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MARKET SHARE ELASTICITIES FOR FUEL AND TECHNOLOGY CHOICE IN HOME HEATING AND COOLING*

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ABSTRACT

A new technique for estimating own- and cross-elasticities of market share for fuel and technology choices in home heating and cooling is presented. We simulate changes in economic conditions and estimate elasticities by calculating predicted changes in fuel and technology market shares. Elasticities are found with respect to household income, equipment capital cost, and equipment operating cost (including fuel price). The method is applied to a revised and extended version of a study by the Electric Power Research Institute (EPRI). Data for that study are drawn primarily from the 1975 - 1979 Annual Housing Surveys.

Results are generally similar to previous studies, although our estimates of elasticities are somewhat lower. We feel the superior formulation of consumer choice and the currency of data in EPRI's work produce reliable estimates of market share elasticities.

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MARKET SHARE ELASTICITIES FOR FUEL AND TECHNOLOGY CHOICE IN HOME HEATING AND COOLING

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1. Introduction

Concern over future energy demand has prompted several researchers to develop predictive computer models of national energy use. A significant factor in such models is the efficiency of home appliances, in particular space heating and cooling equipment, which account for the largest components of residential energy consumption. An example is the Oak Ridge Residential Energy Demand Forecasting Model, which LBL has modified substantially to create the LBL Residential Energy Model [15].

These models require accurate estimates of how the market shares of different fuel choices (electricity, gas, or oil) or technologies (centralized vs. decentralized systems) change in response to changes in the economy. Thus, estimates of the own- and cross-elasticities of the market share of different fuel choices (with respect to capital costs, operating costs, and income) are commonly found as inputs to the modeling effort. This paper describes one method of developing such estimates.

The unique features of our approach are: 1) the derivation of aggregate market share elasticities from individual household data; 2) the linkage of space heating to air conditioning choice; 3) explicit treatment of individual technologies, including heat pumps; and 4) the direct calculation of new market shares due to changes in the explanatory variables. The first three of these features derive from the corresponding aspects of a study by the Electric Power Research Institute [6], which we take as a starting point for our work. The last feature is a direct consequence of the manner in which we develop the elasticities, and is exploited in the LBL Residential Energy Model to provide forecasts of future fuel and technology choices by households. Combining these elasticities with existing techniques for forecasting efficiency choice, thermal integrity of the building shell, and usage behavior results in substantial improvement of the forecasting model.

The study by EPRI has two important features which improve upon previous studies of fuel type market share in residential space heating.

1) It uses a database of individual households and actual technology choices to estimate the coefficients of an equation which relates the probability of selecting among different space heating technologies to household and geographic characteristics. The average of these probabilities over all households can be taken as the predicted market share of that technology. Summing over technologies of a given fuel type provides an estimate of the market share of that fuel. Almost all previous studies of fuel type market share relied on state-level aggregated data, which introduces bias in the estimates.

2) Each household's choice of heating technology is modeled jointly with the choice of central cooling. Since the cost of heating system chosen (central or non-central) depends highly on the presence of central air conditioning, it seems unwise to model the two choices independently. Statistical cross-tabulations verify that heating system choice is correlated with cooling choice. The EPRI study is the first to model that linkage explicitly.

Like most previous studies, our method starts from regression equations. (We use modified versions of the coefficients estimated by EPRI; see Wood, Ruderman, & McMahon [17] for a discussion of this point.) The classical approach to finding elasticities from such equations requires calculating analytic derivatives of market share and using those derivatives (at the mean values of the exogenous variables) in the formula for the elasticity. This approach has two defects, one predictable by theory and one purely practical.

The theoretical defect is that this approach does not determine an overall market elasticity, but rather the elasticity of the "mean" household, which may be completely unrepresentative of the overall market. McFadden and Reid [14] have shown that using only mean values in elasticity calculations neglects the variation of the independent variables and introduces non-trivial bias in the elasticities and all market shares derived from them. The potential effects of this bias are explored in Wood, Ruderman, & McMahon [18], where significant errors are shown to be likely.

The practical defect is that, given the technology-specific choices, the nested logit model, and the normalized exogenous variables used by EPRI, calculating market share elasticities analytically turns out to be so time-consuming as to be judged unprofitable (given the first defect). Instead, we use a "simulation" and "sample enumeration"[1] approach.

We perturb slightly the economic factor for which we want elasticities (e.g., price of electricity, capital cost of a gas system, etc.). For each household in the database, we calculate the new probability (under the perturbed conditions) of choosing each technology, using the regression equation, and determine the change in probability. These changes in probabilities are averaged across households, giving an estimate of the change in market share due to the perturbation of that economic factor. Dividing by the relative size of the perturbation and the original market share gives the appropriate arc elasticity, as a function of the perturbation size. These estimates are approximately quadratic in the perturbation size.

We calculate the arc elasticity as a function of perturbation size for thirteen different perturbations (ranging from -33% to +50%) and fit a least-squares quadratic curve to the results. The intercept of this curve (equivalent to a perturbation size approaching a limit of zero) is taken as our value for the point elasticity. A summary of our own-elasticity results is shown in Table 1.

Table 1

Market Share Own-Elasticities for Space Conditioning Equipment

Equipment	Capital Cost	Energy Cost	Income
Gas Central Heater	-0.47	-0.56	0.06
Gas Non-Central Heater	-1.60	-0.17	-0.57
Oil Central Heater	-1.29	-1.27	-0.27
Oil Non-Central Heater	-1.48	-0.87	-0.59
Electric Central Heater	-0.86	-1.14	-0.09
Electric Non-Central Heater	-1.16	-0.92	-0.25
Conventional Central Cooling	-0.36	-0.24	0.29
Heat Pump	-2.03	-0.79	0.32

[1] "Simulation" refers to the method of perturbing independent variables and noting the changes in predicting market share, hence we "simulate" changes in economic conditions. "Sample enumeration" refers to the calculation of an aggregate market share by calculating a probability of choice for every individual household in the aggregate. The term arises from transportation modeling literature.

The organization of this report is as follows: Section 2 gives a brief survey of several prior studies and a comparison with the results of our method when applied to a revised version of the EPRI study. Section 3 examines the study by EPRI in detail, as it forms the foundation for our work. Section 4 examines our method of deriving elasticities in more detail, with attention to the numerical and modeling issues that arise. Section 5 points out the limitations of our work and associated statistical caveats. There are three appendices: Appendix 1 gives complete results of our estimated elasticities in tabulated form; Appendix 2 discusses several issues related to the derivation and use of a quadratic curve fitted to the arc elasticities; and Appendix 3 explains how these elasticities are incorporated into the LBL Residential Energy Model.

2. Prior Studies

There have been a variety of studies attempting to model consumer space heating fuel choice. This section will briefly review some of the more significant ones. Table 2 shows the elasticities from several of these studies, as well as the comparable results from our work. More extensive literature reviews may be found in Dohrmann [4] and in Hartman [7]. A complete review of issues and techniques in discrete choice modeling is in Amemiya [1].

Anderson [2] modeled fuel type market share by regressing the natural logarithm of the ratio of two fuels' market shares on explanatory variables of household size, fraction of non-urban and single-family housing units, household income, mean December temperature, and prices of the two fuel types. The regression technique was generalized least squares on a database drawn from the 1970 Census. A straight-forward calculation derived in Dohrmann [4] shows how to find elasticities from such a formulation.

Baughman and Joskow [3] used state level data from the 1970 Census to estimate a similar model. Dependent variables were essentially the same as Anderson; explanatory variables were income per capita, mean January temperature, and fuel prices. Both Baughman and Joskow and Anderson used regression techniques which required that all the price cross-elasticities of a given fuel type be equal.

Lin, Hirst and Cohn [10] carried out a study including equipment prices as an explanatory variable. Their dependent variable was the natural logarithm of the ratio of a fuel's market share to all other fuels' shares. Independent variables were fuel prices, equipment prices, income, and heating and cooling degree days. Data used in estimating the parameters were from a cross section of forty-eight states in 1970.

Hartman and Hollyer [8] examined several models and alternative datasets for fuel choice in residential space heating. The elasticities shown in Table 2 are based on state data, pooled from both 1960 and 1970. Like Anderson, and Baughman and Joskow, the model specification used the natural logarithm of the ratio of two fuels' market shares as the dependent variable. Unlike those earlier studies, however, Hartman and Hollyer included the prices of all fuel types as explanatory variables for the market share ratio of any two fuels. Other exogenous variables were the capital cost of the equipment for all fuel types, per capita income and heating degree days averaged for the states, and an index of gas availability for each state.

Dohrmann [4] estimated a model generally similar to those discussed above. The primary element added by his work was the separation of observations (at SMSA level, from Annual Housing Surveys of 1974, '75, and '76) into cells according to fuel availability. Fuel availability distinctions were made for SMSAs with and without fuel oil as an option for residential heating, and for SMSAs which did and did not experience explicit gas restrictions over the period of the study data. Coefficients were estimated separately for each cell. Dependent and independent variables were generally similar to previous work.

Dubin and McFadden [5] modeled appliance utilization simultaneously with appliance choice in a discrete/continuous choice model. The joint model is important for estimating long-run elasticities of fuel consumption. In their words,

> If, as the theory would suggest, the demand for durables and their use are related decisions by the consumer, specifications which ignore this fact will lead to biased and inconsistent estimates of price and income elasticities.[2]

It is somewhat less important, however, for elasticities of market share. Indeed, they estimated market share as a function of exogenous variables in much the same manner as the other studies touched on here. The most significant element of their work for the estimation of market share elasticities was the use of household data for the regressions. Dubin and McFadden modeled the probability of a household choosing an all-electric system of water and space heat versus an all-gas system. Independent variables included the capital and operating costs of the systems, the product

[2] Dubin and McFadden [5], page 345

of capital cost and household income, the availability of gas to the household, the marginal cost of electricity, and a dummy for choice of the all-electric system. Elasticities shown in Table 2 were derived in their paper.

A comparison of the results from prior work reported in Table 2 shows generally similar estimates arising from different studies, but some overall issues and/or notable differences should be discussed.

1) The theoretical underpinning of all of these studies can be traced to a general idea of utility maximization by agents of choice (i.e., households). Thus, for each model there is an implicit utility function to be maximized and an implicit distribution of the uncertain components of utility preference. (It does not matter whether the "error" is taken to be uncertain to the agent of choice, to the econometric model-maker, or both.) In order to be consistent with this formulation, data should be at a level as close to the level of the household as possible. Studies by both EPRI [6] (from which our work follows) and Dubin and McFadden [5] satisfy these criteria.

   Most of the other studies discussed here used some form of aggregated state level data. Hartman and Hollyer used a representative "urban" and "rural" value for each state, and Dohrmann used data by SMSA. Use of aggregated data implies an assumption that all of the consumer choice attributes in the aggregated set can be adequately captured by a single appropriate number. While this is not impossible, Hartman and Hollyer compared results with and without the urban/rural split in their data and suggested that the distinction is important. Unfortunately, they gave no conclusive evidence that the division along the lines they made is sufficient to fully capture the effects of heterogeneity. Lacking evidence showing that households are well represented by data aggregated over some larger unit (i.e., the SMSA or state), studies which estimate coefficients from individual household data are preferred over those that take a more aggregated unit.

2) Related to the issue of using properly disaggregated (i.e., household) data of consumer characteristics and choice is the desirability of properly disaggregating by end use and fuel type. The Baughman and Joskow model sought to model total state level energy consumption and fuel shares. Disaggregation into end uses such as space heating was by relatively crude techniques. As such, its elasticity estimates for individual end uses are somewhat suspect in comparison to estimates based on specific end use data.

3) Also related to the validity of the underlying model of consumer choice is the use of new installations vs. existing housing in the dataset. Consumers may well experience a considerable "inertia" in their choice of heating equipment; the preferred fuel or technology given today's capital and operating costs may be very different from the preferred fuel or technology 15 or 20 years ago. Thus, analysis can most properly capture the elements of utility maximization for new installations if the data is from new installations. Data on market share of fuel types in existing housing stock will provide different (and arguably incorrect) estimates. Baughman and Joskow and some of the formulations used by Hartman and Hollyer can be faulted on this count.

4) The choice of particular explanatory variables in a model can make a significant difference in the results of that model. The first three studies reported in Table 2 failed to include the capital cost of heating equipment as an independent variable, and Dohrmann included only a general index of all equipment prices, without distinguishing among different fuel types. As such, estimates of elasticities, even for the variables which were included, are somewhat suspect in all four studies. Dubin and McFadden limit their choice of data to households with all-electric and all-gas space and water heat, a restriction which eliminates oil as a possibility for space heat. This construction limits the applicability of these elasticities to only a subset of the national market. Dohrmann noted that data drawn from the late 1970's is affected by the curtailment of new residential service by many gas utilities at that time. He suggested that this may account for some of the unanticipated results of sign (gas share elasticity with respect to prices of electricity and oil) in his work. Both Dubin and McFadden and EPRI include specific measures of gas availability as exogenous variables in their studies.

5) Some of the studies failed to find the anticipated sign of elasticities (negative for own-elasticities, positive for cross-elasticities). Examples are Lin, Hirst and Cohn (oil share with respect to the price of electricity), Hartman and Hollyer (the same, plus electric and gas shares with respect to the other's capital cost), and Dohrmann (discussed above). Hartman attributed some of these counter-intuitive results in his work with Hollyer to "coefficients not significantly different from zero."[3]

In summary, we feel that the study by EPRI stands up well in comparison with past studies:

-- it uses an appropriate consumer utility maximization model (space heating and air conditioning chosen jointly), and data on individual households to support it;
-- data are drawn from new construction only;
-- all of the most likely significant explanatory variables are included;
-- it provides for considerable disaggregation by fuel and technology choice for residential space heating.

Elasticities derived from a revised and extended version of the EPRI study are reported in this paper.[4] Our results estimate elasticities that are generally lower than previous studies, i.e., we find that consumers respond to changes in fuel price, income, and other variables less strongly than previous studies have indicated.

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[3] Hartman [7], page 83

[4] These revisions are discussed briefly in Section 3 of this paper, and extensively in Wood, Ruderman, and McMahon [17].

Table 2

Price, Income, and Capital Cost Elasticities for Market Shares of Fuel Types in Home Heating

Anderson [2]

	Elec Price	Gas Price	Oil Price
Electric Share	-2.04	2.21	0.55
Gas Share	0.17	-1.80	0.55
Oil share	0.17	2.21	-1.58

Baughman and Joskow [3]

	Elec Price	Gas Price	Oil Price
Electric Share	-2.08	2.12	3.30
Gas Share	0.23	-1.48	3.30
Oil share	0.23	2.12	-7.21

Lin, Hirst, and Cohn [10]

	Elec Price	Gas Price	Oil Price
Electric Share	-3.19	0.38	1.09
Gas Share	0.57	-1.33	0.03
Oil share	-0.18	2.95	-1.01

Hartman and Hollyer [8] (existing housing stock)

	Elec Price	Gas Price	Oil Price	Income	Cap Cost Elec Heat	Cap Cost Gas Heat	Cap Cost Oil Heat
Electric Share	-3.86	0.28	2.34	0.95	-10.4	-1.82	10.3
Gas Share	1.31	-1.13	0.14	0.09	-2.01	-20.50	21.7
Oil share	-1.47	1.69	-0.52	-0.26	4.45	31.4	-34.4

Dohrmann [4]

	Elec Price	Gas Price	Oil Price	Income
Electric Share	-1.184	0.550	4.733	-2.191
Gas Share	-0.145	-0.749	-1.495	0.371
Oil share	3.476	1.577	-5.787	3.934

Dubin and McFadden [5]

	Elec Price	Gas Price
Electric Share	-0.473	1.41
Gas Share	0.144	-0.43

Lawrence Berkeley Labs / EPRI [this study]

	Elec Price	Gas Price	Oil Price	Income	Cap Cost Elec Heat	Cap Cost Gas Heat	Cap Cost Oil Heat
Electric Share	-1.07	0.59	0.14	-0.14	-0.65	0.54	0.14
Central	-1.14	0.61	0.14	-0.09	-0.65	0.49	0.12
Non-Central	-0.92	0.55	0.14	-0.25	-0.63	0.66	0.18
Gas Share	0.45	-0.56	0.11	0.05	0.23	-0.47	0.13
Central	0.45	-0.56	0.11	0.06	0.23	-0.46	0.13
Non-Central	0.73	-0.17	0.16	-0.57	0.20	-0.71	0.26
Oil share	1.81	0.42	-1.26	-0.28	0.18	0.37	-1.27
Central	1.82	0.42	-1.27	-0.27	0.17	0.37	-1.28
Non-Central	1.35	0.38	-0.87	-0.59	0.20	0.46	-0.94
Central A/C	-0.24	-0.18	0.02	0.29	-0.07	-0.04	0.02
Heat Pump	-0.79	0.67	0.13	0.32	0.40	0.45	0.08

3. Summary of Work by Electric Power Research Institute

In 1982 and 1983 the Electric Power Research Institute (EPRI) undertook a major effort to update the behavioral models embedded in the Residential End-use Energy Planning System (REEPS) model. That work is described in a report by EPRI's principal investigator, Andrew Goett [6]. This section will summarize the principal elements of that report, focusing on the models for space heating, which we have used to derive estimates of fuel and technology market share elasticities.

The REEPS model uses energy consumption estimates by technology, so EPRI created a model of market share of particular technology choices in space heating for new construction. They also model the choice of space heating fuel/technology combinations as being dependent on the choice of whether or not to have central air conditioning. Statistical evidence reported in their study shows an extremely high correlation between the choice of space heating fuel and the presence of central air conditioning, with important consequences to summer electric loads. The most extreme example of this is electric heat pump technology, which is used for both heating and cooling. The following passage from EPRI's report[5] describes the model structure:

> In order to capture interdependence in the appliance choice equations, the space heating and air conditioning decisions are modeled jointly with the dependent variable representing the choice of a space heat/air conditioning combination. The empirical specification is a generalized version of the multinomial logit known as the nested logit model. This functional form allows differential substitution between alternative appliance combinations. The multinomial logit does not allow such differential substitution since the relative odds of any two alternatives are independent of the availability and attributes of other alternatives. This is implausible for some applications. Consider, for example, an air conditioning market in which a consumer could choose between a conventional unit or no unit at all. Suppose a new technology--heat pumps--is introduced to the market. According to the multinomial logit model, the new technology would draw its market share proportionately from non-buyers and buyers of conventional units. Common sense suggests that the heat pump alternative would draw its market share primarily from conventional air conditioning because of their similarity.
>
> In order to capture this differential substitution, the nested logit model groups similar alternatives in a hierarchical structure. In the example of central air conditioning, the multinomial logit representation would treat the three alternatives (no a/c, conventional a/c, and heat pump) equivalently.... [In the nested logit representation,] the two central cooling options are grouped on a separate branch of the probability tree reflecting their close substitutability. Conceptually, the choice is broken down into two levels. At the upper level, the choice alternatives are cooling and no cooling. At the lower level, the alternatives are conventional air conditioning and heat pump, **given** the cooling choice. The decision at each level is represented by a multinomial logit model. The values of the explanatory variables in the lower level choice depend upon the branch of the probability tree. At the upper level, an index of the aggregate characteristics of the lower level alternatives is used as an explanatory variable in the choice model.
>
> For the application of the nested logit to the joint space heating and air conditioning decision, we specify the following general form for utility:
>
> $$U_{ij} = V_i + W_{ij} + \epsilon_{ij}$$
>
> where i indexes central air conditioning alternatives;
> j indexes space heating alternatives;
> U_{ij} is the total utility from a given ij combination;
> V_i is the typical or representative utility from central air conditioning alternative i;
> W_{ij} is the representative utility from air conditioning and space heating alternative ij;
> ϵ_{ij} are random components of utility reflecting unobserved characteristics and random consumer tastes.

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[5] Copyright © 1984, EPRI EA-3409, "Household Appliance Choice: Revision of REEPS Behavioral Models." Reprinted with permission.

Following McFadden [12], if the random terms are assumed to be distributed according to the following form of the Generalized Extremė Value (GEV) distribution:

$$F(\epsilon_{11}, \ldots, \epsilon_{ij}) = \exp\left\{ - \sum_i \left[\sum_j e^{\epsilon_{ij}/(1-\theta)} \right]^{(1-\theta)} \right\}$$

Then the conditional and marginal probabilities $P_{j|i}$ and P_i can be expressed in the following closed forms:

$$P_{j|i} = \frac{e^{W_{ij}/(1-\theta)}}{\sum_{j'} e^{W_{ij'}/(1-\theta)}}$$

and

$$P_i = \frac{e^{V_i + (1-\theta)J_i}}{\sum_{i'} e^{V_{i'} + (1-\theta)J_{i'}}}$$

where

$$0 \leq \theta \leq 1$$

and

$$J_i = \ln\left(\sum_{j'} e^{W_{ij'}/(1-\theta)} \right).$$

The parameters of these models can be estimated sequentially by first estimating the conditional probability models $P_{j|i}$, then calculating the index of the aggregate characteristics at the lower level:

$$J_i = \ln\left(\sum_{j'} e^{W_{ij'}} \right)$$

and using this in the conditional probability model P_i. The parameter estimates are consistent although not fully efficient. The standard errors of the estimates must be adjusted to account for the fact that the value J_i is estimated [by J_i] rather than observed.

The term θ is a measure of the correlation among the error terms. The special case presented here allows correlation among the random components for space heating given air conditioning. If $\theta = 0$, then the error terms are exogenous distributed and the specification is equivalent to the multinomial logit. Positive values indicate correlation among terms. Values outside the unit interval have not been shown consistent with a model of random utility maximization and imply some counterintuitive cross-elasticities.

EPRI uses this specification to model five space heating technologies given central cooling and eight space heating technologies given no central cooling, as detailed in Table 3. As such, their model is very specifically a technology choice model, rather than a fuel choice model as most previous studies have been. This actually makes estimating elasticities with respect to fuel price somewhat more complicated, as discussed in the next section.

The study uses data drawn from the Annual Housing Surveys of 1975 through 1979. Data on individual housing units and the demographic characteristics of their occupants are collected each year. EPRI used only data on new, single-family, owner-occupied housing, with income and the SMSA of each household included. Over 1300 individual households located in more than 120 SMSAs are identified.

For each household, the housing characteristics (e.g., size, insulation) and weather information for that SMSA were input to a thermal load model developed at MIT by McFadden and Dubin [13]. This model gives as output the necessary capacity for heating and cooling the household with each of the modeled technologies, as well as the projected energy consumption. EPRI translates these projections into operating costs using fuel price data, and into capital costs using

Table 3

Technology Choices in EPRI's Model of Space Heating and Cooling	
Central A/C	No Central A/C
1. Gas Forced Air	1. Gas Forced Air 2. Gas Hydronic 3. Gas Non-central
2. Oil Forced Air	4. Oil Forced Air 5. Oil Hydronic 6. Oil Non-central
3. Electric Forced Air 4. Electric Baseboard	7. Electric Forced Air 8. Electric Baseboard
5. Electric Heat Pump	

equipment and installation cost estimates from Means' *Building Construction Cost Data*. Both fuel prices and capital costs are taken for the year of construction (consistent with a modeling assumption of static consumer expectations) and converted to constant 1975 dollars. Capital cost data are calculated as incremental costs, i.e., for heating systems with central cooling, only the additional costs of the heating system are assigned to that choice. For systems that included both central heating and cooling, the cost of ducting is borne by the air conditioning system.

EPRI's model of space heating choice also includes the following variables:

-- Gas service restrictions are modeled by defining binary variables equal to one if the household is located in an SMSA served by a utility that had imposed such a restriction during the time of construction. Three types of restrictions are distinguished: complete prohibition on new hookups, hookups to replace existing customers only, and other restrictions. These variables enter the choice model as interaction terms with the gas space heating alternatives.

-- It is anticipated that there has been some reluctance to accept heat pump technology because of its relatively new and unproven status. This reluctance is expected to disappear over time, as market share grows and the technology is accepted or rejected on its own merits. EPRI felt that this fundamental difference between new and old technologies should be modeled explicitly. They therefore include the lagged market penetration rate of heat pumps in each region as an explanatory variable in the choice model.

-- Since there may be many aspects of a particular technology not captured by operating and capital costs (or which are otherwise quantifiable), the EPRI model includes alternative-specific dummy variables equal to unity only for the alternative specified.

The model of central cooling choice includes the following variables in addition to capital and operating costs:

-- Summer climate is included as an explanatory variable by taking summer dry-bulb temperature minus 82 °F plus the summer wet-bulb temperature minus 65 °F, with a minimum of zero. Since summer discomfort is quite non-linear in the traditional metric of cooling degree-days, this new measure has promise as a useful (i.e., linear) variable.

-- Household income is used both directly and in interaction with the operating cost of heating systems given central cooling.

-- To a certain extent, the choice of central cooling or no central cooling is driven by the utility of the space heating choices available given the cooling choice.

> In the nested logit model, the influence of space heating characteristics upon air conditioning choice is represented by a variable called the "inclusive value." This is an index of the attractiveness of complementary space heating systems upon the air conditioning alternative. Policies or other events which increase the desirability of particular space heating systems will also increase the attractiveness of their complementary air conditioning alternative.[6]

The elasticities reported in this paper are based on a somewhat modified form of the EPRI model, discussed more fully in Wood, Ruderman, and McMahon [17]. These modifications were of several different kinds, including the correction of a simple error in calculation,[7] a respecification of heat pump capital and operating costs, and a redefinition of the "gas restrictions" variables to reflect prior history of gas curtailments. All of the included modifications improved the fit of EPRI's models to the data. The coefficients used in this study are given in Tables 4, 5, and 6. Note that capital and operating costs are "normalized" in the choice models for space heating and air conditioning by the operating cost of an electric baseboard system and the annual air conditioning energy usage, respectively.

[6] EPRI [6] page 3-19

[7] Elasticities based on *only* the correction of the error in calculation are discussed in Wood, Ruderman, and McMahon [18].

Table 4

Space Heating System Choice Model for Households Without Central Air Conditioning

Normalized Capital and Operating Costs

Variable	Estimated Parameter	t-statistic
Installed capital costs/operating cost of an electric baseboard system	-0.5069	-4.738
Annual operating costs/operating cost of an electric baseboard system	-2.234	-5.939
Type 1 gas restrictions in gas alternatives	-3.255	-7.150
Type 2 gas restrictions in gas alternatives	-1.625	-4.268
Type 3 gas restrictions in gas alternatives	-0.5741	-1.933
Dummy for gas hydronic choice	-1.727	-3.525
Dummy for non-central gas choice	-3.530	-8.463
Dummy for oil forced air choice	-1.250	-5.286
Dummy for oil hydronic choice	-0.09038	-0.2202
Dummy for non-central oil choice	-4.165	-6.950
Dummy for electric forced air choice	-1.403	-4.482
Dummy for electric baseboard choice	-1.397	-4.673

Table 5

Space Heating System Choice Model for Households With Central Air Conditioning

Normalized Capital and Operating Costs

Variable	Estimated Parameter	t-statistic
Incremental installed capital costs/operating cost of an electric baseboard system	-0.2870	-6.768
Incremental annual operating costs/operating cost of an electric baseboard system	-3.324	-5.790
Incremental annual operating costs/operating cost of an electric baseboard system × Income ($1000)	-0.03713	-2.340
Type 1 gas restrictions in gas alternative	-2.872	-9.172
Type 2 gas restrictions in gas alternative	-1.111	-2.604
Type 3 gas restrictions in gas alternative	-1.528	-5.483
Lagged regional penetration of heat pumps in heat pump alternative	0.02776	2.407
Dummy for oil forced air choice	-1.828	-8.518
Dummy for electric forced air choice	1.182	4.011
Dummy for electric baseboard choice	-1.158	-3.920
Dummy for heat pump choice	-0.8220	-3.920

Table 6

Central Air Conditioning Choice Model

Normalized Capital and Operating Costs

Variable	Estimated Parameter	t-statistic
Installed capital costs/annual central air conditioning electricity usage ($/1000 BTU)	-4.801	-4.895
Annual operating costs/annual central air conditioning electricity usage ($/1000 BTU)	-128.0	-3.505
Dry bulb summer design temperature - 82 °F + wet bulb summer design temperature - 68 °F if greater than or equal to zero in central cooling alternative	0.1433	10.48
Income ($1000) in air conditioning alternative	0.07195	7.528
Inclusive value from space heating model	0.4028	2.791
Dummy for air conditioning choice	-1.155	-1.988

4. Derivation of Elasticities

This section explains how elasticities were derived from the probability choice model coefficients estimated in the revised EPRI study.

A point elasticity is the ratio of the fractional (or percentage) change in one quantity to a fractional (or percentage) change in another, taken in the limit as the changes become infinitesimally small. Thus, the elasticity of x with respect to y is:

$$\lim_{\Delta y \to 0} \frac{\frac{\Delta x}{x}}{\frac{\Delta y}{y}} = \lim_{\Delta y \to 0} \frac{y}{x} . \frac{\Delta x}{\Delta y} = \frac{y}{x} . \frac{\partial x}{\partial y}$$

Elasticities take on economic significance in assuming that the change in the first quantity is in response to (i.e., is caused by) the change in the other.

The standard technique for estimating point elasticities from the coefficients of an econometric model is to work out the analytic derivatives from the equation relating market share to the explanatory variables. See Dohrmann [4] or Dubin and McFadden [5] for examples of this. In our case, however, several factors make this inconvenient and arguably unwise:

-- The nested logit model of technology choice, however satisfying from the point of view of the actual consumer decision, is more difficult to work with when taking derivatives. The joint probability of a household choosing each particular heating/cooling technology combination is a function of the capital and operating costs of all the technologies, as well as the demographic characteristics of the household. The probability is expressed as the product of a dependent probability and a marginal probability, i.e., the probability of selecting cooling choice i and heating choice j is:

$$P_{ij} = P_{j|i} \cdot P_i \, .$$

Each of these two probabilities is modeled as a multinomial logit, and all the variables from all heating possibilities enter into the cooling choice via the "inclusive value" term. This formulation, plus the use of normalizing factors (e.g., the operating cost of an electric baseboard system), makes taking all but the simplest derivatives a painful, or at best tedious, exercise.

-- The choice of a technology, rather than fuel, model makes it doubly difficult to take a derivative with respect to a particular fuel price. Taking the derivative of market share for a particular technology with respect to fuel price therefore requires calculating and summing n partial derivatives, where n is the number of technologies in the model using that fuel. If we want, say, the elasticity of all gas technologies' market share with respect to the price of electricity, then the problem is confounded, for we have to calculate and sum n partial derivatives separately for each of the gas technologies, and decide on an appropriate way to combine them for the overall elasticity. The same issues arise if we seek a general elasticity with respect to, say, the capital costs of all oil-burning equipment.

-- Even granting the complexity of the derivatives to be found, they are still theoretically tractable. Some are even (relatively) easy: e.g., the derivative of central cooling market share with respect to its own capital cost. In this case, we are working only with the "upper" level of the choice tree, so the "nesting" does not apply and the calculation is far simpler. For any particular household (with specific values of the explanatory variables), the derivative can be shown to equal the coefficient of capital cost from the appropriate level regression, times the probability that household chooses air conditioning, times the probability that the household does not choose air conditioning. From this derivative, the elasticity for that household can be found easily.

What is not clear, however, is whether that easy calculation for a single household translates into accurate elasticities for a national market share. The standard technique calls for substituting "average" or "representative" values for each of the explanatory variables into the calculation of the elasticity. Thus, the "market share" of central cooling (or any other technology) is found as the probability that a representative household (i.e., one whose explanatory variables take on the "average" values) would choose it. There is no compelling reason to support such an approximation. For the EPRI dataset, this technique matches known market shares very poorly compared to the the method we use (described below).[8] While the standard technique may be necessary when used with an aggregated dataset without individual household choices, it seems foolish to fail to use all the information available to us. Furthermore, McFadden and Reid [14] have shown that using only "representative" values neglects the variation of the exogenous variables and thereby introduces bias in the elasticities.

These difficulties led us to our simulation and sample enumeration approach, with the following steps:

1) For each household, we use the measured demographic characteristics (climate, income, etc.) and technology-specific variables (capital and operating costs), together with our revised versions of the coefficients estimated by EPRI (shown in tables 4, 5, and 6 of this paper) to estimate the probability that the household chooses each of the thirteen possible heating/cooling technologies. Averaging with equal weight across households provides estimates of projected market shares. Combining these estimates across common fuel types (gas, oil, electricity) or technology characteristics (central, non-central) gives estimates of market share of these aggregated categories.

2) We select a series of thirteen different perturbation sizes (represented by δ), in the range of -0.333 to +0.500. Most of the values are clustered around zero, although none are equal to it.

3) We use each perturbation in turn to change the values of the input data for the independent variable with respect to which we want to estimate elasticities. Thus, if we want elasticities with respect to the price of electricity, we multiply the operating costs (which are proportional to fuel price) of all systems that use electricity by a factor of $(1 + \delta)$.

4) We calculate from the model the new probabilities (under the perturbed conditions) that each household selects each technology.

5) We use these new probabilities to calculate the average change in market share resulting from the changed conditions. This number can be combined with original market shares and the known size of the perturbation to provide an estimate of the arc elasticity as a function of the perturbation size δ:

$$\frac{\frac{\Delta \text{market share}}{\text{market share}}}{\frac{\Delta \text{operating cost}}{\text{operating cost}}} = \frac{\frac{MS_{new} - MS_{old}}{MS_{old}}}{\frac{OC_{new} - OC_{old}}{OC_{old}}} = \frac{\frac{MS_{new} - MS_{old}}{MS_{old}}}{\frac{(1+\delta)OC_{old} - OC_{old}}{OC_{old}}} = \frac{MS_{new} - MS_{old}}{\delta MS_{old}}$$

Note that the change in market share (the numerator) is averaged over all households in the dataset with equal weights and divided by the original market share (similarly averaged). An alternative method (which we do not use) would be to calculate the arc elasticity for each household separately and then average them. The two methods are exactly equivalent if, in the second method, each household is weighted by the probability that it selects the particular technology whose elasticity is being estimated. In effect, the elasticity of a household which is likely to choose the technology counts more than that of a household unlikely to choose it. This weighted average approach to estimating aggregate elasticities is also used in

[8] This comparison, and other issues relating to aggregation bias, is discussed in Wood, Ruderman, & McMahon [18].

the transportation modal-choice literature (e.g., McCarthy [11]).

6) The logit formulation of the model assures us that each of the components of the ratio to the right of the last equality above can be approximated by a polynomial in δ. Thus the entire ratio (i.e., the arc elasticity as a function of δ) is a polynomial in δ and for small values of δ can be represented by a quadratic by neglecting terms of order δ^3 or higher. We can then get an estimate of the point elasticity (the limit as δ approaches zero) by taking the intercept of the least-squares quadratic curve fitted to the arc elasticity as a function of perturbation size.[9]

   This process is graphically shown in Figure 1 on the following page.

Results of this approach are tabulated in Appendix 1. The elasticities reported there are for the values of the explanatory variables set at their observed values for all households in the dataset, corresponding to the classical approach of reporting elasticities for the independent variables set at their mean values in the dataset.

Our approach to calculating elasticities offers two specific advantages over the classical approach of taking analytic derivatives and evaluating them at mean values of the exogenous variables:

1) The sample enumeration technique (in which an aggregate change in market share is derived from a sum of individual household changes in probability) allows direct estimation of the aggregate market elasticity, without any aggregation bias due to the use of "representative" data.[10]

2) Calculation of a well-fitting quadratic curve to the arc elasticity data means that elasticities over the entire range of perturbations from -33% to +50% can be represented by only three parameters. This simple representation of arc elasticities allows more accurate prediction of new market shares under large relative changes in the independent variables than would be possible using only "constant elasticity" predictions from the point elasticity.

These advantages, and the errors that occur if they are not used, are discussed in Wood, Ruderman, and McMahon [18].

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[9] An arc-elasticity can be viewed as a scaled "arc-derivative" (i.e., a ratio of forward or backward finite differences in two related variables, multiplied by the inverse ratio of the variables themselves). As such, we also found that we could fit an appropriately scaled and shifted transformation of the density function $\partial P/\partial\delta = \alpha P(1-P)$ (where $P = e^{\alpha\delta}/(1+e^{\alpha\delta})$ is a logistic distribution function in δ) and achieve an extremely good fit over a very wide range of values for δ. However, this approach has two drawbacks that lead us to not report it more fully here: 1) it is not linear in the shift and scale parameters to be estimated, and so requires non-linear fitting algorithms, which are not as widely available, and 2) it obscures the simple algebraic relationship between the constant term of a polynomial fit and the point elasticity.

[10] McFadden and Reid [14] discuss the sources and consequences of such bias. Later studies (Reid [16], Landau [9]) offer evidence that aggregation bias is both pervasive and significant.

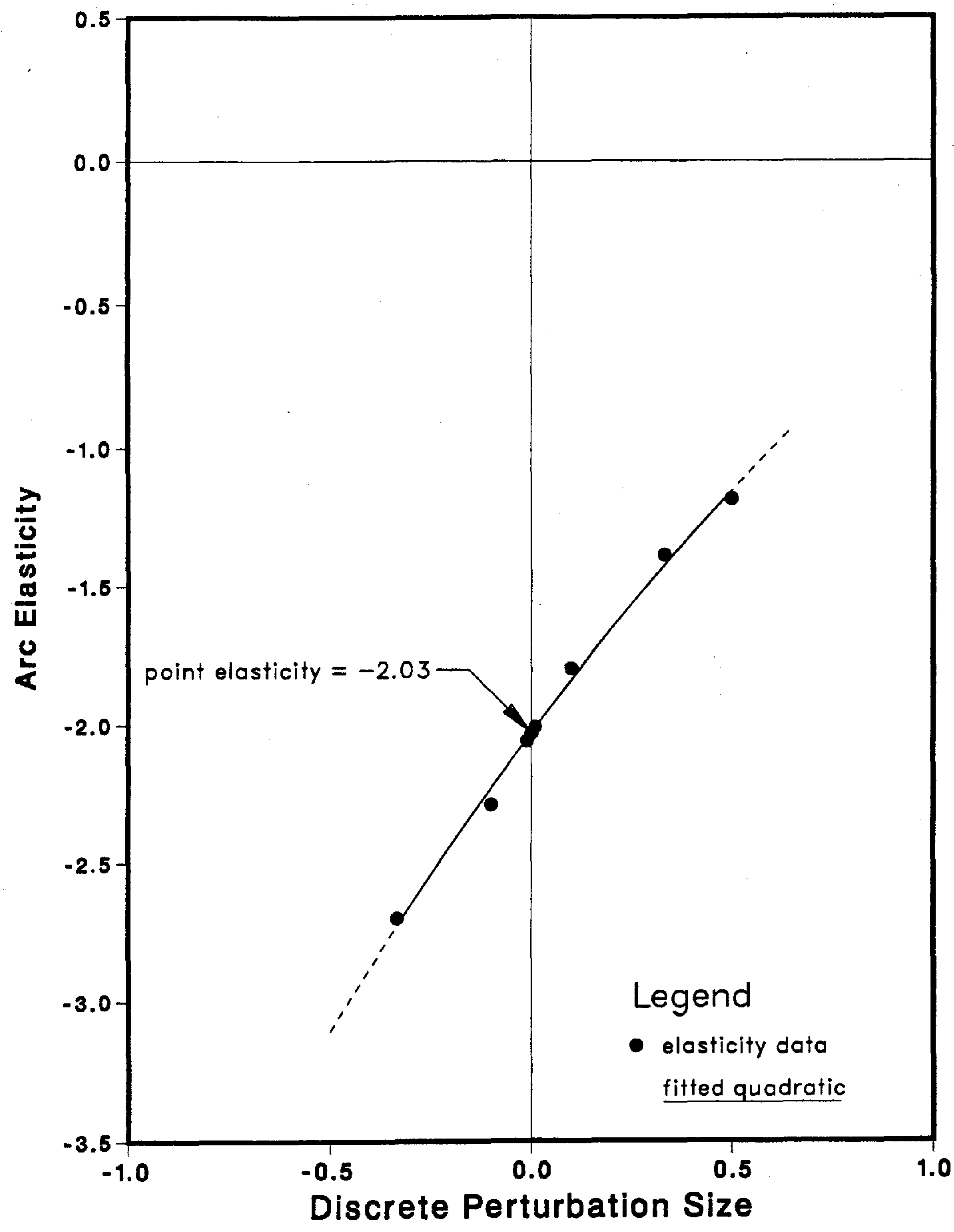

Figure 1. Elasticity of Heat Pumps With Respect To Own Capital Cost.

5. Limitations and Caveats

This section discusses some of the limitations of the model used by EPRI and our simulation approach.

1) The principal fault with elasticities derived from EPRI's work is based on the calculation of operating cost from a price-independent model such as MIT's Thermal Load Model. Operating cost was based on the known household size, the year and type of construction, and the fuel prices in the region at the time of construction. These factors determine the likely thermal integrity of the dwelling, the system capacity necessary for the house under each technology, and the "demand" (the usage) of energy for each technology. But real demand for energy (and thus the real operating cost) are almost certainly price-dependent. Elasticities estimated from a model that treats energy use as constant under changing price conditions may not accurately predict market shares in a real world where demand is elastic.

2) The estimated elasticities reported in Appendix 1 can be taken as statistically unbiased only to the extent that the 1300 or so household observations can be treated as if they were a random draw from the general population. This is only approximately true. The Annual Housing Surveys seek to achieve a balance between the "central city" and "other" in each SMSA examined. So the selection of households is not perfectly random in the raw data. The data was further winnowed by EPRI, in that they used only households which were single-family, owner-occupied, constructed in the year of the Housing Survey, and for which the necessary demographic and weather data were available. There may conceivably be some self-selection in reporting of income or other data, which could affect the randomness of our "representative" sample. While this is certainly true, it is probably not critical. It will be less critical if we limit our results to statements about new, owner-occupied, single-family housing inside SMSAs. For that population, the sample is probably acceptably close to random.

   This "fault" (i.e., the data may not be perfectly representative of the national population) is shared by the classical approach of substituting mean values in an analytic expression for the elasticity, but that approach suffers the addition of aggregation bias.

3) Our arc elasticities resulted from "simulations" using modified coefficients generated in EPRI's study. They do not take into account the variance of those coefficients (we use only the maximum likelihood estimate). The two-step nested logit procedure used by EPRI produces three sets of estimated coefficients (one for the cooling choice and one for each of the heating choices), each of which has a joint normal distribution and an estimated covariance matrix. This uncertainty in the estimated values of the coefficients of EPRI's model is not reflected anywhere in our estimated arc or point elasticities.

   It could be included by true Monte Carlo simulation at another level of the problem: make a series of random draws from the joint normal distributions for the coefficients of the probability model. For each set of coefficients drawn, carry out the elasticity calculations. The elasticities (i.e., the least-squares estimate of the intercept) found in this series of calculations can be averaged, and their mean and standard deviation will reflect the confidence due to uncertain coefficients. It seems reasonable to suspect that the uncertainty found in this manner could be unpleasantly large. Both the fault and its potential solution are shared by the classical approach.

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Appendix 1

This appendix reports the numerical values of the elasticities estimated by our simulation and sample enumeration approach. Each table reports the elasticity of market share for twelve different technologies (or aggregated technologies) with respect to one particular variable.

The first column reports the point elasticity, i.e., the intercept of the quadratic curve fit by least-squares techniques to the arc elasticity as a function of perturbation size. The linear and quadratic terms of that fit are shown in the second and third columns. Inspection of the tables will show that in almost all cross-elasticities the quadratic term is quite small, suggesting that a linear approximation would have been sufficient. This fact gives some credence to the approach we use when estimating new market shares as several variables change at once (discussed in Appendix 2, part B).

Elasticity of 12 Technology Groups wrt Household Income

technology	point elasticity	linear term	quadratic term
central cooling only	0.288	-0.0489	-0.0161
all gas systems	0.0495	0.0217	-0.00993
central gas systems	0.0553	0.0210	-0.0104
non-central gas systems	-0.565	0.0973	0.0313
all oil systems	-0.277	0.0537	0.0174
central oil systems	-0.270	0.0524	0.0169
non-central oil systems	-0.589	0.107	0.0348
all conventional elec	-0.137	-0.0370	0.0153
central electric	-0.0880	-0.0576	0.0200
non-central electric	-0.252	0.0107	0.00471
heat pumps only	0.324	-0.0469	-0.00667
all elec (incl heat pumps)	-0.673×10^{-3}	-0.0399	0.00879

Elasticity of 12 Technology Groups wrt Presence of "Cumulative" Gas Restrictions

technology	point elasticity	linear term	quadratic term
central cooling only	-0.0394	0.0246	-0.00634
all gas systems	-0.184	0.114	-0.0339
central gas systems	-0.184	0.114	-0.0339
non-central gas systems	-0.180	0.112	-0.0383
all oil systems	0.293	-0.181	0.0537
central oil systems	0.293	-0.181	0.0536
non-central oil systems	0.281	-0.177	0.0563
all conventional elec	0.167	-0.103	0.0309
central electric	0.179	-0.112	0.0340
non-central electric	0.139	-0.0828	0.0239
heat pumps only	0.177	-0.110	0.0329
all elec (incl heat pumps)	0.170	-0.105	0.0316

Elasticity of 12 Technology Groups wrt "Weather"

technology	point elasticity	linear term	quadratic term
central cooling only	0.476	-0.255	0.0521
all gas systems	-0.0616	0.0458	-0.0164
central gas systems	-0.0537	0.0415	-0.0156
non-central gas systems	-0.893	0.493	-0.100
all oil systems	-0.499	0.150	0.0509
central oil systems	-0.486	0.144	0.0513
non-central oil systems	-1.034	0.392	0.0492
all conventional elec	0.0794	-0.0139	-0.0157
central electric	0.262	-0.134	0.0230
non-central electric	-0.346	0.265	-0.106
heat pumps only	0.460	-0.281	0.0699
all elec (incl heat pumps)	0.192	-0.0927	0.00951

Elasticity of 12 Technology Groups wrt Capital Cost of Conventional Electric Heat

technology	point elasticity	linear term	quadratic term
central cooling only	-0.0745	-0.0154	0.0235
all gas systems	0.231	-0.0265	-0.0289
central gas systems	0.231	-0.0266	-0.0291
non-central gas systems	0.202	-0.0163	-0.00359
all oil systems	0.175	-0.0209	-0.00610
central oil systems	0.175	-0.0209	-0.00620
non-central oil systems	0.201	-0.0213	-0.00708
all conventional elec	-0.646	0.0483	0.0956
central electric	-0.652	0.106	0.0921
non-central electric	-0.634	-0.0852	0.103
heat pumps only	0.404	0.0160	-0.0976
all elec (incl heat pumps)	-0.336	0.0388	0.0384

Elasticity of 12 Technology Groups wrt Capital Cost of Central Electric Heat

technology	point elasticity	linear term	quadratic term
central cooling only	-0.0764	0.00935	0.0193
all gas systems	0.152	-0.0618	-0.00788
central gas systems	0.153	-0.0620	-0.00807
non-central gas systems	0.0953	-0.0468	0.00625
all oil systems	0.108	-0.0235	-0.818×10^{-4}
central oil systems	0.108	-0.0234	-0.168×10^{-3}
non-central oil systems	0.103	-0.0292	0.00171
all conventional elec	-0.442	0.151	0.0330
central electric	-0.856	0.332	0.118
non-central electric	0.523	-0.272	-0.164
heat pumps only	0.311	-0.0733	-0.0458
all elec (incl heat pumps)	-0.220	0.0846	0.00977

Elasticity of 12 Technology Groups wrt Capital Cost of Non-Central Electric Heat

technology	point elasticity	linear term	quadratic term
central cooling only	0.00200	-0.00695	0.00763
all gas systems	0.0785	-0.0352	-0.00217
central gas systems	0.0783	-0.0352	-0.00210
non-central gas systems	0.107	-0.0365	-0.00804
all oil systems	0.0670	-0.0209	0.00213
central oil systems	0.0663	-0.0207	0.00220
non-central oil systems	0.0972	-0.0290	0.369×10^{-3}
all conventional elec	-0.204	0.0899	0.00912
central electric	0.205	-0.0419	-0.0680
non-central electric	-1.157	0.397	0.189
heat pumps only	0.0932	-0.0455	-0.0144
all elec (incl heat pumps)	-0.116	0.0499	0.00221

Elasticity of 12 Technology Groups wrt Capital Cost of Heat Pumps

technology	point elasticity	linear term	quadratic term
central cooling only	0.418	-0.401	0.0744
all gas systems	0.209	-0.206	0.0739
central gas systems	0.211	-0.207	0.0744
non-central gas systems	0.0429	-0.0359	0.0155
all oil systems	0.117	-0.0598	0.00463
central oil systems	0.119	-0.0607	0.00459
non-central oil systems	0.0489	-0.0252	0.00592
all conventional elec	0.433	-0.426	0.0377
central electric	0.475	-0.444	0.0583
non-central electric	0.336	-0.383	-0.0106
heat pumps only	-2.028	1.952	-0.414
all elec (incl heat pumps)	-0.295	0.277	-0.0960

Elasticity of 12 Technology Groups wrt
Capital Cost of Conventional Central Cooling

technology	point elasticity	linear term	quadratic term
central cooling only	-0.357	-0.0432	-0.0122
all gas systems	-0.0637	-0.0341	-0.00410
central gas systems	-0.0673	-0.0342	-0.00412
non-central gas systems	0.320	-0.0256	-0.00506
all oil systems	0.134	-0.00864	-0.00562
central oil systems	0.129	-0.00867	-0.00529
non-central oil systems	0.346	-0.00801	-0.0123
all conventional elec	-0.204	-0.0789	-0.00360
central electric	-0.299	-0.0487	-0.0256
non-central electric	0.0161	-0.149	0.0479
heat pumps only	0.661	0.343	0.0297
all elec (incl heat pumps)	0.0513	0.0457	0.00638

Elasticity of 12 Technology Groups wrt
Capital Cost of All Gas Heat

technology	point elasticity	linear term	quadratic term
central cooling only	-0.0370	-0.0171	0.0143
all gas systems	-0.465	0.0760	0.00948
central gas systems	-0.463	0.0744	0.0103
non-central gas systems	-0.708	0.246	-0.0807
all oil systems	0.372	0.0394	-0.0154
central oil systems	0.370	0.0388	-0.0153
non-central oil systems	0.459	0.0626	-0.0219
all conventional elec	0.540	-0.121	-0.00469
central electric	0.490	-0.0913	-0.0256
non-central electric	0.656	-0.190	0.0443
heat pumps only	0.447	-0.0714	-0.0185
all elec (incl heat pumps)	0.512	-0.106	-0.00876

Elasticity of 12 Technology Groups wrt Capital Cost of Central Gas Heat

technology	point elasticity	linear term	quadratic term
central cooling only	-0.0391	-0.0171	0.0144
all gas systems	-0.461	0.0794	0.0104
central gas systems	-0.473	0.0780	0.0114
non-central gas systems	0.888	0.231	-0.0921
all oil systems	0.365	0.0334	-0.0172
central oil systems	0.363	0.0330	-0.0170
non-central oil systems	0.448	0.0527	-0.0254
all conventional elec	0.534	-0.125	-0.00532
central electric	0.486	-0.0938	-0.0259
non-central electric	0.646	-0.198	0.0424
heat pumps only	0.445	-0.0722	-0.0183
all elec (incl heat pumps)	0.508	-0.109	-0.00916

Elasticity of 12 Technology Groups wrt Capital Cost of Non-Central Gas Heat

technology	point elasticity	linear term	quadratic term
central cooling only	0.00216	-0.00254	0.00201
all gas systems	-0.00489	0.00682	-0.00669
central gas systems	0.0102	-0.0108	0.00748
non-central gas systems	-1.598	1.868	-1.510
all oil systems	0.00734	-0.00511	0.00277
central oil systems	0.00725	-0.00504	0.00277
non-central oil systems	0.0110	-0.00819	0.00475
all conventional elec	0.00584	-0.00970	0.0100
central electric	0.00397	-0.00591	0.00562
non-central electric	0.0102	-0.0185	0.0200
heat pumps only	0.00180	-0.00245	0.00215
all elec (incl heat pumps)	0.00464	-0.00756	0.00773

Elasticity of 12 Technology Groups wrt Capital Cost of All Oil Heat

technology	point elasticity	linear term	quadratic term
central cooling only	0.0242	-0.0309	0.0220
all gas systems	0.129	-0.125	0.0819
central gas systems	0.127	-0.124	0.0814
non-central gas systems	0.258	-0.237	0.139
all oil systems	-1.273	1.114	-0.699
central oil systems	-1.282	1.130	-0.712
non-central oil systems	-0.941	0.472	-0.175
all conventional elec	0.141	-0.105	0.0596
central electric	0.123	-0.0890	0.0492
non-central electric	0.184	-0.142	0.0839
heat pumps only	0.0808	-0.0623	0.0378
all elec (incl heat pumps)	0.123	-0.0923	0.0532

Elasticity of 12 Technology Groups wrt Capital Cost of Central Oil Heat

technology	point elasticity	linear term	quadratic term
central cooling only	0.0232	-0.0306	0.0220
all gas systems	0.126	-0.124	0.0820
central gas systems	0.125	-0.123	0.0815
non-central gas systems	0.252	-0.235	0.140
all oil systems	-1.246	1.109	-0.702
central oil systems	-1.290	1.141	-0.719
non-central oil systems	0.538	-0.196	-0.00298
all conventional elec	0.138	-0.105	0.0601
central electric	0.120	-0.0892	0.0498
non-central electric	0.179	-0.142	0.0843
heat pumps only	0.0797	-0.0621	0.0380
all elec (incl heat pumps)	0.121	-0.0924	0.0536

Elasticity of 12 Technology Groups wrt Capital Cost of Non-Central Oil Heat

technology	point elasticity	linear term	quadratic term
central cooling only	0.944×10^{-3}	-0.923×10^{-3}	0.600×10^{-3}
all gas systems	0.00286	-0.00306	0.00223
central gas systems	0.00283	-0.00303	0.00218
non-central gas systems	0.00644	-0.00643	0.00445
all oil systems	-0.0272	0.0292	-0.0215
central oil systems	0.00843	-0.00570	0.00259
non-central oil systems	-1.479	1.453	-1.004
all conventional elec	0.00307	-0.00334	0.00255
central electric	0.00239	-0.00244	0.00172
non-central electric	0.00466	-0.00544	0.00443
heat pumps only	0.00112	-0.00112	0.715×10^{-3}
all elec (incl heat pumps)	0.00249	-0.00269	0.00203

Elasticity of 12 Technology Groups wrt Price of Electricity

technology	point elasticity	linear term	quadratic term
central cooling only	-0.242	-0.0286	-0.0460
all gas systems	0.451	-0.624	0.450
central gas systems	0.449	-0.627	0.452
non-central gas systems	0.732	-0.305	0.222
all oil systems	1.811	-0.0125	-0.407
central oil systems	1.822	-0.00535	-0.417
non-central oil systems	1.350	-0.303	0.00168
all conventional elec	-1.072	1.030	-0.728
central electric	-1.137	1.092	-0.768
non-central electric	-0.922	0.886	-0.633
heat pumps only	-0.789	0.264	0.0950
all elec (incl heat pumps)	-0.989	0.803	-0.485

Elasticity of 12 Technology Groups wrt Operating Cost of Central Electric Heat

technology	point elasticity	linear term	quadratic term
central cooling only	-0.380	0.304	0.0766
all gas systems	0.564	-0.623	0.142
central gas systems	0.566	-0.626	0.142
non-central gas systems	0.281	-0.329	0.159
all oil systems	0.636	-0.514	-0.0937
central oil systems	0.641	-0.517	-0.0975
non-central oil systems	0.450	-0.405	0.0606
all conventional elec	-1.755	1.594	0.0625
central electric	-2.877	2.481	0.231
non-central electric	0.860	-0.474	-0.331
heat pumps only	1.250	-0.700	-0.691
all elec (incl heat pumps)	-0.867	0.916	-0.160

Elasticity of 12 Technology Groups wrt Operating Cost of Non-Central Electric Heat

technology	point elasticity	linear term	quadratic term
central cooling only	0.0130	0.195	-0.301
all gas systems	-0.362	-0.119	0.479
central gas systems	-0.364	-0.118	0.480
non-central gas systems	-0.119	-0.169	0.323
all oil systems	0.640	-0.630	0.577
central oil systems	0.652	-0.633	0.574
non-central oil systems	0.187	-0.531	0.668
all conventional elec	0.358	0.858	-1.636
central electric	1.449	-0.883	0.179
non-central electric	-2.183	4.918	-5.865
heat pumps only	0.226	-1.047	1.380
all elec (incl heat pumps)	0.319	0.295	-0.744

Elasticity of 12 Technology Groups wrt Operating Cost of Heat Pumps

technology	point elasticity	linear term	quadratic term
central cooling only	0.670	-0.743	0.0692
all gas systems	0.347	-0.440	0.161
central gas systems	0.350	-0.443	0.161
non-central gas systems	0.0842	-0.115	0.0705
all oil systems	0.294	-0.284	0.0356
central oil systems	0.298	-0.287	0.0347
non-central oil systems	0.128	-0.151	0.0723
all conventional elec	0.656	-0.671	-0.0771
central electric	0.738	-0.732	-0.0678
non-central electric	0.464	-0.528	-0.0986
heat pumps only	-3.293	3.723	-0.539
all elec (incl heat pumps)	-0.512	0.628	-0.214

Elasticity of 12 Technology Groups wrt Operating Cost of Heat Pumps (Heating Side Only)

technology	point elasticity	linear term	quadratic term
central cooling only	0.401	-0.292	0.0410
all gas systems	0.221	-0.193	0.0627
central gas systems	0.223	-0.194	0.0629
non-central gas systems	0.0648	-0.0698	0.0366
all oil systems	0.225	-0.180	0.0336
central oil systems	0.228	-0.182	0.0332
non-central oil systems	0.107	-0.108	0.0468
all conventional elec	0.372	-0.236	-0.00222
central electric	0.438	-0.274	-0.00989
non-central electric	0.218	-0.149	0.0151
heat pumps only	-2.016	1.537	-0.293
all elec (incl heat pumps)	-0.334	0.288	-0.0883

Elasticity of 12 Technology Groups wrt Operating Cost of Heat Pumps (Cooling Side Only)

technology	point elasticity	linear term	quadratic term
central cooling only	0.270	-0.241	0.0270
all gas systems	0.126	-0.104	0.0329
central gas systems	0.127	-0.105	0.0332
non-central gas systems	0.0195	-0.0111	0.00368
all oil systems	0.0694	-0.0307	0.326×10^{-3}
central oil systems	0.0706	-0.0313	0.408×10^{-3}
non-central oil systems	0.0213	-0.00668	0.987×10^{-3}
all conventional elec	0.285	-0.277	0.00243
central electric	0.302	-0.279	0.0339
non-central electric	0.247	-0.274	-0.0714
heat pumps only	-1.281	1.136	-0.149
all elec (incl heat pumps)	-0.178	0.141	-0.0425

Elasticity of 12 Technology Groups wrt Operating Cost of Conventional Central Cooling

technology	point elasticity	linear term	quadratic term
central cooling only	-0.548	-0.209	0.194
all gas systems	-0.0986	-0.0792	0.0227
central gas systems	-0.104	-0.0801	0.0232
non-central gas systems	0.486	0.0198	-0.0343
all oil systems	0.240	-0.00526	-0.0142
central oil systems	0.232	-0.00610	-0.0131
non-central oil systems	0.586	0.0287	-0.0527
all conventional elec	-0.335	-0.238	0.268
central electric	-0.445	-0.232	0.232
non-central electric	-0.0789	-0.253	0.351
heat pumps only	1.043	0.915	-0.727
all elec (incl heat pumps)	0.0721	0.103	-0.0260

Elasticity of 12 Technology Groups wrt Price of Gas

technology	point elasticity	linear term	quadratic term
central cooling only	-0.176	0.0269	0.0265
all gas systems	-0.555	0.0482	0.0719
central gas systems	-0.559	0.0491	0.0724
non-central gas systems	-0.168	-0.0459	0.0117
all oil systems	0.417	0.0172	-0.0290
central oil systems	0.418	0.0171	-0.0291
non-central oil systems	0.377	0.0195	-0.0165
all conventional elec	0.594	-0.0778	-0.0674
central electric	0.614	-0.0585	-0.0960
non-central electric	0.549	-0.123	-0.818×10^{-3}
heat pumps only	0.673	-0.0364	-0.130
all elec (incl heat pumps)	0.618	-0.0656	-0.0859

Elasticity of 12 Technology Groups wrt Operating Cost of Central Gas Heat

technology	point elasticity	linear term	quadratic term
central cooling only	-0.177	0.0268	0.0265
all gas systems	-0.553	0.0499	0.0718
central gas systems	-0.564	0.0495	0.0730
non-central gas systems	0.625	0.0915	-0.0406
all oil systems	0.413	0.0141	-0.0297
central oil systems	0.414	0.0141	-0.0297
non-central oil systems	0.371	0.0151	-0.0176
all conventional elec	0.591	-0.0798	-0.0671
central electric	0.612	-0.0599	-0.0957
non-central electric	0.544	-0.126	-0.376×10^{-3}
heat pumps only	0.672	-0.0372	-0.130
all elec (incl heat pumps)	0.615	-0.0672	-0.0858

Elasticity of 12 Technology Groups wrt Operating Cost of Non-Central Gas Heat

technology	point elasticity	linear term	quadratic term
central cooling only	0.00106	-0.453×10^{-3}	0.296×10^{-4}
all gas systems	-0.00272	0.00176	-0.925×10^{-3}
central gas systems	0.00474	-0.00208	0.737×10^{-3}
non-central gas systems	-0.792	0.408	-0.177
all oil systems	0.00470	-0.00189	0.684×10^{-3}
central oil systems	0.00464	-0.00188	0.749×10^{-3}
non-central oil systems	0.00675	-0.00281	0.966×10^{-3}
all conventional elec	0.00305	-0.00240	0.00144
central electric	0.00213	-0.00152	0.795×10^{-3}
non-central electric	0.00515	-0.00447	0.00319
heat pumps only	0.908×10^{-3}	-0.466×10^{-3}	0.964×10^{-4}
all elec (incl heat pumps)	0.00242	-0.00182	0.00100

Elasticity of 12 Technology Groups wrt Price of Oil

technology	point elasticity	linear term	quadratic term
central cooling only	-0.0164	0.0336	-0.0256
all gas systems	0.115	-0.0940	0.0541
central gas systems	0.114	-0.0941	0.0545
non-central gas systems	0.163	-0.0800	0.0248
all oil systems	-1.263	1.035	-0.590
central oil systems	-1.273	1.052	-0.603
non-central oil systems	-0.868	0.332	-0.0524
all conventional elec	0.141	-0.105	0.0570
central electric	0.142	-0.117	0.0658
non-central electric	0.138	-0.0779	0.0365
heat pumps only	0.133	-0.133	0.0802
all elec (incl heat pumps)	0.139	-0.114	0.0638

Elasticity of 12 Technology Groups wrt Operating Cost of Central Oil Heat

technology	point elasticity	linear term	quadratic term
central cooling only	-0.0172	0.0338	-0.0256
all gas systems	0.113	-0.0935	0.0542
central gas systems	0.112	-0.0936	0.0545
non-central gas systems	0.158	-0.0790	0.0251
all oil systems	-1.242	1.032	-0.592
central oil systems	-1.281	1.059	-0.606
non-central oil systems	0.350	-0.0700	-0.00528
all conventional elec	0.138	-0.106	0.0572
central electric	0.140	-0.117	0.0660
non-central electric	0.134	-0.0779	0.0367
heat pumps only	0.132	-0.133	0.0805
all elec (incl heat pumps)	0.137	-0.114	0.0640

Elasticity of 12 Technology Groups wrt Operating Cost of Non-Central Oil Heat

technology	point elasticity	linear term	quadratic term
central cooling only	0.776×10^{-3}	-0.507×10^{-3}	0.161×10^{-3}
all gas systems	0.00222	-0.00152	0.636×10^{-3}
central gas systems	0.00220	-0.00150	0.608×10^{-3}
non-central gas systems	0.00517	-0.00350	0.00154
all oil systems	-0.0216	0.0153	-0.00736
central oil systems	0.00779	-0.00447	0.00161
non-central oil systems	-1.219	0.821	-0.366
all conventional elec	0.00252	-0.00189	0.976×10^{-3}
central electric	0.00200	-0.00145	0.675×10^{-3}
non-central electric	0.00376	-0.00287	0.00148
heat pumps only	0.924×10^{-3}	-0.614×10^{-3}	0.145×10^{-3}
all elec (incl heat pumps)	0.00204	-0.00151	0.753×10^{-3}

Appendix 2

In this appendix we discuss four issues: a) the reason why our elasticity as a function of perturbation size can be well-approximated by a quadratic; b) the practice of neglecting multiplicative terms when calculating new market shares due to changes in several variables, and instead just allowing the effect of each change to enter additively; c) calculating the point elasticity at values of explanatory variables other than those actually obtained; and d) problems in estimating the uncertainty of the coefficients of the quadratic fit to the arc-elasticity data.

A) Elasticity as a Function of Perturbation Size is Approximately Quadratic

Remember the calculation we use to find our arc elasticity as a function of the perturbation size δ:

$$\frac{\frac{\Delta\text{market share}}{\text{market share}}}{\frac{\Delta\text{operating cost}}{\text{operating cost}}} = \frac{\frac{MS_{new} - MS_{old}}{MS_{old}}}{\frac{OC_{new} - OC_{old}}{OC_{old}}} = \frac{\frac{MS_{new} - MS_{old}}{MS_{old}}}{\frac{(1+\delta)OC_{old} - OC_{old}}{OC_{old}}} = \frac{MS_{new} - MS_{old}}{\delta MS_{old}}$$

Consider only the ratio to the right of the last equality. Each "market share" (MS) component of it is an average of 1300 household probabilities. Each probability is calculated from a conditional logit model, i.e., it is the ratio of an exponential to a sum of exponentials. In a much simplified form, it looks like this:

$$\text{Prob} = \frac{e^{\alpha x+\beta y}}{e^{\alpha x+\beta y} + e^{\alpha w+\beta z}}$$

where x and w are different values (for two different alternatives) of the first independent variable, y and z are different values of the second independent variable, and α and β are estimated coefficients. The term MS_{old} is just a constant as far as δ is concerned. But the term MS_{new} is the average of probabilities which are each a function of δ:

$$\text{Prob}_{new} = \frac{e^{\alpha x+\beta y(1+\delta)}}{e^{\alpha x+\beta y(1+\delta)} + e^{\alpha w+\beta z}}.$$

The numerator of this term can be expressed as a polynomial in δ, as can the denominator. The ratio of two polynomials in δ can be expanded as another polynomial in δ, and summing over households and dividing to get an average market share doesn't affect this. Conveniently, the constant term of the resulting polynomial is exactly equal to MS_{old}, so the subtraction in the numerator of

$$\text{elasticity as a function of } \delta = \eta(\delta) = \frac{MS_{new} - MS_{old}}{\delta MS_{old}}$$

leaves a polynomial in δ without a constant term. Division by δ puts us back to polynomial with a constant, and division by the constant MS_{old} leaves us there.

Thus we can see that the statistic we use to calculate arc elasticity as a function of perturbation size can be expressed as a polynomial in the perturbation size, and we can reasonably neglect terms higher than δ^2 if δ is not too large.[1] This is confirmed by the empirical results that the multiple R^2 statistic obtained by regressing the arc elasticity on a quadratic in perturbation size is commonly 0.98 or better.[2]

[1] Essentially, we have considered the Taylor expansion of Prob_{new} around δ=0 and neglected third- or higher- order terms. The linear term in that expansion (which becomes the constant term after division by δ) has a coefficient which is just the derivative of the market share with respect to the exogenous variable y perturbed by δ, times the exogenous variable y itself.

[2] for perturbation size δ in the range -0.333 to +0.500.

B) Neglecting Multiplicative Terms When Two or More Variables Change

If we have the terms of the quadratic fitted to the arc elasticity as a function of the perturbation size, we can quickly estimate the new market share for any perturbation in the range for which the fit is good. If

$$\eta(\delta) = \frac{MS_{new} - MS_{old}}{\delta MS_{old}} = a_0 + a_1\delta + a_2\delta^2 ,$$

then algebraic rearrangement gives us

$$MS_{new} = MS_{old}[1 + \delta(a_0 + a_1\delta + a_2\delta^2)].$$

If two or more variables, say, energy price (p) and capital cost (K), change simultaneously (by amounts δ_1 and δ_2), we have to carry out an analysis similar to that in part A of this Appendix:

$$MS_{new} = \frac{e^{\alpha x(1+\delta_1)+\beta y(1+\delta_2)}}{e^{\alpha x(1+\delta_1)+\beta y(1+\delta_2)} + e^{\alpha w+\beta z}}.$$

Expansion of the exponentials in this equation gives us an estimate of the arc elasticity $\eta_{p,K}(\delta_1,\delta_2)$ as a polynomial in δ_1 and δ_2. The constant term of this polynomial can be shown to equal the sum of the constant terms in the polynomial expansions of $\eta_p(\delta_1)$ and $\eta_K(\delta_2)$. Thus we can greatly simplify the estimation of new market shares when several variables are changing simultaneously by dropping terms in the product $\delta_1\delta_2$, as well as all terms of order δ^3 or higher. This will result in the effect of each perturbation entering additively, so that

$$\begin{aligned} MS_{new}(\delta_1,\delta_2) &= MS_{old} + [MS_{new}(\delta_1) - MS_{old}] + [MS_{new}(\delta_2) - MS_{old}] \\ &= MS_{new}(\delta_1) + MS_{new}(\delta_2) - MS_{old} \\ &= MS_{old}[1 + \delta_1(a_0 + a_1\delta_1 + a_2\delta_1^2)] + MS_{old}[1 + \delta_2(b_0 + b_1\delta_2 + b_2\delta_2^2)] - MS_{old} \\ &= MS_{old}[1 + \delta_1(a_0 + a_1\delta_1 + a_2\delta_1^2) + 1 + \delta_2(b_0 + b_1\delta_2 + b_2\delta_2^2) - 1] \\ &= MS_{old}[1 + \delta_1(a_0 + a_1\delta_1 + a_2\delta_1^2) + \delta_2(b_0 + b_1\delta_2 + b_2\delta_2^2)] \end{aligned}$$

This is the approach used in LBL's Residential Energy Model.

C) Elasticities When Explanatory Variables Are at New Values

One of the principal advantages of the classical approach (i.e., using analytical derivatives) to getting elasticities from a set of regression coefficients is that it expresses the elasticity as a function of the explanatory variables directly, allowing you to find the point elasticity at values of those variables other than their means. Fortunately, a similar technique is available with our approach as well.

Let

$$MS_{new}(\delta_1) = MS_{old}[1 + \delta_1(a_0 + a_1\delta_1 + a_2\delta_1^2)]$$

and

$$MS_{new}(\delta_2) = MS_{old}[1 + \delta_2(a_0 + a_1\delta_2 + a_2\delta_2^2)]$$

be the predicted market shares of a specific technology at two different perturbations of a particular exogenous variable.[3] Then the point elasticity at the value of the variable perturbed by an

[3] What we are doing is finding the predicted market shares under the conditions that every household in the dataset has the value of that variable perturbed by δ_1 and δ_2. This corresponds to the classical technique where the entire dataset is represented by its mean values, and some shift of those average values is proposed before finding elasticities.

amount δ_1 is:

$$\lim_{\delta_2 \to \delta_1} \frac{MS_{new}(\delta_2) - MS_{new}(\delta_1)}{\frac{(\delta_2 - \delta_1)}{1+\delta_1} MS_{new}(\delta_1)}$$

$$\lim_{\delta_2 \to \delta_1} \frac{MS_{old}[\ a_0(\delta_2 - \delta_1) + a_1(\delta_2^2 - \delta_1^2) + a_2(\delta_2^3 - \delta_1^3)\ ]}{\frac{\delta_2 - \delta_1}{1+\delta_1} MS_{old}[1 \ + \ \delta_1(a_0 + a_1\delta_1 + a_2\delta_1^2)]}$$

$$\lim_{\delta_2 \to \delta_1} \frac{MS_{old}[\ a_0(\delta_2 - \delta_1) + a_1(\delta_2 - \delta_1)(\delta_2 + \delta_1) + a_2(\delta_2 - \delta_1)(\delta_2^2 + \delta_2\delta_1 + \delta_1^2)\ ]}{\frac{\delta_2 - \delta_1}{1+\delta_1} MS_{old}[1 \ + \ \delta_1(a_0 + a_1\delta_1 + a_2\delta_1^2)]}$$

$$\lim_{\delta_2 \to \delta_1} \frac{(1+\delta_1)\ [a_0 \ + \ a_1(\delta_2 + \delta_1) \ + \ a_2(\delta_2^2 + \delta_2\delta_1 + \delta_1^2)]}{1 \ + \ \delta_1(a_0 + a_1\delta_1 + a_2\delta_1^2)}$$

$$\frac{(1+\delta_1)\ [a_0 \ + \ 2a_1\delta_1 \ + \ 3a_2\delta_1^2}{1 \ + \ \delta_1(a_0 + a_1\delta_1 + a_2\delta_1^2)}$$

Thus, the point elasticity at some new value of an explanatory variable can be calculated using no more than the size of the perturbation δ_1 necessary to reach that new value.

D) Some Problems in Estimating the Uncertainty of our Elasticity

We estimate the point elasticity by finding the intercept of a quadratic curve fitted to arc elasticities as a function of perturbation size. The curve is fitted by least-squares techniques, and the quality of that fit is remarkable; we commonly obtain multiple R^2 values for the regressions of 0.98 or better.

It would seem that we have all the components in place for our application of the Gauss-Markov techniques: nearly unbiased estimates with approximately normal error, and analytic evidence that the statistic we are using will follow a polynomial, approximately quadratic relationship to the perturbation size.

Unfortunately, we are lacking one further component to make our application of least-squares techniques completely justified: independence from one data point to the next. The thirteen data points used in each regression were the arc elasticities calculated for different perturbation sizes δ over all 1300 households, the same households for each data point. Thus, although each data point is the best available estimate for the "true" value of the arc elasticity as a function of perturbation size, any error between the calculated statistic and the true value (due to a finite sampling) is likely to occur in all the data. If that error is constant and additive, say, and the true model perfectly quadratic in δ, then the linear and quadratic terms would be estimated without bias, but the estimate of the intercept (i.e., the point elasticity) would be biased. Thus the estimate of the standard error of the intercept, as reported in the tables in Appendix 1, is almost certainly too low. We are actually not that sure about the value of the intercept, because we have a bias of unknown size and direction.

This defect can be cured by calculating each arc elasticity on a different portion of the dataset, where households are distributed into the portions randomly. The estimated arc elasticities are no longer based on the same households for each value of δ, and the requirement of independence among successive errors is met. If we use thirteen different portions with about 100 households in each, then estimate of the quadratic curve intercept will give unbiased results, but the variability of that estimate should increase.

We calculated several elasticities using this method, and obtained pretty much the anticipated results: about the same value for the estimated point elasticity at the intercept, a much larger (and presumably more realistic) estimated standard error of that estimate, and overall greater variability with a corresponding loss of "goodness-of-fit."

This loss of goodness-of-fit is a serious loss, however. It means that the coefficients of the best fitting curve may give a fitted value of

$$MS_{new} = MS_{old}[1 + \delta(a_0 + a_1\delta + a_2\delta^2)].$$

which differs significantly from the best available estimate obtained by using all 1300 households to calculate MS_{new}. Essentially, we are facing the fairly common problem of trading off unbiasedness with large variance (being exactly right, on average) for biasedness with small variance (being almost right most of the time). Because of the importance of estimated market shares fitted by using the coefficients (as above) in LBL's Residential Energy Model, we have chosen to use all the data to estimate each data point. We lose an accurate estimate of our uncertainty, but we gain dependability in predicting changes in market share as economic elements change.

Appendix 3

In this appendix we describe how the elasticity estimates and methodology reported in this paper are incorporated into LBL's Residential Energy Model. We also consider an alternative method, and discuss the circumstances under which the alternative might be preferred.

Concern over future energy demand has prompted several researchers to develop predictive computer models of national energy use. The LBL Residential Energy Model (LBL-REM) is one such model. The model makes long-run projections of future energy demand, given a proposed scenario of economic development, fuel price paths, etc. It was derived from earlier work done at Oak Ridge National Laboratory, with particular improvements that make it highly effective for analyzing changes in the efficiencies of residential appliances. The ORNL model was specifically modified to allow greater disaggregation by end-use and more sophisticated expression of the relationships among the determinants of energy consumption. (See McMahon [15] for a full description of the model.)

Almost all models that simulate future energy demand have to make some accommodation to several problems, including the prediction of market shares for fuels and technologies under changing conditions, and the emergence of new technologies for which consumer preference is unknown.

The first of these is commonly handled by obtaining an estimate of the market share elasticities, perhaps expressed as a function of independent variables. Projected market shares can then be calculated either by straight-line extension of the slope expressed by the elasticity, or by updating the independent variables and recalculating the elasticity periodically. The second method is obviously preferable, with more frequent updating preferred over less frequent. But the expressions used to calculate elasticities are commonly derived from econometric studies which were made in a particular context. The farther one gets from that context, whether in the population surveyed, the technologies offered, or the values of the explanatory variables, the less confidence one should have in the estimated elasticities. It is important that any model be regularly revised to include the best available information in the expressions governing the evolution of the modeled phenomena.

The second problem is difficult to deal with, and different researchers have approached it in different ways. An example of this is heat pump technology, which has claimed a significant market share only in the last five to ten years. In past versions of LBL-REM, heat pumps were considered to be an independently-estimated fraction of all electric systems, with the market share of electricity calculated first and heat pumps broken out afterwards. This approach had obvious drawbacks.

The inclusion of the work reported in this paper will allow improvements both in the methodology of calculating future market shares, and in handling heat pump technology. Based on the quality of the original model by EPRI, we feel the elasticities reported here represent the best available estimates. They are estimated on the proper level of disaggregated data, over all currently available technologies of significance, and using a satisfactory model of the relationship between cooling and heating choices. Furthermore, as discussed in Appendix 2, new market shares due to changing conditions can be calculated directly from the size of the changes in the variables (relative to base values). Heat pumps, being explicitly modeled for the first time in EPRI's work, can be explicitly accounted for in the forecasting model, LBL-REM.

LBL-REM calculates the market shares of each of eight different technologies or technology groups, shown in the table below. Note that central electric systems and non-central electric systems together compose the category "conventional electric," and heat pumps are distinct from this grouping. Note also that "conventional" central cooling is possible in combination with any of the other technologies (with the exception of the heat pump alternative, which is somewhat arbitrarily defined as "non-conventional"). The work reported in this paper has calculated the arc elasticity as a function of perturbation size for each of these technology groups with respect to a number of explanatory variables. Specifically, we use arc elasticities of market share with respect to changes in 1) the operating cost of each of the eight technologies, 2) the capital cost of each of the eight

Technology Groups	
Heating Systems	Cooling Systems
Gas Systems 1. central gas 2. non-central gas Oil Systems 3. central oil 4. non-central oil Conventional Electric Systems 5. central electric 6. non-central electric Non-Conventional Electric Systems 7. heat pumps (includes central cooling)	Conventional Electric Systems 8. central air conditioning

technologies, 3) household income, 4) the presence (or absence) of gas restrictions, and 5) the "weather" exogenous variable.

The first three of these are self-explanatory. The fourth is necessary because the existence of gas curtailments in the late 1970's artificially depressed gas market share, which then "rebounded" after the restrictions were lifted. Similarly, the fifth category, weather, reflects the ongoing general shift of population to the south and west. These non-economic (i.e., independent of traditional price and income measurement) phenomena are essential for accurate modeling of residential energy market shares.

As explained in Appendix 2, we can use the quadratic relationship between arc elasticity and perturbation size to estimate new market shares directly, as the consequence of simultaneous changes in a variety of explanatory variables. For perturbations of size less than or equal to +50%, we use the parameters of the quadratic curve estimated on the data. For perturbations greater than +50% (up to +300%) we use a largely (but not entirely) *ad hoc* procedure of fitting both an exponential and linear model to the data, and selecting coefficients from the better-fitting model.[4]

The model stores values for household income and the capital and operating costs of each of the technologies in the "base year" of the simulation (currently, 1977). Operating costs are calculated by the expression

$$OC = \frac{\text{(fuel price)(usage)(thermal integrity)}}{\text{(equipment efficiency)}}.$$

The algorithm used by LBL-REM for estimating new market shares is described below.

[4] More specifically, we solve two equations: $y = a + b\delta$ and $y = ae^{b\delta}$ for the value of the arc elasticity at the two endpoints of the range $\delta = 0.5$ to 3.0. To insure continuity with the range where $\delta \leq 0.5$, the left endpoint is solved not for the calculated arc elasticity itself, but rather for the fitted value of the quadratic curve derived on the data where $\delta \leq 0.5$. For each of these two equations, the quality of the fit to selected data points in the range can be compared by finding the multiple R^2 value for each equation. The better-fitting equation is then used to predict new market shares when perturbations are in the range $\delta = 0.5$ to 3.0.

LBL-REM Algorithm for Estimating New Market Shares

The following steps for estimating the market shares of space-heating technologies are carried out for each year of a projection:

1) Calculate "current year" values for:

   a) household income;

   b) the capital cost of eight technologies;

   c) the operating cost of eight technologies, given the new fuel prices, efficiencies, thermal integrity, etc.

2) For household income, capital cost, and operating cost of the eight technologies, calculate:

$$\delta = \frac{\text{new value} - \text{base value}}{\text{base value}}.$$

   Call these δ_{inc}, δ_{CC}, and δ_{OC}, respectively.

3) Use these seventeen values of δ (one income effect, eight capital cost effects, and eight operating cost effects) to find the new market shares of each technology.

$$MS_{new} = MS_{old}[\,1 + \text{(income effects)} + \text{(capital cost effects)} + \text{(operating cost effects)}\,]$$

$$\begin{aligned} = MS_{old}[\,1 \; + \; & \delta_{inc}(a_{0,inc} + a_{1,inc}\delta_{inc} + a_{2,inc}\delta^2_{inc}) \; + \\ & \left.\begin{array}{l} \delta_{CC}(a_{0,CC} + a_{1,CC}\delta_{CC} + a_{2,CC}\delta^2_{CC}) \; + \\ \delta_{OC}(a_{0,OC} + a_{1,OC}\delta_{OC} + a_{2,OC}\delta^2_{OC}) \end{array}\right\} \text{terms in first of eight technologies} \\ & \qquad + \; \ldots \; + \\ & \left.\begin{array}{l} \delta_{CC}(h_{0,CC} + h_{1,CC}\delta_{CC} + h_{2,CC}\delta^2_{CC}) \; + \\ \delta_{OC}(h_{0,OC} + h_{1,OC}\delta_{OC} + h_{2,OC}\delta^2_{OC})\,] \end{array}\right\} \text{terms in last of eight technologies} \end{aligned}$$

   Note that each of the seventeen values of δ must be tested for $\delta \leq 0.5$, and the appropriate linear or exponential expression substituted for the quadratic relationship above when the test is not met.

With this procedure, we are treating the effect of simultaneous changes in all the different explanatory variables (income, eight capital costs, and eight operating costs) as being additive in their individual effects. As discussed in Appendix 2, this is accomplished only by a) neglecting all terms of order δ^3 or higher, which seems reasonable if the different perturbations are "small", and b) neglecting cross-product terms of order δ^2, which does not seem so defensible.

We can reduce this error somewhat (at least for operating costs) by using the following argument: Some portion of the change in market share of any given technology is due to simultaneous changes in operating cost of all the other systems. And some portion of the (different) simultaneous changes in the operating cost of all systems using a given fuel is due to the (common) change in fuel price. If we can capture that portion of the change, we could apply the perturbation δ associated with it against an elasticity that was calculated with respect to fuel price, i.e, with respect to simultaneous changes in the operating cost of all the technologies using that fuel. Essentially, we want to find δ_{fuel} and δ_{tech} such that:

$$\delta_{OC} = \delta_{fuel} + \delta_{tech}$$

where *fuel* refers to new fuel prices (and is common to all technologies using that fuel) and *tech* refers to technology-specific changes in efficiency, usage, etc.

That portion of the change which is due to changing fuel prices is used in conjunction with an elasticity calculated by perturbing the operating cost of all technologies using that fuel. As such, it is free of any error caused by neglecting the second-order terms in the product of different perturbation sizes for different operating costs. Only the technology-specific changes δ_{tech} carry that error. However, as we will discuss shortly, this procedure of dividing δ_{OC} into two parts carries its own error. A judgement must be made as to which source of error is likely to be greater for a given economic scenario. Below, we explain an algorithm for carrying out the partial "fix" described in this paragraph.

Alternative Algorithm for Estimating New Market Shares

The following steps for estimating the market shares of space-heating technologies are carried out for each year of a projection:

1) Calculate "current year" values for:
   a) household income;
   b) the capital cost of eight technologies;
   c) the operating cost of eight technologies, given the *new* (i.e., current year) values for fuel prices and *base* (i.e., base year) values for efficiencies, thermal integrity, etc. (call this OC_{fuel});
   d) the operating cost of eight technologies, given the *new* fuel prices, efficiencies, thermal integrity, etc. (call this OC_{new});
2) For household income and the capital cost of the eight technologies, calculate:

$$\delta = \frac{\text{new value} - \text{base value}}{\text{base value}}.$$

3) For the operating cost of the eight technologies, calculate:

$$\delta_{fuel} = \frac{OC_{fuel} - OC_{base}}{OC_{base}}$$

and

$$\delta_{tech} = \frac{OC_{new} - OC_{fuel}}{OC_{base}}$$

where δ_{fuel} represents the perturbation in operating cost (from the base year values) due to changes in fuel prices, and δ_{tech} represents the perturbation due to technology-specific changes in efficiency, thermal integrity, etc. The terms δ_{fuel} and δ_{tech} are calculated so as to satisfy:

$$\delta_{fuel} = \frac{\text{new fuel prices}}{\text{base fuel prices}} - 1$$

and

$$(1 + \delta_{OC})OC_{base} = (1 + \delta_{fuel} + \delta_{tech})OC_{base} = OC_{new}.$$

Clearly, δ_{fuel} will be the same across all technologies using the same fuel, and can take on only three values, represented as δ_{gas}, δ_{oil}, and δ_{elec}.

4) Use these twenty values of δ (one income effect, three fuel effects, eight capital cost effects, and eight technology effects) to find the new market shares of each technology.

$$MS_{new} = MS_{old}[\,1 + (\text{income effects}) + (\text{fuel price effects}) + \cdots$$
$$+ (\text{capital cost effects}) + (\text{technology effects})\,]$$

$$= MS_{old}[\,1 + \delta_{inc}(a_{0,inc} + a_{1,inc}\delta_{inc} + a_{2,inc}\delta_{inc}^2) +$$

$$
\left.\begin{array}{l}\delta_{gas}(a_{0,gas} + a_{1,gas}\delta_{gas} + a_{2,gas}\delta_{gas}^2) \; + \\ \delta_{oil}(a_{0,oil} + a_{1,oil}\delta_{oil} + a_{2,oil}\delta_{oil}^2) \; + \\ \delta_{elec}(a_{0,elec} + a_{1,elec}\delta_{elec} + a_{2,elec}\delta_{elec}^2) \; + \end{array}\right\} \text{fuel effects}
$$

$$
\left.\begin{array}{l}\delta_{CC}(a_{0,CC} + a_{1,CC}\delta_{CC} + a_{2,CC}\delta_{CC}^2) \; + \\ \delta_{tech}(a_{0,tech} + a_{1,tech}\delta_{tech} + a_{2,tech}\delta_{tech}^2) \end{array}\right\} \text{terms in first of eight technologies}
$$

$$
+ \; \ldots \; +
$$

$$
\left.\begin{array}{l}\delta_{CC}(a_{0,CC} + a_{1,CC}\delta_{CC} + a_{2,CC}\delta_{CC}^2) \; + \\ \delta_{tech}(a_{0,tech} + a_{1,tech}\delta_{tech} + a_{2,tech}\delta_{tech}^2)\;] \end{array}\right\} \text{terms in last of eight technologies}
$$

As in the main algorithm, each of the twenty values of δ must be tested for $\delta \leq 0.5$, and the appropriate linear or exponential expression substituted for the quadratic relationship above when the test is not met.

With this procedure, we are essentially trading off between two different sources of error. We have already discussed how, if we treat the effect of simultaneous changes in several variables as being additive in their individual effects, we make an error due to neglect of some second-order terms. Breaking up the change in operating cost into two parts (in the calculation $\delta_{OC} = \delta_{fuel} + \delta_{tech}$), avoids that error for the "fuel price" portion of the change. Unfortunately, breaking up the perturbation into two parts and treating those effects additively requires that we once again neglect a second-order term, this time in the product $\delta_{fuel} \cdot \delta_{tech}$.

There seems to be no practical way out of this dilemma. Treating multiple effects other than additively leads to the requirement of calculating elasticities in all possible combinations of variables that might be perturbed, a combinatorial problem that is unpleasant to contemplate, let alone carry out. Otherwise, we wish to choose a procedure which will make the neglected terms as small as possible. In the main procedure that we propose to use (ignoring the common fuel price effects), the neglected terms are second-order products like $\delta_{1,OC} \cdot \delta_{2,OC}$, where $\delta_{i,OC}$ refers to the perturbation in operating cost of the ith technology, and perturbations are the net of changes in fuel prices, efficiencies, etc. In the alternative procedure (capturing the common fuel price effects), the neglected terms are products like $\delta_{fuel} \cdot \delta_{tech}$.

Clearly, for economic scenarios in which the operating costs of technologies in a particular fuel type tend to move together (i.e., the change in fuel price dominates the technology-specific changes) δ_{tech} will be small relative to δ_{fuel} and their product will be small. If the scenario involves large technology-specific changes above and beyond the fuel price effects, then their product will be (relatively) large. In particular, if fuel prices tend to increase and technology efficiencies improve correspondingly, then the net change in operating costs $\delta_{i,OC}$ may be very small, while δ_{fuel} and δ_{tech} will be large and opposite in sign. For this scenario, the main algorithm will be neglecting terms in the product $\delta_{1,OC} \cdot \delta_{2,OC}$, while the alternative algorithm will neglect the product $\delta_{fuel} \cdot \delta_{tech}$. The main algorithm will be superior. (We note in passing that in scenarios where fuel prices do not change, both algorithms will produce the same results.)

We believe that scenarios of rising fuel prices and efficiencies are more typical of those planned for modeling with LBL-REM, and so we prefer to use the main algorithm. We also intend to produce a working version using the alternative algorithm, to test the significance of the different estimating procedures.

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