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Dynamical systems modeling of the child–mother dyad: Causality between child-directed language complexity and language development

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Abstract

We model the *causal* links between child language (CL) and child-directed language (CDL). We take pairs of sequences of linguistic measurements from a longitudinal study. Each child-mother pair of sequences is considered as an instance of the trajectory of a high-dimensional dynamical system. We then use Multispatial Convergent Cross Mapping to ascertain the directions of causality between the pairs of sequences, that is, whether the complexity of CL drives that of CDL, the complexity of CDL drives that of CL, both, or neither. We find that children are responsive to the amount of speech and the diversity of words produced by their mothers, but not vice-versa. However, the syntactic diversities of the children's utterances drive the syntactic diversity of the mothers' utterances. This is evidence for fine-grained fine-tuning of CDL in response only to the syntax of CL.

Keywords: Causality; Child-Directed Language; Convergent Cross Mapping; Dynamical Systems; Fine-Tuning; Information Theory; Language Acquisition

Introduction

Child-directed language (CDL) –sometimes referred to as "motherese"– is the pattern of language used by parents when talking to young children. It is known to exhibit distinctive characteristics with respect to regular adult language (cf., Snow & Ferguson, 1977): It typically uses shorter utterances, prosody is often exaggerated, redundancy and repetition are higher than normal, and referential context tends to be linked to very immediate contexts. It has been observed that the lexical and syntactic complexity of CDL gradually increases along a child's development (Cross, 1977; Snow, 1989), eventually converging to regular adult language. Whether the simplicity of CDL relative to regular adult language plays a functional role in facilitating language acquisition is a contentious issue in the literature. Some researchers argue that "starting small" (Elman, 1993) is a fundamental aspect that facilitates language acquisition (e.g., Dominey & Dodane, 2004). Others, however, claim that there is no facilitating role played by such simplicity (e.g., Pinker, 1994). This latter group would claim that, to some degree, children might exploit universal aspects of language structure, and their syntactic performance would be unrelated to the input to which they have been exposed. Those researchers advocating for a functional role of the simplified input refer to *fine-tuning* (Snow, 1989, 1995; Sokolov, 1993) as the process by which caregivers adjust the complexity of CDL as a function of the level of complexity of the language produced or understood by the child.

A weak and a strong version of the fine-tuning hypothesis compete in the literature. In the weak interpretation, although parents gradually increase the complexity of their CDL, they do not do so as a direct response to the specific properties of the utterances produced by their children, but rather they adjust to the children's overall level of cognitive development, irrespective of the specificities of the language they produce and understand. In this line, several studies have failed to find a direct link between the complexity of the parent's language and that of the child's (Newport, Gleitman, & Gleitman, 1977; Scarborough & Wycoff, 1986; Valian, 1999). These findings suggest that –if anything– the complexity of CDL might increase as a function of the child's age or overall level of development, but not so much as a direct response to the detailed properties of CL. In contrast, other researchers have found evidence supporting a strong version of the fine-tuning hypothesis: That parents adapt the complexity of CDL in direct response to the specific properties of CL (Huttenlocher, Vasilyeva, Cymerman, & Levine, 2002; Kunert, Fernández, & Zuidema, 2011; Murray, Johnson, & Peters, 1990; Roy, Frank, & Roy, 2009; Snow, 1995; Sokolov, 1993). The strong version of the fine-tuning hypothesis has become the dominant view in the field, considered a well-established fact by influential researchers (e.g., MacWhinney, 2014).

Dynamical systems offer powerful tools for modeling human development (e.g., Smith & Thelen, 2003; van Geert, 1991). These models provide a mathematical framework for implementing the principle that development involves the mutual and continuous interaction of multiple levels of the developing system, which simultaneously unfold over many time-scales. Typically, a dynamical system is described by a system of coupled differential equations governing the temporal evolution of multiple parts of the system and their interrelations. In recent years, it has been noticed that, in human development, such systems extend beyond the individual. In particular, it has been found that the linguistic and behavioral interaction between parent-child dyads can be jointly considered as part of a single dynamical sytem encompassing both the child and the parent (Dale & Spivey, 2006; Steenbeek & van Geert, 2007). In this direction van Geert (1991)

shows that the interaction of components within the child-parent dyad can also be modeled on the time-scale of development itself. One difficulty that arises when trying to model a dynamical system as complex as the joint development of language in a parent–child dyad is that many factors that are important for the evolution of the system might not be available or might not be easily measurable or –even worse– there are additional variables relevant for the system of which the modeler is not even aware. In this respect, a crucial development was the discovery that, in a deterministic coupled dynamical system –even in the presence of noise– the dynamics of the whole system can be satisfactorily recovered using measurements of a single of the system's variables (Takens' Embedding Theorem; Takens, 1981).

The finding above opens an interesting avenue for understanding the processes involved in language acquisition (perhaps more suitably termed language *growth*, following van Geert, 1991). In the same way that systems of differential equations can be used to model the evolution of ecosystems (e.g., predator-prey systems), one could take measurements of the detailed properties of CL and CDL, and build a detailed system of equations capturing the macroscopic dynamics of the process. However, in order to achieve this, it is necessary to ascertain the ways in which different measured variables in the system affect each other. This problem goes beyond estimating correlations (as could be obtained, for instance, using regression models), as one needs to detect asymmetrical *causal* relations between the variables of interest, so that these causal influences can be incorporated into the models.

In this study, we investigate the causal relations between measures describing CL (i.e., number of words produced, lexical diversity, and mean length of utterances) and the equivalent measures in CDL, using the longitudinal data provided in the Manchester Corpus (Theakston, Lieven, Pine, & Rowland, 2001). In order to detect causal relations between the different measures, we make use of state space reconstruction relying on Takens (1981)'s Embedding Theorem, and recently developed techniques for assessing the strength of causal relations in dynamical systems (Multispatial Convergent Cross Mapping; Clark et al., 2015). Our results provide a detailed picture on the presence and effects of fine tuning across different linguistic strata, and provide an important inroad into building a detailed dynamical system jointly considering the co-development of CDL and CL.

Causality Detection in Dynamical Systems

The commonly-held maxim that "correlation does not imply causation" is often misinterpreted to mean that one might have correlated variables that are not involved in any causal relations. Pearl (2000) clarifies that, whenever two variables are correlated, there *must* exist some causal link between them. Namely, if variables A and B are found to be correlated, then one of four possibilities must be true: (a) A causes B, (b) B causes A, (c) A and B form a feedback loop, each causing the other, or (d) there is a third variable C causing both A and B. In order to ascertain a *direct* causal relation between any two variables, one needs to discard possibility (d) above, for which one requires knowledge of all other variables that might be of importance for the system in question. Without knowledge of these possible intervening variables, causal inference using these methods must remain strongly suspect. This problem is, of course, particularly acute when one is dealing with systems as complex as is human language.

The most common technique for assessing the presence of causal relations between time series is Granger-causality (Granger, 1981). It relies on the notion of *separability*, this is, that the information contained by a causal source is unique to it, so that just eliminating that variable from consideration suffices for eliminating the information that it contributes. Purely stochastic systems often exhibit separability. Unfortunately, however, separability is not a property that is exhibited by deterministic non-linear dynamical systems. For studying the interactions of species within ecosystems, Sugihara et al. (2012) introduced *Convergent Cross Mapping* (CCM), a causality-detection technique that is valid for non-separable systems, is capable of identifying weakly coupled variables even in the presence of noise, and –crucially– can distinguish direct causal relations between variables from effects of shared driving variables (i.e., in possibility (d) from the previous paragraph, CCM would *not* find causality).

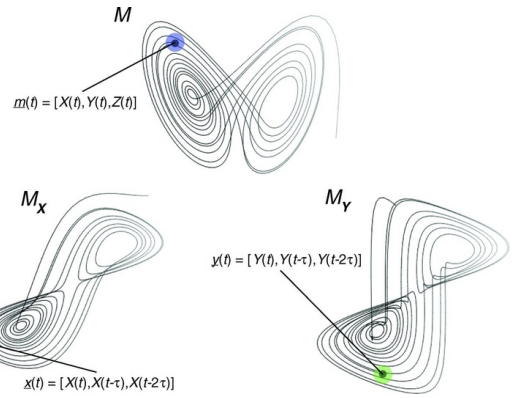

Figure 1: Reconstructed manifold for Lorenz's system (M; top), as well as the shadow manifolds reconstructed considering only X (M_X; bottom-left) and Y (M_Y; bottom-right) (reprinted with permission from Sugihara et al., 2012).

For instance, consider E. Lorenz's often studied dynamical system including three coupled variables $X(t)$, $Y(t)$, and $Z(t)$ whose co-evolution is described by the system of differential equations

$$\begin{cases} \dfrac{\mathrm{d}X}{\mathrm{d}t} = \sigma(Y - X) \\ \dfrac{\mathrm{d}Y}{\mathrm{d}t} = X(\rho - Z) - Y \\ \dfrac{\mathrm{d}Z}{\mathrm{d}t} = XY - \beta Z \end{cases} . \tag{1}$$

The first equation in this system indicates that there is a re-

lation by which Y causes X, as the change in X (i.e., its future value) depends on the value of Y (i.e., the future of X depends on the past of Y even after the past of X itself has been considered), a causal relation whose strength is indexed by parameter σ. The manifold defined by these three variables (Lorenz's famous strange attractor), which we can denote by M, is plotted in the top of Fig. 1. In many circumstances, however, not all variables of the system are available (some might be difficult to measure, or we might not even be aware of their relevance). It is at this point that Takens (1981)'s Embedding Theorem comes into play. Informally speaking, the theorem states that the properties of a coupled dynamical system's attractor can be recovered using only measurements from a single one of its variables. This is achieved by considering multiple versions of the same variable lagged in time, that is, instead of plotting $(X[t], Y[t], Z[t])$, when only measurements of X are available, we can plot $(X[t], X[t+\tau], \ldots, X[t+(E-1)\tau])$. These reconstructed manifolds are termed "shadow" manifolds. M_X denotes the shadow manifold of M reconstructed on the basis of X alone. There are well-studied techniques for finding the appropriate values for the parameters for the lag τ and the number of dimensions E (c.f., Abarbanel, Brown, Sidorowich, & Tsimring, 1993) so that the properties of the original manifold M are recovered by the shadow manifold M_X. Fig. 1 illustrates this point by plotting the shadow manifolds M_X (bottom-left) and M_Y (bottom-right) for the Lorenz system. Notice how both shadow manifolds recover much of the original's structure, using only knowledge of one of its three variables.

Each point in the original manifold M maps onto points in its shadow manifolds, as is illustrated by the points labelled $m(t)$, $x(t)$, and $y(t)$ in Fig. 1. The preservation of the topological properties of the original manifold in its shadow manifolds entails that points that are close-by in the original manifold will also be close-by in its shadow versions. This implies that, for causally linked variables within the same dynamical system, the state of one variable can identify the states of the others. Sugihara et al. (2012) noticed that, when one variable X stochastically drives another variable Y, information about the states of X can be recovered from Y, but not vice-versa. This is the basic insight of the CCM method. To test for causality from X to Y, CCM looks for the signature of X in Y's time series by seeing whether the time indices of nearby points on M_Y can be used to identify nearby points on M_X. Crucially, in order to distinguish causation from mere correlation, CCM requires *convergence*, that is, that cross-mapped estimates improve in estimation accuracy with the sample size (i.e., "library size") used for reconstructing the manifolds. As the library size increases, the trajectories defining the manifolds fill in, resulting in closer nearest neighbors and declining estimation error, which is reflected in a higher correlation coefficient between the points in the neighborhoods of the shadow manifolds. Convergence then becomes the necessary condition for inferring causation. Using both artificial systems and ecological time-series with known dynamics, Sugihara and his colleagues demonstrated that this technique successfully recovers true directional causal relations when these are present, and –crucially– is able to discard spurious causation in the case when both variables are causally driven by a third, unknown, variable, but there is no true direct causation between them.

An inconvenience of CCM, and in general of techniques that rely on manifold reconstruction, is that they generally require that relatively long time-series of the behavior of the system are available. Such long series are, however, very difficult, if not impossible, to obtain in many fields, including of course language acquisition. One can however obtain multiple short time series from different instances of a similar dynamical system. In ecology, for instance, one can obtain short sequences of measurements of the population densities of a group of species measured at different places and times. In language acquisition, we might have multiple, relatively short longitudinal sequences of measurements from different children. With this in mind Clark et al. (2015) developed Multispatial CCM (mCCM), an extension of CCM able to infer causal relations from multiple short time-series measured at different sites, making use of dewdrop regression (Hsieh, Anderson, & Sugihara, 2008) to take the additional heterogeneity into account.

Materials and Methods

We obtained from the CHILDES database (MacWhinney, 2000) the transcriptions contained in the Manchester Corpus (Theakston et al., 2001). This corpus contains annotated transcripts of audio recordings from a longitudinal study of 12 British English-speaking children (6 girls and 6 boys) between the ages of approximately two and three years. The children were recorded at their homes for an hour while they engaged in normal play activities with their mothers. Each child was recorded on two separate occasions in every three-week period for one year. Each recording session is split into two half-hour periods. The annotations include the lemmatized form of the words produced by both the children and their mothers (incomplete words and small word-internal errors were manually corrected in the lemmatization).

In order to increase the sample size in each period, we used a sliding window technique (akin to that used in Moscoso del Prado Martín, 2014), by computing measures for the samples contained in overlapping windows of three consecutive corpus files. In this way, at each point we obtained samples originating from two files from the same recording session, and a file from either the previous or the next recording session. For each child and mother, we recorded the total number of words they produced, the lexical diversity measured as the entropy of the lemmas produced (following the estimation method of Moscoso del Prado Martín, in press, which is demonstrated to be accurate and unbiased for these sample sizes), and the mean length of the utterances (MLU) they produced. Instead of measuring MLU in morphemes (Brown,

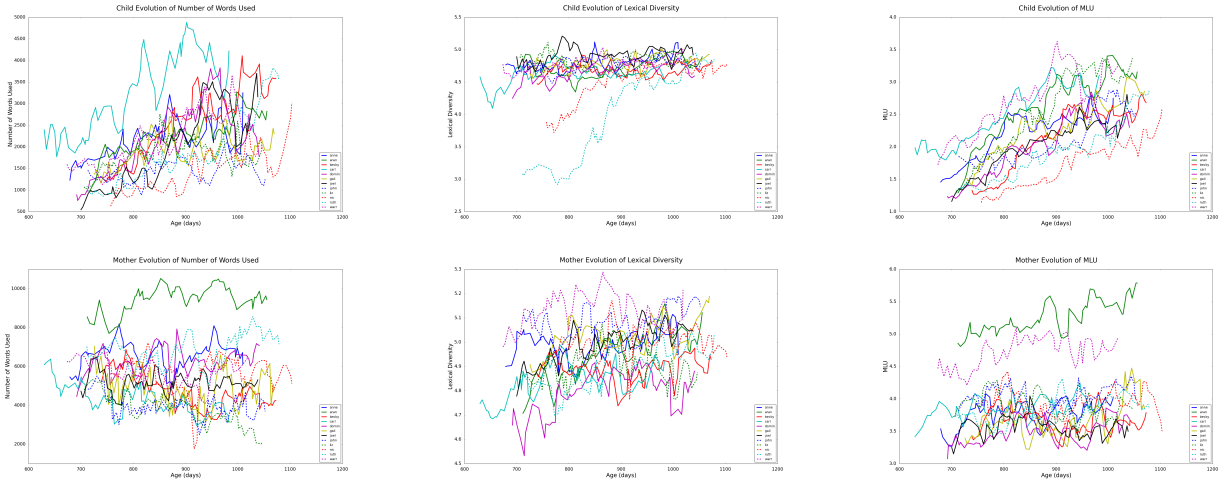

Figure 2: Evolution of the measures under consideration as a function of the children's ages, for the children (top row) and their mothers (bottom row). The total number of words produced are plotted in the left column, the lexical diversities in the middle column, and the MLUs in the right column.

1973), we used the simpler, but equally accurate measure in number of words (Parker & Brorson, 2005). In these ages, MLUs are well known to provide an accurate measure of the syntactic richness of the utterances produced (Brown, 1973), and in fact correlate almost perfectly with explicit measurements of grammatical diversity (Moscoso del Prado Martín, in press).[1] Fig. 2 plots the temporal evolution of the three measures for the children and their mothers.

Table 1: Values of the parameters used for shadow attractor reconstruction.

	Child			Mother		
	Number of Words	Lexical. Diversity	MLU	Number of Words	Lexical Diversity	MLU
τ	3	2	3	3	3	3
E	3	3	4	5	4	4

The optimal time-lags (τ) for constructing the shadow manifolds were estimated as the first local minimum of the lagged self-information in each of the time series (c.f., Abarbanel et al., 1993). The optimal embedding dimensionalities (E) were estimated by optimizing next-step prediction accuracy. The estimates were not found to differ significantly across children or mothers, and therefore for each measure, we used a single estimate of (τ, E) for all children (see Table 1), and a single estimate for all mothers. The time series were checked to ensure that they contained non-linear signal not dominated by noise using a prediction test, and the presence of directional causality between children's and mothers' measures was tested for each of the three variables using mCCM, with 1,000 bootstrapping iterations used to assess the p-values.[2] Finally, to account for our lack of *a priori* predictions on the causal directions to be tested, the p-values were adjusted for multiple comparisons using the false discovery rate for correlated data (FDR; Benjamini & Yekutieli, 2001).

Results and Discussion

Fig. 3 plots the mCCM results for each pair of shadow manifolds. The curves plot how the correlations between nearest neighbours across shadow attractors evolve as one considers increasingly larger library sizes. The p-values report whether these correlation values are significantly increasing (the p-values are obtained by a Monte Carlo method with 1,000 resamplings, and further FDR-corrected for multiple comparisons). Panel (a) shows that convergence indicates a significant causal relation between the number of words produced by the mother, and the number of words produced by her child ($p < .001$) which is not significantly present in the opposite direction ($p = .067$). A similar picture arises in the lexical diversities in panel (b): The richness of the vocabulary produced by the mothers influences the richness of the vocabulary produced by their children ($p = .014$), but the richness of the vocabulary used by children does not significantly affect that of their mothers ($p = .481$). In contrast, panel (c) shows that, in terms of MLU, the language produced by children and their mothers form a feedback loop, with significant causal relations in both directions (child to mother: $p < .001$; mother to child: $p = .005$).

In terms of the amount of speech, or the richness of the vocabulary used, these results indicate that the mothers are not increasing the complexity of CDL in response to the details

[1] We also tested a measure of inflectional diversity (Moscoso del Prado Martín, 2014, in press), which was not found to produce any reliable causal effects, and is therefore not discussed further. However, the presence of these additional tests was nevertheless taken into account when correcting for multiple comparisons.

[2] All computations, except for τ selection, we done using R package `multispatialCCM` (Clark et al., 2015).

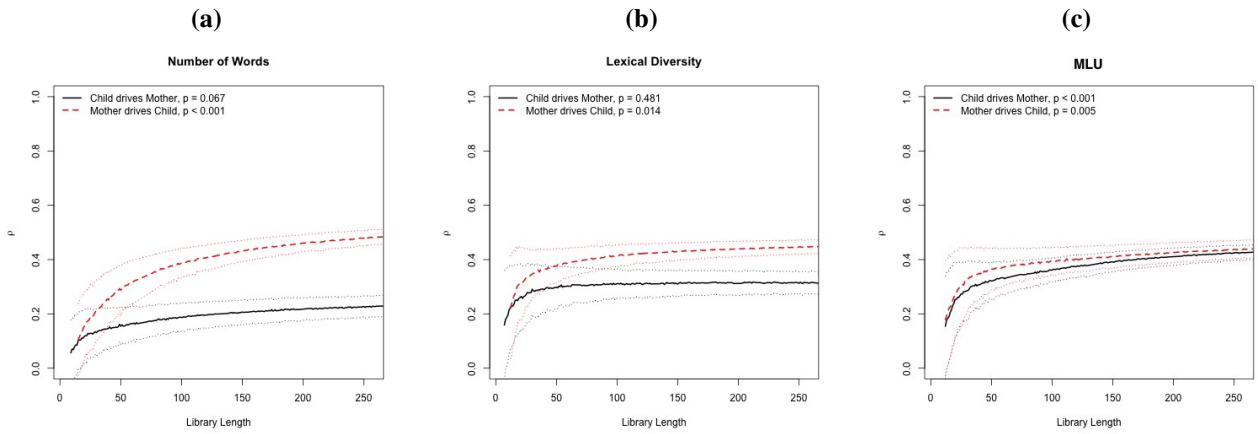

Figure 3: For each of the measures considered, the panels plot the evolution of Pearson's correlation coefficient (ρ) between the predicted and predicting shadow manifolds. The dashed lines denotes the standard deviation of the estimates. The *p*-values indicated in the legend were obtained by the bootstrapping procedure described by Clark et al. (2015). A value of ρ significantly increasing with library length is mCCM's indication of causality between two variables.

of CL, but the children's performance still benefits from the increased quantity and diversity of words. This is evidence that weak lexical fine-tuning serves a functional role. In contrast, when it comes to MLUs, the bidirectional causality provides clear evidence for a strongly coupled system with feedback. As is shown in Moscoso del Prado Martín (in press), MLUs are in fact almost perfectly correlated (i.e., Pearson's $r \approx .96$) with an explicit measure of the diversity of the syntactic structures used in a sample (i.e., the *syntactic diversity* of the sample). Mothers adjust the complexity of their syntactic structures as a direct response to the syntactic complexity of the utterances produced by their children, as is advocated by the strong version of the fine-tuning hypothesis.

Our results provide direct evidence for the fine-tuning hypothesis. For the first time, we have *explicitly* demonstrated that, in all measures studied, the children benefit from the gradual increase in complexity of CDL, as is indicated by the directional causalities found between the measures in CDL and those in CL. In addition, only for the case of syntax, we find direct evidence for the strong version of the fine-tuning hypothesis: The complexity of the syntactic structures produced by mothers are directly caused by those of the syntactic structures produced by their children.

These findings are the first step in building a macroscopic level dynamical system model of language acquisition explicitly considering children jointly with their environment. In order to build such a model, one also needs to test for the explicit causal components of complexity within an individual (e.g., what are the causal connections between increased vocabulary and increased syntactic knowledge?), and those present across individuals and linguistic strata; for instance, it has been reported that the amount of speech produced by parents influences the growth of both the vocabulary (e.g., Hurtado, Marchman, & Fernald, 2008; Weisleder & Fernald, 2013), and the MLUs in the children (Barnes, Gutfreund, Satterly, & Wells, 1983). We anticipate further research in these directions.

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