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Adaptation to Embodied Dynamics: Evidence from Bayes' Ball

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Abstract

In recent years researchers have begun to recognize the remarkable intelligence of even routine interactive behavior—the extent to which humans adapt to and exploit the low-level dynamics of cognitive processes, perception, and motor control without conscious deliberation. We developed an experiment known as Bayes' Ball, in which subjects must estimate short time intervals (< 1000 ms) in order to maximize a point score in a simple ball tracking task. Optimal performance in this task requires that subjects possess intricate knowledge of the dynamics of uncertainty in their estimates of elapsed time (the Weber law of interval timing). The results of our experiment show human performance that approaches, but does not fully match the predictions of an ideal performer. On average, humans differed from the ideal performer by less than 40 ms in their response timing.

Keywords: embodied cognition; time estimation; uncertainty; ideal performer analysis

Introduction

Nearly all of human activity can be described as an intricately coordinated dance of low-level cognitive, perceptual, and motor components, each taking small steps of progress towards a larger goal. The elementary operations performed by these components unfold on the timescale of $1/3$ of a second and form the *embodiment level* of cognition (Ballard, Hayhoe, Pook, & Rao, 1997)—the level of analysis at which the constraints of the physical components first have implications for the ability to achieve goals in the world. The field of psychology has a long history of studying the basic properties of each of these components in isolation, for example, the rate of memory decay as a function of time and practice (Ebbinghaus, 1913) or how movement duration scales with magnitude (Fitts, 1954). However, a related and fascinating question has received considerably less attention. Given a lifetime of experience reaching for objects and remembering facts, to what extent are individuals already and implicitly aware of the performance characteristics of their own embodied cognitive processes?

Given the multitude of varying tasks that humans face over the course of a lifetime, it is notable that the only stable components in these interactions are the internal components themselves. While tasks and demands on cognition are likely to change drastically from minute to minute, the dynamics of memory decay are fairly invariant (and thus predictable) across a wide range of tasks, as are the dynamics of simple reaching movements. Thus from the perspective of efficient skill acquisition, it seems rational that much of what should be learned in routine behavior is an accurate internal model of one's own *embodied dynamics*—the performance characteristics and interactions among cognition, perception, and motor control—rather than knowledge of the task, as this is the

knowledge that is most likely to be relevant in future tasks. This perspective has important implications for research on skill acquisition, as it shifts the explanatory focus from theories of learning a new task, to theories of adapting and exploiting existing internal models to changing environmental circumstances.

The purpose of this research is to explore the extent to which interactive behavior in a novel task is sensitive to the performance dynamics and interactions among simple cognitive, perceptual, and motor processes. The work builds on a growing body of literature demonstrating remarkable sensitivity to low-level dynamics even in the most mundane activities. For example, (Augustyn & Rosenbaum, 2005) presented subjects with two circular targets on a computer screen. The subjects were required to move the mouse cursor to a starting position anywhere along a line connecting the two targets. After choosing a starting position, one of the two targets (randomly chosen) disappeared, and the task for the participant was to move the mouse cursor into the remaining target within a short response interval. Across trials, the width of one of the targets was varied. By making one of the targets larger, it became easier and faster to move the cursor into that target, in accordance with Fitts law. Given the goal of maximizing performance in terms of landing the cursor inside the target within the time period, optimal performance in the task requires accounting for Fitts law, as well as numerous low-level dynamics of the specific task (for example, subjects moved a small wooden disk on a table to control the movement of the cursor). In keeping with their predictions, Augustyn and Rosenbaum found that their participants demonstrated performance that was indistinguishable from an optimal performer.

In a somewhat more complex task, Gray and colleagues (Gray, Sims, Fu, & Schoelles, 2006) examined behavior in a block copying paradigm, where subjects had to memorize and replicate a pattern of eight colored blocks seen in one location of the computer screen at another location. The original pattern of blocks and the workspace area where the pattern was to be reproduced were both normally covered by occluding boxes. To uncover the target or workspace area the subject needed to move the mouse cursor inside the occluding box. As a between-subjects manipulation, the investigators added a lockout time to the target window that varied between 0 and 3.2 seconds, such that the subject had to move the mouse cursor inside the box and wait the lockout duration before the target pattern could be viewed.

Gray *et al* recorded the duration that subjects studied the target pattern, the number of blocks of the pattern that were

successfully copied to the workspace following the first uncovering of the target window, as well as the total number of visits required to complete each trial. Subject performance according to each of these measures systematically varied between conditions, with the finding that as the lockout time increased, subjects spent more time studying the target pattern on each visit, placed more blocks following that visit, and required fewer accesses of the pattern to complete each trial. These adaptations to the lockout period reflected a near-optimal tradeoff between the ability to successfully study and recall all eight blocks on the one hand, and the temporal and motor cost of re-accessing the pattern if some of the blocks are forgotten or not encoded. In this paradigm, the observed performance cannot be attributed to either perceptual-motor or cognitive skill alone, but rather adaptation to the low level dynamics of each, combined with the unique properties of the specific task artifact with which the subjects interacted.

Other studies have found sensitivity and adaptation to the time course of simple arithmetic (Shin & Rosenbaum, 2002), the motor variability associated with rapid pointing movements (Trommershäuser, Maloney, & Landy, 2003)(Maloney, Trommershäuser, & Landy, 2007), and even the detailed mechanical properties of a photocopier machine (Agre & Shrager, 1990). Taken together, they support the idea that humans possess, adapt, and exploit intricate knowledge of their own embodied dynamics in routine interactive behavior. As an attempt at extending these findings, we constructed an experiment to investigate whether humans adapt to and exploit another embodied dynamic; namely the uncertainty associated with estimating short time intervals (< 1000 ms), embedded in the context of a simple perceptual-motor ball tracking task. This paradigm was chosen as it is not intuitively obvious that humans can estimate such short intervals with any accuracy, let alone demonstrate awareness of the uncertainty inherent in their estimates.

Experiment: Bayes' Ball

The Bayes' Ball paradigm consists of a small ball displayed on a computer screen, and an occluding window covering a large portion of the screen (see figure 1). At the beginning of a trial, the ball is visible to the participant. After pressing down a key on a response pad, the ball begins moving from left to right at constant velocity. When the ball reaches the occluding window it becomes hidden from view and continues moving. When the key is released, the ball immediately stops moving and its position is revealed to the participant.

If the subject's goal is to release the key such that the ball stops at a particular location, then it is apparent that the subject must be able to estimate both the velocity of the ball as well as the amount of time that has elapsed since it moved behind the occluder (from elementary physics, $x = v \cdot t$). Whereas ball velocity can be perceptually estimated from its initial movement, elapsed time is fundamentally an intrinsic quantity. More importantly, the human ability to estimate short time intervals is characterized by a lawful pat-

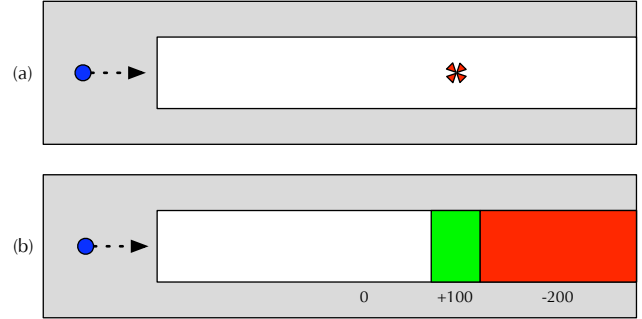


Figure 1: Apparatus used in the Bayes' Ball task. (a) During the training phase subjects attempt to stop the ball directly at the red 'x'. (b) During the test phase, subjects attempt to maximize their point gain by stopping in the reward region (green) and avoiding the penalty region(s) (red).

tern of variability, with the standard deviation of timing accuracy scaling linearly with the duration of the interval to be estimated—a finding referred to as the Weber law of interval timing (Staddon & Higa, 1999).

The Bayes' Ball paradigm is designed to test whether humans are sensitive to this embodied dynamic, by presenting interval estimation under risk as a simple decision-making task. During the test phase of the experiment, subjects must time their responses such that the ball stops in one of three point regions (figure 1b). Some of the regions are worth positive points, while others yield losses. On some trials, the center reward region is surrounded on both sides by penalty regions, while on other trials there is one penalty region and one region worth zero points surrounding the center region. In addition to varying the configuration of reward and penalty regions, the experiment also varies the distance of the targets. Faced with uncertainty about the exact interval of time that has elapsed, the optimal strategy in Bayes' Ball is not to aim for the center of a target, but rather to aim for a location shifted slightly in the direction away from neighboring penalty regions. The magnitude of this compensation should be sensitive to both the uncertainty in the estimated interval as well as the costs and gains associated with the different possible outcomes.

In particular, if subjects have an internal estimate, τ , of the amount of time that has passed since the ball disappeared from view, then according to the Weber law of timing, this estimated time is related to the physical elapsed time t according to a probability distribution $p(t|\tau)$ with standard deviation increasing linearly with τ . In the simplest case, we assume a Gaussian distribution with mean τ and standard deviation $\sigma_0 + \sigma_1\tau > 0$:

$$p(t|\tau) = \frac{e^{-\frac{(t-\tau)^2}{2(\sigma_0 + \sigma_1\tau)^2}}}{(\sigma_0 + \sigma_1\tau)\sqrt{2\pi}}.$$

Assuming the ball's velocity is known with certainty¹, then the location of the ball given physical time t is $p(x|t) = \delta(x - vt)$, where δ is the Dirac delta function. From these two distributions, it follows that the distribution of the ball's position given the internal time estimate is given by

$$p(x|\tau) = \int_{-\infty}^{+\infty} p(x|t)p(t|\tau)dt = \frac{e^{-\frac{(x-v\tau)^2}{2v^2(\sigma_0+\sigma_1\tau)^2}}}{v(\sigma_0 + \sigma_1\tau)\sqrt{2\pi}}.$$

Facing a set of target regions with different point values, the Bayesian optimal decision strategy is therefore to stop the ball at subjective time τ maximizing expected value:

$$U(\tau) = \int_R u(x)p(x|\tau)dx,$$

where the integration is carried out over the set of regions R , and the point value of stopping at a location x is given by $u(x)$. In this case the utility function $u(x)$ is directly specified to the subject as the point value associated with each region. If the boundaries between regions are given by x_1 and x_2 , and the reward values associated with the three regions are r_1 , r_2 , and r_3 , then

$$u(x) = \begin{cases} r_1, & x \leq x_1 \\ r_2, & x_1 < x < x_2 \\ r_3, & x \geq x_2 \end{cases}.$$

The optimal decision time, τ_{opt} , is the value maximizing $U(\tau)$ and is a function of the subject's internal estimation dynamics, the distance to the target on the current trial, as well as the configuration of point regions on the current trial. Presumably, while humans cannot eliminate the uncertainty in their estimation, given opportunity and motivation they should be able to compensate for it by adjusting their intended stopping position closer or farther from the penalty zones. Figure 2 compares the optimal distribution of stopping positions, $p(x|\tau_{opt})$ to actual human performance on that trial configuration. The histogram displays observed human performance for a single subject at a particular distance and penalty configuration, while the smooth curve shows the expected performance of an optimal decision-maker constrained to the estimation uncertainty of the human subject. The reward and penalty configuration is overlaid at the top of the figure. In this example, both the human and ideal performer have shifted their stopping position away from the penalty region, although the magnitude of the shift is slightly smaller for the human than for the ideal performer.

The preceding analysis demonstrates that optimal performance in Bayes' Ball requires estimating not only the passage of time, but the uncertainty inherent in this estimate, and

¹This simplifying assumption is made to keep the equations in a tractable form. In theory, after the practice phase of the experiment a Bayesian observer would have an accurate estimate of velocity, since it is constant on each trial. In practice, any residual uncertainty in velocity is absorbed in fitting the parameters σ_0 and σ_1 to human data.

deploying this information in a perceptual-motor task with task-specific utilities. By comparing human performance in the Bayes' Ball paradigm we can explore the extent to which humans are capable of adapting to and exploiting these low-level dynamics in routine interactive behavior. In conducting the experiment, we held a number of predictions. First, if people have either explicit or implicit knowledge that there is variability in their estimation of elapsed time, then subjects should demonstrate a shift or compensation in their mean stopping position away from penalty regions. Second, if their implicit awareness captures some approximation to Weber's law and not a more crude belief (for example, constant uncertainty independent of target distance), then the magnitude of strategic compensation away from penalty regions should be greater for more distant targets (which require estimating longer intervals) and less for closer targets. Finally, by using an ideal performer analysis for each subject, we can compare the extent to which the observed behavior is optimal.

Method

Participants Twenty three undergraduates volunteered to participate in the experiment for course credit.

Materials The experiment was displayed on an LCD monitor with resolution set to 1280×1024 pixels. Each trial was initiated by pressing and holding a key on a response pad (Cedrus[®] RB-834). The ball travelled at a fixed velocity of 1.0 pixels/ms on each trial ($SD = 0.004$, measured across trials) and stopped as soon as the key was released. The experiment consisted of two phases, a training phase and a test phase. During the training phase, a small red 'x' was displayed at one of three fixed distances (distance = {295, 485, 803} pixels) from the start of the occluding window, shown in Figure 1a. The task for the participant was to time their response (key release) such that the hidden ball stopped at the center of the 'x'. During the test phase subjects attempted to maximize a point score by stopping the ball in one of three regions on the screen. The center region was always worth 100 points. On *both-penalty* trials, the regions to the left and right of the center region yielded a loss of 200 points. On *penalty-before* trials, the region to the left of the center region yielded -200 points while the region to the right was worth 0 points. This mapping was reversed for *penalty-after* trials. In addition, if the ball reached the right side of the screen it was counted as a penalty. The ball's stopping position and point outcome were displayed to the participant after each trial.

Design & Procedure There were no between-subject conditions in the experiment. The practice phase consisted of 750 trials, grouped into blocks of three. Each block contained one trial at each target distance, in random order. During the practice phase, subjects were told to stop the ball as close as possible to the red 'x'. Following the practice phase, subjects completed 900 trials in the test phase. Trials were grouped into blocks of nine, with each block containing one trial at

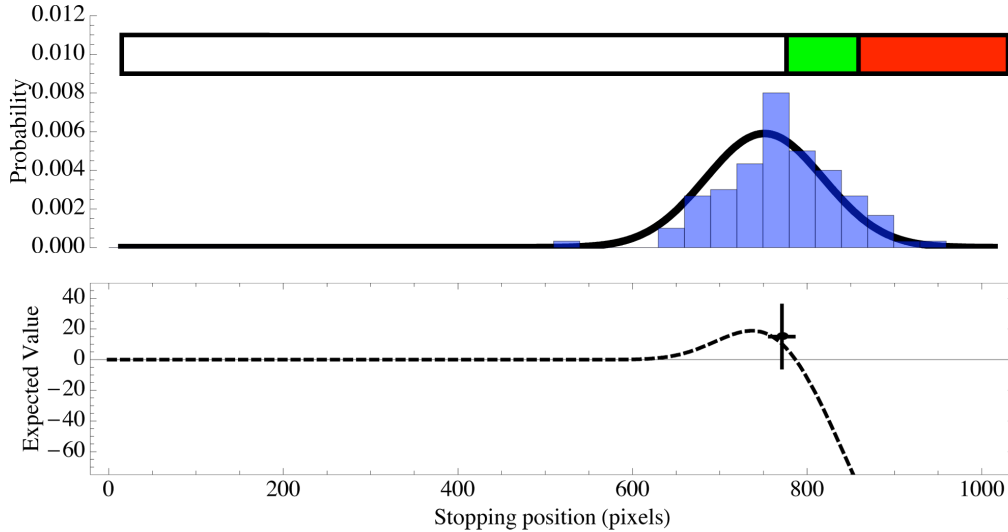


Figure 2: Comparing performance of a single subject to an ideal performer calibrated to that subject. (Top) The smooth curve shows the predicted optimal stopping point distribution, $p(x|\tau_{opt})$, while the histogram shows the observed distribution. (Bottom) Expected value (in points) as a function of mean stopping position. The maximum of the dashed curve corresponds to $U(\tau_{opt})$. The marked point shows the mean stopping position and score for the human subject’s data shown in the histogram at top. Error bars indicate 95% confidence intervals in mean position (x error bars) and score (y error bars).

each combination of target distance and penalty configuration ($\{\text{near, middle, far}\} \times \{\text{penalty-before, both-penalty, penalty-after}\}$), presented in random order. During the test phase, subjects were instructed only to maximize their total point score. Trials were self-paced, and given the velocity of the ball, each trial lasted a maximum of 1200 ms.

Results

In the analyses that follow we concentrate on performance on the test trials. On average, subjects won 13.87 points per trial ($SD = 21.56$), out of a theoretical maximum of 100 points per trial, suggesting that the task was rather challenging for the participants. Indeed, 6 out of the 23 subjects ended the task with negative scores. As far as the main hypotheses of the experiment however, it is possible that even these low-scoring subjects might be optimal in the sense of optimally compensating for a poor ability to accurately estimate intervals.

To address this issue, we examined the ball’s stopping position for each target distance and penalty configuration. To facilitate comparison among the three target distances, we defined the ball’s stopping position on a given trial relative to the center of the reward region. By this measure, positive values indicate stopping to the right of the center of the green reward region, and negative values to the left. The mean target-relative stopping position for each trial distance and configuration is presented in Table 1. A 3×3 ANOVA comparing relative stopping position by distance and penalty configuration yielded a significant interaction between configuration and distance ($F[4, 20254] = 48.558, p < 0.001$). Post-hoc analysis revealed that the interaction stemmed from a greater

Table 1: Target-relative stopping position (pixels) for humans and ideal performers.

Distance	Penalty config.	Mean	$\pm 95\%CI$	Ideal perf.
Near	Penalty-Before	25.94	1.53	11.60
	Both-Penalty	12.65	1.48	-2.94
	Penalty-After	4.40	1.50	-14.97
Middle	Penalty-Before	18.06	2.52	33.48
	Both-Penalty	0.24	2.32	-4.63
	Penalty-After	-13.39	2.25	-32.62
Far	Penalty-Before	12.23	3.49	50.42
	Both-Penalty	-9.11	3.49	-7.38
	Penalty-After	-42.99	3.41	-73.97

effect of distance on relative stopping position for the *penalty-after* than for the *both-penalty* configuration. This interaction was expected, as there should be little or no change in relative stopping position for the *both-penalty* configuration across the three target distances. Unsurprisingly, the main effect of penalty configuration was significant ($F[2, 20254] = 571.825, p < 0.001$). The main effect of target distance was also significant ($F[2, 20254] = 334.592, p < 0.001$).

The main effect of distance was unexpected, as the average stopping position across all three penalty configurations was expected to be close to zero. Relative stopping position on *penalty-before* and *penalty-after* trials should be symmetrical and therefore cancel, while stopping position on the *both-penalty* configurations was expected to be zero (subjects should aim for the center of the target). Across all three

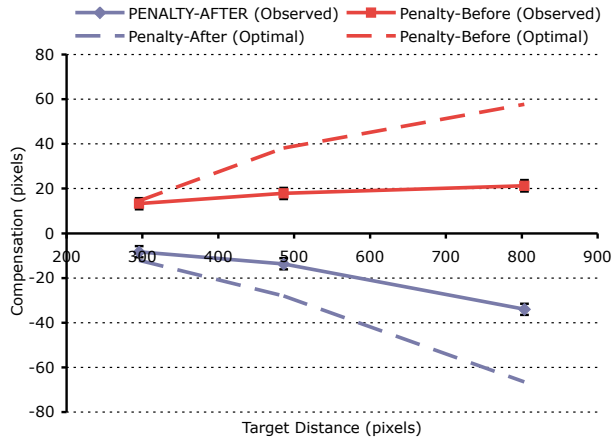


Figure 3: Compensation relative to mean stopping position on *both-penalty* trials for each target distance. Error bars indicate 95% confidence intervals.

penalty configurations however, the relative stopping position shifted from positive for near targets, to negative values for the distant targets. Subjects thus appeared to overshoot the center of the target on the close trials, and undershoot the target on the far trials, regardless of penalty configuration.

To untangle this bias from the main question of interest (compensation due to uncertainty in interval estimation), each subject’s mean stopping position on the *both-penalty* configuration was subtracted from their stopping position on *penalty-before* and subtracted from their stopping position on *penalty-after* trials. This yielded two measures of stopping position, both relative to stopping position on *both-penalty* trials. By looking at relative compensation, we can directly test the effect of uncertainty on stopping position. If subjects are completely ignorant of their uncertainty, then compensation should equal zero for the asymmetrical trials. If they have some awareness of uncertainty in their estimates, but not of the intricacies of the Weber law relation, then compensation might remain constant across target distances. If their performance does reflect implicit awareness of the dynamics of temporal uncertainty, then compensation should scale linearly with distance.

In keeping with the predictions for the experiment, a one-way ANOVA comparing compensation on the *penalty-before* trials showed a significant effect of distance ($F[2, 6731] = 9.653, p < 0.001$). Post-hoc analyses using Tukey’s test revealed a significant increase in compensation between near and middle targets ($p < 0.05$), but no further increase between middle and far targets. Similar analysis of the *penalty-after* trials showed a significant effect of distance ($F[2, 6780] = 113.124, p < 0.001$), with significant increases in compensation across each target distance (all $p < 0.01$). These results are illustrated in Figure 3.

Comparison to an Ideal Performer

The results from the experiment show that humans possess some awareness (implicit or explicit) of the relationship between temporal intervals and temporal uncertainty. To arrive at a more precise assessment of human performance, ideal performer models were calibrated to each participant. Recall that the two key parameters in the ideal performer analysis, σ_0 and σ_1 , determine how uncertainty scales with the magnitude of a temporal interval. Since these parameters cannot be directly measured, they were separately fit to each subject. The parameters σ_0 and σ_1 were held constant for each subject, while τ varied for each target distance and penalty configuration. The parameter values were then chosen that maximized the likelihood $p(x|\tau, \sigma_0, \sigma_1)$ of each participant’s data across all target distances and penalty configurations. In fitting the ideal performer models, it was discovered that the human reaction time data included outliers that could not adequately be captured by a Gaussian distribution. To deal with this, outliers differing more than 2 standard deviations from the mean were removed, and the maximum likelihood estimation was performed using a truncated Gaussian distribution (Ratcliff, 1993).

For the results of the ideal performer analysis to be meaningful, it must be the case that the models accurately capture the constraints on performance for each participant. As one test of this, the predicted total scores were computed for each participant using only the parameters σ_0 , σ_1 , and τ . A paired t -test comparing predicted and observed total scores found no significant difference ($p > 0.05$) between the two. After determining σ_0 and σ_1 for each participant, optimal performance was predicted by computing τ_{opt} , or the stopping criterion that maximizes the expected value $U(\tau)$. The observed human data was then compared to the model’s prediction, given by the distribution $p(x|\tau_{opt})$. Comparisons between the observed data and optimal performers are given in Table 1 (target-relative stopping position) and Figure 3 (compensation relative to *both-penalty* trials). It is immediately apparent that while human subjects demonstrated appropriate compensation due to the asymmetrical penalty configurations, the magnitude of their compensation was well below the Bayesian optimal strategy.

Subjects clearly adapted their behavior to the dynamics of uncertainty in interval estimation and were able to exploit this knowledge to improve performance, though failed to do so in a fully optimal manner. Unfortunately, it is not clear if the failure stems from imperfect knowledge of their variability, or imperfect ability to combine this knowledge with a task-specific utility structure and determine an optimal stopping point. A third possibility is that subjects simply had little to gain by compensating more than they did. To address this possibility, a measure of performance relative to the ideal performer was computed by determining the difference between the subject’s total score and the score expected by adopting an optimal compensation strategy. Across subjects, the mean score difference was -8.96 points ($SD = 5.11$). Thus, despite

their deviation from optimal performance in terms of stopping position, the difference in terms of score was small relative to the potential outcomes of each trial (-200, 0, or +100 points). Taken from another perspective, the largest discrepancy between observed and ideal performance occurred for *penalty-before* trials at the furthest target distance, with the difference equal to 38 pixels. Given the ball's velocity, this discrepancy amounts to an error in timing of less than 40 ms. Given the seemingly high difficulty of adjusting one's response latency by tens of milliseconds, and the relatively small gain to be had for doing so, it seems possible that the observed behavior demonstrates a cost-benefit tradeoff in performance.

Conclusions

Recently, researchers have begun to focus on the profound intelligence of routine interactive behavior—the remarkable human adaptation to the low-level dynamics of perception, cognition, and motor control. Understanding and explaining these findings requires detailed consideration of not just the isolated mechanisms of human cognition, but how humans might routinely acquire and exploit accurate predictive models of their own performance constraints.

Our experiment was designed to explore whether humans could exploit knowledge of the dynamics of uncertainty in time estimation in a simple ball tracking task. Presumably, our subjects all came in to the experiment with a lifetime of experience with events and intervals lasting less than 1,000 ms. It is unlikely, however, that their experience extended to the specific features and utilities of the Bayes' Ball paradigm. Optimal performance in this environment requires adapting behavior to the combination of embodied and task dynamics. In our experiment we found that our subjects demonstrated awareness of the relationship between interval duration and uncertainty known as the Weber law of interval timing. However, human behavior did not fully match that of an ideal performer calibrated to each subject. In particular, humans showed an overall bias—overshooting near targets while undershooting far targets. Further, the magnitude of compensation was roughly half of that predicted by the ideal performer.

While future experiments will be needed to fully explore the deviation from optimal performance observed in this experiment, there are a number of potential explanations. Our experiment utilized a random order of target distances and penalty configurations. It seems highly plausible that our subjects were adopting an 'averaged' strategy, where the stopping point on any given trial was influenced by the preceding trial configuration. This explanation can be tested by comparing performance with trial configurations presented in blocks of consecutive trials rather than randomized. Another possibility is that our experiment simply did not provide enough incentive for subjects to fully adopt Bayesian optimal performance. Matching optimal performance in this experiment would have required adjusting response timing by tens of milliseconds, but only yielded on the order of an extra five points per trial. As the subjects were not given any monetary reward

for their performance, it is also possible that they lacked sufficient motivation (though this explanation is unlikely given that most subjects reported finding the task both challenging and enjoyable). The basic finding from the experiment, however, supports the conclusion that understanding even routine interactive behavior requires theories of skill acquisition that incorporate adaptation to low-level embodied and task dynamics as a basic rather than extraordinary phenomenon. Developing computational models that can learn this capacity given only self-guided experience with a task environment remains a significant challenge for future research.

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