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APPROPRIATING MATHEMATICAL PRACTICES: A CASE STUDY OF LEARNING TO USE AND EXPLORE FUNCTIONS THROUGH INTERACTION WITH A TUTOR

ABSTRACT. This case study uses a sociocultural perspective and the concept of appropriation (Newman, Griffin and Cole, 1989; Rogoff, 1990) to describe how a student learned to work with linear functions. The analysis describes in detail the impact that interaction with a tutor had on a learner, how the learner appropriated goals, actions, perspectives, and meanings that are part of mathematical practices, and how the learner was active in transforming several of the goals that she appropriated. The paper describes how a learner appropriated two aspects of mathematical practices that are crucial for working with functions (Breidenbach, Dubinsky, Nichols and Hawks, 1992; Even, 1990; Moschkovich, Schoenfeld and Arcavi, 1993; Schwarz and Yerushalmy, 1992; Sfard, 1992): a perspective treating lines as objects and the action of connecting a line to its corresponding equation in the form $y = mx + b$. I use examples from the analysis of two tutoring sessions to illustrate how the tutor introduced three tasks (estimating y-intercepts, evaluating slopes, and exploring parameters) that reflect these two aspects of mathematical practices in this domain and describe how the student appropriated goals, actions, meanings, and perspectives for carrying out these tasks. I describe how appropriation functioned in terms of the focus of attention, the meaning for utterances, and the goals for these three tasks. I also examine how the learner did not merely repeat the goals the tutor introduced but actively transformed some of these goals.

How does a newcomer to a mathematical domain learn from a tutor? One response to this question is that the tutors explicitly tell newcomers information and show newcomers their expert skills, and that newcomers acquire new skills through imitation. This study presents an alternative to this scenario using a sociocultural view of how a student learned from a tutor by participating in joint problem solving and appropriating aspects of mathematical practices. The case study shows how a student learned from a tutor not by the explicit teaching, telling, or showing of skills and information but through participation in joint problem solving. I also describe how the student learned important mathematical ideas not by imitation but by appropriating (and thus transforming) some of the tutor's ways of seeing, talking, and acting.

This study describes how appropriation functions in detail and examines how a student appropriated aspects of the mathematical practices crucial for working with functions. The study focuses on two aspects of appropriation: what it is that learners appropriate and how learners actively

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transform what they appropriate. I describe in detail how one student appropriated ways of seeing, talking, and acting (or perspectives, meanings, actions, and goals) that reflect expertise in working with functions: a perspective treating lines as objects and the action of connecting a line to its corresponding equation (in this case in the form $y = mx + b$). This paper builds on previous work on appropriation and addresses questions specific to the appropriation of mathematical practices: What *particular* aspects of mathematical practices does a learner appropriate as they gain expertise in this domain? *How* does a learner appropriate actions, goals, perspectives, and meanings? What are the *central features* of the appropriation process? How does a learner *actively transform* what he or she appropriates?

This case study describes how this learner, by solving problems jointly with a tutor, appropriated the focus of attention for tasks, meanings for utterances, and the actions and goals for carrying out new tasks. The focus of attention, meanings, and goals were not evident in the interactions as isolated pieces of tutor knowledge that were stated explicitly by the tutor. Rather, ways of seeing, talking, and acting were implicitly embedded in mathematical activity as the focus of attention, the meanings of utterances, the perspectives, and the actions and goals used during joint activity. I describe two aspects of mathematical practices, a perspective treating lines as objects and the action of connecting a line to its corresponding equation. These two aspects of expert mathematical practices for working with functions were evident in where the tutor and student focused their attention, in how they interpreted the meanings of utterances (especially questions by the tutor), and in the goals and actions the tutor and student set and carried out to accomplish tasks.

This analysis builds on previous work addressing mathematical practices (Cobb, Stephan, McClain and Gravemeijer, 2001) and describing central perspectives in this domain (Moschkovich et al. 1993). While work using the concept of appropriation (Newman et al., 1989; Rogoff, 1990; Radford, 2001; Rosebery, Warren, and Conant, 1992) provides a foundation, it also leaves room for elaborating on the notion of appropriation by providing more detailed analyses of how learners appropriate particular mathematical practices in different domains. This study elaborates on the notion of appropriation by describing in detail what it was that a learner appropriated as she learned to work with linear functions (where to focus, what utterances mean, how to use new goals, and how to carry out new actions) and how the learner actively transformed several of the goals she appropriated.

The sociocultural perspective used here is a Neo-Vygotskian one that assumes that learning is mediated by social interaction. *Appropriation* (Rogoff, 1990; Wells, 1999) is a central Neo-Vygotskian concept that has been used to describe how learning is mediated by interaction with others and how children learn when adults guide or teach them (Newman et al., 1989; Rogoff, 1990; Radford, 2001; Rosebery et al., 1992; Wells, 1999). Although literature in mathematics education sometimes refers to the appropriation of cultural or mathematical practices, it is not always clear what we mean by 'appropriation' and only a few studies describe how appropriation functions in detail for advanced mathematical topics (for one example, see Radford, 2001).

If we use the notion of 'appropriation' within a sociocultural theoretical framework, then the term has a deeper meaning than the dictionary's definition, 'using something for one's own purposes' and the concept has central characteristics that distinguish it from other constructs for describing learning. Appropriation involves joint productive activity, a shared focus of attention, and shared meanings (Rogoff, 1990). Appropriation also involves taking what someone else produces during joint activity for one's own use in subsequent productive activity while using new meanings for words, new perspectives, and new goals and actions. The analysis provided here uses this meaning of 'appropriation' to describe in detail how a student and a tutor developed a common focus of attention, shared meanings for utterances, and new goals and actions while engaged in joint productive activity working with lines and equations.

I use an interpretation of the concept of appropriation that is compatible with a view of learners as *actively* constructing knowledge. Appropriation "should not be viewed as limited to the process by which the child (novice) learns from the adult (expert) via a static process of imitation, internalizing observed behaviors in an untransformed manner" (Brown et al., 1993, p. 193). Appropriation does not imply that the learner merely repeats or imitates what she appropriates. Rather, learners use appropriated meanings, actions, or goals for their own purposes and are actively involved in appropriation by transforming what they appropriate. The examples of appropriation presented here show that this student did not merely imitate the goals set through interaction with the tutor; instead, she transformed these goals.

The case study examines how a student appropriated two aspects of mathematical practices through interaction with a tutor. The tutor and student worked together as the student explored the algebraic and graphical representations of functions using a computer.[1] The analysis of two videotaped sessions traced how goals were originally set through interaction with the tutor and how the student came to independently initiate, set, and use these goals. This paper focuses on describing the appropriation process and examining how the student was active in transforming goals. This study complements the cognitive analysis presented in Schoenfeld, Arcavi, and Smith (1993) describing learning in this domain. This student learned many things during these tutoring sessions: she corrected previous knowledge, added new pieces of knowledge, and made new connections between pieces of knowledge. However, the focus of this study is not *what* she learned but *how she learned through interactions with the tutor* (for a detailed analysis of what this student learned, see Schoenfeld, et al., 1993). The analysis presented here thus extends this work by describing in detail how this student's learning was mediated by interactions with the tutor. This study also builds on previous work describing two perspectives of linear functions, treating functions as objects and processes (Moschkovich et al., 1993). One difference from this previous work is that these perspectives are not seen as purely cognitive or individual. Instead, they are described as aspects of mathematical practices that a learner appropriates through interaction with a tutor.

Appropriation

This study uses a sociocultural perspective to examine learning in a complex mathematical domain. This sociocultural perspective implies, first, that learning mathematics is viewed as a discursive activity (Forman, 1996) which involves using multiple material, linguistic, and social resources (Greeno, 1994). This perspective assumes that learning is inherently social and cultural "whether or not it occurs in an overtly social context" (Forman, 1996, p. 117). I also assume that representations (such as graphs and equations) have multiple meanings for participants, that these meanings are not inherent to the representations but are embedded in mathematical practices, and that these multiple meanings for representations and inscriptions are negotiated through interactions.

The theoretical framework draws on concepts such as the move from inter-psychological to intra-psychological planes (Vygotsky, 1978, 1979; Wertsch, 1979a, 1979b, 1984, 1985) and the theory of activity as described by Wertsch (1979a). The analysis develops a detailed description of how appropriation (McNamee, 1979; Newman et al., 1989; Rogoff, 1990) functions in teaching/learning interactions. Newman et al. (1989), Brown et al. (1993), and Rogoff (1990) provide definitions and examples of appropriation. While these provide a starting point, they also point to aspects of appropriation that we need to consider in more detail and provide examples in mathematical domains: How do the central aspects of appropriation function during learning/teaching in an advanced mathematical domain? What do learners appropriate? How are learners active during appropriation?

Appropriation has been used to describe how learners use cultural tools from previous generations and resources provided by other people. For example, Rogoff (1990) conceptualizes the process by which individuals profit from social engagement as "appropriation inherent to the process of participation in shared activity" (p. 193). Rogoff uses appropriation as an alternative to internalization and describes it as the product of shared thinking and guided participation. Individuals appropriate "some aspects of activity in which they are already engaged as participants and active observers" (p. 195).

The central features of appropriation as described by Rogoff are that the process involves achieving a shared focus of attention, developing shared meanings, and transforming what is appropriated.[2] Rogoff emphasizes that appropriation is not transmission or imitation, because learners transform information and skills:

> The individual's later use of this shared understanding is not the same as what was constructed jointly; it is an appropriation of the shared activity by each individual that reflects the individual's understanding of and involvement in the activity.
> (p. 195)

Newman et al. (1989) used appropriation to describe in detail how interaction with an adult can affect cognitive change in children. According to Newman et al., during appropriation an expert interprets a student's cognitive product (a statement or an action) within his or her knowledge framework and subsequently engages the student in an activity reflecting this expert understanding of the situation. There are several defining features of appropriation as proposed by Newman et al. (1989). First, there are two individuals engaged in joint productive activity, each with their own understanding of the task. Next, the novice takes some action arising out of her own understanding and the expert provides feedback about the action in terms of an expert understanding of the task. The novice then comes to understand what his or her action meant within the expert framework and learns something about that framework. Lastly, the expert engages the novice in an activity using the expert understanding of the task.

Newman et al. (1989) focus on describing how an expert uses a learner product (an action or a statement), sets a new task that uses the expert interpretation of that action or statement, and engages the learner in that new task. This description of appropriation focuses on how children learn from experts who are simply providing them with an alternative interpretation of their actions. By simply engaging in the second activity the child actively utilizes a new interpretation of the task. Evidence for the success of the appropriation process lies in relating subsequent actions carried out independently by the novice to the expert framework originally revealed through joint productive activity.

While this work provides a foundation for using the concept of appropriation to describe learning mathematics, it also leaves some aspects of appropriation under-specified. First, accounts of appropriation need to describe appropriation as it functions in particular mathematical domains. Second, accounts of appropriation need to clarify and describe in detail what it is that learners appropriate. Learners have been described as appropriating a broad spectrum of things ranging from information or skills, to meanings for words, to interpretations of a task, to ways of acting and thinking, or to discourses and social practices. Framing the notion of appropriation within a sociocultural view presupposes that learners do not appropriate *individual* knowledge, skills, or meanings for words but aspects of mathematical practice that are *social* and that knowledge, skills, meanings, perspectives, actions and goals are not isolated pieces of knowledge which are explicitly told or shown but are embedded in mathematical practices and learned through joint activity.

In mathematics education, Radford (2001) has used the notion of appropriation to focus on the individual appropriation of "technical mathematical expressions" (p. 251) that includes re-creation. Radford's analysis described how students interwove a teacher's words with student's own meanings during what he called 'emergent algebraic thinking.' For example, the word 'rank' first mentioned by the teacher was appropriated and used by students in a way that was not intended by the teacher, so that students used a modified version of the meaning for this term. Radford (2001) emphasizes that appropriation is not a copy – "we adapt the words in terms of our pragmatical needs and particular expressive intentions" – and that students supply "the teacher's words with new meanings" (p. 252). Thus, the appropriation of words described by Radford involves not a copy but an adaptation. In contrast to Radford's (2001) focus on students' appropriation of words and their meanings, in science education Rosebery et al. (1992) described how students appropriate scientific discourse, not only to focus on ways of talking or the meanings of words but also on ways of seeing, acting, and using tools (Gee, 1999).

I use these definitions of appropriation but, rather than focusing only on mathematical ways of talking and meanings for words, I also focus on mathematical ways of seeing through the analysis of the focus of attention and ways of acting through the analysis of goals. I describe the goals introduced by a tutor and the subsequent appropriation of these goals by the student. Appropriated goals are applied for one's own purposes, which may not always be the use intended by the teacher or tutor. This student used goals originally set by the tutor for new and different purposes. Thus, when we look at student independent activity, not only can we 'hear' the echo of the teacher's words in the words of the student, we can also 'see' the echo of goals introduced by the tutor in the independent actions of the student.

Mathematical practices

What are mathematical practices? I use the terms *practice* and *practices* in the sense used by Scribner (1984) for a practice account of literacy to "highlight the culturally organized nature of significant literacy activities and their conceptual kinship to other culturally organized activities involving different technologies and symbol systems" (p. 13). This account of learning to use linear functions focuses on the culturally organized nature of two significant aspects of the mathematical practices for using and exploring functions that involve graphs and equations as symbol systems: a perspective treating functions as objects and the action of connecting a line to its equation (and vice versa).

Mathematical practices are social and cultural, because they arise from communities and mark membership in communities. They are cognitive, because they involve thinking and semiotic, because they involve signs, tools, and meanings. Mathematical practices involve values, points of view, and implicit knowledge.

Cobb et al. (2001) define mathematical practices as the "taken-as-shared ways of reasoning, arguing, and symbolizing established while discussing particular mathematical ideas" (Cobb et al., page 126). In contrast to social norms and socio-mathematical norms, mathematical practices are specific to particular mathematical ideas. While Cobb et al. (2001) see mathematical practices as emergent in classroom activity, I see mathematical practices as simultaneously emergent in ongoing activity and "normative ways of acting that have emerged during extended periods of history" (Cobb et al., 2001). I view the perspective of treating lines as objects and the action of connecting equations to lines as both emergent phenomena during these tutoring interactions as well as already established ways of working with lines within communities that regularly use equations and graphs. I take the view that practices are constituted by actions, goals, perspectives, and meanings and that these goals, actions, perspectives, and meanings are embedded within the mathematical practices that experts participate in when using and exploring functions.

This study shows a learner appropriating two aspects of mathematical practices in the domain of functions, a perspective treating graphs as objects and the action of connecting lines to equations. I describe how a tutor and a student interactionally established goals, meanings, and perspectives that are embedded in tasks. These goals, meanings, actions, and perspectives are part of three tasks the tutor introduced: visually estimating the y-intercept of a line, evaluating the slope of a line, and exploring the parameters in an equation. These tasks reflect two aspects of mathematical practices: a) seeing, talking about, and acting as if a line is an object that can be manipulated and b) seeing, talking about, and acting as if lines are connected to their equations. This student's learning is a consequence of her "insertion into an intellectual practice requiring a social use of signs and the understanding of their meanings" (Radford, 2001, p. 261). I argue that the focus of attention, meanings for utterances, perspectives, actions, and goals that the learner appropriated are significant not as isolated skills, but because the perspective of treating functions as both objects and processes and the action of connecting lines to equations are aspects of mathematical practices central for success in using and exploring functions.

Previous work in the domain of linear functions (Breidenbach et al., 1992; Even, 1990; Moschkovich et al., 1993; Schwarz and Yerushalmy, 1992; Sfard, 1992) has described two perspectives of functions: process and object. From the process perspective, a function is perceived as linking x and y values: for each value of x, the function (or relation) has a corresponding y value. From the object perspective, a function or relation (and any of its representations) are thought of as entities–for example, algebraically as a member of parametrized classes or as graphs that can be manipulated by rotation or translation (Moschkovich et al., 1993). While each of these perspectives or, in Schwarz and Yerushalmy's (1992) words, aspects, may be more or less salient, they are both crucial for learning algebra:

> We see that the symbolic representation of function makes its process nature salient, while the graphical representation suppresses the process nature of the function and thus helps to make the function more entity-like. A proper understanding of algebra requires that students be comfortable with both of these aspects of function. (Schwarz and Yerushalmy, 1992, p. 265)

> Developing competency with linear functions or relations means learning which perspectives and representations can be profitably employed in which contexts, and being able to select and move fluently among them to achieve one's desired ends. (Moschkovich et al., 1993, page 72)

In Moschkovich et al. (1993) we described a framework for connecting representations and perspectives. We described the two perspectives, perhaps implicitly, as individual competencies. In contrast, here I assume that the object perspective is not an individual competency but an important aspect of mathematical practices in this domain.

Rogoff (1990) distinguishes between what she calls ‘skills’ and ‘shifts in perspective.’ She defines skills as “the integration and organization of information and component acts into plans for action under relevant circumstances” and shifts of perspective as involving “giving up an understanding of a phenomenon to take another view contrasting with the original perspective” (p. 142). Following these definitions, I describe two aspects of expert mathematical practices for using and exploring functions: a) actions and goals, such as connecting equations to lines, and b) perspectives (object and process) and shifts between perspectives, such as shifting between the object and process perspectives.

PARTICIPANTS, SETTING, AND TASKS

The student in these tutoring sessions was a 16-year-old female who was participating in a university summer math program for high school students. Students in this program are usually average or high achievers in mathematics courses. Throughout the four sessions she was highly motivated. She had previously taken a self-paced algebra course, a regular geometry course, and at the time of the sessions was taking a summer mathematics course. The tutor was a male educational psychology graduate student with previous mathematics teaching and interviewing experience. The sessions intentionally combined interviewing and tutoring, thus eliciting the student’s own understandings while providing guidance and assistance.

The tutor and student used a computer environment called GRAPHER (Schoenfeld, 1990; Schoenfeld et al., 1993) consisting of a game called Black Blobs (Dugdale, 1982, 1984), which is similar to Green Globs except here the blobs are white or black, and a graphing environment called Dynamic GRAPHER. The game presented the student with a random array of blobs on a Cartesian coordinate system, and the task was to shoot the blobs by entering the equation of a curve. The software was designed to allow learners to explore the connections between lines and equations and to support an object view of curves. The environment and tutor facilitated communication about abstract ideas (lines) and imaginary events (changing lines and equations).[3]

Dynamic GRAPHER allowed the student to change the parameters of a function and see its graph change accordingly. The two analyzed sessions lasted a total of three hours and were six days apart. Session 1 lasted approximately one hour, and Session 2 lasted approximately two hours. Most of the interactions in the analyzed videotapes involved the game except for two episodes (about 20 minutes each in Session 2) where the student used Dynamic GRAPHER. The two sessions considered in this study focused on straight lines except for one 20-minute segment in Session 2 involving parabolas.

The tasks discussed in this paper are generating equations, estimating the y-intercept, evaluating the slope of a line, and exploring parameters. These tasks were the focus of the analysis because most of the interactions in these two sessions involved these tasks, they are crucial for successful performance in either the game or the graphing environment, and they reflect important aspects of expertise in the domain of linear functions. Approximately 75% of the time for Session 1 and 60% of the time for Session 2 revolved around these four tasks. Since the student generated a procedure for calculating a slope independently of tutor assistance, this task is not included in this discussion. The interactions omitted from this analysis largely involved carrying out, discussing, and correcting the student's slope computations.

These four tasks recurred throughout the sessions, and these recurrences are numbered by episodes. Evaluating Slope, Episode 1 refers to the first occurrence of a segment involving evaluating a slope. Evaluating Slope, Episode 2 refers to the second occurrence of a segment involving evaluating a slope and so on.

The analysis of these tutoring sessions focused on the development of the student's goals for solving problems in this domain. Below, I first summarize the analysis of how the student's goals changed, then describe how two aspects of appropriation, a common focus of attention and shared meanings for questions, functioned. In the last section of the paper, I describe how the student was active in transforming the goals initially set through interaction with the tutor.

SUMMARY OF THE ANALYSIS

This section summarizes the results of the micro-genetic analysis (described in Moschkovich, 1989) and provides a global view of the interactions. The analysis of the videotapes focused on characterizing patterns in the interactions. The data was used in a twofold manner to examine in detail how appropriation functioned during the tutor-student interactions and how the student was active in appropriation. The analysis focused on two levels: one coding whether the tutor or the student initiated activity and the other tracing the evolution of new student goals.

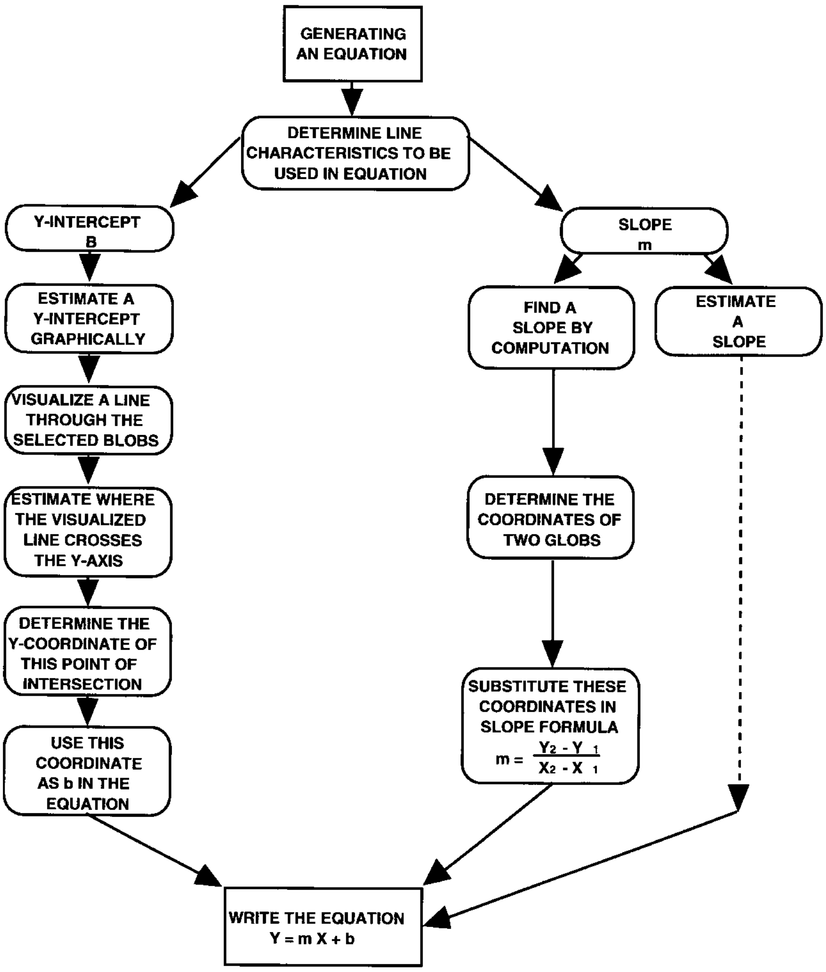

Figure 1. Goals for generating equations.

For determining instances of student-initiated activity each of the two sessions were parsed according to tasks. The task was defined as the general top level goal pursued in the joint actions of tutor and student, for example, ‘hitting a set of blobs.’ Each task usually involved several other tasks utilized for achieving this top-level goal. For example, in order to hit a set of blobs, the student needed to first ‘generate an equation’ (see Figure 1) that would produce a line to hit the blobs. To ‘generate an equation,’ the student needed to ‘find the slope’ and ‘find the y-intercept.’

The tasks coded in these sessions include: hitting a set of blobs, generating an equation, finding a y-intercept algebraically, estimating a y-intercept on the graph, calculating a slope, evaluating a slope calculation, evaluating the slope of a graphed line, modifying an equation (or a line), exploring parameters, and comparing parameters. Instances of each of these tasks were coded according to whether the student or tutor was mainly responsible for initiating them. In this manner the interactions were characterized in terms of which tasks the student or tutor initiated, and a description was developed of how the student's goal setting evolved throughout the sessions.

The second level of analysis focused on the evolution of the student's goals for accomplishing specific tasks. First, those tasks that the student initiated but did not carry out alone (generating equations) were identified. Then those tasks that the student did not initiate were identified (estimating the y-intercept, evaluating the slope of a line, and exploring parameters). Next, the coding identified those tasks that were originally set or carried out through tutor-student interaction and subsequently set or carried out independently by the student. To describe the development of the student's goals and relate this development to interactions with the tutor, the following questions guided the selection and analysis of segments for the case study:

1. Which activities did the student initially not set or carry out independently?
2. Which activities in (1) did the student carry out through interaction with the tutor?
3. Did activities initially set or carried out through interaction with the tutor become independent?
4. How were the goals originally set through interaction with the tutor interpreted and utilized by the student?

These segments were analyzed by searching for actions representing student interpretation and utilization of goals. Student statements or actions reflecting new goals were taken as evidence of student utilization of these goals. The origin of new student goals was traced back to goals originally set through interaction. New student goals were analyzed in terms of whether they were a repetition of or modification of goals originally set through interaction with the tutor.

Most, if not all, problems were set through negotiation between the tutor and the student. Although initially the tutor was more active in setting goals, as the student gained more experience with the software and the subject matter, she increasingly generated and pursued her own problems. Throughout the sessions, the tutor fostered executive control activities such as revising and evaluating that are crucial for competent problem solving in complex domains (Brown, Bransford, Ferrara, and Campione, 1983;

Schoenfeld, 1985). Initially, the tutor functioned as the problem poser, goal setter, critic, and evaluator, asking for and suggesting ideas or plans for possible lines of action and overtly engaging in goal setting, checking, and evaluation. As tutoring proceeded, the student assumed more and more of these regulatory functions, she set new problems, checked results, and evaluated solutions. Thus, the student had the opportunity to appropriate as a set of self-regulatory activities many of the executive control activities first experienced in interaction. She also developed new goals, and changed incomplete or inappropriate goals to reflect more domain knowledge.

The analysis shows that the student appropriated the goals for several tasks initially introduced through joint problem solving with the tutor (generating equations, estimating the y-intercept, evaluating a slope, and exploring parameters). Although in the beginning of the tutoring sessions the student did not set these goals independently, she later initiated and carried out these tasks successfully on her own. Most importantly, the student actively transformed goals introduced by the tutor and used them for her own purposes. For the student, appropriation began with an acceptance of a new goal set by the tutor. Then the student carried out this goal. The next time the student attempted a task, tutor support gradually faded. Appropriation involved the transformation of these goals by the student for her own purposes. Finally the tutor provided no guidance as the student set goals and completed a task independently and successfully.

Appropriating ways of seeing, talking, and acting: Focus of attention, meanings for utterances, and goals

This section summarizes how the student learned to set goals for two tasks, estimating a y-intercept and evaluating a slope, through joint productive activity with the tutor. These examples illustrate two central features of appropriation, developing a shared focus of attention and shared meanings by taking an action or utterance produced by the other person during joint activity and using this action or utterance in subsequent activity. The examples describe the goals for the tasks 'estimating a y-intercept' and 'evaluating a slope.' Both of these tasks involve treating a line as an object and connecting a line to its equation.

In the case of estimating the y-intercept, when the tutor asks the student to 'find *b*,' he is asking her to treat an imagined line as an object that crosses the y-axis and determine the y-coordinates of that point. He is also asking the student to then use that y-coordinate as the *b* in an equation, thus connecting the line and the equation. In the case of evaluating a slope number, the tutor is asking the student to compare the result of a slope calculation

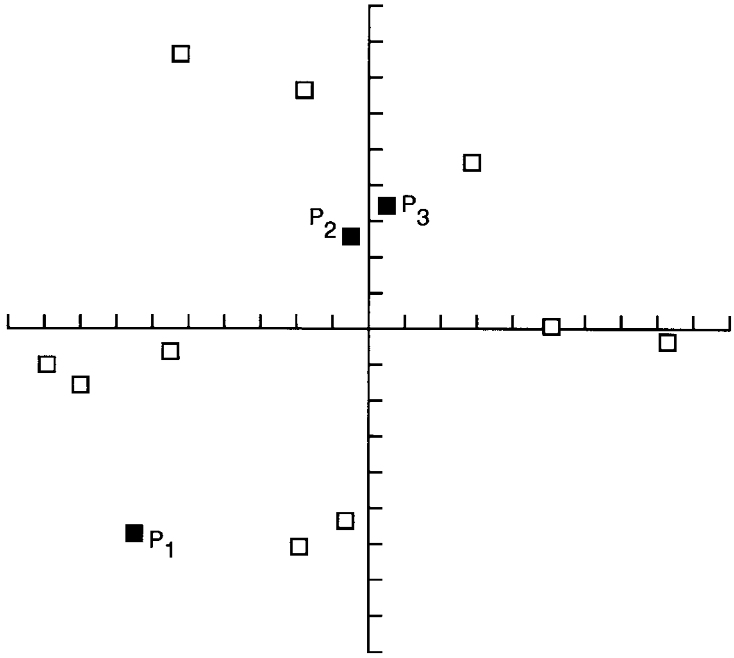

Figure 2. Estimating Y-Intercept episode 1 and evaluating slope episode 2.

with the expected slope for a line, treating the line as an object that can have different orientations on the graph. If the student is comparing the number *m* in an equation with the slope of a line, then she is not only treating a line as an object that can have different orientations on the graph but also connecting an equation to a line.

Estimating Y-intercept

During the student's first attempt to hit a set of blobs in Session 1, the tutor asked the student "Is there any way we can get *y* equals *mx* plus *b*. I mean, can we get *m* and *b*?" He also introduced the task of estimating the y-intercept of a line by looking at the graph and provided the goals for accomplishing this task. The student subsequently used estimation to find all the y-intercepts in these two sessions by estimating where a visualized line would cross the y-axis and determining the y-coordinate of this point.

During her first attempt to generate a line, the student was trying to hit three blobs in Figure 2 (P_1, P_2, and P_3). Two of these blobs straddle the y-axis. P_3 in the first quadrant had coordinates (0.5, 3.5), and P_2 in the fourth quadrant had coordinates (–0.5, 2.5). After calculating the slope for her first equation, the student attempted to find the y-intercept. In response to the tutor's question, "What is the y-intercept?," the student provided the y-coordinates of the two blobs she was trying to hit, 3.5 and 2.5. The tutor used these coordinates to describe where the target line would be, first saying, "3.5 is where the line is on one side of the y-axis. 2.5 is where it is on the other side," and then pointing to the two blobs. Finally, when the tutor put a pen up to the screen where the line would be, the student focused her attention on the point where the line would cross the y-axis. The student then proceeded to determine the y-coordinate of this point and used it to generate an equation. Throughout the rest of Session 1, the student continued to estimate the y-intercept correctly from the graph (Estimating Y-Intercept, Episodes 2-5, not included here), independently setting this goal originally set through interaction with the tutor.

From this interaction, it seems that the student connected the goal of finding a line to generating a number for the slope in the equation by calculating that number using a formula. She was thus connecting a formula for one parameter in the equation, *m*, to the line on the graph. However, she did not independently generate a procedure or number for the *b* slot in the equation. The starting point for this student could be described as making a partial connection between a line and its equation. While she independently generated a procedure for finding the number *m*, she did not independently generate a number for *b*. She did not independently connect the point where a line would cross the y-axis with the number *b* in the equation, and (as described in later episodes) she did not connect the orientation of a line on the graph with the magnitude or sign of the number *m*.

This example illustrates two central features of appropriation, developing a shared focus of attention and shared meanings for utterances. The tutor used the y-coordinates of the two blobs to develop a shared focus of attention, the point where the line would cross the y-axis. Through this shared focus of attention, the tutor and the student also developed a shared meaning for the question "Can we get *b*?" and the actions involved in "getting *b*." This question is ambiguous and can have multiple meanings, for example "Can we calculate the y-intercept using an equation?" or "Can we estimate the y-intercept using an imagined line?" The tutor introduced the second meaning of this question, focused the student's attention on the point where a line would cross the y-axis, asked her to determine the y-coordinate of this point and use that number as *b*.

In order to visually estimate where a line intercepts the y-axis the line has to be treated as an object that can both be imagined and manipulated. The y-intercept must also be seen as existing in two connected representations, the graph and the equation. If one needs to "get *b*" for an equation, one should go see where *b* shows up on the line. The question "Can we get *b*?" when intended to mean "Can we estimate *b* using the graph?" thus presupposes several aspects of an expert view of this domain. First, it presupposes an understanding of the overarching goal "Estimate the y-intercept visually." It also presupposes a perspective of the line as an object that can be manipulated to have different y-intercepts as one moves it up and down the y-axis. It also presupposes acting as if there is a connection between the point where the line crosses the y-axis and the parameter *b* in the equation for that line. By introducing the simple question "Can we get *b*?" the tutor engaged the student in joint problem solving that involves mathematical perspectives, meanings, actions and goals. Through interaction with the tutor, the student came to focus her attention on the y-intercept of an imagined line and to interpret the meaning of the question "Can we get *b*?" as "Can we estimate *b* visually?"

This analysis is an instance of how mathematical practices are *simultaneously* emergent in ongoing activity while at the same time are "normative ways of acting that have emerged during extended periods of history." The perspective of treating lines as objects and the action of connecting equations to lines were both emergent local phenomena during these tutoring interactions, for example enacted in this tutor's particular use of words, gestures, and meanings as well as already established ways of working with lines within communities that regularly use equations and graphs. The actions, goals, perspectives, and meanings that the tutor introduced are simultaneously local, personal, contingent, and emergent and also representative of the mathematical practices experts participate in when using functions.

Evaluating slope

Below I describe how appropriation functioned in detail for one important monitoring task, evaluating slopes (see Figure 3). The example illustrates how the tutor and the student developed shared meanings for utterances and how the student was active in transforming goals introduced by the tutor. The tutor initially used the result of the student's computations (in three different instances) to introduce a new task, evaluating a slope. The student did not initially understand this task. Through interaction with the tutor she first accepted this task when set by the tutor, then came to set the goals for this task herself, and subsequently both initiated and carried out the task independently. This series of excerpts shows that the student did not merely imitate goals originally introduced by the tutor but rather transformed these goals.

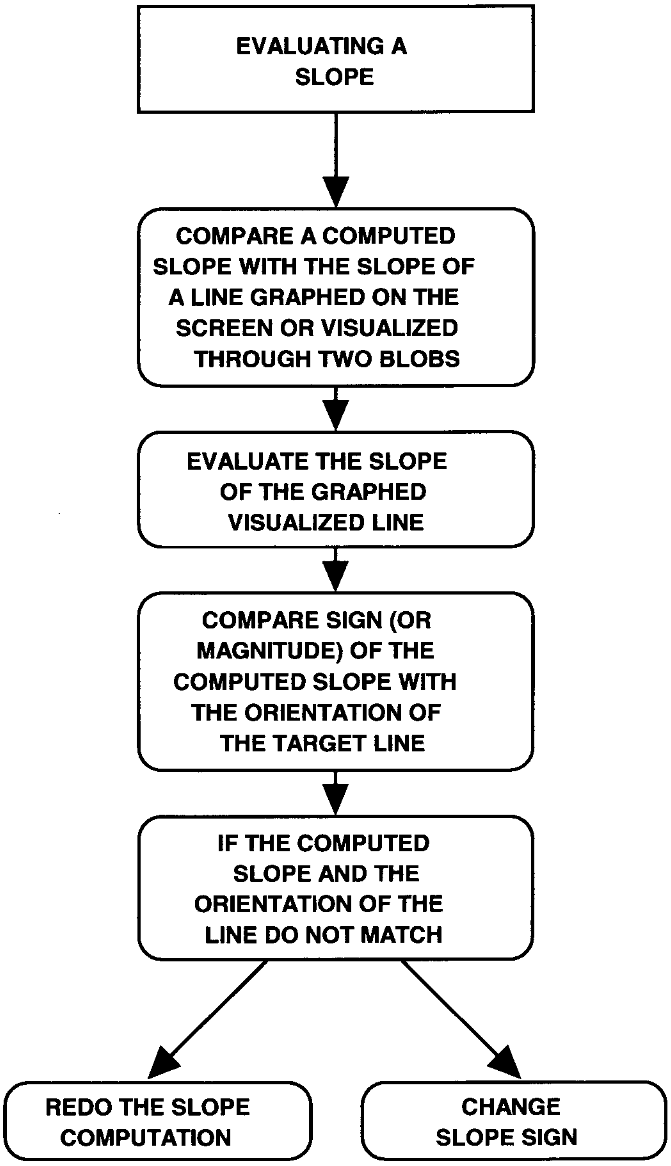

Figure 3. Goals for evaluating the slope of a line.

ES Episode 1

In Session 1, while trying to hit the set of blobs in Figure 2 (P_1, P_2, and P_3), the student first calculated the slope for the line passing through two of these three blobs by using the slope formula. After she produced the number +1 for the slope through computation, the tutor asked her to evaluate this result.

Student:	(Writes down the result of her first slope computation) OK, I think it's negative 1. Negative 1 over negative 1, and so that means that the slope is one!
Tutor:	Now does that look like the right number?
Student:	What do you mean the right number?
Tutor:	Well, do you have any sense of what the slope should come out to be?
Student:	Yes, it should be negative.
Tutor:	Does that look about right?
Student:	No. OK, it can't be right. (She proceeds to check her slope calculation.)

With the question, "Now does that look like the right number?," the tutor used the student's computational result to introduce a new task, evaluating a slope. He directed the student to focus her attention on checking the result of her slope calculation by comparing it to an expectation. He was asking her to base this expectation on knowledge about the orientation of lines, thus treating lines as objects – lines with positive slopes rise to the right and lines with negative slopes rise to the left.

The question, "Now does that look like the right number?," presupposes several aspects of an expert view of this domain. First, an understanding of the task's overarching goal, to compare the result of a computation with an expectation. It also presupposes knowing that when *m* changes in an equation, the orientation of the line changes accordingly. And lastly, it presupposes a perspective of lines as objects that have different orientations. The student did not seem to initially share the tutor's meaning for the question or this interpretation of the overarching goal for the task. She explicitly asked the tutor what he meant by "the right number." The tutor clarified the meaning of his first question with a second question, "Do you have any sense of what the slope should come out to be?" This second question set the goal of comparing the computed number with an expected orientation for the line and provided an opportunity for the student and the tutor to construct a shared meaning for the question, "Does that look like the right number?" By developing a shared meaning for this question, the student also developed an interpretation of the overarching goal for the task that more closely resembles the tutor's interpretation of the task.

Following the tutor's second question, the student accepted the goal set by the tutor and carried it out, first stating her expectation that the slope of a line passing through the selected blobs should be negative and then comparing her expectation to the computed slope. At this point, the student's knowledge of the relationship between the sign of *m* and the orientation of a line was reversed. She described lines rising to the right as having a negative slope and lines rising to the left as having a positive slope. Nevertheless, she evaluated the result of her algebraic computation against her (incorrect) expectation of how the line should look: 'It should be negative." She then proceeded to rework her calculation. This evaluation, based on the comparison of a computed value with the inclination of the expected line through the selected blobs, was thus a goal first set through interaction with the tutor. Appropriation involved developing a shared focus of attention (the orientation of the line on the graph) and a shared meaning for the question "Does that look like the right number?" (Will this number produce a line with the right orientation?). By looking at the student's later actions in Evaluating Slope Episode 3, we will also see that she appropriated the overarching goal for the task, comparing the result of a computation with an expectation.

But first, by looking at Evaluating Slope Episode 2, we can see how the student corrected her understanding of the slope. A few minutes later the student entered the equation $y = x + 3$, producing a line on the screen with slope +1 and hitting two of the selected blobs (P_2 and P_3 in Figure 2). The tutor again prompted the student to compare her expectation for the slope with the orientation of the line on the screen. Through this comparison the student corrected her matching of the slope sign and the orientation of a line.

ES Episode 2

Tutor:	So now what do you think about the fact... your calculation about ah, about... you were expecting, if you were going on your beliefs about positive and negative slopes, you would have expected to see something like this (traces a line with negative slope in front of the screen).
Student:	Yes.
Tutor:	So what do you think about that?
Student:	That I was wrong. That positive slope is this way (pointing to the line on the screen) and negative slope is the other.

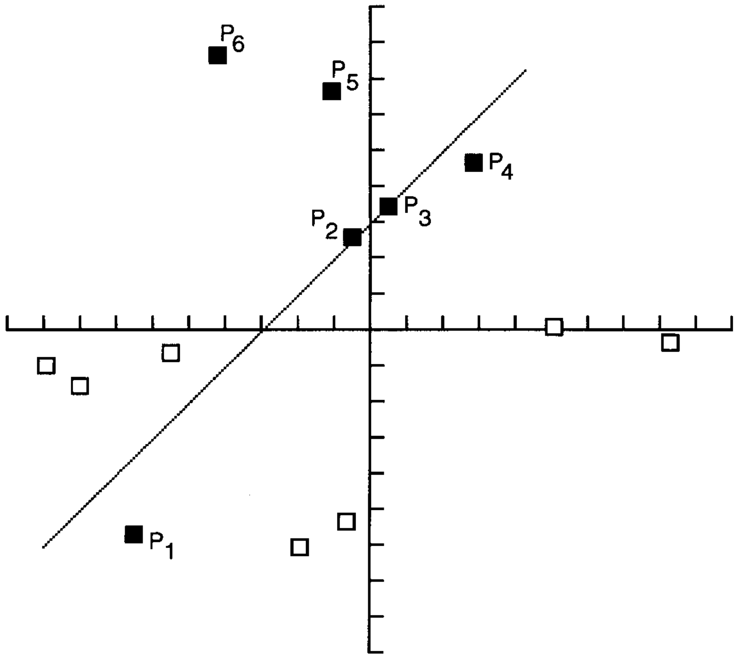

Figure 4. Evaluating slope episode 3.

During the student's next attempt to hit a set of blobs in Session 1 (P_5 and P_6 in Figure 4), the tutor once again used the result of her slope calculation, –1/2, to set the goal of evaluating the slope. After the student calculated the slope of a line through the blobs, the tutor asked her to compare this line with the previous line of slope +1 on the screen. The student had just calculated a slope of –1/2 for the second equation in this session as she was trying to hit three blobs rising up to the left. On the screen was the previous line of slope +1.

ES Episode 3

Student:	OK. So the slope is –1/2.
Tutor:	All right. Does that seem right based on the fact that your slope on the screen is positive?
Student:	No, it's negative.
Tutor:	This one (referring to the slope for the previous line, +1)?
Student:	That one's positive, right...
Tutor:	OK, so does it make sense that this one (the slope for the second line, –1/2) would be negative?

Student: (laughs) Yes. . .
Tutor: OK, good.

In contrast to Evaluating Slope Episode 1, the student now seems to easily understand the tutor's question, "Does that seem right based on the fact that your slope on the screen was positive?" In Evaluating Slope Episode 3, the tutor focused the student's attention on comparing the result of her calculation, –1/2, with the orientation of another line on the screen with slope +1 (instead of comparing a calculation with an expectation as in Evaluating Slope Episode 1). The student responded that it did make sense that the second line should have a negative slope. Although the student did not initiate the evaluation of the slope herself, she seemed to understand the question, readily accepted it as a valid goal, and provided an appropriate response. Her responses show that she and the tutor were now using a shared meaning for the questions, "Does that (your calculation) seem right?" and "Does it make sense that this one would be negative?"

The conditions in Evaluating Slope Episodes 1 and 3 are clearly different in some ways. First, the line in Episode 1 is imagined while the line in Episode 3 is visible on the screen. Second, the tutor's question in Episode 1, "Does that look like the right number?," is different than his question here, "Does that seem right based on the fact that your slope on the screen is positive?" The second question is more specific and explicitly asks the student to consider the fact that the slope on the screen is positive. While it is impossible to tell what the student's response might have been if the two conditions and questions had been identical, what is most significant is not whether the student's actions changed in response to different conditions. What is most significant for an analysis of actions and goals introduced by the tutor and then taken up and carried out independently by the student is the student's subsequent independent actions. After Episode 3 the student proceeded to evaluate slopes independently, whether the lines were visible on the screen or imagined, and in the way the tutor had introduced and thus, implicitly, using the meaning the tutor had intended in these two questions (both the more general first version and the more specific second version).

By Session 2 (Evaluating Slope Episode 4), the student independently initiated the evaluation of a slope by comparing the number she had used for *m* in the equation with the orientation of lines on the screen. In this episode, the student had just entered the equation y = –1.5x + 0 while trying to hit three blobs rising to the right (P_1, P_2, and P_3 in Figure 5) and missed the blobs.

The student independently evaluated the sign of the calculated slope saying, "But still the line is going this way (gesturing in a negative slope direction), and I wanted the line to go the other way (gesturing in a positive

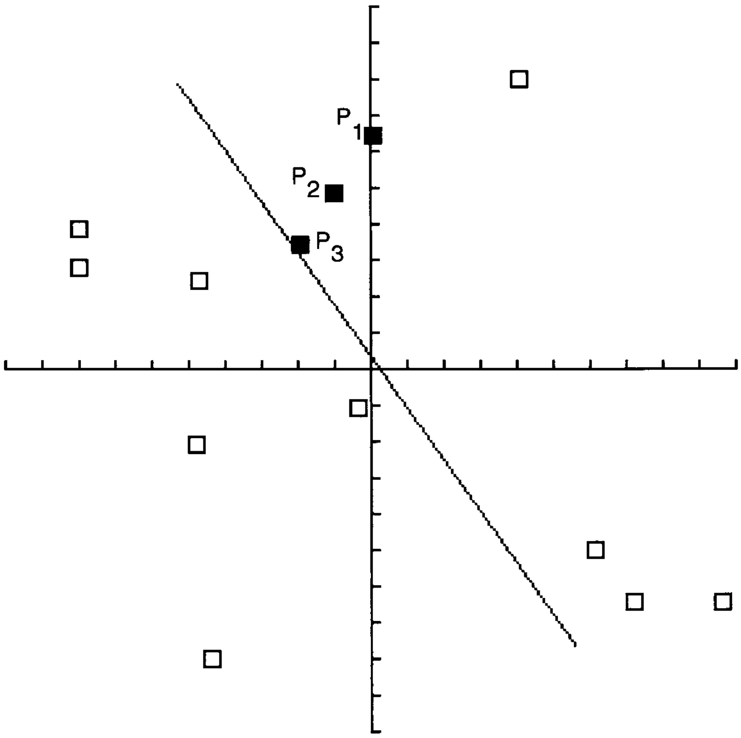

Figure 5. Evaluating slope episode 4.

slope direction). Right? So then... that means the slope isn't negative, it's positive." She independently compared the orientation of the target line with the sign of the slope resulting from her computation. In Evaluating Slope Episode 4 the student initiated this comparison and concluded that she did not want the equation to have a negative slope but a positive one. This time she did not return to her calculation, as she had done in Evaluating Slope Episode 1, but instead changed the sign of *m* in the equation, proceeding to hit the intended blobs with her line.

The analysis of two tasks, estimating y-intercept and evaluating slope, show how interactions with the tutor shifted the learner's focus of attention, supported shared meanings for utterances, and introduced new goals. Through these interactions, the student and the tutor came to jointly focus their attention on the place where a line crosses the y-axis or on the orientation of lines on the screen. They also came to share meanings for utterances such as "Can we get *b*?" and "Does that look like the right number?" Through these interactions the learner also appropriated the goals for carrying out the tasks of estimating y-intercepts and evaluating slopes. However, the learner did not simply repeat the goals that the tutor had introduced. As shown in the next section, the learner was active in transforming some of these goals.

HOW WAS THE LEARNER ACTIVE?

The series of excerpts above show that the student did not merely imitate the goals the tutor had introduced, but rather transformed these goals. The tutor had first used the evaluation of a slope for checking a computation *before* producing a line (Evaluating Slope Episodes 1 and 3). The student, however, independently evaluated the slope of a line already on the screen (Evaluating Slope Episodes 4 and 5) rather than her slope computation. The student thus transformed this goal by using it *after* she had produced a line on the screen, putting this goal to her own use rather than the use originally introduced by the tutor.

The final example below describes how the student appropriated the action of connecting lines to their equations and highlights how the student was active in transforming goals the tutor introduced for exploring parameters (see Figure 6).

Exploring parameters

The student first used the goals set through interaction with the tutor to compare the slopes of two lines. Later, she independently used these goals first to compare the y-intercepts of two lines and then to compare two parabolas.

The tutor first used two lines on the screen to set the goal of exploring changes in the slope.

EP Episode 1

Tutor:	What happens when you make the slope bigger?
Student:	What happens when you make the slope bigger? . . . It turns more . . . what does happen when you make the slope bigger?
Tutor:	Well, here’s one (points to line with slope 1)
Student:	Mhmm.
Tutor:	And this is two.
Student:	Mmhmm.
Tutor:	So what happened from this line to this line?
Student:	This is 1?
Tutor:	This. . .

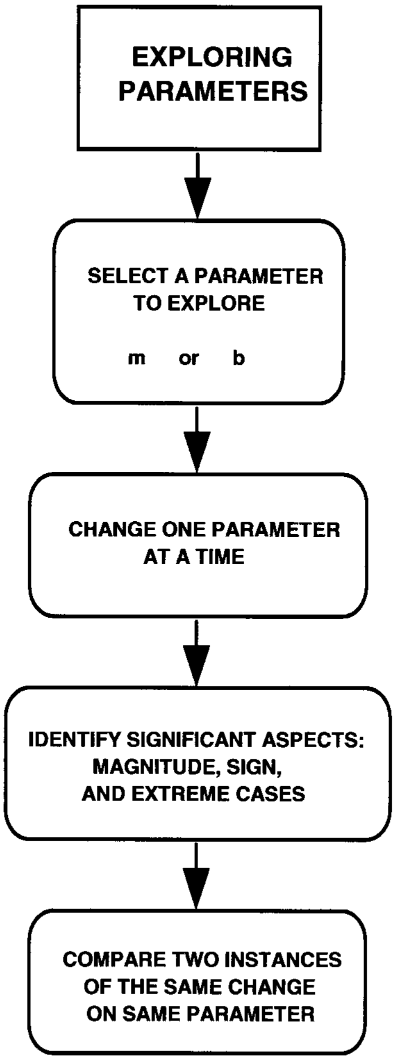

Figure 6. Goals for exploring parameters.

Student: This is 1?

Tutor: Well, point out to me which one was 1. Do you remember now?

Student: Which one was one what?

Tutor: Here's a way we can do that (demonstrates how to make each line flash on and off in the graph window). So what happened between 1 and 2?

Student: It changed a lot, but I don't understand why. I would think it would be going the other way.

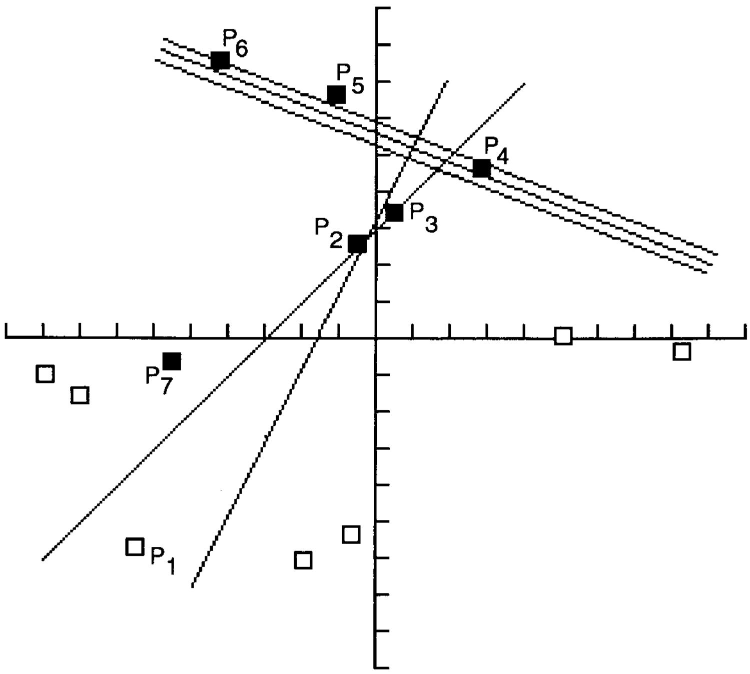

Figure 7. Exploring parameters episode 1.

Tutor:	I know. That's what you were expecting to happen, right?
Student:	Yes.
Tutor:	Well, if this was 1 and this was 2, what does increasing the slope do?
Student:	It makes it go more to the left (waves her hand in a counterclockwise direction) to this side.

The tutor simplified the task by choosing which parameter to explore and which lines to compare. After focusing on comparing the two lines on the screen, the student reached the correct conclusion: increasing the slope "makes it go more to the left (counterclockwise)."

During the second tutoring session the student and the tutor used Dynamic GRAPHER to graph and reflect on equations and their lines. Dynamic GRAPHER allows the student to change the parameters of a function using a slider and see its graph change accordingly. Although the tutor explicitly set the goal of exploring the effect of changes in m and b on the line, the student was initially at a loss for what to do or how to do it. The tutor guided her through a series of actions, asking her to first change only the y-intercept, then only the slope, and then both parameters together. Initially he provided the goals by asking the student to make lines with different y-intercepts and describing where she saw the y-intercept in the graph.

EP Episode 2

Student:	So what do I do?
Tutor:	Well, if you make it . . . make it a y-intercept of 5. (Pause) Now if you look at the graph, where do you see the 5 in the graph?
Student:	Where do I see the 5 in the graph?
Tutor:	Aha.
Student:	On the y-axis.
Tutor:	OK, now make for me . . . a y-intercept of –3 1/2. (Pause) All right so where do you see the –3 1/2?
Student:	On the y-axis.
Tutor:	Now. Why don't we click in the *m* box and see what *m* does? . . . All right make me a slope of –1. (Pause) All right. How could you look at the line and tell it had a –1 slope?
Student:	Hum.
Tutor:	Any way you could do that?
Student:	Oh, it's coming from this side, and it's going down.
Tutor:	Right, right. . . hum does it have the same y-intercept as the old line?
Student:	Yes . . . No . . .Hum . . . yes it does.
Tutor:	How can you see that from the graph?
Student:	Because it's crossing that other line. . . right here (points to where the line crosses the y-axis).
Tutor:	In the same place at the y-axis?
Student:	Yes.

In this episode, the tutor first guided the student to produce several lines and then used these lines to develop a shared focus of attention. Using the goal set by the tutor, "make me a line with ___," the student produced several lines. After the student had produced these lines, the tutor used guiding questions ("Where do you see the 5 in the graph," "Where do you see the –3 1/2 or –3.5?," "How could you look at the line and tell it had a –1 slope?," "How can you see that from the graph?," and "Does the line have the same y-intercept (or slope) as the old line?") to connect the changes in the lines with the changes in the equations. These questions guided the student to focus her attention on connecting the effect that a change in the equation had on the graph.

How are these questions part of the appropriation process? First, each of these questions referred to a line produced by the student, an important defining feature of appropriation as described by Newman et al. (1989).

Second, these questions involve the connection between a line and its equation. Questions that direct the student to locate a specific m or b on the graph or ask for the cause of a change in a line connect graphs to their equations: a change in steepness is connected to a change in m and a change in where the line crosses the y-axis is connected to a change in b.

The tutor thus guided the student to focus her attention on the specific changes that connect an equation to a line. The student was not told explicitly where m or b would show up on the graph or how changes in parameters affect the location, orientation, or steepness of the lines (or vice versa). Instead, the student and tutor participated in joint activity that involves this connection. The student and tutor developed a shared focus of attention on the changes in each parameter and on how particular changes in an equation are evident on the graph. As had occurred for estimating y-intercept and evaluating slope, the tutor first set the goals and later the student set these goals independently.

In two more instances in Session 2 (Exploring Parameters Episodes 3 and 4) the tutor once again guided the student to change m to explore and reflect on the effect of changing that parameter. However, the student did not merely repeat the goals that the tutor had introduced. The tutor had first introduced the goal of exploring the slope by changing the sign of m. The student transformed this goal and used it to also explore and reflect on the y-intercept by changing the sign of b using the slider.

EP Episode 3

Student:	OK, so I change it (the y-intercept) to negative (slides from positive to negative in the b box and back again).
Tutor:	What happens to the line?
Student:	That's making it positive (b = 5), and that's making it negative (b = –1.6), so it goes up and down on the y-axis.

Although in Exploring Parameters Episode 2 the tutor had asked the student to make two lines, one with a y-intercept of +5 and another with a y-intercept of –3.5, he had only asked her one question about where the y-intercept was on the line, "Where do you see the 5 (or –3.5) on the graph,?" but with little reflection on what this change actually did to the line. In contrast, they had a more prolonged conversation reflecting on how changes in the magnitude and sign of m affect the line. Changing the sign was a goal used explicitly by the tutor to explore and reflect on changes in m. Whether this goal is or is not appropriate for exploring changes in b is not as important as the fact that the student transformed the goal the tutor had introduced and used it to explore the other parameter in the equation.

The student transformed the goal "explore changes in the sign of *m*" and used it to explore changes in the sign of *b* as well.

Later on, during the student's second experience on Dynamic GRAPHER, she independently and systematically explored the three parameters in the equation for a parabola $y = ax^2 + bx + c$. She systematically varied both the magnitude and the sign for each parameter and concluded that "the first number makes it grow bigger or smaller," "the second number makes it move to the sides," and "the third number moves it up or down." This is further evidence that she transformed the goals for exploring the parameters of linear equations and used these goals for a different purpose, exploring parabolas.

CONCLUSIONS

The examples illustrate how this tutor introduced meanings, perspectives, actions and goals that are embedded in expert mathematical practices for using and exploring functions. For this student an important result of joint problem solving was that she appropriated mathematical perspectives, meanings, actions, and goals by shifting her focus of attention, developing shared meanings for utterances, and using new goals. The tutor engaged the student in an activity that involved expert understanding of the domain. He provided an expert interpretation of tasks, introduced goals based on this expert interpretation, and provided the student with opportunities to use these interpretations and goals. The examples also show how the student and tutor achieved a shared focus of attention: a) focusing on where a line crosses the y-axis for estimating the y-intercept, b) focusing on how the orientation of a line changes as a result of changing *m*, and c) focusing on the connection between changes in the parameters *m* and *b* in an equation and how lines look on the screen. The tutor and the student also developed shared meanings for questions such as "Can you find *b*?" as they jointly estimated y-intercepts, "Does that look like the right number?" as they jointly evaluated slopes, or "What happens when you make the slope bigger?" as they jointly explored parameters.

What is it that learners appropriate? In particular, what do learners appropriate in this mathematical domain? The analysis shows that this student appropriated new ways of seeing lines and equations by focusing her attention, new ways of talking about lines and equations by coming to share meanings for utterances, and new ways of acting by using new actions and setting new goals.

How did appropriation occur during tutoring in this domain? The examples presented here show that appropriation involved achieving a shared focus of attention, developing shared meanings for utterances, and using new actions and goals. The examples also show how the student was active in transforming the goals she appropriated. The student did not simply imitate the goals set through interaction with the tutor but sometimes transformed these goals. Although the tutor had introduced the goal of evaluating slope for the purpose of evaluating a computation *before* producing a line on the screen, the student used this goal to evaluate her slope calculation *after* a line had been produced. Furthermore, she first used the goal for evaluating a slope to rework her entire calculation, sign and magnitude, and later transformed this goal to changing only the sign of the slope. In the case of estimating the y-intercept, when the student was presented with a new context (a blob *on* the y-axis) she modified one of the tutor's goals. When exploring parameters the student used a goal introduced by the tutor for exploring *m*, "compare positive and negative slopes," to explore the signs of the parameter *b* as well.

Examining how the student transformed goals originally introduced by the tutor for her own purposes has served to include the learner as an active participant in the appropriation process. This case study describes how the ways that the student interpreted and used goals for different purposes are a reflection of how she actively participated in appropriation. Rather than seeing students' transformations of expert goals in unexpected ways as failures to learn, this study presents a view of transformations as a reflection of the learner's active participation in the process of appropriating mathematical perspectives, meanings, actions, and goals. Rather than seeing appropriation as imitation, this case study reminds us that the notion of appropriation includes transformation.

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NOTES

1. The design and impact of the software is described elsewhere (Schoenfeld, 1990) but is not the focus of this article.

2. The term 'shared' could be interpreted to draw on constructivist or Piagetian rather than Vygotskian theories. However, I would like to make a (subtle but important) distinction between the constructivist use of 'shared' and the Vygotskian use of 'shared.' I use 'shared' in the Vygotskian sense used by Rogoff to describe 'joint productive activity' that involves a Vygotskian, rather than Piagetian, conception of intersubjectivity:

   For Vygotsky, shared thinking provides the opportunity to participate in a joint decision-making process from which children may appropriate what they contribute for later use. For Piaget, the meeting of the minds involves two separate individuals, each operating on the other's ideas, using the back and forth of discussion for each to advance his or her own development. This discussion is the product of a two individuals considering alternatives provided socially, rather than the construction of joint understanding between partners. (Rogoff, 1990, p. 149).

3. Rogoff (1990) suggests that intersubjectivity may be especially important for this sort of learning: "Intersubjectivity in problem solving may (also) be important in fostering the development of 'inaccessible' cognitive processes that are difficult to observe or explain – as with shifts in perspective as well as some kinds of understanding and skills" (Rogoff, page 143).

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