UCRL 8394

UNIVERSITY OF CALIFORNIA

Radiation Laboratory

BERKELEY, CALIFORNIA

UCRL-8394

UNIVERSITY OF CALIFORNIA

Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

THEORETICAL INTERPRETATION OF STRANGE-PARTICLE INTERACTIONS

R. H. Dalitz

August 1958

Printed for the U.S. Atomic Energy Commission

THEORETICAL INTERPRETATION OF STRANGE-PARTICLE INTERACTIONS

(Report to the 1958 Conference on High-Energy Physics at CERN)

R. H. Dalitz

Radiation Laboratory
University of California
Berkeley, California

ABSTRACT

Theoretical interpretations of the empirical evidence on the interactions of K mesons and hyperons with nucleons are reviewed, with special attention to those aspects of this evidence which bear on the relative parities of K mesons and hyperons, on the coupling strengths of their interactions, or on the existence of symmetries relating the interactions of different particles. The use of dispersion relations for $K^{\pm}$ scattering appears a very promising approach, although the experimental data are too limited for definite conclusions at the present stage. The theoretical uncertainties in this approach are considered, especially those concerned with the unphysical region in these relations and with the convergence at high energies. The K^--p reaction and scattering data at low energies are analyzed in terms of a short-range interaction for s-wave K^- mesons, characterized by two complex zero-energy interaction lengths; this analysis also gives an adequate account of the K^--p data from emulsion studies up to 100 Mev. Several features of interest in the K^--d capture reactions recently observed are also discussed. Hyperon-nucleon interactions are considered with special reference to the Gell Mann-Schwinger hypothesis of "global symmetry" for the pion-hyperon interaction. Owing to the Σ-Λ mass difference and also to the operation of the Pauli principle for

nucleons, the identification of such a symmetry in the empirical situations for which most data will become available in the near future is not at all direct. For the K^--p capture reactions, the Amati-Vitale inequality required by the pion-hyperon symmetry (with neglect of the Σ-Λ mass difference) is violated according to the present data. The most direct evidence which favors the "global symmetry" hypothesis comes from the binding energies of Λ-hypernuclei. Although the interpretation of these binding energies is made difficult by the possibility of three-body forces between Λ hyperon and nucleons, it appears that the spin dependence, range, and strength of the Λ-nucleon interaction are in general agreement with this hypothesis. Some recent work on the angular correlations observed in the hypernuclear decay of ${}_{\Lambda}He^5$ is also reviewed.

THEORETICAL INTERPRETATION OF STRANGE-PARTICLE INTERACTIONS†

(Report to the 1958 Conference on High-Energy Physics at CERN)

R. H. Dalitz*

Radiation Laboratory
University of California
Berkeley, California

1. Introduction

In the discussion to be given here on the information that has become available on strange-particle interactions, several goals are to be kept in mind. These are the determination of the relative parities of the K particles and the hyperons, and the determination of the coupling parameters for the various interactions between them. In this discussion, the K particle is assumed to have zero spin, and the hyperons spin $\frac{1}{2}$, as is consistent with all the data available. A further feature of interest is the possibility of symmetries between the various interactions, for example whether the hypothesis of a universal pion-baryon coupling as proposed by Gell-Mann[1] and by Schwinger[2] at the Rochester Conference of 1957 is in accord with the data available.

2. K^+-Nucleon Scattering

First the evidence on K^+ scattering is considered briefly. The main features appear as follows:

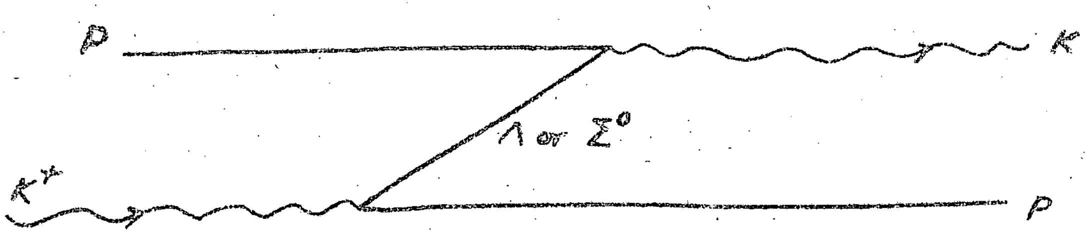
(i) The T=1 interaction is a short-range s-wave repulsion, corresponding to a cross section essentially constant up to about 150 Mev. Beyond this energy, there is some evidence for an increasing T=1 cross section. This rise in cross section may be due to some p-wave interaction, although there is no real evidence for a nonisotropic angular distribution.

* Permanent address: Enrico Fermi Institute for Nuclear Studies, University of Chicago.

† This work was performed under the auspices of the U. S. Atomic Energy

(ii) The T=0 s-wave interaction appears relatively weak, but the evidence for a p-wave interaction appears quite clearly from the appearance of a backward peaking in the K^+-neutron cross section for K^+ energies of 100 Mev and higher. This is also indicated by a marked energy dependence of the charge-exchange scattering by emulsion nuclei, which is rather weak (< 10% of the total scattering) at 50 Mev but reaches a value of about 30% of the total scattering in the range 200 to 300 Mev.

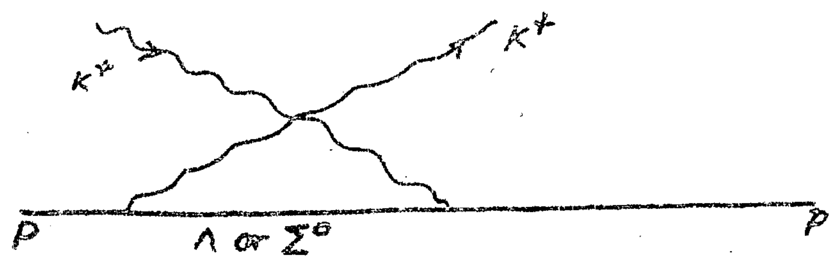
The appearance of both s- and p-wave scattering in the low-energy region seems rather difficult to understand in terms of a scalar K meson, the Λ and Σ hyperons being assumed to have the same parity. However, Ceolin and Taffara[3] have remarked that the appearance of both s and p scattering could be explained in a fairly direct way in terms of pseudoscalar $KN\Lambda$ and $KN\Sigma$ interactions. In lowest-order perturbation theory, a repulsive s-wave interaction for K^+-proton scattering appears naturally through the excitation of virtual antihyperon-nucleon pairs:



The corresponding pair terms for the pion-nucleon system are well known to be very much greater than the observed s-wave scattering allows; however, the suppression of these terms is not well understood for the pion-nucleon case (it may possibly be due to the effects of virtual strange-particle interactions), and it is quite possible that the mechanism for this suppression does not operate in K-nucleon scattering.

The observed K^+-proton cross section of 14 mb is obtained with a value of about 2 or 3 for $(g_\Lambda^2 + g_\Sigma^2)/4\pi$. The T=0 scattering amplitude is proportional to $(g_\Lambda^2 - 3g_\Sigma^2)/4\pi$, so that a weak s-wave scattering in this state may be obtained for suitable choice of the ratio g_Λ/g_Σ. Further, Barshay has suggested the use of a repulsive direct K-pion interaction,[4] of the form $f_{K\pi}\ \bar{K}K\ \pi\cdot\pi$, which would contribute a repulsive T-independent term to the K^+-nucleon scattering amplitude; in this case, weak T=0 s-wave scattering would correspond to a different choice of the relative coupling strengths g_Λ and g_Σ. From this pion-exchange process, one might expect an s-wave amplitude falling off with energy beyond about 100 Mev, but the total K^+-proton cross section might be held up to the observed value by some rising p-wave cross section.

From the positive energy transitions,

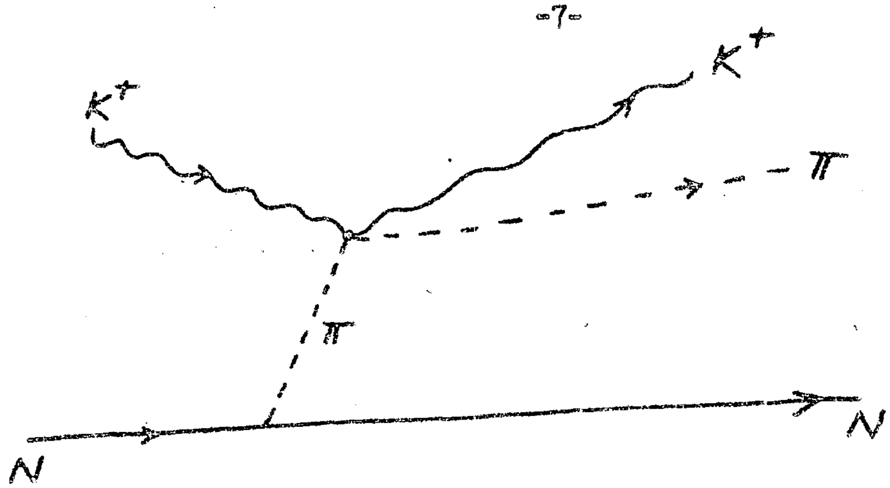


the pseudoscalar coupling gives rise to a pseudovector form of interaction, such as we are now familiar with in pion physics, so that p-wave scattering is also to be expected. Ceolin and Taffara have pointed out that, in lowest approximation, this interaction is attractive in the T=1, $p_{3/2}$ and the T=0, $p_{1/2}$ states.[5] Together with Dallaporta,[5] they have carried out more detailed calculations in the static limit to bring out these qualitative points more clearly; however, there is no detailed agreement with the experimental results.

It must be emphasized that the above remarks are only qualitative and that they could be modified very considerably by the existence of the lighter strongly coupled pion. For this reason we cannot definitely say that the scalar K meson could not account for the evidence, although in lowest approximation the interaction that it leads to is attractive and includes very little p-wave term.* The most cautious statement one could make is that the pseudoscalar interaction is not ruled out, but appears capable of giving the right qualitative behavior.

A new feature of the K^+ data this year is the appearance of π production in K^+ collisions above the theshold at 220 Mev, the rate observed is of the order of 1%. Some calculations have been reported by Ceolin, Dallaporta, and Taffara on this process.[7] Assuming the elastic K^+ scattering to be due to pseudoscalar KYN interactions, they obtain results (a) from a perturbation-theory calculation, which are very much below the production rate actually observed. The second possibility considered, (b), was that the $\bar{K}K\pi\cdot\pi$ interaction was responsible for the elastic scattering. In this case the elastic scattering gives no charge exchange, and $f^2_{K\pi}/4\pi \sim 2$ is needed to give the observed K^+-proton cross section. The $\bar{K}K\pi\cdot\pi$ interaction can then give rise directly to s-wave π production in K^+-N collisions.

* In a paper received at the end of the Conference, Barshay (Phys. Rev. Letters, to be published) has remarked that the appearance of both s- and p-wave scattering could also be explained if only one of the $KN\Lambda$ and $KN\Sigma$ interactions were pseudoscalar, the other being of scalar form. Of course this requires opposite parities for the Λ and Σ hyperons.



Consequently the values obtained for R_π are larger by a factor of 10, but are still well below the number suggested experimentally.

	(a)	(b)
$E-E_{th}$	R_π	
50 Mev	4×10^{-5}	1×10^{-3}
100 Mev	2×10^{-4}	2×10^{-3}

Actually, at 350 Mev K^+ energy, it seems reasonable to expect a rate R_π of at least 10^{-3} per K^+ interaction in complex nuclei, owing to secondary-pion production by recoil nucleons from the backward scattering of K mesons, and this effect may be sufficient to account for the experimental observations. However, it is clear that the observation of π production in K^+-proton collisions would be of interest in giving some information bearing on the relative strengths of the various kinds of K-particle interaction possible.

3. K^--Proton Interactions

Data on K^--proton interactions are now available in some detail from emulsion studies and especially from the work of the Alvarez bubble chamber [illegible] hydrogen, the data have increased greatly from those available a year ago, but are still compatible with the ratios $4 : 2 : 2 : \frac{1}{2}$ for the $\Sigma^- : \Sigma^0 : \Sigma^+ : \Lambda$ reaction ratios (although the Σ^0 and Λ events have not yet been separated in the new data).

The new bubble chamber data on interactions in flight over the energy range of about 5 to 35 Mev is of the greatest interest. These data show no clear indication of any other than s-wave interactions; the angular distributions of the scattering and the reactions are all compatible with an s-wave capture. This appears reasonable for such low incident momenta, the K^--proton interaction being expected to have a range of about $\hbar/m_K c \sim 0.4$ f. There is evidence in the emulsion data that the Σ^-/Σ^+ ratio has changed from the value 1.8 for the low-energy interactions in flight to a value ~ 1 at 100 Mev, but we shall see later that this is not at all incompatible with an s-wave reaction. In discussing these data we shall therefore assume that s-wave capture is predominant. The analysis I shall give is very preliminary and has been carried out only since my arrival at the Conference. I give it here with much hesitancy, but I feel that at least it is instructive and will indicate the kind of additional experimental data we now need.

For discussion of the scattering, two complex phase shifts $\delta_T = \alpha_T + i\beta_T$ --that is four parameters--are needed for the two channels T=0 and T=1. The bubble chamber data have been collected by an averaging based on a reasonable energy dependence for the cross sections to give cross sections evaluated at a mean laboratory momentum of [illegible]35 Mev/c, a K^- energy of 18.5 Mev. The value of $\pi\lambda_c^2$ at this energy [illegible] 60 mb. The elastic cross section is then 64 mb, and the charge-exc[illegible] [illegible] section has been taken as 0.2 $\sigma_{elastic}$, corresponding to the

[illegible]

since some K^0_1 decays may be confused with Λ-decay events; however, the proportion of K^0_1 decays giving such configurations may be expected to be small). To obtain the reaction cross sections for T=0 and T=1, more data are needed than just for the Σ^+ and Σ^- reactions. In fact, in terms of the reaction amplitudes M_0 and M_1 and their relative phase ϕ, the relative proportions for Σ and Λ reactions are

$$\sigma(\Sigma^-) \sim \frac{1}{6} M_0^2 + \frac{1}{4} M_1^2 + \frac{1}{\sqrt{6}} M_0 M_1 \cos\phi , \tag{3.1a}$$

$$\sigma(\Sigma^+) \sim \frac{1}{6} M_0^2 + \frac{1}{4} M_1^2 - \frac{1}{\sqrt{6}} M_0 M_1 \cos\phi , \tag{3.1b}$$

$$\sigma(\Sigma^0) \sim \frac{1}{6} M_0^2 , \tag{3.1c}$$

$$\sigma(\Lambda) \sim \frac{1}{2} N_1^2 . \tag{3.1d}$$

From these expressions, the ratio of T=1 and T=0 reaction cross sections is

$$\sigma_{abs}(T=1)/\sigma_{abs}(T=0) = (M_1^2 + N_1^2)/M_0^2$$
$$= \frac{\left(\sigma(\Sigma^+) + \sigma(\Sigma^-) - 2\sigma(\Sigma^0) + \sigma(\Lambda)\right)}{3\,\sigma(\Sigma^0)} . \tag{3.2}$$

To make progress, we have assumed that the Σ^0, Λ production bears the same ratio to the Σ^- and Σ^+ production as for the K^- capture from rest. This is dangerous, since it is not at all clear how much of this K^--p capture in hydrogen is from the p states. Jackson, Ravenhall, and Wyld[7] have pointed out that the competition of the 2p capture with the 2p-1s radiative transition can be obtained when the p-wave absorption in flight is known. At 100 Mev the absorption cross section appears relatively small, and it is still quite consistent with s-wave capture; if 50% of the observed cross section were p-wave, this would allow only about 30% of the K^- capture from rest to be from the p states. There is, however, some support for this assumption from the Σ^-/Σ^+ ratio, which is 1.8 for the low-energy interactions in flight, quite in accord (within statistics) with the ratio of 2 observed for the captures from rest. In this way a ratio $\sigma_{abs}(T=1)/\sigma_{abs}(T=0)$ of about 0.4 is obtained, the total reaction cross section at 18.5 Mev then being $\frac{1}{2}\left\{\sigma_{abs}(T=1) + \sigma_{abs}(T=0)\right\} = 62$ mb. Since the reaction cross section is directly related to the imaginary part of the phase shift,

$$\sigma_{abs}(T) = \pi \lambda_c^2 \left(1 - \exp(-2\beta_T)\right)^2, \tag{2.3}$$

values may be obtained for β_0 and β_1 from these numbers. Next, the real parts of δ_0 and δ_1 may be obtained from the elastic and charge-exchange cross sections,

$$\sigma_{el} = \pi \lambda_c^2 \left|\left(2 - e^{-2\beta_0} e^{2i\alpha_0} - e^{-2\beta_1} e^{2i\alpha_1}\right)\right|^2, \tag{2.4}$$

$$\sigma_{c.e.} = \pi \lambda_c^2 \left|\left(e^{-2\beta_0} e^{2i\alpha_0} - e^{-2\beta_1} e^{2i\alpha_1}\right)\right|^2 \tag{2.5}$$

There are two distinct solutions for α_0 and α_1; for each of these solutions the relative signs of α_0 and α_1 are determined but their over-all sign is still indeterminate. At this point, it is appropriate to follow the suggestions of Jackson, Ravenhall, and Wyld,[7] and to adopt the zero-range approximation, since even at 100 Mev the wave lengths are still reasonably long relative to the range of the interaction. Zero-energy scattering lengths are then convenient to use and are defined as usual by

$$k \cot \delta_T = \frac{1}{a_T + i b_T} , \qquad (3.6)$$

with neglect of the term $\frac{1}{2}(r_T + i s_T)k^2$ in the usual effective-range treatment. The complex values obtained for these zero-energy scattering lengths are shown in Table I.

TABLE I

K^--p zero-energy scattering amplitudes[a] (10^{-13} cm)		
	$a_0 + ib_0$	$a_1 + ib_1$
Solution A_+	0.28 + i 0.54	1.19 + i 0.22
Solution B_+	1.28 + i 0.71	0.43 + i 0.18

[a] There are also two corresponding solutions A_- , B_- , which are obtained from A_+ , B_+ by reversing the signs of a_0 and a_1.

The following points are now of interest:

(i) a_0, a_1 have the same sign in both these solutions. This follows really from the largeness of the elastic cross section, which requires a reasonably large value of $(a_0 + a_1)$, and the smallness of the charge-exchange scattering, which then requires that a_0 and a_1 have the same sign to give a small value for $(a_0 - a_1)$. This is of interest especially because of the strong argument Ceccarelli[8] gave last year for the conclusion that the K^--nuclear potential is attractive (at least 20 Mev) for low-energy K^- mesons. This conclusion then requires that, since a_0 and a_1 have the same sign, a_0 and a_1 should both correspond to attractive interactions.

(ii) The energy dependence of these cross sections is next of interest. Jackson, Ravenhall, and Wyld[7] have emphasized that the presence of strong absorptive processes gives rise to a downward cusp in the curof energy dependence of $k\sigma_{abs}/4\pi$. There is also a cusp in the elastic cross section. As a result, extrapolation of the data to zero energy must always be done with care. In Fig. 1 the elastic cross sections corresponding to Solutions A and B of Table I are plotted from the expression

$$\sigma_{el} = \pi \left| \frac{a_0 + i b_0}{1 + k b_0 - i k a_0} + \frac{a_1 + i b_1}{1 + k b_1 - i k a_1} \right|^2, \qquad (3.7)$$

and are compared with the emulsion data. Both solutions agree in giving an s-wave cross section that falls rapidly with increasing K^- energy; the two curves agree at 18.5 Mev, of course, but Solution B rises about 30% higher than Solution A at zero energy. Both curves reproduce the general trend of the emulsion data, which reflects the excellent agreement of the emulsion data with the bubble chamber data. In Fig. 2 the absorption cross section $\{\sigma(\Sigma^+) + \sigma(\Sigma^-)\}$ given by 0.71 σ_{abs}, where

$$\frac{k\sigma_{abs}}{4\pi} = \frac{1}{2}\left\{ \frac{b_0}{1 + 2kb_0 + k^2(a_0^2 + b_0^2)} + \frac{b_1}{1 + 2kb_1 + k^2(a_1^2 + b_1^2)} \right\} \qquad (3.8)$$

is compared with the emulsion data. Again the general trend of this cross section is reproduced quite well, although the predicted curve lies a little higher than the emulsion data (owing partly to the somewhat high value of the fitted bubble chamber cross section relative to the general trend of the less well known cross sections from emulsion [illegible]

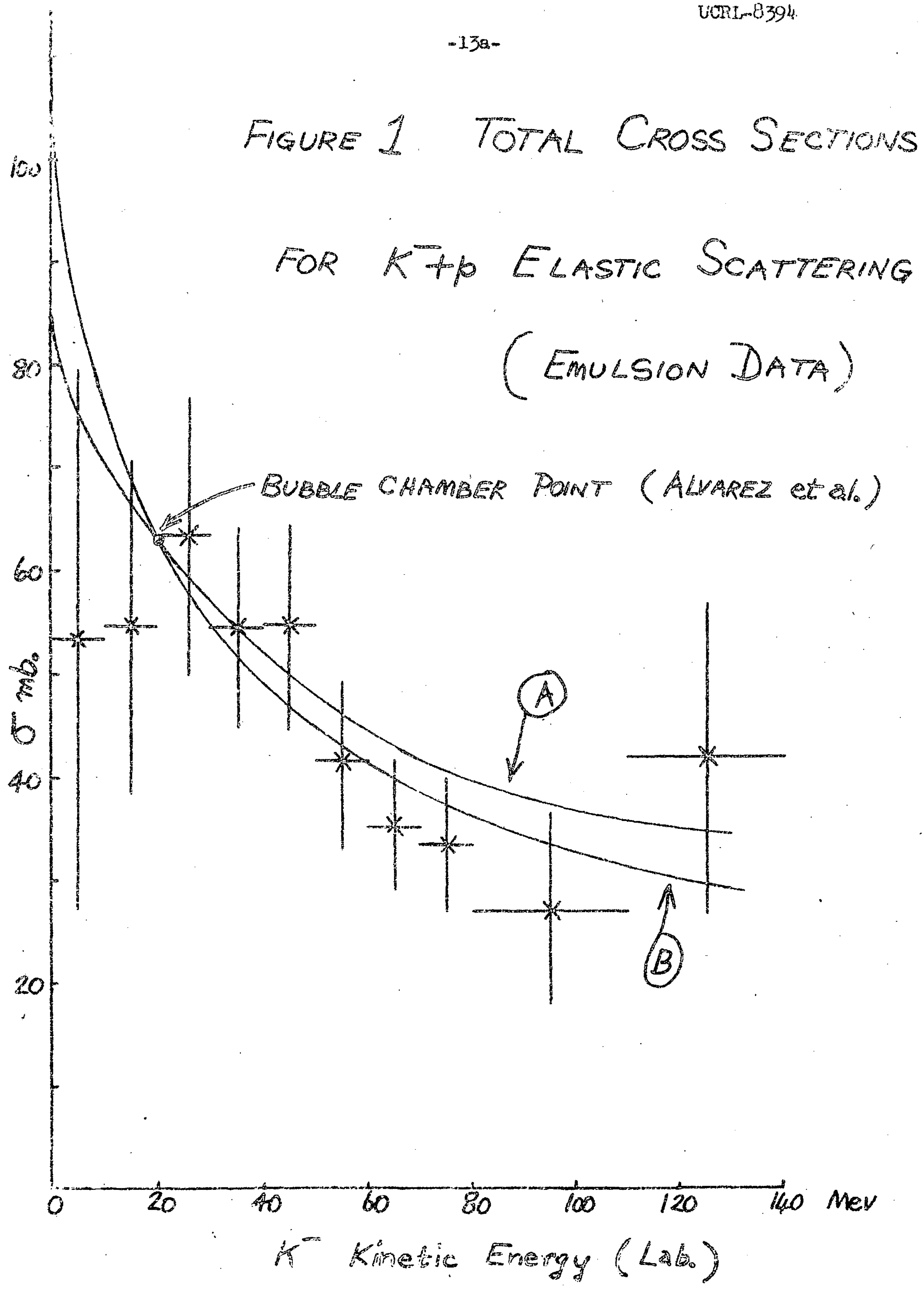


FIGURE 1 TOTAL CROSS SECTIONS
FOR K⁻+p ELASTIC SCATTERING
(EMULSION DATA)
BUBBLE CHAMBER POINT (ALVAREZ et al.)
A
B
σ mb.
100
80
60
40
20
0
20
40
60
80
100
120
140 Mev
K⁻ Kinetic Energy (Lab.)

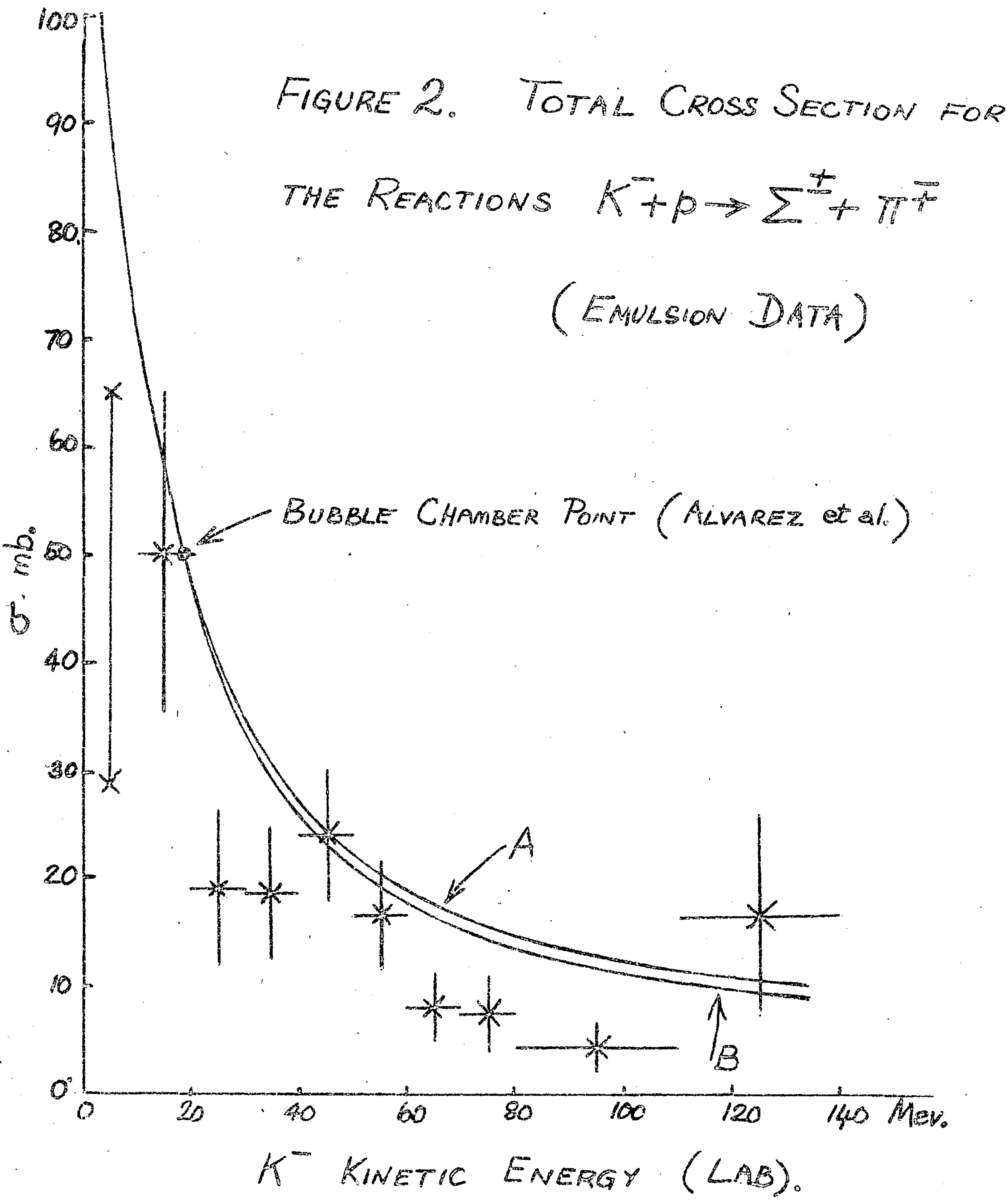


FIGURE 2. TOTAL CROSS SECTION FOR THE REACTIONS $K^- + p \rightarrow \Sigma^\pm + \pi^\mp$
(EMULSION DATA)
BUBBLE CHAMBER POINT (ALVAREZ et al.)
A
B
σ mb.
100
90
80
70
60
50
40
30
20
10
0
0
20
40
60
80
100
120
140
Mev.
K^- KINETIC ENERGY (LAB).

The general agreement found provides some support for this simple interpretation of the data up to 100 Mev in terms of a predominantly s-wave interaction. It will be of interest later to examine how sensitive this fit is to the assumption leading to the relative weight of the T=1 and T=0 absorption processes. Direct evidence on the Σ^0 and Λ production from K^--proton reactions is very desirable at this stage.

(iii) The energy dependences of $\sigma_{abs}(T=0)$ and of $\sigma_{abs}(T=1)$ may be quite different, owing to the fact that their strengths correspond to different values of b. For Solution A, the T=0 and T=1 absorption cross sections have an almost constant ratio, but for Solution B, the T=0 absorption drops by about 50% between 20 Mev and 100 Mev relative to the T=1 absorption. This will be reflected in a corresponding energy dependence of the Σ^-/Σ^+ ratio for Solution B. But even if the ratio $(M_0/M_1)^2$ were independent of energy, the Σ^-/Σ^+ ratio would depend sensitively on the phase angle ϕ; a value $\phi = 70^\circ$ gives the observed Σ^-/Σ^+ ratio at 18.5 Mev and a change to $\phi = 90^\circ$ at 100 Mev would lead to a Σ^-/Σ^+ ratio of unity there. Since the phase is due to the scattering interactions in the K^--p initial state, and especially in the π-hyperon final state, it is not at all unreasonable to find a change in ϕ by $\sim 20^\circ$ between zero energy and 100 Mev.

It is also of interest to mention briefly a reaction which has not yet been observed, although about 3000 K^--proton captures have been examined:

$$K^- + p \longrightarrow \Lambda + \pi^+ + \pi^- . \qquad (3.9)$$

Fujii and Marshak[9] have estimated from perturbation theory that s-state capture of a scalar K meson (which leads to s-wave motions in the final state (Eq. (3.9)) would lead to a branching ratio of several per cent for this reaction. Okun and Pomeranchik[10] give an estimate of about 0.2% for this branching ratio on the basis of phase-space considerations. For a pseudoscalar K meson, these estimates must be reduced by a factor ~50 owing to the p-wave motions which are then necessary in the final state. Okun and Pomeranchuk[10] have pointed out that the energy distribution in the final state of this reaction would be of considerable interest for the determination of KΛ parity if the reaction is ever observed.

4. Dispersion Relations for K Mesons

It appears that the use of dispersion relations for K-nucleon scattering offers a very powerful means for the determination of the KΛ and KΣ parities and coupling constants when the data available on $K^{\pm}$ scattering become more extensive. Their application for this purpose has been discussed recently by Amati and Vitale,[11] by Igi,[12] by Goebel,[13] and by Matthews and Salam.[14] The form of these dispersion relations for K-proton scattering is as follows:

$$D_{+}(\omega) = p_{\Lambda}\frac{X(\Lambda)}{\omega_{\Lambda}+\omega} + p_{\Sigma}\frac{X(\Sigma)}{\omega_{\Sigma}+\omega} + \frac{1}{4\pi^2}\int_{m_K}^{\infty} k'\,d\omega'\left[\frac{\sigma_{+}(\omega')}{\omega'-\omega} + \frac{\sigma_{-}(\omega')}{\omega'+\omega}\right] + \frac{1}{\pi}\int_{\omega_{\Lambda\pi}}^{m_K}\frac{d\omega'\,A_{-}(\omega')}{\omega'+\omega}, \tag{4.1a}$$

$$D_-(\omega) = p_\Lambda \frac{X(\Lambda)}{\omega_\Lambda - \omega} + p_\Sigma \frac{X(\Sigma)}{\omega_\Sigma - \omega}$$

$$+ \frac{1}{4\pi^2} \int_{m_K}^{\infty} k' d\omega' \left[\frac{\sigma_+(\omega')}{\omega' + \omega} + \frac{\sigma_-(\omega')}{\omega' - \omega} \right] + \frac{1}{\pi} \int_{\omega_{\Lambda\pi}}^{m_K} \frac{d\omega' A_-(\omega')}{\omega' - \omega} , \qquad (4.1b)$$

where $\omega_\alpha = (m_\alpha^{\ 2} - m_p^{\ 2} - m_K^{\ 2})/2m_p$, and m_α is the rest energy of the system α. The first terms on the right are the poles at ω_Λ and ω_Σ corresponding to the isolated Λ and Σ particles. The point of special interest here is that the sign of the residue at each pole is proportional to the parity, p_Λ or p_Σ, of the corresponding $K\Lambda$ or $K\Sigma$ pair, whilst the magnitude of the residue is related to the corresponding coupling constants g_Λ and g_Σ . The expressions for the residues are

$$p_\Lambda = +1 : \qquad X(\Lambda) = \frac{g_\Lambda^2}{4\pi} \frac{(m_\Lambda + m_p)^2 - m_K^2}{4\, m_p m_\Lambda} , \qquad (4.2a)$$

$$p_\Lambda = -1 : \qquad X(\Lambda) = \frac{g_\Lambda^2}{4\pi} \frac{m_K^2 - (m_\Lambda - m_p)^2}{4\, m_p m_\Lambda} , \qquad (4.2b)$$

with corresponding expressions for $p_\Sigma = \pm 1$, Λ being replaced everywhere by Σ . It is of interest to add that the K-neutron dispersion relations have the same form except that $X(\Lambda)$ is 0, the term $X(\Sigma)$ is doubled relative to the expression given by the analogue of Eq. (4.2); since the K^-N system has T=1, only the Σ state can contribute a pole, so that the K-neutron dispersion relations relate to the $(K\Sigma)$ parity alone.

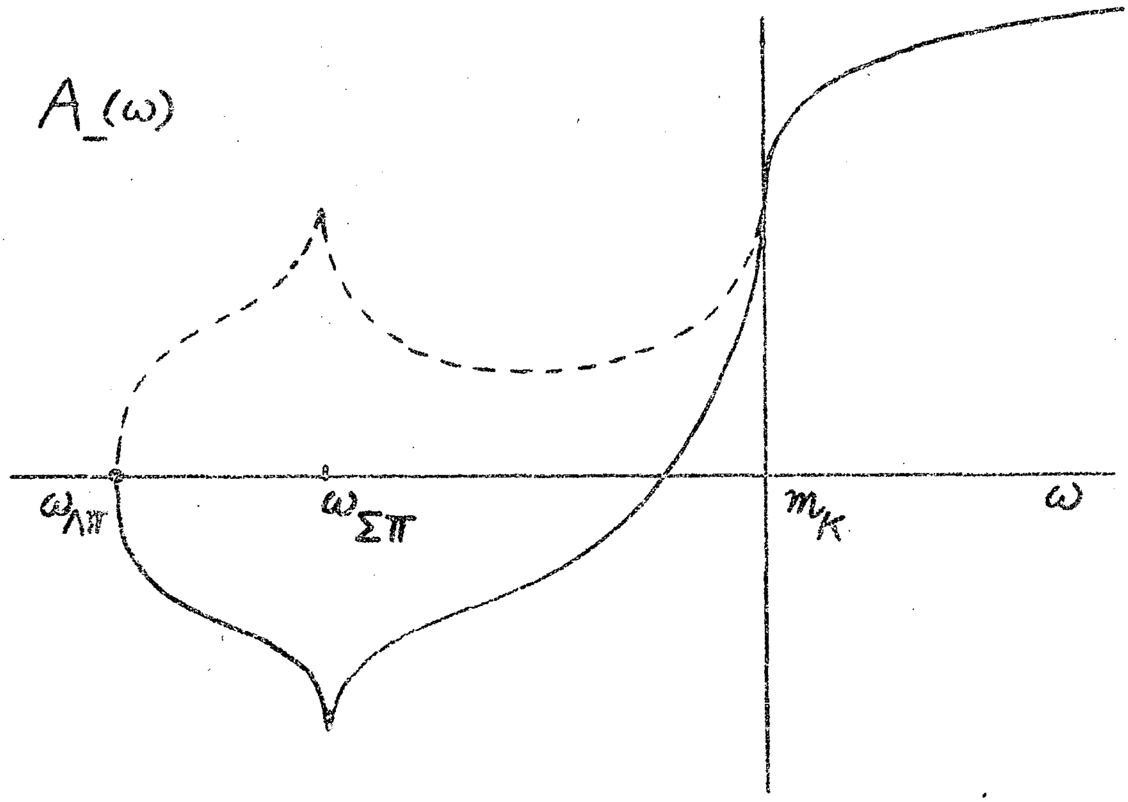
In expressions (4.1), $D_{\pm}$ denote the real parts of the $K^{\pm}$-proton forward scattering amplitude, and A_{-} denotes the imaginary part of the K^{-} forward scattering amplitude. The unphysical region here includes a continuous stretch from $\omega_{\Lambda\pi}$ to m_K, corresponding to the fact that $\Lambda\pi$ and $\Sigma\pi$ states of positive kinetic energy can still be reached from states of unphysical energy for the K^{-}-proton system.

Now let us discuss some of the difficulties in the use of these dispersion relations. Firstly, it has been pointed out to me by S. Tuan (Berkeley) and by R. Oehme (Chicago) that the present techniques for establishing the validity of dispersion relations fail to achieve their purpose for the K-meson case, even for forward scattering. The mass inequalities are only just violated: if the Λ particle were about 5% heavier the necessary condition would be satisfied. However, not all that we know about the possible intermediate states has been put into this calculation and there is every reason to expect that they will be rigorously demonstrated in due course.

Next, the unphysical region may be quite complicated and there is little that we can learn about this from experiments in the physical region. Although A_{-} is required to be positive in the physical region, it is permitted to take on negative values in the unphysical region. We have already mentioned that cusps generally occur at $\omega = m_K$, for

$$A_{-}(\omega) = \frac{1}{2}\left(\frac{b_0 + k(b_0^2 + a_0^2)}{1 + 2kb_0 + k^2(a_0^2 + b_0^2)} + \frac{b_1 + k(b_1^2 + a_1^2)}{1 + 2kb_1 + k^2(a_1^2 + b_1^2)}\right). \quad (4.3)$$

across $\omega = m_K$. For $\omega < \omega_{\Lambda\pi}$, A_- is zero, of course; at $\omega = \omega_{\Lambda\pi}$, A_- has a branch point of type $(\omega - \omega_{\Lambda\pi})^{\frac{1}{2}}$ and may become either positive or negative, and there will also be some kind of cusp in A_- at $\omega = \omega_{\Sigma\pi}$, where the competing $(\Sigma + \pi)$-states begin to play a role. To illustrate these points, two of the many possible curves for $A_-(\omega)$ in the unphysical region are shown on the following figure.



In the form (4.1), the integrals over the cross sections do not converge if the cross sections σ_+ and σ_- approach constant values at infinite energy, and some subtractions will clearly be necessary. Goebel[14] and Matthews and Salam[13] have suggested making use of the form obtained by subtracting Eq. (4.1b) from (4.1a). With $\omega = m_K$, this has the form (assuming Λ and Σ parities to be the same)

$$m_K\left[D_+(m_K)-D_-(m_K)\right]-\frac{m_K^2}{4\pi^2}\int_{m_K}^{\infty}\frac{d\omega'}{k'}(\sigma_+'-\sigma_-')-\frac{m_K^2}{\pi}\int_{\omega_{\Lambda\pi}}^{m_K}\frac{d\omega'}{k'^2}A_-(\omega')$$

$$= -2p\left[X(\Lambda)+X(\Sigma)\right] \tag{4.4}$$

$$\approx \begin{cases} +2(g_\Lambda^2+g_\Sigma^2)/4\pi, & \text{for scalar K meson,} \\ -2(m_K/2m_p)^2(g_\Lambda^2+g_\Sigma^2)/4\pi, & \text{for pseudoscalar K meson.} \end{cases}$$

Here the hope is that $\int d\omega'(\sigma'_+ - \sigma'_-)/k'$ may be convergent. Pomeranchuk[15] has shown that it is reasonable to expect $(\sigma_+ - \sigma_-) \lesssim C \log \omega$, but this is not sufficient to ensure convergence. Even if the integral is convergent, this convergence is slow and its value will then depend on contributions from energies far beyond those for which experimental information exists. Even so, it is of interest to follow a little the arguments of Goebel and of Matthews and Salam. As far as the experiments go, σ_- is larger than σ_+, and it is reasonable to expect this to continue up to fairly high energies, since the K^- interaction has many more reaction channels available than the K^+ interaction; it seems likely that the first integral (if it exists) is positive. Now $D_+(m_K)$ is known to be negative, but fairly small, whereas $D_-(m_K)$ is quite large. The second integral, over the unphysical region, is unknown even in sign, but may be expected to be moderately small (as is the case for simple extrapolations from the physical region). Hence if $D_-(m_K)$ were negative--that is, the K^--proton interaction were repulsive--then the expression (4.4) would almost certainly be positive and the K-meson parity would be even. But if D_- is positive, for an attractive K^--proton interaction, the matter becomes more quantitative, but it appears rather likely that (4.4) would be negative corresponding to a pseudoscalar K meson. However, owing to the question of the validity of the relation (4.4), there is some doubt concerning its use in this way for determination of the K-meson parity.

A modification of the relation (4.4), analogous to the method proposed by Haber-Schaim[16] for the use of pion-nucleon dispersion relations, has been proposed by Igi.[12] After the approximations $\omega_\Lambda = 0.11\ m_K \approx 0$, $\omega_\Sigma = 0.27\ m_K \approx 0$, Igi was led to the expression

$$\frac{1}{2}\omega(D_- - D_+) - \frac{\omega^2}{\pi}\int_{\omega_{\Lambda\pi}}^{m_K}\frac{d\omega' A_-(\omega')}{\omega'^2-\omega^2} - \frac{\omega^4}{4\pi^2}\int_{m_K}^{\infty}\frac{k'd\omega'}{\omega'^2-\omega^2}\left(\frac{\sigma_-'-\sigma_+'}{\omega'^2}\right)$$

$$= -p\left[X(\Lambda)+X(\Sigma)\right] + \frac{\omega^2}{4\pi}\int_{m_K}^{\infty}\frac{k'd\omega'}{\omega'^2}(\sigma_-'-\sigma_+'). \qquad (4.5)$$

If the left-hand side of this expression is plotted as a function of ω (note that the integral over cross sections is now rapidly convergent), it should follow a straight line which may be extrapolated to $\omega = 0$ to obtain the desired indication of the K parity and the K-hyperon coupling constants. However, the unknown integral over the unphysical region will not be unimportant, also the extrapolation to $\omega = 0$ is a very large step relative to the energy range over which experimental data are at present available, so that the use of this interesting relation may be rather uncertain.

However, Igi has also suggested the use of the following form,[12]

$$\omega\left[D_+(\omega) - \tfrac{1}{2}(m_K+\omega)D_+(m_K) - \tfrac{1}{2}(m_K-\omega)D_-(m_K)\right] - \frac{1}{4\pi^2}\int_{m_K}^{\infty}\frac{d\omega'}{k'}\left(\frac{\sigma_+'}{\omega'-\omega} + \frac{\sigma_-'}{\omega'+\omega}\right) - \frac{1}{\pi}\int_{\omega_{\Lambda\pi}}^{m_K}\frac{d\omega' A_-(\omega')}{k'^2(\omega'+\omega)} \qquad (4.6)$$

$$= -p\left[X(\Lambda) + X(\Sigma)\right],$$

which is weighted against contributions from the unphysical region. The cross section integrals again converge rather rapidly and depend on the K^+ cross sections to a far greater extent, owing to the large denominator that goes with σ_-. To pay for these advantages, the formulae are correspondingly more dependent on the energy dependence of the forward scattering amplitude for K^+-p scattering. Igi considers two possibilities:

(a) σ_+ is constant at 15 mb up to $\omega = 4m_K$. In this case he finds that if D_- is attractive then the expression on the left of Eq. (4.6) is positive, corresponding to a pseudoscalar K meson (assuming Σ and Λ to have the same parity) with $(g_\Lambda^2 + g_\Sigma^2)/4\pi \simeq 4$, whereas, if D_- is repulsive, the K meson must be scalar, with $(g_\Lambda^2 + g_\Sigma^2)/4\pi \simeq 0.8$. To obtain these conclusions, the known cross sections had to be extrapolated far beyond our region of knowledge, and the unphysical contribution was estimated by a simple smooth extrapolation into the region $\omega < m_K$, an estimate which did not contribute at all strongly to the final expression. It is difficult to be very definite about how sensitive such calculations are to the cross section assumptions without having had the opportunity of repeating the calculations oneself--some of the data used at low energies deviate considerably (for example, the values for σ^-_{abs}) from our present knowledge at this meeting, but on the other hand, there appears to be some degree of compensation between the various terms when the K^- cross sections in the low-energy region are modified. It is of interest to note that, on the basis of the same cross-section assumptions, together with the additional strong assumption that $(\sigma_- - \sigma_+)$ approaches 0 reasonably quickly with increasing ω, the work of Goebel and Matthews and Salam reached essentially the same conclusion.

(b) a second assumption considered was that σ_+ followed a smooth curve running through the K^+-p scattering cross sections published by the Michigan bubble chamber group,[17] which were consistent with a cross section falling off by a factor of 2 between 50 Mev and zero energy. In this case, the expression was found to be negative for either attractive or repulsive D_-, corresponding to a scalar K meson and $(g_\Lambda^2 + g_\Sigma^2)/4\pi \approx 4$.

My main purpose in discussing this matter in detail when the data are at such a preliminary stage is to stress the urgent need for data on K^+ and K^- cross sections for interaction with protons over wide energy ranges, up to energies very much above those for which data are now available. For the use of Igi's relation (4.6), more accurate studies on K^+-proton scattering cross sections and angular distributions at low energies (say 50 Mev and below) would be very helpful. Also, as Matthews and Salam have emphasized to me, it would be of very great interest to obtain data bearing on the K-neutron cross sections, for example on total cross sections at higher energies by scattering off deuterium; information on K^--neutron forward scattering can be deduced from sufficiently detailed data on K^--proton interactions, on the lines indicated above for the discussion of the K^--proton data now available, but information on K^+-neutron forward scattering will clearly need more direct experiments. From this information it would be possible to draw deductions concerning the $(K\Sigma N)$ interaction alone, which could then be used in conjunction with the K-proton scattering analysis by dispersion relations to lead to clearer conclusions on the parity and strength of the $(K\Lambda N)$ interaction.

5. Capture of K^- Mesons by Deuterium

Next, we turn to discuss some points concerning the new data on K^--deuteron interactions, which Tripp has just reported from the work of the Alvarez bubble chamber group. These data have given us our first clear and very welcome verification of a charge-independence equality. There has not been sufficient time for the measurements to go so far as to distinguish Σ^0 and Λ events in the cases where this may be possible. However, it is of interest to note that there are only $7(\Sigma^- \pi^0 p)$ events recorded compared with $48(Y^0 \pi^- p)$ events. Since charge independence requires the number of $(\Sigma^- \pi^0 p)$ and $(\Sigma^0 \pi^- p)$ to be equal, it appears that the relative production of Σ^0 and Λ from K^--neutron interactions is $\sim 7/41$, so that Λ production is dominant here, in contrast to the situation for K^--proton captures. This is rather reminiscent of the conclusion reported by the Bern group[18] that production appears to be comparable with the total Σ production for K^--interactions in flight at 90 Mev. However, this result for deuterium could quite well arise from secondary interactions in which a Σ particle produced interacted with the neighbor nucleon, transforming to a Λ particle and causing an increase in the Λ/Σ ratio observed.

The Σ^-/Σ^+ ratio appears to be rather lower than that known for K^--proton capture, although some increase might have been expected, owing to the additional neutron-capture events. Since the energy dependence of the 1s capture scarcely comes into play here, it may be necessary to attribute this change (and perhaps part of the increase in Λ production) to an greater rate of K^- capture from the 2p state in deuterium than from the 2p state in hydrogen. This may result from the additional capture channels now available through the neutron interactions, as well as from the larger reduced mass in the K^--deuterium system. For the cases discussed by Fujii and Marshak,[9] it turned out that the rate of 2p absorption in deuterium was about three times as large as the rate of 2p absorption in hydrogen. To clarify the situation, it would be desirable to study in some detail the energy spectra and correlations in K^--d reactions. Okun and Smushkevich have submitted a theoretical study of such correlations on the basis of the impulse approximation, but taking into account elastic scattering between the final baryons;[19] however, there are no data available at present, except in one negative respect, namely that no bound states of type Λp or $\Sigma^- n$ have been detected among the K^--d reaction products. Estimates for the rate of formation of such bound states (if they exist) were made by Pais and Treiman some time ago.[20] Generally speaking, they found that these bound states should be formed about as frequently as the corresponding unbound systems, for a binding energy of 1 Mev. For smaller binding energies B, the branching ratio falls off about as $\sqrt{B}$ On this basis, the absence of these states in the K^--d reactions implies that their binding energies (if positive) are unlikely to exceed several tenths Mev. One important qualification is necessary: if these systems were bound only in the ^1S state (and we shall see reason later to believe that this is the most favorable state for binding), then capture of a pseudoscalar K meson from the 1s state of deuterium could not give rise to this bound state, owing to the selection rules of angular momentum and parity conservation, although capture from the 2p state could.

Finally, the rate of two-nucleon capture modes in deuterium--e.g., $K^- + d \rightarrow \Sigma^- + p$ --is of great interest, in view of the observation that the two-nucleon capture of the K^- meson takes place in perhaps 15% of K^--nuclear capture events in emulsion. From experience with the process of two-nucleon capture for pions, it would seem reasonable to expect this figure to imply that about 2 to 3% of K^- captures in deuterium should involve both nucleons. This is not at all excluded by the present preliminary data; when the data have increased to the point where a statistically significant comparison can be made between the deuterium and the emulsion data on this point, this will be of considerable interest for nuclear physics, since there has been no test to date of the correlation functions in deuterium and in complex nuclei for such large momentum transfer. It is of interest to add that the perturbation calculations of Fujii and Marshak[9] lead to a branching ratio $\sim 0.1\%$ for two-nucleon capture in the case of pseudoscalar K^- capture from s-states, and $\sim 10\%$ for scalar K^- capture; for 2p capture, the proportions were $\sim 1\%$ for pseudoscalar K-meson and $\sim 0.1\%$ for scalar K-meson capture.

6. Hyperon-Nucleon Interactions

Strong interactions between a hyperon and a nucleon may be transmitted by the exchange of K mesons and of pions between them. Exchange of an odd number of K mesons and any number of pions involves the transfer of strangeness between the particles and gives rise to an "exchange" interaction, whereas exchange of pions alone or with an even number of K mesons gives an "ordinary" interaction. Now, since the hyperons are coupled at least moderately strongly to K mesons and nucleons, and the nucleons very strongly with the pions, there must certainly exist pion-hyperon interactions of the type

$$\Lambda \longleftrightarrow \Sigma + \pi \, , \qquad \Sigma \longleftrightarrow \Sigma + \pi \, . \tag{6.1}$$

The forms of these Yukawa-type interactions are determined by charge independence, and the coupling parameters will be denoted by $g_{\Lambda\Sigma}$ and $g_{\Sigma\Sigma}$, respectively. If these interactions (6.1) are reasonably strong, they will dominate over the K-meson interactions in hyperon-nucleon interactions at low energies, owing to the longer range associated with the pion-exchange process.

At this conference last year, Gell-Mann[1] and Schwinger[2] each put forward the hypothesis of a "global symmetry" involving a universal pion-baryon coupling. In order to allow a comparison between the pion coupling for the $T = \frac{1}{2}$ nucleon doublet and the Λ, Σ multiplets of integer isotopic spin, the Λ, Σ states were rearranged into two doublets,

$$Y = \begin{pmatrix} \Sigma^+ \\ (\Lambda - \Sigma^0)/\sqrt{2} \end{pmatrix} , \quad Z = \begin{pmatrix} (\Lambda + \Sigma^0)/\sqrt{2} \\ \Sigma^- \end{pmatrix} . \tag{6.2}$$

Of course this makes sense only if the Λ and Σ multiplets have the same parity, as is here assumed. In terms of these Y and Z doublets, the form of the pion-baryon coupling was

$$G\left\{ \bar{N} \underset{\sim}{\tau} N \cdot \underset{\sim}{\pi} + \bar{Y} \underset{\sim}{\tau} Y \cdot \underset{\sim}{\pi} + \bar{Z} \underset{\sim}{\tau} Z \cdot \underset{\sim}{\pi} + \bar{\Xi} \underset{\sim}{\tau} \Xi \cdot \underset{\sim}{\pi} \right\}, \quad (6.3)$$

where G is the known pion-nucleon coupling parameter and these interaction terms each have the same form in spin and space variables, so that the Y doublet and the Z doublet each behaves in the same way as the nucleon doublet as far as pion interactions are concerned. The use of this form of coupling is equivalent to the choice $g_{\Lambda\Sigma} = g_{\Sigma\Sigma} = G$. It is also possible to consider the choice $g_{\Lambda\Sigma} = g_{\Sigma\Sigma} = -G$, but it appears rather unlikely (see below) that this can be compatible with the experimental facts.

This "global symmetry" obviously cannot correspond completely with the observed facts, of course, since there is a mass difference of $\sim$80 Mev between Σ^0 and Λ states; this symmetry can hold valid only to the approximation that the mass difference $\Delta = m_{\Sigma^0} - m_\Lambda$ can be neglected. Further, if the K couplings were also very strong, they would be expected to distort this symmetry between the pion-bayon couplings quite severely, so that the proposal was put forward that the K couplings might be regarded as a moderately strong interaction whose effects on the pion-hyperon interaction might be neglected as a first approximation. This appears fairly reasonable, as a coupling strength of about one-tenth of that for the pion-nucleon interaction appears reasonable for the KYN interactions, assuming these to be of pseudoscalar form.

It has appeared attractive to seek for a possible symmetry between the strong interactions $N \leftrightarrow \Lambda + K$, $N \leftrightarrow \Sigma + K$ also, for example it was often suggested that $g_{\Lambda K} = g_{\Sigma K} = g$ might hold. This would then lead to the interaction form

$$\sqrt{2}\, g \left\{ \bar{N}(Y K^o + Z K^+) + h.c. \right\}. \tag{6.4}$$

However, it has been pointed out by Pais[21] that this possibility is excluded by the experimental data if the pion-baryon symmetry $g_{\Lambda\Sigma} = g_{\Sigma\Sigma}$ holds. For example, since K^+ is coupled only with Z, and since there are no couplings that mix Z and Y, it is clear that there are no interactions that can lead to the charge-exchange process

$$K^+ + n \rightarrow K^o + p, \tag{6.5}$$

whereas this process is well known. Another counterexample is the reaction

$$K^- + p \longrightarrow \Sigma^+ + \pi^-. \tag{6.6}$$

Here the K^- is coupled only with Z states, and the pion-hyperon couplings cannot modify this. Hence the Σ^+ state (a Y state) cannot be reached from an initial K^- particle and therefore this reaction is predicted to be forbidden, contrary to the evidence. Therefore if the pion-baryon symmetry $g_{\Lambda\Sigma} = g_{\Sigma\Sigma}$ holds, then we have $g_{\Lambda K} \neq g_{\Sigma K}$.

The limitation that this pion-baryon symmetry can be fully effective only in situations for which the Σ^o-Λ mass difference is unimportant severely restricts its applicability to many strange-particle reactions without rather detailed considerations, so that the possibilities for clear-cut tests of this symmetry hypothesis seem relatively few at present. For this reason it is of interest to mention one situation in which this symmetry principle makes a rather clear prediction, although there are no data available yet for a check. This concerns the magnetic moments of the Σ and Λ particles. It appears reasonable to expect these to receive much greater contributions from the pion currents than from the K currents within each baryon. For the terms associated with pion processes alone, the prediction is that $\mu(\Sigma^+) = \mu(Y_+)$ should just be $\mu(p)$, the proton magnetic moment, whereas $\mu(\Sigma^-)$ should equal $-\mu(p)$. In this approximation, the Σ^0 and Λ moments should each be zero and the matrix element for the process $\Sigma^0 \rightarrow \Lambda + \gamma$ should have the form $\mu_t \underset{\sim}{\sigma} \cdot \underset{\sim}{H}$, where the transition moment μ_t is the negative of the neutron magnetic moment.

Now we shall look at several situations in which the pion-baryon interaction is of obvious importance and for which there exists a little empirical evidence. The first of these concerns the interactions between Σ particles and nucleons. In the low-energy region (where the pion interactions tend to be dominant) these interactions may be represented by potentials, to a sufficiently good approximation at present. The neglect of the mass difference Δ in intermediate states will affect the calculation of these potentials relatively little (by less than 10%), since the intermediate states which contribute most have relatively high energies. To this approximation, the Y-N and Z-N potentials are predicted to be identical with the N-N potentials. On this basis one may then hope to proceed phenomenologically and to deduce the hyperon-nucleon potentials from the evidence on nucleon-nucleon scattering. Comparison of these potentials with the data on hyperon-nucleon interactions would then provide a test for the global symmetry hypothesis.

For example, the $\Sigma^+ p$ or $\Sigma^- n$ nuclear forces are predicted to be identical with those for the p-p or n-n systems. For the ^1S state, the $\Sigma^+ p$ or $\Sigma^- n$ potential is predicted to be the same as the ^1S n-p or p-p nuclear force and therefore almost resonant at zero energy (see below), but for the ^3S state, the prediction is by no means unambigous, since the Pauli principle forbids the ^3S state for the p-p or n-n systems. Only triplet states with odd ℓ are permitted for identical particles, and it is therefore necessary to extrapolate the interaction in these states to the triplet states with even ℓ. In principle, this extrapolation is not possible, since it is possible to construct a potential which vanishes for states of low odd ℓ, although finite for states of low even ℓ (note that position-exchange terms are excluded in the potential so far as it is due to pions alone), although such potential terms do not appear very reasonable from the viewpoint of simple meson theory. With a potential limited to simple static and spin-orbit forms, the ^3S Σ^+-p potential may be obtained unambiguously if the odd-ℓ triplet potential is sufficiently well established. Bryan et al.[22] have made calculations of the scattering cross sections and polarization properties for Σ^+-p scattering on the basis of the Marshak-Signell potential, which fits well the p-p scattering below 200 Mev. The main points of physical interest brought out by this calculation are that there is no reason to expect the Σ^+-p angular distribution to be symmetrical about 90°, a forward peaking being a more reasonable expectation, and that quite strong polarization effects are likely to occur in Σ^+-p scattering. However, this calculation also illustrates the practical difficulties of an extrapolation from the observed singlet p-p scattering to the ^3S Σ^+-p potential. Since the triplet p-p scattering is due to P states and higher, it is not sensitive to the form of the triplet potential at short distances, whereas the properties of the S-wave scattering are quite strongly affected by the form of the potential at short distances. The Marshak-Signell potential actually predicts a ^3S bound Σ^+-p state, but it is found possible to modify the triplet-state potential at short distances sufficiently to remove this bound state, without affecting appreciably the fit of the potential for the p-p scattering.

The question whether bound states should be expected to exist for the Σ^--n and Σ^+-p systems is an interesting one. At present there is no clear empirical evidence to indicate the existence of such bound states; in fact, as we have seen above, the evidence from K^--d capture experiments is that a ^3S bound state for Σ^--n is very unlikely, although a ^1S bound state is not yet excluded. On the basis of the global symmetry hypothesis, meson-theoretical calculations by Lichtenberg and Ross[23] and by Ferrari and Fonda[24] indicate that the Σ^--n potential has most attraction in the ^1S state, for which the potential is closely related to the nucleon-nucleon potential. Ferrari and Fonda have pointed out that, with the static limit $G\, \sigma \cdot p / 2M_B$ for the universal interaction $G\gamma_5$, the Σ^--n potential may be expected to be weaker than the N-N potential, since the hyperon mass M_B is about 25% greater than the nucleon mass. This is partly balanced by the appearance of a larger reduced mass in the Σ-N system, but the net effect is that the Σ-N system will be further from binding than the N-N system, as far as forces due to pion exchange are concerned. Ferrari and Fonda emphasize that K-meson exchange would contribute an additional attractive interaction for a pseudoscalar K meson (repulsive for a scalar K meson), so that observation of a Σ^--n bound state would still not conflict with the global-symmetry hypothesis.

When Σ^- particle comes to rest in hydrogen, the Σ^--p capture reaction is observed to lead to the reactions

$$\Sigma^- + p \longrightarrow \Sigma^0 + n, \tag{6.7a}$$

$$\Sigma^- + p \longrightarrow \Lambda + n, \tag{6.7b}$$

in the ratio $\Lambda/\Sigma^0 \approx 2$. Now since the Σ^- particle is a Z particle, the pion-hyperon interactions cannot lead it to the $(\Lambda^0 - \Sigma^0)/\sqrt{2}$ state, which belongs to the Y doublet, so that the global symmetry principle would appear to suggest

$$\langle \Sigma^- p | S | \frac{\Lambda - \Sigma^0}{\sqrt{2}} n \rangle = 0 = \frac{1}{\sqrt{2}} \left\{ \langle \Sigma^- p | S | \Lambda n \rangle - \langle \Sigma^- p | S | \Sigma^0 n \rangle \right\}. \tag{6.8}$$

On this basis, one might expect the matrix elements for the processes (6.7a) and (6.7b) to have the same form, in which case the ratio Λ/Σ^0 would be expected to have the value $(p_\Lambda/p_\Sigma)^{2\ell+1} \approx (4)^{2\ell+1}$ for capture from a state of the Σ^--p atom of orbital angular momentum ℓ, the outgoing momenta in Processes (6.7a) and (6.7b) being $P_\Lambda \sim 280$ Mev/c and $P_\Sigma \sim 70$ Mev/c, respectively. This conclusion does not agree with the data, but the situation is more complicated than this simple argument assumes. To a good approximation, the argument of Eq. (6.8) may be used to deduce that the potentials $\langle \Sigma^- p \,|\, V(r) \,|\, \Sigma^0 n \rangle$ and $\langle \Sigma^- p \,|\, V(r) \,|\, \Lambda n \rangle$ are equal, on the basis of global symmetry. However, the wave length of the outgoing Λ particle is shorter than the range $(\hbar/m_\pi c)$ of these potentials by a factor of about 2, whereas the Σ^0 wave length is longer than this range by about the same factor.

This means that the matrix element $M_\ell(p) \sim \int r^\ell\, V(r)\, j_\ell(pr) d_3 r$ relevant to these processes when the potentials are treated in lowest-order approximation has to be evaluated for quite different momenta in the two cases and may be expected to be a good deal smaller for Process (6.7b) than for (6.7a); in fact, a first-order calculation on these lines leads to a Λ/Σ^0 ratio of less than unity. For capture at rest, the direct application of the global-symmetry argument to the matrix elements for Processes (6.7a) and (6.7b), as given following Eq. (6.8), would be valid only if Δ were less than $\mu^2/2M$. In a more exact treatment of the matrix elements from the potentials, the hyperon-nucleon system must be discussed in terms of states of definite isotopic spin T, since the global symmetry is broken down by the Σ-Λ mass difference. The Σ^--p system is then split into a $T = 3/2$ state, for which the potential is the nucleon-nucleon $T=1$ potential V_1 and for which the transformation (6.7b) is not possible, and a $T = \frac{1}{2}$ state, for which the potential is represented by a matrix between Σ and Λ states, expressible in terms of the nucleon-nucleon potentials V_0 and V_1. For the $T = \frac{1}{2}$ potential, this form is explicitly

$$\begin{pmatrix} V_{\Sigma\Sigma} & V_{\Sigma\Lambda} \\ V_{\Lambda\Sigma} & V_{\Lambda\Lambda} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(V_1 + 3V_0) & \frac{\sqrt{3}}{4}(V_0 - V_1) \\ \frac{\sqrt{3}}{4}(V_0 - V_1) & \frac{1}{4}(3V_1 + V_0) \end{pmatrix} \qquad (6.9)$$

The potential V_1 is strongly attractive, so that there is strong scattering in the $T = \frac{3}{2}$ channel. In the $T = \frac{1}{2}$ channel, the mass difference Δ is important and a pair of simultaneous equations are to be solved, to give the amplitudes of the outgoing Σ^0-n and Λ-n waves for the $T = \frac{1}{2}$ state. If, for example, the Σ^--p $T = \frac{3}{2}$ state were just in resonance at zero energy for the capture state considered, then possible only through the nonresonant $T = \frac{1}{2}$ channel. Detailed calculations of this kind have been carried through by Lichtenberg and Ross[23] and by Weitzner,[25] but from these remarks it will be clear that the conclusions reached will depend a great deal on the precise treatment of the hyperon-nucleon potentials. Also the conclusions will depend a good deal on how much capture occurs from the 2p state of the Σ^--p atom; owing to the long range of the hyperon-nucleon potential, it appears probable that p-state capture may be predominant. The situation is quite complicated and a prediction of the Λ/Σ^0 ratio in Reaction (7) cannot be made on the basis of global symmetry alone, without detailed consideration of the effect of the mass difference Δ. Global symmetry can make a clear statement about these reactions only for Σ^--p reactions of high energy (so that the final kinetic energies are little affected by the mass difference) and of low-momentum transfer (so that the reactions do not explore the region of the potential where K-meson processes contribute).

In a first-order calculation of the Σ-nucleon potential, as given by Lichtenberg and Ross[23] and by Ferrari and Fonda,[24] this potential is found to be attractive in the ^1S and ^3S $T = \frac{3}{2}$ states and in the ^3S $T = \frac{1}{2}$ state for $g_{\Sigma\Lambda} = g_{\Sigma\Sigma} = +G$, but repulsive for the ^1S $T = \frac{1}{2}$ state. With $g_{\Sigma\Lambda} = g_{\Sigma\Sigma} = -G$, the signs of these first-order potentials are to be reversed. Gilbert and White[26] have argued from a comparison of the pion spectra observed in K^- capture in emulsion nuclei with and without an accompanying $\Sigma^{\pm}$ emission that the $\Sigma^{\pm}$-nuclear potential is attractive and about 30 to 40 Mev deep. This conclusion appears difficult to reconcile with coupling parameter $-G$ for the pion-hyperon coupling, and appears to require the choice $+G$ as made by Gell-Mann and Schwinger.

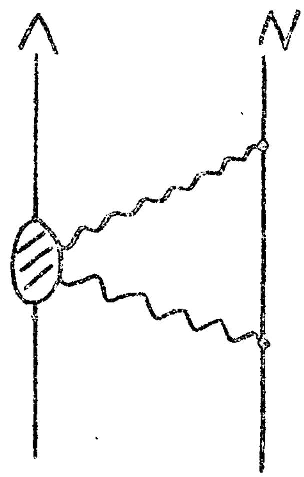
Another situation of interest for the global-symmetry hypothesis is the K^--p capture reaction, since the pion-hyperon interactions in the final state affect the branching ratios for these reactions. The most extensive discussion on this basis has been given by Amati and Vitale.[27] These authors note that the final pion-hyperon system may be expressed in terms of πY and πZ scattering states, and that the scattering properties of the πY and πZ states are identical, being characterized by the same $T = \frac{1}{2}$ and $\frac{3}{2}$ scattering phases (equal to the pion-nucleon phase shifts with $g_{\Lambda\Sigma} = g_{\Sigma\Sigma} = G$). The $T = 0$ $\Sigma + \pi$ state corresponds to $T = \frac{1}{2}$ πY and πZ final states; for $K^- + p$ capture from rest the phase of this amplitude (assuming time-reversal invariance) is precisely the $T = \frac{1}{2}$ phase shift $\delta_{1/2}$. Two orthogonal $T = 1$ pion-hyperon states are then formed, each of which corresponds to $T = \frac{3}{2}$ πY and πZ systems; the two amplitudes leading to these final states then both have the phase $\delta_{3/2}$. After forming the expressions for the branching ratios in terms of these three real amplitudes and the relative phase $(\delta_{1/2} - \delta_{3/2})$, Amati and Vitale show that they imply the inequality

$$(\Sigma^+ + \Sigma^- - 4\Sigma^0)^2 + 4(\Lambda\Sigma^0 - \Sigma^+\Sigma^-) \geq 0 . \tag{6.10}$$

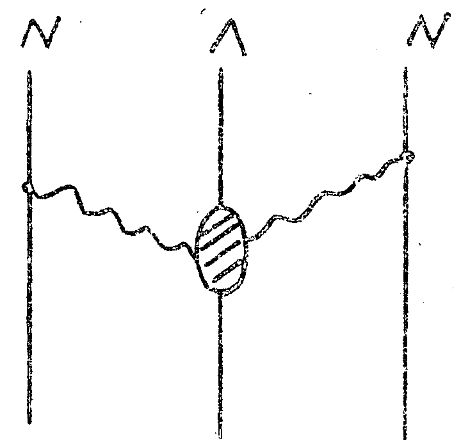
The standard ratios $\Sigma^-:\Sigma^0:\Sigma^+:\Lambda \sim 1:\frac{1}{2}:\frac{1}{2}:\frac{1}{8}$ observed for K^--p capture from rest fail to satisfy this inequality, giving Expression (6.10) the value $-3/2$. This explains in a general way why Kawarabayashi[28] and Yamaguchi,[29] who carried through detailed calculations on the static model for pseudoscalar and for scalar mesons respectively (assuming lowest-order perturbation theory for the K interaction), were unable to find agreement with the data for any values of the free parameters in their calculations.

It is unclear how significant this discrepancy is to be considered. For one thing, the violation of inequality (6.10) is relatively sensitive to the proportion of the Σ^0 reaction, which is relatively poorly known; for another, it is far from clear at present how large an effect the Σ-Λ mass difference will produce in the final state. However, the derivation of the inequality (6.10) does not involve the assumption that the KYN interactions are weak, but only that the capture reaction occurs from a state of definite total angular momentum, either from a bound state or from a continuum state of very low energy.

Finally, we turn to the Λ-nucleon interaction below the Σ threhold, for which there is a good deal of evidence from observations on Λ-hyperfragments. Here the two-body Λ-nucleon forces are due firstly to exchange of two pions, the simplest exchange compatible with T=0 for the Λ particle, and of three pions or more, as well as to the exchange of K mesons with or without pion exchange. The forms to be expected for these potentials have been calculated in the static limit, in varying degrees of approximation and for the various $K\Lambda$, $K\Sigma$, and $\Sigma\Lambda$ parities, by Dallaporta and Ferrari,[30] by Lichtenberg and Ross,[23] and by Ferrari and Fonda.[24] These calculations will be discussed briefly after some remarks on the phenomenological analysis, but we may anticipate this comparison here by the remark that it appears necessary to assume that the forces are due very considerably to the pion-exchange processes, as required by the global-symmetry hypothesis.



(A) Two-body potential arising from two-pion emission.



(B) Three-body potential arising from two-pion emission.

Several authors--Spitzer,[31] Weitzner,[25] and Bach[32]--have pointed out that in this situation, three-body forces may be expected to occur in the same order of approximation of perturbation theory (see above figures). In lowest approximation they will necessarily have the form

$$V_3(1, 2, \Lambda) = \sigma_1 \cdot \sigma_2 \ \tau_1 \cdot \tau_2 \, W(r_{\Lambda 1}, r_{\Lambda 2}) + \text{noncentral terms}, \qquad (6.11)$$

where

$W(r_{\Lambda 1}, r_{\Lambda 2}) \sim \exp(-\mu r_{1\Lambda} - \mu r_{2\Lambda})$ when the separations $r_{\Lambda 1}$, $r_{\Lambda 2}$ are both large. Next we note that if the two nucleons have s-wave relative motion, then $\sigma_1 \cdot \sigma_2 \ \tau_1 \cdot \tau_2 = -3$, so that these three-body forces do not depend on the spin or isotopic spin of the nuclear state for interaction of the Λ particle with a pair of s-shell nucleons. The various calculations reported for these three-body forces have given rather different results. Weitzner's calculation assumes an interaction $F\,\bar{\Lambda}\Lambda\,\underline{\pi}\cdot\underline{\pi}$ to be responsible for both Processes (A) and (B) of the above figures; in this case F must be taken negative to give an attractive Λ-N potential. The central part of the corresponding three-body potential is then repulsive when $r_{\Lambda 1}$ and $r_{\Lambda 2}$ are large relative to r_{12} but, when either $r_{\Lambda 1}$ or $r_{\Lambda 2}$ is small, this three-body potential is attractive over a considerable domain. Spitzer begins from a pseudovector $\Lambda \longleftrightarrow \Sigma + \pi$ interaction and finds three-body forces with an attractive central part, whereas Bach's calculation on a similar basis found a weakly attractive central term. In each calculation, the three-body potential obtained had a complicated noncentral form. The implications of the existence of such three-body potentials are discussed below.

Dalitz and Downs[33] have given a phenomenological analysis of hypernuclear binding energies based on two-body forces alone. The Λ-nucleon potentials were assumed to be central and of Gaussian shape; these simple potentials are to be understood as potentials equivalent to the actual Λ-nucleon potentials as far as their low-energy scattering properties are concerned. For the ${}_{\Lambda}He^5$ system, the result obtained for the total Λ-nucleon potential (see Table II) seems fairly reliable owing to the rigidity of the alpha particle and our detailed knowledge from electron-scattering experiments of the nuclear parameters for He^4. For the ${}_{\Lambda}H^4$, ${}_{\Lambda}He^4$ doublet, the value given is somewhat less certain since there is no direct measure available for the nuclear parameters of H^3 or He^3--the value given for U_3 in the table was obtained by assuming values which seem reasonable for the H^3, He^3 radii. At this point it

TABLE II

Volume integrals U_n of Λ-nuclear potentials (unit Mev$(10^{-13}\ cm)^3$) deduced from the Λ binding energies, for the range parameters $1/m_K$ and $1/2m_\pi$ for the Λ-nucleon two-body potential.

Hypernucleus	B_Λ (Mev)	$U_n(1/m_K)$	$U_n(1/2m_\pi)$	U_n
${}_\Lambda H^3$	~ 0.25	490 → 420	785 → 670	$\frac{3}{2} U_s + \frac{1}{2} U_t$
${}_\Lambda H^4$, ${}_\Lambda He^4$	1.85	600 - 700	820 - 920	$\frac{3}{2} U_s + \frac{3}{2} U_t$
${}_\Lambda He^5$	2.5	695	910	$U_s + 3 U_t$
${}_\Lambda Li^7$	4.5	1650 ± 150	1915 ± 150	
${}_\Lambda Li^8$	5.5	1660 ± 165	1930 ± 180	

Λ-nucleon potentials U_s, U_t (neglecting three-body potentials)

U_s	228 (s = 0.71)	390 (s = 0.71)
U_t	156	174

is of interest to remark that it seems definite that the ${}_{\Lambda}H^4$, ${}_{\Lambda}He^4$ doublet observed has zero spin[34]--this conclusion follows from the high proportion (about 45%) observed for the two-body decay modes among ${}_{\Lambda}H^4$ decay events. This means that the singlet Λ-nucleon potential must be more attractive than the triplet potential; this conclusion is not affected by the possible presence of three-body forces, since to a good approximation these do not depend on the Λ spin.

The hypertriton ${}_{\Lambda}H^3$ is of special interest. It has very low B_{Λ}, not more than a few tenths Mev, so that it has a very open structure. In this situation the three-body forces can have relatively little effect, so that this case should allow a clear estimate of the strength of the two-body force. However, with such light binding, a rather flexible trial wave function is needed and the lower values for U_2 shown in Table II are those obtained recently by Dalitz and Downs for $B_{\Lambda} = 0.25$ Mev,[35] the large values being those obtained earlier with a simpler form for the trial function.[33] These values are substantially below those given elsewhere in the literature, which were generally obtained by using the simplest possible wave functions. Note that the well-depth parameter s for the mean Λ-nucleon potential in the hypertriton is no more than 2/3; of course the mean potential here is a combination of the singlet and triplet potentials, $(3U_s + U_t)/4$. But since the alpha particle gives quite a strong attractive potential, and this potential is given by $(3U_t + U_s)$, the triplet potential cannot be repulsive: from this it follows that the well-depth parameter for the singlet state cannot exceed about 0.9. With the values of $(3U_t + U_s)$ and $(3U_s + U_t)$ given in Table II, the well-depth parameter obtained for the singlet state is actually 0.71, whether the potential is due to pion processes, or to K [illegible]ange, [illegible] no bound state for the Λ-p or Λ-n systems, in accord with the lack of any such evidence. The main assumption here has been the neglect of three-body forces; however, it will be shown that the conclusions cannot be seriously affected by the presence of three-body forces for the Λ particle.

Weitzner has pointed out that these volume integrals U_2, U_3 and U_4 could be fairly well represented by the assumption of a repulsive three-body force, together with a Λ-nucleon two-body potential having only little spin dependence.[25] Including a three-body force, the expressions given above (Table II) for the volume integrals of the Λ-nuclear potentials should be replaced by

$$3U_t + U_s + 6W = U_4 , \quad (6.12a)$$

$$\frac{3}{2}U_t + \frac{3}{2}U_s + 3W = U_3 , \quad (6.12b)$$

$$\frac{1}{2}U_t + \frac{3}{2}U_s + W = U_2 , \quad (6.12c)$$

where W denotes the volume integral of the three-body potential over Λ positions for two nucleons in a distribution with the standard nucleon density. In Eqs.(6.12b) (6.12c), the coefficients of the W term should be modified a little by amounts depending on the average nucleon density in He^3 and H^2, respectively, relative to that for He^4; however, these modifications are not important within the present uncertainties. It is then clear that the three Eqs.(6.12) involve only the quantities U_s and

$(U_t + 2W)$ and that they are consistent only for $U_3 = 3(U_4 + 2U_2)/8$, a condition satisfied by the values given in the table, within their uncertainties. In all cases the singlet potential U_s is given by $(U_3 - \frac{1}{2} U_4)$. However, the best estimate of U_s comes from U_2 and U_4 and is that given in the table, corresponding to a well-depth parameter $s \simeq 0.71$ for either $\hbar/m_K c$ or $\hbar/2m_\pi c$ range. The triplet potential U_t itself cannot be determined from Eqs. (12); as pointed out by Weitzner, it is possible to assume a repulsive three-body potential with W about 72 Mev f^3 and to obtain $U_t \sim U_s$.[25] It appears that the only clear way to decide what are the relative strengths of the two- and three-body potentials is to obtain information directly on Λ-proton scattering for particles of energy much less than 150 Mev. Probably bubble chamber evidence on Λ particles will give some information on the size of these cross sections before too long.

The strength of the Λ-nucleon potential obtained for the singlet state may be compared with the Λ-nucleon potential computed from meson theory by Lichtenberg and Ross, assuming symmetry for the pion-hyperon coupling.[23] The calculated potentials agree with the potentials found empirically in that they predict the stronger attraction to be in the singlet state; the empirical strength for the singlet potential corresponds to a π-hyperon coupling constant a little larger than the pion-coupling constant. Unfortunately it is not possible to make such a direct comparison with the extensive calculations of Ferrari and Fonda,[24] to which I now wish to refer. These last authors have calculated Λ-nucleon forces in the static approximation up to fourth order in the coupling parameters for all combinations of relative parities of K mesons and hyperons, including the exchange of both K mesons and pions. Of course, these potentials are very singular and must be cut off, the core radii being chosen equal to those for the Brueckner-Watson nucleon-nucleon potential. If one wishes to use these potentials directly in a calculation of the Λ-nuclear binding energies, it is clear that the trial functions must be quite complicated. The trial functions they used for the Λ-particle motion were of the form

$$\psi_\Lambda(r) = F(r) \prod_{i=1}^{A-1} \left(1 - \exp(-\beta|\underline{r}-\underline{r}_i|)\right), \quad \text{for all } |\underline{r}-\underline{r}_i| > r_c,$$

$$= 0 \qquad\qquad , \quad \text{for any } |\underline{r}-\underline{r}_i| \leq r_c,$$

where the product is taken over all A-1 nucleons of the core nucleus and β is a variational parameter, the distribution of nucleons in the core nucleus being taken from experiment as far as possible. Ferrari and Fonda find that if they neglect K-meson exchange they can obtain a reasonable value for the binding energies only with the same parity for Σ and Λ particles and a coupling parameter $g_{\Sigma\Lambda}{}^2/4\pi \simeq 16$, perhaps a little larger than the value known for the pion-nucleon coupling. However, if the K-meson exchange is also included, a pseudoscalar K meson gives rise to an additional attraction and a coupling parameter $g_{\Lambda K}{}^2/4\pi = g_{\Sigma K}{}^2/4\pi \simeq 2$ provides sufficient attraction to allow a fit with $g_{\Sigma\Lambda}{}^2/4\pi = 13$. However, since the higher-order pion potentials are not included, nor any estimate of three-body potentials, it is difficult to take this last refinement very seriously, although Ferrari and Fonda remark that scalar K mesons would contribute repulsion, leaving more attraction to be made up by the higher-order terms. With the inclusion of K exchange, Ferrari and Fonda find that they can also obtain a qualitatively satisfactory potential by assuming negative parity for $\Sigma\Lambda$, and either scalar or pseudoscalar $K\Lambda$ parity by suitably choosing the signs and magnitudes of the various coupling parameters; for these cases, however, about half the potential must be provided by K exchange, the fit obtained appears somewhat artificial, and Ferrari and Fonda have not investigated the binding-energy situation in detail for these cases. Some direct evidence that the two-body potential has a range of order $\hbar/2m_\pi c$ is provided by a comparison of the potentials U_6 and U_7 derived from data on the p-shell hypernuclei ${}_\Lambda Li^7$ and ${}_\Lambda Li^8$ with those given in Table II for the light hypernuclei. When parameters recently obtained by the Stanford group for the shape and radius of Li^6 are used, the values obtained for U_6 and U_7 greatly exceed those expected for a Λ-nucleon potential of range parameter $\hbar/m_K c$. Agreement between U_6, U_7, and the U_n of Table I requires a range parameter perhaps 10% larger than $\hbar/2m_\pi c$ for the Λ-nucleon potential, which gives qualitative support for the conclusion that the Λ-nucleon potential arises mainly from the pion processes.

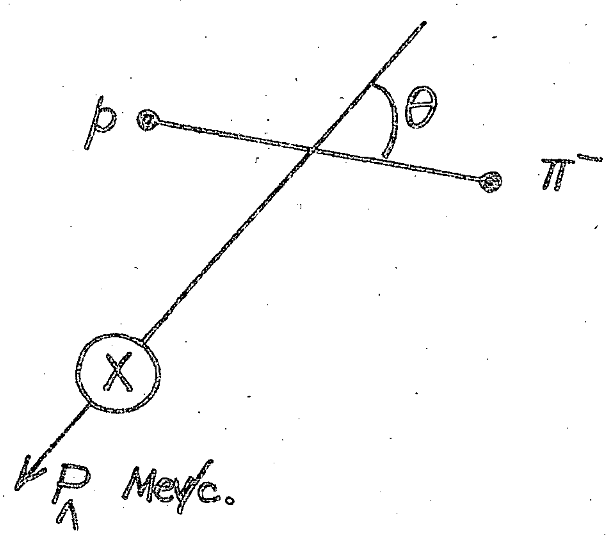
On the basis of the phenomenological analysis it is of interest to note that it appears quite probable that the ${}_\Lambda H^4$ system should have an excited bound state of spin 1. The value of its binding energy will depend on the degree of spin dependence for the Λ-nucleon potential. If there is a repulsive three-body potential, this will reduce the degree of spin dependence necessary to account for the B_Λ data, and therefore make it more certain that this state should be bound. This may be somewhat unfortunate for the K^--He^4 capture experiment,[33] which has looked so promising for the determination of the K-Λ relative parity. For a scalar K meson, the direct formation of ground state ${}_\Lambda H^4$ is strictly forbidden, but the selection rules [illegible] the formation of

$_{\Lambda}H^{4*}$ with spin 1. If the excited state $_{\Lambda}H^{4*}$ is formed in this capture reaction, it will then decay by γ-emission to ground state $_{\Lambda}H^{4}$, so that normal $_{\Lambda}H^{4}$ decay events will be observed, even though the direct formation of $_{\Lambda}H^{4}$ is actually forbidden. This situation may prove quite difficult to sort out.

Finally, there are some calculations to report on a correlation effect which was reported last year in the three-body decay mode for hypernuclei,

$$(X + \Lambda) \rightarrow X + p + \pi^{-} . \qquad (6.13)$$

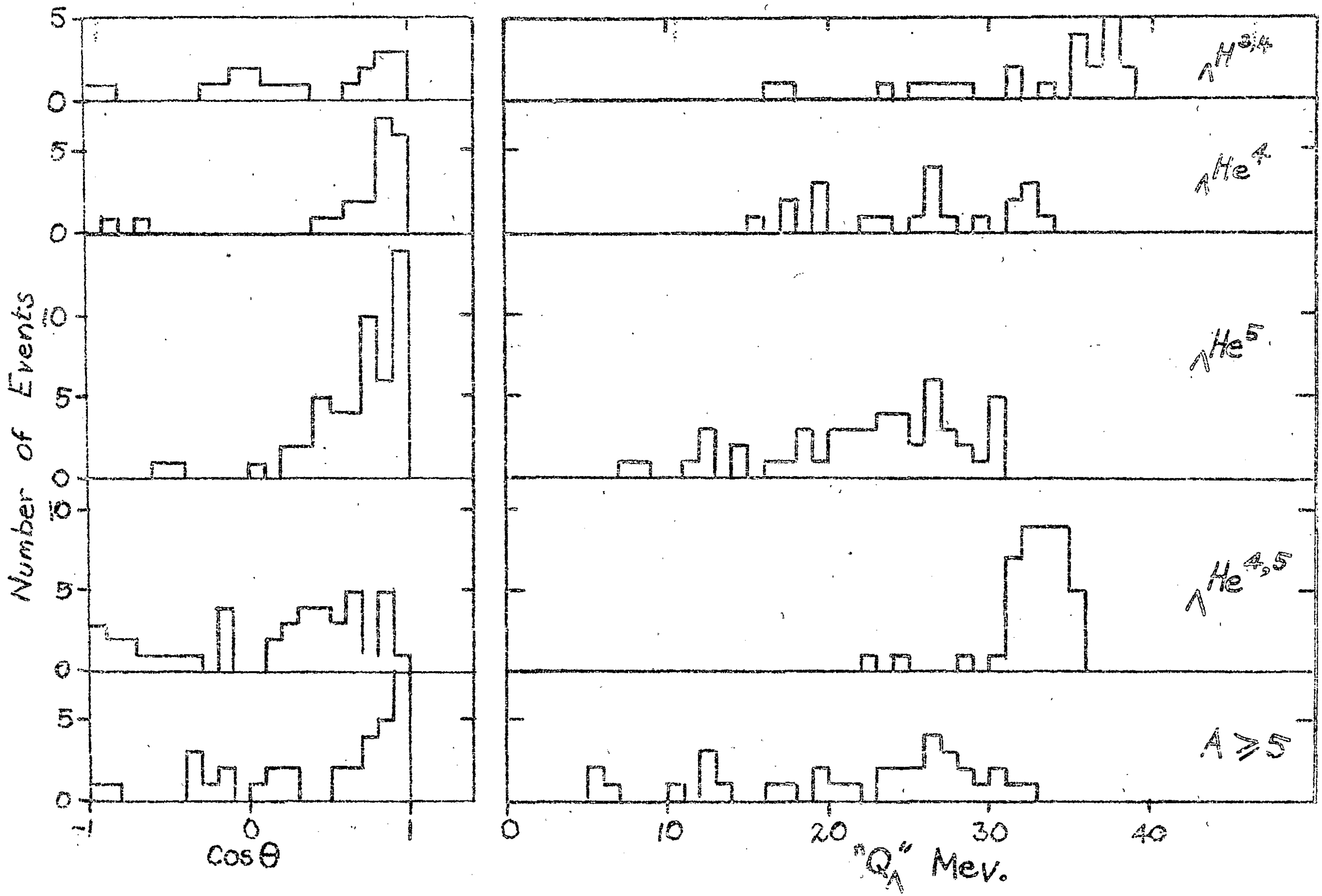
The configuration of this decay mode may be characterized conveniently by giving the recoil momentum "P_{Λ}" of the residual nucleus X and the angle θ, as defined in the figure. The distributions[36] in "P_{Λ}" and cos θ obtained



for various hypernuclei in the recent EFINS - NU collaboration[37] are shown in Fig. 3. The anisotropy in the cos θ distribution, especially for $_{\Lambda}He^{5}$ decay, is very striking, since an isotropic distribution would follow from an s-wave motion for the initial Λ particle if the nuclear interactions between the final proton and the nucleus A could be neglected. For $_{\Lambda}He^{5}$, the effect of this final-state interaction is

FIGURE 3: COMBINED DATA ON COS θ and "Q_Λ" DISTRIBUTIONS IN HYPERNUCLEAR DECAY

FROM EFINS SURVEY (MAY 1958) and EFINS-NU COLLABORATION (JUNE 1958)-- LEVI-SETTI ET AL. (1958).



UCRL-8304

especially strong, owing to the low energy resonance in p-He^4 scattering --as pointed out by Cottingham and Byers[38] who have recently completed a calculation of the distributions to be expected for $_\Lambda He^5$ decay as a result of this resonant interaction. As shown in Fig. 4, their calculation agrees remarkably well with the observed cos θ distribution for $_\Lambda He^5$ decay events. The comparison of the "P_Λ" distribution with experiment for $_\Lambda He^5$ suffers from a bias in the identified $_\Lambda He^5$ events because it becomes very difficult to distinguish between $_\Lambda He^4$ and $_\Lambda He^5$ decay events when the residual He nucleus has a momentum of 100 Mev/c or less. However, this bias against low "P_Λ" values disappears when the data on all $_\Lambda He$ decays of the type of Eq. (6.13) are taken together. The "P_Λ" distribution for about 150 $_\Lambda He$ decay events is given in Fig. 5 and shows some evidence for the double-peaked structure (curve A) predicted for $_\Lambda He^5$ decay by Cottingham and Byers.[38] On this Fig. 5, curve B shows the "P_Λ" distribution expected when the final nuclear interaction is neglected and only the phase space and the momentum distribution in the initial state are taken into account.

FIGURE 4. CosΘ Distribution for $_{\Lambda}He^5 \rightarrow He^4 + p + \pi^-$ Decay Events.

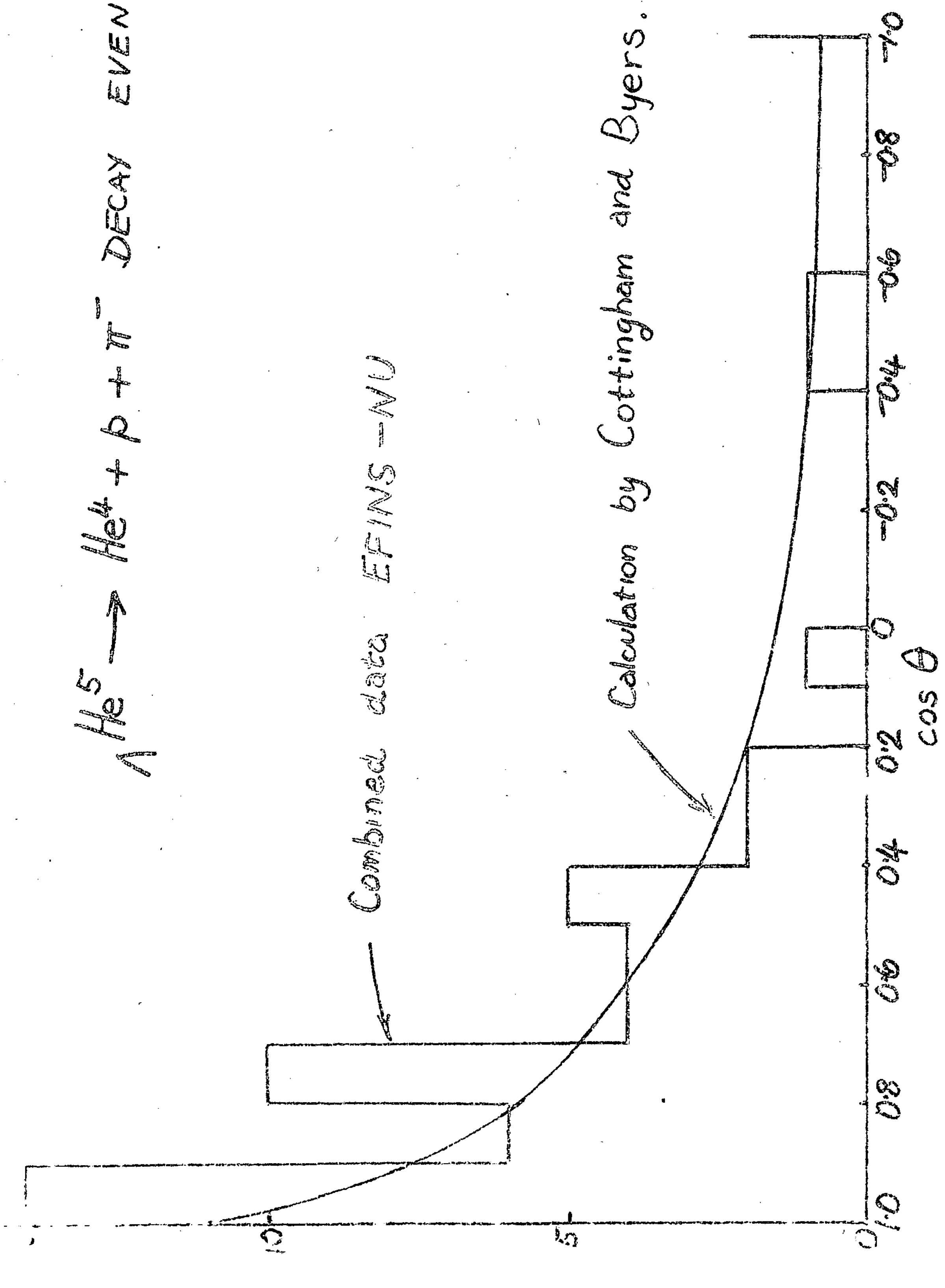
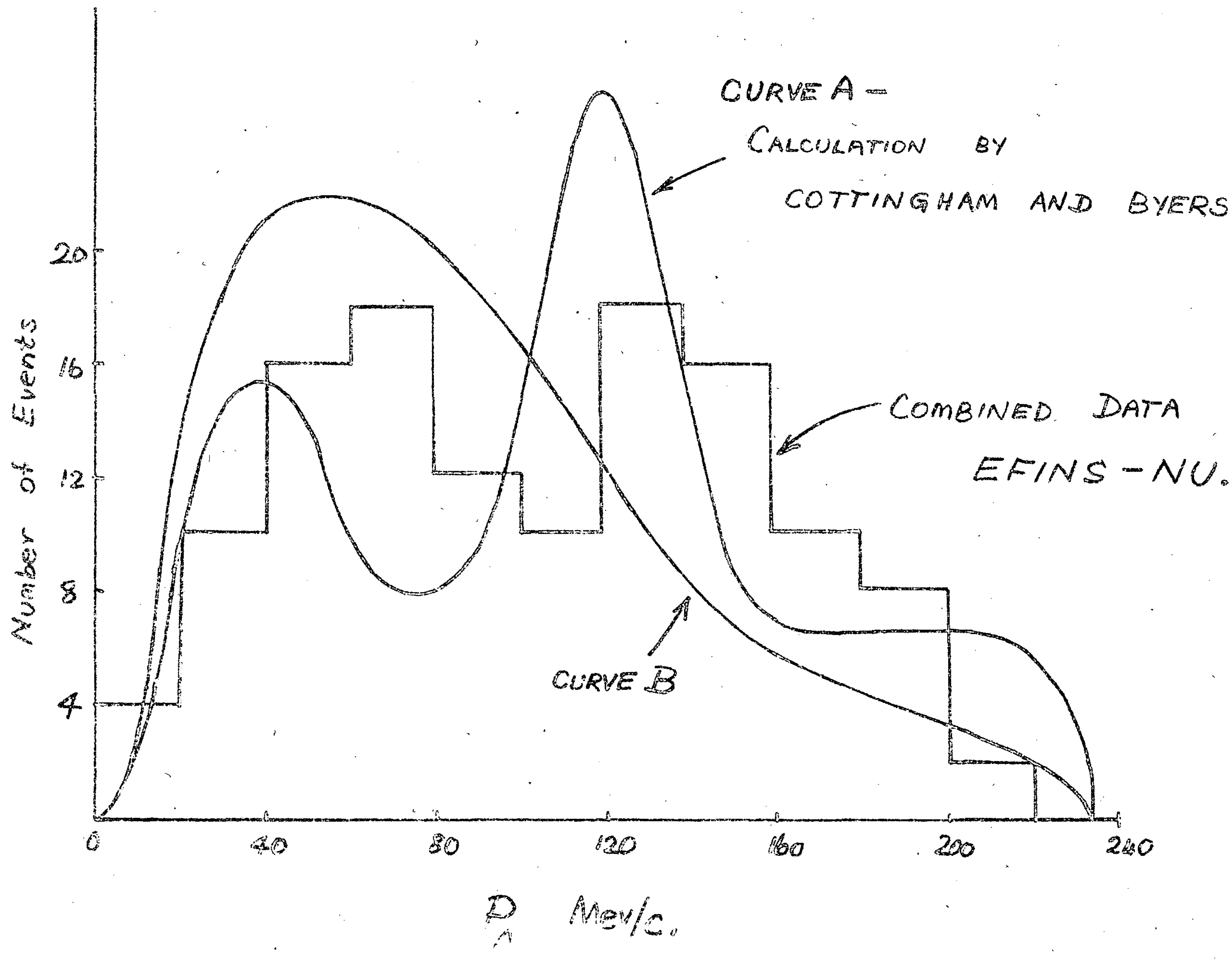


FIGURE 5: THE P_Λ DISTRIBUTION FOR ALL $_\Lambda He^4$ AND $_\Lambda He^5$ DECAY EVENTS OF THE MODE (6.13).



REFERENCES

1. M. Gell-Mann, Phys. Rev. 106, 1296 (1957).

2. J. S. Schwinger, Annals of Physics 2, 407 (1957).

3. C. Ceolin and L. Taffara, Nuovo cimento 5, 435 (1957).

4. S. Barshay, Phys. Rev. 109, 2160 (1958); 110, 743 (1958).

5. C. Ceolin and L. Taffara, Nuovo cimento 5, 435 (1957). Also N. Dallaporta, private communication, 1958.

6. Ceolin, Dallaporta, and Taffara, Pion Production in the K^+-Nucleon Interaction, Nuovo cimento (to be published).

7. Jackson, Ravenhall, and Wyld, Phenomenological Discussion of the Scattering and Absorption of K mesons by Protons, Phys. Rev. (to be published).

8. Alles, Biswas, Ceccarelli, and Crussard, Nuovo cimento 6, 571 (1957).

9. A. Fujii and R. E. Marshak, Nuovo cimento 8, 643 (1958).

10. L. B. Okun and I. Y. Pomeranchuk, On the Determination of the Parity of the K Meson, Zhur. Exptl. i Teoret. Fiz. (to be published).

11. D. Amati and B. Vitale, Nuovo cimento 7, 190 (1958).

12. K. Igi, Progr. Theoret. Phys. (Kyoto) 3, 238 (1957); Dispersion Relation for K-Meson Nucleon Scattering and its Application, Progr. Theoret. Phys. (Kyoto) (to be published).

13. P. T. Matthews and A. Salam, Phys. Rev. 110, 565 and 569 (1958).

14. C. Goebel, Phys. Rev. 110, 572 (1958).

15. I. Y. Pomeranchuk, quoted by G. F. Chew at CERN Conference, 1958.

16 V. Haber-Schaim, Phys. Rev. 104, 1113 (1956).

17. Meyer, Perl, and Glaser, Phys. Rev. 107, 279 (1957).

18. Y. Eisenberg, W. Koch, E. Lohrmann, M. Nikolic, M. Schneeberger, and H. Winzeler, Interactions and Decays of K⁻ Mesons. III, Nuovo cimento (to be published).

19. L. B. Okun and M. I. Smushkevich, Zhur, Exptl' i Teoret. Fiz. 30, 979 (1956).

20. A. Pais and S. Treiman, Phys. Rev. 107, 1396 (1957).

21 A. Pais, Phys. Rev. 110, 574 (1958); Note on Relations Between Baryon Meson Coupling Constants, preprint (1958).

22. Bryan, de Swart, Marshak, and Signell, Phys. Rev. Letters 1, 70 (1958).

23. D. B. Lichtenberg and M. Ross, Phys. Rev. 107, 1714 (1957); 109, 2163 (1958).

24. F. Ferrari and L. Fonda, Nuovo cimento 6, 1027 (1957); The Λ-Hyperon-Nucleon Problem from Meson Theory, Nuovo cimento (to be published).

25. H. Weitzner, Phys. Rev. 110, 593 (1958); Harvard doctoral thesis, 1958.

26. F. C. Gilbert and R. S. White, Phys. Rev. 109, 1770 (1958).

27. D. Amati and B. Vitale, On a Possible Test of Symmetry in Pion-Baryon Interactions, Nuovo cimento (to be published).

28. K. Kawarabayashi, Global Symmetry and the Branching Ratios for K⁻ Capture by Nucleons, Progr. Theoret. Phys. (Kyoto), to be published.

29. Y. Yamaguchi, private communication (1958).

30. N. Dallaporta and F. Ferrari, Nuovo cimento 5, 111 (1957).

31. R. Spitzer, Phys. Rev. 110, 1190 (1958).

32. G. Bach, McGill University, private communication from E. Lomon.

33. R. H. Dalitz and B. W. Downs, Phys. Rev. 110, 958 (1958); 111, (in print).

34. R. H. Dalitz, Hypernuclear Binding Energies and the Λ-Nucleon Interaction, Phys. Rev. (to be published).

35. R. H. Dalitz and B. W. Downs, An Analysis of the Three-Particle Hypernuclear Systems, (in preparation).

36. The distribution shown in Fig. 3 is a plot of the events against "Q_Λ", the kinetic energy of the $\pi^- + p$ system resulting from the decay process (6.13), measured in the $(\pi^- + p)$ rest frame. However, from the equations of conservation of energy and momentum, "Q_Λ" is directly related to "P_Λ" by the expression

$$"Q_\Lambda" = \left\{\left(M_\Lambda - B_\Lambda - "P_\Lambda"^2/2M_X\right)^2 - "P_\Lambda"^2\right\}^{1/2} - M_p - M_{\pi^-}.$$

37. Levi -Setti, Ammar, Slater, Limentani, Roberts, Schlein and Steinberg, Mesic Decays of Hypernuclei from K$^-$ Capture, (1958).

38. N. Byers and N. Cottingham, Birmingham University, private communication (1958).