Title
EFFECTIVE SINGLE-PARTICLE MAGNETIC MOMENTS AROUND 208 Pb

Authors
Maier, K.H.
Nakai, K.
Leigh, J.R.
et al.

Publication Date
1970-10-01

Submitted to Physics Letters

UCRL-19593
Preprint c.2

EFFECTIVE SINGLE-PARTICLE MAGNETIC MOMENTS AROUND ^{208}Pb

K. H. Maier, K. Nakai, J. R. Leigh,
R. M. Diamond, and F. S. Stephens

October 1970

AEC Contract No. W-7405-eng-48

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

DISCLAIMER

EFFECTIVE SINGLE-PARTICLE MAGNETIC MOMENTS AROUND ^{208}Pb*

K. H. Maier†, K. Nakai††, J. R. Leigh, R. M. Diamond, and F. S. Stephens

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

October 1970

An effective magnetic moment operator is empirically derived for shell model states around ^{208}Pb. It reproduces all 8 measured moments to a few tenths of a nuclear magneton, but predicts too high a hindrance for M1-transitions.

- - -

I. STATIC MOMENTS

The well known failure of the shell model to predict the experimental magnetic moments around ^{208}Pb, a region where it is particularly successful otherwise, has stimulated much theoretical work to explain the deviations [1-8]. The main point common to all this work is to consider core excitations caused by the residual interaction in which a nucleon is excited from a filled level in the core to its empty spin-orbit partner orbital [9]. As the effect is linear in the amplitude of the admixture of these core excitations, even small admixtures, hardly detectable in other experiments, can produce significant deviations of the g-factors. This is especially true in the case of the ^{208}Pb core which contains 26 particles in the $\pi\ h_{11/2}$ and $\nu\ i_{13/2}$ orbitals that can

*Work performed under the auspices of the U. S. Atomic Energy Commission.

†NATO fellow on leave from Hahn-Meitner-Institut, Berlin.

††Present address: N. Bohr Institute, Risø, Denmark.

be excited to the empty $\pi\ h_{9/2}$ and $\nu\ i_{11/2}$ levels. Taking these core excitations into account, one can express the magnetic moment for a single particle in the orbital $j = \ell \pm 1/2$:

$$\mu = j \left\{ (g_\ell + \delta g_\ell) \pm \frac{(g_s + \delta g_s) - (g_\ell + \delta g_\ell)}{2\ell + 1} \right\} + g_p \langle (i^2 Y_2 \sigma)_1 \rangle + X \qquad (1)$$

Here X stands for complicated contributions from exchange integrals, while the other terms constitute the direct part. The theoretical studies done so far have calculated the moments starting from a number of residual interactions, but even the most recent and refined calculations [5,7] have not quite succeeded in reproducing the magnitude of the deviations from the Schmidt values, although the trend inward from the latter was always reproduced. Now, however, with the results of the g-factor measurements of Yamazaki, et al. [10-12] on Po isomers and of ours [13] on the 7^- level in ^{206}Pb and of that of the 21/2+ level in ^{207}Bi available, in addition to earlier work, we can try to check the validity of the form of this expression and the importance of the terms in it against the experimental data.

Table 1 (upper part) summarizes the measured magnetic moments of good shell-model levels around ^{208}Pb. The dominant configurations are given in column 3, and for the sake of definiteness we will consider only these configurations, as any admixtures should be small and are not known well enough to allow reliable corrections. The lower part of the table contains the other measured moments that lend themselves only with limitations to this treatment, as they are either based on a ^{206}Pb or ^{210}Po core or serious doubts exist on the purity of the wavefunctions. This will be discussed later.

The first point we can check concerns the importance of the exchange terms (X in eq. (1)). While the direct part obeys the same algebra [14] as the simple single-particle operator, the exchange terms do not [5]. The 8^+ level in ^{210}Po is composed of two $h_{9/2}$ protons (^{209}Bi gs), the $17/2^-$ level in ^{209}Po can be considered as derived from the 8^+ level in ^{210}Po and one $p_{1/2}$ neutron hole (^{207}Pb gs), and the $21/2^+$ level in ^{207}Bi (ref. 15) can be thought of as one $h_{9/2}$ proton and the 7^- level in ^{206}Pb. The g-factors of these three states have been measured. Column 6 of the table gives their moments calculated from the measured values of their constituents according to the rules of the single-particle operator, and hence assuming the dominance of the direct terms. In all three cases, this gives the measured moments within the experimental error of 1, 6, and 2%, respectively. So the exchange terms are probably unimportant, and we will neglect them in the following.

This leaves us with a simple formula containing the parameters δg_s, δg_l, and g_p; these, however, might be different for protons and for neutrons and might depend slightly on the radial wavefunctions. For further analysis we follow the works of Bodenstedt and Rogers [3] and Bohr and Mottelson [4,6], who, in agreement with the conclusions drawn above, omitted exchange terms. Furthermore, they assumed that the only interaction of importance affecting g-factors is of the type $(\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j)$. This interaction polarizes the neutrons and protons of the core oppositely to each other, so that due to their opposite sign of g_s, the effects from neutrons and protons add. For a force of the type $(\sigma_i \cdot \sigma_j)$ that should be of similar strength, the contributions of neutrons and protons are opposite in sign, leading to a ten times smaller effect on the moment. Bodenstedt and Rogers [3] furthermore assume a long-range interaction so that the differences in the radial wavefunctions of the single-particle become unimportant. Thus they get an effective magnetic-moment operator involving only the two parameters δg_s and g_p. They should be equal in magnitude for all levels around one core but opposite in sign for neutrons and protons, as the amplitudes of the admixed wavefunctions differ only in their sign.

The chosen force cannot produce any deviations of g_l. However, Yamazaki, et al. [12] determined $\delta g_l = +0.09 \pm 0.02$ for the proton from a measurement of the g-factor of the $| \pi h_{9/2}, \pi i_{13/2}; 11^- \rangle$ level in ^{210}Po, as the g-factor of this state should be essentially equal to $(g_l + \delta g_l)$. We, therefore, include a term in δg_l. This can be caused by other types of residual interactions [5] and by mesonic effects [12], so that no simple relation between δg_{ln} and δg_{lp} can be assumed a priori, and both parameters must be considered. We now can write the magnetic moment of a single-particle or hole state around the ^{208}Pb core, $j = \ell \pm 1/2$, as

$$\text{protons:} \quad \mu = j \left\{ (g_{\ell p} + \delta g_{\ell p}) \pm \frac{(g_{sp} - \delta g_s) - (g_{\ell p} + \delta g_{\ell p})}{2\ell + 1} \right\} - g_p \langle (i^2 Y_2 s)_1 \rangle \tag{2}$$

$$\text{neutrons:} \quad \mu = j \left\{ (g_{\ell n} + \delta g_{\ell n}) \pm \frac{(g_{sn} + \delta g_s) - (g_{\ell n} + \delta g_{\ell n})}{2\ell + 1} \right\} + g_p \langle (i^2 Y_2 s)_1 \rangle \tag{3}$$

$$\text{with } \langle (i^2 Y_2 s)_1 \rangle = \frac{1}{\sqrt{8\pi}} \langle j \tfrac{1}{2} 10 \mid j \tfrac{1}{2} \rangle \langle j\, j\, 10 \mid j\, j \rangle \cdot \left[1 + (-1)^{j-\ell+1/2} (j + \tfrac{1}{2})\right] \tag{4}$$

If we neglect δg_{1n} at first and take the value of $\delta g_{1p} = 0.09$, we have only the two parameters g_p and δg_s free to fit 5 single-particle moments, three of them being measured directly, and two deduced from the measured moments of a two-particle level and that of the other particle. In view of the fact that the additivity of the moments seems to be really fulfilled, these two should be reliable as well.

The result of a least-square fit is $\delta g_s = +3.14$ nm and $g_p = +3.27$ nm in agreement with estimates given by Bohr and Mottelson [4,6] and Bodenstedt and Rogers [3,16]. The magnetic moments derived in this way are shown in column 7 and reproduce all of the measured moments in column 4 within a few tenths of a nuclear magneton. This agreement is very unlikely to be accidental, especially as we did another fit in which δg_s, g_p, $\delta g_{\ell p}$, and $\delta g_{\ell n}$ were allowed to vary simultaneously without any restraints. The result is $\delta g_s = +3.45$, $g_p = +4.20$, $\delta g_{\ell p} = +0.09$, and $\delta g_{\ell n} = -0.03$ (all in nm). The fact that $\delta g_{\ell p}$ turns out exactly as determined by Yamazaki et al. [12] in a direct way is expected since the same primary data are used; it is however still significant as it shows that this 4-parameter fit gives reasonable results. The results for the other three parameters are reasonable, too. The change of 10% in δg_s and 25% in g_p versus the values found for $\delta g_{\ell n} = 0$ indicates the accuracy with which these parameters can be determined from the present data. As $\delta g_{\ell n}$ is almost exclusively determined by the moment of the ^{206}Pb 7^- level its value depends heavily on the purity of the wavefunction of this state. So we must ascribe a rather large error to it that certainly includes the possibility of $\delta g_{\ell n} = 0$. The magnetic moments resulting from this fit do not differ significantly from those given in column 7. The agreement with the measured values is in both cases better than one might expect from such a simple model.

If we now look at the lower part of the table, we find agreement for the 8^+ levels in ^{208}Po and ^{212}Rn. The $1/2^+$ groundstate of ^{205}Tl and the $13/2^+$ level in ^{205}Pb are reasonably close to the predicted values while they are far from the Schmidt values. These two levels are based on a ^{206}Pb core so one might expect some deviation. In addition it is known that only 70% of the wavefunction of the ^{205}Tl gs is exhausted by the configuration of an $s_{1/2}$ proton hole coupled to the ^{206}Pb ground state [17]. Indeed, Azziz and Covello [17] reproduce the measured magnetic moment exactly with a better wavefunction and using $g_{s\ eff} = 3.45$ which is close enough to our value. Neutron pickup experiments [18,19] indicate that the $13/2^+$ level in ^{205}Pb is less pure than the 7^- level in ^{206}Pb. Based on this, and since the $13/2^+$ level in ^{205}Pb belongs to a ^{206}Pb core rather than a ^{208}Pb one, we used the moment of the $i_{13/2}$ neutron derived from the ^{206}Pb 7^- level for the fit. The energy of the 1^- ground state of ^{210}Bi poses problems in nearly all shell-model calculations, indicating impurities of the wavefunction. The main component of the wavefunction ($\pi\ h_{9/2}$, $\nu\ g_{9/2}$) gives a magnetic moment about 5 times smaller and of opposite sign to that of the most likely admixed components. Therefore this moment cannot be reliably predicted. The measured moment of the 6^- level in ^{206}Pb has the wrong sign for a shell-model level under any reasonable assumption, so that the workers [20] concluded that this level involves proton excitations.

Thus, it is possible to account for all the measured magnetic moments of good shell-model levels around ^{208}Pb using a surprisingly simple effective operator with a few reasonable parameters. This in turn means that the assumptions involved are likely to be valid. They are: (i) The wavefunctions of the levels considered are quite pure, (ii) the exchange terms are unimportant, (iii) the interaction causing the polarization of the core is predominantly of the type $(\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j)$ and of long range. The open question now, of course, is how to explain theoretically this empirically-found operator and the magnitude of the parameters involved. More measurements to further test this expression are certainly indicated, and it would be of interest to have enough data near other doubly-closed shell nuclei, for instance ^{56}Ni, to check if this approach would hold there as well.

II. M1-TRANSITIONS

It is interesting now to consider allowed ($\Delta\ell = 0$) M1-transitions briefly. The wavefunction of a single particle or hole, j, with the admixtures of importance for M1-properties can be written (ref. 7)

$$|j\rangle = |(j, {}^{208}\mathrm{Pb}\ 0^+)_j\rangle + \varepsilon_j^j |(j, {}^{208}\mathrm{Pb}\ 1^+)_j\rangle + \varepsilon_{j'}^j |(j', {}^{208}\mathrm{Pb}\ 1^+)_j\rangle \tag{5}$$

Here j' is the spin-orbit partner of j and the 1^+ level in ^{208}Pb has the structure $(\nu\, i_{13/2}^{-1}, \nu\, i_{11/2})$ or $(\pi\, h_{11/2}^{-1}, \pi\, h_{9/2})$. In first order, the second term leads to the deviations of the static moments without affecting transitions, and only the third term is effective for M1-transitions from j to j' or vice-versa. That is, applying the single-particle M1-operator to the first two terms of eq. (5) and calculating ε_j^j in perturbation theory from the assumed residual interaction yields just the effective operator we have used so far (except for δg_l) [3]. For M1-transitions, the same has to be done for the first and third terms.

It might seem that different operators result for the static case and for transitions, since the relevant admixtures of the wavefunction are different. However, doing the calculations explicitly, as Harada and Pittel have done [7], results in very similar operators. Inserting their eq. (7) into their eq. (8) for moments and into eq. (9) for transitions shows the two correction terms proportional to $\langle s \rangle$ and $\langle i^2(Y_2 s)_1 \rangle$, respectively. The factors in front of these two matrix elements in eq. (8) are equivalent to the δg_s and g_p that we have empirically determined. If one now looks at transitions (eq. (9) of ref. 7), one finds by evaluating $\varepsilon^j_{j'}$, that except for slightly different energy denominators, these factors are identical. Therefore if the approach used so far is really valid the same operator should describe the transitions too if one sets

$$\delta g_s(\text{trans}) = \frac{\Delta^2}{\Delta^2 - \Delta^2_{j,j'}}\ \delta g_s(\text{stat}) \text{ and } g_p(\text{trans}) = \frac{\Delta^2}{\Delta^2 - \Delta^2_{j,j'}}\ g_p(\text{stat})$$

Here Δ is the energy difference between the 1^+ and 0^+ levels in ^{208}Pb which is about 5-8 MeV and $\Delta_{j,j'}$ can be taken directly from the spectrum and is smaller than 2 MeV for the cases to be considered below. Therefore δg_s and g_p should only be increased by a few percent for transitions.

For a transition between single-particle levels from $j_1 = \ell + 1/2$ to $j_2 = \ell - 1/2$ the B(M1)-value calculated with the effective operator becomes (in units of $\frac{3}{4\pi}\left(\frac{e\hbar}{2Mc}\right)^2$):

$$B(M1,\ \ell + \tfrac{1}{2} \rightarrow \ell - \tfrac{1}{2}) = \frac{\ell}{(2\ell + 1)} \left\{ (g_s + \delta g_s) - (g_\ell + \delta g_\ell) + \frac{1}{4\sqrt{2\pi}}\ g_p \right\}^2 \quad (6)$$

The sign in front of δg_s and g_p has been chosen for neutrons; for protons it has to be reversed. For the unperturbed operator the same equation holds with

$\delta g_s = \delta g_\ell = g_p = 0$. Contrary to the static case in which δg_s and g_p give contributions of opposite sign, here they both decrease B(M1), in agreement with the calculations of ref. 7. Using $\delta g_s = 3.14$ nm and $g_p = 3.27$ nm as before, the B(M1)-values for the $f_{7/2} \rightarrow f_{5/2}$ and $p_{3/2} \rightarrow p_{1/2}$ transitions in ^{207}Pb are reduced by a factor of more than 100 relative to the s.p. value, while experimentally these factors are only 4 (ref. 21) and 3 (ref. 22), respectively. For the $1^+ \rightarrow 0^+$ transition in ^{206}Pb, the experimental reduction is 2 to 3 (refs. 23,24). Mottelson [4] has pointed out already that the measured lifetime [25] of the ^{208}Tl $4^+ \rightarrow 5^+$ transition, which shows no hindrance at all, poses a severe problem to the core-polarization picture. This difficulty now seems to be more general and might indicate the limits of this rather simple model. Of course, it should be pointed out that if B(M1) values had also been used in the original parameter determinations, poorer fits would have been achieved for the static moments, but better fits to the transition moments.

We want to thank Dr. J. Quebert for his help and Drs. M. Weigel and M. Redlich for very valuable discussions.

References

1. A. Arima and H. Horie, Progr. Theor. Phys. 11 (1954) 509.
2. R. J. Blin-Stoyle and M. A. Perks, Proc. Phys. Soc. (London) A67 (1954) 885.
3. E. Bodenstedt and J. D. Rogers in, "Perturbed Angular Correlations", eds., E. Karlsson, E. Matthias, and K. Siegbahn, (North-Holland Publishing Co., Amsterdam, 1964) p. 93.
4. B. R. Mottelson, Nordita publications, No. 288, p. 54.
5. J. Blomqvist, N. Freed, and H. O. Zetterström, Phys. Letters 18 (1965) 47.
6. A. Bohr and B. R. Mottelson, Nuclear Structure, Vol. I, p. 336, (Benjamin Inc., 1969, New York, Amsterdam).
7. K. Harada and S. Pittel, Nucl. Phys. A159 (1970) 209.
8. Further references given in 1-7.
9. We will not consider mesonic effects. Articles on this subject are cited in ref. 12.
10. T. Yamazaki and E. Matthias, Phys. Rev. 175 (1968) 1476.
11. T. Yamazaki, T. Nomura, T. Katou, T. Inamura, A. Hashizume, and Y. Tendou, Phys. Rev. Letters 24 (1970) 317.
12. T. Yamazaki, T. Nomura, S. Nagamiya, and T. Katou, Phys. Rev. Letters 25 (1970) 547.
13. K. H. Maier, K. Nakai, F. S. Stephens, and R. M. Diamond, to be published.
14. A. de Shalit and I. Talmi, Nuclear Shell Theory (1963), p. 56 and 449.
15. I. Bergström, C. J. Herrlander, P. Thieberger, and J. Blomqvist, Phys. Rev. 181 (1969) 1642.
16. H. J. Körner, K. Auerbach, J. Braunsfurth, and E. Gerdau, Nucl. Phys. 86 (1966) 395, have determined these two parameters from the ^{207}Pb gs

(ν $p_{1/2}$) and $5/2^-$ level (ν $f_{5/2}$) with very similar results. They stated, however, that these are incompatible with the ^{209}Bi gs (π $h_{9/2}$).

17. N. Azziz and A. Covello, Nucl. Phys. A123 (1969) 681.
18. R. Tickle and J. Bardwick, Phys. Rev. 166 (1968) 1167, and Phys. Rev. 178 (1969) 2006.
19. K. Yagi, T. Ishimatsu, Y. Ishizaki, and Y. Saji, Nucl. Phys. A121 (1968) 161.
20. F. C. Zawislak and J. D. Bowman, Nucl. Phys. A146 (1970) 215.
21. Actually only the M1-E2 mixing ratio has been measured. See ref. 6, p. 344.
22. H. V. Klapdor, P. von Brentano, E. Grosse, and K. Haberkant, Nucl. Phys. A152 (1970) 263.
23. K. Harada and S. Pittel, Phys. Letters 32B (1970) 542.
24. F. R. Metzger, Bull. Am. Phys. Soc. 15 (1970) 546.
25. K. D. Sevier, Nucl. Phys. 61 (1965) 601.

Table I. Magnetic moments of shell-model states around ^{208}Pb. All moments are given in nuclear magnetons. Col. 4 gives the experimental values and the errors exceeding 3%. Col. 5 shows the predictions of the shell-model for the configuration of col. 3 using the free nucleon moments. The values of col. 6 are obtained using the formalism of the single-particle operator from the measured values of the configurations constituing these levels. Col. 7 shows the predictions for the effective operator (Eq. 2-4) with the contributions from the terms in δg_s , g_p, and $\delta g_{\ell p}$ given in co. 8-10.

Level	E_x MeV	Dominant Configuration	μ Experiment	μ Schmidt	μ added	μ Core excitation	Contributions from δg_s	g_p	$\delta g_{\ell p}$
$9/2^-$ ^{209}Bi	gs	$\pi\ h_{9/2}$	+4.08[a]	2.62		3.98	1.28	-.37	.44
$1/2^-$ ^{207}Pb	gs	$\nu\ p_{1/2}^{-1}$	+0.59[a]	0.64		0.57	- .52	.45	0
$5/2^-$ ^{207}Pb	0.570	$\nu\ f_{5/2}^{-1}$	+0.65(5)[b]	1.37		0.63	-1.12	.38	0
		$\nu\ i_{13/2}^{-1}$	(-0.75)[c]	-1.91		-0.62	1.57	-.27	0
		$\pi\ i_{13/2}$	(7.9)[d]	8.79		8.03	-1.57	.27	.54
7^- ^{206}Pb	2.200	$\nu\ i_{13/2}^{-1}, \nu\ p_{1/2}^{-1}$	-0.15[e]	-1.26		-0.05			
8^+ ^{210}Po	~ 1.50	$(\pi\ h_{9/2})^2$	7.30[f]	4.66	7.26	7.07			
11^- ^{210}Po	~ 2.8	$\pi\ i_{13/2}, \pi\ h_{9/2}$	12.0[g]	10.66		12.0			
$17/2^-$ ^{209}Po	~ 1.50	$(\pi\ h_{9/2})^2_{8^+}, \nu\ p_{1/2}^{-1}$	7.48(43)[h]	4.9	7.89	7.64			
$21/2^+$ ^{207}Bi	2.102	$(\nu\ i_{13/2}^{-1}, \nu\ p_{1/2}^{-1})_{7^-}, \pi\ h_{9/2}$	3.41[e]	1.15	3.39	3.39			

(continued)

Table 1. (continued)

Level	E_x MeV	Dominant Configuration	μ Experiment	μ Schmidt	μ added	μ Core excitation	Contributions from δg_s	g_p	$\delta g_{\ell p}$
$1/2^+$ ^{205}Tl	gs	$(\nu\ p_{1/2})^{-2}$, $(\pi\ s_{1/2})^{-1}$	+1.63[a]	2.79		+1.22	-1.57	0	0
$13/2^+$ ^{205}Pb	1.0138	$(\nu\ p_{1/2})^{-2}$, $(\nu\ i_{13/2})^{-1}$	-0.98[i]	-1.91		-0.62	1.57	-.27	0
1^- ^{210}Bi	gs	$\pi\ h_{9/2}$, $\nu\ g_{9/2}$	± .04[a]	+ .08		+ .38			
6^- ^{206}Pb	2.385	$(\nu\ p_{1/2})^{-1}$, $(\nu\ i_{13/2})^{-1}$	+ .13(07)[j]	- .41		- .18			
8^+ ^{208}Po	~ 1.532	$(\pi\ h_{9/2})^2$, $(\nu\ p_{1/2})^{-2}$	7.22[k]	4.66	7.26	7.07			
8^+ ^{212}Rn	~ 1.690	$(\pi\ h_{9/2})^4$	7.05[i]	4.66	7.26	7.07			

[a] C. M. Lederer, J. M. Hollander, and I. Perlman, Table of Isotopes.

[b] Ref. 16.

[c] Derived from the measured values of the $1/2^-$ ^{207}Pb gs and the 7^- ^{206}Pb level assuming a pure configuration.

[d] Derived from the measured values of the $9/2^-$ ^{209}Bi gs and the 11^- ^{210}Po level assuming a pure configuration.

[e] Ref. 13.

[f] Ref. 11.

[g] Ref. 12.

[h] Ref. 10.

[i] K. H. Maier, J. R. Leigh, R. M. Diamond, and F. S. Stephens, to be published.

[j] Ref. 20.

[k] S. Nagamiya, T. Nomura, and T. Yamazaki, Nucl. Phys. A159 (1970) 653.

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720