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Spatial Ultrasonic Wavefront Characterization Using a Laser Parametric Curve Scanning Method

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ABSTRACT

Ultrasonic wavefield imaging (UWI) provides insightful spatial information about ultrasonic wave propagation in planar (2-D) space for nondestructive evaluation and structural health monitoring (NDE-SHM) applications. In all materials, the wavefronts of the incident and reflected waves propagate with unique patterns that may be represented by parametrized polar curves in 2-D geometric space. In this paper, a spatial ultrasonic wavefront characterization method based on a parametric curve laser scan is proposed to characterize the spatial ultrasonic wavefront for both isotropic and anisotropic materials. Three parametric curves (circular, hyperbolic, and cyclic-harmonic curves) were considered. Two wavefront characterization process were carried out, namely (i) deciding the parametric equation of the closed-form geometric plane curve via UWI, and (ii) measuring and updating the ultrasound via laser ultrasonic interrogation system (LUIS) and quantifying the values(s) of the predicted parametric curve equation using a temporal cross-correlation technique. The proposed method was tested on pristine aluminum and cross-ply CFRP plates to characterize the spatial incident and reflected wavefronts of the plates. The non-fiber direction region ($105^\circ \leq \phi_s \leq 165^\circ$) and the fiber direction region ($165^\circ \leq \phi_s \leq 195^\circ$) of the cross-ply CFRP plate were considered in the test. The laser circle scan and the laser cyclic-harmonic curve scan showed the ability to characterize the incident wavefronts of the S0 and A0 modes in the aluminum plate and the CFRP plate, respectively, followed by the laser hyperbolic curve scan. With the promising results obtained in the proposed method, the integration of the parametric curve scanning method into LUIS may provide a new approach to damage detection and useful information for ultrasonic algorithm design in NDE-SHM applications.

Keywords: Ultrasonic wavefield imaging, laser parametric curve scanning, laser ultrasonic techniques, ultrasonic wavefront characterization, spatial ultrasonic wavefront pattern

1. INTRODUCTION

Ultrasonic wavefield imaging in solid media has played a significant role in ultrasonic nondestructive evaluation and structural health monitoring applications. It reveals insightful spatial information about ultrasonic wave propagation in 2-D space. Usually, laser ultrasonic techniques (LUTs) are used with a 2-D laser mirror scanner (LMS) to measure the wavefield data. The visualization of the wavefield happens by playing a sequence of 2-D spatial frames as an animated sequence. As individual images

39 are animated, the guided waves spread outward from the source and subsequently interact with
40 features in the target structure. Therefore, the UWI provides an excellent, high-resolution way to
41 understand of how guided waves propagate and interact with damage such as fatigue cracks [1],
42 impacts [2, 3], and corrosion [4].

43
44 Due to the advantages of the UWI, LUTs have been intensively designed to target isotropic metals
45 and anisotropic composite materials [2, 5-8], owing to their non-contact nature and high spatial
46 resolution. Since the 2-D LMS used in the LUTs is versatile, the LUTs can easily maneuver the laser
47 beam to any desired local point to carry out the ultrasonic measurement. The most common approach
48 used for scanning is to perform a rastering pattern over a region of interest. A circular scanning
49 method has recently been adopted as well [9].

50
51 Nonetheless, LUTs have significant challenges in practical implementation [10, 11]. One of the
52 challenges is slow data acquisition. Although the LUTs have the potential to conduct a large-scale
53 structural inspection with high spatial resolution of full-field ultrasonic data, it may be impractical to
54 perform an inspection when there is no information on the existence of damage, particularly for
55 periodic maintenance services. A raster scan pattern is commonly used in LUTs to generate the
56 ultrasonic wavefield in Cartesian coordinate space because it has better local damage detection. In
57 general, however, the existing LUTs conduct the inspection process by directly performing the raster
58 scanning without knowing whether or not there is damage. It is therefore often costly to carry out an
59 inspection based on the raster scanning method, as the scanning time is proportional to the required
60 inspection area. Several methods have recently been introduced to optimize the inspection time of
61 LUTs by improving the speed of full-field ultrasonic data acquisition. The methods are categorized
62 mainly into two approaches, optimization of LUTs hardware and optimization of laser scanning point.

63
64 In the hardware of a laser ultrasonic generator system, the speed of full-field ultrasonic data
65 acquisition is relatively dependent on the pulse-repetition-rate (PRR) of a Q-switched laser
66 incorporated in a laser scanning system. In Ref [12], a Q-switched laser was used and optimized with
67 the PRR of 20 kHz. The system demonstrated the feasibility of damage detection on a CFRP plate and
68 significantly improved the speed of the full-field ultrasonic data acquisition. However, the total length
69 of the measurement signal was short and limited to 50 μ s because the 20-kHz triggering signal of the
70 Q-switched laser was used to synchronize with the data acquisition system (DAQ). Furthermore, the
71 high laser scanning speed based on the in-plane regime is not suitable for the inspection of metallic
72 structure where the reverberative field has a long dwell time. In Ref [13], the conventional laser
73 ultrasonic system with a 2-D LMS was optimized by employing two 2-D LMSs for multi-area
74 scanning process. Since the single laser beam of a Q-switched laser was splitted into two for the two
75 LMSs, the power of the Q-switched laser for this configuration must be increased in order to have

76 enough energy for the laser beam to generate an ultrasonic signal. Although this approach improved
77 the inspection time by running the scanning process simultaneously on two different scanning areas,
78 but this additional hardware resulted in an increase in the total cost of the laser scanning system.

79
80 Compressive sensing (CS) has recently emerged as a novel signal acquisition paradigm that allows
81 high-dimensional signals to be reconstructed with relatively few measurements. This technique has
82 introduced an expedition to the acquisition process of ultrasonic wavefield data [8, 14-18]. Di Ianni et
83 al. [15] demonstrated that the signal decomposition based on the Fourier domain has better recovery
84 accuracy with less than 34% of the original sampling grid. Harley and Chia [17] introduced a
85 framework to use sparse wavenumber analysis based on CS [18] to create a damage-free model of the
86 wavefield. As a result, these methods improved the scan time of current wavefield imaging. Park et al.
87 [8] proposed an accelerated laser scanning technique based on binary search and compressed sensing
88 to localize and visualize damage with reduced scanning points and scanning time. In any CS
89 technique, the dictionary signal plays a key role as used to reconstruct the high-resolution signal. Thus,
90 the signal reconstruction efficiency depends on dictionary quality. Consequently, it can take a long
91 post-processing time and a high computational burden to obtain the “best” dictionary via a dictionary
92 learning algorithm [19].

93
94 In addition to these two optimization approaches, the optimization of the spatial scanning pattern [9]
95 has recently been introduced, by taking advantage of the flexibility of the laser scanning system as
96 well as the spatial ultrasonic wavefront pattern of the material. The approach focuses on the initial
97 process of diagnosis strategy, damage detection, for SHM applications to detect the anomalies in
98 structures, followed by localization and then characterization (size, type, etc.). Based on this approach,
99 the spatial circle scanning method was proposed by the authors in Ref [9]. In this work, a single circle
100 scan was carried out by the laser scanning system to detect anomaly or damage, instead of using a
101 full-field ultrasonic data measurement method. Since the proposed method was tested on an aluminum
102 plate (isotropic material), the incident wavefronts (S0 and A0 modes) of the circumferential ultrasonic
103 wave arrived at the same time in 2-D space, respectively. Due to the nature of the wave propagation
104 pattern, the spatial ultrasonic wavefront pattern of the incident waves in the isotropic material is
105 formed in a circular shape that can be easily characterized by a parametric circle equation. Hence, the
106 spatial circle scanning method incorporated into the laser scanning system was able to capture the
107 same wavefronts that were defined as a circumferential wavefront. Regards to this, the shape of the
108 circumferential wavefront will remain intact as a circle if there is no anomaly in the directions of
109 wave propagation, and vice versa.

110
111 Therefore, the spatial circle scanning method was developed to exploit the spatial correlation within
112 the circumferential ultrasonic wave propagation and then find the anomaly waves response due to the

113 damage. The covariance-variance matrix was used to evaluate the response of the correlation that
114 acted as a feature for the damage detection. The covariance map [5] of the incident wavefronts
115 demonstrated the ability to detect damage when the damage was located within the area of the
116 wavefront scan point circumference. However, when the damage was located outside the area
117 bounded by the scan circumference, the spatial correlation of the circumferential ultrasonic wave
118 showed low defect detectability. This is because the incident wavefronts remained intact, as there was
119 no interaction with the damage. But the reflected waves that interacted with the damage showed that
120 the damage was still detectable.

121
122 By optimizing the spatial scanning pattern based on the spatial ultrasonic wavefront pattern in an
123 isotropic material, the circular scanning method with the parametric circle equation demonstrated the
124 feasibility of detecting defects. Fundamentally, this is because it is simple to characterize the spatial
125 wavefront pattern as a circle (no circumferential variability in wave propagation). Therefore, it is
126 much more difficult to apply the circular scanning method to an anisotropic material, e.g. a cross-ply
127 CFRP plate, because the ultrasonic wavefront pattern is much more complex; it is unlikely to conform
128 to a circular shape, as the wave speeds now have structured directional variability commensurate with
129 the anisotropy. This results in variable arrival times for the incident wavefronts, resulting in decreased
130 detection performance.

131
132 This paper hypothesizes that even with such anisotropic materials, the wavefront geometries are
133 representable by, and similar to, parametrized polar curves in 2-D geometric space. Since the 2-D
134 laser mirror scanner used in the system used herein [5] has the flexibility to maneuver the laser beam
135 along any scanning curve of interest in 2-D space, this suggests that the spatial ultrasonic wavefront
136 pattern in 2-D space may be characterized by scanning the curve in relation to a parametric curve via
137 LUIS. To test this hypothesis, we propose a parametric curve scanning method to accomplish this.
138 The key purpose of this new proposed method is to overcome the limitations of previous work [9] in
139 the optimization of the spatial scanning pattern and form a generalized approach to perform an
140 optimal scan. The following sections of this article will present the methodology of spatial ultrasonic
141 wavefront characterization, the experimental setup for ultrasonic measurement based on the full-field
142 scanning method and the parametric curve scanning method, and the implementation of the spatial
143 ultrasonic wavefront characterization for incident and reflected wavefronts in both isotropic material
144 (aluminum plate) and anisotropic material (cross-ply CFRP plate).

145 **2. SPATIAL ULTRASONIC WAVEFRONT CHARACTERIZATION METHODOLOGY**

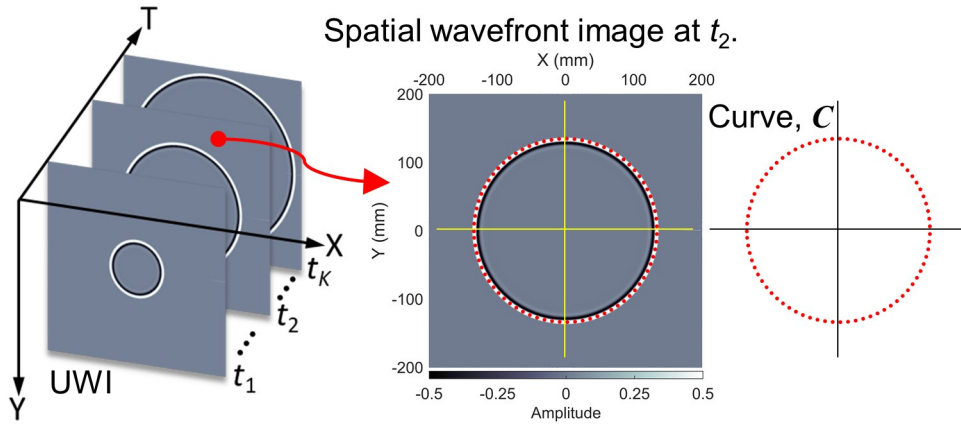
147 In 2-D space, UWI provides a visualization of the propagation patterns of ultrasonic wavefronts. The
148 spatial wavefront patterns observed can reveal important information about both the mechanics
149 (including anisotropy leading to circumferential wave speed variability) and the possible presence of

150 localized anomalies representing defects. Therefore, the characterization of these patterns, based on
 151 modeling them with 2-D geometric plane curves, is proposed and discussed below.

152
 153 **2.1 Spatial Ultrasonic Wavefront Pattern Modeling Concept**

154 Figure 1 shows the schematic diagram illustrating the modeling concept of how the UWI's 2-D planar
 155 spatial wavefront curve may be represented parametrically. To facilitate the understanding of the
 156 concept, the UWI is generated with an artifact single sinusoidal wave propagating progressively in 2-
 157 D space at various times as shown in Fig. 1. Basically, each time frame of the UWI can be viewed as
 158 a single image describing the instantaneous state of the spatial wavefront in the X - Y planar domain.
 159 For example, Fig. 1 shows the spatial wavefront curve with a circular pattern emitting from the center
 160 point and growing as time evolves from t_1 to t_k (one might assume in this trivial example that the
 161 material is isotropic and homogenous). In this case the base wavefront pattern will not change as the
 162 wavefront propagates, unless there is a local impedance change or scatterer along the propagation
 163 direction. Thus, it might be reasonable to parametrize the base wavefront pattern, as in particular the
 164 wavefronts of the incident wave and reflected wave.

165



166
 167 **Figure 1.** Modeling concept of spatial wavefront pattern parameterization.

168
 169 Consider parametrizing the spatial wavefront curve at t_2 in the center of Fig. 1. Since the wavefront
 170 (red-dots in figure) is positioned at a set of coordinates (x, y) in 2-D Euclidean space (\mathbb{R}^2) within the
 171 X - Y domain, the wavefront pattern can thus be described by a planar curve (C) as shown the right-
 172 hand-side of Fig. 1, generally described as

173
$$C(\beta) = \{(g(\beta), h(\beta)) : \beta \in I\}, \quad (1)$$

174 by mapping the parameter $\beta \in \mathbb{R}$ to a point $C \in \mathbb{R}^2$ on the curve, ranging over an interval I . The
 175 coordinate (x, y) of the circular wavefront are defined as functions of an independent parameter β , i.e.,
 176 $x = g(\beta)$ and $y = h(\beta)$, which are the parametric equations describing a planar curve. Consequently, a
 177 spatial wavefront pattern with a set of coordinates (x, y) can be characterized and modeled in a

178 parametric equation when the parameter β is determined for an interval I . To realize this modeling
179 concept, a spatial ultrasonic wavefront characterization process using a laser ultrasonic technique is
180 introduced and discussed next.

181

182 **2.2 Spatial Ultrasonic Wavefront Characterization Process**

183 Figure 2 shows the flowchart of the spatial ultrasonic wavefront characterization process for the 2-D
184 spatial wavefront using the laser parametric curve scanning method. In other terms the ultrasonic
185 wavefront patterns in 2-D space are characterized and defined by the 2-D geometric plane curve
186 parametric equation introduced above. The blue box and the red box in the flowchart are the
187 procedures performed in Matlab™ 2019b software and LUIS, respectively. The characterization
188 process is composed of two primary processes. The first process, named Process 1, at the left-hand
189 side of the flowchart is to decide the closed-form geometric plane curve that describes the wavefront
190 patterns. The second process, named Process 2 at the right-hand-side of the flowchart, is to measure
191 the ultrasound through the LUIS with the parametric curve equation obtained from Process 1 and
192 quantify the accuracy of the wavefront characterization by updating the parameter(s). The process of
193 the parameter(s) updating is repeated until the desired parameter(s) value(s) of the parametric curve
194 equation is(are) reached. The implementation of the proposed method in Fig. 2 does not require two
195 independent platforms (Matlab™ software and LUIS), as they may be easily integrated into the LUIS
196 platform, but that is beyond the scope of this paper.

197

198 The UWI plays a significant role in the characterization process. It provides prior knowledge of the
199 spatial ultrasonic wavefront pattern and the geometric plane curve in relation to the incident and
200 reflected waves of a specimen. Basically, the formation of the spatial ultrasonic wavefront pattern
201 depends on the velocity of the ultrasonic guided waves (Lamb waves) in a plate-like structure, which
202 in turn depends on the material properties and frequency-thickness product. The material of the
203 specimen may be either a metallic (isotropic material) or a composite (anisotropic/quasi-isotropic
204 material). The fundamental symmetric (S0) and asymmetric (A0) modes of Lamb waves are
205 frequently considered in these materials for NDE-SHM applications. So, the characterization process
206 suggested in Fig. 2 focuses only on the incident wavefronts of these two fundamental modes and the
207 reflected wavefronts at a selected frequency range. The frequency range can be either narrowband or
208 broadband, which can be easily done with a signal-condition device as incorporated into LUIS,
209 described in section 3.1. The separation of these two modes in 2-D space must be achieved first so
210 that the S0/A0 mode may be characterized with the right parameter values. To do so, the size of the
211 specimen must be large enough to allow these two modes to have enough time to separate during the
212 wave propagating in 2-D space. Since the velocity of the Lamb waves in a plate-like structure depends
213 on the thickness of the plate, the plate must therefore be relatively flat unless the characterization
214 process is intentionally performed to parameterize the wavefront pattern of a plate with varying

215 thickness. For composite plates, the placement accuracy from local fiber stacking variations can also
 216 affect the accuracy of the characterization performance. Finally, when the incident wave of the S0/A0
 217 mode reflects at a plate boundary, the reflected pattern correlates to the shape of the boundary; the
 218 boundary shape is assumed to be utilized as prior knowledge in order to inform the characterization
 219 process for parameterizing the reflected wave portion. Consequently, the dependent factors for the
 220 characterization process can be summarized as (i) material types, (ii) wavefront types, (iii) plate-like
 221 geometric settings (size, thickness, boundary), and (iv) frequency range, where these settings must be
 222 clearly defined before starting the characterization process.
 223

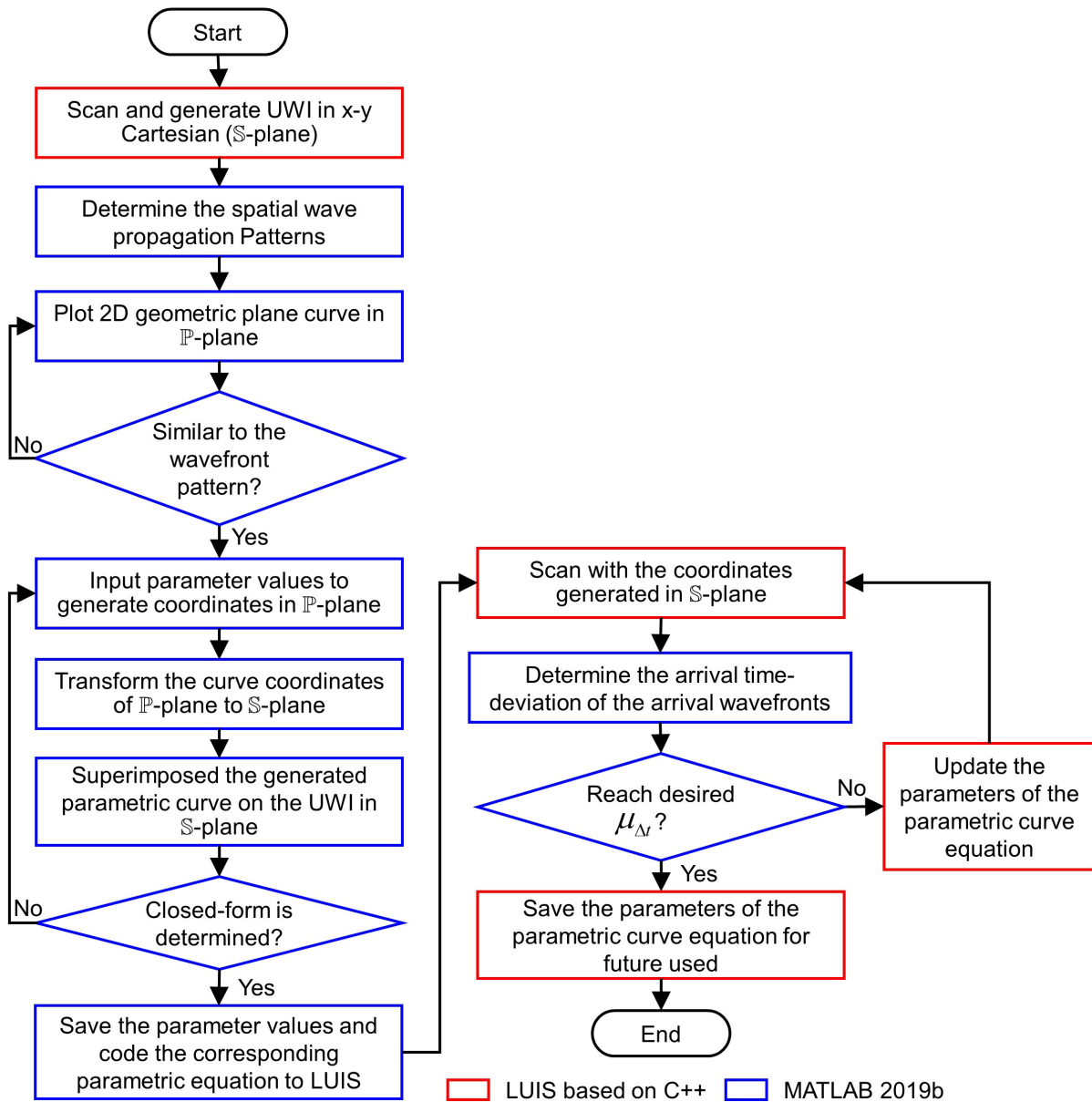


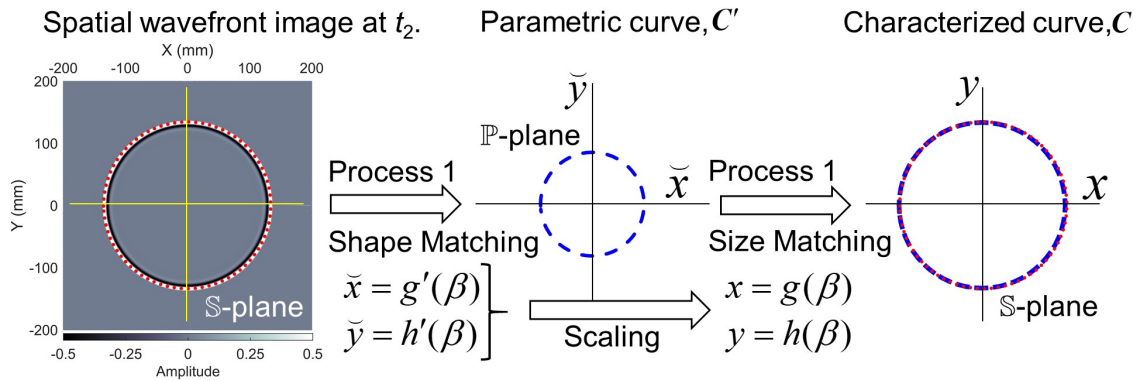
Figure 2. Flowchart of spatial ultrasonic wavefront characterization process. ($\mu_{\Delta t}$: mean arrival time-deviation)

228 The method of characterization in Fig. 2 is implemented with the objective of constructing a database
 229 containing the various sets of parametric equations of the spatial ultrasonic wavefront pattern relative
 230 to materials that ultimately provide a resource for NDE-SHM applications. Furthermore, with this
 231 database, a user may search whether for a particular spatial ultrasonic wavefront pattern
 232 corresponding to a particular material design. If the database does not have the details, the user would
 233 then have to carry out characterization process as shown in Fig. 2 in order to create new parametric
 234 equation(s) for the spatial ultrasonic wavefront of a plate-like structure with the initial settings based
 235 on the dependent factors.

236

237 *Process 1: Identification of Closed-form Curve*

238 Consider the creation of a new parametric equation for a spatial ultrasonic wavefront pattern of a
 239 plate-like specimen. The dependent factors' settings of the specimen are defined, and LUIS starts
 240 performing the full-field ultrasonic measurement to generate the UWI. The experimental setup with
 241 LUIS will be discussed in section 3.1. Once the measurement process is completed, the UWI of the
 242 specimen is generated and represented in the three-dimensional (3-D) volume data with 3-D spatial-
 243 temporal domain as shown in Fig. 1. Each UWI time frame (slice) is represented as a 2-D spatial
 244 wavefront image in the X - Y planar domain (described in this paper as \mathbb{S} -plane), with the horizontal
 245 and vertical axes of x and y respectively. Hence, with the spatial wavefront image at a time instant t ,
 246 the geometric plane curve associated with ultrasonic wavefront pattern may be identified in the \mathbb{S} -
 247 plane as shown in the center of Fig. 1.



248

249 **Figure 3.** Spatial ultrasonic wavefront characterization – Process 1.

250

251 After deciding the targeted spatial wavefront, e.g., the incident wavefront in the spatial wavefront
 252 image at t_2 in Fig. 3, the shape matching process is carried out first in Process 1. The shape matching
 253 process is to match the shape of the parametric curve (blue-shaded circle in the center of Fig. 3) to the
 254 planar curve (red-dots in the left-hand side of Fig. 3) associated to the desired wavefront pattern in the
 255 spatial wavefront image. Various fundamental geometric curves have been studied [20, 21] with well-
 256 defined parametric equations. The incident and reflected wavefront patterns of Lamb waves in a plate-

257 like structure show very reasonable association with these basic parametric curves (e.g. circle,
 258 hyperbola, etc.). Therefore, the shape matching processing can be carried out easily using these basic
 259 parametric curves with the spatial wavefront pattern. The parametric equations and curves are
 260 discussed further in section 3.2.2. Of course, when the complexity of the wavefront pattern is
 261 substantially increased (particularly in reflection patterns in geometrically complex anisotropic
 262 structures), they may not have a corresponding fundamental parametric equation, but parametrization
 263 of the wavefront pattern may still be performed by using a parametric curve fitting method [22, 23].
 264 The complexity of the wavefront pattern is highly affected by local impedance changes or scattering
 265 conditions (e.g. crack, lap-joints, etc.), but the current paper does not consider how the present method
 266 might be used in defect detection.

267
 268 Thus, the parametric curve (blue-shaded circle) can be identified based on the planar curve (red-dots)
 269 on the desired wavefront pattern in the left-hand side of Fig. 3. The corresponding parametric
 270 equations are then determined and expressed as $\tilde{x} = g'(\beta)$ and $\tilde{y} = h'(\beta)$ with the parameter β .
 271 Subsequently, the predicted parametric curve C' with the coordinates (\tilde{x}, \tilde{y}) in the 2-D-plane is
 272 generated and plotted as shown in the center of Fig. 3. The 2-D-plane is defined as \mathbb{P} -plane in this
 273 paper with the horizontal and vertical axes of \tilde{x} and \tilde{y} respectively. Basically, in this stage, the
 274 parametric curve plotted in the \mathbb{P} -plane is employed as a shape reference to assess the shape similarity
 275 to the desired wavefront pattern in \mathbb{S} -plane regardless of the size of the desired wavefront pattern. For
 276 that, the coordinates (\tilde{x}, \tilde{y}) are unitless and generated under the range of $-1 \leq \tilde{x} \leq 1$ and $-1 \leq \tilde{y} \leq 1$ in
 277 \mathbb{P} -plane. If the predicted parametric curve shows a weak resemblance to the desired wavefront pattern,
 278 then the shape matching process will repeat and reassesses the desired wavefront pattern with another
 279 type of parametric curve.

280
 281 After making a match to a target fundamental curve, the next step is to alter the size of the predicted
 282 parametric curve to match the size of the desired wavefront pattern. The two are depicted in separate
 283 planes, the \mathbb{P} -plane and \mathbb{S} -plane as shown in Fig. 3, with different units. Consequently, to execute the
 284 size matching operation, the parametric equations of the predicted parametric curve $C'(\beta)$ in the \mathbb{P} -
 285 plane need to be rescaled to the \mathbb{S} -plane as expressed below

$$286 \quad x = g(\beta) = \alpha_x g'(\beta) \quad \text{and} \quad y = h(\beta) = \alpha_y h'(\beta), \quad (2)$$

287 where the α_x and α_y are the scale factors of x and y , respectively. Therefore, a new curve $C(\beta)$ can be
 288 obtained based on the new parametric equations (Eqn. (2)). This new curve $C(\beta)$ is called a
 289 “characterized curve” in this paper. The parametric equations of the characterized curves, which are

290 corresponding to the incident and reflected waveforms, will be discussed in section 3.2.2. Of course,
291 the size of the incident wavefront increases as it emanates from a point source; nonetheless, the
292 originating pattern of the ultrasonic wavefront will not change, subject to the restrictions above (no
293 local strong defects or other major scatters, for example). The same is true of the reflecting wavefront,
294 although the geometric effects at the boundary will have induced a fundamental change compared to
295 the incident waves, but this will be considered later. Thus, during the matching process, the values of
296 the scale factors of x and y will be altered to fit the wavefront size without altering the expression of
297 the predicted parametric curve (C'). When each time the scale values are modified, the new
298 corresponding coordinates (x, y) of the characterized curve are produced. The characterized curve
299 (blue-shaded circle in the right-hand side of Fig. 3) is then superimposed on the desired wavefronts
300 (red-dots) in the S -plane. This superimposition method is performed to serve as a benchmark for
301 determining the values of the corresponding scale factors of the parametric equation of the
302 characterized curve.

303
304 As illustrated in Fig. 2, if the characterized curve is not matched to the target patterns, the procedure is
305 repeated by adjusting the parameter(s) value(s) accordingly until the shape and size of the
306 characterized curve is matched to some tolerance. If the size of the curve $C(\beta)$ (blue-shaded circle in
307 the right-hand side of Fig. 3) is matched to the planar curve (red-dots) of the desired spatial ultrasonic
308 wavefront pattern, it thus means that the spatial wavefront pattern with the set of coordinates (x, y) is
309 characterized and modeled in the parametric equation (Eqn. (2)) when the parameter β is determined
310 for an interval I . Lastly, the characterized curve is obtained based on this predicted parametric curve
311 equation with the corresponding parameter value(s) and scale factors.

312 313 *Process 2: Quantification of Parameterization*

314 After the characterized curve is obtained, the corresponding predicted parametric equations with the
315 value(s) of the parameter(s) are saved for future reference and coded into LUIS. Using the value(s) of
316 the parameter(s) obtained in Process 1, the LUIS generates the coordinates (x, y) of the characterized
317 curve and runs the scanning process on the specimen. LUIS performs a pixel scanning order that
318 exactly matches the characterized curve obtained in Process 1, which is of course the same as the
319 target spatial wavefront to be characterized by the spatial ultrasonic wavefront image. During the
320 scanning process, each local ultrasonic signal is measured accordingly along the curved scanning path,
321 and the measurements are then used to quantify the accuracy of the characterization process based on
322 the arrival times of the desired spatial wavefronts.

323
324 In general, the type of laser used in LUTs employed for scanning may be either a laser pulse, acting as
325 an actuator (ultrasonic generator), or a laser Doppler vibrometer (LDV), acting as a sensor (ultrasonic

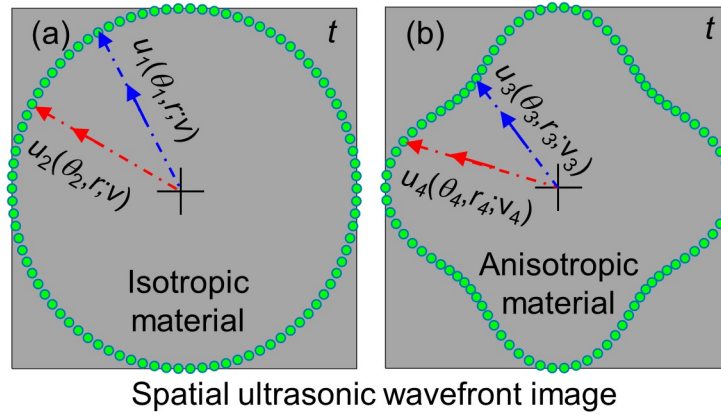
326 receiver). In this paper, LUIS exploits a laser pulse as an ultrasonic generator and a PZT-based sensor
327 as an ultrasonic receiver for Process 2 in the characterization process, but the current approach is not
328 specific to LUIS' architecture; more information on the specific LUIS configuration will be addressed
329 in section 3.1. Based on this selected configuration of LUIS, multiple ultrasonic signals are generated
330 along the characterized curve (C) path and then received at the PZT-based sensor. Usually, the sensor
331 is located at the center of the plate and inside the area bounded by the characterized curve so that
332 maximal lines of sight are maintained.

333
334 When the matched parametric curve scanning process is completed, the measurement set, defined as
335 $u(\theta_s, t)$, is reconstructed in the angle-time domain and depicted in a 2-D image with $M \times N$ grid
336 points, indexed by an angle θ_s (horizontal- θ -direction) and a time t (vertical-direction) respectively.
337 The angle θ_s is the propagation direction of the measured ultrasonic waves relative to a sensor located
338 at the center point in the \mathbb{S} -plane. Each ultrasonic signal $u(\theta_s, t)$ is digitized at a sampling angle $\Delta\theta_s$
339 and a sampling time T_s during the measurement in LUIS. The digitized ultrasound $u(\theta_s, t)$ is thus
340 referred as $u[m, k]$, where m is an index that assigns a number to each angle sample θ_s , ranging from
341 1 to M in θ -direction, and k is an index that assigns a number to each time sample t , ranging from 1 to
342 N . The sample angle θ_s is then expressed as $\theta_s = m\Delta\theta_s$ and the sample time t is expressed as $t = kT_s$.
343 The sample angle and the sample time are then simply represented as m and k , used subsequently in
344 the rest of the paper. The 2-D image representing the $u(\theta_s, t)$ or $u[m, k]$ in the angle-time domain is
345 known in this paper as a parameterized ultrasonic wavefront image (PUWI).

346
347 To summarize the proposed process, any wavefront propagates in a plate-like structure with either a
348 constant wave speed (isotropic material) or a direction-dependent wave speed (anisotropic material).
349 The pattern of the spatial wavefront that forms is unique to that material and is identifiable in the
350 spatial ultrasonic wavefront image as shown in Fig. 1. A snapshot in time, as in Fig. 1, corresponds to
351 a radial direction time-of-arrival time for that locus of points. For example, as shown in Fig. 4, the
352 same spatial wavefront (green dots) arrives at the same (t) in 2-D space for all the propagation
353 directions. Since it is an isotropic material, as shown in Fig. 4(a), the ultrasonic signals, u_1 and u_2 in
354 the 2-D space travel at the same speed (v) and arrive at time t with the same radius (r). But, in the
355 anisotropic material as shown in Fig. 4(b), the u_3 and u_4 in the 2-D space travel at the different speeds
356 (v_3 and v_4) and arrive at time t with different radii (r_3 and r_4). This shows that if the radius of the same
357 spatial wavefront in the spatial ultrasonic wavefront image (independent of material) is estimated
358 correctly, the arrival time of the corresponding wavefront will then be the same for all the propagation

359 directions. And, as mentioned in Process 1, this spatial wavefront is characterized and represented into
 360 a set of coordinates (x, y) which relate to these radii.

361



362

363 **Figure 4.** Spatial ultrasonic wavefront images at a time instant t for (a) isotropic and (b) anisotropic
 364 materials.

365

366 Therefore, the similarity of the laser parametric scanning curve on the specimen to the pattern of the
 367 spatial ultrasonic wavefront at a given time instant reflects the accuracy of the approach. And as such,
 368 if the laser scans with the characterized curve that precisely marks the same spatial wavefront
 369 positions, the resulting arrival times are the same, and the arrival time-deviation in all the propagation
 370 direction is zero. Hence, the mean arrival time-deviation ($\mu_{\Delta t}$) of the measured spatial wavefront
 371 (IWs and RWs) for an angle range (from θ_s^{\min} to θ_s^{\max}) is calculated to evaluate the similarity of the
 372 characterized curve to the actual spatial wavefront pattern. The subscript Δt is referred to as the
 373 arrival time-deviation at an angle θ_s with the condition of $\theta_s^{\min} \leq \theta_s \leq \theta_s^{\max}$. If the $\mu_{\Delta t}$ is not reached
 374 or is larger than the desired preset value (ideally is zero), then the characterized curve pattern has no
 375 clear resemblance to the pattern of the spatial wavefront. Consequently, the smaller the obtained $\mu_{\Delta t}$,
 376 the more accurate the parameter value(s) estimated.

377

378 To calculate $\mu_{\Delta t}$, a temporal cross-correlation technique is employed to calculate the arrival time-
 379 deviation Δt by cross-correlating the reference signal with the test signals. In this paper, the reference
 380 signal is set to the measured ultrasonic signal at the angle range center. The corresponding θ_s^{\min} and
 381 θ_s^{\max} are denoted as m_{\min} and m_{\max} with the conditions of $1 \leq m_{\min} \leq M$ and $1 \leq m_{\max} \leq M$. Then, the
 382 discrete-time cross-correlation is expressed as below:

383

$$C[m, \tau] = \sum_{k=k_{\min}}^{k_{\max}} u_{\text{Ref}}[m_{\text{Ref}}, k] u[m, k + \tau], \quad 0 \leq \tau \leq K - 1 \quad (3)$$

384 Where τ is the lag, and $u_{\text{Ref}}[m_{\text{Ref}}, k]$ and $u[m, k]$ are the reference signal and the test signal,
 385 respectively. The test signal is the measured ultrasounds as plotted on the PUWI. The m is bounded as
 386 $m_{\text{min}} \leq m \leq m_{\text{max}}$, and m_{Ref} is equaled to the center index value between m_{min} and m_{max} . Since the
 387 PUWI provides high visibility of the wavefront propagation in the 2-D angle-time domain, the time
 388 window ranging from t_{min} and t_{max} are defined as k_{min} and k_{max} respectively, and implemented in Eqn.
 389 (3).

390
 391 Once the $C[m, \tau]$ is acquired, the arrival time-deviation $\Delta t[m]$ can be determined by the maximum
 392 argument of $C[m, \tau]$,

$$393 \quad \Delta t[m] = \arg \max_{\tau} \{C[m, \tau]\}, \quad m_{\text{min}} \leq m \leq m_{\text{max}}, \quad (4)$$

394 and subsequently, the $\Delta t[m]$ is used to determine the mean arrival time-deviation $\mu_{\Delta t}$, or

$$395 \quad \mu_{\Delta t} = \frac{1}{M} \sum_{m_{\text{min}}}^{m_{\text{max}}} \Delta t[m]. \quad (5)$$

396 If the mean arrival time-deviation is calculated and deemed unsatisfactory for any reason, the
 397 similarity refining process will be repeated by updating the parameter values of the characterized
 398 curve equation. The refining process is carried out manually in this paper, but the process could be
 399 automated with a robust optimization technique in the future. Lastly, when the desired $\mu_{\Delta t}$ is obtained,
 400 the corresponding parameter(s) value(s), and the characterized curve equation are all stored to the
 401 LUIS for future reference, which may be of benefit to NDE-SHM applications. To align with the
 402 motivation of this paper, the parametric curve equation obtained in this characterization process may
 403 eventually be used to improve the laser ultrasonic inspection time based on the spatial scanning
 404 optimization approach for other materials, especially anisotropic material structure.

405 406 **3. IMPLEMENTATION OF SPATIAL WAVEFRONT CHARACTERIZATION METHOD**

407 **3.1 Experimental Setup**

408 A laser ultrasonic interrogation system (LUIS) described in detail previously [9] was again configured
 409 to perform the scanning on both an aluminum plate (isotropic material) and a cross-ply CFRP plate
 410 (anisotropic material) with the stacking sequence of $[0^\circ/90^\circ/0^\circ]_s$. The dimensions of the aluminum
 411 plate and the CFRP plate were 457.2 mm (H) \times 508.0 mm (W) \times 3.1 mm (T) and 500.0 mm (H) \times
 412 504.0 mm (W) \times 1.2 mm (T), respectively.

413
 414 Two experiments were carried out in this paper. One was performed to measure the propagation of the
 415 full-field ultrasonic wave for the generation of UWI based on the full-field scanning method; the other
 416 was performed to measure the propagation of the ultrasonic wave for the generation of PUWI based

417 on the parametric curve scanning method. During the scanning process, the pulse repetition rate (PRR)
418 of the LUIS was set to 50 Hz and 1 kHz to measure the aluminum plate and the CFRP plate
419 respectively. Since the attenuation coefficient of the aluminum plate is lower than the CFRP plate, the
420 PRR was set 50 Hz for the aluminum plate to avoid reverberation interference during the scanning
421 process. Each generated ultrasound was propagated from the local scan point and received at the
422 piezoelectric (PZT) sensor (S1) which located at the center of the rear scanning surface. The data
423 acquisition system then digitized each local ultrasound. The sampling time for the aluminum and
424 CFRP plates was set to $T_s = 0.2 \mu s$ (5 MHz) with total sample points $K = 1000$ and $K = 2000$,
425 respectively.

426
427 Since both specimens had not been characterized as described in Process 1, the measurement of the
428 full-field ultrasonic wave propagation must be executed first to generate UWI for use in the closed-
429 form curve identification process. For both aluminum and CFRP plates, the in-line bandpass filter was
430 set at 50 kHz low-cutoff frequency and 500 kHz high-cutoff frequency. Circle scanning was used for
431 the full-field measurement instead of raster scanning. The radius was set at the radius interval $\Delta r = 0.5$
432 mm from 20 mm to 220 mm and the angle θ_s was at the angle interval $\Delta\theta_s = 0.13^\circ$ from 0° to 360° .
433 After the scanning process was completed, the full-field ultrasonic data in polar coordinates with
434 $r \times \theta \times t$ matrix grids was converted into Cartesian coordinates with $X \times Y \times t$ matrix grids as shown
435 in Figs. 5 and 6. Because the ultrasonic measurement was performed at the high-resolution scanning
436 angle interval (0.13°) in this paper, there should be no significant variations between the UWI
437 obtained in this circle scanning method and the UWI obtained based on the raster scanning method.

438
439 The LUIS carried out the parametric curve scanning process in Process 2 based on the results of the
440 characterized curve obtained from in Process 1 as shown in Figs. 2 and 3. Two frequency bands of the
441 measured ultrasounds were considered for both plates at 50-500 kHz (broadband) and for only the
442 CFRP plate at 140-150 kHz (narrowband). The implementation of the proposed method (spatial
443 ultrasonic wavefront characterization) is carried out in the next section, and the related findings are
444 evaluated and discussed.

445 446 **3.2 Process 1 – identification of Closed-form Curve**

447 **3.2.1 Spatial Ultrasonic Wavefront Patterns**

448 Based on the flowchart (Fig. 2), the UWI generation is a requirement that must first be considered
449 when the characterized curve of the desired wavefront is not available in the database. So, the
450 ultrasonic wavefront patterns for the closed-form curve recognition process can be identified by
451 taking advantage of the UWI.

452

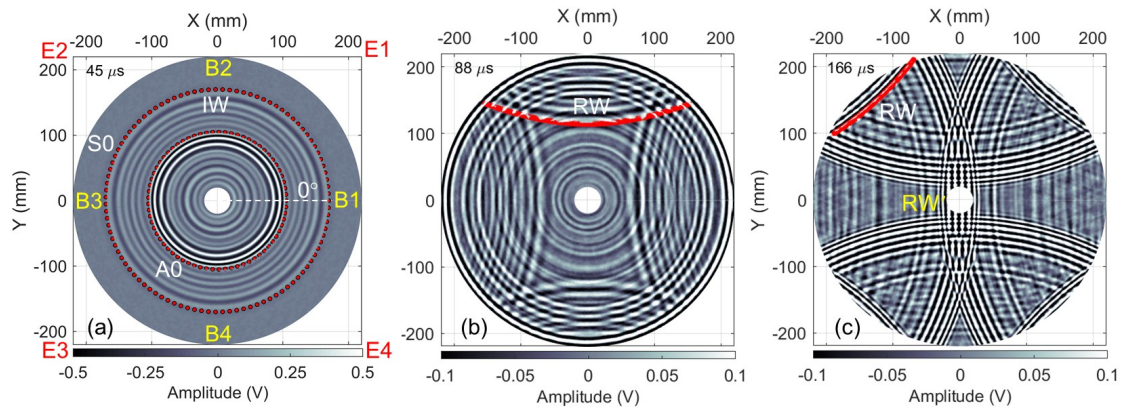
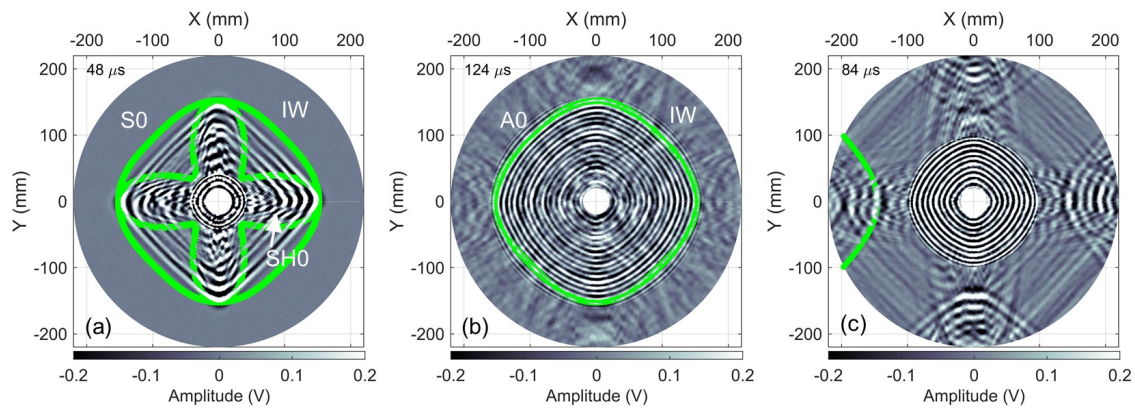


Figure 5. Spatial ultrasonic wavefront images of 3-mm thick aluminum plate at (a) 45 μs , (b) 88 μs , and (c) 166 μs .

As stated in section 3.1, the experimental setup for the full-field ultrasonic measurement was performed to produce the UWIs of the aluminum and CFRP plates, respectively. Figures 5 and 6 show the spatial ultrasonic wavefront images of the aluminum plate UWIs and the CFRP plate, respectively, in \mathbb{S} -plane. Assuming that reciprocity holds in this linear elastodynamic system [11], the ultrasonic waves are visualized as the wavefronts emanate over time from a sensor (“acting” as a point-source). Consequently, Figs. 5(a) and 6(a) show the IWs radially propagated out from the sensor S1 at the center point (0, 0) in the spatial wavefront image for all propagation directions. As both the plates were pristine, only the RWs were observed, which were induced at the boundaries and corners of the plates after the fastest IWs reflected. Commonly, the fastest IW is the fundamental symmetric Lamb mode (S0) followed by the fundamental anti-symmetric Lamb mode (A0) as indicated in Fig. 5(a).

At Fig. 5(a), the IWs (red-dotted circles) of the S0 and A0 modes propagated in the aluminum plate in a circular wavefront pattern. The circular wavefront pattern was shaped since each mode traveled from the sensor S1 at the same wave velocity in all propagation directions. As the S0 mode’s wave velocity is faster than the A0 mode, the S0 and A0 mode’s arrival distance is different at any time instant. Therefore, the IWs of the S0 and A0 modes arrived at a separate distance 170 mm and 105 mm respectively at the time instant 45 μs (Fig. 5(a)).

Then, the RWs induced by the S0 mode were created at 88 μs (Fig. 5(b)) from the boundaries B1, B2, B3, and B4. The B1 and B3 RWs were closer to the S1 (0, 0) than the B2 and B4 RWs because the aluminum plate shape was rectangular, which the B2 and B4 lengths were shorter than the B1 and B3 lengths. Figure 5(c) also shows the RWs induced by A0 mode traveling at 166 μs from the four corners E1, E2, E3, and E4 to the S1. The spatial wavefront image also displayed the reflected wavefronts from B1 (referred to as RW’ in Fig. 5(c)) passing through the S1 and moving to B3 when the time of ultrasonic measurement was long enough. It is found that the RWs’ wavefront patterns from the borders and corners were shaped in patterns of hyperbolic wavefronts.



484

485 **Figure 6.** Spatial ultrasonic wavefront images of cross-ply CFRP plate at (a) 48 μs , (b) 124 μs , and (c)
 486 84 μs .

487

488 Figure 6 shows the spatial ultrasonic wavefront images of the cross-ply CFRP plate ($[0^\circ/90^\circ/0^\circ]_s$).
 489 The CFRP plate is an anisotropic material with the directional-dependent physical properties that
 490 cause directional variation in guided wave propagation in space. Thus, as shown in Fig. 6(a), the
 491 wavefront patterns of the CFRP plate are (not surprisingly) more complex than the wavefront patterns
 492 of the aluminum plate for all the propagation directions. In Figs. 6(a) and (b), the wavefronts of the
 493 fundamental Lamb wave modes (S0, A0, SH0) were observed. The S0 mode had the highest wave
 494 velocity, and the A0 mode had the lowest. The SH0 mode was identified in the spatial wavefront
 495 image as well as between the S0 mode and A0 mode as shown in Fig. 6(a). Figures 6(a) and (b) show
 496 that the IWs (green-dots curves) of the S0 and A0 modes were propagated with the cyclic-harmonic
 497 curve patterns. The RWs propagated in the hyperbolic patterns (Figs. 6(c)) similar to the RWs in the
 498 aluminum plate.

499

500 The wavefront patterns in the spatial ultrasonic wavefront image were identified by reference to the
 501 above observation. The circle curve was found in the aluminum plate and was similar with the IWs.
 502 As for the CFRP plate, it found also that the cyclic-harmonic curve was identical to the IWs. On the
 503 other hand, the hyperbolic curve from the boundaries and corners of both plates was found identical to
 504 the RWs. Based on these observations, the description of the parametric curve equations of these
 505 described wavefront patterns (Figs. 5 and 6) were discussed next to obtain the characterized curves for
 506 the IWs and RWs.

507

508 3.2.2 Parametric Plane Curves

509 In this sub-section, the cyclic-harmonic curve is described first for the IWs of the S0 and A0 modes in
 510 both aluminum and CFRP plates. Then, the hyperbolic curve is described for the RWs.

511

512 *Cyclic-harmonic Curve (CHC)*

513 The parametric equation of cyclic-harmonic curve is selected for the parametric curve for the
 514 wavefronts patterns of the S0 and A0 modes in both aluminum and CFRP plates; and the
 515 corresponding parametric equation of CHC in \mathbb{P} -plane is expressed as

$$516 \quad \tilde{x} = \frac{\cos(\theta_p)}{1+p}(1+p \cos(n\theta_p + \varphi)) \quad \text{and} \quad \tilde{y} = \frac{\sin(\theta_p)}{1+p}(1+p \cos(n\theta_p + \varphi)), \quad (6)$$

517 where n is a number set to the total number of petals in a CHC. The petal is the based pattern of the
 518 CHC. The base pattern of the petal is also can be changed by setting the parameter p and the phase φ .
 519 The phase φ is defined as the rotation angle of the CHC.

520
 521 As stated in previous section 2.2, to obtain the parametric equation of the characterized curve, the
 522 unitless shape of the parametric curve C' must be predicted first. Thus, based on the circular form of
 523 the S0 and A0 modes' IWs in the aluminum plate, as illustrated in Fig. 5(a), the corresponding
 524 parametric equation in \mathbb{P} -plane was found by setting the p to zero in Eqn. (6) and the expression can
 525 be simply expressed as

$$526 \quad \tilde{x} = \cos(\theta_p) \quad \text{and} \quad \tilde{y} = \sin(\theta_p), \quad (7)$$

527 where the \tilde{x} and \tilde{y} are the coordinates of the unit circle with the angle θ_p ranging from 0° to 360° in
 528 the \mathbb{P} -plane. Subsequently, the size of the circle shape is matched to the IWs in \mathbb{S} -plane by fitting the
 529 scale factor to obtain the coordinates (x, y) of the characterized curve with the expression

$$530 \quad x = R \cdot \tilde{x} \quad \text{and} \quad y = R \cdot \tilde{y} \quad (8)$$

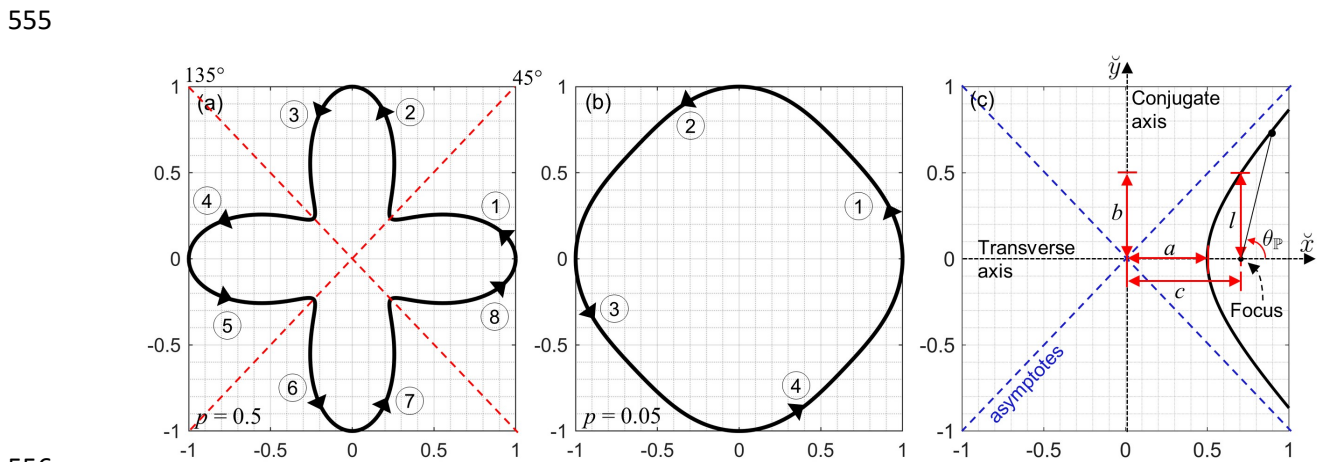
531 where, the R is the scale factor and it is a radius as defined in \mathbb{S} -plane for the desired IW. Then, the
 532 resulting circle is drawn on the desired IWs in the \mathbb{S} -plane to match the size of the produced circle
 533 with the desired IWs in \mathbb{S} -plane.

534
 535 In this work, the spatial ultrasonic wavefront image at $45 \mu\text{s}$ (Fig. 5(a)) was considered, and at this
 536 time the IWs of the S0 and A0 modes arrived at the radius of 170 mm and 100 mm respectively. For
 537 this, the radii of $R = 170 \text{ mm}$ and $R = 100 \text{ mm}$ were set separately into Eqn. (8) to generate two
 538 different coordinates (x, y) of the circles respectively with the values of \tilde{x} and \tilde{y} obtained from Eqn.
 539 (7). The circles (red dots) were then drawn at 170 mm over the IW of the S0 mode and at 100 mm
 540 over the A0 mode as shown in Fig. 5(a). Figure 5(a) demonstrated that, at the desired radius, the
 541 characterized curves fit well with the IWs of the S0 and A0 modes.

542
 543 In the case of the aluminum plate, the characterized curve obtained in Process 1 is simple, where the
 544 parametric equation requires only the parameter θ_p and the scale factor, R , to be considered in this

545 paper. Next, the characterized curves at the R (170 mm and 100 mm) and the corresponding
 546 parametric equations (Eqns. (7) and (8)) were coded and stored for Process 2 to further quantify the
 547 similarity of the characterized curve to the desired spatial wavefront pattern. Process 2
 548 implementation will be described further in section 3.3.

549
 550 Next, as shown in the wavefront patterns in the CFRP plate (Figs. 6(a) and (b)), the cyclic-harmonic
 551 curves (Figs. 7(a) and (b)) are selected as the parametric curves for the wavefronts patterns of the S0
 552 and A0 modes. Figure 7(a) shows that the CHC is revolved at the counterclockwise direction (arrows
 553 from 1 to 8) with four petals ($n = 4$) at $\varphi = 0$. The curve, for $0 < p < 1$, is converged at an offset
 554 distance from the $(0, 0)$ when the θ_p at $\pm 45^\circ$ and $\pm 135^\circ$ as shown in Fig. 7(a).



556
 557 **Figure 7.** Parametric plane curves of cyclic-harmonic curve ((a) $p = 0.5$ and (b) $p = 0.05$) and (c)
 558 hyperbolic curve on the \mathbb{P} -plane.
 559

560 By observing Fig. 6(a), the IWs of the S0 mode in the cross-ply CFRP formed patterns like the shapes
 561 in Figs. 7(a) and (b). Figure 7(a) shows that the CHC at $p = 0.5$ has a strong resemblance to the IWs
 562 of the S0 mode in the fiber direction region (Fig. 6(a)). On the other hand, Fig. 7(b) shows that the
 563 CHC at $p = 0.05$ has a strong resemblance to the IWs of the S0 mode in the non-fiber direction region
 564 (Fig. 6(a)). Then, Fig. 7(b) also shows that the CHC at $p = 0.05$ has the strong resemblance to the IWs
 565 of the A0 mode in non-fiber and fiber direction regions (Fig. 6(b)). As a result, in the next step, the
 566 CHC as defined in Eqn. (6) was further used to match the size in Figs. 7(a) and (b) to the size of the
 567 IWs of the S0 and A0 modes in Figs. 6(a) and (b) to obtained the characterized curve respectively.

568
 569 To match the size of the CHC to the size of the desired IWs, the coordinates (x, y) of the
 570 characterized curve in \mathbb{S} -plane can be generated by using Eqn. (8), except that the \tilde{x} and \tilde{y} obtained
 571 from Eqn. (6) are substituted into Eqn. (8). The radius R of Eqn. (8) is then referred to as the distance
 572 between the center point of the plate and the peak points of the petal at $\pm 90^\circ$ and $\pm 180^\circ$ in the \mathbb{S} -plane.

573 Commonly, a parametric curve equation may be expressed with multiple parameters, like this case.
 574 To ease the confusion, the parameters and scale factors are grouped into a set, denoted as $\mathbf{P}\{\bullet\}$. Then,
 575 $\{\bullet\}$ is the elements of the set representing the parameters and scale factors of the characterized curve
 576 equation. Therefore, the parameters (n, p, φ) and scale factor (R) of the characterized curve are written
 577 as $\mathbf{P}_{\text{CHC}}\{n, p, \varphi, R\}$ with the angle range of $0^\circ \leq \theta_p \leq 360^\circ$.

578
 579 With Eqns. (6) and (8), the characterized curves of the IWs of the S0 and A0 modes at $48 \mu\text{s}$ and 124
 580 μs were obtained with $\mathbf{P}_{\text{CHC}}\{4, 0.5, 0^\circ, 150\}$ and $\mathbf{P}_{\text{CHC}}\{4, 0.05, 0^\circ, 150\}$ as shown in Figs. 6(a) and (b)
 581 respectively. The characterized curves (green-dots) illustrated that the curves were closely aligned
 582 with the wavefronts of S0 and A0 modes. Next, these parametric equations (Eqns. (6) and (8)) with
 583 the \mathbf{P}_{CHC} of the characterized curves were saved and coded in LUIS for Process 2 to further quantify
 584 the similarity performance.

585
 586 *Hyperbolic Curve (HC)*
 587 The patterns of the reflected wavefronts differ from the pattern of the incident wavefronts. Figures
 588 5(b)(c) and 6(c) show that the reflected wavefronts of the aluminum plate and the cross-ply CFRP
 589 plate were both similar to a hyperbolic curve, rather than a cyclic curve. The parametric equation of
 590 the hyperbolic curve is expressed below with the angle θ_p ranging from -90° to 90° :

$$591 \quad \tilde{x} = \left(\frac{l \cos(\theta_p)}{1 - e \cos(\theta_p)} \right) + c \quad \text{and} \quad \tilde{y} = \left(\frac{l \sin(\theta_p)}{1 - e \cos(\theta_p)} \right), \quad (9)$$

592 where, $\tilde{x} \geq 0$, $a > 0$, and

$$593 \quad l = \frac{b^2}{a}, \quad e = \sqrt{1 + \frac{b^2}{a^2}}, \quad \text{and} \quad c = \sqrt{a^2 + b^2}. \quad (10)$$

594 Figure 7(c) shows the hyperbolic plot on the \mathbb{P} -plane with the transverse axis (\tilde{x}) and the conjugate
 595 axis (\tilde{y}). The transverse axis is the chord or the line through the focus, and the conjugate axis is the
 596 line through the center perpendicular to the transverse axis. The a and b are the semi-major and semi-
 597 minor axes on the \tilde{x} and \tilde{y} axes respectively. In this plot (Fig. 7(c)), the a and b are set to 0.5 with
 598 the condition of $-90^\circ < \theta_p < 90^\circ$. The l is known as the semilatus rectum, the e is the eccentricity of
 599 the hyperbola ($e > 1$), and the c is the distance of the focus from the center.

600
 601 To match the size of the RWs in the \mathbb{S} -plane, the base hyperbolic curve $\tilde{x} \geq 0$ in \mathbb{P} -plane as shown in
 602 Fig. 7(c) shall be considered first with the corresponding coordinates (\tilde{x}, \tilde{y}) generated from Eqn. (9)
 603 with the condition of $-90^\circ < \theta_p < 90^\circ$. Then, in order to match the size of the desired RWs, the

604 generated coordinates (\tilde{x}, \tilde{y}) are employed into the parametric equation of the characterized curve in
 605 \mathbb{S} -plane as expressed below:

$$606 \quad \begin{bmatrix} x \\ y \end{bmatrix} = \hat{R}(\Omega) \begin{bmatrix} w_x \tilde{x} + A_{\text{offset}} \\ w_y \tilde{y} \end{bmatrix} \quad \text{and} \quad \hat{R}(\Omega) = \begin{bmatrix} \cos(\Omega) & -\sin(\Omega) \\ \sin(\Omega) & \cos(\Omega) \end{bmatrix}, \quad (11)$$

607 where $\hat{R}(\Omega)$ is the rotation matrix at the rotation angle (Ω) , and A_{offset} is the offset distance of the
 608 concave of the characterized curve along the x -axis in \mathbb{S} -plane. The radius (R) of the center plate at the
 609 concave of the characterized curve is equaled to $aw_x + A_{\text{offset}}$ in \mathbb{S} -plane. In the expression, w_x and w_y
 610 are the half-length of the width and height of the plate respectively. Then, the characterized curve is
 611 generated with the conditions of $|x| \leq w_x$ and $|y| \leq w_y$. In this paper, the parameters and the scale
 612 factor of the characterized curve are grouped and denoted as $\mathbf{P}_H \{a, b, \Omega, A_{\text{offset}}\}$. With the coordinates
 613 (x, y) obtained from Eqn. (11), the corresponding polar coordinates (θ_s, r_s) of the characterized
 614 curve in the \mathbb{S} -plane also can be obtained as well as expressed below:

$$615 \quad \theta_s = \tan^{-1}(y/x) \quad \text{and} \quad r_s = \sqrt{x^2 + y^2}. \quad (12)$$

616
 617 For the aluminum plate, using Eqns. (9) and (11), the characterized curves were obtained at the
 618 $\mathbf{P}_H \{0.5, 0.68, 90^\circ, 0\}$ and $\mathbf{P}_H \{0.5, 1, 132^\circ, 80\}$ for the RWs at the time instants of 88 μs (Fig. 5(b)) and
 619 166 μs (Fig. 5(c)) respectively. Then, for the CFRP plate, the characterized curve was obtained at the
 620 $\mathbf{P}_H \{0.5, 0.35, 180^\circ, 20\}$ for the RWs at the time instants of 84 μs (Fig. 6(c)). Lastly, the parameters and
 621 corresponding parametric equations (Eqns. (9)-(11)) of the characterized curves are saved and coded
 622 in LUIS to further quantify the similarity of the characterized curves of both plates.

623 624 **3.3 Process 2 – Quantification of Parameterization**

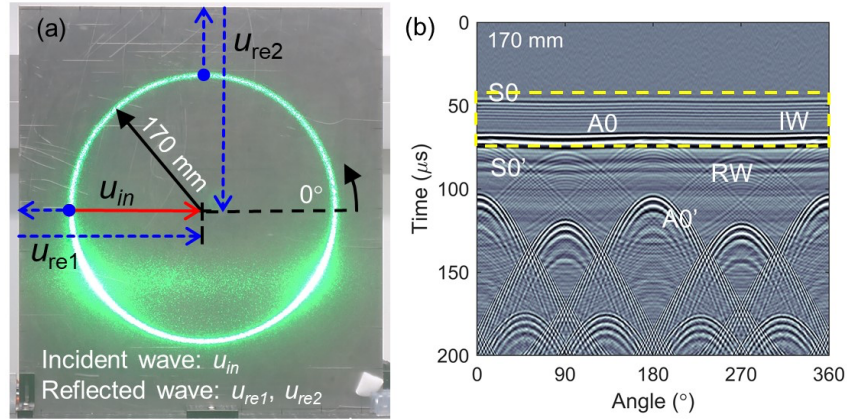
625 **3.3.1 Incident Wavefront Characterization**

626 Previously, the results of Process 1 were discussed, and the parametric equations of the characterized
 627 curves (circle, cyclic-harmonic curve, and hyperbolic curve) were determined and coded into the
 628 LUIS for Process 2. Next, this section will address the outcomes of Process 2.

629 630 *Aluminum Plate*

631 First, the coordinates (x, y) of the characterized curve was regenerated in LUIS based on the results
 632 obtained in Process 1. LUIS used these coordinates as an initial setting and performed the parametric
 633 curve scanning process to measure the ultrasonic signals. Hence, in aluminum plate case, the IWs of
 634 the S0 and A0 modes in the spatial ultrasonic wavefront image at the time instant of 45 μs (Fig. 5(a))

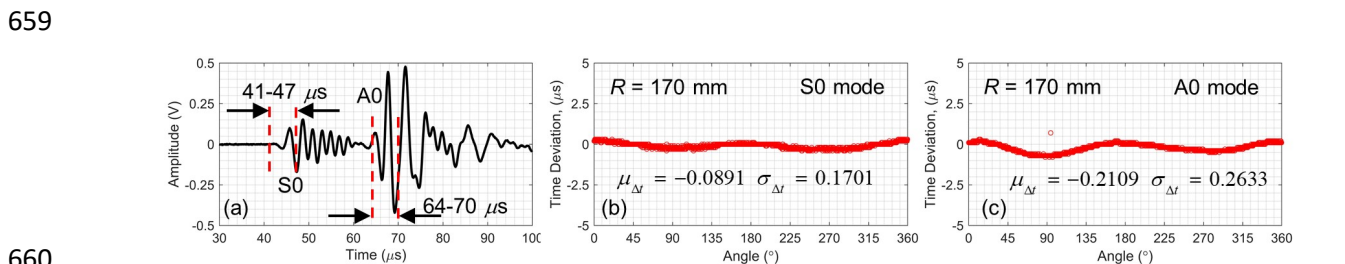
635 was considered in Process 2. Since the characterized curve was identified as a circle in Process 1, the
 636 LUIS performed the circle scan with the parametric curve equations (Eqns. (7) and (8)) at $R = 170$
 637 mm to characterize the IWs of the S0 mode.
 638



639 **Figure 8.** (a) Laser circle scanning pattern and (b) corresponding ultrasonic waves on the aluminum
 640 plate at $R = 170$ mm.
 641
 642

643 Figure 8(a) shows the laser circle scanning pattern on the aluminum plate at the radius of 170 mm.
 644 The circle coordinates (x, y) at $R = 170$ mm were generated based on Eqns. (7) and (8) in LUIS. With
 645 these coordinates, the LMS maneuvered the laser beam to impinge periodically at each scan point on
 646 the plate to generate local ultrasonic waves. The corresponding ultrasonic signals were measured with
 647 the in-line bandpass filter with the low-cutoff frequency of 50 kHz and the high-cutoff frequency of
 648 500 kHz. The low shutter speed of the camera was set during the circle scanning process to provide
 649 enough time to capture a full circle scan as shown in Fig. 8(a). This camera setting was also
 650 configured to capture the other laser scanning curves in later part.

651
 652 After the scanning process, the ultrasonic signals were measured and represented in PUWI with the
 653 angle-time domain as shown in Fig. 8(b). Ideally, if the wavefronts of the incident waves from all the
 654 directions are the same, these wavefronts should also travel at the same speed. Thus, the arrival time-
 655 deviation of the directions will be zero, and the same wavefronts will be displayed in PUWI in
 656 multiple straight lines along the angle-axis in PUWI. As shown in Fig. 8(b), the yellow-shaded box
 657 shows that all IWs arrived in the S0 mode at $45 \mu\text{s}$ and in A0 mode at $64 \mu\text{s}$, and were shaped as the
 658 lines along the angle-axis.



660

661 **Figure 9.** (a) Time history of reference ultrasonic signal ($u_{\text{Ref}}(\theta_{\text{S}}^{\text{Ref}}, t)$) at $\theta_{\text{S}}^{\text{Ref}} = 180^\circ$, arrival time-
662 deviation of the ultrasonic responses ranging from 0° to 360° for (b) S0 mode and (c) A0 mode.
663

664 Based on Eqn. (3), the digitized reference ultrasonic signal $u_{\text{Ref}}[m_{\text{Ref}}, k]$ at the index $m_{\text{Ref}} = 1068$
665 ($\theta_{\text{S}}^{\text{Ref}} = 180^\circ$) was selected, as shown in Fig. 9(a), to calculate the arrival time-deviation for the IWs of
666 the S0 and A0 modes. Figure 9(a) shows the time-history of the u_{Ref} from the PUWI in Fig. 8(b). To
667 measure the arrival time-deviation Δt and mean arrival time-deviation $\mu_{\Delta t}$, the angle range and time
668 window (t_w) were therefore set accordingly as tabulated in Table 1.
669

670 **Table 1.** Summary of angle range ($\Delta\theta_{\text{S}}$) and time window (t_w) assigned to determine arrival time
671 deviation (Δt) and mean arrival time-deviation ($\mu_{\Delta t}$) for IWs characterization in S0 and A0 modes.

Mode	$\Delta\theta_{\text{S}}$ for Δt		Time Window, t_w		$\Delta\theta_{\text{S}}$ for $\mu_{\Delta t}$	
	$\theta_{\text{S}}^{\text{min}}(m_{\text{min}})$	$\theta_{\text{S}}^{\text{max}}(m_{\text{max}})$	$t_{\text{min}}(k_{\text{min}})$	$t_{\text{max}}(k_{\text{max}})$	$\theta_{\text{S}}^{\text{min}}(m_{\text{min}})$	$\theta_{\text{S}}^{\text{max}}(m_{\text{max}})$
S0	0° (1)	360° (2136)	41 (205)	47 (235)	0° (1)	360° (2136)
A0	0° (1)	360° (2136)	64 (320)	70 (350)	0° (1)	360° (2136)

672
673 Figures 9(b) and (c) show that the arrival time-deviations for both S0 and A0 modes were
674 approximated to the zero level. The $\mu_{\Delta t}$ of the S0 mode's IW was obtained at $-0.0891 \mu\text{s}$ with the
675 standard deviation $\sigma_{\Delta t}$ of $0.1701 \mu\text{s}$; and the $\mu_{\Delta t}$ of the A0 Mode's IWs was obtained at $-0.2109 \mu\text{s}$
676 with the standard deviation $\sigma_{\Delta t}$ of $-0.2633 \mu\text{s}$.
677

678 As an aside, in Fig. 8(b) it is noted that the RWs in $0^\circ/360$ and 180° directions traveled faster than the
679 RWs (denoted as S0') in 90° and 270° directions and interfered with the IWs of the A0 mode in the
680 $64\text{-}70 \mu\text{s}$ time window. The difference in arrival times in the reflected waves is because they traveled
681 in a rectangular aluminum plate that had different boundary lengths. It shows that the PUWI can be an
682 alternative to UWI to analyze how reflected waves interact with the boundaries in the angle-time
683 domain, which may be a useful feature to investigate the shape of a target in the NDE-SHM
684 applications.
685

686 Since the ultrasonic wave at 180° was taken as the reference signal for Eqn. (3) to cross-correlate to
687 the test signals, the magnitudes of the arrival time-deviations in the region approximately 90° and
688 270° are slightly higher than the other directions for the S0 and A0 modes (Figs. 9(b) and (c)), which
689 has caused high values in the standard deviations of both cases. In A0 mode, however, the standard

690 deviation is higher than the S0 mode. It is primarily attributable to the wavefronts of the reference
691 signal ($64 \mu\text{s}$ to $70 \mu\text{s}$) at 180° for A0 mode, which has interfered with the reflected waves (S0') and
692 varies from the wavefronts of the A0 mode at about 90° and 270° .

693

694 In this isotropic case (aluminum plate), the characterization of the IWs for both modes is
695 straightforward and only one parameter θ_p and one scale factor R need to be considered. In addition,
696 there is no need repetition of the scanning process with new updated parameters to improve the
697 accuracy of the characterized curve. Subsequently, the laser circle scanning curve can easily
698 characterize the IWs of the S0 and A0 modes with the condition where the sensor (S1) must be in the
699 center of the circle. From a practical point of view, the contact sensor can be easily replaced with a
700 non-contact sensor, such as the laser Doppler vibrometer (LDV) improve the implementation mobility.

701

702 *CFRP Plate*

703 In this subsection, the spatial ultrasonic wavefront characterization of the cross-ply CFRP plate was
704 examined. The parametric CHC scan was performed to quantify the accuracy of the characterized
705 curve with the parameters as predicted in Process 1. The CHC scanning process may be repeated with
706 new updated parameter set to improve the accuracy of the characterized curve. The scanning process
707 was then conducted further to verify the repeatability at various radii. The scanning process thus
708 started in this paper with the radius (R) at first 150 mm, second at 80 mm, and third at 200 mm.

709

710 The experimental setup for the scanning process was configured like the case for the aluminum plate.
711 The LUIS started the scanning process at the initial scanning point of $\theta_s = 0^\circ$ that was in parallel to
712 the fiber direction 90° (yellow arrow in Fig. 10). Each ultrasonic signal with the total signal duration
713 of $400 \mu\text{s}$ was then measured and sampled at 5 MHz. At the same \mathbf{P}_{CHC} , the LUIS did the parametric
714 scan twice to measure the ultrasonic signals in the in-line bandpass filter at two separate settings. The
715 first measurement was set at the narrowband frequency of 140-160 kHz, and the second measurement
716 at the broadband frequency of 50-500 kHz. The narrowband case and the broadband case are referred
717 to as 140-160 kHz and 50-500 kHz respectively and are used in the rest of the discussion in this
718 section. The total rounded scan points (L) of the complete scanned curve are expressed as
719 $L = 2\pi R / \Delta S$, where ΔS is the interval arc-length between the two scan points. Hence, the L of 1885,
720 1005, and 2513 were obtained for the LUIS to conduct the scanning process in relation to the radius R
721 of 150 mm, 80 mm, and 200 mm with the ΔS of 0.5 mm.

722

723 Since the IWs of the S0 and A0 modes in a cross-ply CFRP plate are typically symmetrical in the
724 direction of the fiber (0° and 90°), the IWs in the angle ranges, from 90° to 180° and from 135° to

725 225°, were only considered in this paper to assess the accuracy of the characterized curve for the
726 incident wavefronts. Tables 2 and 3 provide the summary of the variables settings in Eqns. (3) and (4)
727 to calculate the arrival time-deviation accordingly. Then, two regions of the cross-ply CFRP plate,
728 non-fiber direction region ($105^\circ \leq \phi_s \leq 165^\circ$) and fiber direction region ($165^\circ \leq \phi_s \leq 195^\circ$), were
729 preliminary assigned in this paper to determine the mean arrival time-deviation via Eqn. (5). Authors
730 are aware that there may be variations in the angle ranges of both directions as per the cross-ply CFRP
731 plate being made.

732

733 **Table 2.** Summary of angle range $90^\circ \leq \theta_s \leq 180^\circ$ and time window set to determine arrival time-
734 deviation (Δt) at various radius R for IW characterization in S0 and A0 modes. (*Narrowband case
735 and **Broadband case)

R (mm)	Angle Range		Time Window, t_w			
	$\theta_s^{\min} = 90^\circ$	$\theta_s^{\max} = 180^\circ$	S0 Mode		A0 Mode	
	m_{\min}	m_{\max}	$t_{\min}(k_{\min})$	$t_{\max}(k_{\max})$	$t_{\min}(k_{\min})$	$t_{\max}(k_{\max})$
150	471	943	45(225)*	60(300)*	130(650)*	145(725)*
			45(225)**	60(300)**	125(625)**	140(700)**
80	251	503	32(160)*	55(275)*	80(400)*	95(475)*
			32(160)**	45(225)**	75(375)**	90(450)**
200	628	1257	52(260)*	70(350)*	160(800)*	175(875)*
			52(260)**	70(350)**	150(750)**	165(825)**

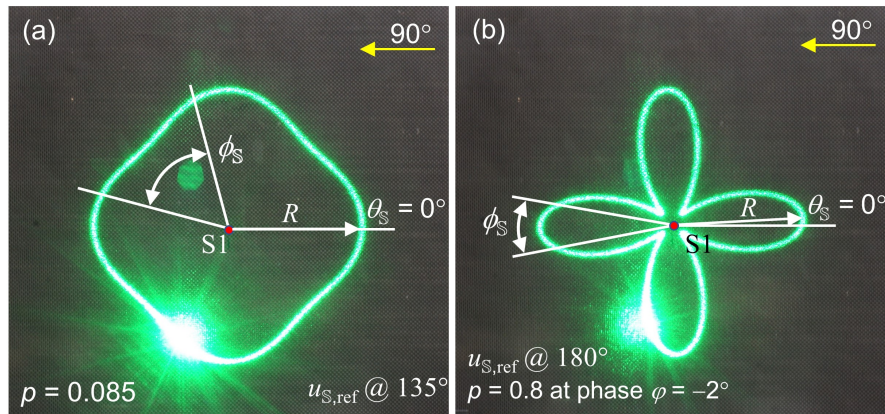
736

737 **Table 3.** Summary of angle range $135^\circ \leq \theta_s \leq 225^\circ$ and time window set to determine arrival time-
738 deviation (Δt) at various radius R for the IW characterization in S0 and A0 modes. (*Narrowband
739 case and **Broadband case)

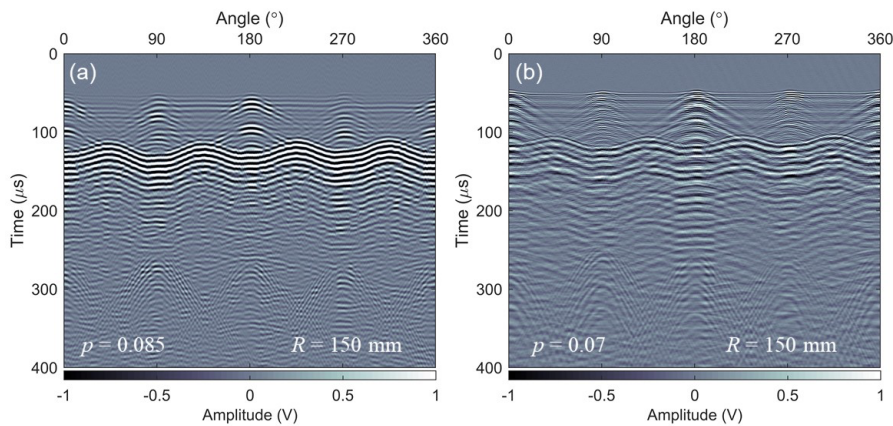
R (mm)	Angle Range		Time Window, t_w			
	$\theta_s^{\min} = 135^\circ$	$\theta_s^{\max} = 225^\circ$	S0 Mode		A0 Mode	
	m_{\min}	m_{\max}	$t_{\min}(k_{\min})$	$t_{\max}(k_{\max})$	$t_{\min}(k_{\min})$	$t_{\max}(k_{\max})$
150	706	1177	45(225)*	65(325)*	125(625)*	140(700)*
			40(200)**	55(275)**	120(600)**	135(675)**
80	377	628	33(165)*	55(275)*	80(400)*	95(475)*
			30(150)**	45(225)**	70(350)**	85(425)**
200	942	1571	50(250)*	70(350)*	163(815)*	178(890)*
			50(250)**	65(325)**	150(750)**	165(825)**

740

741 The LUIS conducted the parametric CHC scanning at the initial scanning process according to the
742 $\mathbf{P}_{\text{CHC}} \{4, 0.05, 0^\circ, 150\}$ and $\mathbf{P}_{\text{CHC}} \{4, 0.5, 0^\circ, 150\}$ of the characterized curves obtained from the Process 1
743 to characterize the IWs of S0 mode in non-fiber and fiber direction regions, respectively. The
744 scanning process was then repeated with a new updated \mathbf{P}_{CHC} until the $\mu_{\Delta t}$ reach a minimum value
745 (ideally, zero). Hence, the lower the mean arrival time-deviation $\mu_{\Delta t}$, the more precise the
746 characterized curve to the desired spatial wavefront pattern is. As mentioned earlier in section 2, the
747 updating process on the \mathbf{P}_{CHC} was done manually. Finally, the updated $\mathbf{P}_{\text{CHC}} \{4, 0.085, 0^\circ, 150\}$ and
748 $\mathbf{P}_{\text{CHC}} \{4, 0.8, -2^\circ, 150\}$ were acquired to characterize the IWs of the S0 mode in non-fiber and fiber
749 direction regions respectively, and the corresponding patterns of the laser scanning curve were
750 captured as shown in Figs. 10(a) and (b).
751



752
753 **Figure 10.** Cyclic-harmonic scanning curves on cross-ply CFRP plate at updated parameters (a)
754 $\mathbf{P}_{\text{CHC}} \{4, 0.085, 0^\circ, 150\}$ and (b) $\mathbf{P}_{\text{CHC}} \{4, 0.8, -2^\circ, 150\}$ for IWs of S0 mode in non-fiber and fiber
755 direction regions respectively.
756

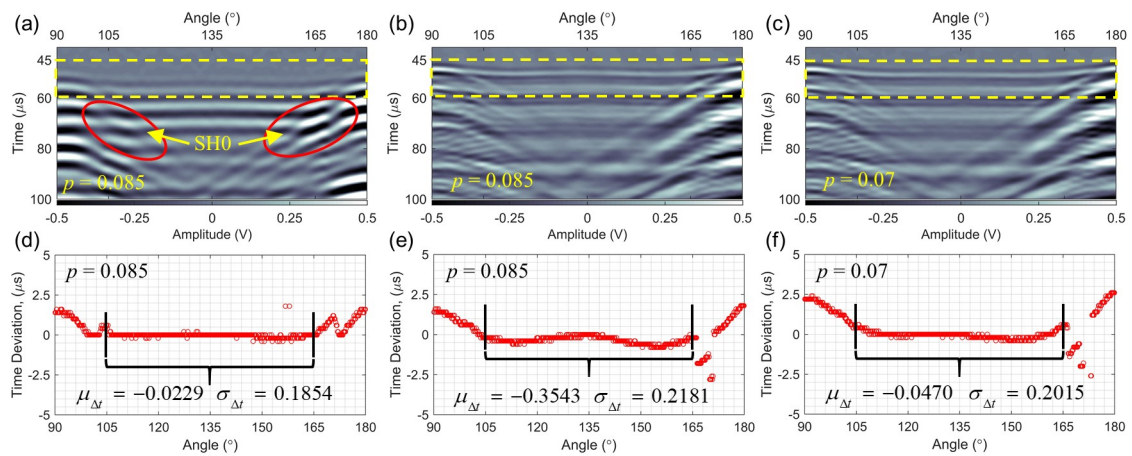


757
758 **Figure 11.** Ultrasonic responses in non-fiber direction region with (a) 140-160 kHz and (b) 50-500
759 kHz based on cyclic-harmonic scanning curves at $\mathbf{P}_{\text{CHC}} \{4, 0.085, 0^\circ, 150\}$ and $\mathbf{P}_{\text{CHC}} \{4, 0.07, 0^\circ, 150\}$
760 respectively.
761

762 Figures 11(a) and (b) show the corresponding measured ultrasounds in the non-fiber direction region
763 with the narrowband frequency and broadband frequency at $\mathbf{P}_{\text{CHC}}\{4,0.085,0^\circ,150\}$ and
764 $\mathbf{P}_{\text{CHC}}\{4,0.07,-2^\circ,150\}$, respectively. Figure 12(a) displays the zoomed PUWI of the ultrasonic
765 responses from 90° to 180° in Fig. 11(a) with the time window from $40\ \mu\text{s}$ to $100\ \mu\text{s}$. By observing
766 the zoomed PUWI, the IWs of the SH0 mode (as indicated in the red ellipses) surface about 105° to
767 165° in the non-fiber direction region. This also indicates that the waves of the S0 mode were starting
768 to separate from the SH0 mode since the S0 mode has higher wave speed than the SH0 mode. Then,
769 the IWs in the fiber direction region were dominated by the S0 mode. Based on Eqns. (3) and (4) with
770 the settings in Table 2, the respective arrival time-deviations with the reference signal u_{Ref} at 135°
771 ($m_{\text{Ref}} = 706$) were determined as shown in Fig. 12(d). The mean arrival time-deviation was then
772 determined by Eqn. (5) and the results were tabulated in Table 4.

773
774 In the case of narrowband inputs, the characterized IWs of the S0 mode at the $\mathbf{P}_{\text{CHC}}\{4,0.085,0^\circ,150\}$
775 were obtained with the $\mu_{\Delta t} = -0.0229\ \mu\text{s}$ and the standard deviation $\sigma_{\Delta t}$ of $0.1854\ \mu\text{s}$. It shows that
776 the laser CHC scan at $\mathbf{P}_{\text{CHC}}\{4,0.085,0^\circ,150\}$ is capable of characterizing the same wavefronts of S0
777 mode in non-fiber direction region with only the maximum absolute deviation (MAD), expressed as
778 $\arg \max \{|\mu_{\Delta t} \pm \sigma_{\Delta t}|\}$, of $0.2083\ \mu\text{s}$ as equivalent to just one sample time (T_s) of the measurement used
779 in the experiment. In contrast, the arrival time-deviations determined in the direction ranges of
780 $90^\circ < \theta_s < 105^\circ$ and $165^\circ < \theta_s < 180^\circ$ were dispersed away from the zero level. Therefore, the laser
781 CHC scan at $\mathbf{P}_{\text{CHC}}\{4,0.085,0^\circ,150\}$ demonstrated better ability to characterize the same IWs of the S0
782 mode in the non-fiber direction region with $105^\circ \leq \phi_s \leq 165^\circ$ compared to fiber-direction region.

783



784

785 **Figure 12.** Ultrasonic responses with frequency bands of (a) 140-160 kHz and (b)(c) 50-500 kHz
786 ranging from 90° to 180°, and corresponding arrival time-deviations (d) and (e)(f) respectively, based
787 on the laser CHC scan at $\mathbf{P}_{\text{CHC}}\{4, 0.085, 0^\circ, 150\}$ and $\mathbf{P}_{\text{CHC}}\{4, 0.07, 0^\circ, 150\}$.
788

789 **Table 4.** Summary of mean arrival time-deviation for IW characterization in S0 mode using laser
790 CHC scanning method with narrowband of 140 kHz-160 kHz.

$\mathbf{P}_{\text{CHC}}\{n, p, \varphi, R\}$	Non-fiber direction region		Fiber direction region		MAD (μs)
	$\mu_{\Delta t}$	$\sigma_{\Delta t}$	$\mu_{\Delta t}$	$\sigma_{\Delta t}$	
$\{4, 0.085, 0^\circ, 150\}$	-0.0229	0.1854	-	-	0.2083
$\{4, 0.085, 0^\circ, 80\}$	-0.0012	0.3522	-	-	0.3534
$\{4, 0.07, 0^\circ, 200\}$	-0.0076	0.1467	-	-	0.1543
$\{4, 0.8, -2^\circ, 150\}$	-	-	-0.0174	0.7256	0.7430
$\{4, 0.8, -2^\circ, 80\}$	-	-	-0.0476	0.6480	0.6956
$\{4, 0.8, -2^\circ, 200\}$	-	-	-0.3286	0.9135	1.2421

791
792 In the case of broadband, the same $\mathbf{P}_{\text{CHC}}\{4, 0.085, 0^\circ, 150\}$ was first used to characterize the IWs of the
793 S0 mode as shown in Fig. 12(b). The corresponding arrival time-deviation was determined as shown
794 in Fig. 12(e) with the $\mu_{\Delta t}$ of $-0.3543 \mu\text{s}$ and the $\sigma_{\Delta t}$ of $0.2181 \mu\text{s}$. It reveals that the laser CHC
795 scanning at $\mathbf{P}_{\text{CHC}}\{4, 0.085, 0^\circ, 150\}$ was less accurate in the broadband case (50-500 kHz) as compared
796 to the narrowband case. Therefore, the scanning process was reiterated and halted at the new updated
797 parameters $\mathbf{P}_{\text{CHC}}\{4, 0.07, 0^\circ, 150\}$ (Fig. 10(b)), and the corresponding ultrasounds were measured as
798 shown in Fig. 11(b). Figures 12(c) and (f) show the zoomed PUWI of Fig. 11(b) and the
799 corresponding arrival time-deviation, respectively.

800
801 At this new updated $\mathbf{P}_{\text{CHC}}\{4, 0.07, 0^\circ, 150\}$, the accuracy of the arrival time-deviation measurement
802 was improved with $\mu_{\Delta t} = -0.047 \mu\text{s}$ and $\sigma_{\Delta t} = 0.2015 \mu\text{s}$ (as tabulated in Table 5), compared to
803 previous results at $\mathbf{P}_{\text{CHC}}\{4, 0.085, 0^\circ, 150\}$ (Fig. 12(e)). This shows that the measurement of the same
804 IWs of the S0 mode in the broadband case (50-500 kHz) was improved by a minor adjustment the
805 parameter of $p = 0.085$ to $p = 0.07$. The MAD was determined at $0.2485 \mu\text{s}$ (Table 5) and showed
806 small difference to the MAD of the narrowband case (Table 4).

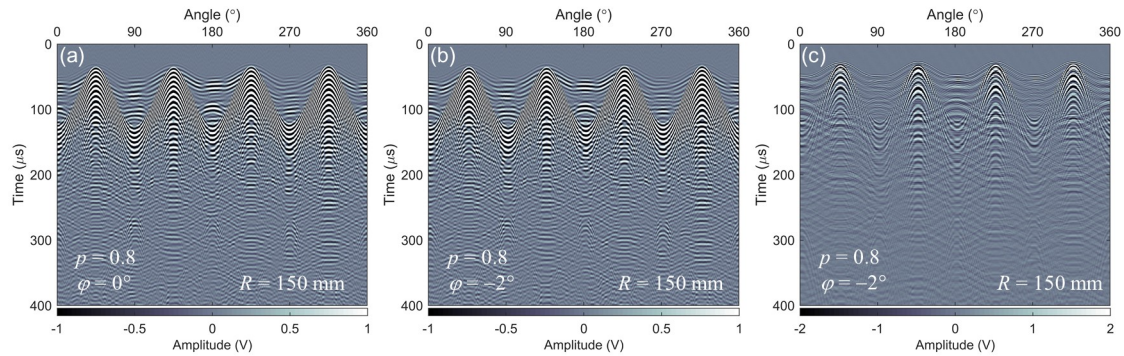
807
808 **Table 5.** Summary of mean arrival time-deviation for IW characterization in S0 mode using laser
809 CHC scanning method with broadband of 50 kHz-500 kHz.

$\mathbf{P}_{\text{CHC}}\{n, p, \varphi, R\}$	Non-fiber direction region		Fiber direction region		MAD (μs)
	$\mu_{\Delta t}$	$\sigma_{\Delta t}$	$\mu_{\Delta t}$	$\sigma_{\Delta t}$	
$\{4, 0.07, 0^\circ, 150\}$	-0.0470	0.2015	-	-	0.2485
$\{4, 0.07, 0^\circ, 80\}$	-0.3148	0.3877	-	-	0.7025
$\{4, 0.07, 0^\circ, 200\}$	-0.1919	0.2761	-	-	0.4680
$\{4, 0.8, -2^\circ, 150\}$	-	-	-0.1165	0.9602	1.0767
$\{4, 0.8, -2^\circ, 80\}$	-	-	-0.0738	0.9718	1.0456
$\{4, 0.8, -2^\circ, 200\}$	-	-	-0.1905	1.1083	1.2988

810

811 The laser CHC scan showed the positive ability not only to characterize the IWs of the S0 mode with
812 the narrowband frequency but also with the broadband frequency in the non-fiber direction region
813 $105^\circ \leq \phi_s \leq 165^\circ$. However, as shown in Figs. 12(d) and (f), in the fiber direction regions
814 $90^\circ \leq \theta_s < 105^\circ$ and $165^\circ < \theta_s \leq 180^\circ$, both \mathbf{P}_{CHC} were unable to characterize the IWs of the S0 mode.
815 Regarding this, in this paper, another parametric scanning curve was considered to characterize the
816 IWs of the S0 mode in fiber direction region $165^\circ \leq \phi_s \leq 195^\circ$. By referring to the spatial ultrasonic
817 wavefront image in Fig. 6(a), the LUIS carried out the CHC scanning curve on the initial parameters
818 $\mathbf{P}_{\text{CHC}}\{4, 0.5, 0^\circ, 150\}$ of the characterized curve obtained in Process 1 and the updating process stopped
819 at the new $\mathbf{P}_{\text{CHC}}\{4, 0.8, -2^\circ, 150\}$.

820



821

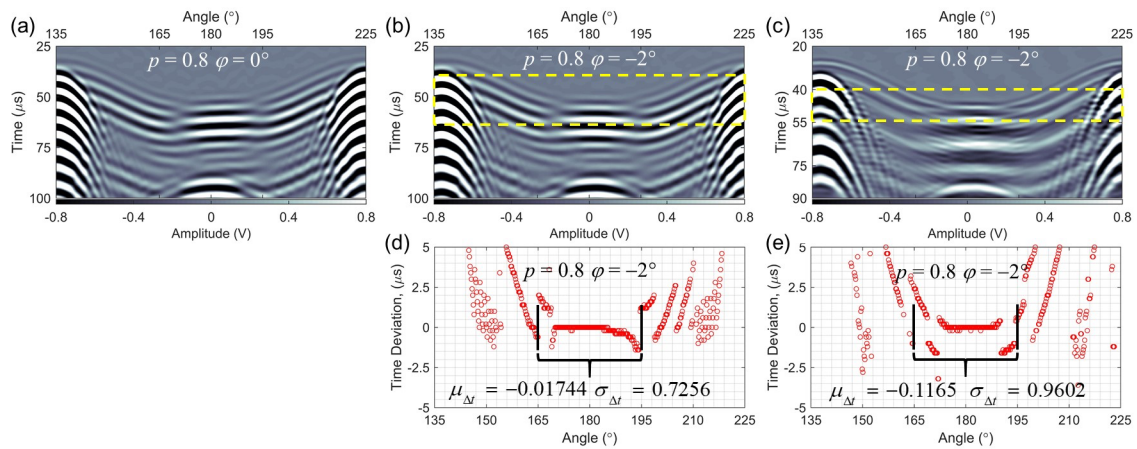
822 **Figure 13.** Ultrasonic responses with the frequency bands of (a)(b) 140-160 kHz and (c) 50-500 kHz
823 based on cyclic-harmonic scanning curves at (a) $\mathbf{P}_{\text{CHC}}\{4, 0.8, 0^\circ, 150\}$ and (b)(c) $\mathbf{P}_{\text{CHC}}\{4, 0.8, -2^\circ, 150\}$
824 respectively.
825

826 Before the measured ultrasound at $\mathbf{P}_{\text{CHC}}\{4, 0.8, -2^\circ, 150\}$ is examined, it is important to discuss the
827 measured ultrasound at $\mathbf{P}_{\text{CHC}}\{4, 0.8, 0^\circ, 150\}$ as shown in Figs. 13(a) and 14(a). Figure 14(a) shows the
828 enlarged Fig. 13(a) ranging from 135° to 225° . The patterns of the measured wavefronts in the fiber

829 direction region were different, compared to previous case in the non-fiber direction region at
830 $\mathbf{P}_{\text{CHC}} \{4, 0.085, 0^\circ, 150\}$ (Fig. 11). Looking at Fig. 14(a), the IWs of the S0 mode did not arrive at the
831 same time in the fiber direction region ($165^\circ \leq \phi_s \leq 195^\circ$). In fact, the arrival time of the same IWs
832 was gradually shorter starting from 165° to 195° and the arrival time-deviations formed a titled line
833 with a slanted angle. This result could imply that there were fiber stacking variations in this local
834 region during the manufacture of the CFRP plate. For this, the $\mathbf{P}_{\text{CHC}} \{4, 0.8, 0^\circ, 150\}$ was updated to
835 $\mathbf{P}_{\text{CHC}} \{4, 0.8, -2^\circ, 150\}$ by altering the phase φ from 0° to -2° to Eqn. (6), and the rotated laser CHC
836 scanning curve by -2° is shown in Fig. 10(b).

837
838 Figure 13(b) shows the corresponding PUWI (narrowband) ranging from 0° to 360° at the updated
839 $\mathbf{P}_{\text{CHC}} \{4, 0.8, -2^\circ, 150\}$, and the zoomed version, ranging from 135° to 225° , is plotted as shown in Fig.
840 14(b). By comparing the wavefronts in Figs. 14(a) and (b), it illustrates that the rotated CHC at $\varphi =$
841 -2° was able to capture the IWs of the S0 mode arrive at the same arrival time. In the case of
842 broadband, the $\mathbf{P}_{\text{CHC}} \{4, 0.8, -2^\circ, 150\}$ was also used in the LUIS for the measurement and the
843 corresponding PUWI is plotted as shown in Figs. 13(c) and 14(c).

844



845
846 **Figure 14.** Ultrasonic responses in narrowband case based on laser cyclic-harmonic curve scan at (a)
847 $\mathbf{P}_{\text{CHC}} \{4, 0.8, 0^\circ, 150\}$ and (b) $\mathbf{P}_{\text{CHC}} \{4, 0.8, -2^\circ, 150\}$. (c) Ultrasonic responses in broadband case. (d)(e)
848 Arrival time-deviations of narrowband and broadband respectively.
849

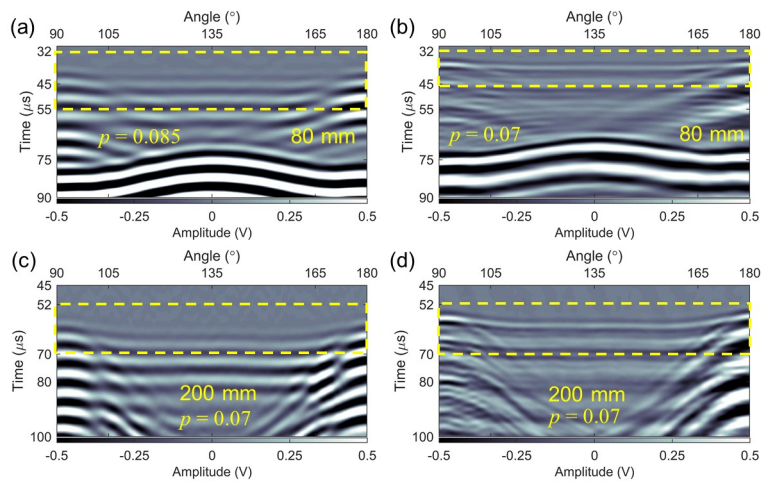
850 For the narrowband and broadband cases, respectively, the u_{Ref} at 180° ($m_{\text{Ref}} = 942$) was selected to
851 assess the arrival time-deviations with the angle range and time window settings as tabulated in Table
852 3. Figures 14(d) and (e) indicate the arrival time-deviations for the cases of narrowband and
853 broadband, respectively. Both cases show that the values of the arrival time-deviations in the fiber
854 direction region (165° to 195°) were approximately zero. The arrival time-deviations between 165°
855 and 195° as shown in Figs. 14(d) and (e) were then introduced into Eqn. (5) to calculate the mean

856 arrival time-deviations. Tables 4 and 5 indicate the results of the mean arrival time-deviation for the
 857 cases of narrowband and broadband, respectively.

858
 859 In the narrowband case, the $\mu_{\Delta t}$ was determined to be $-0.01744 \mu s$ with a $\sigma_{\Delta t}$ of $0.7256 \mu s$. Then, in
 860 the broadband case, the $\mu_{\Delta t}$ was determined to be $-0.1165 \mu s$ with a $\sigma_{\Delta t}$ at $0.9602 \mu s$. The MAD in
 861 Tables 4 and 5 also shows that the narrowband case ($0.7430 \mu s$) has better estimate than the
 862 broadband case ($1.0767 \mu s$). But, both MADs in the case of fiber direction region are higher than the
 863 case of non-fiber region. As a result, the characterization of IWs of S0 mode in the case of fiber
 864 direction region is less accurate compared to the case of non-fiber region.

865
 866 Even though $\mu_{\Delta t}$ was close to zero in the fiber direction region, the corresponding $\sigma_{\Delta t}$ is high in the
 867 cases of narrowband and broadband. The high standard deviations are primarily due to the incorrect
 868 estimate of the arrival time-deviations from 165° to 169.5° and from 185.5° to 195° for the
 869 narrowband case, and from 165° to 174° and from 189° to 195° for the broadband case. These
 870 incorrect estimates are due to the separation of the S0 and SH0 modes at these directions, as shown in
 871 Figs. 14(b) and (c). Thus, for the cases of narrowband and broadband, respectively, the laser CHC
 872 scan with the $\mathbf{P}_{\text{CHC}} \{4, 0.8, -2^\circ, 150\}$ demonstrates the ability to characterize the IWs of the S0 mode in
 873 the fiber direction region ranging only from 169.5° to 185.5° and 174° to 189° in this paper. It reveals
 874 that the parameter $\mathbf{P}_{\text{CHC}} \{4, 0.8, -2^\circ, 150\}$ is able to characterize the S0 mode in the fiber direction
 875 region with the angular range of about 16° and 15° for the narrowband and broadband cases
 876 respectively. It is noted that the angular range can vary as may depend upon the nature of the
 877 specimen being manufactured.

878



879

880 **Figure 15.** Ultrasonic responses with frequency bands of (a)(c) 140-160 kHz and (b)(d) 50-500 kHz
881 ranging from 90° to 180° at the cyclic-harmonic scanning curves at (a) $\mathbf{P}_{\text{CHC}} \{4, 0.085, 0^\circ, 80\}$, (b)
882 $\mathbf{P}_{\text{CHC}} \{4, 0.07, 0^\circ, 80\}$, and (c)(d) $\mathbf{P}_{\text{CHC}} \{4, 0.07, 0^\circ, 200\}$.
883

884 Next, in order to further check the accuracy of the parameters against the change in radius, the same
885 elements of $\mathbf{P}_{\text{CHC}} \{4, 0.085, 0^\circ, R\}$ obtained in the non-fiber direction region (Fig. 12) were tested at the
886 different radius R (80 mm and 200 mm). Hence, the LUIS initially performed the CHC scanning
887 processes with the modified parameters $\mathbf{P}_{\text{CHC}} \{4, 0.085, 0^\circ, 80\}$ and $\mathbf{P}_{\text{CHC}} \{4, 0.085, 0^\circ, 200\}$ with the
888 same experimental setup of 150 mm. Noted that the \mathbf{P}_{CHC} elements were remained the same, but the R
889 was changed to 80 mm and 200 mm for the narrowband and broadband cases, respectively. In the
890 narrowband case, after the updating process for $R = 80$ mm, the \mathbf{P}_{CHC} remained unchanged at
891 $\mathbf{P}_{\text{CHC}} \{4, 0.085, 0^\circ, 80\}$, but in the broadband case, the \mathbf{P}_{CHC} was updated to $\mathbf{P}_{\text{CHC}} \{4, 0.07, 0^\circ, 80\}$. Then,
892 for $R = 200$ mm, the \mathbf{P}_{CHC} was updated to $\mathbf{P}_{\text{CHC}} \{4, 0.07, 0^\circ, 200\}$ for both narrowband and broadband
893 cases.
894

895 Figures 15(a) and (b) show the measured IWs of the S0 mode at 80 mm for the narrowband and
896 broadband frequencies, respectively. With the reference signal u_{Ref} at 135° ($m_{\text{Ref}} = 377$) and the
897 angle range and time windows as tabulated in Table 2, the corresponding arrival time-deviations of
898 the IWs were determined. Then, the mean arrival time-deviations from 105° to 165° were determined
899 and tabulated in Tables 4 and 5.
900

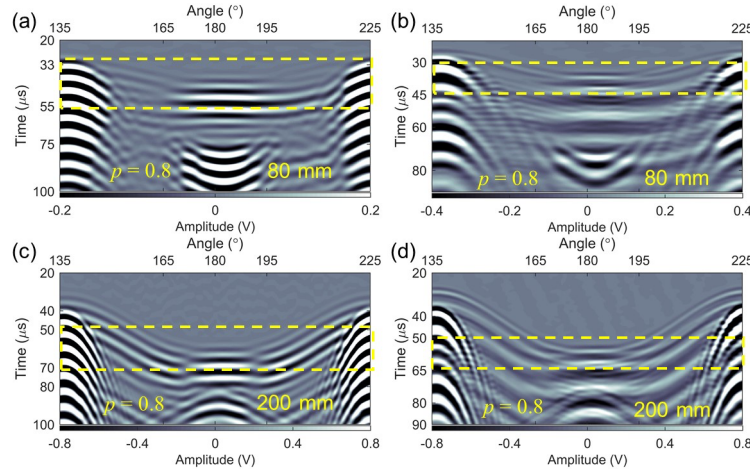
901 In the narrowband case, the $\mu_{\Delta t}$ was obtained to be $-0.0012 \mu\text{s}$ with a $\sigma_{\Delta t}$ of $0.3522 \mu\text{s}$. It shows that
902 the $\mathbf{P}_{\text{CHC}} \{4, 0.05, 0^\circ, 80\}$ was able to measure the same IWs of the S0 mode from 105° to 165° in the
903 non-fiber direction region. In the broadband case, the $\mu_{\Delta t}$ was found to be $-0.3148 \mu\text{s}$ with a $\sigma_{\Delta t}$ of
904 $0.3877 \mu\text{s}$. Both standard deviations are almost the same and are attributed to the wrong measurement
905 in 105° to 110° and 159° to 165°. It is found that the $\mu_{\Delta t}$ was obtained to be $-0.3148 \mu\text{s}$ in the case
906 of broadband that is substantially different from the $\mu_{\Delta t}$ in the case of narrowband at $-0.0012 \mu\text{s}$.
907 Since both standard deviations are about the same (Tables 4 and 5), the high $\mu_{\Delta t}$ obtained in the
908 broadband case is due to the fact that the selected reference signal at 135° traveled faster than the test
909 signals in the non-fiber direction region. As the result, the MAD is obtained by an additional 98.9% at
910 $0.7025 \mu\text{s}$ and higher than the narrowband case.
911

912 Figures 15(c) and (d) show the measured IWs of the S0 mode at 200 mm for the narrowband and
 913 broadband frequencies, respectively. The corresponding arrival time-deviations of the IWs were
 914 determined with the reference signal u_{Ref} at 135° ($m_{\text{Ref}} = 943$) and the angle range and time windows
 915 as shown in Table 2. The mean arrival time-deviations from 105° to 165° were then calculated and
 916 tabulated in Tables 4 and 5.

917
 918 In the case of the narrowband input, the $\mu_{\Delta t}$ was determined to be $-0.0076 \mu\text{s}$ with a $\sigma_{\Delta t}$ at -0.1467
 919 μs , and in the case of broadband, the $\mu_{\Delta t}$ was determined to be $-0.1919 \mu\text{s}$ with a $\sigma_{\Delta t}$ at $0.2761 \mu\text{s}$.
 920 The MADs were obtained at $0.1543 \mu\text{s}$ and $0.4680 \mu\text{s}$ for the narrowband and broadband cases,
 921 respectively. Based on MAD in Tables 4 and 5, in all three separate radii, the narrowband case
 922 demonstrated better characterization of the S0 mode IWs as compared to the broadband case.

923
 924 Next, the IWs of the S0 mode at the radii of 80 mm and 200 mm were considered in the fiber
 925 direction region. Both cases in the narrowband and broadband frequencies were used the same
 926 elements of the parameter ($n = 4$, $p = 0.8$, $\varphi = -2^\circ$) as employed in previous 150 mm case. The
 927 experimental setup for the cases of 80 mm and 200 mm was same to the case of 150 mm except that
 928 the radius in LUIS was altered accordingly.

929



930
 931 **Figure 16.** Ultrasonic responses with frequency bands of (a)(c) 140-160 kHz and (b)(d) 50-500 kHz
 932 ranging from 135° to 225° based on the cyclic-harmonic scanning curves at (a)(b)
 933 $\mathbf{P}_{\text{CHC}} \{4, 0.8, -2^\circ, 80\}$ and (c)(d) $\mathbf{P}_{\text{CHC}} \{4, 0.8, -2^\circ, 200\}$.

934
 935 Figures 16(a) and (b) show the measured IWs of the S0 mode at the $\mathbf{P}_{\text{CHC}} \{4, 0.8, -2^\circ, 80\}$ for the cases
 936 of narrowband and broadband respectively. With the reference signal u_{Ref} at 180° ($m_{\text{Ref}} = 503$), the
 937 corresponding arrival time-deviations of the IWs were determined according to the angle range and
 938 the time window as tabulated in Table 3. The mean arrival time-deviations were then calculated and

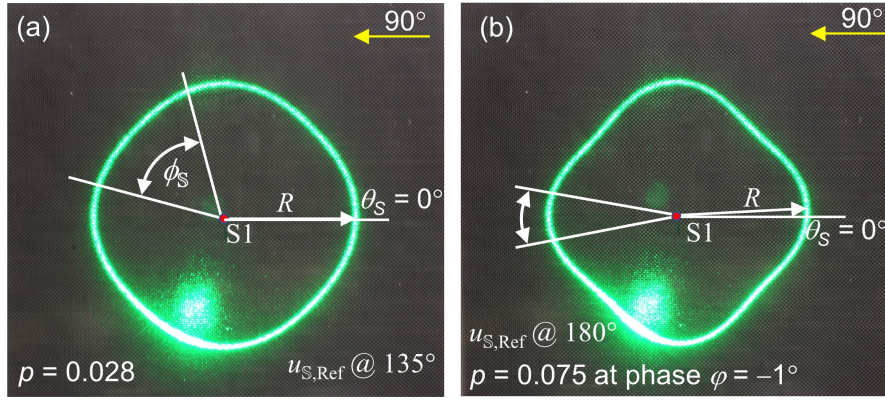
939 tabulated in Tables 4 and 5. The $\mu_{\Delta t}$ was obtained as $-0.0476 \mu s$ with a $\sigma_{\Delta t}$ of $-0.6480 \mu s$ in the case
940 of narrowband and $-0.0738 \mu s$ with a $\sigma_{\Delta t}$ of $-0.9718 \mu s$ in the case of broadband. The corresponding
941 MADs were determined at $0.6956 \mu s$ and $1.0456 \mu s$ for the cases of narrowband and broadband
942 respectively.

943
944 Figures 16(c) and (d) show the measured IWs of the S0 mode at the radius of 200 mm with the
945 narrowband and broadband frequencies respectively at the $\mathbf{P}_{\text{CHC}} \{4, 0.8, -2^\circ, 200\}$. The corresponding
946 arrival time-deviations of the IWs were determined with the reference signal u_{Ref} at 180°
947 ($m_{\text{Ref}} = 1257$), the angle range and the time windows as tabulated in Table 3 for both narrowband and
948 broadband cases respectively. Then, the mean arrival time-deviations were determined and tabulated
949 in Tables 4 and 5. The $\mu_{\Delta t}$ for both frequency bands were obtained to be $-0.3286 \mu s$ and $-0.1905 \mu s$
950 and the standard deviations are $-0.9135 \mu s$ and $-1.1083 \mu s$, respectively. The corresponding MADs
951 were determined at $1.2421 \mu s$ and $1.2988 \mu s$.

952
953 Table 4 shows that the updated \mathbf{P}_{CHC} has a better characterization of the S0 mode IWs in the
954 narrowband frequency compared to the broadband frequency (Table 5), especially in non-fiber
955 direction region. The laser CHC scanning method also shows the ability to characterize the IWs of the
956 S0 mode even at different radius. In addition, the accuracy of the characterization presented in Tables
957 4 and 5 demonstrated that it is highly dependent upon the selected reference signal. The results also
958 show that the laser CHC scan was sensitive to the quality of the wavefronts. If the wavefronts are
959 contaminated due to mode separation (Fig. 12) or change in localized properties (Fig. 14(a)), the
960 arrival time-deviations would be incorrectly obtained. Although the proposed method is sensitive to
961 the conditions of the IWs, it is also beneficial as a guide to show the location of the contaminated
962 wavefronts where it is possible to avoid the placement of the sensor. Based on the parameters
963 obtained for S0 mode in the above studies, the proposed method was able to characterize the spatial
964 wavefront with the promising results based on the mean arrival time-deviation and MAD.

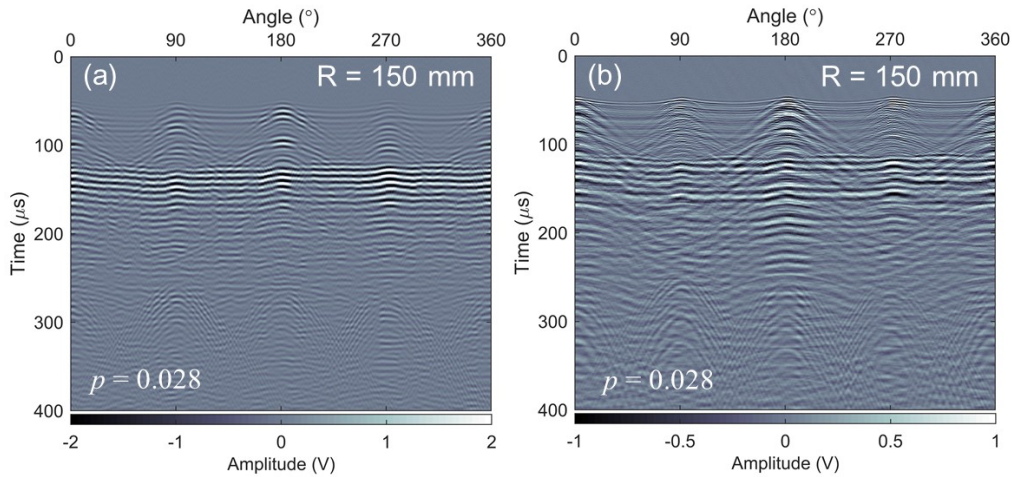
965
966 Next, the proposed method is further test on the spatial wavefronts of the A0 mode. The incident
967 wavefront of the A0 mode in the non-fiber direction region was considered and followed by the
968 incident wavefront of the A0 mode in fiber direction region. These two were also tested for the
969 narrowband and broadband cases at radii 150 mm, 80 mm, and 200 mm. The experimental
970 configurations were not changed and remained the same as performed in the previous case of the S0
971 mode.

972



973
 974 **Figure 17.** Cyclic-harmonic scanning curves on cross-ply CFRP plate at updated parameters (a)
 975 $\mathbf{P}_{\text{CHC}} \{4, 0.028, 0^\circ, 150\}$ and (b) $\mathbf{P}_{\text{CHC}} \{4, 0.07, -1^\circ, 150\}$ for IWs of A0 mode.
 976

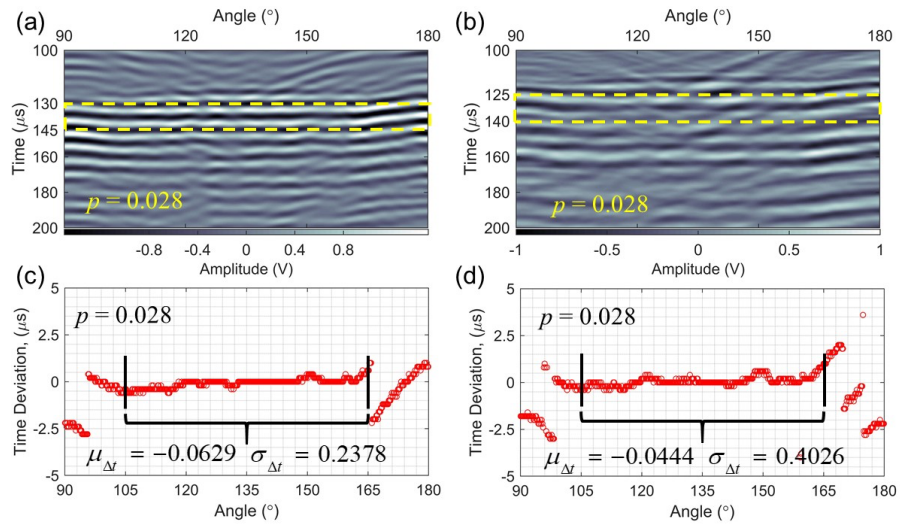
977 Based on Fig. 6(b) in Process 1, the characterized curve with the $\mathbf{P}_{\text{CHC}} \{4, 0.05, 0^\circ, 150\}$ was used as
 978 the initial settings for the LUIS to perform the laser CHC scan to measure the ultrasonic signals. The
 979 LUIS repeated the scanning process and then it stopped at the updated parameter
 980 $\mathbf{P}_{\text{CHC}} \{4, 0.028, 0^\circ, 150\}$ with low mean arrival time-deviation as desired to characterize the IWs of the
 981 A0 mode in the non-fiber direction region (105° to 165°). Figure 17(a) shows the corresponding laser
 982 cyclic-harmonic scanning curves at the updated parameter $\mathbf{P}_{\text{CHC}} \{4, 0.028, 0^\circ, 150\}$.
 983



984
 985 **Figure 18.** Ultrasonic responses with (a) 140-160 kHz and (b) 50-500 kHz based on the cyclic-
 986 harmonic scanning curves at $\mathbf{P}_{\text{CHC}} \{4, 0.028, 0^\circ, 150\}$.
 987

988 Figures 18(a) and (b) show the PUWI at the updated $\mathbf{P}_{\text{CHC}} \{4, 0.028, 0^\circ, 150\}$, and Figs. 19(a) and (b)
 989 show the zoomed PUWI of Figs. 18(a) and (b), respectively. Based on Eqns. (3) and (4), for the
 990 narrowband and broadband cases, the arrival time-deviations were determined with the reference
 991 signal u_{Ref} at 135° ($m_{\text{Ref}} = 706$), the angle range, and the time windows as tabulated in Table 2. Then,

992 the mean arrival time-deviations in the non-fiber direction region were determined via Eqn. (5) and
 993 the results were tabulated in Tables 6 and 7 for both cases.
 994



995
 996 **Figure 19.** Ultrasonic responses with frequency bands of (a) 140-160 kHz and (b) 50-500 kHz
 997 ranging from 90° to 180° , and (c)(d) corresponding arrival time-deviations respectively, based on the
 998 cyclic-harmonic scanning curves at $\mathbf{P}_{\text{CHC}} \{4, 0.028, 0^\circ, 150\}$.
 999

1000 Figure 19(c) shows that the $\mu_{\Delta t}$ was obtained to be $-0.0629 \mu\text{s}$ with a $\sigma_{\Delta t}$ of $0.2378 \mu\text{s}$ in the case of
 1001 narrowband. Although the IWs of the A0 mode at 150 mm were more contaminated than the IWs of
 1002 the S0 mode (Fig. 12), the parameter $\mathbf{P}_{\text{CHC}} \{4, 0.028, 0^\circ, 150\}$ was still capable to characterizing the
 1003 IWs of the A0 mode in the non-fiber direction region. On the other hand, the arrival time-deviation
 1004 began to deviate from the non-fiber direction region to the fiber direction region and it demonstrates
 1005 that the $\mathbf{P}_{\text{CHC}} \{4, 0.028, 0^\circ, 150\}$ was unable to characterize the IWs of the A0 mode in the fiber
 1006 direction region.

1007
 1008 In the broadband case (Fig. 19(d)), the $\mu_{\Delta t}$ was found to be $-0.0444 \mu\text{s}$ with a $\sigma_{\Delta t}$ of $0.4026 \mu\text{s}$ as
 1009 shown in Table 7. The arrival time-deviation shows similar scenarios to the narrowband case, which
 1010 deviated from the non-fiber direction region to the fiber direction region. The standard deviation in
 1011 the broadband case is higher than the narrowband case. This may be due to the dispersive waves
 1012 travelling at various wave speeds where the wave speed of the selected reference signal u_{Ref} may be
 1013 different from the signals of other propagation directions.

1014
 1015 **Table 6.** Summary of mean arrival time-deviation for IW characterization in A0 mode using laser
 1016 CHC scanning method with narrowband of 140 kHz-160 kHz.

$\mathbf{P}_{\text{CHC}} \{n, p, \varphi, R\}$	Non-fiber direction region		Fiber direction region		MAD (μs)
	$\mu_{\Delta t}$	$\sigma_{\Delta t}$	$\mu_{\Delta t}$	$\sigma_{\Delta t}$	
{4, 0.028, 0°, 150}	-0.0629	0.2378	-	-	0.3007
{4, 0.028, 0°, 80}	-0.0308	0.1732	-	-	0.2040
{4, 0.026, 0°, 200}	-0.1338	0.4058	-	-	0.5396
{4, 0.075, -1°, 150}	-	-	-0.0278	0.2075	0.2353
{4, 0.075, -1°, 80}	-	-	-0.0376	0.2059	0.2435
{4, 0.075, -1°, 200}	-	-	-0.3377	0.3769	0.7146

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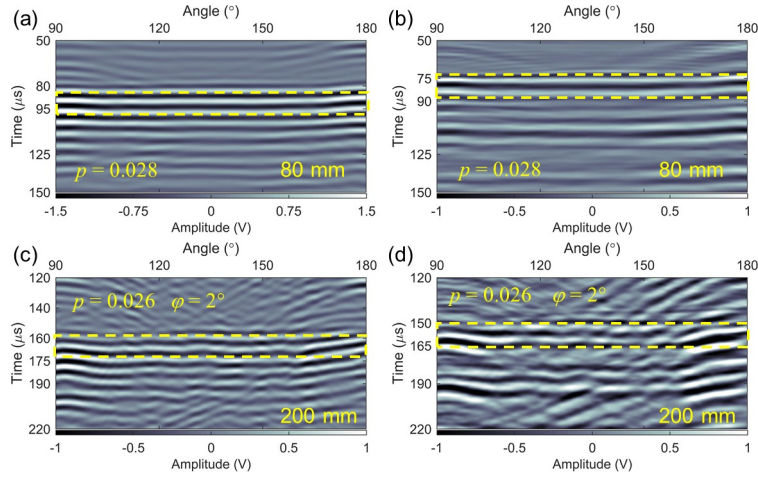
1018 **Table 7.** Summary of mean arrival time-deviation for IWs of A0 mode characterization using laser
1019 CHC scanning method with broadband of 50 kHz-500 kHz.

$\mathbf{P}_{\text{CHC}} \{n, p, \varphi, R\}$	Non-fiber direction region		Fiber direction region		MAD (μs)
	$\mu_{\Delta t}$	$\sigma_{\Delta t}$	$\mu_{\Delta t}$	$\sigma_{\Delta t}$	
{4, 0.028, 0°, 150}	-0.0444	0.4026	-	-	0.4470
{4, 0.028, 0°, 80}	-0.0047	0.2035	-	-	0.2082
{4, 0.026, 2°, 200}	-0.1067	0.8373	-	-	0.9440
{4, 0.075, -1°, 150}	-	-	-0.1051	0.3206	0.4257
{4, 0.075, -1°, 80}	-	-	-0.0690	0.3380	0.4040
{4, 0.075, -1°, 200}	-	-	-0.8774	1.3096	2.1780

1020

1021 Figures 20(a) and (b) show the IWs of the A0 mode measured at $\mathbf{P}_{\text{CHC}} \{4, 0.028, 0^\circ, 80\}$ in the cases of
1022 narrowband and broadband. The corresponding arrival time-deviations of the IWs were determined
1023 with the reference signal u_{Ref} at 135° ($m_{\text{Ref}} = 377$), the angle range, and the time windows as
1024 tabulated in Table 2 respectively. Then, the mean arrival time-deviations were determined as tabulated
1025 in Tables 6 and 7. The $\mu_{\Delta t}$ of the narrowband and broadband cases were found to be $-0.0308 \mu\text{s}$ and
1026 $-0.0047 \mu\text{s}$ with a $\sigma_{\Delta t}$ of $-0.1732 \mu\text{s}$ and $-0.2035 \mu\text{s}$, respectively. It illustrated that the measurement
1027 at 80 mm has better wavefront characterization as compared to the previous case at 150 mm. Looking
1028 at Figs. 20(a) and (b), the IWs of the A0 mode were less contaminated than Figs. 19(a) and (b) as the
1029 reflected waves from the boundaries has yet arrived at 80 mm in all directions from 90° to 180° and
1030 not interfere with the IWs of the A0 mode. This is clearly illustrated in the UWI at the time instant of
1031 $84 \mu\text{s}$ for broadband case in Fig. 6(c).

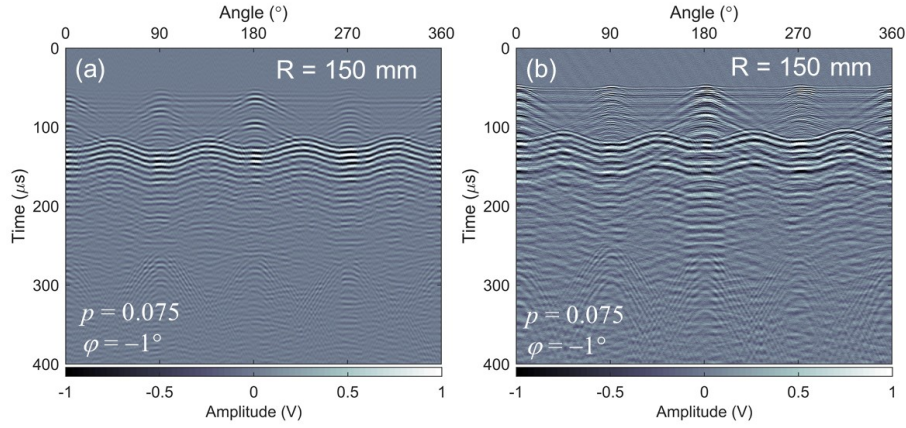
1032



1033
 1034 **Figure 20.** Ultrasonic responses with frequency bands of (a)(c) 140-160 kHz and (b)(d) 50-500 kHz
 1035 ranging from 90° to 180° based on the cyclic-harmonic scanning curves at (a)(b) $\mathbf{P}_{\text{CHC}}\{4, 0.028, 0^\circ, 80\}$
 1036 and (c)(d) $\mathbf{P}_{\text{CHC}}\{4, 0.026, 2^\circ, 200\}$.
 1037

1038 Figures 20(c) and (d) show the IWs of the A0 mode measured at the new updated parameter
 1039 $\mathbf{P}_{\text{CHC}}\{4, 0.026, 2^\circ, 200\}$ after the updating process was performed in LUIS with the initial parameter at
 1040 $\mathbf{P}_{\text{CHC}}\{4, 0.028, 0^\circ, 200\}$. It is noted that the elements $p = 0.026$ and $\varphi = 2^\circ$ were updated. Figures 20(c)
 1041 and (d) show the IWs of the A0 mode for the narrowband and broadband cases respectively. The
 1042 corresponding arrival time-deviations were also obtained with the reference signal at 135°
 1043 ($m_{\text{Ref}} = 943$), the angle range, and time windows as tabulated in Table 2.
 1044

1045 At the $R = 200$ mm, the IWs were contaminated with a lot of interference from the reflected waves
 1046 relative to the previous case of 80 mm. For this reason, the mean arrival time-deviation is higher than
 1047 the cases at 80 mm and 150 mm as summarized in Tables 6 and 7, respectively. For the cases of
 1048 narrowband and broadband, the interference of the reflected waves induced high standard deviations
 1049 at $0.4058 \mu\text{s}$ and $0.8373 \mu\text{s}$ as shown in Figs. 20(c) and (d). The parameters obtained above were
 1050 considered to characterize the IWs of the A0 mode in the region of non-fiber direction but not in the
 1051 region of fiber direction. As a result, the characterization of the IWs of the A0 mode in the fiber
 1052 direction region was investigated and discussed next.
 1053

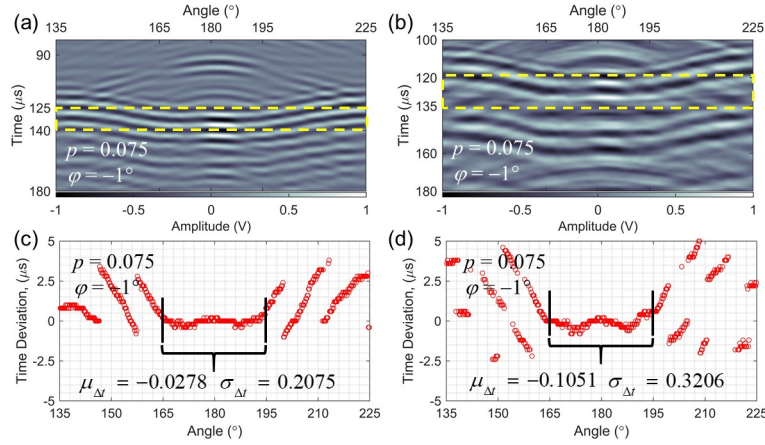


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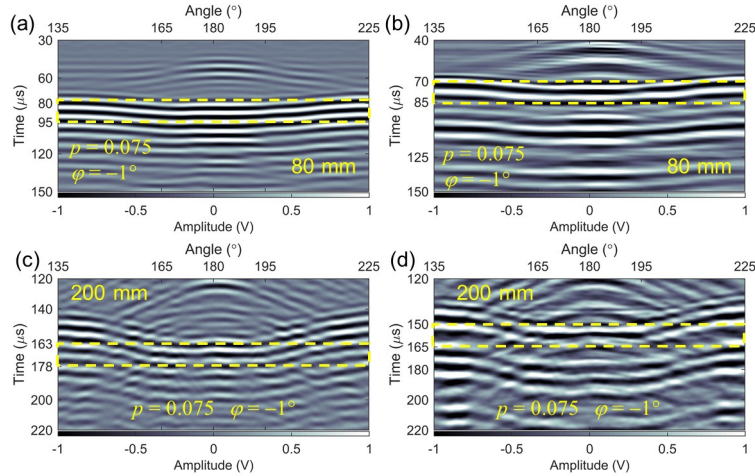
Figure 21. Ultrasonic responses with (a) 140-160 kHz and (b) 50-500 kHz based on the cyclic-harmonic scanning curves at $\mathbf{P}_{\text{CHC}} \{4, 0.075, -1^\circ, 150\}$.

In Fig. 11(b), the ultrasounds were measured at the parameter $\mathbf{P}_{\text{CHC}} \{4, 0.07, 0^\circ, 150\}$, and the patterns of the A0 mode IWs illustrated that the arrival time were about the same in the fiber direction region. For that, in this paper, the $\mathbf{P}_{\text{CHC}} \{4, 0.07, 0^\circ, 150\}$ was chosen as the initial values for the LUIS to perform the updating process for the fiber direction region at 150 mm. As a result, the new parameter $\mathbf{P}_{\text{CHC}} \{4, 0.075, -1^\circ, 150\}$ was obtained, and Fig. 17(b) shows the laser CHC scanning pattern. The corresponding ultrasonic signals were then measured and represented in PUWI as shown in Figs. 21(a) and (b) for narrowband and broadband cases, respectively.

Figures 22(a) and (b) show the zoomed PUWI of Figs. 21(a) and (b) from 135° to 225° for the narrowband and broadband cases, respectively. Repeating the same procedures from the S0 mode to Eqn. (4), the arrival time-deviations of both cases were obtained as shown in Figs. 22(c) and (d). Then, with Eqn. (5), the $\mu_{\Delta t}$ was determined for the narrowband and broadband cases, tabulated in Tables 6 and 7 accordingly. The $\mu_{\Delta t}$ of the narrowband and broadband cases were $-0.0278 \mu\text{s}$ and $-0.1051 \mu\text{s}$, with a $\sigma_{\Delta t}$ of $0.2075 \mu\text{s}$ and $0.3206 \mu\text{s}$, respectively. The parameter shows the ability to characterize the IWs of the A0 mode in the fiber direction region.



1074
1075 **Figure 22.** Ultrasonic responses with frequency bands of (a) 140-160 kHz and (b) 50-500 kHz
1076 ranging from 135° to 225°, and (c)(d) corresponding arrival time-deviations respectively, based on
1077 the cyclic-harmonic scanning curves at $\mathbf{P}_{\text{CHC}} \{4, 0.075, -1^\circ, 150\}$.
1078



1079
1080 **Figure 23.** Ultrasonic responses with frequency bands of (a)(c) 140-160 kHz and (b)(d) 50-500 kHz
1081 ranging from 135° to 225° based on the cyclic-harmonic scanning curves at (a)(b)
1082 $\mathbf{P}_{\text{CHC}} \{4, 0.075, -1^\circ, 80\}$ and (c)(d) $\mathbf{P}_{\text{CHC}} \{4, 0.075, -1^\circ, 200\}$.
1083

1084 On the other hand, Figs. 23(a) and (b) show the measured ultrasound with the parameter
1085 $\mathbf{P}_{\text{CHC}} \{4, 0.075, -1^\circ, 80\}$ at the radius of 80 mm. As discussed in earlier results in Fig. 20, there was no
1086 interference of the IWs of the A0 mode from the reflected waves at 80 mm, as the reflected waves
1087 from the boundary of the plate need longer time to reach a radius of 80 mm from the center of the
1088 plate. Same in this case, the measured IWs of the A0 mode in fiber direction region were also not
1089 affected by the reflected waves. As a result, the mean arrival time-deviations for the narrowband and
1090 broadband cases were obtained to be $-0.0376 \mu\text{s}$ and $-0.0690 \mu\text{s}$ with standard deviations of 0.2059
1091 μs and $0.3380 \mu\text{s}$, respectively.

1092
1093 However, when the magnitude of R is increased, the IWs of the A0 mode are contaminated by the
1094 reflected wave from the S0 mode. For this reason, the mean arrival time-deviations at the radius of

1095 200 mm (Tables 6 and 7) were highest among the other two cases (80 mm and 150 mm). In
1096 narrowband case, the mean arrival time-deviation is obtained $-0.3377 \mu s$ with the standard deviation
1097 of $0.3769 \mu s$, and in broadband case, the mean arrival time-deviation is $-0.8774 \mu s$ and standard
1098 deviation of $1.3096 \mu s$.

1099
1100 In both cases of S0 and A0 modes, the MADs of the non-fiber direction region were lesser than the
1101 MADs of the fiber direction region in both narrowband and broadband. This is because the change
1102 rate of the angular direction in the wavefront pattern was low in the non-fiber direction region as
1103 compared to the fiber direction region. In addition, the MADs in the case of narrowband were lesser
1104 than the MADs in the case of broadband in both fiber and non-fiber directions regions. It is because
1105 the proposed method was sensitive to the complexity of the wavefront pattern. Since, in the case of
1106 broadband, the spatial wavefront of the ultrasound is more complex with multi-mode and multi-
1107 frequency components than the case of narrowband, it is difficult for the laser cyclic-harmonic curve
1108 scan to characterize the complex wavefronts.

1109
1110 The proposed method was also dependent on the purity of the wavefront pattern. As seen in the case
1111 of the A0 mode, as the testing radius increased, the MAD was increased because the IWs of the A0
1112 mode interfered with the reflected waves at further distances from the source. Beyond that, it was also
1113 shown that the dispersive nature of the A0 mode would cause an increase in MAD at the same
1114 parameters' values, e.g. at the radii of 80 mm and 150 mm in Tables 6 and 7.

1115

1116 **3.3.2 Reflected Wavefront Characterization**

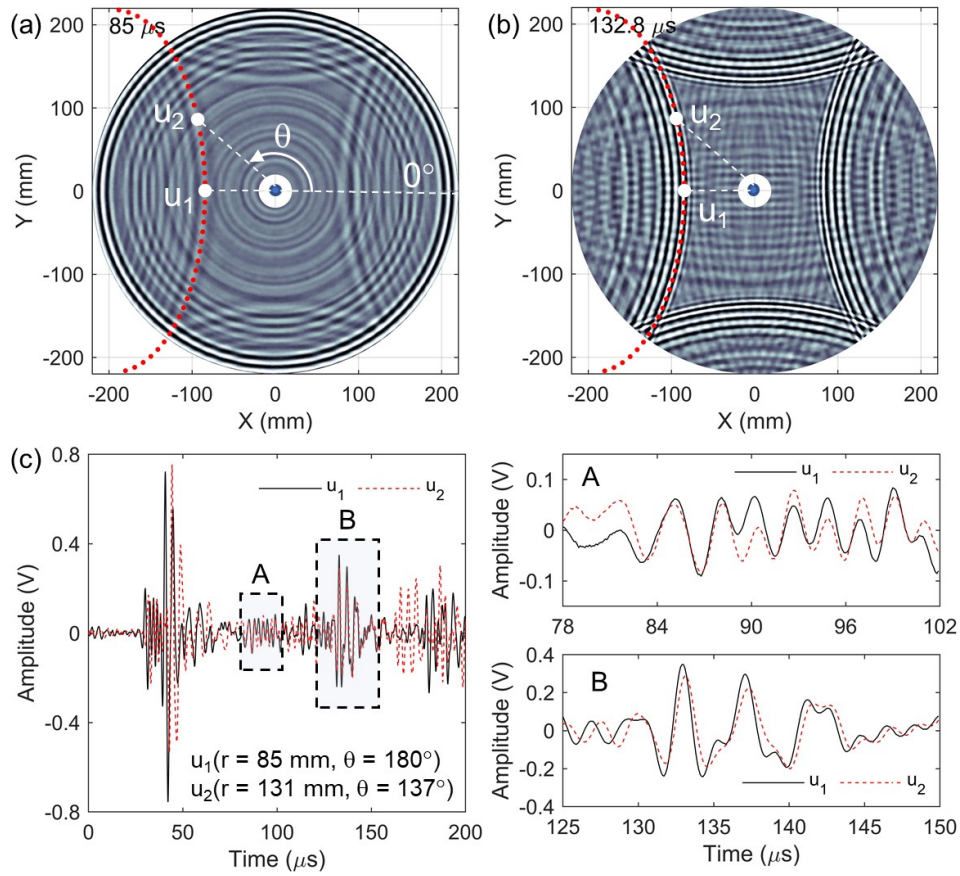
1117 As demonstrated in previous subsection 3.3, the proposed method was reasonably capable of
1118 characterizing the incident wavefronts of the S0 and A0 modes for the aluminum and cross-ply CFRP
1119 plates. In the following subsection, the feasibility of the proposed method is further tested on the
1120 reflected waves of the aluminum and cross-ply CFRP plates.

1121

1122 *Aluminum Plate*

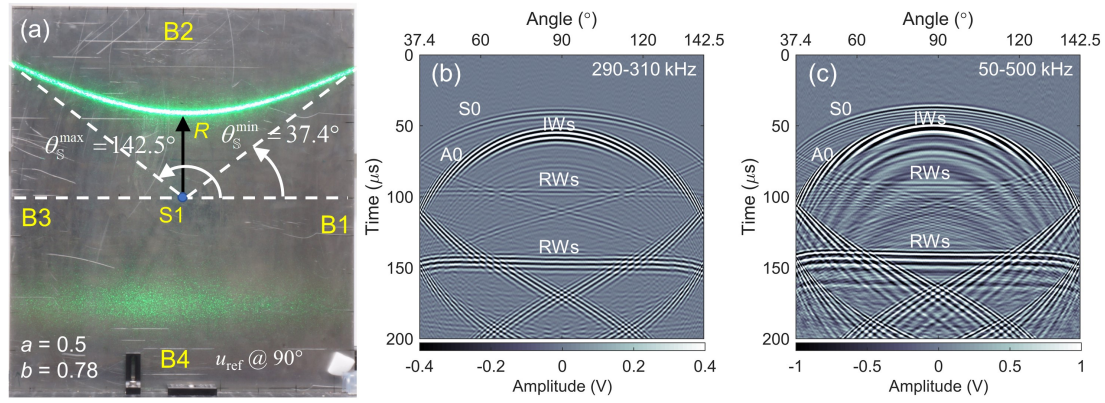
1123 Figures 24(a) and (b) show spatial ultrasonic wavefront images in a 2-D spatial plane at different
1124 time-instants $85 \mu s$ and $132.8 \mu s$. The red-dotted curves on the reflected wavefronts at various times
1125 show the similar curve pattern in hyperbolic curve as discussed in previous subsection 3.2.1. Figures
1126 24(a) and (b) indicate that the same reflected wavefronts propagated through two local points (180° ,
1127 85) and (137° , 131) at two different time instants of $85 \mu s$ and $132.8 \mu s$, respectively. The
1128 corresponding ultrasonic signals, $u_1(180^\circ, 85, t)$ and $u_2(137^\circ, 131, t)$, were considered and the
1129 responses to the time history waves were plotted as shown in Figs. 24(c). By looking at the boxes A

1130 and B as shown on the right side of the Fig. 24(c), the two signals were approximately in phase from
 1131 84 μ s to 88 μ s and from 130 μ s to 134 μ s respectively.
 1132



1133
 1134 **Figure 24.** Aluminum plate UWI at (a) 85 μ s, (b) 132.8 μ s, and (c) local ultrasonic signals
 1135 $u_1(180^\circ, 85, t)$ and $u_2(137^\circ, 131, t)$.
 1136

1137 It infers that the reflected wavefronts may be characterized by obtaining all the corresponding local
 1138 points (liked u_1 and u_2) that are positioned on the coordinates of the same wavefront. For this reason,
 1139 as referred to in the previous subsection 3.2.2, the characterized curve (hyperbolic curve) was
 1140 obtained to characterize the reflected wavefronts of the aluminum and cross-ply CFRP plates are
 1141 discussed next for Process 2.
 1142



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1144

1145

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1147

Figure 25. (a) Laser hyperbolic scanning pattern with parameter $\mathbf{P}_H \{0.5, 0.78, 90^\circ, 0\}$ on aluminum plate and corresponding ultrasounds in PUWI with frequency ranges of (b) 290-310 kHz and (c) 50-500 kHz.

1148

In Process 1, the characterized curve was obtained based on the spatial ultrasonic wavefront image in Fig. 5(b) and the corresponding parameters and scale factors were estimated at $\mathbf{P}_H \{0.5, 0.68, 90^\circ, 0\}$.

1149

1150

As for that, the LUIS performed the laser hyperbolic scan at the initial values of $\mathbf{P}_H \{0.5, 0.68, 90^\circ, 0\}$.

1151

Upon completion of the updating process in Process 2, the updated parameter was obtained at

1152

$\mathbf{P}_H \{0.5, 0.78, 90^\circ, 0\}$. Throughout the scanning process, the laser scanner mirror maneuvered the

1153

pulsed laser beam with the coordinates generated in \mathbb{S} -plane based on the $\mathbf{P}_H \{0.5, 0.78, 90^\circ, 0\}$ as

1154

shown in Fig. 25(a). Since the zero degree was assigned at the line $y = 0$ with the condition of

1155

$0 \leq x \leq w_x$ toward to the boundary B1 (Fig. 25(a)), these θ_s^{\min} and θ_s^{\max} were then determined at

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37.4° and 142.5° respectively, from Eqn. (12) with the consideration of the rotation angle Ω of 90° in

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Eqn. (11) and the interval angle of 0.04° . The radius (R) was calculated at 127 mm from the center

1158

(Fig. 25(a)) based on the expression $a w_x + A_{\text{offset}}$ in section 3.2.2.

1159

1160

Figures 25(b) and (c) show the PUWIs based on the $\mathbf{P}_H \{0.5, 0.78, 90^\circ, 0\}$ for the narrowband (290-310

1161

kHz) and broadband (50-500 kHz) cases, respectively. Both PUWIs show the IWs of the S0 and A0

1162

modes were arrived at various times. Note, on the other hand, that the wavefronts of the reflected

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waves from the boundary B2 had arrived at the same times and formed lines-liked, denoted as RWs in

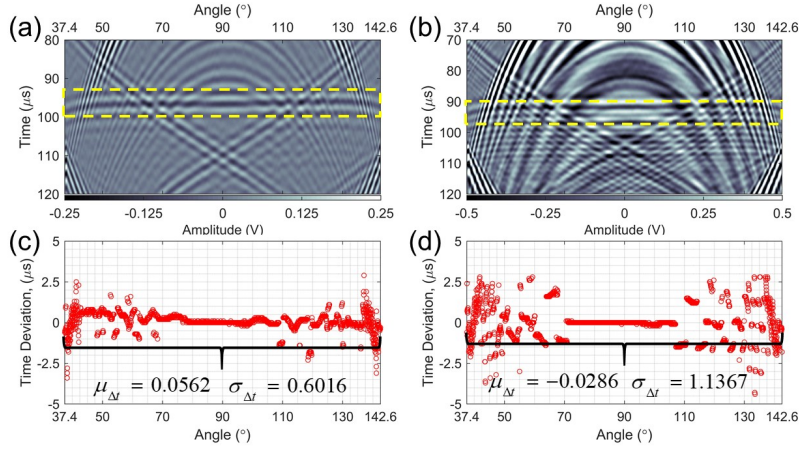
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Figs. 25(b) and (c). Besides than the RWs propagating from B2, the PUWI indicates also the other

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reflected waves propagating from the boundaries B1 and B3.

1166



1167
 1168 **Figure 26.** Zoomed PUWIs with frequency ranges of 290-310 kHz and (b) 50-500 kHz, and (c)(d)
 1169 corresponding arrival time-deviations respectively for RWs due to S0 mode at $\mathbf{P}_H \{0.5, 0.78, 90^\circ, 0\}$.
 1170

1171 Figures 26(a) and (b) show the zoomed PWUI of Figs 25(b) and (c) for the narrowband and
 1172 broadband respectively. The yellow boxes in Figs. 26(a) and (b) show the RWs propagating from the
 1173 boundary B2 after the IWs of S0 mode struck at the B2. The measurement of the ultrasound based on
 1174 $\mathbf{P}_H \{0.5, 0.78, 90^\circ, 0\}$ demonstrated the ability to characterize the RWs from the B2, and these RWs
 1175 were denoted as S_{B2} in this paper. Figures 26(c) and (d) show the arrival time-deviations obtained
 1176 with the reference signal at 90° ($m_{Ref} = 1315$) for both narrowband and broadband, respectively. Then,
 1177 the time windows were set from $92 \mu s$ ($k_{min} = 460$) to $100 \mu s$ ($k_{max} = 500$) for the case of narrowband
 1178 and from $90 \mu s$ ($k_{min} = 450$) to $95 \mu s$ ($k_{max} = 475$) for the case of broadband. Table 8 shows $\mu_{\Delta t}$ was
 1179 determined to be $0.0562 \mu s$ with a $\sigma_{\Delta t}$ of $0.6026 \mu s$ for the narrowband case, and $-0.0286 \mu s$ with a
 1180 $\sigma_{\Delta t}$ of $1.1367 \mu s$ for the broadband case.

1181
 1182 Both mean arrival time-deviations were close to zero, but both MADs were higher than the previous
 1183 IWs cases. It is because their standard deviations were high, especially the arrival time-deviations in
 1184 the directions 37.4° to 75° and 110° to 142.6° , as shown in Figs. 26(c) and (d). By referring to the
 1185 PUWI in Figs. 26(a) and (b), there was a lot of interference from another boundary between S_{B2} and
 1186 other RWs. Despite this interference, the visibility of the S_{B2} pattern was still clearly illustrated in the
 1187 PUWI. Hence, the laser hyperbolic scan at the $\mathbf{P}_H \{0.5, 0.78, 90^\circ, 0\}$ was still able to characterize the
 1188 RWs in the directions ranging from 70° to 110° in this paper.

1189
 1190 **Table 8.** Summary of mean arrival time-deviation for characterization of RWs due to S0 and A0
 1191 modes using laser hyperbolic scanning method with narrowband of 290 kHz-310 kHz and broadband
 1192 50 kHz-500 kHz.

$\mathbf{P}_H \{a, b, \Omega, A_{offset}\}$	Narrowband	Broadband
---	------------	-----------

	$\mu_{\Delta t}$	$\sigma_{\Delta t}$	MAD (μs)	$\mu_{\Delta t}$	$\sigma_{\Delta t}$	MAD (μs)
S0 Mode						
{0.5, 0.78, 90°, 0}	0.0562	0.6026	0.6588	-0.0286	1.1367	1.1653
{0.5, 0.9, 132°, 80}	-	-	-	-	-	-
A0 Mode						
{0.5, 0.78, 90°, 0}	0.0519	0.4894	0.5413	-0.0527	0.8526	0.9053
{0.5, 0.9, 132°, 80}	0.1018	0.3870	0.4888	0.1218	0.3209	0.4427

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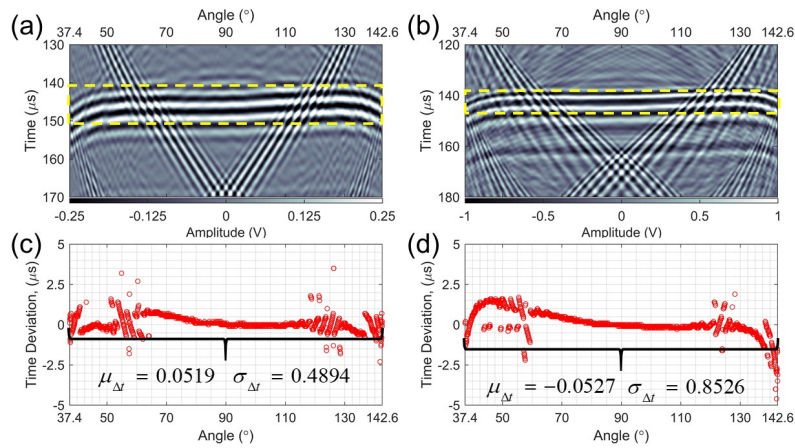
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1202

Figures 27(a) and (b) show another RW A0 mode from boundary B2, denoted as A_{B2} . The arrival time-deviations were obtained with the same settings in the case of S_{B2} , except the time windows. The time windows were set to 142 μs ($k_{\min} = 710$) to 150 μs ($k_{\max} = 750$) for the case of narrowband and 138 μs ($k_{\min} = 690$) and 148 μs ($k_{\max} = 740$) for the case of broadband as shown in Figs. 27(c) and (d) respectively. The mean arrival time-deviations for the narrowband and broadband cases were found to be 0.0519 μs with a $\sigma_{\Delta t}$ of 0.4894 μs and -0.0527 μs with a $\sigma_{\Delta t}$ of 0.8526 μs , as tabulated in Table 8. The MADs of A_{B2} demonstrate greater accuracy than the MADs of S_{B2} . This is because the A_{B2} had less interference than the S_{B2} case in the directions 37.4° to 75° and 110° to 142.6°.



1203

1204

1205

1206

Figure 27. Zoomed PUWIs with frequency ranges of 290-310 kHz and (b) 50-500 kHz, and (c)(d) corresponding arrival time-deviations respectively for RWs due to A0 mode at $\mathbf{P}_H \{0.5, 0.78, 90^\circ, 0\}$.

1207

1208

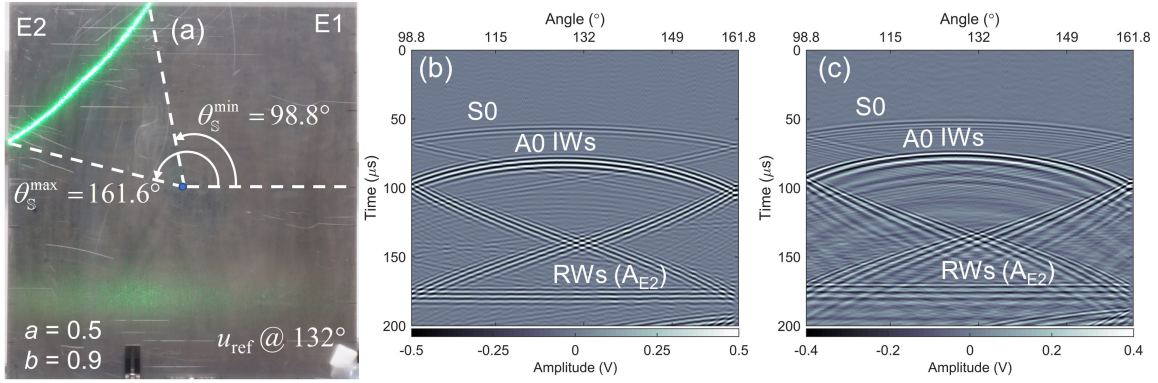
1209

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1211

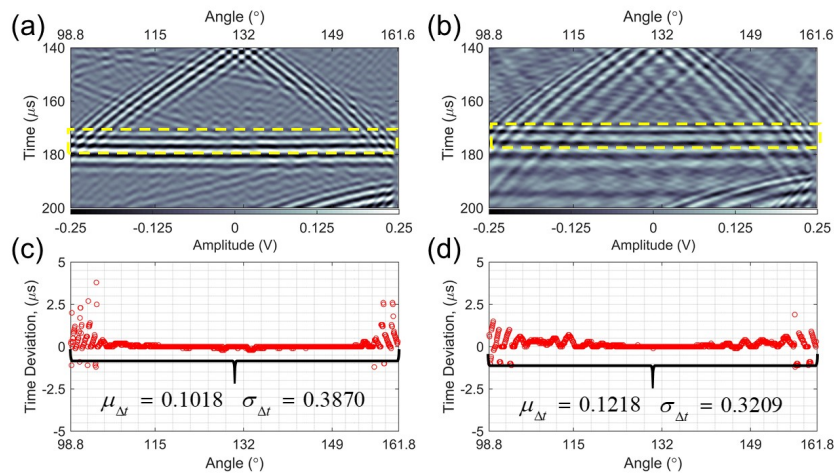
Next, the RWs from the corner of the aluminum plate were considered and characterized. In the previous Process 1, the characterized curve with the parameter of $\mathbf{P}_H \{0.5, 1, 132^\circ, 80\}$ was determined for the RWs propagating from the corner E2 as shown in Fig. 5(c)). The LUIS then performed the laser hyperbolic scan at an angle interval of 0.04° with this parameter as an initial setting. Figure 28(a) shows the laser hyperbolic scanning curve of the updated parameter of $\mathbf{P}_H \{0.5, 0.9, 132^\circ, 80\}$ on the

1212 aluminum plate from $\theta_S^{\min} = 98.9^\circ$ ($m_{\min} = 1$) to $\theta_S^{\max} = 161.6^\circ$ ($m_{\max} = 1568$). Since the aluminum
 1213 plate was rectangular, the angle at the corner E2 was at 132° ($m_{\text{Ref}} = 830$) instead of 135° and the
 1214 ultrasonic signal at 132° was used as the reference signal u_{Ref} .
 1215



1216
 1217 **Figure 28.** (a) Laser hyperbola scanning pattern with parameter $\mathbf{P}_H \{0.5, 0.9, 132^\circ, 80\}$ on aluminum
 1218 plate and corresponding ultrasounds in PUWI with frequency ranges of (b) 290-310 kHz and (b) 50-
 1219 500 kHz.
 1220

1221 Figures 28(b) and (c) show the PUWIs for the cases of narrowband and broadband respectively. It
 1222 notes that the RWs, denoted as A_{E2} , were obtained, generated at corner E2 after the IWs of A0 mode
 1223 impinged there. It was noted, however, that the RWs propagating from E2 due to S0 mode were not
 1224 seen. The low signal-to-noise ratio of the IWs of the S0 mode was suspected to have significantly
 1225 further diminished after hitting corner E2. As a result, the characterization of the RWs (A_{E2}) was only
 1226 tested in this paper.
 1227



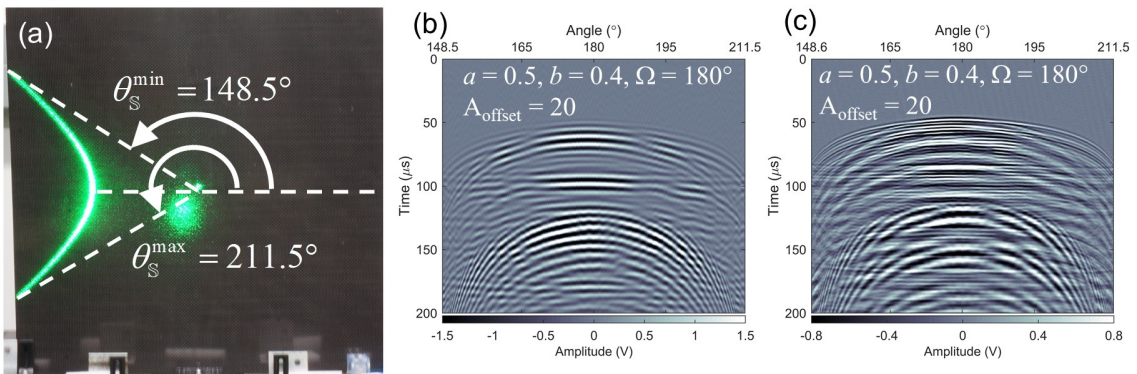
1228
 1229 **Figure 29.** Zoomed PUWIs with frequency ranges of 290-310 kHz and (b) 50-500 kHz, and (c)(d)
 1230 corresponding arrival time-deviations respectively for RWs due to A0 mode at $\mathbf{P}_H \{0.5, 0.9, 132^\circ, 80\}$.
 1231

1232 Figures 29(a) and (b) show the zoomed-in PUWI of Figs. 28(b) and (c), respectively. The arrival time-
 1233 deviations were then obtained with the time windows of $172 \mu\text{s}$ ($k_{\min} = 860$) to $177 \mu\text{s}$ ($k_{\max} = 880$)
 1234 for both narrowband and broadband as shown in Figs. 29(c) and (d). As tabulated in Table 8, the $\mu_{\Delta t}$
 1235 were calculated to be $0.1018 \mu\text{s}$ with a $\sigma_{\Delta t}$ of $0.3870 \mu\text{s}$ for the narrowband case, and to be $0.1218 \mu\text{s}$
 1236 with a $\sigma_{\Delta t}$ of $0.3209 \mu\text{s}$ for the broadband case. The corresponding MADs for the cases of
 1237 narrowband and broadband were obtained at $0.4888 \mu\text{s}$ and $0.4427 \mu\text{s}$ respectively. Among the results
 1238 tabulated in Table 8, the characterization of the RWs from the corner E2 is more accurate than the
 1239 RWs from the boundary B2. It is because of the high interference of B2 over the RWs compared to
 1240 the E2.

1241
 1242 *CFRP Plate*

1243 Next, the laser hyperbolic scan was further tested to the cross-ply CFRP plate. Based on Fig. 6(c), the
 1244 characterized curve (hyperbolic curve) was determined at the parameter of $\mathbf{P}_H\{0.5, 0.35, 180^\circ, 20\}$ in
 1245 Process 1. Subsequently, using $\mathbf{P}_H\{0.5, 0.35, 180^\circ, 20\}$ as an initial setting, the LUIS performed the
 1246 laser hyperbolic scan and the updated parameter was obtained at $\mathbf{P}_H\{0.5, 0.4, 180^\circ, 20\}$. Figure 30(a)
 1247 shows the laser hyperbolic scanning curve from $\theta_S^{\min} = 148.5^\circ$ to $\theta_S^{\max} = 211.5^\circ$ at
 1248 $\mathbf{P}_H\{0.5, 0.4, 180^\circ, 20\}$. Figures 30(b) and (c) show the PUWIs for the narrowband and broadband cases,
 1249 respectively. Looking at the Fig. 6(c), the RWs were generated and they propagated in the directions
 1250 of fiber. The IWs of the S0 mode were suspected as the source of the RWs generation since the S0
 1251 mode traveled faster than the A0 mode as shown in Fig. 6(c). In addition, the spatial ultrasonic
 1252 wavefront image at $84 \mu\text{s}$, as shown in Fig. 6(c), displays that the RWs were interfered with the IWs
 1253 of the S0 mode, but not with the IWs of the A0 mode. Consequently, the updated parameter
 1254 $\mathbf{P}_H\{0.5, 0.4, 180^\circ, 20\}$ was used to characterize the RWs as induced by the IWs of the S0 mode.

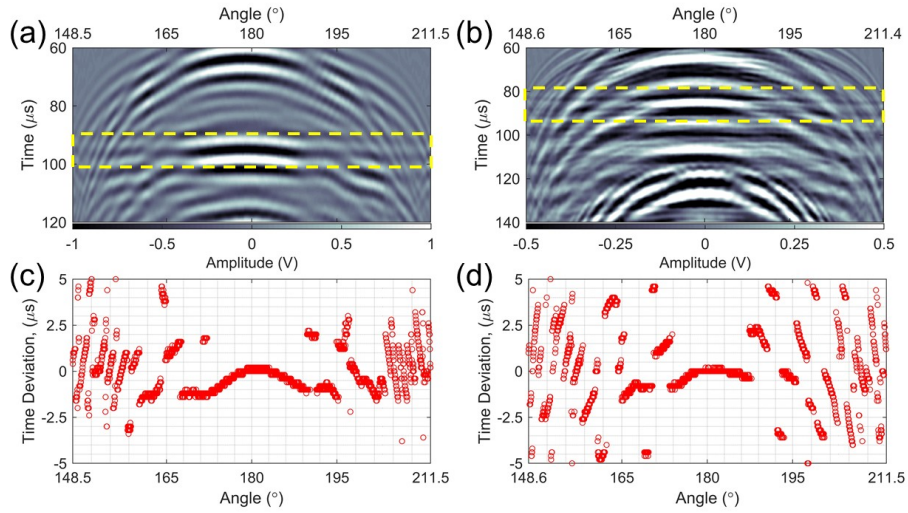
1255



1256

1257 **Figure 30.** (a) Laser hyperbola scanning pattern with parameter $\mathbf{P}_H \{0.5, 0.4, 180^\circ, 20\}$ on aluminum
 1258 plate and corresponding ultrasonics in PUWI with frequency ranges of (b) 140-160 kHz and (b) 50-
 1259 500 kHz.
 1260

1261 Figures 31(a) and (b) show the zoomed PUWIs of Figs. 30 (a) and (b) respectively. Figures 31(c) and
 1262 (d) show that the arrival time-deviations were obtained with the reference signal at 180° ($m_{\text{Ref}} = 789$).
 1263 In the narrowband case, the time window was set to from $90 \mu\text{s}$ ($k_{\text{min}} = 450$) to $102 \mu\text{s}$ ($k_{\text{max}} = 510$),
 1264 and in the broadband case the time window was set to $78 \mu\text{s}$ ($k_{\text{min}} = 390$) to $96 \mu\text{s}$ ($k_{\text{max}} = 480$).
 1265 Although the lines-liked of the RWs can be seen in Figs. 31(a) and (b), but the results of the arrival
 1266 time-deviations showed that the parameter barely characterized the RWs correctly, approximately to
 1267 zero, in the fiber direction region ($165^\circ \leq \phi_s \leq 195^\circ$) for both cases.
 1268



1269 **Figure 31.** Zoomed PUWIs with frequency ranges of 140-160 kHz and (b) 50-500 kHz, and (c)(d)
 1270 corresponding arrival time-deviations respectively for RWs due to S0 mode at $\mathbf{P}_H \{0.5, 0.4, 180^\circ, 20\}$.
 1271
 1272

1273 4. CONCLUSION

1274 In this paper, a new spatial ultrasonic wavefront characterization method based on parametric curve
 1275 equation was proposed and performed to overcome the drawbacks of initial work [9] to accelerate the
 1276 laser ultrasonic inspection time by optimizing the spatial scanning pattern. The proposed method was
 1277 tested on aluminum and cross-ply CFRP plates to characterize the spatial incident and reflected
 1278 wavefronts of the plates. In this paper, the non-fiber direction region ($105^\circ \leq \phi_s \leq 165^\circ$) and the fiber
 1279 direction region ($165^\circ \leq \phi_s \leq 195^\circ$) of the cross-ply CFRP plate were considered in the test. Then, the
 1280 performance of the wavefront characterization was evaluated based on the maximum absolute
 1281 deviation of the mean arrival time-deviation. The results are summarized as follows:

- 1282 • Using ultrasonic images, the incident and reflected wavefronts on rectangular boundary
1283 specimens were identified and shown to have strong correlation to fundamental parametric curves.
1284 The circular curve for the aluminum plate and the cyclic-harmonic curve for cross-ply CFRP plate
1285 were identified in the incident wavefront, and the hyperbolic curve for both plates was identified
1286 in reflected wavefront.
- 1287 • The laser circle scan showed the ability to characterize the incident wavefronts of the S0 and A0
1288 modes of the aluminum plate with the parametric circle equation. The characterization efficiency
1289 was determined at the lowest maximum absolute deviations of $0.2562 \mu\text{s}$ for S0 mode and 0.4742
1290 μs for A0 mode.
- 1291 • The laser cyclic-harmonic curve scan with the parametric cyclic-harmonic curve equation showed
1292 the ability to characterize the incident wavefronts of the S0 and A0 modes at various radii in the
1293 regions of fiber and non-fiber directions. It showed greater characterization efficiency in the non-
1294 fiber direction regions of both modes using a narrowband excitation. The efficiency of the
1295 wavefront characterization was dependent on the change rate of the angular direction, simplicity,
1296 and purity in the wavefront pattern.
- 1297 • The proposed method showed the high versatility and spatial resolution features that scanning
1298 curve can be easily modified by changing the parameter values to increase the efficiency of the
1299 characterization, as proven in Fig. 14. In addition, the suspected local fiber stacking variations
1300 were detected in the CFRP plate in Fig. 14 due to these novel features in the laser cyclic-harmonic
1301 curve scan.
- 1302 • The laser hyperbolic scan with the parametric hyperbolic curve equation showed the ability to
1303 characterize the reflected wavefronts, either from the aluminum plate boundary or corner, that due
1304 to the S0 mode and the A0 mode. In the case of narrowband input, it showed greater
1305 characterization efficiency than the case of broadband input. Nevertheless, in both cases it has
1306 high MAD because of the interference from the other reflected wavefronts from other boundaries
1307 and corners.
- 1308 • The laser hyperbolic curve scan showed ability to parameterize the reflected wavefronts in the
1309 CFRP plate. However, the cross-correlation method was unable to quantify the performance of
1310 the technique because the multiple waves superimposed made the images too complex to
1311 parameterize in a fundamental curve.
- 1312 • In the case of the aluminum plate, the laser hyperbolic curve scan showed the ability to identify
1313 when and localize where the reflected waves were crossed to each other in the PUWI. This feature
1314 may provide useful information in a sparse sensing approach for damage detection, where the
1315 positioning of the sensor can be optimized by avoiding undesirable locations that are not helpful
1316 in the overall detection performance.

1317 The summary above gives a clear picture of the feasibility of the proposed method, but there is plenty
1318 of room for further enquiry and development. First, the temporal cross-correlation technique used in
1319 this paper should be improved or reassessed using another robust technique to quantify the
1320 performance of the wavefront characterization. This is because the cross-correlation coefficient has a
1321 strong relationship to the selected reference signal. Therefore, if the reference signal is distinct from
1322 the test signal, it suffers in generating useful results. As illustrated in the case of the A0 mode (Fig.
1323 9(c)), the selected reference signal (180°) was different from the test signal because it was
1324 contaminated by the reflected wave. Second a wave decomposition technique is required to improve
1325 the quality of the desired wavefront by isolating or filtering the superimposed multiple wavefronts.
1326 Lastly, the repeatability of the proposed method must be evaluated with multiple specimens with the
1327 same controlled manufacturing configurations.

1328

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1331

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