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Consumer Welfare in Online Markets

A dissertation submitted in partial satisfaction
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by

Ryan James Martin

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ABSTRACT OF THE DISSERTATION

Consumer Welfare in Online Markets

by

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Doctor of Philosophy in Economics
University of California, Los Angeles, 2019
Professor Rosa Liliana Matzkin, Chair

I study how to measure consumer welfare changes from demand in online markets. Specifically, I find formulas for calculating *exact* compensating variation and equivalent variation as a function of a *known* demand relation. Consumers are assumed to shop for a single, discrete good. To capture online shopping behavior, I also assume consumers have limited product knowledge. That is, I assume search is costly and consumers thus optimally shop only a subcollection of all available products; I call this consumer-specific subcollection of products a "consideration set." In this framework, I determine formulas to measure consumer welfare changes from price changes and from changes in search result listings. I use simulations and shopping data from an online travel agency to support my analysis.

The dissertation of Ryan James Martin is approved.

Elisabeth Honka

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Jinyong Hahn

Rosa Liliana Matzkin, Committee Chair

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

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CHAPTER 1

Introduction

This dissertation studies how to measure consumer welfare changes in online markets using demand information. Specifically, I find formulas for calculating *exact* compensating variation and equivalent variation as a function of a *known* demand relation. Consumers are assumed to shop for a single, discrete good. To capture online shopping behavior, I also assume consumers have limited product knowledge. That is, I assume search is costly so that consumers optimally shop only a subcollection of all available products; I call this consumer-specific subcollection of products a "consideration set." This setting creates two channels for changes in consumer welfare: (1) price changes and (2) platform changes that affect consideration sets. I explore both in detail in this paper.

My research is motivated by recent antitrust concern over the growing concentration of online platforms.[1] Online, most consumers and sellers find each other through the services of these platforms. Without these platforms, consumers and sellers would have a hard time making new connections: there are no downtown districts to stroll or shelves to peruse online. Thus, a concentrated platform may have great influence over who buys from whom online. This gives rise to several antitrust concerns over search platform conduct. For example, a search platform that also sells its own products, such as Amazon, may be tempted to flex its influence over consumers' search results in negotiations with sellers to extract high proportions of seller revenue on their site and limit seller behavior off their site.[2] Alternatively, a search platform may be tempted to hide search results that would lead consumers

[1]For example, the Federal Trade Commission (FTC) is holding ongoing hearings on Competition and Consumer Protection in the 21st century: https://www.ftc.gov/policy/hearings-competition-consumer-protection that address this topic.

[2]See Khan (2017) for more discussion and evidence of this behavior.

to other search platforms, in order to protect its own market share of searches. This is especially concerning when the search platform represents its organic[3] search results as unbiased. European antitrust authorities fined Google $2.7 billion for this type of conduct.[4] To the extent that this platform conduct influences the distribution of sellers and the ultimate product choice of consumers is to the extent this conduct may also have strong consumer welfare implications. My model framework allows for a rigorous, unrestricted study of these consumer welfare consequences.

In addition to antitrust concern over platform conduct, there are also several traditional consumer welfare questions with results that need to be revised in online markets. In particular, when consumer search results are influenced by price, the welfare consequences of an exogenous price increase may depart significantly from the welfare consequences of a price increase in a traditional market where consumers are expected to have knowledge of all products. My research framework, where consideration sets are accounted for in demand, allows for a detailed exploration of these kind of welfare questions.

My main results are contained in the following three chapters. In Chapter 2, I study consumer welfare changes in response to changing search result lists. I develop a general formula to measure welfare changes using demand information in this environment. I apply this general formula to data from an online travel agency. I find a welfare improvement when the online travel agency goes from random search results to search results ordered in decreasing predicted probability of purchase. I also measure the welfare lost when the top five booking options are removed from consideration sets on the online travel agency.

In Chapter 3, I strengthen my assumptions on the demand information that is available to researchers. In particular, I assume that researchers know the demand for each group of consumers with the same consideration set; I call this "conditional demand." Consideration sets are often seen in click-stream data from online shopping and thus conditional demand may be estimated in online shopping settings. I show many additional welfare results that can

[3]An organic search result is one whose position is not chosen by payment to the search platform, but rather by the search platform's own

[4]http://money.cnn.com/2017/06/27/technology/business/google-eu-antitrust-fine/index.html

be achieved with conditional demand. Results include welfare changes from multiple price increases and welfare changes from changing search lists when preferences are monotonic in money.

In Chapter 4, I identify welfare changes from price increases in online shopping environments. I develop a general formula for welfare changes as functions of average demand. I also establish assumptions unique to the shopping environment that are necessary for accurate results. In particular, I find that when consideration sets are independent of prices, welfare can be estimated with a simple integral of aggregate demand. However, when consideration sets depend on prices, consideration set information is necessary to accurately measure the welfare consequences of price increases. I end with an application to consumers shopping for hotel bookings on an online travel agency. I measure the equivalent variation and compensating variation that results from a 25% and 50% price increase of the most popular booking.

Each chapter starts with an overview of its main results, a literature comparison and a overview of the chapter organization. Proofs are relegated to appendices that immediately follow each chapter's conclusion. Each chapter‘s notation is self-contained.

CHAPTER 2

Identifying Welfare Changes when Online Platforms Change their List of Search Results

Online shopping is often guided by search platforms. Consumers type keywords into query boxes and search platforms deliver a list of products in response. The order that products appear in this list affects the products consumers discover and ultimately purchase. That is, when platforms change the order of products that appear in search results, aggregate demand and welfare are also changed. In this chapter, I study the identification of consumer welfare changes in response to exogenous changes in these product list. I focus on the case of consumers shopping for a single, indivisible product among a collection of substitutes. I show that, in this environment, exact consumer welfare changes—that is, compensating variation and equivalent variation—can be calculated with straightforward integrals of aggregate demand.

This chapter's work is closest to that of J. Hausman (1981) and J. Hausman (1996) and their extensions J. Hausman (1999), Jerry A. Hausman (1997) and Jerry A Hausman and Leonard (2002). The above papers focus on identifying and estimating exact consumer welfare changes from the introduction of a single product or a representative consumer buying a collection of new products according to a price index. All consumers are assumed to have perfect knowledge of the available products. Entry of multiple products is simplified to the entry of one product category. Entry is driven by technological innovation or regulatory decisions. In contrast, this chapter focuses on the role that search result lists have on shaping product knowledge and welfare. Individual search behavior is heterogeneous and the consequences of a changing search list may be heterogeneous and unpredictable across consumers in my environment.

This chapter is also related to Small and Rosen (1981). As in this paper, Small and Rosen (1981) develop tools to estimate welfare changes in discrete choice environments. Small and Rosen (1981) provide tools to estimate welfare changes in response to a change in price, quality or any variable that varies continuously with indirect utility. In contrast, this chapter focuses on changes in consideration sets that, by their very nature, provide discontinuous shifts to indirect utility functions. Thus, the results of this chapter represent a significant extension of the results in Small and Rosen (1981). Indeed, no simple adaption of the techniques used in Small and Rosen (1981) will lead to correct welfare measures in an environment of changing, heterogeneous consideration sets.

Some recent, situation-specific welfare methods have been developed to estimate welfare changes from non-idiosyncratic product removal or exit (not both simultaneously). These include Nevo (2003), Gentzkow (2007), Quan and Williams (2016) and Petrin (2002). My paper provides a generalization of their results, allowing for simultaneous product entry and exit in a flexible utility environment. My results are also shown to be exactly equal to compensating variation or equivalent variation, rather than just approximations. Finally, my methodology allows for recovery of welfare changes caused by unobservable preference matching, rather than just average welfare components.

There is also a growing body of research that is interested in assessing the value of technology, the internet and free (digital) goods and services on GDP. See for example Brynjolfsson, Collis, et al. (2019), E. Diewert W. and Feenstra (2017), W. E. Diewert, Fox, and Schreyer (2018), Feldstein (2017), Groshen et al. (2017), Syverson (2017), Brynjolfsson and Oh (2012) and Greenstein and McDevitt (2011). These papers build on frameworks such as J. Hausman (1996) or Small and Rosen (1981). There is no search component to their models. The focus is on measuring the aggregate welfare consequences of products that are available in the digital economy. In contrast, my paper allows for average welfare changes from idiosyncratic changes in shopping behavior.

Finally, the techniques used to derive welfare formulas in this chapter are similar to those of the previous chapter, which ultimately extend the ideas of Bhattacharya (2015). While Bhattacharya (2015) focused on welfare changes as a result to a single price increase, I extend his environment to measure welfare changes in response to changing consideration sets and exogenous changes in listing rule.

The rest of this chapter is organized as follows. In Section 2.1, I develop notation for this chapter. In Section 2.2, I present my main results: a general formula to measure average welfare changes from platform changes. I also present a simple example illustrating the key ideas captured in the formula. In Section 2.3, I present results on how the formula can be used for counterfactual estimation. In Section 2.5 I present simulation results corroborating my theoretical findings. In Section 2.6, I explore applications of my results to data from an online travel agency (OTA). Finally, in Section 2.7, I conclude.

2.1 Notation

In this section, I develop the notation I need for the rest of the chapter. I start by discussing the role of search platforms in Section 2.1.1. In Section 2.1.2, I discuss product and consumer preference notation. I develop notation and assumptions for consideration sets in Section 2.1.3. I define and develop notation for welfare measures in Section 2.1.4.

2.1.1 Search Listings

A consumer searching for a product online will type keywords into a search platform's query box. From there, the search platform has a *listing rule* α that determines a sequence of products for the consumer to review. The focus of this paper is not on how platforms determine this listing rule; the listing rule is free to depend on observable consumer characteristics and advertising concerns in addition to consumer keywords. Rather, this paper focuses on measuring consumer welfare changes as a response to changes in the listing rule, say from α_0 to α_1. Specific changes in listing rules considered in the empirical section of this paper are as follows: (1) a product listing rule that returns products in a random order changing to a product listing rule that lists products in order of their probability of purchase and (2) a change in listing rule that hides five products from consumers.

2.1.2 Preferences

My setup for products and preferences follows the multinomial choice framework with non-separable utility laid out in Bhattacharya (2015). There is an *observable*[1] *set of products* $\mathcal{J} = \{0, 1, \ldots, J\}$.[2] I denote the *observable vector of market prices* $p^m_{\mathcal{J}} = (0, p^m_1, \cdots, p^m_J)$. The price of the outside product is normalized to 0. When discussing product prices that may differ from their market-values, I will drop the superscript m.

Consumer utility is affected by *observable income* y and *observable attributes* Ψ. Both y and Ψ are fixed for each individual. In examples, I will treat gender as a component of Ψ. I will suppress the notation for Ψ from utility for readability. All identification results should be interpreted as conditional on Ψ.

Consumers have *unobservable preferences* η. I follow Bhattacharya (2015) and do not restrict the dimension of these unobservable preferences.[3]. I restrict utility using the following assumptions.

Assumption 1.A (Monotonicity). *Utility for product j, $u_j(y - p_j, \eta) \in \mathbb{R}$, is strictly increasing and continuous in its first argument for all products $j \in \mathcal{J}$.*

Assumption 1.B (Linearity). *Utility for product j is linear in its first argument. That is,*

$$u_j(y - p_j, \eta) = y - p_j + \tilde{U}_j(\eta)$$

where $\tilde{U}_j(\eta) \in \mathbb{R}$ for all products $j \in \mathcal{J}$.

Linearity is sufficient for Monotonicity. I maintain Monotonicity for the rest of the paper.[4]

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[1]Observable is always used to mean observable to the researcher, not the consumer

[2]It is WLOG to have the products invariant over listing rules. For example, if product M becomes available after a change in listing rule, then we can just disallow M from being in consideration sets under the initial listing rule.

[3] See Bhattacharya (2015) for a discussion on the importance of leaving the dimension of heterogeneity unrestricted in discrete choice preferences

[4]A vector of observable product characteristics $(0, X_1, \ldots, X_J)$ can also be available to the researcher. This information can be treated as suppressed from utility for readability. Unsupressing product characteristics and observable individual characters from utility, we would write $u_j(y - p_j, X_j, \Psi, \eta)$ under Monotonicity and $y - p_j + \tilde{U}_j(\eta; X_j, \Psi)$ under Linearity

2.1.3 Consideration Sets

I assume search is costly and thus consumers do not, in general, view all products returned to them in platform search lists. Instead, each consumer's product knowledge—and therefore demand—is limited to a sub-collection of $\mathcal{J}$, called her consideration set.

Consideration sets are determined through a *consideration function* $\mathcal{C}$. The consideration function $\mathcal{C}$ takes the following arguments: (1) a listing rule α; (2) observable consumer characteristics Ψ (3) unobservable consumer preferences η (4) unobservable non-preference characteristic vector ζ. For readability, as with utility, I will suppress Ψ from notation. Discussion of ζ follows the enumerated definition.

Definition 1 (Consideration Sets)**.** *A consumer with characteristics* (η, ζ) *has consideration set* $\{0\} \subseteq \mathcal{C}(\eta, \zeta, \alpha) \subseteq \mathcal{J}$ *under listing rule* α.[5]

When discussing the consideration set of a fixed consumer (y, η, ζ), I will abbreviate her consideration set under listing rules α_0 and α_1 by $\mathcal{C}_0$ and $\mathcal{C}_1$ respectively.

An individual's unobservable characteristic vector ζ contains those that will affect the products she considers but will not directly enter her utility. For example, a component of ζ captures a consumer's preference for the act of shopping itself. Consumers who like to shop will likely have larger consideration sets than consumers who don't like to shop, even when their product preferences are identical. ζ may also capture product and price beliefs that guide search over different keywords.[6]

For concreteness, consider a consumer searching for a face moisturizer on Amazon.com. Suppose she does not like shopping for very long and chooses to either buy a product from the first page of search results that come up after her keyword search or not buy anything (captured in ζ). She has a preference for branded products (captured in η) and thinks "Biore" is likely to be a relatively inexpensive branded product (captured in ζ). Thus, she types "Biore face moisturizer" into the search box and hits the return key. Some of the

[5] *Unsuprressing* Ψ*, this is* $\mathcal{C}(\Psi, \eta, \zeta, \alpha)$

[6] Depending upon the data set available to the researcher, the keyword query a consumer uses may be part of Ψ or ζ; either way is permissible in this modeling scheme

products in the resulting list are organic and some are sponsored links. Her consideration set is exactly the products listed on this first page of search results, as well as the outside product.

To derive my main results, I rely on the following assumption.

Assumption 2 (Price Independence). *The consideration function $\mathcal{C}$ does not depend on prices p_j^m or income minus prices $y - p_j$ for each j.*

Note that Price Independence does not preclude a consumer from shopping according to her *beliefs* about prices. It only requires that the prices she observes do not cause her to change her shopping behavior.[7]

Price Independence rules out the possibility of a platform removing a product from its lists due to changes in price. It also rules out certain consumer behaviors, such as consumers shopping more after discovering all the products she has discovered are unexpectedly expensive. In principle, some price dependence can be tolerated: the key behavior needed is that a consumers demand for a good falls to 0 for preference reasons before it exits her consideration set due to the search listing rule. That is, as long as the consumers preferences are more sensitive than the search listing rule, the main results still hold.

A consumer (y, η, ζ) purchases product j in $\mathcal{C}(\alpha, \zeta, \eta)$ if

$$u_j(y - p_j^m, \eta) > u_k(y - p_k^m, \eta) \text{ for all } k \in \mathcal{C}(\alpha, \zeta, \eta) \setminus \{ m \}$$

Individual demand is defined by

$$q_j(y, p_j, p_{-j}, \alpha, \eta, \zeta) := \begin{cases} 1 \text{ if } j = \arg\max_{\ell \in \mathcal{C}(\alpha,\eta,\zeta)} u_\ell(y - p_\ell, \eta) \\ 0 \text{ otherwise} \end{cases} \tag{2.1}$$

[7]Although not modeled explicitly, it is perfectly fine for consideration sets to also depend on the non-price product characteristics X_j of products j in their consideration set. In addition, it is perfectly fine for consideration sets to depend on consumers perceptions of their own income level or wealth level, as a part of Ψ. The only complication that might arise here is if certain products purchase changed a consumer's perception of her own wealth level. This must be disallowed, which iswhy consideration sets cannot depend on $y - p_j$

I will simplify notation for *individual demand for product j vs market prices* to

$$q_j^m(y, p_j, \eta, \zeta, \alpha) := q_j(y, p_j, p_{-j}^m, \alpha, \eta, \zeta) \tag{2.2}$$

This simplified form has all product prices, except for product j, fixed at their market prices.

Average demand for product j is

$$Q_j(y, p_j, p_{-j}, \alpha) = \int q_j(y, p_j, p_{-j}, \alpha, \eta, \zeta) dF \tag{2.3}$$

Similarly *average demand for product j vs market prices*

$$Q_j^m(y, p_j, \alpha) = \int q_j(y, p_j, p_{-j}^m, \alpha, \eta, \zeta) dF \tag{2.4}$$

2.1.4 Welfare Measures

The classic welfare measures *compensating variation*, S^{CV}, and *equivalent variation*, S^{EV}, are adapted to my search environment as follows. For an individual with heterogeneity vectors (η, ζ) and income y, S^{EV} is the solution in S to

$$\max_{j \in \mathcal{C}_0} u_j(y - S - p_j^m, \eta) = \max_{j \in \mathcal{C}_1} u_j(y - p_j^m, \eta) \tag{2.5}$$

while S^{CV} is the solution in S to

$$\max_{j \in \mathcal{C}_0} u_j(y - p_j^m, \eta) = \max_{j \in \mathcal{C}_1} u_j(y + S - p_j^m, \eta) \tag{2.6}$$

S^{EV} is the income loss under the initial listing rule that would harm a consumer as much as the damage done by the new listing rule. Compensating variation is the increase in income under the new listing rule that would return a consumer to her utility level under the original listing rule. S^{EV} and S^{CV} are both positive if the consumer is harmed by the new listing rule, relative to the older rule. They are both negative if the consumer is helped by the change in listing rule.

We see from the definitions that S^{EV} and S^{CV} both depend on market prices and both listing rules, as well as the individual's income[8] and unobservable characteristics η and

[8]and suppressed Ψ

ζ. Thus, I denote the functions for equivalent variation and compensating variation by $S^{EV}(y, \eta, \zeta, \alpha_0, \alpha_1, p^m_{\mathcal{J}})$ and $S^{CV}(y, \eta, \zeta, \alpha_0, \alpha_1, p^m_{\mathcal{J}})$ respectively. Similarly, *average compensating variation and equivalent variation* over all consumers is denoted by μ^{CV} and μ^{EV}, and are written as functions $\mu^{CV}(y, \alpha_0, \alpha_1, p^m_{\mathcal{J}})$ and $\mu^{EV}(y, \alpha_0, \alpha_1, p^m_{\mathcal{J}})$; the average is taken over unobservables, η and ζ.[9] When it's clear from context, I will suppress the arguments for listing rules and prices.

When utility is linear in money, $S^{CV} = S^{EV}$ for each individual. I use S^W to represent both S^{CV} and S^{EV} in this case. S^W is simply the difference between utility under the initial listing rule and utility under the final listing rule.

2.2 Main Results: A General Welfare Formula for Changing Consideration Sets

In this section, I determine how to measure welfare changes as a response to search listing changes. I start with my most general result: a formula that measures welfare changes from aggregate demand lines under *Linearity*. I leave the proof for the appendices, but follow up with an example that shows the key ideas.

For succinctness, I first denote *total welfare under listing rule* α_t by Ω_t and define it with the following formula.

$$\Omega_t := \lim_{p_2,\ldots,p_J \to \infty} \int_{p_1^m}^{\infty} Q_1(y, p, p_{-1}, \alpha_t)dp + \lim_{p_3,\ldots,p_J \to \infty} \int_{p_2^m}^{\infty} Q_2(y, p, (p_1^m, p_{-(1,2)}), \alpha_t)dp \tag{2.7}$$

$$+ \cdots + \int_{p_J^m}^{\infty} Q_J(y, p, p_{-J}^m, \alpha_m)dp \tag{2.8}$$

Total welfare under listing rule α_t captures the total value to consumers of the products in their (heterogeneous) consideration sets under listing rule α_t. More precisely, the first term in the sum that defines Ω_t calculates the average value of allowing product 1 to enter all consumers' consideration sets; if a consumer doesn't have product 1 in her consideration set under listing rule α_t, this consumer's contribution to the average is 0. This added value is

[9] Averages would still be functions of consumer observables Ψ, although the researcher could, of course, subsequently average over Ψ if she desired.

relative to consideration sets that only contain the outside product. The second term in the sum that defines Ω_t adds the average value consumers gain by having product 2 in their consideration sets, relative to consideration sets that contained (at most) product 0 and product 1; consumers without product 2 in their consideration set contribute nothing to this value. Consumers with product 2 but not product 1 in their consideration set will contribute average values to the 2nd term that reflect product 2's value relative to the outside product alone. This process of adding in the average value of one more product is continued from the third term until the Jth term in the sum. By the Jth term, all consideration sets will have reached their full size under listing rule α_t. All the terms together give Ω_t the total value of sequentially allowing products 1 to J to enter consideration sets. With this definition in hand, my main welfare result is straightforward.

Theorem 1. *Under Linearity, the average welfare change μ^W for a change in platform listing rule from α_0 to α_1 is*

$$\mu^W = \Omega_0 - \Omega_1$$

In words, by looking at the difference in total welfare created by a change in listing rule across time, we can recover exact average compensating variation. (Exact average compensating variation is also exact average equivalent variation, since the terms coincide under Linearity). Of course, under Linearity, the numbering of products 1 to J can be rearranged arbitrarily and the formula still holds.

Theorem 1 captures several key ideas about welfare analysis in a search environment. The first key is the use of prices. By raising a product's price high enough—beyond consumers' reservation prices—we can effectively turn off the value consumers gain from considering that product. The second key is that, under Linearity, the total value a consumer gains from a purchase can be decomposed into a sum over the value of all products in her consideration set. This holds regardless of her choice of consideration set or the order the sum is performed.

The final key is the importance of the reference product, product 0. Theorem 1 works by building up the utility around the outside product under the different listing rules. If the utility of the outside product is not comparable across listing rules, then we cannot hope to make meaningful welfare comparisons between the two outcomes. If there is greater consistency in consideration sets across listing rules, a larger reference collection of products can be used and the welfare formula can be simplified, as illustrated in the Section 2.3.

The proof of Theorem 1 can be found in the appendix. However, the main intuition behind the above theorem is captured in the following example.

Example: Consider a market with a single consumer (y, η, ζ) who shops a single product, product 1, along with the outside product under listing rule α_0. That is, this consumer has consideration set $\{0, 1\}$ under listing rule α_0. Under listing rule α_1, her consideration set grows to $\{0, 1, 2, 3\}$; she gains *two* additional products in her consideration set. For simplicity, assume all prices are 0 and that utility has the following form:

$$u_0(y, \eta) = y$$

$$u_1(y, \eta) = y + a$$

$$u_2(y, \eta) = y + 10a$$

$$u_3(y, \eta) = y + 10a + \epsilon$$

where $a > 0$ is large while $\epsilon > 0$ is small.

Then, we see the consumer's product choice under listing rule α_0 is 1 and her product choice under listing rule α_1 is 3. Her change in utility is $S^W = u_1(y, \eta) - u_3(y, \eta) = -9a - \epsilon$

Similarly,

$$\Omega_0 - \Omega_1 = -\lim_{p_3 \to \infty} \int_{p_2^m}^{\infty} Q_2(y, p, (0, p_3), \alpha_0) dp - \int_{p_3^m}^{\infty} Q_3(y, p, p_{-3}^m, \alpha_1) dp \tag{2.9}$$

$$= -9a - \epsilon \tag{2.10}$$

as claimed in Theorem 1.

Taking the price of product 3 to infinity makes the consumer behave *as if* product 3 were not in her consideration set. This allows us to measure the value of product 2's addition to her initial consideration set of $\{0, 1\}$. The second integral then takes account of the value of adding product 3 to a consideration set of $\{0, 1, 2\}$. For intuition, consider fig. 2.1 through

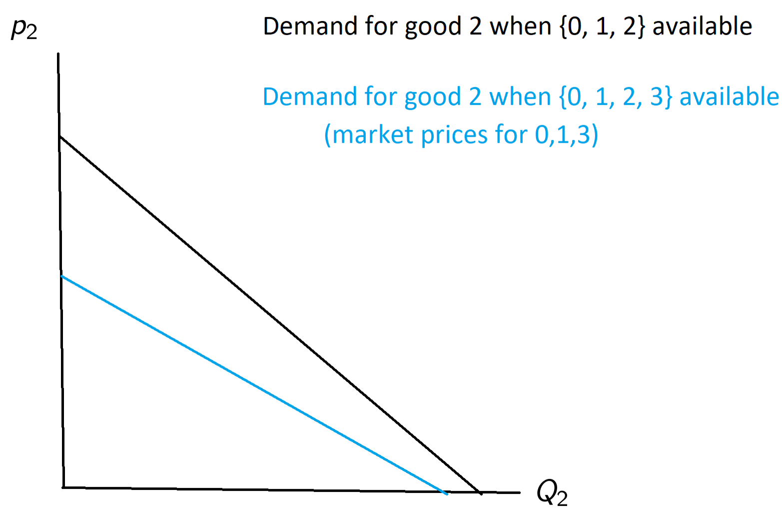


Figure 2.1: Both the blue and black line capture the aggregate quantity demand of product 2, Q_2 vs the price of product 2, p_2 when the prices of all other products are at market prices. The blue line captures demand when consideration sets are all $\{0, 1, 2, 3\}$ whereas the black line captures demand when consideration sets are all $\{0, 1, 2\}$. Necessarily, the blue line is everywhere (weakly) to the left of the black line. As the price of product 3 increases from its market price to infinity, the blue line shifts right, becoming equal to the black line.

fig. 2.2.[10]

♣

2.3 Welfare Results When Listing Rule Consequences Are Predictable

In this section, I look at formulas for measuring changes in welfare from simple changes in listing rules. I demonstrate that the calculations required in Theorem 5 can be significantly simplified in many situations of practical and counterfactual interest. I consider listing rules

[10] Note, if the pricing limit is not included, we have $\int_0^\infty Q_2(y, p, \mathbf{0}, \alpha_1)dp = 0 < \lim_{p_3 \to \infty} \int_0^\infty Q_2(y, p, (0, p_3), \alpha_1)dp = 9a$

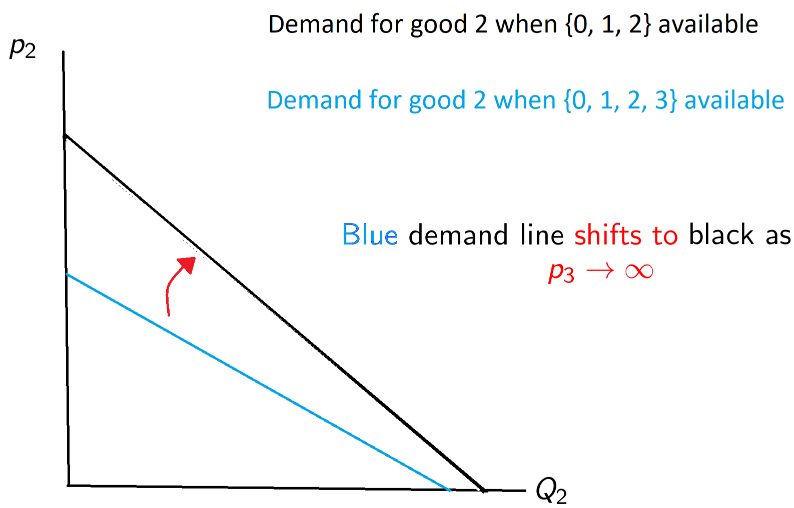


Figure 2.2: Both the blue and black line capture the aggregate quantity demand of product 2, Q_2 vs the price of product 2, p_2 when the prices of all other products are at market prices. The blue line captures demand when consideration sets are all $\{0, 1, 2, 3\}$ whereas the black line captures demand when consideration sets are all $\{0, 1, 2\}$. Necessarily, the blue line is everywhere (weakly) to the left of the black line. As the price of product 3 increases from its market price to infinity, the blue line shifts right, becoming equal to the black line for all finite prices of product 2.

that (1) add or remove single products from the search result list; (2) replace a single product in the list with a new product; or (3) add or remove a collection of products from the search result list.

2.3.1 Listing Rules That Change Search Lists by One product

When changes in search listing rules affect consideration sets by at most one good, welfare changes can be measured with few restrictions on preferences.

Theorem 2. *Suppose that a change in platform listing rule, from α_0 to α_1, has the exact effect of increasing each consumer's probability of shopping product 1.*[11] *Then, under Monotonicity, the average equivalent variation that results is*

$$\mu^{EV}(y, \alpha_0, \alpha_1, p^m_{\mathcal{J}}) = \int_{p_1^m}^{\infty} Q_1^m(y, p, \alpha_0)dp - \int_{p_1^m}^{\infty} Q_1^m(y, p, \alpha_1)dp \tag{2.11}$$

while the average compensating variation is

$$\mu^{CV}(y, \alpha_0, \alpha_1, p^m_{\mathcal{J}}) = \int_{p_1^m}^{\infty} Q_1^m(y + p - p_1^m, p, \alpha_0)dp - \int_{p_1^m}^{\infty} Q_1^m(y + p - p_1^m, p, \alpha_1)dp \tag{2.12}$$

The proof can be found at Section 2.A.3. Intuitively, by looking at the difference, we just find the value gained by the additional consumers shopping product 1.

Analogous results for a probabilistic product loss are as follows.

Theorem 3. *Suppose the listing rule changes exogenously from α_0 to α_1 in a way that makes each consumer more likely to shop product M.*[12] *Then, the average compensating variation of this change in listing rule is*

$$\mu^{CV} = \int_{p_M^m}^{\infty} Q_M^m(y, p_M, \alpha_0)dp_M - \int_{p_M^m}^{\infty} Q_M^m(y, p_M, \alpha_1)dp_M$$

and the average equivalent variation is

$$\mu^{EV} = \int_{p_M^m}^{\infty} Q_M^m(y + p - p_M^m, p, \alpha_0)dp - \int_{p_M^0}^{\infty} Q_M^m(y + p - p_M^1, p, \alpha_1)dp$$

[11] *Specifically, I mean that for every (y, η, ζ), $\mathcal{C}(\alpha_1, \eta, \zeta)$ is either equal to $\mathcal{C}(\alpha_0, \eta, \zeta)$ or equal to $\mathcal{C}(\alpha_0, \eta, \zeta) \setminus \{1\}$*

[12] *Specifically, I mean for every (η, ζ), $\mathcal{C}(y, \alpha_1, \eta, \zeta)$ is either equal to $\mathcal{C}(y, \alpha_0, \eta, \zeta)$ or $\mathcal{C}(y, \alpha_0, \eta, \zeta) \cup \{M\}$*

2.3.2 Single Product Swap

In this section, I present the formula for a welfare change when one product is swapped for another, probabilistically, in all consumer's consideration sets. The formula is a simple integral under the demand curves of the two swapped products.

Theorem 4. *Under Linearity, when the listing rule changes from α_0 to α_1 such that each consumer has a weakly decreased probability of product 1 being in her consideration set and a weakly increased probability of product $M \neq 1$ being in her consideration set, then average welfare change is*

$$\mu^W = \int_{p_1^m}^{\infty} Q_1(y, p, p_{-1}^m, \alpha_0)dp - \int_{p_1^m}^{\infty} Q_1(y, p, p_{-1}^m, \alpha_1)dp \\ - \int_{p_M^0}^{\infty} Q_M(y, p_M, p_{-M}^0, \alpha_0)dp_M + \int_{p_M^0}^{\infty} Q_M(y, p_M, p_{-M}^0, \alpha_1)dp_M$$

In these formulas, product 1 and product M may not both be present in an individual's consideration set contemporaneously. In this case, the comparison works because of indirect welfare comparisons between the reference collection of products, $\mathcal{C}_0 \setminus \{ 1, M \} = \mathcal{C}_1 \setminus \{ 1, M \}$. The necessity of the indirect welfare comparison makes Linearity essential to achieve these results. The proof is in Section 2.A.5

2.3.3 Multiple product Entry

I conclude this section with a formula for welfare changes that result when platform listing rules probabilistically shrink or expand consideration sets.

First, for a collection of products $\mathcal{R} = \{ r_1, \ldots, r_R \}$ define the total value of products in $\mathcal{R}$ by

$$\Gamma_t(\mathcal{R}) := \lim_{p_{r_2},\ldots,p_{r_R} \to \infty} \int_{p_{r_1}^m}^{\infty} Q_{r_1}(y, p, p'_{-r_1}\alpha_t)dp + \lim_{p_{r_3},\ldots,p_{r_R} \to \infty} \int_{p_{r_2}^m}^{\infty} Q_{r_2}(y, p, (p_{r_1}^m, p_{-(r_1,r_2)}), \alpha_t)dp \\ + \cdots + \int_{p_{r_R}^m}^{\infty} Q_{r_R}(y, p, p_{-r_R}^m, \alpha_t)dp$$

Note that $\Gamma_t(\mathcal{J}) = \Omega_t$.

Then, the following formula can be used to calculate exact welfare changes from multiple product entry.

Theorem 5. *Let $\mathcal{R} = \{r_1, \ldots, r_R\}$. Suppose that a change in listing rule from α_0 to α_1 increases the probability that each of the products in $\mathcal{R}$ is shopped. Then, under Linearity*

$$\mu^W = \Gamma_0(\mathcal{R}) - \Gamma_1(\mathcal{R})$$

The reference collection of products is $\mathcal{J} \setminus \mathcal{R}$. Demand for the newly entered products is all that needs to be used. This requires far fewer demand lines to be estimated than Theorem 1. The intuition is very similar same as for Theorem 1.

2.4 Identification with Instruments

Theorem 1 serves as a general tool to identify welfare changes from changes in consideration sets. However, it requires knowledge of each product's demand curve over its own prices and the prices of other products. If a researcher has limited demand knowledge over prices, but instead has knowledge of demand variation over consideration sets, there are alternative ways she can identify welfare changes from the entry of several products.

In particular, suppose consumers can be grouped into two collections $\mathcal{A}_1$ and $\mathcal{A}_2$. Under search rule α_0, the observable and unobservable characteristics of the two groups are identically distributed. Consumer groups $\mathcal{A}_1$ and $\mathcal{A}_2$ are treated differently under listing rule α_1. Denote the action of the platform on group $\mathcal{A}_1$ and $\mathcal{A}_2$ by $\alpha_{1,\mathcal{A}_1}$ and $\alpha_{1,\mathcal{A}_2}$ respectively. Assume this platform action exogenously increases the probability that consumers in group $\mathcal{A}_1$ have product M in their consideration sets and exogenously increases the probability consumers in group $\mathcal{A}_2$ have product M and $M+1$ in their consideration sets. Let $\mu^W(\mathcal{A}_1)$ and $\mu^W(\mathcal{A}_2)$ denote the average welfare effects of the platform's actions on consumer group 1 and 2 respectively. These average welfare effects can be calculated according to Theorem 6.

Theorem 6. *Under Linearity, the average welfare effects of the platform actions described* *above are*

$$\mu^W(\mathcal{A}_1) = \left[\int_{p_M}^{\infty} Q_M^m(y, p_M, \alpha_0)dp - \int_{p_M}^{\infty} Q_M^m(y, p_M, \alpha_{1,\mathcal{A}_1})dp\right]$$

$$\mu^W(\mathcal{A}_2) = \mu^W(\mathcal{A}_1) + \left[\int_{p_{M+1}^m}^{\infty} Q_{M+1}(y, p, \alpha_0)dp - \int_{p_{M+1}^m}^{\infty} Q_{M+1}(y, p, \alpha_{1,\mathcal{A}_2})dp\right]$$

Here, $Q_j(y, p_j, \alpha_1^{\mathcal{A}_k})$ denotes the average demand for product j for consumers in group $\mathcal{A}_k$.

Intuitively, exogenous variation in platform effects across consumers can also be used to estimate the welfare effects of simultaneous product entry. This works when there are groups of consumers who see a single product's increased probability of entering consideration sets and a group who sees both products' increased probability of entering consideration sets. Importantly, measuring the effects on group $\mathcal{A}_2$ requires combining observations of the effects on $\mathcal{A}_2$ and the effects on group $\mathcal{A}_1$. The effects on group $\mathcal{A}_1$ stand in for the area under the demand curve for product M when the price of product p_{M+1} is very large, as in eq. (2.9).

2.5 Simulation Studies

In this section, I present several simulations demonstrating the results of the previous section. First, I present a simulation that extends my example from Section 2.2. Second, I present simulations measuring the average welfare changes under several different consideration function rules $\mathcal{C}$. The first collection of simulations further illustrate the intuition behind Theorem 6. The second set of simulations demonstrate consumer welfare's sensitivity to $\mathcal{C}$ and, by extension, search listing rules.

2.5.1 Welfare and Two Product Entry

In this section, I present results estimating the welfare consequences of the simultaneous entry of two products. I simulate a market with $N = 1000$ members. I assume that each

Formula	Simulation Estimate
$\lim_{p_3\to\infty}\int_0^\infty Q_2(y,p,(\mathbf{0},p_3),\alpha_1)dp$ $+\int_0^\infty Q_3(y,p,\mathbf{0},\alpha_1)dp$	-\$ 91.25
$\int_0^\infty Q_2(y,p,0,p_3),\alpha_1)dp$ $+\int_0^\infty Q_3(y,p,\mathbf{0},\alpha_1)dp$	-\$ 1.41
$\int_0^\infty Q_1(y,p,0,\alpha_0)dp$	\$ 10.00

Table 2.1: The average welfare change μ^W from the entry of product 2 and 3 is given in the top line of this table. The simulated value is very close to the true value. We see that the formula in the second line has a simulated and true value that are very far from μ^W. The third line shows that, initially there was already significant value for product 1 relative to the outside product in the market.

consumer i has utility described by the following rule:

$$\begin{aligned}
u_0(y,\eta_i) &= y+\eta_{i0}\\
u_1(y-p,\eta_i) &= y-p+a+\eta_{i1}\\
u_2(y-p,\eta_i) &= y-p+10a+\eta_{i2}\\
u_3(y-p,\eta_i) &= y-p+10a+\epsilon+\eta_{i3}
\end{aligned}$$

where $\eta_i=(\eta_{i0},\eta_{i1},\eta_{i2},\eta_{i3})$, $a=10$, $\epsilon=1$ and $\eta_{ij}\sim N(0,1)$ for each i and j. At time $t=0$, all consumers shop exactly products 0 and 1. At time $t=1$, all consumers shop products 0, 1, 2 and 3. The market price of all products is normalized to 0 for all time periods. I estimate the welfare consequences of the simultaneous entry of products 2 and 3 on the population of normally distributed individuals using this market's data. This is a generalization of the example from Section 2.2

The main results are presented in table 2.1. We see that compensating variation, which has sample value -\$91.25, is much larger (in absolute value) than the area under the demand curve for product 2 plus the area under the demand curve for product 3, which is only -\$1.41.

2.5.2 Welfare Changes under Different Shopping Behavior

Consideration Rule 1: W Random Products

According to this shopping rule, consumers' consideration sets are independent of their preferences and prices. All consideration sets contain the outside product. Beyond that, W many of the 5 products are randomly assigned to each consumer's consideration set.

Consideration Rule 2: 2 Preferred Products

According to this shopping rule, each consumer's consideration set contains the outside product and the two purchasable product for which she has the highest unobservable preferences. That is, consumer i has the outside product in her consideration set along with the two products j and j' such that $\eta_{ij}, \eta_{ij'} \geq \max_{k \in \{1,\ldots,J\} \setminus \{j,j'\}} \eta_{ik}$. Consumers will always achieve the highest possible welfare according to this rule, despite only shopping two of the five purchasable products.

Consideration Rule 3: 2 Least Preferred Products

According to this shopping rule, each consumer's consideration set contains the outside product and the two purchasable products for which she has the lowest unobservable preferences. This will lead to product choices of purchasable products that are no more than their 3rd least-preferred product. Consumers can only get a relatively favorable product if it happens to coincide with their outside product preference. The total welfare of this consideration rule is much lower than any other rule.

Summary of Simulation Results

In this section, I simulate consumer welfare changes under several different consideration rules. The different consideration rules may be considered driven by different search listing rules. For simplicity, I will assume Linearity. In particular, preferences are assumed to follow

$$u_0(y, \eta_i) = y + \eta_{i0} \tag{2.13}$$

$$u_j(y - p, \eta_i) = y - p + \eta_{ij} \text{ for each } j \in \{1, 2 \ldots 5\}\} \tag{2.14}$$

$$\tag{2.15}$$

Here, $\eta_i = (\eta_{i1}, \ldots, \eta_{iJ})$ and η_{ij} are independent and normally distributed over i and j. For simplicity, all consumers are assumed to have the same income and all market prices are normalized to 0. I simulate $N = 10,000$ consumers. All welfare values, μ^W, are calculated according to Theorem 1. The results are summarized in table 2.2 and table 2.3.

A few lessons can be discerned. First, when consideration sets are filled randomly—that is, according to the W Random Product Consideration Rule–we see that the benefit of shopping more products decreases non-linearly with each additional product. This is because, in this setting, an additional product gets a consumer an additional draw from her unobservable, normally distributed preference distribution. Thus, a consumer's expected utility for randomly shopping W products (including the outside product) is

$$E[\max(Z_0, Z_1, Z_2, \ldots, Z_W)] \text{ where } Z_j \sim N(0,1)$$

while, the expected value of shopping one more product is

$$E[\max(Z_0, Z_1, Z_2, \ldots, Z_{W+1})] - E[\max(Z_1, Z_2, \ldots, Z_W)]$$

which is decreasing in W. This simple pattern is present because product choice is purely over unobservable characteristics. Increasing the number and variance of observable product characteristics will complicate the welfare consequences of more shopping. This example show us the important role that unobservable characteristics may play in final utilities and the value of taking a flexible approach to unobservables in welfare estimation.

Next, consider the welfare outcomes over the random consideration rule with $W = 2$, the 2 preferred products consideration rule, and the 2 least preferred products consideration rule. These simulations show that different consideration functions can produce very different total welfares, even for the same size of consideration sets. We see in table 2.3

Consideration Rule	Ω_0
2 Least Preferred Products	0.257
1 Random Product	.568
2 Random Products	.851
2 Preferred Products	1.289
3 Random Products	1.055
4 Random Products	1.177
All Products	1.289

Table 2.2: Total welfare Ω_0 is simulated under a variety of shopping behaviors.

that consumers who shop only two purchasable products can have total welfares ranging from .257 to .851 to 1.289. The first case is when consumers shop according to the Least Preferred Product rule. The second is under a 2 Random Product shopping rule. The final is under the 2 Most Preferred Product shopping rule. We can conclude that a search listing rule that provides some direction to consumers may significantly improve welfare over a consumer getting random results. These results highlight the value of recovering not just how many products each consumer shops, but also how the products shopped correlated with preferences when performing welfare analysis. The results in Theorem 1 automatically take these unobservable preferences into account.

2.6 Data Application

2.6.1 Data Overview

In this section, I estimate welfare changes from listing rule changes for a data set that details the click and purchase behavior of a collection of consumers booking hotels using an online

Consideration Rule Change	μ^W	$\lvert\mu^W\rvert/\Omega_0$
1 Random Product to 2 Random Products	-\$0.283	.498
2 Random Products to 2 Preferred Products	-\$0.438	.515
2 Random Products to 2 Least Preferred Products	\$0.594	.698
2 Random Products to 3 Random Products	-\$0.204	.240
3 Random Products to 4 Random Products	-\$0.122	.116
4 Random Products to All Products	-\$0.112	.095

Table 2.3: This table shows the average compensating variation, both in dollars and as a fraction of initial total welfare, Γ_0, over a variety of consideration function changes. Γ_0 is the total welfare from the consideration rule to the left of the "to." From the table, we see that consumer welfare grows by about 49.8% when a second product is added to a consideration set containing only 1 random product (and the outside product). This is nearly the same percent growth in average welfare as going from 2 random products in consideration sets to full information, 51.5%. (The welfare outcome for two preferred products is the same as the full information case, shopping all 5 purchasable products.) Each additional random product increases welfare, but as a decreasing fraction of initial total welfare. As consumers go from shopping two random products to their two least preferred product, average welfare falls by an amount of 69.8% their initial total.

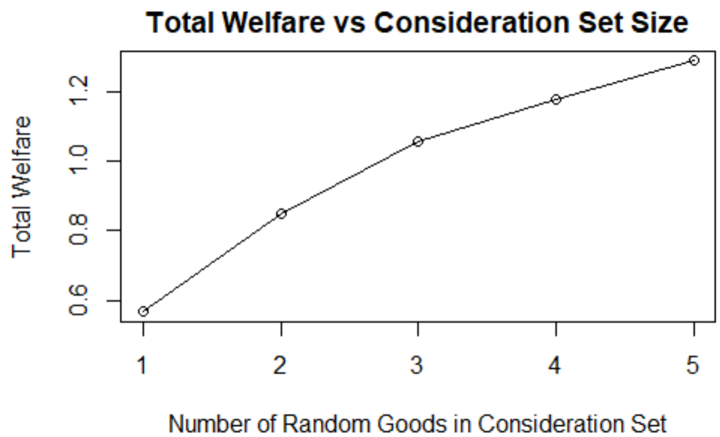


Figure 2.3: Each random product added to a consideration set increases total welfare. Since consideration sets are independent of preferences for this simulation, the increased welfare reflects increased unobservable preferences for the chosen product as consumers search longer. Indeed, this graph can be calculated by simply estimating $E(\max_{j\in\{1,\ldots,k\}}\eta_{ij}-\eta_{i0})$ for each number of random products in consideration sets k.

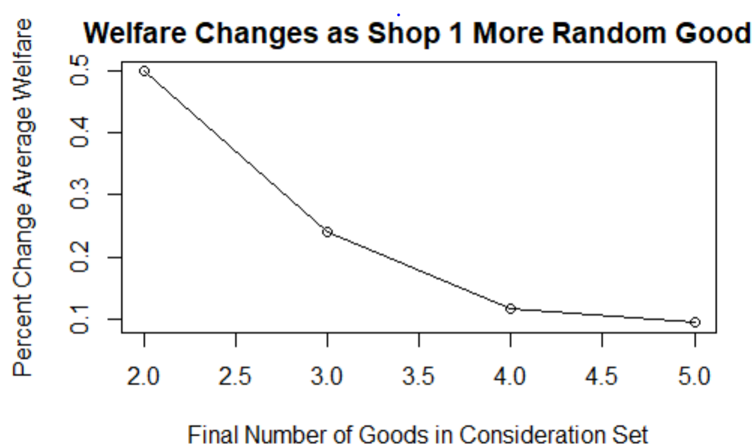


Figure 2.4: Each new product added to consideration sets increases utility, but by a shrinking amount. This is because the expected increased utility for one more draw from η_{ij} goes to 0 as the size of the number of products already in the consideration set grows.

travel agent (OTA). The data is from the 2013 data challenge for the IEEE's[13] International Conference on Data Mining (ICDM).[14] The competition was open to the public[15] through the online data science community Kaggle. Data for the contest was provided by Expedia, a large OTA.

The data is centered around a collection of *search impressions* that OTA users interacted with, primarily at *Expedia.com*. To understand the term search impression, first consider a consumer searching on *Expedia.com* for vacation accommodation in 2013. This consumer would initially face a page as pictured in fig. 2.6. Here, the OTA user would enter her vacation destination, the days she planned to spend in her vacation destination, the number of rooms she would like to book and the number of adults and children that she will be traveling with. All this information, pictured in the blue boxes in fig. 2.6, is collected by Expedia and used to produce a sequence of listings of available hotel rooms. The user is promptly directed to this listing sequence upon entering her information.

An example of a single hotel listing is given in fig. 2.7. The blue boxes identify information that Expedia collects and that is included in the data set for each hotel listing. Each search is likely to produce several listings. The number of individual listings will vary depending upon the destination city and the availability of hotel rooms at the given date. In the data, the number of listings on the first page of results vary between 1 and 34. A *search impression* is then defined to be the first page of search listings for a given user query. In addition to the information in blue boxes, Expedia also provides information on the listings clicked and booked for each search impression, as well as a vector of characteristics and past behavior for each consumer. Figure 2.5 shows the distribution of search impression length in the data.

When an OTA user clicks on a listing, a new page opens with more hotel details. In particular, information such as the size of available hotel beds, parking fees, pictures of room interiors, availability of free breakfast, room amenities and any hidden fees becomes

[13] IEEE is the Institute of Electrical and Electronics Engineers

[14] This is considered the world's premier research conference in data mining. Data challenges are typically held annually

[15] Competition rules only barred employees of online travel agencies from competing.

available to a user who clicks on a hotel listing. Booking cannot be done without clicking on the listing first.

There are a few features of this data set that make it particularly amenable to analyzing the relationship between consideration sets and prices. First, the data tells us information on the entire first page of search results each consumer faces. It also tells us the listing each consumer clicked as well as the listing the consumer eventually booked (if any). Thus, if we define the products that enter a consideration set as the products a consumer clicks, the data set tells us exactly the prices of all products the consumer could add to her consideration set and the products the consumer does add to her consideration set. Moreover, Expedia even provides us with data from one of their experiments: the data set includes search impressions where the hotel listing order was determined by Expedia's proprietary ranking algorithm as well as search impressions that had listings ordered randomly. This provides us with a few examples of how consideration set formation may change as platform listing rules change.

While the data set provides an excellent opportunity to study the relationship between prices and consideration sets, a few important caveats should be pointed out. First, search impressions only list the first page of results for each user query. Thus, a consumer who searches beyond the first page of listing results (should there be additional listings) will not have her search behavior correctly tied to her behavior on the first page of results. Her behavior on the second page of results would be treated as a separate (unassociated) search impression, if included at all. The same would be true for a consumer who went back to the first page of results and changed her search query. Thus, to the extent that consumers viewed multiple search result pages or considered alternative booking dates, the results of this study will underestimate the size of individual OTA user consideration sets. Ursu (2017) provides some evidence from a companion data set that more than 40% of Expedia.com users only look at the first page of results. Thus, assuming each consumer has consideration set exactly equal to the search impression is reasonable for a large fraction of Expedia.com users.

For competitive reasons, Expedia would not verify how representative the sample was. However, Ursu (2017)used a companion dataset from the Wharton Customer Analytics Initiative on consumer searches for hotels on a popular OTA in her study of this same Expedia

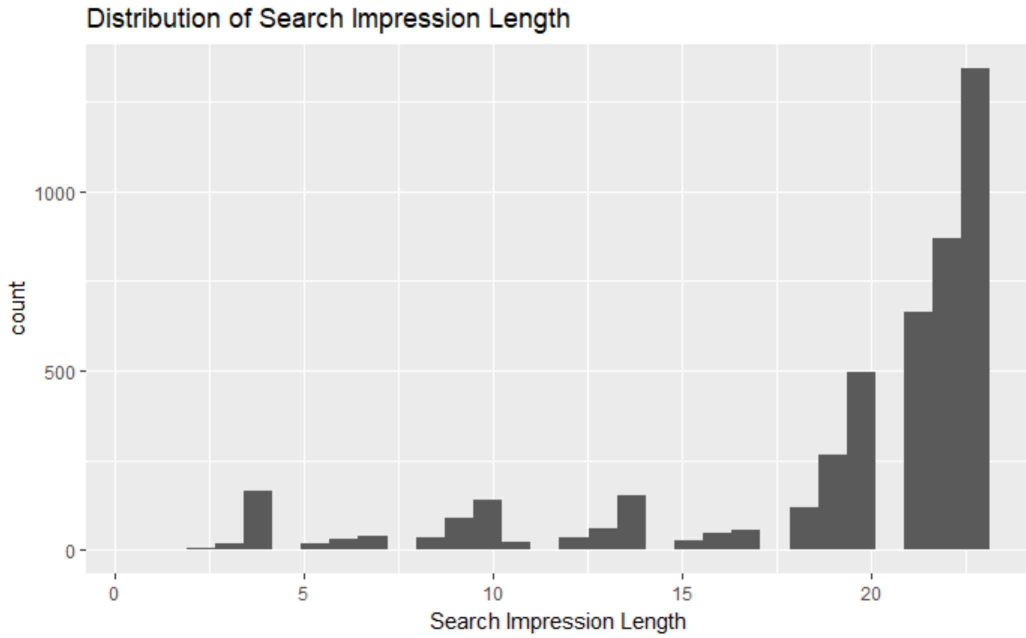


Figure 2.5: This figure shows the variation of search impression length over all consumers

data set. This companion study verified that the Expedia data set was representative of the largest shopping groups on Expedia.

2.6.2 Demand Estimation Strategy

I fit a model of utility where, for product j in consumer i's consideration set at time t, we have

$$u_{ijt} = \alpha(y - p_{ijt}) + \beta' X_j + \eta_{ijt}$$
$$u_{i0t} = \alpha y + \eta_{i0t}$$

where η_{ijt} is a standard Type I extreme value distributed random variable independent over j and t given i's consideration set. I assume each search impression is a unique user and that consideration sets are exactly the products listed in the search impression. This conclusion is supported in previous literature using the data set (namely, Ursu (2017)). Based on previous studies (Liu et al. 2013), I include property star rating, property branding information, the property location score and an indicator variable for promotions in X_j as strong predictors of product choice. I run the regression in R (R Core Team 2017) using the `mlogit` package, Croissant (2019). Results are displayed in table 2.4. All of the included variables are highly significant. As expected for demand, the coefficient on price is negative and highly significant.

Table 2.4

	Dependent variable:
	Hotel Booked
property star rating	0.513^{***}
	(0.048)
property brand boolean	0.418^{***}
	(0.053)
property location score 1	-0.922^{***}
	(0.043)
price in usd	-0.010^{***}
	(0.001)
promotion flag	0.266^{***}
	(0.047)
Observations	4,694
Note:	*p<0.1; **p<0.05; ***p<0.01

2.6.3 Welfare Change from Random Rankings to Purchase Rankings

As discussed in the data overview, the data contains information over two different listing rules. The first listing rule is a "random" listing rule. Under the random listing rule, the list order are filled in random order of relevant products, except for a few sponsored listings that also take slots in the list. The second listing rule ranks products using a proprietary algorithm known to Expedia.com. The rule ranks products (at least in part) by their relevance (or probability of purchase). Indeed, the goal of the data challenge was to produce an algorithm that could Learn To Rank the products in order of their purchase and click likelihood; these are called LeToR algorithms.

In order to estimate the change in welfare between the two listing rules, I use demand estimates from the previous section. Given the consideration groups observed over the two listing rules, I can predict average demand for each group and over all prices. Using eq. (2.7) and Theorem 1, I find total welfare under the random listing rule to be \$104.81 and total welfare under the proprietary ranking rule to be \$113.65. Thus, I conclude welfare is improved by the proprietary ranking person by an average of \$8.84 per person.

2.6.4 Welfare Changes from Removing the Top 5 Products

In this section I estimate the counterfactual welfare loss from a new listing rule that hides the top 5 products from consideration sets. The top 5 products are the products with the largest market share in the sample data. These market shares are listed in table 2.5. Together, these firms account for about 20% of the observed bookings. The market is not dominated by any one hotel: even the hotel with the largest market share accounts for less than 6% of total sales.

Given my demand estimates from earlier in this section, I calculate welfare using Theorem 1. The calculation amounts to removing product A then B then C, et cetera. The results are given in table 2.5. The total welfare loss from the 5 products averages to a value of \$20.51 per person. Each additional product reduces consumer welfare by \$2-\$6.

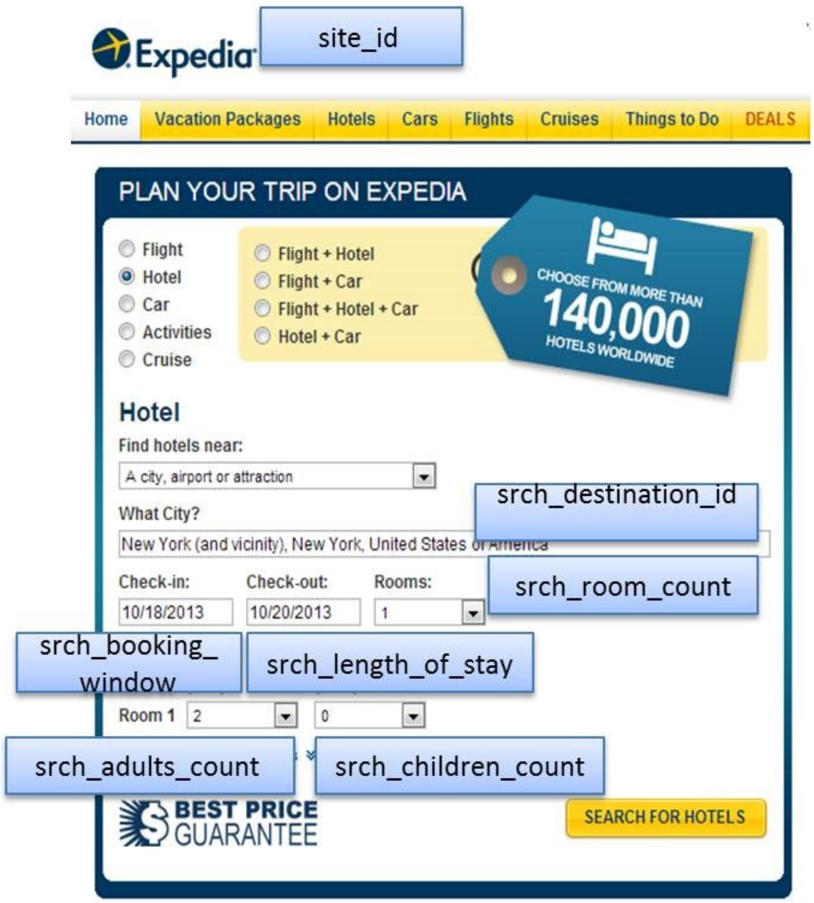


Figure 2.6: First page encountered by users of Expedia.com. Users select their travel destination, the number of rooms they wish to book, the days they wish to spend at the destination, and the number of adults and children who will be staying in the room. All of this information, highlighted by the blue boxes, is collected by Expedia.com and taken to the website

Table 2.5: This table estimates the welfare loss from removing product A then B, then C, etc. until E is removed last. Products A through E are the most frequently purchased products in the data set.

Product	Market Share	Estimated Market Share	Estimated Welfare Loss
A	.05646	.021283	$2.27
B	.04772	.030695	$3.37
C	.04261	.043229	$4.98
D	.04026	.045443	$5.54
E	.039625	.034399	$4.34
Total	.2267	.1750	$20.51

2.7 Conclusion

I have presented several formulas for measuring consumer welfare changes that result from an online shopping platform changing the way it lists search results. Welfare changes can be recovered with straightforward integrals of aggregate demand. I have also provided formulas for estimating counterfactual welfare changes under certain, simple listing rule changes. Applications of the formulas to demand data from an online travel agency yield reasonable welfare results.

Figure 2.7: This picture shows a typical listing that Expedia users would have observed during the experimental period. Each search impression contains between 1 and 34 of these listings.

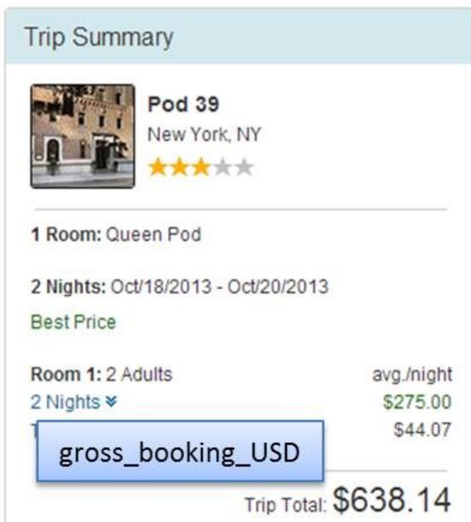


Figure 2.8: Picture detailing the final choice made by the consumer

2.A Proofs from chapter 2

For this appendix, I define *inverse utility for product j*, denoted $u_j^{-1}(\bar{u}, \eta)$, as the solution in y to

$$u_j(y, \eta) = \bar{u} \tag{2.16}$$

By Monotonicity, $u_j^{-1}(\cdot, \eta)$ is a well-defined, continuous and strictly increasing function.

First, I consider the case of a single good's entry or exit from. The results are straightforward extensions of the results from chapter 4.

Theorem 7. *Suppose good 1 was available under α_0 but is no longer available under α_1. Then, under Monotonicity, the average equivalent variation from this loss is exactly*

$$\mu^{EV} = \int_{p_1^m}^{\infty} Q_1(y, p, p_{-1}^m, \alpha_0) dp$$

while the average compensating variation is exactly

$$\mu^{CV} = \int_{p_1^m}^{\infty} Q_1(y + p - p_1^m, p, p_{-1}^m, \alpha_0) dp$$

The proof is in Section 2.A.2

Theorem 8. *Suppose good M is unavailable for consideration or purchase under α_0 but becomes available under α_1. Then, under Assumption 9.A, the average compensating variation of this good's availability is exactly*

$$\mu^{CV} = \int_{p_M^m}^{\infty} Q_M(y, p, p_{-M}^m, \alpha_1) dp$$

while the average equivalent variation is exactly

$$\mu^{EV} = \int_{p_M^m}^{\infty} Q_1(y + p - p_M^1, p, p_{-M}^m, \alpha_1) dp$$

This formula does not require all consumers to shop good M once it becomes available. Good M can be shopped idiosyncratically under α_1. It only requires that no consumer has good M in her consideration under α_0. This formula can be interpreted in terms of a price decrease of good M, from infinity down to its market price.

The asymmetric formulas across compensating variation and equivalent variation for the good addition and good removal cases are a direct consequence of the asymmetric formulas for single price increases and single price decreases. See Section 4.2 for more details.

2.A.1 Proof of Theorem 1

Fix an arbitrary consumer (y, η, ζ) and let ω_t be defined by

$$\omega_t := \lim_{p_2^m, \ldots, p_J^m \to \infty} \int_{p_1^m}^{\infty} q_1(y, p, p_{-1}^m, \eta, \zeta, \alpha_t) dp + \lim_{p_3^m, \ldots, p_J^m \to \infty} \int_{p_2^m}^{\infty} q_2(y, p, p_{-2}^m, \eta, \zeta, \alpha_t) dp \tag{2.17}$$

$$+ \cdots + \int_{p_J^m}^{\infty} q_J(y, p, p_{-J}^m, \eta, \zeta, \alpha_t) dp \tag{2.18}$$

By Linearity and Tonnelli's Theorem, it suffices to prove that

$$\omega_t = \max_{j \in \mathcal{C}_t} u_j(y - p_j^m, \eta) - u_0(y, \eta)$$

because then

$$\omega_0 - \omega_1 = \max_{j \in \mathcal{C}_0} u_j(y - p_j^m, \eta) - \max_{j \in \mathcal{C}_1} u_j(y - p_j^m, \eta) = S^W$$

and

$$\Omega_0 - \Omega_1 = \int \omega_0 dF - \int \omega_1 dF = \mu^W$$

To that end, note that

$$\begin{aligned}
\omega_t = & \lim_{p_2^m, \ldots, p_J^m \to \infty} \int_{p_1^m}^{\infty} q_1(y, p, p_{-1}^m, \eta, \zeta, \alpha_t) dp + \lim_{p_3^m, \ldots, p_J^m \to \infty} \int_{p_2^m}^{\infty} q_2(y, p, p_{-2}^m, \eta, \zeta, \alpha_t) dp \\
& + \cdots + \int_{p_J^m}^{\infty} q_J(y, p, p_{-J}^m, \eta, \zeta, \alpha_t) dp \\
= & \mathbb{1}\left(1 = \arg \max_{\mathcal{C}_t \setminus \{2,3,\ldots,J\}} \tilde{U}_j(\eta) - p_j^m\right) \left[\max_{j \in \mathcal{C}_m \setminus \{2,3,\ldots,J\}} \tilde{U}_j(\eta) - p_j^m - \left(\max_{j \in \mathcal{C}_t \setminus \{1,2,\ldots,J\}} \tilde{U}_j(\eta) - p_j^m\right)\right] \\
& + \mathbb{1}\left(2 = \arg \max_{\mathcal{C}_t \setminus \{3,\ldots,J\}} \tilde{U}_j(\eta) - p_j^m\right) \left[\max_{j \in \mathcal{C}_t \setminus \{3,4,\ldots,J\}} \tilde{U}_j(\eta) - p_j^m - \left(\max_{j \in \mathcal{C}_t \setminus \{2,3,\ldots,J\}} \tilde{U}_j(\eta) - p_j^m\right)\right] \\
& + \cdots + \mathbb{1}\left(J = \arg \max_{\mathcal{C}_t} \tilde{U}_j(\eta) - p_j^m\right) \left[\max_{j \in \mathcal{C}_t} \tilde{U}_j(\eta) - p_j^m - \left(\max_{j \in \mathcal{C}_t \setminus \{J\}} \tilde{U}_j(\eta) - p_j^m\right)\right]
\end{aligned}$$

I now conclude the proof with induction on $J \in \mathbb{N}$.

Note that for $J = 1$, $\mathcal{C}_t \subseteq \{0, 1\}$ and

$$\begin{aligned}
\omega_t &= \mathbb{1}(1 = \arg\max_{\mathcal{C}_t}) \left[\tilde{U}_1(\eta) - p_1^m - \tilde{U}_0(\eta)\right] \\
&= \max_{j \in \mathcal{C}_t} \left[U_j(\eta) - p_j^m\right] - \tilde{U}_0(\eta)
\end{aligned}$$

which proves the base case. Now suppose it holds for the collection of goods $\{0, 1, \ldots, K\}$. Then, for $J = K + 1$

$$\begin{aligned}
\omega_t = &\left[\max_{j \in \mathcal{C}_t \setminus \{J\}} \tilde{U}_j(\eta) - p_j^m\right] - \tilde{U}_0(\eta) \qquad \text{(by inductive hypothesis)} \\
&+ \mathbb{1}\left(J = \arg\max_{\mathcal{C}_t} \tilde{U}_j(\eta) - p_j^m\right) \left[\max_{j \in \mathcal{C}_t} \tilde{U}_j(\eta) - p_j^m - \left(\max_{j \in \mathcal{C}_t \setminus \{J\}} \tilde{U}_j(\eta) - p_j^m\right)\right] \\
= &\max_{j \in \mathcal{C}_t} \left[\tilde{U}_j(\eta) - p_j^m\right] - \tilde{U}_0(\eta)
\end{aligned}$$

which concludes the proof.

2.A.2 Proof of Theorem 7

Fix a consumer (y, η, ζ). There are two cases for this consumer. In case 1, she did not purchase good 1 under α_0. In case 2, she did.

In case 1, the consumers purchase behavior does not change before and after the change in platform behavior. Thus, $S^{CV} = 0 = S^{EV}$. Moreover, the consumer has 0 demand for good 1 at every price above p_1^m by Monotonicity. She also has 0 demand for good 1 as the prices of other goods in her consideration set are lowered while holding the price of good 1 fixed. Thus,

$$\int_{p_1^m}^{\infty} q_1^m(y, p, \eta, \zeta, \alpha_0) dp = 0 = \int_{p_1^m}^{\infty} q_1^m(y + p - p_1^m, p, \eta, \zeta, \alpha_0) dp$$

Next consider case 2. The consumer initially purchased good 1 but cannot purchase it after the change in platform behavior. Thus, S^{EV} is such that.

$$\max_{j \in \mathcal{C}_0} u_j(y - p_j^m - S^{EV}, \eta) = \max_{j \in \mathcal{C}_0 \setminus \{1\}} u_j(y - p_j^m, \eta)$$

Since the consumer is worse off after the change, $S^{EV} \geq 0$. Moreover, for all S such that $0 < S < S^{EV}$ we have

$$\max_{j \in \mathcal{C}_0} u_1(y - p_j^m - S, \eta) > \max_{j \in \mathcal{C}_0 \setminus \{1\}} u_j(y - p_j^m, \eta) > \max_{j \in \mathcal{C}_0 \setminus \{1\}} u_j(y - p_j^m - S, \eta)$$

Thus, S^{EV} solves

$$u_1(y - p_1^m - S^{EV}, \eta) = \max_{j \in \mathcal{C}_0 \setminus \{1\}} u_j(y - p_j^m, \eta)$$

Thus, if $\bar{p}$ is the price of good 1 such that this consumer switches from choosing good 1 to her second-best option, then

$$u_1(y - p_1^m - S^{EV}, \eta) = \max_{j \in \mathcal{C}_0 \setminus \{1\}} u_j(y - p_j^m, \eta) = u_1(y - \bar{p}, \eta)$$

So,

$$S^{EV} = \bar{p} - p_1^m = \int_{p_1^m}^{\infty} q_1^m(y, p, \eta, \zeta, \alpha_0) dp$$

Aggregating over both cases and all individuals we see

$$\begin{aligned} \mu^{EV} &= \int \int_{p_1^m}^{\infty} q_1^m(y, p, \eta, \zeta, \alpha_0) dp dF \\ &= \int_{p_1^m}^{\infty} Q_1(y, p, p_{-1}^m, \alpha_0) dp \end{aligned}$$

which finishes the proof for the case of equivalent variation.

Next, note that S^{CV} is also positive and solves

$$\max_{j \in \mathcal{C}_0} u_j(y - p_j^m, \eta) = \max_{j \in \mathcal{C}_0 \setminus \{1\}} u_j(y - p_j^m + S^{CV}, \eta)$$

Thus

$$\begin{aligned} S^{CV} &= \int_0^{\infty} \mathbb{1}(S < S^{CV}) dS \\ &= \int_0^{\infty} \mathbb{1}(u_1(y - p_1^m, \eta) > \max_{j \in \mathcal{C}_0 \setminus \{1\}} u_j(y - p_j^m + S, \eta)) dS \\ &= \int_0^{\infty} q_1^0(y + p - p_1^m, p, \eta, \zeta, \alpha_0) dp \end{aligned}$$

Aggregating as above, over individuals and both cases, yields

$$\mu^{CV} = \int_{p_1^m}^{\infty} Q_1(y + p - p_1^m, p, p_{-1}^m, \alpha_0) dp$$

which concludes the proof.

2.A.3 Proof of Theorem 2

I will prove the result for the μ^{CV}. The proof for μ^{EV} is analogous.

Fix a consumer (y, η, ζ). There are two cases. In case 1, the consumer makes the same good choice under α_0 as under α_1. In case 2, her choice changes. In case 1, S^{CV} must be zero, and we see

$$\int_{p_1^m}^{\infty} q_1^m(y, p, \eta, \zeta, \alpha_0)dp - \int_{p_1^m}^{\infty} q_1^m(y, p, \eta, \zeta, \alpha_1)dp = 0$$

In case 2, she must have purchased good 1 at time $t = 0$. We know from Section 2.A.2 that

$$S^{CV} = \int_{p_1^m}^{\infty} q_1^m(y, p, \eta, \zeta, \alpha_0)dp$$

Moreover, for this consumer, since $1 \notin \mathcal{C}_1$ (otherwise she would have purchased it,

$$0 = \int_{p_1^m}^{\infty} q_1^m(y, p, \eta, \zeta, \alpha_1)dp$$

Thus, for this case as well,

$$S^{CV} = \int_{p_1^m}^{\infty} q_1^m(y, p, \eta, \zeta, \alpha_0)dp - \int_{p_1^m}^{\infty} q_1^m(y, p, \eta, \zeta, \alpha_1)dp$$

Aggregating over consumers and cases and then switching integration order by Tonelli's theorem yields the desired result.

2.A.4 Proof of Theorem 5

This is a corollary of Theorem 1. As discussed in Section 2.A.1, the ordering of the goods doesn't matter. So, let the goods $1, \ldots, J = 1, 2 \ldots, r_1, \ldots, r_R$. That is, good $J = r_R$, good $J - 1 = r_{R-1}, \ldots, J - R + 1 = r_1$. Then

$$\Omega_t = \lim_{p_2^m, \ldots p_J^m} \int_{p_1^0}^{\infty} Q_1(y, p, p_{-1}^m, \alpha_t)dp + \cdots + \lim_{p_{r_1-1}^m, \ldots p_J^m} \int_{p_{r_1-1}^t}^{\infty} Q_{r_1-1}(y, p, p_{-(r_1-1)}^m, \alpha_t)dp + \Gamma_t r$$

Since

$$\Delta_t := \lim_{p_2^m, \ldots p_J^m} \int_{p_1^m}^{\infty} Q_1(y, p, p_{-1}^m, \alpha_t)dp + \cdots + \lim_{p_{r_1-1}^m, \ldots p_J^m} \int_{p_{r_1-1}^t}^{\infty} Q_{r_1-1}(y, p, p_{-(r_1-1)}^m, \alpha_t)$$

is invariant to goods $\{ r_1, \dots, r_R \}$ inclusion in consideration sets or not–the price limits make these Δ_t independent of them–we see $\Delta_1 = \Delta_0$ and thus

$$\Omega_0 - \Omega_1 = \Gamma_0(\mathcal{R}) - \Gamma_1(\mathcal{R})$$

2.A.5 Proof of Theorem 4

Fix a consumer (y, η, ζ). I start by showing

$$S^W = \int_{p_1^m}^{\infty} q_1^m(y,p,\eta,\zeta,\alpha_0)dp - \int_{p_1^0}^{\infty} q_1^m(y,p,\eta,\zeta,\alpha_1)dp - \int_{p_M^m}^{\infty} q_M^m(y,p,\eta,\zeta,\alpha_0)dp + \int_{p_M^m}^{\infty} q_M^m(y,p,\eta,\zeta,\alpha_1)dp$$

In the cases where only good 1 exits this consumers consideration set or only good M enters this consumer's consideration set or her purchase behavior does not change, the result is clear from previous results. This leaves only the case where the consumer purchased good 1 under α_0 and purchases good M under α_1. In this case, $\mathcal{C}_0 \setminus \{ 1 \} = \mathcal{C}_1 \setminus \{ M \}$

Thus,

$$\begin{aligned}
&\int_{p_1^m}^{\infty} q_1^m(y,p,\eta,\zeta,\alpha_0)dp - \int_{p_1^m}^{\infty} q_1^m(y,p,\eta,\zeta,\alpha_1)dp - \int_{p_M^m}^{\infty} q_M^m(y,p,\eta,\zeta,\alpha_0)dp + \int_{p_M^0}^{\infty} q_M^m(y,p,\eta,\zeta,\alpha_1)dp \\
&\quad = u_1(y - p_1^m, \eta) - \max_{j \in \mathcal{C}_0 \setminus \{ 1 \}} u_j(y - p_1^m, \eta) - \left[u_M(y - p_M^m, \eta) - \max_{j \in \mathcal{C}_1 \setminus \{ M \}} u_j(y - p_j, \eta) \right] \\
&\qquad\qquad \text{(Theorem 7, Theorem 3 and Linearity)} \\
&\quad = u_1(y - p_1^m, \eta) - u_M(y - p_M^m, \eta) \\
&\quad = S^W
\end{aligned}$$

CHAPTER 3

Conditional Demand And Welfare

In this chapter, I continue my study of consumers shopping for a discrete good online. However, in this chapter I assume that consideration sets and individual purchase decisions are observable at market prices. Thus, consumers can be grouped by the products they shop and buy. In addition, I assume the researcher knows the demand curve for each of these groupings; I call this "conditional demand." In this setting, I identify consumer welfare changes—specifically, compensating variation and equivalent variation—as a response to search platform behavior changes and price changes.

In my model, consumers are allowed two categories of unobservable heterogeneity: unobservable preferences and unobservable search characteristics. I do not restrict the dimension of either of these unobservables. When utility is linear in money, I determine consumer welfare changes with few restrictions on how consumers search. I also find that welfare changes can be predicted when a welfare change shrinks the collection of goods consumers search. With nonparametric utility, a few more, but mostly testable, restrictions on shopping behavior and data quality are required to identify welfare changes.

In the econometric literature, my paper is most closely related to that of Bhattacharya (2015). Bhattacharya (2015) determines closed-form solutions for the distribution of equivalent variation and compensating variation as a function of aggregate demand. He does not consider demand identification itself. Welfare changes arise because of the exogenous price change of a single good. In his environment, good choices are discrete and consumers have knowledge of all products. Bhattacharya (2015) makes no assumptions on the dimension of unobservable preferences and only weak monotonicity and continuity assumptions on utility functions. Utility is not assumed separable in unobservables. Bhattacharya (2015) shows that *exact* formulas for average welfare changes can be determined from aggregate demand alone; there is no need to find compensated (Hicksian) demand. This holds even though utility is not linear in money.

As in Bhattacharya (2015), I deal with the point identification of welfare changes given demand in a discrete choice setting. My demand assumptions generalize his. Consumers in my model have limited product knowledge. Hence, I assume the researcher has demand conditional on consumers' observable attributes and the products they search. With this demand, I determine exact formulas for compensating variation and equivalent variation in response to an exogenous price change, an exogenous increase or decrease of consideration sets by one good and an exogenous single-good swap. For all results but the single-good swap, my preference model is identical to that of Bhattacharya (2015). For the single good swap, I assume preferences are linear in money. To the best of my knowledge, my paper is the first to consider welfare changes in such a general preference framework and without a specific game-theoretic search model. Especially, my formulas for welfare changes in response to changes in consideration sets significantly generalize the technique and applicability of Bhattacharya (2015). All my solutions nest the full-information markets considered in Bhattacharya (2015).

In addition to welfare changes from a single good's exogenous price change, I point identify formulas for welfare changes as a result of multiple exogenous price changes occurring simultaneously with changes in consideration sets. Doing so requires strengthening my data assumptions to include demand given consideration sets and product choices. That is, I must condition demand on a more specific group to get these results. To the best of my knowledge, I am the first to identify exact formulas for equivalent variation and compensating variation from multiple price changes or multiple exogenous changes in consideration sets when preferences are nonseparable over discrete goods.

While my paper assumes discrete goods, there is a significant literature studying welfare with continuous choice. Continuous choice allows consumers to choose consumption quantities from the nonnegative real line. Most recently, Jerry A. Hausman and W. K. Newey (2016) researched welfare identifiability in markets without search under preference and data assumptions very similar to those of Bhattacharya (2015). However, Jerry A. Hausman and

W. K. Newey (2016) show that average equivalent variation cannot be identified with continuous choice in this general preference setting. Instead, they offer welfare bounds through the solution of a differential equation.

Work in discrete and continuous choice welfare analysis goes back farther than Bhattacharya (2015) and Jerry A. Hausman and W. K. Newey (2016). Other results in discrete choice welfare analysis include Domencich and D. McFadden (1975), Small and Rosen (1981), Herriges and Kling (1999), Dagsvik and Karlström (2005) and Berry and Haile (2014). Other recent results in welfare analysis with continuous choice include Hoderlein and Vanhems (2011), Blundell, Horowitz, and Perry (2012), Lewbel (2013). These results all rely on either stricter preference assumptions, stricter assumptions on unobservables or approximations to continuous choice models. None of these results limit consumer knowledge to consideration sets.

It is important to note that I follow Bhattacharya (2015) and Jerry A. Hausman and W. K. Newey (2016) in point identifying welfare changes as a function of (conditional) demand. This can be contrast with the (full) identification of welfare changes starting from observational price and quantity data. For markets with either discrete or continuous choice and no search, there is an established literature on identifying and estimating demand from observational data. See for example, R. Matzkin (1993), R. L. Matzkin (1993), R. W. Blundell and Powell (2004), and Chernozhukov, Fernandez-Val, and W. Newey (2018) in discrete choice and R. W. Blundell and Powell (2004), R. Matzkin (2015), Richard Blundell, Kristensen, and R. Matzkin (2014), Richard Blundell, Kristensen, and R. Matzkin (2013) in continuous choice. However, conditional nonparametrtic demand identification and estimation in search markets is a relatively new research agenda. To the best of my knowledge, the only paper on conditional demand identification or estimation from observational data without a game-theory based model of consumer search is Amano, Rhodes, and Seiler (2017); Amano, Rhodes, and Seiler (2017) assumes utility given consideration sets is linear with additively separable, Type I Extreme Value unobservables. Since my research interests are first in consumer welfare, my analysis has started assuming ideal demand data. Nonparametrically identifying conditional demand from micro search data is still an open problem.

Comparison With Literature on Consumer Welfare in Search Markets

Game-theoretic research of markets where consumers search goes at least as far back as the early 1960s with Stigler (1961). From then until now, researchers have steadily written more nuanced game-theoretic models examining markets with consumers who search (e.g. Diamond (1971), Rothschild (1974), Weitzman (1979), Burdett and Judd (1983), Morgan and Manning (1985), McAfee (1995), Chade and Smith (2006) and G. Ellison and S. F. Ellison (2009)). In the 2000s, when advances in computer science made large, detailed micro search datasets available, an outpouring of research began adapting these game-theoretic models to empirical questions. A large number of influential empirical papers studying consumer surplus changes in search markets have been written. Studied topics include welfare changes as a result of search ranking changes (e.g. Ursu (2017) and Athey and G. Ellison (2011)), the welfare effects of platform changes (e.g. Lewis and Wang (2013), Dinerstein et al. (2017), Fradkin (2015)), the welfare effects of advertising and search (e.g. Honka, Hortaçsu, and Vitorino (2017) and Seiler and Yao (2017)), and the welfare effects of changing search costs (e.g. Honka (2014), Ershov (2016) and Moraga-Gonzalez, Sándor, and Wildenbeest (2017)).

While much has been learned, the results depend on strong modeling assumptions. Namely, the results depend on parametric utility forms, additively separable and parametric unobservables, and strong, model-based assumptions on the dependence between consideration sets and preferences. The results of this paper can help empirical papers make more robust conclusions. The examples in this paper can serve as a guide to the sources of confoundment in consumer welfare analysis in search markets. Finally, the framework in this paper can provide a foundation for the development of statistical tests on the presence of confoundment in welfare models. In particular, this paper can provide the foundation for a statistical test on the dependence structure between consideration sets and preferences.

The dependence I allow between consideration sets and preferences is a significant generalization of existing results. To the best of my knowledge, strong model-based dependence structures are used in all existing results on consumer surplus changes in markets where consumers search. Importantly, incorrectly assumed dependence structures can seriously jeopardize the validity of consumer surplus results; see Section 3.5 of this paper for an example. This paper's focus on conditional demand sidesteps the dependence issues that may arise in papers that directly model the search process. The method used in this paper allows for the correct identification of consumer welfare changes under essentially arbitrary consideration set and preference dependence. Instead, I require that the researcher has access to conditional demand for fixed consideration sets and that consideration sets do not change with prices; see Section 3.1 of this paper for the details. Assuming consideration sets do not vary with prices is a standard technique used in this literature. See Amano, Rhodes, and Seiler (2017) for an empirical application of conditional demand estimation.

The results of this paper are also related to studies of the welfare effects of product and platform entry and exit. Markets with entry and exit can be thought of as special cases of the search markets I consider in this paper. For example, a market with product entry has each consumer's consideration set increase by one good. Thus, the results of this paper are related to this literature as well. See Heckman (1974), J. Hausman (1996), Nevo (2003), Gentzkow (2007), Quan and Williams (2016) and Petrin (2002) for influential papers on estimating consumer welfare changes in markets with product or labor supply entry.

Content Organization

The rest of this paper is organized as follows. In Section 3.1, I introduce the notation I will use for the remainder of the paper and discuss my data assumptions. In Section 3.2, I consider welfare changes from exogenous price changes. In Section 3.3, I consider welfare changes from arbitrary, exogenous changes in consideration sets and prices; these are the main results of the paper. In Section 3.4, I consider welfare changes from adding goods to consideration sets, removing goods from consideration sets or swapping one good for another; these results can be used for counterfactual welfare estimation when platforms can significantly influence consideration sets. In Section 3.5, I show how structure imposed by game-theoretic search models, unnecessary with a conditional demand approach, may strongly bias welfare estimates. In Section 3.6, I conclude. Proofs are given in the appendices. All figures are presented at the end in Section 3.E.

3.1 Notation and Model Overview

In this section, I develop the notation I need for the rest of the chapter. I start by looking at consumer preferences in Section 4.1.1. I develop notation and assumptions for consideration sets in Section 4.1.2. I define and develop notation for welfare measures in Section 4.1.3.

3.1.1 Preferences

My setup for goods and preferences follows the multinomial choice framework with nonseparable utility laid out in Bhattacharya (2015). There are *two time steps* $t = 0, 1$. There is a *set of goods* $\mathcal{J} = \{0, 1, \ldots, J\}$.[1] *Goods* have *temporally invariant attributes* $(0, X_1, \ldots, X_J)$. Good attributes may vary with the platform; different websites will have different ratings. Thus, each product is platform specific. Since good attributes are unimportant for the results in this paper, I will suppress their notation. All results should be interpreted as conditional on good attributes. All prices may vary with time. *Prices at time step t* are $p^t_{\mathcal{J}} = (0, p^t_1, \cdots, p^t_J)$. The price of the outside good is fixed at 0 for both periods. Goods, prices and attributes are all observable to the researcher at all time periods.

Consumers have *observable income y* and *observable attributes* Ψ that affect their utility. Both y and Ψ are fixed for all individuals over time. In examples, I will treat gender as a component of Ψ. All identification results should be interpreted as conditional on y and Ψ. I will suppress the notation for Ψ from utility for readability.

Consumers have *unobservable preferences η*. I follow Bhattacharya (2015) and do not restrict the dimension of these unobservable preferences.[2] Instead, I assume that η is tempo-

[1] It is WLOG to have the goods invariant over time. For example, if good M becomes available at time step 1 but isn't available at time step 0, then we can just disallow M from being in initial consideration sets.

[2] See Bhattacharya (2015) for a discussion on the importance of leaving the dimension of heterogeneity unrestricted in discrete choice preferences

rally invariant for each consumer. In addition, I restrict utility using the following assumptions.

Assumption 3.A (Monotonicity)**.** *Utility for good j, $u_j(y - p_j, \eta; X_j, \Psi) \in \mathbb{R}$, is strictly increasing and continuous in its first argument for all goods $j \in \mathcal{J}$.*

Assumption 3.B (Linearity)**.** *Utility for good j is determined by*

$$u_j(y - p_j, \eta; X_j, \Psi) = y - p_j + \tilde{U}_j(\eta; X_j, \Psi)$$

where $\tilde{U}_j(\eta) \in \mathbb{R}$ for all goods $j \in \mathcal{J}$.

Suppressing Ψ and X_j from utility notation leaves us with abbreviated utility forms $u_j(\tilde{y} - p_j, \eta)$ and $y - p_j + \tilde{U}_j(\eta)$. Linearity is sufficient for Monotonicity. I maintain Monotonicity for the rest of the paper. Linearity will only be used for certain results.

I define *inverse utility for good j*, denoted $u_j^{-1}(\bar{u}, \eta)$, as the solution in y to

$$u_j(y, \eta) = \bar{u} \tag{3.1}$$

By Monotonicity, $u_j^{-1}(\cdot, \eta)$ is a well-defined, continuous and strictly increasing function.

3.1.2 Consideration Sets

Consumers have imperfect knowledge of the products in $\mathcal{J}$. The shopping process is how consumers acquire product information. I will denote a *consumer's consideration function* by $\mathcal{C}$. Consumers have perfect knowledge of the prices, attributes and utilities of all the goods in their consideration sets. Consumers are uncertain of the prices, attributes, and hence utilities of all goods not in their consideration sets. Further, consumers cannot buy goods that are not in their consideration sets.

Consider a consumer searching for sunglasses online. She may have an idea about the characteristics and prices of the sunglasses she wants. However, in order to buy a pair, she will first have to find a place that will sell them to her. As she navigates to websites of sunglass sellers and discovers sunglasses she can buy, her uncertainty about the products she discovers is resolved. The sunglasses she discovers become part of her consideration set.

The sunglasses in her consideration set will depend on her observable characteristics Ψ. For example, female shoppers will likely prefer sunglasses designed for women, or at least unisex sunglasses. Moreover, the sunglasses a consumer shops also depends on unobservable preferences η. A consumer who prefers polarized sunglasses will likely have more polarized sunglasses in her consideration set, having included "polarized" in her keyword search.

An individual's consideration set is also determined by her *unobservable, non-preference characteristics* ζ. For example, ζ captures a consumer's preference for the act of shopping itself or a consumer's familiarity with internet shopping tools. If a consumer enjoys shopping, she will likely have a larger consideration set. If a consumer has limited knowledge of browser plugins,[3] comparison shopping engines,[4] or platform-specific tools of refining search results,[5] it will affect her consideration set. Together, ζ and η will capture a consumer's *price and product characteristic beliefs*. I assume ζ is temporally invariant. I denote ζ and η's *joint distribution* by F. I do not restrict the dependence structure between ζ and η.

Finally, a consumer's consideration set is affected by the online intermediary behavior. This includes the way platforms return search results, the way advertising appears on platforms and other features of the platforms' websites. I call this *platform behavior* and denote it by α^t. I assume α^t is observed for both t. Despite the specific name, it should be understood to include the behavior of any relevant intermediary who can affect a consumer's consideration set.

Platform behavior changes are central to my research question. Exogenous changes in platform behavior provide a channel for changes in consumer welfare without changes in prices.

[3] Browser plugins are tools that can be added to your browser. There are several browser plugins designed specifically for online shopping. For example, Honey, at https://www.joinhoney.com/ searches the internet for coupon codes to apply to your order. When shopping on Amazon.com, it will also search across sellers within Amazon to find the lowest priced seller of whatever you are browsing.

[4] Comparison shopping engines are websites that allow cross-platform comparisons of goods. They are a special case of search aggregators. Examples of comparison shopping engines include Google Shopping, Nextag and PriceGrabber. Sophisticated comparison shopping engines have market research tools that track prices over times and will offer price predictions.

[5] For example, most platforms will allow you to refine search results by average consumer rating, price, brand and more.

Definition 2 (Consideration Sets). *A consumer with attributes* Ψ*,* ζ*, and* η *is able to purchase and has full information on goods from* $\mathcal{C}(\eta, \zeta, \alpha^t; \Psi) \subseteq \mathcal{J}$*. The zero good is always in* $\mathcal{C}(\eta, \zeta, \alpha^t; \Psi)$*.*

When it is clear from context, I will use $\mathcal{C}_t$ to denote the variable for a consumer's consideration at time t. A consumer with consideration set $\mathcal{C}_t$ purchases product $m \in \mathcal{C}_t$ if

$$u_m(y - p_m, \eta) > u_j(y - p_j, \eta) \text{ for all } j \in \mathcal{C}_t \setminus \{ m \}$$

Empirical papers typically include detailed models of the search process. For example, Koulayev (2014) and Honka (2014) model consumer beliefs in the search process. The results of this paper do not exclude such detailed modeling, but rather abstract around them. Since welfare changes are still identified without modeling the search process explicitly, it is without loss of generality to leave it out. Not modeling search directly also lets this paper's model nest many search frameworks. For a more detailed exploration of how a consumer's consideration set could generally be formed, consider the discussion in this paper's supplementary appendix or see Morgan and Manning (1985). For references on papers that model consideration set formation, see this paper's introduction.

I will need an assumption on consideration sets.

Assumption 4 (Price Exogeneity). *Consideration sets* $\mathcal{C}_t$ *are independent of prices and income.*

The difference in ability to recover consumer welfare from a simple area under a demand curve hinges critically on Price Independence. Note that Price Independence does not preclude a consumer from shopping according to her *beliefs* about prices. It only requires that the prices she observes do not cause her to change her shopping behavior. This is true in the case of consumers performing simultaneous search, a type of search that has been determined more likely than sequential search in studies such as Honka and Chintagunta (2016) and Honka, Hortaçsu, and Vitorino (2017). Under Price Independence, a price change would leave $\mathcal{C}_0 = \mathcal{C}_1$ for all individuals but would still have welfare consequences.

Since goods are platform specific, Price Independence rules out the possibility of a platform removing a product due to its change of price. It also rules out a consumer shopping more after discovering all the products she has discovered are unexpectedly expensive.

In principle, requiring consideration sets to be independent of income is not completely necessary. It would be fine to allow consumers' consideration sets to depend on their own perceived wealth level, as a component of Ψ, as long as a good's purchase doesn't change consumers' own perceived wealth level. This assumption is made mostly for convenience.

Individual demand is defined by

$$q_j(y, p_j, p_{-j}, \eta, \zeta, \alpha) := \begin{cases} 1 \text{ if } j = \arg\max_{\ell \in \mathcal{C}(\alpha,\eta,\zeta)} u_\ell(y - p_\ell, \eta) \\ 0 \text{ otherwise} \end{cases} \tag{3.2}$$

At time t, I will simplify notation for *individual demand for product j at time t* to

$$q_j^t(y, p_j, \eta, \zeta; \mathcal{C}_t) := q_j(y, p_j, p_{-j}^t, \eta, \zeta, \alpha^t; \mathcal{C}) \tag{3.3}$$

This simplified form has all good prices, except for good j, fixed at their market prices at time t.

Aggregate (or average) demand for good j is

$$Q_j(y, p_j, p_{-j}, \alpha) = \int q_j(y, p_j, p_{-j}, \alpha, \zeta, \eta) dF \tag{3.4}$$

The following random variables specify each consumer's choice at time step t. A consumer's *first choice at time t FC^t* is the consumer's first choice given her consideration set. A consumer's *second choice at time t SC^t* is her second best option at time t.

$$FC^t(y, \mathcal{C}_t, \eta; p_{\mathcal{C}_t}^t) = \arg\max_{j \in \mathcal{C}_t} u_j(y - p_j^t, \eta) \tag{3.5}$$

$$SC^t(y, \mathcal{C}_t, FC^t, \eta, p_{\mathcal{C}_t}^t) = \arg\max_{j \in \mathcal{C}_t \setminus \{FC^t\}} u_j(y - p_j^t, \eta) \tag{3.6}$$

Functions FC^t and SC^t are given in *indirect form* above. When useful, I will abuse notation and write them in *direct form*: $FC^t(y, \eta, \zeta, \alpha^t, p_{\mathcal{J}}^t)$ and $SC^t(y, \eta, \zeta, \alpha^t, p_{\mathcal{J}}^t)$.

Knowledge of $\mathcal{C}_t$ need not specify its owner's heterogeneity parameters ζ and η. However, it does provide important information about these parameters. Let $A, B \subseteq \mathcal{J}$ be consideration sets. Fix goods $i \in A$ and $j \in B$. I will define three *consumer groups*. The first consumer group is a *consideration group at time t*, defined by

$$\mathcal{C}^{-1}(y, \alpha^t, A) := \{ (\eta, \zeta) : \mathcal{C}(y, \alpha^t, \eta, \zeta) = A \} \tag{3.7}$$

The second is a *purchase group at time t*, defined by

$$\mathcal{C}^{-\star}(y, i, \alpha^t, A, p^t_{\mathcal{J}}) := \{ (\eta, \zeta) : \mathcal{C}(y, \alpha^t, \eta, \zeta) = A \text{ and } FC^t = i \} \tag{3.8}$$

The final group, called a *welfare group*, is defined by

$$\mathcal{C}^{-1}_{0,1}(y, \alpha^0, \alpha^1, i, j, A, B, p^0_{\mathcal{J}}, p^1_{\mathcal{J}}) := \{ (\eta, \zeta) : FC^0 = i, FC^1 = j, \mathcal{C}_0 = A, \mathcal{C}_1 = B \} \tag{3.9}$$

In my micro search environment, the probabilities of each consumer group are observable. The groups increase in specificity from eq. (3.7) to eq. (3.9). A consideration group at time t is related by common consideration set at time t. Their consideration sets may differ at time $1 - t$. Consumers in a purchase group at time t share both consideration sets and product choice at time t. A welfare group is a collection of consumers whose purchase groups coincide at time $t = 0$ and $t = 1$. Only the last group requires linking individual consumers over time.

I will often talk about averages over consumer groups. To write these averages compactly, for all functions $h(\eta, \zeta)$ integrable with respect to F and all collections of consumers $\mathcal{A}$ with positive F measure, define

$$⨍_{\mathcal{A}} h dF := \frac{1}{P(\mathcal{A})} \int_{\mathcal{A}} h dF \tag{3.10}$$

It is also useful to define individual consumer *reservation prices for good j given demand restricted to A* as

$$\bar{P}_j(y, p_{-j}, A, \eta) := \inf \{ p_j \in \mathbb{R} : Q_j(y, p_j, p_{-j}, A, \eta) = 0 \} \tag{3.11}$$

This is the smallest price of good j such that a consumer considering A will not choose j. I will also define *simple reservation prices for good j at time t* by

$$\bar{p}^t_j(y, p_{-j}, \eta, \zeta; \mathcal{C}_t) := \inf \{ p_j \in \mathbb{R} : q^t_j(y, p_j, \eta, \zeta; \mathcal{C}_t) = 0 \} \tag{3.12}$$

$\bar{p}_j^t$ is then the smallest price of good j such that a consumer will choose another item $k \neq j$ in her consideration set at its price at time t, p_k^t, over good j. If a consumer has $p_j^t < \bar{p}_j^t$, this consumer must choose good j. By Monotonicity, reservation prices are finite for simple and restricted demands.

Finally, I assume the researcher has knowledge of *(aggregate) conditional demand given consumer group* $\mathcal{A}$. The more specific the consumer group, the stronger the assumption

Assumption 5 (Demand Conditioned on Consumer Group $\mathcal{A}$)**.** *The researcher knows the conditional demand of group $\mathcal{A}$ at time t*

$$Q_j(y, p_j, p_{-j}, \alpha^t, \mathcal{A}) := \int_{\mathcal{A}} q_j(y, p_j, p_{-j}, \eta, \zeta, \alpha^t) dF \tag{3.13}$$

for all income y, all goods j, all price vectors $p \in \{0\} \times \mathbb{R}^J$ and the observed platform behaviors α^0 and α^1.

Similarly, we can define *average conditional demand at time t for a consumer group $\mathcal{A}$* as

$$\bar{Q}_j(y, p_j, p_{-j}, \alpha^t, \mathcal{A}) := \fint_{\mathcal{A}} q_j(y, p_j, p_{-j}, \eta, \zeta, \alpha^t) dF \tag{3.14}$$

The interpretation of the above assumption is as follows. Fix the consumers in an observed consideration group or purchase group at either time t, or fix the consumers in an observed welfare group across time periods. Then, we can observe the purchase decisions of each of these groups over all prices and all incomes. For most results, I will only need to assume we observe purchase decisions in the special case demand income agrees with consideration group income: $\tilde{y} = y$.

Making these assumptions follows the strategy employed in Bhattacharya (2015). He assumes the researcher knows aggregate demand—in the form of structural choice probabilities—for all prices and incomes. He then uses this data to determine compensating variation and equivalent variation. Since consumers have limited information in my environment, Assumption 5 is the analogous assumption. Indeed, if all consumers have the same consideration set $\mathcal{J}$, then Assumption 5 for consideration groups is identical to the demand assumption of Bhattacharya (2015). Assumption of demand knowledge for purchase groups and welfare groups is a stronger assumption and will only be made when necessary.

Assuming conditional demand knowledge as in Assumption 5 is stronger than assuming observational micro search data. Famously, the simultaneity of supply decisions and demand decisions makes an instrument necessary for extracting demand data from observational market data in classical markets; see the literature discussion in my introduction for specific references. In a search environment, extracting demand data from observational data would require dealing with even more challenges. Prices and consideration sets will be jointly determined by consumer preferences, platform behavior and seller pricing decisions. Thus, more complicated endogeneity issues would need to be handled. To the best of my knowledge, no work has been done to nonparametrically identify conditional demand from observational data in consumer search markets. See Amano, Rhodes, and Seiler (2017) for a reference on parametrically identifying and estimating conditional demand in welfare markets.

The ideal data set would be developed as follows. A shopping platform which can choose product prices observes consumers' initial consideration sets and product choices. They then experiment varying prices of their goods and[6], observe the product choices of all consumers. Next, the platform changes its behavior α^0 to α^1 by, for example, changing its search algorithm or changing the way it displays advertisements. The search platform now observes final consideration sets and product choices. From here, they again experimentally vary the prices of their goods to observe final consumer product choices as a function of prices. If the distribution of consumer observables and unobservables is the same over different geographic regions, variation of prices and platform behavior over geography may be used in place of time. Online, approximate customer location can be inferred from the customer's IP address. Approximate income can be inferred from matching location to census data; census data reports average incomes by geographic location. If demand by over group over incomes is also desired, the platform can check if the distribution of consideration sets varies over income when all other observables are fixed. Tracking consumers over time can be done with cookies.[7]

[6]Note, the competition structure or its timing is implicitly being assumed to allow firms to observe their own demand over a change in price while the competitors' prices remain fixed.

[7]More sophisticated tracking techniques that do not require cookies are also possible. One example is computer fingerprinting. See the discussions in Hoofnagle et al. (2012) or the more general discussion in

While conditional demand may require better data to calculate, it may also be significantly more accurate than traditional, unconditioned demand. Even in traditional markets, most consumers do not have access to the full collection of market goods. For example, a consumer's decision on what breakfast cereal to buy will be limited by what is available at her local grocery stores. Health food stores and traditional grocery stores may offer very different products; each makes its stocking decisions based on the kinds of customers it expects to receive. We conclude, a consumer shopping at a health food store is more likely to prefer whole-grain cereal than a consumer shopping at a regular grocery store. Online, when keyword searches may include brand and store preferences, the dependence is likely even stronger. When sufficient data is available, finding demand conditioned on consideration sets may lead to much more accurate preference and welfare estimates than traditional, unconditioned demand.

3.1.3 Welfare Measures

The classic welfare measures *compensating variation*, S^{CV}, and *equivalent variation*, S^{EV}, are adapted to my search environment as follows. For an individual with heterogeneity vector (η, ζ) and income y, S^{EV} is the solution in S to

$$\max_{j \in \mathcal{C}_0} u_j(y - S - p_j^0, \eta) = \max_{j \in \mathcal{C}_1} u_j(y - p_j^1, \eta) \tag{3.15}$$

while S^{CV} is the solution in S to

$$\max_{j \in \mathcal{C}_0} u_j(y - p_j^0, \eta) = \max_{j \in \mathcal{C}_1} u_j(y + S - p_j^1, \eta) \tag{3.16}$$

S^{EV} is the income loss that would harm a consumer as much as the damage done by price and platform behavior changes. Compensating variation is the increase in income that would return a consumer to her original utility level after the price and platform behavior change. If S^{EV} or S^{CV} is positive, then the consumer's utility increased over the change.

Since $\max_{j \in \mathcal{C}_1} u_j(y - p_j^1, \eta) = u_{FC^1}(y - p_{FC^1}^1, \eta)$, we see that S^{EV} is an indirect function of the entire initial consideration set $\mathcal{C}_0$ but only FC^1 from $\mathcal{C}_1$. As we take income S^{EV} from

Acquisti, Taylor, and Wagman (2016) for more details.

consumers, they are allowed to switch goods from FC^0 to any other good in their initial consideration set. To capture this indirect relationship, I will write the *indirect equivalent variation function* as $S^{EV}(y, \eta, \mathcal{C}_0, FC^1, p^0_{\mathcal{C}_0}, p^1_{FC^1})$. Similarly, I will write the indirect compensating variation as $S^{CV}(y, \eta, FC^0, \mathcal{C}_1, p^0_{FC^0}, p^1_{\mathcal{C}_1})$ for a consumer's *indirect compensating variation function.* With a slight abuse of notation, I will refer to their *direct functional forms* as $S^{EV}(y, \eta, \zeta, \alpha^0, \alpha^1, p^0_{\mathcal{J}}, p^1_{\mathcal{J}})$ and $S^{CV}(y, \eta, \zeta, \alpha^0, \alpha^1, p^0_{\mathcal{J}}, p^1_{\mathcal{J}})$. When it's clear from context, I will suppress prices and platform behavior from function references. *Average compensating variation and equivalent variation* over all consumers is denoted by μ^{CV} and μ^{EV}, respectively. These, collectively, are my primary quantities of interest.

All averages are functions of income, prices and platform behavior. Thus, for $K \in \{ CV, EV \}$, I will write the total average function as $\mu^K(y, \alpha^0, \alpha^1, p^0_{\mathcal{J}}, p^1_{\mathcal{J}})$. When it's clear from context, I will suppress the arguments for platform behavior and prices.

When utility is linear in money, $S^{CV} = S^{EV}$ for each individual. I use S^W to represent both S^{CV} and S^{EV} in this case. S^W is simply the difference between final and initial utility in this case.

With conditional demand, I can now define consumer welfare changes averaged over specific consumer groups. Specifically, I denote *compensating variation and equivalent variation averages over consumer groups* $\mathcal{A}$ by $\mu^{CV}(y, \alpha^0, \alpha^1, p^0_{\mathcal{J}}, p^1_{\mathcal{J}}, \mathcal{A})$ and $\mu^{EV}(y, \alpha^0, \alpha^1, p^0_{\mathcal{J}}, p^1_{\mathcal{J}}, \mathcal{A})$. When its clear from context, I will suppress price and platform behavior notation, referring to the functions simply as $\mu^{CV}(y, \mathcal{A})$ and $\mu^{EV}(y, \mathcal{A})$

Identifying welfare changes by consumer groups gives us refined welfare measures. We may expect welfare changes to vary considerably over different consumer groups. For example, the welfare consequences of a change in platform behavior for consumers who shop the items on the first page of platform results may be very different from consumers who shop specific items regardless of their location on a search result listing. Obviously,

$$\mu^{CV}(y) = \sum_{\mathcal{A}} P(\mathcal{A}) \mu^{CV}(y, \mathcal{A})$$

3.2 Welfare Changes When Prices Change

In this section, I derive a formula for welfare changes under an exogenous change in the prices of a collection of goods without a change in intermediate behavior (i.e. $\alpha^0 = \alpha^1$). For intuition, I start with the simple case of a single good price increase. I show my solution coincides with that of Bhattacharya (2015) when $\mathcal{C}_t = \mathcal{J}$ for all individuals. These results are a straightforward extension of Bhattacharya (2015), but with a novel, simplified derivation. For comparison, I also give results on price decreases in this papers supplementary appendix; these were not derived in Bhattacharya (2015) but completely analogous. Finally, I find results for welfare changes under multiple price changes; multiple, simultaneous price changes creates analysis problems that significantly complicate the problem relative to the single price results from Bhattacharya (2015).

3.2.1 Welfare Changes When One Price Changes

Fix $A \subseteq \mathcal{J}$ and income y. WLOG, suppose the price of good 1 is the only price that changes. That is $p_1^1 > p_1^0$ and $p_j^0 = p_j^1$ for all $j \neq 1$. In addition, assume $\alpha^0 = \alpha^1$. Then, under Price Independence consumers' consideration sets are unchanged. Thus, $\mathcal{C}^{-1}(y, \alpha^0, A) = \mathcal{C}_1^{-1}(y, \alpha^1, A)$. We then have the following result.

Lemma 1. *For any consumer with income y, consideration set $\mathcal{C}_0$ containing 1 and unobservables ζ and η in the environment described above, S^{EV} can be determined by*

$$S^{EV}(y, \eta, \zeta, \alpha^0, \alpha^1, p^0_{\mathcal{J}}, p^1_{\mathcal{J}}) = \int_{p_1^0}^{p_1^1} q_1^0(y, p, \eta; \mathcal{C}_0) dp \tag{3.17}$$

The proof of Lemma 1 is given in Section 3.A.2. However, the intuition can be easily understood from a simple observation and then a close look at individual consumer demand $q_1^0(y, p, \eta)$ under a few cases.

For the fact, note that for a consumer who initially prefers good 1 and any $S > 0$

$$\max_{k \in \mathcal{C}_0 \setminus \{1\}} u_k(y - S - p_k^0, \eta) < u_{SC^0}(y - p^0_{SC^0}, \eta)$$

In words, taking income away from our consumers makes all their initial choices except for good 1 worse than the utility they get from SC^0 at it's initial price. Thus, S^{EV} for a consumer who initially chooses good 1, can be simplified to the solution in S of

$$u_1(y - S - p_1^0, \eta) = \max \{ u_{SC^0}(y - p_{SC^0}^0, \eta), u_1(y - p_1^1, \eta) \}$$

In particular, the income reduction over *all goods* on the LHS of the definition of S^{EV} can be reduced to the price effect of good 1. Thus, we only need consider the demand for good 1 over its own prices. Demand is depicted in fig. 3.1. The cases are (1) $\bar{p}_1^0 > p_1^1$, (2) $p_1^1 \geq \bar{p}_1^0 \geq p_1^0$ and (3) $\bar{p}_1^0 < p_1^0$.

In the first case, the price change is small enough that the consumer still prefers good 1 after the market change. Therefore $q_1^0(y, p, \eta) \equiv 1$ for $p \in (p_1^0, p_1^1)$. Further, S^{EV} solves

$$u_1(y - S^{EV} - p_1^0, \eta) = u_1(y - p_1^1, \eta)$$

so we can plug in and verify $S^{EV} = p_1^1 - p_1^0 = \int_{p_1^0}^{p_1^1} q_1^0(y, p, \eta; \mathcal{C}_0) dp$. See fig. 3.2 for an illustration of this integral.

In the second case, the price change is sufficiently large to force the consumer to switch to her second best option to good 1. As soon as she switches goods, her utility stops changing with the price increase. This is depicted in fig. 3.3. In this case, $S^{EV} = \bar{p}_1^0 - p_1^0$, since

$$u_1(y - \bar{p}_1, \eta) = u_{SC^0}(y - p_{SC^0}^0, \eta)$$

$$\Rightarrow u_{SC^0}(y - p_{SC^0}^0, \eta) = u_{FC^1}(y - p_{FC^1}^0, \eta) = u_1(y - S^{EV} - p_1^0, \eta)$$

$$\Rightarrow S^{EV} = \bar{p}_1 - p_1^0 = \int_{p_1^0}^{p_1^1} q_1(p; y, \eta) dp$$

which also agrees with eq. (3.17).

In the third case, $FC^0 \neq 1$. Our consumer does not change her choice, so $S^{EV} = 0$. In agreement, $q_1^0(y, p, \eta) \equiv 0$ on (p_1^0, p_1^1) and eq. (3.17) is 0. Since the consumer is guaranteed welfare at least as high as $u_{SC^0}(y - p_{SC^0}^0, \eta)$ both before and after the price change, the good that maximizes her welfare will be either 1 or SC^0 if she changes goods.

Averaging welfare changes over all consumers with income y and consideration set $\mathcal{C}_0 = A$ gives our first result.

Theorem 9. *The average equivalent variation over consideration group* $\mathcal{C}^{-1}(y, \alpha^0, A)$ *in response to a price increase of good* 1 *is*

$$\mu^{EV}(y, \mathcal{C}^{-1}(y, \alpha^0, A)) = \int_{p_1^0}^{p_1^1} \bar{Q}_1(y, p, p_{-1}^0, \alpha^0, \mathcal{C}^{-1}(y, \alpha^0, A))dp \tag{3.18}$$

if $1 \in A$ *and 0 otherwise. This is identified through Assumption 5.*

For a proof, see Section 3.A.3. Intuitively, the result is clear. We integrate S^{EV} over all individuals in the consumer group. Since demand is nonnegative the integral order can be swapped by Tonelli's theorem.

The integrand is now aggregate demand for good 1 at time 0 conditioned on consumer group $\mathcal{C}^{-1}(y, \alpha^0, A)$. Thus, our average welfare change is just the area under the aggregate demand curve between p_1^0 and p_1^1 for consumers with income y and initial consideration set A. This last line is in exact agreement with that of (21) in Bhattacharya (2015) in the special case that $\mathcal{C}_0 = \mathcal{J}$ for all consumers.

This strategy, of finding individual welfare changes in terms of individual demand and then aggregating to get average welfare formulas, will work for the remainder of the paper. For brevity, I will leave most of the individual welfare formulas in the appendices. However, all results are a straightforward application of this process.

Average compensating variation from a single price increase is a more complicated integral than the single price integral needed for equivalent variation. Revealed preference does not allow us to simply the RHS in the definition of S^{CV} to the price effect of one good. As S increases from 0 to S^{CV}, a consumer may optimally choose any good from $\mathcal{J}$, not just 1 or SC^0. While this complicates our intuition, it actually simplifies the derivation of the results: we can integrate over income and prices moving simultaneously to construct an integral that looks exactly like the formula for S^{CV}. Varying income has the added benefit of working around the constancy of the price of the outside good.

Theorem 10. *Under Monotonicity and Price Independence*

$$\mu^{CV}(y, \mathcal{C}^{-1}(y, \alpha^0, A)) = \int_{p_1^0}^{p_1^1} \bar{Q}_1(y + p - p_1^0, p, p_{-1}^0, \alpha^0, \mathcal{C}^{-1}(y, \alpha^0, A))dp$$

The proof is given in Section 3.A.4. Intuitively, for an arbitrary consumer

$$q_1^0(y+p-p_1^0,p,\eta)=\begin{cases}1 \text{ if } u_1(y-p_1^0,\eta)\geq \max_{k\neq 1} u_k(y+p-p_k,\eta)\\ 0 \text{ otherwise}\end{cases}$$

for $p\in(p_1^0,p_1^1)$. Thus, the $p^\star$ that sends $q_1^0(y+p^\star-p_1^0,p,\eta)$ to 0 is exactly S^{CV} when S^{CV} is between p_1^0 and p_1^1. If p makes it to p_1^1 with $q_1^0(y+p-p_1^0,p,\eta)$ still 1, then we must conclude $S^{CV}=p_1^1-p_1^0$, since this compensation will leave the consumer choosing good 1 at its original purchase price, perfectly compensated.

For a price decrease, the opposite is the case: average equivalent variation calculations requires an integral over a simultaneous price change in all prices but good 1 while compensating variation can be calculated as an integral along a line of price decrease for good 1 alone.

3.2.2 Welfare Changes Under Many Price Changes

Dealing with multiple price changes is much more complicated than the single price case. This is, in part, because we cannot tell if an individual consumer's welfare went up or down in some cases. If the price changes clearly indicate welfare's direction, we can get exact welfare formulas. When price changes do not clearly indicate welfare's direction, we will need stronger assumptions to get more than welfare bounds. Also, we need stronger data assumptions to be sure to isolate the welfare effects across different product choices.

I give the results for average equivalent variation first.

Theorem 11. *Under Monotonicity and Price Independence, for consumers in $\mathcal{C}_1^{-\star}(y,j,B)$, if $p_j^1>p_j^0$ consumers are hurt and the average equivalent variation is*

$$\mu^{EV}(y,\mathcal{C}_1^{-\star}(y,j,B))=\int_0^\infty\left[1-\bar{Q}_j^0(y-S,p-p_j^0,p_{-j}^0,\alpha^0,\mathcal{C}_1^{-\star}(y,\alpha^1,j,B))\cdot 1(S+p_j^0\geq p_j^1)\right]dS$$

If instead $p_j^1\leq p_j^0$, then individual consumers may be hurt or helped, but the following average *bounds hold*

$$\int_0^\infty \left[1 - \bar{Q}_j(y - S, p - p_j^0, p_{-1}^0, \alpha^0, \mathcal{C}_1^{-\star}(y, \alpha^1, j, B)) \cdot 1(S + p_j^0 \geq p_j^1)\right] dS$$

$$\geq \mu^{EV}(y, \mathcal{C}_1^{-\star}(y, \alpha^1, j, B)) \geq -\int_{p_j^1 - p_j^0}^{0} \bar{Q}_j(y - S, p_j^1 - S, p_{-1}^0, \alpha^0) dS \tag{3.19}$$

where the lower bound in eq. (3.19)*is an equality when the prices of all goods weakly decreases.*

For the proofs, see Section 3.A.6.

To calculate average equivalent variation, we use consumers' demand for their most preferred good at time 1 over all other goods at time 0. Since each consumer only chooses one good at time $t = 1$ but may prefer many goods at time $t = 1$ prices to goods at $t = 0$ prices, averaging over consideration groups would result in double counting welfare gains and loses. For example, if the price of good 1 and good 2 both fall, a consumer may have

$$u_1(y - p_1^1, \upsilon) > u_2(y - p_2^1,) > \max_{j \in \mathcal{C}_0} u_j(y - p_j^0, \upsilon).$$

Thus, this consumer would have positive areas under two demand curves:

$$\int_{p_2^1 - p_2^0}^{0} q_2^0(y - S, p_2^1 - S, \eta) dS > 0, \ \textit{and} \int_{p_1^1 - p_1^0}^{0} q_1^0(y - S, p_1^1 - S, \eta) dS > 0.$$

However, since purchase groups are subsets of consideration sets and since integrals of positive integrands are larger when you integrate over larger areas, we can bound average equivalent variations using demand conditioned on consideration groups.

Corollary 1. *Average equivalent variation, in response to multiple price changes, for consumers in consideration group $\mathcal{C}_1^{-\star}(y, \alpha^0, j, B)$ can be bound by*

$$\frac{1}{P(\mathcal{C}_1^{-\star}(y, \alpha^0, j, B))} \int_0^\infty \Big[P(\mathcal{C}_1^{-1}(y, \alpha^0, B)) -$$

$$Q_j(y - S, p - p_j^0, p_{-j}^0, \alpha^0, \mathcal{C}_1^{-1}(y, \alpha^1, B)) \cdot 1(S + p_j^0 \geq p_j^1) \Big] dS$$

$$\geq \mu^{EV}(y, \mathcal{C}_1^{-\star}(y, \alpha^1, j, B))$$

$$\geq \frac{-1}{P(\mathcal{C}_1^{-\star}(y, \alpha^1, j, B))} \int_{p_j^1 - p_j^0}^{0} Q_j(y - S, p_j^1 - S, p_{-j}^0, \alpha^0, \mathcal{C}_1^{-1}(y, \alpha^1, B)) dS$$

The analogous results for compensating are left to Theorem 20 and Corollary 4 in the appendix.

3.3 Welfare Changes When Consideration Sets and Prices Change

In this section, I consider welfare identification under simultaneous, exogenous changes in platform behavior and prices. In Section 3.3.1, I show μ^W can be identified over all exogenous price and platform changes under Linearity with the most specific consumer groups, welfare groups. In Section 3.3.2, I extend these results to the weaker preference assumptions of Monotonicity and Income Consistency. Under Monotonicity and Income Consistency, identification is achieved when consideration sets are "semi-stable" over the change in prices and platform behavior. Without semi-stability, neither compensating variation nor equivalent variation can be recovered.

The results in this section all require observing individual consumer shopping behavior both before and after the price and platform behavior changes; that is, all averages are over welfare groups. This is because I do not restrict search behavior with a specific model of search. I also do not assume platforms can perfectly influence consideration sets in this section. For example, my assumptions allow the partial failure of a platform's efforts to add products $\{8, 9, 10\}$ to a consumer group's consideration sets. The consumers who add all products to their consideration sets will be in a different welfare group than consumers who only add a subset of the products from $\{8, 9, 10\}$ to their consideration set. I show tracking consumer decisions before and after the platform and price changes can lead to exact average welfare group identification in cases like this with few additional assumptions.

3.3.1 Welfare Changes Under Linearity

Under Linearity, $S^{CV} = S^{EV} = S^{W}$. In this environment, both average compensating variation and average equivalent variation are μ^W. This average can be recovered from demand conditioned on welfare groups as a response to arbitrary, exogenous changes in all goods' prices and arbitrary, exogenous changes in platform behavior.

To derive these results, it helps to define *counterfactual conditional demand* for an individual (y, η, ζ). Given a consideration set A, this is defined as

$$q_j(y, p_j, p_{-j}, \alpha, A) := \begin{cases} 1 \text{ if } j = \arg\max_{k \in A} u_k(y - p_k, \eta) \\ 0 \text{ otherwise} \end{cases} \tag{3.20}$$

This is simply the individual's product choice if the she were forced to choose from consideration set A.

Lemma 2. *Under Linearity, for any individual with unobserved heterogeneity (η, ζ) and income y, S^W can be determined by*

$$S^W = \int_{p^0_{FC^0}}^{\infty} q_{FC^0}(y, p, 0, \{FC^0, 0\})dp - \int_{p^1_{FC^1}}^{\infty} q_{FC^1}(y, p, 0, \{FC^1, 0\}, \eta)dp \tag{3.21}$$

The proof is included in the appendices in Section 3.B.1.1, but the main idea is as follows. When utility is linear in money, S^W can be decomposed into the difference between the initial purchase's utility above the outside good and the final purchase's utility above the outside good. That is,

$$\begin{aligned} S^W &= \left[-p^0_{FC^0} + \tilde{U}_{FC^0}(\eta)\right] - \left[-p^1_{FC^1} + \tilde{U}_{FC^1}(\eta)\right] && \text{(by definition)} \\ &= \left[(-p^0_{FC^0} + \tilde{U}_{FC^0}(\eta)) - \tilde{U}_0(\eta)\right] - \left[(-p^1_{FC^1} + \tilde{U}_{FC^1}(\eta)) - \tilde{U}_0(\eta)\right] && (3.22) \end{aligned}$$

Since the outside good is guaranteed to be in both $\mathcal{C}0$ and $\mathcal{C}1$, this is a relative comparison that can always be made. The integrals in eq. (3.21) are exactly this difference.

Since our formula is the difference of two unbounded integrals, I have to assume Finite Differences to ensure the aggregation is well-defined. Finite Differences requires at least one of the aggregate integrals is finite. This is similar to Assumption 8 from the single good swap formulas of Section 3.4.2.

Assumption 6 (Finite Differences)**.** *For all welfare groups $\mathcal{C}^{-1}_{0,1}(y, i, j, A, B)$ either*

$$\int_{p^f_j}^{\infty} \int_{\mathcal{C}^{-1}_{0,1}(y,i,j,A,B)} q_j(y, p, p_j, \{i, j\}, \eta)dF dp$$

or

$$\int_{p^0_i}^{\infty} \int_{\mathcal{C}^{-1}_{0,1}(y,i,j,A,B)} q_i(y, p, p_i, \{i, j\}, \eta)dF dp$$

is finite.

This assumption is very weak. It is met by all the empirical workhorse models. It requires that either the fraction of consumers in $\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)$ preferring good i over j disappears "fast enough" as the price of good i goes to infinity or vice versa as the price of good j goes to infinity. If no consumer would choose good i over good j when i's price is \$100 trillion and j's price is its market price, the assumption is met. For Theorem 12, it only needs to hold when either i or j is 0. However, the assumption is mathematically necessary because I have made so few assumptions on preferences. A demonstration of the necessity of this assumption is given in this paper's supplemental appendix.

Theorem 12. *Under Linearity and Finite Differences, the average of S^W for consumers in $\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)$ is*

$$\begin{aligned}\mu^W(y,\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)) = \lim_{\substack{p_\ell\to\infty & \forall \ell\in A\setminus\{0,i\}}}\Bigg[\int_{p_i^0}^{\infty} Q_i(y,p,p_{-i},\alpha^0,\mathcal{C}_{0,1}^{-1}(y,i,j,A,B))dp \\ - \lim_{\substack{p_\ell\to\infty & \forall \ell\in B\setminus\{0,j\}}}\int_{p_j^1}^{\infty} Q_j(y,p,p_{-j},\alpha^1,\mathcal{C}_{0,1}^{-1}(y,i,j,A,B))dp\end{aligned} \tag{3.23}$$

Further, since eq. (3.23) *is the difference of two quantities assumed observed in Assumption 5, this formulas identifies $\mu^W(y,\mathcal{C}_{0,1}^{-1}(y,i,j,A,B))$.*

For the proof see Section 3.B.1.2. The key idea is that we can remove the influence of all goods that are unimportant for welfare by taking their prices to infinity. Since utility is monotonic in money, the limit can be passed through the integral by the monotonic convergence theorem. This is similar to ideas I use in Section 3.4.

3.3.2 Welfare Changes under Monotonicity

In this subsection, I consider welfare changes as a response to arbitrary, exogenous price changes and platform changes under Monotonicity. However, we run into the following problem. Calculating equivalent variation requires calculating a consumer's tradeoff between her final good choice FC^1 and all goods in her entire initial consideration set $\mathcal{C}_0$. Thus, in order to calculate S^{EV} for an individual, we need her demand as a function of a consideration set $\mathcal{C} \supseteq \mathcal{C}_0 \cup \{ FC^1 \}$. This means, either consideration sets expand or shrink: $\mathcal{C}_0 \subseteq \mathcal{C}_1$ or $\mathcal{C}_1 \subseteq \mathcal{C}_0$. Similarly, calculating compensating variation for a consumer requires measuring her tradeoff between her initial good choice FC^0 and all goods in her final consideration set $\mathcal{C}_1$. Again, this means consideration sets have to expand or shrink.

To allow richer consideration set behavior, additional assumptions are needed to reduce the dependence of S^{EV} and S^{CV} from entire consideration sets of products to only a few products. Short of assuming Assumption 9.B, we can do so with a combination of Assumption and the Income Consistency:

Assumption 7 (Income Consistency)**.** *If an individual prefers good j in set $A \subseteq \mathcal{J}$ at income y, then she prefers it at any income $\tilde{y}$. That is, if*

$$u_j(y - p_j, \eta) \geq u_k(y - p_k, \eta) \text{ for all } k \in A$$
$$\text{then } u_j(\tilde{y} - p_j, \eta) \geq u_k(\tilde{y} - p_k, \eta) \text{ for all } k \in A \text{ and } \tilde{y} \in \mathbb{R}$$

Income Consistency requires that consumers' ranking of goods does not change as their incomes rise. Linearity is sufficient for Income Consistency. Income Consistency is weaker in that it does not require utility differences to remain constant over income.

Under income consistency S^{EV} solves

$$u_{FC^0}(y - S - p^0_{FC^0}, \eta) = u_{FC^1}(y - p^1_{FC^1}, \eta)$$

while S^{CV} solves

$$u_{FC^0}(y - p^0_{FC^0}, \eta) = u_{FC^1}(y + S - p^1_{FC^1}, \eta)$$

This means, calculating equivalent or compensating variation only requires looking at the tradeoff between FC^0 and FC^1.

Lemma 3. *Under Income Consistency and Monotonicity, any individual with income y and heterogeneity (η, ζ), has individual compensating variation S^{CV} determined by*

$$S^{CV} = \int_{-\infty}^{p^1_{FC^1}} q_{FC^0}(y, p^0_{FC^0}, p_{FC^1}, \{ FC^1, FC^0 \}, \eta) dp_{FC^1} - \int_{p^1_{FC^1}}^{\infty} q_{FC^1}(y, p_{FC^1}, p^0_{FC^0}, \{ FC^1, FC^0 \}, \eta) dp_{FC^1} \tag{3.24}$$

when $FC^1 \neq 0$. S^{EV} *is determined by*

$$S^{EV} = \int_{p^0_{FC^0}}^{\infty} q_{FC^0}(y, p_{FC^0}, p^1_{FC^1}, \{FC^0, FC^1\}, \eta)dp_{FC^0} - \int_{-\infty}^{p^0_{FC^0}} q_{FC^1}(y, p^1_{FC^1}, p_{FC^0}, \{FC^0, FC^1\}, \eta)dp_{FC^0} \quad (3.25)$$

when $FC^0 \neq 0$. *If* $FC^0 = 0$ *and* $FC^1 = 0$, $S^{EV} = 0 = S^{CV}$.

See Section 3.B.1.3 for the proof. The intuition for Lemma 3 is simple. A consumer's demand restricted to choices FC^0 and FC^1 captures her willingness to substitute across her initial and final choices. From here, applying the same logic from case (2) of the discussion of Lemma 1 yields the result. For example, in eq. (3.24) the first integral will only be nonzero if the consumer was hurt by the change in prices and platform behavior. In this case, the integral will be the price for good FC^1 that makes this consumer indifferent between goods FC^0 and good FC^1 subtracted from the initial price of good FC^1. On the other hand, the second integral will be nonzero only if the consumer benefits from the change in prices and change in platform behavior. In this case, the integral will subtract the final price of good FC^1 from the price that makes her indifferent between FC^0 and FC^1 at FC^1's initial price. Demand has a star on it in case $FC^0 = FC^1$.

Lemma 3 cannot be used for all cases. The integrals are over prices; we cannot integrate over the price of the outside good, which is fixed at 0. We need to use income variation to identify compensating variation when the outside good is a welfare group's final good choice. We also need income variation to identify equivalent variation when the outside good is a welfare group's initial choice. The cases missing from Lemma 3 are covered by Lemma 4 using this idea.

Lemma 4. *Under Income Consistency and Monotonicity, for any consumer with unobservables* (ζ, η) *income* y, *and* $FC^1 = 0$,

$$S^{CV}(y, \eta, FC^0, 0; p^0_{FC^0}, 0) = \int_{p^0_{FC^0}}^{\infty} q_{FC^0}(y + p - p^0_{FC^0}, p, 0, \{FC^0, 0\}, \eta)dp$$

If instead $FC^0 = 0$, *then*

$$S^{EV}(y, \eta, 0, FC^1; 0, p^1_{FC^1}) = \int_{-\infty}^{p^1_{FC^1}} q_{FC^1}(y + p - p^1_{FC^1}, p, 0, \{FC^1, 0\}, \eta)dp$$

The proof of Lemma 4 is in Section 3.B.1.4 in the appendices. Additionally, in the supplemental appendix, I show the necessity of income information under Monotonicity when calculating either compensating variation or equivalent variation with either FC^0 or $FC^1$the outside good.

The intuition for the necessity of income information is straightforward. For example, in identifying S^{CV}, we need to find the S such that $u_0(y+S,\eta) = u_{FC^0}(y - p^0_{FC^0}, \eta)$. With only price variation along the conditional demand line, we can only vary the RHS of this equation for the special case that $S = 0$. Therefore we cannot recover any information on $u_0(y+S,\eta)$ for any $S \neq 0$. The intuition for not being able to identify S^{EV} is very similar.

In order to identify average welfare changes from Lemma 3 and Lemma 4, we will need a guarantee that FC^0 and FC^1 will be together in either $\mathcal{C}_0$ or $\mathcal{C}_1$. When this happens, I say *consideration sets are semi-stable over time.* There are two ways this can happen.

Definition 3 (Semi-Stability)**.** *Fix a consumer group $\mathcal{A}$. A change in consideration sets for this group is weakly expanding if $FC^0 \in \mathcal{C}_1$ for all consumers in $\mathcal{A}$ It is weakly contracting if $FC^1 \in \mathcal{C}_0$ for all consumer in $\mathcal{A}$. A change in consideration sets over time that is either weakly contracting or weakly expanding is semi-stable.*

If platform behavior changes to make search less costly, we may expect consideration sets to weakly expand; consumers still explore their previous purchase but may explore many others. If a platform removes a product listing from its search results, we may expect consideration sets to weakly shrink in response.

Theorem 13. *Under Finite Differences, Income Consistency, Price Independence and Monotonicity, the average compensating variation for welfare group $\mathcal{C}^{-1}_{0,1}(y,i,j,A,B)$ such that i,j in same consideration set at time $t^\star$ (Semi-Stability), and $j \neq 0$ is*

$$\mu^{CV}(y, \mathcal{C}^{-1}_{0,1}(y,i,j,A,B)) = \lim_{\substack{p_\ell \to \infty \\ \forall \ell \in C \setminus \{i,j,0\}}} \Bigg[\int_{-\infty}^{p_j^1} Q_i(y, p_i^0, p_{-i}, \alpha^{t^\star}, \mathcal{C}^{-1}_{0,1}(y,i,j,A,B)) dp_j$$

$$- \int_{p_j^1}^{\infty} Q_j(y, p_j, (p_i^0, p_{-\{i,j\}}), \alpha^{t^\star}, \mathcal{C}^{-1}_{0,1}(y,i,j,A,B)) dp_j \Bigg] \tag{3.26}$$

and the average compensating variation for $i \neq 0$ ($j = 0$ is fine) is

$$\mu^{EV}(y, \mathcal{C}_{0,1}^{-1}(y,i,j,A,B)) = \lim_{p_\ell \to \infty\ \forall \ell \in C \setminus \{i,j,0\}} \left[\int_{p_i^0}^{\infty} Q_i(y, p_i, (p_j^1, p_{-\{i,j\}}), \alpha^{t^\star}, \mathcal{C}_{0,1}^{-1}(y,i,j,A,B)) dp_i \right. \tag{3.27}$$

$$\left. - \int_{-\infty}^{p_i^0} Q_j(y, p_j^1, p_{-j}, \alpha^{t^\star}, \mathcal{C}_{0,1}^{-1}(y,i,j,A,B)) dp_i \right]$$

Similarly, extending Lemma 4, we have the following result.

Theorem 14. *Under Monotonicity, Income Consistency and Price Independence, the average compensating variation for welfare group $\mathcal{C}_{0,1}^{-1}(y,i,0,A,B)$ is*

$$\mu^{CV}(y, \mathcal{C}_{0,1}^{-1}(y,i,0,A,B)) = \lim_{p_\ell \to \infty \text{ for all } \ell \in A \setminus \{i,0\}} \int_{p_i^0}^{\infty} Q_i(y + p - p_i^0, p, p_{-i}, \alpha^0, \mathcal{C}_{0,1}^{-1}(y,i,j,A,B)) dp \tag{3.28}$$

and the average compensating variation for this group is

$$\mu^{EV}(y, \mathcal{C}_{0,1}^{-1}(y,0,j,A,B)) = \lim_{p_\ell \to \infty \text{ for all } \ell \in B \setminus \{j,0\}} \int_{-\infty}^{p_j^1} Q_j(y + p - p_j^1, p, p_{-j}, \alpha^1, \mathcal{C}_{0,1}^{-1}(y,0,j,A,B)) dF dp \tag{3.29}$$

Both are identified under Assumption 5.

3.4 Welfare Changes When Platform Behavior Changes

In this section, I determine welfare changes when consideration sets shrink or expand without a price change. The changes in consideration sets are the result of exogenous platform behavior changes. Only Monotonicity is assumed of preferences for these results. I also find welfare changes when one product is swapped for another in all consideration sets; dealing with product swaps requires Linearity. More general swap results are considered in Section 3.3.

The results of this section give formulas that predict welfare changes under strong assumptions on platform power. When a consumer cannot find a product she normally shops, we may generally expect her to try other search platforms or experiment with new products. Moreover, we may expect consumers to ignore some of the extra products suggested to them by platforms. The assumptions in this section do not allow consumers any of these re-searching responses. They assume that consumers' search decisions are to just accept the changes the platform makes. Thus, the results of this section should be applied with caution. In general, a platform's ability to change consumers' consideration sets without consumers re-searching should be justified with arguments and data specific to the market being studied. Results that allow for weaker platform power are considered in Section 3.3.

At the same time, a platform need not have strong power for the results of this section to be useful. In situations where predictions are important and data quality is limited, such as with many anti-trust cases, this section's results may give useful "worst case", "best case" or "baseline" identification strategies. For example, consider a search platform that replaces first page sunglass results for rival stores with 10th page sunglass results from a partner business's store. In this case, the single good swap formula may provide a useful "worst-case" formula to predict consumer damage. This, in turn, may be enough to justify anti-trust authority intervention.

3.4.1 Welfare Changes When Consideration Sets Shrink or Expand

First, I look at welfare changes when platforms remove (WLOG) good 1 from all consideration sets.

Theorem 15. *Under Monotonicity, the average equivalent variation over consideration group $\mathcal{C}^{-1}(y, \alpha^0, A)$ from the removal of good 1 from each consumer's consideration set is*

$$\begin{aligned}&\mu^{EV}(y, \mathcal{C}^{-1}(y, \alpha^0, A))\\&= \int_{p_1^0}^{\infty} \bar{Q}_1(y, p, p_{-1}^0, \alpha^0, \mathcal{C}^{-1}(y, \alpha^0, A))dp\end{aligned} \tag{3.30}$$

if $1 \in A$ *and 0 otherwise. The average compensating variation over consideration group*

$\mathcal{C}^{-1}(y, \alpha^0, A)$ from the removal of good 1 from consideration sets is

$$\mu^{CV}(y, \mathcal{C}^{-1}(y, \alpha^0, A)) = \int_{p_1^0}^{\infty} \bar{Q}_1(y + p - p_1^0, p, \alpha^0, \mathcal{C}^{-1}(y, \alpha^0, A))dp \tag{3.31}$$

if $1 \in A$ and 0 otherwise. Both $\mu^{EV}(y, \mathcal{C}^{-1}(y, \alpha^0, A))$ and $\mu^{CV}(y, \mathcal{C}^{-1}(y, \alpha^0, A))$ are identified through Assumption 5.

The proof is given in Section 3.C.1.1. Intuitively, this is analogous to welfare changes from a price increase of good 1, when the new price of the good 1 is infinity. Sending p_1^1 to infinity in Theorem 10 and eq. (3.17) gets these formulas. The analogous relationship between formuls for product entry and price decreases from infinity also holds.

Theorem 16. *Under Monotonicity, when consumers in $\mathcal{C}^{-1}(y, \alpha^0, A)$ have good $M = J + 1$ exogenously added to their consideration sets, average compensating variation is*

$$\mu^{CV}(y, \mathcal{C}_1^{-1}(y, \alpha^1, A \cup \{ M \})) = -\int_{p_M^1}^{\infty} \bar{Q}_M(y, p_M, p_{-M}^1, \alpha^1, \mathcal{C}_1^{-1}(y, \alpha^1, A \cup \{ M \}))dp_M$$

The average equivalent variation is

$$\mu^{EV}(y, \mathcal{C}_1^{-1}(y, \alpha^1, A \cup \{ M \})) = -\int_{p_M^1}^{\infty} \bar{Q}_M^1(y + p - p_M^1, p, p_{-M}^1, \alpha^1, \mathcal{C}_1^{-1}(y, \alpha^1, A \cup \{ M \}))dp$$

The proof of this result is given in this paper's supplemental appendix.

Next, suppose a platform is able to remove a collection of goods $R \subseteq \mathcal{J}$ from all individual's consideration sets. If consumers then make product decisions without searching more, they will have (weakly) lost welfare. As before, compensating and equivalent variation can be found as areas under conditional demand curves.

Theorem 17. *Let $B = A \setminus R$ where $R \subseteq \mathcal{J}$. Under Monotonicity and Assumption 10, the average equivalent variation for consumers in collection $\mathcal{C}_1^{-\star}(y, \alpha^1, j, B)$ from exogenously removing products in R from their consideration sets is*

$$\mu^{EV}(y, \mathcal{C}_1^{-\star}(y, \alpha^1, j, B)) = \int_0^{\infty} [1 - \bar{Q}_j y - S, p_j^0 - S, p_{-j}^0, \alpha^1, \mathcal{C}_1^{-\star}(y, \alpha^1, j, B))]dS \tag{3.32}$$

while the average compensating variation for group $\mathcal{C}_0^{-\star}(y,\alpha^0,i,A)$is

$$\mu^{CV}(y,\mathcal{C}_0^{-\star}(y,\alpha^0,i,A)) = \lim_{p_\ell^0\to\infty:\forall\ell\in(A\cap R)\backslash\{\,i\,\}} \int_0^\infty \bar{Q}_i(y+S,S+p_i^0,p_{-i}^0,\alpha^0,\mathcal{C}_0^{-\star}(y,\alpha^0,i,A))dp \tag{3.33}$$

The proof is in Section 3.C.1.2. The results are derived very similarly to those of Theorem 11 and Theorem 20.

The formulas in Theorem 17 can be used for welfare change predictions. This is because they only require knowledge of conditional demand at time $t=0$. If researchers only have demand conditional on consideration groups, they may still bound welfare changes using the following result.

Corollary 2. *Let $B = A\backslash R$ Under Monotonicity and Price Independence, the average equivalent variation for consumers in collection $\mathcal{C}_1^{-1}(y,\alpha^0,A) = \mathcal{C}_1^{-1}(y,\alpha^1,B)$ from exogenously removing products in $R\subseteq\mathcal{J}$ from their consideration sets can be bounded by*

$$0 \le \mu^{EV}(y,\mathcal{C}_1^{-1}(y,\alpha^1,B)) \le \sum_{j\in R}\int_0^\infty [1-\bar{Q}_j(y-S,p_j^0-S,p_{-j}^0,\alpha^1,\mathcal{C}_1^{-1}(y,\alpha^1,B))]dS \tag{3.34}$$

$$0 \le \mu^{CV}(y,\mathcal{C}_0^{-1}(y,\alpha^0,A)) \le \sum_{i\in R}\lim_{p_\ell^0\to\infty:\forall\ell\in(A\cap R)\backslash\{\,i\,\}}\int_0^\infty \bar{Q}_i(y+S,S+p_i^0,p_{-i}^0,\mathcal{C}_0^{-1}(y,\alpha^0,A))dp \tag{3.35}$$

The analogous results for adding a collection of goods R to each consideration set are as follows.

Theorem 18. *Let $B = A\cup R$. Under Monotonicity and Price Independence , the average equivalent variation for consumers in collection $\mathcal{C}^{-\star}(y,\alpha^0,j,B)$ from exogenously adding products in $R\subseteq\mathcal{J}$ to their consideration sets is*

$$\mu^{EV}(y,\mathcal{C}^{-\star}(y,\alpha^1,j,B)) = -\lim_{p_\ell^0\to\infty:\forall\ell\in R\backslash\{\,j\,\}}\int_{-\infty}^0 \bar{Q}_j(y-S,p_j^0-S,p_{-j}^1,\alpha^0,\mathcal{C}^{-\star}(y,\alpha^0,j,B)))dS \tag{3.36}$$

while the average compensating variation for memebers of group $\mathcal{C}_0^{-\star}(y,\alpha^0,i,A)$is

$$\mu^{CV}(y,\mathcal{C}_0^{-\star}(y,\alpha^0,i,A)) = -\int_{-\infty}^0 [1-\bar{Q}_i(y+S,S+p_i^0,p_{-i}^1,\mathcal{C}_0^{-\star}(y,\alpha^0,i,A))]dp \tag{3.37}$$

The proof is given in Section 3.C.1.3. The welfare changes above are calculated with respect to conditional demand at time 1. When consideration sets shrink, we can predict the welfare change using conditional demand data at time $t = 0$. When consideration sets grow, we can "predict the past" using conditional demand data at time $t = 1$. Demand given the largest consideration set can always be used to give information on more restricted demand through pricing limits.

Just as with the product removal case, we can bound welfare when only demand conditioned on consideration sets is known.

Corollary 3. *Let $B = A \cup R$ Under Monotonicity and Price Independence, the average equivalent variation for consumers in collection $\mathcal{C}^{-1}(y, \alpha^1, B)$ from exogenously adding products in $R \subseteq \mathcal{J}$ from their consideration sets is*

$$0 \geq \mu^{EV}(y, \mathcal{C}^{-1}(y, \alpha^1, B)) \geq \tag{3.38}$$
$$-\sum_{j \in R \cup A} \lim_{p_\ell^0 \to \infty : \forall \ell \in R \setminus \{j\}} \int_{-\infty}^{0} \bar{Q}_j(y - S, p_j^0 - S, p_{-j}^1, \alpha^1, \mathcal{C}^{-1}(y, \alpha^1, B)))dS$$

while the average compensating variation for consumers in $\mathcal{C}^{-1}(y, \alpha^0, A)$is bounded by

$$0 \geq \mu^{CV}(y, \mathcal{C}^{-1}(y, \alpha^0, A)) \geq -\sum_{i \in R} \int_{-\infty}^{0} [1 - \bar{Q}_i(y + S, S + p_i^0, p_{-i}^1, \alpha^0, \mathcal{C}^{-1}(y, \alpha^0, A))]dp \tag{3.39}$$

3.4.2 Welfare Changes From Swapping One Good For One Other in Consideration Sets

There are two surprises to the welfare consequences of a platform exogenously and simultaneously removing (WLOG) good 1 and adding a new good $M = J + 1$ to all consideration sets. The first is that under Monotonicity neither average compensating variation nor equivalent variation can be identified given even demand conditioned on welfare groups, the strongest data assumption I make. The second surprise is that under Linearity both average compensating variation and equivalent variation can be identified using only demand conditioned on consideration groups, the weakest data assumption I make. We do not need to condition on consumers initial or final choices to get a correct welfare formula.

The first surprise is explained by the fact that we do not observe demand for good 1 and good M together. Thus, we cannot make a direct welfare calculation. Moreover, Monotonicity prevents us from making an indirect comparison through an alternative, fixed good. In contrast, under Linearity indirect comparisons can be made. For example, suppose a consumer initially chooses good 1 and then, after the swap, chooses good M. Under Linearity, the welfare in going from good 1 to good M could be calculated by going from good 1 to good 0 and then good 0 to good M.

The second surprise is explained by the fact that the easiest indirect comparison that can be made, through each consumers second most preferred goods E^0 and E^1, exactly balances in all cases. The details of this balancing are given after the theorem's statement.

Theorem 19. *Suppose the search platform removes good 1 from all consumers choice sets and adds new good $M = J+1$ to all consideration sets. Then, under Linearity and Assumption 8, the average welfare change over the consideration group $\mathcal{C}^{-1}(y, \alpha^0, A)$ is*

$$\mu^W(y, \mathcal{C}^{-1}(y, \alpha^0, A)) = \int_{p_1^0}^{\infty} Q_1(y, p, p_{-1}^0, \alpha^0, \mathcal{C}^{-1}(y, \alpha^0, A))dp \tag{3.40}$$

$$- \int_{p_M^1}^{\infty} Q_M(y, p, p_{-M}^1, \alpha^0, \mathcal{C}^{-1}(y, \alpha^0, A))dp \tag{3.41}$$

The formal proof is given in Section 3.C.2. Intuitively, the first integral in eq. (3.40) measures the average welfare lost in going from good 1 to the second most preferred good in $\mathcal{C}0$, E^0. All consumers who do not initially prefer good 1 contribute nothing to the area under this curve. Similarly, the second integral in eq. (3.40) measures the average welfare gained for each consumer in going from her second most preferred good in $\mathcal{C}1$, E^1, to M. Consumers who do not choose good M at time $t = 1$ contribute nothing to this second integral.

The result follows from considering individual demand in four cases: (1) $FC^0 = 1$ and $FC^1 = M$, (2) $FC^0 \neq 1$ and $FC^1 = M$, (3) $FC^0 = 1$ and $FC^1 \neq M$ and (4) $FC^0 \neq 1$ and $FC^1 \neq M$. In case (1), the second most preferred good to good 1 must also be the second most preferred good to good M. That is, $E^0 = E^1 \in \mathcal{C}0, \mathcal{C}1$. The integrals in eq. (3.40) then calculate the distance from 1 to E^0 and E^0 to M, which cancels the E^0s from the formula and leaves the correct difference. In case (2), $FC^0 = E^1 \neq 1$. Thus, her first choice at time $t = 0$ is still available at time $t = 1$ and has become her second choice. The second integral, measuring the distance welfare distance from E^1 to M coincides with the distance from FC^0 to M. The first integral is 0. Thus, welfare from individuals in this case will be correctly calculated. Case (3) is the opposite of case (2) and has analogous reasoning. In case (4), the consumer does not prefer 1 or M, so her favorite good is always available and always chosen. This consumer has no welfare change and her contribution to both integrals will be 0.

Finally, note that Assumption 8 must be assumed to ensure the difference in Theorem 19 is well-defined. Assumption 8 simply requires the average welfare loss from good 1 or the average welfare gain from good M is finite.

Assumption 8. *For all consideration groups $\mathcal{C}^{-1}(y, \alpha^0, A)$ either*

$$\int_{p_1^0}^{\infty} \int_{\mathcal{C}^{-1}(y,\alpha^0,A)} q_1^0(y, p, \eta) dF dp$$

or

$$\int_{p_M^1}^{\infty} \int_{\mathcal{C}^{-1}(y,\alpha^0,A)} q_M^1(y, p, \eta) dF dp$$

is finite.

3.5 Preferences and Consideration Set Dependence

In this section, I show how improper modeling of the dependence between consideration sets and η can make welfare estimation fail poorly. This is true even when standard empirical assumptions hold; serious failure can happen even when utility is linear in all arguments and unobserved preferences are iid Type I Extreme Value random variables. In contrast, using conditional demand to calculate welfare changes produces accurate welfare measures regardless of the dependence structure between consideration sets and η.

To make my point, I offer the following two examples. In both examples, the researcher knows each individual's utility form. Thus, in each example, the researcher may estimate average welfare changes by estimating utility before and after the market change. The only incorrect part of the researcher's analysis is the relationship she assumes between consideration sets and unobservable preferences, which allows her to predict counterfactual demand and product choices. In the first case, her estimate of average welfare changes can be made arbitrarily large while the true welfare change is 0. In the second example, her estimate of average welfare change is 0 while the true change can be made arbitrarily large.

Setup for Examples 1 and 2. Consider a market with only two goods, 0 and 1. Suppose utility is known to be

$$u_j(y - p_j, \eta) = y - p_j + \beta_j + \eta_j$$

with $\eta = (\eta_0, \eta_1)$ and the η_j iid standard Gumbel errors for each $j = 0, 1$.[8] Suppose that the researcher knows $\beta_0 = 0$. For simplicity, assume prices are 0 for all goods. Suppose that half of all consumers in this market have consideration sets $\mathcal{C}_0 = \{\, 0 \,\}$ and the other half have consideration sets $\mathcal{C}_0 = \{\, 0, 1 \,\}$. Finally, notice that if all consumers had perfect market knowledge, i.e. $\mathcal{C}_0 = \{\, 0, 1 \,\}$, we would have

$$\begin{aligned} P(j \text{ chosen}) &= \frac{e^{y - p_j + \beta_j}}{\sum_{k \in \{\, 0,1 \,\}} e^{y - p_k + \beta_k}} \\ &= \frac{e^{\beta_j}}{1 + e^{\beta_1}} \end{aligned} \tag{3.42}$$

for each j, as derived in McFadden (1974).

Example 1. In this example, the researcher assumes that consideration sets are independent of preferences. She assumes the reason half the consumers search $\mathcal{C}_0 = \{\, 0, 1 \,\}$ and half search $\mathcal{C}_0 = \{\, 0 \,\}$ is because of heterogeneous distaste for search. In particular, individuals with $\mathcal{C}_0 = \{\, 0 \,\}$ don't like to search, while individuals with $\mathcal{C}_0 = \{\, 0, 1 \,\}$ are relatively willing to search.

Suppose that β_1 and β_0 are both 0. Define the random variable ζ by

$$\zeta(\omega) = -p_1 + \beta_1 + \eta_1(\omega) - (-p_0 + \beta_0 + \eta_0(\omega)) = \eta_1(\omega) - \eta_0(\omega)$$

[8] I am abusing notation and letting η and ζ be random variables in examples 4 and 5, rather than the realizations of random variables

for all $\omega \in \Omega$.[9] Assume consideration sets are determined by the rule

$$\mathcal{C}_0 = \begin{cases} \{\,0, 1\,\} & \text{if } \zeta > 0 \\ \{\,0\,\} & \text{if } \zeta \leq 0 \end{cases}$$

The interpretation is that only those consumers who prefer product 1 bother to shop product 1.

In this market,

$$\begin{aligned} P(\zeta > 0) &= \frac{e^{\beta_0}}{e^{\beta_0} + e^{\beta_1}} \\ &= \frac{1}{2} \end{aligned}$$

is the fraction of shopping consumers, as required in the initial setup.

Since a consumer searches $\{\,0, 1\,\}$ if and only if she prefers good 1, we have

$$\begin{aligned} P(1 \text{ Chosen } | \mathcal{C}_0 = \{\,0, 1\,\}) &= 1 \\ &\neq \frac{e^{\beta_1}}{\sum_{k \in \{\,0,1\,\}} e^{\beta_k}} \end{aligned} \tag{3.43}$$

Thus, a researcher who correctly models utility but incorrectly assumes that unobservable preferences are independent of consideration sets will incorrectly estimate her model. In particular, estimating a standard logit model will lead her estimate of β_1, denoted $\hat{\beta}_1$, to have a very large, positive bias. That is, $\hat{\beta}_1 >> \beta_1 = 0$.[10] (Only $\hat{\beta}_1 = \infty$ would achieve equality in eq. (3.43)).

Next, suppose the researcher wants to estimate the counterfactual welfare effects of reducing search difficulty. The researcher supposes search difficulties are reduced to where all consumers search all three products. The researcher further supposes there is no accompanying price change. Since the researcher knows each consumer's utility function's form, she

[9] Here I am using the standard $(\Omega, \mathcal{F}, P)$ notation for my probability space.

[10] Here, $>>$ can be read as "much larger than."

can estimate welfare effects directly. She knows S^W is determined by

$$\begin{aligned} S^W(\omega) &= p_{FC^1} - p_{FC^0} + \beta_{FC^0} - \beta_{FC^1} + \eta_{FC^0}(\omega) - \eta_{FC^1}(\omega) \\ &= \beta_{FC^0} - \beta_{FC^1} + \eta_{FC^0}(\omega) - \eta_{FC^1}(\omega) \quad \text{(Since all prices assumed 0 for simplicity)} \end{aligned}$$

Thus, the average counterfactual welfare change for these consumers would be

$$\begin{aligned} \mu^W &= \int_{\mathcal{C}_0^{-1}(y,\alpha^0,\{0\})} (\beta_{FC^0} - \beta_{FC^1} + \eta_{FC^0}(\omega) - \eta_{FC^1}(\omega))dP \\ &= 0 \end{aligned} \tag{3.44}$$

The first line is true because only consumers whose consideration sets grew could have nonzero S^{CV}. The second line comes from the fact that $FC^0 = FC^1 = 0$ for all consumers with consideration set $\mathcal{C}_0 = \{0\}$, by construction.

However, the estimated value of μ^W, denoted by $\hat{\mu}^W$ will not be 0. I will assume the researcher approximates β_j with $\hat{\beta}_j$ to estimate average welfare change $\hat{\mu}_W$.

Claim 1. *$\hat{\mu}^W$ goes to negative infinity as $\hat{\beta}_1$ goes to infinity while $\mu^W = 0$ regardless.*

The proof can be found at Section 3.D.1 in the appendices. Looking at eq. (3.44) and sending β_1 to infinity suggests the proof, however. Thus, we expect the researcher's estimate $\hat{\mu}^W$ to be much less than 0. The researcher finds a large increase in average equivalent variation due to a reduction in search difficulty. This is despite there being no real welfare change. The researcher's incorrect conclusion is driven by her failure to account for the dependence between consideration sets and the fact that she lacks counterfactual product choices.

♣

Example 2. This case will be the opposite of Example 1. Suppose $\beta_1 >> 0$. In this case, the researcher assumes information and consumer search cost barriers are insignificant. Instead, the researcher assumes that consumers who don't want good 1 don't bother to shop it. That is, the researcher assumes the correct relationship between preferences and consideration sets in Example 1 is also the correct relationship in this example. The researcher claims that a large advertising campaign to encourage more consumer search would have no effect on welfare. The researcher's assumptions also lead her to estimate demand "as if" everyone had $\mathcal{C}_0 = \{ 0, 1 \}$, finding $\hat{\beta}_1$ according to the aggregate market share equation

$$\frac{1}{2} = \frac{e^{\hat{\beta}_1}}{1 + e^{\hat{\beta}_1}}$$

Thus, the researcher determines $\hat{\beta}_1 = 0 = \beta_0$ under her assumptions.

However, correctly modeling the dependence between consideration sets and welfare would lead to a very different conclusion. Indeed, we see an advertising campaign that spurred all consumers to search all products would lead to welfare gains

Claim 2. μ^W *tends to negative infinity as* β_1 *goes to infinity. However,* $\hat{\mu}^W = 0$ *regardless.*

For the proof, see Section 3.D.2. The reasoning is essentially the reverse of the previous examples. Thus, the researcher estimates 0 average equivalent variation despite the fact that the market change may result in arbitrarily large average welfare gains. This failure is in spite of the researcher knowing the functional form of utility, including the distribution of unobservables.

♣

The first example has preferences determining consideration sets. That is, people who prefer good 1 shop it, but people who do not prefer good 1 only shop the empty set. This example is motivated by markets for gender-specific products on college campuses. For the most part, only female college students know the price of feminine products; young men typically don't shop (or know the prices of) feminine products because they don't have a use for them. Young men's market welfare would not improve if they were made to shop feminine products.

The second example has preferences independent of consideration sets. Instead, consideration sets are determined by some exogenous random variable. All people want the product if available, just some don't have access for reasons that do not depend on preferences. This can be motivated by the market for a new vaccine or medicine for a serious, common illness. While many will benefit from treatment, the newness of the product makes it so many consumers don't know to make an appointment and request it from their doctors.

The dependence between consideration sets and preferences can be arbitrarily complicated. To the best of my knowledge, no other paper that models the consumer search process has allowed arbitrary dependence between preferences and consideration sets. My use of conditional demand allows me to sidestep the difficulties of modeling search directly but still allows me to account for this dependence.

3.6 Conclusion

I have identified compensating variation and equivalent variation in discrete choice markets where consumers search. The identifying formulas correctly measure welfare changes from both price changes and shopping behavior changes, given conditional demand. This significantly generalizes classical demand-based welfare measures; classical measures required price changes to calculate welfare changes and did not allow consumer's product knowledge to change with the market. My results place few restrictions on how consumers search. By identifying welfare through conditional demand, I have avoided many of the difficulties that come out of modeling search behavior. Importantly, my results allow arbitrary dependence between preferences and consideration sets. When utility is linear in money, the welfare formulas can be applied to arbitrary market changes. For nonparametric utility, welfare changes are identifiable if the market changes are weakly expanding or weakly contracting.

3.A Proofs from Section 3.2

3.A.1 Additional Results from Section 3.2

Theorem 20. *Under Monotonicity and Price Independence, for consumers in $\mathcal{C}_0^{-\star}(y, \alpha^0, i, A)$, if $p_i^1 < p_i^0$, individual equivalent variation is negative and average equivalent variation is*

$$\begin{aligned}&\mu^{CV}(y, \mathcal{C}_1^{-\star}(y, \alpha^0, i, A))\\&\qquad = -\int_{-\infty}^{0} \left[1 - \bar{Q}_i(y + S, p_i^0 + S, p_{-i}^1, \alpha^0, \mathcal{C}_0^{-\star}(y, \alpha^0, i, A)) \cdot 1(S \leq p_i^1 - p_i^0)\right] dS\end{aligned}$$

If instead $p_i^1 \geq p_i^0$, then individual compensating variation may have gone up or down but the following average bounds hold

$$\begin{aligned}&-\int_{-\infty}^{0} \left[1 - \bar{Q}_i(y + S, p_i^0 + S, p_{-i}^1, \alpha^0, \mathcal{C}_0^{-\star}(y, \alpha^0, i, A)) \cdot 1(S \leq p_i^1 - p_i^0)\right] dS\\&\qquad \leq \mu^{CV}(y, \mathcal{C}_0^{-\star}(y, \alpha^0, i, A)) \leq \int_0^{p_i^1 - p_i^0} \bar{Q}_i(y + S, p_i^0 + S, p_{-1}^1, \alpha^0, \mathcal{C}_0^{-\star}(y, \alpha^0, i, A)) dS\end{aligned}$$

where the first inequality is an exact equality in the case that the price of all goods weakly increases.

The proof can be found in Section 3.A.7. The corresponding corollary for compensating variation is given in Corollary 4.

Corollary 4. *Average compensating variation, in response to multiple price changes, for consumers in consideration group $\mathcal{C}_1^{-\star}(y, \alpha^1, j, B)$ can be bound by*

$$\begin{aligned}&\frac{-1}{P(\mathcal{C}_0^{-\star}(y, \alpha^0, i, A))} \int_{-\infty}^{0} \Big[P(\mathcal{C}_0^{-1}(y, \alpha^0, A))\\&\qquad - Q_i(y + S, p_i^0 + S, p_{-i}^1, \alpha^0, \mathcal{C}_0^{-1}(y, \alpha^0, A))) \cdot 1(S \leq p_i^1 - p_i^0)\Big] dS\\&\leq \mu^{CV}(y, \mathcal{C}_0^{-\star}(y, \alpha^0, i, A))\\&\qquad \leq \frac{1}{P(\mathcal{C}_0^{-\star}(y, \alpha^0, i, A))} \int_0^{p_i^1 - p_i^0} Q_i(y + S, p_i^0 + S, p_{-i}^1, \alpha^0, \mathcal{C}_0^{-1}(y, \alpha^0, A)) dS\end{aligned}$$

3.A.2 Proof of Lemma 1

Proof. Consider the three cases

1. $\bar{p}_1^0 \geq p_1^1$

2. $p_1^1 > \bar{p}_1^0 > p_1^0$

3. $\bar{p}_1^0 \leq p_1^0$

Note that these three cases partition all possible outcomes for our arbitrary consumer. It is therefore sufficient to show

$$S^{EV}(y, \eta, \zeta, \alpha^0, \alpha^1, p_{\mathcal{J}}^0, p_{\mathcal{J}}^1) = \int_{p_1^0}^{p_1^1} q_1^0(y, p, \eta; \mathcal{C}_0) dp$$

holds in each case.

In the first case, $q_1^0(y, p, \eta) \equiv 1$ for $p \in (p_1^0, p_1^1)$ by definition of $\bar{p}_1^0$. Further, for $S \in (0, p_1^1 - p_1^0)$

$$u_1(y - S - p_1^0, \eta) \geq u_1(y - p_1^1, \eta) \geq \max_{j \in \mathcal{C}_0 \setminus \{1\}} u_j(y - p_j^0, \eta) \geq \max_{j \in \mathcal{C}_0 \setminus \{1\}} u_j(y - S - p_j^0, \eta).$$

Thus

$$u_1(y - S^{EV} - p_1^0, \eta) = u_1(y - p_1^1, \eta)$$

so we can plug in and verify $S^{EV} = p_1^1 - p_1^0 = \int_{p_1^0}^{p_1^1} q_1^0(y, p, \eta; \mathcal{C}_0) dp$.

In the second case, I claim $SC^0 = \arg\max_{j \in \mathcal{C}_0} u_j(y - p_j^1 - S^{EV}, \eta)$. Since $S^{EV} < p_1^1 - p_1^0$, we cannot have $1 = \arg\max_{j \in \mathcal{C}_0} u_j(y - p_j^1 - S^{EV}, \eta)$. Further, since revealed preference tells us

$$\begin{aligned} u_1(y - p_j^0, \eta) \geq u_{SC^0}(y - p_j^0, \eta) &\geq \max\{\, u_1(y - p_j^1, \eta), \max_{j \in \mathcal{C}_0 \setminus \{1, SC^0\}} u_j(y - p_j^0, \eta)\,\} \\ \Rightarrow u_1(y - p_j^0 - S^{EV}, \eta) &= \max_{j \in \mathcal{C}_0} u_j(y - p_j^0 - S^{EV}, \eta) = u_{SC^0}(y - p_j^0, \eta) \end{aligned}$$

Thus, $S^{EV} = \bar{p}_1^0 - p_1^0$, since

$$u_1(y - \bar{p}_1, \eta) = u_{SC^0}(y - p_{SC^0}^0, \eta)$$

$$\begin{aligned} \Rightarrow u_{SC^0}(y - p_{SC^0}^0, \eta) = u_{FC^1}(y - p_{FC^1}^0, \eta) = u_1(y - S^{EV} - p_1^0, \eta) \\ \Rightarrow S^{EV} = \bar{p}_1 - p_1^0 = \int_{p_1^0}^{p_1^1} q_1(p; y, \eta) dp \end{aligned}$$

where the last line follows from the definition of $\bar{p}_1^0$

In the third case, trivially $S^{EV} = 0$ and $q_1^0(y, p, \eta) \equiv 0$ on (p_1^0, p_1^1) by definition of $\bar{p}_1^0$. Thus

$$\int_{p_1^0}^{p_1^1} q_1(p; y, \eta) dp = 0 = S^{EV}.$$

□

3.A.3 Proof of Theorem 9

Proof. If $1 \in A$

$$\begin{aligned}
\mu^{EV}(y, \mathcal{C}^{-1}(y, \alpha^0, A)) &:= \fint_{\mathcal{C}_0^{-1}(y,\alpha^0,A)} S^{EV}(y, \eta, \zeta) dF \\
&= \fint_{\mathcal{C}_0^{-1}(y,\alpha^0,A)} \int_{p_1^0}^{p_1^1} q_1^0(y, p, \eta) dp dF && \text{(by Lemma 1)} \\
&= \int_{p_1^0}^{p_1^1} \fint_{\mathcal{C}_0^{-1}(y,\alpha^0,A)} q_1^0(y, p, \eta) dF dp && \text{(Tonelli's Theorem)}
\end{aligned}$$

If $1 \notin A$, then there is no change observed by the consumers in $\mathcal{C}^{-1}(y, \alpha^0, A)$ and therefore no welfare change: $\mu^{EV}(y, \mathcal{C}^{-1}(y, \alpha^0, A)) = 0$ □

3.A.4 Proof of Theorem 10

Proof. I claim

$$S^{CV}(y, \eta, \zeta, \alpha^0, \alpha^1, p_{\mathcal{J}}^0, p_{\mathcal{J}}^1) = -\int_{p_1^1}^{p_1^0} q_1^1(y + p - p_1^0, p, \eta) dp \tag{3.45}$$

This can be seen in two cases. First consider a consumer with $FC^0 = 1$. Then $S^{CV} \in [0, p_1^1 - p_1^0]$ by Monotonicity and since

$$u_1(y - p_1^0) = u_1(y + (p_1^1 - p_1^0) - p_1^1, \eta) \leq \max_{j \in \mathcal{C}_1} u_j(y + (p_1^1 - p_1^0) - p_j^1, \eta)$$

Thus, in this case

$$\begin{aligned} S^{CV} &= \int_0^{S^{CV}} 1 dp \\ &= \int_0^{p_1^1 - p_1^0} 1\left\{\, u_1(y - p_1^0, \eta) \geq \max_{j \in \mathcal{C}_1} u_j(y + \delta - p_j^1, \eta) \,\right\} d\delta \\ &= \int_0^{p_1^1 - p_1^0} q_1^1(y + \delta, \delta - p_1^0, \eta, \zeta) d\delta \\ &= \int_{p_1^0}^{p_1^1} q_1^1(y + p - p_1^0, p, \eta, \zeta) dp \end{aligned}$$

In the case the consumer does not choose $FC^0 = 1$, $1 \neq \arg\max_{j \in \mathcal{C}_1} u_j(y - p_j^1, \eta)$. Thus we see $S^{CV} = 0$ and $q_1^1(y + \delta, \delta - p_1^0, \eta, \zeta) = 0$ for $\delta \in (p_1^0, p_1^1)$ which confirms the integral holds again. This completes all possible cases for individual welfare.

Aggregating, we thus see

$$\begin{aligned} \mu^{CV}(y, \mathcal{C}^{-1}(y, \alpha^0, A)) &= \int_{\mathcal{C}^{-1}(y, \alpha^0, A)} \int_{p_1^0}^{p_1^1} q_1^0(y + p - p_1^0, p, \eta) dp dF \\ &= \int_{p_1^0}^{p_1^1} \int_{\mathcal{C}^{-1}(y, \alpha^0, A)} q_1^0(y + p - p_1^0, p, \eta) dF dp \qquad \text{(Tonelli's Theorem)} \end{aligned}$$

$\square$

3.A.5 Proofs for Welfare Changes When Many Prices Change

For an arbitrary consumer, define

$$g(S) := \max_{k \in \mathcal{C}_0} u_k(y - S - p_k^0, \eta) - u_{FC^1}(y - p_{FC^1}^1, \eta) \tag{3.46}$$

and

$$h(S) := \max_{k \in \mathcal{C}_1} u_k(y + S - p_k^1, \eta) - u_{FC^0}(y - p_{FC^0}^0, \eta) \tag{3.47}$$

Then, we have the following lemmas

Lemma 5.A. *g has the following properties*

1. *monotonically decreasing in* S

2. *unique zero at* S^{EV}

3. $g(p^1_{FC^1} - p^0_{FC^1}) \geq 0$

4. $1(g(S) > 0) = 1 - q^0_{FC^1}(y - S, p^1_{FC^1} - S, \eta) \times 1(p^0_{FC^1} + S > p^1_{FC^1})$ *when* $FC^1 \in \mathcal{C}_0$ *except for perhaps on a set of Lebesgue measure 0 when*

5. $1(g(S) < 0) = q^0_{FC^1}(y - S, p^1_{FC^1} - S, \eta) \times 1(p^0_{FC^1} + S \geq p^1_{FC^1}) = 1$ *when* $FC^1 \in \mathcal{C}_0$ *except for perhaps on a set of Lebesgue measure 0*

Lemma 5.B. *h has the following properties*

1. *monotonically increasing in* S

2. *unique zero at* S^{CV}

3. $h(p^1_{FC^0} - p^0_{FC^0}) \geq 0$

4. $1(h(S) > 0) = 1 - q^1_{FC^0}(y + S, p^0_{FC^0} + S, \eta) \cdot 1(S < p^1_{FC^0} - p^0_{FC^0})$ *when* $FC^0 \in \mathcal{C}_1$ *except for perhaps on a set of Lebesgue measure 0*

5. $1(h(S) < 0) = q^1_{FC^0}(y + S, p^0_{FC^0} + S, \eta) \cdot 1(S < p^1_{FC^0} - p^0_{FC^0})$ *when* $FC^0 \in \mathcal{C}_1$ *except for perhaps on a set of Lebesgue measure 0*

Proof of Lemma 5. I first prove the results for g

1. This is clear from Monotonicity and since the max of monotonically decreasing functions is monotonically decreasing.

2. This is clear from the definition of S^{EV}

3. From

$$\begin{aligned} g(p^1_{FC^1} - p^0_{FC^1}) &= \max_{k \in \mathcal{C}_0} u_k(y - p^1_{FC^1} + p^0_{FC^1} - p^0_k, \eta) - u_{FC^1}(y - p^1_{FC^1}, \eta) \\ &\geq u_{FC^1}(y - p^1_{FC^1} + p^0_{FC^1} - p^0_{FC^1}, \eta) - u_{FC^1}(y - p^1_{FC^1}, \eta) \\ &= 0 \end{aligned}$$

4. From

$$g(S) > 0 \Leftrightarrow \max_{k \in \mathcal{C}_0} u_k(y - S - p^0_k, \eta) > u_{FC^1}(y - p^1_{FC^1}, \eta)$$

$$\Leftrightarrow \max_{k \in \mathcal{C}_0 \setminus \{ FC^1 \}} u_k(y - S - p^0_k, \eta) > u_{FC^1}(y - p^1_{FC^1}, \eta) \text{ or } p^0_{FC^1} + S < p^1_{FC^1}$$

$$\Leftrightarrow q^0_{FC^1}(y - S, p^1_{FC^1} - S, \eta) = 0 \text{ or } 1(p^0_{FC^1} + S > p^1_{FC^1}) = 0$$

except perhaps at points of S that make consumers indifferent between two or more goods. By Monotonicity, these points have Lebesgue measure 0. Thus

$$1(g(S) > 0) = 1 - q^0_{FC^1}(y - S, p^1_{FC^1} - S, \eta) \times 1(p^0_{FC^1} + S > p^1_{FC^1}) \text{ Lebesgue a.e. in S}$$

5. Immediate from (4): $1(g(S) < 0) = 1 - 1(g(S) > 0)$ Lebesgue a.e.

For h, I just prove 3 and 5.

3. From

$$h(p^1_{FC^0} - p^0_{FC^0}) \geq u_{FC^0}(y - p^0_{FC^0}, \eta) - u_{FC^0}(y - p^0_{FC^0}, \eta) = 0$$

5. From

$$h(S) < 0 \Leftrightarrow$$

$$u_k(y + S - p^1_k, \eta) \leq u_{FC^0}(y - p^0_{FC^0}, \eta) \; \forall k \in \mathcal{C}_1 \setminus \{ FC^0 \} \text{ and } S - p^1_{FC^0} < -p^0_{FC^0}$$

$$\Leftrightarrow q^1_{FC^0}(y + S, p^0_{FC^0} + S, \eta) = 1 \text{ and } 1(S < p^1_{FC^0} - p^0_{FC^0}) = 1$$

Thus

$$q^1_{FC^0}(y + S, p^0_{FC^0} + S, \eta) \cdot 1(S < p^1_{FC^0} - p^0_{FC^0}) 1(h(S) < 0) =$$

□

3.A.6 Proof of Theorem 11

Proof. Fix an arbitrary consumer (ζ, η) in $\mathcal{C}_0^{-\star}(y, \alpha^1, j, B))$.

First, suppose $p_j^1 \geq p_j^0$. Then $S^{EV} \geq 0$ (consumer hurt) because from Lemma 5.A, $g(p_j^1 - p_j^0) \geq 0$, g is monotonically decreasing and $g(S^{EV}) = 0$ Thus $S^{EV} \geq p_j^1 - p_j^0 \geq 0$. Therefore

$$\begin{aligned}
S^{EV} &= \int_0^{S^{EV}} 1 dS \\
&= \int_0^{\infty} 1(g(S) > 0) dS \\
&= \int_0^{\infty} \left[1 - q^0_{FC^1}(y - S, p^1_{FC^1} - S, \eta) \times 1(p^0_{FC^1} + S > p^1_{FC^1})\right] dS && \text{(Lemma 5.A)}
\end{aligned}$$

Averaging over all consumers and switching the integrals by Tonelli's Theorem gives the equality result.

Next, consider the case of a price decrease in good j: $p_j^1 < p_j^0$. Here, we cannot tell if S^{EV} is positive or negative. If it is negative, then

$$\begin{aligned}
S^{EV} &= -\int_{-\infty}^{0} 1(g(S) < 0) dS \\
&= -\int_{p_j^1 - p_j^0}^{0} q_1^0(y - S, p_j^1 - S, \eta) dS && \text{(Lemma 5.A)}
\end{aligned}$$

Moreover, if $S^{EV} \geq 0$, then $\int_{-\infty}^{0} 1(g(S) < 0) = 0$ by part 1 of Lemma 5.A. Thus, we see

$$\begin{aligned}
S^{EV} &= S^{EV} 1(S^{EV} < 0) + S^{EV} 1(S^{EV} > 0) \\
&\geq= S^{EV} 1(S^{EV} < 0) \\
&= -\int_{p_j^1 - p_j^0}^{0} q_1^0(y - S, p_j^1 - S, \eta) dS
\end{aligned}$$

The second line is an exact equality if no consumers have positive S^{EV}. All prices falling is sufficient for this.

Similarly, the other bound follows from the fact that

$$S^{EV} \cdot 1(S^{EV} > 0) = \int_0^{\infty} \left[1 - q^0_{FC^1}(y - S, p^1_{FC^1} - S, \eta) \times 1(p^0_{FC^1} + S > p^1_{FC^1})\right] dS$$

because the integral on the RHS is zero for conusmers with $S^{EV} < 0$.

$\square$

3.A.7 Proof of Theorem 20

Proof. Fix an arbitrary consumer (ζ, η) in $\mathcal{C}_0^{-\star}(y, \alpha^0, i, B))$. First, consider a price decrease of good i: $p_i^1 < p_i^0$. In this case, by Lemma 5.B, $S^{CV} \leq 0$. Thus

$$\begin{aligned} S^{CV} &= -\int_{S^{CV}}^{0} dS \\ &= \int_{-\infty}^{0} 1(h(S) > 0) dS \\ &= \int_{-\infty}^{0} \left[1 - q_i^1(y+S, p_i^0+S, \eta) \cdot 1(S < p_i^0 - p_i^1)\right] dS \qquad \text{(Lemma 5.B)} \end{aligned}$$

Averaging and applying Tonelli's theorem acheives the first result.

Similarly, if $p_i^1 \geq p_i^0$, the consumer may have $S^{CV} \geq 0$ or $S^{CV} \leq 0$. In the case that $S^{CV} \geq 0$, we see

$$S^{CV} \cdot 1(S^{CV} \geq 0) = \int_0^{p_i^1 - p_i^0} q_i^1(y+S, S+p_i^0, \eta) dS$$

Since

$$\begin{aligned} S^{CV} &= S^{CV} 1(S^{CV} \geq 0) + S^{CV} 1(S^{CV} \leq 0) \\ &\leq S^{CV} 1(S^{CV} \geq 0) \\ &= \int_0^{p_i^1 - p_i^0} q_i^1(y+S, S+p_i^0, \eta) dS \end{aligned}$$

averaging and applying Tonelli's theorem gets the first inequality. Since all prices weakly increasing is enough to guarantee $S^{CV} \geq 0$ for all consumers, the inequality is an equality in this case.

Similarly, noting

$$S^{CV} 1(S^{CV} \leq 0) = \int_{-\infty}^{0} \left[1 - q_i^1(y+S, p_i^0+S, \eta) \cdot 1(S < p_i^0 - p_i^1)\right] dS$$

since the integral on the RHS is zero for consumers with $S^{CV} > 0$ gives the other bound. □

3.B Proofs from Section 3.3

In the following lemmas, both prices and platform behavior may be changing from $t = 0$ to $t = 1$

Lemma 6. *Under Linearity,*

$$S^W = u_{FC^0}(y - p^0_{FC^0}, \eta) - u_{FC^1}(y - p^1_{FC^1}, \eta)$$

In particular, income changes will not lead consumers to switch good choice.

Proof. Under Linearity,

$$\max_{j \in \mathcal{C}_0} u_j(y - p^0_j - S, \eta) = \max_{j \in \mathcal{C}_0} u_j(y - p^0_j, \eta) - S = u_{FC^0}(y - p^0_{FC^0}, \eta) - S$$

□

3.B.1 Proofs from Section 3.3.1

3.B.1.1 Proof of Lemma 2

Proof. At $\bar{P}_{FC^0}(y, 0, \{ FC^0, 0 \}, \eta)$, we have

$$\begin{aligned} &u_{FC^0}(y - \bar{P}_{FC^0}(y, 0, \{ FC^0, 0 \}, \eta), \eta) = u_0(y, \eta) \\ &\Rightarrow \bar{P}_{FC^0}(y, 0, \{ FC^0, 0 \}, \eta), \eta) = \tilde{U}_{FC^0}(\eta) - \tilde{U}_0(\eta) \end{aligned} \tag{3.48}$$

The last line follows from Linearity. Similarly,

$$\begin{aligned} &u_{FC^1}(y - \bar{P}_{FC^1}(y, 0, \{ FC^1, 0 \}, \eta), \eta) = u_0(y, \eta) \\ &\Rightarrow \bar{P}_{FC^1}(y, 0, \{ FC^0, 0 \}, \eta) = \tilde{U}_{FC^1}(\eta) - \tilde{U}_0(\eta) \end{aligned} \tag{3.49}$$

By revealed preference, we have

$$\begin{aligned} &\int_{p^0_{FC^0}}^{\infty} q_{FC^0}(y, p, 0, \{ FC^0, 0 \}, \eta) dp - \int_{p^1_{FC^1}}^{\infty} q_{FC^1}(y, p, 0, \{ FC^1, 0 \}, \eta) dp \\ &= \bar{P}_{FC^0}(y, 0, \{ FC^0, 0 \}, \eta) - p^0_{FC^0} - \left(\bar{P}_{FC^1}(y, 0, \{ FC^1, 0 \}, \eta) - p^1_{FC^1} \right) \\ &= -p^0_{FC^0} + \tilde{U}_{FC^0}(\eta) - \tilde{U}_0(\eta) - \left[-p^1_{FC^1} U_{FC^1}(\eta) - \tilde{U}_0(\eta) \right] \quad \text{(Using eq. (3.49) and eq. (3.48))} \\ &= S^W \quad \text{(by eq. (3.22))} \end{aligned}$$

□

3.B.1.2 Proof of Theorem 12

Proof. First note that for any consumer $(\eta, \zeta) \in \mathcal{C}_{0,1}^{-1}(y, i, j, A, B)$,

$$q_i(y, p_i, p_{-i}, \eta, \zeta, \alpha^0) = \not{q}_i(y, p_i, p_{-i}, A, \eta) \text{ and}$$

$$q_j(y, p_j, p_{-i}, \eta, \zeta, \alpha^1) = \not{q}_j(y, p_j, p_{-i}, B, \eta)$$

for all price vectors $p_{\mathcal{J}} \in \{0\} \times \mathbb{R}^J$ by Price Independence.

Further, note that by Monotonicity

$$\lim_{\substack{p_\ell \to \infty \\ \forall \ell \in A \setminus \{i, 0\}}} q_i(y, p_i, p_{-i}, \eta, \zeta, \alpha^0) = \not{q}_i(y, p_i, 0, \{FC^0, 0\}, \eta) \tag{3.50}$$

$$\lim_{\substack{p_\ell \to \infty \\ \forall \ell \in B \setminus \{j, 0\}}} q_j(y, p_j, p_{-j}, \eta, \zeta, \alpha^1) = \not{q}_j(y, p_j, 0, \{FC^1, 0\}, \eta) \tag{3.51}$$

Therefore

$$
\begin{aligned}
\pi^W(y,i,j,A,B) &= \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} \int S^W dF \\
&= \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} \Big[\int_{p^0_{FC^0}}^{\infty} \not{q}_{FC^0}(y,p,0,\{\, FC^0, 0 \,\},\eta)dp \\
&\qquad - \int_{p^1_{FC^1}}^{\infty} \not{q}_{FC^1}(y,p,0,\{\, FC^1, 0 \,\},\eta)dp \Big] dF && \text{(by Lemma 2)} \\
&= \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} \int_{p^0_i}^{\infty} \not{q}_i(y,p,0,\{\, i, 0 \,\},\eta)dp \\
&\qquad - \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} \int_{p^1_j}^{\infty} \not{q}_j(y,p,0,\{\, j, 0 \,\},\eta)dpdF \\
&&& \text{(by group characteristics and Finite Differences)} \\
&= \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} \int_{p^0_i}^{\infty} \lim_{\substack{p_\ell \to \infty \\ \forall \ell \in A \setminus \{\, i,0 \,\}}} q_i(y,p_i,p_{-i},\eta,\zeta,\alpha^0)dp \\
&\qquad - \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} \int_{p^1_j}^{\infty} \lim_{\substack{p_\ell \to \infty \\ \forall \ell \in B \setminus \{\, j,0 \,\}}} q_j(y,p,p_{-j},\eta,\zeta,\alpha^1)dpdF \\
&&& \text{(by eq. (3.50) and eq. (3.51))} \\
&= \lim_{\substack{p_\ell \to \infty \\ \forall \ell \in A \setminus \{\, i,0 \,\}}} \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} \int_{p^0_i}^{\infty} q_i(y,p_i,p_{-i},\eta,\zeta,\alpha^0)dp \\
&\qquad - \lim_{\substack{p_\ell \to \infty \\ \forall \ell \in B \setminus \{\, j,0 \,\}}} \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} \int_{p^1_j}^{\infty} q_j(y,p,p_{-j},\eta,\zeta,\alpha^1)dpdF \\
&&& \text{(Monotone Convergence Thm.)} \\
&= \lim_{\substack{p_\ell \to \infty \\ \forall \ell \in A \setminus \{\, i,0 \,\}}} \int_{p^0_i}^{\infty} \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} q_i(y,p_i,p_{-i},\eta,\zeta,\alpha^0)dFdp \\
&\qquad - \lim_{\substack{p_\ell \to \infty \\ \forall \ell \in B \setminus \{\, j,0 \,\}}} \int_{p^1_j}^{\infty} \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} q_j(y,p,p_{-j},\eta,\zeta,\alpha^1)dFdp \\
&&& \text{(Tonelli's Theorem)}
\end{aligned}
$$

□

3.B.1.3 Proof of Lemma 3

Proof. I start by proving the formula for S^{EV}

We have, by definition of $\bar{P}_{FC^0}$ and S^{EV}

$$u_{FC^0}(y - \bar{P}_{FC^0}(y, p^1_{FC^1}, \{\, FC^0, FC^1 \,\}, \eta), \eta) = u_{FC^1}(y - p^1_{FC^1}, \eta) = u_{FC^0}(y - S^{EV} - p^0_{FC^0}, \eta)$$

Therefore,

$$S^{EV} = \bar{P}_{FC^0}(y, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta) - p^0_{FC^0}$$

The proof follows by considering three cases

Case 1: $S^{EV} \geq 0$ and $FC^0 \neq FC^1$

Case 2: $S^{EV} < 0$ and $FC^0 \neq FC^1$

Case 3: $FC^0 = FC^1$

First, consider case 1. Here, $\bar{P}_{FC^0}(y, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta) \geq p^0_{FC^0}$. Thus,

$$q^\star_{FC^0}(y, p_{FC^0}, p^1_{FC^0}, \{ FC^0, FC^1 \}, \eta) = \begin{cases} 1 \text{ when } p_{FC^0} \in (p^0_{FC^0}, \bar{P}_{FC^0}(y, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta)) \\ 0 \text{ when } p_{FC^0} > \bar{P}_{FC^0}(y, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta) \end{cases}$$

while

$$q^\star_{FC^1}(y, p^1_{FC^0}, p'_{FC^0} \{ FC^0, FC^1 \}, \eta) = 1 - q^\star_{FC^0}(y, p_{FC^0}, p^1_{FC^0}, \{ FC^0, FC^1 \}, \eta) = 0$$

for $p_{FC^0} \in (-\infty, p^0_{FC^0})$. Thus

$$\begin{aligned} S^{EV} &= \bar{P}_{FC^0}(y, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta) - p^0_{FC^0} \\ &= \int_{p^0_{FC^0}}^{\bar{P}_{FC^0}(y, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta)} 1 dp \\ &= \int_{p^0_{FC^0}}^{\infty} q^\star_{FC^0}(y, p_{FC^0}, p^1_{FC^0}, \{ FC^0, FC^1 \}, \eta) dp_{FC^0} - 0 \\ &= \int_{p^0_{FC^0}}^{\infty} q^\star_{FC^0}(y, p_{FC^0}, p^1_{FC^0}, \{ FC^0, FC^1 \}, \eta) dp_{FC^0} \\ &\quad - \int_{-\infty}^{p^0_{FC^0}} q^\star_{FC^1}(y, p^1_{FC^0}, p'_{FC^0} \{ FC^0, FC^1 \}, \eta) dp_{FC^0} \end{aligned}$$

Second, consider case 2. Here, $\bar{P}_{FC^0}(y, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta) \leq p^0_{FC^0}$. Thus,

$$q^\star_{FC^0}(y, p_{FC^0}, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta) = 0$$

for $p_{FC^0} \in (p^0_{FC^0}, \infty)$ while

$$
\begin{aligned}
q^{\star}_{FC^1}(y, p^1_{FC^1}, p_{FC^0}, \{ FC^0, FC^1 \}, \eta) &= 1 - q^{\star}_{FC^0}(y, p_{FC^0}, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta) \\
&= \begin{cases} 1 \text{ when } p_{FC^0} \in (\bar{P}_{FC^0}(y, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta), p^0_{FC^0}) \\ 0 \text{ when } p_{FC^0} < \bar{P}_{FC^0}(y, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta) \end{cases}
\end{aligned}
$$

Thus

$$
\begin{aligned}
S^{EV} &= \bar{P}_{FC^0}(y, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta) - p^0_{FC^0} \\
&= -\int_{\bar{P}_{FC^0}}^{p^0_{FC^0}} q_{FC^1}(y, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta) 1 dp \\
&= 0 - \int_{-\infty}^{p^0_{FC^0}} q_{FC^1}(y, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta) 1 dp \\
&= \int_{p^0_{FC^0}}^{\infty} q^{\star}_{FC^0}(y, p_{FC^0}, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta) dp_{FC^0} \\
&\qquad - \int_{-\infty}^{p^0_{FC^0}} q^{\star}_{FC^1}(y, p^1_{FC^1}, p_{FC^0}, \{ FC^0, FC^1 \}, \eta) dp_{FC^0}
\end{aligned}
$$

Finally, in case 3,

$$
\begin{aligned}
&\int_{p^0_{FC^0}}^{\infty} q^{\star}_{FC^0}(y, p_{FC^0}, p^1_{FC^1}, \{ FC^0, FC^1 \}, \eta) dp_{FC^0} \\
&\qquad - \int_{-\infty}^{p^0_{FC^0}} q^{\star}_{FC^1}(y, p^1_{FC^1}, p_{FC^0}, \{ FC^0, FC^1 \}, \eta) dp_{FC^0} \\
&= \int_{p^0_{FC^0}}^{\infty} \mathbb{1}(p_{FC^0} < p^1_{FC^0}) dp - \int_{-\infty}^{p^0_{FC^0}} [1 - \mathbb{1}(p_{FC^0} < p^1_{FC^0})] dp_{FC^0} \\
&= (p^1_{FC^0} - p^0_{FC^0}) \mathbb{1}(p^1_{FC^0} > p^0_{FC^0}) - (p^0_{FC^0} - p^1_{FC^0}) \mathbb{1}(p^1_{FC^0} \leq p^0_{FC^0}) \\
&= p^1_{FC^0} - p^0_{FC^0} \\
&= S^{EV}
\end{aligned}
$$

□

3.B.1.4 Proof of Lemma 4

Proof. The proof is analogous to that of Theorem 10

□

3.B.1.5 Proof of Theorem 13

Proof. I start with the π^{EV} case

$$\begin{aligned}
\pi^{EV} &= \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} \Big[\int_{p^0_{FC^0}}^{\infty} q^\star_{FC^0}(y, p_{FC^0}, p^1_{FC^0}, \{ FC^0, FC^1 \}, \eta) dp_{FC^0} \\
&\quad - \int_{-\infty}^{p^0_{FC^0}} q^\star_{FC^1}(y, p^1_{FC^0}, p_{FC^0} \{ FC^0, FC^1 \}, \eta) dp_{FC^0} \Big] && \text{(Lemma 3)} \\
&= \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} \int_{p_i^0}^{\infty} \lim_{p_\ell \to \infty \ \forall \ell \in C \setminus \{ i,j,0 \}} q_i^\star(y, p_i, (p_j^1, p_{-\{ i,j \}}), \eta, \zeta, \alpha^{t^\star}) dp_{FC^0} \\
&\quad - \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} \int_{-\infty}^{p_i^0} \lim_{p_\ell \to \infty \ \forall \ell \in C \setminus \{ i,j,0 \}} q_j^\star(y, p_j^1, p_{-j}, \eta, \zeta, \alpha^{t^\star}) dp_{FC^0} \\
& && \text{(Monotonicity and Finite Differences)} \\
&= \lim_{p_\ell \to \infty \ \forall \ell \in C \setminus \{ i,j,0 \}} \int_{p_i^0}^{\infty} \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} q_i^\star(y, p_i, (p_j^1, p_{-\{ i,j \}}), \eta, \zeta, \alpha^{t^\star}) dp_{FC^0} \\
&\quad - \lim_{p_\ell \to \infty \ \forall \ell \in C \setminus \{ i,j,0 \}} \int_{-\infty}^{p_i^0} \fint_{\mathcal{C}_{0,1}^{-1}(y,i,j,A,B)} q_j^\star(y, p_j^1, p_{-j}, \eta, \zeta, \alpha^{t^\star}) dp_{FC^0} \\
& && \text{(Monotone Convergence theorem and Tonelli for switching the bounds.)}
\end{aligned}$$

The case of π^{CV} is very similar. □

3.B.1.6 Proof of Theorem 14

Proof. Both results follow from a straightforward application of Lemma 4, monotone convergence theorem and Tonelli's theorem, as has been done above. □

3.C Proofs from Section 3.4

3.C.1 Proofs for Welfare Changes When Consideration Sets Shrink or Expand

3.C.1.1 Proof for Theorem 15

Proof. Set $p_1^1 = \infty$ and it is clear the proof of single price increase reduces to this case □

3.C.1.2 Proof of Theorem 17

Proof. Recalling our definitions of $g(S)$ and and $h(S)$ from eq. (3.46) and eq. (3.47) respectively, but restricting $p_k^0 = p_k^1$ for all k and noting that $FC^1 \in \mathcal{C}_0$ because consideration sets reduced in size, we see

$$\begin{aligned} S^{EV} &= \int_0^\infty 1(g(S) > 0)dS \\ &= \int_0^\infty [1 - q^0_{FC^1}(y - S, p^0_{FC^1} - S, \eta) \times 1(p^0_{FC^1} + S > p^0_{FC^1})]dS \\ &= \int_0^\infty [1 - q^0_{FC^1}(y - S, p^0_{FC^1} - S, \eta)]dS && \text{(Since } S > 0) \end{aligned}$$

From here, continuing the rest of the proof of the case $S > 0$ from Section 3.A.6 achieves the result.

For S^{CV}, note that $FC^0 \notin \mathcal{C}_1$ and therefore

$$\begin{aligned} 1(h(S) < 0) &= 1(u_{FC^0}(y - p^0_{FC^0}) > \max_{k \in \mathcal{C}_1} u_k(y + S - p^0_k, \eta)) \\ &= q_{FC^0}(y + S, S + p^0_{FC^0}, p^0_{-FC^0}, \mathcal{C}_1 \cup FC^0, \eta) \\ &= \lim_{p^0_\ell \to \infty \;\; \forall \ell \in R \setminus \{FC^0\}} q^0_{FC^0}(y + S, S + p^0_{FC^0}, \eta) \end{aligned}$$

Next, note that $1(h(S) < 0) = 1 - 1(h(S) > 0)$ Lebesgue a.e. in S by Monotonicity. Therefore, following Section 3.A.6, we have

$$\begin{aligned} S^{EV} &= \int_0^\infty 1(h(S) > 0)dS \\ &= \int_0^\infty \lim_{p^0_\ell \to \infty \;\; \forall \ell \in R \setminus \{FC^0\}} q^0_{FC^0}(y + S, S + p^0_{FC^0}, \eta)dS \\ &= \lim_{p^0_\ell \to \infty \;\; \forall \ell \in R \setminus \{FC^0\}} \int_0^\infty q^0_{FC^0}(y + S, S + p^0_{FC^0}, \eta)dS \\ & && \text{(Monotonic Convergence Theorem)} \end{aligned}$$

Doing the usual aggregating, passing the limits through integral by the Monotonic Convergence theorem again and switching integrals by Tonnelli's theorem gives the result □

3.C.1.3 Proof of Theorem 18

Proof. This proof is exactly analogous to Section 3.C.1.2 □

3.C.2 Proofs for Welfare Changes When One Good is Swapped

The proof of uses the following lemma. I prove Lemma 7 first, then proceed to the proof of Theorem 19.

Lemma 7. *Let $B = (A \setminus 1) \cup M)$ where $M \notin A$ and $1 \in A$. Then, under Linearity, the welfare change for a consumer with initial consideration set A and final consideration set B is*

$$S^W = \int_{p_1^1}^{\infty} q_1(y, p_1, p_{-1}, A, \eta)dp - \int_{p_M^1}^{\infty} q_M(y, p, p_{-M}, B, \eta)dp$$

Proof of Lemma 7. The proof proceeds by considering the cases. There are four: (i) $FC^0 = 1$ and $FC^1 = M$, (ii) $FC^0 \neq 1$ and $FC^1 = M$, (iii) $FC^0 \neq 1$ and $FC^1 \neq M$ and (iv) $FC^0 = 1$ and $FC^1 \neq M$. I verify the formula holds in each case.

Case i. Since $FC^0 = 1$ and $FC^1 = M$, it must be that $E^0 = E^1$. That is, the second favorite to one remains the second favorite to M after good 1 is gone. Then note that, under *Monotonicity*

$$\begin{aligned}
S^W &= -p_1^1 + \tilde{U}_1(\eta) - (-p_M^1 + \tilde{U}_M(\eta)) \\
&= \left[-p_1^1 + \tilde{U}_1(\eta) - (-p_{E^1}^1 + \tilde{U}_{E^1}(\eta))\right] - \left[-p_M^1 + \tilde{U}_M(\eta) - (-p_{E^1}^1 + \tilde{U}_{E^1}(\eta))\right] \\
&= \int_{p_1^1}^{\infty} q_1(y, p, p_{-1}, A, \eta)dp - \int_{p_M^1}^{\infty} q_M(y, p, p_{-M}, B, \eta)dp
\end{aligned}$$

which verifies the formula in this case

Case ii. In this case, $D^0 = E^1 \neq 1$ and we are left with the 1 good expansion formula derived earlier

$$S^W = \int_{p_M^1}^{\infty} q_M(y, p, p_{-1}, B, \eta)dp$$

However, note that in this case, by revealed preference,

$$\int_{p_1^1}^{\infty} q_1(y, p, p_{-1}, A, \eta)dp = 0$$

which verifies the formula holds in this case.

Case iii. Since $FC^1 \neq M$ and $FC^0 \neq 1$, the consumer's product choice is unchanged with the swap. Therefore, $S^W = 0$. Likewise, by revealed preference,

$$\int_{p_1^1}^{\infty} q_1(y, p, p_{-1}, A, \eta) dp = 0 = \int_{p_M^1}^{\infty} q_M(y, p, p_{-1}, B, \eta) dp$$

Case iv. In this case $FC^1 = E^0$. The addition of good M does not affect the consumer's surplus. Thus, the utility is the same as in the single good removal case proved earlier.

$$S^W = \int_{p_1^1}^{\infty} q_1(y, p, p_{-1}, A, \eta) dp$$

At the same time, by revealed preference,

$$\int_{p_M^1}^{\infty} q_M(y, p, p_{-1}, B, \eta) dp = 0$$

This verifies the formula holds for this case.

Since these four cases are exhaustive, this concludes the proof. □

Proof of Theorem 19. Under Assumption 8, the difference

$$\int_{p_1^1}^{\infty} \fint_{\mathcal{C}_0^{-1}(y,\alpha^0,A)} q_1(y, p_1, p_{-1}, A, \eta) dF dp - \int_{p_M^1}^{\infty} \fint_{\mathcal{C}_0^{-1}(y,\alpha^0,A)} q_M(y, p, p_{-M}, B, \eta) dF dp$$

is well defined. Thus, we see

$$\begin{aligned}
&\int_{p_1^1}^{\infty} \fint_{\mathcal{C}_0^{-1}(y,\alpha^0,A)} q_1(y, p_1, p_{-1}, A, \eta) dF dp - \int_{p_M^1}^{\infty} \fint_{\mathcal{C}_0^{-1}(y,\alpha^0,A)} q_M(y, p, p_{-M}, B, \eta) dF dp \\
&= \fint_{\mathcal{C}_0^{-1}(y,\alpha^0,A)} \int_{p_1^1}^{\infty} q_1(y, p_1, p_{-1}, A, \eta) dp dF - \fint_{\mathcal{C}_0^{-1}(y,\alpha^0,A)} \int_{p_M^1}^{\infty} q_M(y, p, p_{-M}, B, \eta) dp dF \\
&\hspace{30em} \text{(by Tonelli)} \\
&= \fint_{\mathcal{C}_0^{-1}(y,\alpha^0,A)} S^W dF
\end{aligned}$$

which concludes the proof

□

3.D Proofs from Section 3.5

My proofs will make use of the following fact.

Claim 3.

$$\int_{-\infty}^{a} \frac{(x-a)e^{-x}}{(1+e^{-x})^2} dx = -a - \ln(e^{-a}+1)$$

Claim 3 can be verified with any symbolic integrator[11], or with the substitution $u = e^{-x}$, integration by parts and some algebra.

Let $\hat{P}$ denote the researcher's assumed probability distribution. Under $\hat{P}$, consideration sets are independent of preferences and explained by search ease.

3.D.1 Proof of Claim 1

Proof. Putting all this together, we see

$$\begin{aligned}
\hat{\mu}^W(y) &= \int_{\mathcal{C}N^{-1}(y,\{0\})} (\hat{\beta}_{FC^0} - \hat{\beta}_{FC^1} + \Upsilon_{FC^0}(\omega) - \Upsilon_{FC^1}(\omega)) d\hat{P} \\
&= \int \mathbb{1}(FC^1 = 1, FC^0 = 0)(0 - \hat{\beta}_1 + \Upsilon_0(\omega) - \Upsilon_1(\omega)) \times \mathbb{1}(\mathcal{C}N = 0) d\hat{P} \\
&= \hat{P}(\mathcal{C}N = \{0\}) \times \int (\Upsilon_0(\omega) - \Upsilon_1(\omega) - \hat{\beta}_1)\mathbb{1}(\Upsilon_0 < \hat{\beta}_1 + \Upsilon_1) d\hat{P} \\
&\qquad \text{(by independence under } \hat{P}) \\
&= \frac{1}{2} \times \int (W - \hat{\beta}_1)\mathbb{1}(W < \hat{\beta}_1) d\hat{P} \\
&\qquad \text{(Letting } W = \Upsilon_0 - \Upsilon_1,\ W \sim \mathit{Logistic}(0,1) \text{ under } \hat{P}) \\
&= \frac{1}{2} \times \int_{-\infty}^{\hat{\beta}_1} (w - \hat{\beta}_1) \frac{e^{-w}}{(1+e^{-w})^2} dw \qquad \text{(Using Logistic pdf)} \\
&= \frac{1}{2}[-\hat{\beta}_1 - \ln(e^{-\hat{\beta}_1} + 1)] \qquad \text{(using Claim 3)} \\
&\to -\infty \text{ as } \hat{\beta}_1 \to \infty
\end{aligned}$$

□

[11] For example, here https://www.symbolab.com

3.D.2 Proof of Claim 2

Proof.

$$
\begin{aligned}
\mu^W(y) &= \int_{\mathcal{C}N^{-1}(y,0)} (\beta_{FC^0} - \beta_{FC^1} + \Upsilon_{FC^0}(\omega) - \Upsilon_{FC^1}(\omega))dP \\
&= \int_{\mathcal{C}N^{-1}(y,0)} \mathbb{1}(\beta_1 + \Upsilon_1 > \beta_0 + \Upsilon_0)(\beta_0 - \beta_1 + \Upsilon_0(\omega) - \Upsilon_1(\omega))dP \\
&= P(\mathcal{C}N = \{\, 0 \,\}) \int_{-\infty}^{\beta_1} [w - \beta_1] \frac{e^{-w}}{(1+e^{-w})^2} dw \\
&= \frac{1}{2}[-\beta_1 - \ln(e^{-\beta_1} + 1)]
\end{aligned}
$$

which go to negative infinity as β_1 goes to infinity. $\square$

3.E Figures

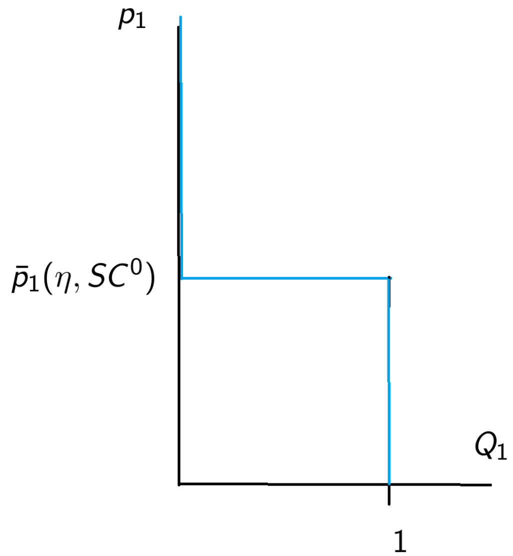


Figure 3.1: Demand for good 1 with constant consideration set $\mathcal{C}_0$. Note that the height of the demand line depends on the consumer's utility for her next best option to good 1 in $\mathcal{C}_0$. Lower utility for her next best option means a higher $\bar{p}_1$.

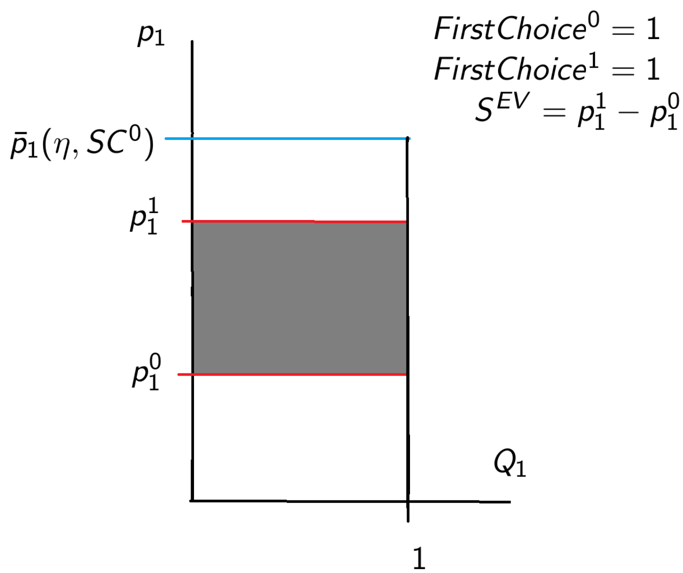


Figure 3.2: This figure illustrates the case of a price change that does not change a consumer's product choice. In order to return the consumer back to her original utility, she must be compensated by exactly the price increase.

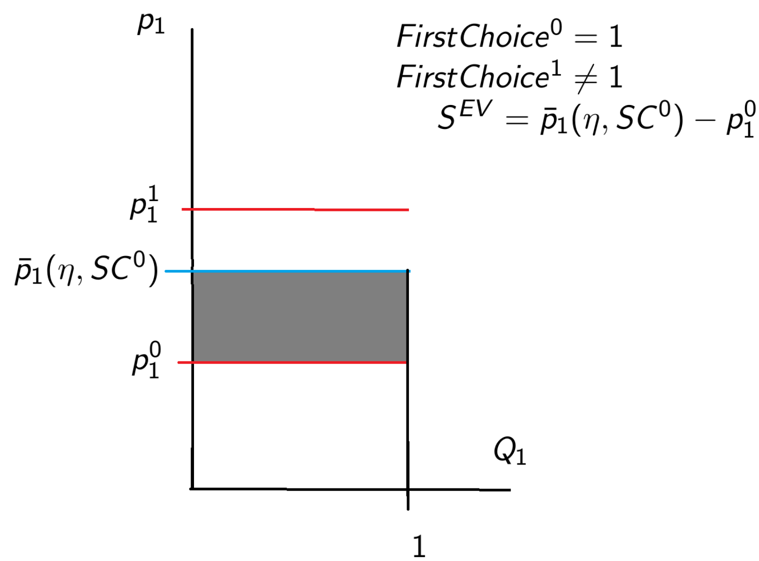


Figure 3.3: This figure illustrates the case of a price increase that changes the product a consumer purchases. The price increase takes surplus away from the consumer. Since the consumer switches away from 1, the harm she experiences must be less than $p_1^1 - p_1 0$. Switching product choices ends the harm she experiences from a price increase.

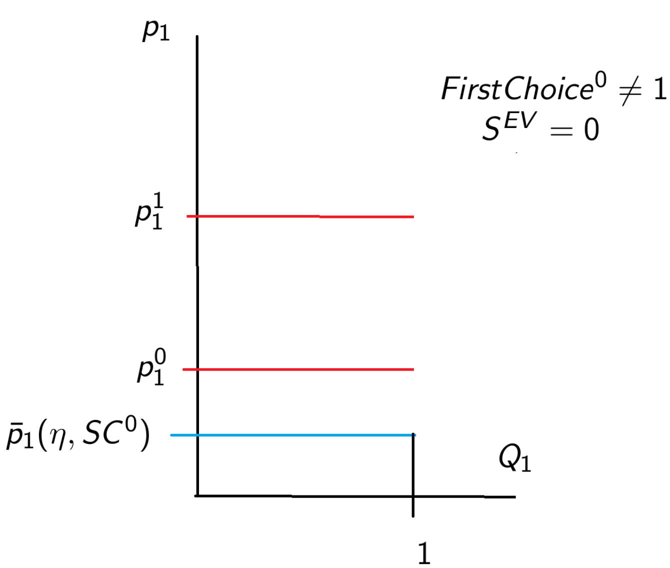


Figure 3.4: Case 3 shows an inconsequential price change. Since the consumer never wanted good 1, her welfare isn't hurt by its price increase

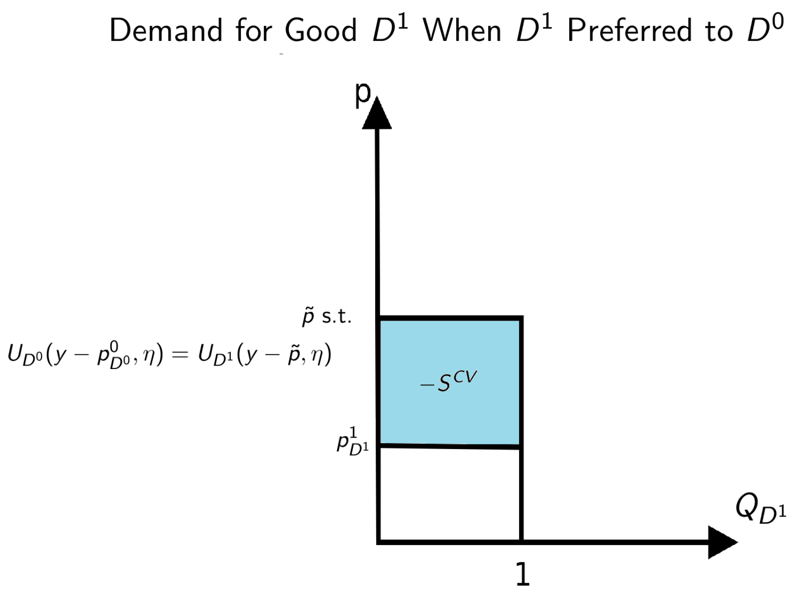


Figure 3.5: This graph shows a consumer who benefited from a market change. Her preferences are linear in money. She prefers her final market outcome over her initial market outcome. Her compensating variation can be recovered from integrating the conditional demand function $Q_{FC^1}(y, p, p^0_{FC^0}, \{FC^0, FC^1\}, \eta)$ from $p^1_{FC^1}$ to infinity. The area between $p^1_{FC^1}$ and the top of the conditional demand function for FC^1 is exactly the negative of $S^{CV} = S^{EV} = S^W$ under Linearity.

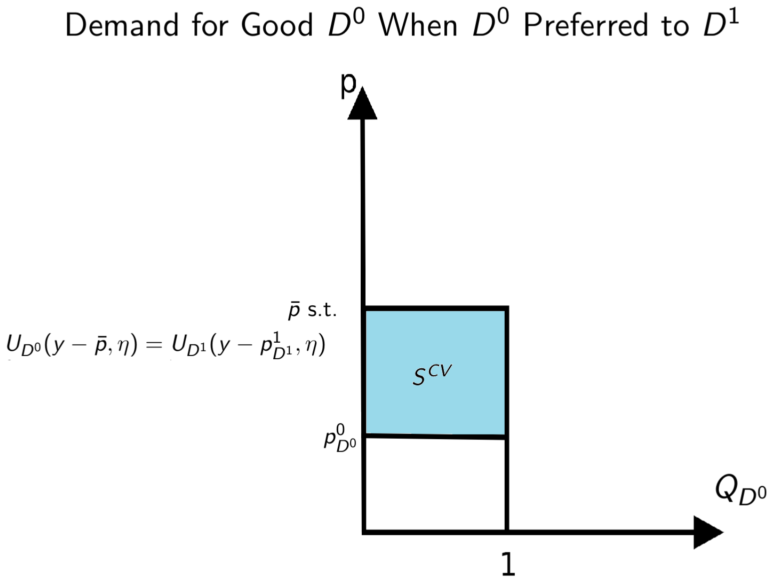


Figure 3.6: This graph shows a consumer who was hurt by a market change. Her preferences are linear in money. She prefers her initial market choice at $p^0_{FC^0}$ over her final market choice at $p^1_{FC^1}$. Her compensating variation can be recovered from integrating the conditional demand function $Q_{FC^0}(y, p, p^f_{FC^1}, \{ FC^0, FC^1 \}, \eta)$ from $p^0_{FC^0}$ to infinity. The area between $p^0_{FC^0}$ and the top of the conditional demand function for FC^0 is exactly $S^{CV} = S^{EV} = S^W$ under Linearity.

CHAPTER 4

Welfare and Price Changes in a Search Environment

In this chapter, I study the consumer welfare effects of exogenous price changes in an online shopping environment. When consideration sets are independent of prices—that is, when Price Independence holds—classical welfare results extend to the online shopping environment without issue: exact welfare consequences of price changes can be measured with integrals of aggregate demand. When Price Independence fails, however, additional information is needed to accurately measure exact welfare changes.

This chapter is most closely related to that of Bhattacharya (2015). Bhattacharya (2015) determines closed-form solutions for the distribution of equivalent variation and compensating variation as a function of aggregate demand in an environment without shopping consumers. He does not consider demand identification itself. Welfare changes arise because of the exogenous price change of a single good. In his environment, good choices are discrete and consumers have knowledge of all products. Bhattacharya (2015) makes no assumptions on the dimension of unobservable preferences and only weak monotonicity and continuity assumptions on utility functions. Utility is not assumed separable in unobservables. Bhattacharya (2015) shows that *exact* formulas for average welfare changes can be determined from aggregate demand alone; there is no need to find compensated (Hicksian) demand. This holds even though utility is not linear in money.

I extend Bhattacharya (2015)'s results to an environment where consumers have idionsyncratic consideration sets. I show that Bhattacharya (2015)'s results continue to hold in this environment, as long as consideration sets are independent of prices. However, when consideration sets depend on prices, the results no longer hold: when consideration sets depend on prices, an increase in price may cause discontinuous shifts in demand that render areas under curves misleading or worse.

The results of this chapter can be understood in the context of Small and Rosen (1981). Small and Rosen (1981) find formulas for consumer welfare changes as a response to both price changes and quality changes. They show the welfare changes with respect to any variable x can be straightforward as long as compensated demand varies continuously with x. When consideration sets are independent of prices, compensated demand varies continuously with prices under standard assumptions on preferences and welfare can still be calculated. However, when consideration sets depend on prices, compensated demand varies discontinuously with prices and classic welfare results need significant adapting.

While this thesis focuses on discrete goods, there is a significant literature studying welfare with continuous choice. Continuous choice allows consumers to choose consumption quantities from the nonnegative real line. Most recently, Jerry A. Hausman and W. K. Newey (2016) researched welfare identifiability in markets without search under preference and data assumptions very similar to those of Bhattacharya (2015). However, Jerry A. Hausman and W. K. Newey (2016) show that average equivalent variation cannot be identified with continuous choice in this general preference setting. Instead, they offer welfare bounds through the solution of a differential equation.

Work in discrete and continuous choice welfare analysis goes back farther than Bhattacharya (2015) and Jerry A. Hausman and W. K. Newey (2016). Other results in discrete choice welfare analysis include Domencich and D. McFadden (1975), Small and Rosen (1981), Herriges and Kling (1999), Dagsvik and Karlström (2005) and Berry and Haile (2014). Other recent results in welfare analysis with continuous choice include Hoderlein and Vanhems (2011), Blundell, Horowitz, and Perry (2012), Lewbel (2013). These results all rely on either stricter preference assumptions, stricter assumptions on unobservables or approximations to continuous choice models. None of these results limit consumer knowledge to consideration sets.

It is important to note that I follow Bhattacharya (2015) and Jerry A. Hausman and W. K. Newey (2016) in point identifying welfare changes as a function of (conditional) demand. This can be contrast with the (full) identification of welfare changes starting from observational price and quantity data. For markets with either discrete or continuous choice and no search, there is an established literature on identifying and estimating demand from observational data. See for example, R. Matzkin (1993), R. L. Matzkin (1993), R. W. Blundell and Powell (2004), and Chernozhukov, Fernandez-Val, and W. Newey (2018) in discrete choice and R. W. Blundell and Powell (2004), R. Matzkin (2015), Richard Blundell, Kristensen, and R. Matzkin (2014), Richard Blundell, Kristensen, and R. Matzkin (2013) in continuous choice. However, conditional nonparametrtic demand identification and estimation in search markets is a relatively new research agenda. To the best of my knowledge, the only paper on conditional demand identification or estimation from observational data without a game-theory based model of consumer search is Amano, Rhodes, and Seiler (2017); Amano, Rhodes, and Seiler (2017) assumes utility given consideration sets is linear with additively separable, Type I Extreme Value unobservables. Since my research interests are first in consumer welfare, my analysis has started assuming ideal demand data. Nonparametrically identifying conditional demand from micro search data is still an open problem.

In Section 4.2, I derive welfare formulas under Price Independence. In Section 4.3, I show how these formulas fail when Price Independence fails. In Section 4.4, I examine tests for Price Independence. In Section 4.5, I estimate welfare changes from price increases using click-stream data from a major online travel agency. In Section 4.6, I conclude this chapter.

4.1 Notation

In this section, I develop the notation I need for the rest of the chapter. I start by looking at consumer preferences in Section 4.1.1. I develop notation and assumptions for consideration sets in Section 4.1.2. I define and develop notation for welfare measures in Section 4.1.3.

4.1.1 Preferences

My setup for goods and preferences follows the multinomial choice framework with nonseparable utility laid out in Bhattacharya (2015). There is a *set of goods* $\mathcal{J} = \{0, 1, \ldots, J\}$.

Goods have *temporally invariant attributes* $(0, X_1, \ldots, X_J)$. Good attributes may vary with the platform; different websites will have different ratings. Thus, each product is platform specific. Since good attributes are unimportant for the results in this paper, I will suppress their notation. All results should be interpreted as conditional on good attributes. All prices may vary with time. *Prices at time step t* are $p^t_{\mathcal{J}} = (0, p^t_1, \cdots, p^t_J)$ where $t \in \{0, 1\}$. The price of the outside good is fixed at 0 for both periods. Goods, prices and attributes are all observable to the researcher at all time periods.

Consumers have *observable income* y and *observable attributes* Ψ that affect their utility. Both y and Ψ are fixed for all individuals over time. In examples, I will treat gender as a component of Ψ. All identification results should be interpreted as conditional on y and Ψ. I will suppress the notation for Ψ from utility for readability.

Consumers have *unobservable preferences* η. I follow Bhattacharya (2015) and do not restrict the dimension of these unobservable preferences.[1] Instead, I assume that η is temporally invariant for each consumer. In addition, I restrict utility using the following assumptions.

Assumption 9.A (Monotonicity)**.** *Utility for good j, $u_j(y - p_j, \eta; X_j, \Psi) \in \mathbb{R}$, is strictly increasing and continuous in its first argument for all goods $j \in \mathcal{J}$.*

Assumption 9.B (Linearity)**.** *Utility for good j is determined by*

$$u_j(y - p_j, \eta; X_j, \Psi) = y - p_j + \tilde{U}_j(\eta; X_j, \Psi)$$

where $\tilde{U}_j(\eta) \in \mathbb{R}$ for all goods $j \in \mathcal{J}$.

Suppressing Ψ and X_j from utility notation leaves us with abbreviated utility forms $u_j(\tilde{y} - p_j, \eta)$ and $y - p_j + \tilde{U}_j(\eta)$. Linearity is sufficient for Monotonicity. I maintain Monotonicity for the rest of the paper. Linearity will only be used for certain results.

I define *inverse utility for good j*, denoted $u_j^{-1}(\bar{u}, \eta)$, as the solution in y to

$$u_j(y, \eta) = \bar{u} \tag{4.1}$$

[1] See Bhattacharya (2015) for a discussion on the importance of leaving the dimension of heterogeneity unrestricted in discrete choice preferences

By Monotonicity, $u_j^{-1}(\cdot, \eta)$ is a well-defined, continuous and strictly increasing function.

4.1.2 Consideration Sets

Consumers have imperfect knowledge of the products in $\mathcal{J}$. The shopping process is how consumers acquire product information. I will denote a *consumer's consideration function* by $\mathcal{C}$. Consumers have perfect knowledge of the prices, attributes and utilities of all the goods in their consideration sets. Consumers are uncertain of the prices, attributes, and hence utilities of all goods not in their consideration sets. Further, consumers cannot buy goods that are not in their consideration sets.

Consider a consumer searching for sunglasses online. She may have an idea about the characteristics and prices of the sunglasses she wants. However, in order to buy a pair, she will first have to find a place that will sell them to her. As she navigates to websites of sunglass sellers and discovers sunglasses she can buy, her uncertainty about the products she discovers is resolved. The sunglasses she discovers become part of her consideration set.

The sunglasses in her consideration set will depend on her observable characteristics Ψ. For example, female shoppers will likely prefer sunglasses designed for women, or at least unisex sunglasses. Moreover, the sunglasses a consumer shops also depends on unobservable preferences η. A consumer who prefers polarized sunglasses will likely have more polarized sunglasses in her consideration set, having included "polarized" in her keyword search.

An individual's consideration set is also determined by her *unobservable, non-preference characteristics* ζ. For example, ζ captures a consumer's preference for the act of shopping itself or a consumer's familiarity with internet shopping tools. If a consumer enjoys shopping, she will likely have a larger consideration set. If a consumer has limited knowledge of browser plugins,[2] comparison shopping engines,[3] or platform-specific tools of refining search results,[4] it will affect her consideration set. Together, ζ and η will capture a consumer's *price and product characteristic beliefs.* I assume ζ is temporally invariant. I denote ζ and η's *joint distribution* by F. I do not restrict the dependence structure between ζ and η.

Definition 4 (Consideration Sets). *A consumer with attributes Ψ, ζ, and η is able to purchase and has full information on goods from $0 \subseteq \mathcal{C}(\eta, \zeta; \Psi) \subseteq \mathcal{J}$.*

When it is clear from context, I will use $\mathcal{C}_t$ to denote the variable for a consumer's consideration at time t. A consumer with consideration set $\mathcal{C}_t$ purchases product $m \in \mathcal{C}_t$ if

$$u_m(y - p_m, \eta) > u_j(y - p_j, \eta) \text{ for all } j \in \mathcal{C}_t \setminus \{ m \}$$

Empirical papers typically include detailed models of the search process. For example, Koulayev (2014) and Honka (2014) model consumer beliefs in the search process. The results of this paper do not exclude such detailed modeling, but rather abstract around them. Since welfare changes are still identified without modeling the search process explicitly, it is without loss of generality to leave it out. Not modeling search directly also lets this paper's model nest many search frameworks. For a more detailed exploration of how a consumer's consideration set could generally be formed, consider the discussion in this paper's supplementary appendix or see Morgan and Manning (1985). For references on papers that model consideration set formation, see this paper's introduction.

I will need an assumption on consideration sets.

Assumption 10 (Price Independence). *Consideration sets $\mathcal{C}_t$ are independent of prices and income.*

[2]Browser plugins are tools that can be added to your browser. There are several browser plugins designed specifically for online shopping. For example, Honey, at https://www.joinhoney.com/ searches the internet for coupon codes to apply to your order. When shopping on Amazon.com, it will also search across sellers within Amazon to find the lowest priced seller of whatever you are browsing.

[3]Comparison shopping engines are websites that allow cross-platform comparisons of goods. They are a special case of search aggregators. Examples of comparison shopping engines include Google Shopping, Nextag and PriceGrabber. Sophisticated comparison shopping engines have market research tools that track prices over times and will offer price predictions.

[4]For example, most platforms will allow you to refine search results by average consumer rating, price, brand and more.

The difference in ability to recover consumer welfare from a simple area under a demand curve hinges critically on Price Independence. Note that Price Independence does not preclude a consumer from shopping according to her *beliefs* about prices. It only requires that the prices she observes do not cause her to change her shopping behavior. This is true in the case of consumers performing simultaneous search, a type of search that has been determined more likely than sequential search in studies such as Honka and Chintagunta (2016) and Honka, Hortaçsu, and Vitorino (2017). Under Price Independence, a price change would leave $\mathcal{C}_0 = \mathcal{C}_1$ for all individuals but would still have welfare consequences.

In principle, requiring consideration sets to be independent of income is not completely necessary. It would be fine to allow consumers' consideration sets to depend on their own perceived wealth level, as a component of Ψ, as long as a good's purchase doesn't change consumers' own perceived wealth level. This assumption is made mostly for convenience.

Individual demand is defined by

$$q_j(y, p_j, p_{-j}, \eta, \zeta) := \begin{cases} 1 \text{ if } j = \arg\max_{\ell \in \mathcal{C}(\eta,\zeta)} u_\ell(y - p_\ell, \eta) \\ 0 \text{ otherwise} \end{cases} \tag{4.2}$$

At time t, I will simplify notation for *individual demand for product j at time t* to

$$q_j^t(y, p_j, \eta, \zeta) := q_j(y, p_j, p_{-j}^t, \eta, \zeta) \tag{4.3}$$

This simplified form has all good prices, except for good j, fixed at their market prices at time t.

Average demand for good j is

$$Q_j(y, p_j, p_{-j}) = \int q_j(y, p_j, p_{-j}, \zeta, \eta) dF \tag{4.4}$$

4.1.3 Welfare Measures

The classic welfare measures *compensating variation*, S^{CV}, and *equivalent variation*, S^{EV}, are adapted to my search environment as follows. For an individual with heterogeneity vector

(η, ζ) and income y, S^{EV} is the solution in S to

$$\max_{j \in \mathcal{C}_0} u_j(y - S - p_j^0, \eta) = \max_{j \in \mathcal{C}_1} u_j(y - p_j^1, \eta) \tag{4.5}$$

while S^{CV} is the solution in S to

$$\max_{j \in \mathcal{C}_0} u_j(y - p_j^0, \eta) = \max_{j \in \mathcal{C}_1} u_j(y + S - p_j^1, \eta) \tag{4.6}$$

Equivalent variation is the income loss at time 0 that would harm a consumer as much as the damage done by price increase. Compensating variation is the increase in income that would return a consumer to her original utility level after the price and platform behavior change. If S^{EV} or S^{CV} is positive, then the consumer's utility increased over the change.

Since $\max_{j \in \mathcal{C}_1} u_j(y - p_j^1, \eta) = u_{FC^1}(y - p_{FC^1}^1, \eta)$, we see that S^{EV} is an indirect function of the entire initial consideration set $\mathcal{C}_0$ but only FC^1 from $\mathcal{C}_1$. As we take income S^{EV} from consumers, they are allowed to switch goods from FC^0 to any other good in their initial consideration set. To capture this indirect relationship, I will write the *indirect equivalent variation function* as $S^{EV}(y, \eta, \mathcal{C}_0, FC^1, p_{\mathcal{C}_0}^0, p_{FC^1}^1)$. Similarly, I will write the indirect compensating variation as $S^{CV}(y, \eta, FC^0, \mathcal{C}_1, p_{FC^0}^0, p_{\mathcal{C}_1}^1)$ for a consumer's *indirect compensating variation funcation*. With a slight abuse of notation, I will refer to their *direct functional forms* as $S^{EV}(y, \eta, \zeta, p_{\mathcal{J}}^0, p_{\mathcal{J}}^1)$ and $S^{CV}(y, \eta, \zeta, p_{\mathcal{J}}^0, p_{\mathcal{J}}^1)$. When it's clear from context, I will suppress prices from function references. *Average compensating variation and equivalent variation* over all consumers is denoted by μ^{CV} and μ^{EV}, respectively. These, collectively, are my primary quantities of interest.

All averages are functions of income, prices and platform behavior. Thus, for $K \in \{CV, EV\}$, I will write the total average function as $\mu^K(y, p_{\mathcal{J}}^0, p_{\mathcal{J}}^1)$. When it's clear from context, I will suppress the arguments for platform behavior and prices.

When utility is linear in money, $S^{CV} = S^{EV}$ for each individual. I use S^W to represent both S^{CV} and S^{EV} in this case. S^W is simply the difference between final and initial utility in this case.

4.2 Welfare Measures Under Price Independence

In this section, I derive consumer welfare formulas for single price increases and decreases. Under Price Independence, many of the classic welfare results hold. First, I look at the welfare consequences of a price increase.

Theorem 21. *If the price of good 1 increases from p_1^0 to p_1^1, under Price Independence the average equivalent variation is*

$$\mu^{EV} = \int_{p_1^0}^{p_1^1} Q_1(y, p, p_{-1}^0)dp \tag{4.7}$$

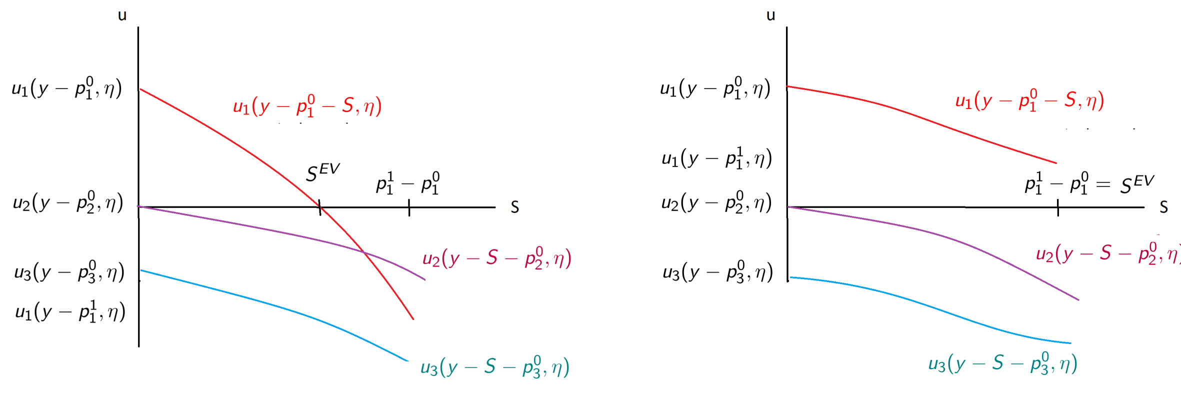
and the average compensating variation is

$$\mu^{CV} = \int_{p_1^0}^{p_1^1} Q_1^0(y + p - p_1^0, p, p_{-1}^0)dp \tag{4.8}$$

The exact equivalent variation of a price increase is simply the area under the (regular) demand from good 1's initial price to it's final price. Compensated demand is not necessary for this calculation; regular demand is enough. Similarly, exact compensating variation can be found by looking at an area under the demand curve for good 1. However, for compensating variation, the price increase of good 1 follows a specific parametric path. The line simultaneously lowers the price of all goods except and measures the demand good 1 loses as consumers switch to their alternative goods.

These formulas do not require that the researcher observed individual consideration sets. As long as consideration sets are stable over price changes, i.e. Price Independence holds, then exact welfare can be found as an integral under uncompensated, aggregate demand. Also, these results agree exactly with the formulas in Bhattacharya (2015). That is, the fact that consumers' knowledge is restricted to consideration sets does not change the formulas for equivalent variation or compensating variation in response to a single price change. Of course, everything else the same, the aggregate demand functions themselves will be different across the two environments since consumers are likely to have different product choices when their product knowledge is limited.

The asymmetry in the formulas can be understood through fig. 4.1 and fig. 4.2. Suppose at time 0 prices that a consumer prefer good 1 to good 2 and good 2 to good 3. For equivalent



(a) $S^{EV} < p_1^1 - p_1^0$ (b) $S^{EV} = p_1^1 - p_1^0$

Figure 4.1: This figure considers a consumer's equivalent variation S^{EV} due to a price increase of good 1 when the consumer's initial choice was good 1. Here, either $S^{EV} = p_1^1 - p_1^0$ and the consumer does not switch goods or, $0 \leq S^{EV} < p_1^1 - p_1^0$ and the consumer switches to her second most preferred good. The good this consumer chooses at time 1 must have the same utility as her compensated utility for good 1 at time 0.

variation, the price increases of good 1 means her period 1 utility will be at least as good as her utility for good 2 at p_2^0 $(= p_2^1)$. Thus, the price increase will need to compensate her at most up to her utility for good 2 at time 2. But taking away more income from good 2 and good 3 will leave her with lower utility than even that. Thus, the equivalent variation in this case will always be the amount of price increase to good 1 that makes her indifferent to good 2 or exactly equals the full price increase of good 1. These two cases are shown in fig. 4.1a and fig. 4.1b.

In contrast, for a price increase, simultaneously lowering the price of all goods at time 1 does not guarantee the consumer will be happiest with good 1 or even good 2. This is illustrated in fig. 4.2. In this picture, the consumer prefers good 1 at time 0 and good 2 at time 1. However, lowering the price of all goods incrementally by ΔS at $time 1$, we find that the consumer is fully compensated buying good 3 with the smallest compensation.

For welfare changes in response to a price decrease, the results are reversed. Now, compensating variation is a simple integral over a single goods price change while exact equiva-

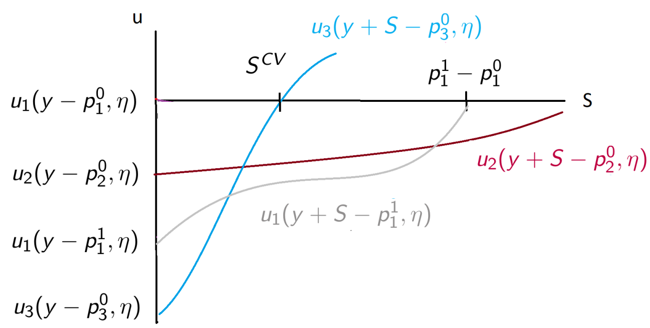


Figure 4.2: This figure considers a consumer's compensating variation S^{CV} due to a price increase of good 1 when the consumer's initial choice was good 1. Here, either $0 \leq S^{CV} \leq p_1^1 - p_1^0$. However, under Monotonicity, we cannot be certain that the good the consumer purchases after the price increases at time $t = 1$ is the same good that maximizes her compensated utility.

lent variation requires an integral along a line where the prices of all goods but good 1 are decreased simultaneously

Theorem 22. *Under Price Independence, the average equivalent variation in response to a price decrease of good* 1 *is*

$$\mu^{EV}(y, p^0_{\mathcal{J}}, p^1_{\mathcal{J}}) = -\int_{p_1^1}^{p_1^0} Q_1(y + p - p_1^0, p, p^0_{-1})dp \tag{4.9}$$

and the average compensating variation is

$$\mu^{CV}(y, p^0_{\mathcal{J}}, p^1_{\mathcal{J}}) = -\int_{p_1^1}^{p_1^0} Q_1(y, p, p^0_{-1})dp \tag{4.10}$$

4.3 Measuring Welfare Without Price Independence

When Price Independence fails, it is not possible to measure the welfare changes from a goods price change using that goods aggregate demand alone. This is because the effects of a price increase on consideration sets may exacerbate the price increase's negative welfare effects. For example, if a product becomes much harder to find after a small price increase, the effective welfare loss would be closest to the product's price increasing to infinity. On the other hand, if the platform replaces the products listing position with an alternative the consumer likes even better, her welfare will go up after the price increase. Similarly, a price increase may inspire a consumer to search more. The find goods she prefers even to the her original choice at its original price. Thus, a price increase may actually improve a consumer's welfare when Price Independence fails. To illustrate these two caess formally, I will develop two different examples.

Example. A consumer always shops exactly the first good a search platform returns to her. The platform's results are sensitive to the prices of good 1. If the price of good 1 is below $p_1^\star$, the platform returns an ordering that favors good 1: good 1 occupies the first spot, followed by good 2. However, if good 1's price is above $p^\star$, good 1 is placed at the end of the list: in this case, the list reads good 2 then good 1.

Suppose the consumer's utility is as follows:

$$
\begin{aligned}
u_0(y,\eta) &= y \\
u_1(y-p_1^t,\eta) &= y - p_1^t + \beta_1 \\
u_2(y-p_2^t,\eta) &= y - p_2^t + \beta_2
\end{aligned}
$$

Putting this together, the consumer's demand for good 1 is

$$
q_1(y,p_1) = \begin{cases} 1 & \text{if } \beta_1 > p_1 \text{ and } p_1 < p_1^\star \\ 0 & \text{otherwise} \end{cases}
$$

Suppose $p_1^0 < p_1^\star < p_1^1$ so that the consumer's initial consideration set is $\{0,1\}$ and her final consideration set is $\{0,2\}$. Further, suppose $\beta_2 > p_2^1$ and $\beta_1 > p_1^0$, so the consumer always purchases good 1 and then good 2.

The area under her demand curve for good 1 between initial and final prices is $p_1^\star - p_1^0$.

However, it's easy to see that her equivalent variation is actually

$$\beta_2 - \beta_1 + p_1^0 - p_2^1$$

Moreover, we can easily choose the prices and parameters β_2 and β_1 so that her actual equivalent variation is very different from $p_1^\star - p_1^0$. In particular, she may prefer good 2 at price p_2^1 over good 1 at price p_1^1, so that her equivalent variation is negative and the price increase benefits her. It may also be that she strongly prefers good 1 at price p_1^1 to good 2 at price p_2^1, so that

$$\beta_2 - \beta_1 + p_1^0 - p_2^1 > p_1^1 - p_1^0 > p_1^\star - p_1^0$$

In words, the area under her demand curve may underestimate the welfare damage of the price increase.

4.4 Testing for Price Independence

The previous two sections highlight how critical Price Independence is in correctly concluding the welfare consequences of a price change. In this section, I provide a heuristic to measure the dependence between consideration sets and prices. I demonstrate the efficacy of this heuristic using simulated data.

I run two simple simulations to demonstrate the efficacy of my heuristic. Suppose $J = 5$; consumers choose among 5 goods and an outside good in a market. Suppose that preferences are determined for individual i by

$$u_{ijm} = \begin{cases} \beta_j - p_{jm} + \eta_{ijm} \text{ if } j \neq 0 \\ \eta_{i0m} \text{ if } j = 0 \end{cases}$$

where $\beta_j = 4$ for $j \neq 0$. For simplicity, assume each consumer is in her own market and that $p_{jm} \sim^{iid} Unif(0, 5)$ while $\eta_{ijm} \sim^{iid} N(0, 1)$. Finally consider the following two shopping methods: (1) consumers shop the outside good and two random products from $\{1, 2, 3, 4, 5\}$ and (2) consumers shop the outside good and the two cheapest products from $\{1, 2, 3, 4, 5\}$.

In the first case, a good heuristic will show no dependence between consideration sets. In the second case, a good heuristic should show a strong dependence.

I simulate draws from 10,000 customers according to above details. I then run the heuristicThe results are reported in table 4.1 and table 4.2

	Dependent variable:
	Good 1 Included in Consideration Set
Price of Good 1	0.017 (0.044)
Price of Good 2	0.024 (0.044)
Price of Good 3	−0.003 (0.044)
Price of Good 4	−0.008 (0.044)
Price of Good 5	0.048 (0.044)
Constant	-0.324^{***} (0.050)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 4.1: Results for regression of product one's inclusion in consideration sets on prices. In this regression, shopping behavior is simulated as follows: consumers shop a random sub-collection of two products, in addition to the outside good, from the available five. Shopping is independent of prices. The insignificance of the coefficients of price in this regression correctly relay the information that consideration sets do not depend on prices

4.5 Data Example: Online Travel Agents

In this section, I estimate welfare changes from price changes for a data set that details the click and purchase behavior of a collection of consumers booking hotels using an online travel agent (OTA). The data is from the 2013 data challenge for the IEEE's[5] International

[5]IEEE is the Institute of Electrical and Electronics Engineers.

	Dependent variable:
	Good 1 Included in Consideration Set
Price of Good 1	-9.804^{***} (0.216)
Price of Good 2	3.153^{***} (0.106)
Price of Good 3	2.993^{***} (0.105)
Price of Good 4	3.031^{***} (0.105)
Price of Good 5	3.088^{***} (0.104)
Constant	-2.243^{***} (0.099)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 4.2: Results for regression of product one's inclusion in consideration sets on prices. In this regression, shopping behavior is simulated as follows: consumers shop the two cheapest goods in addition to the outside good. The regression results correctly capture good 1's inclusion on all good prices

Conference on Data Mining (ICDM).[6] The competition was open to the public[7] through the online data science community Kaggle. Data for the contest was provided by Expedia, a large OTA.

The data is centered around a collection of *search impressions* that OTA users interacted with, primarily at *Expedia.com*. To understand the term search impression, first consider a consumer searching on *Expedia.com* for vacation accommodation in 2013. This consumer would initially face a page as pictured in fig. 2.6. Here, the OTA user would enter her vacation destination, the days she planned to spend in her vacation destination, the number of rooms she would like to book and the number of adults and children that she will be traveling with. All this information, pictured in the blue boxes in fig. 2.6, is collected by Expedia and used to produce a sequence of listings of available hotel rooms. The user is promptly directed to this listing sequence upon entering her information.

An example of a single hotel listing is given in fig. 2.7. Again, the blue boxes are all information that Expedia collects and are included in the data set. Each search is likely to produce several listings. The number of individual listings will vary depending upon the destination city and the availability of hotel rooms at the given date. In the data, the number of listings on the first page of results vary between 1 and 34. A *search impression* is then defined to be the first page of search listings for a given user query. In addition to the information in blue boxes, Expedia.com also provides information on the listings clicked and booked for each search impression, as well as a vector of characteristics and past behavior for each consumer. A detailed description of all covariates provided in the data can be found in the appendix.

When an OTA user clicks on a listing, a new page opens with more hotel details. In particular, information such as the size of available hotel beds, parking fees, pictures of room interiors, availability of free breakfast, room amenities and any hidden fees becomes available to a user who clicks on a hotel listing. Booking cannot be done without clicking on

[6]This is considered the world's premier research conference in data mining. Data challenges are typically held annually

[7]Competition rules only barred employees of online travel agencies from competing.

the listing first.

There are a few features of this data set that make it particularly amenable to analyzing the relationship between consideration sets and prices. First, the data tells us information on the entire first page of search results each consumer faces. It also tells us the listing each consumer clicked as well as the listing the consumer eventually booked (if any). Thus, if we define the products that enter a consideration set as the products a consumer clicks, the data set tells us exactly the prices of all products the consumer could add to her consideration set and the products the consumer does add to her consideration set. Moreover, Expedia even provides us with data from one of their experiments: the data set includes search impressions where the hotel listing order was determined by Expedia's proprietary ranking algorithm as well as search impressions that had listings ordered randomly. This provides us with a few of how consideration set formation may change as platform behavior changes.

While the data set provides an excellent opportunity to study the relationship between prices and consideration sets, a few important caveats should be pointed out. First, search impressions only list the first page of results for each user query. Thus, a consumer who searches beyond the first page of listing results (should there be additional listings) will not have her search behavior correctly tied to her behavior on the first page of results. Her behavior on the second page of results would be treated as a separate (unassociated) search impression, if included at all. The same would be true for a consumer who went back to the first page of results and changed her search query. Thus, to the extent that consumers viewed multiple search result pages or considered alternative booking dates, the results of this study will underestimate the size of individual OTA user consideration sets. Ursu (2017) provides some evidence from a companion data set that more than 40% of Expedia users only look at the first page of results.

For competition reasons, Expedia would not verify how representitive the sample was. However Ursu (2017)used a companion dataset from the Wharton Customer Analytics Initiative on consumer searches for hotels on a popular OTA in her study of this same Expedia data set. This companion study verified that the Expedia data set was representative of the largest shopping groups on Expedia.

There are a few caveats to the ranking systems used. Namely, Expedia allowed some advertisements on both the randomly ranked listings and Expedia's own ranked listings. Therefore, Expedia's own rankings are not exactly those of a Learning To Rank algorithm and the randomly ranked listings are not fully random. Moreover, the data set does not provide an obvious indicator of which listings are random and which are not.[8] However, the vast majority of the listings are not advertisements and there is significant variation in between Expedia's proprietary ranking listings and the random listings. Therefore, there is still much that can be learned from comparing the characteristics of the two different groups.

Finally, the last caveat that should be mentioned is that the data set is missing income data. If the location data of shoppers wasn't anonymized, we could compare locations with census data on average incomes per area. This would allow us add an income estimate to each consumer's product choice. Assuming Linearity can also be used, as income differences out of the relative utility relationship and is therefore not needed for estimating demand under Linearity.

4.5.1 Demand Estimation

I estimate a model of discrete choice where individual i's utility for good j at time t is

$$u_{ijt} = \begin{cases} P(y - p_j^t) + \beta' X_j + \eta_{ijt} \text{ for } j \neq 0 \\ P(y) + \eta_{i0t} \text{ for } j = 0 \end{cases}$$

Here, η_{ijt} are standard Type I Extreme Value distributed random variables and, given consideration sets, are independent over i, j and t. P is a polynomial function, chosen to give a flexible fit to the utility for money. X_j is a vector of characteristics about property j. Based on previous studies of this data set (Liu et al. 2013), I include property star rating, property branding information, the property location score and an indicator variable for promotions in X_j as strong predictors of product choice.

To compare equivalent variation and compensating variation under Monotonicity, I fit

[8] The data providers did suggest we might be able to infer which listings are random by looking for anomalies in search lists.

polynomials of degree one and two to the money term of utility. Third degree polynomials were considered, however price coefficients of the third degree term were statistically insignificant. Since I do not observe income[9], I assume interaction terms between income and price have coefficient zero. Since income is constant, all of its polynomial terms differ out of the choice equation.

I run the regressions in R (R Core Team 2017) using the `mlogit` package (Croissant 2019). Results are displayed in table 4.3 and table 4.4. All of the included variables are highly significant in both regression. As expected for demand, the coefficient on price is negative in both regressions.

4.5.2 Results

I look at the welfare consequences of exogenously increasing the price of the the market's top property. I am defining the top property as the property with the largest market share in the sample data. This market share is 9.2%, only a few percentage points away from the next four products: 7.78%, 6.95%, 6.58% and 6.46%.[10]. I consider a price increases of 25% and 50% the top product's average market price. For reference, the average price of the top product is \$117.76 whereas the average price over all products is \$129.29.

I use the given demand estimates and Theorem 21 to estimate changes in consumer welfare. There are three cases. When utility is linear in money (that is, when the polynomial term on money in utility has order one), compensating variation and equivalent variation coincide. I call this the *linear-in-money* case. When higher order money terms are considered, compensating variation and equivalent variation disagree. The values listed for compensating variation and equivalent variation refer to this higher order polynomial case.

For ease of understanding, I plotted the integrands for all 3 cases in fig. 4.3. The inte-

[9]Expedia provided integers to denote the region the shoppers were shopping from, but did not provide a code to link the regions with named place. If geographic information were decoded, geographic income estimates could be used for shoppers. The link between income and region could be made with census data.

[10]These market shares are without considering the outside good. Since 38.6% of search impressions do not result in a purchase, only 5.6% of customers make a reservation with the top product

Table 4.3

	Dependent variable:
	Hotel Booked
property star rating	0.513***
	(0.048)
property brand boolean	0.418***
	(0.053)
property location score	−0.922***
	(0.043)
price in usd	−0.010***
	(0.001)
promotion flag	0.266***
	(0.047)
Observations	4,694
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 4.4

	Dependent variable:
	Property booked
property star rating	0.581***
	(0.050)
property brand boolean	0.417***
	(0.053)
property location score 1	−0.917***
	(0.043)
price in usd	−0.015***
	(0.001)
promotion flag	0.266***
	(0.047)
square of price in usd	0.00001***
	(0.00000)
Observations	4,694
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 4.5: This table shows the average welfare changes when the top products price increases by \$29.44 or \$58.88.

Price Increase	μ^{CV}	μ^{EV}	μ^{W} (Linear In Money)
\$29.44[12]	\$0.52	\$0.55	\$0.54
\$58.88[13]	\$0.85	\$1.02	\$0.95

grand curves for the linear-in-money case and equivalent variation case show the relationship between average demand for the top property vs a price increase of its own price. The integrand curve for compensating variation reflects the average demand for the top property while decreasing the price of all goods except the top property's price.

The welfare results are summarized in table 4.3 and table 4.6. Table 4.3 shows the welfare consequences averaged over all users. Table 4.6 rescales the averages in table 4.5 by the number of search impressions per estimated buyer of the top product at market prices.[11] All the welfare change measures are relatively close for a price increase of 25% the average price. The results start to diverge more significantly under the larger 50% price increase. Pictures of the integrals are depicted in fig. 4.4, fig. 4.5 and fig. 4.6.

Figure 4.3 shows clearly how the three welfare measures diverge for larger prices. We see that the integrand for the equivalent variation case is significantly steeper than the integrand for the linear in money case. In turn, the integrand for the linear in money case is steeper than the integrand for the compensating variation case. Thus, it appears the linear in money case acts as a good balance between the two different measures for this data set.

4.6 Conclusion

In a search environment, demand depends on prices and consumers' consideration sets. When price changes do not affect consumers' consideration sets, classic welfare interpretations of

[11]The number of estimated purchases of the top product in the linear in money case is one person higher than the number of estimated purchases of the top product in the CV and EV cases. This is because of squared price term in the demand prediction for the CV and EV cases.

Table 4.6: This table shows the average welfare change rescaled by the number of search impressions N per estimated buyer of the top product $\hat{N}_1$.

Price Increase	$\mu^{CV} \times \frac{N}{\hat{N}_1}$	$\mu^{EV} \times \frac{N}{\hat{N}_1}$	$\mu^{W} \times \frac{N}{\hat{N}_1}$ (Linear In Money)
\$29.44	\$24.68	\$26.29	\$25.48
\$58.88	\$40.79	\$ 48.75	\$44.40

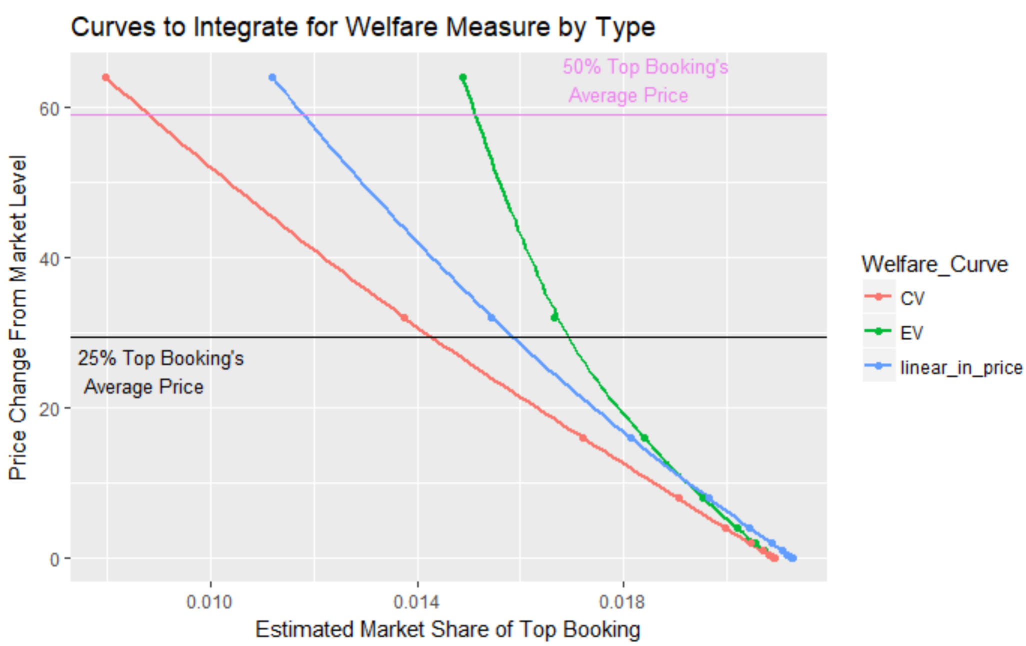


Figure 4.3

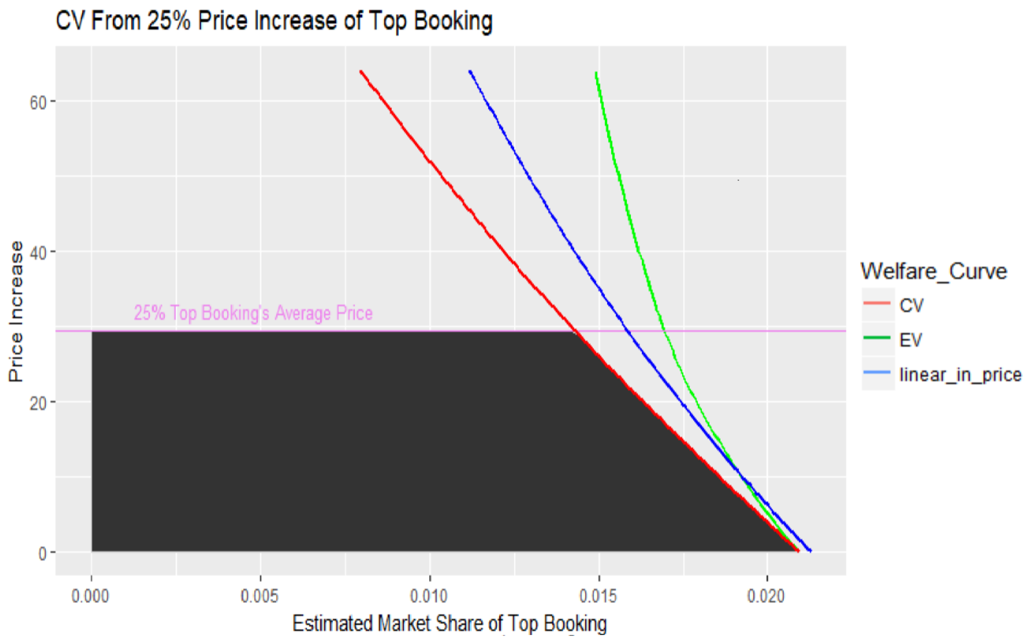


Figure 4.4

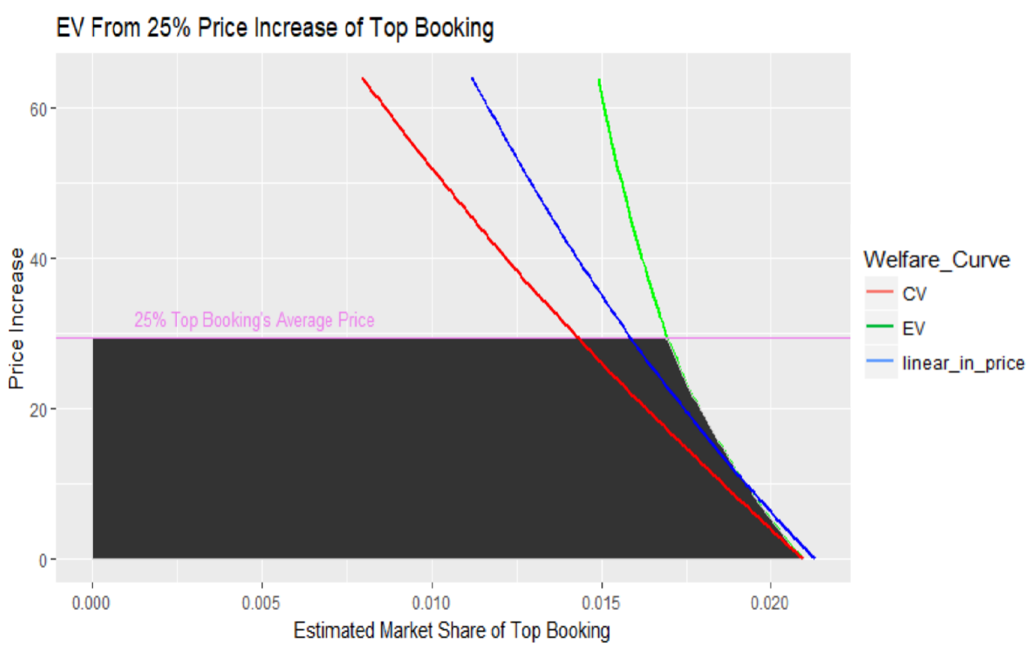


Figure 4.5

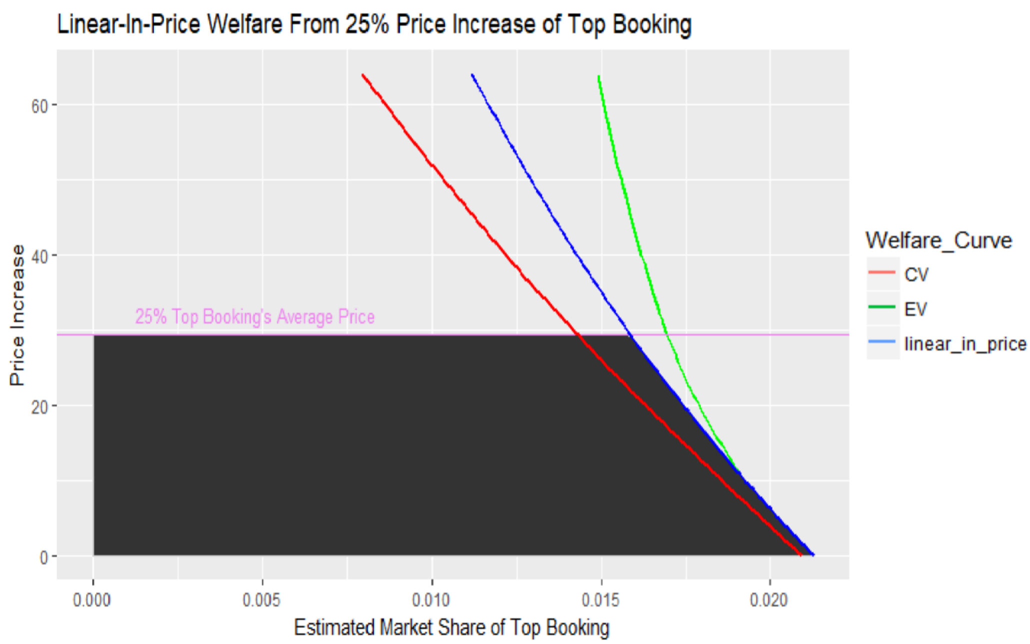


Figure 4.6

areas under demand curves still stand. In this case, the exact consumer welfare harm of an exogenous price increase can be found using a simple integral under the demand curve. I provide simulation studies and an empirical example using data from an online travel agency to demonstrate this. However, when consideration sets are sensitive to prices, the researcher needs to observe consideration set responses to price changes in order to accurately measure the welfare consequences of an exogenous price increase.

4.A Proofs from chapter 4

4.A.1 Proof of Theorem 21

The proof is analogous to the proof of Theorem 22 below. A proof of Theorem 21, using conditional demand, can also be found in Section 3.A

4.A.2 Proof of Theorem 22

I start with the simpler case, eq. (4.10). Since the price decreases, $p_1^1 < p_1^0$. Fix a consumer, (y, η, ζ). By Price Independence, $\mathcal{C}_0 = \mathcal{C}_1$. Thus, there are three cases. In case 1, the consumer buys good 1 before and after the price decrease. In case 2, the consumer buys the good only after the price decrease. In case 3, the consumer never buys the good.

Trivially, for case 3,

$$S^{CV}(y, \eta, \zeta, p_{\mathcal{J}}^0, p_{\mathcal{J}}^1) = 0 = \int_{p_1^1}^{p_1^0} q_1^0(y, p, \eta, \zeta) dp$$

Since $q_1^0(y, p, \eta, \zeta)$ for all $p \in (p_1^1, p_1^0)$.

For case 1 and 2, revealed preference tells us

$$\max_{j \in \mathcal{C}_0} u_j(y - p_j^0, \eta) < \max_{j \in \mathcal{C}_1} u_j(y - p_j^1, \eta) = u_1(y - p_1^1, \eta)$$

so $S^{CV} < 0$. But for $S < 0$ and for all $j \neq 1$ we have

$$u_j(y - p_j^0, \eta) = u_j(y - p_j^1, \eta) > u_j(y - p_j^1 + S, \eta)$$

That is, for all S such that $S^{CV} < S < 0$,

$$\begin{aligned}
\max_{j \neq 1, j \in \mathcal{C}_1} u_j(y - p_j^1, \eta) &\leq \max_{j \in \mathcal{C}_0} u_j(y - p_j^0, \eta) \\
&< \max_{j \in \mathcal{C}_1} u_j(y - p_j^1 + S, \eta) \\
&= \max\left(\max_{j \in \mathcal{C}_1, j \neq 1} u_j(y - p_j^1 + S, \eta), u_1(y - p_1^1 + S, \eta) \right)
\end{aligned}$$

Thus, S^{CV} is simply the solution in S to

$$\max_{j \in \mathcal{C}_0} u_j(y - p_j^0, \eta) = u_1(y - p_1^1 + S, \eta) \tag{4.11}$$

We can thus get a simple, explicit solution for S in case 1 and 2. In particular, for case 1 we have

$$S^{CV}(y, \eta, \zeta, p_{\mathcal{J}}^0, p_{\mathcal{J}}^1) = p_1^0 - p_1^1 = -\int_{p_1^1}^{p_1^0} q_1^0(y, p, \eta, \zeta) dp$$

where the first inequality can be verified by plugging into eq. (4.11) and the second inequality from *Monotonicity* and the definition of case 1.

Finally, for case 2, note that at price $\bar{p}$ such that the consumer becomes indifferent between good 1 and his choice at time 0, we have

$$u_1(y-\bar{p},\eta) = \max_{j\in\mathcal{C}_0} u_j(y-p_j^0,\eta) = u_1(y-p_1^0+S^{CV},\eta)$$

plugging in from eq. (4.11) of S^{CV}. Thus,

$$S^{CV} = \bar{p}-p_1^0 = \int_{p_1^0}^{p_1^1} q_1^0(y,p,\eta,\zeta)dp = -\int_{p_1^1}^{p_1^0} q_1^0(y,p,\eta,\zeta)dp$$

This is the same formula in all cases and the cases are exhaustive.

Putting this all together gives

$$\begin{aligned}\mu^{CV} &= -\int\int_{p_1^1}^{p_1^0} q_1^0(y,p,\eta,\zeta)dpdF \\ &= -\int_{p_1^1}^{p_1^0} Q_1(y,p,p_{-1}^0)dp \qquad\qquad \text{(by Tonnelli's Theorem)}\end{aligned}$$

This concludes the proof for compensating variation. For equivalent variation, note that $S^{EV}<0$ and for any individual

$$\begin{aligned}\max_{j\in\mathcal{C}_0} u_j(y-S^{EV}-p_j^0,\eta) &= \max_{j\in\mathcal{C}_1} u_j(y-p_j^1,\eta) \\ \Rightarrow S^{EV} &= 0-(-S^{EV}) \\ &= -\int_{-\infty}^{0} \mathbb{1}(\max_{j\in\mathcal{C}_0} u_j(y-S-p_j^0,\eta) > \max_{j\in\mathcal{C}_1} u_j(y-p_j^1,\eta))dS \\ &= -\int_{p_1^1-p_1^0} q_1^0(y-S,p_1^1-S,\eta,\zeta)dS \\ &= -\int_{p_1^1}^{p_1^0} q_1(y+p-p_1^1,p,\eta,\zeta)dp\end{aligned}$$

Extensions to the average over all individuals can now be done exactly as above in the case of S^{CV}. This proves eq. (4.9).

Acquisti, Alessandro, Curtis Taylor, and Liad Wagman (2016). "Economics of Privacy". In: *Journal of Economic Literature* 54.2. http://dx.doi.org/10.1257/jel.54.2.442, pp. 442–492.

Amano, Tomomichi, Andrew Rhodes, and Stephan Seiler (2017). "Large-scale Demand Estimation with Search Data". In: *Working Paper*. URL: https://people.stanford.edu/sseiler/sites/default/files/large_scale_demand_nov2017.pdf.

Athey, Susan and Glenn Ellison (2011). "Position Auctions with Consumer Search*". In: *The Quarterly Journal of Economics* 126.3, pp. 1213–1270. DOI: 10.1093/qje/qjr028. URL: http://dx.doi.org/10.1093/qje/qjr028.

Berry, Steven T. and Philip A. Haile (2014). "Identification in Differentiated Products Markets Using Market Level Data". In: *Econometrica* 82.5, pp. 1749–1797. DOI: 10.3982/ECTA9027. URL: https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA9027.

Bhattacharya, Debopam (2015). "Nonparametric Welfare Analysis for Discrete Choice". In: *Econometrica* 83.2, pp. 617–649. DOI: 10.3982/ECTA12574.

Blundell, R, J. Horowitz, and M. Perry (2012). "Measuring the Price Responsiveness of Gasoline Demand: Economic Shape Restrictions and Nonparametric Demand Estimation". In: *Quantitative Economics* 3.1, pp. 29–51.

Blundell, Richard W. and James L. Powell (2004). "Endogeneity in Semiparametric Binary Response Models". In: *The Review of Economic Studies* 71.3, pp. 655–679. DOI: 10.1111/j.1467-937X.2004.00299.x. URL: http://dx.doi.org/10.1111/j.1467-937X.2004.00299.x.

Blundell, Richard, Dennis Kristensen, and Rosa Matzkin (2013). "Control Functions and Simultaneous Equations Methods". In: *American Economic Review* 103.3, pp. 563–69. URL: https://EconPapers.repec.org/RePEc:aea:aecrev:v:103:y:2013:i:3:p:563-69.

— (2014). "Bounding quantile demand functions using revealed preference inequalities". In: *Journal of Econometrics* 179.2, pp. 112–127.

Brynjolfsson, Erik, Avinash Collis, et al. (Mar. 2019). *GDP-B: Accounting for the Value of New and Free Goods in the Digital Economy*. Working Paper 25695. National Bureau of Economic Research. DOI: 10.3386/w25695. URL: http://www.nber.org/papers/w25695.

Brynjolfsson, Erik and JooHee Oh (2012). "The Attention Economy: Measuring the Value of Free Digital Services on the Internet". In: *ICIS*.

Burdett, Kenneth and Kenneth L. Judd (1983). "Equilibrium Price Dispersion". In: *Econometrica* 51.4. http://www.jstor.org/stable/1912045, pp. 955–969.

Chade, Hector and Lones Smith (2006). "Simultaneous Search". In: *Econometrica* 74.5, pp. 1293–1307.

Chernozhukov, Victor, Ivan Fernandez-Val, and Whitney Newey (2018). "Nonseparable Multinomial Choice Models in Cross-Section and Panel Data". In: *arXiv:1706.08418v2*. URL: https://arxiv.org/abs/1706.08418v2.

Croissant, Yves (2019). *Estimation of multinomial logit models in R: The mlogit Packages*. Version 0.4-2.

Dagsvik, John K. and Anders Karlström (2005). "Compensating Variation and Hicksian Choice Probabilities in Random Utility Models that are Nonlinear in Income". In: *Review of Economic Studies* 72.1, pp. 57–76. URL: https://EconPapers.repec.org/RePEc:oup:restud:v:72:y:2005:i:1:p:57-76.

Diamond, Peter A. (1971). "A Model of Price Adjustment". In: *Journal of Economic Theory* 3, pp. 156–168.

Diewert W, Erwin and Robert Feenstra (Dec. 2017). *Estimating the Benefits and Costs of New and Disappearing Products*. Microeconomics.ca working papers. Vancouver School of Economics. URL: https://ideas.repec.org/p/ubc/pmicro/tina_marandola-2017-12.html.

Diewert, W. Erwin, Kevin Fox, and Paul Schreyer (2018). *The Digital Economy, New Products and Consumer Welfare*. Economic Statistics Centre of Excellence (ESCoE) Discussion Papers ESCoE DP-2018-16. Economic Statistics Centre of Excellence (ESCoE). URL: https://EconPapers.repec.org/RePEc:nsr:escoed:escoe-dp-2018-16.

Dinerstein, Michael et al. (2017). "Consumer Price Search and Platform Design in Internet Commerce". In: *Working*. http://web.stanford.edu/ leinav/wp/Search.pdf.

Domencich, T and D. McFadden (1975). *Urban Travel Demand - A Behavioral Analysis*. North-Holland: Oxford. Chap. 4. URL: https://eml.berkeley.edu/~mcfadden/travel.html.

Ellison, Glenn and Sara Fisher Ellison (2009). "Obfuscation, and Price Elasticities on the Internet". In: *Econometrica* 77. DOI: 10.3982/ECTA5708, pp. 427–452.

Ershov, Daniel (2016). "The Effect of Consumer Search Costs on Entry and Quality in the Mobile App Market". In: *University of Toronto Job Market Paper*. URL: http://www.law.northwestern.edu/research-faculty/searlecenter/events/internet/documents/Ershov_Consumer_Search.pdf.

Feldstein, Martin (May 2017). "Underestimating the Real Growth of GDP, Personal Income, and Productivity". In: *Journal of Economic Perspectives* 31.2, pp. 145–64. DOI: 10.1257/jep.31.2.145. URL: http://www.aeaweb.org/articles?id=10.1257/jep.31.2.145.

Fradkin, Andrey (2015). "Search Frictions and the Design of Online Marketplaces". In: *Working Paper*.

Gentzkow, Matthew (2007). "Valuing New Goods in a Model with Complementarity: Online Newspapers". In: *American Economic Review* 97.3, pp. 713–744. URL: https://EconPapers.repec.org/RePEc:aea:aecrev:v:97:y:2007:i:3:p:713-744.

Greenstein, Shane and Ryan McDevitt (2011). "The broadband bonus: Estimating broadband Internet's economic value". In: *Telecommunications Policy* 35.7, pp. 617–632. URL: https://EconPapers.repec.org/RePEc:eee:telpol:v:35:y:2011:i:7:p:617-632.

Groshen, Erica L. et al. (May 2017). "How Government Statistics Adjust for Potential Biases from Quality Change and New Goods in an Age of Digital Technologies: A View from the Trenches". In: *Journal of Economic Perspectives* 31.2, pp. 187–210. DOI: 10.1257/jep.31.2.187. URL: http://www.aeaweb.org/articles?id=10.1257/jep.31.2.187.

Hausman, Jerry (1981). "Exact Consumer's Surplus and Deadweight Loss". In: *American Economic Review* 71.4, pp. 662–76. URL: https://EconPapers.repec.org/RePEc:aea:aecrev:v:71:y:1981:i:4:p:662-76.

— (1996). "Valuation of New Goods under Perfect and Imperfect Competition". In: *The Economics of New Goods*. National Bureau of Economic Research, Inc, pp. 207–248. URL: https://EconPapers.repec.org/RePEc:nbr:nberch:6068.

Hausman, Jerry (Apr. 1999). "Cellular Telephone, New Products, and the CPI". In: *Journal of Business & Economic Statistics* 17.2, pp. 188–194. URL: https://ideas.repec.org/a/bes/jnlbes/v17y1999i2p188-94.html.

Hausman, Jerry A. (1997). "Valuing the Effect of Regulation on New Services in Telecommunications". In: *Brookings Papers on Economic Activity* 28.1997 Micr, pp. 1–54. URL: https://ideas.repec.org/a/bin/bpeajo/v28y1997i1997-3p1-54.html.

Hausman, Jerry A. and Whitney K. Newey (2016). "Individual Heterogeneity and Average Welfare". In: *Econometrica* 84.3, pp. 1225–1248. ISSN: 1468-0262. DOI: 10.3982/ECTA11899. URL: http://dx.doi.org/10.3982/ECTA11899.

Hausman, Jerry A and Gregory K Leonard (Sept. 2002). "The Competitive Effects of a New Product Introduction: A Case Study". In: *Journal of Industrial Economics* 50.3, pp. 237–263. URL: https://ideas.repec.org/a/bla/jindec/v50y2002i3p237-63.html.

Heckman, James (1974). "Shadow Prices, Market Wages, and Labor Supply". In: *Econometrica* 42.4, pp. 679–94. URL: https://EconPapers.repec.org/RePEc:ecm:emetrp:v:42:y:1974:i:4:p:679-94.

Herriges, J and C. Kling (1999). "Nonlinear Income Effects in Random Utility Models". In: *Review of Economics and Statistics* 81.1, pp. 62–72.

Hoderlein, Stefan and Anne Vanhems (Jan. 2011). *Welfare analysis using nonseparable models*. CeMMAP working papers CWP01/11. Centre for Microdata Methods and Practice, Institute for Fiscal Studies. URL: https://ideas.repec.org/p/ifs/cemmap/01-11.html.

Honka, Elisabeth (2014). "Quantifying Search and Switching Costs in the US auto insurance industry". In: *RAND Journal of Economics* 45.4, pp. 847–884.

Honka, Elisabeth and Pradeep Chintagunta (2016). "Simultaneous or Sequential? Search Strategies in the US Auto Insurance Industry". In: *Marketing Science*. URL: http://dx.doi.org/10.1287/mksc.2016.0995.

Honka, Elisabeth, Ali Hortaçsu, and Maria Ana Vitorino (2017). "Advertising, consumer Awareness and Choice: Evidence from the U.S. Banking Industry". In: *RAND Journal of Economics* 48.3, pp. 611–646.

Hoofnagle, Chris Jay et al. (2012). “Behavioral Advertising: The Offer You Can’t Refuse”. In: *Harvard Law and Policy Review* 273. URL: http://scholarship.law.berkeley.edu/facpubs/2086.

Khan, Lina M. (Jan. 2017). “Amazon’s Antitrust Paradox”. In: *Yale Law Yournal* 126.3, pp. 564–907. URL: http://www.yalelawjournal.org/article/amazons-antitrust-paradox.

Koulayev, Sergei (2014). “Search for Differentiated Products: Identification and Estimation”. In: *RAND* 45.3, pp. 553–575.

Lewbel A an Pendakur, K (2013). “Unobserved Preference Heterogeneity in Demand Using Generalized Random Coefficients”. In: *Report, Boston College*.

Lewis, Gregory and Albert Wang (2013). “Who benefits from improved search in platform markets”. In: *SSRN*. URL: http://ssrn.com/abstract=2249816.

Liu, Xudong et al. (2013). “Combination of Diverse Ranking Models for Personalized Expedia Hotel Searches.” In: *CoRR* abs/1311.7679. URL: http://dblp.uni-trier.de/db/journals/corr/corr1311.html#LiuXZYPLSW13.

Matzkin, Rosa (1993). “Nonparametric identification and estimation of polychotomous choice models”. In: *Journal of Econometrics* 58.1-2, pp. 137–168. URL: https://EconPapers.repec.org/RePEc:eee:econom:v:58:y:1993:i:1-2:p:137-168.

— (2015). “Estimation of Nonparametric Models With Simultaneity”. In: *Econometrica* 83, pp. 1–66. URL: https://EconPapers.repec.org/RePEc:wly:emetrp:v:83:y:2015:i::p:1-66.

Matzkin, Rosa L. (1993). “Nonparametric identification and estimation of polychotomous choice models”. In: *Journal of Econometrics* 58, pp. 137–168.

McAfee, Preston R. (1995). “Multiproduct Equilibrium Price Dispersion”. In: *Journal of Economic Theory* 67, pp. 83–105.

McFadden, D (1974). “Conditional Logit Analysis of Qualitative Choice Behavior”. In: *Frontiers in Econometrics. Academic Press: New York*. Ed. by P Zarembka, pp. 105–142.

Moraga-Gonzalez, José Luis, Zsolt Sándor, and Matthijs R. Wildenbeest (2017). “Nonsequential Search Equilibrium with Search Cost Heterogeneity”. In: *International Journal of Industrial Organization* 50. DOI:10.1016/j.ijindorg.2016.06.009, pp. 392–414.

Morgan, Peter and Richard Manning (1985). “Optimal Search”. In: *Econometrica* 53.4. http://www.jstor.org/stable/1912661, pp. 923–944.

Nevo, Aviv (May 2003). “New products, quality changes, and welfare measures computed from estimated demand systems”. English (US). In: *Review of Economics and Statistics* 85.2, pp. 266–275. ISSN: 0034-6535. DOI: 10.1162/003465303765299792.

Petrin, Amil (2002). “Quantifying the Benefits of New Products: The Case of the Minivan”. In: *Journal of Political Economy* 110.4, pp. 105–130.

Quan, Thomas W. and Kevin R. Williams (2016). “product variety, across-market demand heterogeneity and the value of online retail”. In: *Cowles Foundation Working Paper*. URL: https://cowles.yale.edu/sites/default/files/files/pub/d20/d2054.pdf.

R Core Team (2017). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing. Vienna, Austria. URL: http://www.R-project.org/.

Rothschild, Michael (1974). “Searching for the Lowest Price When the Distribution of Prices is Unknown”. In: *Journal of Political Economy* 82.4. http://www.jstor.org/stable/1837141, pp. 689–711.

Seiler, Stephan and Song Yao (2017). “The Impact of Advertising along hte Conversion Funnel”. In: *Quantitative Marketing and Economics* 15.3, pp. 241–278.

Small, Kenneth A. and Harvey S. Rosen (1981). “Applied Welfare Economics with Discrete Choice Models”. In: *Econometrica* 49.1, pp. 105–130. URL: http://www.jstor.org/stable/1911129.

Stigler, George J. (1961). “Economics of Information”. In: *Journal of Political Economy* 69.3, pp. 213–225.

Syverson, Chad (May 2017). “Challenges to Mismeasurement Explanations for the US Productivity Slowdown”. In: *Journal of Economic Perspectives* 31.2, pp. 165–86. DOI: 10.1257/jep.31.2.165. URL: http://www.aeaweb.org/articles?id=10.1257/jep.31.2.165.

Ursu, Raluca M. (2017). “The Power of Rankings”. In: *Working Paper*. May 6, 2017 version.

Weitzman, M. L. (1979). “Optimal Search for the best alternative”. In: *Econometrica* 47, pp. 641–654.