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A NOTE ON THE MICROECONOMICS
OF MIGRATION

BY

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CENTER FOR REAL ESTATE AND URBAN ECONOMICS
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A NOTE ON THE MICROECONOMICS OF MIGRATION*

by

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1. Introduction

It is the purpose of this note to demonstrate in a very simple model that an individual's migration from a small town to a large city may be rationalized purely by a consumption motive, rather than the motive of obtaining a higher income. More specifically, it is shown that in a large city an individual may derive a higher utility from spending a given amount of income than in a small town. This may be so *even if the prices for commodities obtained at both locations are higher in the large city than in the small town.* That difference in attainable utility levels may induce our individual to spend a positive amount of income in order to migrate.

In the model presented here, this behavior is based on the realistic assumption that the set of commodities, or more precisely, the set of commodity varieties marketed in large cities, is larger than in small towns.[1] Thus, whenever we assume as usual that the typical consumer's utility index is strictly quasi concave, the utility derived from consuming a commodity bundle chosen from a larger variety of commodities is higher than that derived from a commodity bundle involving fewer commodities, provided that at given income and prices prevailing in the large city, the consumer's budget set will cut the level set defined by the utility level attainable in the small town. This will be the case, for instance, if prices for the commodities marketed at both locations are identical and prices of commodities marketed at the large marketplace

only are not so high as to drive our consumer's demand for all of these commodities to zero. Other combinations of prices at which the result holds can be thought up at ease. In fact, it emerges quite naturally from the model that *the smaller the elasticity of substitution between the commodities from which the consumer derives utility, the larger the space of price vectors under which the result holds.* It follows that the lower that elasticity of substitution, the larger the price differentials our consumer is willing to bear and still will be willing to incur a given cost of migration; or, interpreted the other way around, the larger the cost of migration he will be willing to incur in order to avail himself of more extended consumption possibilities.

The remainder of this note is organized as follows: The formal model is developed and the principal results are derived in section 2. A graphical example, given in section 3, illustrates matters. The note concludes with some remarks on some theoretical and empirical implications of the model structure and its results. It is finally concluded that the argument applies with similar force to the migration of firms.

2. The Model[2]

Consider a consumer who is endowed with income R and a preference ordering over a vector of N commodities i , $i \in N$, $N = \{1,...N\}$, $N \geq 2$. We assume that his preferences are representable by a utility index $u(\underline{x})$ where $\underline{x} = (x_1,...x_N)$ denotes nonnegative quantities of the N commodities. The index u is assumed to satisfy the usual properties, that is,

(i) u is differentiable once and strictly increasing in each

x_i, $x_i \geq 0$, $i = 1,\ldots N$.

(ii) u is strictly quasi concave.

Thus, indifference levels between any two commodities are strictly convex.

Furthermore, we suppose that only a proper subset of the commodities is essential in the sense that whenever commodity i belongs to this subset our consumer cannot survive without consuming a positive quantity of this commodity, whereas he will do so if he is unable to consume any positive quantity of the (inessential) commodities not belonging to this subset. Denoting this subset by E, and letting $\underline{\hat{x}}$ denote a vector within which one or more x_i, $i \in E$ assume a zero value

(iii) $\hat{x}_i = 0 \Rightarrow \nexists \underline{x}$ such that $u(\underline{x}) < u(\underline{\hat{x}})$.

Thus if we choose to represent graphically our consumer's utility index and select i to be an essential and j an inessential commodity, then indifference levels will cut the x_i, but not the x_j axis.

Assumption (iii) is natural within the context discussed here: Were all commodities essential, would the consumer have to be availed of all commodities at all locations in order to survive. In this unrealistic case, the argument would fall apart.

We finally assume our consumer to be a utility maximizer. Thus, letting

$$B(\underline{p}, R) = \{\underline{x} \in R_+^N \mid \underline{p} \cdot \underline{x} \leq R\}$$

denote his budget set, we suppose that the consumer chooses the bundle $\hat{\underline{x}}$ maximizing $u(\underline{x})$ subject to $\underline{x} \in B(\underline{p}, R)$.

Suppose that our consumer is in the situation called state A to purchase and consume only the subset E--or, more generally, a proper subset of N including E, for example, N_1--of the commodities at prices p_i^A, $i \in N_1$.[3] Without loss of generality, let the set of commodities be $N_1 = \{1,\ldots N_1\}$, $E \leq N_1 < N$, where $E \equiv \#E$. Thus, in state A, the consumer maximizes his utility under the additional restriction that $x_i = 0$, $i = N_1 + 1,\ldots,N$. Denote by $(\underline{x}_1^A, \underline{0})$ the consumer's optimal choice, and by $u_A = u(\underline{x}_1^A, \underline{0})$ the associated utility level. By assumptions (i) and (ii), the consumer's optimal allocation will be unique and will exhaust all income R.

We now wish to show that an extension of the set of commodities available will increase the attainable utility level, implying conversely a decrease in the expenditures incurred in order to obtain u_A, as long as the prices of the commodities in the extension are not too high. Without loss of generality, we may consider an extension such that the set of all N commodities is available. Call this state B, and let in that state the vector of strictly positive prices $\underline{p}^B = (\underline{p}_1^A, \underline{p}_2^B) = (p_1^A,\ldots,p_{N_1}^A, p_{N+1}^B,\ldots,p_N^B)$ prevail.[4] Denote by $(\underline{x}_1^B, \underline{x}_2^B)$ and $u_B = u(\underline{x}_1^B, \underline{x}_2^B)$ the consumer's optimal choice at these prices and the associated utility level, respectively.

Proposition: Suppose (i) to (iii) hold. Let $\underline{p}^B = (\underline{p}_1^A, \underline{p}_2^B)$ prevail. Then $u(\underline{x}_1^B, \underline{x}_2^B) \geq u(\underline{x}_1^A, \underline{0})$, and $u(\underline{x}_1^B, \underline{x}_2^B) > u(\underline{x}_1^A, \underline{0})$ provided that

$p_i^B < \tilde{p}_i$ for some $i \in N \setminus N_1$ with

$$\tilde{p}_i = p_j^A \frac{\frac{\partial u}{\partial x_i}(\underline{x}_1^A, \underline{0})}{\frac{\partial u}{\partial x_j}(\underline{x}_1^A, \underline{0})} \quad \text{for some } j \in N_1.$$

Proof: Observe first that $(\underline{x}_1^A, \underline{0})$ is feasible no matter which prices $\underline{p}_2^B$ prevail. Thus $(\underline{x}_1^A, \underline{0}) \in B(\underline{p}_1^A, \underline{p}_2^B, R)$. Hence $u(\underline{x}_1^B, \underline{x}_2^B) \geq u(\underline{x}_1^A, \underline{0})$. Second, note that if $p_i < \tilde{p}_i$,

$$\tilde{p}_i = p_j^A \frac{\frac{\partial u}{\partial x_i}(\underline{x}_1^A, \underline{0})}{\frac{\partial u}{\partial x_j}(\underline{x}_1^A, \underline{0})},$$

then under $B(\underline{p}_1^A, \underline{p}_2^B, R)$ the optimal choice $(\underline{x}_1^B, \underline{x}_2^B) \neq (\underline{x}_1^A, \underline{0})$. A statement to the contrary would be inconsistent with the relevant Kuhn-Tucker conditions. By assumptions (i) and (ii), however, $(\underline{x}_1^B, \underline{x}_2^B)$ is the unique optimal choice. Hence $u(\underline{x}_1^B, \underline{x}_2^B) > u(\underline{x}_1^A, \underline{0})$. ∎

We now will show by a simple extension of the proposition that state B may be preferred to A even if $\underline{p}_1^B \gg \underline{p}_1^A$, and/or if, in state B, our consumer's income R is reduced by a fixed (migration) cost $K > 0$.

Let $u(\underline{x'}_1^B, \underline{x'}_2^B)$ denote the optimal utility attainable if $\underline{x} \in B(\underline{p}_1^B, \underline{p}_2^B, R)$ for $\underline{p}_1^B = \varepsilon \cdot \underline{p}_1^A$ and $u(\underline{x''}_1^B, \underline{x''}_2^B)$ the one if $\underline{x} \in B(\underline{p}_1^A, \underline{p}_2^B, R - K)$.

Corollary: Under the conditions of the proposition,

(i) there is an $\varepsilon > 1$ such that

$$u(\underline{x'}_1^B, \underline{x'}_2^B) > u(\underline{x}_1^A, \underline{0})$$

(ii) there is a K > 0 such that

$$u(\underline{x}''^B_1, \ \underline{x}''^B_2) > u(\underline{x}^A_1, \ \underline{0}).$$

<u>Proof</u>: This follows trivially from the proof of the proposition and the observation that by assumption (ii), $\underline{x}(\underline{p}, R)$ is continuous in all variables. ∎

3. A Graphic Example

We maintain the notation and assumptions introduced before and consider a case $N = 2$, $N_1 = \{1\}$, $N \setminus N_1 = \{2\}$. Suppose that in state A only commodity 1 is offered at price p^A_1. In the diagram drawn below, the utility maximizing commodity bundle is denoted by $(x^A_1, 0) = \left(\dfrac{R}{p^A_1}, \ 0\right)$ and the associated utility level by $u(x^A_1, \underline{0})$.
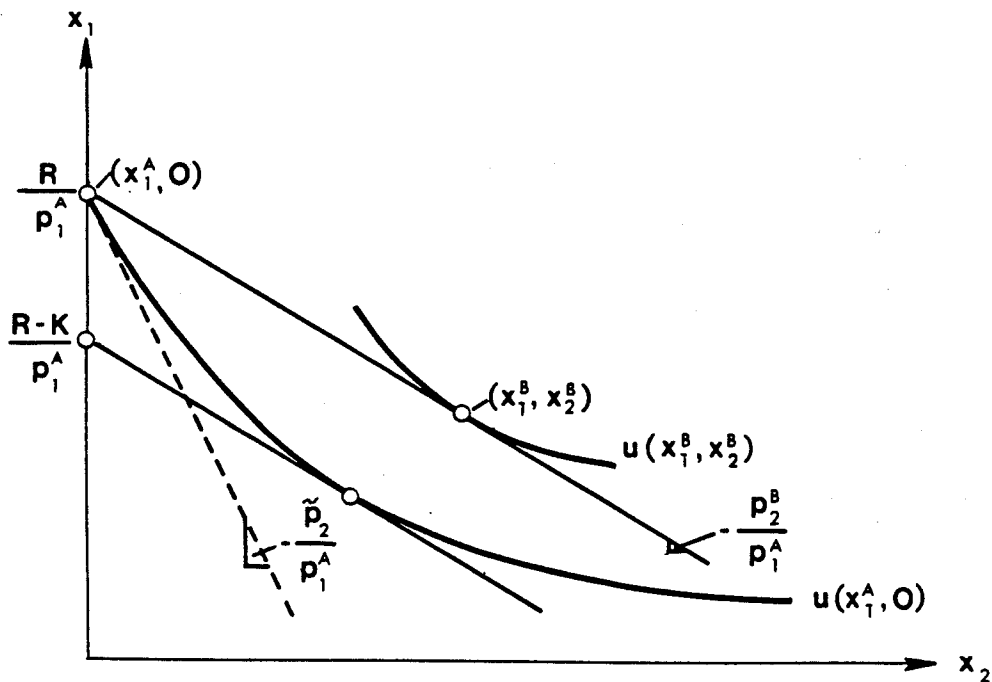
Suppose now that our consumer alternatively is confronted with the option of purchasing both commodities at prices $p_1^B = p_1^A$ and p_2^B. His utility maximizing commodity bundle then will involve positive quantities of both commodities as long as, for given p_1^A, $p_2^B < \tilde{p}_2$

$$\text{where} \quad \tilde{p}_2 = p_1^A \; \frac{\dfrac{\partial u}{\partial x_2}\left(\dfrac{R}{p_1^A}, \; 0\right)}{\dfrac{\partial u}{\partial x_1}\left(\dfrac{R}{p_1^A}, \; 0\right)} \; . \quad \text{This bundle is denoted by} \quad (x_1^B, \; x_2^B).$$

It follows also that $u(x_1^B, x_2^B) > u(x_1^A, 0)$ whenever $p_2^B < \tilde{p}_2$. Because of this, our consumer would be willing to incur a migration cost (converted to an annuity) up to K in order to be availed of the enlarged consumption possibilities. In fact, $u(x_1^B, x_2^B) > u(x_1^A, 0)$ holds for any price ratio $\pi \in \left(0, \dfrac{\tilde{p}_2}{p_1^A}\right)$, in particular also for prices $p_1 > p_1^A$.

For the sake of simplicity, assume finally that the consumer's utility function is CES with elasticity of substitution greater than unity. As the elasticity of substitution between the two commodities *decreases*, the interval of price ratios at which positive combinations of the two commodities are preferred *increases*, and so does--at given prices--the maximal expenditure on migration our consumer is willing to incur in order to better himself. Thus, the likelihood that the consumption motive induces him to migrate increases with decreasing elasticity of substitution between the two commodities.

4. Concluding Remarks

The motive to migrate discussed here is by no means new: It is mentioned in several places in the literature on migration. What is new is that this motive is given precision within the standard model of consumer behavior. Such precision is needed for several reasons: First, when explaining individual and aggregate behavior in analytical models, and evaluating it in terms of efficiency and distributional objectives; furthermore, when tracing, in such analytical models, the possible impacts of policy proposals; finally, as a theoretical basis for empirically quantifying behavioral hypotheses and environments of choice.

Within the former (analytical) context, the little model provided here, while of value on its own, should be thought of as an element in a global model involving the interaction of many agents. Within the latter (empirical) context, the model teaches us, for instance, that in the development of local cost of living indices, attention should be paid to the fact that attainable levels not only vary with prices, but with the commodity varieties that can be purchased at given nominal income, and that these do significantly vary among agglomerations of different size. Disregarding this phenomenon in the present context could render incorrect the conclusions drawn from conventional estimates of local cost of living indices.

Let us be a little bit more precise on the latter point: Any empirically computed cost of living index must obviously be computed on the basis of the commodity bundle E. Suppose now that, as commonly

argued, the price level for commodities in E increases with city size. In this case, any empirically computed index tends to *overestimate* the increase in the cost of living, or conversely: tends to *underestimate* the attractiveness of large cities with respect to the consumption motive.

A further comment is on the conceptualization of the model. It is at variance with the way space is treated within the Arrow-Debreu framework. There, the commodity space is simply extended by the location of availability of commodities. This, as is well known, may easily lead to nonconvexities in the consumption set, as well as the consumer's preferences. Here, commodities differ, in view of the consumer, only by physical characteristics, and not by location of availability. This removes the nonconvexities that exclusively are of a technical nature and allows us to concentrate on modeling the impact of the nonconvexity in the budget set arising from overcoming space.

Finally, it is worth mentioning that the arguments developed in this note apply with similar force to the migration of firms. In fact, the argument can be translated *directly* for a one-product firm that is a price taker in input and output markets at both locations.

FOOTNOTES

[1] It is hardly necessary to justify that assumption A rationalization in microeconomic language runs as follows: The marketing of commodities is subject to decreasing average cost. However, cost schedules vary across commodities, implying that varieties with comparatively large average marketing costs are marketed only at locations where aggregate demand is sufficiently large at given prices.

[2] This is a mere reinterpretation of a model used in a forthcoming paper on Transportation Nonconvexities and the Location of Markets in Space.

[3] We exclude the possibility that positive amounts of all commodities not marketed in the small town may be obtained from commuting to some distant market place. A sufficient but by no means necessary condition is that the cost of obtaining all such commodities would exceed the feasibility constraints of the typical consumer. Incidentally, the case where larger sets of commodities may be obtained when commuting is treated in the paper mentioned in footnote 2.

[4] We neglect differences in the transactions cost between the two states of obtaining some commodity. As it will become obvious from the corollary below, this does not affect the principal reasoning.

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