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Bayesian Methods and Markov Switching Models for the Analysis of U.S. Postwar Business Cycle Fluctuations

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Bayesian Methods and Markov Switching Models for the Analysis of U.S. Postwar Business Cycle Fluctuations

A Dissertation submitted in partial satisfaction
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in

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by

Jie Li

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ABSTRACT OF THE DISSERTATION

Bayesian Methods and Markov Switching Models for the Analysis of U.S. Postwar Business Cycle Fluctuations

by

Jie Li

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Dr. Marcelle Chauvet , Chairperson

This dissertation consists of five chapters addressing analytically and empirically U.S. Postwar business cycle fluctuations. Markov Switching models and Bayesian estimation methods are used to investigate United States macroeconomic dynamics in the last 60 years. Chapter 1 introduces the structure of this dissertation. Chapter 2 proposes a dynamic stochastic general equilibrium (DSGE) model with Markov Switching and heteroskedastic shocks to examine the role of agents' beliefs separately from changes in monetary policy in explaining inflation fluctuations. Bayesian analysis is conducted with Markov Switching to support regime switches in the private sector, in the implementation of monetary policy and in the volatility of shocks in the U.S. Postwar economy, which are related to the "Great Inflation", the "Great Moderation" and the 2008 financial crisis. A counterfacutal analysis found that if agents maintained a weak response to macroeconomic dynamics over time, there would be lower inflation during the "Great Inflation". In addition, irrespectively to monetary policy regimes, supply shocks are the main driver of inflation fluctuations, while demand shocks are the main source of changes in the output gap. However, when agents maintain a higher risk aversion towards consumption with a higher slope in the Phillips curve, demand

shocks also play a role in driving inflation, even though supply shocks are still the main driver of inflation. Chapter 3 emphasizes on the monetary policy with an investigation on the assumption that policymakers commit to a Taylor rule, using a time-varying inflation-unemployment dynamic model on U.S. economy. This chapter is based on the conjecture that potential policymakers' misperception may be originated from unobserved deviations of unemployment from its natural rate. Five processes are proposed for policymakers' belief under commitment to inflation and unemployment and compare them with a baseline autoregressive process without commitment. The models are estimated using Bayesian techniques. Empirical results are as follows: First, policymakers' belief is very persistent even when it commits to a Taylor-type policy rule. Second, the run-up of U.S. inflation around 1980 can be mostly attributed to policymakers' misperception while the peak surge of inflation in 1974 is possibly a result of non-policy shocks. Third, models with commitment dominate models without commitment, especially in periods of large oscillations in inflation. In particular, when policymakers are committed to respond to a Taylor-type policy rule, the average loss function is considerably reduced over time, thus effectively lessening potential misperceptions. Chapter 4 introduces a simple version of adaptive expectation to a dynamic stochastic general equilibrium (DSGE) model to evaluate the goodness of fitness and forecasting performance on U.S. macroeconomic indicators. Analytical maximum likelihood estimation results represent a DSGE model with adaptive expectation outperforms a DSGE model with rational expectation. In addition to providing a better fit of inflation and output gap in the U.S. Postwar macro economy, a DSGE model with adaptive expectation also leads to redundant lagged inflation in fitting inflation dynamics. Chapter 5 concludes and proposes future extension.

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Chapter 1

Introduction

U.S. Postwar business cycle fluctuations and macroeconomic dynamics has been studied extensively over 60 years. Recent research started from theoretical exploration such as dynamic stochastic general equilibrium(DSGE) models to empirical investigation; from monetary policy analysis to exogenous, non-policy shocks observation; from maximum likelihood estimation to Bayesian approaches. The common recognition of previous literature is that United States postwar business cycle experienced three periods: the “Great Inflation” period, the “Great Moderation” period and our recent financial crisis starting from 2007 December. Simultaneously, arguments arise for the explanation of sources resulting in “Great Inflation”, “Great Moderation” and 2008 financial crisis as well as for important policy implication. This dissertation shows analytically and empirically further investigation on U.S. postwar macroeconomic dynamics using Markov Switching approaches and Bayesian estimations.

The “Great Inflation”, the ”Great Moderation” and 2008 financial crisis are related to regime switches in the private sector, in the conduct of monetary policy and in the volatility of shocks in the U.S. Postwar economy. Such a conclusion is represented in Chapter 2. Chapter 2, "Regime Switching, Monetary Policy and Agents' Beliefs in U.S. Business Cycles" investigates the evolution of inflation and output dynamics in the United States over the last 60 years. In particular, it proposes a dynamic stochastic general equilibrium (DSGE) with Markov Switching and heteroskedastic shocks to examine the role of agents' beliefs separately from changes in monetary policy in explaining inflation fluctuations. The model is estimated using Bayesian techniques and results shed light on the driving sources to inflation and output gap fluctuations in the last 60 years.

How to avoid a large oscillation in inflation is also a critical issue in the "Great Inflation" period. Chapter 3, "How Largely the Commitment can Beat Policymakers' Misperceptions?" analyzes this question in a perspective of monetary authority. This chapter investigates the assumption that policymakers commit to a Taylor rule, using a time-varying inflation-unemployment dynamic model for the U.S. economy. Our model is based on the conjecture that potential policymakers' misperception may be originated from unobserved deviations of unemployment from its natural rate. Five processes in this chapter include a time-invariant Taylor rule in which policymakers can only observe previous inflation and unemployment, a time-varying Taylor rule in which policymakers adjust their commitment each period according to available information, a Taylor rule in which commitment switches between high and low inflation and unemployment phases, following a Markov Switching process, a Taylor rule in which commitment is changed as a response to different regimes in unemployment, a Taylor rule with commitment adjusted according to low or high inflation regimes only. The models are estimated using Bayesian techniques. Our empirical results shows policymakers' belief is very persistent even when it commits to a Taylor-type policy rule. In addition, the reason of peak surge of U.S. inflation during 1974 and 1980 is also illustrated. With the measurement of average loss, models with commitment are found to be superior to others.

Chapter 4 "Adaptive Expectations and Inflation Persistence" investigates inflation persistence by proposing a dynamic stochastic general equilibrium(DSGE) model with adaptive expectation. The proposed model is compared with a DSGE model including rational expectation. Model fit and out-of-sample forecasting show DSGE model with adaptive expectation outperforms others. Findings are summarized in the conclusion section of this dissertation, that is, Chapter 5.

Chapter 2

Regime Switching, Monetary Policy and Agents' Beliefs in U.S. Business Cycles

2.1 Introduction

It has been extensively documented in the literature that the evolution of inflation and output dynamics is considerably different before and after the mid 1980s. Such distinct phases have been denoted the "Great Inflation" and the "Great Moderation". There is a current controversial debate on the sources of the differences across these periods. Recent work has examined whether these differences are due to changes in the structure of the economy or to changes in the size of exogenous shocks.

Following previous literature, the purpose of this paper is to investigate the role of the private sector, the Federal Reserve, and the volatility of exogenous shocks in explaining the dynamics of U.S. inflation and output over the last 60 years. In particular, it proposes a small scale Dynamic Stochastic General Equilibrium model (DSGE) with Markov Switching to investigate the role of agents separately from changes in monetary policy - under homoskedastic or heteroskedastic shocks - in explaining inflation fluctuations.

The "Great Inflation" period in the 1970s was characterized by an upsurge in inflation, accompanied by high volatility in the real and nominal sectors. Inflation was brought to moderate levels in the early 1980s with a tight monetary policy by the Federal Reserve under the chairmanship of Paul Volcker. From the mid-1980s until the 2008 financial crisis, the U.S. economy had been operating on a stable track with relatively low inflation levels and unemployment rate, and a much more stable growth in real output. Despite the fact that the economy experienced recessions in the early 1990s and 2000s, these contractions were considerably milder and shorter than the previous ones. The remarkable economic stability from the mid-1980s to 2007 was named the "Great Moderation". However, the meltdown in the U.S. housing sector spread to the financial system and to the real economy since 2007, with strong economic consequences worldwide. This has led to a renewal interest in understanding potential underlying causes of the changes in stability in the economic system across the last decades.

There is a large literature investigating the sources to the "Great Inflation" and the "Great Moderation". Some paper finds that the "Great Moderation" is the result of a substantial change in the monetary policy under the Fed chairmanship of Paul Volcker and Alan Greenspan such as Judd and Rudebusch (1998), Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004) and Boivin and Giannoni (2008), among others. An alternative strand finds that the decline of inflation and output variability since the mid 1980s is the result of reduced volatilities of exogenous non-policy shocks. Some of the representative papers are McConnell and Perez-Quiros (2000), Stock and Watson

(2003), Cogley and Sargent (2005), Sims and Zha (2006) or Liu, Waggoner, and Zha (2008), among others. In particular, Sims and Zha (2006) show that time variation in structural disturbance variances is the main driver of the macroeconomic stabilization during the "Great Moderation".

More recently, there has been some important related research that develops a methodology to estimate Markov Switching models under rational expectation. In particular, Farmer, Waggoner, and Zha (2008)(FWZ) purposed a methodology to calculate minimal state variable(MSV) solutions, and provided sufficient and necessary conditions for their existence. Bianchi (2009), Liu, Waggoner, and Zha (2009), among others, have applied this method to examine the sources of the "Great Inflation" and the "Great Moderation" using a DSGE model with Markov Switching. On the other hand, Davig and Leeper (2007) proposed a generalized Taylor principle that allows the reaction coefficients in the monetary policy rule to switch across regimes according to a Markov process. Eo (2008) and Davig and Doh (2009) used this method to examine the role of regime switching in explaining inflation fluctuations, giving a micro foundation interpretation.

In general, all the previous literature has focused on the role of changes in the behavior of monetary policy under heteroskedastic shocks. This paper differs from the existing literature in that it investigates separately the changes in agents' beliefs across regimes to shed light on the role of changes in monetary policy, changes in the private sector, or changes in the volatility of shocks in explaining the "Great Inflation" and the "Great Moderation" phases.

The proposed Markov Switching DSGE model is considered under four assumptions [1]. First, it is assumed that the response of household and firm (agents' beliefs)

[1]Our small-scale DSGE model is comprised of an intertemporal Euler equation derived from house-

to economic fluctuation may have changed over the last 60 years. In this case, only the structural parameters of the Euler equation and the Phillips curve are allowed to follow Markov regime switching processes, while the parameters in the monetary policy function are fixed. Second, changes in agents' beliefs are considered under heteroskedastic shocks to investigate the hypothesis that high inflation in the 1970s may have been driven by larger exogenous shocks. In this case, two independent Markov chains are considered to control the structural parameters and the disturbance variances. Third, in order to capture changes in monetary policy separately from changes in the private sector, only the structural parameters in the monetary policy function are permitted to switch across regimes. Finally, it is assumed that the Federal Reserve faces shocks that display stochastic volatility.

The methodology proposed in FWZ (2008) is used to obtain the model's solution, which entails a VAR with time dependent coefficients. The system is cast in state space model and the Gibbs sampling combined with Metropolis-Hasting is used to solve the system.

Four main findings stand out from the model estimation. First, our results support regime switches in the private sector, in the conduct of monetary policy and in the volatility of shocks in the U.S. postwar economy, which are related to the "Great Inflation", the "Great Moderation", and the 2008 financial crisis. During the "Great Inflation", consumers displayed a higher risk aversion to time variation in consumption and preferred a more stable consumption path. Meanwhile, firms were adjusting prices more flexibly due to a lower adjustment cost, as represented by a higher slope in the

hold's optimal decision on consumption and bond holdings, a Phillips curve describing a monopolistically competitive firm facing a downward sloping demand curve for its differentiated goods, and a monetary policy function depicting the response of monetary authority to the deviation of the inflation and output from their steady states respectively. The model considers three different exogenous shocks: demand, supply (or technology shock) and monetary shocks.

Phillips curve. These observations lead to the conclusion that agents responded more strongly to the economy in the 1970s. On the other hand, the Federal Reserve maintained the output gap closed to zero at a cost of higher inflation in the 1970s, while this was reversed from the 1980s on, with the Fed willing to accept a recession to keep inflation low. In addition, the volatility of shocks was much higher during the "Great Inflation" than that during the "Great Moderation".

Second, the impulse response functions across different regimes indicate that when agents respond strongly to economic dynamics, fluctuations in inflation and output are much more accentuated. Such an observation does not occur when the Federal Reserve responds more strongly to inflation.

Third, the paper also finds that, irrespectively to monetary policy regimes, supply shocks are the main driver of inflation fluctuations, while demand shocks are the main source of changes in the output gap. However, when agents maintain a higher risk aversion towards consumption with a higher slope in the Phillips curve, demand shocks also play a role in driving inflation, even though supply shocks are still the main driver of inflation.

Finally, a counterfactual analysis finds that if agents maintained a weak response to the economy over time, there would be lower inflation during the "Great Inflation" period. On the other hand, if a hawk regime dominated the whole sample with a strong response by the Federal Reserve to inflation, inflation fluctuations would be dampened in the 1970s, with the sacrifice of a reduced output gap.

This paper is organized as follows: Section 2.2 briefly describes Markov chains and state space models. Section 2.3 presents a benchmark DSGE model with fixed parameters and the proposed Markov Switching DSGE models. This section also reviews the minimum state variable solutions (MSV) proposed by Farmer, Waggoner, and Zha

(2008). Section 2.4 discusses the estimation algorithms. The estimation results and analysis are presented in section 2.5, and section 2.6 concludes. The prior distribution for the DSGE model parameters and a description of the DSGE model is provided in the Appendix.

2.2 Preliminary: Markov Chains and State Space Models

Understanding the dynamics of the U.S. postwar macroeconomic fluctuation has always been an important topic of economic research. Some popular empirical ways to investigate potential changes over time is to divide the sample into subperiods based on some major policy changes or the tenure of chairman of the Federal Reserve. The subsample method has the advantage that is easy to implement. Nevertheless, it is subjective and exogenous to the model proposed, which has as drawbacks the possibility of neglecting underlying unobservable factors for the possible changes. An alternative is to allow for the possibility of endogenous Markov switching, governed by estimated transition probabilities. This method has become widespread both in empirical and theoretical macroeconomic research since the seminar paper by Hamilton (1989). Below we review some simple definitions of and applications of Markov chains, which serve as the basis for the model proposed in this paper.

Definition 1 *A Markov chain is an integer time process, $\{\xi_t, t \geq 0\}$ for which each random variable ξ_t, $t \geq 1$ is interger valued and depends on the past only through the most recent random variable ξ_{t-1}. i.e., for all integer $t \geq 1$ and all integer $i, j, k, \ldots, m$,*

$$Pr[\xi_t = j \mid \xi_{t-1} = i, \xi_{t-2} = k, \ldots, \xi_0 = m] = Pr[\xi_t = j \mid \xi_{t-1} = i]$$

$Pr[\xi_t = j \mid \xi_{t-1} = i]$ *depends only on* i *and* j *(not* t*) and is denoted by*

$$Pr[\xi_t = j \mid \xi_{t-1} = i] = p_{ij}{}^2$$

A Markov chain in which each ξ_t has a finite set of possible sample values is a finite-state Markov chain. Note ξ_t could be explained as the state of the chain at time t. The possible values for the state at time t could be $\{1, 2, \ldots, m\}$ or $\{0, 1, \ldots\}$[3]. p_{ij} is the probability of going to state j given that the previous state is i. The new state conditional on the previous state is independent of all earlier states. For example, a four-state Markov chain with probability $p_{i,j}, i, j = 1, \ldots, 4$ can be described in figure 2.1, where p_{ij} represents the probability switching to state j from state i. p_{23} is the probability of transiting into state 3 given that the previous state is 2. [4]

A probability matrix that only takes into account changes from one state to the alternative is defined as a one-step transition probability matrix as following:

Definition 2 *The matrix containing* p_{ij}*, the transition probabilities*

$$H = \begin{bmatrix} p_{11} & p_{12} & \cdots \\ p_{21} & p_{22} & \cdots \\ \vdots & \vdots & \vdots \end{bmatrix}$$

is called the one-step transition probability matrix of the process.

[2]To distinguish P in the state space model, in the section 2.4, we will use H stand for the probability matrix and p_{ij} as the transition probability.

[3]In this paper, we consider the state starting from 1 instead of 0.

[4]The matrix describing figure 2.1 is

$$H = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix}$$

Similarly, n-step transition probability matrix could be defined as $H^n = \underbrace{H \times H \times \ldots \times H}_{n \text{ times}}$, where when $n = 1$, H^n becomes one-step transition probability matrix.

The switching from one state to the alternative is not easily observed since it requires observations of the underlying Markov chains. The hidden Markov models(HMM) are, thus, widely used. In a HMM model, there are two types of states: the observable and the hidden/unobservable states. The detailed interpretation is enclosed in the appendix.

The state space model is one of the tools that can be used to estimate the unobservable states of hidden Markov models. The state space model starts with defining a vector of real-valued observed series $\{Y_t\}$ under the assumption that observations were generated from a sequence of hidden state vectors $\{S_t\}$, as represented in figure 2.2. It is assumed that given the unobserved S_t, the observation vector Y_t is conditionally independent from all other variables and S_t is conditionally independent from $S_1, \ldots, S_{t-2}$.

The general state space model could be written as

$$Y_t = D + ZS_t + \upsilon_t \tag{2.1}$$

$$S_t = TS_{t-1} + R\epsilon_t \tag{2.2}$$

where $\upsilon_t \sim N(0, U)$, $\epsilon_t \sim N(0, Q)$. Equation (2.1) is defined as the measurement equation and Equation (2.2) as the transition equation. Figure 2.2 illustrates the general state space model.

The combination of the state space model with hidden Markov chains can be obtained by specifying a distribution over observations $\{Y_t\}$ and $\{S_t\}$ at each time step t given a discrete hidden state (Markov chain ξ_t), and the probability of transiting from one hidden state to another. If there is only one unobserved state $\{S_t\}$, the Kalman filter can be applied to calculate the maximum likelihood in its standard form. However, when we have ξ_t that is affecting S_t simultaneously, Kalman filter can not work directly since now S_t is not unique. We use an approximation proposed by Kim (1994), which basically combines the Kalman filter and Hamilton's (1989) filter. The algorithm will be introduced in the section 2.4.2 and for a detailed description of state space model see Kim and Nelson (1998).

The combination of the state space model with a Markov chain and transition probability matrix could also be represented by figure 2.3. [5]

As observed, Y_t depends on S_t and ξ_t. S_t is also affected by an independent Markov chain ξ_t. We will use this model embedded in a small scale DSGE model that tracks the underlying shifting of monetary policy and agents' beliefs.

2.3 A small scale DSGE model

This paper considers a popular prototypical New Keynesian monetary DSGE model, which is tested by Lubik and Schorfheide (2004). The details about it could be found in King(2000) and Woodford (2003).

2.3.1 Benchmark model: Fixed parameters

The benchmark model starts with fixed parameters. After log linearizing around the steady states, a small scale DSGE model can be summarized by the fol-

[5] The model which is described by figure 2.3 is:

$$Y_t = D + Z(\xi_t)S_t + \upsilon_t$$
$$S_t = T(\xi_t)S_{t-1} + \epsilon_t$$

lowing three equations:

$$\tilde{y}_t = E_t[\tilde{y}_{t+1}] - \tau(\tilde{R}_t - E_t[\tilde{\pi}_{t+1}]) + g_t \tag{2.3}$$

$$\tilde{\pi}_t = \beta E_t[\tilde{\pi}_{t+1}] + \kappa(\tilde{y}_t - z_t) \tag{2.4}$$

$$\tilde{R}_t = \rho_R \tilde{R}_{t-1} + (1 - \rho_R)(\gamma_1 \tilde{\pi}_t + \gamma_2 \tilde{y}_t) + \epsilon_{R,t} \tag{2.5}$$

where $\tilde{y}_t, \tilde{\pi}_t$ and $\tilde{R}_t$ are defined, respectively, as output, quarterly inflation and nominal interest rate. The tilde denotes percentage deviations from a steady state or, in the case of output, from a trend path.

Equation (2.3) represents an intertemporal Euler equation derived from the households' optimal choice of consumption and bond holdings. Since the small scale DSGE model considered does not include investment, output is proportional to consumption, and the exogenous process g_t captures the net effects of these exogenous shifts on the Euler equation. The parameter $0 < \beta < 1$ is the households' discount factor and $\tau > 0$ can be interpreted as the intertemporal substitution elasticity.

The inflation dynamics are characterized by equation (2.4) describing a continuum of monopolistically competitive firms facing a downward-sloping demand curve for its differentiated product. Here the sticky price is due to quadratic adjustment costs in nominal prices or a Calvo-style rigidity allowing only a fraction of firms to adjust their prices. The expectational Phillips curve (2.4) has a slope κ, which corresponds to a positive value for $\tilde{y}_t$ in the boom.

The behavior of the monetary authority is characterized by a tradeoff between inflation and output in the equation (2.5). The central bank adjusts its instrument to deviations of inflation and output from their respective target levels by controlling a nominal interest rate. The policy implementation error or the unanticipated deviation from the systematic component of the monetary policy rule is represented by $\epsilon_{R,t}$ with standard error as σ_R.

$$g_t = \rho g_{t-1} + \epsilon_{g,t} \tag{2.6}$$

$$z_t = \rho z_{t-1} + \epsilon_{z,t} \tag{2.7}$$

where we assume zero correlation ρ_{gz} between the innovations $\epsilon_{g,t}$ and $\epsilon_{z,t}$ with standard deviation σ_g and σ_z.

The process z_t in equation (2.7) and g_t in equation (2.6) are assumed to follow univariate AR(1) processes with coefficients ρ_g and ρ_z. Note that z_t captures exogenous shifts on the marginal costs of production, and could be interpreted as a technology shock. Similarly, g_t summarizes changes in preferences or a time-varying government spending, and can be considered as a demand shock.

The system could be solved using gensys.[6] The linear rational expectations model comprised of equations (2.3) to (2.7) can be rewritten in the canonical form

$$\Gamma_0 S_t = \Gamma_1 S_{t-1} + C + \Psi \epsilon_t + \Pi \eta_t \tag{2.8}$$

where

$$S_t = [\tilde{y}_t, \tilde{\pi}_t, \tilde{R}_t, g_t, z_t, E_t(\tilde{y}_{t+1}), E_t(\tilde{\pi}_{t+1})]'$$

$$\epsilon_t = [\epsilon_{R,t}, \epsilon_{g,t}, \epsilon_{z,t}]'$$

$$\eta_t = [(\tilde{y}_t - E_{t-1}[\tilde{y}_t]), (\tilde{\pi}_t - E_{t-1}[\tilde{\pi}_t])]'$$

[6]The matlab code gensys could be downloaded from: http://sims.princeton.edu/yftp/gensys

Gensys will return a first order VAR in the state variable:

$$S_t = T(\theta)S_{t-1} + R(\theta)\epsilon_t \tag{2.9}$$

Where the vector θ collects the parameters of the loglinearized DSGE model as:

$$\theta = [\tau, \beta, \kappa, \psi_1, \psi_2, \rho_R, \rho_g, \rho_z, \sigma_R, \sigma_g, \sigma_z]'$$

To solve the unobserved variables such as g_t, $E_t(\tilde{y}_{t-1})$, a state space model is deployed to solve the law of motion of the DSGE model.[7]

$$Y_t = D(\theta) + ZS_t + \upsilon_t$$

$$S_t = T(\theta)S_{t-1} + R(\theta)\epsilon_t$$

$$\upsilon_t \sim N(0, U),\ U = diag(\sigma_y^2, \sigma_\pi^2, \sigma_r^2)$$

$$Y_t = \begin{bmatrix} y_t \\ \Delta \ln P_t \\ \ln R_t^A \end{bmatrix} \quad D(\theta) = \begin{bmatrix} 0 \\ \ln \pi^* \\ 4(\ln \pi^* + \ln r^*) \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where υ_t is a vector of measurement errors.[8] y_t, $\Delta \ln P_t$ and $\ln R_t^A$ describe the output

[7]The benchmark model is closed to the one used in Lubik and Schorfheide (2004) and Bianchi (2009).

[8]Note that this vector autoregression will be singular so long as the number of shocks is less than the number of variables in the system. Sometimes, it will also be singular even when the number of shocks is equal to the number of variables. In order to reconcile the singular equilibrium from the model with the clearly non-singular nature of the data, Diebold and Rudebusch (1996) suggested introducing the errors in the measurement equation. In the technical part, we drop the solution when the vector

gap, quarterly inflation, and the nominal interest rate respectively. The likelihood of the DSGE model is calculated as $\ell(\theta, \sigma_\zeta \mid Y^T)$.

2.3.2 Solving MS-DSGE model

A linear dynamic stochastic general equilibrium model with constant parameters has limited ability in addressing changes in the dynamics of the economy. Parameter switching linear rational expectation model is an extension that dates back to Hamilton(1989, 1994). In an AR process described in these papers, a Markov chain with a transition matrix can capture different states of the condition mean, which can capture fluctuations in the economic activity. The extended autoregressive representation with constant state-independent parameters is extended by Farmer, Waggoner, and Zha (2008) to solve the regime switching models with rational expectation. The idea is to compute a minimal state variable(MSV) solution to an expanded state space of a Markov switching model. By writing an equivalent model with fixed parameters in this expanded space, the authors prove that when a unique equilibrium exists, it is in a class of minimum state variable solutions. Their paper also provides the sufficient and necessary conditions for the existence of the MSV solution. Based on these conditions, they show equivalence between an MSV solution to the original model, and an MSV solution to the expanded state space.

The calculation of a MSV solution and the solution method is outlined below [9].

autoregression is singular.

[9] Please refer to Farmer et al (2008) for details.

Let equations (2.3) to (2.5) be controlled by a Markov chain ξ_t as:

$$\tilde{y}_t = E_t[\tilde{y}_{t+1}] - \tau(\xi_t)(\tilde{R}_t - E_t[\tilde{\pi}_{t+1}]) + g_t \tag{2.10}$$

$$\tilde{\pi}_t = \beta E_t[\tilde{\pi}_{t+1}] + \kappa(\xi_t)(\tilde{y}_t - z_t) \tag{2.11}$$

$$\tilde{R}_t = \rho_R \tilde{R}_{t-1} + (1 - \rho_R)(\gamma_1(\xi_t)\tilde{\pi}_t + \gamma_2(\xi_t)\tilde{y}_t + \epsilon_{R,t}) \tag{2.12}$$

Thus equation (2.10) to (2.12) represent a Markov Switching Dynamic Stochastic General Equilibrium (MS-DSGE) model with some parameters fixed and others following Markov switching processes. A state space model covering equation (2.10) to (2.12) could be represented as follows:

$$\overset{\Gamma_0(\xi_t)}{\begin{bmatrix} \underset{(n-l)\times n}{\Gamma_{0,1}(\xi_t)} \\ \underset{l\times n}{\Gamma_{0,2}} \end{bmatrix}} \underset{n\times 1}{S_t} = \overset{\Gamma_1(\xi_t)}{\begin{bmatrix} \underset{(n-l)\times n}{\Gamma_{1,1}(\xi_t)} \\ \underset{l\times n}{\Gamma_{1,2}} \end{bmatrix}} \underset{n\times 1}{S_{t-1}} + \overset{\Psi(\xi_t)}{\begin{bmatrix} \underset{(n-l)\times k}{\psi(\xi_t)} \\ \underset{l\times k}{0} \end{bmatrix}} \underset{k\times 1}{\epsilon_t} + \overset{\Pi}{\begin{bmatrix} \underset{(n-l)\times n}{0} \\ \underset{l\times n}{\pi} \end{bmatrix}} \underset{l\times 1}{\eta_t} \tag{2.13}$$

where ξ_t follows an m-state Markov chain. $\xi_t \in M \equiv \{1, \ldots, m\}$. The Markov chain evolves according to a stationary transition matrix H that defines the probability of moving from one state to another as:

$$Pr[\xi_t = i \mid \xi_{t-1} = j] = H = p_{ij}$$

In equation (2.13), n is the number of endogenous variables($n = 7$ in this case), k is the number of the number of exogenous shocks($k = 3$), l is the number of the rational expectations forecast errors($l = 2$). The fundamental equations of (2.13) are allowed to switch across regimes but the parameters $\Gamma_{0,1}$, $\Gamma_{1,1}$ and Π, which define the non-fundamental shocks, do not depend on ξ_t.

To solve the equation (2.13), FWZ provides a pair of bounded stochastic processes $\{S_t, \eta_t\}$ such that it is consistent for all realizations of $\{\xi_t, \epsilon_t\}$. We firstly rewrite (2.13) to an expanded state vector $\overline{S}_t$ with fixed parameters:

$$\overline{\Gamma}_0 \overline{S}_t = \overline{\Gamma}_1 \overline{S}_{t-1} + \overline{\Psi} u_t + \overline{\Pi} \eta_t \tag{2.14}$$

where

$$\underset{nm \times nm}{\overline{\Gamma}_0} = \begin{bmatrix} diag(a_1(1), \ldots, a_1(m)) \\ a2, \ldots, a2 \\ \Phi \end{bmatrix} \tag{2.15}$$

$$\underset{nm \times nm}{\overline{\Gamma}_1} = \begin{bmatrix} diag(b_1(1), \ldots, b_1(m))(H \otimes I_n) \\ b_2, \ldots, b_2 \\ 0 \end{bmatrix} \tag{2.16}$$

$$\underset{nm \times l}{\Pi} = \begin{bmatrix} 0 \\ \pi \\ 0 \end{bmatrix}, \quad \underset{(l-1)m \times nm}{\Phi} = \begin{bmatrix} \mathrm{e}_2' \otimes \phi_2 \\ \vdots \\ \boldsymbol{e}_m' \otimes \phi_m \end{bmatrix} \tag{2.17}$$

$$\underset{nm \times (k+m-l)m}{\Psi} \begin{bmatrix} \boldsymbol{I}_{(n-l)m} & diag(\psi(1), \ldots, \psi(m)) \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{2.18}$$

$$\overline{S}_t = \begin{bmatrix} \iota_{(\xi_t = 1)} S_t \\ \vdots \\ \iota_{(\xi_t = m)} S_t \end{bmatrix}$$

The vector of error terms u_t is defined as

$$u_t = \begin{bmatrix} \Xi_{\xi_t}(\boldsymbol{e}_{\xi_{t-1}} \otimes (\mathbf{1}_m' \otimes \boldsymbol{I}_n)\overline{S}_{t-1}) \\ \boldsymbol{e}_{\xi_t} \otimes \epsilon_t \end{bmatrix}$$

where

$$\underset{(n-l)p \times np}{\Xi_i} = (diag[b_1(1), \ldots, b_1(m)]) \times [(\boldsymbol{e}_i \mathbf{1}_m' - H) \otimes \boldsymbol{I}_n]$$

Where $\boldsymbol{I}_n$ denotes the $n \times n$ identity matrix, $\boldsymbol{e}_i$ denotes the i^{th} column of $\boldsymbol{I}_n$, $\mathbf{1}_m$ is the m-dimensional column vector of ones. There are two kinds of shocks in the extended state space. FWZ(2008) refer to them as switching shocks and normal shocks. In their definitions, the switching shocks $\Xi_{\xi_t}(\boldsymbol{e}_{\xi_{t-1}} \otimes (\mathbf{1}_h' \otimes \boldsymbol{I}_n)S_{t-1})$ turn on or off the appropriate blocks of the model to represent the Markov dynamics, while the normal shocks $\boldsymbol{e}_{\xi_t} \otimes \epsilon_t$ carry the fundamental errors that hit the structural equations, distributed to the appropriate block of the expanded system. They show that both shocks have expectation equal to zero[10].

Definition 3 *A solution to Equation* (2.14) *is a stochastic process* $\{\overline{S}_t, \eta_t\}_{t=1}^{\infty}$ *such that:*

(1) $\{\overline{S}_t, \eta_t\}_{t=1}^{\infty}$ *jointly satisfy Equation* (2.14).

(2) The endogenous stochastic process $\{\eta_t\}$ *satisfies the property,* $E_{t-1}[\eta_t] = 0$.

(3) S_t *is bounded in expectation in the sense that* $\|E_t[\overline{S}_{t+s}]\| < M_t$ *for all* $s > 0$.

In a MS-DSGE model the sunspot solutions are pervasive, and FWZ focuses on a class of minimal state variable(MSV) solutions. By introducing Φ, they prove the equivalence between the MSV to the original model and the MSV to the expanded

[10]For details of Definition 3 and 4, please refer to Farmer et al (2008). Bianchi(2009) also summarized definitions of Farmer et al.

fixed coefficient model. To obtain a MSV solution, a matrix Z is defined such that $Z'\overline{S}_t = 0$. The introduction of Z considers the impact of different regimes and makes up the zero restrictions on the variables. Suppose regime 1 occurs, the third block of equation (2.14) will impose a series of zero restrictions on the variables referring to regimes $i = 2 \ldots m$. These restrictions will constraint the corresponding element of $\overline{S}_t$ to zero by incorporating with the one arising from the first block of equations. If regime $i = 2, \ldots m$ occurs, Φ will serve as a similar block of zero restrictions on regime 1. The following definition of the unstable Γ_0 and Γ_1 is designed to lead up to a theorem that enables us to compute Φ.

Definition 4 *Let $QSZ = \Gamma_0$ and $QTZ = \Gamma_1$ be the QZ-decomposition of $\{\Gamma_0, \Gamma_1\}$. Reorder the upper triangular matrices $S = (s_{i,j})$ and $T = (t_{i,j})$ in such a way that $t_{i,i}/s_{i,i}$ is in an increasing order. Let $q \in \{1, 2, \ldots m\}$ be the integer such that $t_{i,i}/s_{i,i} < 1$ if $i \leq q$ and $t_{i,i}/s_{i,i} > 1$ if $i > q$. Let Z_u, partitioned as $Z_u = [z_1, \ldots, z_m]$ be the last $nk - q$ rows of Z. Beginning with a set of matrices $\{\phi_i^0\}_{i=2}^m$ and generate the associated matrix Γ^0. Next, calculate Z_u^0 by computing the QZ decomposition of $\{\Gamma_0^0, \Gamma_1\}$ and set $\phi_i^1 = z_i^1$. Repeat this procedure until convergence.*

If it converges, definition 4 implies that the solution $\{S_t, \eta_t\}$ to equation (2.14) is consistent with equation (2.13). A VAR process can, thus, be written with time dependent coefficients as:

$$S_t = T(\xi_t, \theta)S_{t-1} + R(\xi_t, \theta)\epsilon_t \tag{2.19}$$

The law of motion of the DSGE states depends on the structural parameters θ and the regimes ξ_t. Note ξ_t could follow a m-state Markov chain where m could be greater than 2.

2.3.3 Alternative solution methods

There are alternative solution methods to a MS-DSGE model. Notice that FWZ do not discuss the determinacy, but some other papers have addressed this issue. Davig and Leeper (2007) and Davig and Doh (2009) map endogenous variables into policy choices in terms of a generalization of the Taylor principle. Their solution method makes use of the monotone map method, based on Coleman(1991). The algorithm requires a discretized state space and a set of initial decision rules that reduce the model to a set of nonlinear expectational first-order difference equations. They show a condition that can rule out indeterminate equilibria in a version of the New Keynesian model, where parameters of the policy rule follow a Markov switching process. A solution consists of a set of functions that map the minimum set of state variables into values for the endogenous variables. The solution method is discussed by Farmer, Waggoner, and Zha (2008), who argues that this model suffers from an incomplete subset of fundamental equilibria missed by their condition. Furthermore, at the stage local uniqueness of a solution must be proved perturbing the equilibrium decision rules.

Another solution algorithm for a large class of linear-in-variable regime switching models is developed by Svensson and Williams(2005). Their method is closely related to the class presented by FWZ. However, due to lack of a diagnostics for conditions of a unique solution, their algorithm is considered to converge to a unique solution, to one of a set of indeterminate solutions, or to an unbounded stochastic difference equation that goes against the appropriate transversality conditions.

Bikbov and Chernov (2008) generalizes a method proposed by Cho and Moreno (2006) for fixed coefficient New-Keynesian models to the case of regime switching dynamics. The solution proposed by Bikbov and Chernov (2008) is achieved by working directly on the original model through an iteration procedure. They show in the case of a unique stationary solution, their method delivers the same solution as obtained with the QZ decomposition method. If the rational expectations solution is not unique, the method yields the minimum state variable solution. It is not quite clear whether a similar argument applies to the case with Markov Switching dynamics and how to check if a unique stationary equilibrium exists.

Cho (2009) provides a technical foundation for a fairly general class of Markov Switching General Equilibrium (MSGE) models by developing a conceptually straightforward and technically simple solution methodology to those models, based on the forward method of Cho and Moreno (2009). They show a sufficient solution selection criterion within the class of fundamental solutions, regardless of model determinacy.

We choose to apply the method proposed by FWZ due to its computation efficiency. In addition, although the uniqueness of the MSV solution does not imply uniqueness in a larger class of solutions, it provides necessary conditions to establish existence and boundness of the minimum state variable solution. As for the problem of indeterminacy/determinacy in a MS-DSGE model, this is a very complicated issue that has not yet been solved in the literature. Davig and Leeper(2007) proposed a condition to rule out indeterminate equilibria. However, as Farmer et al(2008) shows, their condition rules out a subset of indeterminate equilibrium but does not establish uniqueness of the fundamental equilibrium.

2.4 Estimation Strategies

The solutions solved by FWZ on a small-scale MS-DSGE model return a VAR with time dependent coefficients. We combine the VAR with the measurement equation that is cast in a state space form:

$$Y_t = D(\theta^{ss}) + ZS_t + \upsilon_t \tag{2.20}$$

$$S_t = T(\xi_t^{sa})S_{t-1} + R(\xi_t^{sa})\epsilon_t \tag{2.21}$$

$$\epsilon_t \sim N(0, Q(\xi_t^{er})),\ Q(\xi_t^{er}) = diag(\theta^{er}(\xi_t^{er})) \tag{2.22}$$

$$\upsilon_t \sim N(0, U),\ U = diag(\sigma_x^2, \sigma_\pi^2, \sigma_R^2) \tag{2.23}$$

$$H^{sa}(\cdot, i) \sim D(a_{ii}^{sa}, a_{ij}^{sa}), H^{er}(\cdot, i) \sim D(a_{ii}^{er}, a_{ij}^{er}) \tag{2.24}$$

where ξ_t^{sa} is an unobserved state capturing the agents' beliefs regime and ξ_t^{er} denotes an unobserved state that describes the evolution of the stochastic volatility regime [11]. It is easy to estimate the likelihood of a state space model with fixed parameters using Kalman filter and then calculate the posterior combined with prior distribution, using Bayesian methods. However, notice that in a MS-DSGE model, the underlying DSGE state vector S_t is not unique given an observation for Y_t due to the uncertainty of the Markov state. Hamilton's filter, which is usually used for evaluating the likelihood of Markov Switching models, can not be applied directly here when Markov states are not history independent. The probability controlled by a Markov state depends on the value of S_{t-1} and its distribution relies on $\{\xi_s\}_{s=1}^{t-1}$.

However, if we can observe $\xi^{sa,T}$ and $\xi^{er,T}$, the Kalman filter could be applied to capture the distribution of S_t given Y_t and it will be possible to return an unequivocal

[11] ξ_t^{sp} defines the state of monetary regime.

state of S_t. At the same time, if S^T is now observable, then Hamilton filter could be used to extract the probability of a Markov state. The Gibbs sampling algorithm combined with Metropolis-Hasting, suggested in Bianchi(2009), is considered as one way to calculate the posterior in a MS-DSGE model. However, Bianchi(2009) does not address the case in which the solution does not converge. This paper follows Lubik and Schofheide(2004), which assigns a very small likelihood in the absence of convergence so that only converged solutions will be kept. Here we will still introduce Gibbs sampling and Kim's approximation in this section, as illustrated in Bianchi(2009).

Note that the posterior density function is non-Gaussian and complicated in shape, it is very important to find the appropriate posterior mode. We base our computation of likelihood on Kim's approximation(Kim (1994)) whose filter for the state space model entails a combination of Kalman filter and Hamilton filter, along with an approximation. The algorithm is illustrated in section 2.4.2.

The algorithm used here is as follows:

2.4.1 Gibbs sampling

The basic algorithm of Gibbs sampling is summarized as follows:

At the beginning of iteration n we have: θ^{sa}_{n-1}, θ^{ss}_{n-1},θ^{er}_{n-1}, S^T_{n-1},$\xi^{sa,T}_{n-1}$,$\xi^{er,T}_{n-1}$,H^{sa}_{n-1},H^{er}_{n-1}.

1. Given θ, H^{sa}_{n-1} and H^{er}_{n-1}, use Kim's filter to get a filtered estimate of the Markov switching states and then use the backward drawing method to get $\xi^{sa,T}_n$ and $\xi^{er,T}_n$.

2. Given $\xi^{sa,T}_n$ and $\xi^{er,T}_n$, draw probability matrix H^{sa}_n and H^{er}_n according to a Dirichlet distribution.

3. Draw ϑ^{sa}, ϑ^{ss} and ϑ^{er} as new θ from the proposed distribution. Assign a very small likelihood if the solution solved by FWZ doesn't converge. The updated

parameters of θ are accepted or rejected according to a Metropolis-Hasting algorithm with a probability $\min\{1, r\}$ where

$$r = \frac{\ell\left(\vartheta^{sa}, \vartheta^{er}, \vartheta^{ss} \mid Y^T, \xi_{n-1}^{sa,T}, \xi_{n-1}^{er,T}\right) p\left(\vartheta^{sa}, \vartheta^{er}, \vartheta^{ss}\right)}{\ell\left(\theta_{n-1}^{sa}, \theta_{n-1}^{er}, \theta^{ss} \mid Y^T, \xi_{n-1}^{sa,T}, \xi_{n-1}^{er,T}\right) p\left(\theta_{n-1}^{sa}, \theta_{n-1}^{er}, \theta_{n-1}^{ss}\right)}$$

We can also observe the filtered estimates of the DSGE states: $\tilde{S}_n^t$ in each period.

4. if $n < n_{sim}$, go back to 1, otherwise stop, where n_{sim} is the desired number of iterations.

In the algorithm described above, the likelihood is approximated when maximizing the posterior mode. The detailed algorithm is described in section 2.4.2 with Kim's approximation.

2.4.2 Kim's approximation

In this section we describe Kim's approximation of the likelihood function. To simplify, we combine the Markov Switching states of structural parameters and of heteroskedastic shocks in a unique chain ξ_t with m states. ξ_t could be explained to hold m different values with $m = m^{sp} \times m^{er}$, and evolves according to a transition matrix $H = H^{sa} \otimes H^{er}$ with two independent Markov chains H^{sa} and H^{er}. For a given set of parameters, and some assumptions about the initial DSGE state variables and Markov Switching latent variables, we can recursively run the following filter.

$$S_{t|t-1}^{(i,j)} = T_j S_{t-1|t-1}^{i}$$

$$T_j = T(\xi_t = j)$$

$$P_{t|t-1}^{(i,j)} = T_j P_{t-1|t-1}^{i} T_j^{'} + R_j Q_j R_j^{'}$$

$$Q_j = Q(\xi_t = j),\ R_j = R(\xi_t = j)$$

$$e_{t|t-1}^{(i,j)} = y_t - D - Z S_{t|t-1}^{(i,j)}$$

$$f_{t|t-1}^{(i,j)} = Z P_{t|t-1}^{(i,j)} Z^{'} + U$$

We update $S_{t-1|t}$ and $P_{t-1|t}$ by adding more information till time t.

$$S_{t|t}^{(i,j)} = S_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)} Z^{'} \left(f_{t|t-1}^{(i,j)}\right)^{-1} e_{t|t-1}^{(i,j)}$$
$$P_{t|t}^{(i,j)} = P_{t|t-1}^{(i,j)} - P_{t|t-1}^{(i,j)} Z^{'} \left(f_{t|t-1}^{(i,j)}\right)^{-1} Z P_{t|t-1}^{(i,j)}$$

Some approximations are introduced to make the above Kalman operational. At the end of iteration, the $M \times M$ posteriors $S_{t|t}^{(i,j)}$ and $P_{t|t}^{i,j}$ are collapsed into M posteriors $S_{t|t}^{j}$ and $P_{t|t}^{j}$ to complete the Kalman filter.

$$S_{t|t}^{j} = \frac{\sum_{t=1}^{M} Pr[\xi_{t-1} = i, \xi_t = j \mid Y_t] S_{t|t}^{(i,j)}}{Pr[S_t = j \mid Y_t]}$$
$$P_{t|t}^{j} = \frac{\sum_{i=1}^{M} Pr[S_{t-1} = i, S_t = j \mid Y_t] \left(P_{t|t}^{(i,j)} + \left(S_{t|t}^{j} - S_{t|t}^{(i,j)}\right)\left(S_{t|t}^{j} - S_{t|t}^{(i,j)}\right)^{'}\right)}{Pr[\xi_t = j \mid Y_t]}$$

Finally, we can calculate the likelihood density of observation y_t as:

$$\ell(y_t \mid Y_{t-1}) = \sum_{j=1}^{m}\sum_{i=1}^{m} f(y_t \mid \xi_{t-1} = i, \xi_t = j, Y_{t-1}) Pr[\xi_{t-1} = i, \xi_t = j \mid Y_t]$$

$$f(y_t \mid \xi_{t-1} = i, \xi_t = j, Y_{t-1}) = (2\pi)^{-N/2} \mid f_{t|t-1}^{(i,j)} \mid^{-1/2} \exp\left\{-\frac{1}{2} e_{t|t-1}^{(i,j)\prime} f_{t|t-1}^{(i,j)} e_{t|t-1}^{(i,j)}\right\}$$

2.5 Empirical results

The section reports empirical results for five models considered. The first one is taken as a benchmark model with all parameters fixed. The other four specifications extend the constant parameters model to allow for underlying Markov switching processes. The first extension allows structural parameters of expectational Phillips curves and intertemporal Euler equation to change across regimes according to the Markov process ξ_t^{sa}. The second extension adds heteroskedastic shocks to the first extension with parameters and heteroskedastic shocks evolving according to two independent Markov chains ξ_t^{sa} and ξ_t^{er}. In the third extension, policymakers' response to the inflation and output gap in the Taylor Rule is assumed to switch in across different regimes following the Markov chain ξ_t^{sp} whereas the last extension consider heteroskedastic shocks with switching structural parameters controlled by two independent Markov chains ξ_t^{sp} and ξ_t^{er}. The five models are summarized by table 2.1.

The data used are from 1954Q3 to 2009Q2. The series are obtained from the website of Federal Reserve of Saint Louis. Output gap is measured as the percentage deviations of real per capita GDP from a trend obtained with the HP filter. Inflation is the quarterly percentage change of the CPI(Urban, all items). Nominal interest rate is the Effective Federal Funds Rate.

2.5.1 Parameters estimates and regime probabilities

The DSGE model with constant parameters is taken as the start of the investigation on macroeconomic dynamics. The priors are specified according to those in the previous literature, which are summarized in the appendix A.1. It is important to notice that in the case of the fixed coefficient DSGE model, only when the Federal Reserve reacts strongly to deviations of inflation from its target will this allow for determinacy. In order to avoid the impact of indeterminancy, we restrict γ_1 in equation (2.5) to have a mean of 1.5 and a standard deviation of 0.5. This implies the central bank raises the nominal rate by 1.5 percent in response to a 1-percent discrepancy between actual and desired inflation. We set the prior of output gap targeting γ_2 as 0.8 and interest smoothing ρ_R as 0.7. The estimation of the benchmark model is reported in the table 2.2.

Over the whole sample, the Federal Funds Rate reacts strongly to deviations of inflation from its target with $\gamma_1 = 2.17$ while output gap receives less weight with the coefficient of 0.89. The coefficient of the Phillips curve is estimated as only, $k = 0.073$, which may imply a relatively flat Phillips curve in which the firm faces a stickier price in the good market. The inflation target in the benchmark model is 0.946, implying a target for annual inflation around 4%. $\tau = 0.419$ represents the risk aversion of consumers towards the time variation of consumption is around 2[12]. The technology shock is estimated as being much higher than monetary shock and demand shock.

There is a great interest in clarifying what happens when the consumers change their attitude towards current consumption and the firms alter the rigidity or stickiness of prices. Here I allow structural parameters τ and κ to switch across regimes evolving

[12]The risk aversion of consumers is represented by τ^{-1}.

according to a Markov chain ξ^{sa}[13]. The transition matrix H^{sa} in this model used by agents to change their beliefs is assumed to be a one-step transition probability matrix.

Table 2.3 reports means and 90% error bands for DSGE parameters and transition matrices when we have τ and κ switching across regimes, which corresponds to the second extension proposed. In this model, the remaining structural parameters are kept constant and the Federal Funds Rate is assumed with a strong response to deviation of inflation from its target in order to obtain a determinacy solution [14]. The estimation indicates striking differences between two regimes. Regarding parameters of the private sector, we observe that under regime 1 (ξ_t^{sa}=1), the household has a lower intertemporal substitution elasticity with $\tau(\xi^{sa} = 1) = 0.121$, which implies that the consumer is more averse to variations in consumption and prefer a stable consumption path. That is, she will be less willing to substitute consumption intertemporally. On the other hand, the elasticity of output κ has a larger value under regime 1. A larger slope-coefficient on the output gap in the Phillips curve indicates that the firm is facing a more flexible price in the good market with a lower cost of price adjustment. Prices are considered more flexible in the U.S. during the high and volatile inflation of the 1970 as discussed in Gali and Gertler (1999), Cogley and Sbordone (2005), Fernandez-Villaverde and Rubio-Ramarez (2007). Figure 2.4 shows (filtered) probabilities assigned to $\xi_t^{sa} = 1$ takes place around 1958, during the "Great Inflation" and around 2008. Regime 1 is characterized as the one in which agents respond more strongly to the economy. In this regime, agents have a higher risk aversion and a steeper Phillips curve.

[13]Note Clarida, Gali, and Gertler (2000) mention that there is no widespread consensus on the value κ and values found in the literature range from 0.05 to 1.22. To be consistent with Lubik and Schorfheide (2004), I assume there are smaller τ and higher κ under the first regime with potential larger volatility at that time.

[14]An interesting thing is if we allow indeterminacy in the model 2, the regime 1 will be much wider than my current result. Agents have to maintain their beliefs much longer if they accept a monetary authority with passive monetary policy

In addition to switching in agents' beliefs, we allow for heteroskedastic shocks in the model. Structural parameters of the private sector and heteroskedastic shocks are controlled by two independent Markov chains. Following Bollen, Gray, and Whaley (2000), a convenient way to represent two independent Markov chains is to introduce a four-state probability transition matrix. The process is described as follows:

Firstly, define ξ^{sa} as the agents' beliefs switching regime with the probability $Pr[\xi_t^{sa} = j \mid \xi_{t-1}^{sa} = i] = p_{ij},\ i, j = 1, 2$ and ξ^{er} as the variance change regime with the probability $Pr[\xi_t^{er} = j \mid \xi_{t-1}^{er} = i] = q_{ij},\ i, j = 1, 2$. Both ξ_t^{sa} and ξ_t^{er} evolve according to a first-order one-step Markov scheme with transition matrix

$$H^{sa} = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix}$$

for the agents' beliefs regime, and

$$H^{er} = \begin{bmatrix} q_{11} & 1 - q_{22} \\ 1 - q_{11} & q_{22} \end{bmatrix}$$

for the variance regime. Now define a regime indicator variable that spans the regime space for both agents variables and variance regimes as

$$\xi_t = \begin{cases} 1, \xi^{sa} = 1, \xi^{er} = 1 \\ 2, \xi^{sa} = 2, \xi^{er} = 1 \\ 3, \xi^{sa} = 1, \xi^{er} = 2 \\ 4, \xi^{sa} = 2, \xi^{er} = 2 \end{cases}$$

where ξ_t evolves according to a first-order Markov process with the transition probability matrix

$$\Pi = \begin{bmatrix} p_{11}q_{11} & (1-p_{11})q_{11} & p_{11}(1-q_{11}) & (1-p_{11})(1-q_{11}) \\ (1-p_{11})q_{11} & p_{22}q_{11} & (1-p_{11})(1-q_{22}) & p_{22}(1-q_{22}) \\ p_{11}(1-q_{11}) & (1-p_{22})(1-q_{11}) & p_{11}q_{22} & (1-p_{22})q_{22} \\ (1-p_{11})(1-q_{11}) & p_{22}(1-q_{11}) & (1-p_{11})q_{22} & p_{22}q_{22} \end{bmatrix}$$

The transition parameters p_{22} and q_{11} can be easily obtained from the above transition probability matrix. Table 2.4 presents the results with two Markov Switching processes. The results in table 2.4 are similar to those in table 2.3 with a higher τ and smaller κ under regime 1, and the opposite under regime 2. The difference between κ across regimes becomes slightly greater compared to the case with only agents' beliefs changing. Another difference in this model is that the response of the Federal Reserve to the inflation is much smaller– there is a lower inflation target. Regime 1 ($\xi^{er} = 1$) for heteroskedastic shocks is assigned higher standard deviation in the priors of the model. The estimation results show a clear distinction between the two volatility regimes. Similar to the model 1, technology shock is still the highest one.

The top panel in figure 2.5 shows that regime 1 of agents' beliefs occurs around 1957, 1960, during the "Great Inflation", and around 2008. The bottom panel represents the low volatility of shocks which takes place elsewhere except during the "Great Inflation", the early of 1990s, the early 2000s and around 2008. There is a reduction in the size of exogenous shocks during the "Great Moderation" compared with other periods. Note that the figure captures large shocks that hit the U.S. economy in the last two years.

The next extension of the benchmark model investigated is the case in which policy changes across regimes. Table 2.5 reports the results when we have different regimes for policy parameters. Under regime $1(\xi_t^{sp} = 1)$, the Federal Funds Rates react more strongly to inflation with $\gamma_1 = 2.17$ while the output gap has lower weight ($\gamma_2 = 0.758$). We define this as the *"Hawk regime*", that is, the one describing the situation in which the government reacts more than one-for-one to a change in inflation. Similarly, when there is a smaller reaction of the Federal Reserve to inflation with $\gamma_1 = 0.885$, we have the alternative regime ($\xi_t^{sp} = 2$) denominated the *"Dove Regime"*. There are not great differences between coefficients on output gap across the regimes. The behavior of the Federal Reserve is mainly reflected by a change of response to inflation. Figure 2.6 shows the probability of policy switching. The figure depicts a passive monetary policy by the Federal Reserve from 1955 to 1958, from 1974 to 1980, and from 2005 to 2008. There is a large literature showing evidence that during the "Great Inflation", policymakers were relatively unresponsive to deviations of inflation from the target, following a loose monetary policy. On the other hand, during the Post-Volcker period the Fed showed a stronger commitment to bring the economy back to the steady state in order to achieve low inflation with a cost of reduced output.

In the last proposed extension, we assume that the Fed faces heteroskedastic shocks. Compared with the case of only policy parameters switching, in this case these estimates indicate that there is a slightly higher response to inflation and a smaller inflation target over the sample.

The probability in the top panel of figure 2.7 represents the policy switching regimes. It is well-known that the Federal Reserve accommodated the inflation around the 1970s and started to raise the interest rate in the early year of 1980s. The interesting thing is there were some changes before our current crisis - the estimates indicate that Federal Reserve was following a loose monetary policy around 2005. The probability of lower volatility over the sample is shown in the lower panel. A higher probability is assigned to the period after the year of 1984. We also notice there was high volatility of shocks around the 2007.

2.5.2 Impulse response analysis

Figure 2.8 shows impulse response functions to different shocks. Since there are four states, we plot the regime of high volatility and low volatility together in red and blue, respectively. Thus, the first two rows of figure 2.8 show the impulse responses to a monetary shock under the strong response and weak response of agents, respectively. Both output and inflation decrease following an increase in the Federal Funds Rate. Under regime 1, output drops initially by almost 0.2 percent and inflation decreases by 0.3 percent to an unanticipated tightening of monetary policy. In this regime, agents respond more strongly to keep the output gap close to zero at a cost of higher inflation.

The third and the fourth rows show the impulse responses to a demand shock. Output and inflation increase under both regimes but are more volatile under the regime 1. In particular, the response of inflation is higher under regime 1 pushed by a demand shock. It is easy to understand the movement by observing the firm will adjust the prices more frequently when facing a demand shock.

The last two rows contain impulse responses to an adverse supply shock, i.e. to an unexpected decrease in z_t. The fluctuation of inflation and output is stronger under the regime corresponding to strong agents' beliefs. This is also due to more flexible prices and stable consumption preference during this regime.

There is a larger literature that investigates the role of unfavorable supply-side shocks as causes of the high inflation of the 1970s. Output decreases and becomes negative under both regimes, but at a larger extent under regime 1. Inflation fluctuates more in response to a supply shock under the regime 1 than regime 2. This result is similar to Lubik and Schorfheide (2004). The conjecture that the 1970s were characterized by important supply shocks is confirmed by the variance decomposition in section 2.5.3, discussed below.

Figure 2.9 summarizes the impulse responses to different shocks when policy parameters switch with heteroskedastic shocks for both high and low volatility regimes. A positive monetary policy will reduce the output gap and inflation, and raise the interest rate. The difference between the hawk and dove regime lies in the magnitude of the response. The impulse response of inflation is lower in the dove region. The result is consistent with the view that monetary policy is more effective in an environment with a low inflation target(Bernanke and Mishkin (1997), Mishkin (2007), Goodfriend and King (2005)). As shown in Figure 2.9, a demand shock pushes output to a higher level, and also generates a higher inflation. The responses are stronger under the Dove regime. This is consistent with the response of the Federal Funds rate that is larger under the Hawk regime. The dynamics of the variables are similar across two regimes, and thus the Fed does not face any trade-off when deciding how to respond to a demand shock. A positive supply shock will increase the output gap under Dove regime and decrease it under the Hawk regime. The result is also consistent with Bianchi (2009), who interprets how the behavior of the Federal Reserve differs across two regimes. Under the Hawk regime the Fed is willing to accept a recession in order to control inflation. On the contrary, under the Dove regime the policy response is much weaker because the Fed tries to keep the output gap around zero, at the cost of higher inflation. If the expected inflation is very high, a negative real interest rate will boost the economy in the short run.

2.5.3 Variance decomposition

Variance decompositions for output gap, inflation, and interest rates are summarized in figure 2.10, which represents contributions of structural shocks to the volatility of macroeconomic variables for all possible combinations of agents' beliefs regimes and volatility regimes. Such a decomposition enables better understanding of what kind of shock contributes more in different regimes. The first two values, on the left of the blue dashed line, refer to the high volatility regime, while the third and the fourth value assume that the low volatility regime is in place. In each sub-group, the first point marks the standard deviation under the stronger reaction regime of agents' beliefs and the second point marks the agents' weak response. The figure presents the variance decomposition for four possible regime combinations. Output is mainly driven by the demand shock, especially under the low volatility regime. We can note that change of agents' beliefs does matter in explaining the volatility of inflation. When the firm is facing more flexible prices, the inflation could be driven by the demand shock while at the same time the supply shock still play a dominant role.

Figure 2.11 shows variance decompositions under switching policy parameters and heteroskedastic shocks. Similarly to figure 2.10, the dashed blue line separates subgroup as high and low volatility regime. The first point in each sub-group refers to the hawk region. As we can observe, in the case of output a large fraction of volatility comes from the demand shock, especially under the Dove-High regime. Supply shocks are mainly responsible for the volatility of inflation while monetary policy shocks play a marginal role. Supply shocks explain more under the hawk region. When dove regime is in effect, the contribution of demand shocks is basically null. We arrive at the same conclusion as the evidence from impulse response functions: under the Dove regime, the

Fed accommodates supply shocks to dampen the inflation. When we consider policy switching, monetary policy shocks explain a small fraction of the Federal Funds Rate volatility under hawk regime, considering the Fed has a stronger incentive to bring the economy back on track.

2.5.4 Counterfactual analysis

This section implements counterfactual analysis in the extended regime switching models, in order to simulate what would have happened if regime changes had not occurred. This kind of analysis brings us more meaningful interpretation in the context of the MS-DSGE model. The entire law of motion changes in a way that is consistent with new assumptions around the behavior of switching. Since we can observe different switching probabilities , we can investigate what would have happened if agents' beliefs or monetary policy about the probability of moving across regimes had been different. This has important implications for counterfactual simulations in which a regime is assumed to have been in place throughout the sample because the expectation mechanism and the law of motion are consistent with the fact that no other regime would have been observed. By implementing counterfactual analysis, two main conclusions can be drawn according to results of this section. First, inflation would be lower during the "Great Inflation" if agents persist in a weak response with more sticky prices over the sample. Second, if the hawk regime had been in place through the entire regime, a little would have changed for the dynamics of inflation at the sacrifice of the production.

Two main conclusions can be drawn from the results of implementing counterfactual analysis. First, inflation would be lower during the "Great Inflation" if agents displayed a weak response with more sticky prices over the sample. Second, if the hawk regime had been in place through the entire regime, little would have changed with respect to inflation dynamics at the sacrifice of output.

Three analysis were undertaken in the counterfactual analysis. Figure 2.12 shows the results when consumers always have a lower risk aversion and firms adjust the price less frequently. It is noticeable that if agents had always behaved in this manner, inflation would have been lower during the “Great Inflation” period.

Figure 2.13 is the result under the assumption that the hawk region is always in place. The higher interest rate in the 70s causes output to decrease slightly while it is difficult to curb inflation. In order to further study the behavior of the Federal Reserve, we consider augmenting the policy parameters by doubling the values in the Hawk region. Figure 2.14 shows the results of this experiment. In order to reduce inflation, the Federal Reserve needs to react much more strongly, while at the same time there is a risk of a recession.

2.5.5 Model comparison

This section reports the maximum likelihood of each model. As in table 2.7, regime switching models display similar maximum likelihood values, while the DSGE model with fixed parameters displays a much lower one.

2.6 Conclusions

This paper investigates a small-scale dynamic stochastic general equilibrium model with regime switches on structural parameters. It extends the benchmark model of fixed parameters by considering the role of agents’ beliefs, changes in the behavior of the Federal Reserve and the stochastic volatility of exogenous shocks. A Gibbs-sampling with Metropolis-Hasting algorithm is implemented as the estimation strategy after solving the minimum state variable solutions. In an application to postwar U.S. inflation and output dynamics, the results indicate that there were substantial different shifts in monetary policy, agents' beliefs, and in the volatility of non-policy shocks in the last 60 years. If the agents always maintain a weak response to economic dynamics, inflation would have been lower during the 1970s. The more intense response to inflation by the Federal Reserve would also have helped mitigate the great inflation. Supply shocks are found as the main drivers of the inflation if there were no switches in the agents' beliefs. Finally, there were substantial changes in agents' beliefs, and in the volatility of shocks in the recent years. Such an occurrence could be potential early signals of the 2008 financial crisis.

Table 2.1: DSGE Models with Regime Switching Descriptions

Model	Regime-Switching parameters
$\mathcal{M}_1$	None
$\mathcal{M}_2$	τ and κ
$\mathcal{M}_3$	τ, κ, σ_R, σ_g and σ_z
$\mathcal{M}_4$	γ_1 and γ_2
$\mathcal{M}_5$	γ_1, γ_2, σ_R, σ_g and σ_z

Table 2.2: Benchmark model

Parameter	Estimation	Parameter	Estimation
γ_1	2.174 (2.096 , 2.233)	r^*	0.246 (0.201 , 0.254)
γ_2	0.893 (0.821 , 0.979)	π^*	0.946 (0.899 , 1.000)
ρ_R	0.848 (0.754 , 0.978)	σ_R	0.391 (0.297 , 0.501)
τ	0.419 (0.419 , 0.462)	σ_g	0.354 (0.191 , 0.468)
β	0.922 (0.842 , 1.000)	σ_z	1.973 (1.946 , 2.000)
κ	0.073 (0.030 , 0.114)	σ_y	0.413 (0.000 , 0.668)
ρ_g	0.831 (0.805 , 0.862)	σ_p	0.440 (0.338 , 0.563)
ρ_z	0.899 (0.871 , 0.960)	σ_r	0.073 (0.000 , 0.182)

Benchmark model: Means and 90 percent error bands of the DSGE parameters

Table 2.3: Only agents' beliefs change

Parameter	$\xi_t^{sa}=1$	$\xi_t^{sa}=2$
τ	0.121 (0.037 , 0.176)	0.533 (0.408 , 0.667)
κ	0.681 (0.455 , 0.934)	0.085 (0.059 , 0.103)
β	0.982 (0.962, 0.998)	
γ_1	1.964 (1.748, 2.189)	
γ_2	0.789 (0.657, 0.903)	
ρ_R	0.774 (0.753, 0.795)	
ρ_g	0.860 (0.803, 0.918)	
ρ_z	0.880 (0.846, 0.900)	
r^*	0.383 (0.121, 0.654)	
π^*	0.836 (0.715, 0.958)	

Parameter	$\bar{\xi}_t^{er}=1$
σ_R	0.445 (0.351 , 0.581)
σ_g	0.334 (0.203 , 0.431)
σ_z	1.833 (1.667 , 1.998)
σ_y	0.141 (0.007 , 0.323)
σ_p	0.498 (0.245 , 0.742)
σ_r	0.209 (0.016 , 0.462)

$diag(H^{er})$
0.928 (0.834, 1.000)
0.984 (0.971, 1.000)

Only agents' beliefs change: Means and 90 percent error bands of the DSGE and transition matrix parameters

Table 2.4: Different regimes on agents' beliefs and stochastic volatilities

Parameter	$\xi_t^{sa}=1$	$\xi_t^{sa}=2$
τ	0.128 (0.126 , 0.131)	0.577 (0.575 , 0.583)
κ	0.672 (0.671 , 0.675)	0.052 (0.053 , 0.054)
β	0.858 (0.858, 0.859)	
γ_1	2.161 (2.177, 2.192)	
γ_2	0.962 (0.962, 0.963)	
ρ_R	0.739 (0.739, 0.740)	
ρ_g	0.836 (0.836, 0.836)	
ρ_z	0.952 (0.954, 0.956)	
r^*	0.462 (0.462, 0.463)	
π^*	0.754 (0.754, 0.755)	

Parameter	$\xi_t^{er}=1$	$\xi_t^{er}=2$
σ_R	0.707 (0.715 , 0.723)	0.379 (0.387 , 0.394)
σ_g	0.576 (0.575 , 0.579)	0.390 (0.389 , 0.392)
σ_z	1.677 (1.645 , 1.739)	0.497 (0.495 , 0.501)
σ_y	0.017 (0.019, 0.020)	
σ_p	0.310 (0.300, 0.328)	
σ_r	0.394 (0.408, 0.421)	

$diag(H^{sa})$	$diag(H^{er})$
0.963 (0.950, 0.968)	0.974 (0.963, 0.977)
0.982 (0.976, 0.981)	0.925 (0.928, 0.965)

Different regimes on agents' beliefs and stochastic volatilities: Means and 90 percent error bands of the DSGE and transition matrix parameters.

Table 2.5: Only policy changes

Parameter	$\xi_t^{sp}=1$	$\xi_t^{sp}=2$
γ_1	2.170 (2.099 , 2.228)	0.758 (0.668 , 0.881)
γ_2	0.885 (0.836 , 0.943)	0.630 (0.510 , 0.714)
ρ_R	0.737 (0.692, 0.778)	
τ	0.772 (0.675, 0.865)	
β	0.955 (0.925, 0.981)	
κ	0.124 (0.090, 0.145)	
ρ_g	0.797 (0.770, 0.838)	
ρ_z	0.874 (0.856, 0.892)	
r^*	0.589 (0.386, 0.714)	
π^*	0.873 (0.748, 0.999)	

Parameter	$\bar{\xi}_t^{er}=1$
σ_R	0.519 (0.370 , 0.610)
σ_g	0.398 (0.322 , 0.468)
σ_z	1.782 (1.482 , 1.993)
σ_y	0.371 (0.010 , 0.780)
σ_p	0.453 (0.378 , 0.522)
σ_r	0.029 (0.000 , 0.073)

$diag(H^{er})$
0.889 (0.785, 0.992)
0.956 (0.913, 0.996)

Only policy changes: Means and 90 percent error bands of the DSGE and transition matrix parameters.

Table 2.6: Different regimes on policy and stochastic volatilities

Parameter	$\xi_t^{sp}=1$	$\xi_t^{sp}=2$
γ_1	2.215 (2.165 , 2.289)	0.985 (0.971 , 1.000)
γ_2	0.885 (0.871 , 0.900)	0.853 (0.831 , 0.875)
τ	0.562 (0.480, 0.652)	
β	0.987 (0.965, 1.000)	
κ	0.068 (0.049, 0.082)	
ρ_R	0.730 (0.703, 0.752)	
ρ_g	0.851 (0.818, 0.890)	
ρ_z	0.928 (0.894, 0.962)	
r^*	0.505 (0.450, 0.546)	
π^*	0.826 (0.799, 0.846)	

Parameter	$\xi_t^{er}=1$	$\xi_t^{er}=2$
σ_R	0.776 (0.655 , 0.869)	0.383 (0.371 , 0.398)
σ_g	0.831 (0.719 , 0.906)	0.354 (0.340 , 0.362)
σ_z	1.837 (1.699 , 1.953)	0.505 (0.450 , 0.546)
σ_y	0.057 (0.016, 0.083)	
σ_p	0.278 (0.232, 0.337)	
σ_r	0.035 (0.015, 0.054)	

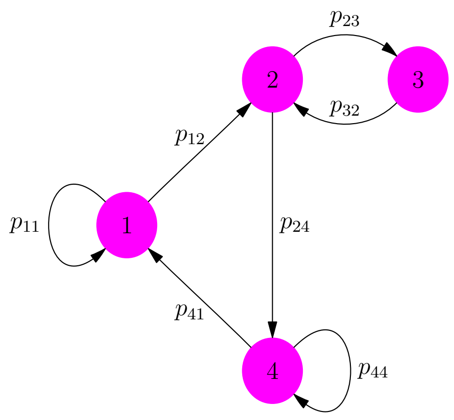
$diag(H^{sp})$	$diag(H^{er})$
0.970 (0.946, 0.995)	0.955 (0.912, 0.998)
0.965 (0.938, 0.996)	0.895 (0.811, 0.980)

Different regimes on policy and stochastic volatilities: Means and 90 percent error bands of the DSGE and transition matrix parameters

Table 2.7: DSGE models with Maximum Likelihood

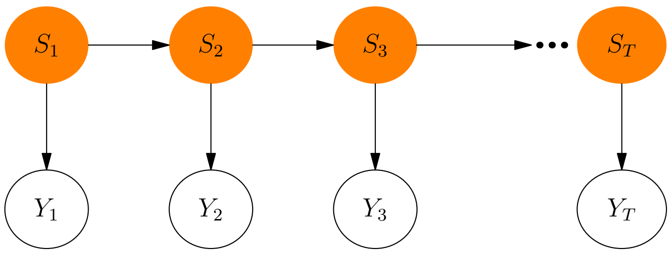
Model	Regime Switching parameters	Maximum Likelihood
$\mathcal{M}_1$	None	-1334
$\mathcal{M}_2$	τ and κ	-828.96
$\mathcal{M}_3$	τ, κ, σ_R, σ_g and σ_z	-716.44
$\mathcal{M}_4$	γ_1 and γ_2	-734.69
$\mathcal{M}_5$	γ_1, γ_2, σ_R, σ_g and σ_z	-817.75

Figure 2.1: Graphical representation of a 4-state Markov Chain



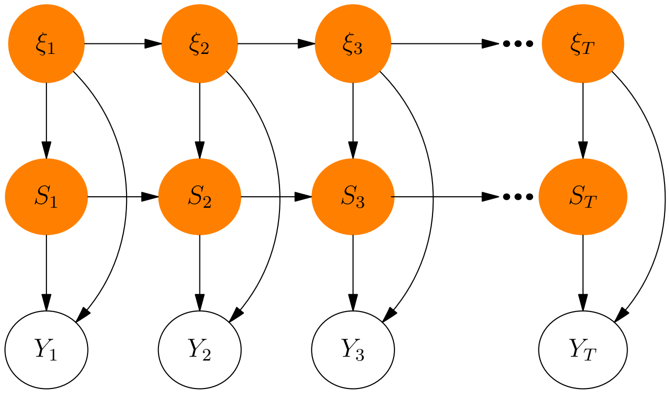
Note: Some directed arcs from i to j are included in the graph. $p_{ij} > 0$

Figure 2.2: State space model specifying conditional independence relations



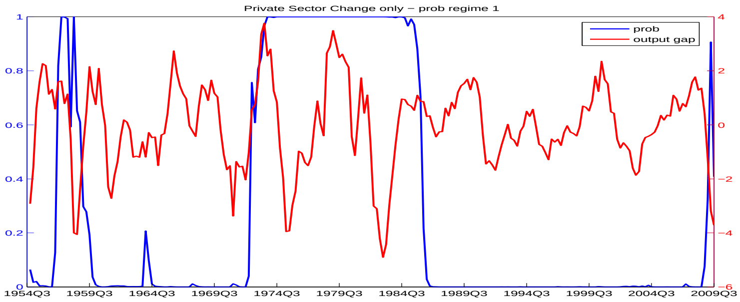
Note: Y_t is conditionally independent from all other variables given the state S_t, S_t only depends on S_{t-1}. The tinted nodes in orange represent hidden variables and unfilled nodes represent observed variables.

Figure 2.3: A figure of switching state space model



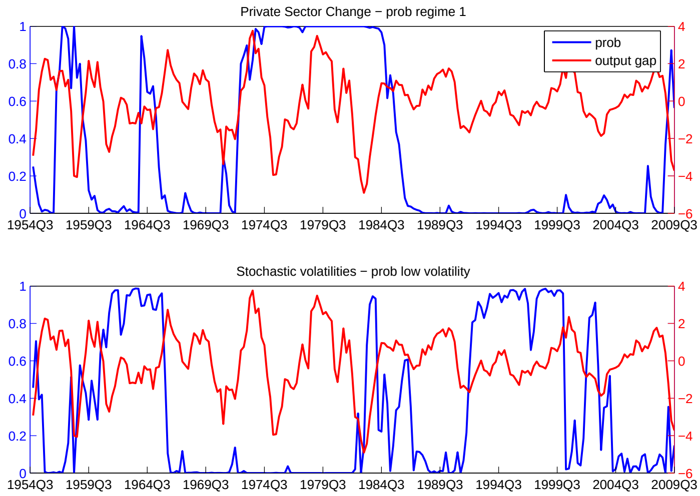
Note: Both the measurement equation and transition equation evolve according to a Markov chain. The tinted nodes in orange represent hidden variables and unfilled nodes represent observed variables.

Figure 2.4: MS-DSGE model: Only Private Sector changes



Probability of regime 1 at posterior mode estimates for the private sector structural parameters(Agents respond more strongly to the economy).

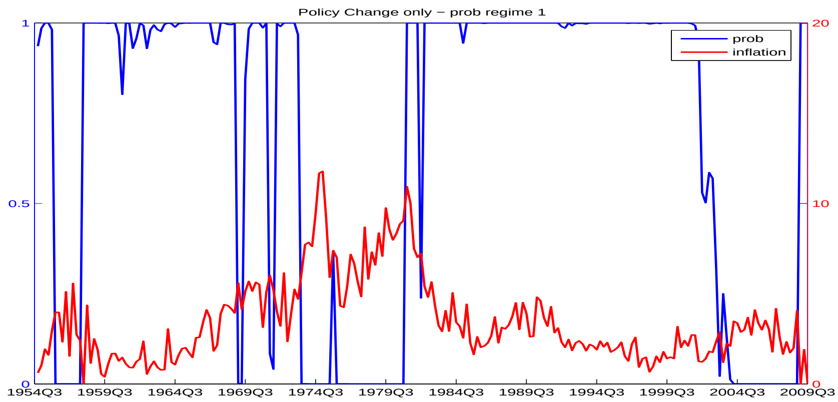
Figure 2.5: MS-DSGE model: Agents' beliefs and stochastic volatilities



Top panel: probability of regime 1 at posterior mode estimates for the private sector structural parameters.

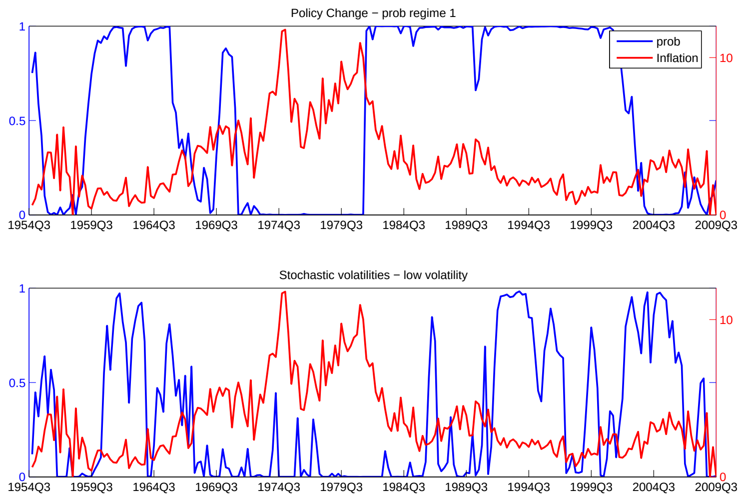
Lower panel: probability of regime 2 for variances–lower stochastic volatility.

Figure 2.6: MS-DSGE model: Only Policy changes.



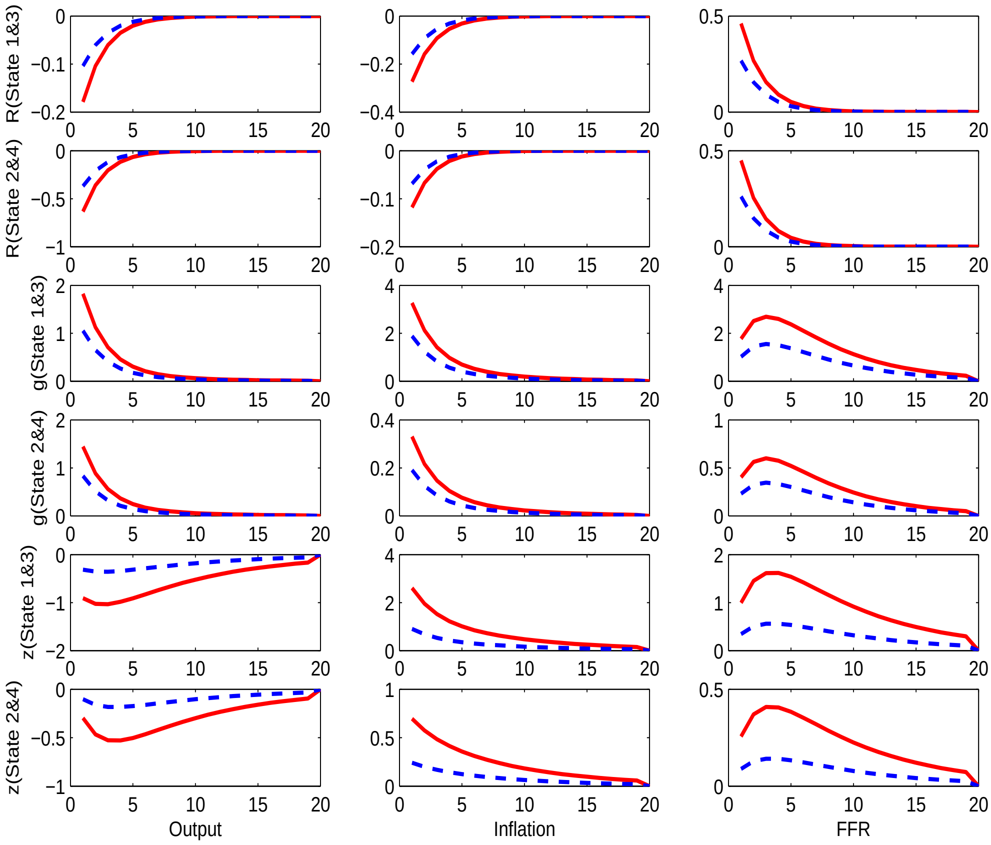
Probability of regime 1 at posterior mode estimates for monetary policy parameters (Hawk regime).

Figure 2.7: MS-DSGE model: Monetary policy and stochastic volatilities.



Top panel, probability of regime 1 for the policy structural parameters, Hawk regime.
Lower panel, probability of regime 2 for the stochastic volatilities, low volatility regime.

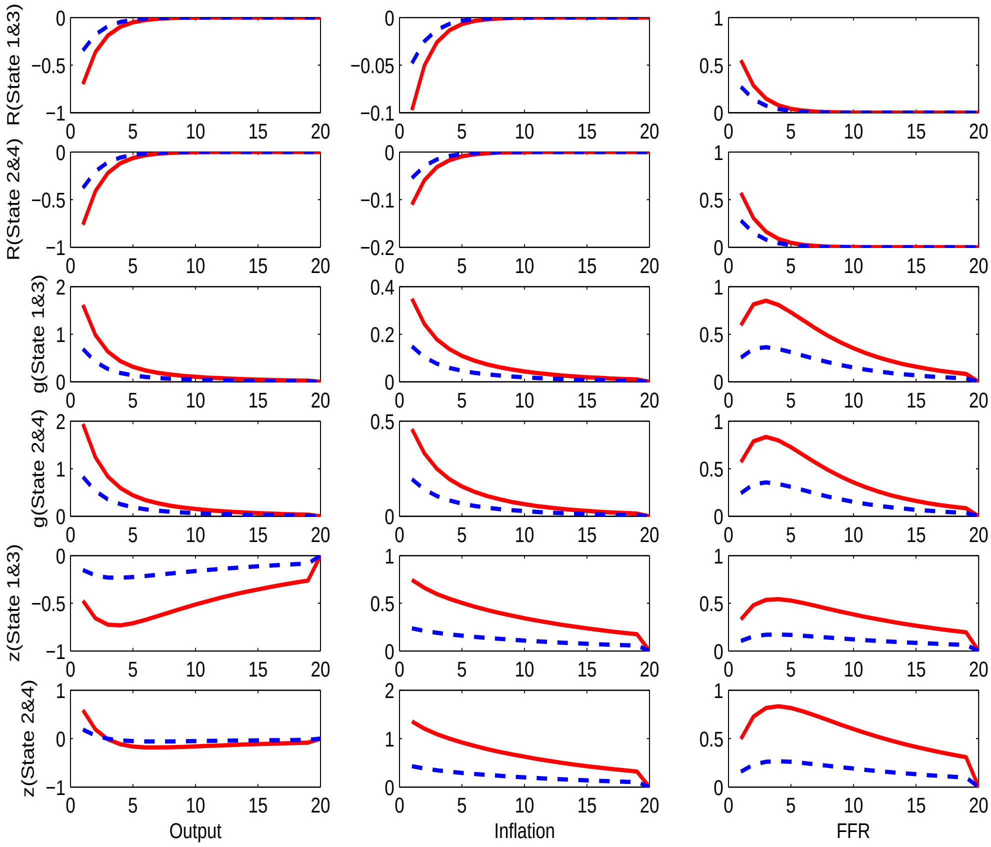
Figure 2.8: Impulse responses functions. Agents' beliefs and stochastic volatilities.



1-[strong response, high volatility]; 2-[weak response, high volatility], 3-[strong response, low volatility], 4-[weak response, low volatility]

R: Monetary shock; g: Demand shock; z: Adverse supply shock.

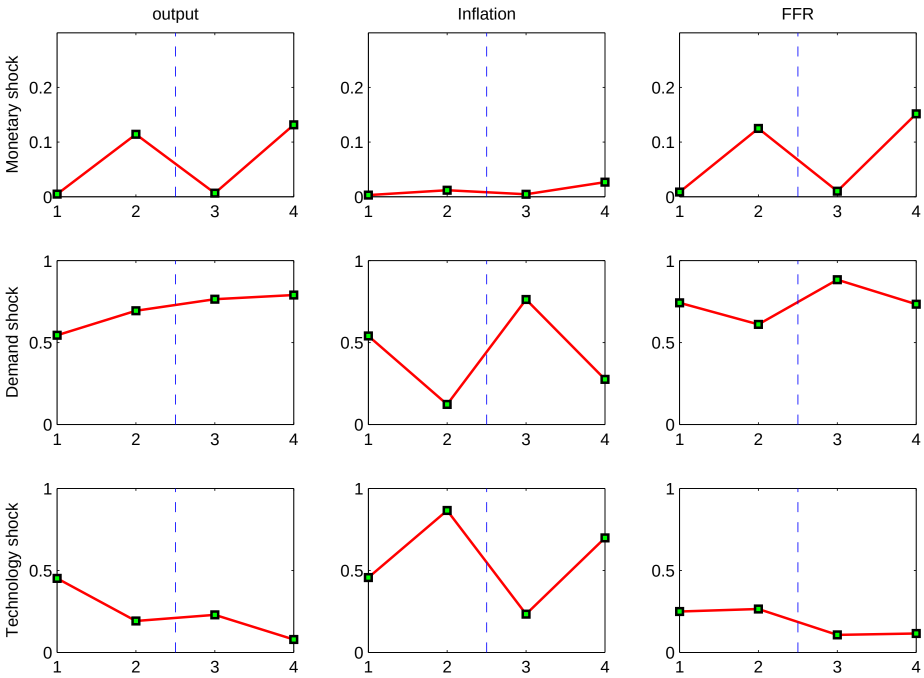
Figure 2.9: Impulse responses functions. Monetary policy and stochastic volatilities.



1-[hawk regime, high volatility], 2-[dove regime, high volatility], 3-[hawk regime, low volatility], 4-[dove regime, low volatility]

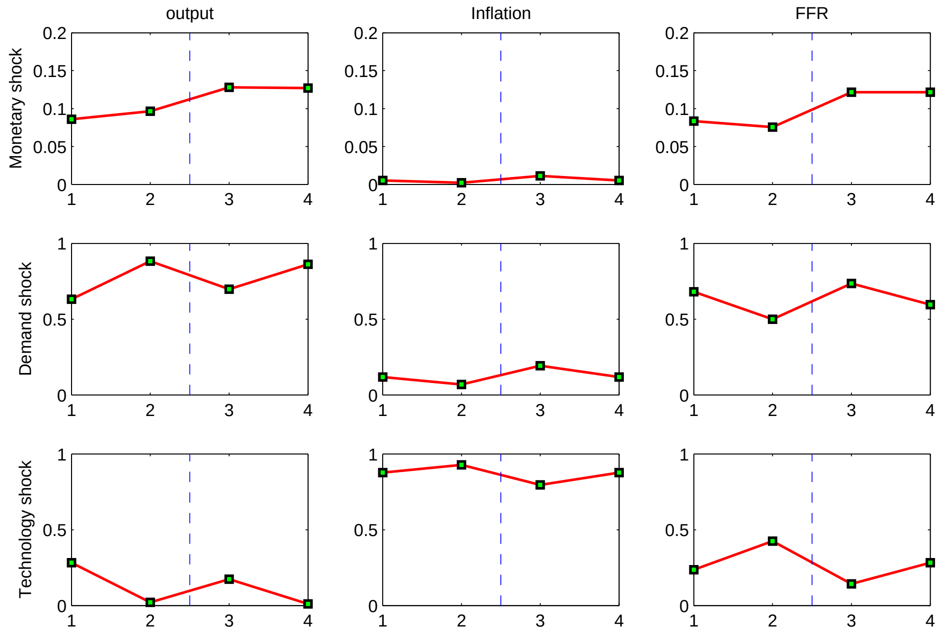
R: Monetary shock; g: Demand shock; z: Adverse supply shock.

Figure 2.10: Contributions of the different structural shocks: Agents' beliefs switching



Contributions of the different structural shocks to the volatility of the macroeconomic variables for different regime combinations: Agents' beliefs switching.
1-[strong response, high volatility]; 2-[weak response, high volatility], 3-[strong response, low volatility], 4-[weak response, low volatility]

Figure 2.11: Contributions of the different structural shocks: Policy switching



Contributions of the different structural shocks to the volatility of the macroeconomic variables for different regime combinations: policy switching.
1-[hawk regime, high volatility], 2-[dove regime, high volatility], 3-[hawk regime, low volatility], 4-[dove regime, low volatility]

Figure 2.12: Counterfactual simulation: Low risk aversion and Phillips curve slope

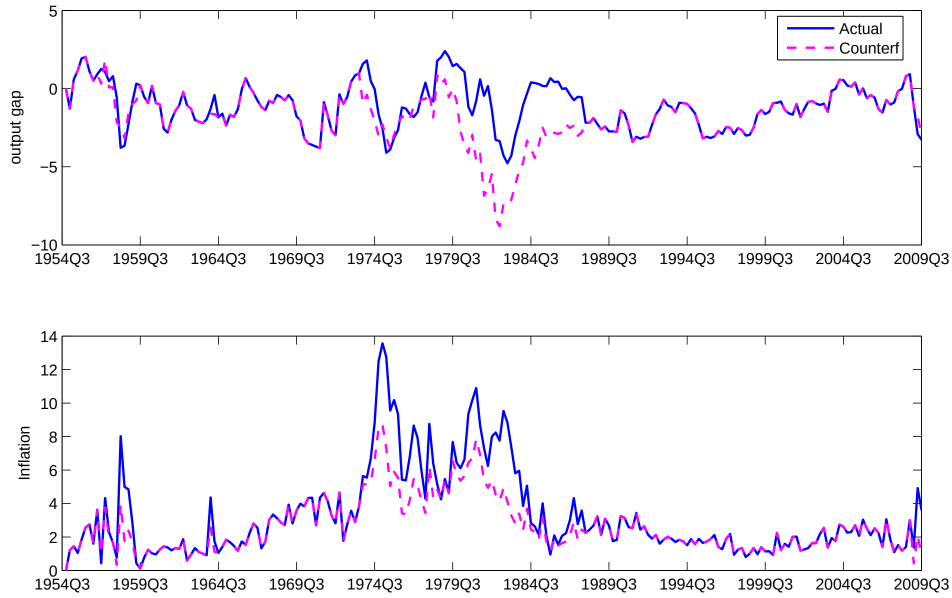


Figure 2.13: Counterfactual simulation: Always in the Hawk regime

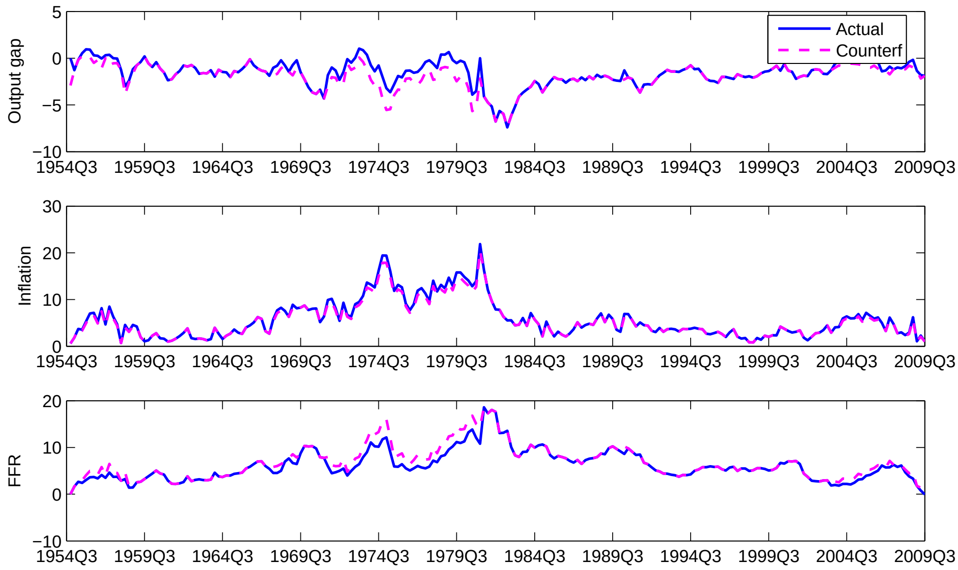
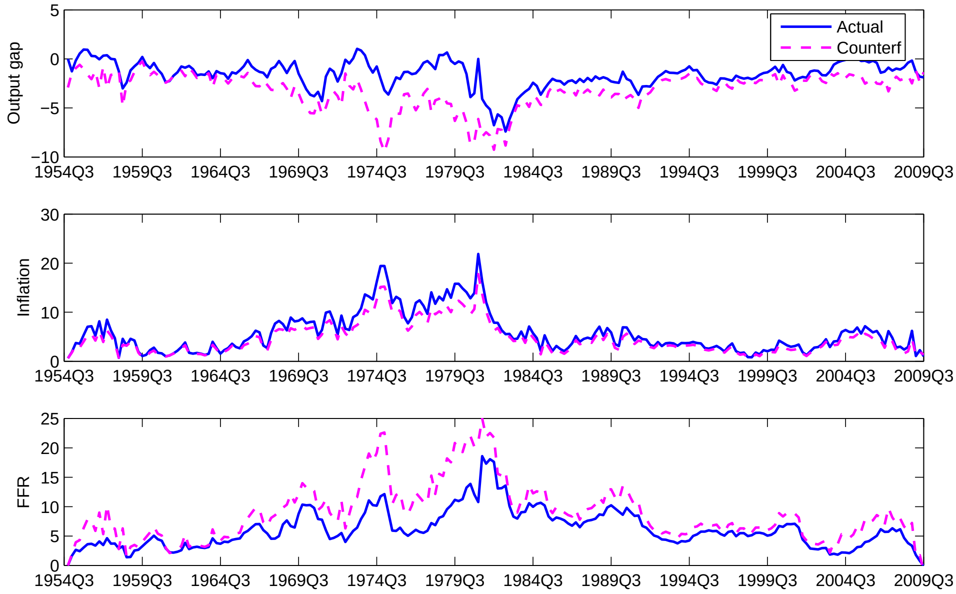


Figure 2.14: Counterfactual simulation: Augmented Hawk regime



Chapter 3

How Largely the Commitment can Beat Policymakers' Misperceptions?

3.1 Introduction

Policymakers have a trade-off on inflation and unemployment. This episode, is described as "a discretionary policymaker can create surprise inflation, which may reduce unemployment and raise government revenue" in Barro and Gordon (1983). The 1960s to 1980s saw such an occurrence known as the "Great Inflation" while the following 26 years distinguished itself in a certain state of low inflation and shallow recessions named as the "Great Moderation". These stylized facts caused much interest which attempted to explain the behavior of inflation and unemployment, especially the reason resulting in the "Great Inflation". Previous studies could be basically categorized as "bad luck" view, "lack of commitment" view and "policy mistakes" view. While previous research focuses more on the reason of inflation fluctuation, our paper investigates how to alleviate such a bad situation in the economy. Since it is difficult to avoid bad luck originating from exogenous and non-policy shocks, this paper aims to model the evolution of policymakers' belief, which combines their commitment with policy mistakes. Results of this paper demonstrate that a commitment to the Taylor-type monetary rule can effectively reduce policymakers' misperceptions and thus contribute to a stabilization policy.

The source of "bad luck" is well stated as higher exogenous, non-policy shocks occurring in the 1960s and 1970s. Ahmed, Levin, and Wilson (2002) stated that reduced innovation variances resulted in largely decline in aggregate output volatility. Cogley and Sargent (2005) achieved similar conclusion that innovation variances fluctuated substantially larger in the late 1970s. Other research such as Kim and Nelson (1998), Sims and Zha (2006) and Stock and Watson (2003) also supported the findings of greater disturbances during the "Great Inflation". The argument "lack of commitment" originates from such a consideration that policymakers have no incentive to keep inflation low. Barro and Gordon (1983) referred it as a "long-term contracts between the government and the private sector". This conclusion was also supported by research from Christiano and Fitzgerald (2003), Christiano and Gust (2000) and etc. Moreover, Bullard and Eusepi (2005) assumed that monetary policymakers were committed to a Taylor-type policy rule in a general equilibrium, sticky price economy and found a substantial increase in inflation during the 1970s attributed to this source. The last explanation to the "Great Inflation" is the "policy mistakes", in which addresses U.S. monetary policy was less responsive to inflationary in the 1960s and 1970s. Policymakers under the Fed chairmanship of Arthur Burns failed to respond to inflation as appropriately as those under Paul Volcker and Alan Greespan (for example, Boivin and Giannoni (2002), Clarida, Gali, and Gertler (2000) and Orphanides (2002)). Primiceri (2005) found when policymakers underestimated both the natural rate of unemployment and the persistence of inflation in the Phillips curve, a prolonged high inflation ending with rapid disinflation will occur. Similarly, although Ahmed, Levin, and Wilson (2002) marked reduced innovation contributed to the decline in aggregate output volatility, they admitted that good policy played a more important role in explaining the post-1984 decline in the volatility of inflation.

Our paper investigates the assumption that policymakers commit to a Taylor rule, using an inflation-unemployment dynamic model for the US economy. Our approach differs from previous work as we model policymakers' belief as a latent variable rather than as represented by observed nominal interest rate. Our paper is based on the conjecture that policymakers' misperceptions originate from unobserved deviations of unemployment from its natural rate. We propose four processes for policymakers: belief under commitment to inflation and unemployment and compare them with a baseline autoregressive process without commitment. The models are: 1) a time-invariant Taylor rule in which policymakers can only observe previous inflation and unemployment; 2) a time-varying Taylor rule in which policymakers adjust their commitment each period according to available information; 3) a Taylor rule in which commitment switches between high and low inflation and unemployment phases, following a Markov switching process; 4) a Taylor rule with commitment adjusted according to low or high inflation regimes only. 5) a Taylor rule in which commitment is changed as a response to different regimes in unemployment only. We specify a loss function derived from a constrained minimization of the divergence in inflation and unemployment that also penalizes shifts in the policy variables.

The models are estimated using Bayesian techniques. We find that our estimated belief performs the role of real interest rate. Our empirical results are as follows.

First, policymakers' belief is very persistent even when it commits to a Taylor-type policy rule. Second, the run-up of U.S. inflation around 1980 is mostly attributed to policymakers' misperception while the peak in the end of 1974 is possibly a result from large non-policy shocks. Third, models with commitment dominate models without commitment, especially in periods of large oscillations in inflation. When policymakers are committed to respond to a Taylor-type policy rule, the average loss function is efficiently reduced over the time, thus effectively lessening their misperception.

Results obtained from the proposed specifications illustrate the extent to which different commitments mitigate policymakers' misperceptions given the loss function. Our results have important policy implications as they indicate how and when it is appropriate for policymakers to choose a commitment in their reaction to inflation and unemployment. In particular, the models indicate that a flexible or more activist policy is more appropriate in reacting to inflation when there is high unemployment target, whereas a policy that is consistent over time is more suitable under low unemployment target. Moreover, our results shed light on the source of the two large rises in inflation during the "Great Inflation" period and the prevalence of low inflation during the early 1980s.

The paper is organized as follows. Section 3.2 presents the theoretical model of the evolution of policymakers' misperception. Section 3.3 discusses how to estimate the model in a Bayesian approach. Section 3.4 provides the empirical results and a model fit. Section 3.5 closes this paper with the concluding remarks.

3.2 Model the Policymakers' misperception

3.2.1 The benchmark Model

We start from a simple rational expectation model that describes the dynamics of inflation and unemployment in the private sector part. The inflation rate π_t is equal to inflation expectation $E_t\pi_{t+1}$, plus a component that depends on lag polynomial of deviation of unemployment from its "natural rate", $u_{t-1} - u^N_{t-1}$,

$$\pi_t = E_t\pi_{t+1} + \tilde{\theta}(L)(u_{t-1} - u^N_{t-1}) + \varepsilon_t \tag{3.1}$$

where ε_t is a random shock with the distribution of i.i.d. $N(0, \sigma^2_\varepsilon)$.

We assume policymakers' misperception originates from the aggregate demand equation. Policymakers adjust their belief based on the deviation of unemployment from its natural rate. Equation (3.2) describes such a dynamic evolution as follows.

$$(u_t - u^N_t) = \rho(L)(u_{t-1} - u^N_{t-1}) + V_{t-1} + \eta_t \tag{3.2}$$

V_t is a control variable of policymakers and η_t is a random innovation with i.i.d $N(0, \sigma^2_\eta)$ distribution. The detailed description of V_t, as Primiceri (2005) is "unemployment deviates from the natural rate either because of a random shock or because of policymakers' decisions about stabilization policy".

The natural rate, u^N_t, is assumed to fluctuate over time in response to a shock τ_t according to the autoregressive process [1]

$$u^N_t = \gamma u^N_{t-1} + \tau_t \tag{3.3}$$

[1] We ignore a constant term here to simplify the model.

where τ_t is serially uncorrelated and normally distributed with mean zero and standard deviation σ_τ.

Equation (3.1) to (3.3) describe the dynamic process of inflation and unemployment augmented with a control policy variable. In the aggregate demand curve, V_{t-1} performs the role of real interest rate as the one $i_t - E_t\pi_{t+1}$. V_{t-1} is also considered capturing the joint effect of monetary and fiscal policy.

A rational expectation is introduced in equation (3.1), which describes a dynamic process between inflation and unemployment, considered as a Phillips curve. In our model, we assume that some agents are fully rational, while the rest of them form their expectations adaptively.

$$E_t\pi_{t+1} = (1 - \tilde{\alpha}(1))E_{t-1}\pi_t + \tilde{\alpha}(L)\pi_{t-1} \tag{3.4}$$

where $\tilde{\alpha}(L)$ is a lag polynomial. By substituting (3.4) into (3.1), we can get a backwarding Phillips curve as follows:

$$\pi_t = \alpha(L)\pi_{t-1} - \theta(L)(u_{t-1} - u_{t-1}^N) + \varepsilon_t \tag{3.5}$$

where $\alpha(L) = \tilde{\alpha}(L)/\tilde{\alpha}(1)$ and $\theta_L = \tilde{\theta}_L/\tilde{\alpha}(1)$. Equation (3.5) captures the inflation fluctuation originating from a "cost-push" shock or the deviation from the natural rate.

The policy-controlled variable V_t affects deviation of unemployment from the natural rate each period. V_t is chosen by policymakers based on available information, that is, the inflation rate, the unemployment rate and their belief of last period.[2]

Policymakers are assumed to know the joint dynamic process of inflation and

[2]Note the natural rate of unemployment is unobserved but could be estimated by policymakers.

unemployment very well. They estimate the natural rate of unemployment and all coefficients in each period. Simultaneously, they respond to previous belief and adjust it according to available information.

We assume policymakers follow a loss function derived from a constrained minimization of the divergence in inflation and unemployment that also penalizes shifts in the policy variables.

$$E_{t-1}\{(1/2)(u_t - ku_t^N) + (b/2)\pi_t^2 + (1/2)(V_t - V_{t-1})^2\} \tag{3.6}$$

where $1 \geq k \geq 0$ and $b > 0$. $k = 1$ could be taken as an efficient criterion which penalizes any departures of u_t from u_t^N. In this case, the target becomes the natural rate of unemployment. $k < 1$ represent the possibility that the natural rate has intention to exceed the efficient level when there exists distortion. When $k = 0$, policymakers ignore the natural rate of unemployment. In this situation, unemployment target disappears and policymakers' time-consistency problem becomes most powerful.

Many authors solve the problem of loss function by minimizing policy variable such as Ireland (1999), Primiceri (2005), Reis (2003)) and among others. That is, when we have a loss function as

$$\min_{V_t} E_{t-1}\{(1/2)(u_t - ku_t^N) + (b/2)\pi_t^2 + (1/2)(V_t - V_{t-1})^2\} \tag{3.7}$$

subject to

$$\pi_t = \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} - \theta_1(u_{t-1} - u_{t-1}^N) - \theta_2(u_{t-2} - u_{t-2}^N) + \varepsilon_t \tag{3.8}$$

and

$$(u_t - u_t^N) = \rho_1(u_{t-1} - u_{t-1}^N) + \rho_2(u_{t-2} - u_{t-2}^N) + V_{t-1} + \eta_t \tag{3.9}$$

Since V_t doesn't enter directly into constraints, the functional form of V_t is hard to express accurately. At the same time, it is unrealistic for policymakers to react to many coefficients and variables in the solved form of V_t[3]. To distinguish from other research, we investigate policymakers' belief by firstly assuming they follow an autoregressive process as

$$V_t = \phi V_{t-1} + \xi_t \tag{3.10}$$

where ξ_t is assumed as an i.i.d $N(0, \sigma_\xi^2)$. A state-space model covering (3.3), (3.8), (3.9) and (3.10) could be written as

$$d_t = CF_t \tag{3.11}$$

$$F_t = AF_{t-1} + Be_t \tag{3.12}$$

where $d_t = [\pi_t, u_t]'$ and $F_t = [\pi_t, \pi_{t-1}, u_t, u_{t-1}, V_t, u_t^N, u_{t-1}^N]'$.

In this model, policymakers choose V_t based on previous belief V_{t-1}. The greater ϕ in equation (3.10) is, the more persistent policymakers' belief is. If policymakers only respond to previous belief, equation (3.10) approximates a random walk with ϕ close to the unity. ϕ is expected smaller when the policymakers synthesize more information into their reaction equation. In the Phillips curve (3.8), θ_1 and θ_2 determine whether policymakers perceive a very costly inflation-unemployment trade-off and whether they would like accept a higher unemployment for a limited relief from inflation.

An autoregressive process in policymakers' belief is obviously too simple to

[3] The optimized V_t in the equation (3.7) is policymakers' previous belief V_{t-1}, however, to further explore, V_t is actually affected by $(u_t - u_t^N) - \rho_1(u_{t-1} - u_{t-1}^N) - \rho_2(u_{t-2} - u_{t-2}^N) - \eta_t$.

model the complicated decision evolution. In addition to previous belief, there is lagged value of inflation and unemployment that could be observed by policymakers. Based on this information, we introduce five extensions in the following subsection.

3.2.2 Five Extensions

We firstly assume policymakers commit to a time-invariant Taylor rule. In this commitment, policymakers can only observe previous inflation and unemployment[4].

$$V_t = \phi V_{t-1} + (1-\phi)[\alpha \pi_{t-1} + \delta u_{t-1}] + \xi_t \tag{3.13}$$

Equation (3.13) represents policymakers can observe previous inflation and unemployment, and thus adjust current belief based on them. Meanwhile, they are also able to estimate the natural rate of unemployment corresponding to (3.8) and (3.9).

A more powerful commitment is a time-varying Taylor rule. Policymakers update their belief each period as follows:

$$V_t = \phi V_{t-1} + (1-\phi)[\alpha_{t-1} \pi_{t-1} + \delta_{t-1} u_{t-1}] + \xi_t \tag{3.14}$$

One hand, this commitment depicts a very quick response of policymakers each period. On the other hand, a quick adjustment may lead to unstable performance of monetary policy. Thus, except a time-varying commitment, we are also interested in observing policymakers responding to inflation and unemployment across different regimes. The introduction of regime switching is considered as follows:

$$V_t = \phi V_{t-1} + (1-\phi)[\alpha_{s_{t-1}} \pi_{t-1} + \delta_{s_{t-1}} u_{t-1}] + \xi_t \tag{3.15}$$

[4]Since V_t is an unobservable variable. To distinguish parameters and latent variables, we only let parameters on inflation or unemployment follow a time-varying approach or regime-switching.

where

$$\alpha_{s_{t-1}} = \alpha_0(1 - s_{t-1}) + \alpha_1 s_{t-1}$$

$$\delta_{s_{t-1}} = \delta_0(1 - s_{t-1}) + \delta_1 s_{t-1}$$

$s_{t-1} = 0$ or 1 represents regime 0 or 1 respectively. This means, under regime 1, policymakers' belief is controlled by α_1 and δ_1, while under regime 0, it is reacted by α_0 and δ_0.

One issue arises when we assume inflation and unemployment perform across same regimes. Figure 3.1 shows the evolution inflation and unemployment from 1954Q3 to 2007 Q4. Inflation obviously performs a leading indicator compared to unemployment. Such an stylized fact is captured by equation (3.16) and (3.17), which assume only inflation or unemlpoyment switches across regimes.

$$V_t = \phi V_{t-1} + (1 - \phi)[\alpha_{s_{t-1}}\pi_{t-1} + \delta u_{t-1}] + \xi_t \tag{3.16}$$

$$V_t = \phi V_{t-1} + (1 - \phi)[\alpha\pi_{t-1} + \delta_{s_{t-1}} u_{t-1}] + \xi_t \tag{3.17}$$

Estimation results of five extensions from equation (3.13) to (3.17) will be discussed in section 3.4.

3.3 Bayesian Estimation

This section describes the estimation method of Bayesian approaches. A state-space model covering equation (3.11) and (3.12) is estimated by a random walk Metropolis-Hastings algorithm. In equation (3.11) and (3.12), matrices A, B and C are functions of structural parameters Δ:

$$\Delta = [\alpha 1, \alpha 2, \theta 1, \theta 2, \rho 1, \rho 2, \phi, \alpha, \delta, \gamma, \sigma_{\varepsilon}, \sigma_{\eta}, \sigma_{\xi}, \sigma_{\tau}]'$$

Then the likelihood of the model can be obtained via an application of the Kalman filter

$$p(d_t \mid d_{t-1}, \Delta) = \prod_{t=1}^{T} p(d_t \mid d_{t-1}, \Delta), t = 1, 2, ..., T \tag{3.18}$$

To simulate posterior distribution of the parameter, we reparameterize Δ as Δ^*. We draw Δ^* from specified distribution [5] and calculate the acceptance rate as

$$min \left\{ \frac{p(\Delta^*) p(d_T \mid \Delta^*)}{p(\Delta^{(i-1)}) p(d_T \mid \Delta^{(i-1)})}, 1 \right\} \tag{3.19}$$

where $i = 1, 2, ..., N$ and N is the number of draws from the posterior distribution of Δ.

In the regime switching model, transition probabilities are also calculated by a Bayesian approach. We calculate posterior distribution as follows:

$$p(\Delta) p(s_T \mid \Delta) p(d \mid \Delta, s_T) \tag{3.20}$$

To solve $p(s_T \mid d_T, \Delta)$, we draw whole sequence from $p(s_T \mid d_T, \Delta)$ using backward (in time) sequential partition $s_T \sim p(s_T \mid d_T, \Delta)$. $p(s_T \mid d_T, \Delta)$ could be calculated as

$$p(s_T \mid d_T, \Delta) = p(s_T \mid d_T, \Delta) \times \prod_{t=1}^{T-1} p(s_t \mid s_{t+1}, \ldots, s_T, d_T, \Delta) \tag{3.21}$$

[5] The prior distribution is represented in Table 3.1 and 3.2

Given Markov property of unobservable variables, we have

$$p(s_t \mid s_{t+1}, \dots, s_T, d_T, \Delta) = p(s_t \mid s_{t+1}, d_t, \Delta) \tag{3.22}$$

Using Bayes' Theorem, the transition probability could be calculated as

$$p(s_t = i \mid s_{t+1} = j, d_t, \Delta) \tag{3.23}$$

$$= \frac{p_{ij} \times \pi_{i,t|t}}{\pi_{j,t+1|t}} \tag{3.24}$$

where $p_{ij} = p(s_{t+1} = j \mid s_t = i, \Delta)$ and $\pi_{i,t|t} = p(s_t = i \mid d_t, \Delta)$. The filtered probability is calculated by iteration as

$$\pi_{j,t+1|t} = p(s_{t+1} = j \mid d_t, \Delta) \tag{3.25}$$

$$= \sum_{i=1}^{m} p(s_{t+1} = j, s_t = i \mid d_t, \Delta) \tag{3.26}$$

$$= \sum_{i=1}^{m} p(s_{t+1} = j \mid s_t = i, d_t, \Delta) \times p(s_t = i \mid d_t, \Delta) \tag{3.27}$$

$$= \sum_{i=1}^{m} p_{ij} \times \pi_{i,t|t} \tag{3.28}$$

and we update predicted probability by Bayes Theorem as follows

$$\pi_{j,t+1|t+1} = p(s_{t+1} = j \mid d_{t+1}, \Delta) \tag{3.29}$$

$$= p(s_{t+1} = j \mid d_t, d_{t+1}, \Delta) \tag{3.30}$$

$$= \frac{\pi_{j,t+1|t} \times p(d_{t+1} \mid s_{t+1=j,\Delta})}{p(d_{t+1} \mid d_t, \Delta)} \tag{3.31}$$

Time-varying parameter(TVP) model is similar to a Markov switching (MS) model but unobservable variable is a continuous variable. The posterior simulation of time-varying model is conceptually similar to that of Markov switching model: we draw unobservable variables conditional on data and parameters, and draw parameters given data and unobservable variables.

Since we have

$$V_t = \phi V_{t-1} + (1-\phi)[\alpha_{t-1}\pi_{t-1} + \delta_{t-1}u_{t-1}] + \xi_t$$

We assume varying parameters follow a random walk with error terms

$$\alpha_t = \alpha_{t-1} + \varepsilon_t^{\alpha} \tag{3.32}$$

$$\delta_t = \delta_{t-1} + \varepsilon_t^{\delta} \tag{3.33}$$

where $\varepsilon_t^{\alpha} \sim N(0, H^{\alpha})$ and $\varepsilon_t^{\delta} \sim N(0, H^{\delta})$.

Let $\tilde{\beta} = [\alpha, \delta]'$, we follow Carter and R.Hohn (1994) and factorize the full conditional posterior distribution of the state vectors belonging to the sample period as:

$$p(\tilde{\beta_T} \mid d_T, \Delta) = p(\tilde{\beta_T} \mid d_T, \Delta)\prod_{t=1}^{T}(\tilde{\beta_t} \mid d_T, \Delta) \tag{3.34}$$

By iteration, we have

$$\tilde{\beta}_{t+1} = \tilde{\beta}_{t+1|t} + e_{t+1|t} \tag{3.35}$$

$$= \tilde{\beta}_{t+1|t} + e_{t|t} \tag{3.36}$$

where $e_{t|t}$ is simulated each time based on extracted variables by Kalman filter and the acceptance rate of equation (3.19).

3.4 Empirical Results

The section represents empirical results of our model. Quarterly changes in seasonally adjusted GDP deflator, converted as $400 * log(P_t/P_{t-1})$, serve to measure inflation. The seasonally adjusted civilian unemployment rate averaged over each three months is taken as quarterly unemployment rate here. The whole sample is from 1954:Q3 to 2007:Q4.

In this section, we compare and investigate different assumptions of policymakers' misperception, estimated by a Bayesian approach. Four main conclusions are obtained: first, policymakers' misperception is very persistent, even though it commits to a Taylor rule. Second, policymakers' misperception was observed highest around 1979. The run-up of U.S. inflation around 1980 is mostly attributed to policymakers' misperception while the peak in the end of 1974 is possibly a result from large non-policy shocks. Severe oil shocks could be considered as one of main reasons resulting in fierce inflation fluctuation in 1974. Third, models with commitment dominate models without commitment, especially in periods of large oscillations in inflation. Over the whole sample, the time-invariant commitment can reduce policymakers' misperception highest to 8.5%. The time-varying commitment can reduce much more when there is a high target on unemployment. Policymakers are expected to keep consistent with a commitment to the Taylor rule, especially in periods of large inflation fluctuation. Last, we subtract misperception from nominal interest rate. The left component is quite similar to Survey of Professional Forecasters(SPF) that underestimates inflation in the 1970s while overestimates inflation in the 1980s and 1990s. The estimation of policymakers' misperception in our model, captures the characteristics of the real interest rate.

Table 3.1 and 3.2 represents Bayesian estimations of the model without commitment and with time-invariant commitment[6]. As observed, the estimated natural rate of unemployment and policymakers' misperception are very persistent since ϕ and γ are quite close to the unity. γ in the model without commitment is slightly greater than that of time-invariant commitment. Other estimated parameters are similar although difference is observed. For both models, α_1 is slightly greater than α_2 and θ_1, which demonstrates inflation responds more to the last period inflation instead of unemployment. However, a large estimate of θ_2 reveals that inflation was greatly influenced by unemployment in the last two period. Such a observation confirms that inflation performs as a leading indicator in U.S. economy. In the IS curve, the dynamics of unemployment originates more from the previous period since we have a greater estimate of ρ_1. Moreover, in our estimation, demand shock in the Phillips curve dominates other shocks.

Table 3.3 reports average loss over the whole sample from 1954:Q3 to 2007:Q4. We compute different average loss given different b and k. When k is as small as 0.5, there is no big difference between models without commitment and with commitment. If policymakers choose to put less weight on the target of unemployment, it won't hurt them much without commitment. However, in previous literature, k was estimated more than 0.8 (for instance, Barro and Gordon (1983), Primiceri (2005)). When k is greater than 0.8, it means a slight departure of u_t from u_t^N will be penalized. As we can observe in table 3.3, a commitment to a Taylor rule will help policymakers reduce their loss effectively. Models with time-varying commitment as well as Markov switching commitment perform the best. Compared to the model without commitment, model with Markov switching on inflation will decrease average loss over 25%. Table 3.6 summarizes maximum likelihood for all proposed models. Among them, model with

[6]More results are available upon request.

time-varying commitment returns the largest maximum likelihood.

We also investigate estimation in different subsamples. Results are represented in table 3.4 and 3.5 respectively. Table 3.4 represents results of Pre-Volcker period from 1954:Q3 to 1979:Q4 whereas table 3.5 represents results of Post-Volcker period from 1980:Q1 to 2007:Q4. Average loss in table 3.5 in comparison to table 3.4 is smaller in the Post-Volcker period. Such a result comes not only from low inflation rate in the Post-Volcker period but also from less deviation from natural rate of unemployment. Models with time-varying commitment and Markov switching commitment still dominate others when k is over 0.8 in both subsamples. Similarly, we can observe when policymakers respond flexibly to inflation, the average loss will be reduced more than 35% during Post-Volcker period. When k is equal to 0.5, that is, when policymakers have a low target of unemployment, it is difficult for model with commitment to beat model without commitment.

Figure 3.2 and 3.3 depict the misperception of policymakers in different models in comparison with inflation and unemployment. During periods of large oscillations in inflation and unemployment, misperceptions from models with time-varying commitment and Markov switching commitment are obviously smaller than others. When inflation rate is low, there is no big difference between different models. We also note that during the "Great Inflation" period, misperception around 1979 is greater than that around 1974. Obviously, policymakers' misperception could not completely interpret the high peak in 1974. The run-up of U.S inflation around 1979 is mostly attributed to policymakers' misperception while the peak in the end of 1974 is possibly a result from large non-policy shocks such as oil shocks.

Figure 3.4 analyze misperception in comparison with interest rate. The relationship between nominal interest rate i_t, inflation π_t and real interest rate r_t is as follows:

$$i_t - \pi_t = r_t$$

Then inflation could be represented as

$$\pi_t = i_t - r_t$$

The effective federal funds rate averaged over each three months is taken as quarterly nominal interest rate i_t and we model inflation as $i_t - V_t$, that is, interest rate subtracts misperception. The modeled inflation with interest rate is plotted in figure 3.4 and compared with SPF. As we can observe, in a model without considering interest rate, the modeled inflation is very similar to SPF, which underestimates inflation around 1974, catches the peak in the 1979 and overestimates in the 1980s. The predicted misperception in our model actually performs the role of real interest rate.

Figure 3.5 shows a model fit of inflation and unemployment. Our model provides a better fit for unemployment.

3.5 Concluding Remarks

There is always a trade-off between inflation and unemployment faced by policymakers. They expect to have a stable economy without violent fluctuation on inflation and unemployment in the world. During their reaction to inflation and unemployment, it is unavoidable for policymakers to make mistakes. Thus, how to avoid serious mistakes is proceeded to the agenda. Our models demonstrate that policymakers should commit to a Taylor rule, which in a large extent, can prevent them from serious mistakes. In addition, our results have important policy implications as they indicate how and when it is appropriate for policymakers to choose a commitment in their reaction to inflation and unemployment. Our models indicate that a flexible or more activist policy is more appropriate when there is high unemployment target while a more consistent policy is more suitable under low unemployment target. The distinguished interpretation on two large rises in inflation in 1974 and 1979 also contributes to the investigation of sources to the "Great Inflation".

Table 3.1: Time invariant model without the commitment

Coefficients	Initial Value	Prior			Posterior		
		Density	Mean	std	Mean	std	[25%, 95%]
α_1	0.7	B	0.4	0.2	0.1211	0.0253	[0.08, 0.190]
α_2	0.5	B	0.3	0.2	0.0862	0.0139	[0.06, 0.11]
θ_1	0.4	B	0.1	0.05	0.0590	0.0087	[0.05, 0.09]
θ_2	-0.2	B	0.3	0.2	0.5947	0.0319	[0.53, 0.68]
ρ_1	0.7	B	0.6	0.2	0.3564	0.0284	[0.31, 0.41]
ρ_2	-0.5	B	0.3	0.2	0.1088	0.0186	[0.05, 0.13]
ϕ	0.8	B	0.9	0.2	0.9814	0.0026	[0.97, 0.99]
γ	0.5	B	0.9	0.2	0.9996	0.0001	[0.9993, 0.9997]
σ_ϵ	0.8	G	0.15	0.15	1.2126	0.0400	[1.17, 1.28]
σ_η	0.8	G	0.15	0.15	0.0282	0.0274	[0.015, 0.12]
σ_υ	0.8	G	0.15	0.15	0.2339	0.0226	[0.21, 0.29]
σ_τ	0.8	G	0.15	0.15	0.2320	0.0155	[0.21, 0.26]

Prior densities and posterior estimates: Time invariant model without the commitment.

Table 3.2: Time invariant model under the commitment

Coefficients	Initial Value	Prior			Posterior		
		Density	Mean	std	Mean	std	[25%95%]
α_1	0.7	B	0.4	0.2	0.1106	0.0319	[0.050.17]
α_2	0.5	B	0.3	0.2	0.0969	0.0283	[0.050.15]
θ_1	0.4	B	0.1	0.05	0.0835	0.0222	[0.050.14]
θ_2	-0.2	B	0.3	0.2	0.6046	0.0780	[0.450.75]
ρ_1	0.7	B	0.6	0.2	0.4479	0.0870	[0.300.60]
ρ_2	-0.5	B	0.3	0.2	0.1222	0.0537	[0.060.23]
ϕ	0.8	B	0.9	0.2	0.9856	0.0048	[0.970.99]
α	1.5	N	1.5	0.25	1.0978	0.1951	[0.751.56]
δ	0.7	N	0.5	0.25	-0.6229	0.1446	$[-0.98 - 0.32]$
γ	0.5	B	0.9	0.2	0.9994	0.0002	[0.9990.9995]
σ_ϵ	0.8	G	0.15	0.15	1.1674	0.0661	[1.061.30]
σ_η	0.8	G	0.15	0.15	0.0318	0.0085	[0.0180.05]
σ_υ	0.8	G	0.15	0.15	0.1616	0.0321	[0.110.23]
σ_τ	0.8	G	0.15	0.15	0.2660	0.0264	[0.210.31]

Prior densities and posterior estimates: Time invariant model under the commitment.

Table 3.3: Mean Value of Loss Function

	k=0.5			k=0.8			k=1		
	b=0.5	b=1	b=2	b=0.5	b=1	b=2	b=0.5	b=1	b=2
No commitment	5.4157	9.8996	18.868	7.7839	12.268	21.236	14.5378	19.0217	27.9896
TIV commitment	5.3856	9.8696	18.837	7.1695	11.653	20.621	13.2908	17.7747	26.7426
TVP commitment	5.5358	10.0197	18.9876	6.5634	11.0473	20.0152	11.8270	16.3110	25.2788
MS commitment	5.3461	9.83	18.7979	7.1706	11.6545	20.6224	13.344	17.828	26.796
MS on inflation parameter	5.7574	10.241	19.209	5.7406	10.225	19.192	9.8164	14.3	23.268
MS on unemployment parameter	5.3435	9.8274	18.795	7.4252	11.909	20.877	13.879	18.363	27.331
Relative TIV	0.9944	0.9970	0.9984	0.9211	0.9499	0.9711	0.9142	0.9344	0.9554
Relative TVP	1.0222	1.0121	1.0064	0.8432	0.9005	0.9425	0.8135	0.8575	0.9031
Relative MS	0.9871	0.9930	0.9963	0.9212	0.9500	0.9711	0.9179	0.9372	0.9574
Relative MS on inflation	1.0631	1.0345	1.0181	0.7375	0.8335	0.9038	0.6752	0.7518	0.8313
Relative MS on unemployment	0.9867	0.9927	0.9962	0.9539	0.9708	0.9831	0.9547	0.9574	0.9765

Note: Here we use the loss function as follows: $Loss = E_{t-1}\{(1/2)(u_t - ku_t^n)^2 + (b/2)\pi_t^2 + (1/2)(V_t - V_{t-1})^2\}$

Relatived TIV is calculated as (TIV commitment/no commitment).

Table 3.4: Mean Value of Loss Function: Pre-Volcker subsample

	k=0.5			k=0.8			k=1		
	b=0.5	b=1	b=2	b=0.5	b=1	b=2	b=0.5	b=1	b=2
No commitment	6.9755	12.951	24.903	9.7527	15.729	27.68	16.381	22.357	34.309
TIV commitment	6.7762	12.752	24.704	9.2523	15.228	27.18	15.601	21.577	33.529
TVP commitment	6.8747	12.851	24.802	8.5688	14.545	26.496	14.046	20.022	31.973
MS commitment	6.7855	12.761	24.713	9.1481	15.124	27.076	15.372	21.348	33.299
MS on inflation parameter	7.0297	13.006	24.957	7.5956	13.571	25.523	11.811	17.787	29.739
MS on unemployment parameter	6.7996	12.775	24.727	9.4264	15.402	27.354	15.933	21.909	33.861
Relative TIV	0.9714	0.9846	0.9920	0.9487	0.9681	0.9819	0.9524	0.9651	0.9773
Relative TVP	0.9855	0.9923	0.9959	0.8786	0.9247	0.9572	0.8575	0.8956	0.9319
Relative MS	0.9728	0.9853	0.9924	0.9380	0.9615	0.9782	0.9384	0.9549	0.9706
Relative MS on inflation	1.0078	1.0042	1.0022	0.7788	0.8628	0.9221	0.7210	0.7956	0.8668
Relative MS on unemployment	0.9748	0.9864	0.9929	0.9665	0.9792	0.9882	0.9727	0.9800	0.9869

Note: Here we use the loss function as follows: $Loss = E_{t-1}\{(1/2)(u_t - ku_t^n)^2 + (b/2)\pi_t^2 + (1/2)(V_t - V_{t-1})^2\}$

Relatived TIV is calculated as (TIV commitment/no commitment).

Pre-Volcker period: 1954:Q3-1979:Q4.

Table 3.5: Mean Value of Loss Function: Post-Volcker subsample

	k=0.5			k=0.8			k=1		
	b=0.5	b=1	b=2	b=0.5	b=1	b=2	b=0.5	b=1	b=2
No commitment	3.8429	6.8103	12.745	5.7387	8.7061	14.641	12.485	15.452	21.387
TIV commitment	4.1078	7.2208	13.4468	5.2555	8.3686	14.5946	11.1675	14.2805	20.5065
TVP commitment	4.1701	7.1375	13.072	4.5149	7.4823	13.417	9.4875	12.455	18.39
MS commitment	3.8859	6.8533	12.788	5.1328	8.1002	14.035	11.151	14.118	20.053
MS on inflation parameter	4.4531	7.4205	13.355	3.8542	6.8216	12.756	7.7318	10.699	16.634
MS on unemployment parameter	3.8676	6.835	12.77	5.3589	8.3263	14.261	11.649	14.616	20.551
Relative TIV	1.0333	1.0188	1.0100	0.8774	0.9192	0.9519	0.8681	0.8934	0.9230
Relative TVP	1.0851	1.0480	1.0257	0.786	0.8594	0.9164	0.7599	0.8060	0.8599
Relative MS	1.0112	1.0063	1.0034	0.8944	0.9304	0.9586	0.8932	0.9137	0.9376
Relative MS on inflation	1.1588	1.0896	1.0479	0.6716	0.7835	0.8713	0.6193	0.6924	0.7778
Relative MS on unemployment	1.0064	1.0036	1.0020	0.9338	0.9564	0.9740	0.9330	0.9459	0.9609

Note: Here we use the loss function as follows: $Loss = E_{t-1}\{(1/2)(u_t - ku_t^n)^2 + (b/2)\pi_t^2 + (1/2)(V_t - V_{t-1})^2\}$

Relatived TIV is calculated as (TIV commitment/no commitment).

Post-Volcker period: 1980:Q1-2007:Q4.

Table 3.6: Maximum Likelihood in different models

Model	Maximum Likelihood
No commitment	-704.27
TIV commitment	-654.78
TVP commitment	-496.24
Markov Switching commitment	-672.31
Markov Switching on Inflation	-541.39
Markov Switching on Unemployment	-520.79

Figure 3.1: Inflation and Unemployment

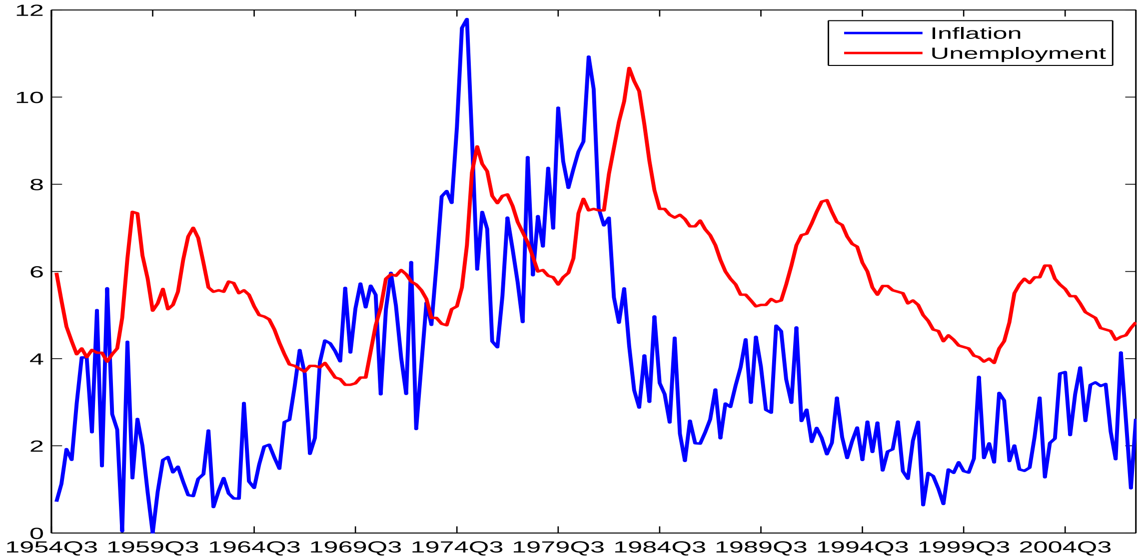
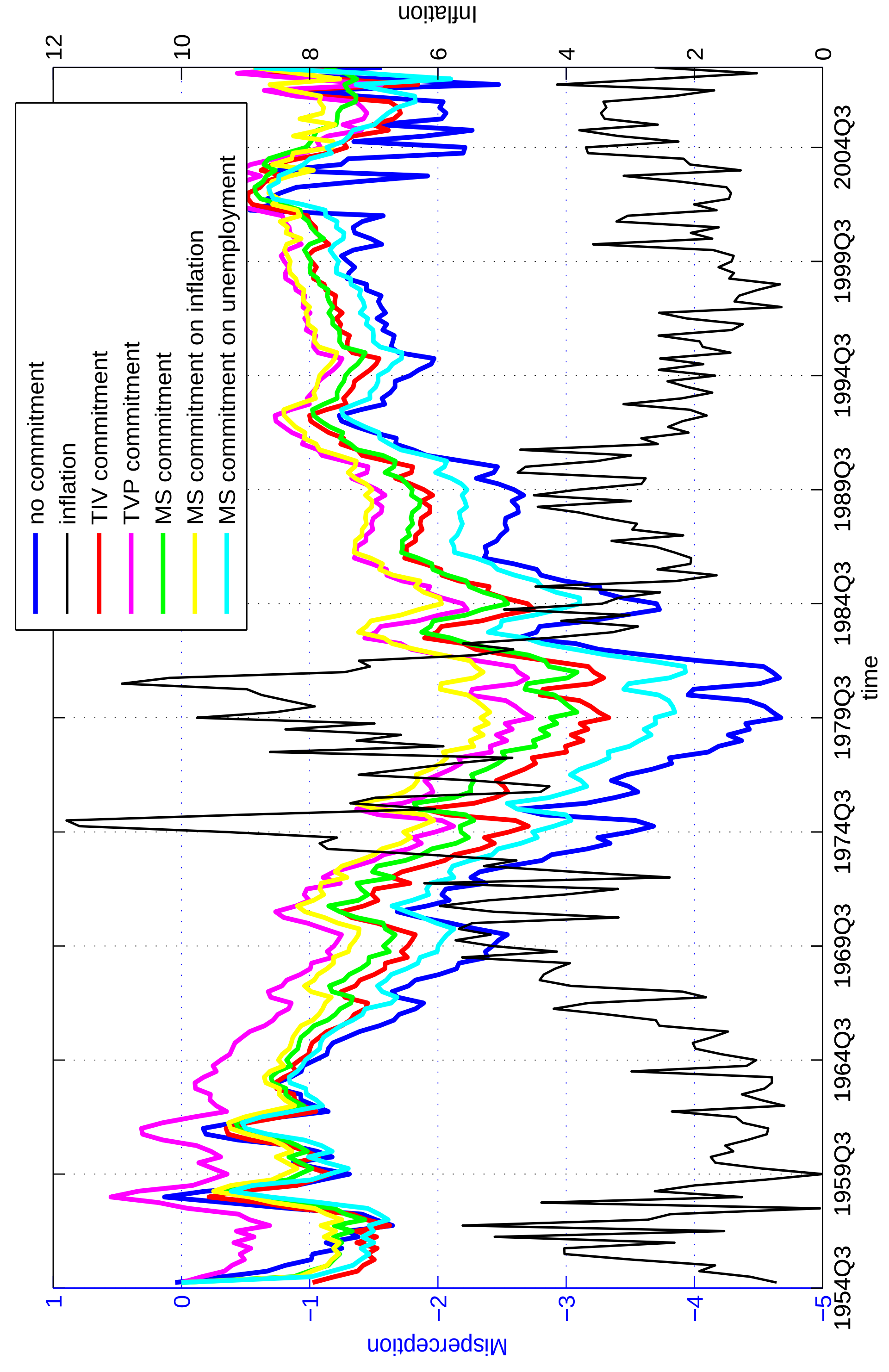


Figure 3.2: Misperceptions in different cases in comparison with inflation



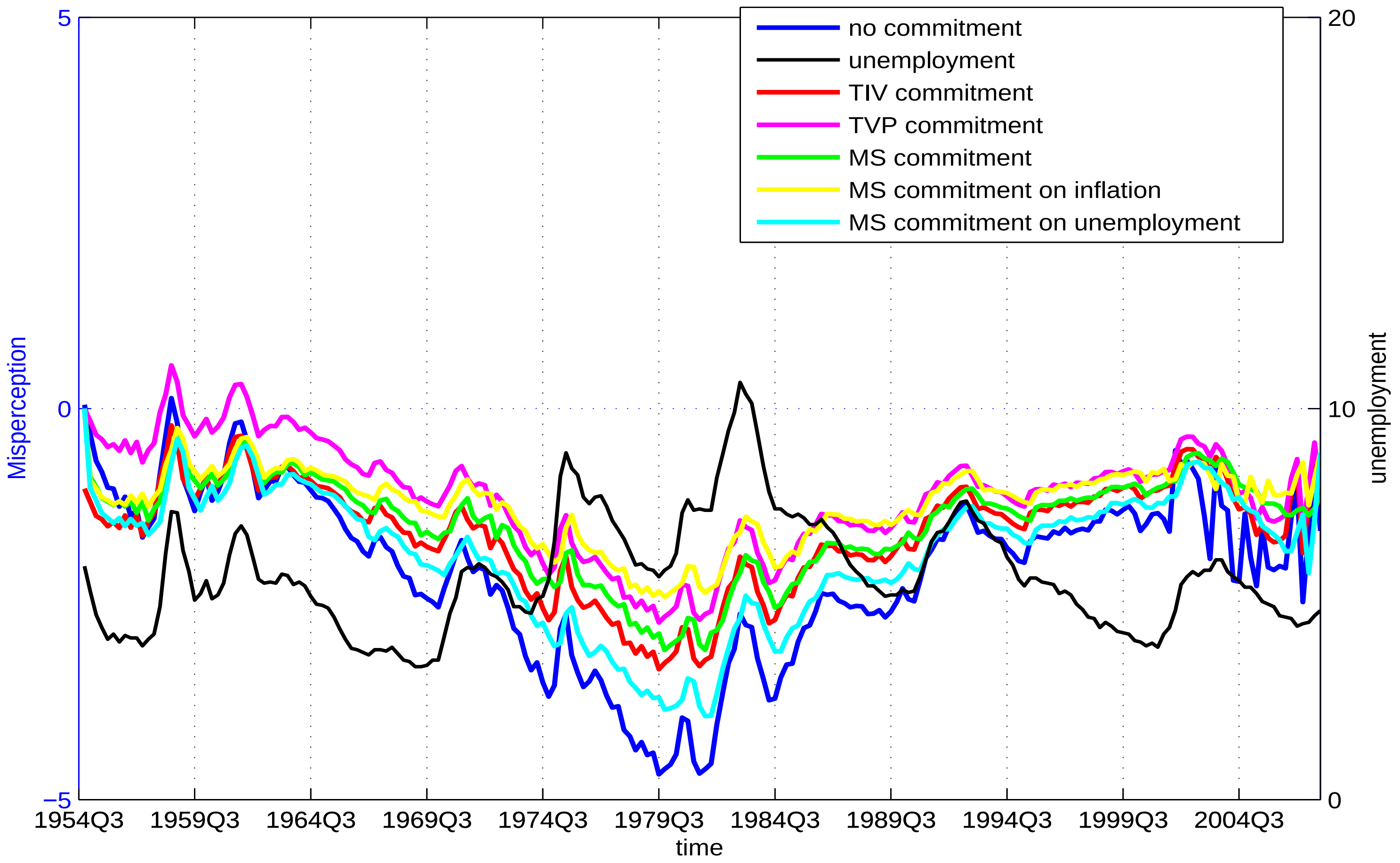
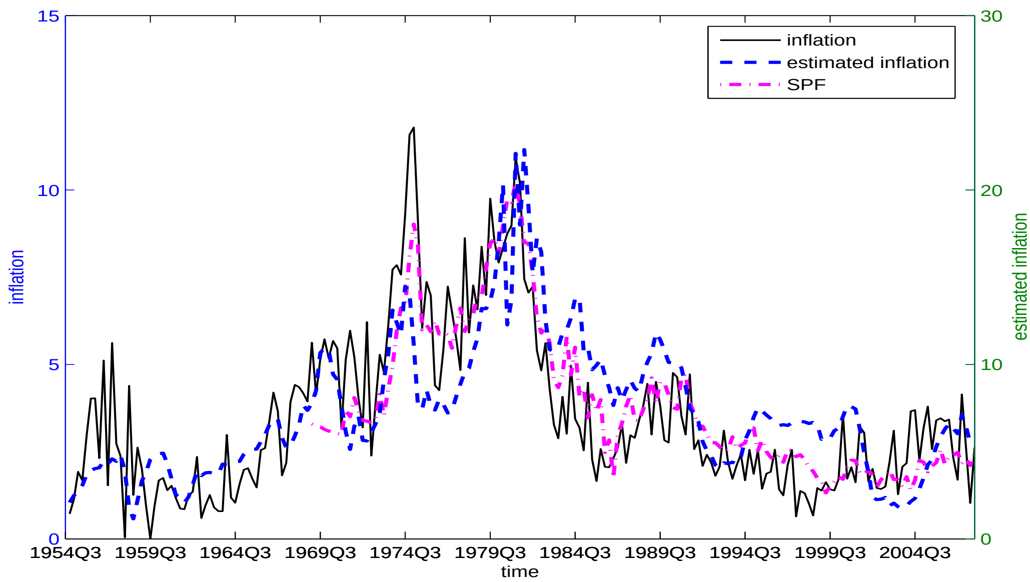


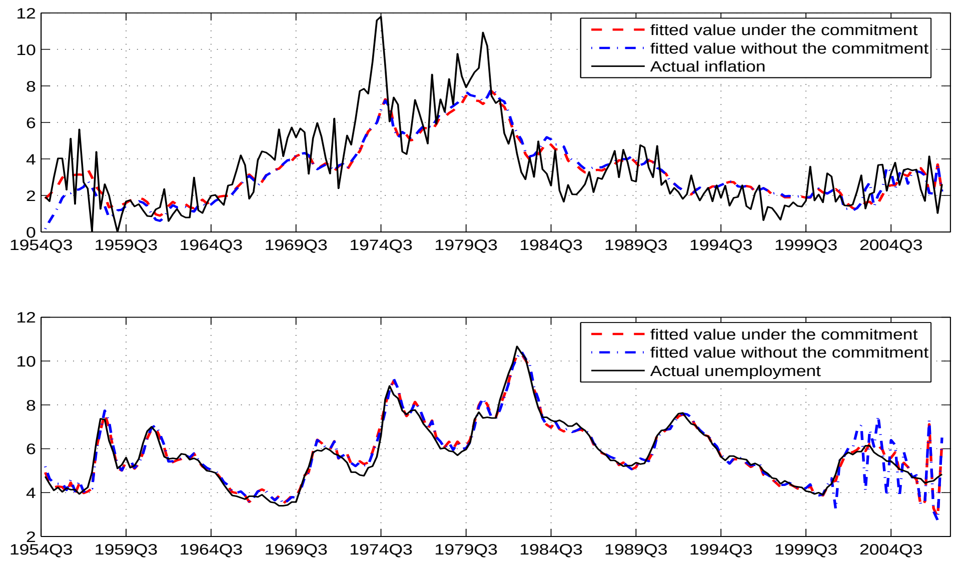
Figure 3.3: Misperceptions in different cases in comparison with unemployment

Figure 3.4: Inflation, Interest Rate and SPF



Note: Estimated Inflation = Nominal Interest Rate - Misperception

Figure 3.5: Fitted and Actual Value



Chapter 4

Adaptive Expectations and Inflation Persistence

4.1 Introduction

In the literature, the most common way to relax rational expectations is to assume that agents learn about parameters using constant gain, least squares learning and then predict future variables with estimated parameters. In particular, Milani (2007) estimated a DSGE model by assuming that agents behave as econometricians to forecast future inflation and output gaps with the reduced form of an economic model, and learn the reduced form parameters over time. This paper deviates from this model of learning by employing the simple adaptive expectations proposed by Cagan (1956), in which expectations evolve by correcting errors in the expectations themselves. We propose a small-scale DSGE model with a simple version of adaptive expectations (see, e.g., Cagan, 1956) to evaluate the goodness of fitness and forecasting performance. Adaptive expectations allow us not only to estimate the DSGE model directly without solving rational expectation problems, but also to estimate inflation and output expectations, since expectations can be treated as state variables in our model. Estimation is implemented by the maximum likelihood estimation (MLE).

Our results indicate that estimated expectations outperform VAR forecasts in forecasting future inflation and output gaps. Furthermore, estimated expectations appear to perform very well even though we assume that agents formulate their expectations based on their imperfect and naive knowledge about the true economy. We show that replacing rational expectations with adaptive expectations leads to an increase in likelihood, supporting our learning over the rational expectations model in fitting the U.S. data.

This paper also investigates whether adaptive expectations could replace the role of lagged inflation or output gap with respect to persistence. The purely forward-looking new Keynesian models are well known for their failure to generate persistence in the key macroeconomic variables. For example, Gali and Gertler (1999) and Christiano, Eichenbaum, and Evans (2005) incorporate lagged inflation into the Phillips curve to generate persistence as well as to improve the goodness-of-fit of inflation dynamics. To incorporate a lagged inflation term into the Phillips curve, they assume that a fraction of firms adjust their prices by automatic indexation to the past period's inflation rate. For output gap persistence, habit in consumption, as proposed by Fuhrer and Moore (1995), is often used to incorporate a lagged output gap term into the IS curve in a model economy. However, it is still controversial whether lagged inflation and output gap terms are necessary to generate persistence in the Keynesian models. In particular, Sbordone (2005) favors the purely forward-looking Phillips curve in terms of its goodness-of-fit. Chari, Kehoe, and McGrattan (2009) doubt the hybrid new Keynesian Phillips curve due to its failure to provide micro-foundations. Milani (2007) examines this issue within a DSGE model allowing agents to learn about parameters in the reduced form of a

DSGE model, and concludes that indexation and habit formation are redundant. We investigate this issue and find that adaptive expectations could limit the role of lagged inflation in fitting inflation dynamics. But a lagged output gap term turns out to be necessary even after the introduction of adaptive learning.

Simulations show that our model generates plausible responses to demand, cost-push and interest rate shocks. We introduce expectations shocks to the model, which can affect inflation and the output gap. One of the important features in the survey of inflation expectations reflecting people's beliefs is that expectations lag behind variables. Our simulation results demonstrate that the lag-behind property can be generated by adaptive learning.

The contents of the paper are as follows. Section 4.2 investigates the hybrid new Keynesian Phillips curve under adaptive expectations. Section 4.3 sets up a DSGE model under learning and examines the issues of inflation and output persistence. In Section 4.4, we evaluate the goodness-of-fit of DSGE models under adaptive expectations and rational expectations. Section 4.4 also presents estimation results of the DSGE model with respect to inflation and output persistence. We evaluate forecasting performance in terms of the root mean squared error. Section 4.5 provides impulse response functions. The last section concludes the paper.

4.2 Inflation Expectations and Inflation Persistence

The new Keynesian Phillips curve (NKPC) relates current inflation to both the output gap and inflation expectations in a profit-maximizing framework featuring staggered price contracts and forward-looking behavior in price-setting. However, the NKPC is often criticized due to its inability to generate inflation persistence, and therefore costly disinflation. In response to this challenge, Gali and Gertler (1999) and Christiano, Eichenbaum, and Evans (2005) try to correct this failure of the NKPC by adding a lagged inflation term to the model. They incorporate lagged inflation into the NKPC by assuming that a fraction of firms adjust their prices by rule-of-thumb or automatic indexation to the past period's inflation rate. The hybrid NKPC, derived by assuming that there are two types of agents, forward- and backward-looking agents, is of the form

$$\pi_t = \mu_\pi E_t \pi_{t+1} + (1 - \mu_\pi)\pi_{t-1} + \lambda y_t + \varepsilon_t^\pi, \tag{4.1}$$

where π_t and y_t denote inflation and the output gap, and ε_t^π represents the cost-push shock, which has heteroskedasticity, $\varepsilon_t^\pi | \Omega_{t-1} \sim N(0, h_t^2)$. We model time-varying variance with GARCH(1,1), of the form

$$h_t = \alpha_1 + \alpha_2 (\varepsilon_{t-1}^\pi)^2 + \alpha_3 h_{t-1}. \tag{4.2}$$

The use of lagged inflation to generate inflation persistence is still controversial in terms of its goodness-of-fit and micro-foundations. After investigating the closed-form solution of the purely forward-looking Phillips curve, Sbordone (2005) concludes that lagged inflation is not necessary for the goodness-of-fit. Gali and Gertler (1999) report that lagged inflation plays a limited role in accounting for inflation dynamics. In addition, Woodford (2007), Rudd and Whelan (2007), Cogley and Sbordone (2008), Benati (2008), and Chari, Kehoe, and McGrattan (2009) consider the addition of lagged inflation to the Phillips curve by indexation or by rule-of-thumb to be an ad hoc solution. The source of the inflation persistence remains open to question. This paper explores whether a simple version of adaptive expectations is able to replace the role of lagged inflation in explaining inflation dynamics. When we set $E_t\pi_{t+1} \equiv \beta_t^\pi$ for the state space model, then the adaptive learning is of the form

$$\beta_t^\pi = \beta_{t-1}^\pi + \tilde{\alpha}_\pi(\pi_{t-1} - \beta_{t-1}^\pi) + \omega_t^\pi, \tag{4.3}$$

where $\omega_t^\pi \sim N(0, \sigma_{\omega\pi}^2)$. We denote by ω_t^π the expectation shock. In correcting forecast errors to formulate inflation expectations, we use lagged inflation rather than current inflation in this section, as current inflation may not be observed (see, e.g., Sargent, Williams, and Zha (2009)). On the other hand, one may object to using lagged inflation and prefer current inflation in correcting errors (see, e.g., Evans and Ramey (2006)). For this reason, we also adopt current inflation to correct forecast errors in the next section.

We construct a state-space model, because the path of inflation expectations is not observed. One of the convenient features of adaptive expectations is that they allow us to estimate both inflation expectations and model parameters directly, by using the Kalman filter. In estimating the model, we assume that ε_t^π follows GARCH(1,1). The state space model can be written as follows:

$$\pi_t = [\mu_\pi \quad 1] \begin{bmatrix} \beta_t^\pi \\ \varepsilon_t^\pi \end{bmatrix} + (1 - \mu_\pi)\pi_{t-1} + \lambda y_t \tag{4.4}$$

$$\begin{bmatrix} \beta_t^\pi \\ \varepsilon_t^\pi \end{bmatrix} = \begin{bmatrix} 1 - \tilde{\alpha}_\pi & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_{t-1}^\pi \\ \varepsilon_{t-1}^\pi \end{bmatrix} + \tilde{\alpha}_\pi \begin{bmatrix} \pi_{t-1} \\ 0 \end{bmatrix} + \begin{bmatrix} \omega_t^\pi \\ \varepsilon_t^\pi. \end{bmatrix} \tag{4.5}$$

When we assume that agents formulate their expectations using adaptive learning rather than rational expectations, we can estimate the hybrid NKPC and the path of inflation expectations using the state space model in Equations (4.4) and (4.5). For the estimation, we use the Congressional Budget Office (CBO) output gap measure. The output gap is defined as the percentage deviation of real GDP from potential output as measured by the Congressional Budget Office. The implicit GDP deflator is adopted to calculate the inflation rate.

Table 4.1 displays the maximum likelihood estimation results. The coefficient associated with inflation expectations is estimated to be around 1. This result suggests that adaptive expectations make the role of lagged inflation unnecessary in explaining inflation dynamics. The finding thus indicates that inflation persistence may result from adaptive expectations rather than lagged inflation. The value of $\tilde{\alpha}_\pi$ is estimated to be 0.60, with standard error, 0.06, suggesting that the learning model is supported by the data. The estimates of α_2 and α_3 are also significantly different from zero, supporting heteroskedasticity in the error term of the Phillips curve.

Figure 4.1 displays inflation expectations from the Survey of Professional Forecasters (SPF), realized one-quarter-ahead inflation and estimated inflation expectations from the state space model. Estimated inflation expectations seem to provide a good approximation of people's beliefs about future inflation that are observed from the SPF. Estimated inflation expectations capture the well-known feature that the observed inflation expectations "lag behind" inflation. This behavior is clearly observed in the 1970s and the early 1980s, when inflation was changing drastically. The second figure presents the heteroskedasticity, which is measured by GARCH(1,1). The uncertainty of inflation started to increase in the early 1970s and was staying continuously high by the early 1980s.

4.3 A DSGE model under adaptive expectations

This section introduces a small-scale DSGE model under adaptive expectations. We employ Equation (4.1) for the Phillips curve, but assume that the cost-push shock follows a first order autoregressive process, $\varepsilon_t^\pi = \delta_\pi \varepsilon_{t-1}^\pi + v_t^\pi$, as in the literature, and that v_t^π is $N(0, \sigma_\pi^2)$. We employ the same adaptive expectations shown in Equation (4.3), but we replace the lagged inflation with current inflation, assuming that economic agents adjust their expectations using current inflation rather than lagged inflation. We employ the IS curve of the form

$$y_t = \mu_y E_t y_{t+1} + (1 - \mu_y) y_{t-1} - \sigma(i_t - E_t \pi_{t+1}) + \varepsilon_t^y, \tag{4.6}$$

where i_t denotes the nominal interest rate, ε_t^y represents the demand shock, and is assumed to follow an AR(1) process, and $\varepsilon_t^y = \delta_y \varepsilon_{t-1}^y + v_t^y$. v_t^y is assumed to be $N(0, \sigma_y^2)$. The baseline IS curve, as a special case ($\mu_y = 1$) of Equation (4.6) derived from a utility-maximization problem, is able to generate a "jump" in the output gap in response to a change in the expected future output gap while output gap measures are persistent. As addressed by Fuhrer (2000), this problem can be cured by the inclusion of habit formation in consumption. It plays an important role in incorporating a lagged output gap term in the IS curve as a source of output persistence. Empirical studies of DSGE models under rational expectations have often reported significant degrees of habit formation in consumption (e.g., Smets and Wouters (2007) and Christiano, Eichenbaum, and Evans (2005)). On the other hand, Milani (2007) provides empirical evidence that allowing agents to learn about parameters makes habit formation redundant. In this respect, we reinvestigate whether Cagan's adaptive expectations mechanism is able to replace the role of habit formation in consumption. Adaptive expectations for the output gap take the form

$$\beta_t^y = \beta_{t-1}^y + \tilde{\alpha}_y(y_t - \beta_{t-1}^y) + \omega_t^y, \tag{4.7}$$

where $E_t y_{t+1} \equiv \beta_t^y$. Equations (4.3) and (4.7) impose dynamics in expectations and therefore allow us to treat expectations as state variables, so we can directly estimate expectations without solving rational expectation problems. We assume that ω_t^π is $N(0, \sigma_{\omega\pi}^2)$.

We adopt a monetary policy rule of the form,

$$i_t = \rho i_{t-1} + (1-\rho)(\alpha_\pi \pi_t + \alpha_y y_t) + \varepsilon_t^i, \tag{4.8}$$

where the interest rate shock follows an AR(1) process, $\varepsilon_t^i = \delta_i \varepsilon_{t-1}^i + v_t^i$, and $v_t^i \sim N(0, \sigma_i^2)$. The coefficient associated with the lagged nominal interest rate measures the degree of smoothing in monetary policy. The Taylor rule shows that the nominal interest rate adjusts in response to economic activity and inflation.

The state-space form can be written as

$$d_t = CS_t \tag{4.9}$$

$$S_t = AS_{t-1} + Be_t, \tag{4.10}$$

where $d_t = [\pi_t, y_t, i_t]'$ and $s_t = [\pi_t, y_t, i_t, E_t\pi_{t+1}, E_t y_{t+1}, \varepsilon_t^\pi, \varepsilon_t^y, \varepsilon_t^i]'$. The parameters of the model are collected in the parameter vector Θ_L={λ, μ_π, μ_y, σ, ρ, α_π, α_y, $\tilde{\alpha_\pi}$, $\tilde{\alpha_y}$, $\sigma_{\omega\pi}$, $\sigma_{\omega y}$, σ_π, σ_y, σ_i, δ_π, δ_y, δ_i} of the learning model. We also estimate the DSGE model under rational expectations, in which the parameters of the model are included in the following vector, Θ_{RE}={λ, μ_π, μ_y, σ, ρ, α_π, α_y, σ_π, σ_y, σ_i, δ_π, δ_y, δ_i}.

4.4 Empirical Results

This section presents the estimates of the DSGE model under adaptive expectations. Our data, covering the time period from 1954Q3 to 2007Q4 includes inflation, the CBO output gap and the interest rate. The effective federal funds rate is adopted for the interest rate.

For comparison, Table 4.2 reports the estimates of the DSGE model under both rational expectations and adaptive expectations. The results are obtained using the maximum likelihood estimator.[1] The findings indicate that the coefficient on inflation expectations in the learning model is approximately 0.89, which reveals that the role of lagged inflation is quite limited in explaining inflation dynamics. Compared to the estimate of μ_π from Section 4.2, the coefficient appears to be lower. However, both results suggest that a lagged inflation term is not important when adaptive expectations are introduced. Turning to the rational expectations model, the coefficient (μ_π) on inflation expectations is estimated to be 0.74, which is lower than the estimate from the learning model. The estimated coefficient is consistent with the findings of Gali and Gertler (1999).[2] On the other hand, a model introduced by Christiano, Eichenbaum, and Evans (2005) implies that this coefficient is closer to 0.5. In the learning model, a higher estimate of μ_π reveals that the lagged inflation intended to generate inflation persistence could be replaced with adaptive learning.

The estimates of μ_y from the learning and rational expectations models are very similar, which implies that the role of the lagged output gap is still necessary in generating persistence. That is, the introduction of adaptive learning does not make the

[1] Table 4.1 and 4.2 are estimates of the model without inflation targets. We also check the estimation results of the model with inflation targets in the Taylor rule. Although we do not report, estimation results are similar.

[2] Gali and Gertler (1999) employ the labor's share of income represented by as a proxy for the real marginal cost in estimating the hybrid new Keynesian Phillips curve.

lagged output gap unnecessary. Habit formation in consumption seems to be required to generate output gap persistence. This finding is different from that of Milani (2007). In his model, in which agents forecast future variables using the reduced form of a DSGE model and learn about the reduced form parameters over time, a lagged output gap term is reported to be redundant in generating persistence. In our model, the role of adaptive output gap expectations does not seem to be sufficient, on the grounds that the estimated coefficient $\tilde{\alpha_y}$ measuring the speed of correcting errors is not statistically significant. Therefore, the expectations are modeled as a random work process. In contrast to the estimate of $\tilde{\alpha_y}$, the estimated value of $\tilde{\alpha_\pi}$ is statistically significant and robust to the model specifications, that is, in both the DSGE model and the Phillips curve.

For the monetary policy parameters, the parameter estimate of ρ capturing the degree of interest rate smoothing is 0.92. Under the assumption of rational expectations, the coefficient on lagged interest rate is estimated to be 0.68. This parameter is often reported to be around 0.75 in the literature (e.g., Clarida, Gali, and Gertler (2000)). The coefficient on inflation expectations in the Taylor rule is estimated to be 1.67 for the adaptive learning and 1.95 for the rational expectations model. In the learning model, the response of the Fed to inflation seems to be less aggressive. The parameter α_y displaying the Fed's response to the economic activity is obtained similarly from the learning and rational expectations models.

It is worth highlighting that the introduction of the adaptive learning leads to an increase in likelihood from -1027.6 to -968.8, suggesting our model provides a better performance than the rational expectations model in fitting the data. Turning to the variances of shocks, we could conjecture the source of this improvement. When adaptive expectations are employed, the estimate of σ_π is reported to be 0.50, which is lower than 0.77 from rational expectations model. On the other hand, the parameter σ_y is estimated to be higher in learning model. While adaptive learning plays a crucial role in reducing errors from the Phillips curve, it does not seem to perform better than rational expectations model in the IS curve. The variance of interest rate shock σ_i is estimated to be lower with learning model employed. Overall, when considering a higher likelihood, the improvement of the goodness-of-fit from the Phillips curve and the Taylor rule may dominate that from the IS curve.

Table 4.3 provides simple descriptions of the root mean square error of the estimated adaptive expectations in the learning model and the one step ahead out-of-sample VAR forecast respectively. The evaluation period for VAR forecast is from 1954Q1 to 1959Q4 and we forecast the inflation and the output gap of next period recursively. To compare the forecast performance of the adaptive expectations with VAR forecast, we choose the same window size from 1960Q1 to 2007Q4. The results in Table 4.3 present the better performance of the learning model with a much smaller root mean square error.

The upper panel of Figure 4.2 reports the developments of adaptive inflation expectations and VAR forecast with the actual series. The estimated expectations provide a good approximation of realized future inflation, closely following each turning point of inflation without underestimating or overestimating the observed inflation. The lower panel of Figure 4.2 depicts the dynamic behavior of output gap expectations and the VAR forecast with the realized value of the output gap. As we observe here, the VAR forecast is quite close to people's estimated expectations. However, as confirmed in Table 4.3, adaptive expectations seem to outperform the VAR forecast. Both adaptive learning and VAR forecast capture each turning point of the output gap although there is a lag from the realized series.

Figure 4.3 displays actual and predicted values for inflation and the output gap. As we confirm from the likelihoods, the model with adaptive expectations fits the data very well in general. In particular, the estimated Phillips curve performs quite well in tracking inflation, even predicting each turning point. The predicted output gap also provides a good approximation of the output gap. Overall, the predicted values appear to suggest that adaptive learning works to match the data quite well.

This section arrives at four conclusions. First, the higher likelihood in the learning model demonstrates its better performance in the estimation compared to rational expectations model. Second, adaptive expectations are confirmed as a major source of inflation persistence that is confirmed by the relatively larger coefficient on inflation expectation in the learning model. On the other hand, similar estimates on output gap expectations in both learning and rational expectation model reveal the continuing importance of the lagged output gap even though we introduce adaptive expectations into the model. Third, although we assume that agents have naive knowledge about the economy, people's beliefs, measured by adaptive expectations outperform VAR forecasts in terms of the root mean square errors. Finally, the predicted inflation and the output gap work very well in fitting the actual data, which are presented in Figure 4.3. Overall, estimation results and figures confirm the importance of adaptive expectation, especially in modeling inflation, which is an indispensable part of DSGE models. In the next section, we simulate the model based on estimation results.

4.5 Impulse Response Functions

This section explores whether the learning model is able to generate plausible responses to shocks. We employ the estimates from Table 4.2 to obtain impulse response functions with two exceptions. We set σ to the value of 1, as in Clarida, Gali, and Gertler (2000) because the estimate of σ is not statistically significant. The estimate of ρ is higher than that reported in the literature. We therefore set ρ to be 0.79, which is consistent with the estimate of Clarida, Gali, and Gertler (2000) for the Volcker-Greenspan period.

Figure 4.4 displays impulse response functions to one-standard-deviation shocks. Red lines present the effects of each shock on the endogenous variables and blue lines show responses of expectations to each shock. The learning model has the property that expectations tend to move with a lag behind variables due to adaptive learning, as is often observed in the surveys of inflation expectations.

The first row presents the responses of inflation, the output gap, the interest rate, and the dynamics of expectations in response to a cost-push shock. The shock leads to an immediate increase in inflation. The effect of the cost-push shock on inflation starts to decrease over time. Following a cost-push shock, inflation expectations initially rise before reaching a peak and then start to decrease. After several quarters, inflation and inflation expectations seem to be on a similar path. The figure in the first row and second column shows the response of the output gap and its expectations to a cost-push shock. After a slight increase, the output gap starts to decrease due to the increase in the interest rate that results from the Fed's response to stabilize inflation. Output gap expectations move with a lag behind the output gap. In response to a cost-push shock, the nominal interest rate initially increases due to rising inflation and then it starts to fall in response to the decrease in inflation and the output gap.

Figures in the second row display responses of variables to a demand shock. The shock has a positive, persistent effect on inflation. Inflation expectations appear to follow the inflation path with a lag. This feature could be observed in the surveys of inflation expectations, such as the SPF and Livingston survey. A positive demand shock causes the output gap to increase, and the effect of the shock on output sharply decreases over time. Since inflation and the output gap increase in response to a demand shock, the interest rate increases.

The third row presents the impact of an interest rate shock on the variables and expectations. Following an interest rate shock, both the actual inflation and its expectations show hump-shaped responses. Despite the limited role of lagged inflation in the Phillips curve, the model is able to generate a hump-shared response of inflation to an interest rate shock. The interest rate has a negative effect on inflation, leading to a persistent decrease over time before reaching its bottom. The pattern of response of inflation expectations is very similar to that of inflation. An interest rate shock leads to a decrease in the output gap and its expectations.

Figure 4.5 reports the responses of key economic variables to one-standard-deviation expectation shocks. An inflation expectation shock causes inflation to increase, as does the cost-push shock. The patterns of responses of the variables are very similar to those generated in response to a cost-push shock. Turning to an output gap expectation shock, as reported in the second row, a positive output gap expectation shock raises the inflation and output, therefore leading to an increase in the interest rate.

4.6 Conclusion

This paper examines the role of adaptive expectations in inflation and output gap dynamics in the DSGE model. Interestingly, our estimates of inflation and output gap expectations using naive adaptive learning appear to outperform VAR forecasts. These results are meaningful, since DSGE models under rational expectations can be expressed as VAR systems with parameter restrictions. Our findings also indicate that the learning model outperforms the rational expectations DSGE model in terms of its goodness of fitness. Finally, the main contribution of this paper is to show that adaptive expectations can replace the role of lagged inflation in generating inflation persistence. When the model includes learning, the coefficient on lagged inflation in the Phillips curved is estimated to be around zero. However, the lagged output gap seems to play a role in generating persistence, regardless of adaptive expectations.

Table 4.1: Maximum Likelihood for a Phillips Curve Estimation: A single equation

Parameters	Estimates	Standard Errors
λ	0.325	0.049
μ_π	1.005	0.085
$\tilde{\alpha_\pi}$	0.603	0.064
α_1	0.154	0.099
α_2	0.201	0.103
α_3	0.674	0.155
σ_π	0.000	0.127
$Maximum Likelihood$	-313.159	

Note: This Table shows maximum likelihood estimates of the DSGE model from Equation (4.4) to (4.5), including dynamic systems of inflation, output gap, interest rate and expectation dynamic process. Different output gaps including linearly detrended output and quadratically detrended output are also estimated and the results are similar. Here we only report the results of the output detrended using the CBO measure of potential output. The sample period is 1954:Q3-2007:4Q. The estimated equations are as follows:

$$\pi_t = \mu_\pi E_t \pi_{t+1} + (1 - \mu_\pi)\pi_{t-1} + \lambda y_t + \varepsilon_t^\pi$$

$$E_t \pi_{t+1} = E_{t-1}\pi_t + \tilde{\alpha}_\pi(\pi_{t-1} - E_{t-1}\pi_t) + \omega_t^\pi$$

$$h_t = \alpha_1 + \alpha_2(\varepsilon_{t-1}^\pi)^2 + \alpha_3 h_{t-1}$$

Table 4.2: Maximum Likelihood Estimation in a DSGE model

Parameters	learning		Rational Expectations	
	Estimates	Standard Error	Estimates	Standard Error
λ	0.040	0.024	0.009	0.005
μ_π	0.888	0.253	0.738	0.164
μ_y	0.780	0.238	0.774	0.075
σ	0.032	0.039	0.044	0.008
ρ	0.922	0.043	0.678	0.077
α_π	1.670	0.320	1.945	0.350
α_y	1.088	0.961	1.054	0.274
$\tilde{\alpha_\pi}$	0.536	0.159	-	-
$\tilde{\alpha_y}$	0.283	0.862	-	-
$\sigma_{\omega\pi}$	0.512	0.562	-	-
$\sigma_{\omega y}$	0.318	0.406	-	-
σ_π	0.503	0.382	0.767	0.052
σ_y	0.601	0.444	0.407	0.001
σ_i	1.352	0.092	1.746	0.124
δ_π	0.000	0.010	-0.001	0.085
δ_y	0.413	0.084	0.552	0.099
δ_i	0.020	0.090	0.507	0.090
Maximum Likelihood	-969.8		-1027.6	

Note: This table shows the maximum likelihood estimates of a complete DSGE model in Equation (4.3) and (4.6) – (4.10). The sample period is 1954:Q3-2007:Q4. The equations of the learning model are as follows:

$$\pi_t = \mu_\pi E_t \pi_{t+1} + (1-\mu_\pi)\pi_{t-1} + \lambda y_t + \varepsilon_t^\pi$$

$$y_t = \mu_y E_t y_{t+1} + (1-\mu_y) y_{t-1} - \sigma(i_t - E_t \pi_{t+1}) + \varepsilon_t^y$$

$$E_t \pi_{t+1} = E_{t-1}\pi_t + \tilde{\alpha}_\pi(\pi_t - E_{t-1}\pi_t) + \omega_t^\pi$$

$$E_t y_{t+1} = E_{t-1} y_t + \tilde{\alpha}_y (y_t - E_{t-1} y_t) + \omega_t^y$$

$$i_t = \rho i_{t-1} + (1-\rho)(\alpha_\pi \pi_t + \alpha_y y_t) + \varepsilon_t^i$$

$$\varepsilon_t^\pi = \delta_\pi \varepsilon_{t-1}^\pi + v_t^\pi$$

$$\varepsilon_t^y = \delta_y \varepsilon_{t-1}^y + v_t^y$$

$$\varepsilon_t^i = \delta_i \varepsilon_{t-1}^i + v_t^i$$

Table 4.3: Root Mean Square Error: Learning vs. one-step ahead VAR Forecasts

RMSE	Learning	VAR
Inflation	0.3179	1.1537
Output Gap	0.2351	0.7824

Figure 4.1: Inflation and Expectation in the Phillips Curve: A single equation

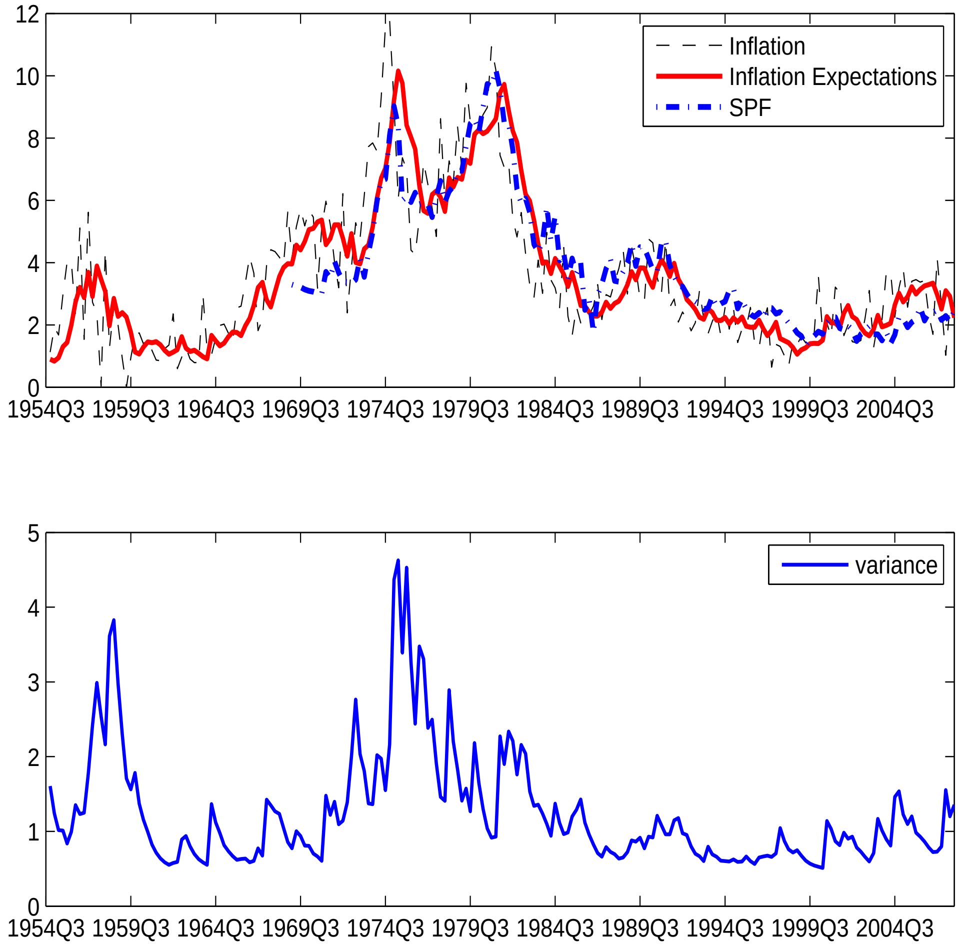


Figure 4.2: Inflation, Output gap and their Expectations in a DSGE model

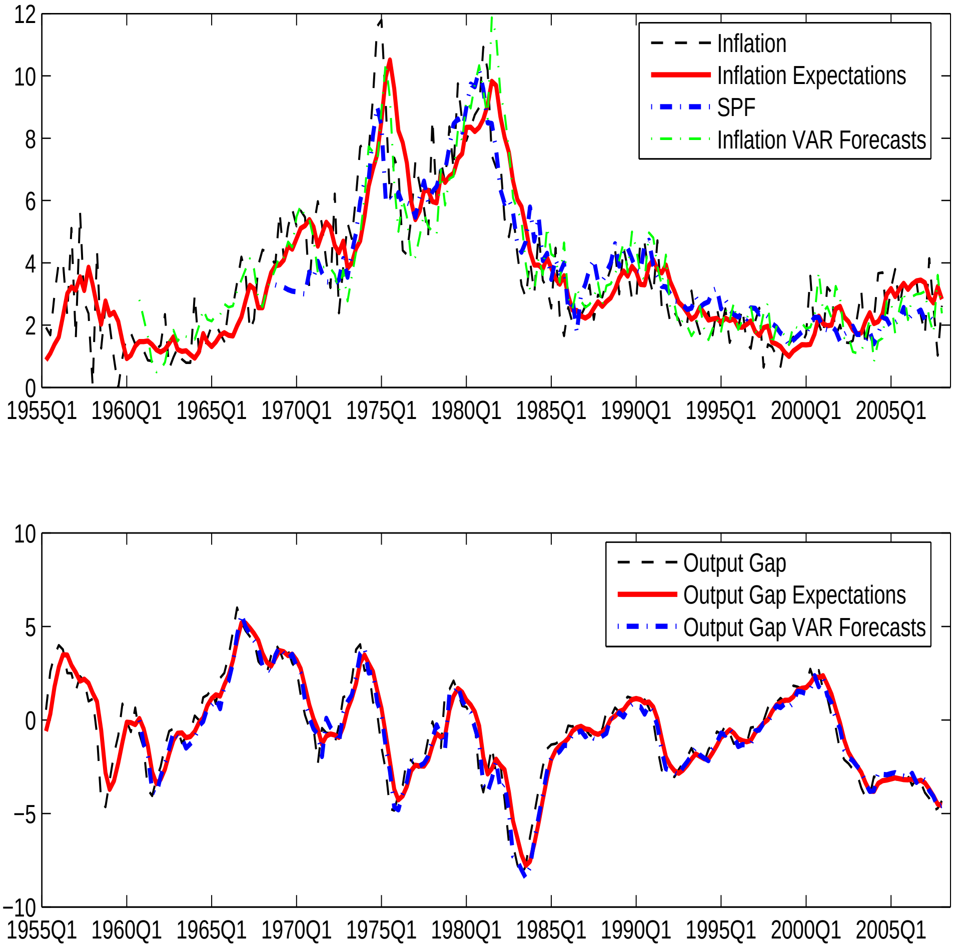


Figure 4.3: Fitted values of Inflation and Output Gap

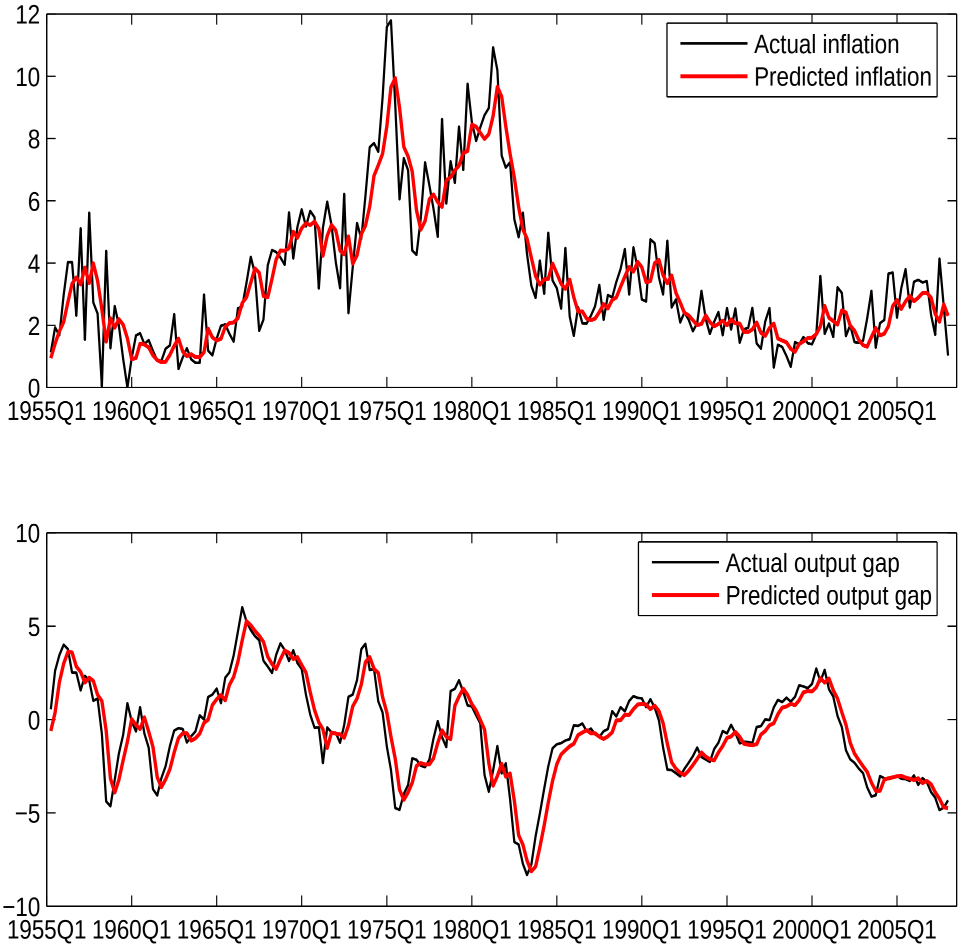


Figure 4.4: Shocks and Impulse Response Functions

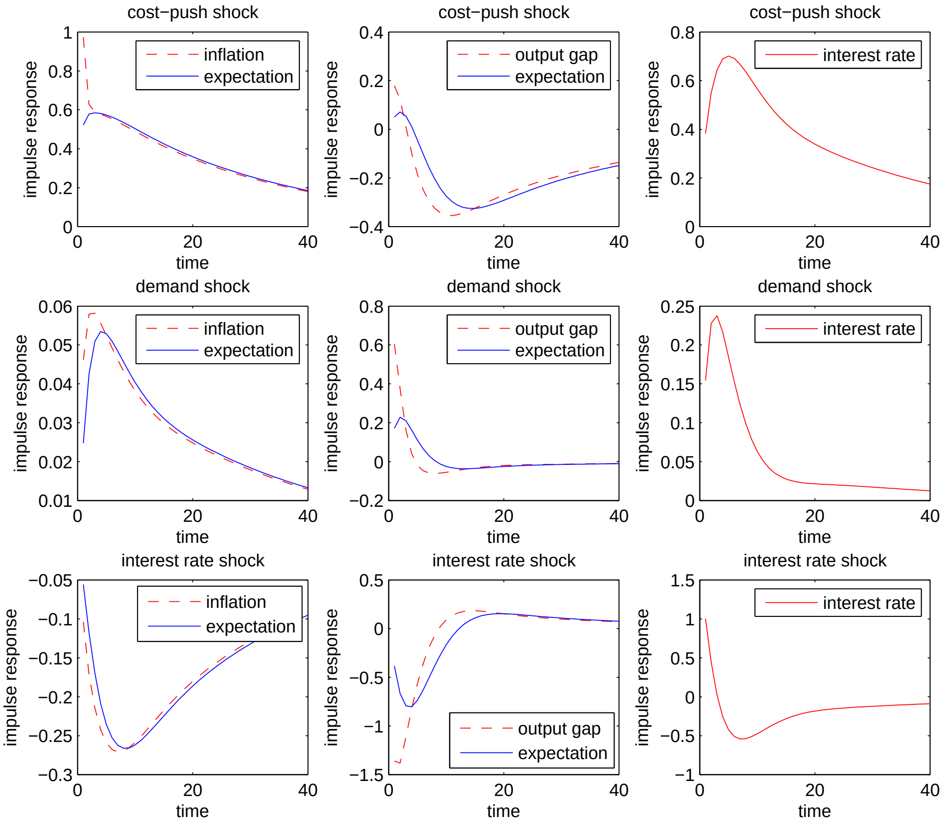
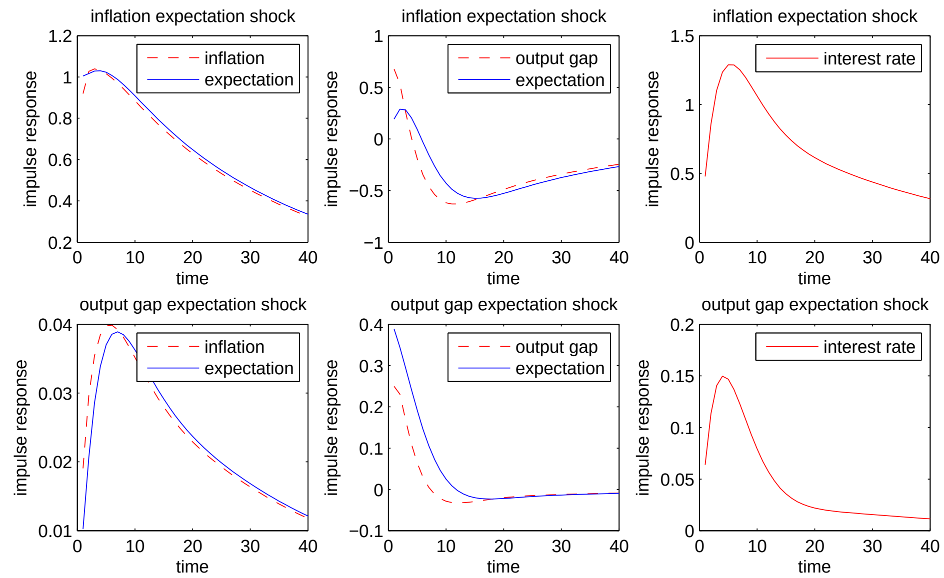


Figure 4.5: Expectation Shocks and Impulse Response Functions



Chapter 5

Conclusions

This dissertation contributes to the literature on the analysis of United States Postwar business cycle fluctuations in a perspective of Markov Switching and Bayesian estimation. First, in Chapter 2, we propose a dynamic stochastic general equilibrium (DSGE) model with the assumption of regime switches in the private sector, in the conduct of monetary policy and in the volatility of shocks. A Gibbs-sampling with Metropolish-Hasting algorithm is used to estimate this model. Our estimation results support changes occurring in the private sector, in the monetary policy and in the volatility of exogenous, non-policy shocks which are related to the "Great Inflation", the "Great Moderation" and the 2008 financial crisis. Variance decomposition and impulse response functions are conducted to reveal the role of different shocks. Our results find irrespectively to monetary policy regimes, supply shocks are the main driver of inflation fluctuations, while demand shocks are the main source of changes in the output gap. If there were no switches in the agents' beliefs, supply shocks are also main drivers of the inflation. This chapter also demonstrates that if the agents always maintain a weak response to economic dynamics, inflation would have been lower during the 1970s. The more intense response to inflation by the Federal Reserve would also have helped mitigate the great inflation. In addition, there were substantial changes in agents' beliefs, and in the volatility of shocks in the recent years. Such an occurrence could be potential early signals of the 2008 financial crisis.

Second, in Chapter 3, we propose a baseline dynamic model with five extensions to investigate policymakers' commitment to inflation and unemployment. Those models are based on the conjecture that potential policymakers' misperception may be originated from unobserved deviations of unemployment from its natural rate. Five processes are purposed, which have been illustrated in the introduction part of this dissertation. In addition to five processes, this chapter specifies a loss function derived from a constrained minimization of the divergence in inflation and unemployment that also penalizes shifts in the policy variables. Bayesian estimation is also applied in this chapter. Findings support our estimated belief performs the role of real interest rate. Empirical results are summarized as follows: 1) policymakers' belief is very persistent even when it commits to a Taylor-type policy rule. 2) the run-up of U.S. inflation around 1980 is mostly attributed to policymakers' misperception while the peak in the end of 1974 is possibly a result from large non-policy shocks. 3) models with commitment dominate models without commitment, especially in periods of large oscillations in inflation. When policymakers are committed to respond to a Taylor-type rule, the average loss function is efficiently reduced over the time, thus effectively lessening their misperception. In addition to those findings, this chapter also contributes to important policy implication as it indicates how and when it is appropriate for policymakers to choose a commitment in their reaction to inflation and unemployment. That is, a flexible or more activist policy is more appropriate in reacting to inflation when there is high unemployment target, whereas a policy that is consistent over time is more suitable under low unemployment target. Moreover, our results shed light on the source of the two large rises in inflation during the "Great Inflation" period and the prevalence of low inflation during the early 1980s.

Third, in Chapter 4, we investigate the inflation persistence based a dynamic stochastic general equilibrium (DSGE) model with adaptive expectation. We observe in this chapter that model with adaptive expectation makes lagged inflation redundant in fitting inflation dynamics. Simultaneously, maximum likelihood estimation supports a DSGE model with adaptive expectation is superior to a DSGE model with rational expectation in model fit and out-of-sample forecasting.

Overall, our analytical and empirical analysis based on three small dynamic stochastic general equiilbrium (DSGE) models with Markov Switching contributes to interpret the United States Postwar business cycle fluctuations, especially for the evolution of inflation, unemployment, output gap and interest rate. This dissertation identifies sources in driving inflation, unemployment and output gap as well as provides important policy implications. Further analysis and investigation on medium and large DSGE models in explaining U.S. macroeconomic dynamics as well as how to improve the model fit and forecasting performance based on Bayesian estimation will be interesting topics for future research.

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Appendix A

Appendix of Chapter 2

A.1 Priors

Prior distribution for DSGE parameters and transition matrices are summarized in Table $A.1$.

Parameter	Density	Range	Mean	Std.deviation
τ	Gamma	$\mathbb{R}^+$	0.6	0.5
κ	Gamma	$\mathbb{R}^+$	0.1	0.1
β	Beta	$[0,1)$	0.9	0.1
γ_1	Normal	$\mathbb{R}^+$	1.5	0.5
γ_2	Normal	$\mathbb{R}^+$	0.8	0.1
ρ_R	Beta	$[0,1)$	0.5	0.2
ρ_g	Beta	$[0,1)$	0.8	0.1
ρ_z	Beta	$[0,1)$	0.7	0.1
r^*	Gamma	$\mathbb{R}^+$	0.6	0.3
π^*	Normal	$\mathbb{R}^+$	0.75	0.17
σ_R	Inv.Gamma	$\mathbb{R}^+$	0.25	0.14
σ_g	Inv.Gamma	$\mathbb{R}^+$	0.4	0.3
σ_z	Inv.Gamma	$\mathbb{R}^+$	1	0.5
σ_y	Inv.Gamma	$\mathbb{R}^+$	0.15	0.1
σ_p	Inv.Gamma	$\mathbb{R}^+$	0.15	0.1
σ_r	Inv.Gamma	$\mathbb{R}^+$	0.1	0.05

Table A.1: Prior distribution for DSGE model parameters

Assume Markov Switching transition matrices follow a Dirichlet distribution:

$$H^{sp}(\cdot, i) \sim D(a_{ii}^{sp}, a_{ij}^{sp})$$

$$H^{sa}(\cdot, i) \sim D(a_{ii}^{sa}, a_{ij}^{sa})$$

$$H^{er}(\cdot, i) \sim D(a_{ii}^{er}, a_{ij}^{er})$$

Similar to Bianchi(2009), we choose $a_{ii}^{sp} = a_{ii}^{sa} = a_{ii}^{er} = 10$ and $a_{ij}^{sp} = a_{ij}^{sa} = a_{ij}^{er} = 1$.

A.2 Hidden Markov Models

Given the hidden states $\Xi = \{\xi_1, \xi_2, \ldots, \xi_N\}$, the state at the length t as Q_t and the observation symbols $V = \{v_1, v_2, \ldots, v_M\}$ and the symbol at the length t as O_t, we can have the state transition probability distribution $[A]_{ij} = \{a_{ij}\}$ where

$$a_{ij} = P(Q_{t+1} = s_j \mid Q_t = s_i), \quad 1 \leq i, j \leq N$$

Similarly, we can define the observation symbol probability distribution as $[B]_{jk} = \{b_j(v_k)\}$, where

$$b_j(v_k) = P(O_t = v_k \mid O_t = s_j), \quad 1 \leq j \leq N, 1 \leq k \leq M$$

A.3 The model

We consider a prototypical New Keynesian monetary DSGE model in which details can be found in King(2000) and Woodford(2003). Details of model description could be referred to Bianchi(2009).

Consider a continuum of monopolistic firms, a representative household, and a monetary policy authority in our economy. The household maximizes the following utility function:

$$E_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\left(\frac{C_s^{1-\tau}-1}{1-\tau}+\chi\log\frac{M_s}{P_s}-h_s\right)\right] \tag{A.1}$$

subject to

$$C_t+\frac{B_t}{P_t}+\frac{M_t}{P_t}+\frac{T_t}{P_t}=W_th_t+\frac{M_{t-1}}{P_t}+R_{t-1}\frac{B_{t-1}}{P_t}+D_t \tag{A.2}$$

For each monopolistically competitive firm, a quadratic adjustment cost is derived from a downward-sloping demand curve:

$$Y_t(j)=\left(\frac{P_t(j)}{P_t}\right)^{-1/\upsilon}Y_t \tag{A.3}$$

and we have quadratic adjustment cost as

$$AC_t(j)=\frac{\varphi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)}-\pi\right)^2Y_t(j) \tag{A.4}$$

Firms use a linear production function as:

$$Y_t(j)=A_th_t(j) \tag{A.5}$$

where total factor productivity A_t evolves according to a random walk:

$$\ln A_t = \ln \gamma + \ln A_{t-1} + \tilde{a}_t \tag{A.6}$$

$$\tilde{a}_t = \tilde{a}_{t-1} + \epsilon_{a,t} \tag{A.7}$$

Firms maximize the present value of future profits by choosing the price $P_t(j)$:

$$E_t \left[\sum_{s=t}^{\infty} Q_s \left(\frac{P_s(j)}{P_s} Y_s(j) - W_s h_s(j) - \frac{\varphi}{2} \left(\frac{P_s(j)}{P_{s-1}(j)} - \pi \right)^2 \right) Y_s(j) \right] \tag{A.8}$$

where Q_s is the marginal value of a unit of the consumption good: $Q_s/Q_t = \beta[u_c(s)/u_c(t)] = \beta^{s-t}(C_t/C_s)^{\tau}$.

The nominal interest rate is affected by deviations of inflation and output from their target levels:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\rho_R} \left[(\frac{\pi_t}{\pi^*})^{\psi_1} (\frac{Y_t}{Y_t^*})^{\psi_2} \right]^{(1-\rho_R)} e^{\epsilon_{R,t}} \tag{A.9}$$

In addition, government expenditure follows a stationary AR(1) process as follows:

$$\tilde{g}_t = \rho_g \tilde{g}_{t-1} + \epsilon_{g,t} \tag{A.10}$$

Therefore $\epsilon_{g,t}$ can be interpreted as a shock to Government expenditure. The government collects a lump-sum tax(or provides a subsidy) to balance the fiscal deficit:

$$\zeta_t Y_t + R_{t-1} \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} = \frac{B_t}{P_t} + \frac{M_t}{P_t} + \frac{T_t}{P_t} \tag{A.11}$$