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# IMPLICATIONS OF THE EFFECTIVE NUMBER OF PARTIES FOR CABINET FORMATION

*Rein Taagepera*

## ABSTRACT

This note helps to explain how cabinet-level concentration of power is constrained by party level concentration of seats. Arend Lijphart's *Patterns of Democracy* (1999) measures concentration of executive power by the frequency of 'minimal winning and one-party cabinets' (MW/OP), and party concentration by the effective number (N) of legislative parties. In his factor analysis, these highly correlated indices are the central features that distinguish consensual from majoritarian systems. The present study establishes a quantitative logical relationship leading from N to the major component of MW/OP, so as to explain the reasons behind Lijphart's empirical observation. The note analyses separately the frequency of various types of cabinet coalitions that Lijphart's book has lumped together. It also offers a new way to visualize the effective number of legislative parties, as twice the minimal number of parties needed to form a minimal winning coalition.

KEY WORDS ■ consensual and majoritarian systems ■ effective number of parties  
■ types of cabinet coalitions

How does the number of parties in a representative assembly affect the type of cabinets formed? The question is important because cabinet durations and working styles tend to differ depending on whether one has, say, a multi-party minority cabinet or a one-party majority cabinet – and the latter looks more likely when the assembly includes few parties.

The issue can be investigated on various levels, depending on the specificity of information taken into account. When the seat shares of all parties are known, a game theoretical analysis for this particular constellation is possible, the more so when the parties' ideological stands are also available. But taking into account only a single major indicator, the effective number of parties,<sup>1</sup> can yield important generalizations across many countries. This

is the approach taken here, to explain how cabinet-level concentration of power is constrained by party level concentration of seats.

In his *Patterns of Democracy* (1999), Arend Lijphart finds that the frequency of 'minimal winning and one-party cabinets' (which I abbreviate as MW/OP) emerges as the central variable in factor analysis that distinguishes majoritarian from consensus democracies. According to Lijphart, MW/OP reflects the concentration versus sharing of executive power. Its loading is as high as 0.93 on the joint-power dimension or Factor I (1999: 246), which includes electoral disproportionality, the number of parties, cabinet life and interest group pluralism. Almost as central is the effective number of legislative parties (N). Its loading is -0.90. Correspondingly, the correlation between N and MW/OP is very high ( $R^2 = 0.76$ ; p. 245), as can be seen in the graph on page 112.

Lijphart accepts this relationship as an empirical one. It makes of course intuitive sense that the frequency of one-party cabinets decreases as the number of parties goes up. But why does it decrease precisely at the rate observed? By clarifying the link between N and MW/OP, one would gain insights into the meaning of both measures.

This note points out a quantitative logical relationship leading from the effective number of parties to the major component of MW/OP, so as to give a theoretical basis to Lijphart's empirical observation. It also analyses separately the frequency of various types of coalitions that Lijphart's book has lumped together. Finally, it offers a new way to visualize the effective number of legislative parties. While the effective number has become the standard way to measure the number of parties, its meaning is sometimes hard to sense when some parties are very large and others are small. It will be shown that N can be visualized as twice the minimal number of parties needed to form a minimal winning coalition.

### Logical Constraints on One-Party Majority Cabinets

In cabinet formation, Lijphart highlights two features: whether a cabinet consists of one or several parties and whether it is minimal winning. 'Minimal winning' is a concept originating among game theorists, but useful also in other contexts. It means that a coalition has majority (more than 50 percent), but would lose majority if any one of the partners defected. In contrast, 'oversized' cabinets have more members than needed for reaching majority. Accordingly, Lijphart distinguishes five cabinet types (the abbreviated symbols being mine): minimal winning one-party cabinets ( $MW_1$ ); minimal winning multiparty cabinets ( $MW_m$ ); one-party minority cabinets ( $m_1$ ); multiparty minority cabinets ( $m_m$ ); and oversized cabinets (OS).<sup>2</sup>

Lijphart (1999: 91) posits that one-party majority cabinets represent pure majoritarianism, while one-party minority cabinets and multiparty MW cabinets are 'in an intermediary position'. This leaves the oversized and

multiparty minority cabinets as representing pure consensus philosophy. Accordingly, Lijphart's measure for majoritarianism in cabinet formation (1999: 110–11) boils down to the following:<sup>3</sup>

$$MW/OP = MW_1 + 1/2 [MW_m + m_1].$$

As graphically documented in Lijphart's Fig. 6.1 (1999: 112), the relationship between MW/OP and N is extremely strong. The empirical best-fit line in that graph decreases from MW/OP = 100 percent at about N = 1.4 to MW/OP = 0 percent at N = 5.7. Why do these particular coordinates prevail? The answer is straightforward for the core of the degree of majoritarianism ( $MW_1$ ), because the effective number of parties imposes the following two constraints on  $MW_1$ .

### *N*<2

A value of  $N < 2$  cannot occur unless one party has >50 percent of the seats (e.g. 51–49 yields  $N = 1.999$ ). Hence  $MW_1$  is always possible when  $N < 2$  – and hence  $MW_m$  is impossible. Mild shortfalls from  $MW_1 = 100$  percent can occur when (1) the major party forms an oversized coalition by taking along a smaller partner (like Australia's Liberals and Country Party), or (2) the majority party desists and allows minor party/parties to form a minority cabinet (which is hardly likely).

An artifactual possibility must be added when the value of  $N$  is the *mean* for many elections. Even while the mean  $N$  is below 2.0, the country may have  $N$  above 2.0 for some individual elections, so that the largest party's share could drop below 50 percent. Empirically, no country with  $N < 2$  has MW/OP less than 99 percent in Lijphart's graph (1999: 112).<sup>4</sup> However, his empirical regression line corresponds to MW/OP = 87 percent at  $N = 2$  and hence falls below what is logically expected and actually observed.

### *N*>4

At the other extreme,  $MW_1$  becomes impossible when  $N$  exceeds 4. Indeed, with the largest party's share above 50 percent,  $N$  cannot reach 4 even when all other parties have only one seat each (e.g. for a 200-seat assembly distributed as 101–1–1–1–1– . . .  $N = 3.88$ ). Thus the non-zero values of MW/OP at  $N > 4$  – and there are 9 such countries in Lijphart's graph (1999: 112) – are due to multiparty MW and one-party minority cabinets ( $MW_m$  and  $m_1$ ), except for artifactual exceptions due to the use of the means of many elections.<sup>5</sup>

For  $2 > N > 4$ , any values of  $MW_1$  are logically possible. One would expect the average  $MW_1$  to be close to 100 percent for  $N$  barely above 2 and close to 0 percent for  $N$  approaching 4, but for individual countries anything goes. When one draws a line from  $N = 2$ , MW/OP = 100 to  $N = 4$ , MW/OP = 0 in Lijphart's graph (1999: 112), only Austria, Germany and Mauritius (all with frequent OS cabinets) plus Australia fall below this line.

Since  $MW_1$ ,  $MW_m$  and  $m_1$  are not listed separately in the book, the logical constraints cannot be checked on that basis. However, Arend Lijphart has graciously allowed me to use his more detailed data, and this brings us to the next task.

### Frequency of Cabinet Types at Various Effective Numbers of Parties

Table 1 gives the average frequencies of the five cabinet types at various ranges of effective number of parties.<sup>6</sup> Figure 1 shows the same data graphically, in a cumulative format. Before considering this overall picture, the more detailed patterns for each coalition type will be discussed separately.

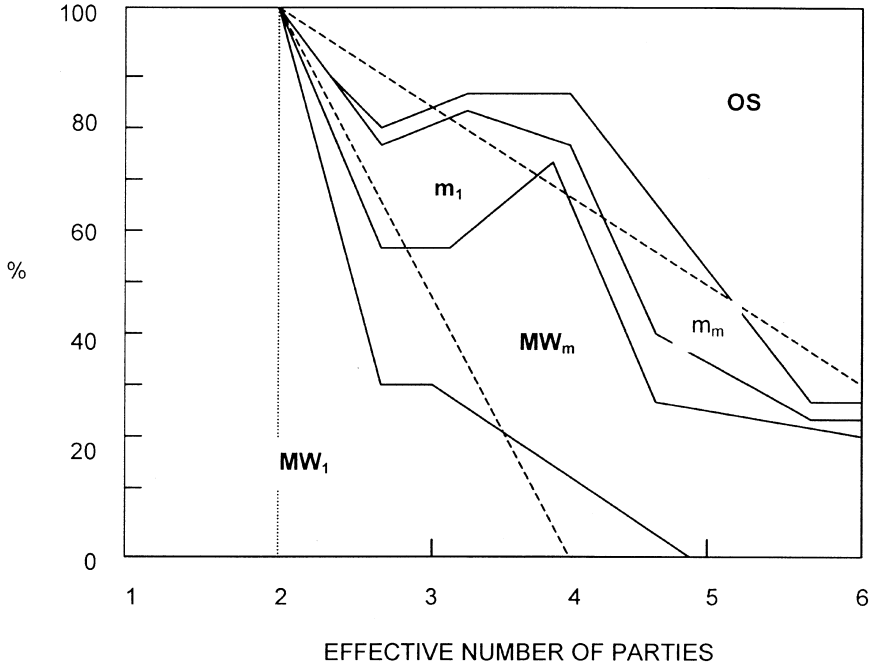
*One-party majority cabinets (MW<sub>1</sub>)* represent close to 100 percent of all cabinets for  $N < 2$ , falling below 100 percent only artifactually (the lowest being Trinidad with 98.1 percent). Their share remains high for  $2.0 < N < 2.5$ , and then drops sharply. Beyond  $N > 4$ , only the artifactual cases of India and Belgium occur. The line  $MW_1 = 50(4-N)$ , going from  $N = 2$ ,  $MW_1 = 100$  to  $N = 4$ ,  $MW_1 = 0$ , approximates the average pattern, but is on the high side for  $2.5 < N < 3.0$ , where Mauritius (0 percent  $MW_1$ ) and Germany (1.5 percent) are the lowest; all other countries (Austria, Ireland, Spain) are also low.

*Multiparty MW cabinets (MW<sub>m</sub>)* appear, as expected, as soon as  $N$  surpasses 2.0, reaching a peak of about 50 percent of all cabinets at  $N = 4.0$ . Their share decreases thereafter but remains appreciable (around 20 percent) even at the very highest number of parties. The scatter is enormous, especially at  $3 < N < 4$  (88 percent for Iceland and Luxembourg, 7 percent for Columbia and 0 percent for Venezuela). There are no logical limits, at  $N > 2$ . One can only venture that at  $N < 3$  the possibility of  $MW_1$  still reduces the likelihood of  $MW_m$ . When  $N > 5$ , so many parties are needed to form an MW coalition that the addition of one more party may represent a minor cost.

**Table 1.** Average frequencies (%) of five cabinet types at various effective numbers of parties

Range of $N$	No. of countries	$MW_1$	$MW_m$	$m_1$	$m_m$	OS
1 to 1.99	7	99.6	0.3	0.07	0.04	0
2.00 to 2.49	6	72.6	8.9	8.8	0.3	9.3
2.50 to 2.99	5	31.2	26.3	19.9	2.8	19.7
3.00 to 3.49	7	31.3	26.6	23.3	3.8	17.1
3.50 to 4.49	5	16.6	50.5	8.2	9.6	15.2
4.50 to 4.99	4	0.02	25.2	13.3	19.8	41.7
5.00 to 5.99	3	0	22.9	3.6	3.1	70.3

*Data source:* Lijphart, private communication. MW with more than 80% seats are counted as OS.



**Figure 1.** Cumulative percent frequencies of cabinet types versus effective number of legislative parties. The dashed lines correspond to approximations  $MW_1=50(4-N)$  and  $OS+m_m=35(0.5N-1)$

*Single-party minority cabinets ( $m_1$ )* start to occur as soon as  $N$  surpasses 2 and reach a mean peak of about 23 percent at  $N = 3$ , heavily influenced by high values for Norway (48 percent), Spain (54 percent) and Sweden (66 percent). But even around  $N = 5$ , Finland and Italy have 10 percent of such cabinets. At the other extreme, half the countries at any  $N$  exhibit no  $m_1$ . No logical restraints can be specified, at  $N > 2$ . As long as  $N < 3.5$ , the largest party, even when it falls short of majority, may still be close to 50 percent and might be able to balance off the opposite extremes. For  $N > 4.5$ , even the largest party is likely to fall appreciably short of majority, making formation and survival of single party cabinet difficult. One could well conclude that at that stage multiparty minority cabinets might become more likely, and this is indeed the case.

*Multiparty minority cabinets ( $m_m$ )* hardly exist for  $N < 2.8$ , as one might expect: Even if the largest party does not have a majority, the addition of a single second party readily makes it MW. The mean frequency of  $m_m$  peaks around  $N = 4.5$  at a modest 20 percent, boosted by Denmark (40 percent). The total for all minority cabinets ( $m_1+m_m$ ) holds fairly steady around 25 percent from  $N = 2.7$  to  $N = 5$ .

Finally, the mean frequency of *oversized cabinets (OS)* grows fairly steadily as  $N$  increases beyond 2. The line  $OS = 35(0.5N-1)$  expresses the

average trend, from 0 at  $N = 2$  to 70 percent at  $N = 6$ . Once again, no logical constraints are apparent, and individual variations are huge. Mauritius (70 percent OS, at  $N = 2.7$ ), Switzerland (92 percent, at  $N = 5.4$ ), Austria (grand MW coalitions counted as OS by Lijphart), and Israel are relatively high, while Denmark ( $N = 4.5$ ), Norway ( $N = 3.4$ ) and Sweden ( $N = 3.3$ ) have avoided OS completely, preferring minority cabinets.<sup>7</sup>

The overall distribution of cabinet types at varying  $N$  is shown in Figure 1. In a very approximate way, pure majoritarianism is delimited by the aforementioned equation  $MW_1 = 50(4-N)$ , when  $2 < N < 4$ . Pure consensualism is delimited by  $OS + m_m = 35(0.5N-1)$ , a line that fits the combination of OS and  $m_m$  even better than OS alone, at  $N > 2$ . This leaves for the intermediary category  $MW_m + m_1 = 65(0.5N-1)$  when  $2 < N < 4$ , and  $MW_m + m_1 = 135 - 17.5N$  for  $N > 4$ . These zones are approximate, but they answer the following important question.

Results of measurements at times implicitly tell us 'You should have measured in a somewhat different way'. Is this the case here? It makes sense to take  $MW_1$  as pure majoritarianism and OS as pure consensualism. But what about the other cases? Are  $MW_m$  and  $m_1$  both really exactly half way from the pure extremes? And is  $m_m$  as pure consensualism as OS? One could replace their coefficients (1/2 and 1) in the defining equation by undetermined parameters:

$$\text{Degree of majoritarianism} = MW_1 + aMW_m + bm_1 + cm_m$$

and, correspondingly,

$$\text{Degree of consensualism} = OS + (1-c)m_m + (1-a)MW_m + (1-b)m_1.$$

One could then determine the values of  $a$ ,  $b$  and  $c$  so as to minimize scatter in a graph of  $MW/OP$  vs.  $N$  (such as Lijphart's Fig. 6.1). A glance at Figure 1 suggests that short of going to intricate curve fitting, with no theoretical basis by which to justify it, it is hard to disentangle the zone of  $m_1$  from that of  $MW_m$ . The two seem to represent, indeed, a common zone that expands with increasing  $N$ . To a lesser degree the same applies to  $m_m$  and OS. What this means is that there is no reason to deviate, in the equations above, from the values  $c = 0$  and  $a = b$  implied by Lijphart. Do  $m_1$  and  $MW_m$  truly represent exact middle ground ( $a = b = 0.5$ ) between  $MW_1$  and OS? We have no reason to argue for a different value.

Now we proceed to a different issue. Rather than visualizing cabinet types in terms of effective numbers of parties the reverse approach is taken.

### **A New Visualization of the Effective Number of Legislative Parties**

When a seats constellation like 45-29-21-5 leads to  $N = 3.00$ , it is easy to visualize it as vaguely equivalent to three equal-sized parties. But what about 53-15-10-10-10-2, which also yields  $N = 3.00$  (Taagepera and Shugart,

1989: 259)? In what sense can this constellation be even vaguely akin to three equal-sized parties? I will now show the following:

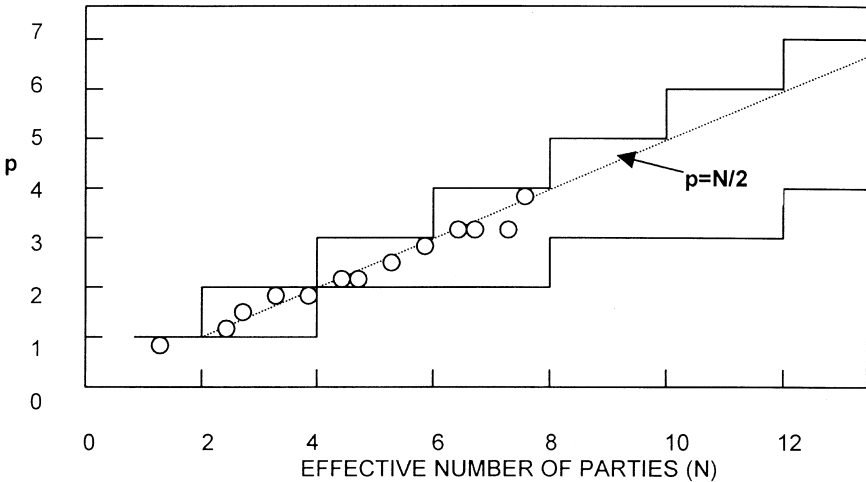
One-half of the effective number of parties approximates the minimal number of parties required to form an MW coalition.

In the two examples above,  $N/2 = 1.5$ . So the minimal number of partners may be either 1 (as is the case for 53-15-10-10-10-2) or 2 (as is the case for 45-29-21-5).

MW cabinets with more than the minimal number of parties are of course possible, and may even be more likely. For 48-48-1-1-1-1 the resulting  $N = 2.17$ ,  $N/2 = 1.08$  allows for a one-party cabinet or a two-party MW coalition, and 48-48 fits the bill. Yet a four-party coalition 48-1-1-1 is much more likely than such a grand coalition (which by Lijphart's 80+ criterion would be considered oversized). But here we are concerned with the conceptual minimum number ( $p$ ) of MW partners needed. It can be shown that this minimal number (1) cannot fall below the integer part of  $N/4+0.5$ :  $p > I[(N+2)/4]$  and (2) cannot exceed  $N/2$  rounded up to the next integer:  $p < I[(N+1)/2]$ .

These limits are shown in Figure 2 (disregard for the moment the observed points). For  $N < 2$  it means of course that the only MW is a one-party cabinet. For  $N$  ranging from 2 to 4, minimal-partner MW has either one or two parties. For  $4 < N < 6$ , it is either 2 or 3 parties. For larger  $N$  the range widens. Thus, for  $N$  ranging from 6 to 8, minimum-partner MW can have 2, 3 or 4 parties. Effective numbers larger than 8 are exceedingly rare.<sup>8</sup>

The relationship  $p = N/2$  is shown as a dotted line in Figure 2. It is the best-fit straight line between the higher and lower logical limits between  $N = 2$  and  $N = 6$ . For  $N > 6$  it is an overestimate.



**Figure 2.** Minimum number of partners ( $p$ ) needed to form a minimal-winning coalition, depending on the effective number ( $N$ ) of legislative parties: theoretical limits and averages of actual elections (from Table 2)



**Table 2.** Minimum number of partners ( $p$ ) needed to form a minimal-winning coalition, depending on the effective number ( $N$ ) of legislative parties in actual elections

<i>Range of N</i>	<i>No. of cases at</i>		<i>Mean</i>	<i>Mean</i>	<i>Difference</i>
	$I[N/2]$	$I[N/2]+1$	$p$	$N/2$	
1 to 1.49	4	–	1	NA	
1.5 to 1.99	78	–	1	NA	
2 to 2.49	87	26	1.23	1.12	+ .12
2.5 to 2.99	37	72	1.66	1.37	+ .29
3 to 3.49	8	85	1.91	1.62	+ .29
3.5 to 3.99	0	54	2.00	1.87	+ .13
4 to 4.49	40	1	2.02	2.12	– .10
4.5 to 4.99	33	3	2.08	2.37	– .29
5 to 5.49	16	10	2.38	2.62	– .24
5.5 to 5.99	3	8	2.7	2.87	– .2
6 to 6.49	8	0	3.0	3.12	– .1
6.5 to 6.99	3	0	3.0	3.37	– .4
7 to 7.49	4	0	3.0	3.62	– .6
7.5 to 7.99	0	1	4.0	3.87	+ .1

*Data:* Mackie and Rose (1991), all elections from 1900 on. Total: 581 elections.

The round symbols in Figure 2 indicate the mean values of  $p$  for a large sample of actual elections (see data in Table 2). These points meander around the  $p = N/2$  line, hugging the value of  $p = 2$  when  $N$  goes from 3 to 5, and then hugging  $p = 3$  when  $N$  goes from 5.5 to 7.

In sum, the integer closest to  $N/2$  is usually the minimal number of partners needed for MW, but both closest integers are possible. At  $N > 6$  even lower values can theoretically occur but have not been observed.<sup>9</sup> Thus the effective number of legislative parties can be roughly visualized as twice the minimal number of partners needed to form a minimal winning coalition. Note that this visualization cannot be extended to the effective number of electoral parties – votes do not form coalitions, only seats do.

## Conclusions

Lijphart's (1999: 110) measure of 'MW cabinets and one-party cabinets' (MW/OP) may look confusing and artificial but can be reworded – and one is hard put to find a more suitable single index to express the distribution of cabinet types. Its strong correlation with the effective number of legislative parties is not an empirical happenstance, but is logically imposed to an appreciable degree. In this sense, the effective number of parties is the independent variable driving MW/OP. In turn, going in the opposite direction, MW/OP supplies a way to visualize the meaning of the effective

number for constellations where the image of equivalence to some number of equal-sized parties becomes stretched. This conceptually reinforced logical tie between the two core indices to measure the degree of majoritarianism/consensualism of a country adds credibility to both of them.

### Notes

- 1 Laakso and Taagepera (1979) effective number of legislative parties is  $N = 1/\sum s_i^2$ , where  $s_i$  is the seat share of the  $i$ -th party and summation is over all seat-winning parties.
- 2 Lijphart (1999: 107) classifies those multiparty MW cabinets which have 80 percent support or more in the legislature among the oversized (OS) rather than  $MW_m$ , because they are far above the minimum size needed; e.g., for a constellation 40–40–20, a 40–40 coalition widely exceeds 40–20, which is also available in principle.
- 3 Lijphart's own way (1999: 110–11) of introducing MW/OP is sufficiently convoluted for one reviewer of his book (van der Kolk, 2000) to ask why he doesn't simply use  $MW_1$  plus one-half of  $MW_m$  and  $m_1$ , without realizing that this is exactly what Lijphart's indirect procedure boils down to. Also, Lijphart's collapsed label for the combined majoritarian category ('percent minimal winning, one-party cabinets'), such as given in his Fig. 6.1 (1999: 112) risks being mistaken for  $MW_1$  alone, especially when the comma is accidentally dropped (as is indeed the case on pp. 139, 244, 245, 246, 312 and 315). This is why I have used a slash instead of a comma in my abbreviation: MW/OP.
- 4 In Lijphart's Fig. 6.1 (1999: 112) two countries with mean  $N < 2$  have  $MW/OP < 100$  percent due to the artifactual reason: Trinidad,  $N = 1.82$ ,  $MW/OP = 99.1$  percent and New Zealand,  $N = 1.96$ ,  $MW/OP = 99.5$  percent. For some individual elections, both countries have reached  $N$  around 2.2 (1999: 77), so that the largest party share may fall below 50 percent.
- 5 Artifactual exceptions can occur for *mean*  $N$  above 4, when  $N$  for some individual elections involved is under 4. In Lijphart's country set this is the case for India (mean  $N = 4.11$  but individual values of  $N$  for the 6 elections ranging from as low as 2.51 to as high as 6.53) and Belgium (mean 4.32, range 2.45 to 6.51) (1999: 76).
- 6 Table 1 gives half-unit brackets of  $N$ , except at 1 to 1.99, where only  $MW_1$  is possible (except for artifacts due to using mean  $N$ ), and 3.50 to 4.49 and 5.00 to 5.99, so as to have at least 3 countries averaged.
- 7 The effective number of parties imposes few boundaries on pure consensus formats (OS and  $m_m$ ). Oversized coalitions can occur at any  $N$ . Even for constellation 99–1, an OS coalition (indeed a 'grand coalition!') is logically possible. And multiparty minority cabinets become logically possible the moment three parties are represented in the assembly – even for 98–1–1 a multiparty minority cabinet 1–1 isn't *logically* impossible! However, any such development would be so much at the mercy of the large party that we would not expect pure consensus formats (OS and  $m_m$ ) to occur in a significant way at  $N < 2$  – and this is the case empirically.
- 8 Actually, these limits apply when the smallest parties are infinitesimally splintered. When the lowest seat share is one seat, the limits are narrower. For a 100-seat

assembly, 51-1-1-1- . . . is the upper limit for having a one-party MW, and it yields  $N = 3.77$ . Already with 50-1-1-1- . . . at least 2 parties are needed for MW, while the resulting  $N = 3.92$  is still shy of  $N = 4$ . Similarly, two-party MW is possible for  $S = 100$  only up to 26-25-1-1-1- . . . ( $N = 7.41$ ), as 25-25-1-1-1- . . . already requires at least 3 partners, with  $N = 7.69$  well short of  $N = 8$ . At the upper limit, a two-party MW can be reached right at  $N = 2.00$  when  $S = 100$ , with the constellation 50-50. But when  $S = 101$ , 51-50 has  $N < 2$  ( $N = 1.9998$ ) while a minimal-partner MW of 2 parties can be first reached only with 50-50-1, which means  $N = 2.398$ , beyond  $N = 2.00$ .

- 9 While  $p = 2$  is in principle possible at  $6 < N < 8$ , no such cases were observed in practice. Indeed, it would take an unlikely constellation such as 26-25-8-8-8-8-5-4 to reach  $N = 6.01$  and still have the possibility of a 2-party MW.

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