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Improved Method to Estimate the 3dB Power Bandwidth of a Fabry-Pérot Cavity Antenna Covered by a Thin Frequency Selective Surface

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Abstract — **In this paper, an analytical method is proposed to estimate the 3dB power bandwidth of a Fabry-Pérot Cavity antenna covered by a thin frequency selective surface. The 3dB power bandwidth of the antenna is found using the broadside radiation power density of the antenna calculated based on the reciprocity theorem along with a transmission line model of the antenna and the circuit model of the FSS layer. The accuracy of the proposed method is verified by numerical simulations.**

Keywords-Fabry-Pérot cavity (FPC) antenna; leaky-wave antenna; EBG antenna; bandwidth

I. INTRODUCTION

The concept of a Fabry-Pérot cavity antenna (FPC) was employed by von Trentini in [1] to improve the radiation gain of an antenna. Gain enhancement methods were also investigated by using multiple superstrates in [2-3]. Zhao et al. discussed how the FPC antenna is a type of leaky-wave antennas (LWA) and investigated LWAs formed from a substrate covered by a frequency selective surface (FSS) made of periodic metallic patches or slots as discussed in [4-5]. In [6], the fundamental properties and optimization of broadside uniform LWAs were studied. However, LWA also has been investigated in more practical designs [7-8]. In [7], two layers of periodic alumina-rods were placed above a patch antenna to increase the directivity. An optimized partially reflective surface was placed on top a waveguide-opening to achieve a high radiation gain [8]. The bandwidth behavior of LWAs covered by different types of FSS layers has been investigated as function of the FSS admittance model in [9].

In this work, we consider a FPC antenna consisting of a grounded substrate layer of thickness h with parameters ε_r and μ_r, on top of which is an FSS layer that forms the top of the cavity. Reciprocity is used to find the broadside radiation power density of the antenna as a function of the transmission-line model of the antenna and the FSS admittance model. This is used to determine an approximate but accurate formula for the bandwidth of the antenna. In order to arrive at this formula the power density as a function of frequency is approximated by using a second-order Taylor expansion. The bandwidth is then found by calculating the frequencies for which the radiated power density is down by 3 dB from the broadside radiation power density of the antenna at the central frequency. The accuracy of the proposed method is verified with numerical simulations.

II. BROADSIDE POWER DENSITY OF A FPC ANTENNA

A FPC antenna can be fed either by an electric dipole in the middle of cavity at a distance of h_s from its ground plane or by a magnetic dipole (slot) on its ground plane, as shown in Figs.1(a) and (b) respectively.
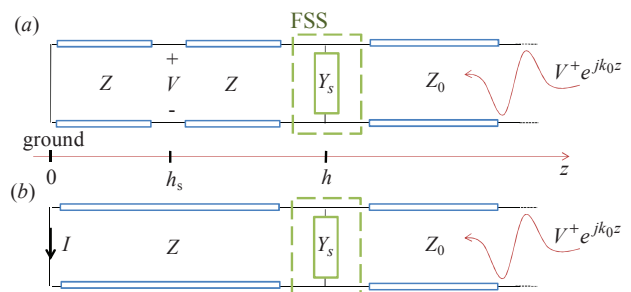

Figure 1. The TEN model of a FPC antenna, (a) fed by an electric dipole in the middle of the cavity at distance h_s, (b) fed by a magnetic dipole (slot) on the ground plane of the antenna.

Using the reciprocity theorem as discussed in [10], the electric far-field radiation power density of an arbitrary antenna is equal to the electric field induced at its feeding point in the receiving mode when the antenna is illuminated by a plane-

wave $E_q^{inc} = E_0 e^{jk_0 z}$ where $E_0 = -j\eta_0 k_0 /(4\pi)$ (notably, the plane-wave is propagating in the -z direction where z is assumed to be the broadside radiation direction of the antenna).

On other hand, based on the transverse equivalent network (TEN) transmission-line model of the FPC antenna, also discussed in [10], all fields in the structure of the antenna can be expressed as a voltages and currents and the mentioned plane-wave is modeled as an incident voltage wave $V^+ e^{jk_0 z}$ (where $V^+ = E_0$) as shown in Fig. 1, where $Z = Z_0/\xi_r$, Z_0 is the free-space characteristic impedance and $\xi_r = \varepsilon_r/\mu_r$. Using the transmission-matrix formulas, it can be seen that $V = jZ\sin(kh_s)I$ where $k = k_0\sqrt{\mu_r \varepsilon_r}$ and k_0 is the free-space wavenumber at the operating frequency. In this work all of the calculations are for a FPC antenna fed by a magnetic dipole (slot) on its ground plane but without loss of generality, it is also valid for the antenna fed by an electric dipole in the middle of the cavity. Using the TEN model of the antenna shown in Fig. 1(b), the current I induced at the short circuit (modeling the tangential magnetic field at the ground plane in the actual structure, when illuminated by the incident plane wave) can be found, which models the radiated far field by reciprocity. The broadside radiation power density of the antenna can then be found as

$$P = \frac{2\xi_r^2 |E_0|^2 Y_0}{\sin^2(kh)}\left(\frac{1}{1+\overline{B}_{tot}^2}\right), \tag{1}$$

where $\overline{B}_{tot} = \overline{B}_s - \xi_r \cot(kh)$, $Y_s = j\overline{B}_s/\eta_0$ and h is the height of the cavity (thickness of the substrate). An approximate expression for the optimum frequency that maximized the power density radiated at broadside is

$$\cot(k_{op}h) = \frac{1}{\xi_r}\overline{B}_s(\omega_{op}), \tag{2}$$

where ω_{op} is the optimum radian frequency of the antenna and k_{op} is the corresponding wavenumber inside the substrate. At the optimum frequency, an approximate expression for the power density radiated at broadside is

$$P_{max} = \frac{2\xi_r^2 |E_0|^2 Y_0}{\sin^2(k_{op}h)}, \tag{3}$$

where E_0 is calculated at ω_{op}.

I. POWER BANDWDITH OF THE ANTENNA

The power bandwidth is defined by the radian frequencies $\omega_{3dB}^\pm$ at which $P(\omega_{3dB}^\pm) = P_{max}/2$. Assuming that the frequency variation of E_0 over the antenna bandwidth is negligible, the half-power bandwidth condition is simplified as

$$F_1 + F_2 = 2\sin^2(k_{op}h), \tag{4}$$

where

$$F_1 = \sin^2\left(k_{3dB}^\pm h\right), \quad F_2 = \left[\overline{B}_s \sin\left(k_{3dB}^\pm h\right) - \xi_r \cos\left(k_{3dB}^\pm h\right)\right]^2 \tag{5}$$

and $k_{3dB}^\pm$ are the wavenumbers in the substrate at the band edges. Assuming now that $\Delta^\pm = \omega_{3dB}^\pm - \omega_{op}$ is a small fraction of ω_{op}, the functions F_1 and F_2 can be approximated through a Taylor expansion around ω_{op}; taking also into account that $B_{tot}(\omega_{op}) \approx 0$ and that, for high values of $\overline{B}_s$, it follows from (2) that $\cos(k_{op}h) \cong -1$, it is found that

$$F_1 \cong \sin^2(k_{op}h) - 2\sin(k_{op}h)\frac{h}{c_d}\Delta + \left[1 - \sin^2(k_{op}h)\right]\left(\frac{h}{c_d}\right)^2\Delta^2 \tag{6}$$

$$F_2 \cong \overline{B}_{tot}'^2(\omega_{op})\Delta^2,$$

where c_d is the speed of light in the substrate. Furthermore, the sine function in (6) can be expressed in terms of $\overline{B}_{s,op} = \overline{B}_s(\omega_{op})$ through (2). By letting again $\cos(k_{op}h) \cong -1$, it follows that $\sin(k_{op}h) \cong -\xi_r/\overline{B}_{s,op}$. From (6) the following polynomial approximation of (4) can then be obtained:

$$\left[c_1 + \left(\omega_{op}\overline{B}_{s,op}'\right)c_2 + \left(\omega_{op}\overline{B}_{s,op}'\right)^2 c_3\right]\Delta^2 + c_4\omega_{op}\Delta - \xi_r^2\omega_{op}^2 = 0, \tag{7}$$

where $\overline{B}_{s,op}' = \overline{B}_s'(\omega_{op})$ and

$$c_1 = \left(k_{op}h\right)^2\left[\overline{B}_{s,op}^4 + \overline{B}_{s,op}^2\left(2\xi_r^2 + 1\right) + \left(\xi_r^4 - \xi_r^2\right)\right]$$
$$c_2 = 2\left(k_{op}h\right)\xi_r\left(\overline{B}_{s,op}^2 + \xi_r^2\right)$$
$$c_3 = \xi_r^2$$
$$c_4 = 2\left(k_{op}h\right)\xi_r\overline{B}_{s,op}. \tag{8}$$

Solving (7) for $\Delta^\pm$ and letting the fractional 3-dB power bandwidth be defined as $BW_{3dB} = \left(\Delta^+ - \Delta^-\right)/\omega_{op}$ yields

$$BW_{3dB} = \frac{\sqrt{c_4^2 + 4\xi_r^2\left[c_1 + \left(\omega_{op}\overline{B}_{s,op}'\right)c_2 + \left(\omega_{op}\overline{B}_{s,op}'\right)^2 c_3\right]}}{c_1 + \left(\omega_{op}\overline{B}_{s,op}'\right)c_2 + \left(\omega_{op}\overline{B}_{s,op}'\right)^2 c_3}. \tag{9}$$

The fractional power bandwidth of an FPC antenna is thus expressed as a function of ξ_r, ω_{op}, $\overline{B}_{s,op}$, and $\overline{B}_{s,op}'$. In the next part, using (9), various FPC antennas are investigated by considering different circuit models for a thin FSS. For high-gain FPC antennas $\overline{B}_{s,op} \gg 1$ (hence from (2) it follows that

$k_{op}h \cong \pi$). In (9), the dominant term both inside the square-root and in the denominator is then proportional to $\overline{B}_{s,op}^4$; neglecting all other terms, (9) can be approximated as

$$\mathrm{BW}_{3\mathrm{dB}} = \frac{2\xi_r}{\pi \overline{B}_{s,op}^2} \qquad (10)$$

which is the formula proposed in [4-6] for the antenna fractional power bandwidth.

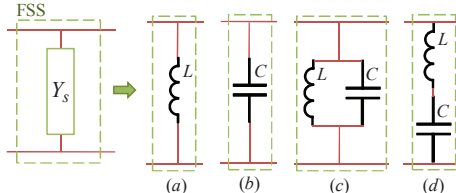## II. BANDWIDTH BEHAVIOR OF A FPC ANTENNA COVERED BY THIN FSS MODELED BY AN LC CIRCUIT

Figure 2.   A simple LC model of a thin FSS layer.

For an FSS layer modeled as an inductor, $\overline{B}_s = -1/(\omega L Y_0)$. If modeled as a capacitor, $\overline{B}_s = \omega C/Y_0$. In these cases (called cases (a) and (b)) the first derivative of $\overline{B}_s$ at the optimum frequency of the antenna ($\overline{B}'_{s,op}$) can be expressed, respectively, as

$$\omega_{op}\overline{B}'_{s,op} = \frac{1}{\omega_{op}L Y_0} = -\overline{B}_{s,op}, \ \omega_{op}\overline{B}'_{s,op} = \frac{\omega_{op}C}{Y_0} = \overline{B}_{s,op}. \ (11)$$

By using (11) in (9), the result (10) is obtained for large values of $\left|\overline{B}_{s,op}\right|$. Fig. 3 and 4 show the fractional 3 dB power bandwidth of a FPC antenna covered by inductive and capacitive FSS layers respectively, along with a comparison between the exact values calculated from the half-power equation, the estimated ones from the new formula (9) and the simple high-gain approximate formula (10) for a cavity filled with air ($\varepsilon_r = \mu_r = 1$). For an FSS modeled as a series LC network (model c shown in Fig. 2) $\overline{B}_s = -\omega C Z_0/(\omega^2 L C - 1)$; hence, by letting $\omega_{LC} = 1/\sqrt{LC}$ and $\chi = \omega_{op}/\omega_{LC}$, it follows that

$$\overline{B}_{s,op} = -\frac{\omega_{op}C}{Y_0(\chi^2 - 1)} \ , \ \omega_{op}\overline{B}'_{s,op} = -\overline{B}_{s,op}\frac{\chi^2 + 1}{\chi^2 - 1} \qquad (12)$$

For $\chi \gg 1$ the FSS layer is inductive and the second of (12) reduces to (11-1); hence the fractional bandwidth of the antenna is the same as in model (a). Similarly, for $\chi \ll 1$ the FSS layer is capacitive, the second of (12) reduces to (11-2),

and hence the bandwidth is the same as in model (b). For both of these cases, the high-gain approximation (10) is valid for large values of $\left|\overline{B}_{s,op}\right|$.
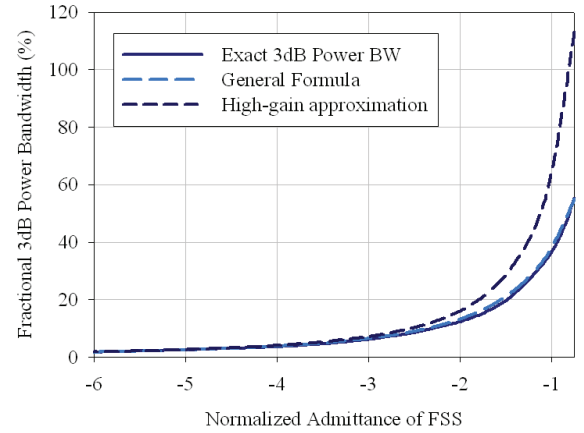
Figure 3.   Fractional 3 dB power bandwidth of an FPC antenna with an FSS modeled as an inductor (model a): comparison between the exact value and those obtained with (9) (*general formula*) and (10) (*high-gain approximation*).
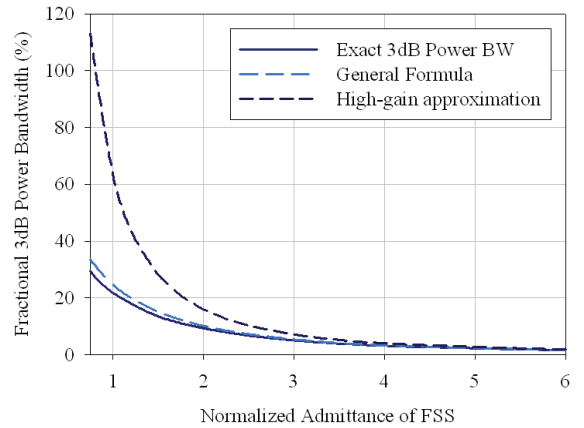
Figure 4.   Fractional 3 dB power bandwidth of an FPC antenna with an FSS modeled as a capacitor (model b): comparison between the exact value and those obtained with (9) (*general formula*) and (10) (*high-gain approximation*).

For χ very close to one (i.e., when the operating frequency ω_{op} is very close to the FSS resonance frequency ω_{LC}) $\left|\omega_{op}\overline{B}'_{s,op}\right| \gg \left|\overline{B}_{s,op}\right|$; thus in (9) $\left(\omega_{op}\overline{B}'_{s,op}\right)^2$ is the most significant term. Neglecting all other terms, (9) can then be simplified as

$$\mathrm{BW}_{3\mathrm{dB}} = \frac{2}{\left|\overline{B}_{s,op}\right|}\left|\frac{\chi^2 - 1}{\chi^2 + 1}\right| \cong 2\left|\frac{\chi - 1}{\overline{B}_{s,op}}\right|. \qquad (13)$$

Finally, for an FSS modeled as a parallel LC network (model d), $\overline{B}_s = (\omega^2 L C - 1)/(\omega L Y_0)$; hence it follows that

$$\overline{B}_{s,op} = \frac{\chi^2 - 1}{\omega_{op} L Y_0} \ , \ \omega_{op} \overline{B}'_{s,op} = \overline{B}_{s,op} \frac{\chi^2 + 1}{\chi^2 - 1} \qquad (14)$$

As in the previous case (*c*), for $\chi \ll 1$ the FSS layer is inductive and the fractional bandwidth of the antenna is the same as in model (*a*); for $\chi \gg 1$ the FSS layer is capacitive and the bandwidth is the same as in model (*b*). For both of these cases, the high-gain approximation (10) is accurate for large values of $\left| \overline{B}_{s,op} \right|$. As with model (*c*), when χ is very close to unity, $\left| \omega_{op} \overline{B}'_{s,op} \right| \gg \left| \overline{B}_{s,op} \right|$ and (10) can be approximated through (13). A numerical comparison is made between the exact fractional 3 dB power bandwidth values of a FPC antenna covered by a FSS layer modeled either by model (*c*) or (*d*) shown in Fig. 2, and the estimated bandwidth values calculated using (13) and the values calculated using the high-gain approximation formula (10) are shown for cases where the central frequency of the antenna is selected very close to the resonance frequency of the FSS structure ($\chi \approx 1$). The comparisons are made for different values of $\overline{B}_{s,op}$ as shown in Tables I-IV. The accuracy of the previous formula (10) is limited by the type of FSS (inductive and capacitive FSS layers) and also the maximum gain of the FPC, (accurate for high-gain FPCs). The proposed formula (9) can estimate the power bandwidth of a FPC antenna covered by any arbitrary thin FSS with no constraints on its reflectivity, type of FSS FSS, and FSS resonance frequency (i.e., χ may be arbitrary). Furthermore, (9) distinguishes between capacitive and inductive FSS covers of the FPC, which (10) does not.

III. CONCLUSION

The main result of this paper, namely formula (9) for estimating the fractional power bandwidth of a Fabry-Pérot cavity antenna covered with a thin FSS, has been validated in the previous section considering both inductive (slots) and capacitive (dipoles) FSSs. In particular, (9) is able to account for FSSs operating either far or close to their resonance. Compared with the previously reported formula (10), given in [4-6], the new formula (9) takes into account the frequency dependence of the FSS equivalent susceptance B_s and remains accurate for relatively low values of B_s, i.e., for FPC antennas with low to moderate directivity.

TABLE I.   NORMALIZED 3 dB POWER BANDWIDTH OF AN FPC ANTENNA WITH FSS MODELED AS SERIES LC CIRCUIT (INDUCTIVE, $\chi = 1.001$)

$\overline{B}_{s,op}$	EXACT BW$_{3dB}$	APPROX. BW$_{3dB}$ (13)	HIGH-GAIN APPROX. BW$_{3dB}$ (10)
-4	0.053%	0.050%	4%
-6	0.034%	0.033%	1.77%
-8	0.025%	0.025%	1%
-10	0.020%	0.020%	0.64%

TABLE II.   NORMALIZED 3 dB POWER BANDWIDTH OF AN FPC ANTENNA WITH FSS MODELED AS SERIES LC CIRCUIT (CAPACITIVE, $\chi = 0.999$)

$\overline{B}_{s,op}$	EXACT BW$_{3dB}$	APPROX. BW$_{3dB}$ (13)	HIGH-GAIN APPROX. BW$_{3dB}$ (10)
4	0.053%	0.050%	4%
6	0.034%	0.033%	1.77%
8	0.025%	0.025%	1%
10	0.020%	0.020%	0.64%

TABLE III.   NORMALIZED 3 dB POWER BANDWIDTH OF AN FPC ANTENNA WITH FSS MODELED AS PARALLEL LC CIRCUIT (INDUCTIVE, $\chi = 1.001$)

$\overline{B}_{s,op}$	EXACT BW$_{3dB}$	APPROX. BW$_{3dB}$ (13)	HIGH-GAIN APPROX. BW$_{3dB}$ (10)
4	0.049%	0.050%	4%
6	0.033%	0.033%	1.77%
8	0.025%	0.025%	1%
10	0.019%	0.020%	0.64%

TABLE IV.   NORMALIZED 3 dB POWER BANDWIDTH OF AN FPC ANTENNA WITH FSS MODELED AS PARALLEL LC CIRCUIT (CAPACITIVE, $\chi = 0.999$)

$\overline{B}_{s,op}$	EXACT BW$_{3dB}$	APPROX. BW$_{3dB}$ (13)	HIGH-GAIN APPROX. BW$_{3dB}$ (10)
-4	0.049%	0.050%	4%
-6	0.033%	0.033%	1.77%
-8	0.024%	0.025%	1%
-10	0.019%	0.020%	0.64%

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