Gunrock: GPU Graph Analytics

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For large-scale graph analytics on the GPU, the irregularity of data access and control flow, and the complexity of programming GPUs, have presented two significant challenges to developing a programmable high-performance graph library. "Gunrock", our graph-processing system designed specifically for the GPU, uses a high-level, bulk-synchronous, data-centric abstraction focused on operations on a vertex or edge frontier. Gunrock achieves a balance between performance and expressiveness by coupling high performance GPU computing primitives and optimization strategies with a high-level programming model that allows programmers to quickly develop new graph primitives with small code size and minimal GPU programming knowledge. We characterize the performance of various optimization strategies and evaluate Gunrock's overall performance on different GPU architectures on a wide range of graph primitives that span from traversal-based algorithms and ranking algorithms, to triangle counting and bipartite-graph-based algorithms. The results show that on a single GPU, Gunrock has on average at least an order of magnitude speedup over Boost and PowerGraph, comparable performance to the fastest GPU hardwired primitives and CPU shared-memory graph libraries such as Ligra and Galois, and better performance than any other GPU high-level graph library.

CCS Concepts: •**Theory of computation → Graph algorithms analysis;** •**Computing methodologies → Parallel algorithms;** •**Computer systems organization → Single instruction, multiple data;**

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1 INTRODUCTION

Graphs are ubiquitous data structures that can represent relationships between people (social networks), computers (the Internet), biological and genetic interactions, and elements in unstructured meshes. Many practical problems in social networks, physical simulations, bioinformatics, and other applications can be modeled in their essential form by graphs and solved with appropriate graph primitives. Various types of such graph primitives that compute and exploit properties of particular graphs are collectively known as graph analytics. In the past decade, as graph problems have grown larger in scale and become more computationally complex, the research of parallel graph analytics has raised great interest in order to overcome the computational resource and memory bandwidth limitations of single processors. In this paper, we describe "Gunrock," our graphics processor (GPU)-based system for graph processing that delivers high performance in computing graph analytics with its high-level, data-centric parallel programming model. Unlike previous GPU graph programming models that focus on sequencing computation steps, our data-centric model's

key abstraction is the *frontier*, a subset of the edges or vertices within the graph that is currently of interest. All Gunrock operations are bulk-synchronous and manipulate this frontier, either by computing on values within it or by computing a new frontier from it.

At a high level, Gunrock targets graph primitives that are iterative, convergent processes. Among the graph primitives we have implemented and evaluated in Gunrock, we focus in this paper on breadth-first search (BFS), single-source shortest path (SSSP), betweenness centrality (BC), PageRank, connected components (CC), and triangle counting (TC). Though the GPU's excellent peak throughput and energy efficiency [40] have been demonstrated across many application domains, these applications often exploit regular, structured parallelism. The inherent irregularity of graph data structures leads to irregularity in data access and control flow, making an efficient implementation on GPUs a significant challenge.

Our goal with Gunrock is to deliver both performance and programmability. Gunrock's performance is similar to customized, complex GPU hardwired graph primitives, and its high-level programming model allows programmers to quickly develop new graph primitives. To do so, we must address the key challenge in a highly parallel graph processing system: managing irregularity in work distribution. Gunrock integrates sophisticated load-balancing and work-efficiency strategies into its core. These strategies are hidden from the programmer; the programmer instead expresses *what* operations should be performed on the frontier rather than *how* those operations should be performed. Programmers can assemble complex and high-performance graph primitives from operations that manipulate the frontier (the "what") without knowing the internals of the operations (the "how").

Our contributions, extending those from our previous work [82], are as follows:

(1) We present a novel data-centric abstraction for graph operations that allows programmers to develop graph primitives at a high level of abstraction while delivering high performance. This abstraction, unlike the abstractions of previous GPU programmable frameworks, is able to elegantly incorporate profitable optimizations—kernel fusion, push-pull traversal, idempotent traversal, and priority queues—into the core of its implementation.
(2) We design and implement a set of simple and flexible APIs that can express a wide range of graph processing primitives at a high level of abstraction (at least as simple, if not more so, than other programmable GPU frameworks).
(3) We describe several GPU-specific optimization strategies for memory efficiency, load balancing, and workload management that together achieve high performance. All of our graph primitives achieve comparable performance to their hardwired counterparts and significantly outperform previous programmable GPU abstractions.
(4) We provide a detailed experimental evaluation of our graph primitives with performance comparisons to several CPU and GPU implementations.

Gunrock is currently available to external developers in an open-source repository at http://gunrock.github.io/, under an Apache 2.0 license.

2 RELATED WORK

This section discusses the research landscape of large-scale graph analytics frameworks in four fields:

(1) Single-node CPU-based systems, which are in common use for graph analytics today, but whose serial or coarse-grained-parallel programming models are poorly suited for a massively parallel processor like the GPU;

(2) Distributed CPU-based systems, which offer scalability advantages over single-node systems but incur substantial communication cost, and whose programming models are also poorly suited to GPUs;
(3) GPU "hardwired," low-level implementations of specific graph primitives, which provide a proof of concept that GPU-based graph analytics can deliver best-in-class performance. However, best-of-class hardwired primitives are challenging to even the most skilled programmers, and their implementations do not generalize well to a variety of graph primitives; and
(4) High-level GPU programming models for graph analytics, which often recapitulate CPU programming models. The best of these systems incorporate generalized load-balance strategies and optimized GPU primitives, but they generally do not compare favorably in performance with hardwired primitives due to the overheads inherent in a high-level framework and the lack of primitive-specific optimizations.

2.1 Single-node CPU-based Systems

Parallel graph analytics frameworks provide high-level, programmable, high-performance abstractions. The Boost Graph Library (BGL) is among the first efforts towards this goal, though its serial formulation and C++ focus together make it poorly suited for a massively parallel architecture like a GPU. Designed using the generic programming paradigm, the parallel BGL [29] separates the implementation of parallel algorithms from the underlying data structures and communication mechanisms. While many BGL implementations are specialized per algorithm, its breadth_first_visit pattern (for instance) allows sharing common operators between different graph algorithms. Stanford Network Analysis Platform (SNAP) is another general purpose network analysis and graph mining library which contains C++ and Python implementations of various graph algorithms and a dataset collection of large networks [46]. It is one of the most important benchmarks in the field of high performance graph analytics.

2.2 Distributed CPU-based Systems

Traditional distributed data processing system such as MapReduce fits poorly for graph processing tasks which usually contain highly irregular workloads. Thus people have developed distributed systems for graph processing. There are two major framework families for CPU-based large-scale graph processing system: Pregel and GraphLab. Pregel [50] is a Google-initiated programming model and implementation for large-scale graph computing that follows the Bulk-Synchronous Parallel (BSP) model [79]. A typical application in Pregel is an iterative convergent process consisting of global synchronization barriers called super-steps. The computation in Pregel is vertex-centric and based on message passing. Its programming model is good for scalability and fault tolerance. However, standard graph algorithms in most Pregel-like graph processing systems suffer slow convergence on large-diameter graphs and load imbalance on scale-free graphs. Apache Giraph is an open source implementation of Google's Pregel. It is a popular graph computation engine in the Hadoop ecosystem initially open-sourced by Yahoo!. GraphLab [49] is an open-source large scale graph processing library that relies on the shared memory abstraction and the gather-apply-scatter (GAS) programming model. It allows asynchronous computation and dynamic asynchronous scheduling. By eliminating message-passing, its programming model isolates the user-defined algorithm from the movement of data, and therefore is more consistently expressive. PowerGraph [24] is an improved version of GraphLab for power-law graphs. It supports both BSP and asynchronous execution. For the load imbalance problem, it uses vertex-cut to split high-degree vertices into equal degree-sized redundant vertices. This exposes greater parallelism in scale-free

graphs. GraphChi [44] is a centralized system that can process massive graphs from secondary storage in a single machine. It introduces a novel graph partitioning method called Parallel Sliding Windows (PSW), which sorts the edges by their source node IDs to provide load balancing.

Beyond these two families, several other graph libraries have influenced the direction of graph analytics research, including shared-memory-based systems and domain-specific languages. Ligra [71] is a CPU-based graph processing framework for shared memory. It uses a similar operator abstraction to Gunrock for doing graph traversal. Its lightweight implementation is targeted at shared memory architectures and uses CilkPlus for its multi-threading implementation. Galois [57, 62] is a graph system for shared memory based on a different operator abstraction that supports priority scheduling and dynamic graphs and processes on subsets of vertices called active elements. However, their model does not abstract the internal details of the iterating process from the user. Users have to generate the active elements set directly for different graph algorithms. Green-Marl [36] is a domain-specific language for writing graph analysis algorithms on shared memory with built-in breadth-first search (BFS) and depth-first search (DFS) primitives in its compiler. Its language approach provides graph-specific optimizations and hides complexity. However, the language does not support operations on arbitrary sets of vertices for each iteration, which makes it difficult to use for traversal algorithms that cannot be expressed using a BFS or DFS. GraphX [25] is a distributed graph computation framework that unifies graph-parallel and data-parallel computation. It provides a small, core set of graph-parallel operators expressive enough to implement the Pregel and PowerGraph abstractions, yet is simple enough to be cast in relational algebra. Help is a library that provides high-level primitives for large-scale graph processing [65]. Using the primitives in Help is more intuitive and faster than using the APIs of existing distributed systems.

2.3 Specialized Parallel Graph Algorithms

Recent work has developed numerous best-of-breed, hardwired implementations of many graph primitives.

BFS. Breadth-first search is among the first few graph primitives researchers developed on the GPU due to its representative workload pattern and its fundamental role as the building block primitive to several other traversal-based graph primitives. Harish and Narayanan [33] first proposed a quadratic GPU BFS implementation that maps each vertex's neighbor list to one thread. Hong et al. [37] improved on this algorithm by mapping workloads to a series of virtual warps and letting an entire warp cooperatively strip-mine the corresponding neighbor list. Merrill et al.'s linear parallelization of the BFS algorithm on the GPU [55] had significant influence in the field. They proposed an adaptive strategy for load-balancing parallel work by expanding one node's neighbor list to one thread, one warp, or a whole block of threads. With this strategy and a memory-access-efficient data representation, their implementation achieves high throughput on large scale-free graphs. Beamer et al.'s recent work on a very fast BFS for shared memory machines [6] uses a hybrid BFS that switches between top-down and bottom-up neighbor-list-visiting algorithms according to the size of the frontier to save redundant edge visits. Enterprise [47], a GPU-based BFS system, introduces a very efficient implementation that combines the benefits of direction optimization, Merrill et al.'s adaptive load-balancing workload mapping strategy, and a status-check array. BFS-4K [12] is a GPU BFS system that improves the virtual warp method to a per-iteration dynamic one and uses dynamic parallelism for better load balancing.

CC. Connected components can be implemented as a BFS-based primitive. The current fastest connected-component algorithm on the GPU is Soman et al.'s work [75] based on a PRAM connected-component algorithm [30] that initializes each vertex in its own component, and merges component IDs by traversing in the graph until no component ID changes for any vertex.

BC. Several parallel betweenness centrality implementations on the GPU are based on the work from Brandes [8]. Pande and Bader [60] proposed the first BC implementation on the GPU using a quadratic BFS. Sariyüce et al. [67] adopted PowerGraph's vertex-cut method (which they termed vertex virtualization) to improve load balancing and proposed a stride-CSR representation to reorganize adjacency lists for better memory coalescing. McLaughlin and Bader [52] developed a work-efficient betweenness centrality algorithm on the GPU that combines a queue-based linear multi-source BFS-based work-efficient BC implementation with an edge-parallel BC implementation. It dynamically chooses between the two according to frontier size and scales linearly up to 192 GPUs.

SSSP. Davidson et al. [17] proposed a delta-stepping-based [56] work-efficient single-source shortest path algorithm on the GPU that explores a variety of parallel load-balanced graph traversal and work organization strategies to outperform previous parallel methods that are either based on quadratic BFS [33] or the Bellman-Ford algorithm (LonestarGPU 2.0 [10]).

TC. Green et al.'s GPU triangle counting algorithm [28] computes set intersections for each edge in the undirected graph, and uses a modified intersection path method based on merge path [26], which is considered to have the highest performance for large-sorted-array set intersection on the GPU.

After we discuss the Gunrock abstraction in Section 3, we will discuss how to map these specialized graph algorithms to Gunrock and the differences in Gunrock's implementations.

2.4 High-level GPU Programming Models

Several works target the construction of a high-level GPU graph processing library that delivers both good performance and good programmability. We categorize these into two groups by programming model: (1) BSP/Pregel's message-passing framework and (2) the GAS model.

In Medusa [86], Zhong and He presented their pioneering work on parallel graph processing using a message-passing model called Edge-Message-Vertex (EMV). It is the first high-level GPU graph analytics system that can automatically execute different user-defined APIs for different graph primitives, which increases the programmability to some extent. However, its five types of user-defined API—namely, *ELIST, EDGE, MLIST, MESSAGE, VERTEX*—are still vertex-centric and need to be split into several source files. Moreover, Medusa does not have a fine-grained load balancing strategy for unevenly distributed neighbor lists during graph traversal, which makes its performance on scale-free graphs uncompetitive compared to specialized graph primitives.

Two more recent works that follow the message-passing approach both improve the execution model. Totem [22] is a graph processing engine for GPU-CPU hybrid systems. It either processes the workload on the CPU or transmits it to the GPU according to a performance estimation model. Its execution model can potentially solve the long-tail problem (where the graph has a large diameter with a very small amount of neighbors to visit per iteration) on GPUs, and overcome GPU memory size limitations. Totem's programming model allows users to define two functions—*algo_compute_func* and *msg_combine_func*—to apply to various graph primitives. However, because its API only allows direct neighbor access, it has limitations in algorithm generality. Frog [70] is a lock-free semi-asynchronous parallel graph processing framework with a graph coloring model. It

has a preprocessing step to color the graph into sets of conflict-free vertices. Although vertices with the same color can be updated lock-free in a parallel stage called color-step, on a higher level the program still needs to process color-steps in a BSP style. Within each color-step, their streaming execution engine is message-passing-based with a similar API to Medusa. The preprocessing step of the color model does accelerate the iterative convergence and enables Frog to process graphs that do not fit in a single GPU's memory. However, the CPU-based coloring algorithm is inefficient. Because of the limitation of its programming model, which requires that it visit all edges in each single iteration, its performance is restricted.

The GAS abstraction was first applied on distributed systems [24]. PowerGraph's vertex-cut splits large neighbor lists, duplicates node information, and deploys each partial neighbor list to different machines. Working as a load balancing strategy, it replaces the large synchronization cost in edge-cut into a single-node synchronization cost. This is a productive strategy for multi-node implementations. GAS offers the twin benefits of simplicity and familiarity, given its popularity in the CPU world. VertexAPI2 [19] is the first GPU high-level graph analytics system that strictly follows the GAS model. It defines two graph operators: *gatherApply*, the combination of GAS's gather step and apply step, and *scatterActivate*, GAS's scatter step. Users build different graph primitives by defining different functors for these operators. VertexAPI2 has four types of functors: *gatherReduce*, *gatherMap*, *scatter*, and *apply*. Underneath their abstraction, VertexAPI2 uses state-of-the-art GPU data primitives based on two-phase decomposition [5]. It shows both better performance and better programmability compared to message-passing-based GPU libraries. MapGraph [20] echoes VertexAPI2's framework and integrates both moderngpu's load-balanced search [5] and Merrill et al.'s dynamic grouping workload mapping strategy [55] to increase its performance. CuSha [42] is also a GAS model-based GPU graph analytics system. It solves the load imbalance and GPU underutilization problem with a GPU adoption of GraphChi's PSW. They call this preprocessing step "G-Shard" and combine it with a concatenated window method to group edges from the same source IDs.

nvGRAPH[1] is a high-performance GPU graph analytics library developed by NVIDIA. It views graph analytics problems from the perspective of linear algebra and matrix computations [41], and uses semi-ring SpMV operations to express graph computation. It currently supports three algorithms: PageRank, SSSP, and Single Source Widest Path.

Table 1 compares Gunrock with the above high-level GPU-based graph analytics systems on various metrics. Our novel data-centric programming model and its efficient implementation makes Gunrock the only high-level GPU-based graph analytics system with support for both vertex-centric and edge-centric operations, as well as runtime fine-grained load balancing strategies, without requiring any preprocessing of input datasets.

From a programmability perspective, Gunrock aims to 1) define a programming model that can abstract most common operations in graph analytics at a high level; 2) be flexible enough to express various types of graph analytics tasks; and 3) match the GPU's high-throughput, data-centric, massively parallel execution model, balancing on the generality of the model with its performance.

From a performance perspective, Gunrock attempts to 1) build its low-level optimizations on top of the state-of-the-art basic data parallel primitives on the GPU and 2) design optimizations to allow the usage of different combinations and parameter-tuning methods in our graph operations. Together these goals enable performance that is comparable to specialized GPU implementations.

[1] nvGRAPH is available at https://developer.nvidia.com/nvgraph.

Metrics	Medusa	Totem	Frog	VertexAPI2	MapGraph	CuSha	Gunrock
programming model	m-p	m-p	m-p	GAS	GAS	GAS	data-centric
operator flexibility	v-c, e-c	v-c	v-c	v-c	v-c	v-c	v-c, e-c
load balancing	no	limited	limited	fine-grained	fine-grained	fine-grained	fine-grained
preprocessing	no	no	coloring	no	no	G-shard	no
execution model	BSP	BSP,hybrid	semi-async	BSP	BSP	BSP	BSP

Table 1. Detailed comparison of different high-level GPU graph analytics systems. m-p means message-passing based model, v-c and e-c mean vertex-centric and edge-centric respectively. Note that part of load balancing work in Frog and CuSha are done offline (the coloring model for Frog and G-shard generation process for CuSha). This table focuses on graph-centric abstractions, so we leave out libraries that follow a linear algebra abstraction such as nvGRAPH.

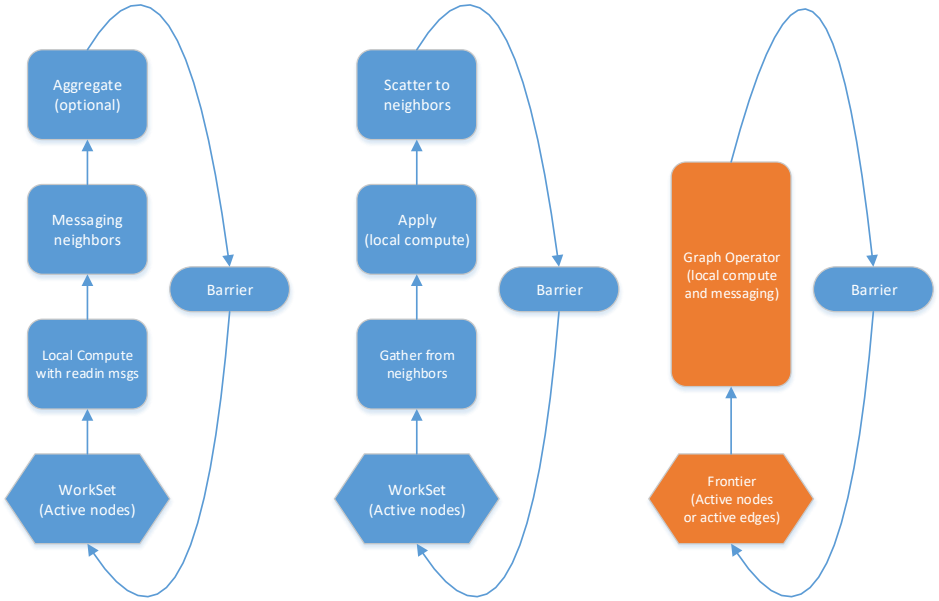


Fig. 1. Iterative convergence process presented using Pregel's message passing model, PowerGraph's GAS model, and Gunrock's data-centric model.

3 DATA-CENTRIC ABSTRACTION

A common finding from most CPU and GPU graph analytics systems is that most graph analytics tasks can be expressed as iterative convergent processes (Figure 1). By "iterative," we mean operations that may require running a series of steps repeatedly; by "convergent," we mean that these iterations allow us to approach the correct answer and terminate when that answer is reached. Mathematically, an iterative convergent process is a series of operations $f_0, f_1, ..., f_{n-1}$ that operate on graph data G, where $G_i = f(G_{i-1})$, until at iteration i, the stop condition function $C(G_i, i)$ returns true.

Both Pregel and PowerGraph focus on sequencing steps of *computation*. Where Gunrock differs from them and their variants is our abstraction. Rather than focusing on sequencing steps of *computation*, we instead focus on manipulating a data structure, the *frontier* of vertices or edges. The frontier represents the subset of vertices or edges that is actively participating in the computation.

Gunrock's data-centric framework design not only provides the features of other frameworks but also provides high performance. It is flexible enough to be expanded by new graph operators, as long as they operate on one or more input frontiers with graph data and generate one or more output frontiers. Because of this design, we claim that thinking about graph processing in terms of manipulations of frontiers is the right abstraction for the GPU. We support this statement qualitatively in this section and quantitatively in section 7.

One important consequence of designing our abstraction with a data-centered focus is that Gunrock, from its very beginning, has supported both node and edge frontiers, and can easily switch between them within the same graph primitive. We can, for instance, generate a new frontier of neighboring edges from an existing frontier of vertices. In contrast, gather-apply-scatter (PowerGraph) and message-passing (Pregel) abstractions are focused on operations on vertices and either cannot support edge-centric operations or could only support them with heavy redundancy within their abstractions.

In our abstraction, we expose bulk-synchronous "steps" that manipulate the frontier, and programmers build graph primitives from a sequence of steps. Different steps may have dependencies between them, but individual operations within a step can be processed in parallel. For instance, a computation on each vertex within the frontier can be parallelized across vertices, and updating the frontier by identifying all the vertices neighboring the current frontier can also be parallelized across vertices.

The graph primitives we describe in this paper use three traversal operators: *advance*, *filter*, and *segmented intersection*. They may also use one compute operator, which is often fused with one of the traversal operators (Figure 2). Each graph operator manipulates the frontier in a different way. The input frontier of each operator contains either node IDs or edge IDs that specify on which part of the graph we are going to perform our computations. The traversal operators traverse the graph and generate an output frontier. Within a traversal operator, each input item can potentially map to zero, one, or more output items; efficiently handling this irregularity is the principal challenge in our implementation. In this section, we discuss the functionality of the operators, then discuss how we implement them in Section 5.

Advance An *advance* operator generates a new frontier from the current frontier by visiting the neighbors of the current frontier. Each input item maps to multiple output items from the input item's neighbor list. A frontier can consist of either vertices or edges, and an advance step can input and output either kind of frontier. Advance is an irregularly-parallel operation for two reasons: 1) different vertices in a graph have different numbers of neighbors and 2) vertices share neighbors. An efficient advance is the most significant challenge of a GPU implementation.

The generality of Gunrock's advance allows us to use the same advance implementation across a wide variety of interesting graph operations. According to the type of input frontier and output frontier, Gunrock supports 4 kinds of advance: V-to-V, V-to-E, E-to-V, and E-to-E, where E represents an edge frontier and V represents a vertex frontier. For instance, we can utilize Gunrock advance operators to 1) visit each element in the current frontier while updating local values and/or accumulating global values (e.g., BFS distance updates); 2) visit the node or edge neighbors of all the elements in the current frontier while updating source vertex, destination vertex, and/or edge values (e.g., distance updates in SSSP); 3) generate edge frontiers from vertex frontiers or vice versa (e.g., BFS, SSSP, SALSA, etc.); or 4) pull values from all vertices 2 hops away by starting from an edge frontier, visiting all the neighbor edges, and returning the far-end vertices of these neighbor edges.

Filter A *filter* operator generates a new frontier from the current frontier by choosing a subset of the current frontier based on programmer-specified criteria. Each input item maps to zero or one output item. Though filtering is an irregular operation, using parallel scan for efficient filtering is well-understood on GPUs. Gunrock's filters can either 1) split vertices or edges based on a filter (e.g., SSSP's 2-bucket delta-stepping), or 2) compact out filtered items to throw them away (e.g., duplicated vertices in BFS or edges where both end nodes belong to the same component in CC).

Segmented Intersection A *segmented intersection* operator takes two input node frontiers with the same length, or an input edge frontier, and generates both the number of total intersections and the intersected node IDs as the new frontier. We call input items with the same index in two input frontiers a pair. If the input is an edge frontier, we treat each edge's two nodes as an input item pair. One input item pair maps to multiple output items, which are the intersection of the neighbor lists of two input items. Segmented intersection is the key operator in TC, with segment being defined as nodes that belong to one neighbor list. It is useful for both global triangle counting and computing the clustering coefficient. The output frontier could be combined with the two input frontiers to enumerate all the triangles in the graph.

Compute A *compute* operator defines an operation on all elements (vertices or edges) in its input frontier. A programmer-specified compute operator can be used together with all three traversal operators. Gunrock performs that operation in parallel across all elements without regard to order. Note that it is the user's responsibility to handle the potential data race (the usual way is to use atomic operation). The computation can access and modify global memory through a device pointer of a structure-of-array (SoA) that may store per-node and/or per-edge data. Because this parallelism is regular, computation is straightforward to parallelize in a GPU implementation. Many simple graph primitives (e.g., computing the degree distribution of a graph) can be expressed with a single Gunrock computation operator.

Gunrock primitives are assembled from a sequence of these four operators, which are executed sequentially: one step completes all of its operations before the next step begins. Typically, Gunrock graph primitives run to convergence, which on Gunrock usually equates to an empty frontier; as individual elements in the current frontier reach convergence, they can be filtered out of the frontier. Programmers can also use other convergence criteria such as a maximum number of iterations or volatile flag values that can be set in a computation step.

Example: Expressing SSSP in programmable GPU frameworks. Now we use an example to show how different programmable CPU/GPU frameworks express a graph primitive to further study the key difference between Gunrock's data-centric abstraction and other frameworks. We choose SSSP because it is a reasonably complex graph primitive that computes the shortest path from a single node in a graph to every other node in the graph. We assume weights between nodes are all non-negative, which permits the use of Dijkstra's algorithm and its parallel variants. Efficiently implementing SSSP continues to be an interesting problem in the GPU world [11, 17, 18].

The iteration starts with an input frontier of active vertices (or a single vertex) initialized to a distance of zero. First, SSSP enumerates the sizes of the frontier's neighbor list of edges and computes the length of the output frontier. Because the neighbor edges are unequally distributed among the frontier's vertices, SSSP next redistributes the workload across parallel threads. This can be expressed within an advance operator. In the final step of the advance operator, each edge adds its weight to the distance value at its source value and, if appropriate, updates the distance value of

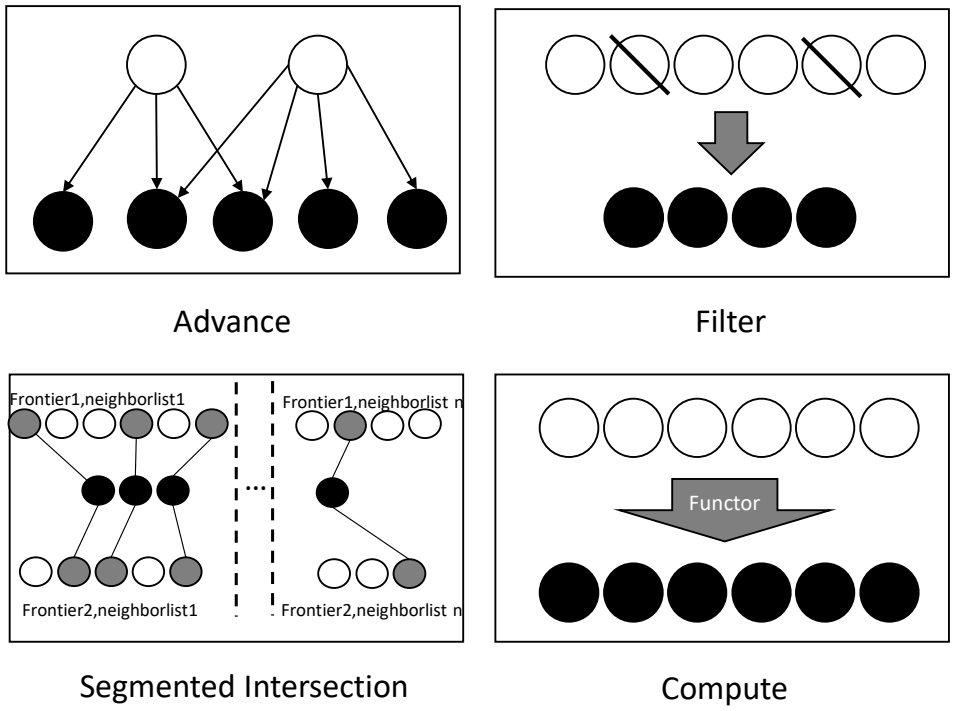


Fig. 2. Four operators in Gunrock's data-centric abstraction convert a current frontier (in white) into a new frontier (in black).

its destination vertex. Finally, SSSP removes redundant vertex IDs (specific filter), decides which updated vertices are valid in the new frontier, and computes the new frontier for the next iteration.

Gunrock maps one SSSP iteration onto three Gunrock operators: (1) *advance*, which computes the list of edges connected to the current vertex frontier and (transparently) load-balances their execution; (2) *compute*, to update neighboring vertices with new distances; and (3) *filter*, to generate the final output frontier by removing redundant nodes, optionally using a two-level priority queue, whose use enables delta-stepping (a binning strategy to reduce overall workload [17, 56]). With this mapping in place, the traversal and computation of path distances is simple and intuitively described, and Gunrock is able to create an efficient implementation that fully utilizes the GPU's computing resources in a load-balanced way.

3.1 Alternative Abstractions

In this section we discuss Gunrock's design choices compared to several alternative abstractions designed for graph processing on various architectures (shown in figure 3).

Gather-apply-scatter (GAS) abstraction Recently, Wu et al. [84] compared Gunrock vs. two GPU GAS frameworks, VertexAPI2 and MapGraph, demonstrating that Gunrock had appreciable performance advantages over the other two frameworks. One of the principal performance differences they identified comes from the significant fragmentation of GAS programs across many kernels that we discuss in more detail in section 5. Applying automatic kernel fusion [58] to GAS+GPU implementations could potentially help close their performance gap.

ALGORITHM 1: Single-Source Shortest Path, expressed in Gunrock's abstraction.

Function *Set_Problem_Data* (*G*, *P*, *root*); ▷ G:graph data structure,P:problem data structure.

    Set *P.labels* to ∞;
    Set *P.preds* to -1;
    P.labels[*root*] ← 0;
    P.preds[*root*] ← -1;
    Insert *root* to *P.frontier*;

Function *Update_Label* (*s_id*, *d_id*, *e_id*, *P*); ▷ s_id,d_id,e_id:source ID,destination ID,edge ID.

    new_label ← *P.labels*[*s_id*]+*P.weights*[*e_id*];
    return *new_label* < *atomicMin(P.labels[d_id],new_label)*;

Function *Set_Pred* (*s_id*, *d_id*, *P*)

    P.preds[*d_id*] ← *s_id*;
    P.output_queue_ids[*d_id*] ← *output_queue_id*;

Function *Remove_Redundant* (*node_id*, *P*)

    return *P.output_queue_ids[node_id]* == *output_queue_id*;

Function *SSSP_Enactor* (*G*, *P*, *root*)

    Set_Problem_Data (*G*, *P*, *root*);
    while *P.frontier.Size* () > 0 **do**
        Advance (*G*, *P*, *Update_Label*, *Set_Pred*);
        Filter (*G*, *P*, *Remove_Redundant*);
    end

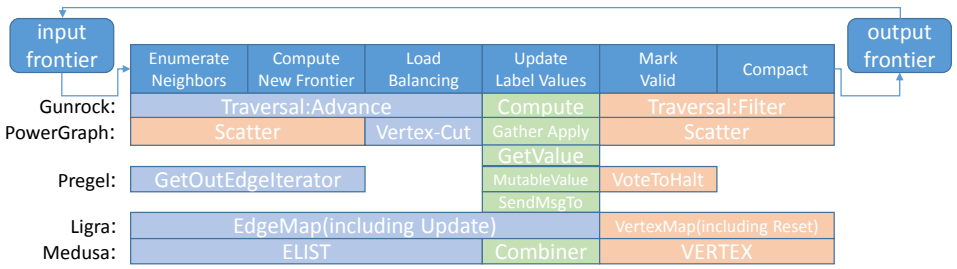


Fig. 3. Operations that make up one iteration of SSSP and their mapping to Gunrock, PowerGraph (GAS) [24], Pregel [50], Ligra [71], and Medusa [86] abstractions.

At a more fundamental level, we found that a compute-focused programming model like GAS was not flexible enough to manipulate the core frontier data structures in a way that enabled powerful features and optimizations such as direction-optimizing traversal and two-level priority queues; both fit naturally into Gunrock's abstraction. We believe bulk-synchronous operations on frontiers are a better fit than GAS for forward-looking GPU graph programming frameworks.

Message-passing Pregel [50] is a vertex-centric programming model that only provides data parallelism on vertices. For graphs with significant variance in vertex degree (e.g., power-law graphs), this would cause severe load imbalance on GPUs. The traversal operator in Pregel is general enough to apply to a wide range of graph primitives, but its vertex-centric

design only achieves good parallelism when nodes in the graph have small and evenly-distributed neighborhoods. For real-world graphs that often have an uneven distribution of node degrees, Pregel suffers from severe load imbalance. The Medusa authors note the complexity of managing the storage and buffering of these messages, and the difficulty of load-balancing when using segmented reduction for per-edge computation. Though they address both of these challenges in their work, the overhead of *any* management of messages is a significant contributor to runtime. Gunrock prefers the less costly direct communication between primitives and supports both push-based (scatter) communication and pull-based (gather) communication during traversal steps.

CPU strategies Ligra's powerful load-balancing strategy is based on CilkPlus, a fine-grained task-parallel library for CPUs. Despite promising GPU research efforts on task parallelism [13, 78], no such equivalent is available on GPUs, thus we implement our own load-balancing strategies within Gunrock. Galois, like Gunrock, cleanly separates data structures from computation; their key abstractions are ordered and unordered set iterators that can add elements to sets during execution (such a dynamic data structure is a significant research challenge on GPUs). Galois also benefits from speculative parallel execution whose GPU implementation would also present a significant challenge. Both Ligra and Galois scale well within a node through inter-CPU shared memory; inter-GPU scalability, both due to higher latency and a lack of hardware support, is a much more manual, complex process.

Help's primitives Help [65] characterizes graph primitives as a set of functions that enable special optimizations for different primitives at the cost of losing generality. Its Filter, Local Update of Vertices (LUV), Update Vertices Using One Other Vertex (UVUOV), and Aggregate Global Value (AGV) are all Gunrock filter operations with different computations. Aggregating Neighbor Values (ANV) maps to the advance operator in Gunrock. We also successfully implemented Form Supervertices (FS) in Gunrock using two filter passes, one advance pass, and several other GPU computing primitives (sort, reduce, and scan).

Asynchronous execution Many CPU and GPU frameworks (e.g., Galois, GraphLab, and Frog) efficiently incorporate asynchronous execution, but the GPU's expensive synchronization or locking operations would make this a poor choice for Gunrock. We do recover some of the benefits of prioritizing execution through our two-level priority queue (details in section 5.1.5).

Gunrock's software architecture is divided into two parts. Above the traversal-compute abstraction is the application module. This is where users define different graph primitives using the high-level APIs provided by Gunrock. Under the abstraction are the utility functions, the implementation of operators used in traversal, and various optimization strategies.

Gunrock programs specify three components: the *problem*, which provides graph topology data and an algorithm-specific data management interface; the *functors*, which contain user-defined computation code and expose kernel fusion opportunities that we discuss below; and an *enactor*, which serves as the entry point of the graph algorithm and specifies the computation as a series of graph operator kernel calls with user-defined kernel launching settings.

Given Gunrock's abstraction, the most natural way to specify Gunrock programs would be as a sequence of bulk-synchronous steps, specified within the enactor and implemented as kernels, that operate on frontiers. Such an enactor is in fact the core of a Gunrock program, but an enactor-only program would sacrifice a significant performance opportunity. We analyzed the techniques that hardwired (primitive-specific) GPU graph primitives used to achieve high performance. One of their principal advantages is leveraging producer-consumer locality between operations by integrating

multiple operations into single GPU kernels. Because adjacent kernels in CUDA or OpenCL share no state, combining multiple logical operations into a single kernel saves significant memory bandwidth that would otherwise be required to write and then read intermediate values to and from memory. In the CUDA C++ programming environment, we have no ability to automatically fuse neighboring kernels together to achieve this efficiency.

We summarize the interfaces for these operations in figure 4. Our focus on kernel fusion enabled by our API design is absent from other programmable GPU graph libraries, but it is crucial for performance.

```
// functor interfaces
static __device__ __forceinline__ bool
    AdvanceFunctor(
  VertexId s_id,  // source node ID
  VertexId d_id,  // destination node ID
  DataSlice *d_data_slice,
  SizeT edge_id,  // edge list ID
  LabelT label, // label value
  SizeT input_pos,  // input queue idx
  SizeT &output_pos); // output queue idx
static __device__ __forceinline__ bool
    FilterFunctor(
  VertexId node,  // node ID
  DataSlice *d_data_slice,
  LabelT label, // label value
  SizeT input_pos,  // input queue idx
  SizeT output_pos);  // output queue idx

// operator interfaces

// advance
template<typename AdvancePolicy, typename
     Problem, typename Functor, typename
    AdvanceType>
__global__ void Advance(
SizeT queue_length,
VertexId *input_frontier,
VertexId *output_frontier,
DataSlice *data_slice);
```

```
// filter
template<typename FilterPolicy, typename
    Problem, typename Functor, bool
    ComputeFlag>
__global__ void Filter(
SizeT queue_length,
VertexId *input_frontier,
VertexId *output_frontier,
DataSlice *data_slice);

// neighborhood reduction
template<typename NeighborhoodPolicy,
    typename Problem, typename Functor,
    typename ReduceOp>
__global__ void Neighborhood(
SizeT queue_length,
VertexId *input_frontier,
VertexId *output_frontier,
DataSlice *data_slice);

// segmented intersection
template<typename IntersectionPolicy,
    typename Problem, typename Functor>
__global__ long Intersection(
SizeT queue_length,
VertexId *input_frontier1,
VertexId *input_frontier2,
VertexId *output_frontier,
DataSlice *data_slice);
```

Fig. 4. Gunrock's Graph Operator and Functor APIs. The Operator APIs divide the whole workloads into load-balanced per-edge or per-node operations and fuse the kernel with a functor that defines one such operation.

To conclude this section, we list the benefits of Gunrock's data-centric programming model for graph processing on the GPU:

- The data-centric programming model allows more flexibility on the operations, since the frontier is a higher level abstraction of streaming data, which could represent nodes, edges, or even arbitrary sub-graph structures. In Gunrock, every operator can be vertex-centric or edge-centric for any iteration. As a result, we can concentrate our effort on solving one problem—implementing efficient operators—and see that effort reflected in better performance on various graph primitives.
- The data-centric programming model decouples a compute operator from traversal operators. For different graph primitives, compute operators can be very different in terms of complexity and the way they interact with graph operators. Decoupling Gunrock's compute operator from its traversal operators gives Gunrock more flexibility in implementing a general set of graph primitives.
- The data-centric programming model allows an implementation that both leverages state-of-the-art data-parallel primitives and enables various types of optimizations. The result is

an implementation with better performance than any other GPU graph-processing system, achieving comparable performance to specialized GPU graph algorithms.
- The data-centric programming model uses high level graph operator interfaces to encapsulate complicated implementations. This makes Gunrock's application code smaller in size and clearer in logic compared to other GPU graph libraries. Gunrock's Problem class and kernel enactor are both template-based C++ code; Gunrock's functor code that specifies per-node or per-edge computation is C-like device code without any CUDA-specific keywords.
- The data-centric programming model eases the job of extending Gunrock' single-GPU execution model to multiple GPUs. It fits with other execution models such as a semi-asynchronous execution model and a multi-GPU single-node/multi-node execution model. Our multi-GPU graph processing framework [59] is built on top of our data-centric programming model with an unchanged core single-GPU implementation coupled with advanced communication and partition modules designed specifically for multi-GPU execution.

4 EFFICIENT GRAPH OPERATOR DESIGN

In this section, we analyze the implementation of each graph operator, and briefly discuss how these implementations enable optimizations (with more details in section 5). Figure 5 shows the sample graph we use to illustrate our graph operator design, and figure 6 shows the corresponding arrays for graph storage in compressed sparse row (CSR) format (discussed in section 5.4) in Gunrock.

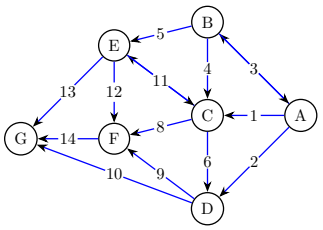


Fig. 5. A sample directed graph with 7 nodes and 15 edges.

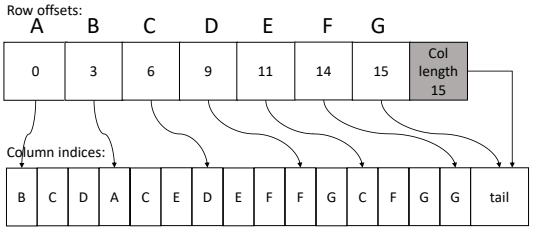


Fig. 6. CSR format of sample graph in Gunrock.

4.1 Advance

The advance operator takes the input frontier, visits the neighbor list of each item in the input frontier, then writes those neighbor lists to the output frontier (figure 7). The size of these neighbor lists may differ significantly between different input items. Thus an efficient implementation of advance needs to reduce the task granularity to a homogeneous size and then evenly distribute these smaller tasks among threads [55] so that we can efficiently parallelize this operator.

At a high level, advance can be seen as a vectorized device memory assignment and copy, where parallel threads place dynamic data (neighbor lists with various lengths) within shared data structures (output frontier). The efficient parallelization of this process requires two stages: 1) for the allocation part, given a list of allocation requirements for each input item (neighbor list size array computed from row offsets), we need the scatter offsets to write the output frontier; 2) for the copy part, we need to load-balance parallel scatter writes with various lengths over a single launch.

The first part is typically implemented with prefix-sum. For the second part, there are several implementation choices, which differ in two primary ways:

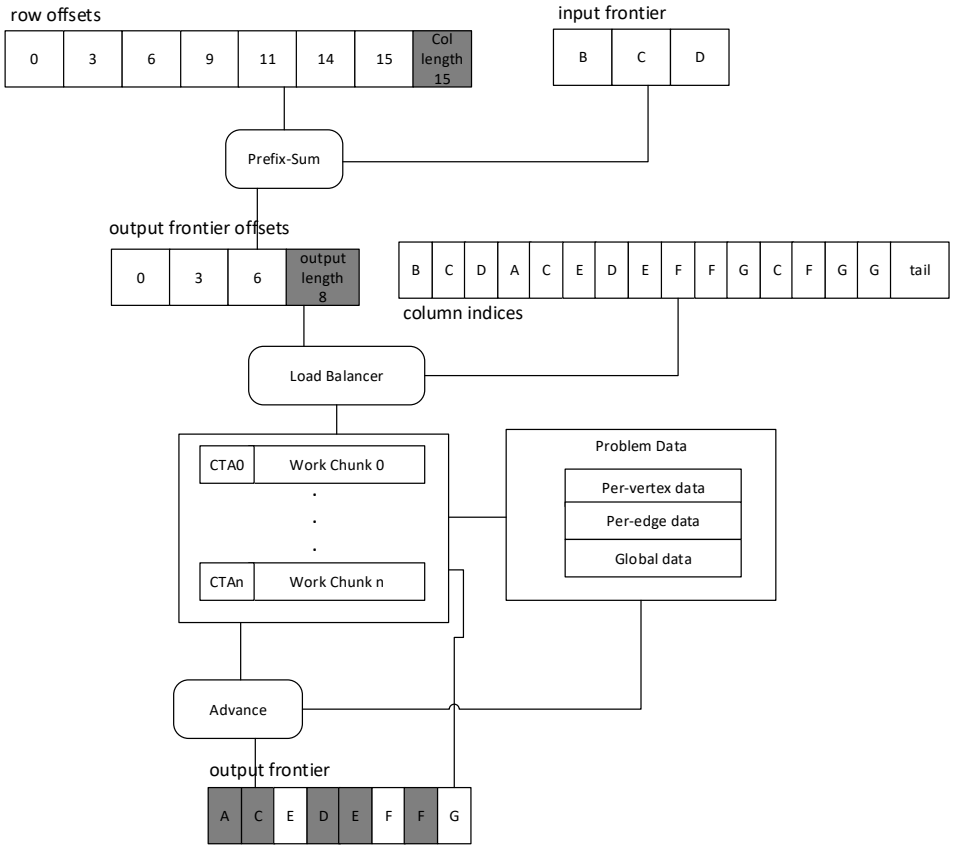


Fig. 7. Both push-based advance and pull-based advance have a prefix-sum part and a load balancer part.

load balancing A coarse-grained load-balancing strategy divides the neighbor lists by size into multiple groups, then processes each group independently with a strategy that is optimized for the sizes in that group. A fine-grained load-balancing strategy instead rebalances work so that the same number of input items or the same number of output items are assigned to a thread or group of threads.

traversal direction A push-based advance expands the neighbor lists of the current input frontier; a pull-based advance instead intersects the neighbor lists of the unvisited node frontier with the current frontier.

We provide more details in section 5.1.

4.2 Filter

Gunrock's filter operator is in essence a stream compaction operator that transforms a sparse representation of an array (input frontier) to a compact one (output frontier), where the sparsity comes from the different returned values for each input item in the validity test function (figure 8). (In graph traversal, multiple redundant nodes will fail the validity test.) Efficient stream compaction implementations also typically rely on prefix-sum; this is a well-studied problem [7, 34]. In Gunrock, we adopt Merrill et al.'s filtering implementation [55], which is based on local prefix-sums with

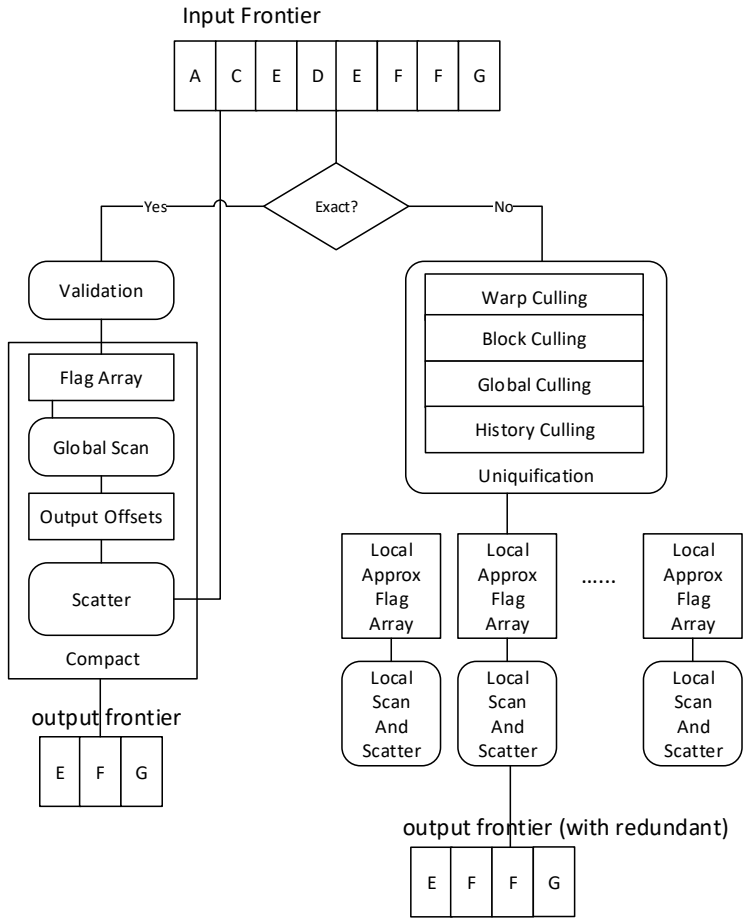


Fig. 8. Filter is based on compact, which uses either a global scan and scatter (for exact filtering) or a local scan and scatter after heuristics (for inexact filtering).

various culling heuristics. The byproduct of this implementation is a uniquification feature that does not strictly require the complete removal of undesired items in the input frontier. We provide more detail in Section 5.2.

4.3 Segmented Intersection

Gunrock's segmented intersection operator takes two input frontiers. For each pair of input items, it computes the intersection of two neighbor lists, then outputs the intersected items (figure 9). It is known that for intersection computation on two large frontiers, a modified merge-path algorithm would achieve high performance because of its load balance [4]. However, for segmented intersection, the workload per input item pair depends on the size of each item's neighbor list. For this reason, we still use prefix-sum for pre-allocation, then perform a series of load-balanced intersections according to a heuristic based on the sizes of the neighbor list pairs. Finally, we use a stream compaction to generate the output frontier, and a segmented reduction as well as a global reduction to compute segmented intersection counts and the global intersection count.

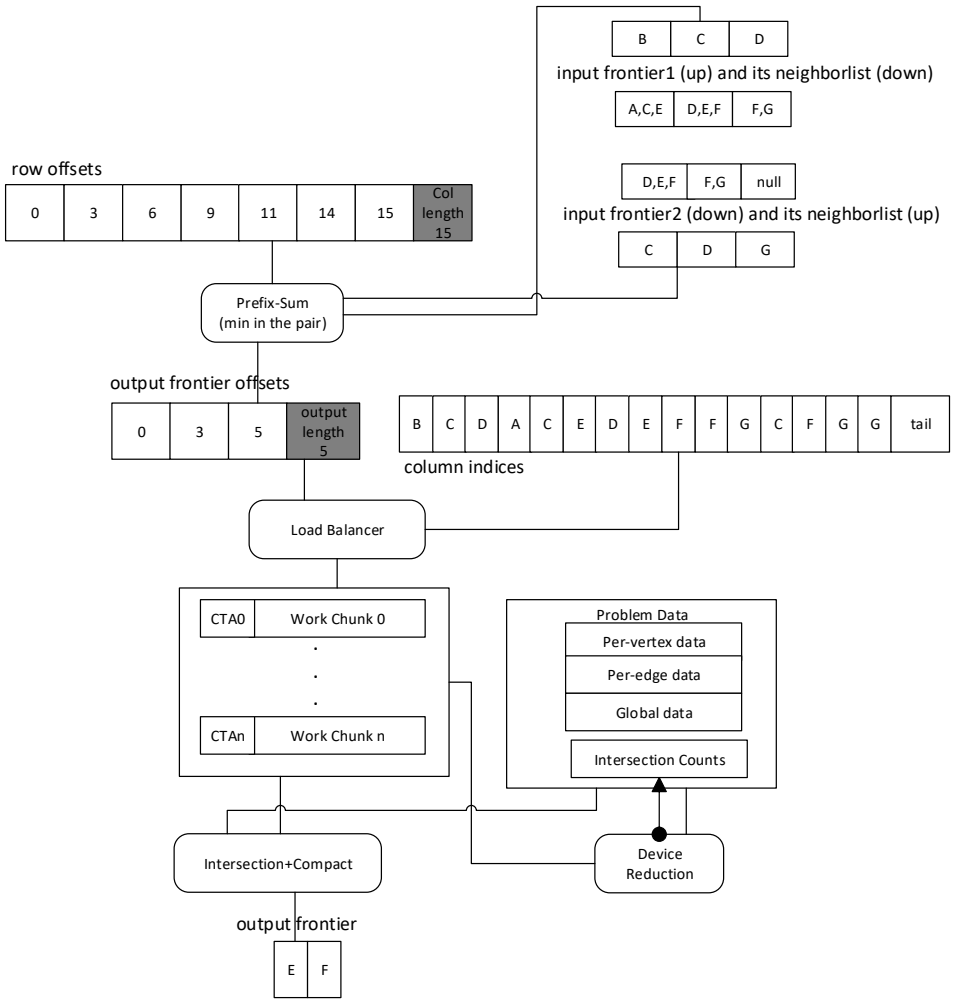


Fig. 9. Segmented intersection implementation that uses prefix-sum, compact, merge-based intersection, and reduction.

High-performance segmented intersection requires a similar focus to high-performance graph traversal: effective load-balancing and GPU utilization. In our implementation, we use the same dynamic grouping strategy proposed in Merrill's BFS work [55]. We divide the edge lists into two groups: 1) small neighbor lists and 2) one small and one large neighbor list. We implement two kernels (Two-Small and Small-Large) that cooperatively compute intersections. Our TwoSmall kernel uses one thread to compute the intersection of a node pair. Our SmallLarge kernel starts a binary search for each node in the small neighbor list on the large neighbor list. By using this 2-kernel strategy and carefully choosing a threshold value to divide the edge list into two groups, we can process intersections with the same level of workload together to gain load balancing and higher GPU resource utilization. Currently, if two neighbor lists of a pair are both large ones, we use the SmallLarge kernel. Ideally a third kernel that utilizes all threads within a block to work on two large neighbor lists would yield better performance.

Optimization Strategy	Module Name in Gunrock
Static Workload Mapping	ThreadExpand
Dynamic Grouping Workload Mapping	TWC_FORWARD
Merge-based Load-Balanced Partitioning Workload Mapping	LB, LB_LIGHT, and LB_CULL
Pull Traversal	Inverse_Expand

Table 2. Four graph traversal throughput optimization strategies and their corresponding module names in the Gunrock implementation, where LB_LIGHT processes load balance over an input frontier, LB processes load balance over an output frontier, and LB_CULL combines LB and LB_LIGHT with a follow-up filter into a fused kernel.

5 SYSTEM IMPLEMENTATION AND OPTIMIZATIONS

Choosing the right abstraction is one key component in achieving high performance within a graph framework. The second component is optimized implementations of the primitives within the framework. One of the main goals in designing the Gunrock abstraction was to easily allow integrating existing and new alternatives and optimizations into our primitives to give more options to programmers. In general, we have found that our data-centric abstraction and our focus on manipulating the frontier have been an excellent fit for these alternatives and optimizations, compared to a more difficult implementation path for other GPU computation-focused abstractions. In this section, we offer examples by discussing optimizations that help increase the performance in four different categories:

- Graph traversal throughput
- Synchronization throughput
- Kernel launch throughput
- Memory access throughput

5.1 Graph Traversal Throughput Optimizations

One of Gunrock's major contributions is generalizing different types of workload-distribution and load-balance strategies. These strategies previously only appeared in specialized GPU graph primitives. We implement them in Gunrock's general-purpose advance operators. As a result, any existing or new graph primitive that uses an advance operator benefits from these strategies.

In this section, we define the workload for graph primitives as per-edge and/or per-node computation that happens during graph traversal. Gunrock's advance step generates an irregular workload. Consider an advance that generates a new vertex frontier from the neighbors of all vertices in the current frontier. If we parallelize over input vertices, graphs with a variation in vertex degree (with different-sized neighbor lists) will generate a corresponding imbalance in per-vertex work. Thus, mapping the workload of each vertex onto the GPU so that all vertex work can be processed in a load-balanced way is essential for efficiency. Table 2 shows our traversal throughput optimization strategies and their corresponding module name in Gunrock implementation. Later in this section, we summarize the pros and cons, with guidelines for usage, of these optimizations in Table 3.

The most significant previous work in this area balances load by cooperating between threads. Targeting BFS, Hong et al. [37] map the workload of a single vertex to a series of virtual warps. Merrill et al. [55] use a more flexible strategy that maps the workload of a single vertex to a thread, a warp, or a block, according to the size of its neighbor list. Targeting SSSP, Davidson et al. [17] use two load-balanced workload mapping strategies, one that groups input work and the other that groups output work. The first load-balances over the input frontier, the second load-balances

over the output frontier. Every strategy has a tradeoff between computation overhead and load-balance performance. We show in Gunrock how we integrate and redesign these two strategies, use heuristics to choose from them to achieve the best performance for different datasets, and generalize this optimization to various graph operators.

5.1.1 Static Workload Mapping Strategy. One straightforward approach to map the workload is to map one frontier vertex's neighbor list to one thread. Each thread loads the neighbor list offset for its assigned node, then serially processes edges in its neighbor list. Though this simple solution will cause severe load imbalance for scale-free graphs with unevenly distributed degrees and extremely small diameter, it has the significant advantage of negligible load balancing overhead and works well for large-diameter graphs with a relatively even degree distribution. Thus in Gunrock, we keep this static strategy but provide several improvements in the following aspects:

cooperative process We load all the neighbor list offsets into shared memory, then use a block of threads to cooperatively process per-edge operations on the neighbor list.

loop strip mining We split the neighbor list of a node so that multiple threads within the same SIMD lane can achieve better utilization.

This "ThreadExpand" method performs better when used for large-diameter graphs with a relatively even degree distribution since it balances thread work within a block, but not across blocks. For graphs with a more uneven degree distribution (e.g., scale-free social graphs), we turn to a second strategy.

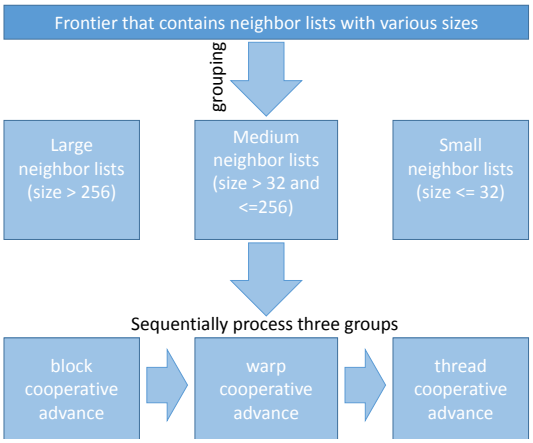


Fig. 10. Dynamic grouping workload mapping strategy [55].

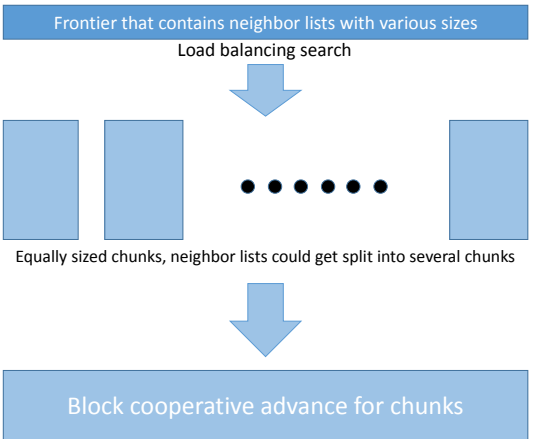


Fig. 11. Merge-based load-balanced partitioning workload mapping strategy [17].

5.1.2 Dynamic Grouping Workload Mapping Strategy. Gunrock uses a load balancing strategy called TWC (Thread/Warp/CTA Expansion) based on Merrill et al.'s BFS implementation [55] but with more flexible launch settings and user-specific computation functor support. As a graph operator building block, it now can be generalized to support the traversal steps in other traversal-based graph primitives. It is implemented to solve the performance bottleneck when ThreadExpand is applied to frontiers with significant differences in neighbor list sizes. Like Merrill et al., we directly address the variation in size by grouping neighbor lists into three categories based on their size, then individually processing each category with a strategy targeted directly at that size. Our three sizes are 1) lists larger than a block, 2) lists larger than a warp (32 threads) but smaller than a

block, and 3) lists smaller than a warp. We begin by assigning a subset of the frontier to a block. Within that block, each thread owns one node. The threads that own nodes with large lists arbitrate for control of the entire block. All the threads in the block then cooperatively process the neighbor list of the winner's node. This procedure continues until all nodes with large lists have been processed. Next, all threads in each warp begin a similar procedure to process all the nodes with medium-sized lists. Finally, the remaining nodes are processed using our ThreadExpand method. As Merrill et al. noted in their paper, this strategy can guarantee a high utilization of resource and limit various types of load imbalance such as SIMD lane underutilization (by using per-thread mapping), intra-thread imbalance (by using warp-wise mapping), and intra-warp imbalance (by using block-wise mapping) (Figure 10).

5.1.3 Merge-based Load-Balanced Partitioning Workload Mapping Strategy. The specialization of dynamic grouping workload mapping strategy allows higher throughput on frontiers with a high variance in degree distribution, but at the cost of higher overhead due to the sequential processing of the three different sizes. Also, to deal with intra-block load imbalance, additional scheduling and work stealing method must be applied, which further adds to the load-balancing overhead. A global one-pass load balancing strategy would potentially solve the intra-block load imbalance problem and thus bring better performance. Davidson et al. [17] and Gunrock improve on the dynamic grouping workload mapping strategy by globally load-balancing over either the input frontier or the output frontier (figure 11). This introduces load-balancing overhead, but significantly increases the traversal throughput for scale-free graphs.

Input frontier load balance maps the same number of input items to a block, then puts the output offset for each input item computed by prefix-sum into shared memory. Just as in ThreadExpand, Gunrock uses cooperative processing and loop strip mining here as well. All threads within a single block will cooperatively visit all the neighbor lists of the input items that belong to this block. When a thread starts to process a new neighbor list, it requires a binary search to find the corresponding source node ID.

Output frontier load balance first uses a global prefix-sum to compute all output offsets, then forms an arithmetic progression of $0, N, 2N, \ldots, |Input|$ where N is the number of edges each block processes. A global sorted search [5] of this arithmetic progression in the output offset array will find the starting indices for all the blocks within the frontier. After organizing groups of edges into equal-length chunks, all threads within one block cooperatively process edges. When we start to process a neighbor list of a new node, we use binary search to find the node ID for the edges that are going to be processed. Using this method, we ensure both inter-and-intra-block load-balance (Figure 11).

At a high level, Gunrock makes its load-balancing strategy decisions depending on graph topology. We note that our load-balancing workload mapping method performs better on social graphs with irregularly distributed degrees, while the dynamic grouping method is superior for graphs where node degrees are evenly distributed. For this reason, in Gunrock we implement a hybrid of both methods on both vertex and edge frontiers, using the dynamic grouping strategy for nodes with relatively smaller neighbor lists and the load-balancing strategy for nodes with relatively larger neighbor lists. We pick average degree as the metric for choosing between these two strategies. When the graph has an average degree of 5 or larger, we use the load-balancing strategy, otherwise we use dynamic grouping strategy. Within the load-balancing strategy, we set a static threshold. When the frontier size is smaller than the threshold, we use coarse-grained load balance over nodes (load balance on the input frontier), otherwise coarse-grained load balance over edges (load balance on the output frontier). We have found that setting this threshold to 4096 yields

consistent high performance for tests across all Gunrock-provided graph primitives. Users can also change this value easily in the Enactor module for their own datasets or graph primitives. Superior load balancing is one of the most significant reasons why Gunrock outperforms other GPU frameworks [84].

5.1.4 Push vs. Pull Traversal. Certainly other GPU programmable graph frameworks also support an advance step. However, because they are centered on vertex operations on an implicit frontier, they generally only support "push"-style advance: the current frontier of active vertices "pushes" active status to its neighbors, which creates a new frontier of newly active vertices. We show this in figure 12.Beamer et al. [6] described a "pull"-style advance on CPUs (figure 13): instead of starting with a frontier of active vertices, pull starts with a frontier of *unvisited* vertices, filters it for vertices with neighbors in the current frontier, and generates a new frontier with the output of the filter.

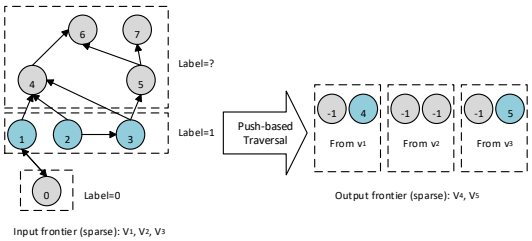


Fig. 12. Push-based graph traversal. -1 means invalid label.

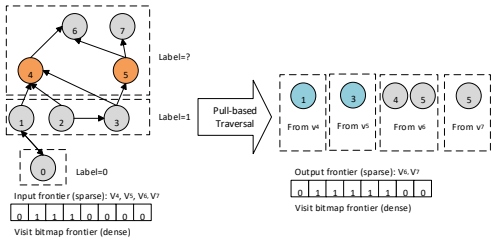


Fig. 13. Pull-based graph traversal.

Beamer et al. showed this approach is beneficial when the number of unvisited vertices drops below the size of the current frontier. Vertex-centered GPU frameworks have found it challenging to integrate this optimization into their abstraction due to the lack of a flexible frontier representation. Compared to them, our data-centric abstraction is a much more natural fit, because we can easily perform more flexible operations on frontiers. Gunrock achieves this with two frontiers. During the "push" phase, it uses the active frontier as usual in the advance step. When switching to the "pull" phase, it first generates a unvisited frontier with all the unvisited nodes, Then it uses the unvisited frontier in the advance step, visiting all unvisited nodes that have visited predecessors, and generates both a new active frontier and a new unvisited frontier. The capability of keeping two active frontiers differentiates Gunrock from other GPU graph processing programming models. For better performance, Gunrock may use per-node bitmaps to indicate whether a node has been visited.

For pull-based traversal on the GPU, we modified Beamer et al.'s shared-memory-CPU-based heuristics to switch between push and pull-based traversal. Given the number of edges to check from the frontier (m_f), the number of vertices in the frontier (n_f), the edges to check from unexplored vertices (m_u), and two tuning parameters α and β, Beamer et al. define two equations:

$$m_f > \frac{m_u}{\alpha} = C_{TB} \tag{1}$$

$$n_f < \frac{n}{\beta} = C_{BT} \tag{2}$$

where n is number of nodes in the graph and C_{TB} and C_{BT} are thresholds for the push-to-pull-based traversal switch and the pull-to-push-based traversal switch separately. However, on the GPU, because computing m_f and m_u requires two additional passes of prefix-sum, we estimate them as

ALGORITHM 2: Direction optimizing BFS

```
Input: G;                                          ▷ Csr format of graph storage
Input: I_active;                                   ▷ The input active frontier
Input: I_unvisited;                                ▷ The input unvisited frontier
Output: O_active;                                  ▷ The output active frontier
Output: O_unvisited;                               ▷ The output unvisited frontier
if DirectionDecision() == push then
    Advance(G, I_active, O_active);
    advance_mode ← push;
end
else
    if advance_mode == push then
        I_unvisited ← GenerateUnvisitedFrontier(G.labels, V);
    end
    Reverse_Advance(G, I_unvisited, O_active, O_unvisited);
    advance_mode ← pull;
end
```

follows:

$$m_f = \frac{n_f \times m}{n} \tag{3}$$

$$m_u = \frac{n_u \times n}{n - n_u} \tag{4}$$

where m is the number of edges in the graph, n_f is the current frontier length, and n_u is the number of unvisited nodes. Instead of directly using the size of $I\_unvisited$, we keep subtracting the size of I_active from n for each iteration to calculate n_u, because $I\_unvisited$ is not available during the "push" phase. We also modified the switching points:

$$m_f > m_u \times do\_a = C_{TB} \tag{5}$$

$$m_f < m_u \times do\_b = C_{BT} \tag{6}$$

With proper selection of do_a and do_b, our new heuristics find the optimal iteration to switch.

With this optimization, we see a large speedup on BFS for scale-free graphs. In an abstraction like Medusa, with its fixed method (segmented reduction) to construct frontiers, it would be a significant challenge to integrate a pull-based advance.

5.1.5 Two-Level Priority Queue. A straightforward BSP implementation of an operation on a frontier treats each element in the frontier equally, i.e., with the same priority. Many graph primitives benefit from prioritizing certain elements for computation with the expectation that computing those elements first will save work overall (e.g., delta-stepping for SSSP [56]). Gunrock generalizes the approach of Davidson et al. [17] by allowing user-defined priority functions to organize an output frontier into "near" and "far" slices. This allows the GPU to use a simple and high-performance split operation to create and maintain the two slices. Gunrock then considers only the near slice in the next processing steps, adding any new elements that do not pass the near criterion into the far slice, until the near slice is exhausted. We then update the priority function and operate on the far slice.

Optimization Strategy	Summary
Static Workload Mapping	Low cost for load balancing; bad for varying degree distributions. Should use to traverse graphs with a relatively uniform edge distribution per vertex.
Dynamic Grouping Workload Mapping	Moderate cost for load balancing; bad for scale-free graphs. Should use to traverse mesh-like graphs with large diameters when frontier size gets larger than 1 million.
Merge-based Load-Balanced Partitioning Workload Mapping	High cost for load balancing on power-law graphs, but low cost for regular graphs; shows consistently better performance on most graphs. Should use as a default traversal strategy choice.
Pull Traversal	Has one-time frontier conversion/preparation cost; good for scale-free graphs with a large amount of common neighbors. Should not use on regular graphs and when the number of unvisited vertices is either too large or too small (for more detail, refer to section 7.4.)

Table 3. Four graph traversal throughput optimization strategies, with their pros, cons, and guidelines for usage in Gunrock.

Like other graph operators in Gunrock, constructing a priority queue directly manipulates the frontier data structure. It is difficult to implement such an operation in a GAS-based programming model since that programming model has no explicit way to reorganize a frontier.

Gunrock's two-level priority queue implementation is a modified filter operator, which uses two stream compactions to not only form the output frontier of input items with true flag values, but also to form a "far" pile of input items with false flag values.

Currently Gunrock uses this specific optimization only in SSSP. However, we believe a workload reorganization strategy based on a more general multisplit operator [2], which maps one input frontier to multiple output frontiers according to an arbitrary number of priority levels, would fit nicely into Gunrock's data-centric programming model. By dividing a frontier into multiple subfrontiers and making the computation of each subfrontier not conflict with the computation of others, we can run the computation of each subfrontier asynchronously. This, in turn, offers more opportunity to exploit parallelism between subfrontiers and will potentially increase the performance of various types of algorithms, such as node ranking, community detection, and label propagation, as well as those on graphs with small "long tail" frontiers.

5.2 Synchronization Throughput Optimization

For graph processing on the GPU, the bottlenecks of synchronization throughput come from two places:

concurrent discovery In a tree traversal, there can be only one path from any node to any other node. However in graph traversal, starting from a source node, there could be several redundant visits if no pruning is conducted. Beamer et al. [6] categorize these redundant visits into three types: 1) visited parents; 2) being-visited peers; and 3) concurrently discovered children. Concurrent discovery of child nodes contributes to most synchronization overhead when there is per-node computation.

dependencies in parallel data-primitives Sometimes the computation during traversal has a reduction (Pagerank, BC) and/or intersection (TC) step on each neighbor list.

For synchronization overhead brought by concurrent discovery, the generalized pull-and-push based traversal discussed in section 5.1.4 will reduce it. In this section we present another optimization that solves this problem from the perspective of the idempotence of a graph operation.

5.2.1 Idempotent vs. non-idempotent operations. Multiple elements in the frontier may share a common neighbor. This has two consequences: 1) it will cause an advance step to generate an output frontier that has duplicated elements; 2) it will cause any computation on the common neighbors to run multiple times. The second consequence causes a potential synchronization problem by producing race conditions. A general way to avoid race conditions is to introduce atomic operations. However, for some graph primitives with *idempotent* operations (e.g., BFS's visit status check), repeating a computation causes no harm. In such a case, Gunrock's advance step will avoid (costly) atomic operations, repeat the computation multiple times, and output all redundant items to the output frontier. Again, users need to be responsible for data race in the per-edge computation (see section 3). Gunrock's filter step has incorporated a series of inexpensive heuristics [55] to reduce, but not eliminate, redundant entries in the output frontier. These heuristics include a global bitmask, a block level history hash table, and a warp level hash table. The sizes of each hash table is adjustable to achieve the optimal tradeoff between performance and redundancy-reduction rate.

5.2.2 Atomic Avoidance Reduction Operations. To reduce the synchronization overhead of reduction, we either 1) reduce the atomic operations by hierarchical reduction and the efficient use of shared memory on the GPU or 2) assign several neighboring edges to one thread in our dynamic grouping strategy so that partial results within one thread can be accumulated without atomic operations.

5.3 Kernel Launch Throughput Optimization

Gunrock and all other BSP-model-based GPU graph processing libraries launch one or several GPU kernels per iteration, and often copy a condition check byte from device to host after each iteration to decide whether to terminate the program. Pai and Pingali also noted this kernel launch overhead and recently proposed several compiler optimizations to reduce it [58]. In Gunrock, we implemented two optimizations that target this overhead:

Fuse computation with graph operator Specialized GPU implementations fuse regular computation steps together with more irregular steps like advance and filter by running a computation step (with regular parallelism) on the input or output of the irregularly-parallel step, all within the same kernel. To enable similar behavior in a programmable way, Gunrock exposes its computation steps as *functors* that are integrated into all its graph operator kernels at compile time to achieve similar efficiency. We support functors that apply to {edges, vertices} and either return a Boolean value (the "cond" functor), useful for filtering, or perform a computation (the "apply" functor). These functors will then be integrated into Gunrock's graph operator kernel calls, which hide any complexities of how those steps are internally implemented.

Fuse filter step with traversal operators Several traversal-based graph primitives have a filter step immediately following an advance or neighborhood-reduction step. Gunrock implements a fused single-kernel traversal operator that launches both advance and filter steps. Such a fused kernel reduces the data movement between double-buffered input and output frontiers.

5.4 Memory Access Throughput Optimization

For graph problems that require irregular data accesses, in addition to exposing enough parallelism, a successful GPU implementation benefits from the following application characteristics: 1) coalesced memory access, 2) effective use of the memory hierarchy, and 3) reducing scattered reads and writes. Our choice of graph data structure helps us achieve these goals.

By default, Gunrock uses a compressed sparse row (CSR) sparse matrix for vertex-centric operations. Gunrock also allows users to choose coordinate list (COO) representation for edge-centric operations. CSR uses a column-indices array, C, to store a list of neighbor vertices and a row-offsets array, R, to store the offset of the neighbor list for each vertex. It provides compact and efficient memory access, and allows us to use prefix-sum to reorganize sparse and uneven workloads into dense and uniform ones in all phases of graph processing [55]. In terms of data structures, Gunrock represents all per-node and per-edge data as structure-of-array (SOA) data structures that allow coalesced memory accesses with minimal memory divergence. In terms of graph operators, Gunrock implements carefully designed for loops in its kernel to guarantee coalesced memory access. We also efficiently use shared memory and local memory to increase memory access throughput in the following ways:

In dynamic grouping workload mapping Gunrock moves chunks of input frontiers into local memory, and uses warp scan and warp streaming compaction.

In load-balanced partition workload mapping Gunrock uses shared memory to store the resulting indices computed by merge-based load balanced search.

In filter operator Gunrock stores two types of hash tables (block-wise history hash tables and warp-wise history hash tables) in shared memory.

6 GRAPH APPLICATIONS

One of the principal advantages of Gunrock's abstraction is that our advance, filter, segmented intersection and compute steps can be composed to build new graph primitives with minimal extra work. For each primitive in Figure 14, we describe the hardwired GPU implementation to which we compare, followed by how we express this primitive in Gunrock. Section 7 compares the performance between hardwired and Gunrock implementations.

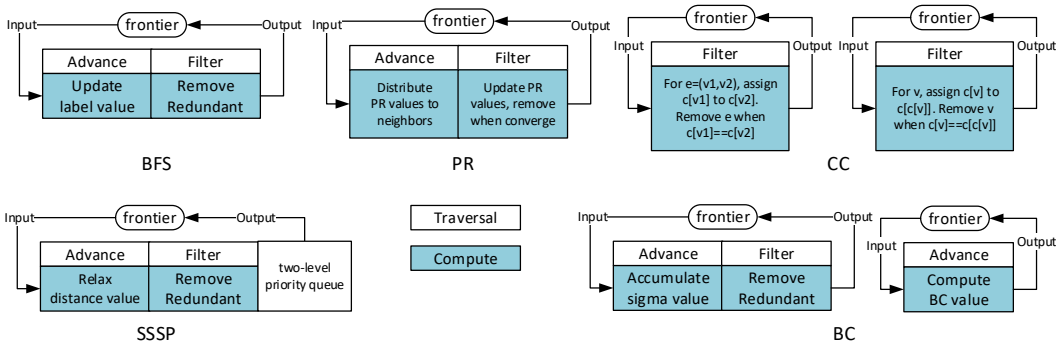


Fig. 14. Operation flow chart for selected primitives in Gunrock (a black line with an arrow at one end indicates a while loop that runs until the frontier is empty).

6.1 Breadth-First Search (BFS)

BFS initializes its vertex frontier with a single source vertex. On each iteration, it generates a new frontier of vertices with all unvisited neighbor vertices in the current frontier, setting their depths and repeating until all vertices have been visited. BFS is one of the most fundamental graph primitives and serves as the basis of several other graph primitives.

Hardwired GPU Implementation The well-known BFS implementation of Merrill et al. [55] achieves its high performance through careful load-balancing, avoidance of atomics, and heuristics for avoiding redundant vertex discovery. Its chief operations are expand (to generate a new frontier) and contract (to remove redundant vertices) phases.

Gunrock Implementation Merrill et al.'s *expand* and *contract* map nicely to Gunrock's advance operator and filter operator separately. During advance, we set a label value for each vertex to show the distance from the source, and/or set a predecessor value for each vertex that shows the predecessor vertex's ID. We implement efficient load-balancing (section 5.1.2 and section 5.1.3) and both push- and pull-based advance (section 5.1.4) for more efficient traversal. Our base implementation uses atomics during advance to prevent concurrent vertex discovery. When a vertex is uniquely discovered, we set its label (depth) and/or predecessor ID. Gunrock's BFS uses the idempotent advance operator to avoid the cost of atomics and uses heuristics within its filter that reduce the concurrent discovery of child nodes (section 5.2.1).

6.2 Single-Source Shortest Path

Single-source shortest path finds paths between a given source vertex and all other vertices in the graph such that the weights on the path between source and destination vertices are minimized. While the advance mode of SSSP is identical to BFS, the computation mode differs.

Hardwired GPU Implementation We implement the GPU SSSP algorithm from the work from Davidson et al. [17]. They provide two key optimizations in their SSSP implementation: 1) a load-balanced graph traversal method and 2) a priority queue implementation that reorganizes the workload. Gunrock generalizes both optimization strategies into its implementation, allowing them to apply to other graph primitives as well as SSSP. We implement Gunrock's priority queue as an additional filter pass between two iterations.

Gunrock Implementation We start from a single source vertex in the frontier. To compute a distance value from the source vertex, we need one advance and one filter operator. On each iteration, we visit all associated edges in parallel for each vertex in the frontier and relax the distance's value (if necessary) of the vertices attached to those edges. We use an AtomicMin to atomically find the minimal distance value we want to keep and a bitmap flag array associated with the frontier to remove redundant vertices. After each iteration, we use a priority queue to reorganize the vertices in the frontier.

6.3 Betweenness Centrality

The BC index can be used in social network analysis as an indicator of the relative importance of vertices in a graph. At a high level, the BC for a vertex in a graph is the fraction of shortest paths in a graph that pass through that vertex. Brandes's BC formulation [8] is most commonly used for GPU implementations.

Hardwired GPU Implementation Brandes's formulation has two passes: a forward BFS pass to accumulate sigma values for each node, and a backward BFS pass to compute centrality values. Jia et al. [38] and Sariyüce et al. [67] both use an edge-parallel method to

implement the above two passes. We achieve this in Gunrock using two advance operators on an edge frontier with different computations. The recent (hardwired) multi-GPU BC algorithm by McLaughlin and Bader [52] uses task parallelism, dynamic load balancing, and sampling techniques to perform BC computation in parallel from different sources on different GPU streaming multiprocessors.

Gunrock Implementation Gunrock's implementation also contains two phases. The first phase has an advance step identical to the original BFS and a computation step that computes the number of shortest paths from source to each vertex. The second phase uses an advance step to iterate over the BFS frontier backwards with a computation step to compute the dependency scores. We achieve competitive performance on scale-free graphs with the latest hardwired BC algorithm [54]. Within Gunrock, we have not yet considered task parallelism since its execution model does not fit Gunrock's current framework, but it is an interesting area for future work.

6.4 Connected Component Labeling

The connected component primitive labels the vertices in each connected component in a graph with a unique component ID.

Hardwired GPU Implementation Soman et al. [75] base their implementation on two PRAM algorithms: hooking and pointer-jumping. Hooking takes an edge as the input and tries to set the component IDs of the two end vertices of that edge to the same value. In odd-numbered iterations, the lower vertex writes its value to the higher vertex, and vice versa in the even numbered iteration. This strategy increases the rate of convergence. Pointer-jumping reduces a multi-level tree in the graph to a one-level tree (star). By repeating these two operators until no component ID changes for any node in the graph, the algorithm will compute the number of connected components for the graph and the connected component to which each node belongs.

Gunrock Implementation Gunrock uses a filter operator on an edge frontier to implement hooking. The frontier starts with all edges and during each iteration, one end vertex of each edge in the frontier tries to assign its component ID to the other vertex, and the filter step removes the edge whose two end vertices have the same component ID. We repeat hooking until no vertex's component ID changes and then proceed to pointer-jumping, where a filter operator on vertices assigns the component ID of each vertex to its parent's component ID until it reaches the root. Then a filter step removes the node whose component ID equals its own node ID. The pointer-jumping phase also ends when no vertex's component ID changes.

6.5 PageRank and Other Node Ranking Algorithms

The PageRank link analysis algorithm assigns a numerical weighting to each element of a hyper-linked set of documents, such as the World Wide Web, with the purpose of quantifying its relative importance within the set. The iterative method of computing PageRank gives each vertex an initial PageRank value and updates it based on the PageRank of its neighbors until the PageRank value for each vertex converges. PageRank is one of the simplest graph algorithms to implement on GPUs because the frontier always contains all vertices, so its computation is congruent to sparse matrix-vector multiply; because it is simple, most GPU frameworks implement it in a similar way and attain similar performance.

In Gunrock, we begin with a frontier that contains all vertices in the graph and end when all vertices have converged. Each iteration contains one advance operator to compute the PageRank

value on the frontier of vertices, and one filter operator to remove the vertices whose PageRanks have already converged. We accumulate PageRank values with AtomicAdd operations.

Bipartite graphs. Geil et al. [21] used Gunrock to implement Twitter's who-to-follow algorithm ("Money" [23]), which incorporated three node-ranking algorithms based on bipartite graphs (Personalized PageRank (PPR), Stochastic Approach for Link-Structure Analysis (SALSA), and Hyperlink-Induced Topic Search (HITS)). Their implementation, the first to use a programmable framework for bipartite graphs, demonstrated that Gunrock's advance operator is flexible enough to encompass all three node-ranking algorithms, including a 2-hop traversal in a bipartite graph.

6.6 Triangle Counting

Hardwired GPU Implementation The extensive survey by Schank and Wagner [68] shows several sequential algorithms for counting and listing triangles in undirected graphs. Two of the best performing algorithms, *edge-iterator* and *forward*, both use edge-based set intersection primitives. The optimal theoretical bound of this operation coupled with its high potential for parallel implementation make this method the core idea behind several GPU implementations [28, 63].

Gunrock Implementation In Gunrock, we view the TC problem as a set intersection problem [81] by the following observation: An edge $e = (u, v)$, where u, v are its two end nodes, can form triangles with edges connected to both u and v. Let the intersections between the neighbor lists of u and v be $(w_1, w_2, \ldots, w_N)$, where N is the number of intersections. Then the number of triangles formed with e is N, where the three edges of each triangle are $(u, v), (w_i, u), (w_i, v), i \in [1, N]$. In practice, computing intersections for every edge in an undirected graph is redundant. We visit all the neighbor lists using advance. If two nodes u and v have two edges (u, v) and (v, u) between them, we only keep one edge that points from the node with larger degree to the node with smaller degree (for nodes with identical degree number, we use vertex ID as the tie breaker). This halves the number of the edges that we must process. Thus, in general, set intersection-based TC algorithms have two stages: (1) forming edge lists; (2) computing set intersections for two neighbor lists of an edge. Different optimizations can be applied to either stage. Our GPU implementation follows the *forward* algorithm and uses advance, filter, and segmented-intersection operators.

6.7 Subgraph Matching

The subgraph matching primitive finds all embeddings of a graph pattern q in a large data graph g. It can also be extended to handle graph homomorphism problems, which can be used in database pattern searching.

Hardwired GPU Implementation Existing subgraph matching algorithms are mostly based on a backtracking strategy. First, vertices that cannot contribute to the final solutions are filtered out based on their labels. Then these candidates are passed to a recursive procedure to be further pruned in different matching orders based on the query graph and the sub-region where the candidate is located in the data graph. The order can significantly affect the performance. The recursive subroutine is hard to map efficiently to GPUs for two reasons. First, because of different matching orders for candidate vertices, warp divergence is a problem. Second, due to irregular graph patters, we see uncoalesced memory accesses. Some existing GPU implementations parallelize the backtracking method to join candidate edges in parallel to form partial solutions and repeat the method until the query graph

pattern is obtained. However, these implementations generate more intermediate results, which makes the problem memory-bounded.

Gunrock Implementation Our implementation using Gunrock [81] follows a filtering-and-joining procedure. In the filtering phase, we use a filter operator on a vertex frontier to prune out vertices based on both vertex labels and vertex degrees. Vertices with degree less than a certain query node's degree or with a different label cannot be that node's candidate. After that, we use advance and filter operators to collect the candidate edges. Then we do a join using our optimized segmented intersection operator.

7 PERFORMANCE CHARACTERIZATION

We first show overall performance analysis of Gunrock on nine datasets including both real-world and generated graphs; the topology of these datasets spans from regular to scale-free.

We ran all experiments in this paper on a Linux workstation with 2×Intel Xeon E5-2637 v2 CPUs (3.50 GHz, 4-core, hyperthreaded), 256 GB of main memory, and an NVIDIA K40c GPU with 12 GB on-board memory. GPU programs were compiled with NVIDIA's nvcc compiler (version 8.0.44) with the -O3 flag. The BGL and PowerGraph code were compiled using gcc 4.8.4 with the -O3 flag. Ligra was compiled using icpc 15.0.1 with CilkPlus. For Ligra, elapsed time for best possible sub_algorithm was considered (for example, BFSCC was considered instead of CC when it was the faster implementation for some particular datasets). All PageRank implementations were executed with maximum iteration set to 1. All results ignore transfer time (both disk-to-memory and CPU-to-GPU). All Gunrock tests were run 10 times with the average runtime and MTEPS used for results.

Datasets. We summarize the datasets we use for evaluation in Table 4. Soc-orkut (soc-ork), soc-livejournal1 (soc-lj), and hollywood-09 (h09) are three social graphs; indochina-04 (i04) is a crawled hyperlink graph from indochina web domains; rmat_s22_e64 (rmat-22), rmat_s23_e32 (rmat-23), and rmat_s24_e16 (rmat-24) are three generated R-MAT graphs with similar vertex counts. All seven datasets are scale-free graphs with diameters of less than 30 and unevenly distributed node degrees (80% of nodes have degree less than 64). Both rgg_n_24 (rgg) and roadnet_USA (roadnet) datasets have large diameters with small and evenly distributed node degrees (most nodes have degree less than 12). soc-ork is from the Stanford Network Repository; soc-lj, i04, h09, and roadnet are from the UF Sparse Matrix Collection; rmat-22, rmat-23, rmat-24, and rgg are R-MAT and random geometric graphs we generated. For R-MAT, we use 16 as the edge factor, and the initiator parameters for the Kronecker graph generator are: $a = 0.57, b = 0.19, c = 0.19, d = 0.05$. This setting is the same as in the Graph 500 Benchmark. For random geometric graphs, we set the threshold parameter to 0.000548. The edge weight values (used in SSSP) for each dataset are uniform random values between 1 and 64.

Measurement methodology. We report both runtime and traversed edges per second (TEPS) as our performance metrics. (In general we report runtimes in milliseconds and TEPS as millions of traversals per second [MTEPS].) Runtime is measured by measuring the GPU kernel running time and MTEPS is measured by recording the number of edges visited during the running (the sum of neighbor list lengths of all visited vertices) divided by the runtime. When a library does not report MTEPS, we use the following equations to compute it for BFS and BC: $\frac{|E|}{t}$ (BFS) and $\frac{2\times|E|}{t}$ (BC), where E is the number of edges visited and t is runtime. For SSSP, since one edge can be visited multiple times when relaxing its destination node's distance value, there is no accurate way to estimate its MTEPS number.

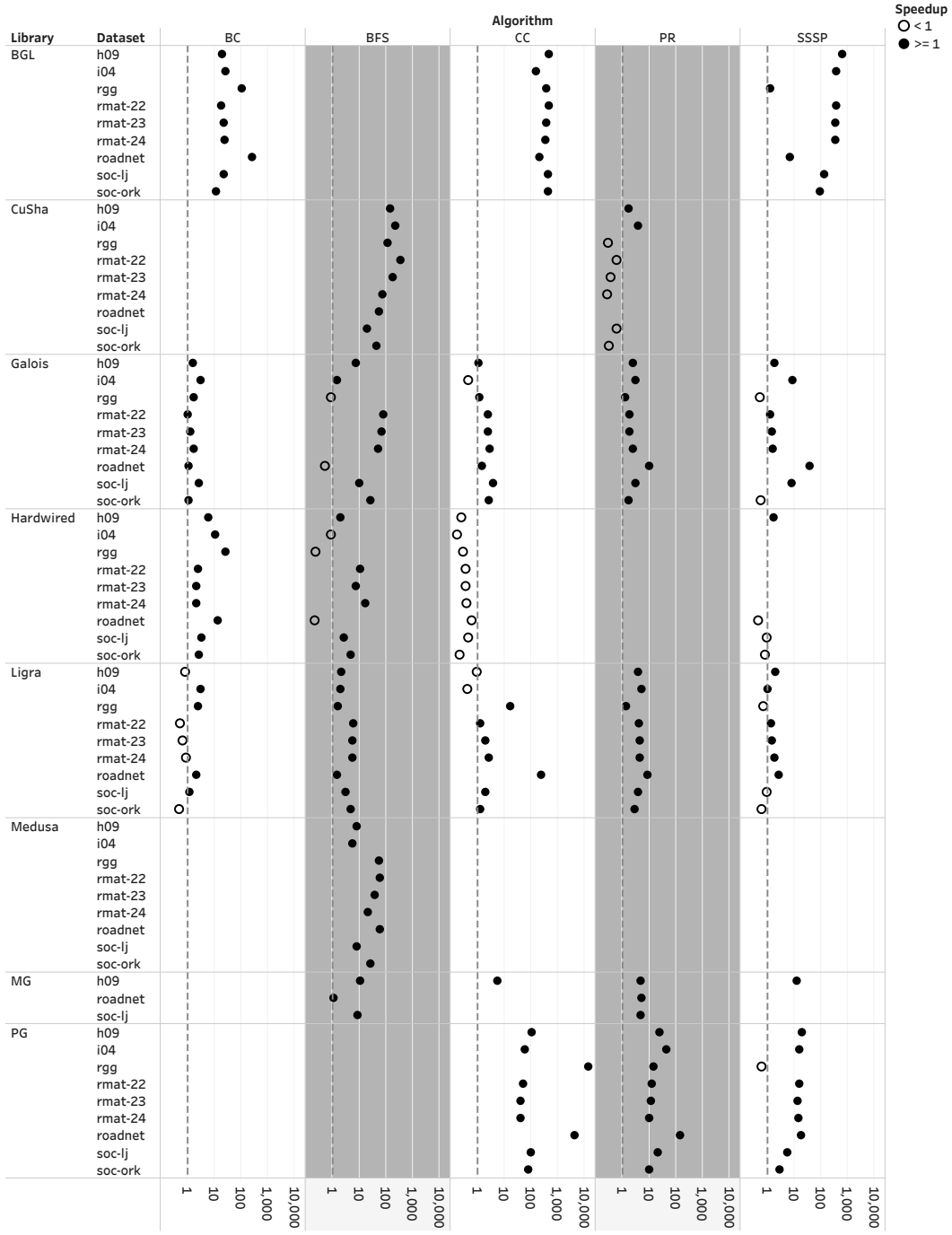


Fig. 15. Execution-time speedup for Gunrock vs. five other graph processing libraries/hardwired algorithms on nine different graph inputs. Data is from Table 6. Black dots indicate Gunrock is faster, white dots slower.

Dataset	Vertices	Edges	Max Degree	Diameter	Type
soc-orkut	3M	212.7M	27,466	9	rs
soc-Livejournal1	4.8M	85.7M	20,333	16	rs
hollywood-09	1.1M	112.8M	11,467	11	rs
indochina-04	7.4M	302M	256,425	26	rs
rmat_s22_e64	4.2M	483M	421,607	5	gs
rmat_s23_e32	8.4M	505.6M	440,396	6	gs
rmat_s24_e16	16.8M	519.7M	432,152	6	gs
rgg_n_24	16.8M	265.1M	40	2622	gm
roadnet_USA	23.9M	57.71M	9	6809	rm

Table 4. Dataset Description Table. Graph types are: r: real-world, g: generated, s: scale-free, and m: mesh-like. All datasets have been converted to undirected graphs. Self-loops and duplicated edges are removed.

Algorithm	Galois	BGL	PowerGraph	Medusa
BFS	8.812	—	—	22.49
SSSP	2.532	99.99	8.058	2.158*
BC	1.57	32.12	—	—
PageRank	2.16	—	17.73	2.463*
CC	1.745	341.1	182.7	—

Table 5. Geometric-mean runtime speedups of Gunrock on the datasets from Table 4 over frameworks not in Table 6. *Due to Medusa's memory limitations [86], its SSSP and PageRank comparisons were measured on four smaller datasets from Medusa's original paper.

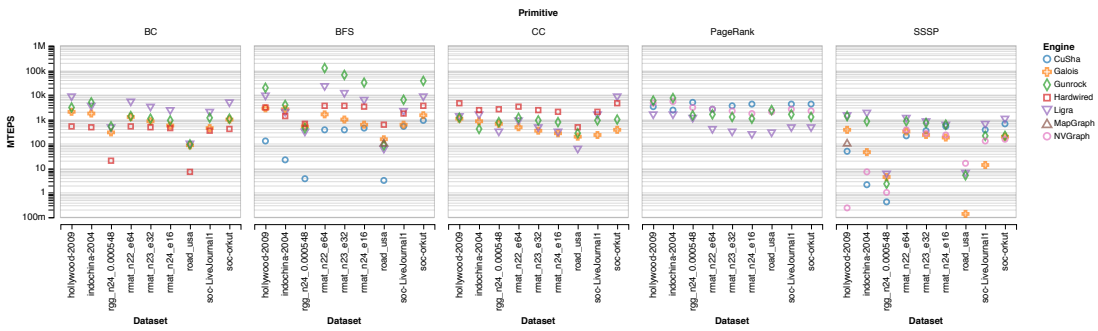


Fig. 16. Performance in MTEPS for Gunrock vs. six other graph processing libraries/hardwired algorithms on nine different graph inputs. Data is from Table 6.

7.1 Performance Summary

Tables 5 and 6, and Figures 15, 16, and 17, compare Gunrock's performance against several other graph libraries and hardwired GPU implementations. In general, Gunrock's performance on BFS-based primitives (BFS, BC, and SSSP) shows comparatively better results when compared to other graph libraries on seven scale-free graphs (soc-orkut, soc-lj, h09, i04, and rmats), than on two small-degree large-diameter graphs (rgg and roadnet). The primary reason is our load-balancing strategy during traversal and particularly our emphasis on good performance for highly irregular

		Runtime (ms) [lower is better]					Edge throughput (MTEPS) [higher is better]				
Alg.	Dataset	CuSha	MapGraph	Hardwired GPU	Ligra	Gunrock	CuSha	MapGraph	Hardwired GPU	Ligra	Gunrock
BFS	soc-ork	244.9	OOM	25.81	26.1	5.573	868.3	OOM	12360	8149	38165
	soc-lj	263.6	116.5	36.29	42.4	14.05	519.5	1176	5661	2021	6097
	h09	855.2	63.77	11.37	12.8	5.835	131.8	1766	14866	8798	19299
	i04	17609	OOM	67.7	157	77.21	22.45	OOM	8491	1899	3861
	rmat-22	1354	OOM	41.81	22.6	3.943	369.1	OOM	17930	21374	122516
	rmat-23	1423	OOM	59.71	45.6	7.997	362.7	OOM	12971	11089	63227
	rmat-24	1234	OOM	270.6	89.6	16.74	426.4	OOM	—	5800	31042
	rgg	68202	OOM	138.6	918	593.9	3.887	OOM	2868	288.8	466.4
	roadnet	36194	763.4	141	978	676.2	3.189	151	1228	59.01	85.34
SSSP	soc-ork	—	OOM	807.2	595	981.6	—	OOM	770.6	—	216.7
	soc-lj	—	—	369	368	393.2	—	—	1039	—	217.9
	h09	—	1069	143.8	164	83.2	—	—	1427	—	1354
	i04	—	OOM	—	397	371.8	—	OOM	—	—	801.7
	rmat-22	—	OOM	—	774	583.9	—	OOM	—	—	827.3
	rmat-23	—	OOM	—	1110	739.1	—	OOM	—	—	684.1
	rmat-24	—	OOM	—	1560	884.5	—	OOM	—	—	587.5
	rgg	—	OOM	—	80800	115554	—	OOM	—	—	2.294
	roadnet	—	OOM	4860	29200	11037	—	OOM	25.87	—	5.229
BC	soc-ork	—	—	1029	186	397.8	—	—	413.3	4574	1069
	soc-lj	—	—	492.8	180	152.7	—	—	347.7	1904	1122
	h09	—	—	441.3	59	73.36	—	—	510.3	7635	3070
	i04	—	—	1270	362	117	—	—	469	3294	5096
	rmat-22	—	—	1867	399	742.6	—	—	517.5	4840	1301
	rmat-23	—	—	2102	646	964.4	—	—	481.3	3130	1049
	rmat-24	—	—	2415	978	1153	—	—	430.3	2124	901.2
	rgg	—	—	26938	2510	1023	—	—	19.69	422.5	518.4
	roadnet	—	—	15803	2490	1204	—	—	7.303	92.7	95.85
PageRank	soc-ork	52.54	OOM	—	476	173.1					
	soc-lj	33.61	250.7	—	200	54.1					
	h09	34.71	93.48	—	77.4	20.05					
	i04	164.6	OOM	—	210	41.59					
	rmat-22	188.5	OOM	—	1250	304.5					
	rmat-23	147	OOM	—	1770	397.2					
	rmat-24	128	OOM	—	2180	493.2					
	rgg	53.93	OOM	—	247	181.3					
	roadnet	—	123.2	—	209	24.11					
CC	soc-ork	—	OOM	46.97	260	211.7					
	soc-lj	—	OOM	43.51	184	93.27					
	h09	—	547.1	24.63	90.8	96.15					
	i04	—	OOM	130.3	315	773.7					
	rmat-22	—	OOM	149.4	563	429.8					
	rmat-23	—	OOM	212	1140	574.3					
	rmat-24	—	OOM	256.7	1730	664.1					
	rgg	—	OOM	103.9	6000	355.2					
	roadnet	—	OOM	124.9	50500	208.9					

Table 6. Gunrock's performance comparison (runtime and edge throughput) with other graph libraries (CuSha, MapGraph, Ligra) and hardwired GPU implementations on a Tesla K40c GPU . All PageRank times are normalized to one iteration. Hardwired GPU implementations for each primitive are Enterprise (BFS) [47], delta-stepping SSSP [17], gpu_BC (BC) [67], and conn (CC) [75]. OOM means out-of-memory. A missing data entry means either there is a runtime error, or the specific primitive for that library is not available.

graphs. As well, graphs with uniformly low degree expose less parallelism and would tend to show smaller gains in comparison to CPU-based methods. Table 7 shows Gunrock's scalability. In general, runtimes scale roughly linearly with graph size for BFS, but primitives with heavy use of atomics on the frontier (e.g., BC, SSSP, and PR) show increased atomic contention within the frontier as graph sizes increase and thus do not scale ideally. For CC, the reason for non-ideal scalability is mainly the increase of race conditions when multiple edges try to hook their source node to a common destination node, which happens more often when power-law graphs get bigger and degree numbers become more unevenly distributed.

7.2 vs. CPU Graph Libraries

We compare Gunrock's performance with four CPU graph libraries: the Boost Graph Library (BGL) [73], one of the highest-performing CPU single-threaded graph libraries [51]; PowerGraph,

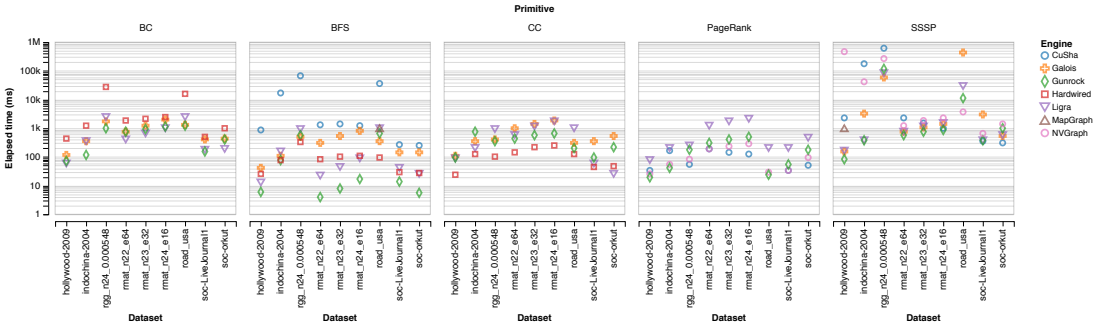


Fig. 17. Performance in runtime for Gunrock vs. six other graph processing libraries/hardwired algorithms on nine different graph inputs. Data is from Table 6.

a popular distributed graph library [24]; and Ligra [71] and Galois [57, 62], two of the highest-performing multi-core shared-memory graph libraries. Against both BGL and PowerGraph, The speedup of Gunrock on average on all primitives ranges from 6x to 337x. Compared to Ligra, Gunrock's performance is generally comparable on most tested graph primitives; note Ligra results are on a 2-CPU machine. The performance inconsistency for SSSP vs. Ligra is due to comparing our delta-stepping-based method [56] with Ligra's Bellman-Ford algorithm. Our SSSP's edge throughput is smaller than BFS but similar to BC because of similar computations (atomicMin vs. atomicAdd) and a larger number of iterations for convergence. The performance inconsistency for BC vs. Ligra on four scale-free graphs is because Ligra applies pull-based traversal on BC while Gunrock has not yet done so. Compared to Galois, Gunrock shows more performance advantage on traversal-based graph primitives (BFS, SSSP, and BC) and less on PageRank and CC, due to their dense computation and more regular frontier structures.

7.3 vs. Hardwired GPU Implementations and GPU Libraries

Compared to hardwired GPU implementations, depending on the dataset, Gunrock's performance is comparable or better on BFS, BC, and SSSP. For CC, Gunrock is 5x slower (geometric mean) than the hardwired GPU implementation due to irregular control flow because each Gunrock iteration starts with full edge lists in both hooking and pointer-jumping phases. The alternative is extra steps to perform additional data reorganization. This tradeoff is not typical of our other primitives. While still achieving high performance, Gunrock's application code is smaller in size and clearer in logic compared to other GPU graph libraries.

Gunrock's problem class (that defines problem data used for the graph algorithm) and kernel enactor are both template-based C++ code; Gunrock's functor code that specifies per-node or per-edge computation is C-like device code without any CUDA-specific keywords. Writing Gunrock code may require parallel programming concepts (e.g., atomics) but neither details of low-level GPU programming nor optimization knowledge.[2]

Gunrock compares favorably to existing GPU graph libraries.

vs. MapGraph MapGraph is faster than Medusa on all but one test [20] and Gunrock is faster than MapGraph on all tests: the geometric mean of Gunrock's speedups over MapGraph on BFS, SSSP, PageRank, and CC are 4.679, 12.85, 3.076, and 5.69, respectively.

[2]We believe this assertion is true given our experience with other GPU libraries when preparing this evaluation section, but freely acknowledge this is nearly impossible to quantify. We invite readers to peruse our annotated code for BFS and SALSA at http://gunrock.github.io/gunrock/doc/annotated_primitives/annotated_primitives.html.

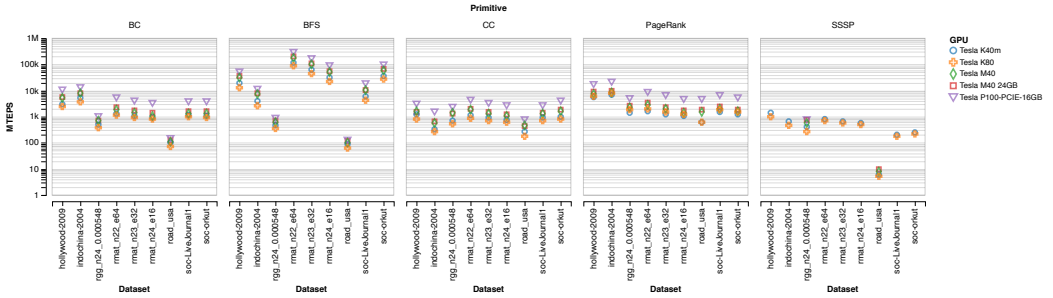


Fig. 18. Gunrock's performance on different GPU devices. The Tesla M40 24GB is attached to a Haswell CPU and has a higher boost clock (1.3 GHz). The Tesla M40 is attached to an Ivy Bridge CPU with a boost clock of 1.1 GHz.

vs. CuSha Gunrock outperforms CuSha on BFS and SSSP. For PageRank, Gunrock achieves comparable performance with no preprocessing when compared to CuSha's G-Shard data preprocessing, which serves as the main load-balancing module in CuSha.

vs. Totem The 1-GPU Gunrock implementation has 1.83x more MTEPS (4731 vs. 2590) on direction-optimized BFS on the soc-LiveJournal dataset (a smaller scale-free graph in their test set) than the 2-CPU, 2-GPU configuration of Totem [66].

vs. nvGRAPH For SSSP, nvGRAPH is faster than Gunrock on the roadnet dataset, but slower on the other datasets. Gunrock in general performs better on scale-free graphs than it does on regular graphs. For PageRank, nvGRAPH is faster than Gunrock on six datasets and slower on three (h04, i09, and roadnet). nvGRAPH is closed-source and thus a detailed comparison is infeasible.

All three GPU BFS-based high-level-programming-model efforts (Medusa, MapGraph, and Gunrock) adopt load-balancing strategies from Merrill et al.'s BFS [55]. While we would thus expect Gunrock to show similar performance on BFS-based graph primitives to these other frameworks, we attribute our performance advantage to two reasons: 1) our improvements to efficient and load-balanced traversal that are integrated into the Gunrock core, and 2) a more powerful, GPU-specific programming model that allows more efficient high-level graph implementations. 1) is also the reason that Gunrock implementations can compete with hardwired implementations; we believe Gunrock's load-balancing and work distribution strategies are at least as good as if not better than the hardwired primitives we compare against. Gunrock's memory footprint is at the same level as Medusa and better than MapGraph (note the OOM test cases for MapGraph in Table 6). Our data footprint is $\alpha|E| + \beta|V|$ for current graph primitives, where $|E|$ is the number of edges, $|V|$ is the number of nodes, and α and β are both integers where α is usually 1 and at most 3 (for BC) and β is between 2 to 8.

Figure 18 shows the performance of Gunrock's v0.4 release on four different GPUs: Tesla K40m, Tesla K80, Tesla M40, and Tesla P100. Programs are compiled by nvcc (8.0.44) with -O3 flag and GPU SM versions according to the actual hardware. Across different GPUs, Gunrock's performance generally scales with memory bandwidth, with the newest Tesla P100 GPU demonstrating the best performance.

7.4 Optimization Strategies Performance Analysis

Figures 19 and 20 show how different optimization strategy combinations and different workload mapping strategies affect the performance of graph traversal.

Dataset	Runtime (ms)					Edge throughput (MTEPS)		
	BFS	BC	SSSP	CC	PR	BFS	BC	SSSP
kron_g500-logn18 ($v = 2^{18}, e = 14.5$M)	1.319	12.4	13.02	9.673	3.061	10993	2339	1113
kron_g500-logn19 ($v = 2^{19}, e = 29.7$M)	1.16	26.93	26.59	20.41	6.98	25595	2206	1117
kron_g500-logn20 ($v = 2^{20}, e = 60.6$M)	2.355	67.57	56.22	43.36	19.63	25723	1793	1078
kron_g500-logn21 ($v = 2^{21}, e = 123.2$M)	3.279	164.9	126.3	97.64	60.14	37582	1494	975.2
kron_g500-logn22 ($v = 2^{22}, e = 249.9$M)	5.577	400.3	305.8	234	163.9	44808	1248	817.1
kron_g500-logn23 ($v = 2^{23}, e = 505.6$M)	10.74	900	703.3	539.5	397.7	47077	1124	719

Table 7. Scalability of 5 Gunrock primitives (runtime and edges traversed per second) on a single GPU on five differently-sized synthetically-generated Kronecker graphs with similar scale-free structure.

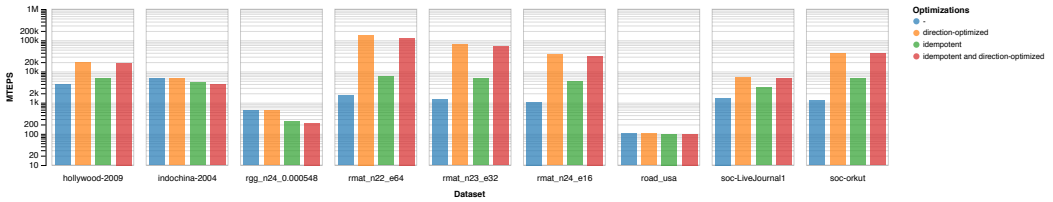


Fig. 19. Gunrock's performance with different combinations of idempotence and direction-optimized traversal.

Without losing generality, for our tests on different optimization strategies, we use BFS and fix the workload mapping strategy to LB_CULL so that we can focus on the impact of different optimization strategies. Our two key optimizations—idempotence and direction-optimized traversal—both show performance increases compared to the baseline LB_CULL traversal, except for rgg and roadnet, for which idempotence does not increase performance because the inflated frontiers cancel out the performance increase for avoiding atomics. Also, our experiment shows that when using LB_CULL for advance, enabling both direction-optimized and idempotence always yields worse performance than with only direction-optimized enabled. The reason behind this is that enabling idempotence operation in direction-optimized traversal iteration causes additional global data accesses to a visited-status bitmask array. We also note that for graphs with very small degree standard deviation (< 5), using idempotence will actually hurt the performance. The reason is for these graphs, the number of common neighbors is so small that avoiding atomic operations will not provide much performance gain, but using idempotence introduces an additional pass of filter heuristics, and the cost of that pass outweighs the performance gain from avoiding atomic operations.

Our tests on different workload mapping strategies (traversal mode) shows that LB_CULL constantly outperforms other two strategies on these 9 datasets. Its better performance compared to TWC is due to its switching between load-balance over the input frontier and load- balance over the output frontier according to the input frontier size (Section 5.1.3). Its better performance compared to LB is due to its kernel fusion implementation, which reduces the kernel launch overhead and also some additional data movement between operators. However, without a more thorough performance characterization, we cannot make the conclusion that LB_CULL will always have better performance. For instance, in our SSSP tests on two mesh-like graphs with large diameters and small average degrees, TWC shows better performance. In general, we currently predict which strategies will be most beneficial based only on the degree distribution; many application scenarios may allow pre-computation of this distribution and thus we can choose the optimal strategies before we begin computation.

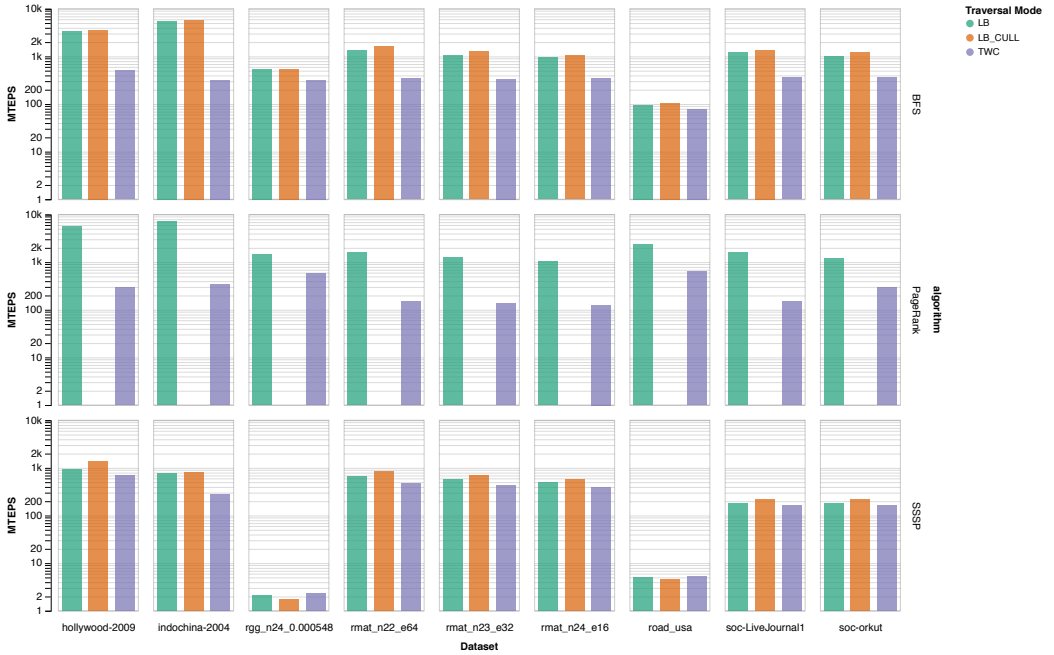


Fig. 20. BFS, SSSP, and PR's performance in Gunrock using three different workload mapping strategies: LB, LB_CULL, and TWC. For PR, since its current operation order in the enactor is an advance followed by a filter, so LB_CULL is not valid.

Figure 21 shows performance as a function of the direction-optimized parameters *do_a* and *do_b* (Section 5.1.4). In general, R-Mat graphs, social networks, and web graphs show different patterns due to their different degree distributions. But every graph has a rectangular region where performance figures show a discontinuity from the region outside that rectangle. This is because our two direction-optimized parameters indirectly affect the number of iterations of push-based traversal and pull-based traversal. The number of iterations is discrete and thus forms the rectangular region. As shown in figure 21, with one round of BFS execution that starts with push-based traversal, increasing *do_a* from a small to a large value speeds up the switch from push-based to pull-based traversal, and thus always yield better performance at first. However, when *do_a* is too large, it causes pull-based traversal to start too early and performance drops because usually in early iterations, small-sized frontiers show better performance on push-based traversal than pull-based traversal. Parameter *do_b* controls when to switch back from pull-based to push-based traversal. On most graphs, regardless of whether they are scale-free or mesh-like graphs, keeping a smaller *do_b* so that the switch from pull-based to push-based traversal never happens would help us achieve better performance. However, this is not always true: in indochina-2004, the location of its rectangular region is at the lower right, which means either keeping both a larger *do_a* and *do_b* will yield better performance on this dataset, or the parameter range we have chosen is not wide enough to show the complete pattern for this dataset. At any rate, it is clear that the same parameters do not deliver the best performance for all datasets.

Figures 22 and 23 show Gunrock's per-iteration advance performance as a function of input and output frontier size. The input frontier figure is noisier because for some scale-free datasets, a small input frontier can generate a very large output frontier, causing an outlier in the scatter plot. In

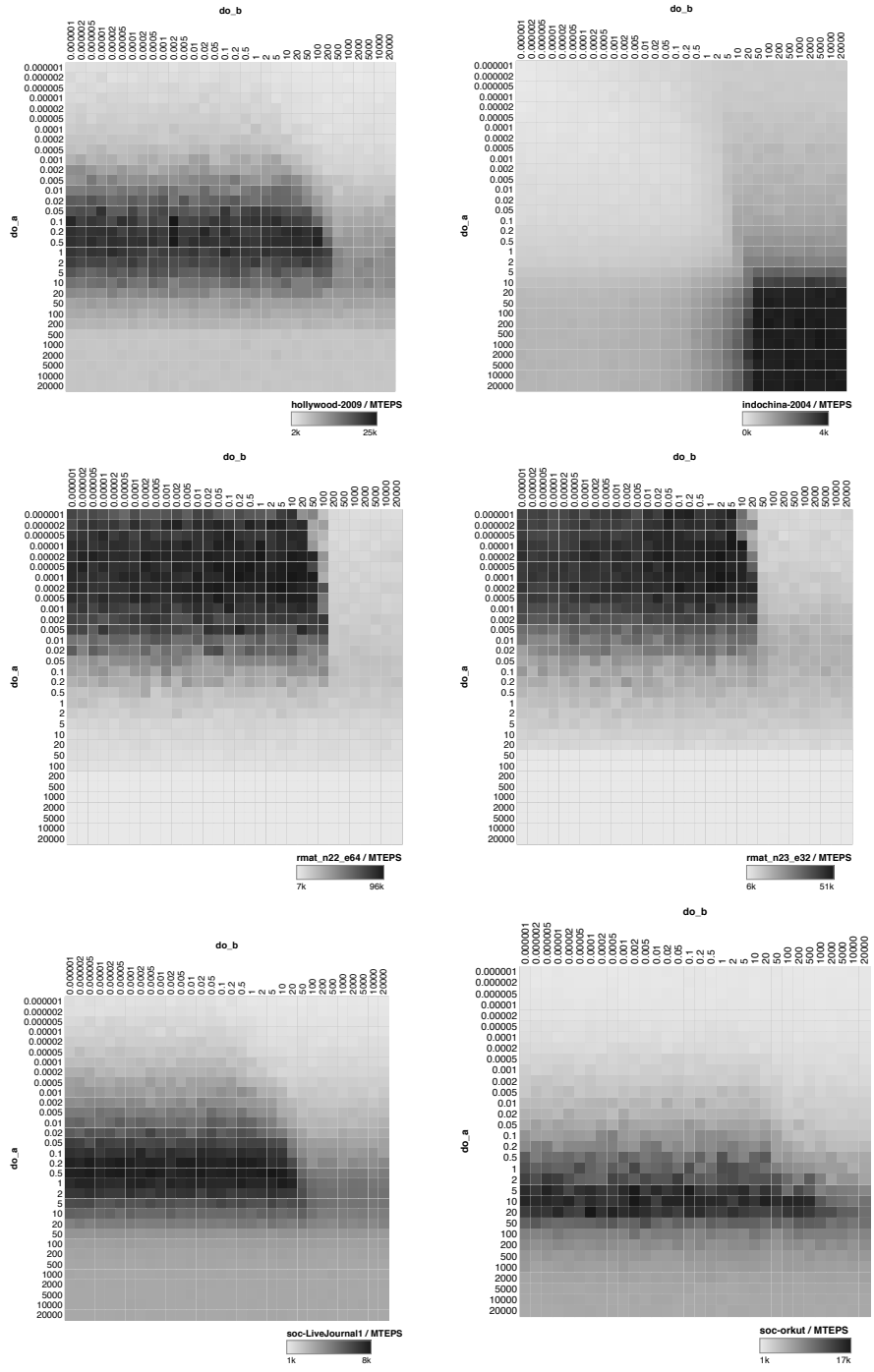


Fig. 21. Heatmaps show how Gunrock's two direction-optimized parameters, **do_a**, and **do_b**, affect BFS performance. Each square represents the average throughput of 25 BFS runs, each starting at a random node. Darker color means higher TEPS . Datasets on the top row are hollywood-2009 and indochina-2004; on the second row, rmat-n22 and rmat-n23; and on the bottom row, soc-livejournal and soc-orkut.

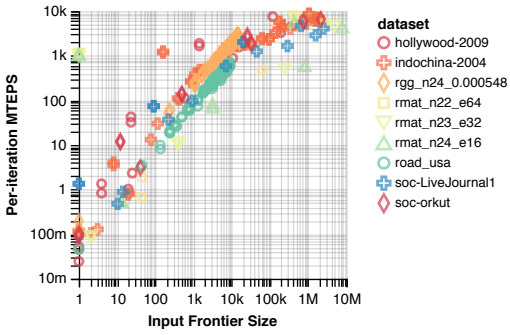


Fig. 22. Per-iteration advance performance (in MTEPS) vs. input frontier size.

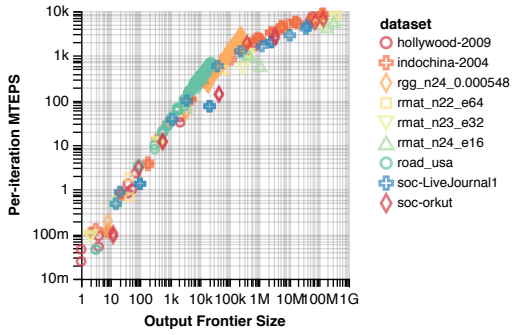


Fig. 23. Per-iteration advance performance (in MTEPS) vs. output frontier size.

Table 8. Experimental datasets for GPU Who-To-Follow algorithm.

Dataset	Vertices	Edges
wiki-Vote	7.1k	103.7k
twitter-SNAP	81.3k	2.4M
gplus-SNAP	107.6k	30.5M
twitter09	30.7M	680M

these plots, for the two roadnet graphs (roadnet and rgg), we are using our TWC strategy, while the other datasets use LB_CULL. This creates two different types of curve. For datasets that use the LB_CULL strategy, the input frontier performance curve reaches a horizontal asymptote when the input frontier size is larger than 1 million, and the output frontier performance curve keeps growing even for a billion-element output frontier. However, the performance increase for datasets that use LB_CULL slows down when the output frontier size is larger than one million. For the two datasets that use the TWC strategy, both the input frontier performance curve and the output frontier performance curve are linear, and present better performance than datasets using LB_CULL when the output frontier size is larger than 1 million. The results of this experiment demonstrate that in general, to achieve good performance, Gunrock needs a relatively large frontier to fully utilize the computation resource of a GPU.

7.5 GPU Who-To-Follow Performance Analysis

We characterize our who-to-follow implementation (based on Goel et al.'s algorithm [23]) with different datasets (Tables 8). Table 9 shows the runtimes of this primitive on these datasets. Runtimes are for GPU computation only and do not include CPU-GPU transfer time. The wiki-Vote dataset is a real social graph dataset that contains voting data for Wikipedia administrators; all other datasets are follow graphs from Twitter and Google Plus [43, 45]. Twitter09 contains the complete Twitter follow graph as of 2009; we extract 75% of its user size and 50% of its social relation edge size to form a partial graph that can fit into the GPU's memory.

7.5.1 Scalability. In order to test the scalability of our WTF-on-GPU recommendation system, we ran WTF on six differently-sized subsets of the twitter09 dataset. The results are shown in Figure 24. We see that the implementation scales sub-linearly with increasing graph size. As we

Table 9. GPU WTF runtimes for different graph sizes.

Time (ms)	wiki-Vote	twitter	gplus	twitter09
PPR	0.45	0.84	4.74	832.69
CoT	0.54	1.28	2.11	51.61
Money	2.70	5.16	18.56	158.37
Total	4.37	8.36	26.57	1044.99

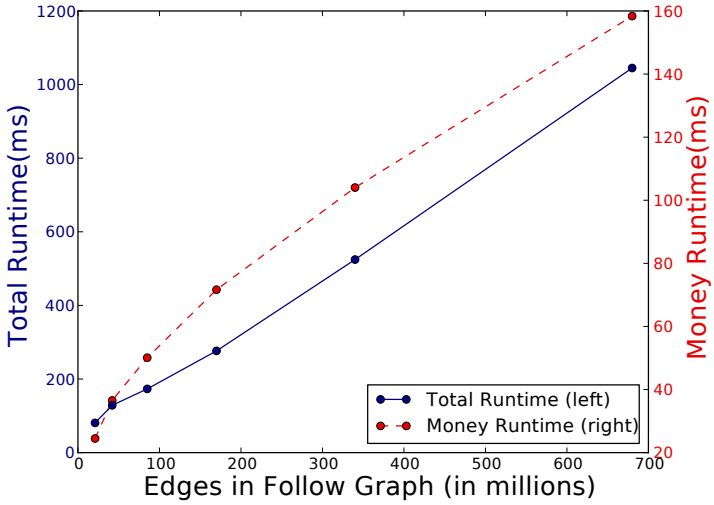


Fig. 24. Scalability graph of our GPU recommendation system.

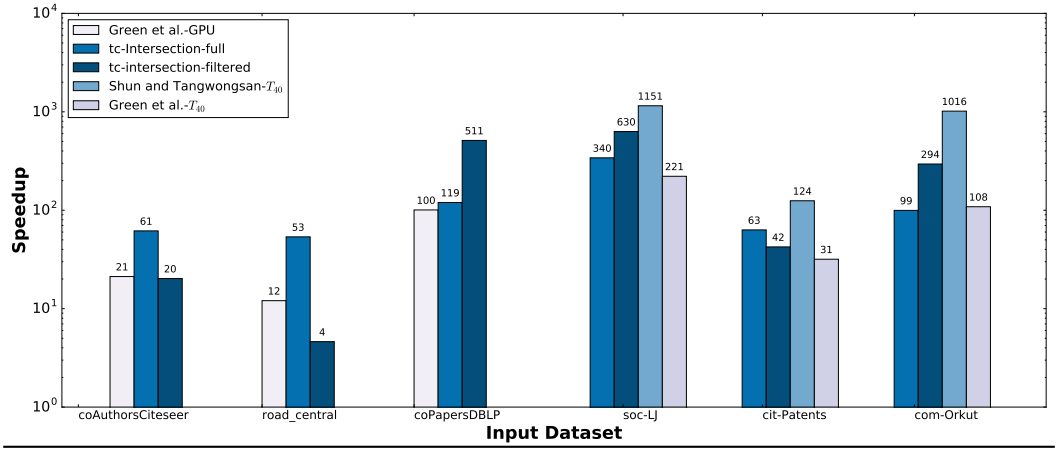
double the graph size, the total runtime increases by an average of 1.684x, and the runtime for Money increases by an average of 1.454x. The reason lies in our work-efficient parallel implementation. By doing per-vertex computation exactly once and visiting each edge exactly once, our parallel algorithm performs linear $O(m + n)$ work. The reason that we have better scalability for the Money algorithm is that although we are doubling the graph size each time, we always prune the graph with a fixed number of nodes that we computed via personalized PageRank (PPR) and we call "Circle of Trust" (CoT). In our implementation we set the size of CoT to 1000 to match the original Who-To-Follow algorithm.

7.5.2 Comparison to Cassovary. We chose to use the Cassovary graph library for our CPU performance comparison. The results of this comparison are shown in Table 10. Cassovary is a graph library developed at Twitter. It was the library Twitter used in their first WTF implementation [31].

We achieve speedups of up to 1000x over Cassovary for the Google Plus graph, and a speedup of 14x for the 2009 Twitter graph, which is the most representative dataset for the WTF application. One difference between the GPU algorithm and the Cassovary algorithm is that we used the SALSA function that comes with the Cassovary library, instead of using Twitter's Money algorithm for the final step of the algorithm. Both are ranking algorithms based on link analysis of bipartite graphs,

Table 10. GPU WTF runtimes comparison to Cassovary (C).

	wiki-Vote		twitter		gplus		twitter09	
Step (runtime)	C	GPU	C	GPU	C	GPU	C	GPU
PPR (ms)	418	0.45	480	0.84	463	4.74	884	832.69
CoT (ms)	262	0.54	2173	1.28	25616	2.11	2192	51.61
Money (ms)	357	2.70	543	5.16	2023	18.56	11216	158.37
Total (ms)	1037	4.37	3196	8.36	28102	26.57	14292	1044.99
Speedup		235.7		380.5		1056.5		13.7



Dataset	coAuthorsCiteseer	road_central	coPapersDBLP	soc-LJ	cit-Patents	com-Orkut
time (s)	0.28	3.254	100.7	564.3	9.856	1951

Fig. 25. Execution-time speedup (top figure) for our Gunrock TC implementations ("tc-intersection-full" and "tc-intersection-filtered"), Green et al.'s GPU implementation [28] ("Green et al.-GPU"), Shun and Tangwongsan's 40-core CPU implementation [72] ("Shun and Tangwongsan-T_{40}") and Green et al.'s 40-core CPU implementation [27] ("Green et al.-T_{40}"). All are normalized to a baseline CPU implementation [68] on six different datasets. Baseline runtime (in seconds) is given in the table.

and in the original Who-To-Follow paper [31], Gupta et al. use a form of SALSA for this step, so this is reasonable for a comparison.

7.6 GPU Triangle Counting Performance Analysis

We compare the performance of our GPU TC to three different exact triangle counting methods: Green et al.'s state-of-the-art GPU implementation [28] that runs on an NVIDIA K40c GPU, Green et al.'s multicore CPU implementation [27], and Shun and Tangwongsan's multicore CPU implementation [72]. Both of the state-of-the-art CPU implementations are tested on a 40-core shared memory system with two-way hyper-threading; their results are from their publications. Our CPU baseline is an implementation based on the *forward* algorithm by Schank and Wagner [68].

In general, Gunrock's TC implementation shows better performance than the state-of-the-art GPU implementations because of our key optimizations on workload reduction and GPU resource

utilization (figure 25). It achieves comparable performance to the fastest shared-memory CPU TC implementation. Gunrock's TC implementation with only the simple per-thread batch set intersection kernel achieves a 2.51× and 4.03× (geometric-mean) speedup as compared to Green et al.'s CPU [27] and GPU [28] implementations respectively. We believe our speedup is the result of two aspects of our implementation: 1) using filtering in edge list generation, and reforming the induced subgraph with only the edges not filtered, effectively reducing five-sixths of the workload; and 2) dynamic grouping that helps maintain a high GPU resource utilization. In practice we observe that for scale-free graphs, the last step of optimization to reform the induced subgraph show a constant speedup over our intersection method without this step. However, for road networks and some small scale-free graphs, the overhead of using segmented reduction will cause a performance drop. We expect further performance gains from future tuning of launch settings for the large-neighbor-list intersection kernel, which we do not believe is quite optimal yet. These optimizations are all transparent to our data-centric programming model and apply to our segmented intersection graph operator, which we hope to use in other graph primitives in the future.

8 CONCLUSION

Our work on the data-centric programming model for graph processing on the GPU has enabled us to build a highly programmable, high-performance graph analytics system. It also opens up various interesting yet challenging research opportunities.

8.1 Limitations

Single-GPU Our current data-centric programming model is designed and optimized for a single-GPU architecture. We have extended our implementation to multiple GPUs on one node [59], but have not yet addressed larger machines [61].

Fits-in-memory Our current single-GPU implementation does not support computation on graphs that exceed the size of a GPU's global memory.

Static Graph Our current programming model targets static graphs; we have not yet considered streaming graphs or graphs with dynamically changing topology.

Limited Compiler Optimizations Our current implementation does not support extensive compiler optimizations such as kernel fusion and AST (abstract syntax tree) generation [58].

8.2 Future Work

Moving forward, there are several aspects of Gunrock we could improve, including architecture, execution model, meta-linguistic abstraction, performance characterization, core graph operators, new graph primitives, graph datatype, and usability.

8.2.1 Architecture and Execution Model. To enable large-scale graph analysis where the data do not fit in single GPU device memory, we see three ways to scale:

Scale-Out Gunrock is designed to serve as a standalone framework that can be integrated into a multi-GPU single-node graph processing system [59]. When we built this system, we identified two interesting research problems: 1) the impact of different partitioning methods on the performance, and 2) tradeoffs between computation and communication for inter-GPU data exchange. We hope to extend this framework to multiple nodes and explore these two problems more completely, and also investigate dynamic repartitioning for load balancing, vertex cut, and heterogeneous processing for large-diameter graphs.

Scale-Up CPU-GPU memory bandwidth is still limited compared to intra-GPU and inter-GPU bandwidth. Thus we need to identify the situations where out-of-core approaches are necessary and/or useful. The current Gunrock framework has not been designed to support

out-of-core directly, but several other projects [44, 70] offer interesting conclusions that would influence Gunrock's development in terms of graph data representation, partitioning, and communication reduction to make it suitable for out-of-core computation.

Streaming Recent work maps the streaming model to GPUs by partitioning graphs into edge streams and processing graph analytics in parallel [64, 69]. Supporting streaming requires more significant changes than out-of-core and raises interesting research questions such as how to partition the graph and when to synchronize. At its core, we expect Gunrock's data-centric programming model maps naturally to streaming process where a frontier is an edge stream.

We see two interesting future directions for Gunrock's execution model:

Asynchronous The work of Wang et al. [80] views an asynchronous execution model in BSP as the relaxation of two synchrony properties: 1) isolation, meaning within an iteration, newly generated messages from other vertices can not be seen; and 2) consistency, meaning each thread waits before it proceeds to the next iteration until all other threads have finished the process for the same iteration. To enable asynchronous execution on current Gunrock, we could 1) use a priority queue in a single-node GPU, allowing the merging of multiple remote frontiers in multi-node GPU, or adding multi-pass micro iterations within one iteration to relax the consistency property; or 2) allow per-vertex/per-edge operations to propagate results to others to relax the isolation property. In terms of the impact to our data-centric programming model, asynchronous execution could significantly accelerate convergence in primitives such as SSSP, CC, PR, and LP, by intelligently ordering vertex updates and incorporating the most recent updates.

Higher-level task parallelism Higher-level task parallelism can bring more parallelism for several graph primitives. Preliminary work [48, 53] in this direction executes multiple BFS passes simultaneously on GPUs. Applications that benefit from adding this additional level of parallelism at the streaming multiprocessor level include all-pairs shortest paths and BC. Usually such higher-level task parallelism requires keeping multiple active frontiers and specifying different groups of blocks to handle graph workloads on different active frontiers. Gunrock's data-centric abstraction can be used to implement this with some additional modifications to frontier management and workload mapping.

8.2.2 Meta-Linguistic Abstraction.

Gunrock as a Backend Gunrock currently offers C/C++-friendly interfaces that make Gunrock callable from Python, Julia, and other high-level programming languages. The future work along this path is to identify and implement support for Gunrock as a back end to a higher-level graph analytics framework (such as TinkerPop or NetworkX) or create our own graph query/analysis DSL on top of Gunrock.

Add More Backends to Gunrock Gunrock's current implementation is not the only way to implement our data-centric programming model. Another interesting research direction is to make other back ends support our data-centric programming model. Candidates include GraphBLAS (a matrix-based graph processing framework) [41], VertexAPI2 (a GAS-style graph processing framework) [19], and the EmptyHeaded Engine [1] (a Boolean algebra set-operation-based graph query library).

8.2.3 Core Graph Operators. Every graph analytics framework implements its abstraction as a set of supported operators on a graph. Which operators to choose may be a function of the hardware architecture and programming language. What is the right set of graph operators that Gunrock should support? To Gunrock's original three operators—advance, filter, and compute—we have

added segmented intersection. We see several other operators as candidates for future Gunrock implementation:

Neighborhood Reduction Neighborhood reduction will visit the neighbor list of each item in the input frontier and perform a user-specified reduction over each neighbor list. It shares many similarities to our advance operator. An efficient neighborhood reduction operator would accelerate several graph primitives that need to reduce over neighborhoods, such as maximal independent set, coloring, and PageRank.

Priority Queue Using multi-split [2], we can create priority queues with more than one priority level. This can be applied to the delta-stepping SSSP algorithm and can also serve as an alternative frontier manipulation method in an asynchronous execution model.

Sampling A sampling operator can be viewed as an extension to the standard filter. Currently its applications include approximated BC, TC, and more approximated graph primitives.

Forming Supervertex In our current minimum-spanning-tree primitive, we have implemented a supervertex-forming phase using a series of filter, advance, sort, and prefix-sum. This could be integrated into a new operator. Note that currently, we form supervertices by rebuilding a new graph that contains supervertices. This can be used in hierarchical graph algorithms such as clustering and community detection.

An alternative direction would be to offer a set of lower-level data-parallel primitives including both general primitives such as prefix-sum and reduction, as well as graph-specific primitives such as load-balanced search (for evenly distributing edge expanding workload), and use them to assemble our higher-level graph operators.

Which of these two approaches—implementing many graph operators and specific optimizations for each operator, and building up graph operators on top of lower-level data-parallel primitives—we use in a GPU graph analytics programming model will affect both the programmability and performance of the system. In Gunrock, we use a mixed model where we share common components such as load-balanced search, prefix-sum, and compact for advance, filter and neighborhood reduction, while we also include specialized optimizations for advance, filter, and segmented intersection. An interesting future research direction is to think of the graph operator abstraction at a more meta level, and try to build up a hierarchical framework for designing graph operators that includes not only graph traversal and computation building blocks such as all five graph operators we have in Gunrock today, but also memory- and workload-optimization building blocks that can be plugged into and replaced in our higher-level graph operator implementation.

8.2.4 New Graph Primitives. Our current set of graph primitives are mostly textbook-style algorithms. One future work is to expand this list to more complex graph primitives spanning from network analysis, machine learning, data mining, and graph queries.

Graph Coloring and Maximal Independent Set Efficient graph matching and coloring tasks [15] can be computed by Gunrock more efficiently than previous work. Maximal independent set (MIS) algorithms follow the same path. Both primitives can be implemented using Gunrock's current operator set with new operators such as neighborhood reduction and multi-level priority queue potentially improving their performance. These primitives can serve as the building blocks for asynchrony and task-level parallelism.

Strongly Connected Components A directed graph is strongly connected if any node can reach any other node. This algorithm is traditionally a problem related to depth-first search, which is considered unsuitable for GPUs. However, Slota et al.'s work [74] improved on Barnat et al.'s DFS-based work [3] and avoids DFS by combining BFS and graph coloring.

More Traversal-based Algorithms To make use of our efficient graph traversal operators, we can modify our BFS to implement several other similar primitives: 1) *st*-connectivity, which simultaneously processes two BFS paths from s and t; 2) A* search as a BFS-based path finding and search algorithm; 3) Push-Relabel algorithm as a BFS-based maximum flow algorithm [35]; 4) {reverse} Cuthill-McKee algorithm, a sparse matrix bandwidth reduction algorithm based on BFS [16]; 5) belief propagation, a sum-product message passing algorithm based on BFS; and finally 6) radii estimation, a k-sample BFS-based algorithm for estimating the maximum radius of the graph.

Subgraph Isomorphism Most optimized subgraph matching algorithms on CPUs [32] are based on backtracking strategies, which follow a recursive routine and cannot efficiently be adapted to GPUs. Recent subgraph matching on large graphs on the GPU [77] and distributed memory cloud [9, 76], both of which use graph traversal extensively, and could fit nicely and turn out to work more efficiently in Gunrock's framework. However, these methods are memory-bound which suggests a focus on more effective filtering and joining methods. Extensions to our current segmented intersection operator more flexible input format support for query graphs will potentially make subgraph matching in Gunrock perform better.

***k*-SAT** k-SAT is a type of Boolean satisfiability problem that can be represented as an iterative updating process on a bipartite graph. Combining the sampling operator and our efficient traversal operator, it potentially fits nicely into Gunrock.

SGD and MCMC Both Stochastic Gradient Descent in matrix completion and Markov Chain Monte Carlo have large amounts of parallelism. The former can be presented as either an iterative bipartite graph traversal-updating algorithm [39] or a conflict-free subgraph parallel updating algorithm. In order to decrease the number of conflict updates and increase the data re-usage rate, the incomplete matrix is divided into sub-matrices where graph coloring can help identify the conflict-free elements in those small sub-matrices. Graph coloring can take advantage of Gunrock's efficient traversal operator to achieve better performance. MCMC's iterative dense update makes it a good candidate for matrix-based graph algorithms. However, Gunrock operators can also represent this problem.

8.2.5 New Graph Datatypes. All graph processing systems on the GPU use CSR and edge lists internally. We see significant opportunity in designing suitable graph datatypes and data structures to enable graph primitives on mutable graphs and matrix-typed graph primitives.

Mutable Graphs The meaning of mutable is twofold: mutable by primitive, and mutable by input data. *Mutable by primitive* means that the graph primitive changes the graph structure, and includes primitives such as MST, community detection, mesh refinement, and Karger's mincut. The operations related include simple ones such as adding/removing nodes/edges, and more complex ones such as forming supervertices. Currently there is no good solution for handling general graph mutations on GPUs with efficiency. *Mutable by input data* means that we process our algorithms on input datasets that change over time (so we would like to both incrementally update the graph data structure and incrementally update a solution given the incremental change in the data structure). We need to provide either approximated results or the capability of doing incremental computation.

Adjacency Matrix Form Matrix-based graph algorithms are also widely used for graph processing such as BFS and PageRank. Our sparse-matrix sparse-vector operator and its application to BFS [85] has shed light on more applications in matrix form such as MIS, PageRank, SSSP, and several spectral methods, which are used in algorithms like collaborative filtering.

Optimized Graph Topology Format The work of Wu et al. [83] shows how data reorganization can help with memory uncoalescing. However, for graph analytics, we need to design better strategies since the memory access indices (node IDs in the frontier) change dynamically every iteration. Both CuSha [42] and Graphicionado [14] proposed optimizations on memory access via edge-list grouping according to source node ID and destination node ID. Such optimizations, and a well-designed cache framework, would reduce random memory access during graph analytics. It is also an open question if bandwidth reduction algorithms such as reverse Cuthill-McKee would bring better memory locality for graph analytics.

Rich-data-on-{vertices,edges} Complex networks often contain rich data on vertices and/or edges. Currently Gunrock puts all the information on edges/vertices into a data structure on the GPU, which is not ideal. Adding the capability of loading partial edge/vertex information onto the GPU could enable graph query tasks and allow us to support several network analysis primitives that use this rich information during their computation.

8.3 Summary

Gunrock was born when we spent two months writing a single hardwired GPU graph primitive. We knew that for GPUs to make an impact in graph analytics, we had to raise the level of abstraction in building graph primitives. With this work, we show that with appropriate high-level programming model and low-level optimizations, parallel graph analytics on the GPU can be both simple and efficient. More specifically, this work has achieved its two high-level goals:

- Our data-centric, frontier-focused programming model has proven to map naturally to the GPU, giving us both good performance and good flexibility. We have also found that implementing this abstraction has allowed us to integrate numerous optimization strategies, including multiple load-balancing strategies for traversal, direction-optimal traversal, and a two-level priority queue. The result is a framework that is general (able to implement numerous simple and complex graph primitives), straightforward to program (new primitives only take a few hundred lines of code and require minimal GPU programming knowledge), and fast (on par with hardwired primitives and faster than any other programmable GPU graph library).
- Our open-sourced GPU graph processing library Gunrock provides a graph analytics framework for three types of users: 1) data scientists who want to take the advantage of the GPU's superior computing power in big data applications; 2) algorithm designers who want to use the existing efficient graph operators in Gunrock to create new graph algorithms and applications; and 3) researchers who want to reproduce the results of our research, or make improvements to our core components. We hope that in the future, Gunrock will serve as a standard benchmark for graph processing on the GPU.

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REFERENCES

[1] Christopher R. Aberger, Andres Nötzli, Kunle Olukotun, and Christopher Ré. 2015. EmptyHeaded: Boolean Algebra Based Graph Processing. *CoRR* abs/1503.02368 (2015). http://arxiv.org/abs/1503.02368

[2] Saman Ashkiani, Andrew A. Davidson, Ulrich Meyer, and John D. Owens. 2016. GPU Multisplit. In *Proceedings of the 21st ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP 2016)*. 12:1–12:13. DOI: http://dx.doi.org/10.1145/2851141.2851169

[3] Jiri Barnat, Petr Bauch, Lubos Brim, and Milan Ceska. 2011. Computing Strongly Connected Components in Parallel on CUDA. In *Proceedings of the 2011 IEEE International Parallel & Distributed Processing Symposium (IPDPS '11)*. IEEE Computer Society, Washington, DC, USA, 544–555. DOI: http://dx.doi.org/10.1109/IPDPS.2011.59

[4] Sean Baxter. 2013. Modern GPU Multisets. (2013). https://nvlabs.github.io/moderngpu/sets.html.

[5] Sean Baxter. 2013–2016. Moderngpu: Patterns and Behaviors for GPU Computing. (2013–2016). http://moderngpu.github.io/moderngpu.

[6] Scott Beamer, Krste Asanović, and David Patterson. 2012. Direction-Optimizing Breadth-First Search. In *Proceedings of the International Conference on High Performance Computing, Networking, Storage and Analysis (SC '12)*. Article 12, 10 pages. DOI: http://dx.doi.org/10.1109/SC.2012.50

[7] Markus Billeter, Ola Olsson, and Ulf Assarsson. 2009. Efficient Stream Compaction on Wide SIMD Many-core Architectures. In *Proceedings of the Conference on High Performance Graphics 2009 (HPG '09)*. ACM, New York, NY, USA, 159–166. DOI: http://dx.doi.org/10.1145/1572769.1572795

[8] Ulrik Brandes. 2001. A faster algorithm for betweenness centrality. *Journal of Mathematical Sociology* 25, 2 (2001), 163–177. DOI: http://dx.doi.org/10.1080/0022250X.2001.9990249

[9] M. Bröcheler, A. Pugliese, and V. S. Subrahmanian. 2010. COSI: Cloud Oriented Subgraph Identification in Massive Social Networks. In *Proceedings of the 2010 International Conference on Advances in Social Networks Analysis and Mining (ASONAM '10)*. IEEE Computer Society, Washington, DC, USA, 248–255. DOI: http://dx.doi.org/10.1109/ASONAM.2010.80

[10] Martin Burtscher, Rupesh Nasre, and Keshav Pingali. 2012. A Quantitative Study of Irregular Programs on GPUs. In *Proceedings of the 2012 IEEE International Symposium on Workload Characterization (IISWC) (IISWC '12)*. IEEE Computer Society, Washington, DC, USA, 141–151. DOI: http://dx.doi.org/10.1109/IISWC.2012.6402918

[11] Martin Burtscher, Rupesh Nasre, and Keshav Pingali. 2012. A Quantitative Study of Irregular Programs on GPUs. In *IEEE International Symposium on Workload Characterization (IISWC-2012)*. 141–151. DOI: http://dx.doi.org/10.1109/IISWC.2012.6402918

[12] F. Busato and N. Bombieri. 2015. BFS-4K: An Efficient Implementation of BFS for Kepler GPU Architectures. *IEEE Transactions on Parallel and Distributed Systems* 26, 7 (July 2015), 1826–1838. DOI: http://dx.doi.org/10.1109/TPDS.2014.2330597

[13] Daniel Cederman and Philippas Tsigas. 2008. On Dynamic Load-Balancing on Graphics Processors. In *Graphics Hardware 2008*. 57–64. DOI: http://dx.doi.org/10.2312/EGGH/EGGH08/057-064

[14] Yunji Chen, Tao Luo, Shaoli Liu, Shijin Zhang, Liqiang He, Jia Wang, Ling Li, Tianshi Chen, Zhiwei Xu, Ninghui Sun, and Olivier Temam. 2016. Graphicionado: A High-Performance and Energy-Efficient Accelerator for Graph Analytics. In *Proceedings of the 49th Annual IEEE/ACM International Symposium on Microarchitecture (MICRO-49)*. IEEE Computer Society, Washington, DC, USA.

[15] Jonathan Cohen and Patrice CastonGuay. 2012. Efficient Graph Matching and Coloring on the GPU. *GPU Technology Conference* (March 2012). http://on-demand.gputechconf.com/gtc/2012/presentations/S0332-Efficient-Graph-Matching-and-Coloring-on-GPUs.pdf

[16] E. Cuthill and J. McKee. 1969. Reducing the Bandwidth of Sparse Symmetric Matrices. In *Proceedings of the 1969 24th National Conference (ACM '69)*. ACM, New York, NY, USA, 157–172. DOI: http://dx.doi.org/10.1145/800195.805928

[17] Andrew Davidson, Sean Baxter, Michael Garland, and John D. Owens. 2014. Work-Efficient Parallel GPU Methods for Single Source Shortest Paths. In *Proceedings of the 28th IEEE International Parallel and Distributed Processing Symposium (IPDPS 2014)*. 349–359. DOI: http://dx.doi.org/10.1109/IPDPS.2014.45

[18] Daniel Delling, Andrew V. Goldberg, Andreas Nowatzyk, and Renato F. Werneck. 2010. PHAST: Hardware-accelerated shortest path trees. *J. Parallel and Distrib. Comput.* 73 (Sept. 2010), 940–952. DOI: http://dx.doi.org/10.1016/j.jpdc.2012.02.007

[19] Erich Elsen and Vishal Vaidyanathan. 2013. A vertex-centric CUDA/C++ API for large graph analytics on GPUs using the Gather-Apply-Scatter abstraction. (2013). http://www.github.com/RoyalCaliber/vertexAPI2.

[20] Zhisong Fu, Michael Personick, and Bryan Thompson. 2014. MapGraph: A High Level API for Fast Development of High Performance Graph Analytics on GPUs. In *Proceedings of the Workshop on GRAph Data Management Experiences and Systems (GRADES '14)*. Article 2, 6 pages. DOI: http://dx.doi.org/10.1145/2621934.2621936

[21] Afton Geil, Yangzihao Wang, and John D. Owens. 2014. WTF, GPU! Computing Twitter's Who-To-Follow on the GPU. In *Proceedings of the Second ACM Conference on Online Social Networks (COSN '14)*. 63–68. DOI: http://dx.doi.org/10.1145/2660460.2660481

[22] Abdullah Gharaibeh, Tahsin Reza, Elizeu Santos-Neto, Lauro Beltrao Costa, Scott Sallinen, and Matei Ripeanu. 2014. Efficient Large-Scale Graph Processing on Hybrid CPU and GPU Systems. *CoRR* abs/1312.3018, 1312.3018v2 (Dec. 2014). arXiv:1312.3018v2

[23] Ashish Goel, Pankaj Gupta, John Sirois, Dong Wang, Aneesh Sharma, and Siva Gurumurthy. 2015. The Who-To-Follow System at Twitter: Strategy, Algorithms, and Revenue Impact. *Interfaces* 45, 1 (Feb. 2015), 98–107. DOI: http://dx.doi.org/10.1287/inte.2014.0784

[24] Joseph E. Gonzalez, Yucheng Low, Haijie Gu, Danny Bickson, and Carlos Guestrin. 2012. PowerGraph: Distributed Graph-Parallel Computation on Natural Graphs. In *Proceedings of the 10th USENIX Conference on Operating Systems Design and Implementation (OSDI '12)*. USENIX Association, 17–30.

[25] Joseph E. Gonzalez, Reynold S. Xin, Ankur Dave, Daniel Crankshaw, Michael J. Franklin, and Ion Stoica. 2014. GraphX: Graph Processing in a Distributed Dataflow Framework. In *Proceedings of the 11th USENIX Conference on Operating Systems Design and Implementation (OSDI'14)*. USENIX Association, Berkeley, CA, USA, 599–613. http://dl.acm.org/citation.cfm?id=2685048.2685096

[26] Oded Green, Robert McColl, and David A. Bader. 2012. GPU Merge Path: A GPU Merging Algorithm. In *Proceedings of the 26th ACM International Conference on Supercomputing (ICS '12)*. ACM, New York, NY, USA, 331–340. DOI: http://dx.doi.org/10.1145/2304576.2304621

[27] Oded Green, Lluís-Miquel Munguía, and David A. Bader. 2014. Load Balanced Clustering Coefficients. In *Proceedings of the First Workshop on Parallel Programming for Analytics Applications (PPAA '14)*. 3–10. DOI: http://dx.doi.org/10.1145/2567634.2567635

[28] Oded Green, Pavan Yalamanchili, and Lluís-Miquel Munguía. 2014. Fast Triangle Counting on the GPU. In *Proceedings of the Fourth Workshop on Irregular Applications: Architectures and Algorithms (IA3 '14)*. 1–8. DOI: http://dx.doi.org/10.1109/IA3.2014.7

[29] Douglas Gregor and Andrew Lumsdaine. 2005. The Parallel BGL: A Generic Library for Distributed Graph Computations. In *Parallel Object-Oriented Scientific Computing (POOSC)*.

[30] John Greiner. 1994. A Comparison of Parallel Algorithms for Connected Components. In *Proceedings of the Sixth Annual ACM Symposium on Parallel Algorithms and Architectures (SPAA '94)*. 16–25. DOI: http://dx.doi.org/10.1145/181014.181021

[31] Pankaj Gupta, Ashish Goel, Jimmy Lin, Aneesh Sharma, Dong Wang, and Reza Zadeh. 2013. WTF: The Who to Follow Service at Twitter. In *Proceedings of the International Conference on the World Wide Web*. 505–514.

[32] Wook-Shin Han, Jinsoo Lee, and Jeong-Hoon Lee. 2013. Turboiso: Towards Ultrafast and Robust Subgraph Isomorphism Search in Large Graph Databases. In *Proceedings of the 2013 ACM SIGMOD International Conference on Management of Data (SIGMOD '13)*. ACM, New York, NY, USA, 337–348. DOI: http://dx.doi.org/10.1145/2463676.2465300

[33] Pawan Harish and P. J. Narayanan. 2007. Accelerating large graph algorithms on the GPU using CUDA. In *Proceedings of the 14th International Conference on High Performance Computing (HiPC'07)*. Springer-Verlag, Berlin, Heidelberg, 197–208. DOI: http://dx.doi.org/10.1007/978-3-540-77220-0_21

[34] Mark Harris, John D. Owens, Shubho Sengupta, Yao Zhang, and Andrew Davidson. 2009–2016. CUDPP: CUDA Data Parallel Primitives Library. (2009–2016). http://cudpp.github.io/.

[35] Zhengyu He and Bo Hong. 2010. Dynamically Tuned Push-Relabel Algorithm for the Maximum Flow Problem on CPU-GPU-Hybrid Platforms. In *Proceedings of the 2010 IEEE International Symposium on Parallel and Distributed Processing (IPDPS 2010)*. Georgia Institute of Technology, Atlanta, United States. DOI: http://dx.doi.org/10.1109/IPDPS.2010.5470401

[36] Sungpack Hong, Hassan Chafi, Edic Sedlar, and Kunle Olukotun. 2012. Green-Marl: A DSL for Easy and Efficient Graph Analysis. In *Proceedings of the Seventeenth International Conference on Architectural Support for Programming Languages and Operating Systems (ASPLOS XVII)*. 349–362. DOI: http://dx.doi.org/10.1145/2189750.2151013

[37] Sungpack Hong, Sang Kyun Kim, Tayo Oguntebi, and Kunle Olukotun. 2011. Accelerating CUDA Graph Algorithms at Maximum Warp. In *Proceedings of the 16th ACM Symposium on Principles and Practice of Parallel Programming (PPoPP '11)*. 267–276. DOI: http://dx.doi.org/10.1145/1941553.1941590

[38] Yuntao Jia, Victor Lu, Jared Hoberock, Michael Garland, and John C. Hart. 2011. Edge v. Node Parallelism for Graph Centrality Metrics. In *GPU Computing Gems Jade Edition*, Wen-mei W. Hwu (Ed.). Morgan Kaufmann, Chapter 2, 15–28. DOI: http://dx.doi.org/10.1016/B978-0-12-385963-1.00002-2

[39] Rashid Kaleem, Sreepathi Pai, and Keshav Pingali. 2015. Stochastic Gradient Descent on GPUs. In *Proceedings of the 8th Workshop on General Purpose Processing Using GPUs (GPGPU 2015)*. ACM, New York, NY, USA, 81–89. DOI: http://dx.doi.org/10.1145/2716282.2716289

[40] Stephen W. Keckler, William J. Dally, Brucek Khailany, Michael Garland, and David Glasco. 2011. GPUs and the Future of Parallel Computing. *IEEE Micro* 31, 5 (Sept. 2011), 7–17. DOI: http://dx.doi.org/10.1109/MM.2011.89

[41] Jeremy Kepner, Peter Aaltonen, David Bader, Aydın Buluç, Franz Franchetti, John Gilbert, Dylan Hutchison, Manoj Kumar, Andrew Lumsdaine, Henning Meyerhenke, Scott McMillan, Jose Moreira, John D. Owens, Carl Yang, Marcin Zalewski, and Timothy Mattson. 2016. Mathematical Foundations of the GraphBLAS. In *Proceedings of the IEEE High Performance Extreme Computing Conference*. DOI: http://dx.doi.org/10.1109/HPEC.2016.7761646

[42] Farzad Khorasani, Keval Vora, Rajiv Gupta, and Laxmi N. Bhuyan. 2014. CuSha: Vertex-centric Graph Processing on GPUs. In *Proceedings of the 23rd International Symposium on High-performance Parallel and Distributed Computing (HPDC '14)*. 239–252. DOI: http://dx.doi.org/10.1145/2600212.2600227

[43] Haewoon Kwak, Changhyun Lee, Hosung Park, and Sue Moon. 2010. What is Twitter, a social network or a news media?. In *Proceedings of the International Conference on the World Wide Web*. 591–600. DOI: http://dx.doi.org/10.1145/1772690.1772751

[44] Aapo Kyrola, Guy Blelloch, and Carlos Guestrin. 2012. GraphChi: Large-scale Graph Computation on Just a PC. In *Proceedings of the 10th USENIX Conference on Operating Systems Design and Implementation (OSDI'12)*. USENIX Association, Berkeley, CA, USA, 31–46. http://dl.acm.org/citation.cfm?id=2387880.2387884

[45] Jure Leskovec. 2009–2016. SNAP: Stanford Large Network Dataset Collection. (2009–2016). http://snap.stanford.edu/data/.

[46] Jure Leskovec and Rok Sosič. 2016. SNAP: A General-Purpose Network Analysis and Graph-Mining Library. *ACM Transactions on Intelligent Systems and Technology (TIST)* 8, 1 (2016), 1.

[47] Hang Liu and H. Howie Huang. 2015. Enterprise: Breadth-first Graph Traversal on GPUs. In *Proceedings of the International Conference for High Performance Computing, Networking, Storage and Analysis (SC '15)*. ACM, New York, NY, USA, Article 68, 12 pages. DOI: http://dx.doi.org/10.1145/2807591.2807594

[48] Hang Liu, H. Howie Huang, and Yang Hu. 2016. iBFS: Concurrent Breadth-First Search on GPUs. In *Proceedings of the 2016 International Conference on Management of Data (SIGMOD '16)*. ACM, New York, NY, USA, 403–416. DOI: http://dx.doi.org/10.1145/2882903.2882959

[49] Yucheng Low, Joseph Gonzalez, Aapo Kyrola, Danny Bickson, Carlos Guestrin, and Joseph M. Hellerstein. 2010. GraphLab: A New Parallel Framework for Machine Learning. In *Proceedings of the Twenty-Sixth Annual Conference on Uncertainty in Artificial Intelligence (UAI-10)*. 340–349.

[50] Grzegorz Malewicz, Matthew H. Austern, Aart J. C. Bik, James C. Dehnert, Ilan Horn, Naty Leiser, and Grzegorz Czajkowski. 2010. Pregel: A System for Large-scale Graph Processing. In *Proceedings of the 2010 ACM SIGMOD International Conference on Management of Data (SIGMOD '10)*. 135–146. DOI: http://dx.doi.org/10.1145/1807167.1807184

[51] Robert Campbell McColl, David Ediger, Jason Poovey, Dan Campbell, and David A. Bader. 2014. A Performance Evaluation of Open Source Graph Databases. In *Proceedings of the First Workshop on Parallel Programming for Analytics Applications (PPAA '14)*. 11–18. DOI: http://dx.doi.org/10.1145/2567634.2567638

[52] Adam McLaughlin and David A. Bader. 2014. Scalable and High Performance Betweenness Centrality on the GPU. In *Proceedings of the International Conference for High Performance Computing, Networking, Storage and Analysis (SC14)*. 572–583. DOI: http://dx.doi.org/10.1109/SC.2014.52

[53] A. McLaughlin and D. A. Bader. 2015. Fast Execution of Simultaneous Breadth-First Searches on Sparse Graphs. In *2015 IEEE 21st International Conference on Parallel and Distributed Systems (ICPADS)*. 9–18. DOI: http://dx.doi.org/10.1109/ICPADS.2015.10

[54] A. McLaughlin, J. Riedy, and D. A. Bader. 2015. A fast, energy-efficient abstraction for simultaneous breadth-first searches. In *2015 IEEE High Performance Extreme Computing Conference (HPEC)*. 1–6. DOI: http://dx.doi.org/10.1109/HPEC.2015.7322466

[55] Duane Merrill, Michael Garland, and Andrew Grimshaw. 2012. Scalable GPU Graph Traversal. In *Proceedings of the 17th ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP '12)*. 117–128. DOI: http://dx.doi.org/10.1145/2145816.2145832

[56] U. Meyer and P. Sanders. 2003. Δ-stepping: a parallelizable shortest path algorithm. *Journal of Algorithms* 49, 1 (Oct. 2003), 114–152. DOI: http://dx.doi.org/10.1016/S0196-6774(03)00076-2 1998 European Symposium on Algorithms.

[57] Donald Nguyen, Andrew Lenharth, and Keshav Pingali. 2013. A Lightweight Infrastructure for Graph Analytics. In *Proceedings of ACM Symposium on Operating Systems Principles (SOSP '13)*. 456–471. DOI: http://dx.doi.org/10.1145/2517349.2522739

[58] Sreepathi Pai and Keshav Pingali. 2016. A Compiler for Throughput Optimization of Graph Algorithms on GPUs. *SIGPLAN Not.* 51, 10 (Oct. 2016), 1–19. DOI: http://dx.doi.org/10.1145/3022671.2984015

[59] Yuechao Pan, Yangzihao Wang, Yuduo Wu, Carl Yang, and John D. Owens. 2016. Multi-GPU Graph Analytics. *CoRR* abs/1504.04804, 1504.04804v3 (April 2016). arXiv:cs.DC/1504.04804v3

[60] Pushkar R. Pande and David A. Bader. 2011. Computing Betweenness Centrality for Small World Networks on a GPU. In *2011 IEEE Conference on High Performance Embedded Computing*.

[61] Roger Pearce, Maya Gokhale, and Nancy M. Amato. 2014. Faster Parallel Traversal of Scale Free Graphs at Extreme Scale with Vertex Delegates. In *Proceedings of the International Conference for High Performance Computing, Networking, Storage and Analysis (SC '14)*. IEEE Press, Piscataway, NJ, USA, 549–559. DOI: http://dx.doi.org/10.1109/SC.2014.50

[62] Keshav Pingali, Donald Nguyen, Milind Kulkarni, Martin Burtscher, M. Amber Hassaan, Rashid Kaleem, Tsung-Hsien Lee, Andrew Lenharth, Roman Manevich, Mario Méndez-Lojo, Dimitrios Prountzos, and Xin Sui. 2011. The Tao of Parallelism in Algorithms. In *Proceedings of the 32nd ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI '11)*. 12–25. DOI: http://dx.doi.org/10.1145/1993498.1993501

[63] A. Polak. 2016. Counting Triangles in Large Graphs on GPU. In *2016 IEEE International Parallel and Distributed Processing Symposium Workshops (IPDPSW)*. 740–746. DOI: http://dx.doi.org/10.1109/IPDPSW.2016.108

[64] Amitabha Roy, Ivo Mihailovic, and Willy Zwaenepoel. 2013. X-Stream: Edge-centric Graph Processing using Streaming Partitions. In *Proceedings of the Twenty-Fourth ACM Symposium on Operating Systems Principles*. ACM, 472–488.

[65] Semih Salihoglu and Jennifer Widom. 2014. HelP: High-level Primitives For Large-Scale Graph Processing. In *Proceedings of the Workshop on GRAph Data Management Experiences and Systems (GRADES '14)*. Article 3, 6 pages. DOI: http://dx.doi.org/10.1145/2621934.2621938

[66] Scott Sallinen, Abdullah Gharaibeh, and Matei Ripeanu. 2015. Accelerating Direction-Optimized Breadth First Search on Hybrid Architectures. *CoRR* abs/1503.04359, 1503.04359v1 (March 2015). arXiv:cs.DC/1503.04359v1

[67] Ahmet Erdem Sariyüce, Kamer Kaya, Erik Saule, and Ümit V. Çatalyürek. 2013. Betweenness Centrality on GPUs and Heterogeneous Architectures. In *Proceedings of the 6th Workshop on General Purpose Processor Using Graphics Processing Units (GPGPU-6)*. 76–85. DOI: http://dx.doi.org/10.1145/2458523.2458531

[68] Thomas Schank and Dorothea Wagner. 2005. Finding, Counting and Listing All Triangles in Large Graphs, an Experimental Study. In *Proceedings of the 4th International Conference on Experimental and Efficient Algorithms (WEA'05)*. 606–609. DOI: http://dx.doi.org/10.1007/11427186_54

[69] Hyunseok Seo, Jinwook Kim, and Min-Soo Kim. 2015. GStream: A Graph Streaming Processing Method for Large-scale Graphs on GPUs. In *Proceedings of the 20th ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP 2015)*. ACM, New York, NY, USA, 253–254. DOI: http://dx.doi.org/10.1145/2688500.2688526

[70] Xuanhua Shi, Junling Liang, Sheng Di, Bingsheng He, Hai Jin, Lu Lu, Zhixiang Wang, Xuan Luo, and Jianlong Zhong. 2015. Optimization of Asynchronous Graph Processing on GPU with Hybrid Coloring Model. In *Proceedings of the 20th ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP 2015)*. 271–272. DOI: http://dx.doi.org/10.1145/2688500.2688542

[71] Julian Shun and Guy E. Blelloch. 2013. Ligra: a lightweight graph processing framework for shared memory. In *Proceedings of the 18th ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP '13)*. 135–146. DOI: http://dx.doi.org/10.1145/2442516.2442530

[72] J. Shun and K. Tangwongsan. 2015. Multicore Triangle Computations Without Tuning. In *IEEE 31st International Conference on Data Engineering*. 149–160. DOI: http://dx.doi.org/10.1109/ICDE.2015.7113280

[73] Jeremy G. Siek, Lie-Quan Lee, and Andrew Lumsdaine. 2001. *The Boost Graph Library: User Guide and Reference Manual*. Addison-Wesley.

[74] G. M. Slota, S. Rajamanickam, and K. Madduri. 2014. BFS and Coloring-Based Parallel Algorithms for Strongly Connected Components and Related Problems. In *2014 IEEE 28th International Parallel and Distributed Processing Symposium*. 550–559. DOI: http://dx.doi.org/10.1109/IPDPS.2014.64

[75] Jyothish Soman, Kothapalli Kishore, and P J Narayanan. 2010. A Fast GPU Algorithm for Graph Connectivity. In *24th IEEE International Symposium on Parallel and Distributed Processing, Workshops and PhD Forum (IPDPSW 2010)*. 1–8. DOI: http://dx.doi.org/10.1109/IPDPSW.2010.5470817

[76] Zhao Sun, Hongzhi Wang, Haixun Wang, Bin Shao, and Jianzhong Li. 2012. Efficient Subgraph Matching on Billion Node Graphs. *Proc. VLDB Endow.* 5, 9 (May 2012), 788–799. DOI: http://dx.doi.org/10.14778/2311906.2311907

[77] Ha-Nguyen Tran, Jung-jae Kim, and Bingsheng He. 2015. Fast Subgraph Matching on Large Graphs using Graphics Processors. In *Database Systems for Advanced Applications*, Matthias Renz, Cyrus Shahabi, Xiaofang Zhou, and Muhammad Aamir Cheema (Eds.). Lecture Notes in Computer Science, Vol. 9049. Springer International Publishing, 299–315. DOI: http://dx.doi.org/10.1007/978-3-319-18120-2_18

[78] Stanley Tzeng, Brandon Lloyd, and John D. Owens. 2012. A GPU Task-Parallel Model with Dependency Resolution. *IEEE Computer* 45, 8 (Aug. 2012), 34–41. DOI: http://dx.doi.org/10.1109/MC.2012.255

[79] Leslie G. Valiant. 1990. A Bridging Model for Parallel Computation. *Commun. ACM* 33, 8 (Aug. 1990), 103–111. DOI: http://dx.doi.org/10.1145/79173.79181

[80] Guozhang Wang, Wenlei Xie, Alan J. Demers, and Johannes Gehrke. 2013. Asynchronous Large-Scale Graph Processing Made Easy. In *CIDR*. www.cidrdb.org.

[81] Leyuan Wang, Yangzihao Wang, Carl Yang, and John D. Owens. 2016. A Comparative Study on Exact Triangle Counting Algorithms on the GPU. In *Proceedings of the 1st High Performance Graph Processing Workshop (HPGP '16)*. 1–8. DOI: http://dx.doi.org/10.1145/2915516.2915521

[82] Yangzihao Wang, Andrew Davidson, Yuechao Pan, Yuduo Wu, Andy Riffel, and John D. Owens. 2016. Gunrock: A High-Performance Graph Processing Library on the GPU. In *Proceedings of the 21st ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP 2016)*. 11:1–11:12. DOI: http://dx.doi.org/10.1145/2851141.2851145

[83] Bo Wu, Zhijia Zhao, Eddy Zheng Zhang, Yunlian Jiang, and Xipeng Shen. 2013. Complexity Analysis and Algorithm Design for Reorganizing Data to Minimize Non-coalesced Memory Accesses on GPU. In *Proceedings of the 18th ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP '13)*. ACM, New York, NY, USA, 57–68. DOI: http://dx.doi.org/10.1145/2442516.2442523

[84] Yuduo Wu, Yangzihao Wang, Yuechao Pan, Carl Yang, and John D. Owens. 2015. Performance Characterization for High-Level Programming Models for GPU Graph Analytics. In *IEEE International Symposium on Workload Characterization (IISWC-2015)*. 66–75. DOI: http://dx.doi.org/10.1109/IISWC.2015.13

[85] Carl Yang, Yangzihao Wang, and John D. Owens. 2015. Fast Sparse Matrix and Sparse Vector Multiplication Algorithm on the GPU. In *Graph Algorithms Building Blocks (GABB 2015)*. 841–847. DOI: http://dx.doi.org/10.1109/IPDPSW.2015.77

[86] Jianlong Zhong and Bingsheng He. 2014. Medusa: Simplified Graph Processing on GPUs. *IEEE Transactions on Parallel and Distributed Systems* 25, 6 (June 2014), 1543–1552. DOI: http://dx.doi.org/10.1109/TPDS.2013.111

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