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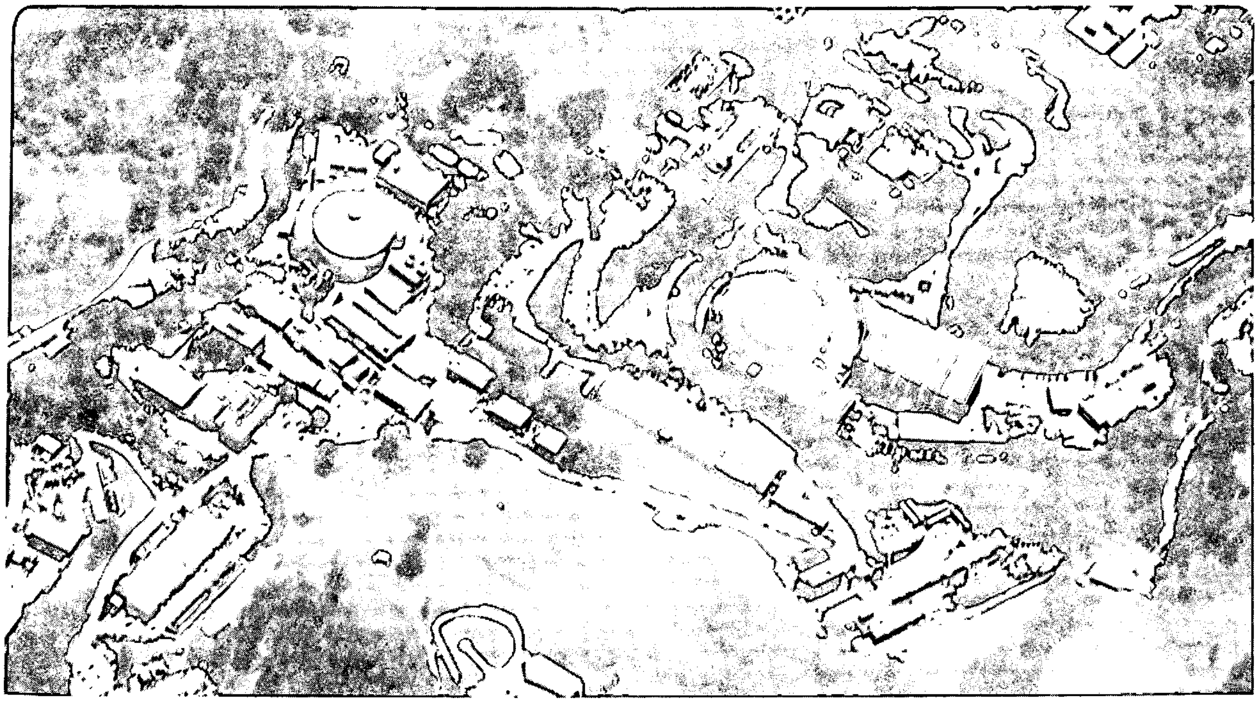

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Survey of Composite Particle Models of Electroweak Interaction

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SURVEY OF COMPOSITE PARTICLE MODELS OF ELECTROWEAK INTERACTIONS

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ABSTRACT

Models of composite weak bosons, the top-condensate model of electroweak interaction and related models are surveyed. Composite weak bosons must be tightly bound with a high compositeness scale in order to generate approximate gauge symmetry dynamically. However, naturalness argument suggests that the compositeness scale is low at least in toy models. In the top-condensate model, where a composite Higgs doublet is formed with a very high scale, the prediction of the model is insensitive to details of the model and almost model-independent. Actually, the numerical prediction of the t-quark and Higgs boson masses does not test compositeness of the Higgs boson nor condensation of the t-quark field. To illustrate the point, a composite t_R-quark model is discussed which leads to the same numerical prediction as the top-condensate model. However, different constraints are imposed on the structure of the Higgs sector, depending on which particles are composite. The attempt to account the large t-b mass splitting by the high compositeness scale of the top-condensate model is reinterpreted in terms of fine tuning of more than one vacuum expectation value. It is difficult to lower, without a fourth generation, the t-quark mass in the composite particle models in general because the Yukawa coupling of the t-quark to the Higgs boson, $f_t^2/4\pi = 0.1$ for $m_t = 200$ GeV, is too small for a coupling of a composite particle.

1. Introduction

The Standard Model contains many elementary particles. Vector bosons fit nicely to the gauge bosons of $SU(3)_c \times SU(2)_L \times U(1)$. No experimental evidence has so far been found against the vector bosons being the gauge bosons. However, real test for $W^\pm$ and Z being gauge bosons will be made by observation of their self-couplings in $W^\pm$-pair production. For quarks and leptons, there are too many of them, and little has been understood about their masses at a fundamental level. It is therefore tempting to postulate some structure for the quarks and leptons. The most plausible is compositeness of Higgs bosons. They are introduced solely for the purpose of breaking gauge and chiral symmetries spontaneously. Being spinless, their masses are not protected from quadratic divergence. In order to have the Higgs-boson mass much below the Planck scale, an extremely fine tuning is required unless some cutoff is brought in by broken supersymmetry or by compositeness of the Higgs particle. The technicolor model is the best known example of composite Higgs bosons, but it faces difficulty in flavor-changing neutral interactions. Meanwhile, as the experimental lower limit on the t-quark mass has inched up, the proximity of the t-quark mass to the electroweak symmetry breaking scale recently prompted the top condensate model. The model breaks the symmetry by condensation of the t-quark field forming a composite Higgs boson. It is more *economical* than the technicolor model in the sense that no new particles need to be introduced. It also produces interesting numerical predictions. However, a close look at the model reveals not only attractive features but also disappointing ones.

In this survey, I will present my understanding of the issues involved in compositeness of the vector bosons and the Higgs particle, and also of the quarks and leptons in connection to the top-condensate model. In Section 2, I will focus on the model of composite W and Z in which gauge symmetry is approximate and dynamical in origin. In Section 3, the top-condensate model is reviewed and analyzed critically. I will argue that the numerical prediction of the model based on the renormalization-group analysis is virtually model-independent. To make my point, I will present in Section 4 a toy model of a composite right-handed *t*-quark which, by renormalization group analysis, leads to the same low-energy prediction as the top condensate model. In Section 5, It is pointed out that constraints on model building are quite different from one type of models to another. Much stronger constraints are imposed on the Higgs sector of the composite t_R model than on the top condensate model. In Section 6, reinterpretation of the socalled *critical instability* is made in terms of fine-tuned vacuum-expectation-values.

2. Electroweak Vector Bosons

2.1. Composite Vector Bosons and Dynamical Gauge Symmetry

The photon is an abelian gauge boson. There are enough experimental evidences for that gluons are gauge bosons of the unbroken color-SU(3) symmetry. Asymptotic freedom and confinement are the convincing ones. In contrast to the photon and gluons, $W^{\pm}$ and Z are massive. They are interpreted as gauge bosons which acquire mass by spontaneous symmetry breaking. The issue I will address here is whether these massive vector bosons *can be* composite or not. More specifically, I ask whether the electroweak gauge symemtry can arise as an approximate dynamical symmetry from a Lagrangian which violates it *explicitly*. Though composite gauge bosons do not solve any outstanding phenomenological problem, it is an interesting question theoretically and phenomenologically. In the 1950's Heisenberg tried to generate everything from a four-fermion interaction[1]. The first attempt focusing on a composite photon was made by Bjorken[2] in 1963. A clear, comprehensible picture was introduced by Eguchi and Sugawara[3,4] in the 1970's. They are all based on four-fermion interaction models. Eguchi's work[4] was used to interpret the gauge bosons of the Standard Model as composite[5,6]. I first sketch the essence of the argument for dynamical generation of gauge symmetry.

The strategy of the model is to implant a global symmetry and turn it into a local symmetry. Here let us choose a model Lagrangian of the Nambu-Jona-Lasinio type[7] with a vectorial SU(n) flavor symmetry,

$$L_{\psi} = \sum_a (i\overline{\psi}^a \partial\!\!\!/ \psi_a - m\overline{\psi}^a\psi_a) - \sum_\alpha (G/2)(\overline{\psi}\gamma^\mu\Lambda_\alpha\psi)(\overline{\psi}\gamma_\mu\Lambda_\alpha\psi), \tag{2.1}$$

where $\Lambda_\alpha(\alpha = 1,2\cdots\cdots n^2\text{-}1)$ are the $n \times n$ generator matrices of SU(n), and the fundamental fermons $\psi_a(a = 1,2\cdots\cdots n)$, often referred to as the preons, transform according to the fundamental representaiton of SU(n). In addition, if we wish to justify the chain diagram approximation to be made below, we introduce another global symmetry of "color" $SU(N_c)$ with $N_c \to \infty$. The QCD colors may be part of these "colors". The preons ψ are in the fundamental representation of $SU(N_c)$ too. The "color" summation is understood for the fermion bilinears in Eq.(2.1). It is easy to extend the vectorial SU(n) to the chiral symmetry $SU(n)_L$ or $SU(n)_R$ by replacing ψ by a left- or right-handed field $\psi_{L,R}$ and dropping the mass term. The strong attractive coupling G can generate bound states in the spin-one, "color"-singlet channels of the SU(n) adjoint representation. We attempt to identify them with composite gauge bosons.

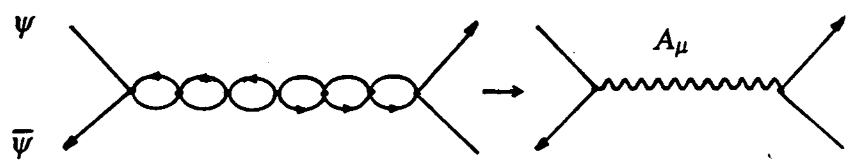

Fig.1. Formation of a composite vector boson by a fermionic preon pair.

Introducing the auxiliary fields $A_\mu{}^\alpha$ for the composite vector bosons, we can turn the action of the Lagrangian (2.1) into

$$S = \iint D\bar{\psi}\, D\psi \exp[i\int L_\psi d^4x],$$

$$= \iiint D\bar{\psi} D\psi D A_\mu{}^\alpha \exp[i\int L d^4x], \tag{2.2}$$

where

$$L = \sum_a (i\bar{\psi}^a \partial\!\!\!/ \psi_a - m\,\bar{\psi}^a\psi_a) + \sum_\alpha (\mu_0{}^2/2) A^\mu{}_\alpha A_{\mu,\alpha} - g_0 \sum_\alpha (\bar{\psi}\gamma^\mu \Lambda_\alpha \psi) A_\mu{}^\alpha, \tag{2.3}$$

with $g_0{}^2 = \mu_0{}^2 G$. By the covariant derivative $D_\mu \equiv \partial_\mu + ig_0 A_\mu$ where $A_\mu = \sum_\alpha \Lambda_\alpha A_{\mu,\alpha}$ with $tr(\Lambda_\alpha \Lambda_\beta) = \delta_{\alpha\beta}/2$, this Lagrangian is written in the compact form

$$L = \bar{\psi}(iD\!\!\!\!/ - m)\psi + \mu_0{}^2 tr(A^\mu A_\mu), \tag{2.4}$$

where the summation over the flavor SU(n) indices has also been suppressed. This Lagrangian (2.4) is gauge invariant up to the mass term of A_μ. The next step is to generate the kinetic energy term from the preon loop diagrams of Fig.2.

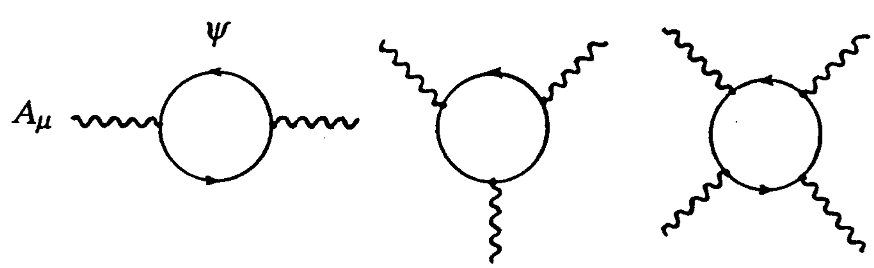

Fig.2. The leading N_c diagrams which generate the $G_{\mu\nu}G^{\mu\nu}$ and $A_\mu A^\mu$ terms. The same result is obtained by literally summing up infinite series of chain diagrams.

The result is

$$\Delta L_A = - Z_A \mathrm{tr}(G_{\mu\nu}G^{\mu\nu})/2 + \delta\mu_o{}^2 A_\mu A^\mu, \tag{2.5}$$

where

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig_o[A_\mu, A_\nu]. \tag{2.6}$$

It is not a miracle that the derivatives of A_μ appear only in the combination of $G_{\mu\nu}$. It is guaranteed by the local invariance of the preon part of the Lagrangian L in Eq.(2.4) because the gauge noninvariant term $\delta\mu_o{}^2 A_\mu A^\mu$ does not enter the preon loops of Fig.2. The constant Z_A is logarithmically divergent and renormalized away by

$$\sqrt{Z_A} A_\mu \rightarrow A_\mu,$$

$$g_o = \sqrt{Z_A}\, g,$$

$$\mu_o{}^2 + \delta\mu_o{}^2 = Z_A\, \mu^2. \tag{2.7}$$

An important issue regarding composite gauge bosons is whether the composite vector bosons can be made massless in this model. It is possible to make the composite vector bosons massless by letting the unrenormalized coupling to infinity[2]. It amounts to setting the cutoff Λ equal to ∞ so that $\mu^2 = (\mu_o{}^2 + \delta\mu_o{}^2)/Z_A \rightarrow 1/\infty = 0$. However, if we insist that Λ cannot be larger than the Planck scale, setting literally $\Lambda = \infty$ is not an acceptable option. Then, is it possible to make an *exact tuning* $\mu_o{}^2 + \delta\mu_o{}^2 = 0$ in the last relation of Eq.(2.7) ? If we could[4], $L + \Delta L_A$ would have a perfect local SU(n) symmetry with A_μ being its massless gauge bosons. It would be amazing since a perfect local symmetry arises from a Lagrangian which breaks it explicitly (through the kinetic energy term $i\bar{\psi}\partial\!\!\!/\psi$ in L_ψ). It is not just *spontaneous generation* of gauge symmetry, but perfect screening of gauge symmetry violation.

A close look at the hypothesis of $\mu_o{}^2 + \delta\mu_o{}^2 = 0$ raises a serious question. Since the fundamental Lagrangian L_ψ possesses the global SU(n) symmetry, the vacuum polarization $\Pi_{\mu\nu}(q)$ of the currents $\bar{\psi}\gamma^\mu\Lambda_\alpha\psi$ must be transverse:

$$\Pi_{\mu\nu}(q) = (- g_{\mu\nu} + q_\mu q_\nu/q^2)\Pi(q^2) \tag{2.8}$$

by *global current conservation.* The vacuum polarization $\Pi(q^2)$ can be evaluated explicitly in the the large N_c limit. It vanishes like q^2 near $q^2 = 0$:

$$\Pi(q^2) = \text{const.} \times q^2 \quad \text{as } q^2 \rightarrow 0. \tag{2.9}$$

Therefore the self-energy of the auxiliary vector bosons vanishes at $q^2 = 0$:

$$\delta\mu_o{}^2 = g^2\Pi(0) = 0\,. \tag{2.10}$$

Then there is no chance to cancel the mass term $\mu_o{}^2$ (= $g_o{}^2/G$) with $\delta\mu_o{}^2$. If one found

$\Pi(0) \neq 0$ by explicit evaluation of the integral, it would be simply an artifact due to neglect of global current conservation (like a photon mass by gauge noninvariant regularization). If a global SU(n) symmetry breaking is introduced in the original Lagrangian L_ψ, we can generate a nonvanishing $\delta\mu_0{}^2$ to allow for the exact tuning. But such a breaking enters the kinetic energy term of A_μ through the preon-loop diagrams and destroys the gauge-invariant form $\mathrm{tr}(G_{\mu\nu}G^{\mu\nu})/2$ in ΔL_A.

The lesson of this exercise is that if we start with a Lagrangian with an explicit gauge-symmetry breaking, the closest thing that we can get to gauge symmetry is the massive Yang-Mills theory†:

$$L = \bar{\psi}(i\not{D} - m)\psi - \mathrm{tr}(G_{\mu\nu}G^{\mu\nu})/2 + \mu^2 A^\mu A_\mu. \qquad (\mu \neq 0) \qquad (2.11)$$

Our conclusion can be reinforced by the theorem of Case and Gasiorowicz[9]. It states that:

If a theory has a Lorentz-covariant conserved current, it cannot have a massless particle of spin >1/2 that carries the charge associated with this current.

The theorem can be easily proved by helicity conservation in the brickwall frame. It is valid whether a particle is elementary or composite. Of course, the theorem does not apply to genuine nonabelian gauge bosons since a Lorentz-covariant conserved current does not exist. Recall that the Lorentz-covariant gauge currents cannot be written unless the unphysical polarizations are introduced. In contrast, in the type of models that we have considered here, such currents ($\bar{\psi}\gamma^\mu\Lambda_\alpha\psi$) do exist because a global symmetry is built in. Kugo later states explicitly[10] that *no gauge symmetry can arise unless it exists from the outset.*

2.2. Model Building

We can build an electroweak model of the composite *massive* gauge bosons[11] by borrowing the idea of Hung and Sakurai[12] and of Bjorken[13]. We need composite SU(2) bosons A_μ with coupling g, and an *elementary* Abelian gauge boson B_μ with coupling g'. The latter mixes with the neutral ($I_3 = 0$) component of the SU(2) bosons A_μ through preon pairs and forms the physical photon and the Z boson:

$$\gamma = B\cos\theta + A^{(3)}\sin\theta\,, \qquad Z = -B\sin\theta + A^{(3)}\cos\theta. \qquad (2.12)$$

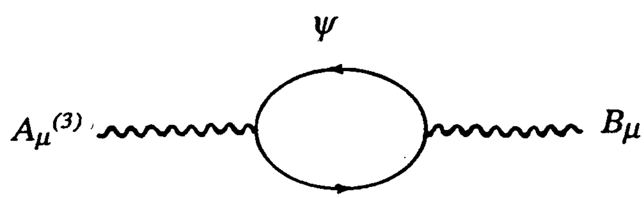

Fig.3. Mixing of $A^{(3)}$ and B into Z and γ by mixing through a preon pair.

† Gauge symmetry is sometimes not violated but invisible unless a right set of fields is chosen[9]. The CP^{N-1} model, where the composite A_μ are made of scalar bosons, is an example. Whether a viable electroweak model can be built with such a theory or not is another question. I thank to T. Kugo for bringing my attention to the CP^{N-1} model.

The mixing angle θ of γ and Z is correctly related to the gauge couplings g and g' through $\tan\theta = \tan\theta_W \equiv g'/g$, as was already pointed before[12,13]. Therefore, at the tree-diagram level, the model is identical with the electroweak gauge sector of the Standard Model.

However, the models of composite W and Z encounter a stiff phenomenological challenge. Let us examine the magnitude of the SU(2) gauge coupling of this model. For $N_c \to \infty$, the SU(2) gauge coupling is calculable. The result is

$$g^2 = 24\pi^2/[N_c ln(\Lambda^2/m^2)], \qquad (2.13)$$

where m is the preon mass. In order to have the quarks and leptons couple with A_μ through the correct V - A interaction, we must add the left-handed quarks and leptons to the preon doublets of the electroweak SU(2) symmetry. This implies a large "color" symmetry encompassing the left-handed quarks of three QCD colors, the left-handed leptons, and the other preons[14]. Then Eq.(2.13) for the gauge coupling g^2 remains valid provided N_c should represent the multiplicity of all fermion doublets. Mass difference among different fermion doublets can be taken into account in Eq.(2.13). An unpleasant feature of the composite weak-boson models is in the magnitude of g^2 given by Eq.(2.13). In order to reproduce the experimental value $g^2/4\pi \approx 1/30$ at the electroweak scale, we need to postulate a very large value for N_c. If the logarithm in the denominator is no larger than ≈ 10, N_c must be $\gtrsim 50 \sim 60$. There are 12 doublets of quarks and leptons in the three generations, which contribute to N_c only by a half of 12 since the quark and lepton doublets are left-handed. The rest must come from the other preon doublets.

The magnitude of the gauge coupling is a fundamental issue in the composite models of the electroweak gauge bosons. In general, dimensionless couplings $g^2/4\pi$ of composite particles come out to be roughly O(1) apart from the $1/N_c$ suppression. If we accept the naturalness argument to be discussed below, we do not expect a strong suppression due to the inverse logarithm $[ln(\Lambda^2/m^2)]^{-1}$. Therefore, we must resort to the large number of preon doublets to suppress the gauge coupling.

What about the scale of the W and Z masses in this model ? The only explicit scale of dimension in the original Lagrangian is in the four-preon coupling G. In addition, the cutoff Λ enters as a physical parameter. After the wavefunction renormalization of $\mathbf{A}_\mu$, the W and Z masses are given by

$$M_W{}^2 = M_Z{}^2\cos^2\theta_W = 24\pi^2/[GN_c\log(\Lambda^2/m^2)]. \qquad (2.14)$$

The electroweak scale $v = 247$ GeV is identified with $2G^{-1/2}$ by comparing Eqs.(2.13) and (2.14) with $m_W \equiv g^2v^2/4$. If the four-fermion interaction of the preons is supposed to be a low-energy limit of more fundamental interactions, one natural choice of the ultraviolet cutoff is $G = O(1/\Lambda^2)$. With the SU(2) gauge coupling of Eq.(2.13), this sets the scale of the W and Z masses equal to $O(g\Lambda)$. If $G = O(16\pi^2/\Lambda^2)$ instead, the scale is $O(g\Lambda/4\pi)$. In either case, this naturalness argument does not allow us to choose a value for Λ much larger than v. For comparison, the naive dimension-analysis *neglecting* the preon current conservation would estimate the vector-boson self-energy $\delta\mu^2$ to be $O(g^2N_c\Lambda^2/16\pi^2)$. This is a little embarrassing.

The smaller the composite vector-boson masses are, the better the dynamical gauge symmetry is in the Lagrangian (2.11). We therefore prefer a model with $m_{W,Z} \ll \Lambda$. The difficulty in lowering the W and Z masses far below Λ is due to naturalness of the magnitude of G. If we can find some argument to introduce two scales such that $\Lambda \gg G^{-1}$, the problem with experiment will be alleviated. We cannot resort to fine tuning of the W and Z masses since the model is free from quadratic divergence in the leading N_c order. Because of the absence of quadratic divergence, the model is more natural than the top-condensate model, but the naturalness prevents us from setting the W and Z masses many orders of magnitude below the compositeness scale.

Phenomenologically the composite weak-boson models of this type closely resemble the Standard Model with the physical Higgs mass of $O(\Lambda)$ at the tree-diagram level. However, the gauge interaction has a form factor even at the tree-diagram level typically like

$$F(q^2) = F(0)/(1 - q^2/\Lambda^2), \tag{2.15}$$

where q_μ is the momentum transfer. This form factor damping is an effective gauge symmetry breaking. The experimental lower bounds on Λ for composite fermions are currently in the range of a few TeV, but the scale Λ for composite W and Z has been little known and will be probed through the self-couplings of W, Z, and γ. The larger the cutoff Λ is, the better the dynamical gauge symmetry is. From this viewpoint, it appears difficult to have even an approximate gauge symmetry in the confinement-type models[16] of W and Z. The form factor damping generates deviation from the Standard Model through radiative corrections too. Can the model withstand the recent precision measurement of the electroweak parameters at LEP ? Though no thorough study has been made to analyze radiative corrections[15], the situation is precarious. The reason is that the region above the scale Λ in the loop-momentum is governed by the fundamental Lagrangian L_ψ which violates gauge symmetry explicitly. Though the low-energy effective Lagrangian $L + \Delta L_A$ contains this explicit breaking only in the vector boson mass μ, the magnitude of μ is not many orders smaller than Λ.

3. Composite Higgs Bosons

The infinity produced by loop diagrams is actually finite because the cutoff energy cannot possibly exceed the Planck mass. Since the gauge couplings of Standard Model are small at high energies, there is nothing unnatural in hiding logarithmic *infinity* by renormalization. However, divergence is quadratic for mass of a spinless boson, so an extremely fine tuning is required if we want to keep the Higgs boson mass at the electroweak energy scale. There exist two known ways to avoid this fine tuning. One is to introduce a broken supersymmery whose scale of breaking acts as an effective cutoff by the nonrenormalization theorem of supersymmetry. Some reason must still be found as to why a bare mass is many orders of magnitude smaller than the Planck mass. The other option is to abandon *elementary* Higgs bosons. The idea of composite Higgs bosons is therefore well motivated. After all, there is no elementary Higgs boson in the Bardeen-Cooper-Schrieffer model of superconductors to which the basic idea of the Standard Model can be traced back.

Among the composite Higgs boson models, the technicolor model[17,18] is the oldest and best known example. In the technicolor model, a strong vectorial gauge interaction binds preons, namely the techniquarks, into Higgs bosons. At the scale of

O(1 TeV), the binding force becomes so strong as to drive the symmetric Higgs-mass-square negative. Then a techniquark-antiquark pair condenses to stabilize the vacuum and breaks the electroweak gauge symmetry of $SU(2) \times U(1)$. These composite Higgs bosons do not interact with the quarks and leptons directly. In order to give mass to the quarks and leptons by the techniquark condensate, one must add yet another new interaction, called the extended technicolor interaction, at an even higher energy scale, Λ_{ETC}. The large *t*-quark mass requires a small Λ_{ETC}, while the stringent experimental upper bounds on flavor-changing neutral interactions cannot tolerate a small Λ_{ETC}. The allowed region of Λ_{ETC} is squeezed out completely for the original technicolor model. The *walking technicolor* model is an attempt to find the allowed region for Λ_{ETC} by postulating an irregular energy dependence for the techniquark condensate. Another class of models of composite Higgs bosons, called the supercolor model, was studied extensively by Georgi and his collaborators[19]. This model breaks simultaneously the $SU(2) \times U(1)$ gauge symmetry and the chiral symmetry in order to avoid flavor-changing neutral interactions.

An interesting model of composite Higgs bosons was recently proposed by Miransky, Tanabashi, and Yamawaki[20] and by Nambu[21]. The model was motivated by the steady rise of the experimental lower limit of the *t*-quark mass. Since the t-quark mass is now not far from the electroweak scale $v = 247$ GeV, the *t*-quark may be able to play *the role of the techniquarks* for symmetry breaking. In this model, the composite Higgs bosons provide mass directly to both the gauge bosons and the quarks (and the leptons, if generalized). The preons of the Higgs bosons are the third-generation quarks in its simplest version. One artificial aspect is that the binding force of four-fermion interaction must be introduced in an *ad hoc* manner at a high energy scale and finely tuned to generate the electroweak scale.

3.1. Top-Condensate Model

Since the essence of the model consists in the Higgs-fermion sector, we first focus on that part by taking the limit that all fermions are massless except for the *t*-quark. The model Lagrangian is

$$L_q = i\, \overline{\psi}_L \partial\!\!\!/ \psi_L + i\, \overline{t}_R \partial\!\!\!/ t_R + G(\overline{\psi}_L t_R)(\overline{t}_R \psi_L), \tag{3.1}$$

where $\psi_L = (t_L, b_L)^T$, $\psi_{L,R} = (1 \pm \gamma_5)\psi/2$, and color summation is understood in all quark bilinears. The color number N_c is taken to infinity to justify the chain diagram approximation below. By iterating the chain diagrams in the spin-0 channel of ψ_L and $\overline{t}_R$, we obtain a composite Higgs doublet Φ and its conjugate $\Phi^\dagger$.

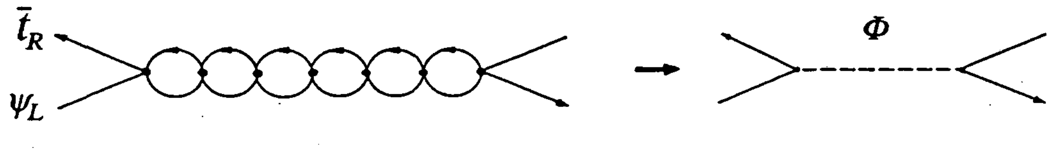

Fig.4. Formation of composite Higgs bosons by a quark-antiquark pair.

Since the diagrams are quadratically divergent, a cutoff is made at Λ. The Lagrangian L_q is regarded as an effective Lagrangian below the scale Λ. The four-fermion interaction may come from a more fundamental renormalizable Lagrangian if $\Lambda \ll M_{Planck}$ or may be intrinsically unrenormalizable if $\Lambda = O(M_{Planck})$. When the coupling G is increased beyond a critical value $G_c = 8\pi^2/N_c\Lambda^2$, the mass square of Φ turns negative. Then a symmetric vacuum becomes unstable, and Φ develops a vacuum-expectation-value, *i.e.*, the t-$\bar{t}$ pair forms a condensate, $\langle \bar{t}t \rangle \neq 0$. For this reason we refer to this type of models as the top-condensate model. The original fermionic Lagrangian L_q of Eq.(3.1) can be turned into an effective Lagrangian with the auxiliary fields Φ and $\Phi^\dagger$ as

$$L = i\bar{\psi}_L \partial\!\!\!/ \psi_L + i\bar{t}_R \partial\!\!\!/ t_R - f_o(\bar{\psi}_L\Phi)t_R - f_o\bar{t}_R(\Phi^\dagger\psi_L). \tag{3.2}$$

Computing the kinetic energy and self-interaction terms from the quark-loop diagrams of Fig.5, just as in the composite vector bosons in Section 2, we find

$$\Delta L_\Phi = Z_\Phi\, \partial_\mu\Phi^\dagger\partial^\mu\Phi - \kappa_o^2\Phi^\dagger\Phi - \lambda_o(\Phi^\dagger\Phi)^2/2, \tag{3.3}$$

where

$$Z_\Phi = N_c f_o^2 \, ln(\Lambda^2/m_t^2)/16\pi^2,$$

$$\lambda_o = N_c f_o^4 \, ln(\Lambda^2/m_t^2)/8\pi^2. \tag{3.4}$$

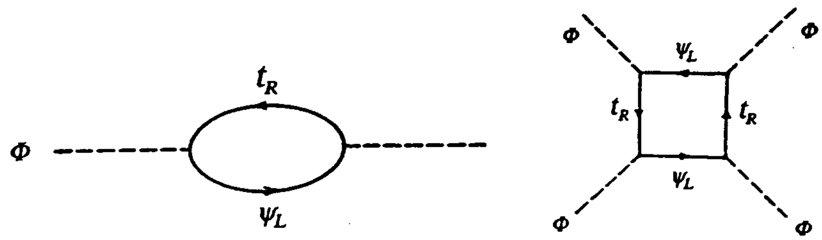

Fig.5. Generation of the kinetic energy and self-interaction terms of the Higgs doublet fields Φ.

Rescaling the Higgs fields as $\sqrt{Z_\Phi}\Phi \rightarrow \Phi$, we obtain

$$L = i\bar{\psi}_L \partial\!\!\!/ \psi_L + i\bar{t}_R \partial\!\!\!/ t_R + \partial_\mu\Phi^\dagger\partial^\mu\Phi - \kappa^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2/2$$

$$- f(\bar{\psi}_L\Phi)t_R - f\bar{t}_R(\Phi^\dagger\psi_L), \tag{3.5}$$

where

$$\kappa^2 = \kappa_o^2/Z_\Phi,$$

$$f^2 = f_o^2/Z_\Phi = 16\pi^2/[N_c \, ln(\Lambda^2/m_t^2)],$$

$$\lambda = \lambda_o/Z_\Phi^2 = 32\pi^2/[N_c \, ln(\Lambda^2/m_t^2)]. \tag{3.6}$$

When G is larger than the critical value $G_c = 8\pi^2/N_c\Lambda^2$, κ_o^2 is negative, so the symmetric vacuum is unstable. As G increases beyond G_c, the t-quark starts having mass, which compensates the increase in the effective binding force and prevents the zero-mass poles in the ($\bar{\psi}_L t_R$, $\bar{t}_R \psi_L$) channels from moving into the negative mass square region. The zero mass poles represent the three Nambu-Goldstone modes to be absorbed by the electroweak gauge bosons. Meanwhile, the pole in the channel $\bar{t}t = \bar{t}_L t_R + \bar{t}_R t_L$ moves up in the positive direction, giving the massive physical Higgs boson. The *t*-quark mass by condensation of the *t*-quark field is a sign of spontaneous breakdown of the chiral symmetry and also of the electroweak gauge symmetry. The value of the *t*-quark mass is determined by the self-consistency condition or the *gap equation* (see Fig. 6),

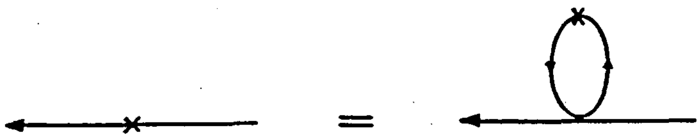

Fig.6. The gap equation for the *t*-quark mass. The cross denotes m_t.

$$1 = 2N_cG\int[d^4k/(2\pi)^4]/(k^2 + m_t^2), \quad (3.7)$$

where k_μ is the Euclidian loop-momentum. The integral in Eq.(3.7) is quadratically divergent. The natural choice of Λ is $O(1/\sqrt{G})$. In order to keep m_t much smaller than Λ, we must make a fine tuning $(G - G_c)/G_c = O(m_t^2/\Lambda^2)$. This is admittedly unnatural, but *thanks to this unnaturalness, there is a large room for the allowed values of Λ in contrast to the composite vector-boson models.* There is another benefit of a large Λ. When Λ is many orders of magnitude larger than the electroweak scale, flavor-changing neutral interactions, which enter the binding force when three generations are incorporated, are suppressed by $O(E^2/\Lambda^2)$ and totally harmless at low energies. However, a crucial weakness of the model is that a large Λ makes a decisive experimental test of the model virtually impossible, as I will elaborate later.

This simple model gives a striking prediction on the ratio of the physical Higgs mass to the t-quark mass,

$$m_H/m_t = \sqrt{\lambda}v/(fv/\sqrt{2}) = 2, \quad (3.8)$$

according to Eqs.(3.6). Furthermore, once a value of the cutoff Λ is chosen, the values of f^2 and λ are fixed so that m_H amd m_t themselves are obtained. For $\Lambda = 10^{15}$ GeV, for instance, the *t*-quark mass computed with f of Eq.(3.6) with $N_c = 3$ is about 160 GeV. It increases slowly as Λ decreases, reaching 250 GeV for $\Lambda = 10^8$ GeV. Therefore, the cutoff must be chosen very high in order for m_t to be consistent with the "experimental" upper bound. If a fourth generation of quarks is introduced,[22,23] the major contribution to the condensate comes from those heavier quarks, so the *t*-quark mass can be made smaller with a lower Λ. Otherwise the high cutoff Λ is the one of the main feature of the model.

3.2. Extended Top-Condensate Models

Generalizing the model to include the other quark masses is straightforward[20,24]. Replace the Lagrangian L_ψ of Eq.(3.1) by

$$L_\psi = \Sigma_\alpha i\bar{\psi}^{(\alpha)}{}_L \partial\!\!\!/ \psi^{(\alpha)}{}_L + \Sigma_j i\bar{u}^{(j)}{}_R \partial\!\!\!/ u^{(j)}{}_R + \Sigma_j i\bar{d}^{(j)}{}_R \partial\!\!\!/ d^{(j)}{}_R + GJ^\dagger J,$$

with

$$J = \Sigma_{\alpha,j}\, c_j{}^{(\alpha)}\bar{u}^{(j)}{}_R\psi^{(\alpha)}{}_L + \Sigma_{\alpha,j} c'_j{}^{(\alpha)}\, \bar{d}^{(j)}{}_R{}^c\psi^{(\alpha)}{}_L{}^c, \tag{3.9}$$

where the summation in α is over the three generations of left-handed doublets and the summation in *j* is over the right-handed quarks, *e.g.*, $u^{(3)}{}_R = t_R$. The superscript *c* denotes a charge-conjugated field, and the normalization condition $\Sigma_{\alpha,j}[|c_j{}^{(\alpha)}|^2 + |c'_j{}^{(\alpha)}|^2] = 1$ is imposed in Eq.(3.9).

The particular linear combination appearing in *J* is the eigenchannel in which a composite Higgs doublet is formed. For $G > G_c$, the symmetry breaks down spontaneously to generate the quark mass matrices for (u,c,t) and for (d,s,b):

$$M_{ab} = c_b{}^{(a)} f v/\sqrt{2} \quad \text{and} \quad M'_{ab} = c'_b{}^{(a)} f v/\sqrt{2}, \tag{3.10}$$

respectively, where f^2 is given by Eq.(3.6) in the leading $\log\Lambda$ order. Since the coefficients $c_b{}^{(a)}$ and $c'_b{}^{(a)}$ are arbitrary complex numbers, M_{ab} and M'_{ab} are the most general quark-mixing matrices. Unfortunately the model has no predictive power on the quark mass pattern. It is trivial to include the lepton mass.

The model Lagrangian of Eq.(3.9) generates a single Higgs doublet which feeds both the up-quark masses and the down-quark masses. It is easy to build a model which has two composite Higgs doublets Φ_1 and Φ_2, one for the up-quark masses and the other for the down-quark masses. One consequence of such a model[24] is that the the *t-b* quark mass difference must be explained by the difference in the two vacuum-expectation-values $\langle\phi_1{}^o\rangle$ and $\langle\phi_2{}^o\rangle$, namely, by the difference between the $\langle\bar{t}\, t\rangle$ and $\langle\bar{b}b\rangle$ condensates, not by the Yukawa couplings of the two Higgs doublets. It is easy to see that the Yukawa couplings of Φ_1 and Φ_2 are equal to each other and given by the first of Eq.(3.6) in the leading $\log\Lambda$ approximation. This equality is quite general for the two-doublet models and later will be understood in terms of an infrared fixed point of renormalization group.

3.2. Beyond Toy Model

The model is so far a toy model. The chain diagram approximation is justified only in the limit of $N_c = \infty$. The gauge interactions of $SU(3)_C \times SU(2)_L \times U(1)$ must be added to make a model for the real world. Nice predictions such as $m_H = 2m_t$ of the toy model are modified by radiative corrections of the gauge interactions and by finiteness of $N_c(= 3)$. One must take into account these effects somehow. The loop-diagram calculation beyond a few loops are cumbersom when gauge interactions are added. Bardeen, Hill, and Lindner[22] proposed to use the renormalization group for computation of the gauge interaction corrections and the finite N_c effects.

It is easy to deduce the running couplings $f(\mu)^2$ and $\lambda(\mu)$ of the toy model from Eq.(3.6). They are given by

$$f(\mu)^2 = f_o{}^2/Z_\Phi(\mu) = 16\pi^2/[N_c\, ln(\Lambda^2/\mu^2)],$$

$$\lambda(\mu) = \lambda_0/Z_\Phi(\mu)^2 = 32\pi^2/[N_c ln(\Lambda^2/\mu^2)], \tag{3.11}$$

where

$$Z_\Phi(\mu) = N_c f_0{}^2 ln(\Lambda^2/\mu^2)/16\pi^2. \tag{3.12}$$

If the Higgs doublet is composite, its field Φ ought to disappear from the Lagrangian at its scale of compositeness Λ. This does happen in the toy model (*cf.* Eq.(3.12)). Therefore, Bardeen *et al*[22] incorporated

$$Z_\Phi(\mu) \rightarrow 0 \quad \text{as } \mu \rightarrow \Lambda \tag{3.13}$$

as the condition of compositeness for Φ in solving the renormalization group. For the running coupling $f(\mu)^2$ and $\lambda(\mu)$, the compositeness condition (3.13) requires

$$f(\mu)^2,\ \lambda(\mu) \rightarrow \infty \quad \text{as } \mu \rightarrow \Lambda. \tag{3.14}$$

Bardeen *et al* integrated numerically the one-loop renormalization-group equations from Λ down to the electroweak scale with the boundary condition Eq.(3.14), including all interactions of the Standard Model with $N_c = 3$. Then $f(\mu)$ at the electroweak scale determines the *t*-quark mass. The values of m_t and m_H depend on the running distance $ln(\Lambda/v)$. The relation $m_H = 2m_t$ is no longer true. Their result of calculation is summarized in Table 1.

Table 1. The values of m_t, m_H, and m_H/m_t as functions of Λ, predicted by Bardeen *et al.*[22] The infinity for $f(\Lambda)^2$ and $\lambda(\Lambda)$ to be set equal to was replaced by some large finite number in the actual calculation. The uncertainty in the numerical values in the Table is typically from ~1% at $\Lambda = 10^{19}$GeV to ~10% at $\Lambda = 10^{4\sim5}$GeV.

Λ(GeV)	10^{19}	10^{17}	10^{15}	10^{13}	10^{11}	10^{10}	10^{9}	10^{8}	10^{7}	10^{6}	10^{5}	10^{4}
m_t(GeV)	220	225	231	239	250	257	266	279	295	320	362	458
m_H(GeV)	241	248	258	270	287	298	312	331	356	394	458	609
m_H/m_t	1.10	1.10	1.12	1.13	1.15	1.16	1.17	1.19	1.21	1.23	1.27	1.33

The predicted values of m_t are only marginally consistent with the theoretical upper bound ($\lesssim$ 200 GeV) on m_t which has been deduced from the electroweak parameter measurement. Nevertheless, it is interesting that the concrete numerical prediction comes out of the model in the right ballpark.

Aside from the closeness of the predicted values to the expected values for m_t, the prediction is very stable against variation of Λ and of the initial values $f(\Lambda)^2$ and $\lambda(\Lambda)$. In the numerical integration of the one-loop renormalization-group equations done by Hill *et al*[25] long before the top-condensate model, essentially any pair of the boundary values, $f(\Lambda)^2$ and $\lambda(\Lambda)$, larger than 1 converges to the small neighborhood of the point $(f(v)^2, \lambda(v)) = (2.2, 1.2)$. That is, once the light particle content is specified, the numerical prediction is very insentive to what happens at the compositeness scale Λ.

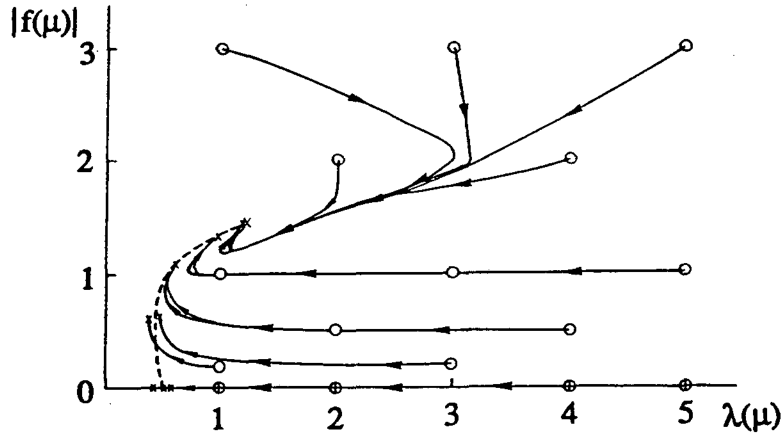

Fig.7. The flow diagram of the running couplings $|f(\mu)|$ and $\lambda(\mu)$ from Hill *et al.*[25] $|f|$ and λ start at open circles with $\Lambda = 10^{15}$ GeV

In the language of renormalization group, insensitivity of $f(v)^2$ and $\lambda(v)$ is due to the infrared stability of the couplings $f(\mu)^2$ and $\lambda(\mu)$. The same is true for the coupling ratio $f(\mu)^2/\lambda(\mu)$. Let us elaborate this point.

The one-loop renormalization-group equations read

$$16\pi^2\, \mu df^2 d\mu = (2N_c + N_f + 1)f^4 + (\text{terms of } g_c{}^2f^2,\ g_f{}^2f^2,\ g_1{}^2f^2),$$

$$16\pi^2\, \mu d\lambda/d\mu = 4N_cf^2\lambda - 4N_cf^4 + 2(N_f + 4)\lambda^2 + (\text{terms of } g_f\lambda^2,\ g_1{}^2f^2),$$

$$16\pi^2\, \mu dg_c{}^2/d\mu = -\,(22N_c/3 - 8)g_c{}^4,$$

$$16\pi^2\, \mu dg_f{}^2/d\mu = -\,(22N_f/3 - 25/3)g_f{}^4,$$

$$16\pi^2\, \mu dg_1{}^2/d\mu = |c|g_1{}^4, \qquad (3.15)$$

where the subscript f refers to the electroweak $SU(N_f)$. $N_c = 3$ and $N_f = 2$ in the real world. The positive constant $|c|$ in the last line depends on the U(1) charge assignment. For the standard assignment with $N_c = 3$ and $N_f = 2$, $|c| = 41/3$ for three generations with one Higgs doublet. It is well known that $f^2 = \lambda = 0$ is the infrared fixed point when the gauge couplings are turned off. Therefore, $f(\mu)^2$ and $\lambda(\mu)^2$ evolve very slowly toward zero at low energies, which is the reason for insensitivity to Λ when Λ is large. To understand the stability of $f(\mu)^2/\lambda(\mu)$ at low energies, we should examine the renormalization-group equation for this ratio of the couplings $\rho(\mu) \equiv f(\mu)^2/\lambda(\mu)$ for $N_c = 3$ and $N_f = 2$:

$$16\pi^2\mu d\rho/d\mu = 12f^2(\rho + 0.88)(\rho - 1.13) + (\text{gauge coupling terms}). \qquad (3.16)$$

Without the gauge couplings, the ratio $\rho(\mu)$ is attracted to the infrared-fixed point $\rho = 1.13$ in the low-energy limit. When Λ is large, ρ actually reaches a close neighborhood of this fixed point. That is the reason why for large Λ the low-energy ratio $f(v)^2/\lambda(v)$ ($= 2m_t{}^2/m_H{}^2$) is very insensitive to the scale Λ and to its initial value at Λ. The infrared-fixed value $\rho = 1.13$ in the zero gauge-coupling approximation gives $m_t/m_H =$

0.79, while the numerical calculation by Bardeen *et al*[22] including the gauge couplings gives $m_t/m_H = 0.91$. For the original toy model[20,21] for which $N_c \to \infty$ and $g_c^2 = g_f^2 = g_1^2 = 0$, the renormalization-group equation for $\rho(\mu)$ is

$$16\pi^2 \mu d\rho/d\mu = 4N_c\lambda(\rho - 1/2). \quad (3.17)$$

This infrared-fixed value $\rho = 1/2$ leads to the toy-model prediction $m_H = 2m_t$.

We can build a physical picture of the composite Higgs boson from the program of Bardeen *et al.* In the coordinate space, the composite Higgs boson is made of a small core of size $1/\Lambda$ at the center in which the binding force acts. Outside the core, a cloud of particles extends over a long distance. The property of this cloud was analyzed by renormalization group. The low-energy values of $f(\mu)^2$ and $\lambda(\mu)$ are determined almost entirely by the property of the cloud. They are hardly affected by the size of the core $1/\Lambda$ nor by the values of $f^2(\Lambda)$ and $\lambda(\Lambda)$ at the core. When something is varied at the core, the particle cloud always copmpensates the variation according to infrared stability of renormalization group. Therefore we cannot *see through* the core behind the cloud.

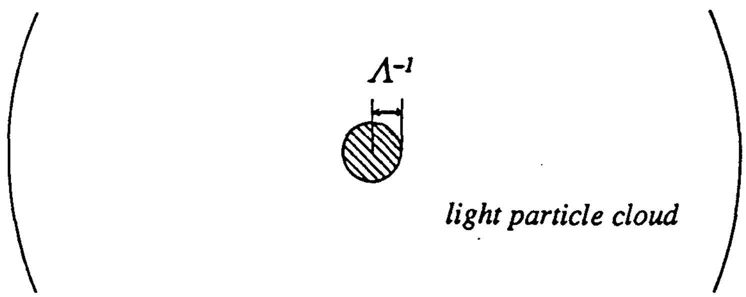

Fig.8. The picture of the composite Higgs boson. The size of the core is blown up in scale.

It has been pointed out that if binding forces of higher dimension are added to the simplest model of the Lagrangian L_ψ in Eq.(3.1), for instance[26],

$$L_{int} \to J^\dagger J; \qquad J = (g/\Lambda)(\bar{t}_R\psi_L) + (f/\Lambda^3)(\partial^\mu\bar{t}_R\partial_\mu\psi_L), \quad (3.18)$$

the toy-model prediction can be altered. In fact, the prediciton of the top-condensate model can be changed arbitrarily in principle.[27,28] That is, once more general interacitons are included, the model ceases to have a predictive power. However, in order to change the prediction substantially, one has to overcome infrared stability. We can do so only if we add higher-dimensional interactions with very large coupling constants and keep the running distance $ln(\Lambda/v)$ fairly short. In the toy model prior to the renormalization-group analysis, the masses and the couplings can be changed with the higher-dimensional interactions but not in the leading order of $\log\Lambda$.[26,27] In this sense, the numerical prediction by Bardeen *et al*[22] is *practically unique* or *the most natural one*[29] as long as the scale Λ is very high.

This stability against additional interactions is a nice feature of the model. However, this same feature has a disturbing side from a different viewpoint. Being so stable and insensitive to dynamics of binding, the prediction which results from the renormalization-group analysis tells practically nothing about dynamics of binding. In fact, the numerical prediciton of m_t and m_H does not decisively test whether or not the Higgs fields are composite nor whether the *t*-quark condensation actually occurs. To make the point, I will present a model where the right-handed *t*-quark is composite of the left-handed quarks and of *elementary* Higgs bosons. By renormalization-group analysis, this model predicts the same values for m_H and m_t as those of the top-condensate model.

4. Composite Fermions

4.1 Composite t_R Model

Simple soluble models of composite fermions can be built with fermionic preons and bosonic preons[30]. In order to compare with the top-condensate model, we present a model in which the right-handed *t*-quark is composite of the left-handed quark doublet $\psi_L = (t_L, b_L)^T$ and an elementary Higgs doublet, *i.e.*, $t_R \sim \Phi\psi_L$. The electroweak symmetry is broken by a vacuum expectation value of the Higgs boson field through the standard Higgs potential $V(\Phi^\dagger\Phi)$. To make the model solvable, we modify the electroweak $SU(2)_L$ into $SU(N)_L$ and take the large N limit. Let us denote the electroweak symmetry by $SU(N_f)_L$. The $SU(3)_c \times SU(N_f)_L \times U(1)$ transformation properties of the particles are

$$\psi_L = (3, N_f, Y/2),$$

$$\Phi = (1, \overline{N_f}, Q_t - Y/2), \qquad t_R = (3, 1, Q_t). \tag{4.1}$$

We introduce an appropriate binding force necessary to form a bound state in the $\Phi\psi_L$ channel (see Fig.9). For instance, the unrenomalizable interaction

$$L_{int} = -i\Sigma_{a,b}G[\overline{\psi_L}^a\Phi_a{}^\dagger(\partial\Phi^b)\psi_{Lb}] + h.c., \tag{4.2}$$

serves this purpose. The indices *a* and *b* refer to the N_f flavors of $SU(N_f)$ and the color summation is understood in Eq.(4.2). By tuning the coupling G to a value of $O(\Lambda^{-2})$, we can generate a light composite singlet-quark ξ with the quantum numbers

$$\xi_R \sim (\partial\Phi)\psi_L = (3, 1, Q_t),$$

$$\xi_L \sim \Phi\psi_L = (3, 1, Q_t). \tag{4.3}$$

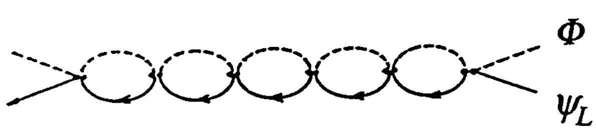

Fig.9. Formation of a composite singlet-quark ξ from a left-handed quark doublet ψ_L and an elementary Higgs doublet Φ.

The field ξ_L is not wanted for the Standard Model. In order to remove it from the light particle spectrum,† we introduce another elementary quark η_R and a singlet Higgs field ρ which interact with $\Phi\psi_L$ through the interaction $\sim(1/\Lambda)\overline{\psi}_L\Phi^\dagger\rho\eta_R$. When the composite ξ is formed, this interaction generates the effective Yukawa interaction

$$L_{int} = - f_\rho \overline{\xi}_L \rho \eta_R . \tag{4.4}$$

If we so arrange the potential of the field ρ as to generate a large vacuum-expectation-value $\langle\rho\rangle = O(\Lambda)$, the interaction Eq.(4.4) generates a large Dirac mass $M_\eta = O(f_\rho\Lambda)$ between ξ_L and η_R. Then the mass matrix among ξ_R, $\xi_L{}^c$, and η_R is of the form

$$M = \begin{pmatrix} 0 & m_\xi & 0 \\ m_\xi & 0 & M_\eta \\ 0 & M_\eta & 0 \end{pmatrix} . \tag{4.5}$$

One linear combination of ξ_R and η_R is a massless eigenmode which is identified with the right-handed *t*-quark:

$$t_R = \xi_R \cos\alpha - \eta_R \sin\alpha \qquad (\tan\alpha = m_\xi/M_\eta), \tag{4.6}$$

The mode orthogonal to t_R is a supermassive singlet quark with the mass $\sqrt{M_\eta{}^2 + m_\xi{}^2}$. Let us tune m_ξ to a value much smaller than Λ, for instance, the electroweak scale. Then the t_R-quark is made of the composite particle ξ_R up to $\tan^2\alpha = O(v^2/\Lambda^2)$ in probability. The low-energy effective Lagrangian of this model is identical with the Standard Model Lagrangian except that the parameters associated with the t_R-quark are calculable as functions of the cutoff Λ.

The light particle content of our model is identical with that of the Standard Model. In contrast to the top-condensate model, this toy model does not predict the physical Higgs boson mass even in the large N_f limit since Higgs bosons are elementary. However, we will later see that renormalization-group analysis leads us to the same prediction on m_H as that of the top-condensate model. The Yukawa coupling of the *t*-quark in this model is calculable from the infinite seiries of chain diagrams in Fig.9:

$$f(\mu)^2 = 32\pi^2/[N_f ln(\Lambda^2/\mu^2)]. \tag{4.7}$$

The same result follows by the auxiliary field method for ξ as

$$f(\mu)^2 = f_o{}^2/Z_\xi(\mu) \tag{4.8}$$

with

$$Z_\xi(\mu) = 32\pi^2 f_o{}^2 N_f ln(\Lambda^2/\mu^2). \tag{4.9}$$

† It is interesting to note that η_R is precisely what we would need to satisfy the anomaly matching condition of 'tHooft.[31] However, since our model is unrenormalizable, the nonrenormalization theorem of the anomaly is not guaranteed. It is not clear what significance the simple anomaly matching has in this model.

If $N_f = 2$ is substituted, Eq.(4.7) would yield a value for the *t*-quark mass which is $\sqrt{3}$ times larger than the corresponding value of the top-condensate model. The predicted value for m_t would be too large. However, it is no more than a toy-model prediction. To make a prediction for the real world, we must include the gauge interactions with $N_f = 2$ instead of $N_f \to \infty$. If we follow the prescription[22] followed in the top-condensate model, we should use the renormalization group by feeding the compositeness condition

$$Z_\xi(\mu) \to 0 \quad \text{as } \mu \to \Lambda,$$

i.e.,

$$f(\mu)^2 \to \infty \text{ as } \mu \to \Lambda, \tag{4.10}$$

as the boundary condition. In the composite t_R-model, the bare coupling λ_0 of the Higgs self-interaction is a free parameter. However, when we move from the large N_f limit to the real world of $N_f = 2$, the renormalized coupling $\lambda(\mu)$ communicates with $f(\mu)$ below Λ and grows with $f(\mu)$ as μ approaches Λ. More precisely, the large $f(\mu)$ enters $\lambda(\mu)$ through the quark-box diagram of Fig.10 and pushes $\lambda(\Lambda)$ to ∞ as $\mu \to \Lambda$.

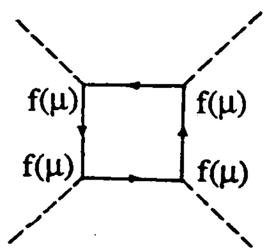

Fig.10. The quark-box diagram which enhances $\lambda(\mu)$ with $f(\mu)^4$ just below Λ

In terms of the one-loop renormalization-group equation for λ, $4N_cf^2\lambda - 4N_cf^4$ in the right-hand side of Eq.(3.15) enhance $\lambda(\mu)$ like $1/\ln(\Lambda^2/\mu^2)$ below $\mu = \Lambda$. Therefore, even though the bare coupling λ_0 is arbitrary, the boundary conditions on $f(\mu)^2$ and $\lambda(\mu)$ at Λ, as approached from below, are the same as in the top-condensate model:

$$f(\Lambda)^2,\ \lambda(\Lambda) \to \infty. \tag{4.11}$$

Since the low-energy particle spectrum of this model is identical with that of the top-condensate model, the renormalization-group equations are also the same. In fact, as we expect, the renormalization group-equations for the top-condensate model and the composite t_R-model in the toy-model version are the different large-N-limits of the complete one-loop equations. In the top-condensate model ($N_c \to \infty$), the renormalization-group equations for $f(\mu)^2$ and $\lambda(\mu)$ are obtained from the two equaitons in Eq.(3.11):

$$16\pi^2\, \mu df^2/d\mu = 2N_cf^4,$$

$$16\pi^2\, \mu d\lambda/d\mu = 4N_cf^2\lambda - 4N_cf^4. \tag{4.12}$$

For the composite t_R-model ($N_f \to \infty$), the running coupling Eq.(4.7) gives the renormalization-group equation

$$16\pi^2 \mu df^2/d\mu = N_f f^4. \tag{4.13}$$

Eqs.(4.12) and (4.13) are nothing other than the large N_c and N_f limits, respectively, of the complete one-loop renormalization-group equations Eq.(3.15) with the gauge couplings turned off.

To summarize, when the running distance $\ln(\Lambda/v)$ of the renormalization group is long, the low-energy values of couplings converge to an infrared fixed point of the renormalization group. Then the top-condensate model and the composite t_R model predict the same values for the low-energy parameters. That is, we cannot distinguish between the two models by looking only at the low-energy parameters when the compositeness scale is high. It is possble to build a model[30] of a composite ψ_L from elementary t_R and Φ which leads to the same low-energy predictions as these two models if the compositeness scale is very high. Such a model is hardly surprising.

5. Model Dependence of Structure of the Higgs Sector

We have argued that when the scale of compositeness is very high, our knowledge in low-energy physics does not provide a useful information as to which particles are composite or whether any particle is composite. The numerical prediction at low energies rests entirely on the single input that the running couplings of composite particles blow up at the compositeness scale. Compositeness at Λ certainly leads to $f(\Lambda)^2 = \infty$ in one-loop, but the converse may not necessarily be true. The relation $f(\Lambda)^2 = \infty$ in one-loop may simply mean a *strongly interacting Higgs sector at* Λ.[32-34] Nevertheless, if the t-quark mass and the Higgs boson mass are found in future experiment at the values predicted by Bardeen *et al*,[22] the composite Higgs-boson scenario is clearly the most attractive possibility. Are we sure that there is no clue at low energies to distinguish among different models ? In fact, there is some though it is tenuous. In this section, I show that different constraints are imposed on the structure of the Higgs boson sector, depending on which particle is composite.[35] Different Higgs sectors are required when we try to give mass to the quarks other than the t-quark and to the leptons.

5.1 Top-Condensate Model

In the top-condensate model, a single Higgs-doublet can give mass to all fermions, as was explained in Sec.3. If two doublets are introduced, the difference in the up-fermion masses and the down-fermion masses must be accounted for by differing the two vacuum-expectation-values. The reason is that the low-energy Yukawa couplings of the up-fermions and of the down-fermions are equal to each other because an infrared fixed point is located at this ratio equal to one. Let us elaborate this point.

When two Higgs doubelts are formed as composite particles, each of them must be made of ψ_L and either the right-handed up-quarks or the right-handed down-quarks in order to avoid flavor-changing neutral interactions.[36] We will hereafter denote the two doublets by Φ_t and Φ_b:

$$\Phi_t \sim \Sigma_{\alpha,j} c_j^{(\alpha)} \overline{u}^{(j)}{}_R \psi^{(\alpha)}{}_L, \quad \Phi_b \sim \Sigma_{\alpha,j} c'_j{}^{(\alpha)} \overline{d}^{(j)}{}_R{}^c \psi^{(\alpha)}{}_L{}^c, \tag{5.1}$$

where

$$\Sigma_{\alpha,j}|c_j^{(\alpha)}|^2 = \Sigma_{\alpha,j}|c'_j{}^{(\alpha)}|^2 = 1. \qquad (5.2)$$

The charged lepton term may be added to Φ_b, or to Φ_t, with a suitable modification of the normalization condition on the coefficients. The running Yukawa couplings f_t and f_b of Φ_t and Φ_b to the quarks can be computed easily in the toy model. In the limit of $m_t \gg m_c$, m_u and $m_b \gg m_s$, m_d, they are given by

$$f_t(\mu)^2 = f_b(\mu)^2 = 16\pi^2/[N_c ln(\Lambda^2/\mu^2)]. \qquad (5.3)$$

The compositeness condition of Φ_t and Φ_b is

$$f_t(\mu)^2, \ f_b(\mu)^2 \to \infty \ \text{as} \ \mu \to \infty. \qquad (5.4)$$

When Eq.(5.4) is imposed as the boundary condition in the renormalization-group equations including the gauge couplings with $N_c = 3$, the low-energy values of the Yukawa couplings are obtained. Their values are equal to each other up to a tiny perturbation of the U(1) gauge coupling,

$$f_b(v)^2 = f_t(v)^2 \qquad (5.5)$$

and the common value is very close to the $f(v)^2$ of the *minimal* top-condensate model. It is seen in a numerical calculation[37] that the equality of Eq.(5.5) holds with a high precision. The reason is that $f_b^2/f_t^2 = 1$ is an infrared fixed point up to the U(1) coupling preturbation:

$$16\pi^2\mu d(f_b/f_t)/d\mu = 3f_t f_b(f_b^2/f_t^2 - 1) + g_1^2 f_b/f_t. \qquad (5.6)$$

Consequently the difference of the *t*- and *b*-quark masses must be accounted for by that of the two vacuum-expection-values, $v_t = \sqrt{2}\langle\phi_t^o\rangle$ and $v_b = \sqrt{2}\langle\phi_b^o\rangle$:

$$m_b/m_t = f_b(v)\langle\phi_b^o\rangle/f_t(v)\langle\phi_t^o\rangle = v_b/v_t. \qquad (5.7)$$

Since the only constraint on v_t and v_b is $m_w^2 = g^2(v_t^2 + v_b^2)/4$, it is free to adjust the ratio v_b/v_t and to produce any value for m_b/m_t. If we arrange appropriately the coefficients $c_j^{(\alpha)}$ and $c'_j{}^{(\alpha)}$ in Eq.(5.1), we can reproduce other quark masses too.

Therefore, in the top-condensate model, there is no constraint on the Higgs sector stronger than the constraint imposed on the Standard Model.

5.2. Composite t_R-Model

The situation is different in the extention of the composite t_R-quark model. First of all, we cannot give the right masses to both the *t*-quark and the *b*-quark by a single Higgs doublet Φ, if t_R and b_R are composite. The reason is as follows: The composite b_R must be formed of $\Phi^\dagger$ and ψ_L. The Yukawa couplings in the toy model are[30]

$$f_t(\mu)^2 = f_b(\mu)^2 = 32\pi^2/[N_f ln(\Lambda^2/\mu^2)]. \qquad (5.8)$$

Solving the renormalization-group equations, we find that at the electroweak scale, the Yukawa coupling ratio reaches the close neighborhood of its infrared fixed point

$$f_t(v)^2/f_b(v)^2 = 1. \tag{5.9}$$

When there is only one vacuum-expectation-value $\langle\phi^o\rangle = \langle\phi^{o\dagger}\rangle = v/\sqrt{2}$, the relation $m_b = m_t$ is an inevitable consequence. Only by introducing two elementary Higgs doublets, Φ_t and Φ_b, can we avoid $m_b = m_t$.

Therefore, in the composite right-handed quark models, it is not an option but a necessity to introduce more than one Higgs doublet in order to generate the *t*- and *b*-masses correctly. With the two doublets Φ_t and Φ_b, the t_R- and b_R-quarks are formed by $\Phi_t\psi_L$ and $\Phi_b\psi_L$, respectively. Once Φ_t and Φ_b are introduced, we can generate any value for m_b/m_t by differing the two vacuum-expectation-values through the Higgs potential. For the τ-lepton, we probably need not introduce a third doublet Φ_τ. The equality $m_\tau = m_b$ at a high scale Λ can be made consistent with the observed mass ratio $m_\tau/m_b \approx 1/3$ at our energies: We learned it in grand unified theories that the gluon corrections can account for the ratio of 3 if the gluon cloud is integrated from Λ to low energies.[38]

If we try to make the the first- and second-generation quarks also composite, we must introduce more Higgs doublets. The reason is that all of the Yukawa couplings have a common infrared fixed value up to the small U(1) coupling effect.[35] This infrared fixed value is equal to $f(v)^2$ of the minimal top-condensate model. Therefore, we need as many vacuum-expectation-values as the number of quarks and leptons. This proliferation of Higgs fields causes a serious problem of flavor-changing neutral interactions. Therefore, the simplest or least unnatural composite right-handed-fermion model is the one in which only the third generation fermions are composite with two Higgs doublets Φ_t and Φ_b. Treating the third generation different from the rest is not so ugly as you might think. After all, the motivation of the top-condensate model was that the third generation, in particular the *t*-quark, is somehow special.

6. Critical instability

In the top-condensate model and its entention, the magnitude of the fermion mass generated through the gap equation is extremely sensitive to the value of the cutoff Λ. The reason is that the gap equation is written in terms of the two dimensionless variables, m^2/Λ^2 and $G^2\Lambda^2$. To get a fermion mass much smaller than Λ, one has to tune finely the coupling G near the critical value $G_c = 8\pi^2/N_c\Lambda^2$. A tiny perturbation to the binding force results in a large shift in the fermion mass. If two separate gap equations exist for the *t*- and *b*-quarks, their masses are very sensitive to the two couplings G_t and G_b of four-fermion binding forces. It is therefore tempting to attribute the large *t-b* mass splitting to this sensitivity, called the *critical instability*.[20,39-41] To make this idea work, one postulates that the fundamental four-fermion binding forces be strictly equal for the *t*- and *b*-quarks. Then the only difference between the two binding forces is due to the U(1)-gauge coupling. Though this gauge coupling effect is normally tiny, the large cutoff Λ enhances it enormously to account for the desired *t-b* mass splitting. We argue in this section that generating the *t-b* mass splitting by the critical instability is equivalent to introducing two composite Higgs doublets Φ_t and Φ_b and making a fine tuning of the vacuum-expectation-values of the two Higgs fields.

Let us examine the fine tuning problem first in the case of a single Higgs doublet with the only one quark mass $m_t \neq 0$. The gap equation Eq.(3.7) can be rewritten into

$$(G - G_c)/G = (8\pi^2/N_c)v^2/\Lambda^2, \qquad (6.1)$$

where $G_c = 8\pi^2/N_c\Lambda^2$ and $m_t^2 = f^2v^2/2$ with $f^2 = 16\pi^2/[N_c ln(\Lambda^2/m_t^2)]$ have been used. In the low-energy effective theory, the coupling G is hidden in v. It is transparent in the form of Eq.(6.1) that the fine tuning of G means that of v. The gap equation not only determines m_t, but also is the condition that the Nambu-Goldstone bosons appear in the $\bar{t}_R\psi_L$ channels.

It is obvious from this simple observation that if two gap equations exist, there ought to be two composite Higgs-doublets Φ_t and Φ_b. This conclusion can be easily verified in a simple solvable model.[42] The two gap equations can be obtained in the toy model when there are strong enough binding forces in the two eigenchannels of quark-antiquark scattering, $\bar{t}_R\psi_L$ and $\bar{b}_R\psi_L$:

$$L_{int} = G_t(\bar{\psi}_L t_R)(\bar{t}_R\psi_L) + G_b(\bar{\psi}_L b_R)(\bar{b}_R\psi_L), \qquad (6.2)$$

where G_t and G_b are tuned near the critical value $G_c = 8\pi^2/N_c\Lambda^2$. The resulting gap equations are

$$1 = 2N_cG_t\int[d^4k/(2\pi)^4]/(k^2 + m_t^2), \qquad (6.3)$$

$$1 = 2N_cG_b\int[d^4k/(2\pi)^4]/(k^2 + m_b^2). \qquad (6.4)$$

As we can check easily, the gap equation (6.3) is the condition that a massless bound state be formed as the Nambu-Goldstone boson in the $i\bar{t}\gamma_5 t$ channel. It is a component of the composite Higgs doublet $\Phi_t \sim \bar{t}_R\psi_L$ whose neutral component develops vacuum-expectation-value $\langle\phi_t^o\rangle = v_t/\sqrt{2}$. Similarly Eq.(6.4) is the condition that the Nambu-Goldstone boson be formed in the $i\bar{b}\gamma_5 b$ channel, which belongs to the second doublet Φ_b with the vacuum-expectation-value $\langle\phi_b^o\rangle = v_b/\sqrt{2}$. For the charged modes, one linear combination $\phi_t\cos\beta + \phi_b\sin\beta$ ($\tan\beta \equiv v_b/v_t$) and its conjugate are the Nambu-Goldstone modes. The two massless neutral bosons appear in this simplified model since there are two global U(1) symmetries in the Lagrangian (6.2):

$$U(1)_t: \quad t_R \rightarrow e^{i\delta}t_R,$$

$$U(1)_b: \quad b_R \rightarrow e^{i\delta'}b_R. \qquad (6.5)$$

They lead to the Peccei-Quinn U(1) symmetry[43] for the low-energy effective Lagrangian. If one wishes to avoid it, one should break $U(1)_t \times U(1)_b$ to the electroweak U(1) by adding to L_{int} the term

$$\Delta L_{int} = G_{tb}(\bar{\psi}_L t_R)(\overline{b_R{}^c}\psi_L{}^c). \qquad (6.6)$$

For simplicity I will keep on working with $G_{tb} = 0$. The two gap equations Eq.(6.3) and (6.4) can be rewritten, just like Eq.(6.1), into

$$(G_t - G_c)/G_t = (8\pi^2/N_c)v_t^2/\Lambda^2, \quad (6.7)$$

$$(G_b - G_c)/G_b = (8\pi^2/N_c)v_b^2/\Lambda^2, \quad (6.8)$$

where $G_c = 8\pi^2/N_c\Lambda^2$. Tuning G_t and G_b is equivalent to tuning v_t and v_b. The Yukawa couplings of Φ_t and Φ_b are free from quadratic divergence and independent of G_t and G_b. In the low-energy effective Lagrangian, the quark masses are given by $m_t = f_t v_t/\sqrt{2}$ and $m_b = f_b v_b/\sqrt{2}$ with $v^2 = v_t^2 + v_b^2$ where $f_t^2 = 16\pi^2/[N_c ln(\Lambda^2/m_t^2)]$ and $f_b^2 = 16\pi^2/[N_c ln(\Lambda^2/m_b^2)]$.

The idea of critical instability postulates $G_t = G_b$. The perturbation of the gauge interactions can be incorporated approximately[39-41] by adding to G_t and G_b the terms

$$\Delta G_t \approx 4g_3^2/3\Lambda^2 + g_1^2/9\Lambda^2, \quad (6.9)$$

$$\Delta G_b \approx 4g_3^2/3\Lambda^2 - g_1^2/18\Lambda^2 \quad (6.10)$$

in the case of $N_c = 3$. The small difference $\Delta G_t - \Delta G_b \approx g_1^2/6\Lambda^2$ and the *t-b* mass splitting are related by[42]

$$m_t^2 - m_b^2 \approx 3g_1(\Lambda)^2 f^2\Lambda^2/(16\pi^2)^2, \quad (6.11)$$

or

$$v_t^2 - v_b^2 \approx 6g_1^2(\Lambda)\Lambda^2/(16\pi^2)^2. \quad (6.12)$$

To generate the desired *t-b* mass difference with the known U(1) coupling, the value of the cutoff Λ is preferred to be O(100 TeV). However, this value of Λ is much too low to give the *t*-quark mass in the right range (*cf.* Table 1).

One easy way out of this difficulty is to introduce the fourth-generation quarks, t' and b'. Then there are *four* gap equations, one each for (t, b, t', b'). The effective four-fermion couplings are

$$G_t = G_3 + \Delta G_t, \quad G_b = G_3 + \Delta G_b,$$

$$G_{t'} = G_4 + \Delta G_t, \quad G_{b'} = G_4 + \Delta G_b, \quad (6.13)$$

where both G_3 and G_4 are tuned near G_c with $G_4 > G_3$, while ΔG_t and ΔG_b are given by Eqs.(6.9) and (6.10), respectively. The four gap equations mean the four composite Higgs doublets, Φ_t, Φ_b, $\Phi_{t'}$, and $\Phi_{b'}$, made of $\overline{t}_R\psi_L$, $\overline{b}_R\psi_L$, $\overline{t'}_R\psi_L$, and $\overline{b'}_R\psi_L$, respectively. For the overall scale of the quark masses, the values predicted in Table 1 are applicable to $(m_t^2 + m_b^2 + m_{t'}^2 + m_{b'}^2)^{1/2}$ instead of m_t. The relation $G_t - G_b = G_{t'} - G_{b'}$ which results from Eq.(6.13) predicts[41]

$$m_{t'}^2 - m_{b'}^2 = m_t^2 - m_b^2. \quad (6.14)$$

This is a genuine *prediction* of the critical instability. However, $m_{t'}^2 + m_{b'}^2$ cannot be predicted, but is *tuned* by G_4.

6. Conclusion

The predictions of composite particle models with a very high compositeness scale are virtually model-independent. They depend only on the low-energy particle spectrum of the model, not on how composite particles are built nor on which particles are composite. If we want to build a model which can be tested unambiguously by experiment, a model must have a low compositeness scale. What options do we have if we want the third-generation quarks to be the preons of the Higgs bosons? Aside from introducing a fourth generation, there are not so many options. Several models have been proposed since the top-condensate model. It would be the most economical if we could generate the Higgs doublet Φ from ψ_L and $\bar{t}_R$ through exchange of Φ, but this is impossible because chirality conservation and $SU(2)_L$ invariance conflict with each other in such a model. The simplest binding force is probably spin-one-boson exchange between ψ_L and $\bar{t}_R$. Whether the spin-one-boson is abelian or nonabelian, vector-boson-exchange generates forces in spin-one channels of $\bar{\psi}_L\psi_L$ and $\bar{t}_R t_R$ as well as in spin-zero channels of $\bar{\psi}_L t_R$. If the most-attractive-channel analysis is made in the Born approximation, the force is stronger in the spin-zero channel. Therefore, if bound states are formed, they will first appear in the channels of the Higgs doublet Φ, and, if the coupling is strong enough, in the spin-one channels.

In my semi-quantitative analysis so far made, it is very difficult to obtain a Yukawa coupling of a composite Higgs particle small enough to lead to $m_t < 200$ GeV. The reason is as follows: If one solves a bound-state problem in any known method, *e.g.*, Bethe-Salpeter equation, the N/D method, *etc*, the coupling of a bound state computed from the residue of the pole is usually of the order of $f^2/4\pi = 1$ or more often a little larger. In fact $f/4\pi^2 \approx 1$ is more natural. The old-timers familiar with the S-matrix bootstrap calculations probably remember that the attempt to get the experimental value $f_{\rho\pi\pi}^2/4\pi \approx 2.5$ for the ρ meson was not successful; $f_{\rho\pi\pi}^2/4\pi$ almost always came out to be larger than 2.5. It is true that the Yukawa coupling of the *t*-quark to the Higgs boson is strong, but *not strong enough* to be easily generated in a dynamical calculation of a composite particle. For the ρ meson, the narrowness of the decay width was finally understood by the quark picture: The ρ meson is a $^3S_1$ bound state of quark-antiquark, *not* a *p*-wave $\pi\pi$ resonance: the ρ meson mass should be understood as twice the constituent quark mass, not computable in terms of the attractive $\pi\pi$ interaction force in the *p*-wave channel. Since the ρ meson is made of $q\bar{q}$, it decays *reluctantly* into the only open channel, which is $\pi\pi$. If we look into the inside of the ρ meson, its $\pi\pi$ composition is much less than 100% of the all hadron compositions. If we follow this lesson, we may introduce new constituents and try to make a composite Higgs state which interacts less strongly with the *t*-quark. This will take us back to the fourth-generation extention or the technicolor model.

If a desired model can be built, what does it look like and how can it be tested ? The low-energy limit of Higgs-boson dynamics is completely fixed by chiral invariance, so it tells nothing about compositeness of Φ. Only when the $\Phi\Phi$ collision energy becomes substantially above the threshold, do we expect some difference among different models of Φ. It is yet to be studied what distinguishes conclusively between the Standard Model and the low-scale, non-technicolor composite model of Φ, if any.

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