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Essays on Competition: Contests, Personnel Economics, and Corporate Citizenship

By

Dylan Blu Minor

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

Business Administration

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor John Morgan, Chair
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Abstract

Essays on Competition: Contests, Personnel Economics, and Corporate Citizenship

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Professor John Morgan, Chair

I explore competition in three different settings. First, I examine how a contest designer can increase total efforts through softening incentives. In particular, softer incentives are called for when the following is met: contestants have more convex costs, there are many contestants, or designers have (sufficient) concave valuation of effort. In the extreme, more total effort can be generated from offering a larger second than first prize.

Next, I test this theory experimentally in the lab under the framing of personnel economics. I find the general comparative statics given above hold. However, on an individual basis, people depart from the theoretical predictions. Workers use a heuristic of going "all in" or abstaining from work when they find themselves above or below, respectively, some private target level of ability. Additionally, most workers take on different roles-slackers, quitters, or consistent workers--depending on the institutional setting (i.e., degree of competition and incentives structure). The consequence of such behavior is the comparative statics for optimal organizational design are softened.

Finally, I study empirically and theoretically how firms compete on the dimension of corporate social responsibility (CSR). More efficient firms are able to use CSR as an insurance mechanism. That is, some firms invest *ex-ante* in higher levels of CSR, which enables them to better withstand the tumult of negative business shocks; stakeholders give higher CSR firms the benefit of the doubt in terms of the cause of an adverse event (i.e., the cause is more often attributed to bad luck over bad management), resulting in an insurance like benefit to CSR. This finding is validated empirically by studying S&P500 firms over a 16 year period.

Part I

Competition via Contests

In our first section we study competition in the setting of contests. We find when designing incentives for heterogeneous agents facing competition there is a conflicting interaction: as the more able are incentivized the less able are disincentivized. I label the former the "incentive effect" and the latter the "discouragement effect." Such adverse interaction becomes severe in the face of participants having convex costs of effort or capacity constraints, larger contests, contestants with similar levels of ability, and contest designers with concave benefit over participant effort. Indeed, in such a world, the "discouragement effect" dominates the "incentive effect," prescribing the optimal incentives to be flat or possibly even inverted—offering a larger prize to second place than to first place. In short, providing greater benefit to the lesser able can elicit more total effort than having greater benefit awarded to the most able. These findings are explored for both all-pay and winner-pay contests.

1 Introduction

Designing optimal incentives within employment relationships has been an important and well researched topic. A fundamental lesson from this work has been the prescription of sharp incentives within the firm. Consider the canonical principal (employer) and agent (employee) model. Assuming risk neutrality of both parties the trivial solution is to "sell the store" to the employee, yielding first best from maximally powered incentives. But when employees are different the problem becomes more complex.

Instead, we will argue, interacting sharp, competitive incentives with heterogeneous ability can in fact destroy effort. In particular, the less able are less likely to win and thus "give up." We dub this the "discouragement effect." Meanwhile, the most able do increase their efforts when facing sharper incentives, which we dub the "incentive effect." However, the interaction of these two effects can become so severe that the "discouragement" effect dominates the "incentive effect," thus prescribing soft incentives.

The intuition for shifting the top performer's reward to the lesser performers is actually quite simple: we lose some effort from the most able, who are most likely to receive the first place reward. However, we receive increased effort from all the

rest as a result of their more likely earned second place reward being larger. If this increased effort overcomes the most able's lessoned effort, total effort is increased.

We naturally have in mind broader applications of softened incentives than personnel economics. In fact, whenever we encounter people or firms competing for a prize or prizes, our results will often apply. Indeed, many economic settings can be cast as a contest. Whether firms are competing for business or to avoid a regulator, nonprofits are vying for donors' dollars or even politicians seeking election, we have multiple agents competing after prizes. Therefore, for most of our analysis, we will refer to competing agents more generally as contestants or participants. The firm or beneficiary of the agents' effort is simply called the designer of the contest.

We are certainly not the first to suggest it might be optimal to offer a portion of a prize to second place over a winner-takes-all (WTA) scheme. Moldovanu and Sela (2001), hereafter MS, in their seminal paper find a designer ought to offer some fraction of the total prize to second place if the contestant's cost function has the "right" curvature. In particular, if the curvature of the cost function is convex "enough" the result follows. Contrarily, they find, if the cost function is linear or concave, a winner-takes-all scheme dominates.

My paper begins by generalizing MS's analysis by removing their restriction on monotonic prize ordering. We create a new mechanism dubbed the Generalized Second Prize Contest that allows non-monotonic prize allocations. We can then provide conditions of when equal prizes, or even a larger second prize, is preferred, in addition to asking when we want to offer any fraction of a second prize. We can then also determine for the case of an *indivisible* prize if it is best to reward a single prize to first or second place. This new mechanism then weakly dominates MS's mechanism in terms of total revenue generated from offering two prizes.

We next study an important class of contests—linear contestant costs with capacity constraints–to extend our results and address four questions that MS do not consider. First, what is the best prize division when the degree of contestant heterogeneity changes? One can imagine some settings where there is a group of mostly very skilled contestants, while some other settings witness contestants with a wide distribution of ability. When a designer faces a group of similarly high skilled competitors, intuition suggests offering a WTA contest would result in Bertrand price competition and maximal prize dissipation, yielding maximum effort for the designer. In contrast, when facing a group with diverse skill, extant contest theory tells us total effort tends to be depressed since the lesser give up due to little prospect of winning and in response the most able exert less effort. Thus, the previous intuition from the "incentive" and "discouragement" effect would suggest we ought to offer more of a second prize to induce effort from the lesser able and then force effort from the

most able, creating a "race to the top." However, in turns out both these intuitions are wrong. In fact, it is actually when competition is the fiercest—when contestants have similar levels of ability, even if very high levels of ability—that it is critical to soften incentives by flattening the prize distribution (i.e., moving from a WTA to equal prize contest).

Second, we explore how a designer should construct a contest if she values effort across contestants in a manner other than perfect substitutes—extant literature usually assumes perfect substitutes. However, one can imagine a setting where workers have complementary work inputs. Another example is the designer that has the goal of contestant proficiency. These examples mean the curvature of the *designer's* benefit function over effort is *concave*. Under such a setting, it turns out it is often best to offer more of a second prize, *even* when contestant cost functions are linear with no constraints. That is, linear participant costs is no longer sufficient to recommend a WTA contest, as it was in the setting of MS.

Third, we provide some clear empirical predictions. It is often difficult to observe cost functions in data, let alone to measure their curvature. However, with our class of contests, we are able to provide some sharp predictions that do not rely on cost function curvature and instead are based on more readily observable factors.

Finally, we show how our predictions and comparative statics also apply to an English auction in the setting when a divisible good is being auctioned off to bidders with independent private values. This then also provides an analog for the winner-pay contest where only winner(s) pay their bid.

2   Related Literature

A thorough review of providing incentives within the firm can be found in Prendergast (1999). The end result is much of the theoretical literature has prescribed sharp employee incentives. However, as he points out, the empirical literature, at best, finds mixed support that such high powered schemes are witnessed in practice. In short, it seems firms should generally be offering very sharp incentives, but they do not. We argue this inconsistency is reconciled by accounting for the *interaction* effect of incentives and competition.

The contest literature, meanwhile, can be divided into three main strands. One of the first, and earliest, strands was initiated by Tullock (1980). He set the problem up as players having a chance of winning a contest as a function of a particular contestant's effort vis-a-vis all other contestants' effort level. Much of the focus of this literature is the degree of rent dissipation through rent seeking. That is, determining what percent of the prize is exerted in effort to obtain such a prize.

Another strand has to do with casting a contest as an all-pay auction (e.g., see Bulow & Klemperer (1999)). Here we find we can analyze contest outcomes and participant behavior by drawing on the rich auction literature. However, it is almost always assumed effort costs are linear, as in auctions where a bidder's cost of a bid is most often linear. Nonetheless, in practice, and in many economic applications the firm or individual's cost function is assumed to be convex. An important exception of assumed cost linearity (though only under complete-information) is the recent contribution of Siegel (2009). However, he is concerned with player behavior and equilibrium payoffs, and does not explore contest design.

The third strand has to do with designing contests, moving from the focus on participant behavior to how to design an optimal contest. That is, from the perspective of a contest designer, deciding how much to allocate between multiple prizes or deciding between single or multiple stage contests to maximize contest revenue or effort. Moldovanu and Sela (2001) is eponymous of this work. As mentioned previously, Moldovanu and Sela make a seminal contribution in this literature of allowing participant costs to be convex. See also Moldovanu and Sela (2006) and Moldovanu et al. (2007) for more examples of contest design.

The early work of Lazear and Rosen (1981) can also be thought of as from the perspective of a contest designer. They analyze using output rank order payments to maximize worker effort. We begin our analysis by generalizing MS in terms of prize ordering and designer goals, before moving on to some entirely new questions.

3   Contestant Effort Choices

Our general model consists of k agents that commonly value $n < k$ prizes as $V_1, V_2, \cdots V_n$. However, in contrast to past literature, we do not require any ordering on the value of prizes. In addition, each agent has private information of their cost of effort. In particular, their cost of effort level e is assumed to be $c\gamma(e)$, where c is drawn from some F with lower support $\underline{c}$ bounded away from zero (to assume away costless or negative cost of effort) and upper support $\bar{c}$. Our cost function $\gamma(e)$ is assumed endowed with $\gamma'(e) > 0$, $\gamma''(e) \geq 0$, and $\gamma(0) = 0$. Hence, the objective function of each agent is:

$$\max_e P_1(e, \mathbf{e}_{-i}) \times V_1 + ... + P_n(e, \mathbf{e}_{-i}) \times V_n - c\gamma(e)$$

Each $P_i(e, \mathbf{e}_{-i})$ is then the probability effort level e induces for winning the ith prize given the strategy of all the other players. However, using the revelation

principle we can rewrite the agent's problem as simply choosing a type $\widehat{c}$ to declare himself, yielding:

$$\max_{\widehat{c}} F_1(\widehat{c}) \times V_1 + ... + F_n(\widehat{c}) \times V_n - c\gamma(b(\widehat{c}))$$

Here we have $b(\widehat{c})$ as the equilibrium bidding function and $F_i(\widehat{c})$ the probability of placing i given her declaration of being type $\widehat{c}$. Now since we assume each agent's cost type is also unknown to the contest designer, the designer has the following problem, as he wants to maximize total expected agent effort:

$$\max_{(V_1, ..., V_n)} k \int h\left(b(c, V_1, ..., V_n)\right) F'(c)dc$$

subject to $V_1 + \cdots + V_n \leq V$ total prize mass

The designer is simply choosing the values of the prizes $V_1, ..., V_n$ such that the expected revenue is maximized. For example, if $h(x) \equiv x$ then this is simply the expected effort of any particular agent multiplied by k, the number of agents[1]. We thus generally say expected revenue over effort since $h(\cdot)$ may not be linear, and in particular we will sometimes assume it is concave to allow for different designer goals. Based on our setup, we can now find the equilibrium bidding function $b(\cdot)$. Note we will use the term bidding and effort function interchangeably, so we can think of the optimal effort of a particular agent as their effort cost bid for the given prize.

3.1 The Effort Function

We now focus our analysis on two prizes, which will be sufficient to provide our results and intuition. We leave generalizing to an arbitrary number of prizes to future work.

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[1] At first blush, the notion of choosing different prize mass distributions for optimal effort output seems similar to so called handicapping. That is, under handicapping the designer forces certain participant(s) to essentially get partial credit for their effort, thus causing different outcomes. This can be sensible under a complete information setting, where much of the handicapping literature resides. However, in our setting, where a designer does not know each participant's cost of effort and such cost types are not correlated, handicapping is not practical; the designer does not know who has a particular cost type. Consequently, handicapping in this incomplete information world means the designer arbitrarily designates one contestant (s) with a handicap. However, doing so necessarily means *less* revenue for the designer, and thus we do not consider this scenario as we are studying contest design from a revenue maximization perspective.

Following MS, we denote the value of 2nd prize as α and $1 - \alpha$ the value of the first prize, giving a normalized total prize mass of 1. However, we will relax their constraint of $\alpha \in [0, \frac{1}{2}]$, instead allowing for any distribution of first and second prize: $\alpha \in [0, 1]$. That is, we now will solve for the optimal prize allocation given the designer's choice of any prize distribution over two prizes.

The inverse of our contestant cost (of effort) function $\gamma(x)$ is $g(x)$. As outlined above, we assume $\gamma(x)$ is weakly convex. First assuming $V_1 \geq V_2$, it is then routine to find the bidding function of each participant by integrating "down" the first order condition of contestants (i.e., their differential equations) with the initial condition of the highest cost type providing zero effort. However, MS provide their bidding function in a particularly helpful form, defining a participant's bid as a convex combination of two objects based on the distribution of prize mass:

$$b(c) = g(A(c)(1 - \alpha) + B(c)(\alpha))$$

These two objects $A(c)$ and $B(c)$ represent the optimal bid by a cost type c with linear costs of effort under the case of there only being a first prize and second prize, respectively. They are defined thus:

$$A(c) \equiv (k - 1) \int_c^{\overline{c}} \frac{1}{a}(1 - F(a))^{k-2} \times F'(a) da$$

$$B(c) \equiv (k - 1) \int_c^{\overline{c}} \frac{1}{a}(1 - F(a))^{k-3} \times [(k - 1)F(a) - 1] \times F'(a) da$$

In Figure 1, we now consider the equilibrium effort as a function of contestant type in the face of a single prize of \$1 and two equal prizes, each worth \$.50. The red curve, representing effort under a single prize, is the highest for the lowest cost (i.e., most able types), but then is lower for the top 80% of cost types compared with the blue effort curve, which is effort under equal prizes. Thus, we see equal prizes elicit less effort from the most able, but more from all the rest. In fact, there is a crossing point at about 20% of the most able population (i.e., the cost type $c \approx .62$). Hence, to the left of this point, a single prize incentivizes greater effort from the most able, thus we label this the "Incentive Effect"–i.e., all the area between the blue and red curves for the most able type. However, to the right of this dividing cost type point we lose effort from all participants and thus label this area of difference as the "Discouragement Effect"–i.e., incentivizing the most able means discouraging over

80% of the population, resulting in their reduced effort. The intuition is since the top 80% cost types now have to be best rather than just second best to get a prize, they start giving up, as their chances for such achievement are dismal.

Figure 1: Incentive and Discouragement Effect
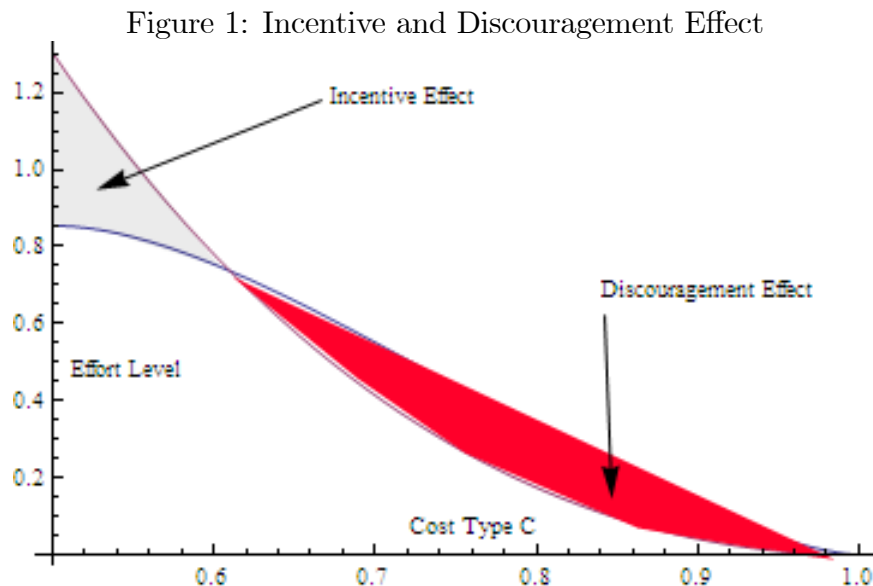
Now we want to consider what would happen if we actually offer a second prize *larger* than first prize. That is, in our notation, we want to explore allowing $\alpha > .5$. When we do allow $\alpha > .5$, we run into the problem (for an incomplete contest setting) that the best response function then becomes non-monotonic in type, as we prove in our next lemma. Thus, we will need to provide a mechanism to correct for this.

Lemma 1 *If $\alpha > .5$, the contestant best response function becomes single peaked with a maximum at $\widehat{c}$ such that $F(\widehat{c}) = \frac{2\alpha - 1}{k\alpha - 1}$*

Proof: see appendix.

We now turn to another example in Figure 2 of increasing the value of the second prize compared with the first prize, as well as having a second prize larger than first prize. Here, as we assumed above, we have $\gamma(x) = x^2$, $c \in U[.5, 1]$, and $k = 5$ participants:

Our horizontal axis on the above figure represents cost type c. The vertical axis is best response effort for a given type. The blue (solid) line is the bidding function

Figure 2: Effort of Each Type for Different Prize Divisions
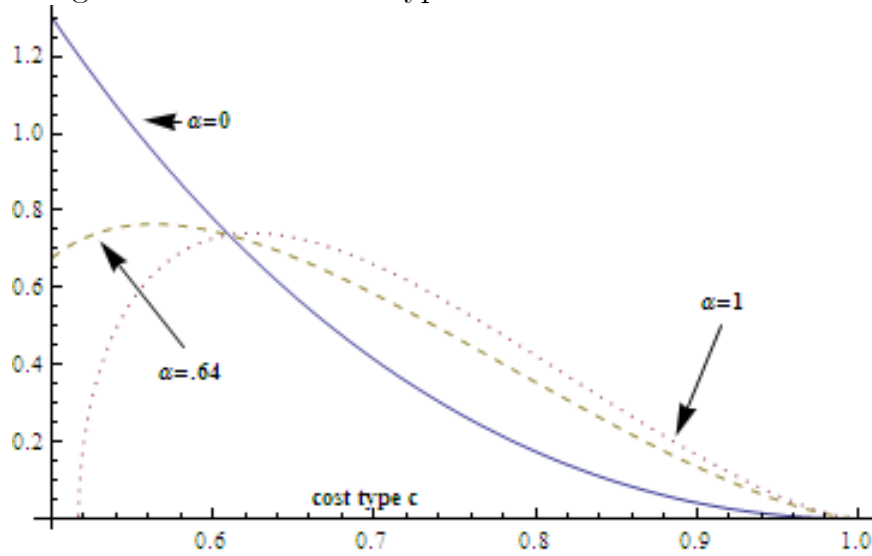
assuming $\alpha = 0$ (i.e., only a 1st prize is offered) and the red (dotted) shows bidding under $\alpha = 1$ (i.e., only a second prize is offered). These two lines again show the trade off between offering more of a first versus second prize. The 1st prize always increases the effort of the lowest cost types until about type .61. However as the cost becomes greater for a given type, then it is the second prize that creates more effort. Hence, offering more of a second prize increases the effort of the roughly top 80% of cost types, but reduces the effort of the bottom 20% of cost types. Thus, where the marginal revenue increase of raising the 2nd prize equals the marginal loss in reducing the 1st prize, we find our optimal α^*.

Finally, the yellow (dashed) line traces the bidding function of $\alpha \approx .64$, which is the optimal α for this example. Of course, even though $\alpha \approx .64$ yields the highest total expected effort, it is not feasible[2] due to its effort function non-monotonicity. We now turn to making such prize allocation feasible.

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[2]Here we mean not feasible in the contract theory sense. That is, since types are private information for each contestant, we must have each type's local IC met, which is violated with a non-monotonic bidding function.

4   Generalized 2nd Prize Contest

We propose the generalized second prize contest (GSPC), which then fixes the non-monotonicity of the bidding function for $\alpha > .5$. We "iron out" the non-monotonic part of the bidding function by creating a pooling interval. In particular, we find some maximal effort level e^* at which pooling will occur endogenously by participants. Any exerting effort below this level will be ranked by effort, as before, to determine prize allocation. However, any contestants at e^* are pooled. If there is only one such contestant, they receive 1st prize. The next highest effort contestant with effort below e^* will get second prize. If there are two or more contestants in the effort pooling interval, first and second prize will be randomly allocated with equal chance among contestants along the pooling interval. For example, if 3 people pool, each of them has a separate $1/3$ chance of getting 1st prize and a $1/3$ chance of receiving 2nd prize. Hence, there is a $1/9$ chance a contestant receives both first and second prize.

To see an example of the GSPC mechanism, we continue our last bidding function example and add the location of the pooling interval, as seen in Figure 3:

Figure 3: The GSPC Mechanism
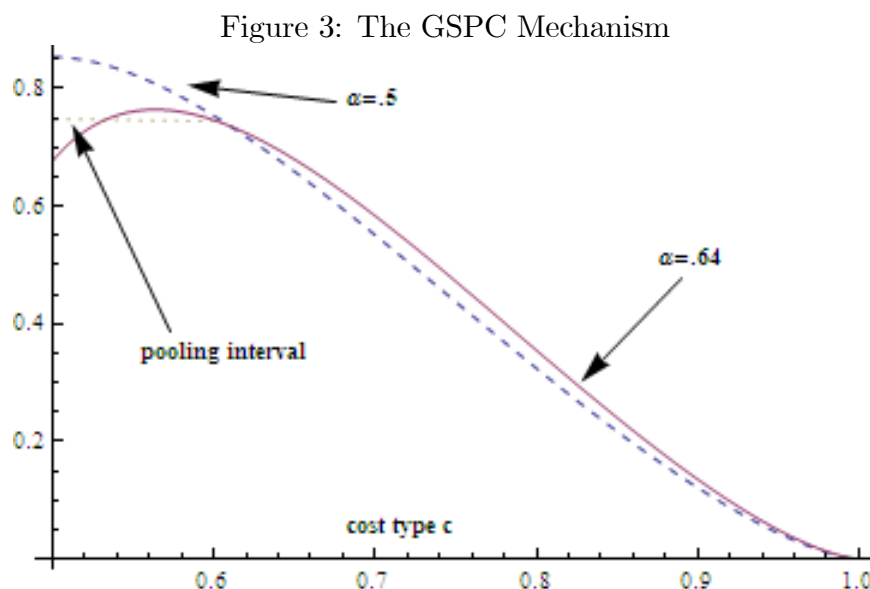
Here the blue (dashed) line represents effort if we instead set $\alpha = .5$, whereas the red (solid) line shows $\alpha \approx .64$. The yellow (dotted) line then shows the effort level

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of the pooling interval. Note if the area between the blue and red line but below the yellow line is greater than the area above the yellow line and below the blue line, then the GSPC generates more total effort than a contest constraining $\alpha = .5$.

It turns out we can always find a symmetric equilibrium, as our next proposition gives:

Proposition 1 *The generalized 2nd prize contest mechanism exists, meets all incentive compatibility constraints, and induces a (weakly) monotonic bidding function.*

Proof:
See Appendix.

The idea of the proof is we can find a unique contestant type that is indifferent between pooling and participating under the non-pooling contest. We then show that everyone in the pooling interval (i.e., all cost types lower than the indifferent cost type) prefers not to deviate up or down. Next we see everyone not pooling (i.e., everyone with greater cost than the indifferent cost type) strictly prefers to remain as they are. Finally, we then show the pooling interval always arises beyond the single peak of the original non-monotonic best response function, ensuring the new mechanism induces a weekly monotonic bidding structure. We also note if $\alpha \leq .5$, then the GSPC has a pooling interval with zero mass and thus collapses to a strictly monotonic bidding function. That is, the extant literature's constrained contest is nested within this one. Additionally, we now see that our generalization allows us to offer a larger second then first prize, but it also allows us to offer *only* a second prize, as we return to later.

4.1 GSPC With Divisible Prizes

First note In the case of divisible prizes it is then natural to solve for an optimal α^* such that we maximize expected contest revenue. We are using the term revenue over effort to again accommodate that under concave designer benefit functions, it is total revenue and not effort *per se* that we are maximizing. That is, we solve under linear designer benefits:

$$\max_{\alpha \in [0,1]} R(\alpha) = k \int_{\underline{c}}^{\overline{c}} g(A(c) + \alpha(B(c) - A(c))) \times F'(c) dc$$

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However, if we allow the designer's benefit function to be non-linear, we then have:

$$\max_{\alpha \in [0,1]} R(\alpha) = k \int_{\underline{c}}^{\bar{c}} h(g(A(c) + \alpha(B(c) - A(c)))) \times F'(c)dc$$

Unfortunately, α^* must generally be solved for numerically, thus requiring a given set of parameters for a given problem. Here are some examples of the optimal α given the total participants k and pair (w, z), where the contest designer's benefit function is y^w and the contestant's *total* cost function is $c \cdot x^z$. Throughout our examples we assume c is distributed uniform such that $c \in U[.5, 1]$:

Optimal Second Prize $\alpha^* \in [0,1]$				
benefit/cost exponents	k participants			
(w, z)	3	4	5	10
$(1, 2)$	.38	.54	.64	.84
$(1, 3)$	.50	.65	.74	.89
$(.5, 2)$	.54	.69	.77	.91

For example, with $k = 3$ participants, functional form of participant cost $c\gamma(x) = cx^2$ and designer concave benefit of $y^{.5}$, the optimal prize allocation is 54% to second and 46% to first. This gives us a ratio of prizes of $\frac{.54}{.46}$, or about a 17% greater second prize.

Thus, with a bit more convexity of cost, concavity of designer benefit or more participants, we can quickly get the optimal allocation being greater than 50% allocated to second prize. We now state in general when we want to offer a larger second than first prize.

Proposition 2 *It is optimal to offer a larger second prize than first prize through our GSPC if our sufficient condition is met:*

$$k \int_{\underline{c}}^{\bar{c}} h' \left(g(\frac{1}{2}(A(c) + B(c))) \right) \times g'(\frac{1}{2}(A(c) + B(c)))(B(c) - A(c)) \times F'(c)dc > 0$$

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Proof: see appendix.

Note this sufficient condition is only an assumption on the primitives: the convexity of the participant cost function (which determines its inverse $g(\cdot)$), the concavity of the designer benefit function $h(\cdot)$, number of participants k, and the distribution $F(\cdot)$ of cost types c. If these four factors are combined in a sufficient manner, then our above condition is met.

Corollary 1 *The GSPC always yields (weakly) more total revenue than the constrained (MS) contest.*

This corollary, which follows immediately from our previous Proposition, gives that whenever we must offer equal prizes under the constrained contest (i.e., $\alpha \leq .5$ binds), the GSPC will provide more total effort with a larger second prize. Meanwhile, when the optimal $\alpha^* < .5$, then the two contests agree, providing the same revenue. Thus, in short, the GSPC dominates the constrained mechanism of MS.

Now many prizes in practice aren't readily divided up into equal (or even multiple prizes). For example, consider the position of CEO. A firm would not (likely) want to divide this into 10 smaller equal positions due to (presumed) synergy of the CEO multi-tasking. Also, we could think of certain prizes costing the designer in terms of both a fixed and variable cost for each prize unit offered. With sufficient fixed costs, the designer will want to limit the number of prizes, maybe even offering only a single prize. We now explore the question *when is it better under an indivisible prize to offer it to second place over first place.*

4.2 GSPC With an Indivisible Prize

From our previous analysis of divisible prizes, we have an obvious condition for offering only a second prize versus first prize being optimal if $\widetilde{R}(1) > R(0)$. However, this is more than is needed. There may be some $0 < \alpha^* < 1$ such that $\widetilde{R}(\alpha^*) > \widetilde{R}(1)$ and yet we still have $\widetilde{R}(1) > \widetilde{R}(0)$. Thus, in the spirit of our divisible prize results, we can make similar assumptions on the primitives to assure a sole second prize is a preferred to a first prize.

We now consider some examples comparing the revenue of offering only a first prize versus only a second prize. We as before assume total cost is cx^z for effort x and cost type $c \in U[.5, 1]$. The designer's benefit function is simply y^w, where y is a contestant's total effort. We report the increased revenue in the table below.

Strikingly, total revenue is increased some 10% to 20% once we have four or five contestants by offering a prize only to second place over first place. The intuition is through shifting the prize from first to second, the most able (i.e., lowest cost), which are most likely to win first prize, have a lower marginal benefit and thus exert less effort (recall there is still the possibility of pooling, so there is a chance the most able can get second place if at least one other contestant pools). However, others that are less able (i.e., higher cost) are more likely to win second, which is now quite large, thus causing them to exert greater effort. As long as the latter group's increased effort overcomes the former group's reduced effort, total effort is increased.

Increased Revenue from Offering a Sole 2nd Prize				
benefit/cost	k participants			
(w, z)	3	4	5	10
$(1, 2)$	1.9%	6.4%	9.4%	16.4%
$(1, 3)$	6.4%	11.3%	14.5%	21.2%
$(.5, 2)$	7.2%	11.9%	14.9%	21.3%

We now turn to considering in more detail a designer with different goals.

5 Designers with Different Effort Goals

Consider the professor that has a student moving from a 98% to a 100% grade and another student moving from 68% to 70%. The latter 2% change is likely more valued than the former 2% change. This suggests a class of contests where the designer does not value effort as perfect substitutes: proficiency. If a teacher is educating students with the primary goal of helping them all reach a level of proficiency, then the designer has diminishing valuation of effort across participants. Similarly, consider the regulator who wants to move firms to a certain standard of environmental care. As yet another example, consider contestants that have complementary effort inputs. If this is so, then again the designer values effort across any particular worker in a diminishing manner. In other words, each of these classes are such that the designer has concave benefit over the effort of contestants. One could even argue this concave valuation over effort case is more the rule than the exception in the real world. Concavity of effort valuation provides a nice parallel to the role of convexity of contestant costs in inducing greater revenue from offering a larger second prize. Now we can have linear contestant costs and still want to offer a second prize if the designer has concave valuation of effort, in contrast to MS

(2001) finding linear costs always prescribe a WTA structure. In fact, we can state the following link between contestant costs and designer benefits.

Lemma 2 *For every (invertible) convex cost function $\gamma(e)$, there exists a concave designer benefit function $h(x)$, such that with linear benefits and linear costs, respectively, revenue is the same.*

$$\max_{\alpha \in [0,1]} R(\alpha) = k \int_{\underline{c}}^{\overline{c}} g(A(c) + \alpha(B(c) - A(c))) \times F'(c) dc$$

where $g(x) \in \{\gamma(e)^{-1}, h(x)\}$

Proof. This result can be seen immediately by writing out the revenue function under linear designer benefits and strictly convex participant costs and then again with strictly concave designer benefits and linear contestant costs. If we define $h(x) \equiv \gamma(e)^{-1}$, the result follows. ∎

It should be clear if both the contestant has convex costs and the designer concave valuation of effort, then these two effects only amplify one another, even more readily pushing the optimal prize mass down from the top performer to second place. However, we still have the problem of disentangling the precise effect of a particular primitive on whether or not to offer more of a second prize. We thus next turn to a class of cost functions that allows us to make some clear prescriptions.

6   Effort Capacity Constraints

Though we will give up some of the generality of our contestant cost function, in return we will be able to disentangle the effects of our primitives, as well as garner some intuition of how it works for general convex cost functions. In particular, we will assume contestants have linear costs of effort but face some effort capacity constraint. We could instead assume a common budget constraint or even monotonically decreasing constraints (or budget constraints) in cost type. It should be clear only minor modification of our proofs are necessary to show the same results for these settings. However, to ease exposition we will assume a common effort capacity constraint.

In addition to simplifying analysis, the notion of capacity constraints on effort is quite realistic in some settings. For example, lobbying caps are imposed or lawyers are limited on maximal award amounts. Workers can only work 24 hours a day and

are subject to some maximal physical strength. All we require for the analysis to be interesting is these constraints still bind for the most able.
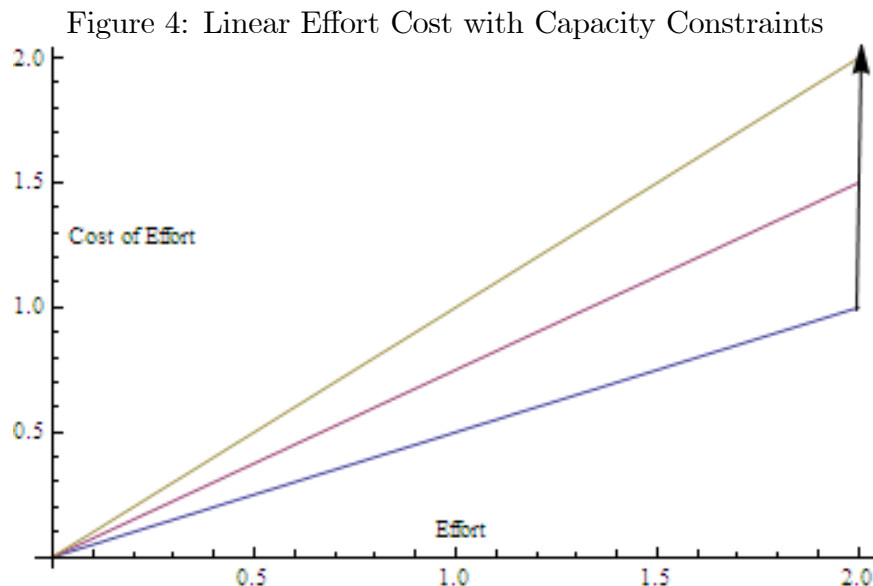
Consider the class of linear cost functions with cost type c, effort e, and capacity constraint $\widehat{e}$[3]:

$$c\gamma(e) = ce \text{ if } 0 \leq e < \widehat{e}$$

$$c\gamma(e) = +\infty \text{ if } e \geq \widehat{e}$$

We will call the capacity threshold $\widehat{e}$ *maximal effort*.

Thus we next show in Figure 4 the total cost function for type $c \sim U(.5, 1)$, taking the lowest, highest, and mean cost type, and $\widehat{x} = 2$ is:

Figure 4: Linear Effort Cost with Capacity Constraints

We can also imagine an analog where the *designer* has a constraint on the return to effort. That is, rather than having a smooth, concave valuation over effort as before, assume the designer has a sharp cutoff from the value of any particular player's production of effort. This can be though of as a simplified version of capturing say a teacher that gets rewarded for the number of students passing some standard as

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[3]See Megidish and Sela (2009) for example of linear costs and floor constraints. That is, contestants must put in a minimal amount of effort.

opposed to rewarded for their mean score. Alternatively, imagine a group of workers operating in some dangerous environment where they only remain safe if *all* engage in some several safety steps.

6.1   Equal vs. WTA Contest

We will only consider how our primitives must change to ensure an equal prize contest provides more revenue than a WTA contest. However, it should be apparent in the proofs, the arguments can be used to show the results apply as a comparative static as well, calling for a greater second prize, but not necessarily equal prizes. To proceed it is first helpful to prove a Lemma.

Lemma 3 *There exists a unique type c^{**} that provides the same level of effort regardless of α, the share of second prize. As $\alpha \nearrow$, types $c < c^{**}$ reduce effort and types $c > c^{**}$ increase effort.*

Proof. MS (2001) prove there exists some unique c^{**} such that $A(c^{**}) = B(c^{**})$, where $A(c) > B(c)$ when $c < c^{**}$, and $A(c) < B(c)$ when $c > c^{**}$. Combining this result with bidding $b(\alpha, c) = (1 - \alpha)A(c) + \alpha B(c)$ means increasing the share of second prize α increases effort for all cost types $c > c^{**}$ and reduces effort for all types $c < c^{**}$. ∎

This Lemma tells us there is some unique type that is indifferent to any prize structure. That is, she provides the same effort regardless of the distribution of prizes. Meanwhile, everyone below this cost type *reduces* effort as second prize is increased and everyone with greater cost *increases* effort. Hence, we need only compare the difference in these two changes as we increase the second prize to determine the superior prize structure. Now we can prove our next Proposition.

Proposition 3 *Fix a binding capacity constraint $\widehat{e}$ and assume linear participant costs and some atomless distribution of ability $F(c)$ with strictly positive lower support. Offering two equal prizes over a winner-takes-all (WTA) contest produces more total revenue given one of the following:*
   1) Enough of a contestant's capacity constraint $\widehat{e}$
   2) Enough of a designer's constraint over the return of individual effort e
   *3) Enough participants k*

Proof. 1) Define the effort of the type $c^{**} : b(\alpha, c^{**}) = e^{**}$, where c^{**} is defined in Lemma 3. Set capacity constraint $\widehat{e} = e^{**}$. This means as we increase $\alpha \to \frac{1}{2}$,

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revenue is strictly increased. This is because $\widehat{e}$ is binding for all types subject to the incentive effect (i.e., $c < c^{**}$). That is, these types would like to produce more effort but are unable for all values of $\alpha \in [0, \frac{1}{2}]$. However, all the other higher cost types are subject to the discouragement effect (i.e., $c > c^{**}$) and thus increase effort as $\alpha \to \frac{1}{2}$. Hence, there is no tradeoff as $\alpha \to \frac{1}{2}$. Thus, there exists some $\widehat{e} > e^{**}$ such that the tradeoff of increasing α is an equal tradeoff at $\alpha = \frac{1}{2}$.

2) Set the designer's constraint over individual effort at e^{**} and the argument is identical to (1).

3) Fix some k and binding $\widehat{e}$. As the participants k increase, c^{**} and thus e^{**} changes. In particular, as shown in the appendix, as $k \longrightarrow \infty$, $c^{**} \longrightarrow \underline{c}$ and $b(0, \underline{c})$ increases. But this means, the maximal bid for an arbitrary k is eventually surpassed by c^{**} as $k \longrightarrow \infty$. This also means there exists some finite k^* such that $e^{**} = \widehat{e}$. The argument then proceeds as in (1). ∎

The intuition for this proof is best seen by the following figure:
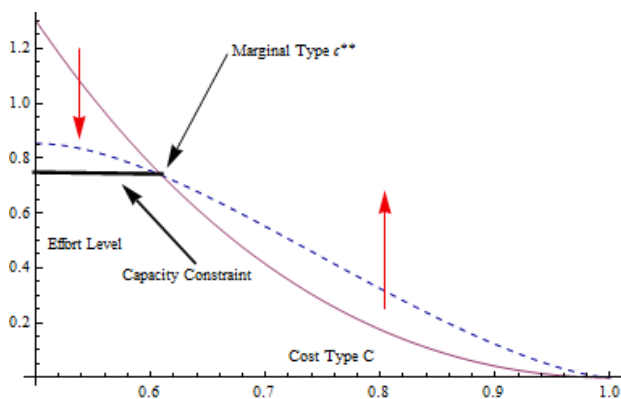
Capacity Constraints and Optimal Prizes

The marginal type c^{**} is key to the proof. If the capacity constraint is set to $\widehat{e} = e^{**}$, where e^{**} is the bid of the marginal type c^{**}, there is no tradeoff in increasing α from zero to $\frac{1}{2}$. Without a constraint, we would be losing effort from all types $c < c^{**}$ while gaining effort from all types $c > c^{**}$. However, with the constraint $\widehat{e}$, we do not lose any effort from $c < c^{**}$ because they would like to provide more effort but are unable for all values $\alpha \in [0, \frac{1}{2}]$. We could then chose some $\widehat{e} > e^{**}$ such that the tradeoff is just equalized at $\alpha = \frac{1}{2}$. This then also provides intuition of why convex costs in general can induce greater effort through flattening the prize structure: rather than starkly removing the upper portion of effort from the most

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able it simply distorts such effort downward to a greater extent than when the most able's effort is much lower under an equal prize contest. In other words, convexity of cost *blunts* the incentive effect.

The intuition is similar for the designer with concave valuation over effort. However, now rather than distorting down the effort of the most able, it is distorting down the *value* of effort exerted from the most able. Thus, concave valuation of effort *reduces the return* from the incentive effect.

As we increase the number of participants, we also increase e^{**}, which then means it surpasses our original $\widehat{e}$ for some k. In other words, the discouragement effect has relatively more impact as $k \longrightarrow \infty$. We can say that increasing number of participants *amplifies* the discouragement effect. Now we consider the role of skill heterogeneity on prize structure.

6.2 Heterogeneity of Skill and Prize Structure

First we note some results derived in the appendix that can be used for estimating linear bids. The first pair of expressions puts a lower and upper bound on the bid of an equal prize contest, and the latter pair for a WTA:

$$\frac{1}{2}\underline{(A(c) + B(c))} = \frac{1}{2\overline{c}}\left((F(c) \times (k-2) + 1) \cdot (1 - F(c))^{k-2}\right)$$
$$\frac{1}{2}\overline{(A(c) + B(c))} = \frac{1}{2\underline{c}}\left((F(c) \times (k-2) + 1) \cdot (1 - F(c))^{k-2}\right)$$

$$\underline{A}(c) = \frac{1}{\overline{c}}(1 - F(c))^{k-1}$$

$$\overline{A}(c) = \frac{1}{\underline{c}}(1 - F(c))^{k-1}$$

Now we calculate the expected bid under a WTA under incomplete information as $\underline{c}$ and $\overline{c}$, the bounds of our distribution of types, collapse to the mean type $\widetilde{c}$. We then get:

$$\frac{1}{\widetilde{c}} \int (1 - F(c))^{k-1} f(c) dc = \frac{1}{k} \times \frac{1}{\widetilde{c}}$$

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Hence, as usually assumed in the complete information case, with $\widetilde{c} = 1$, we simply get the expected bid of $\frac{1}{k}$ and thus expected revenue is $\frac{1}{k} \times k = 1$, which means 100% rent dissipation.

In other words, as we approach (perfect) homogeneity with linear costs and no capacity constraints, the expected bid in a WTA contest is simply $\frac{1}{k}$ per each contestant, which is precisely the same as the *expected bid* under a complete information case (and symmetric equilibrium) with mixed strategies. Note, however, that under incomplete information the strategies are unique pure strategies as opposed to the mixed strategies of the complete information case.

We now consider the expected bids under incomplete information and two equal prizes (i.e., $\alpha = .5$). As the (incomplete info) contest approaches homogeneity, we write it thus:

$$\int_{\underline{c}}^{\overline{c}} \frac{1}{2}(A(c) + B(c)) \times F'(c)dc$$

$$\rightarrow \quad \frac{1}{\widetilde{c}} \int \left((F(c) \times (k-2) + 1) \cdot (1 - F(c))^{k-2} \right) f(c)dc = \frac{1}{k} \times \frac{1}{\widetilde{c}}$$

This then provides the same result for equal and WTA case, as well as the complete information case. In other words, as we approach homogeneity both in the case of complete information and incomplete information we get the same expected revenue of 1 regardless if we have a WTA or equal prizes. An easy extension to our above analysis shows this relationship between the complete and incomplete information contest is true for any $\alpha \in [0, 1]$.

In the spirit of the purification theorem (i.e., Harsanyi (1973)), we have a found a sequence of pure strategies (i.e., the sequence of unique bids for each type from a distribution that converges to its mean type) under the incomplete information game that converges to the (symmetric) mixed strategy equilibrium of the same game under complete information. That is, we could consider our found relationship as a refinement of the multiple equilibria of a complete information contests—only the symmetric equilibria survive. And we also see it only takes an ε of uncertainty over the type space to move from a (non-unique) mixed strategy equilibrium to a symmetric pure strategy equilibrium. We will now use this relationship for another proposition:

Proposition 4 *If an effort capacity constraint binds for some measure of the type space, given enough homogeneity of types, an equal prize contest provides more revenue than a WTA contest*

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Proof. We already showed the expected revenue of the WTA contest converges to that of the equal prize contest as we approach homogeneity (with no capacity constraints). Again assuming the mean type $\widetilde{c}$, using our estimates found in the appendix, as we approach homogeneity the maximal bid (i.e., of the lowest cost type) in the WTA contest is

$$\frac{1}{\widetilde{c}} \times (1 - F(\underline{c}))^{k-1} = \frac{1}{\widetilde{c}}$$

With equal prizes we find, in the limit, the lowest cost type bidding

$$\frac{1}{2\widetilde{c}} \left((F(\underline{c}) \times (k-2) + 1) (1 - F(\underline{c}))^{k-2} \right) = \frac{1}{2\widetilde{c}}$$

Thus, with a capacity constraint $\widehat{e} \in \left[\frac{1}{2\widetilde{c}}, \frac{1}{\widetilde{c}} \right]$, effort is reduced only under a WTA contest. Thus, with sufficient homogeneity, the equal contest provides strictly greater revenue, as revenue was roughly equal before any capacity constraints. ∎

To understand what is happening under homogeneity Figure 5 below assumes cost type distribution of $c \in [.9, 1.1]$:

Figure 5: Approaching Homogeneity of Cost Type
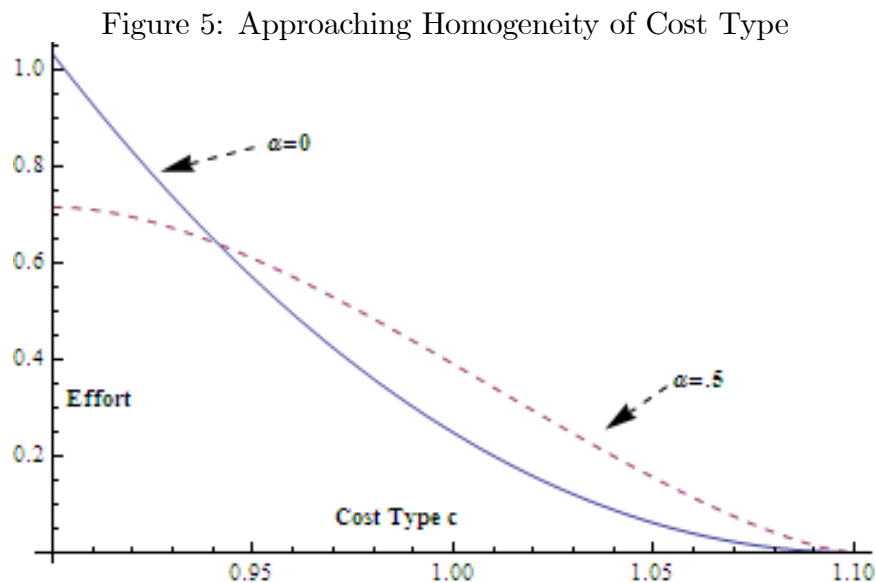
As we approach cost type homogeneity, we have $\int^{\overline{c}} (B(c) - A(c)) \times$ $F'(c)dc \to 0$, which means the area under the curve (i.e., the expected revenue) for

both $\alpha = 0$ and $\alpha = .5$ is the same. But this means setting a capacity constraint of $e = .8$, for example, results in the equal prize contest providing strictly more revenue than the WTA contest. That is, doing so reduces the area of the former without affecting the area of the latter.

This proof also provides nice intuition of why with convex costs having a second prize is superior to a WTA contest once types become homogeneous enough: as types are sufficiently homogeneous, any combination of first and second prize provide roughly the same revenue. However, under a WTA contest, the lowest cost types are providing much greater effort than under an equal prize. Hence, introducing convex costs distorts downward these greatest effort levels more than the lower effort levels these types provide under an equal prize scheme. The net result is again less expected total revenue from a WTA over equal prize contest.

6.3   Empirical Predictions

Ideally, we would like to take our predictions to the data. However, the results of MS and our earlier results that generalize theirs are difficult to test. This is because we all rely on the curvature of the contestant cost function. Cost functions are difficult to observe and measure in practice, let alone their degree of curvature.

Fortunately, our last class of contests—linear costs with capacity constraints— provide some sharp empirical predictions that do not rely on measuring the curvature of cost functions. Indeed, it should be clear that if we now introduce convex costs coupled with capacity constraints, this only strengthens the above comparative statics. Hence, regardless of the curvature of the cost function, the comparative statics still hold in the face of capacity constraints.

Thus, we can predict as follows:

When participants (or firms) have a meaningful limitation on their maximal effort choice, we can say the following regarding optimal incentive structure:

1) As the range of ability *decreases*, incentives flatten

2) As the designer values effort in a complementary manner or has the purpose of incentivizing agents to reach a given standard or proficiency, incentives flatten

3) As the number of competitors increase, incentives flatten

4) As capacity limitations become more severe, incentives flatten

If we are able to observe total output, then we can replace all the above predictions' statements that "incentives flatten" with "revenue increases with flatter incentives."

We now turn to the English auction for not only further intuition by means of order statistics, but also to show this interaction of the "incentive" and "discouragement" effect applies more broadly to other settings. In fact, an English Auction can be though of as a winner-pay contest where contestants compete but only the winner(s) pay his(their) bid(s). Yates (2010) provides some real world examples of these kinds of contests, including versions where contest losers are reimbursed for their efforts or bids *ex-post*.

7   Intuition via the English Auction

To begin we consider linear participant cost contests (with no constraint) and one versus two equal prizes, and link these to an English Auction (EA), showing their revenue equivalence. To correlate the two forms, we assume participants in the EA only want, or are able, to acquire one unit and participants have independent private valuations over objects. For the former, using some results from Moldovanu et al (2008) and some further analysis, we write the revenue of an all-pay auction (AP), which is equivalent to a contest with linear participant costs and no constraints, as the following, normalizing the total prize mass to 1:

$$R_{AP}(\alpha) = (1 - 2 \cdot \alpha) \times E(k - 1, k) + 2 \cdot \alpha \times E(k - 2, k)$$

We then see the revenue from an all-pay auction is a convex combination of the 2nd and 3rd order statistics. Thus, with a WTA auction (i.e., $\alpha = 0$) this then collapses to $E(k - 1, k)$, the expected value of the second most valuing type. Through the revenue equivalence theorem, we know this is also the same as the English auction with a single prize being auctioned off (with linear costs).

Now with two equal prizes, we get $R_{AP}(\frac{1}{2}) = E(k - 2, k)$,the third most valuing type. We could appeal to Krishna (2002), for a multi-unit revenue equivalence theorem to show that we then obtain the same revenue from a (generalized) English auction. However, it is more instructive to explicitly find the revenue from the (generalized) English auction. We first note once the third to last participant drops out of the auction, both remaining contestants will immediately drop out: since first and second prize are the same, there is no value in further bidding. Hence, ex-ante, for the auctioneer, the expected revenue for two equal prizes, or objects, is simply:

$$R_{EA}\left(\frac{1}{2}\right) = \frac{1}{2} \times E(k - 2, k) + \frac{1}{2} \times E(k - 2, k) = E(k - 2, k)$$

That is, the first and second most valuing type drop out immediately after the

third most valuing type drops out, which for her is when her net expected return is zero, as remaining in until the bid reaches ones valuation is the dominant strategy. This can be expressed as $\frac{1}{2} \times A_i - b_i = 0 \Rightarrow b_i = \frac{1}{2} \times A_i$, where A_i is the ith order statistic. Taking the expectation then gives the third most valuing type dropping out at a bid over equal objects each worth $\frac{1}{2}$ as $\frac{1}{2} \times E(k-2, k)$. Since after she drops our there remain two contestants to pay for two objects, total revenue is then $2 \times \frac{1}{2} \times E(k-2, k) = E(k-2, k)$, just the same as an AP auction.

Now when we introduce convex participant costs in the auction (or contest), the revenue equivalence theorem fails since convex costs are equivalent to assuming risk averse bidders. With convexity of costs we have $\frac{1}{2} A_i - \gamma(b_i) = 0$, which can be rewritten as $g\left(\frac{1}{2} \times A_i\right) - b_i = 0$, where $\gamma(\cdot)^{-1} \equiv g(\cdot)$, and $\gamma(\cdot)$ is the convex cost function. If instead bidder costs are linear, having the auctioneer value individual bids as $g(\cdot)$ shows the equivalence.

Again, with this auction format, the bidding strategy is simple in both a single good or equal good auction: remain in the auction until your value is reached.

Thus, for the single prize, we get $A_i - \gamma(b_i) = 0$, which means the second most valuing type drops out. The expected ex-ante revenue from this is then:

$$R_{EA}(0) = \int g(a) \times f_2^k(a) \times da < g\left(\int a \times f_2^k(a) \times da\right) = g(E(k-1, k))$$

$f_2^k(a)$ is the pdf of the distribution of the second order statistic and integration is over the support of types with k total contestants. The inequality follows from Jensen's inequality.

Similarly, we then find for the equal prize expected revenue is:

$$R_{EA}\left(\frac{1}{2}\right) = 2 \times \int g\left(\frac{1}{2} \times a\right) \times f_3^k(a) \times da$$

We can now compare the two prize distribution revenues to determine which garners more total (expected) revenue than the other. First, note as before, when costs are linear we get $g(x) = x$:

$$R_{EA}(0) = E(k-1, k) > E(k-2, k) = R_{EA}\left(\frac{1}{2}\right)$$

Thus, it is again always best to only offer a single 1st prize when costs are linear. However, now consider what happens as convexity increases. Here we mean convexity of the cost function $\gamma(\cdot)$ increases and thus the concavity of its inverse $g(\cdot)$ increases in the Arrow-Pratt sense: $\frac{-g(\cdot)''}{g(\cdot)'} \to z$. This then means, in the limit,

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we get $g(\cdot) \to c$, some constant c. Hence, we have equal prizes producing revenue in the limit of:

$$\lim_{\frac{-g(\cdot)''}{g(\cdot)'} \to z} R_{EA}\left(\frac{1}{2}\right)$$

$$= 2 \times \int \lim_{\frac{-g(\cdot)''}{g(\cdot)'} \to z} \left[g\left(\frac{1}{2} \times a\right) \right] \times f_3^k(a) \times da$$

$$= 2 \times \int c \times f_3^k(a) \times da = 2 \times c \times 1 = 2 \times c$$

.

Thus, we have:

$$\lim_{\frac{-g(\cdot)''}{g(\cdot)'} \to z} R_{EA}\left(\frac{1}{2}\right)$$

$$= 2 \times c > c = \lim_{\frac{-g(\cdot)''}{g(\cdot)'} \to z} R_{EA}(0)$$

Thus, there exists some degree of convexity such that an equal good English auction provides more revenue than a single good auction.

The intuition of how convexity causes equal goods to dominate a single good is simple: with increased convexity, the differential in revenue garnered from offering a \$.50 versus \$1 prize becomes increasingly small. However, under equal goods we are getting two participants paying this revenue rather than just one under a WTA. In other words, convexity starts limiting how much more of a bid a larger good elicits. We then reach a crossing point where although the bid is less for a \$.5 over \$1 good, it is not less than *half* the greater bid offered for the \$1 good. Thus, under an English auction with (sufficient) participant convex costs, if an auctioneer could divide an object into two equal parts, auctioning them off simultaneously would provide greater expected revenue than auctioning it off as a single object. This intuition then also follows for a contest setting: increased convexity starts limiting the value of having a larger first prize, thus allowing the *two* slightly less incentivizing prizes to garner more *total* contestant effort. Hence, with enough convexity, an equal prize contest yields more total revenue than a WTA contest.

It is then also immediate how instead with participant linear costs but designer concave valuation of individual bids, given enough concavity of benefit, two prizes dominates a single prize contest: convex costs and linear benefits is mathematically

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equivalent to linear costs and concave benefits. With both convex costs and concave benefits, the need to offer equal prizes is only strengthened.

The English auction also provides intuition on how increasing the number of participants makes an equal prize auction more likely to provide more revenue than a single prize auction. Recall under linear costs we have the revenue of allocating the prize mass as:

$$R_{AP}(\alpha) = (1 - 2 \cdot \alpha) \times E(k - 1, k) + 2 \cdot \alpha \times E(k - 2, k)$$

Thus, as $k \to \infty$, we have $E(k-1, k) \to E(k-2, k)$. This means in the limit of a large contest, we receive the same revenue regardless of prize distribution. Nonetheless, in a finite population it is still always best to only auction a single first prize with linear participant costs. However, when we introduce *strict* participant cost convexity, we are assured there exists some finite k such that offering equal prizes yields more revenue than offering only a first. This is immediate from our above analysis of strict convexity causing the expected revenue garnered from a $.5 prize to be strictly greater than revenue garnered from a $1 prize. That is, given large enough k, $E(k-1, k)$ and $E(k-2, k)$ become sufficiently similarly valued to provide greater total revenue from auctioning equal goods over a single good. Thus the size of the auction and the degree of cost convexity amplify one another: more of one requires then less of the other to still be assured we optimally auction off two equal goods over a single good.

We can also now see the role of heterogeneity of prize valuation. As the support of the distribution of types approaches a single type, the 1st and 2nd order statistic converge to one another—i.e., $E(k-1, k) \to E(k-2, k)$. Hence, following our argument above for increasing the number of participants, once we fix the convexity of the cost function and number of participants, sufficiently *decreasing* the heterogeneity of valuation will also result in equal goods providing more revenue than a single good to the auctioneer.

We summarize our above analysis in the following proposition:

Proposition 5 *Fix the size (number of participants k), convexity of participant bid costs, concavity of auctioneer benefit over bids, and degree of heterogeneity of valuation in an English auction with independent private values.*

1) There exists some $k^* \geq k$ such that for all $k \geq k^*$ offering two equal goods each worth V provides greater total revenue than offering a single good worth $2V$

2) There exists some degree of convexity of bid cost such that $c^* \equiv \frac{-h(\cdot)''}{h(\cdot)'} \geq \frac{-g(\cdot)''}{g(\cdot)'}$ yields for all $\frac{-h(\cdot)''}{h(\cdot)'} \geq c^*$ offering equal goods dominates a single good (where our cost function $\gamma^{-1}(\cdot) = g(\cdot)$).

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3) There exists some degree of concavity of benefit over bids such that offering equal goods dominates offering a single good

4) There exists some decreased level of heterogeneity of valuation such that offering equal goods dominates offering a single good

This then means we are given four levers to assure more revenue from auctioning equal prizes over a single prize: the size of the auction, the convexity of participant costs, the concavity of designer benefit, and the degree of valuation homogeneity. We only need to increase one to provide our result. We can again think of the English auction as an analog for a winner pays contest. That is, a contest where only the winner(s) has(have) to pay her(their) bid. Thus, the above results then follow immediately for a winner-pay contest.

8   Conclusion

Whether it be business, politics, or even academics, much is actually a contest. As such, an important task is to consider how to best design a contest. Central to this problem is accounting for the interaction of the "incentive effect" and "discouragement effect." This interaction arises with the combination of competition and heterogeneous ability. With only one of these factors, there is no tradeoff. Similarly, If only living in a world of linear costs and benefits and no capacity constraints, this interaction means little. Though it is convenient to study only one of these interacting factors in isolation, seldom does this characterize the real world. Instead, the presence of these dual forces is more the norm than the exception.

And in a world with both forces, once we face contestants with capacity constraints or convex costs, larger contests, contestants of similar ability, or designers with marginal decreasing benefits over effort, this interaction can become severe: the "discouragement" effect dominates the "incentive" effect calling for optimal incentives to be flat or possibly even inverted—second prize should be larger than first prize.

However, bidding under an inverted incentive scheme becomes non-monotonic. For such a problem we designed a mechanism we dubbed the generalized second prize contest (GSPC) mechanism, which nests in it the constrained contest that restricts a weakly greater first prize. We then found the GSPC (weakly) dominates the constrained contest in terms of total revenue generated for the contest designer.

In addition, we studied an alternative class of contests—contestants with linear costs and capacity constraints—that has the characteristic of providing sharp and measurable empirical predictions that can be taken to the data.

Finally, we studied a different setting of an English auction and find all the comparative statics applied in this setting, as well. The English auction is also an analog for another important class of contests: winner-pay-contests, and thus the results additionally apply here.

We do note that we only considered the case of two prizes. It would be interesting to expand our analysis and consider the case when we can offer n prizes with $n < k$ contestants; which prize should be largest? What about if prizes are indivisible— which place should receive the sole prize? We suspect we will find a k prize analog of our results.

Part II

Competition via Personnel Economics

We next test the above theory experimentally in the setting of personnel economics. We vary both the degree of competition and the bonus structure. We find,as theory suggests, that softening incentives tends to increase overall effort. However, we also find that individuals *behave* quite differently than expected. Instead of exerting effort in a smooth fashion declining in lessened ability, players mostly use a threshold heuristic—working excessively or not at all, depending on their ability in comparison to some threshold type. We also find that worker types are not immutable: different institutional designs cause workers to become different types: some reasonable and hard workers, but others slackers and quitters. We find in this setting, we maximize total effort by providing higher levels of worker competition but rewarding the best performer less, even giving the largest bonus to the *second* best performer. This structure elicits roughly 62% more total effort per bonus dollar over our baseline winner-takes-all bonus case.

9   Introduction

Many economic models prescribe sharp incentives. Incentives in the "real world," however, tend to be softer. For example, to align incentives and maximize performance we ought to make employees residual claimants of a firm. However, we see few examples of material employee ownership of firms. In fact, Kim & Ouimet (2009) report a mere 18% of all workers own any of their employing firm's shares. Similarly, although performance based labor contracts should usually be preferred over flat salaries, we find little evidence of such contracts in the workplace. Lemieux, McCleod and Parent (2009) report only 14% of workers receive any type of performance pay—which includes bonuses, piece rates, or commissions.

Labor tournaments with steep incentives offer another approach to maximizing effort. However, this high powered structure is not so apparent in practice. Two employees of similar activities but varying effort can both advance to subsequent positions. Seniority can sometimes trump performance for advancement. And a higher position, though accompanied with higher pay, when paired with greater

responsibility may not be such a steep jump in reward, if any.[4]

Why do soft incentives abound? We argue that it stems from the intersection of employee heterogeneity and competition. In particular, if a firm sharpens its incentives—by giving a greater share of the bonuses to the top performer(s) or increasing the number of workers competing over a set of bonuses—the *most able* are incentivized but the rest give up due to discouragement. Recent theoretical developments suggest this latter effect can overcome the former (e.g., see Moldovanu & Sela (2001) and Minor (2010)). But this again is theorizing. Ideally, we would like to manipulate firm compensation and ownership to deduce why incentives are soft in practice. In addition to the difficulty of having firms agree to our running experiments on their workers, even with such permission, it would be difficult to manipulate worker ability, let alone know its precise value. This is important because this newer strand of literature prescribing soft incentives relies heavily on precisely knowing worker types. We can instead use the laboratory to simultaneously test the effects of incentives and competition. We can then not only vary reward structure and degree of competition, but can also vary private information and ability among participants, all the while knowing their precise values.

In this spirit, we explore experimentally the effects of different institutional structures on effort. By institutional structure we mean the workers' degree of competition and the "steepness" of the bonuses being faced. The greater the fraction of bonuses given to the highest performers over lesser ones, the "steeper" the bonus structure. We thus manipulate these factors, as well as varying the ability of employees, to uncover when soft incentives actually do increase overall effort.

Our results are as follows:

1) As theory predicts, softer incentives tend to increase total effort—up to 62% over our baseline winner-takes-all case.

2) Individuals behave differently than expected: contestants choose levels of effort in a binary fashion based on a threshold heuristic.

3) Worker types are not immutable. Instead, the institutional structure—the degree of competition and bonus structure—creates different types of workers, which has important implications for organizational design.

4) We propose a simple model with a single parameter that captures subject behavior, and not only is a good fit for our data, but also explains well the data from other experimental studies.

Thus, the general prediction of softer incentives increasing overall effort is correct. However, workers make unexpectedly simple effort decisions. In particular, employ-

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[4]See Prendergast (1999) for further examples of surprisingly soft incentives within the firm.

ees have some cutoff of ability type, which varies across people. When a worker has ability above her cutoff, she exerts great levels of effort, and when she is below it she exerts little or no effort.

Further, we find there is a close interaction between a person and his environment. Specifically, we find the location of the cutoff point is influenced by the institutional structure, which means worker types are not immutable. In all of our institutional settings we find about a third of the workers have very low ability thresholds, and thus act as Strivers: overconfident types who work at capacity unless they are the very lowest of ability. Meanwhile, when the bonus is reserved for only the top performer and competition is fierce, the balance of workers tends to have very high ability thresholds, thus becoming Quitters: unless the most able of types, they simply give up due to discouragement. If instead, competition is reduced and more than the best performer receives a bonus, Quitters become Slackers—rather than giving up, workers put in minimal effort, hoping to capture an easy payday by gaming the system. Finally, when we maintain high competition but reward the typical worker more than the top performer, we witness many Realists—having a threshold of average ability, these people work very hard when they are above average ability and minimally when below average. It is under this setting that Quitters are encouraged to become Realists and Realists are discouraged from becoming Slackers. And it is this setting that produces the greatest total worker effort, some 62% greater than our baseline case per dollar of bonuses awarded.

The balance of our paper is organized as follows. Our first section reviews some relevant literature. The second section provides our theoretical predictions for our class of experiments. The next two sections review our experimental design and results. Section 5 provides the behavioral model to explain our observed threshold bidding. The next two sections calibrate and test this model. We conclude with a discussion and prescriptions for organizational design.

10   Literature Review

As mentioned in the introduction, Prendergast (1999) provides a survey of the disharmony between theory and empirics in the realm of firm incentives. In addition to existing empirical analyses, there have been some field experiments that explore varying work incentives. For example, Bandiera et al. (2005) study the effects of relative pay and piece rate pay on seasonal worker effort. Lazear (2000) examines the effects of a particular company adopting a piece rate scheme and finds that it

increases productivity, though relative pay is not explored. Neither of these explore the interaction of competition and incentive structure.

There has been considerable interest in exploring contests experimentally. Many of these studies have been the special case of the complete information Tullock contest (see Morgan et al. (2010) for a summary). The focus of these studies is bidder behavior versus theoretical predictions using the Tullock success function. Of particular interest in many of these studies is the degree of rent dissipation over optimal design of incentives. We are instead interested in the optimal design of contests over contestant behavior *per se.* We are also interested in contests where all players' abilities are not perfectly known, as they generally are in the above literature.

A new literature does explore different contest and incentive designs, and their effect on effort. For example, Cason et al. (2010) compare a winner-takes-all scheme to alternative schemes of piece rates and sharing prize mass proportionally based on output. Additionally, Freeman & Gelber (2010) study a winner-takes-all versus multiple prize contest. They find that when contestants know their type and the realized types of their competitors, a multi prize contest compared to a winner-takes-all contest elicits more effort from the less able and less effort from the most able. Finally, Muller & Schotter (2010), study private value contests with a constant degree of competition and compare equal to winner-takes-all prizes. This latter paper tests the constrained prize structure from Moldovanu & Sela (2006), which means it is testing a suboptimal contest as shown in Minor (2010). More importantly, Muller & Schotter (2010) do not vary competition, which as outlined below, is the central cause of incentive trade-offs. Consequently, they do not fully identify subject strategy and behavior.

Whatever the case, a consistent theme in this recent experimental work is identifying a "discouragement" effect. That is, as incentives become sharper, the most able try harder, but the less able try less, or even give up, creating a sharp discontinuity in effort degradation. Though these findings are consistent, they remain largely unexplained. One main purpose of this paper is to identify *why* we witness this behavior and study explicitly not only the "discouragement" effect but also identify the tradeoff with incentivizing via sharper incentives.

These past studies also generally hold the level of competition constant and thus are not able to identify how this interacts with the incentive and discouragement effect. Hence, another purpose of this paper is to determine the interaction effect of competition and incentives. We find when we optimally design incentives, we must determine not only reward structure but also the best degree of competition. Additionally, we find by varying the intensity of competition, we identify another behavior in addition to quitting; softened incentives interacted with decreased com-

petition, even though theoretically optimal, create a class of contestants that try to "game the system" through regular but minimal effort—i.e., slacking.

We will provide a simple behavioral model that predicts all three of the observed effects. In addition, not only does this model explain our data, we are able to obtain data from two other similar studies that will also be explained by our model. In fact, in some ways our model even better explains this "out of sample" data than our own data. But first we review extant theory.

11   Theory

Assume a setting where workers exert effort and are awarded a bonus in rank order based on effort. We are concerned with those settings where it is difficult to precisely measure employee effort, but it is possible to rank them. For example, consider a team setting where employee activities are not easily quantifiable but the manager has a good sense of how much each employee relatively contributes to the project.

The cost of effort x is linear. Employees have privately known heterogeneous effort cost types c drawn from a uniform distribution with support $[.5, 1]$. Workers face one or two bonuses, depending on the treatment, with common values V_1 and V_2. For simplicity, we assume a base wage of zero throughout. Although V_1 is awarded for the greatest effort and V_2 for the second greatest effort, they need not be monotonically ordered in terms of value. A worker then faces the following objective function:

$$\max_x F_1(b(x)^{-1}) \times V_1 + F_2(b(x)^{-1}) \times V_2 - c \cdot x$$

The CDF $F_1(x)$ $(F_2(x))$ is the probability of placing 1st (2nd) given effort x. The term $b(x)^{-1}$ is the inverse of the equilibrium effort function. The general procedure for finding this effort function is simply taking the first order condition and then integrating down to an arbitrary cost type c. The equilibrium effort function can be pinned down assuming the highest cost type in equilibrium exerts zero effort.

Assuming $V_1 \geq V_2$, we get the following equilibrium effort, as originally outlined in Moldovanu and Sela (2001):

$$b(c) = A(c) \cdot V_1 + B(c) \cdot V_2$$

We define

$$A(c) \equiv (N-1) \int_c^1 \frac{1}{a}(1 - \frac{c - .5}{1})^{N-2} \times 2da$$

and

$$B(c) \equiv (N-1) \int_c^1 \frac{1}{a}(1 - \frac{c - .5}{1})^{N-3} \times [(N-1)\frac{c - .5}{1} - 1] \times 2da$$

with $N \in \{3, 6\}$ contestants.

To examine the full contract space of two prizes, we want to allow for the possibility of a larger second prize. Minor (2010) develops a mechanism to accommodate this case. In particular, when we have $V_1 < V_2$, a pooling interval endogenously emerges where some measure of the most able types $c \in [.5, c^*]$ all exert the same effort up to some marginal cost type c^* that is indifferent to pooling or providing effort as under $V_1 \geq V_2$. All types $c \in [c^*, 1]$ still provide effort levels as given by $b(c)$ above.

We also want to allow for effort capacity constraints. Many settings—whether it be maximal physical strength or a maximum of 24 hours in a day—have some maximum of possible effort. As Minor (2010) shows, with an effort capacity constraint and linear costs, we similarly have a pooling interval where some measure of the most able types $c \in [.5, \bar{c}]$ all provide the capacity effort level κ up to some upper type $\bar{c}$, and then all higher costs types exert effort according to $b(c)$. As shown, in Minor (2010), once we introduce these capacity constraints the optimal prize structure can become much flatter than without such constraints. Indeed, with linear costs and *no* constraints, a winner-takes-all contest dominates. However, modest capacity constraints quickly make such a scheme a poor design choice.

Below we show a chart for two experimental treatments of $N = 3$ and one with a single bonus for the greatest effort and another with equal bonuses for first and second greatest effort. Compared with an equal bonus scheme, the winner-takes-all (i.e., WTA) setting induces the greatest effort for the lowest cost types (i.e., the most able) but less effort for all other costs types. Hence, we see an immediate trade-off in designing rewards: as we sharpen the incentives via shifting the bonus mass to first place, we increase the effort of the most able, but disincentivize all the others. We term the former the "incentive effect" and the latter the "discouragement effect."
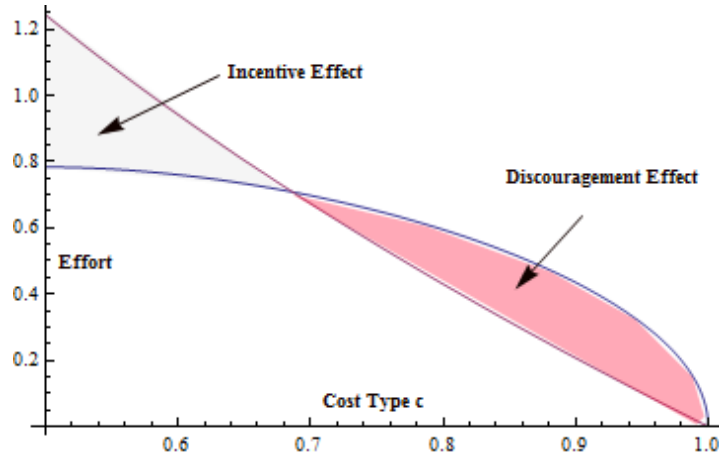
We explore both of these effects, as well as contestant behavior and contest design in more general terms through controlled laboratory experiments, which we turn to now.

12   Experimental Design

In total, we conducted 8 experimental sessions during February 2010. Our 141 Subjects were undergraduate and graduate students from UC Berkeley. Sessions lasted approximately 45 minutes from reading instructions to subject payment, which averaged approximately \$15 per subject. Subjects were not allowed to participate more than one time. The experiments were programmed and conducted with the software *z-Tree* developed by Fischbacher (2007).

We designed the experiments to operationalize the notion of workers competing for bonuses under varying bonus allocations and degrees of competition, as we are not only interested in these individual effects but also their interaction effects.

We did not frame the experiments as workers and bonuses. Instead, we simply awarded "prizes" for "effort." We wanted to keep the framing more generic to make sure we were capturing incentive and discouragement effects that were not colored by subjects' perceptions of the type of experiment being conducted.

We also used a chosen effort design. The reason for this is two-fold. First, theory suggesting softer incentives can increase total effort relies heavily on knowing precise ability types, which can be carefully done in a chosen effort setting. Second, we view this study as a first step to fully testing these ideas of increasing effort through soft incentives. If we are unable to obtain this result in a controlled laboratory

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environment, we have little hope of doing so in a field experiment, let alone the "real world."

Sessions varied on the dimensions of bonus allocation, number of workers (i.e., degree of competition), and the presence of effort capacity constraints. Since these contest games are complex and require learning, we allowed subjects to compete over 30 periods. For the bonus allocations, we simply changed the distribution of a constant $400 (in experimental units) bonus mass, which amounted to a per period bonus mass of $2 US dollars. The winner takes all (WTA) treatment awarded $400 per period to the most chosen effort. The equal prize (EP) was then $200 awarded to each of the top two workers in terms of effort. Finally, the larger second prize (SP) contest awarded $260 to second greatest effort and $140 to the greatest. We also varied treatments across 3 and 6 worker groups to study the effects from the degree of competition.

Finally, all but the first two sessions had a total effort capacity constraint of 240 per period. This was done to allow us to explore how constraints affect optimal incentive structure. Theory predicts when workers have no constraint, such as 24 hours per day or some maximal physical strength, the optimal structure is to only offer a WTA bonus structure. However, once workers have limits, it is best, in theory, to offer softer incentives. Table 1 shows a summary of all 8 sessions and the corresponding treatment.

Table 1: Comparing Different Treatments Across Sessions						
Treatment	Session	Subjects	Group Size	1st Prize	2nd Prize	Constraint
WTA	1	15	3	$400	$0	none
Equal	2	18	3	$200	$200	none
Const. WTA	3	18	3	$400	$0	240
Const. Equal	4	18	3	$200	$200	240
Large WTA	5&6	36	6	$400	$0	240
1st< 2nd Prize	7&8	36	6	$140	$260	240

After instructions were read, play began. Subjects were given an endowment of $300 experimental units each period. Each period a subject was randomly assigned a cost type c, which was drawn from a uniform distribution with support $[.5, 1]$. Each worker knew only their own cost type and then the distribution from which her other 2 (or 5) contestants cost types were drawn. After learning their cost type, workers chose how much effort to exert. In addition to his cost type, the subject's endowment value, the value of the bonus (or just one bonus if the contest was a WTA), and a calculator should he need to calculate his cost of effort, were all displayed. From this screen, the worker would then enter his effort. After all subjects submitted their

chosen effort, the next screen revealed all three (or six) submitted chosen levels of effort, as well as showing the calculation of the subject's winnings for that particular period:

$$V - c \times e + W$$

Where, V is the value of any bonus awarded, c his cost type, e his chosen units of effort, and W is that period's endowment.

Play continued to the next period where all subjects would again be randomly assigned another cost type from the same distribution and independent of past periods of play, and independent of the other players. In the end, subjects, as they were told before play began, were paid based on six randomly chosen periods. This was done to limit the chances of any type of dynamic strategy and to limit income effects. At the end of the 30 periods of play, subjects took a short questionnaire and risk test. The risk test was a coarse version of Holt & Laury (2002). Rather than providing all 10 lotteries from their original study, we offered subjects half of these, as we had the twin purpose of parsimony and tracing a subject's risk attitude. Our aim was not to have a precise measure of risk aversion but instead be able to identify risk neutral from risk averse and risk seekers, as will be discussed later.

One concern is our study, as any experimental study, suffers from a degree of external validity. However, in our case, although we did not actually test workers in firms, it does have the feature that the subjects (i.e., students) are typically workers or have the experience of being workers in real firms. Whatever the case, we knew if we could not generate theorized incentive effects in a controlled laboratory environment, we have no hope of doing so in the "wild" real world. Thus, these experiments are an important first step in testing theory. We now turn to our experimental test predictions.

12.1 Experimental Predictions

Our next table reports the Nash Equilibrium effort bidding predictions across all our sessions.

Table 2: Nash Equilibrium Effort Predictions Across All Sessions				
Treatment	Expected Effort	Type $< c^*$ Effort	Type $\geq c^*$ Effort	Subjects
WTA	164.28	347.97	70.59	15
Equal	156.04	227.66	120.22	18
Const. WTA	126.06	237.33	70.43	18
Const. Equal	155.88	227.20	120.22	18
Large WTA	63.39	238.83	36.40	36
1st<2nd Prize	86.43	230.57	64.26	36

We report in Table 2 not only the expected effort as given by theory, but also the expected effort conditional on being below or above the marginal type c^*. This marginal type c^* is indifferent between any distribution of bonuses. Thus, those lower cost types than c^* are subject to the "incentive effect" and those higher cost types to the "discouragement effect." This can be seen when comparing session 1 (i.e., WTA) to session 2 (i.e., Equal). When we move from session 2 to 1, the most able types (i.e., $c < c^*$) increase their expected effort from 227.66 to 347.97, for roughly 120 increased effort units. However, the less able decrease their effort from 120.22 to 70.59, or about 50 reduced units of effort. In this example, it still provides more total revenue, as seen by the expected effort figure of 164.28 to provide a WTA contest. That is, the "incentive effect" overcomes the "discouragement effect." However, this is not true of the other sessions. These later sessions are designed through capacity constraints such that softening incentives to equal prizes or in the extreme even inverting incentives—making second place bonuses larger, as we did in sessions 7&8—actually increases effort over a WTA scheme. Now we consider our experimental results.

13   Experimental Results

Table 3 summarizes the results per session in terms of total effort. As can be seen, our theoretical predictions are poor, and we can reject our theoretical effort predictions as correct at the 5% level for all treatments. However, the direction of misprediction depends on the treatment. In particular, we find theory under-predicts the total effort of the winner takes all contests (WTA) but over-predicts effort in the equal bonus contests. However, for the large contests (i.e., sessions 5-8), theory systematically under predicts total effort for both treatments.

What theory does get right, however, is the comparative static of softening incentives in the face of effort capacity constraints: when players face capacity constraints on effort (i.e., sessions 2-8), softening incentives does increase effort, and increasingly so for larger contests.

Comparing WTA to Constrained WTA shows us our competition effect: total effort for the same total bonus dollars increases over 200. Next, comparing Larger Second Prize to Large WTA shows the effect of softening the incentives via not only now offering a second prize, but a larger one: effort is increased yet another 50 units. This means in combination, these two levers of competition and softening incentives increases effort over our baseline (i.e., constrained) winner-takes-all structure by some 62%.

Table 3: Nash Equilibrium Effort Predictions vs. Actual				
Treatment	Predicted Effort	Actual Effort	Δ Effort	P Value
WTA	492.84	607.65	114.81	.0017
Equal	468.12	398.63	−69.49	.0000
Const. WTA	378.18	421.33	43.15	.0384
Const. Equal	467.64	434.36	−33.28	.0074
Large WTA	380.34	634.40	254.06	.0000
1st<2nd Prize	518.58	681.43	162.85	.0000

13.1 The Nature of Effort Disparity

To examine the source of our disparate results, we further divide effort exerted into effort cost types that are below and above our marginal cost type c^{**} that is (in theory) indifferent to prize structure. The cost types $c < c^{**}$ we call "low cost" and the balance of types we dub "high cost." Table 4 summarizes each of these types for each session over the last 15 periods of exerting effort.

Table 4: Predictions vs. Actual For High and Low Cost Types				
	Nash Predicted Effort		Actual Effort	
Treatment	Type $< c^*$	Type $\geq c^*$	Type $< c^*$	Type $\geq c^*$
WTA	370.20	81.41	401.10(.07)	114.73(.01)
Equal	229.72	128.67	149.01(.00)	125.96(.31)
Const. WTA	238.13	82.00	186.95(.00)	120.51 (.00)
Const. Equal	228.97	128.67	157.63(.00)	139.28(.05)
Large WTA	238.73	38.21	165.17(.00)	97.46(.00)
1st<2nd Prize	230.59	67.61	201.71(.00)	101.30(.00)
P Values of Differences in Parentheses				

We find a general theme of the "low cost," most able types under exerting compared with theory and the "high cost" types over-exerting. That is, these observed effort patterns are *flatter* than the Nash Equilibrium prediction. This is in contrast to past all-pay-auction experiments where contestants generally have *steeper* effort behavior. That is, with this past behavior, players over-exert vis-a-vis the Nash Equilibrium bid when a low cost type and under-exert when a high cost type. This type of behavior is in line with augmenting the risk neutral theory above with players being risk averse (see Fibich et al. (2006)). For our findings of flatter effort exertion, this same logic would mean our contestants were *risk seekers*. However, this possibility is rejected based on subject risk test questionnaires, which report that subjects are generally risk neutral to slightly risk averse. What are participants doing?

13.2 Actual Effort Behavior

It must be that at least some participants are doing something different than theory predicts—there is some force flattening their effort choices. To identify this force, we must then examine effort exertion on an *individual* level. It is then we notice that there seems to be some cutoff cost type for each participant. When she receives a cost type draw below this cutoff type, she exerts almost capacity effort and when her realization is above this cost type, she all but abstains (i.e., exerts zero effort or close to it).

For example, the subject, as reported in figure 1, exerts maximal effort—i.e., effort is at the same level as the solid line, which is the capacity effort level, when she realizes cost types just below the average cost type of .75, and then provides no effort at type realizations above this. Actual effort exerted is denoted by the diamonds. The dashed line is the Nash Equilibrium predicted effort, labeled NEBid. We also

report the last 15 periods of play, as by then workers had stabilized their strategy, as we discuss in our econometric section.
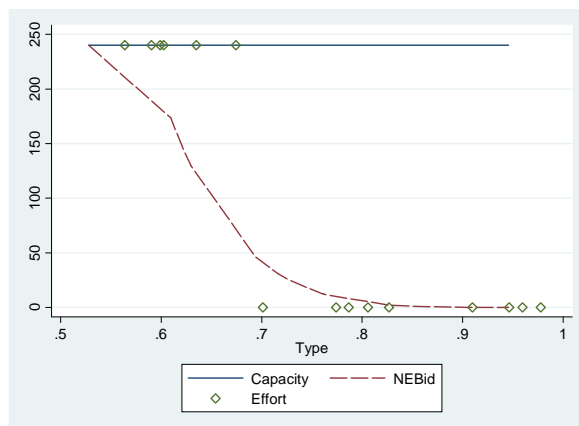

Figure 1: Session 5, Subject 2

Next, in figure 2 we witness a worker with a very low threshold. Indeed, for any cost type realization below around .60, he provides very high effort. However, above this he provides no effort.
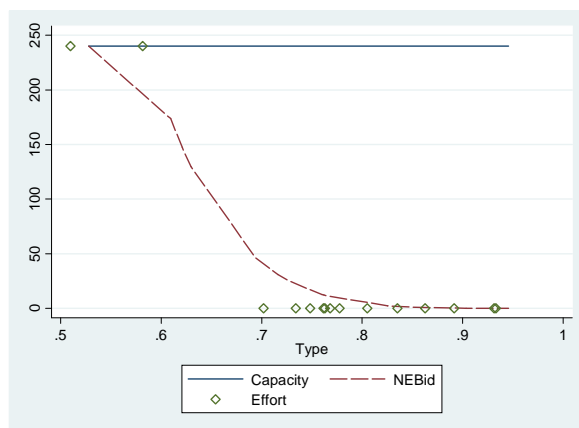

Figure 2: Session 5, Subject 11

Finally, in figure 3 we have an example of a high threshold contestant—here she is approximately always exerting high effort if she realizes a cost type below .9. There is only one period where this is not true, where she exerts no effort.
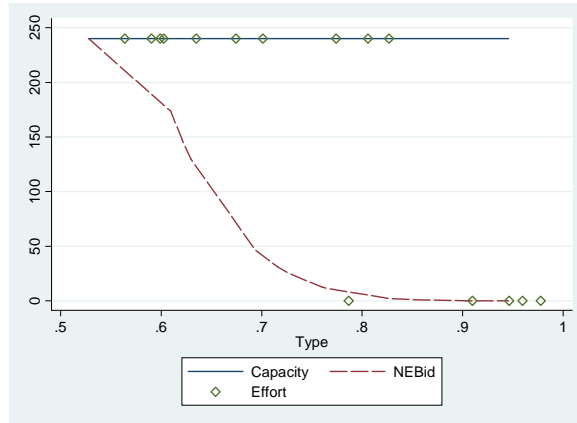
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Figure 3: Session 6, Subject 14

This threshold bidding behavior can then be thought of as many bidders with different step functions. Since these step functions have different cutoffs, when we aggregate them together, we can then get a flatter bidding structure, as noted in our previous section.

Viewing such graphs as shown above for all 141 subjects, it seems for most sessions, 1/2 to 2/3 of the workers exhibit this threshold behavior. The balance of workers seem to be close to Nash Equilibrium predicted effort or more often simply a noise worker—randomly choosing a bid between zero and the maximal effort. To verify this subjective observation quantitatively, a simple threshold effort strategy, as outlined explicitly in our next section, compared to actual effort, carries a mean squared error of 1070.96 across all sessions. The Nash Equilibrium predicted effort compared to actual effort carries a mean squared error of 3430.06. We now develop a model to understand this threshold strategy.

14   A Threshold Model

For a player, calculating a best response over continuous type space is a complex calculation. Indeed, expected effort given in Table 2 must be calculated numerically. It is thus plausible that contestants use some type of heuristic to calculate a best response. An intuitive heuristic would simply be to best respond *as if* all other players are the average cost type, here denoted $\bar{c}$.

Thus, suppose a player optimized against an average person. A model describing what average people do is captured by a complete information contest with all of

mean cost type $\bar{c}$ and effort capacity constraints. We can then find a mixed strategy[5] equilibrium. The capacity constraint is denoted $\kappa < \frac{V_{\max}}{\bar{c}}$, where $V_{\max}$ is the greatest valued prize. To our knowledge, equilibria have not yet been identified in the case of capacity constraints, and so we first turn to this analysis.

What we find is if all players are the average type $\bar{c}$, they will go "all in," exerting the full capacity effort level κ some fraction of them time, denoted p^*. The other $1-p^*$ of the time the player will play a mixed strategy drawn from a distribution $F(x)$ with support $x \in [0,\delta]$, as outlined in our appendix. That is, in addition to exerting full capacity effort κ, the worker will periodically exert anything from 0 up to $\delta < \kappa$ of effort (where δ is defined in our appendix). We state this formally in our next Proposition:

Proposition 6 *Assume a contest with $N \geq 3$ homogeneous players, up to 2 prizes, and binding effort capacity constraint κ. The symmetric equilibrium requires players play κ with probability p^* and bid with probability $1-p^*$ from distribution $F(x)$ with support $x \in [0,\delta]$, where $\delta < \kappa$. See the appendix for expressions for p^* and $F(x)$.*

Proof: See Appendix.

This equilibrium is such that the expected profit from exerting either κ or effort levels from $F(x)$ is zero. Hence, all workers are indifferent between mixing between providing effort κ and providing some effort x according to $F(x)$. There is no deviation to exert effort in the gap between δ and κ, as there is no increased chance of winning a bonus and yet there is increased cost in doing so.

When such workers do not have a binding constraint (i.e., $\kappa > \frac{V_{\max}}{\bar{c}}$),as is the case for session 2 (note for session 1 the periodic endowment of 300 causes a binding κ), an easy extension of Baye et al. (1996), yields the equilibrium of mixing over a distribution of some CDF $F(x)$ with upper support $\frac{V_{\max}}{c}$, where $V_{\max}$ is the largest prize value. Thus, we see for all sessions, the upper support for effort exerted is the lesser of $\frac{V_{\max}}{c}$ or κ.

With the above results, we can now find the best response if playing as if all other players are type $\bar{c}$. Clearly, if a worker herself draws $\bar{c}$, she is indifferent between exerting $x \in [0,\delta]$ and κ, as we are in the homogeneous game as given above. However, if she draws $c < \bar{c}$, her strict best response is to exert κ, as now her expected profit is maximally positive. Contrarily, if she draws $c > \bar{c}$, her best response is to exert

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[5]As can easily be shown, no pure strategy equilibrium exists. Suppose one does exist. If such a pure strategy bid is less than $\frac{V_{\max}}{\bar{c}}$, one player can bid ε more and capture the prize for sure, and for a profit. Thus the only possible pure strategy is for all to bid $\frac{V_{\max}}{\bar{c}}$. However, the expected profit is negative at this level. Hence, any equilibrium must be mixed.

zero, as her expected profit is strictly negative otherwise. This then gives us our next proposition:

Proposition 7 *Assume an employee exerts effort as if other employees are the mean type $\bar{c}$. An employee with effort cost level c then best responds with effort e as follows, where $M \equiv \min\{\kappa, \frac{V_{\max}}{\bar{c}}\}$:*

$$c < \bar{c} \text{ exert } e = M$$

$$c > \bar{c} \text{ exert } e = 0$$

$$c = \bar{c} \text{ exert } e \in [0, M]$$

Hence, using the simple heuristic that one exerts effort as if others are of average ability, yields a simple strategy: exert effort M if realizing a below average cost type and 0 otherwise.

Now we allow workers to have varying degrees of confidence[6]. Malmendier & Tate (2005) and Heaton (2002) model overconfidence as a worker assuming their project has greater expected return than its true expected return. The former model it as the expected value being magnified by some factor greater than one, while the latter assumes that managers attach a greater probability to success than is warranted. In our setting, these two approaches are identical. Hence, we write the expected payoff to worker i as follows:

$$E[\pi_i] = E[win] \cdot \gamma_i - \bar{c} \cdot x$$

The only difference from Proposition 2 is now the introduction of our confidence parameter γ_i, which represents the degree of confidence. If $\gamma_i = 1$, we have, as under Proposition 2, a perfectly rational employee. If $\gamma_i > 0$ the worker is over confident

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[6]Alternatively, we get the same result as in Proposition 3 if we instead allow employees to have varying degrees of risk aversion. However, this approach in our view does not make as much sense conceptually. As shown later, this would mean that varying degrees of competition and bonus structure affect a subject's risk preferences.

and if $\gamma_i > 0$ the employee is underconfident. For notation simplicity we have above defined

$$
\begin{aligned}
E[win] \equiv\ & (1 - p^*)^{N-1} \cdot F(x)^{N-1} \times V_1 \\
& + (N-1) \cdot (1 - p^*)^{N-1} \cdot F(x)^{N-2} \cdot (1 - F(x)) \times V_2 \\
& + (N-1) \cdot (1 - p^*)^{N-2} \cdot p^* \cdot F(x)^{N-2} \times V_2
\end{aligned}
$$

If we additionally define $\widetilde{c}_i \equiv \gamma_i \cdot \overline{c}$ and replace $\overline{c}$ with $\widetilde{c}_i$, we have as in Proposition 2 the same p^* and a threshold strategy as a best response. However, now the threshold strategy is based on the cutoff cost type $\widetilde{c}_i$ rather than $\overline{c}$. This provides our next Proposition:

Proposition 8 *Assume an employee exerts effort as if all other employees are the mean type $\overline{c}$. An employee with effort cost level c and degree of confidence γ_i then best responds with effort e as follows, where $M \equiv \min\{\kappa, \frac{V_{\max}}{\overline{c}}\}$ and $\widetilde{c}_i \equiv \gamma_i \cdot \overline{c}$:*

$$
c < \widetilde{c}_i \text{ exert } e = M
$$

$$
c > \widetilde{c}_i \text{ exert } e = 0
$$

$$
c = \widetilde{c}_i \text{ exert } e \in [0, M]
$$

In summary, we have generated a best response strategy for contestants that simplifies an admittedly very complex optimization problem over continuous type space into simply two states of the world: when realizing a cost type below the threshold type, exert very high levels of effort and when realizing a cost type below, exert zero effort. To explore how well this heuristic strategy describes our actual subject behavior, we first estimate $\widetilde{c}$.

15   Estimating the Threshold Model

Allowing for a distribution of γ_i over workers, we estimate $\widetilde{c}_i$ separately for each worker. Note then each worker's effort function is a step function. However, since each contestant is allowed to have his own γ_i, in aggregate the session wide bidding

function can look like a flatter Nash Equilibrium effort function, as it is an amalgam of step functions with varying step points.

To estimate $\widetilde{c}_i$, we will use what we call an Absolute Error Minimization (AEM) algorithm. This process is simply minimizing the following loss function over 30 periods of effort for a given contestant i:

$$\min_{\widetilde{c}_i} \left[|\widehat{b}_{i,t} - (I_{\widehat{c}_i < \widetilde{c}_i} \times M_i)| \right]$$

The term $\widehat{b}_{i,t}$ is the observed effort. When $\widehat{c}_i > \widetilde{c}_i$, the expected effort is zero. Since $M \equiv \min\{\kappa, \frac{V\max}{\bar{c}_i}\}$, M varies across sessions and individuals, but not periods. Note, however, $M = \kappa$ for all sessions, except session 2, where $M = \frac{V\max}{\bar{c}_i} = 266.67$. For sessions 3-8, we get $\kappa = 240$. Finally, for session 1, through the periodic capacity constraint, we have $\kappa = 400$.

We use the absolute value loss function rather than squared deviations since the former is more robust to outliers. Further, this estimation procedure identifies the true $\widetilde{c}_i$ if we have subjects making effort exertion errors of no more than half the maximal effort, as our next proposition proves.

Proposition 9 *If the exertion error $\xi_t < \frac{M}{2}$ for all periods t, then our AEM algorithm identifies the true threshold type $\widetilde{c}_i$*

Proof: See appendix.

This proposition says as long as the contestant is "close enough" to using the threshold heuristic with true cutoff type $\widetilde{c}$, which means not over or under exerting by $1/2$ of the maximal effort, our algorithm identifies the true $\widetilde{c}$ of such a worker.

15.1 Workers Take on Different Roles

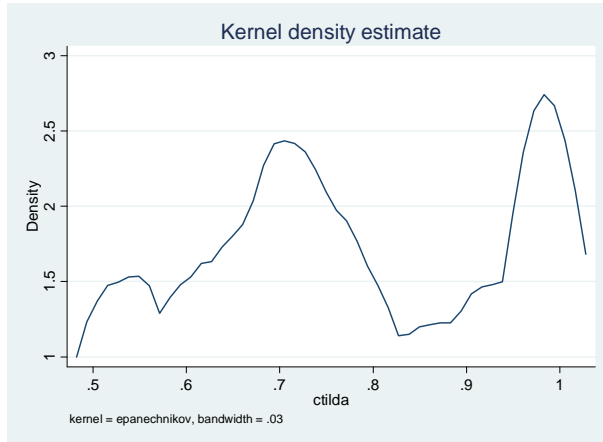Based on our above algorithm, we report the empirical distribution of our $\widetilde{c}_i$ estimates:

Figure 4: Disribution of $\widetilde{c}_i$

Interestingly, as Figure 4 shows, there are three groupings of $\widetilde{c}$ estimates. A large mass of players are just about the mean of the cost types at $\widetilde{c}_i = .75$. We again call these Realists. There is also a substantial grouping just below $\widetilde{c}_i = 1$. These are overconfident workers we dub Strivers—they are working as if all others are almost the worst type. Finally, there is a smaller group of workers close to $\widetilde{c}_i = .5$, playing as if all others are close to the most able, lowest cost type. This means they exert little or no effort most of the time.

Having a low $\widetilde{c}_i$ can mean one of two things: either the player is very pessimistic of her chance at winning (i.e., under-confident) or she simply wants to puts forth minimal effort, hoping to win something for almost nothing—i.e., gaming the system. We again dub these threshold types as Quitters or Slackers, respectively.

These Slackers are prevalent in sessions 2 and 4— treatments with two bonuses for only three contestants. In these sessions, if we consider players with below average $\widetilde{c}_i$ (i.e., $\widetilde{c}_i < .75$), we get the mean effort to be 89.13 when they realize a cost type draw $c > \widetilde{c}_i$. Our theory model, however, suggests this should be zero, though we should allow for some noise. In contrast, the other six sessions do approximate this, as there mean effort is 29.55 for below average $\widetilde{c}_i$ workers when realizing an above $\widetilde{c}_i$ cost type draw. This difference in low efforts is significant at the 1% level. Meanwhile, when this same group of below average $\widetilde{c}_i$ threshold workers realizes cost type $c < \widetilde{c}_i$, they should aggressively exert effort. And that is exactly what all do for the sessions other than 2 and 4: the mean effort put in for sessions other than 2 and 4 is 235.6. However, session 2 and 4 players exert an effort of only 110.5. This difference is again significant at the 1% level. Recall also with a low $\widetilde{c}_i$ threshold, a worker does not often realize a cost type draw $c < \widetilde{c}_i$. Hence, the Slacker mostly exerts effort at modest levels and generally avoids high levels of effort. Meanwhile,

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Quitters all but give up, only exerting effort when receiving the most able of all type draws.

15.2 Threshold Model Goodness of Fit

Now having estimated $\widetilde{c}_i$ and explored its heterogeneity across workers, we consider a simple measure of fit of our best response bidding: mean squared error of predicted versus actual effort. Table 5 records the mean squared errors for both bidding models, divided per session. We combine sessions 5 and 6 and then 7 and 8, as they are the same treatment. For the first session, forced to choose between the threshold model and the Nash Equilibrium model, the latter is a better fit for describing the aggregate data. Nonetheless, for all seven other sessions, the threshold bidding model is a much better fit.

Table 5: Mean Squared Errors of Predicted Effort per Session, per Model						
	1	2	3	4	5&6	7&8
Nash Equilibrium	2331.75	2867.50	2555.16	3008.69	5326.93	2920.22
Threshold Model	2757.62	1384.794	1230.37	793.16	1046.89	665.47

One might be concerned that our identified threshold behavior are somehow isolated to UC Berkeley undergraduate students. However, we were able to obtain data from the Noussair & Silverman (2006) study. Their subjects were from Emory University. Additionally, they had an entirely different framing—that of a private values all-pay auction. Thus, players in their setting realized valuations as opposed to cost types. Players also received an initial endowment as opposed to our periodic endowment. Finally, their study was a paper and pencil experiment versus ours which was computerized. Despite all these differences, we found a similarity to their individual bidding results and ours. Within their treatments, for the last 15 periods of bidding, the mean squared error of actual bidding to the Nash prediction is 42,086.74 (note this error is much higher than our treatments because their bidding ranges from zero to 1,000 versus ours is zero to 600 for the most able type (and 300 for the least able type). In comparison, our threshold model yields an error of 23,847.69, roughly half the Nash prediction error.

In addition to NS (2006), Muller & Schotter (2010) find the same threshold behavior as this study. Their setting is a private values contest with equal or winner-takes-all prizes and a constant group size of four contestants. They also have quadratic vs. linear as opposed to our capacity constraints vs. linear. In addition to threshold behavior identified in their data, our model can also predict their realized revenues which were again incongruent with extant theory. We summarize this in Table 6 below.

Treatment	ctilda	Max Bid	Threshold Predicted Rev	Nash Predicted Rev	Actual Rev
LC-1	0.71	1.330	2.234	1.452	2.391
LC-2	0.78	0.670	1.501	1.164	1.452
QC-1	0.67	1.155	1.570	1.859	1.524
QC-2	0.81	0.816	2.025	1.944	1.963

Table 6: Predicting Contest Revenue in Muller & Schotter (2010)

For example, for treatment LC-1 (i.e., linear costs with a single WTA prize), the Nash Equilibrium predicts 1.452 of revenue versus their actual revenue average of 2.391. Meanwhile, if we input their data into our threshold model, we predict average revenue of 2.234. Interestingly, their model does an even better job of predicting their out-of-sample revenue than our own.

Though it seems many are engaged in this threshold behavior, it would be naive for us to assume that this is what the entire subject pool does. We now consider what proportion of players best represents our data—allowing not just for threshold behavior but also Nash and noise workers (i.e., randomly selecting effort levels over the entire effort support).

15.3 Worker Roles Depend on Institutional Structure

It would be tempting to consider these worker types as exogenous—some are just Quitters, Realists, some Slackers, and some Strivers. However, it turns out workers take on these different roles as a function of the reward structure and not simply based on some fixed characteristic. This becomes apparent when we examine the distribution of $\widetilde{c}_i$ as a function of the institutional structure—i.e., degree of competition and bonus structure.

To see this, we formally designate a Striver as having $\widetilde{c}_i > .85$ (i.e., the top 30% of the type support). This means these players are almost always exerting full effort. Symmetrically, we dub those players with $\widetilde{c}_i < .65$ (i.e., the bottom 30% of the type support) as either a Slacker or Quitter. The former is one that puts in at least 20% of the capacity constraint effort (i.e., 48 units of effort). The latter puts in less than this amount. Results do not materially change using alternative cutoffs. The fundamental idea is when $\widetilde{c}_i < .65$, the player is rarely putting in much effort. If they are exerting effort close to zero when they are below there cutoff type, we call it Quitting, whereas a more material but low effort level is Slacking—hoping to win a prize on minimal effort. The remaining players are called Realist. In Figure 5

below we report the proportion of each of these types as a function of the reward structure.
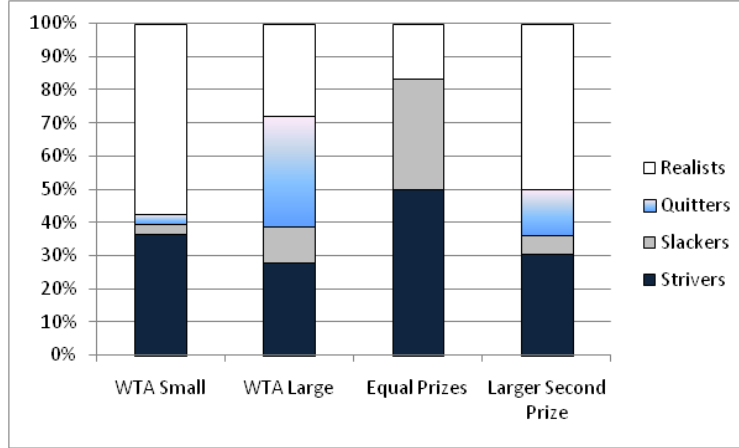

Figure 5: Worker Types vs. Reward Structure

As can be seen in Figure 5, for any reward structure, there is always a healthy population of Strivers. These overconfident types tend to represent around 1/3 of workers. However, once we increase competition (i.e., move from 3 to 6 workers), as the case in WTA large, about 1/3 of the workers become Quitters—giving up due to discouragement. If instead, we keep competition low but soften incentives, as in Equal Prize treatment, instead of Quitters, workers become Slackers—about 1/3 try to game the system through minimal effort. Finally, if we make incentives moderate by offering 1/3 of the workers a bonus and inverting the bonus structure—giving an even larger bonus to the second best, as in the case of Larger Second prize—we again get over 50% of workers becoming Realists. When they are above average, they work at capacity and abstain when below average. Having a large number of Realists interacted with increased competition, also means the greatest total revenue, as shown above in Table 3. Thus we see with threshold bidding, we want to increase competition because we get more overall effort from the most able. However, we must soften (but not too much) the bonus structure to make sure everyone besides the top performer continues to put in healthy levels of effort. This works because moderate incentives discourages Realists from becoming Slackers and encourages Quitters not to quit.

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16   Estimating the Proportion of Player Types

Our final analysis is another robustness check to determine how well our threshold model explains our individual choice data over alternative models. In particular, we next estimate from the entire subject pool what proportion of players are best described as choosing effort according to the Nash Equilibrium prediction versus the threshold effort strategy versus simply being a noise worker.

To do so, we use a non-linear least squares pooled panel regression. To account for potential serial correlation and idiosyncratic errors, we cluster the standard errors on subject. In particular, our model is as follows:

$$\min_{\gamma_1, \gamma_2} Q \equiv \sum \left[effort_{i,t} - \gamma_1 \left(I_{\widehat{c_{i,t}} < \widetilde{c}_i} \times M_i \right) + \gamma_2 \left(b_{NE}(\widehat{c_{i,t}}) \right) + (1 - \gamma_1 - \gamma_2) \cdot n_t + e_{i,t} \right]^2$$

Again, we have $M \equiv \min\{\kappa, \frac{V \max}{\widetilde{c}_i}\}$. The term $I_{\widehat{c_{i,t}} < \widetilde{c}_i}$ is an indicator function taking on the the value 1 when $\widehat{c_{i,t}} < \widetilde{c}_i$ and zero otherwise. Each $\widetilde{c}_i$ is estimated in a first step per our algorithm outlined in the previous section. We denote $b_{NE}(\widehat{c}_i)$ as the Nash Equilibrium predicted effort given realized cost type $\widehat{c}_i$. Our third term n_t is simply the mean of the effort support, for the given session, for a mean type— i.e., the expected effort of the mean noise worker. For the first two sessions this is $n_1 = \frac{300}{2 \cdot .75} = 200$ and $n_2 = \frac{200}{2 \cdot .75} = 133.33$. However, for all other sessions due to the binding capacity constraint of 240, we have $n_t = \frac{240}{2} = 120$. Finally, we have an error term $e_{i,t}$, which we cluster on individuals.

We are estimating the parameters γ_1 and γ_2 to predict the proportion of threshold and Nash Equilibrium workers, respectively. This then implies $1 - \gamma_1 - \gamma_2$ is the proportion of predicted noise workers. Regression results show the population of threshold workers to represent about 56% of all workers, the Nash Equilibrium workers, about 19%, and noise workers the balance. When estimating these parameters on an individual session basis, for all but the first session, we cannot reject the null hypothesis that the proportion of Nash Equilibrium workers is zero. Meanwhile, the proportion of threshold workers is always highly significant.

For above estimates we are using the last 15 periods of exerted effort, as the first 15 periods are much nosier. Indeed, if we use all 30 periods we estimate 50% being threshold workers and just 11% being Nash Equilibrium workers. This means some subjects that were early flagged as noise workers learn to be threshold workers or Nash Equilibrium workers for the last 15 periods. In the end, it seems clear that the vast majority, if not all, subjects for some sessions are engaged in this threshold heuristic.

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17   Discussion and Conclusion

We set out to explore simple economic models and their relationship to real world incentives. There has been disharmony between these two, as the former has predicted much sharper incentives than witnessed in practice. However, recent theory is now suggesting softening incentives can increase overall effort. And so it is this line of thought that we pursued experimentally. What we find is it is true. At least in the laboratory, we can increase total effort with softer incentives. However, we also discovered workers do not work as we thought they would—in a sophisticated optimization over continuous type space. Instead, we find that workers greatly simplify individual effort choices with a heuristic. In particular, there is some threshold type for players such that they exert capacity effort if realizing a cost type better than this, and all but abstain otherwise. This has important implications in organization design: workers tend to over- and under-exert themselves depending on their sense of ability relative to their peers.

The most surprising finding is we also discovered institutional structure—the degree of competition and "steepness" of bonuses—actually affects the location of the threshold for many players. All structures had the presence of Strivers—overconfident types that almost always worked at capacity. However, having incentives too soft with low competition or too high with high competition transformed the balance of workers into Slackers or Quitters, respectively. Finally, offering moderate incentives in terms of a larger second prize and interacting this with greater competition resulted in a lot of Realists—those that work very hard when above average and very little when below. It was then this setting that also created the greatest total effort.

The designer must be careful since institutional design ultimately determines worker behavior, which means workers are not immutable types. For example, a firm finding itself with a long list of Slackers producing minimal effort might make the wrong decision by firing them and hiring new workers. Indeed, firing the Slackers and hiring new workers could result in simply more slackers, as the real problem is the institutional design and not the type of workers. Instead, the firm may need to increase the degree of competition. In a sales organization, for example, this would mean making sales regions over which salespeople compete for bonuses larger (but not too large), resulting in more salespeople competing for the same bonuses. As another example, if a firm's turnover is very high, instead of the cause being unskilled workers that cannot cut it, it could be that the firm needs to provide more rewards to other than the top performer(s).

Hence, we see the organizational designer actually has two tools: the degree of

competition and the prize structure. It is then a delicate balance between these two that ultimately generates the most effort: a blend of competition and bonuses to adequately spread the work and rewards among workers.

Part III

Competition via Corporate Citizenship

Our final section explores how firms can compete on the dimension of corporate citizenship. In particular, I posit that corporate citizenship, as obtained through Corporate Social Responsibility (CSR), is being used by some firms as an insurance mechanism, helping them better withstand the tumult of negative business shocks. Guided by this theory, I empirically test CSR as insurance in the setting of product markets. I find higher CSR type firms enjoy \$600 million of saved firm value after an adverse event compared with low types. In addition, as theory predicts, I find higher type firms experience events less often.

18   Introduction

> "CSR is best seen as the management of risk, as the avoidance of damages
> to the company's reputation." *Financial Times*, July 7, 2004.

There have been a plethora of past studies examining the relationship of a firm's financial performance with its level of corporate social responsibility (CSR). In short, the studies show there is little relation between the two (see Elfenbein (2007) for an extensive survey). Meanwhile, CSR seems to be increasingly important to firms. Indeed, a recent survey by the *Economist* magazine[7] reports some 56% of managers consider CSR as a "high" or "very high" priority. This compares with roughly 34% three years ago and an expected 69% three years hence. Further, they report 87% of firms now have a CSR firm program. Echoing firm sentiment, many MBA program ranking schemes now include a standalone category for CSR. Why are firms so concerned with CSR?

Recent work has offered various reasons from managers seeking "warm" glow from their CSR activities (e.g., Fisman et al. 2006) to firms simply appeasing the demands of NGOs to prevent boycott, or even forestalling looming governmental regulation (e.g., Baron & Diermeier 2007). Additionally, it has been suggested that firms use CSR to signal a whole swath of different messages, most revolving around

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[7]January 7th, 2008

the trustworthiness of the firm and its effect on *current* revenue (e.g., Goyal 2006), or even firms that use CSR as a form of penance, offsetting the firm's past irresponsible behavior (e.g., Kotchen & Moon 2007).

However, *when managers are actually asked why they engage in CSR*, they claim it is to secure a better brand and reputation. However, only some 6.5% of managers report that CSR increases revenue. What is the value of increasing (brand) reputation if in the end it does not increase revenue? Similarly, some 63% of managers say their adoption of sustainability practices either does not change or even decreases profit. How does this make business sense? *We posit that a primary value of CSR is that of an insurance mechanism for the firm's value.* That is, investing in CSR can help build social reputation that softens the blow of future business shocks on a firm's value. The primary benefit comes *after* an event. Thus, CSR is engaged in not wishing to increase a firm's value *but rather to protect it.* This is much in the spirit of Hermalin (2008) who shows that higher levels of corporate governance are a result of firms wanting to protect their profits as opposed to better governance yielding higher profits. But, whereas he is concerned with corporate governance protecting current profits, we are concerned with protecting the value of the firm through a *contingent* future benefit when facing shocks.

The mechanism by which CSR investment is expected to "payoff" during an event is at least twofold. First, there are NGOs that will make demands of firms in terms of CSR commitments. To the extent such firms meet these requests, it is expected they will receive limited wrath from these NGOs *after* a "bad" event occurs (see Vogel (2005))[8]. A second reason could be that when firms invest in CSR, they indicate their level of CSR related issue effort (e.g., environmental effort), helping improve investors and regulators' posteriors after a negative issue related event.

For example, if a firm overtly incurs extra expense to have superior environmental management systems, when it faces an environmental accident, such an *ex-ante* commitment could help tip the scale from the view of the investor and regulator that such an event was really due to bad luck instead of negligence. Hence, assuming that negligence is more costly than noise, the firm's market value would be less punished and it would be less likely to be pursued by a regulator than had the firm not shown such commitment. For this paper, we examine the mechanism of this latter form.

For a very recent concrete example of this mechanism consider British Petroleum (hereafter, BP) and Johnson and Johnson (hereafter, J&J). Leading up to its recent devastating oil spill on April 20th, 2010, BP had begun developing a reputation of carelessness through outsourcing safety and cost cutting, providing few actions that showed a genuine concern for environmental safety. Indeed, as publicly noted by the

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[8]See also Baron and Diermeier (2007) for a further exploration of this possibility.

US Chemical safety board, BP was "cutting the costs for safety and maintenance to increase profits[9]." Consequently, on April 20th, 2010 while analysts best estimates of *total* spill costs were around $3 to $12 billion, the stock market wiped out $32 billion of firm value[10]. Investors weighed the scale of negligence versus noise and it tipped heavily toward negligence.

In contrast, Johnson and Johnson (J&J) faced a product recall of various over-the-counter children's medicines based on serious manufacturing problems. However, J&J had already begun hiring outside experts to help improve its quality control in the problem plant, as it detected a need to do so. Thus, while J&J will certainly have to pay for faulty manufacturing, J&J showed it was taking substantive steps to improve safety and quality, reducing total expected long run firm cost. When a recall was triggered investors instead tipped the scale more in the direction of noise and not gross negligence[11], providing nominal change in firm value.

We thus view the contribution of this paper as twofold. To our knowledge, this is the first paper to formally[12] examine CSR as an insurance mechanism. Second, this is the first paper to empirically test the notion of CSR as reputation insurance. In short, we aim to open the black box of CSR decision making by examining CSR as reputation insurance, examining the efficacy of the primary reason given by managers for engaging in CSR.

Our paper is organized as follows. In the first section we present our model of CSR investment, exploring the mechanism of CSR benefit and identifying when it is expected to payoff. The next section provides our empirical analysis, which suggests CSR does generally provide a substantial insurance benefit. Our final section provides a concluding discussion.

19   CSR as Insurance

The central idea of CSR as an insurance mechanism is, again, that a firm first makes a CSR investment to obtain a higher CSR reputation. By CSR reputation we mean the firm's reputation of how conscientiously it goes about the production and selling of its goods, which in turn then creates a sense of how trustworthy the firm is.

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[9]See Wall Street Journal story "Drilling Down: A Troubled Legacy in Oil" May 1,2010.

[10]See http://money.cnn.com story "BP loses $32 billion in value on spill."

[11]See Wall Street Journal story "J&J Lapses Are Cited in Drugs for Kids" on 5/27/2010.

[12]For an informal discussion of CSR as an insurance mechanism for firm value see Minor and Morgan (2010). In additon, see Peloza (2006).

For example, consider the realm of product markets. Product recalls can provide shocks to firm value and reputation. Now CSR in product markets can be thought of as a firm's superior reputation earned from the conscientious creation, marketing, and distribution of its products. A firm with high CSR would, for example, embrace superior quality assurance procedures in the development and production of its products, conduct ethical marketing campaigns, provide products with extra social value, provide products to disadvantaged demographic groups, and generally face product recalls voluntarily. In contrast, a bad CSR firm could often be involved in regulation fights, suffer safety violation fees, accept lower product safety standards, and conduct limited due diligence on their supply chain. One key distinguishing characteristic between high and low CSR activities is that the latter will tend to be less costly in the short term, and particularly less costly if the bad event never occurs, which, for the current example, is a product recall. Since some of the low CSR type activities are unobservable by the public, if an event never occurs, it is difficult to know the full extent of how responsible or irresponsible the firm is being. Nevertheless, to the extent that these unobservable activities (e.g., limited supply chain due diligence) are correlated with other activities that *are* observable (e.g., regulatory fines), it is still possible to develop reasonable ex-ante reputations.

Now if a bad business shock should occur, the firms with higher CSR reputations will not be punished as badly. This net result is similar to an example: consider a group of workers with typical incomes of $50,000 and homeowner insurance premiums of $500. The insured and uninsured would only have a 1% difference in annual income, which would likely be difficult to distinguish statistically. However, upon an event (e.g., a house fire), there would be a very significant difference between an uninsured and insured homeowner in terms of cash flow via the insurance benefit. Hence, to find out if the homeowners insurance acted like insurance, we would again have to make sure we capture negative events and not simply a time series of ordinary cash flows.
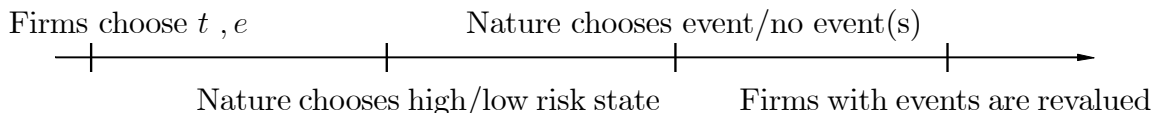
Therefore, here the expected CSR insurance benefit, just as with homeowners insurance, is both a function of the probability of the event occurring and the expected net benefit received, conditional on the event occurring. Also, CSR is not expected to offer 100% insurance. Typically, with insurance there is a deductible and/ or coinsurance. Thus, a firm will still likely lose value during an adverse event, but they will lose less value than had they not been insured.

Therefore, in our formulation, it is not expected that CSR will ever payoff in a current revenue or profit sense, just as with traditional insurance. In fact, we conjecture a slight negative contemporaneous effect since CSR investment is costly,

though there may be some offsetting of current value (e.g., high CSR might attract or retain higher quality employees). Moreover, if a higher CSR reputation lessens the chance of an adverse event (and thus its cost), this too does not show in a time series. This explains how the past literature could have had such difficulty establishing a statistical relationship between CSR and accounting for financial performance— most firms for a typical time series experience no CSR benefit. To find the value of CSR, we must instead examine time periods of unlikely events.

Now the precise benefit of CSR reputation during an event is the differential of lost firm value between a higher and lower CSR type firm. This differential is driven by how investors ex-post asses the probability that the firm willfully caused the event versus the event simply being due to bad luck, which will be determined by the firm's ex-ante CSR reputation. This probability assessment then affects how likely the event will be found to result from negligence. The firm's punishment then increases in the likelihood it is found to be negligent, which is determined by an enforcer. We now turn to formally modeling this idea.

19.1   A Model of CSR as Reputation Insurance

Firms choose t , e Nature chooses event/no event(s)

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        Nature chooses high/low risk state   Firms with events are revalued

    We examine a model of CSR as reputation insurance where firms choose their level of CSR activity as well as direct effort to manage business risk. The above diagram provides the timeline of the game. Formally, suppose that a firm chooses its CSR type, $t \in \{R, N\}$, which is *publicly observed*. R (N) denotes a responsible (negligent) firm. While it costs zero for a firm to be a negligent type, choosing to be a responsible type costs a firm c, where this cost is the realization of a privately observed random variable having support $[0, \infty)$ over an atomless distribution function $F(\cdot)$. A firm also chooses its level of effort $e \geq 0$, which is *unobservable*. Effort costs are increasing and convex. For simplicity, suppose that the cost of e units of effort is $C(e) = \frac{1}{2}e^2$.

    The combination of CSR activity and effort determine the business risk state $\theta \in \{h, l\}$ of the firm. With probability $\gamma_t e$ the low risk state is realized while the high risk state occurs with complementary probability. The parameter γ_t represents the influence of CSR activity on the business risk state where $0 < \gamma_N < \gamma_R$. In other words, responsible CSR activity reduces the chance of being in the high risk state. Following the state realization, nature then determines whether an adverse event occurs. With probability p_θ an event occurs in state θ where $0 < p_l < p_h$. That is, events are less likely to occur if the firm is operating in the low risk state than in the high risk state.

    Stakeholders *only observe* whether an event, E, has occurred and the firm's level of CSR activity (i.e., γ_t). A firm's direct effort is unobservable. Thus, stakeholders make an assessment of the likelihood that the firm was operating in a high risk state conditional on an event occurring. Define

$$\rho(t) = \Pr[\theta = h | E, t]$$

That is, $\rho(t)$ denotes the (equilibrium) beliefs of stakeholders that the firm was in the high risk state conditional on an event occurring and the firm's level of CSR activity. A firm triggering an event from a high risk state can be thought of as having been "at fault," whereas if the event was triggered from a low risk state the firm is not "at fault." A firm then suffers losses K, scaled by $\rho(t)$, stakeholders' belief the firm was operating in a high risk state. That is, being "at fault" is more costly than not being "at fault."

To summarize, the expected profits of a firm with cost parameter c choosing type t and effort e are

$$\pi\left(c,t,e\right) = \pi_0 - cI_{t=R} - \frac{1}{2}e^2 - \left\{\left(1 - \gamma_t e\right)p_h + \gamma_t e p_l\right\}\rho\left(t\right)K$$

Here, I is an indicator function which equals one if the firm chooses a high level of CSR activity (i.e., a responsible type). We denote status quo profit π_0.

To summarize the various states, below is a probability tree of possible outcomes:
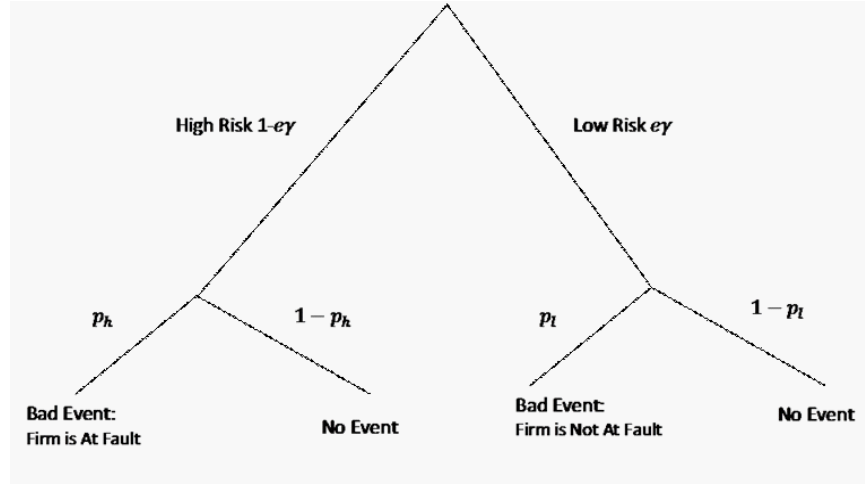
There exist unique values (e_N^*, e_R^*) corresponding to the equilibrium effort of a firm of type t. To see this, notice that, the optimal effort choice for a firm of type t is a globally concave problem having as its solution:

$$e_t^* = \gamma_t\left(p_h - p_l\right)\rho\left(t\right)K \tag{1}$$

To close the model, it remains to determine equilibrium beliefs. Recall stakeholders know a firm's type and can then deduce its equilibrium effort. Hence, from Bayes' rule, upon an event, we have belief

$$\rho\left(t\right) = \frac{\left(1 - \gamma_t e_t^*\right)p_h}{\left(1 - \gamma_t e_t^*\right)p_h + \gamma_t e_t^* p_l} \tag{2}$$

Thus, any equilibrium effort levels simultaneously solve

$$
\begin{aligned}
e_N^* &= \frac{\left(1 - \gamma_N e_N^*\right)p_h \gamma_N\left(p_h - p_l\right)K}{\left(1 - \gamma_N e_N^*\right)p_h + \gamma_N e_N^* p_l} \\
e_R^* &= \frac{\left(1 - \gamma_R e_R^*\right)p_h \gamma_R\left(p_h - p_l\right)K}{\left(1 - \gamma_R e_R^*\right)p_h + \gamma_R e_R^* p_l}
\end{aligned}
\tag{3}
$$

Finally, to assure beliefs are well defined, we require K to be bounded as $0 < K < \frac{1}{\gamma_R^2(p_h - p_l)}$. This also means then e_t^* is bounded from above.

Not also, in equilibrium, for some unique c^* we have

$$\pi(0, N, e_N^*) = \pi(c^*, R, e_R^*) \tag{4}$$

This can readily be seen by noting $\pi(\cdot)$ is strictly decreasing in c and $\pi(0, R, e_R^*) > 0$ while $\pi(c, R, e_R^*) < 0$ as $c \to +\infty$.

This hitherto analysis provides our first proposition:

Proposition 10 *In the unique equilibrium, responsible (negligent) firms exert effort e_R^* (e_N^*) as given in (3) and have CSR cost $c \leq c^*$ ($c > c^*$),where c^* solves equation (4).*

Proof. From our above analysis, we know the pair (e_N^*, e_R^*) are the result of a global concave problem and thus unique. By (4) we know there exists some unique c^* such that all firms below (above) this cost are responsible (negligent) types. ∎

The following lemma proves useful in identifying properties of optimal effort.

Lemma 4 *In any equilibrium, $\gamma_N e_N^* < \gamma_R e_R^*$.*

Proof. Suppose to the contrary that $\gamma_N e_N^* \geq \gamma_R e_R^*$. Equivalently $\gamma_N e_N^* / \gamma_R e_R^* \geq 1$. Using equation (1), we have

$$\frac{\gamma_N e_N^*}{\gamma_R e_R^*} = \left(\frac{\gamma_N}{\gamma_R}\right)^2 \frac{\rho(N)}{\rho(R)}$$

and, substituting using equation (2), we obtain

$$\frac{\gamma_N e_N^*}{\gamma_R e_R^*} = \left(\frac{\gamma_N}{\gamma_R}\right)^2 \left\{ \frac{\frac{(1-\gamma_N e_N^*)p_h}{(1-\gamma_N e_N^*)p_h + \gamma_N e_N^* p_l}}{\frac{(1-\gamma_R e_R^*)p_h}{(1-\gamma_R e_R^*)p_h + \gamma_R e_R^* p_l}} \right\} \tag{5}$$

Next, notice that the function

$$\phi(x) = \frac{(1-x)p_h}{(1-x)p_h + xp_l}$$

is strictly decreasing in x. Therefore, the expression in the curly brackets in equation (5) is at most one. Since $\frac{\gamma_N}{\gamma_R} < 1$, it then follows that the RHS equation (5) is fractional, which contradicts the hypothesis that $\gamma_N e_N^* / \gamma_R e_R^* \geq 1$. ∎

Though we could solve for e_t^* explicitly using the quadratic formula, it is much simpler to use that fact we always have $e_N^* \gamma_N < e_R^* \gamma_R$ in equilibrium. That is, we know the responsible firm is less likely to find itself in a high risk state than a negligent firm. Indeed, this inequality is all that is necessary to prove our next proposition, which then provides our core empirical implications. To ease notation, we define $\Pr(E \mid t) \equiv (1 - \gamma_t e) \, p_h + \gamma_t e p_l$ and $E[C \mid t] \equiv \rho(t) \, K$ for $t \in \{N, R\}$.

Proposition 11 *Responsible firms have events less often than negligent firms (i.e., $\Pr(E \mid R) < \Pr(E \mid N)$) and also face a lesser expected penalty upon an event (i.e., $E[C \mid R] < E[C \mid N]$).*

Proof. Invoking Lemma 1, in equilibrium $e_N^* \gamma_N < e_R^* \gamma_R$. But this means $\Pr(E \mid R) < \Pr(E \mid N)$ by definition. Similarly, $E[C \mid R] < E[C \mid N]$. ∎

This proposition tells us responsible firms face fewer events and lesser penalties upon events. This then implies responsible firms *also* experience a lesser change in firm value upon an event, as we show in our next section.

20 Empirical Examination

20.1 General Strategy

Our primary empirical aim is to test the notion that CSR can work as reputation insurance. That is, high CSR type (i.e., responsible) firms enjoy a buffering of their firm value vis-a-vis a low-type (i.e., negligent) firm upon an event. From Proposition 2, we have that the high type firms are imposed with a lesser expected cost upon an event compared to a low type firm. This also implies high CSR type firms will experience a lesser change in firm value compared with a low CSR type firm, as we now show.

In particular, we can write out explicitly the expected change in firm value. First, a firm's value is the present value of all future *expected* cash flows. This means we have:

$$\text{Firm Value} = \sum_{k=0}^{\infty} \frac{\pi}{(1+r)^k}$$

$$= \sum_{k=0}^{\infty} \frac{\pi_0 - cI_{t=R} - \frac{1}{2}e^2 - \frac{1}{2}\gamma_t^2 - \Pr(E \mid t) \times E[C \mid t]}{(1+r)^k}$$

$$= \sum_{k=0}^{\infty} \frac{\pi_0 - \frac{1}{2}e^2 - \frac{1}{2}\gamma_t^2}{(1+r)^k} - \sum_{k=0}^{\infty} \frac{\Pr(E \mid t) \times E[C \mid t]}{(1+r)^k}$$

Now once an event happens we replace $\Pr(E \mid t)$ with 1. The change in firm value then becomes the difference between the value with a current period event (i.e., $\Pr(E \mid t) = 1$) and an uncertain period event (i.e., $\Pr(E \mid t) < 1$):

$$\left(\sum_{k=0}^{\infty} \frac{\pi_0 - \frac{1}{2}e^2 - \frac{1}{2}\gamma_t^2}{(1+r)^k} - \sum_{k=0}^{\infty} \frac{1 \times E[C \mid t]}{(1+r)^k} \right)$$

$$- \left(\sum_{k=0}^{\infty} \frac{\pi_0 - \frac{1}{2}e^2 - \frac{1}{2}\gamma_t^2}{(1+r)^k} - \sum_{k=0}^{\infty} \frac{\Pr(E \mid t) \times E[C \mid t]}{(1+r)^k} \right)$$

$$= -(1 - \Pr(E \mid t)) \times E[C \mid t]$$

As outlined below, in our data, our parameters are such that $1 - \Pr(E \mid t)$ is 97% or 98%, when a firm is a negligent or responsible type, respectively. However, the average event costs $E[C \mid t]$ are roughly \$1 billion or \$.5 billion for negligent and responsible types, respectively. Hence, the high type firms should be losing much less in firm value.

We are then going to estimate the insurance benefit between different CSR type firms. This amounts to the difference between the change in value of a responsible and negligent firm upon an event:

$$\begin{aligned} &\text{insurance benefit} \\ = \quad &(1 - \Pr(E \mid R))\rho(R)K \\ &-(1 - \Pr(E \mid N))\rho(N)K \end{aligned}$$

It should also be clear from the previous arguments, we can readily expand our two types to three (or more) types to get the same predictions in monotonic ordering[13].

[13]For example, for three types, assign the medium CSR type cost $c_{MR} \in [0, \infty)$ across firms. Thus, now the medium responsibility level firm versus the negligent firm analysis is just as before when comparing just two types. However, adding a high responsibility type cost as $c_{HR} = ac_{MR}$ for some $a > 1$, can provide a partition of three levels of c_t such that we have one of each three types of firms in equilibrium, depending of a firm's particular cost c_{MR}.

That is, the higher the type, the lower the event rate and the lesser the change in firm value. Here we will empirically study three different types, as we describe in detail below.

Our empirical setting is product markets where the event is a product recall. These events are often seen by the investment committee as a potential shock to a firm's value and reputation due to their signaling nature (Davidson and Worrell (1992) provide a review of past product recall literature. See also Hartman (1987) for a hedonic model treatment of recalls).

CSR reputation in product markets can be thought of as a firm's superior reputation earned from the conscientious creation, marketing, and distribution of its products. "Good" CSR will typically mean a firm will embrace superior quality assurance procedures in the development and production of its products, conduct ethical marketing campaigns, provide products with extra social value, provide products to disadvantaged demographic groups, and generally face product recalls voluntarily. In contrast, "Bad" CSR means firms are usually involved in regulation fights, suffer safety violation fees, accept lower product safety standards, and conduct limited due diligence on their supply chain.

Thus, we categorize our three firm types as follows. The lowest type, which we will call "Irresponsible" types, are involved in "Bad" things, ex-ante an event. The next type, "Responsible" types, are not involved with "Bad" things, but neither are they involved in "Good" things—they are simply responsible corporate citizens. Finally, there are some exceptional firms that not only avoid being involved in "Bad" things, but are also participating in some extra "Good" things. These firms we dub "Stellar" types. This typology aligns with the notion that it is costly to move from one to the other: as a firm becomes more conscientious in its activities it moves from Irresponsible to Responsible, and then with even further conscientiousness it becomes a Stellar type.

Our empirical strategy is to first calculate the abnormal change in firm values (i.e., after controlling for firm heterogeneity) during an event and to regress these varying percent changes in firm value on the level of ex-ante CSR reputation, as well as various time, financial, and industry controls. We begin by reviewing our data characteristics and then turn to our event study methodology and regression model.

20.2 Data

Our Data consist of three components. The first part is the abnormal returns of various firms during our product recalls, which we describe in detail in the next

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section. The event returns are then merged with Compustat, our second set of data. For firm control data of the S&P 500 firms we have: annual sales ("Sales (net)"), asset value ("Assets-total"), market value ("common shares outstanding"×"price-calender year-closing"), and percent of profits per share ("EPS (Basic) - Exclude Extra. Items "÷"price-calender year-closing"). Actual product recall events were obtained from manual collection of product recall events of S&P 500 firms as indexed by the Wall Street journal from 1991 through 2006. Although this categorization of product recalls is certainly not perfect, it is the primary source used by past product recall literature. Further, we wanted to have an ex-ante fixed criteria of selecting recalls to prevent subjective inclusion or exclusion on the part of the researcher. We do note recalls included in the Wall Street Journal press announcements are biased towards larger event recalls. However, our theory predicts it is these large scale recalls where we will see any effects, if any exist, from ex-ante CSR reputation.

Occasionally some firms had more than one event announcement in a year, most often a later press announcement related to the same event. For our data collection, we simply summed the abnormal returns together, following the methodology as shown below, by summing abnormal returns over event window days. Having more than one event in a year for a given firm occurred for 25 of the firm/ event years for an average of 1.5 additional events for each occurrence. This excludes autos, and so is out of a total of 147 firm event years. In other words, roughly 17% of the firm years had multiple events, each averaging an additional 1.5 events.

An important exception was automobile firms (GM, Ford, and Chrysler pre-1999). These firms are very different in that they have a product recall every year, and typically multiple recalls in a given year. Hence, we dummy for these three firms since the probability of recall is 100% every year versus less than a 3% chance of recall for all firms. Our results are also robust to simply dropping automobiles from the data.

Our final component of data is CSR ratings from KLD analytics. KLD is considered the "gold standard" of CSR ratings by social investment firms. It is also most commonly used in past related academic studies (see Chatterji et al. (2007) for a review). KLD conducts proprietary research to assign annual CSR ratings to publicly held firms across various dimensions .

For KLD's CSR ratings on the product dimension, analyst's score a firm on four areas of positive (i.e., "product strengths") and negative (i.e., "product concerns") CSR. The four areas of CSR strengths include "Product Quality," "R&D," "Benefits economically disadvantaged people," and "Other. " The concerns areas include "Product Safety," "Marketing Controversy," "Antitrust Concerns," and "Other." One can think of this rating scheme as a latent variable model: each firm is rated by

analysts on various factors unobserved by the econometrician. Once a firm has a value above some threshold, they receive an outcome of one for each of 8 categories (i.e., four strengths and four concerns), and zero otherwise. Finally, KLD then provides a Product Strengths and Product Concerns rating that are each simply coded 0,1,2,3 or 4, measuring the number of ones earned in each of the respective categories. Now we consider the probability of event for an "Irresponsible," "Responsible," and "Stellar" firm. "Irresponsible" is a firm that has at least total product concerns of 1 or greater before an event occurs. A "Responsible" firm has avoided bad marks (i.e., no product concerns marks) but neither does it have any exceptional marks (i.e., product strengths marks). Finally, "Stellar" firms have avoided bad marks while additionally obtaining exceptional marks. The empirically likelihood[14] (i.e., probability) of a Irrepsonsible (I), Responsible (R), and Stellar (S) type firm of having an event over the entire 15 year period is $\Pr(\text{event}|\gamma_I) = 3.5\%$, $\Pr(\text{event}|\gamma_R) = 2.2\%$, $\Pr(\text{event}|\gamma_S) = 2.1\%$, respectively. Both of the higher types (i.e., Responsible and Stellar types) are statistically different from the Irresponsible type. However, the higher types are not statistically different from each other. Nonetheless, the monotonic ordering theory suggests in event rates is preserved. We now turn to the estimation of abnormal event returns for the core of our study.

20.3   Financial Event Studies

The particular event study methodology we use is a financial events study[15]. The

idea behind a (financial) event study is to measure the effect of an event on firm value. This approach relies on finance theory's notion of market efficiency: firms are priced based on all currently available public information. Thus, once new public information is released it is almost immediately absorbed into the value of the firm via its stock price.

The event study methodology procedurally has the first step of estimating how a particular company's stock price changes in relation to various market factors *before* the event occurs. The particular factor model we use is the most commonly used Fama/ French model. Expected return in this setting is estimated as an OLS specified by:

$$R_{i,t} = \alpha_i + \beta_i R_{M,t} + S_i SMB_t + H_i HML_t + \varepsilon_{i,t}$$

[14]We exclude the Auto industry, as previously mentioned it has exceptionally high recall rates compared with all other firms.

[15]For a thorough review see MacKainly (1997).

That is, the return of the stock equals a firm fixed effect , plus a sensitivity to the general market return R_M, sensitivity to small stocks versus large stocks (SMB), and finally a sensitivity to high versus low book to market type stocks. Coefficients are estimated from a time series just before but disjoint to the particular event of interest; here, following common practice, the estimation period begins 8 months prior and ends 30 days prior to the event. These coefficient estimates are then used to predict the return during the event period. That is, our predicted return around the event period becomes:

$$\widehat{R}_{i,t} = \widehat{\alpha}_i + \widehat{\beta}_i R_{M,t} + \widehat{S}_i SMB_t + \widehat{H}_i HML_t$$

The next step is to then use this estimated returns model from the first step to predict what the expected returns are during the event of interest and then calculate the "abnormal return," defined as the difference in *actual* return from the *predicted* return: $AR_{i,t} = R_{i,t} - \widehat{R}_{i,t}$. The cumulative abnormal return is then simply the sum of these returns. For our study, we used the day before and the day of the event announcement as our "event window." This is the most stringent of windows; however, we wanted to minimize the effect of any other previous or subsequent news confounds. We begin the window the day before, as is practice, to capture any "news leakage" the day before the event announcement. Thus cumulative abnormal return is then simply: $CAR_i = \sum_{t=-1}^{0} AR_{i,t}$, where 0 is the event day. Below is a graph of the event study estimation method.
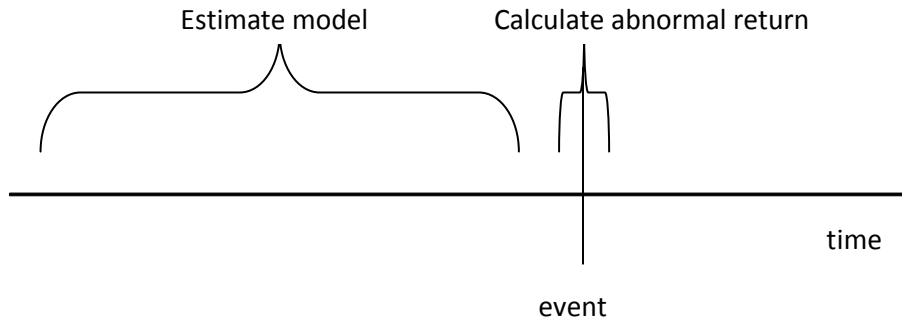
Financial Event Study Methodology

Our next figure reports the distribution of our CARs in relation to an estimated normal distribution of the data. As expected, the actual data has a fatter negative tail than a normal distribution. That is, we expect there to be more severe negative events and fewer severe positive events than a normal distribution would predict,

since recalls are generally costly. There is also a much greater grouping of events at the center of just below zero, which seems to be close to the direct cost of a typical event. Hence, it seems there are routine recalls that are presumably of an exogenous type, thus only changing firm value by close to the direct recall cost effect. However, there a good number of events that have huge negative costs, much beyond routine recall expenses. It is these events against which we hypothesize superior CSR reputation will help protect firm value. We now turn to examining the relationship of event returns to ex-ante CSR reputation.
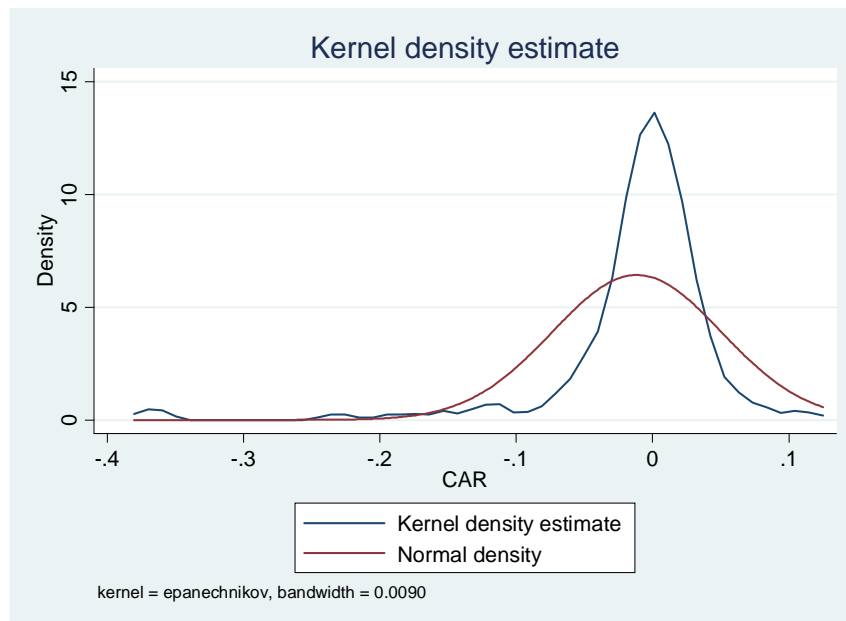
Actual Distribution of CAR versus Normal Distribution

20.4 Regression Model and Results

Once we calculate our abnormal returns (CAR) through a financial event study on every firm facing an adverse event, our final step is to examine any relationship between the ex-ante product CSR level and the respective CAR via a cross sectional regression. In particular, we specify the following:

$$CAR_i = \alpha + \beta_1 Auto_i + \beta_2 L.responsible_i + \beta_3 L.stellar_i + \beta_4 \overrightarrow{YEAR}_i + \beta_5 L.\overrightarrow{FIRM}_i + \varepsilon_i$$

CAR_i is again the cumulative abnormal return for firm i as calculated in the previous section. $Auto_i$ is simply a dummy for if the firm is an automobile manufacture. $L.responsible_i$ is a dummy for an average product CSR type, as defined in our former section, but its value is for the year *prior* to the event. Similarly, $L.stellar_i$ denotes the firm was a high type firm the year *before* the event. Thus, our low types, Irresponsible firms, are our baseline. That is, the former two dummies will tell us how much better compared with a low type the higher types fare under an adverse event.

We added year fixed effects for some specifications, denoted by the vector $\overrightarrow{YEAR_i}$. Finally, our firm financial and industry controls captured in the vector $L.\overrightarrow{FIRM_i}$, again valued based on year prior to an event.

Industry controls are made by dummying the NAICS industry code to the 2 digit level. We do not dummy at the 3 digit level because most dummies could not be estimated since we only have 184 firm event years over 15 years. Even with 2 digit level industry codes about half of our dummies cannot be estimated due to a paucity of observations. Below we report our regression results. When we control for lagged financials we lose some observations due to their not being in the index previously or recent mergers.

		Dependent Variable: Cumulative Abnormal Return			
Variable	**None**	**Time**	**Financial**	**Industry**	**All**
Responsible Type	.0279***	.0287***	.0305***	.0240**	.0283**
	(.0091)	(.0095)	(.0108)	(.0098)	(.0134)
Stellar Type	.0274***	.0251***	.0314***	.0190*	0.0163
	(.0097)	(.0097)	(.0104)	(.0105)	(.0158)
Year Control	NO	YES	NO	NO	YES
Firm Control	NO	NO	YES	NO	YES
Industry Control	NO	NO	NO	YES	YES
N	184	184	156	184	156
R Squared	0.0719	0.1413	0.0855	0.1191	0.2266

Notes: *,**,*** represent statistical significance at 90%,95%,and 99% confidence levels.   Standard errors reported in parentheses.

The baseline type is irresponsible. All specification include a dummy for Auto.

Each column of results differs based on controls included. As can be seen, if a firm is able to carefully avoid being involved in bad activities (i.e., a Responsible type), it will save close to 3% of abnormal firm value should it face an event compared

with the losses of a low type firm. This effect is also economically significant, as it amounts to an average saved firm value of over \$600 million for the median firm (market) value of \$23 billion. Meanwhile, if a firm is one of those exceptional firms that not only is careful to avoid bad activities, but also is involved in good ones—a Stellar type, the savings in firm value is similar to the Responsible firm type. Even for the final column, using all controls, the difference in coefficients is not statistically different.

What this says is, it really pays to carefully avoid harmful activities in building reputation. However, it does not seem to pay, at least in an insurance sense, to build reputation of going the extra mile and providing some additional "good" activities.

Of course, there could be contemporaneous benefits in provisioning some extra "good" activities. However, as past literature has found, there has not been any financial benefit identified in doing so. In short, at least financially speaking, it really pays to be responsible by avoiding bad, but neither does it pay to also be exceptionally good.

It might seem curious that adding time, financial, and industry controls does not change our CSR coefficient estimates. However, recall in the event study, we already controlled for time and financial characteristics as a well as a fixed firm effect for each firm. The main effect of adding all the controls is to introduce more noise while not affecting coefficient estimates. Indeed, when estimating all our coefficients (i.e., the final column) we are estimating a parameter per fewer than every 5 observations. Thus not surprisingly, our final column of estimates have our coefficients less significant.

One concern in estimating these that abnormal returns is it could be the case abnormal return is simply the expected direct cost (e.g., the cost of replacing faulty automobile tires) of the product recall. If ex-ante CSR level is related with actual event cost, this in itself would be interesting, as it indicates that CSR predicts the level of an event. Nonetheless, it would not support our CSR reputation story where there is uncertainty over the degree of negligence and its punishment that the market must price in immediately after an event. Unfortunately, the expected direct cost of a product recall is seldom made public (nor is it commonly disclosed ex-post). However, for our sample, roughly 10% of the announcements were accompanied by estimates of the direct event costs. For this subsample, the direct costs explain roughly 16% of the variation in CAR. Further, when a loss is sustained by a firm (i.e., a negative CAR), the direct costs represent 38% of the total loss on average. In absolute value terms (i.e., because sometimes a firm has a positive CAR during an event), direct costs represent 26% of the value of CAR. Thus, while this sub-sample is only a small portion of the events, it suggests it is not the expected direct cost of

an event driving differences in CAR. Further, the direct costs have a very narrow band of cost difference, whereas the change in abnormal firm value varies widely, suggesting there is much more than just product recall direct cost embedded in the CAR. Our theory suggests the CAR should be a combination of direct recall loss and (expected) financial loss if negligent.

In the end, the evidence is quite suggestive that ex-ante CSR reputation affects ex-post value of a firm. But it is not so much being involved with exceptionally good things as it is in carefully avoiding bad things. Thus, effective CSR in insurance terms seems to be more about making sure a firm is being careful to avoid harmful behavior— i.e., simply being a responsible corporate citizen rather than an exceptional one.

One can even estimate what a firm should pay for such carefulness. As far as benefit, firms moving from Irresponsible to Responsible (or Stellar), saves some $600 million of firm value. With an incidence rate of roughly 2.5%, this means a risk neutral firm should be willing to pay over $15 million per annum to be a higher type.

21   Concluding Discussion

We have offered a (partial) solution to the puzzle of why firms invest in CSR when it has no apparent effect on current profits or firm value, yet it is a costly activity. In particular, we proposed that firms use CSR as a reputation insurance mechanism. We developed a model that showed we generally expect a firm with high ex-ante CSR to better weather a shock to firm value, as well as experience such events less often.

Empirically, we also found support for CSR acting as insurance, though the results are nuanced. In particular, it really pays to be responsible as a firm to avoid bad behavior—this tends to save over $600 million of abnormal should a firm face an adverse event. However, then becoming an *exceptional* corporate citizen by engaging in additional stellar behavior, does not seem to pay additional dividends in an insurance sense or financial sense.

Hence, with many business schools and firms focusing on "doing well by doing good," a better mantra might be "doing well by carefully avoiding bad." There is perhaps a more succinct way to put it: *primum non nocere*—first do no harm. In fact, if a manager becomes too focused on doing good, she very well could miss the seemingly more important task of avoiding harm.

This is the very advice BP missed as it spent significant sums of money to rebrand itself with its sunburst logo and the tagline "Beyond Petroleum." It was involved with some "good" environmental projects. However, it completely missed the call to avoid harm, to be careful to avoid bad events. Consequently, as its number came up, and nature drew a bad event for BP, investors and regulators swung the scale of responsibility firmly to negligent. And so it is, completely uninsured, BP will have to pay substantial sums of money.

> "Most of the rhetoric on CSR may be about doing the right thing and trumping competitors, but much of the reality is plain risk management. It involves limiting the damage to the brand and the bottom line that can be inflicted by a bad press and consumer boycotts, as well as dealing with the threat of legal action."
>
> *Economist , January 7,2008*

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22   Appendix

22.1   Proofs of Lemmas and Propositions

Lemma 1: *If $\alpha > .5$, the contestant best response function becomes single peaked with a maximum at $\widehat{c}$ such that $F(\widehat{c}) = \frac{2\alpha - 1}{k\alpha - 1}$*

Proof: We first write the bidding function as $b(\alpha, c) = g((1 - \alpha)A(c) + \alpha B(c))$, where $g(\cdot)^{-1} = \gamma(\cdot)$ (i.e., $g(\cdot)$ is the inverse of the cost function). Now note $\frac{d}{dc}g((1 - \alpha)A(c) + \alpha B(c)) = \underbrace{g'((1 - \alpha)A(c) + \alpha B(c))}_{>0}[(1 - \alpha)A'(c) + \alpha B'(c)]$. The former term is always positive for $c \in [\underline{c}, \overline{c})$ since $g(\cdot)$ is strictly increasing and $(1 - \alpha)A(c) + \alpha B(c)$ is always positive. The latter term, we will see, is single peaked, thus making our entire expression single peaked. Expanding $(1 - \alpha)A'(c) + \alpha B'(c)$, we get:

$$(1 - \alpha)(-(k - 1)\frac{1}{c}(1 - F(c))^{k-2} \times F'(c)$$

$$+\alpha((k - 1)\frac{1}{c}(1 - F(c))^{k-3} \times [(1 - (k - 1)F(c)] \times F'(c)$$

Rearranging terms then yields:

$$\underbrace{(k - 1)\frac{1}{c}(1 - F(c))^{k-3} \times F'(c)}_{>0} \times [(F(c) - 1)(1 - \alpha) + \alpha(1 - (k - 1)F(c)]$$

The former term is always positive so we only focus on the latter term, which further rearranging gives:

$$F(c) - \alpha F(c) - 1 + \alpha + \alpha - k\alpha F(c) + \alpha F(c)$$
$$= -k\alpha F(c) - (1 - F(c)) + 2\alpha$$

First note at the lowest cost type $\underline{c}$, we get simply $2\alpha - 1$, which is always positive for $\alpha > .5$ at $c = \underline{c}$. That is, our bidding function is increasing at the lowest cost type. Similarly, with the highest cost type, we get $-k\alpha + 2\alpha$, which is always negative for $k \geq 3$. Thus, our bidding function is decreasing at the highest cost type.

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Now solving the above for a unique zero gives:

$$-k\alpha F(c) - (1 - F(c)) + 2\alpha \equiv 0$$

$$\Rightarrow \quad (k\alpha - 1)F(c) = 2\alpha - 1 \Rightarrow F(c) = \frac{2\alpha - 1}{k\alpha - 1}$$

.

Define then $\widehat{c}$ such that $F(\widehat{c}) = \frac{2\alpha-1}{k\alpha-1}$. Now when $c \in [\underline{c}, \widehat{c})$ we have $a \equiv F(c)$:

$$\underbrace{(1 - k\alpha)a}_{<0} + 2\alpha - 1$$

Once we fix α and k, we see the above expression, which then determines the sign of the derivative of the bidding function, is strictly *decreasing* in a. At $a = \frac{2\alpha-1}{k\alpha-1}$, the above expression equals zero. Meanwhile, with $a \in [\underline{c}, \widehat{c})$ the expression is positive and with $a \in (\widehat{c}, \overline{c}]$ the expression is negative. Hence, the bidding function is single peaked at $\widehat{c}$. Thus, our type $\underline{c} < \widehat{c} < \overline{c}$ provides the highest effort over all types. $\square$.

Proposition 1 *The generalized 2nd prize contest mechanism exists, meets all incentive compatibility constraints, and induces a (weakly) monotonic bidding function.*

Proof: We first show how the mechanism meets all incentive compatibility constraints. For contestants who would optimally provide effort below e^*, this problem is just as before so their bidding function remains as under a contest with no pooling, which we will call *no pool* bidding or *no pool* contest, depending on the context. Call this cutoff c^* such that for all $c \in [c^*, \overline{c}]$ these participants provide their effort below e^* as under *no pool*. Now all that have costs of effort $c \in [\underline{c}, c^*]$ are to be in the pooling effort interval. For this pooling group we now must make sure such effort interval is incentive compatible against the deviation of exerting more or less effort than e^*, make sure such c^* is beyond $\widehat{c}$ (i.e., the peak of the non-modified bidding function) to induce a (weakly) monotonic bidding function, and also make sure at c^* such participant is indifferent between optimal effort solved under *no pool* and the pooling interval payoff. We check each of these necessary conditions in turn.

Let the total prize be worth 1, as before. Let $\alpha > \frac{1}{2}$ be the second place share with the remainder being the first place share. Suppose there are k players. Let p be the measure of types in the pooling interval. Then, the expected payoff from bidding in the pooling interval is

$$\pi_{pool} = \underbrace{\sum_{i=1}^{k-1} \binom{k-1}{i} \frac{1}{i+1} p^i (1-p)^{k-1-i}}_{\text{1 or more other contestants pool}} + \underbrace{(1-p)^{k-1}(1-\alpha)}_{\text{No other contestant pool}}$$

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We first need to verify the contestant c^* at the end of the pooling interval (i.e., type c^* where $p = F(c^*)$) is indifferent between pooling or exerting the identical effort e^* under *no pool* bidding. When we set $p \equiv F(c^*)$, the payoff for c^* under a *no pool* contest is as follows:

$$\pi_{no\ pool} = (k-1)\alpha p (1-p)^{k-2} + (1-\alpha)(1-p)^{k-1}$$

Hence, we require $\pi_{pool} = \pi_{no\ pool}$, thus we solve for the indifferent value of p

$$\sum_{i=1}^{k-1} \binom{k-1}{i} \frac{1}{i+1} p^i (1-p)^{k-1-i} + (1-\alpha)(1-p)^{k-1}$$
$$= (k-1)\alpha p (1-p)^{k-2} + (1-\alpha)(1-p)^{k-1} \iff$$

$$\sum_{i=1}^{k-1} \binom{k-1}{i} \frac{1}{i+1} p^i (1-p)^{k-1-i} = (k-1)\alpha p (1-p)^{k-2}$$

Now divide by $(1-p)^{k-1}$ to obtain

$$\sum_{i=1}^{k-1} \binom{k-1}{i} \frac{1}{i+1} \left(\frac{p}{1-p}\right)^i = (k-1)\alpha \frac{p}{1-p}$$

Now, with a change of variable, let $z = \frac{p}{1-p}$ to obtain

$$\sum_{i=1}^{k-1} \binom{k-1}{i} \frac{1}{i+1} (z)^{i-1} = (k-1)\alpha$$

Now fix α and k. Then the LHS of the equality is strictly increasing in z, which is strictly increasing in p. Further at the limit as $p \to 0 \Rightarrow z \to 0$, the LHS converges to $\frac{k-1}{2}$, only the $i = 1$ term remains[16]. The RHS is then a greater finite number $(k-1)\alpha > \frac{k-1}{2}$ with $\alpha > .5$ as $p \to 0$.

Oppositely, as $p \to 1$ the LHS approaches $+\infty$, whereas the RHS is again a finite number. Hence, there exists a unique $p^* \in (0, 1)$ that solves the above equation. Solving for p^* then determines both e^* and c^*. Hence, once we fix α and k, we

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[16]This can also be seen by taking the limit of the generating function of $\sum_{i=1}^{k-1} \binom{k-1}{i} \frac{1}{i+1} (z)^{i-1} \equiv \frac{(1+z)^k - (k)z - 1}{(k)z^2}$ as $z \to 0$.

can always find our needed c^* *uniquely.* Further, by meeting the above equality we have actually also met the IC constraint, which we call IC_{down}, for preventing pooling types from deviating down; thus, we see IC_{down} binds. Note also p^* is increasing in α. However, p^* can be either increasing or decreasing in k depending on the parameterization, as k affects both α and p (holding α constant) in a complex way.

Once we have e^* and c^* we already know any $c \in [c^*, \bar{c}]$ does not want to deviate, as they are already choosing their optimal effort per the *no pool* bidding structure. Meanwhile, any $c \in [\underline{c}, c^*)$ will not want to deviate by providing less effort than the pooling effort level because if it was not worth it for the c^* type to do so, then it certainly is not worth it for the lower cost types. That is, in considering whether to exert less effort, the c^* type trades off the saved cost of less effort with a reduced expected gross benefit. Thus, if the c^* type's cost savings did not justify less effort, it certainly will not be justified for those with lower cost (savings) facing the same reduced expected benefit.

Now we need to check that a participant in the pooling interval does not want to deviate up, as doing so would guarantee a first prize. The payoff from deviating up is thus:

$$\pi_{up} = 1 - \alpha$$

Hence, we require that

$$\pi_{pool} - \pi_{up} \geq 0$$

Substituting.

$$\pi_{in} - \pi_{up} = \sum_{i=1}^{k-1} \binom{k-1}{i} \frac{1}{i+1} p^i (1-p)^{k-1-i} - \left(1 - (1-p)^{k-1}\right)(1-\alpha)$$

Now, recall by the binomial theorem:

$$1 - (1-p)^{k-1} = \sum_{i=1}^{k-1} \binom{k-1}{i} p^i (1-p)^{k-1-i}$$

Hence

$$\pi_{in} - \pi_{up} = \sum_{i=1}^{k-1} \binom{k-1}{i} \left(\frac{1}{i+1} - (1-\alpha)\right) p^i (1-p)^{k-1-i}$$

Clearly, if $\frac{1}{k} > (1-\alpha) \iff \alpha \geq 1 - \frac{1}{k}$, IC_{up} is met, as it means all sums being added above are positive. This requirement simply says the second prize share α

needs to be weakly greater than 1 minus the inverse the number of participants. Thus, this requirement increases in k; however, the optimal α^* is also increasing in k. It is meanwhile trivial if $\alpha = 1$ (i.e., there is only a second prize), IC_{up} is met. However, this sufficient condition is obviously more than needed.

The precise requirement is readily found by solving the generating function of $\sum_{i=1}^{k-1} \binom{k-1}{i} \left(\frac{1}{i+1} - (1-\alpha) \right) p^i (1-p)^{k-1-i}$:

$$\frac{(1-p)^{k-1}(p-1-kp\alpha + (\frac{1}{1-p})^{k-1}(1+kp(\alpha-1)))}{kp}$$

The term of interest is $(p-1-kp\alpha + (\frac{1}{1-p})^{k-1}(1+kp(\alpha-1)))$, as this determines if the entire equation is (weakly) positive and thus IC_{up} is met. We can then solve for when this term is (weakly) greater than zero:

$$(p-1-kp\alpha + (\frac{1}{1-p})^{k-1}(1+kp(\alpha-1))) \geq 0 \Rightarrow$$

$$kp\alpha\left((\frac{1}{1-p})^{k-1} - 1\right) \geq 1-p+(kp-1)(\frac{1}{1-p})^{k-1} \Rightarrow$$

$$1 \geq \alpha \geq \frac{1-p}{kp((\frac{1}{1-p})^{k-1}-1)} + \frac{(kp-1)(\frac{1}{1-p})^{k-1}}{kp((\frac{1}{1-p})^{k-1}-1)}$$

The middle term is our first sufficient condition. Since the designer gets to choose α, this condition can regardless always be met since any $\alpha \in [\frac{1-p}{kp((\frac{1}{1-p})^{k-1}-1)} + \frac{(kp-1)(\frac{1}{1-p})^{k-1}}{kp((\frac{1}{1-p})^{k-1}-1)}, 1]$ will satisfy IC_{up}. Additionally, we will consider indivisible prizes, which means we again have $\alpha = 1$ or $\alpha = 0$. Finally, when we do allow for divisible prizes, we could allow that the designer to simply declare any observed effort greater than e^* is still counted as e^*. Since effort is costly, no player would ever exert greater than e^*.

Lastly, we also need to check the type $c^* \geq \widehat{c}$. That is, we need to make sure the indifference point from where we end the pooling interval is *after* the single peak of the *no pool* bidding function; otherwise, we still have not solved the non-monotonicity problem. Recall our IC_{down} condition was the following being (weakly) positive:

$$\sum_{i=1}^{k-1} \binom{k-1}{i} \frac{1}{i+1} (z)^{i-1} - (k-1)\alpha$$

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We then substitute in $z = \frac{F(\widehat{c})}{1 - F(\widehat{c})}$, where $F(\widehat{c}) = \frac{2\alpha - 1}{k\alpha - 1}$ as found in our first Lemma. If the expression is negative, it means $c^* > \widehat{c}$, since IC_{down} is not yet met at $\widehat{c}$. That is, we need to choose a larger $p^* > F(\widehat{c})$ (since the above is strictly increasing in z, which is strictly increasing in p) to meet IC_{down}. But this then means we get $c^* > \widehat{c}$.

To see we always have $c^* > \widehat{c}$, first note $\frac{dF(\widehat{c})}{d\alpha} = \frac{\partial}{\partial\alpha} \frac{2\alpha - 1}{k\alpha - 1} = \frac{k(1 - 2\alpha) - 1}{(k\alpha - 1)^2} < 0$ (for $\alpha \geq .5$). This then means $\frac{\partial z}{\partial\alpha} < 0$ when evaluated at $c = \widehat{c}$ since z is strictly increasing in $p \equiv F(c)$. Now we take the derivative of our IC_{down} condition with respect to α and consider its value when evaluated at $c = \widehat{c}$:

$$\frac{d}{d\alpha}(\sum_{i=1}^{k-1} \binom{k-1}{i} \frac{1}{i+1} (z)^{i-1} - (k-1)\alpha)$$

$$= \sum_{i=1}^{k-1} \binom{k-1}{i} \frac{1}{i+1} (i-1)(z)^{i-2} \underbrace{\frac{\partial z}{\partial\alpha}}_{<0} + \underbrace{-(k-1)}_{<0} < 0$$

$$\underbrace{\phantom{\sum_{i=1}^{k-1} \binom{k-1}{i} \frac{1}{i+1} (i-1)(z)^{i-2} \frac{\partial z}{\partial\alpha}}}_{\leq 0}$$

Hence, our IC_{down} condition is strictly decreasing in α. This means if we can show such expression is non positive at $\alpha = .5$, we are done. Recall, as we already showed, when $\alpha = .5$, we get $z = 0 \Rightarrow \sum_{i=1}^{k-1} \binom{k-1}{i} \frac{1}{i+1} (z)^{i-1} \rightarrow \frac{k-1}{2}$. But this then means $\sum_{i=1}^{k-1} \binom{k-1}{i} \frac{1}{i+1} (z)^{i-1} - (k-1)\frac{1}{2} = \frac{k-1}{2} - \frac{k}{2} + \frac{1}{2} = 0$ with $\alpha = .5$. Hence, since $\sum_{i=1}^{k-1} \binom{k-1}{i} \frac{1}{i+1} (z)^{i-1} - (k-1)\alpha$ is strictly decreasing in α, it has to be the case for any $\alpha > .5$ we get $c^* > \widehat{c}.\square$

Lemma 5 R and $\widetilde{R}$ are concave in α for $\alpha \in [0, \frac{1}{2}]$ and $\alpha \in [0, \frac{1}{2})$, respectively

Proof: Recall $R(\alpha) = k \int_{\underline{c}}^{\overline{c}} g(A(c) + \alpha(B(c) - A(c))) \times F'(c)dc$.

Taking the first derivative with respect to α yields:

$$R'(\alpha) = k \int_{\underline{c}}^{\overline{c}} g'(A(c) + \alpha(B(c) - A(c))) \times (B(c) - A(c)) \times F'(c)dc$$

Taking the derivative again with respect to α yields:

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$$R''(\alpha) = k \int_{\underline{c}}^{\overline{c}} g''(A(c) + \alpha(B(c) - A(c))) \times (B(c) - A(c)))^2 \times F'(c)dc$$

Since $g''(\cdot) < 0$ (i.e., because $g(\cdot)$ is concave), we get $R''(\alpha) < 0$, as desired. Extending this to the GSPC, note $\widetilde{R}(\alpha)$ is the same as $R(\alpha)$ for $\alpha \in [0, \frac{1}{2})$. $\square$

Lemma 6 $\widetilde{R}'(\frac{1}{2}) > R'(\frac{1}{2})$

First note $\widetilde{R}'(\alpha) = R'(\alpha)$ for all $\alpha \in [0, \frac{1}{2})$. Also recall for all $\alpha \in [0, \frac{1}{2}]$ we have $\widetilde{R}(\frac{1}{2}) = R(\frac{1}{2})$, the revenue is the same (and the functions precisely the same) since there is no pooling interval until $\alpha > \frac{1}{2}$. It would then be tempting to immediately assert $\widetilde{R}'(\frac{1}{2}) = R'(\frac{1}{2})$. However, once we increase α by an ε a pooling interval develops, and thus we must account for this to determine $\widetilde{R}'(\frac{1}{2})$. As we do increase α by an ε, all we do is shift the bottom support of the integral comprising $R'(\frac{1}{2})$ to some $c > \underline{c}$ just greater than $\underline{c}$. Thus, we want to show $\frac{d}{dc} R(\frac{1}{2}) > 0$, and we will be done:

$$\frac{d}{dt}\bigg|_{\underline{c}} \left[k \int_{t}^{\overline{c}} g'(\frac{1}{2}(A(c) + B(c))) \times (B(c) - A(c))) \times F'(c)dc \right] > 0$$

Hence, we get

$$\frac{d}{dt} \left[k \int_{t}^{\overline{c}} g'(\frac{1}{2}(A(c) + B(c))) \times (B(c) - A(c))) \times F'(c)dc \right]$$

$$= -k \cdot g'(\frac{1}{2}(A(t) + B(t))) \times (B(t) - A(t)) \times F'(t)$$

However, we know $(B(t) - A(t)) < 0$ when $t = \underline{c}$. Thus, since we always have $g'(\cdot) > 0$ and $F'(t) > 0$, the entire expression is then strictly positive. In addition, we have now added a pooling interval that induces positive total revenue value once $\alpha > \frac{1}{2}$ that is in addition to the revenue related to the above expression.

But both these facts then mean $\widetilde{R}'(\frac{1}{2}) > R'(\frac{1}{2})$, as desired. $\square$

Proposition 2 It is optimal to offer a larger second prize than first prize through

our GSPC if our sufficient condition is met:

$$k \int\limits_{\underline{c}}^{\overline{c}} h' \left(g(\tfrac{1}{2} \left(A(c) + B(c) \right)) \right) \times g'(\tfrac{1}{2} \left(A(c) + B(c) \right))(B(c) - A(c)) \times F'(c)dc > 0$$

Proof:

First take the derivative of our revenue function with respect to α :

$$\frac{d}{d\alpha} R(\alpha) = \frac{d}{d\alpha} k \int\limits_{\underline{c}}^{\overline{c}} h(g(A(c) + \alpha(B(c) - A(c)))) \times F'(c)dc$$

$$= \; k \int\limits_{\underline{c}}^{\overline{c}} h' \left(g(A(c) + \alpha(B(c) - A(c))) \right) \times g'(A(c)$$

$$+ \alpha(B(c) - A(c)))) \times (B(c) - A(c)) \times F'(c)dc$$

No we evaluate this expression at $\alpha = 5$:

$$\frac{d}{d\alpha} R(.5) = k \int\limits_{\underline{c}}^{\overline{c}} h' \left(g(\tfrac{1}{2} \left(A(c) + B(c) \right)) \right) \times g'(\tfrac{1}{2} \left(A(c) + B(c) \right))(B(c) - A(c)) \times F'(c)dc$$

Now if $\frac{d}{d\alpha} R(.5) > 0$, we know then that $\widetilde{R}'(\tfrac{1}{2}) > R(.5) > 0$, per the previous Lemma . But this then means it is optimal to offer a larger second prize via the GSPC. $\square$

23 Estimating Some Convex Combinations of $A(c)$ and $B(c)$

We first find upper and lower bounds of $A(c)$.

First recall $A(c) \equiv (k-1) \int\limits_{c}^{\overline{c}} \tfrac{1}{a}(1 - F(a))^{k-2} \times F'(a)da.$

Hence,

$$(k-1)\int_c^{\overline{c}} \frac{1}{a}(1-F(a))^{k-2} \times F'(a)da \;>\; \frac{1}{\overline{c}}\int_c^{\overline{c}} (k-1)(1-F(a))^{k-2} \times F'(a)da$$

$$= \frac{1}{\overline{c}}\left[-(1-F(a))^{k-1}\right]_c^{\overline{c}}$$

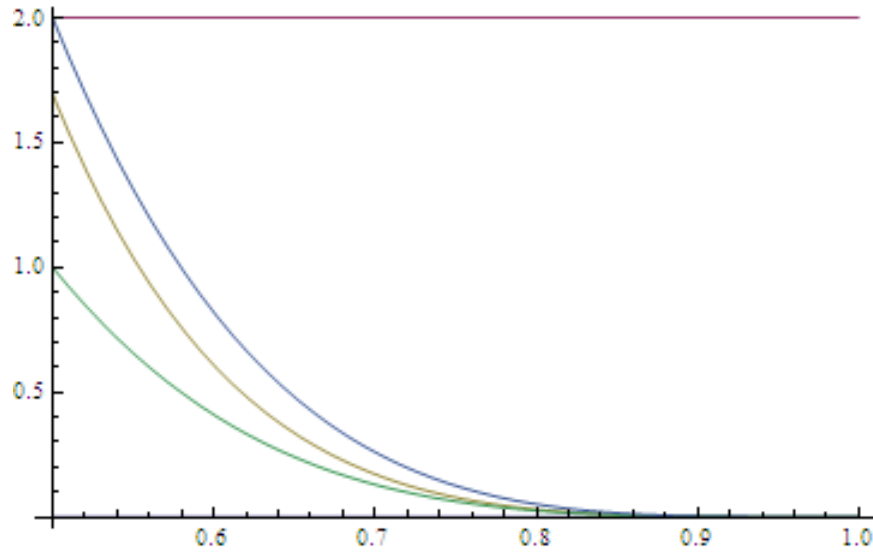$$= \frac{1}{\overline{c}}(1-F(c))^{k-1} = \underline{A}(c).$$

Thus we have:

$$\underline{A}(c) = \frac{1}{\overline{c}}(1-F(c))^{k-1}$$

$$\overline{A}(c) = \frac{1}{\underline{c}}(1-F(c))^{k-1}$$

That is, we have $\underline{A}(c) < A(c) < \overline{A}(c)$.

Here is a plot of our estimates with $k = 5$, quadratic costs, and $c \in U[.5, 1]$:

We next solve for some values of the above expression individually.

By integration of parts we have

$$\int_c^{\bar{c}} (1 - F(a))^{k-3} \times F(a) \times F'(a) da$$

$$= \quad [F(a) \times -\frac{1}{k-2}(1 - F(a))^{k-2}]_c^{\bar{c}} - \int_c^{\bar{c}} -\frac{1}{k-2}(1 - F(a))^{k-2} \times F'(a) da$$

$$= \quad F(c) \times \frac{1}{k-2}(1 - F(c))^{k-2} - \left[\frac{1}{(k-2)(k-1)} \times (1 - F(a))^{k-1} \right]_c^{\bar{c}}$$

$$= \quad F(c) \times \frac{1}{k-2}(1 - F(c))^{k-2} + \frac{(1 - F(c))^{k-1}}{(k-2)(k-1)}$$

.

That is, we get:

$$\int_c^{\bar{c}} (1 - F(a))^{k-3} \times F(a) \times F'(a) da$$

$$= \quad F(c) \times \frac{1}{k-2}(1 - F(c))^{k-2} + \frac{(1 - F(c))^{k-1}}{(k-2)(k-1)}$$

$$= \quad \frac{(k-2) \cdot F(c) + 1}{(k-2)(k-1)} \times (1 - F(c))^{k-2}$$

Secondly, we note through similar analysis:

$$\int_c^{\bar{c}} (1 - F(a))^{k-3} \times F'(a) da = (1 - F(c))^{k-2} \times \frac{1}{k-2}$$

Now we estimate $\frac{1}{2}(A(c) + B(c))$.
Again after some basic calculations we have

$$\frac{1}{2}(A(c) + B(c)) = \frac{1}{2}(k-1)(k-2) \int_c^{\bar{c}} \frac{(1 - F(a))^{k-3}}{a} \times F(a) \times F'(a) da$$
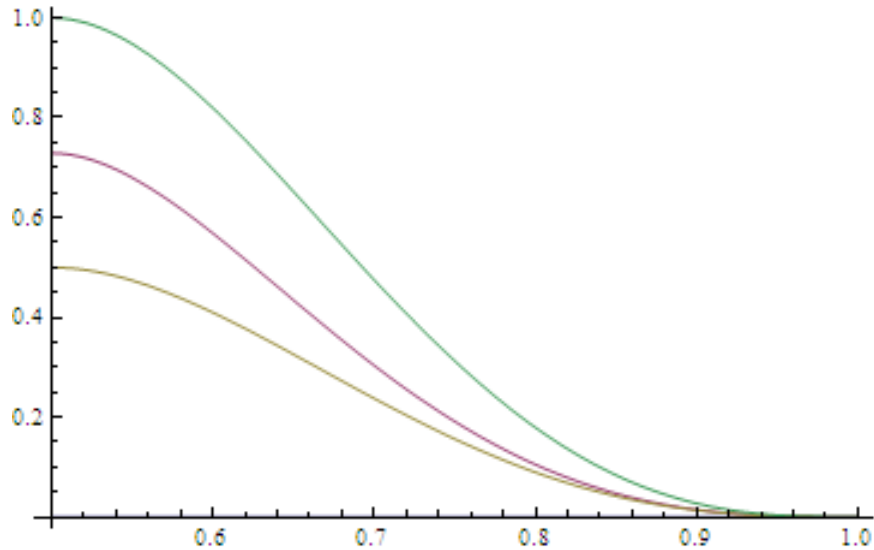
We then get using our results from above:

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$$\frac{1}{2}(k-1)(k-2)\left(\int_c^{\overline{c}} \frac{(1-F(a))^{k-3}}{a} \times F(a) \times F'(a)da\right)$$

$$> \frac{1}{2\overline{c}}(k-1)(k-2)\left(\frac{(k-2)\cdot F(c)+1}{(k-2)(k-1)} \times (1-F(c))^{k-2}\right)$$

$$= \frac{1}{2\overline{c}}\left((F(c) \times (k-2) + 1)\cdot(1-F(c))^{k-2}\right)$$

Hence, we have:

$$\frac{1}{2}\underline{(A(c) + B(c))} = \frac{1}{2\overline{c}}\left((F(c) \times (k-2) + 1)\cdot(1-F(c))^{k-2}\right)$$

$$\frac{1}{2}\overline{(A(c) + B(c))} = \frac{1}{2\underline{c}}\left((F(c) \times (k-2) + 1)\cdot(1-F(c))^{k-2}\right)$$

This gives us $0 < \frac{1}{2}\underline{(A(c) + B(c))} < \frac{1}{2}(A(c) + B(c)) < \frac{1}{2}\overline{(A(c) + B(c))}$.

Here is a plot of these upper and lower bounds around the true value plotted as the red curve:

Now we solve for upper and lower bounds of $B(c) - A(c)$.

First, we have by definition and some trivial calculations:

$$B(c) - A(c) = (k-1) \int_c^{\bar{c}} \frac{(1 - F(a))^{k-3}}{a} \times (k \cdot F(a) - 2) \, F'(a) da$$

This means we know

$$B(c) - A(c)$$
$$< (k-1) \int_{F^{-1}\left(\frac{2}{k}\right)}^{\bar{c}} \frac{(1 - F(a))^{k-3}}{F^{-1}\left(\frac{2}{k}\right)} \times (kF(a) - 2) \, F'(a) da$$
$$+ (k-1) \int_{\underline{c}}^{F^{-1}\left(\frac{2}{k}\right)} \frac{(1 - F(a))^{k-3}}{F^{-1}\left(\frac{2}{k}\right)} \times (kF(a) - 2) \, F'(a) da$$

Thus, we have

$$\overline{B(c) - A(c)}$$
$$= \frac{(k-1)}{F^{-1}\left(\frac{2}{k}\right)} \left(k \times \frac{(k-2)F(c) + 1}{(k-2)(k-1)} \times (1 - F(c))^{k-2} - (1 - F(c))^{k-2} \times \frac{2}{(k-2)} \right)$$

$$= \frac{1}{F^{-1}\left(\frac{2}{k}\right)} \left(k \times \frac{(k-2) \cdot F(c) + 1}{(k-2)} \times (1 - F(c))^{k-2} - (1 - F(c))^{k-2} \times \frac{2(k-1)}{(k-2)} \right)$$

$$= \frac{1}{F^{-1}\left(\frac{2}{k}\right)} \frac{k(k-2)F(c) + k - 2(k-1)}{(k-2)} \times (1 - F(c))^{k-2}$$

$$\implies \overline{B(c) - A(c)} = \frac{1}{F^{-1}\left(\frac{2}{k}\right)} \left(k \cdot F(c) - 1 \right) \times (1 - F(c))^{k-2}$$

This then implies we have $F(c^{**}) > \frac{1}{k}$, where $A(c^{**}) = B(c^{**})$ (see MS (2001) for a proof there exists a unique $c^{**} \neq 1$ such that $A(c^{**}) = B(c^{**})$). This follows from noting $\overline{B(c) - A(c)} = 0$ when $c = F^{-1}(\frac{1}{k})$. However, since $\overline{B(c) - A(c)}$ is an upper bound, we know we have $B(c) - A(c) < 0$ at $c = F^{-1}(\frac{1}{k})$ (since $B(c) - A(c) < 0$ for all $c < c^{**}$). But this then also means it must be that $F(c^{**}) > \frac{1}{k}$. Thus, we now know:

$$\frac{1}{k} < F(c^{**}) < \frac{2}{k}$$

Note this implies $c^{**} \longrightarrow \underline{c}$ **as** $k \longrightarrow \infty$.

Next, we solve for $\underline{B(c) - A(c)}$.
We want

$$B(c) - A(c) \;>$$

$$(k-1) \int_{F^{-1}\left(\frac{2}{k}\right)}^{\overline{c}} \frac{(1 - F(a))^{k-3}}{\overline{c}} \times (k \cdot F(a) - 2)\, F'(a) da$$

$$+ (k-1) \int_{c}^{F^{-1}\left(\frac{2}{k}\right)} \frac{(1 - F(a))^{k-3}}{\underline{c}} \times (k \cdot F(a) - 2)\, F'(a) da$$

$\underline{\text{First assuming } c \geq F^{-1}\left(\frac{2}{k}\right)}$, we can solve the above sum as simply the analog to $\overline{B(c) - A(c)}$, replacing $\underline{c}$ with $\overline{c}$:

$$\frac{(k-1)}{\overline{c}} \int_{c}^{\overline{c}} (1 - F(a))^{k-3} \times (k \cdot F(a) - 2)\, F'(a) da$$

$$= \; \frac{1}{\overline{c}} \left(k \cdot F(c) - 1\right) \times \left(1 - F(c)^{k-2}\right)$$

Now when $c < F^{-1}\left(\frac{2}{k}\right)$ we must solve for the latter part of the sum, which we find as:

$$(k-1) \int_{\underline{c}}^{F^{-1}\left(\frac{2}{k}\right)} \frac{(1 - F(a))^{k-3}}{\underline{c}} \times (k \cdot F(a) - 2)\, F'(a) da$$

$$= \; \frac{1}{\underline{c}} \left(-(\frac{k-2}{k})^{k-2} + (k \cdot F(c) - 1) \times (1 - F(c))^{k-2} \right)$$

Thus, combining both the sums we have

$$\underline{B(c) - A(c)}$$

$$= \frac{1}{\overline{c}} \left(k \cdot \left(\frac{2}{k}\right) - 1 \right) \times \left(1 - \left(\frac{2}{k}\right)^{k-2}\right) + \frac{1}{\underline{c}} \left(-(\frac{k-2}{k})^{k-2} + (k \cdot F(c) - 1) \times (1 - F(c))^{k-2} \right)$$

$$= \frac{1}{\overline{c}}(\frac{k-2}{k})^{k-2} + \frac{1}{\underline{c}} \left(-(\frac{k-2}{k})^{k-2} + (kF(c) - 1) \times (1 - F(c))^{k-2} \right)$$

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Thus, in total we have:

$$\underline{B(c) - A(c)} \;=\; \frac{1}{c}\left((k \cdot F(c) - 1) \times (1 - F(c)^{k-2})\right)$$
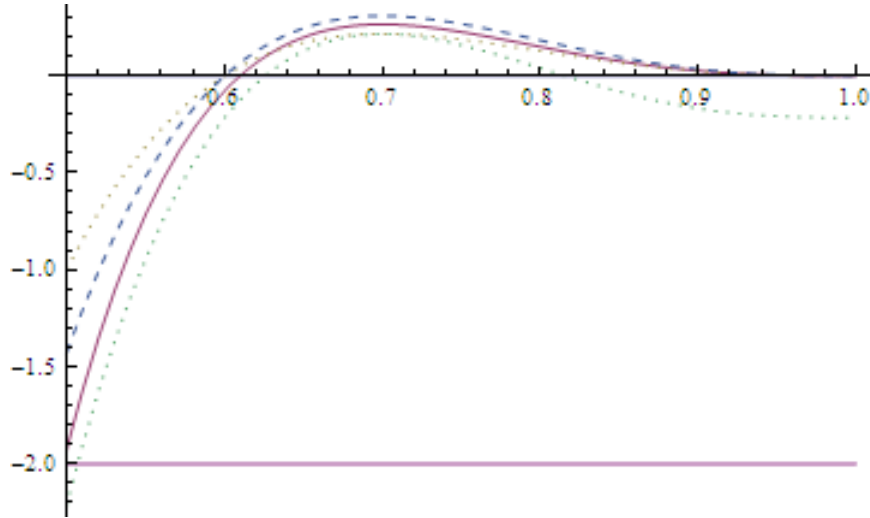
$$\text{when } c \;\geq\; F^{-1}\left(\frac{2}{k}\right)$$

$$\underline{B(c) - A(c)} \;=\; \frac{1}{c}(\frac{k-2}{k})^{k-2} + \frac{1}{c}\left((kF(c) - 1) \times (1 - F(c))^{k-2} - (\frac{k-2}{k})^{k-2}\right)$$

$$\text{when } c \;\leq\; F^{-1}\left(\frac{2}{k}\right)$$

Note also at $F^{-1}(\frac{2}{k})$, we have

$$\frac{1}{c}\left((kF(c) - 1) \times (1 - F(c)^{k-2})\right)$$

$$= \frac{1}{c}(\frac{k-2}{k})^{k-2} + \frac{1}{c}\left((kF(c) - 1) \times (1 - F(c))^{k-2} - (\frac{k-2}{k})^{k-2}\right)$$

Here is a graph of our upper and lower bounds- red is the actual values (for upper bound we have blue and for the lower bound we switch plots from yellow to green at $c = .7$):

Lemma 7 *For any distribution F with strictly lower positive support, we have:*

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1) A lower bound for $B(\underline{c}) - A(\underline{c})$ is $\frac{1}{3} \times \left(\frac{1}{\overline{c}} - \frac{1}{\underline{c}} \right) - \frac{1}{\underline{c}}$, where $\underline{c}$ is the lowest cost type and $\overline{c}$ the highest

2) A lower bound for the bid of the lowest cost type with equal first and second prizes (i.e., $\frac{1}{2} \left(B(\underline{c}) - A(\underline{c}) \right)$) is $\frac{1}{2\overline{c}}$

Proof: We can estimate this with the following lower bound:

$$\underline{B(\underline{c}) - A(\underline{c})} = \frac{1}{\overline{c}} (\frac{k-2}{k})^{k-2} + \frac{1}{\underline{c}} \left((k \cdot F(\underline{c}) - 1) \times (1 - F(\underline{c}))^{k-2} - (\frac{k-2}{k})^{k-2} \right)$$

$$= \frac{1}{\overline{c}} (\frac{k-2}{k})^{k-2} + \frac{1}{\underline{c}} \left((-1) \times (1)^{k-2} - (\frac{k-2}{k})^{k-2} \right) = \left(\frac{1}{\overline{c}} - \frac{1}{\underline{c}} \right) (\frac{k-2}{k})^{k-2} - \frac{1}{\underline{c}}$$

As can be readily verified, this increases in k to a finite value in the limit. Thus with $k = 3$ (we assume at least 3 participants), $\underline{B(\underline{c}) - A(\underline{c})}$ has a minimal lower bound of $\left(\frac{1}{\overline{c}} - \frac{1}{\underline{c}} \right) \cdot (\frac{3-2}{3})^{3-2} - \frac{1}{\underline{c}} = \frac{1}{3} \times \left(\frac{1}{\overline{c}} - \frac{1}{\underline{c}} \right) - \frac{1}{\underline{c}} \equiv \underline{B(c) - A(c)}$. That is, recall also $B(c) - A(c)$ is most negative at $B(\underline{c}) - A(\underline{c})$; thus, $\frac{1}{3} \times \left(\frac{1}{\overline{c}} - \frac{1}{\underline{c}} \right) - \frac{1}{\underline{c}}$ is the absolute lower bound of $B(c) - A(c)$. For example, with $c \sim U(.5, 1)$, we get $\underline{B(c) - A(c)} = -\frac{7}{3}$, regardless of the number of participants.

Now we find a lower bound for the lowest cost type's bid when we have equal first and second prizes (i.e., $\alpha = .5$) and linear cost functions. Previous analysis shows a lower bound for $\frac{1}{2}\underline{(A(c) + B(c))} = \frac{1}{2\overline{c}} \left((F(c) \times (k-2) + 1)(1 - F(c))^{k-2} \right)$. For the lowest cost type $\underline{c}$, this becomes:

$$\frac{1}{2\overline{c}} \left((F(\underline{c}) \times (k-2) + 1)(1 - F(\underline{c}))^{k-2} \right) = \frac{1}{2\overline{c}}$$

For example, with $c \sim U(.5, 1)$, we get the lower bound of the lowest cost type's bid to be $\frac{1}{2\overline{c}} = \frac{1}{2}$, for any number of contestants . $\square$

Thus, for our first portion of our Lemma, we have a found a simple lower bound on $B(\underline{c}) - A(\underline{c})$, the *maximal dis-incentivizing* effect of increasing the second prize for the lowest cost type $\underline{c}$, which is again also the most disincentivized type. In particular, if we shift ε of the prize mass from first to second prize, the lowest cost type will reduce effort by no more than $\left(\frac{1}{3} \times \left(\frac{1}{\overline{c}} - \frac{1}{\underline{c}} \right) - \frac{1}{\underline{c}} \right) \times \varepsilon$ (and all other low cost types $c < c^{**}$ will reduce effort by a lesser amount).

The second part of the Lemma is simply finding a lower bound on the bid of the lowest cost type, regardless of the number of participants. That is, we know the lowest cost type $\underline{c}$ will always bid at least $\frac{1}{2\overline{c}}$.

Proposition 6 *Assume a contest with $N \geq 3$ homogeneous players, up to 2 prizes, and binding effort capacity constraint κ. The symmetric equilibrium requires players play κ with probability p^* and bid from distribution $F(x)$ with probability $1 - p^*$ and support $x \in [0, \delta]$, where $\delta < \kappa$.*

Proof: We will find a symmetric equilibrium where bidders have an atom at κ, the maximal bid, and spend the balance of their bidding mass mixing over a continuous bidding support that is a strict subset of the entire possible bidding space $x \in [0, \kappa]$ with CDF $F(x)$. We will show bidding either κ or from $F(x)$ both produce an expected profit of zero.

Thus, we first want a player to be indifferent between bidding 0 and κ.

We denote $\begin{pmatrix} N-1 \\ j \end{pmatrix}$ to mean of the $N-1$ other players, j bid zero. V is the total prize mass for an arbitrary $K < N$ prizes (i.e., $V = \sum_k V_k$). Thus given the probability p of playing κ, we have the needed equality as :

$$\begin{pmatrix} N-1 \\ 0 \end{pmatrix} \cdot p^{N-1} \cdot \frac{V}{N} + \begin{pmatrix} N-1 \\ 1 \end{pmatrix} \cdot (1-p) \cdot p^{N-1} \cdot \frac{V}{N-1} + \ldots$$

$$+ \begin{pmatrix} N-1 \\ N-1 \end{pmatrix} (1-p)^{N-1} \cdot V_1 - \overline{c} \cdot \kappa = 0$$

The LHS are all the possibilities of prize sharing, where the first term is the expected prize if all $N-1$ players and player i bid the maximum. This final term is if all bid 0 save bidder i, who bids κ. This then means player i receives the first prize V_1.

We know the above expression has at least one zero since clearly the LHS is continuous and we have $p = 0 \Rightarrow V_{\max} - \overline{c} \cdot \kappa > 0$ (since $\kappa < \frac{V_{\max}}{\overline{c}}$) and with $p = 1 \Rightarrow \frac{V}{N} - \overline{c} \cdot \kappa < 0$ (since $\frac{V}{N \cdot \overline{c}} < \kappa$). Denote p^* as a solution to above.

We next find the mixing strategy drawn from $F(x)$. We write expected profit of mixing from $F(x)$ as:

$$E(\pi) = (1 - p^*)^{N-1} \cdot F(x)^{N-1} \times V_1$$

$$+(N-1) \cdot (1-p^*)^{N-1} \cdot F(x)^{N-2} \cdot (1-F(x)) \times V_2$$

$$+(N-1) \cdot (1-p^*)^{N-2} \cdot p^* \cdot F(x)^{N-2} \times V_2 - \bar{c} \cdot x = 0$$

We can first find the bidding support by setting $F(x) = 1$ and $F(x) = 0$. We also need to show $F(x)$ is increasing in x over the bidding interval since it is a CDF, which is readily verified via the implicit function theorem.

Hence, we have found a symmetric equilibrium where each player bids κ with probability p^* and then draws a bid from $F(x)$ with probability $1 - p^*$. $\square$

Proposition 9 *If the chosen effort error $\xi_t < \frac{M}{2}$ for all periods t, then our AEM algorithm identifies the true threshold type $\widetilde{c}_i$*

Proof: Assume our contestant's true threshold type is $\widetilde{c}$. By assumption, a contestant makes all effort choices with errors $\xi_t < \frac{M}{2}$. Consider now minimizing the sum of absolute estimation errors $\sum_{Ti} |\varepsilon_t|$, where we define $\varepsilon_t = \widehat{b}_t - b_t(\widetilde{c})$. We have $\widehat{b}_t$ as the actual effort at time t and $b_t(\widetilde{c})$ as the predicted effort given a posited $\widetilde{c}$ as the true representative type and no effort choice error. Define some arbitrary $\widehat{c} < \widetilde{c}$ from the type space. Define $A = A_1 \oplus A_2 \oplus A_3$, the disjoint partition, such that all $\varepsilon_t \in A_1$ when $c \geq \widetilde{c}$, $\varepsilon_t \in A_2$ when $\widetilde{c} > c \geq \widehat{c}$, and $\varepsilon_t \in A_3$ when $c < \widehat{c}$. Denote the number of elements in A_i as $N_i \in \mathbb{N}$ for $i \in \{1, 2, 3\}$. By assumption, if we chose posited threshold to be $\widetilde{c}$, we know for any such $\varepsilon_t \in A$, $|\varepsilon_t| < \frac{M}{2}$, since $\xi_t < \frac{M}{2}$. Thus the total error is $\sum_T |\varepsilon_t| < (N_1 + N_2 + N_3) \cdot \frac{M}{2}$.

Now consider changing our posited threshold point from $\widetilde{c}$ to $\widehat{c}$. Since our sets are disjoint, $\sum_T |\varepsilon_t|$ over the regions A_1 and A_3 are just as before. However, now $\sum_T |\varepsilon_t|$ over the region A_2 is greater. This is because whereas when assuming $\widetilde{c}$, we have $\sum_T |\varepsilon_t| < N_2 \times \frac{M}{2}$ summed over the region A_2, when we instead choose $\bar{c}$ we get $\sum_T |\varepsilon_t| \geq N_2 \times \frac{M}{2}$ over the region A_2. But this means now our total error $\sum_T |\varepsilon_t|$ is larger than before. Since $\widehat{c}$ is arbitrary and the argument is virtually identical for choosing $\widehat{c} > \widetilde{c}$, this completes the proof. $\square$

In addition to being robust against frequent but moderate errors, our algorithm is also robust against large but infrequent errors. To illustrate this point, consider

a player that exerts effort perfectly save one effort choice that should have been M but was instead entered as 0. As long as the deviation is more than one cost type step (i.e., recall there are discrete number of realized cost types) from the threshold $\widetilde{c}$,the estimation error minimization choice of $\widetilde{c}$ is still the true $\widetilde{c}$. This is because by choosing a $\widehat{c} \neq \widetilde{c}$, we now add to our total summed error $m \times |M|$, where m is the number of elements between $\widetilde{c}$ and $\widehat{c}$. Thus with more than one element (i.e., $m > 1$), we get $m \times |M| > |M|$. Hence, we will again chose the true $\widetilde{c}$ as our estimate. If $\widehat{c}$ is one step away from $\widetilde{c}$ (i.e., the next element below or above $\widehat{c}$),then we simply estimate $\widetilde{c}$ as one step from the true $\widetilde{c}$. In short, if our algorithm gets it wrong it is likely the subject is not acting like a threshold bidder.