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**Exchange Risk Management
And Corporate Capital Structure**

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Exchange Risk Management and Corporate Capital Structure

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Abstract

We analyze hedging policies for a corporation that generates a foreign currency cash flow that is not known with certainty. We first show that if investors care about real rather than nominal cash flows, there are no “free lunches” due to the Siegel Paradox. The profile of cash flows that accrue to the equityholders of the firm in various states is completely determined by the *contractual* value of the firm’s foreign currency liabilities alone: the optimality of any hedging policy thus does not depend on the seniority structure of the domestic and foreign currency debts and the foreign currency liability could always be chosen to be a forward contract to deliver the foreign currency. We obtain an intriguing result that the probability of bankruptcy for a firm that attempts to minimize this probability is *lower* when there is some uncertainty in the exchange rates than when there is no uncertainty in the exchange rates: the firm reduces the probability of bankruptcy by borrowing *more* than its financing needs through foreign currency borrowing alone and by investing the excess funds in domestic risk free securities. Our result provides a rationale for why firms may choose not to invoice their products in a currency that has no inflation variability; this may also explain why firms choose to write contracts in nominal rather than real terms.

1 Introduction

The volatility in foreign exchange rates increased dramatically after the breakdown of the Bretton Woods system of fixed exchange rates [See Smith, Smithson and Wilford (1990)]. The short term movements in exchange rates are often not accompanied by changes in all prices in the two countries. As a result, nominal changes in exchange rates do cause unexpected changes in relative prices. All firms whose value in real terms is affected by changes in relative prices thus face an exchange risk [Levi (1983)]. In this paper, we analyze hedging policies for a corporation that generates a foreign currency cash flow whose *real* value is affected by *nominal* changes in the exchange rate.

The exchange rate uncertainty associated with the value of a cash flow at a future date that is denominated in the foreign currency can be hedged perfectly in the forward market, provided that the foreign currency value of the cash flow is known with certainty. In this paper, we analyze hedging policies when the foreign currency cash flow is not known with certainty. Any hedging policy essentially attempts to alter the profile of cash flows that accrue to the equityholders of the firm in various states that are characterized by the size of the foreign currency cash flow and the level of the exchange rate. The profile of cash flows that accrue to various claimholders depends on the seniority as well as the currency denomination of various claims. Therefore, in general, a choice of a hedging policy also implies a choice of corporate capital structure.

We first show that, in perfect capital markets, the *real* expected value of the residual cash flow to the equityholders, even in the presence of exchange rate uncertainty, is identical for any choice of capital structure and therefore for any choice of hedging policy. Thus, unlike the results in Black (1990) which focuses on the *nominal* values of the cash flows, our results are not being driven by “free lunches” due to the Siegel Paradox [Siegel (1972)] in the presence of exchange rate uncertainty. We then show that the profile of cash flows that accrue to the equityholders of the firm in various states is completely determined by the *contractual* value of the firm’s foreign currency liabilities alone. The implication of this result is that the optimality of any hedging policy does not depend on the seniority structure of the domestic and foreign currency debts but depends only on the contractual value of the foreign currency liabilities. We show that this also implies that the foreign currency liability could always be chosen to be a forward contract to deliver the foreign currency

even though the foreign currency cash flow for the firm is uncertain.

Of course, in perfect capital markets, corporations need not hedge exchange risk at all since investors can do it on their own [Aliber (1978)]. Market imperfections, such as taxes, agency problems and dead-weight costs associated with financial distress, however, may provide incentives for corporations to hedge the exchange risk [Dufey and Srinivasulu (1983), Stulz (1984), Shapiro and Titman (1985) and Smith and Stulz (1985)]. Because there are large dead-weight costs associated with bankruptcy, the firms may choose hedging policies that minimize the probability of bankruptcy [Hodder and Senbet (1990)].

We obtain an intriguing result that the probability of bankruptcy for the firm with an uncertain foreign currency cash flow, if it attempts to minimize this probability, is *lower* when there is some uncertainty in the exchange rate than when there is no uncertainty in the exchange rate. We show that when there is some uncertainty in the exchange rate, the firm would borrow *more* than its debt financing needs through foreign currency borrowing alone and invest the excess funds in domestic risk free securities. The intuition for this result is as follows. Suppose that the firm borrows nothing domestically but with foreign currency loans borrows an amount that is exactly equal to its financing needs. Then depending on the realized value of the foreign currency cash flow, the firm will either have a deficit or a surplus, the domestic currency value of which depends on the exchange rate. If the exchange rate (expressed as the domestic currency price of the foreign currency) is high, the domestic currency value of the deficit or the surplus is high: if the exchange rate is low the deficit or the surplus is low. Now suppose the firm takes a gamble with zero expected value that is contingent on the exchange rate such that its payoff is positive if the exchange rate is low and is negative if the exchange rate is high. With this gamble the deficit in the high deficit states gets even larger and the surplus in the low surplus states gets larger: but that does not affect the probability of bankruptcy. However, the deficit in the low deficit states gets smaller which reduces the probability of bankruptcy. On the other hand, the surplus in the high surplus states gets smaller which increases the probability of bankruptcy. But since a relatively small amount of domestic funds are required to pull the firm out of bankruptcy from low deficit states but a relatively large amount of domestic funds are required to drag the firm into bankruptcy from high surplus states, a gamble with symmetric payoffs would reduce the overall probability of bankruptcy. By borrowing in foreign currency an amount in excess of its financing needs, investing the proceeds in domestic risk free securities, the

firm essentially takes such a gamble.

Clearly then, if there were no variation in the exchange rates, no such gamble is possible and the probability of bankruptcy would be higher. This result suggests that firms attempting to minimize the probability of bankruptcy will choose not to invoice their products in currencies of countries that have no exchange rate uncertainty. The intuition, of course, is applicable to a more general context of inflation uncertainty. Our results suggest that firms attempting to minimize the probability of bankruptcy may choose to price their products in *nominal* rather than *real* terms, borrow more than the financing needs through *nominal* debt contracts and invest the excess funds in *real* assets.

2 The Model

2.1 Exchange Risk

We analyze a very simple one period model in which a firm based in the U.S. has a single project that generates a single cash flow $x^f \geq 0$, denominated in a foreign currency denoted FX, at the end of the period. For simplicity, let us normalize the current exchange rate to be $\$1/\text{FX}$. Let $s > 0$ denote the exchange rate, expressed in dollars for each unit of FX, at the end of the period. Let $f(x^f)$ and $g(s)$ denote the density functions of x^f and s respectively.

Let us first analyze an example, described in Black (1990), that attempts to show that in the presence of exchange rate uncertainty, there exist profit opportunities from pure speculation. Let us consider a case in which the exchange rate at the end of the period, s , could either be $\$2/\text{FX}$ with probability one half or $\$0.50/\text{FX}$ with probability one half. Consider two assets, 1 and 2. Asset 1 pays $\$1$ for sure at the end of the period and asset 2 pays FX 1 for sure at the end of the period. If everything is symmetric asset 1 and asset 2 should have a relative price of one-for-one. A person holding asset 1 gets a sure payoff of $\$1$ at the end of the period. But if he trades asset 1 for asset 2, his payoff is FX 1 which is equivalent to $\$2$ with probability one half and $\$0.50$ with probability one half, the expected value of which is $\$1.25$ which is greater than the payoff from holding asset 1! This is known as the Siegel Paradox.

On closer examination, however, the Siegel Paradox disappears if the exchange rate at

the end of the period, s , is assumed to be determined by the Purchasing Power Parity relationship between the general price levels in the two countries and that investors care about the *real* rather than the *nominal* value of the cash flows. In the example that we considered above, the exchange rate at the end of the period, s , could be either $\$2/\text{FX}$ with probability one half because the price level in the U.S. doubles and the price level in the foreign country stays the same or be $\$0.50/\text{FX}$ with probability one half because the price level in the U.S. stays the same and the price level in the foreign country doubles. Now, a person holding asset 1, receives $\$1$ for sure at the end of the period, the *real* value of which, in current dollars, is either 0.50 with probability one half if the price level in the U.S. doubles or is equal to 1 with probability one half if the price level in the U.S. stays the same; the *real* expected value in current dollars is thus equal to 0.75. If he trades asset 1 for asset 2, he receives $\text{FX } 1$ for sure which is equivalent to $\$2$ with probability one half when the price level in the U.S. has doubled or is equivalent to $\$0.50$ with probability one half when the price level in the U.S. has stayed the same; the *real* expected value in current dollars is thus equal to 0.75 in this case also. Therefore, there is no expected benefit from trading asset 1 for asset 2.¹

In order to ensure, that our results are not being driven by “free lunches” due the Siegel Paradox we make the following two assumptions.

Assumption 1 *All agents care only about the real, rather than the nominal, value of the cash flows.*

Assumption 2 *The exchange rate at the end of the period, s , is determined by the Purchasing Power Parity relationship between the general price levels in the two countries.*²

Let us also normalize the expected *real* rate of return to be zero in both countries.³ For

¹The fact that reasoning in real terms disposes of the Siegel Paradox has been discussed previously in the literature. For instance, see Adler and Dumas (1983), footnote 60, page 955.

²One might argue that there seems to be ample evidence the exchange rates seem to violate the Purchasing Power Parity relationship between the general price levels in the two countries [for instance, see Mussa (1979)]. But then, one has to model these violations in an equilibrium framework. In a simple model with perfect capital markets, such as the one we consider in this paper [and also the one in Black (1990)], exchange rate movements that violate the Purchasing Power Parity relationship would be inconsistent with equilibrium and may of course imply presence of expected profit opportunities. Our goal in this paper is not to develop an equilibrium model that allows for violations of the Purchasing Power Parity but to illustrate the main insights of the paper in a simple but consistent framework. Our analysis abstracts away from issues arising from linkages between currency risk and relative price risk that are discussed in Shapiro (1984).

³Equilibrium with risk neutral agents (see Assumption 4) in financial markets ensures that expected real rates are equal across countries.

simplicity, we assume that the general price level in the U.S. stays the same. This implies that the *nominal* riskless rate of return in the U.S. is also zero. Since we have assumed that the price level in the U.S. does not change, variation in s is caused only by the variation in the general price level in the foreign country. This implies that the real value of any cash flow denominated in FX can be obtained by evaluating the corresponding value of the cash flow in dollars, *i.e.*, by multiplying the FX cash flow by the prevailing $\$/\text{FX}$ exchange rate s . Of course, by construction, the nominal and real values of cash flows in dollars are identical.⁴ Let us also normalize the expected value of s to be equal to one. This implies that the nominal rate of return in the foreign country is also zero.

Definition 1 *A firm is said to face an exchange risk if nominal changes in exchange rates cause changes in real cash flows or real value of the firm.*

A change in the exchange rate s , in our model, is caused only because of changes in the general price level in the foreign country to maintain the Purchasing Power Parity between the general price levels in the two countries. However, the short term movements in exchange rates are often not accompanied by changes in all prices in the two countries. As a result, nominal changes in exchange rates do cause unexpected changes in relative prices which may subject firms to exchange risks. If the FX cash flow x^f changes one to one in response to the change in general price level, then the real value of the cash flow is unchanged by the change in the exchange rate and the firm does not face any exchange risk [Cornell (1980)]. To ensure that the firm does face some exchange risk, we must therefore assume that the FX cash flow x^f does not change one to one with the general price level. We formalize this, in a simple way, by making the following assumption.

Assumption 3 *The firm's FX cash flow x^f and the exchange rate s at the end of the period, are stochastically independent.*

The real value of the FX cash flow x^f is simply equal to sx^f . Intuitively, it is clear that the above assumption implies that the firm faces an exchange risk since a change in the nominal exchange rate s is not accompanied by any corresponding change in x^f and

⁴This normalization would allow us to interpret our results more generally to a case of inflation uncertainty.

therefore the real value of the FX cash flow sx^f is affected by a change in s .⁵ Of course, this assumption is stronger than what is needed to ensure that the firm faces an exchange risk. All that is needed is that a change in s is not accompanied by an offsetting change in x^f such that sx^f is unaltered because then the firm does not face any exchange risk.⁶ We make the stronger assumption only for simplicity.

2.2 Hedging Policies

Let c denote the part of the financing need which needs to be financed by issuing debt; the rest is financed by equity. Let y and y^f denote the proceeds from a dollar loan and an FX loan respectively. Recall that the current exchange rate is normalized to \$1/FX. The proceeds from the two loans must be at least as large as c , *i.e.*, the following financing constraint must be satisfied.

$$y + y^f \geq c. \quad (1)$$

We assume that any excess funds $y + y^f - c$ are invested in risk-free securities in the U.S. (which, we earlier assumed, pay an interest rate of zero).

Let z denote the face value (in \$) of the dollar loan and let z^f denote the face value (in FX) of the foreign currency loan to be paid at the end of the period.

The real value of the cash flow (which in our case is simply the value of the cash flow in dollars) at the end of the period, then is $sx^f + y + y^f - c$. The values of z and z^f , in equilibrium, depend on the seniority structure of the loans.

Suppose first that the FX loan is senior to the dollar loan. If the cash flow at the end of the period $sx^f + y + y^f - c$ is sufficient, the lenders of the FX loan receive the entire face value of the loan whose value in dollars is sz^f . Otherwise, the firm goes bankrupt and the FX lenders receive the entire cash flow $sx^f + y + y^f - c$. The cash flow that accrues to the FX lenders can therefore be written as $\text{Min} \{ sx^f + y + y^f - c, sz^f \}$.

We now make another simplifying assumption that allows us to price these loans.

Assumption 4 *All loans are priced competitively in a market with risk neutral agents.*

⁵It is easy to check that this assumption implies that the measure of economic exposure given by $\text{Cov}(sx^f, s)/\text{Var}(s)$ [See Hodder (1982) and Adler and Dumas (1984)] is positive.

⁶The measure of economic exposure given by $\text{Cov}(sx^f, s)/\text{Var}(s)$ is equal to zero in that case.

The value of the FX loan y^f , then, is simply the expected value of the cash flow that accrues to the FX lenders, *i.e.*,

$$y^f = E \left[\text{Min} \left\{ sx^f + y + y^f - c, sz^f \right\} \right]. \quad (2)$$

The residual cash flow after paying off the FX lenders is $\text{Max} \left(0, sx^f + y + y^f - c - sz^f \right)$. If this cash flow is sufficient, the lenders of the junior dollar loan receive the face value of the dollar loan, z , otherwise the firm goes bankrupt and the lenders of the dollar loan receive the entire residual cash flow. The cash flow that accrues to the lenders of the dollar loan therefore can be written as $\text{Min} \left\{ \text{Max} \left(0, sx^f + y + y^f - c - sz^f \right), z \right\}$ and the value of the dollar loan y is given by

$$y = E \left[\text{Min} \left\{ \text{Max} \left(0, sx^f + y + y^f - c - sz^f \right), z \right\} \right]. \quad (3)$$

Similarly if the dollar loan is senior to the FX loan, the senior dollar loan and the junior FX loan then are priced as follows.

$$y = E \left[\text{Min} \left\{ sx^f + y + y^f - c, z \right\} \right]. \quad (4)$$

$$y^f = E \left[\text{Min} \left\{ \text{Max} \left(0, sx^f + y + y^f - c - z \right), sz^f \right\} \right]. \quad (5)$$

Regardless of the seniority structure of the two loans, the dollar value of the residual cash flow to the equityholders, in either case is

$$\text{Max} \left\{ 0, sx^f + (y + y^f - c) - (sz^f + z) \right\}. \quad (6)$$

Given a distribution for the exchange rate s and the FX cash flow x^f , the distribution of the residual cash flow to the equityholders, in general, depends on the levels of the two debts y and y^f and the corresponding face values z and z^f of these debts which are determined by the seniority structure of the two debts. Since, any hedging policy essentially attempts to alter this profile of cash flows that accrue to the equityholders of the firm in various states that are characterized by different realizations of s and x^f , we define a hedging policy as follows.

Definition 2 *The selection of the levels of debt in each currency y and y^f together with the seniority structure of the debts constitutes a hedging policy.*

Notice that all claims are priced given a hedging policy in place (or equivalently given a capital structure). We are thus ruling out any transfers of wealth among the various debt and equity holders of the firm.

Definition 3 *Two hedging policies are equivalent if and only if for any given realizations of s and x^f , the dollar value of the residual cash flow to the equityholders is identical for the two policies.*

Adding (2) and (3), we get

$$y + y^f = E \left[\text{Min} \left\{ sx^f + (y + y^f - c), sz^f + z \right\} \right]. \quad (7)$$

It is easy to check that adding (4) and (5) also results in an identical equation. Intuitively, the debtholders receive either the entire contractual value of their loans $sz^f + z$ or the entire cash flow of the firm $sx^f + (y + y^f - c)$ if the firm cannot meet its contractual obligations fully. The expected value of the cash flow that accrues to the debtholders must equal the market value of the loans $y + y^f$ regardless of the seniority of the two loans.

The above equation (7) can be rewritten as follows.

$$E \left[\text{Min} \left\{ sx^f - c, sz^f - (y + y^f - z) \right\} \right] = 0. \quad (8)$$

Lemma 1 *The expected value of the residual cash flow to the equityholders is identical for any choice of a hedging policy (or equivalently, for any choice of capital structure).*

Proof: The expected value of the residual cash flow to the equityholders can be expressed as

$$\begin{aligned} & E \left[\text{Max} \left\{ 0, sx^f + (y + y^f - c) - (sz^f + z) \right\} \right] \\ &= E(sx^f - c) + E \left[\text{Max} \left\{ -(sx^f - c), -(sz^f - (y + y^f - z)) \right\} \right] \\ &= E(sx^f - c) - E \left[\text{Min} \left\{ sx^f - c, sz^f - (y + y^f - z) \right\} \right] \\ &= E(sx^f) - c \quad [\text{From (8)}] \end{aligned}$$

which is simply the present value of the project which is independent of any hedging policy. ■

This rather obvious result is reassuring, nevertheless, because it confirms our earlier assertion that results are not being driven by “free lunches” due to the Siegel Paradox. Our results, thus, are different from the results in Black (1990).

We now state our first important result.

Lemma 2 *All hedging policies with identical face values z^f of the FX loan are equivalent.*

Proof: Notice that from (8), choosing a value for z^f sets a corresponding value for $y + y^f - z$ which [from (6)] completely determines the residual cash flow to the equityholders.■

This result is quite obvious for the case when the FX loan is junior to the dollar loan. In this case, given a value of z^f , $y - z$ as well as y^f are uniquely determined. To see this, notice that from financing constraint (1) we know that $y + y^f - c \geq 0$. Suppose now that $y + y^f - c = 0$. Any increase in y increases the surplus cash flow $y + y^f - c$ by exactly that amount since all excess funds are invested in domestic risk free securities. Since the dollar loan is senior, the face value of the dollar loan z also increases by exactly the same amount since the incremental dollar loan is riskless. So $y - z$ remains unchanged and therefore the residual cash flow after paying the dollar loan is also unchanged. Consequently, the value of the junior FX loan y^f is also unchanged. Clearly, given a value of z^f , the residual cash flow to the equityholders is uniquely determined.

What is surprising is that this result also holds when the FX loan is senior. Consider the case, when $y + y^f = c$. Given a value of z^f , as the junior dollar loan y is increased, since the increased surplus is used to payoff the senior FX liability first, it increases the value of the the FX loan y^f . But it also increases the face value of the junior dollar loan z . What Lemma 2 guarantees is that the combined increase in the value of the two loans $y + y^f$ is exactly offset by an equal increase in the face value of the dollar loan z such that $y + y^f - z$ is unaltered. Consequently, the residual cash flow to the equityholders is unchanged.

2.3 Optimal Hedging Policies

The result in Lemma 2 is analytically significant because it reduces the problem of determining an optimal hedging policy to simply choosing the optimal level of a single variable, z^f . The economic significance of this result is expressed in the following two propositions.

Proposition 1 *The optimality of any hedging policy does not depend on the seniority structure of the domestic and the foreign currency debt.*

Proof: Let z^f denote the face value of the FX loan in an optimal hedging policy. The seniority structure only determines the value of the proceeds y^f that are raised from the FX loan with the face value z^f . Therefore, the only thing we have to make sure is that the hedging policy remains feasible for any seniority structure of debts. In other words, the financing constraint (1) must be satisfied for any seniority structure. If for some seniority structure the financing constraint is violated, funds from the dollar loan y can be increased sufficiently to satisfy the financing constraint. From Lemma 2, we know that the optimality remains unaltered since z^f stays the same.■

Proposition 2 *An optimal hedging policy in which the foreign currency liability is a forward contract to deliver the foreign currency is always feasible.*

Proof: Let z^f denote the face value of the FX loan in an optimal hedging policy. A forward contract to deliver z^f units of FX is equivalent to carrying out a riskless FX loan of the same amount. We can ensure that the FX loan is deemed riskless by the market by making the FX loan senior and by increasing the proceeds from the dollar loan y sufficiently. This is possible since the financing constraint (1) only places a lower bound on y and from Lemma 2 we know that the policy continues to remain optimal as long as z^f remains unchanged. Since the excess cash $y + y^f - c$ is invested in riskfree U.S. securities, choosing a sufficiently high value of y would ensure that the senior FX loan is deemed riskless. This is equivalent to posting a performance bond for a forward contract.■

If there were no certainty about x^f then it is, of course, obvious that the optimal hedging policy can be carried out with a forward contract to deliver $z^f = x^f$ units of FX. The result in Proposition 2 guarantees that an optimal hedging policy can still be carried out with a forward contract even if the FX cash flow x^f is uncertain.

Notice that, so far, we have not specified the objective function that the firm optimizes.

2.3.1 Minimizing the Probability of Bankruptcy

If there are large dead-weight costs associated with bankruptcy then an optimal hedging policy may minimize the probability of bankruptcy. We say that the firm is bankrupt if, at the end of the period, it is not able to meet its contractual debt obligations, *i.e.*, if

$$sx^f + (y + y^f - c) < sz^f + z.$$

We first provide a simple numerical example that illustrates the formal results in this section.

Suppose

$$x^f \in \text{FX}\{1, 2, 3, 4\} \quad \text{with equal probability,}$$

$$s \in \$ \left\{ \frac{1}{2}, 1\frac{1}{2} \right\} / \text{FX} \quad \text{with equal probability,}$$

so there are eight possible states, each with a probability equal to one-eighths. Let

$$c = \$2.$$

Suppose that the firm borrows an amount exactly equal to its financing needs through foreign currency borrowing and borrows nothing in the domestic currency. In other words,

$$y = 0, y^f = c = \$2.$$

It is easy to check from Table 1 that the face value z^f of $\text{FX}2\frac{1}{2}$ is consistent with the debt pricing condition. In this case, and the firm will be bankrupt in four out of eight states, so that the probability of bankruptcy is equal to one-half.

Notice that in this case the probability of bankruptcy does not depend on the variation in exchange rate. The states for which $x^f \in \text{FX}\{1, 2\}$, the firm will be bankrupt since $x^f < z^f = \text{FX}2\frac{1}{2}$. The states for which $x^f \in \text{FX}\{3, 4\}$, the firm will be solvent since $x^f > z^f = \text{FX}2\frac{1}{2}$. So the probability of bankruptcy is one-half regardless of any variation in s .

Table 1

State	1	2	3	4	5	6	7	8
x^f (in FX)	1	1	2	2	3	3	4	4
s (in \$/FX)	$\frac{1}{2}$	$1\frac{1}{2}$	$\frac{1}{2}$	$1\frac{1}{2}$	$\frac{1}{2}$	$1\frac{1}{2}$	$\frac{1}{2}$	$1\frac{1}{2}$
sx^f	$\frac{1}{2}$	$1\frac{1}{2}$	1	3	$1\frac{1}{2}$	$4\frac{1}{2}$	2	6
y^f	2	2	2	2	2	2	2	2
$-c$	-2	-2	-2	-2	-2	-2	-2	-2
$sx^f + (y^f - c)$	$\frac{1}{2}$	$1\frac{1}{2}$	1	3	$1\frac{1}{2}$	$4\frac{1}{2}$	2	6
sz^f	$1\frac{1}{4}$	$3\frac{3}{4}$	$1\frac{1}{4}$	$3\frac{3}{4}$	$1\frac{1}{4}$	$3\frac{3}{4}$	$1\frac{1}{4}$	$3\frac{3}{4}$
$\text{Min}[sx^f + (y^f - c), sz^f]$	$\frac{1}{2}$	$1\frac{1}{2}$	1	3	$1\frac{1}{4}$	$3\frac{3}{4}$	$1\frac{1}{4}$	$3\frac{3}{4}$
$sx^f + (y^f - c) - sz^f$	$-\frac{3}{4}$	$-2\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$2\frac{1}{4}$
$\text{Max}[0, sx^f + (y^f - c) - sz^f]$	0	0	0	0	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$2\frac{1}{4}$

Now let us suppose that the firm increases its foreign currency borrowing y^f by $\Delta y^f = \text{FX}\frac{1}{2} = \$\frac{1}{2}$ to say $y^{f'}$. If there is no variation in exchange rate, it is easy to see that the probability of bankruptcy does not change because of any further borrowing since all additional borrowing is invested in risk free assets and would revert back to the debtholders at the end of the period. But if the exchange rates vary as postulated, the following table (Table 2) shows that the face value z^f increases by $\Delta z^f = \text{FX}\frac{1}{2}$ to say $z^{f'}$. It is easy to check from Table 2 that the face value $\Delta z^f = \text{FX}\frac{1}{2}$ for a corresponding increase in the FX loan $\Delta y^f = \text{FX}\frac{1}{2} = \$\frac{1}{2}$ is indeed consistent with the debt pricing condition. The intuition for this is that the additional loan amount of $\$ \frac{1}{2}$ is invested in domestic risk free assets. Since the firm is bankrupt in the first four states, the entire amount of the additional loan of $\$ \frac{1}{2}$ goes to the debtholders. In the remaining four states the cash flow is sufficient to pay the debtholders the increased contractual value of the loan completely [$\$ \frac{1}{4}$ in the low s states and $\$ \frac{3}{4}$ in the high s states]. So the debtholders are either able to recoup the entire amount of the additional loan or are paid fully the contractual amount, the increase in the face value z^f is equal to the increase in the value y^f of the FX loan.

Table 2

State	1	2	3	4	5	6	7	8
$sx^f + (y^f - c)$	$\frac{1}{2}$	$1\frac{1}{2}$	1	3	$1\frac{1}{2}$	$4\frac{1}{2}$	2	6
Δy^f	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$sx^f + (y^{f'} - c)$	1	2	$1\frac{1}{2}$	$3\frac{1}{2}$	2	5	$2\frac{1}{2}$	$6\frac{1}{2}$
sz^f	$1\frac{1}{4}$	$3\frac{3}{4}$	$1\frac{1}{4}$	$3\frac{3}{4}$	$1\frac{1}{4}$	$3\frac{3}{4}$	$1\frac{1}{4}$	$3\frac{3}{4}$
$s\Delta z^f$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$
$sz^{f'}$	$1\frac{1}{2}$	$4\frac{1}{2}$	$1\frac{1}{2}$	$4\frac{1}{2}$	$1\frac{1}{2}$	$4\frac{1}{2}$	$1\frac{1}{2}$	$4\frac{1}{2}$
$\text{Min}[sx^f + (y^{f'} - c), sz^{f'}]$	1	2	$1\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{1}{2}$	$4\frac{1}{2}$	$1\frac{1}{2}$	$4\frac{1}{2}$
$sx^f + (y^f - c) - sz^f$	$-\frac{3}{4}$	$-2\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$2\frac{1}{4}$
$\Delta y^f - s\Delta z^f$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$
$sx^f + (y^{f'} - c) - sz^{f'}$	$-\frac{1}{2}$	$-2\frac{1}{2}$	0	-1	$\frac{1}{2}$	$\frac{1}{2}$	1	2
$\text{Max}[0, sx^f + (y^{f'} - c) - sz^{f'}]$	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1	2

The important thing to notice is that by borrowing in foreign currency more than its financing need c , the firm is able to reduce the probability of bankruptcy to three-eighth since it is now able to meet its debt obligations fully in five out of eight states.

By borrowing an additional $\frac{1}{2}$, the firm has taken an incremental gamble that takes $\frac{1}{4}$ away from states in which s , and therefore the surplus or the deficit, $|s(x^f - z^f)|$, is high. The gamble pays off $\frac{1}{4}$ in states in which s , and therefore the surplus or the deficit, $|s(x^f - z^f)|$, is low. An increase of $\frac{1}{4}$ is sufficient to pull the firm out of bankruptcy from state 3 (and reduce the deficit in state 1) but a decrease of $\frac{1}{4}$ is not enough to drag the firm into bankruptcy in state 6 or state 8. Notice that the deficit in states 2 and 4 becomes even larger but that does not alter the probability of bankruptcy since the firm was bankrupt in those states anyway. Also the surplus in states 5 and 7 becomes larger which does not alter the probability of bankruptcy either.

We now show these results formally.

The condition for bankruptcy can be rewritten as follows.

$$q \equiv sx^f + (y + y^f - c) - (sz^f + z) < 0.$$

Thus q denotes the difference between the end of period dollar cash flow and the dollar value of the contractual payment to the debtholders. The dollar value of the residual cash flow to the equityholders then is $\text{Max}\{0, q\}$.

If the firm attempts to minimize the probability of bankruptcy then it will choose z^f that minimizes the probability π that $q < 0$. By a simple change of variables, the density function of q can be shown to be

$$h(q) = \int_0^\infty f\left(z^f - \frac{(y + y^f - c) - z - q}{s}\right) \frac{1}{s} g(s) ds. \quad (9)$$

Then,

$$\pi \equiv \text{Prob}[q < 0] = \int_{-\infty}^0 h(q) dq. \quad (10)$$

Lemma 3 *If the firm has no domestic debt and its foreign currency loan y^f is exactly equal to its financing needs c , then a small incremental increase in the foreign currency loan amount increases the face value of the foreign currency loan by the same amount. Formally, if $y = 0$ then $\frac{dy^f}{dz^f} = 1$ for that value of z^f for which $y^f = c$.*

Proof: See the Appendix.

The intuition for this result is as follows. With $y = 0$ and $y^f = c$, the firm's cash flow at the end of the period is sx^f . The contractual value of the debt payment is sz^f . So, the firm is bankrupt if and only if $x^f < z^f$. For all states for which $x^f > z^f$ the original debt was being paid off completely and there was some excess cash left for equityholders. For these states, a slight increase in the contractual payment could still be met completely. For all states $x^f < z^f$ the firm was originally bankrupt, so the slight amount of excess funds that the firm now has in addition to c would go to the debtholders completely for these states. Since the debtholders are either able to recoup the entire amount of the additional loan or are paid fully the contractual amount, the marginal increase in the face value z^f is equal to the marginal increase in the value y^f of the FX loan.

We use the result in Lemma 3 to now prove the following result.

Lemma 4 *If the firm has no domestic debt and its foreign currency loan y^f is exactly equal to its financing needs c , then a small incremental increase in the foreign currency loan amount decreases the probability of bankruptcy for the firm. Formally, if $y = 0$ then $\frac{d\pi}{dz^f} < 0$ for that value of z^f for which $y^f = c$.*

Proof: See the Appendix.

The intuition for this result is as follows. The firm chooses a value of the FX debt y^f that is slightly larger than c , and invests the excess funds, say Δy^f , in U.S. riskfree securities. From Lemma 3, the increase in the face value the FX loan Δz^f is equal to Δy^f units of FX, the dollar value of which is equal to $s\Delta y^f$. So, essentially the firm takes an incremental gamble whose payoff, in dollars, is equal to $\Delta y^f(1 - s)$. The payoff from this gamble is positive if $s < 1$, negative if $s > 1$ and the expected value of this gamble is zero. When s is high (> 1), the value of the surplus or the value of the deficit, $|s(x^f - z^f)|$, is high. When s is low (< 1) the value of the surplus or the value of the deficit, $|s(x^f - z^f)|$, is low. So, for any given x^f , the gamble takes funds away from high surplus (deficit) states and puts them in low surplus (deficit) states. As a result the deficit in the high deficit states gets even larger but that does not change the probability of bankruptcy. Also, the surplus in the low surplus states gets larger and that does not alter the probability of bankruptcy either. But the deficit in the low deficit states becomes smaller which reduces the probability of bankruptcy. However, the surplus in the high surplus states becomes smaller which increases the probability of bankruptcy. Since a relatively small amount of domestic funds need to be infused in the low deficit states to pull the firm out of bankruptcy but a relatively large amount of funds need to be extracted from high surplus states to drag the firm into bankruptcy, the overall effect of the gamble is to reduce the probability of bankruptcy.⁷ (See Figure 1)

⁷Another way to think about the intuition behind this result is as follows. The payoff from the incremental gamble expressed not in real (dollar) terms but in terms of units of foreign currency FX is $\Delta y^f(\frac{1}{s} - 1)$. The expected value of this payoff, in terms of FX, is positive [from Jensen's Inequality]. The condition for bankruptcy if the firm borrows just enough to meet its financing need is whether or not $x^f < z^f$. So, the deficit, if the firm is bankrupt, or the surplus if the firm is solvent, expressed in units of FX is equal to $(z^f - x^f)$. Since this deficit (or surplus) in units of FX is uncorrelated with the payoff (also in units of FX) from the incremental gamble $\Delta y^f(\frac{1}{s} - 1)$, taking this incremental gamble reduces the probability of bankruptcy because the expected payoff of this gamble (in units of FX) is positive. I am grateful to Sheridan Titman for pointing this out to me.

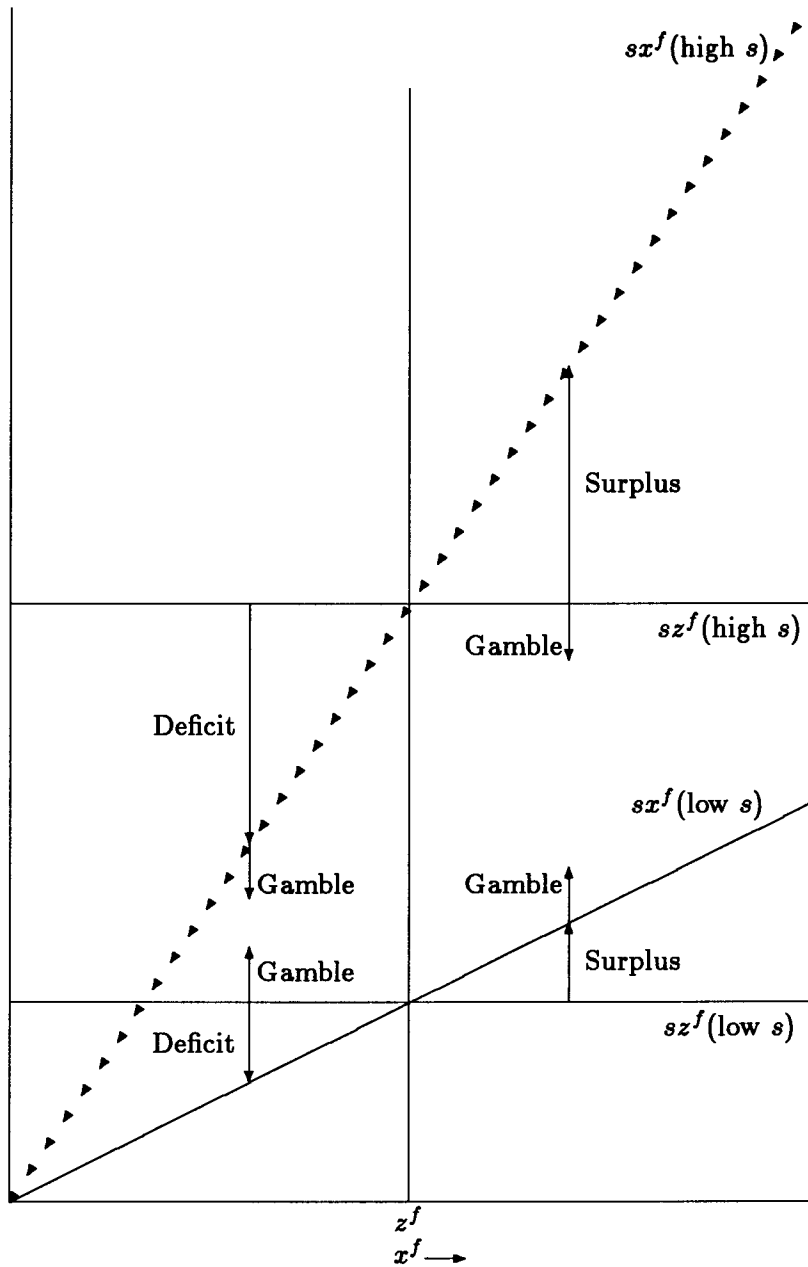


Figure 1

What we have shown in Lemma 5 is that if the firm does not borrow domestically at all then its foreign currency borrowing *strictly* exceeds its financing needs c , if the firm attempts to minimize its probability of bankruptcy. The following figure provides a numerical illustration of how the probability of bankruptcy π varies as the size of the foreign currency borrowing is increased.

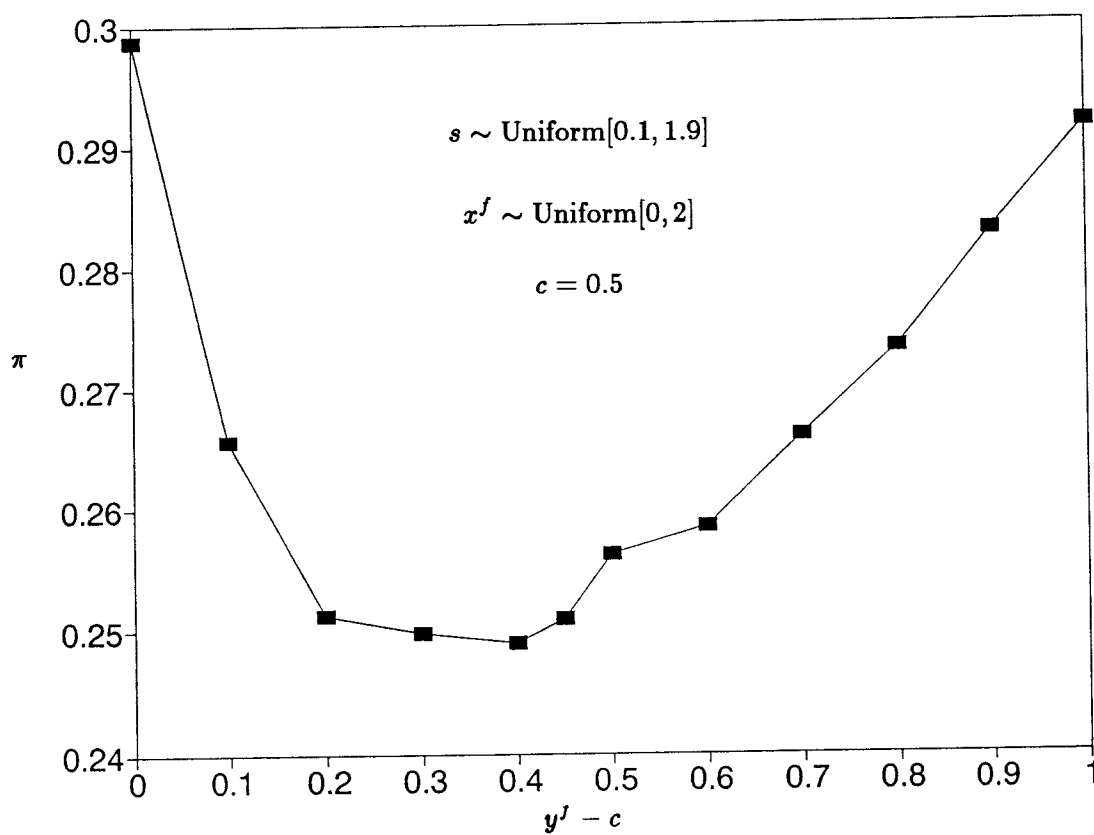


Figure 2

The following Proposition demonstrates that even if the firm borrows domestically also, its foreign currency borrowing alone would still exceed its financing need c .

Proposition 3 *If the firm attempts to minimize the probability of bankruptcy, it would borrow more than its financing need c through foreign currency borrowing alone. Formally, y^{f*} is always strictly greater than c for $c > 0$.*

Proof: In Lemma 4, we showed that if $y = 0$ then π is decreasing in z^f at $y^f = c$ which implies that the optimal level of the FX loan $y^{f*} > c$. Let z^{f*} denote the value of z^f for $y^f = y^{f*}$ and $y = 0$. From Lemma 2 we know that any other hedging policy with the face value $z^f = z^{f*}$ would also be optimal. Now consider if the firm also borrows domestically, *i.e.*, let $y > 0$. There are two possibilities.

1. The dollar loan is senior. Since $y^{f*} > c$ with $y = 0$, any additional funds raised with the dollar loan would stay in risk free securities, so that the dollar loan could be paid off in full. Clearly for a given value of z^{f*} the value of the FX loan would stay the same.
2. The dollar loan is junior. Any additional funds raised with the dollar loan could be used to pay off the FX loan first which reduces the riskiness of the FX loan. For a given face value $z^f = z^{f*}$ of the FX loan the corresponding value of y^f would increase with an increase in y . Since the value of the FX loan y^f was strictly greater than c even when $y = 0$, the value of y^f for $y > 0$ would increase even further. ■

Since for all optimal hedging policies $y^{f*} > c$, without loss of generality, we can now restrict our attention to hedging policies for which $y = 0$.

Lemma 5 *If $y^f = c$ then the probability of bankruptcy π does not depend on the variation in the end of period exchange rate s .*

Proof: For $y = 0$ and $y^f = c$, we have $q = sx^f - sz^f$. Therefore,

$$\pi \equiv \text{Prob}[q < 0] = \text{Prob}[sx^f < sz^f] = \text{Prob}[x^f < z^f].$$

Clearly, the probability that $x^f < z^f$ does not depend on the variation in s . ■

The intuition for this result is straightforward. If the firm borrows just enough to meet the costs at the beginning of the period, the dollar value of the cash flow at the end of the period is sz^f . The dollar value of the contractual payment is sz^f . Any variation in s affects the cash flow sz^f and the contractual payment sz^f by the same factor so that the condition for bankruptcy ($x^f < z^f$) stays unaltered regardless of any variation in s .

We now prove the main result of the paper.

Proposition 4 *The probability of bankruptcy for the firm if it attempts to minimize this probability is lower when there is some uncertainty in the end of period exchange rate s than when there is no uncertainty in s .*

Proof: We have seen in Lemma 4 and Proposition 3 that the probability of bankruptcy when there is some variation in s is smaller at the optimal value z^{f*} which implies that $y^{f*} > c$ than for a case when $y^f = c$. But from Lemma 5 we know that $y^f = c$ gives us the probability of bankruptcy when there is no variation in s . ■

The result in Proposition 4 appears counterintuitive at first. But as we have seen in Lemma 4, the variation in the exchange rate allows the firm to write debt contracts in such a way that it takes a gamble whose payoffs are symmetric and contingent on the value of the exchange rate in such a way that it transfers funds from high deficit states into low deficit states and from high surplus states into low surplus states. But, since a relatively small amount of domestic funds need to be infused in the low deficit states to pull the firm out of bankruptcy and a relatively large amount of funds need to be extracted from high surplus states to drag the firm into bankruptcy, the overall effect of the gamble is to reduce the probability of bankruptcy.

The result in Proposition 4 also suggests an intriguing implication. Suppose the firm had a choice of the currency in which to invoice its products. It could choose to invoice its products in terms of the foreign currency which is the case we have analyzed in this paper. But suppose the firm were to invoice its products in dollars. The firm would face the uncertainty in terms of the size of its cash flow, possibly due to the uncertainty in the quantity of the sales of its product. But in terms of the exchange rate uncertainty, the firm could act as if there were no exchange rate variation since its cash flow is in terms of dollars. But from Proposition 4 we know that the probability of bankruptcy in this case is higher

than in the case in which it invoices its products in units of FX. This result suggests that firms attempting to minimize the probability of bankruptcy would choose not to invoice their products in dollars. Since in our model dollar represented the currency of the country with no inflation, the general interpretation of this result is that firms would choose not to invoice their products in currencies of countries that have no inflation uncertainty.

The intuition behind our results is applicable to a more general case of inflation uncertainty. In our framework, the U.S. did not have any inflation. So assets denominated in dollars could be thought of as real assets. In our model, the real value of the assets (or cash flows) denominated in the foreign currency depended on the exchange rate which moved entirely because of changes in the general price level in the foreign country. These could be thought of as assets denominated in nominal terms. The entire analysis in the paper could then be translated in these terms. The implications, of course, are that firms that have written nominal contracts for its products would attempt to hedge the inflation risk by writing nominal debt contracts. Our result suggests that firms attempting to minimize the probability of bankruptcy may choose to price their products in *nominal* rather than *real* terms, borrow more than the financing needs through *nominal* debt contracts and invest the excess funds in *real* assets.

3 Concluding Remarks

We analyzed hedging policies for corporations when the foreign currency cash flow is not known with certainty. We showed that the optimality of any hedging policy does not depend on the seniority structure of the domestic and foreign currency debts: it depends only on the contractual value of the foreign currency liability which could always be chosen to be a forward contract to deliver the foreign currency.

We obtained an empirical implication that predicts that if there is some exchange rate variability, firms attempting to minimize the probability of bankruptcy would borrow *more* than their financing needs through foreign currency borrowing alone and invest the excess funds in domestic risk free securities. We obtained an intriguing result that the probability of bankruptcy for a firm that attempts to minimize this probability is *lower* when there is some uncertainty in the exchange rates than when there is no uncertainty in the exchange rates. This result suggests that firms attempting to minimize the probability of

bankruptcy would choose not to invoice their products in currencies of countries that have no inflation uncertainty. The intuition, of course, is applicable to a more general context of inflation uncertainty. Our result suggests that firms attempting to minimize the probability of bankruptcy may choose to price their products in *nominal* rather than *real* terms, borrow more than the financing needs through *nominal* debt contracts and invest the excess funds in *real* assets.

We did not explore the use of instruments such as options in our analysis [Giddy (1983)]. One would suspect that if the firms use a richer set of instruments, they could reduce the probability of bankruptcy even further. We provide a numerical example that suggests that this may indeed be the case.

Consider the example discussed in Table 1. Here the firm is bankrupt in four out of the eight states. We showed in Table 2 that if the firm increases its foreign currency borrowing by $\$ \frac{1}{2}$, it is bankrupt in only three out of eight states. But now suppose that we allow the firm to contract any claim that is contingent on s .⁸ In particular, suppose the firm writes three call options on unit of foreign currency with an exercise price equal to \$1. The payoff to the holder of each call is equal to $\Delta'(s) \equiv \text{Max}\{0, s - 1\}$. Since investors are risk neutral, the price of this call option Δ is equal to

$$\Delta = E \Delta'(s) = E[\text{Max}\{0, s - 1\}] = \$ \frac{1}{4}.$$

Let

$$\delta(s) \equiv \Delta - \Delta'(s).$$

The following table (Table 3) shows that if the firm writes three call options, its cash flow in each state will be sufficient to make the payments to the holders of the calls.

It is easy to check that the face value $z^f = \text{FX}2\frac{1}{2}$ of the FX loan of $y^f = \text{FX}2 = \$2$ continues to be consistent with the debt pricing condition.

Notice however that the firm is now able to meet its contractual obligations in six out of eight states and the probability of bankruptcy goes down to one-fourth. The firm has now taken a gamble whose payoff $\delta(s)$ is such that it transfers $\$ \frac{3}{4}$ from high s states to low s states. This pulls the firm out of bankruptcy from states 1 and 3. It reduces the surplus in

⁸Notice that we cannot allow contracts that are contingent on x^f since x^f may not be a publicly verifiable variable. This is perhaps the reason why we needed debt contracts in the first place. See Diamond (1984) and Townsend (1979).

states 6 and 8 just enough to keep it solvent in both states. Of course, the deficit in states 2 and 4 becomes worse and the surplus in states 5 and 7 becomes better, but that does not alter the probability of bankruptcy.

Table 3

State	1	2	3	4	5	6	7	8
$sx^f + (y^f - c)$	$\frac{1}{2}$	$1\frac{1}{2}$	1	3	$1\frac{1}{2}$	$4\frac{1}{2}$	2	6
3Δ	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
$-3\Delta'$	0	$-1\frac{1}{2}$	0	$-1\frac{1}{2}$	0	$-1\frac{1}{2}$	0	$-1\frac{1}{2}$
$3\delta \equiv 3(\Delta - \Delta')$	$\frac{3}{4}$	$-\frac{3}{4}$	$\frac{3}{4}$	$-\frac{3}{4}$	$\frac{3}{4}$	$-\frac{3}{4}$	$\frac{3}{4}$	$-\frac{3}{4}$
$sx^f + (y^f - c) + 3\delta$	$1\frac{1}{4}$	$\frac{3}{4}$	$1\frac{3}{4}$	$2\frac{1}{4}$	$2\frac{1}{4}$	$3\frac{3}{4}$	$2\frac{3}{4}$	$5\frac{1}{4}$
sz^f	$1\frac{1}{4}$	$3\frac{3}{4}$	$1\frac{1}{4}$	$3\frac{3}{4}$	$1\frac{1}{4}$	$3\frac{3}{4}$	$1\frac{1}{4}$	$3\frac{3}{4}$
$\text{Min}[sx^f + (y^f - c) + 3\delta, sz^f]$	$1\frac{1}{4}$	$\frac{3}{4}$	$1\frac{1}{4}$	$2\frac{1}{4}$	$1\frac{1}{4}$	$3\frac{3}{4}$	$1\frac{1}{4}$	$3\frac{3}{4}$
$sx^f + (y^f - c) + 3\delta - sz^f$	0	-3	$\frac{1}{2}$	$-1\frac{1}{2}$	1	0	$1\frac{1}{2}$	$1\frac{1}{2}$
$\text{Max}[0, sx^f + (y^f - c) + 3\delta - sz^f]$	0	0	$\frac{1}{2}$	0	1	0	$1\frac{1}{2}$	$1\frac{1}{2}$

The numerical example above suggests that the optimal set of contracts that minimize the probability of bankruptcy may well be non-linear contracts whose payoffs are contingent on the value of random but publicly observable variables such as the exchange rate. We leave it to future research to explore these issues further.

Appendix

Proof of Lemma 3

Clearly $z = 0$ since $y = 0$. Substituting $y = z = 0$ in (7), we get

$$\begin{aligned}
 y^f &= \mathbf{E} \left[\text{Min} \left\{ sx^f + (y^f - c), sz^f \right\} \right] \\
 &= \mathbf{E}(sz^f) + \mathbf{E} \left[\text{Min} \left\{ sx^f + (y^f - c) - sz^f, 0 \right\} \right] \\
 &= z^f \mathbf{E}(s) + \mathbf{E} [\text{Min} \{q, 0\}] \\
 &= z^f + \int_{-\infty}^0 qh(q) dq \\
 &= z^f + \int_{-\infty}^0 \int_0^{\infty} qf \left(z^f - \frac{(y^f - c) - q}{s} \right) \frac{1}{s} g(s) ds dq \quad [\text{From (9)}]. \tag{11}
 \end{aligned}$$

Differentiating (11) with respect to z^f and rearranging, we get

$$\frac{dy^f}{dz^f} = \frac{1 + \int_{-\infty}^0 \int_0^{\infty} qf' \left(z^f - \frac{(y^f - c) - q}{s} \right) \frac{1}{s} g(s) ds dq}{1 + \int_{-\infty}^0 \int_0^{\infty} qf' \left(z^f - \frac{(y^f - c) - q}{s} \right) \frac{1}{s^2} g(s) ds dq}. \tag{12}$$

Substituting $y^f = c$ and a change of variable $\theta = q/s$ in (12), we get

$$\begin{aligned}
 \frac{dy^f}{dz^f} &= \frac{1 + \int_{-\infty}^0 \int_0^{\infty} s\theta f' (z^f + \theta) g(s) ds d\theta}{1 + \int_{-\infty}^0 \int_0^{\infty} \theta f' (z^f + \theta) g(s) ds d\theta} \\
 &= \frac{1 + \int_{-\infty}^0 \theta f' (z^f + \theta) \left(\int_0^{\infty} sg(s) ds \right) d\theta}{1 + \int_{-\infty}^0 \theta f' (z^f + \theta) \left(\int_0^{\infty} g(s) ds \right) d\theta} \\
 &= 1. \blacksquare
 \end{aligned}$$

Proof of Lemma 4

Substituting for $h(q)$ from (9) and substituting $y = z = 0$ in (10), we get

$$\pi = \int_{-\infty}^0 \int_0^{\infty} f \left(z^f - \frac{(y^f - c) - q}{s} \right) \frac{1}{s} g(s) ds dq.$$

Differentiating π with respect to z^f we get

$$\frac{d\pi}{dz^f} = \int_{-\infty}^0 \int_0^{\infty} f' \left(z^f - \frac{(y^f - c) - q}{s} \right) \left(1 - \frac{1}{s} \frac{dy^f}{dz^f} \right) \frac{1}{s} g(s) ds dq. \tag{13}$$

Substituting $y^f = c$, $\frac{dy^f}{dz^f} = 1$ [from Lemma 3] and the change of variable $\theta = q/s$ in (13),

we get

$$\begin{aligned}\frac{d\pi}{dz^f} &= \int_{-\infty}^0 \int_0^{\infty} f'(z^f + \theta) \left(1 - \frac{1}{s}\right) g(s) ds d\theta \\ &= \left[\int_{-\infty}^0 f'(z^f + \theta) d\theta \right] \left[\int_0^{\infty} \left(1 - \frac{1}{s}\right) g(s) ds \right] \\ &= f(z^f) \left[1 - \mathbf{E} \left(\frac{1}{s} \right) \right] \\ &< 0 \quad \text{[From Jensen's Inequality].} \blacksquare\end{aligned}$$

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