycle/Premanent Income Hypothesis, *The Quarterly Journal of Economics*, Vol. CXII, Issue 1, 1-55.

_____, (2000), Portfolios of the Rich, working paper, Johns Hopkins University.

Chan, Y. L., and L. Kogan, (2002), Catching Up with the Joneses: Heterogeneous Preferences and the Dynamics of Asset Prices, *The Journal of Political Economy*, Volume 110, Issue 6, 1255-1285.

Claus, J. and J. Thomas, (2001), Equity Premia as Low as Three Percent? Evidence from Analysts' Earnings Forecasts for Domestic and International Stock Markets, *The Journal of Finance,* Volume LVI, No. 5, 1629-1666.

Cochrane, J. H., (1997), Where is the market going? Uncertain facts and novel theories, *Economic Perspective*, Federal Reserve Bank of Chicago, 3-37.

Cogley, T., (2002), Idiosyncratic risk and the equity premium: evidence from the consumer expenditure survey, *Journal of Monetary Economics*, Volume 49, 309-334.

Cohn, R. A., W. G. Lewellen, R. C. Lease and G. G. Schlarbaum, Individual Investor, (1975), Risk Aversion and Investment Portfolio Composition, *The Journal of Finance,* Volume 39, 605-620.

Constantinides, G. M, (2003), Rational Asset Prices, working paper, University of Chicago.

Constantinidies, G. M., J. B. Donaldson, and R. Mehra, (2002), Junior Can't Borrow: A New Perspective on the Equity Premium Puzzle, working paper, University of Chicago.

Cooley, T. F. (ed.), (1995), *Frontier of Business Cycle Research*, Princeton: Princeton University Press.

Cooley, T. F. and M. Ogaki, (1996), A Time Series Analysis of Real Wages, Consumption, and Asset Returns, *Journal of Applied Econometrics*, Volume 11, Issue 2, 119-134.

Davis, S. J. and Willen, P., (2000), Using Financial Assets to Hedge Labor Income Risks: Estimating the Benefits, working paper, University of Chicago.

Deaton, A., (1991), Saving and Liquidity Constraints, *Econometrica*, Volume LIX, No. 5, 1221-1248.

Donaldson, R. G., M. J. Kamstra, and L. A. Kramer, (2003), Stare down the barrel and center the crosshairs: Targeting the ex ante equity premium, working paper, Federal Reserve Bank of Atlanta.

Duffie D. and G. M. Constantinides (1996), Asset Pricing with Heterogeneous Consumers, *The Journal of Political Economy*, Volume 104, No. 2, 219-241.

Epstein, L. G. and S. E. Zin, (1989), Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework, *Econometrica*, Volume 57, Issue 4, 937-969.

Fama, E. F. and K. R. French, (2002), The Equity Premium, *The Journal of Finance* Volume LVII, No. 2, 637-659.

Fernández-Villaverde, J. and D. Krueger, (2002), Consumption over the Life Cycle: Some Facts from Consumer Expenditure Survey Data, working paper, University of Pennsylvania.

Friedman, M., (1957), *A Theory of Consumption Function*, Princeton, N.J.: Princeton University Press.

_____, (1963), Windfalls, the 'Horizon' and Related Concepts in the Permanent Income Hypothesis, in *Measurement Economics: Studies in Mathematical Economics and Econometrics*, edited by Carl F. Christ et al., Stanford, CA: Stanford University Press.

Friend, I. And M. E. Blume, (1975), The Demand for Risky Assets, *The American Economic Review*, Volume 65, Issue 5, 900-922.

Gist, J. R., (2000), Wealth Distribution in 1998: Findings from the Survey of Consumer Finances, memo, Public Policy Institute AARP.

Griliches, Z., G. Maddala, R. Lucas, and N. Wallace, (1962), Notes on Estimated Aggregate Quarterly Consumption Function, *Econometrica*, Volume 30, 491-500.

Haliassos, M. and C. Bertaut (1995), Why Do So Few Hold Stocks, *Economic Journal*, Volume 105, 1110-1129.

Hansen, G. D and A. Imrohoroglu, (1992), The Role of Unemployment Insurance in an Economy with Liquidity Constraints and Moral Hazard, *The Journal of Political Economy*, Volume 100, Issue 1, 118-142.

Heaton, J. and D. J. Lucas, (1996), Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing, *The Journal of Political Economy*, Vol. 104, Vo. 3, 443-487.

——————, (1997), Market Frictions, Savings Behavior, and Portfolio Choice, *Macroeconomic Dynamics*, 1, 76-101.

——————, (1999), Market Frictions, Stock prices and fundamentals, in Ben Bernanke and Julio Rotemberg, eds., *Macroeconomics Annual 1999*, NBER (MIT Press, Cambridge, MA).

——————, (2000), Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk, *The Journal of Finance*, Volume 55, No. 3, 1163-1198.

Hong, H., J. D. Kubik, and J. C. Stein, (2001), Social Interaction and Stock-Market Participation, working paper, Stanford University.

Hsu, J. C., (2001), More Than Our Wildest Expectation: the Equity Premium Puzzle as Money Falling out of the Sky, working paper, ULCA.

——————, (2003), Equity Premium for Households with Heterogeneous Skill and Risky Labor Income, working paper, ULCA.

Hubbard, R. G., J. Skinner and S. P. Zeldes, (1995), Precautionary Saving and Social Insurance, *Journal of Political Economy*, Volume 103, No. 2., 360-399.

Imrohoroglu, Ayse, (1989), Cost of Business Cycles with Indivisibilities and Liquidity Constraints, *The Journal of Political Economy*, Volume 97, No. 6, 1364-1383.

Jagannathan, R. and N. R. Kocherlakota, (1996), Why Should Older People Invest Less in Stocks Than Younger People?, *Federal Reserve Bank of Minneapolis Quarterly Review*, 11-23.

Kennickell, A., (2000), Recent Changes in U.S. Family Finances: Results from the 1998 Survey of Consumer Finances, *Federal Reserve Bulletin*.

Kimball, M. S., (1990a) Precautionary Saving and the Marginal Propensity to Consume, NBER working paper no. 3403.

——————, (1990b) Precautionary Saving in the Small and in the Large, *Econometrica*, Volume LVIII, 53-73.

Krusell, P. and A. A. Smith Jr., (1998), Income and Wealth Heterogeneity in the Macroeconomy, *Journal of Political Economy*, Volume 106, No. 5, 867-896.

Liu, J. and E. Neis, (2002), Endogenous Retirement and Wealth Shocks, working paper, UCLA.

Lucas, D. J., (1994), Asset pricing with undiversifiable income risk and short sales constraints: Deepening the equity premium puzzle, *Journal of Monetary* Economics, Volume 34, 325-341.

NYSE Fact Book 2001, (2002), New York Stock Exchange, Inc., Library of Congress, No. 56-10699.

Mankiw, N. G. and S. Zeldes, (1991), The consumption of stockholders and nonstockholders, *Journal of Financial Economics*, Volume 29, 97-112.

Mohabbat, K. A. and E. O. Simos, (1977), Consumer Horizon: Further Evidence, *The Journal of Political Economy*, Volume 85, No. 4, 851-858.

Morin, R. A. and A. Fernandez Suarez, (1983), Risk Aversion Revisited, *The Journal of Finance*, Volume 38, No. 4, 1201-1216.

Piazzesi, M., M. Schneider and S. Tuzel, (2003), Housing, Consumption and Asset Pricing, working paper, UCLA.

Rabin, M., (2000), Risk Aversion and Expected-Utility Theory: A Calibration Theorem, *Econometrica*, Volume 68, No. 5, 1281-1292.

Starr-McCluer M., (1998), Stock Market Wealth and Consumer Spending, working paper, Federal Reserve Board of Governors.

Storesletten, K., C. I. Telmer, and A. Yaron, (2001), Asset Pricing with Idiosyncratic Risk and Overlapping Generations, working paper, Stockholm University.

VanDerhei, J., R. Galer, C. Quick and J. Rea, (1999), 401(k) Plan Asset Allocation, Account Balances and Loan Activity, *EBRI Issue Brief*, Issue 205.

Vissing-Jorgensen, A., (1999), Limited Stock Market Participation and the Equity Premium Puzzle, working paper, University of Chicago.

_____, (2002a), Towards an explanation of household portfolio choice heterogeneity: Nonfinancial income and participation cost structures, working paper, University of Chicago.

——————, (2002b), Limited Asset Market Participation and the Elasticity of Intertemporal Substitution, *The Journal of Political Economy*, Volume 110, No. 4, 825-853.

Weil, D. N., (1994), The Saving of the elderly in Micro and Macro Data, *The Quarterly Journal of Economics*, Volume 109, No. 1, 55-81.

Weil, P., (1989), The Equity Premium Puzzle and the Risk-Free Rate Puzzle, *Journal of Monetary Economics*, Volume 24, Issue 2, 401-421.

Weil, P., (1993), Precautionary Savings and the Permanent Income Hypothesis, *The Review of Economic Studies*, Volume 60, Issue 2, 367-383.

Yoo, P. S., (1994), Age Dependent Portfolio Selection, working paper, Federal Reserve Bank of Saint Louis.

Zeldes, S. P., (1989), Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence, *Quarterly Journal of Economics*, Volume CIV, 275-98.

Figure 1. Age-Skill Cohort Consumption Plan and the Implied RRA
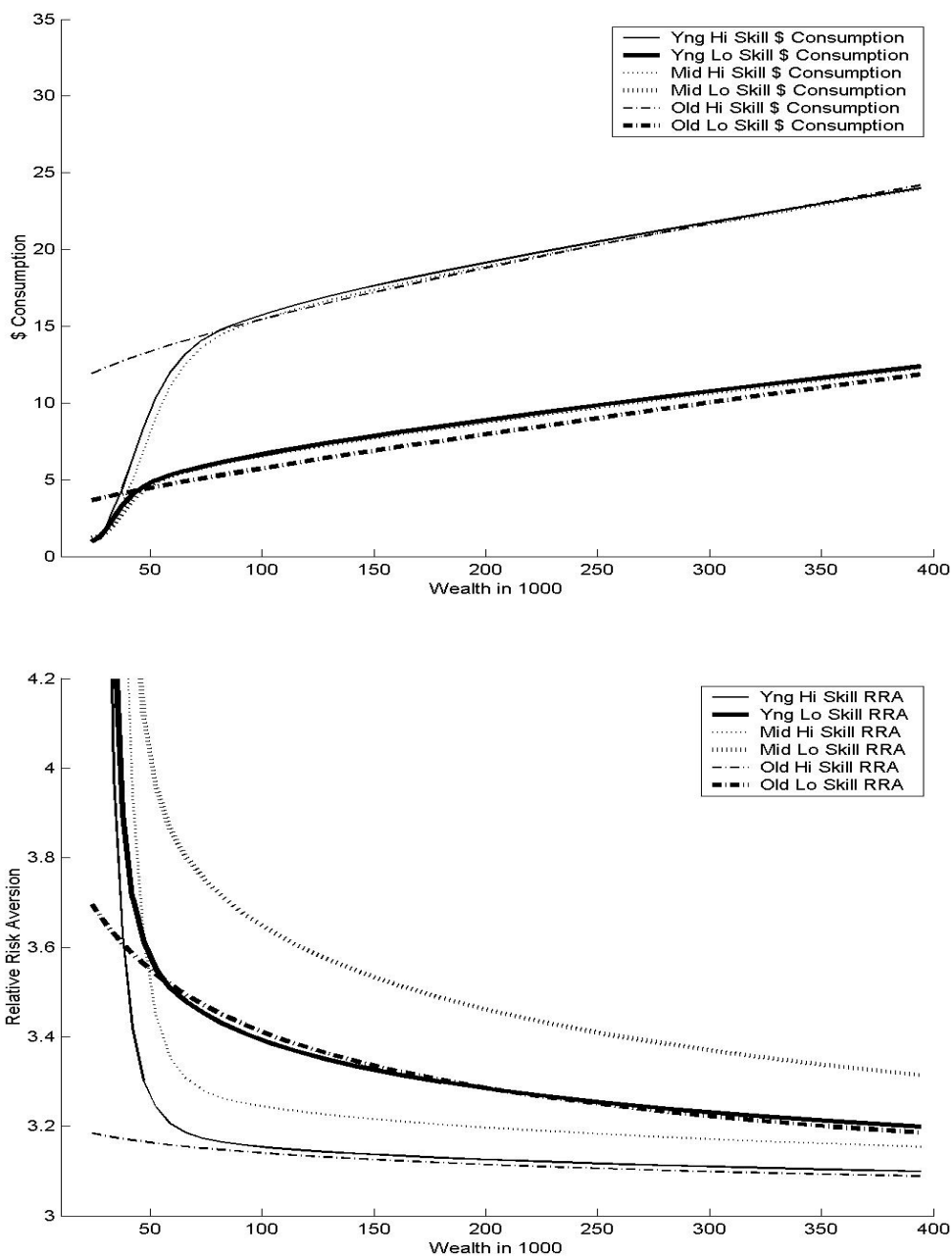
The top graph plots the average consumption plan for each age-skill cohort (young-high skill, young-low skill, etc.). The bottom graph plots the implied relative risk aversion (RRA) associated with the consumption plan. Note that the high skill group almost always has higher steady state consumption and thus lower RRA at every level of wealth. Within the same skill group, the retirees (old) are less relatively risk averse than the young; this is because retirees received guaranteed pension income. The young, in turn, are less relatively risk averse than the middle-aged; this is because the middle-aged households have a high subsistence consumption level (due to an increase in household size).

Figure 2. Age-Skill Cohort Quarterly Savings Plan
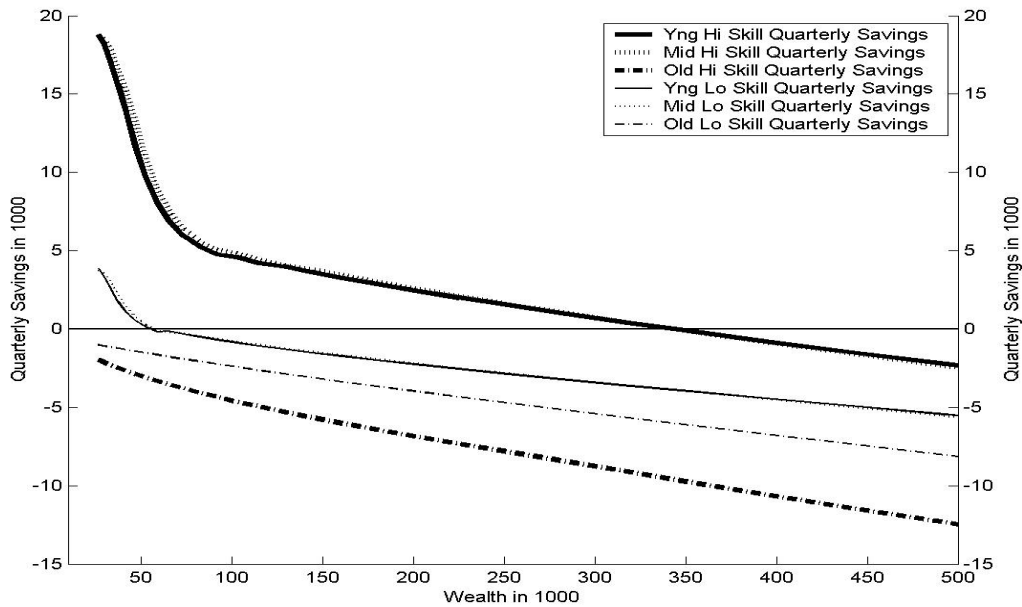

The quarterly savings plans are plotted for the different age-skill cohort (young and high skill, young and low skill, etc.). For the most part, the young and middle-aged high skill households have very similar savings pattern; this is similarly true for the young and middle-aged low skill households. Note that the retirees do not save in general. This graph helps us identify graphically the lifecycle wealth targets for the two skill groups; the wealth target occurs at where the savings plan crosses zero (on the y-axis). Non-retired high skill households have an average wealth target around $290K, while non-retired low skill households have an average wealth target around $50K. Retirees from both skill groups have wealth targets of zero.

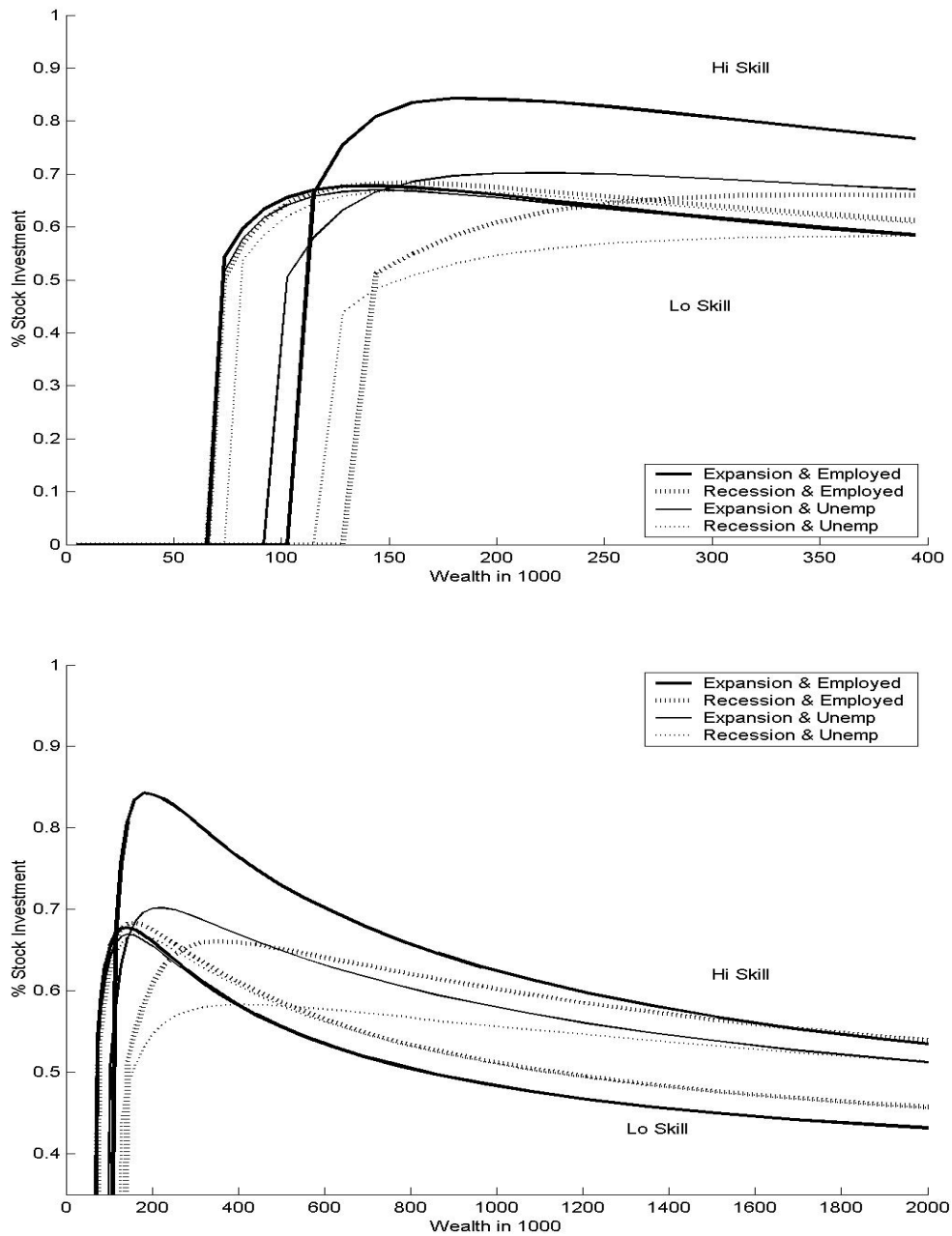**Figure 3a. Conditional Stock Allocation for Young Households**
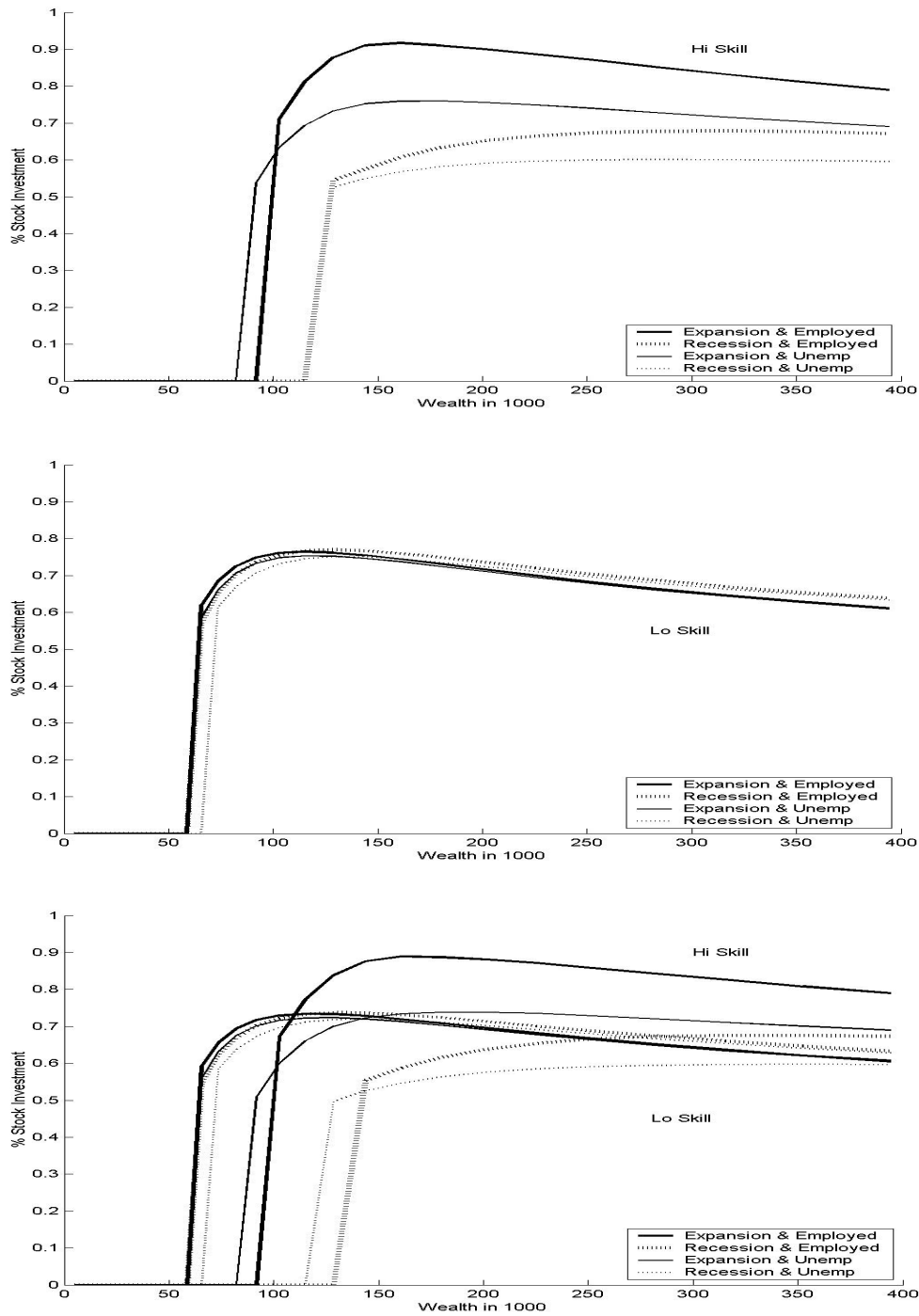
The stock allocation policy is plotted for the high skill young household vs. the low skill young household. We see that the low skill household is willing to enter the equity market at a lower wealth level than the high skill household. This is due to the lower covariance between low skill wage rate fluctuations and equity returns. The high skill household's portfolio allocation is also more sensitive to business cycle fluctuations.

**Figure 3b. Conditional Stock Allocation for Young Households**
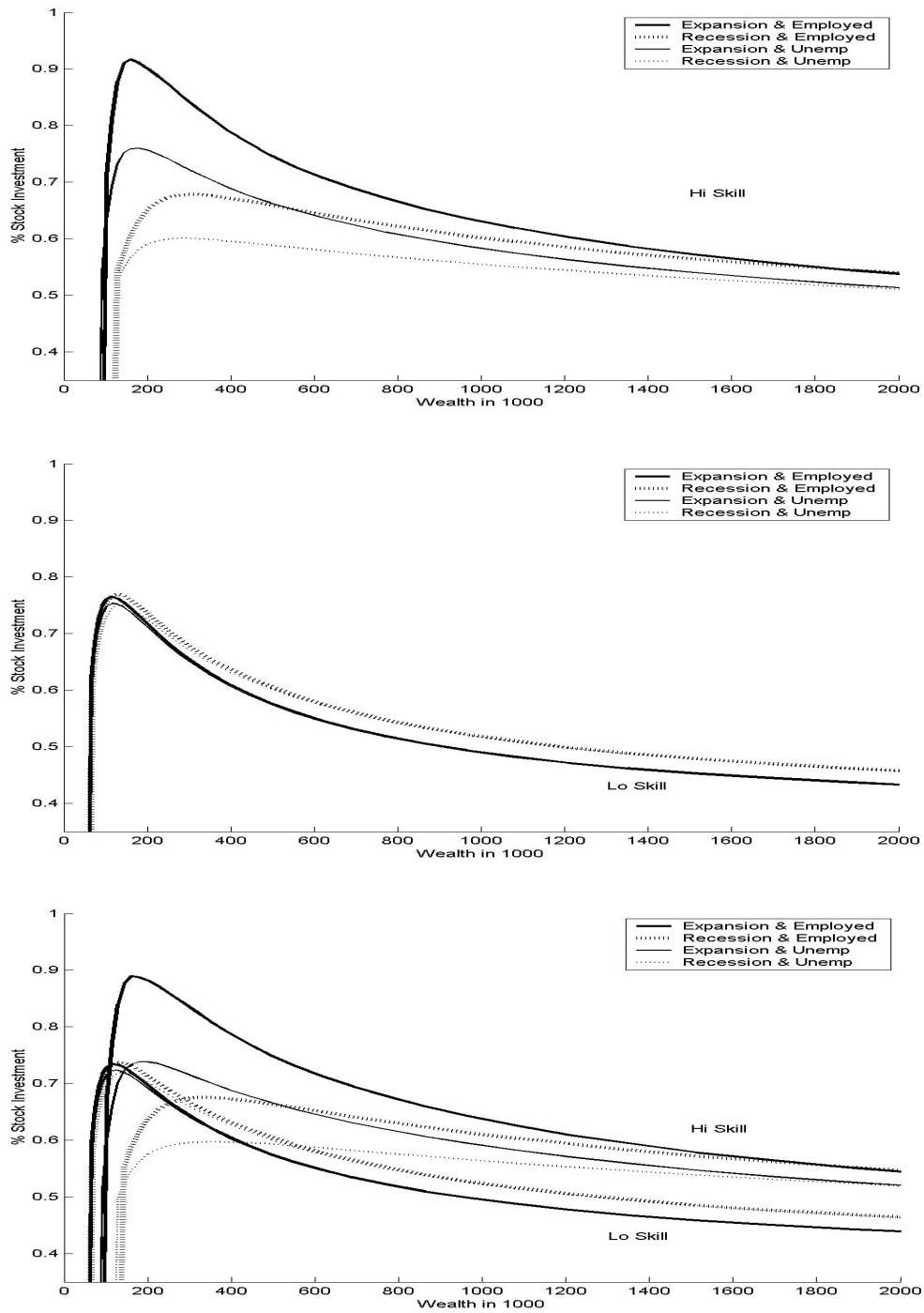
The stock allocation policy is plotted for the high skill young household vs. the low skill young household for an extended range of wealth levels. We see that the high skill household has more aggressive stock allocation while employed and more conservative stock allocation while unemployed when compared to the low skill household. This is because the high skill household faces the additional risk of job displacement when it is unemployed.

**Figure 4. Conditional Stock Allocation for Middle-Aged Households**

The stock allocation policy is plotted for the high skill middle-aged household vs. the low skill middle-aged household. The comparison between the high skill and low skill households is similar to what is presented for Figure 3 for the young households. Contrasting with the portfolio policy for the young in Figure 3, we see that middle-aged households of both skill types hold less aggressive stock allocations than their younger counterparts. This is because households invest in stocks to diversify their human capital portfolio. Since young households have the greatest amount of human capital, they desire a greater equity exposure to hedge their human capital risk.

**Figure 5. Conditional Stock Allocation for Retired Households**
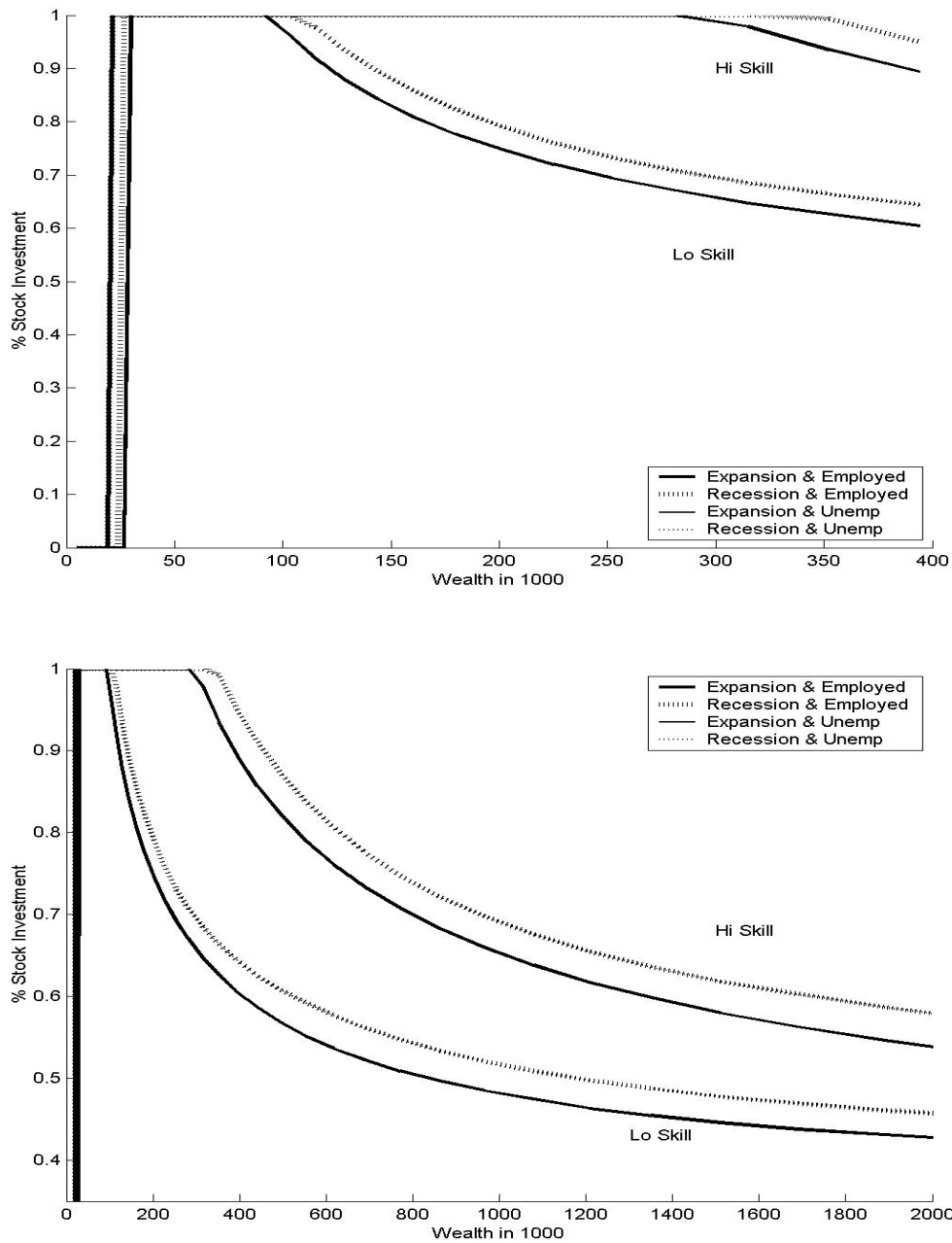
The stock allocation policy is plotted for the high skill retired household vs. the low skill retired household. Note that retired households no longer face labor income risk; they receive a fixed pension payment every period until they die. Without labor income risk, retirees' conditional consumption growth volatility is significantly reduced compared to their younger counterparts; as a result they are able to bear more equity risk. In particular, the high skill retirees find the 4.5% expected equity return overwhelmingly more attractive than the 1% risk free return and do not hold any bank deposits at wealth levels below $270K.

Figure 6. Portfolio Allocations when there is Fixed Cost vs. No Fixed Cost
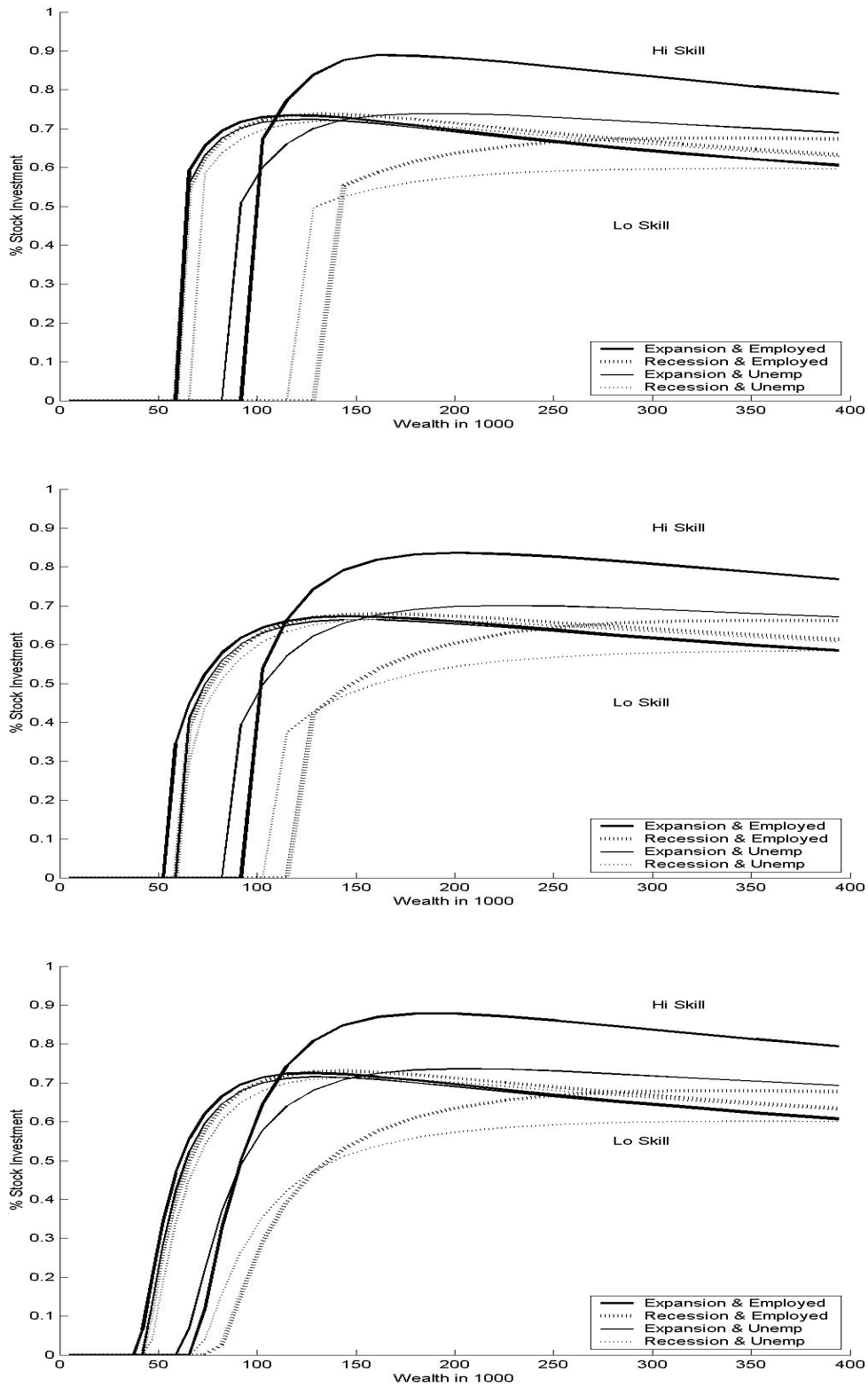
The graphs plot, respectively, portfolio allocations (for young households) with a fixed cost of stock ownership equaling $400, $200 and $40 per annum. Note that at wealth levels below $40K, the demand for stock is negligible.

Figure 7. Lifecycle Target Wealth Level vs. Equity Market Participation Cutoff Wealth Level
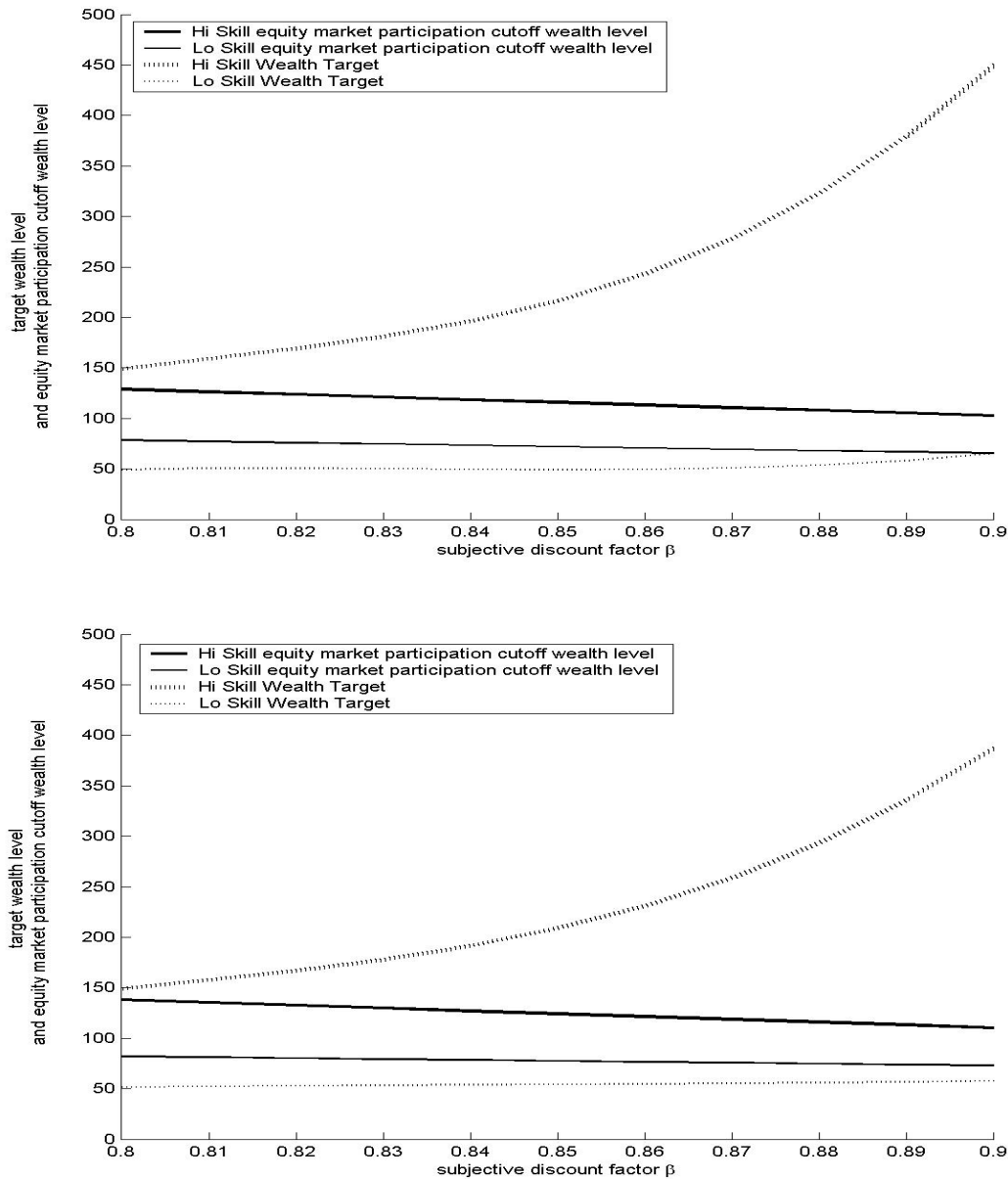
The top plot plots the target wealth level and the equity market participation cutoff wealth level for young high skill vs. low skill households for various values of the subjective discount factor β. The bottom plot shows the same for middle-aged households. Note that we can use the observed median household wealth level for the two groups to help us select a reasonable value for β. From the 1998 Survey of Consumer Finance, the median household wealth for the top 20% income households and bottom 80% income households are respectively \$273.8K and \$37.6K. This suggest that $\beta = 0.88$ would be appropriate. The participation cutoff wealth level depends on the assumed fixed cost of stock ownership. We select the fixed cost to ensure that low skill households do not participate.