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NONCONVEXITY, EFFICIENCY AND EQUILIBRIUM IN EXHAUSTIBLE RESOURCE DEPLETION

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Abstract: We reconsider the problem of inefficiency and nonexistence of a competitive equilibrium in exhaustible resource markets where extraction costs are nonconvex. The existence of a backstop technology (which induces a flat portion of the industry demand curve) restores both existence and efficiency, provided that the backstop price is sufficiently low. If firms face even a small amount of uncertainty regarding their rivals' stocks, a backstop technology is sufficient to restore existence of competitive equilibrium, even if the backstop price is very high. In this case, however, the competitive equilibrium is not efficient.

Key words: exhaustible resource, nonconvex costs, existence of competitive equilibrium

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NONCONVEXITY, EFFICIENCY AND EQUILIBRIUM IN EXHAUSTIBLE RESOURCE DEPLETION

1. Introduction

A competitive equilibrium may not exist or may be incapable of reproducing a socially efficient ("optimal") allocation if production costs are nonconvex. A common form of nonconvexity, U-shaped average cost curves, does not give rise to problems of existence or optimality of competitive equilibrium in static models, provided that the minimum efficient scale (the level of output that minimizes costs) is small relative to market demand. Indeed, the assumption that costs are U-shaped figures large in most economics textbooks. Two recent studies [Eswaran, Lewis and Heaps - hereafter ELH - (1983) and Hartwick, Kemp and Long - hereafter HKL - (1986)] have, however, pointed out that U-shaped costs are likely to make existence of competitive equilibrium problematic in nonrenewable resource markets. This recognition has undermined the theory based on the seminal work of Hotelling (1931). Under quite general circumstances this theory predicts that price rises and industry output falls over time. At some point output becomes very small, and it is no longer the case that the minimum efficient scale is small relative to aggregate demand. In this circumstance the nonconvexity problem cannot be finessed as in static models. Since equilibrium in nonrenewable resource markets requires that supply equal demand at all points in time, a failure to achieve equilibrium over the final part of an extraction trajectory means that the entire trajectory cannot be an equilibrium.

A number of ways in which an equilibrium might be restored are mentioned by ELH and in succeeding literature. These include: costless entry and exit by firms [Schultze (1974) and Mumy (1984)]; heterogeneous firms, possibly with market power [Kimmel (1984)]; the possibility of "chattering controls"; uncertain prices [Mason (1990)]. This paper proposes a simple resolution to the existence and optimality question: the presence of a backstop technology.[1] A backstop technology implies that the rate of production (and sales) of the nonrenewable resource need not become small as exhaustion of the resource approaches. Therefore, the requirement that supply equal demand does not require firms to produce at a level below their minimum efficient scale (i.e., where costs are nonconvex).

The other key contribution of the paper is a rigorous derivation of the conditions that characterize optimal extraction at the terminal date for the case in which a competitive equilibrium does not exist. The result here is that extraction ceases not at the point of minimum average cost, as generally believed [see for example Conrad and Clark (1987)], but to the left of minimum efficeint scale, i.e. where average cost exceeds marginal cost. A final contribution of the paper is to show how equilibrium, but not optimality, is retrieved by the introduction of a very weak assumption about uncertainty: each firm in uncertain, not about its own stock, but about the size of the stock held by others.

The next section derives the optimal trajectory in the presence of a backstop technology and a constant flow cost of production; the latter is one source of nonconvex costs. We show that these two features have the same formal effect on a standard welfare function. Section 3 shows that the two features have however very different implications for a competitive equilibrium: the constant flow cost exacerbates the existence problem while the backstop technology ameliorates it. If the price at which the backstop is supplied is sufficiently low, in a sense to be made precise in what follows, a competitive equilibrium exists and reproduces the optimum. In section 4 we show that just the presence of a backstop, even at a high price, is sufficient to ensure equilibrium if each firm is uncertain

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[1] For a discussion of other aspects of the influence on resource depletion of the existence of a backstop, see Dasgupta and Heal (1979).

about the size of the stock held by others. The problem is "convexified" by attaching probabilities to the dates when others' stocks might be exhausted. However, the competitive equilibrium will typically not reproduce the optimal trajectory; existence but not optimality is restored.

2. The Efficient Extraction Trajectory

The industry cost function for $Q>0$ is assumed to be $G + H(Q)$, where Q is industry output; cost is 0 for $Q=0$. In order to compare the efficient trajectory and the competitive equilibrium (if it exists) with a fixed number of firms, we require that the technology in the two cases be the same. If there are n firms this implies that a representative firm's cost of extracting at rate $q>0$ is $g + h(q)$, where $G=nq$ and $H(nq) \equiv nh(q)$.[2] Upper case letters denote aggregate quantities or industry cost functions, whereas lower case letters denote the individual firm's corresponding output or cost function. The remainder of this section uses only industry level variables. The minimum efficient scale, denoted Q^*, equates average and marginal costs: $[G + H(Q^*)]/Q^* = H'(Q^*)$. We assume that Q^* is finite and that costs are convex for $Q>Q^*$.

If the consumption of the backstop is B, the social utility of total consumption is given by $U(Q+B)$, the area under the inverse demand function; hence $U'(Q+B) \equiv p(Q+B)$, the market price. We suppose that there exists a price $\overline{p}$ at which a perfectly elastic supply of a

[2] We want to study the symmetric equilibrium with a fixed number of firms, since the cases of asymmetric equilibria and frictionless entry and exit have already been investigated; see citations in the Introduction. Both the assumptions of frictionless entry and exit and of a fixed number of firms are interesting as limiting cases of a more general model. We note, however, a difficulty with frictionless entry and exit. If firms cease to behave as price takers when there are just a few left, the competitive equilibrium breaks down. Noncompetitive behavior near the final part of the trajectory alters the equilibrium during the earlier phase; thus even when there are many firms in the industry, the equilibrium extraction path may not lie close to the efficient path.

substitute for the resource is available; $\bar{p}$ is the backstop price. Thus, the residual demand curve facing the resource industry has a flat portion at $\bar{p}$. The quantity $\bar{Q}$ is defined by the relation $U'(\bar{Q}) = \bar{p}$, or $\bar{p}\bar{Q} + k = U(\bar{Q})$. In the second equality k is the area between the total demand curve and price $\bar{p}$, from Q=0 to $Q=\bar{Q}$.

The social utility of consuming the resource at rate Q is

$$U^{o} = \begin{cases} U(Q) \quad for \quad Q \geq \bar{Q} \\ U(\bar{Q}) - \bar{p}(\bar{Q} - Q) \quad for \quad Q < \bar{Q} \end{cases}$$

The efficient trajectory solves the problem

$$\max_{\{Q\},T} \int_0^T e^{-rt}[U^{o}(Q) - H(Q)]dt - \int_0^T e^{-rt}Gdt + e^{-rT}\frac{k}{r}$$

$$subject\ to \quad \dot{S} = -Q$$

where r is the discount rate. The last two terms of the maximand can be rewritten as $e^{-rT}(G+k)/r - G/r$. This simplification is the basis for our remark above concerning the formal equivalence between a constant flow cost (G) and a backstop technology (which gives rise to a positive value of k). The present value of ceasing extraction at time T is the discounted flow of the reduction in the constant cost, plus the value of consuming the backstop.

We define the optimal rate of extraction at T, the time at which the resource is exhausted, as Q_T. The relation between Q_T, Q^* and $\bar{Q}$ is given by

Proposition 1. We distinguish the following three cases: (i) $Q^* < \bar{Q}$, (ii) $Q^* = \bar{Q}$, and (iii) $Q^* > \bar{Q}$. The corresponding optimal terminal rate of extraction satisfies (i) $Q_T = Q^* < \bar{Q}$, (ii) $Q_T = Q^* = \bar{Q}$, and (iii) $Q^* > Q_T > \bar{Q}$.

The proof is in the Appendix. The Proposition states that if the backstop price is sufficiently low that the market demand at that price is no less than the minimum efficient scale [i.e., cases (i) and (ii)], then it is optimal for the terminal rate of extraction to be at the minimum efficient scale: the conventional result. If, on the other hand - the backstop price is sufficiently high - which of course includes the case of no backstop, that the demand at that price is lower than the minimum efficient scale [case (iii)], the optimal terminal rate of extraction is lower than the minimum efficient scale. In this case it is optimal to extract over an interval when industry costs are concave. The economic interpretation is as follows. Suppose that a unit of the resource were held over and sold after T. Since consumption after T is smaller than before T, the marginal utility of consumption is higher after T; thus, the transfer increases the utility of consumption. However, since the rate of extraction is lower after T (it is 0) than before T, the transfer increases the costs of extracting the unit. The optimal plan balances the costs and benefits. There is no trade-off in cases (i) and (ii), so it is optimal to stop extracting at the minimum efficient scale. We emphasize that it is meaningful to speak of the backstop price as being high or low only in relation to the level of the price at which quantity demanded equals the minimum efficient scale.

3. The Competitive Industry

If firms have U-shaped cost curves, as we have assumed, and if they are price takers, then the necessary conditions to their profit maximization problem require that their terminal extraction rate is q^* (see ELH or HKL). Thus, in a symmetric equilibrium the terminal rate of aggregate extraction must be $nq^* = Q^*$. We noted above that this is also the optimal terminal extraction rate if and only if the backstop price is sufficiently low. Moreover, it is straightforward to verify that the remaining necessary conditions to the firms' profit maximization problems, which determine the rate of change of extraction before the exhaustion time, reproduce the necessary conditions of the social optimization problem. Therefore, under the maintained assumption that the first and second order necessary conditions are sufficient for a global maximum, we have

Corollary 1. A competitive equilibrium exists and reproduces the efficient extraction path if and only if $Q^* \leq \bar{Q}$ [cases (i) and (ii)].

The "if" part of the corollary follows from a comparison of the extraction trajectories that satisfy the necessary conditions to the firms' and the planner's problems. The "only if" part follows from the fact that in case (iii) the desired terminal rates are different; therefore a competitive equilibrium cannot reproduce the efficient trajectory. In addition, a competitive trajectory fails to exist, since it would involve a jump in price at T. Such a jump cannot be part of an equilibrium, because if firms are price takers and anticipated the jump, they would want to hold on to some of their stock. This last observation is the basis for ELH's and HKL's nonexistence result in the absence of a backstop technology.

4. Existence Without Optimality: Depletion Under Uncertainty

We have shown that the presence of a backstop may, but need not, restore existence and optimality of a competitive equilibrium. It is not particularly surprising that a competitive equilibrium may fail to be efficient, but the possibility of nonexistence is more troubling. Since something has to happen in real-world markets, models with no equilibria are unsatisfactory.

It is reasonable to look for a simple modification that guarantees existence of a competitive equilibrium. To this end, we assume that firm i is uncertain about the aggregate stock of its rivals, and therefore views the trajectory of future prices as random.[3] We denote the stock of firm i as s_i and the aggregate stock of the remaining firms as S_{-i}. Firm i knows its own stock with certainty, but it regards S_{-i} as a random variable, $\tilde{S}_{-i}$. Denote by $\pi(S,C_t)$ firm i's subjective probability at time t that the remaining stock of the other firms is at least S, given that their cumulative extraction up to that time has been C_t. Firms may have rational expectations; for example, the initial stocks of each firm may be an independent draw from the same distribution, and $\pi(S,0)$ may give the probability that the sum of n-1 draws is no less than S.[4] The degree of uncertainty may be small, but we assume that under no circumstances will all firms who have not yet exhausted their stock be certain about their (remaining) rivals' stocks.[5]

As in the previous section, we assume that there is a backstop technology, but allow the backstop price, $\bar{p}$, to be arbitrarily high, subject to the technology being viable. "Viability" means that the price is low enough such that some of the substitute will be demanded, so there is a flat portion of the industry residual demand curve for low levels of production of the resource; the flat portion can be very small, but not infinitesimal. In order

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[3] Mason (1990) also considers the possibility that the presence of uncertainty results in the existence of a competitive equilibrium. He assumes that firms (incorrectly) ignore the possibility of a price jump and concludes that uncertainty does not resolve the existence problem.

[4] We could allow firms to have different subjective probabilities, but in order to remain close to the deterministic (symmetric) model we assume that the function π is common to all firms. Firms may have the same initial stock, so that in equilibrium they exhaust at the same time. Nevertheless, if firms are not certain about their rivals' stock size, they view the price trajectory as random.

[5] That is, we assume that the probability of a "nontrivial perfect information state" is 0; such a state is one in which there is more than one firm who has not exhausted, and all such firms know the aggregate stock. If such a state occurred with positive probability, we would need to construct the equilibrium that ensues from that state in order to construct the full equilibrium. But we know that under perfect certainty there is no equilibrium in case (iii).

to insure that price-taking behavior is consistent with the situation where a single firm remains with positive stock, we require that n is large enough so that the minimum efficient scale of firm i, $q^*_i = Q^*/n$ satisfies $p(q^*_i) > \bar{p}$, or $q^*_i < n\bar{q}$[6]. (If firm i were the only firm, and produced at its minimum efficient scale, demand for the backstop would be positive.) If a single firm remains with positive stocks there is no uncertainty (given earlier assumptions), so a competitive equilibrium in this situation requires that $q^*_i < n\bar{q}$. This restriction would be unnecessary if firms were uncertain about their own stocks; in that case, the presence of backstop is not required.

If at some time all of i's competitors had exhausted their stock and i still had a stock size of s, it would extract the resource in order to maximize the present discounted stream of profits. We denote as V(s) the value function to this optimization problem.[7] Since i takes its competitors' extraction trajectory as given, we can replace the function $\pi(S,C_t)$ with $\pi^*(\tau,C_t)$, defined as the subjective probability at t that rivals will still be extracting at time $\tau > t$, given that their cumulative extraction has been C_t. The probability that firm i's rivals exhaust their stocks over a small interval dt is given by the "hazard rate" $-\partial\pi^*/\partial\tau = -(\partial\pi/\partial S)\partial S/\partial\tau = -[\partial\pi/\partial S]Q_{-i}(\tau)$, where $Q_{-i}(\tau)$ is the aggregate extraction of i's rivals at τ, conditional on not having previously exhausted. Each firm takes the price path as given,

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6 This inequality does not rule out case (iii). Recall the definition $p(\bar{Q}) = \bar{p}$; for large n, $Q^* > \bar{Q} > Q^*/n$ is certainly possible.

7 If i's remaining stock is small - which can be guaranteed by fixing the initial aggregate stock and making n large - then $p_t = \bar{p}$ during the time that i is the only remaining firm. Firm i will have no market power in this case, because of the backstop technology. If i's remaining stock were large, it might exercise market power by setting price below $\bar{p}$. Both possibilities are consistent with the description of equilibrium.

prior to its rivals' exhaustion. By defining firm i's costs as c(q), where c(q) = 0 for q=0 and c(q) = g+h(q) for q>0, we can write i's optimization problem at time 0 as[8]

$$\max_{\{q\}} \int_0^\infty - [\int_0^\tau e^{-r\tau} (p_t\, q_t - c(q_t))dt + V(s)\, e^{-r\tau}] \frac{\partial \Pi^*}{\partial \tau} d\tau$$
$$subject\ to\ \dot{s} = -q,\ q \geq 0,\ s \geq 0,\ s_o\ given.$$

Integrating by parts the first term

$$\int_0^\infty \int_0^\tau e^{-r\tau}(p_t q_t - c(q_t))dt \frac{\partial \Pi^*}{\partial \tau} \partial\tau,$$

and making a change of variables from τ to t yields the optimization problem

$$\max_{\{q\}} \int_0^\infty e^{-rt}\{[p_t\, q_t - (c(q_t)]\pi*(t,0) - \frac{\partial\ \pi*(t,0)}{\partial t} V(s_t)\}dt$$
$$subject\ to\ \dot{s} = -q,\ q \geq 0,\ s \geq 0,\ s_o\ given.$$

The firm's objective is thus made up of two parts: the present value of extracting when its rivals are still extracting (up to date t) times the probability that they are still extracting at that date, plus the present value of extracting after they have run out, times the probability that they will run out at t. Note that by attaching these probabilities we have convexified the

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[8] See Kemp (1976), note 3, for an explanation of why, in problems of this sort, it is unnecessary to allow for revision at t>0.

problem. All firms then solve problems of this form. They anticipate a jump in the price at a random time.[9]

The nonexistence of equilibrium in the deterministic model arises because firms can predict when a jump would occur (and not simply because a jump must occur at some time). In the model with uncertainty firms do not know precisely when the jump will occur; the anticipation of a jump at a random time is no impediment to the existence of a competitive equilibrium. A firm that had exhausted its stock just before the jump would wish that it had been a bit more conservative, and retained some of the resource to sell after the jump. This regret is the product of hindsight, but is not due to the lack of foresight.

Although the presence of uncertainty restores the existence of a competitive equilibrium, it may result in the failure of that equilibrium to be socially efficient (e.g., in the absence of a complete set of contingent markets). The planner whose objective is to maximize the expectation of the discounted flow of social surplus faces the familiar problem of how to eat a cake of unknown size [Kemp(1976), Gilbert (1979)]. Even if the planner and the "representative firm" have the same information about aggregate stocks, the planner recognizes that the probability of exhausting the aggregate stock is endogenous. (In other words, the planner treats the hazard rate as $-(\partial\pi/\partial S)Q$, and recognizes its dependence on the extraction decision; the individual firm treats the hazard rate as $\partial\pi^*/\partial t$, which it takes as given.) Thus, when firms behave atomistically, there is an informational externality which causes their decisions to differ from those of the planner.

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[9] The model can be made more descriptive by allowing firms to consider the possibility that there is more than one jump, as different groups of rivals exhaust their stocks. This extension complicates the notation but adds no additional insight.

For the deterministic model we noted above that with the exception of the boundary conditions, the planner's and the representative firm's optimality conditions are equivalent. With uncertainty, however, the costate equations to the two problems differ; this is a reflection of the informational externality. In cases (i) and (ii) identified in Proposition 1, the boundary conditions to the two problems are the same under certainty, and nonconvexity ceases to be an issue. With uncertainty, the informational externality remains. Thus, it may be the case that uncertainty restores existence of a competitive equilibrium to the model, but it is also possible that uncertainty renders a competitive equilibrium inefficient.

4. Conclusion

The likelihood that a competitive equilibrium does not exist when costs are U-shaped challenges the theory of nonrenewable resources. This challenge appears much more severe than in the case of static markets; there the problem can be resolved by choosing an appropriate number of firms. A parallel resolution in the case of nonrenewable resources requires a continuously declining number of firms—which as we have noted poses a difficulty for the attainment of a competitive equilibrium. We have suggested an alternate resolution, based on the existence of a backstop technology. If the backstop price is sufficiently low a competitive solution exists and is efficient. If the backstop price is high, a competitive equilibrium exists, but is in general not efficient, provided that there is even a small amount of uncertainty regarding aggregate stocks.

Appendix: Proof of Proposition 1

The planner's Hamiltonian, $\boldsymbol{H}$ and necessary conditions for an interior optimum are

$$\boldsymbol{H} = U^{o} - H(Q) - \lambda Q \tag{1}$$

$$U^{o\prime}(Q) - H^{\prime}(Q) - \lambda = 0 \tag{2}$$

$$\dot{\lambda} = r\lambda \tag{3}$$

$$U^{o}(Q_T) - H(Q_T) - \lambda_T Q_T - (G + k) = 0 \tag{4}$$

$$U^{o\prime\prime}(Q) - H^{\prime\prime}(Q) < 0 \tag{5}$$

where equation (1) defines the Hamiltonian, λ is the costate variable, equation (4) is the boundary condition at T, and equation (5) is the second order condition for maximization of the Hamiltonian. We now define

$$f(Q) \equiv U^{o}(Q) - H(Q) - [U^{o\prime}(Q) - H^{\prime}(Q)]Q - (G + k)$$

From (5), $f'(Q) > 0$ and (2) and (4) imply $f(Q_T) = 0$. For case (i) we have $f(Q^*) = 0 < f(\bar{Q})$, so $Q_T = Q^* < \bar{Q}$. For case (ii) $f(Q^*) = f(\bar{Q}) = 0$, so $Q_T = Q^* = \bar{Q}$. For case (iii) $f(\bar{Q}) < 0 < f(Q^*)$, so $\bar{Q} < Q_T < Q^*$.

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