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Essays on Dynamic Information and Incentives Management

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by

Dawei Jian

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ABSTRACT OF THE DISSERTATION

Essays on Dynamic Information and Incentives Management

by

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Dr. Long Gao, Chairperson

This dissertation consists of three essays that explore dynamic information and incentives management. Chapter 1 serves as an introduction. In Chapter 2, I study how a manufacturer should sell products through an online retail channel. It is a new class of channel contracting problem, where the retailer can privately observe and control the evolving market conditions. I characterize the optimal contract which unifies the classic first- and second-best policies: it resembles the classic second-best in the short run, but converges to the dynamic first-best in the long run. The result highlights the dual role of network effects: although network effects can improve channel surplus by expanding market size, they can also exacerbate information friction by enhancing the retailer's ability to manipulate the market. Furthermore, I provide new practical guidance: the private information per se need not hurt channel efficiency.

Chapter 3 focuses on a new class of product line design problems, inspired by the growing popularity of personalized subscriptions. Here, consumers' future preferences are determined endogenously by past purchases, current valuation, and random shocks. The

optimal design resolves a dynamic tradeoff between preventing cannibalization, extracting surplus, and exploiting consumer habituation. The results shed new lights on product line design: the classic downward distortion principle may no longer work, firms can practice first-degree price discrimination after initial sales. This chapter sheds new light on the increasing demand for personalized subscriptions.

In Chapter 4, I investigate the joint design of compensation and self-directed training schedules, motivated by the trend of self-directed learning in training practice. In the model, the salesperson privately observes his skills, exert effort in selling season, and he can self-invest hiddenly to enhance skills when training; the firm can learn from the salesperson's choice, update the training schedules and revise the sales targets over time. I present a simple implementation of this complex design, and emphasize how training exacerbates the agency problem by providing extra opportunities for the salesperson to manipulate their skills hiddenly. Therefore, I recommend that the firm downgrade training, particularly in the early stages, and implement a penetration-skimming training schedule, where the optimal training level initially increases and then declines towards the end.

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Chapter 1

Introduction

The three essays focus on the role of dynamic information and incentives in marketing. The central question is what management can achieve, when the information necessary for decision making is dispersed, privately held, and evolving dynamically over time. The agenda is driven by business practices in product diffusion, distribution channels, product-line design, and salesforce compensation. The primary machinery is dynamic programming, game theory, and mechanism design.

In particular, I study how to manage bilateral relationships under dynamic information asymmetry. The existing analytical literature predominantly focuses on the static framework with fixed information endowment. That framework cannot capture progressive information revelation and sequential decision making—two critical aspects of many businesses. As such, when and how to use dynamic private information remains an open question. The three essays attempt to address this question in the dynamic principal-agent framework, e.g., how to use dynamic information to improve channel efficiency, design the subscription services plan, and incentivize effort implicitly. This entails detailed treatment of information evolution and business specifics. The main goal is to characterize the intertemporal effects and policy implications of dynamic information.

In Chapter 2, titled "Managing Channel Profit with Network Effects", I investigates how a manufacturer should distribute network goods through an online retail channel. Network goods (e.g., smart phones and game consoles) exhibit network externalities and drive the market growth: the larger the past sales, the bigger the future market for the good. I first validate the micro-foundation empirically with real data. I then study this new class of channel contracting problem, where the retailer privately observes and controls the evolving market conditions. By formulating the problem as a dynamic principal-agent model, I use first-order approach to derive the optimal long-term channel contract. (i) It resembles the classic second-best in the short run, but converges to the dynamic first-best in the long run. This structure is driven by the dynamic interplay of persistent network effects and vanishing information friction, However, without considering network effects, classical contracts over emphasize the second-best contracts. (ii) This chapter characterizes the dual role of network effects. Although network effects can improve channel profit by expanding market size, they can also exacerbate the information friction by enhancing the retailer's ability to manipulate the market. As such, previous studies may have overestimated the profit improvement of network effects. (iii) The results provide new practical guidance: private information per se need not hurt channel efficiency, since the manufacturer can use recursive advance selling to extract new information for free. However, previous studies may have overstated the harm of information friction, ignoring the information endogeneity. The results also shed light on when and why manufacturers should overproduce supply, mitigate network effects, prefer long-term contracts, favor incumbent retailers, and improve retailer information capability, despite information friction. By highlighting how the manufacturer responses to the endogenous information friction, this study sharpens the understanding of channel theory and practice.

In Chapter 3, "Inside the Subscription Box: Product Line Design with Consumer Habituation", I seek to understand how firms can strategically exploit the information endogeneity. The motivation comes from the popular personalized subscriptions service design, where a firm faces the challenge on how to learn customers' evolving preferences, and personalize product offerings. The consumer's future preferences are shaped by past purchases—consumer habituation. By studying the new class of product line design problems, this chapter shows the optimal design differs substantially from the classic solution of second-degree price discrimination: it resolves a dynamic tradeoff between preventing cannibalization, extracting surplus, and exploiting consumer habituation strategically; when the consumers are satiating about their habits, the optimal design may entail upward distortion—even beyond and above the first-best level—to homogenize future consumer preferences. The results sharpens the understanding of product line design theory: the classic downward-distortion principle may no longer work, firms can practice first-degree price discrimination after initial sales. Practically, this study helps explain the rising popularity of personalized subscriptions: they can help firms to leverage consumer uncertainty to relax the cannibalization constraints, internalize the welfare gain from consumer habituation, and improve social welfare by reducing consumer heterogeneity. The result also calls for caution on marketing promotions: in a subscription context, although promotions can enhance consumer valuation, they can also exacerbate cannibalization by enhancing the consumer's ability to extract the information rent. As such, excessive promotions can hurt firm profit and social welfare—a stark contrast to the conventional view.

Besides the explicit monetary incentives, I also investigate how the marketing participants explore dynamic incentives with implicit incentives. Chapter 4, "Salesforce Compensation with Self-Directed Training", investigates how the firm should design compensation with self-directed training, a problem of dynamic adverse selection and intertemporal moral hazard. In the model, the salesperson privately observes his skills, exert effort in selling season, and he can self-invest hiddenly to enhance skills when training; the firm can learn from the salesperson's choice, update the training schedules and revise the sales targets over time. The optimal scheme and training schedule resolve a dynamic tradeoff between triggering implicit incentives, screening information, and maximizing efficiency. I find the optimal compensation differs from existing one but with simple implementations: (i) Quota-commission structure controls the contemporaneous adverse selection and moral hazard. (ii) Deferred compensation mitigates the dynamic adverse selection. (iii) Front-load training allowance alleviates intertemporal moral hazard. I emphasize the dark side of self-directed training: despite the efficiency gain due to skills enhancing, training exacerbates the agency problem by enhancing the salesperson's ability to manipulate the skills. Hence I recommend an inverted U-shaped training schedule: the optimal training level elevates initially and then declines till the end. The results inform practice on why firms "hires for talend and trains for skills", prefer self-directed training. I also call two cautions that firms shouldn't schedule training aggressively in the short-run, and trust the matured salespeople indefinitely in the long-run. By highlighting the role of self-directed training, this study sharpens the understanding of salesforce training and compensation theory, as well as the practice.

Chapter 2

Managing Channel Profits with Network Effects

2.1 Introduction

Network effects are the engine of many product markets; e.g., video games, e-readers, PC products and services (Nair, 2007; Tellis et al., 2009; Dubé et al., 2010; Li, 2019). They drive the market growth: the larger the past sales (installed base), the more valuable the product, the higher the future demand (Xie and Sirbu, 1995). In such markets, retailers not only hold superior knowledge of market conditions, but also control the pace of the market growth. To sell network products, how should manufacturers contract with retailers?

Two factors complicate contract design. The first is the agency problem. In a distribution channel, the two parties are strategic players pursuing divergent interests. Their relationship is often strained by information asymmetry: the retailer enjoys information advantage over the manufacturer, due to his expertise and control of consumer market conditions. The information is critical for a wide range of channel decisions (e.g., production, promotion, and pricing). Yet without right incentives, the retailer is unwilling to forgo his information advantage: although he can share the information to improve channel efficiency, he can also abuse it to extract *information rent*. As such, the agency problem arises and the channel efficiency suffers (Arya and Mittendorf, 2004).

The second complication is network effects. Despite the potential for market growth, network effects can inflict contractual headaches. First, network effects systematically change the retailer's market condition and preference over time. The larger the retail market size, the better contract terms the retailer can demand. In response, the manufacturer must dynamically adjust the price and quantity based on newly revealed information, taking into account how her adjustment affects future retailer behavior. Second, network effects endogenize information asymmetry, producing a sequence of private information. In each period, the retailer can gain new information advantage, control its magnitude through sales, thereby enhancing his bargaining position. In response, the manufacturer must screen information sequentially, pay higher information rent, and intensify sales distortion. These requirements impose sequential incentive constraints, involving multi-dimensional private information that arises endogenously over time. A nontrivial task for contract design.

The channel literature is largely silent on how to write such a contract. The existing literature has mainly focused on static settings with exogenous information asymmetry; it offers limited guidance on how to sell network goods through distribution channels over time. In this paper, I seek to understand how network effects drive channel contacting and long-run performance. I address three questions: (i) What is the optimal contract for a network good? (ii) How do network effects change the existing insights? (iii) What are the managerial implications?

I model the bilateral channel relationship as a dynamic game with endogenous information asymmetry. The retailer has superior knowledge about evolving conditions of his retail market, as driven by network effects and random shocks. At the outset, the manufacturer offers a long-term contract that governs the channel for multiple periods. Both parties can learn over time. In each period, the retailer can learn new market information and expand market size through sales. From the retailer's choice, the manufacturer can also infer new market information; she can then update her belief and revise production schedule. Both parties are strategic, forward-looking, and profit maximizing.

This work makes three contributions. The first is to characterize the optimal contract. The main conceptual challenge is how to *price* the retailer's information advantage: he can privately observe and control the evolving market condition. Exploiting this advantage, he can misreport for higher profit. To dissuade him, the manufacturer must pay the potential gain the retailer expects from all the misreporting opportunities. I show each act of selling has both carryover and network effects, which measure the marginal impact of current condition and sales on future market conditions. Using these two notions, it is able to pin down the information rent as the sum of the weighted misreporting gains in all future periods, where the weight is the carryover effect (adjusted for network effects). Hence, the information rent—the price for truthful information sharing—is precisely the option value of all the misreporting opportunities during the entire relationship.

I find the optimal contract differs substantially from conventional ones in structure and performance. It resolves a dynamic tradeoff between exploiting network effects, screening new information, and optimizing channel efficiency. (i) To exploit network effects, the manufacturer should set aggressive sales schedules and payment terms. The purpose is to motivate the retailer to work harder, sell more, and thus spur market growth. (ii) To screen new information, the manufacturer should offer advance selling recursively: she should charge expected future rent in the current period, and refund it later contingent on future market condition. This recursive mechanism ensures that the retailer has a stake in future channel efficiency, thereby committing him to sharing information truthfully over time. (iii) To optimize channel efficiency, the manufacturer should price discriminate across type and over time: she should sell and pay more for a retailer with larger market size, and she should adjust the sales dynamically based on newly revealed information. The payment differential ensures that the retailer is willing to sell to the best of his market potential. The dynamic adjustment helps the manufacturer tailor production to actual market size, thereby adapting to changing market conditions (He et al., 2008).

As a result, the optimal contract unifies the classic first- and second-best policies: in the short run, it resembles the second-best, because the information friction is still severe; in the long run, it converges to the first-best (adjusted for network effects), because information friction vanishes but network effects persist. The pace of convergence depends on the rate of market carryover across time. The long-run performance is primarily driven by market carryover and network effects: indeed, the prior distribution of the retailer's initial condition (type)—the main driver of the classic second-best—plays no role in the long run.

The second contribution is to characterize the dual role of network effects. The first is the well-known *efficiency role*: by increasing market size, network effects can improve channel surplus. The second is the novel *agency role*: by enhancing the retailer's ability to manipulate future markets, network effects can either alleviate or aggravate information friction (asymmetry). As such, network effects induce *counterveiling incentives*. Depending on how they shape the future market distribution, the manufacturer may either encourage market growth by promoting sales, or moderate market growth by restricting sales. The sales restriction tends to occur in the early stage of the relationship, when information friction is still severe. As such, network effects can *reduce* channel efficiency—a stark contrast to the conventional view.

Ignoring network effects, however, conventional policies can mislead contract design. For example, the classic second-best policy is suboptimal for network goods. If the manufacturer follows it, the concern of information asymmetry would dictate her contract design, resulting in perpetual rent payment and sales distortion. But this outcome can be misguided, unnecessary, and costly. In practice, a channel relationship is usually long-term, involving two-sided learning. Through repeated interactions, the retailer can learn manufacturer-specific information (type), while the manufacturer can also learn retailer-specific information, thereby reducing the information friction. If the contract is properly structured, the information friction will be short-lived. Indeed, under the optimal contract, the manufacturer needs to pay the information rent only for extracting the initial information; thereafter, she can use recursive advance selling to extract all future private information at no cost. Hence, information asymmetry should not dictate long-term contract design. To the extent channel relationships are long-term, previous studies may have overstated the impact of information asymmetry.

The third contribution is to provide practical guidance. This work identifies when and why firms may restrict network effects, overproduce output, offer recursive advance selling, favor incumbent retailers, and improve retailer forecasting capability, despite information asymmetry. More importantly, this work helps rationalize the prevalence of long-term contracts. Intuitively, supply and demand conditions change stochastically over time. As such, the manufacturer should prefer short-term contracts, because they provide more flexibility to manage supply and demand risks. Yet long-term contracts are prevalent. Why?

I propose a new explanation. The central argument is that long-term contracts can mitigate two deficiencies short-term contracts suffer: (i) without a long-term perspective, a short-term contract orders too little, traps the channel in a low-efficiency equilibrium, and hence wastes the potential of the retail market; (ii) because it is renewed every period, the short-term contract allows the retailer to retain real information advantage over time; this deficiency prolongs output distortion and rent payment, thereby perpetuating efficiency loss. By contrast, long-term contracts can mitigate both growth and information deficiencies. Hence they can outperform short-term contracts in a wide range of situations. The improvement is substantial, when the relationship is durable and the growth rate is high. In these circumstances, long-term contracts should prevail.

2.2 Literature Review

This work studies a new class of channel problems. It connects contract design with network effects and advance selling. Advance selling decouples purchase from consumption: it allows sellers to book sales long before customer consumption. The literature provides several justifications for advance selling (Xie and Shugan, 2009). For example, advance selling can segment the market for price discrimination (Dana, 1998), hedge demand uncertainty (Subramanian et al., 1999), divert excess demand off peak times (Gale and Holmes, 1993), mitigate capacity shortage (Desiraju and Shugan, 1999), leverage buyer uncertainty (Xie and Shugan, 2001), and profit from customer cancellations (Xie and Gerstner, 2007). This work offers a new justification—advance selling can also serve as a screening device to alleviate adverse selection. To my knowledge, the screening role of advance selling is new to the literature.

Network effects arise when consumer demand increases in past sales (network size). As demand-side economies of scale, they are a defining feature of many product markets (Parakhonyak and Vikander, 2019); for comprehensive reviews, please see Farrell and Klemperer (2007), Liu and Chintagunta (2009), and Nair (2019).[1] A central issue is how to price

[1]The empirical studies are extensive, including e.g., video games (Dubé et al., 2010; Prieger and Hu, 2012; Lee, 2013; Chao and Derdenger, 2013; Derdenger, 2014), game consoles (Nair, 2007; Liu, 2010), PDAs (Nair et al., 2004), PC products and services (Tellis et al., 2009), digital TVs (Gupta et al., 1999; Bhaskaran and Gilbert, 2005), network services (Katona et al., 2011), razors and blades (Hartmann and Nair, 2010), movies, books and e-readers (Li, 2019). See Liu and Chintagunta (2009), and Nair (2019) for comprehensive reviews.

network goods over time; the fundamental tradeoff is intertemporal, between investing in new customers (penetration pricing) and harvesting existing consumers (skimming pricing); the theoretical underpinning is dynamic programming; see, e.g., Dhebar and Oren (1985), Dhebar and Oren (1986), Bensaid and Lesne (1996), Xie and Sirbu (1995), Cabral et al. (1999), Chien and Chu (2008), and Radner et al. (2014).[2] Two main results are: (i) network effects usually improve system efficiency and benefit firms; (ii) to exploit network effects, the optimal output should exceed the level specified by marginal-cost pricing (the static first-best).

This work enriches this literature in two ways. First, this literature mainly focuses on business-to-consumer (B2C) settings. Yet in practice, business-to-business (B2B) transactions are also common (Lilien, 2016). I show once the retailer is in the game, the new strategic behaviors arise, and they can change the existing insights considerably. Second, this literature usually assumes complete information, revealing only the bright side of network effects. Network effects also have a dark side: once information asymmetry is considered, network effects *can* exacerbate agency cost, reduce efficiency, and hurt firms; the optimal output can go either above the classic first-best or below the second-best. Ignoring the dark side, previous studies may have overestimated the benefit of network effects.

The channel literature is extensive. A central theme is how to design contracts to reduce inefficiencies. Two culprits are *double marginalization* and *information asymmetry* (Tirole, 1988).[3] Double marginalization arises when contracts fail to internalize vertical

[2] There is also a stream of literature that studies *static pricing* under network effects. See, e.g., Leibenstein (1950), Rohlfs (1974), Katz and Shapiro (1985), Sundararajan (2003), Economides (1996), Sun et al. (2004), Li (2005), Chen and Xie (2007), Jing (2007), Csorba (2008), Prasad et al. (2010), He et al. (2012), He et al. (2017), and Veiga (2018). By contrast, I study *dynamic pricing* of network goods in a distribution channel.

[3] Conceptually, double marginalization dates back to Spengler (1950), the first-best stems from Pigou (1947)'s analysis of externalities, and the second-best goes back to Mirrlees (1971).

externalities, e.g., as in wholesale pricing (Kolay and Shaffer, 2013). The literature proposes various coordinating contracts, e.g., quantity discount, two-part tariff, and revenue sharing. A main insight is the *internalization principle* (Hermalin, 2009): by internalizing vertical externalities, these contracts can achieve the first-best under full information.[4] The full-information assumption simplifies the analysis by avoiding incentive compatibility constraints, but it is often inadequate for modeling realistic channel relationships (Chu, 1992; Arya and Mittendorf, 2004; Jiang et al., 2016; Dukes et al., 2017). Indeed, the assumption implies that all parties are willing and able to communicate and act upon all the information fully, costlessly, and instantaneously. Such a *frictionless* world has no place for strategic uncertainty. But practitioners do not operate in a frictionless world—they must act with limited information. Once information asymmetry is considered, however, strategic uncertainty sets in, new interactions emerge, and the second inefficiency arises.

The channel literature on information asymmetry builds on adverse selection and signaling paradigms (Sudhir and Datta, 2008).[5] A central theme is how to reduce inefficiencies of information asymmetry.[6] The basic idea is simple: to elicit private information,

[4]The full-information-coordination result has also been extended to dynamic settings; see, e.g., Shugan (1985); Jørgensen (1986); Eliashberg and Jeuland (1986); Chintagunta and Jain (1992); Jørgensen and Zaccour (2003); Chiang (2012). The coordination instruments include quantity discounts (Jeuland and Shugan, 1983), two-part tariff (Moorthy, 1987), franchise agreements (Desai and Srinivasan, 1995), forward buying (Desai et al., 2010), return allowance (Padmanabhan and Png, 1997; Arya and Mittendorf, 2004; Wang et al., 2020), bargaining power (Iyer and Villas-Boas, 2003), dual channels (Arya et al., 2007), revenue sharing (Cachon and Lariviere, 2005), trust (Özer et al., 2011), fairness (Cui et al., 2007; Katok et al., 2014), and bounded rationality (Ho and Zhang, 2008). They all implement the *internalization principle*, so that firms alter preferences (price sensitivity) to internalize vertical externalities (Hermalin, 2009).

[5]The channel literature on information asymmetry is extensive. See, e.g., Desai and Srinivasan (1995); Iyer (1998); Villas-Boas (1998), Desai (2000); Li (2002), Arya and Mittendorf (2004), Mishra and Prasad (2005), He et al. (2008); Gal-Or et al. (2008), Guo (2009); Guo and Iyer (2010), Dukes et al. (2011), Jiang et al. (2011), Mittendorf et al. (2013), Jiang et al. (2016), Wang et al. (2019), Wang et al. (2020).

[6]There is also a vast economics literature on information asymmetry. See, e.g., Mirrlees (1971), Mussa and Rosen (1978), Maskin and Riley (1984), Baron and Besanko (1984), Laffont and Tirole (1986), Laffont (1993), Courty and Hao (2000), Battaglini (2005), Pavan et al. (2014). For book-length treatment, see Laffont and Martimort (2001) and Bolton and Dewatripont (2005).

the principal (manufacturer) should reward good outcomes and punish bad ones. Such a discriminatory scheme requires a payoff wedge between outcomes (types) to ensure *incentive compatibility* (IC). I focus on adverse selection. This literature has three general insights (Mussa and Rosen, 1978): (i) the agent (retailer) should benefit from his private information; (ii) the principal should pay information rent and distort output (sales); (iii) the first-best is unattainable. These insights assume a static context with exogenously fixed private information. They rule out the possibility to control and respond to new information arising gradually over time—a fundamental function of the channels selling network goods.

My model endogenizes information asymmetry with network effects. I show once network effects are considered, the three general insights may no longer hold: the agent need not benefit from private information, the principal need not distort all the outputs, and private information need not hurt efficiency. In particular, the first-best is attainable when private information arrives independently over time. Hence, conventional insights are not robust to the assumption of fixed private information: ignoring information endogeneity, previous studies may have overestimated the harm of information asymmetry. To my knowledge, this work is the first attempt to design an optimal channel contract for network goods, where private information evolves endogenously over time.

2.3 Model

The starting point is Jeuland and Shugan (1983). I consider a distribution channel, where the upstream manufacturer (she) produces and sells a network product through the downstream retailer (he) over T periods. Both parties are strategic, forward-looking, and profit-maximizing, with a discount factor $\delta \in (0,1)$. To ensure business continuity, the manufacturer writes a long-term contract ϕ that governs the relationship for entire T periods. The retailer has superior knowledge of evolving market conditions $\theta_t \in \Theta \equiv [\underline{\theta}, \bar{\theta}]$, while the manufacturer only knows the prior distribution F of initial θ_1. All other parameters are common knowledge.

The dynamic game plays out as follows. (i) At the outset, the manufacturer offers the retailer a contract $\phi = (q_t, T_t)_{t=1}^{T}$, which specifies the lump-sum payment T_t for supplying quantity q_t in period t (with wholesale price T_t/q_t). (ii) Each period upon observing market condition θ_t, the retailer orders quantity q_t. (iii) The manufacturer produces the order q_t at constant marginal cost c, and gets payment T_t. (iv) The retailer sells the product for revenue $R(\theta_t, q_t) \equiv \big(\theta_t - q_t\big)q_t$. (v) The market condition evolves to θ_{t+1} and the game advances to period $t+1$. Table 2.1 defines all the notations.

2.3.1 Endogenous Market Evolution with Network Effects

The markets for network goods have three key features (Liu and Chintagunta, 2009). First, the market condition can persist, because of the *carryover effect* (Dekimpe et al., 2008). In reality, market conditions $(\theta_t)_{t\geq 1}$ are often serially correlated, because of consumers' habit persistence, forward-looking behavior, and dynamic responses to marketing variables. As such, similar market conditions tend to persist for a while before changing substantially to another one (Chintagunta et al., 2006). Second, the market can grow endogenously, because of *network effects*. The greater the current sales, the more attractive

Table 2.1: Notation

Symbol	Description
$\equiv$	Equal by definition
α	Market carryover rate, $\alpha \in [0, 1)$
β	Network effect (growth) rate, $\beta \in [0, \infty)$
δ	Discount rate, $\delta \in (0, 1)$
θ_t	Retailer's realized market condition $\theta_t \in \Theta = [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$
θ^t	Retailer's realized market condition history upon time t, $\theta^t \equiv (\theta_1, \theta_2, \ldots, \theta_t)$
$\hat{\theta}^t$	Retailer's reported market condition history upon time t, $\hat{\theta}^t \equiv (\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_t)$
$\mathcal{P}$	Contracting environment
ϕ	Contracts, $\phi = (q_t, T_t)_{t=1}^T$ with quantity q_t and payment T_t
$\mathbb{E}^\phi$	Expectation taken with respect to the stochastic process induced by contract ϕ
$F(\cdot)$	CDF of θ_1, with density $f(\cdot)$
$F(\cdot\|\theta_{t-1}, q_{t-1})$	Conditional CDF of θ_t, with conditional density $f(\cdot\|\theta_{t-1}, q_{t-1})$
ε_t	Market shock in period t, $\mathbb{E}\varepsilon_t = \mu$
$G(\cdot)$	Distribution of shock ε_t
$h(\theta)$	Inverse hazard ratio, $h(\theta) = \frac{1-F(\theta)}{f(\theta)}$
c	Manufacturer's unit production cost
$R(\theta_t, q_t)$	Retailer's revenue function in period t
$J_t(\theta^t)$	Manufacturer's continuation profit for period t onward
$U_t(\theta^t)$	Retailer's continuation profit for period t onward
$w_t(\theta_t, q_t)$	Flow channel surplus in period t, $w_t(\theta_t, q_t) \equiv R(\theta_t, q_t) - cq_t$
$W_t(\theta^t)$	Continuation channel surplus for period t onward
$\lambda_t(\theta_t)$	Marginal surplus gain from network effects
$\rho_t(\theta_t)$	Marginal information cost after period t

the product becomes, the greater the future market size (Parakhonyak and Vikander, 2019; Kamada and Öry, 2020). Third, the market condition can fluctuate, because of the *random effect*. For example, the fluctuation can arise from consumer preference shift, competitive moves, and business cycles (Villas-Boas, 1999; Acemoglu, 2008).

To capture those three features, I follow the empirical literature and specify the market dynamics by

$$\theta_{t+1} = \alpha\theta_t + \beta q_t + \varepsilon_{t+1}. \tag{2.1}$$

In this model, (i) $\alpha \in [0,1)$ is the market carryover rate, which measures how current market condition affects the next one; (ii) $\beta \geq 0$ is the network effect (growth) rate, which measures how current sales stimulate future purchase—the intensity of network effects; (iii) $\varepsilon_{t+1} \sim G$ is the IID random shock, which measures the uncertainty beyond the channel's control. Given current condition θ_t and sales q_t, the future market θ_{t+1} follows the conditional distribution $F(\theta_{t+1}|\theta_t, q_t) = G(\theta_{t+1} - \alpha\theta_t - \beta q_t)$. Hence, the market conditions $(\theta_t)_{t\geq 1}$ are a Markov process controlled by sales $(q_t)_{t\geq 1}$, endogenously.[7]

Importantly, network effects endogenize information asymmetry. Because of his expertise and direct contract with consumers, the retailer is better informed about initial market condition θ_1; after each round of sales q_t, he can also observe *new information* ε_{t+1}. Given the information advantage, he can game the manufacturer in two ways: (i) by under-reporting θ_t, he can project a grim future θ_{t+1} and secure better price from the manufacturer; (ii) by manipulating sales q_t, he can partially control future market condition

[7] Specifically, θ_1 is drawn from a prior distribution F with support Θ and density f. The market shocks $(\varepsilon_t)_{t\geq 2}$ follow IID distribution $G(\cdot)$ with density $g(\cdot)$. Thus θ_t is drawn from the conditional distribution $F(\theta_t|\theta_{t-1}, q_{t-1}) = G(\theta_t - \alpha\theta_{t-1} - \beta q_{t-1})$. To avoid negative market conditions, I assume $\theta_t \geq 0$ almost surely.

$\theta_{t+1} = \alpha\theta_t + \beta q_t + \varepsilon_{t+1}$, and hence his future payoff. It turns out, these two intertemporal linkages are key to contract design.

2.3.2 Contract Design

Following Mussa and Rosen (1978), I frame the contracting problem as one of mechanism design. In this framework, the *revelation principle* simplifies the search for optimal contracts to direct truthtelling mechanisms (Myerson, 1986).[8] Because of self-selection and revealed preference, ordering quantity $\hat{q}_t$ is equivalent to *reporting* market conditions $\hat{\theta}^t \equiv (\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_t)$: they both produce the same outcome $\big(\hat{q}_t = q_t(\hat{\theta}^t), \hat{T}_t = T_t(\hat{\theta}^t)\big)$.[9] I call the retailer who has time-t history of market conditions $\theta^t = (\theta_1, \ldots, \theta_t)$ the *retailer-θ^t*. The long-term contract $\phi = (q_t, T_t)_{t=1}^T$ then reduces to a sequence of quantity-payment rules (functions), $q_t, T_t : \Theta^t \to \mathbb{R}_+$.

I first specify the retailer's behavior. Given contract ϕ, the retailer finds his best response $(\hat{q}_t, \hat{T}_t)_{t=1}^T$ by solving

$$U_1(\theta_1) = \max_{(\hat{q}_t, \hat{T}_t)_t \in \phi} \mathbb{E}\left[\textstyle\sum_{t\geq 1} \delta^{t-1}\big((\theta_t - \hat{q}_t)\hat{q}_t - \hat{T}_t\big) \;\big|\; \theta_1\right],$$

[8] The *revelation principle* states that, under full commitment, any mechanism that depends on the private information, can also be implemented by a direct mechanism in which the parties are induced to truthfully report their information (Myerson, 1986).

[9] The equivalence is well-known (Tirole, 1988): in each period t, the retailer *self-selects* order quantity $\hat{q}_t$, within the range $\mathcal{Q}$ of agreed quantities. From order $\hat{q}_t$, the manufacturer *infers* type $\hat{\theta}_t = q_t^{-1}(\hat{q}_t) : \mathcal{Q} \to \Theta^t$, supplies $\hat{q}_t$, and pays $\hat{T}_t = T_t(\hat{\theta}^t)$. Both self-selection and type-reporting procedures produce the same outcome $\big(\hat{q}_t = q_t(\hat{\theta}^t), \hat{T}_t = T_t(\hat{\theta}^t)\big)$. Following the convention (Mussa and Rosen, 1978; Moorthy, 1984; Arya and Mittendorf, 2004), I adopt the type-reporting procedure for exposition, with the understanding that I use *reporting* in the metaphorical sense.

where the expectation $\mathbb{E}[\cdot|\theta_1]$ is taken with respect to the process $(\theta_t)_{t\geq 1}$ controlled by θ_1 and $(\hat{q}_t)_{t\geq 1}$. This entails dynamic optimization: in each period t, retailer-θ^t self-selects quantity $q_t(\hat{\theta}^t)$ from the contract ϕ (or reports $\hat{\theta}^t$), to maximize his continuation payoff

$$\tilde{U}_t(\hat{\theta}^t;\theta^t) = R\big(\theta_t, q_t(\hat{\theta}^t)\big) - T_t(\hat{\theta}^t) \ + \ \delta\mathbb{E}\big[U_{t+1}(\hat{\theta}^t, \theta_{t+1})|\theta_t, q_t(\hat{\theta}^t)\big],$$

where $U_{t+1}(\theta^{t+1}) \equiv \tilde{U}_{t+1}(\theta^{t+1};\theta^{t+1})$ is his equilibrium payoff under the *truthtelling strategy* ($\hat{\theta}^t = \theta^t, \forall t$). He will accept the deal ϕ only if it is more profitable than his outside option (normalized to zero):

$$U_1(\theta_1) \geq 0. \tag{IR}$$

He will report truthfully if truthtelling is in his best interest:

$$U_t(\theta^t) = \max_{\hat{\theta}^t} \tilde{U}_t(\hat{\theta}^t;\theta^t), \quad \forall \theta^t. \tag{IC_t}$$

Anticipating the retailer's best response over time, the manufacturer seeks to maximize his payoff

$$\tilde{J}(\phi) = \mathbb{E}\left[\textstyle\sum_{t\geq 1} \delta^{t-1}\big(T_t(\theta^t) - c\cdot q_t(\theta^t)\big)\right],$$

by solving

$$J_1 = \max\{\ \tilde{J}(\phi) : \phi \in \Phi\ \}, \quad \text{where } \Phi = \{\phi : IR, IC_t, \forall t\}. \tag{$\mathcal{P}$}$$

Hence Φ is the set of all feasible contracts that respect participation and sequential incentive constraints.

2.3.3 Key Assumptions

In a parsimonious way, this model captures the essence of the problem—network effects and endogenous private information. The key assumptions are consistent with the practice and literature. (i) I assume the manufacturer can commit to a long-term contract, a common practice in B2B transactions (Lilien and Grewal, 2012). (ii) I assume the (inverse) hazard rate $h(\theta_1) \equiv \frac{1-F(\theta_1)}{f(\theta_1)}$ is non-increasing, a standard assumption in the literature.[10] (iii) To rule out triviality, I assume unit production cost c is sufficiently small so that sales are positive. (iv) I assume linear consumer demand, with retail price $P_t = \theta_t - q_t$. This standard assumption helps derive closed-form solutions (Tirole, 1988). (v) To ease exposition, I use the lump-sum payment T_t; the corresponding wholesale price is T_t/q_t. Lump-sum payments are also common in practice, e.g., in the form of fixed term contract, trade promotion, slotting pay-to-stay fees, and slotting allowances (Kuksov and Pazgal, 2007). (vi) The single stage game follows Mussa and Rosen (1978), Jeuland and Shugan (1983), and Arya and Mittendorf (2004); what set us apart are the forward-looking behavior and endogenous market dynamics—the heart of the problem.

I assume no specific contract forms. Indeed, the contract space Φ is fairly general, allowing optimal contracts in arbitrary forms. This approach avoids the pitfall of optimizing over restrictive classes (e.g., wholesale and linear contracts), which amount to limiting the manufacturer's bargaining power, tying her hands for coordination, and imposing extra inefficiencies. Instead, I first identify the general optimal contract, and then implement it with commonly used instruments, such as quantity discount and revenue sharing.[11]

[10] The monotone hazard rate assumption is standard in the screening literature (Laffont and Martimort, 2001). It removes the case where multiple agents will select the same contract (Fudenberg and Tirole, 1991). It admits many commonly used distributions; e.g., uniform, normal, logistic, exponential, and Gamma (Bagnoli and Bergstrom, 2005).

[11] As Shugan (2005) pointed out, much of the marketing research focuses on "*analyzing the given rules of the game*", but few seek to "*design the rules of the game*". A drawback of "*analyzing the given rules*" is that it says little about the best outcome one can achieve. By contrast, I seek to *design the optimal rules* of the game $\mathcal{P}$.

In this model, both parties engage in sequential *Bayesian learning*. As time goes by, they learn new information, update posterior beliefs, and forecast future market conditions with increasing precision. To elaborate, I fix a contract ϕ. Let $f^t(\theta_{t+1}|\theta_1)$ the t-step conditional distribution of θ_{t+1} given θ_1, and $f_{t+1}(\theta_{t+1})$ be the marginal (density) distribution of θ_{t+1}. Hence, $f^1(\theta_2|\theta_1) \equiv f(\theta_2|\theta_1)$, $f^t(\theta_{t+1}|\theta_1) \equiv \int f^{t-1}(\theta_{t+1}|\theta_2) \cdot f(\theta_2|\theta_1)\,\mathrm{d}\theta_2$, $f_1(\theta_1) = f(\theta_1)$, and $f_{t+1}(\theta_{t+1}) \equiv \int f(\theta_{t+1}|\theta_t) \cdot f_t(\theta_t)\,\mathrm{d}\theta_t$. At time 1, the manufacturer forecasts the retailer's current type θ_1 with prior density distribution $f(\theta_1)$ and his future type θ_{t+1} with *marginal distribution* $f_{t+1}(\theta_{t+1})$; by contrast, the retailer knows his type θ_1 precisely (i.e., point mass δ_{θ_1} at θ_1), but predicts his future type θ_{t+1} with *conditional distribution* $f^t(\theta_{t+1}|\theta_1)$. In period $t+1$, upon observing θ_{t+1}, the retailer updates his prediction of type θ_{t+1} from $f(\theta_{t+1}|\theta_t)$ to θ_{t+1} (i.e., point mass $\delta_{\theta_{t+1}}$ at θ_{t+1}), and updates his belief about future type θ_s from $f^{s-t}(\theta_s|\theta_t)$ to $f^{s-t-1}(\theta_s|\theta_{t+1})$—a step closer to the revelation of θ_s. The manufacturer can learn from interacting with the retailer: using a screening contract, she can elicit the new information θ_{t+1} and update her belief accordingly. As such, this model captures the *ex ante* and *ex post* knowledge of each party in each period, as well as their updating mechanisms. This is the standard treatment in the literature; see, e.g., Bolton and Dewatripont (2005, ch. 9).

To focus on network effects, I abstract away from other well-studied issues; e.g., inventory, bargaining, and competition. These issues are important, but they are beyond the scope of this paper. Indeed, the problem $\mathcal{P}$ itself is a complex game with compound effects. To explicate each effect, I consider four regimes in sequel (Table 2.2), with increasing complexity in information structure and market dynamics. For these games, the proper solution concept is *perfect Bayesian equilibrium* (PBE).

Table 2.2: Four Regimes

	No Network Effect ($\beta = 0$)	Network Effect ($\beta > 0$)
Full Information	$\bar{\mathcal{P}}^n$	$\bar{\mathcal{P}}$
Asymmetric Information	$\mathcal{P}^n$	$\mathcal{P}$

Note: I use bar $\bar{(\cdot)}$ to denote full information, and superscript n to denote "*n*o network effects".

2.4 Full Information Benchmarks

To appreciate the role of information asymmetry, I first establish full-information benchmarks. In the regime $\bar{\mathcal{P}}$, the manufacturer has *perfect* visibility and control. She can observe the retailer's behavior, monitor his market condition θ_t, and dictate his sales q_t. Such tight control preempts any manipulation. With perfect visibility, the manufacturer only needs to ensure retailer participation. Her problem becomes

$$\bar{J}_1 = \max\{\ \tilde{J}(\phi) :\ IR\ \}. \tag{$\bar{\mathcal{P}}$}$$

To ensure (IR), it suffices to charge sales revenue $T_t = R(\theta_t, q_t)$. Let $w_t(\theta_t, q_t) \equiv R(\theta_t, q_t) - cq_t$ be the flow channel surplus in period t. The problem $\bar{\mathcal{P}}$ then reduces to a centralized control that maximizes the net present value of channel surplus: $\bar{J}_1 = \max_{q_1,\ldots,q_T} \mathbb{E}\big\{ \sum_{t=1}^{T} w_t(\theta_t, q_t) \big\}$.

The key issue is how to set sales (production) schedules to exploit network effects. The exploitation requires extra current sales to stimulate future markets. Doing so improves future revenue but may hurt current profit. The main tradeoff is *intertemporal*, between current cost overrun and future network gain. To solve $\bar{\mathcal{P}}$, I reformulate it as a Markov decision process, with Bellman equations

$$J_t(\theta^t) = \max_{q_t \ge 0} \Big\{ R(\theta_t, q_t) - cq_t \;\; + \;\; \delta \mathbb{E} J_{t+1}(\theta^t, \alpha\theta_t + \beta q_t + \varepsilon_{t+1}) \Big\}, \quad \forall t \le T, \tag{2.2}$$

and boundary condition $J_{T+1}(\theta^{T+1}) \equiv 0$.[12] I find:

Proposition 2.1 *(a) In regime $\bar{\mathcal{P}}^n$ without network effects, the optimal contract $\bar{\phi}^n$ is*

$$\bar{q}_t^n(\theta_t) = \tfrac{1}{2}(\theta_t - c), \qquad \bar{T}_t^n(\theta_t) = R\big(\theta_t, \bar{q}_t^n(\theta_t)\big), \quad \forall t, \theta_t. \tag{2.3}$$

(b) In regime $\bar{\mathcal{P}}$ with network effects, the optimal contract $\bar{\phi}$ is

$$\bar{q}_t(\theta_t) = a_t\theta_t - b_t c + d_t\mu, \qquad \bar{T}_t(\theta^t) = R\big(\theta_t, \bar{q}_t(\theta_t)\big), \quad \forall t, \theta_t, \tag{2.4}$$

where a_t, b_t, and d_t are constants decreasing in t.[13]

(c) Under full information, all else equal, network effects increase sales and channel surplus:

$$\bar{q}_t(\theta_t) - \bar{q}_t^n(\theta_t) \; = \; \tfrac{\delta}{2}\lambda_t(\theta_t), \quad W_1(\bar{\phi}) \ge W_1(\bar{\phi}^n),$$

where $\lambda_t(\theta_t) = \beta\mathbb{E}\big[\bar{q}_{t+1}(\theta_{t+1}) - \frac{\alpha}{\beta}\left(\theta_{t+1} - 2\bar{q}_{t+1}(\theta_{t+1}) - c\right) \big| \theta_t\big]$.

Part (a) is the first-best without network effects. In the regime $\bar{\mathcal{P}}^n$, the retailer faces exogenous market conditions, $\theta_{t+1} = \alpha\theta_t + \varepsilon_{t+1}$. His current sales q_t have no bearing on the future. To control him, the manufacturer solves a myopic optimization: the optimal quantity $\bar{q}_t^n$ equalizes the marginal revenue $\frac{\partial}{\partial q_t} R(\theta_t, q_t) = \theta_t - 2q_t$ with the marginal cost

[12] Without network effects ($\beta = 0$), the problem $\bar{\mathcal{P}}$ reduces to $\bar{\mathcal{P}}^n$.

[13] The constants are defined recursively by $a_t = \frac{1+[a_{t+1}(\delta\beta+2\alpha\delta)-\alpha\delta]\alpha}{2-[a_{t+1}(\delta\beta+2\alpha\delta)-\alpha\delta]\beta}$, $b_t = \frac{1+[b_{t+1}(\delta\beta+2\alpha\delta)-\alpha\delta]}{2-[a_{t+1}(\delta\beta+2\alpha\delta)-\alpha\delta]\beta}$, $d_t = \frac{[(a_{t+1}+d_{t+1})(\delta\beta+2\delta\alpha)-\delta\alpha]}{2-[a_{t+1}(\delta\beta+2\alpha\delta)-\alpha\delta]\beta}$, $a_T = \frac{1}{2}$, $b_T = \frac{1}{2}$, and $d_T = 0$.

c. This is a demanding offer: it allows the manufacturer to perfectly match supply with demand, fully extract the *channel surplus* $W_1 = \mathbb{E}\big[\sum_{t\geq 1}\delta^{t-1}w_t(\theta_t, q_t)\big]$, and leave the retailer with zero profit (rent).

Part (b) is the first-best with network effects. The key notion is the (marginal) surplus gain from network effects, $\lambda_t(\theta_t) \equiv \frac{\partial}{\partial_{q_t}}\mathbb{E}[W_{t+1}|\theta_t]$. In regime $\bar{\mathcal{P}}$, the retailer has *endogenous* condition $\theta_{t+1} = \alpha\theta_t + \beta q_t + \varepsilon_{t+1}$. His current sales q_t have both short- and long-term consequences: by selling $\mathrm{d}q_t$ more than the first-best $\bar{q}_t^n$, the channel will suffer short-term profit loss $[(\theta_t - 2q_t) - c]\cdot \mathrm{d}q_t$, but it can also gain from long-term network effects $\lambda_t(\theta_t)\cdot \mathrm{d}q_t$. The gain comes from expanding future market size (at rate β), enhancing future revenue ($R_\theta \geq 0$), and increasing future surplus ($\lambda_t(\theta_t) \geq 0$). The optimal contract $\bar{\phi}$ balances short-term loss with long-term gain, resulting in penetration pricing. Given perfect visibility, the manufacturer can still extract the entire surplus, and the retailer still has no ability to make profits.

Part (c) shows how to leverage network effects. First, relative to the myopic first-best $\bar{q}_t^n$, the manufacturer in $\bar{\mathcal{P}}$ should sell more aggressively. The upward adjustment $(\bar{q}_t - \bar{q}_t^n) = \frac{\delta}{2}\lambda_t$ is to *invest* in market growth; once the growth materializes, she can then *harvest* the increased market size θ_{t+1} by expanding sales to $\bar{q}_{t+1}$. Second, the adjustment is driven by three factors: the greater the market carryover α, the higher the growth rate β, the longer the relationship T, the bigger the upward adjustment. When network effects vanish, so does the upward adjustment, i.e., $\bar{q}_t^n = \lim\limits_{\beta\to 0}\bar{q}_t$ uniformly.

The key takeaway is that, the network effects *enhance surplus.* By increasing future market size, network effects improve the value of the option to expand future sales for higher margin (Dixit, 1994). The policy implications are immediate. Under full information, the manufacturer should raise output in early periods to internalize the option value of market growth; she should use *penetration pricing* with decreasing price path; absent information friction, she can expropriate the entire gain from network effects.

These are classical results (Liu and Chintagunta, 2009). They assume full (complete) information and simplistic behavior—firms are omniscient with perfect visibility. This framework simplifies analysis by avoiding *strategic uncertainty*:[14] Each firm can read the rival's mind, observe his internal operations, predict his future maneuvers, all in perfect precision. But such simplification also produces unrealistic predictions. For example, the manufacturer can extract the entire channel surplus, while the retailer has no ability to make any profits. This prediction is hard to square with reality. Most retailers do make profits. For example, Apple's retailers enjoy 4.5% profit margin on iPhone X sales (Aulakh, 2017); the video game retailers register 5% profit margin on Nintendo Switch consoles (Iggy, 2017).

The discrepancy here is driven by the full-information assumption. In reality, a manufacturer only has limited visibility into a retailer's internal operations: she may have a reasonably good forecast (distribution) about the retailer's market condition and behavior, but not in perfect precision. Such strategic uncertainty can change her calculation in a fundamental way (MORRIS, 1995): the rational manufacturer must form the prior $F(\theta_1)$, screen new information θ_t, adjust output q_t, and update her belief $\theta_{t+1} \sim F(\cdot|\theta_t, q_t)$ dynamically over time. Yet the full-information framework $\bar{\mathcal{P}}$ has no place for such strategic

[14]Strategic uncertainty is the uncertainty concerning the purposeful behavior of players in an interactive decision situation (Brandenburger, 1993).

uncertainty—the core of this problem. To make credible predictions, I must model information asymmetry and strategic behavior it entails.

2.5 Contracting under Dynamic Information Asymmetry

When the retailer has superior information on market condition θ_t, new interactions arise. Behind the veil of information asymmetry, he can manipulate market information to secure a better price. This threat compels the manufacturer to pay information rent and distort sales—the classical solution for adverse selection. As a result, the retailer profits from information advantage, and the channel efficiency suffers (Arya and Mittendorf, 2004).

When the retailer can also use network effects to manipulate market growth, he becomes much harder to control. First, network effects enhance his information advantage. Beyond the initial condition θ_1, he can also observe new conditions $(\theta_2, \ldots, \theta_T)$ arising gradually over time. Second, network effects provide a new way to extract rent: by manipulating current sales q_t, the retailer can control the distribution $F(\cdot|\theta_t, q_t)$ of future market condition θ_{t+1}, endogenize information asymmetry, and shape his future profits. Therefore, network effects enhance the retailer's ability to manipulate.

In response, the manufacturer must entertain new solutions. First, she must ensure truthtelling in every period, because information asymmetry persists over time. Second, she must consider how current sales affect the retailer's future behavior, because network effects endogenize information asymmetry. The key is how to set contingent sales targets, across type and over time. To better understand information friction and network effects, I examine two regimes $\mathcal{P}^n$ and $\mathcal{P}$ in sequel.

2.5.1 Regime $\mathcal{P}^n$: The Price for Sequential Information Sharing

In regime $\mathcal{P}^n$, only information friction is at work ($\beta = 0$). The market condition θ_t evolves *exogenously* over time. Due to proximity and expertise, the retailer can privately observe multiple market signals: before contracting, he observes θ_1; after each sales q_t, he observes new information ε_{t+1} and infers $\theta_{t+1} = \alpha\theta_t + \varepsilon_{t+1}$. The manufacturer cannot observe $(\theta_t)_{t\geq 1}$, so she must screen them to better match supply with demand. Screening requires sequential incentive constraints $(IC_t)_{t\geq 1}$ to guarantee truthtelling in all periods. The central issue is how to price the retailer's information advantages. The contracting problem becomes

$$J_1^n = \max\{\ \tilde{J}(\phi):\ IR, IC_t, \forall t\,\}. \tag{$\mathcal{P}^n$}$$

Proposition 2.2 *In regime $\mathcal{P}^n$, the optimal contract ϕ^n set quantity and payment as*

$$q_t^n(\theta^t) = \tfrac{1}{2}(\theta_t - c) - \tfrac{1}{2}h(\theta_1)\alpha^{t-1}, \qquad T_t^n(\theta^t) = R\big(\theta_t, q_t^n(\theta^t)\big) - U_t(\theta^t) + \delta\mathbb{E}[U_{t+1}(\theta^{t+1})|\theta_t],$$

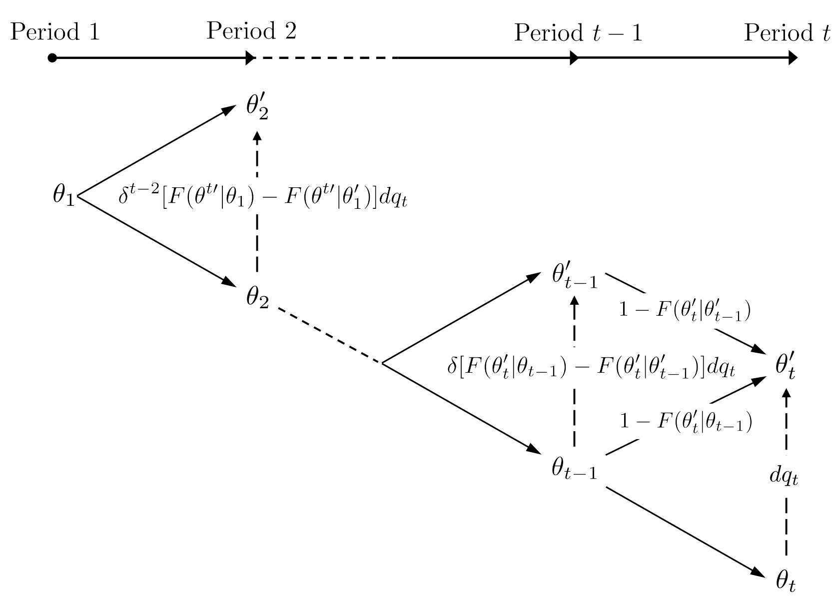
where $U_t(\theta^t) = \int_{\underline{\theta}}^{\theta_t} \mathbb{E}\big[\sum_{\tau\geq t}\delta^{\tau-t}\alpha^{\tau-t}\cdot q_\tau^n(\theta^\tau)|\tilde{\theta}_t\big]\cdot \mathrm{d}\tilde{\theta}_t$ is the profit (information rent) for retailer-θ^t.

In regime $\mathcal{P}^n$, the manufacturer has imperfect information. She offers the contract for two purposes: to screen information and to extract surplus. When the firms are myopic ($\delta = 0$), their interaction is one-shot: in this *c*lassic world $\mathcal{P}^c$, the optimal solution ϕ^c is to pay information rent and restrict sales. The solution allows the retailer-θ_1 to profit $U_1(\theta_1)$ from the market advantage $R_\theta\,\mathrm{d}\tilde{\theta}_1 = q_1^n\,\mathrm{d}\tilde{\theta}_1$ he has over the low type peers. Instead of truthtelling, he has the *option* to underreport θ_1 and pocket in $q_1^n\,\mathrm{d}\tilde{\theta}_1$. To keep him honest, the manufacturer must pay the corresponding gain (rent) from the misreporting option.

The resulting rent $U_1(\theta_1) = \int_{\underline{\theta}}^{\theta_1} q_1^n \, d\tilde{\theta}_1$ drives the payoff wedge (gap) between retailer-θ_1 and $\underline{\theta}$. Moreover, restricting sales q_1^n for $\tilde{\theta}_1 < \theta_1$ can reduce rent payment $U_1(\theta_1)$. This is the central idea of the classic second-best ϕ^c (Mussa and Rosen, 1978).

When the firms are *forward-looking* ($\delta > 0$), they interact repeatedly (Che et al., 2007). The classic solution is no longer sufficient. As fresh information $(\theta_t)_{t\geq 2}$ arises over time, the retailer gains new information advantages over the manufacturer. Because of market carryover ($\alpha > 0$), his advantage also spreads over time: not only does he enjoy the market advantage today, but also he tends to enjoy it tomorrow. To keep him honest, the manufacturer must discipline him sequentially: she must pay for all the future advantages, in every period t affected by initial θ_1. As such, the total rent $U_1(\theta_1)$ must *price in* all the misreporting opportunities, across type and over time. A challenging task.

Figure 2.1: The evolution of the market condition and misreporting incentives



The core of the task is to compute the present value of market-θ_1's carryover effect. Along each path θ^t, the initial condition θ_1 has the residual effect $\frac{\partial}{\partial\theta_1}\mathbb{E}[\theta_t|\theta_1] = \alpha^{t-1}$. This intertemporal linkage confers market advantage $R_\theta \, \mathrm{d}\tilde{\theta}_1 = q_t^n \, \mathrm{d}\tilde{\theta}_1$, resulting in the revenue gain $\alpha^{t-1} \cdot q_t^n \, \mathrm{d}\tilde{\theta}_1$. Driven by initial θ_1, this is the requisite wedge for enforcing IC_t. Taking all such gains (wedges) into account, the manufacturer must "average" them over time and across type, paying retailer-θ_1 the extra rent $\mathbb{E}\big[\sum_{t\geq 1} \delta^{t-1}\alpha^{t-1}q_t^n|\tilde{\theta}_1\big]\,\mathrm{d}\tilde{\theta}_1$. The extra rent is the sum of α-weighted revenues that retailer-θ_1 can gain from misreporting during the entire relationship. There are $[\underline{\theta}, \theta_1)$ such misreporting opportunities, so the manufacturer must pay retailer-θ_1 the total rent

$$U_1(\theta_1) = \int_{\underline{\theta}}^{\theta_1} \mathbb{E}\big[\textstyle\sum_{t\geq 1} \delta^{t-1}\alpha^{t-1} \cdot q_t^n(\theta^t) \,|\, \tilde{\theta}_1\big] \cdot \mathrm{d}\tilde{\theta}_1. \tag{2.5}$$

The rent payment is the *shadow price* for enforcing sequential truthtelling $(IC_t)_{t\geq 1}$. It is the price that the manufacturer must pay to dissuade the retailer from exercising his misreporting options over time. These options stem from the manufacturer's uncertainty (across type) and retailer's market carryover (over time). As such, the rent payment is the *externalities* imposed by uncertainty and carryover, through incentive constraints $(IC_t)_{t\geq 1}$ on contract design. The greater the manufacturer's uncertainty (large $h(\theta_1)$), the greater the market advantage ($R_\theta = q_t^n$), the stronger the market carryover α^{t-1}, the larger the rent payment. I can simplify the expected rent as[15]

$$\mathbb{E}U_1(\theta_1) = \mathbb{E}\left[\textstyle\sum_{t\geq 1}\delta^{t-1}\alpha^{t-1}q_t^n(\theta^t)\cdot h(\theta_1)\right] = \mathbb{E}\big[U_1'(\theta_1)\cdot h(\theta_1)\big]. \tag{2.6}$$

It is the expected product of marginal rent $U_1'(\theta_1)$ and hazard rate $h(\theta_1)$, depending on θ_1 only. This suggests that the rent is paid entirely for screening initial θ_1. As I shall show in §4, all new information $(\theta_t)_{t\geq 2}$ can be screened at no cost.

The rent structure (2.5) guides production planning. It suggests that the manufacturer can reduce rent $U_1(\theta_1)$ by restricting quantity $q_t^n(\theta_1', \theta_2^t)$ for lower type retailers $\theta_1' < \theta_1$. The restriction reduces the efficiency of low type retailer-θ_1', but it helps reclaim the rents the manufacturer would otherwise concede to high type retailers. This is the key idea behind *quantity discount* (Moorthy, 1987). What complicates production planning is the dynamic information structure: unlike the static case, initial information θ_1 in $\mathcal{P}^n$ has long-term effects, spreading over multiple periods; moreover, the new information $(\theta_t)_{t\geq 2}$ arises over time. These dynamics allow retailer-θ_1 multiple opportunities to misreport. To discipline him, the quantity restriction should also be dynamic: not only the initial quantity for retailers $\theta_1' < \theta_1$, but also their future quantity along path (θ_1', θ_2^t) should be properly restricted.

[15]By Fubini's theorem (Dudley, 2002) and Eq. (2.5), I can exchange the order of integration, integrate by parts, and conclude:

$$\begin{aligned}\mathbb{E}U_1(\theta_1) &= \int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{\theta}}^{\theta_1} U_1'(\tilde{\theta}_1)\,\mathrm{d}\tilde{\theta}_1\cdot f(\theta_1)\,\mathrm{d}\theta_1 = \int_{\underline{\theta}}^{\bar{\theta}}\int_{\tilde{\theta}_1}^{\bar{\theta}} U_1'(\tilde{\theta}_1)f(\theta_1)\,\mathrm{d}\theta_1\cdot\mathrm{d}\tilde{\theta}_1\\ &= \int_{\underline{\theta}}^{\bar{\theta}}[1-F(\tilde{\theta}_1)]U_1'(\tilde{\theta}_1)\cdot\mathrm{d}\tilde{\theta}_1 = \int_{\underline{\theta}}^{\bar{\theta}}\frac{1-F(\tilde{\theta}_1)}{f(\tilde{\theta}_1)}U_1'(\tilde{\theta}_1)\cdot f(\tilde{\theta}_1)\,\mathrm{d}\tilde{\theta}_1 = \mathbb{E}[h(\tilde{\theta}_1)U_1'(\tilde{\theta}_1)] = \mathbb{E}[U_1'(\theta_1)\cdot h(\theta_1)].\end{aligned}$$

I now specify the optimal quantity q_t^n with a perturbation argument. In Fig. 2.1, consider an infinitesimal increase $\mathrm{d}q_t$ at retailer-θ^t. The perturbation ignites two countervailing effects. The first is the direct *surplus gain* $\frac{\partial}{\partial_{q_t}} w_t \cdot \mathrm{d}q_t = [(\theta_t - 2q_t^n) - c] \cdot \mathrm{d}q_t$ at θ^t, which discounts back to the extra profit at θ_1:

$$\text{surplus gain } = \delta^{t-1} \cdot \underbrace{f(\theta^t|\theta_1) \cdot f(\theta_1)}_{\text{density } f(\theta_1, \theta_2^t)} \cdot [(\theta_t - 2q_t^n) - c] \cdot \mathrm{d}q_t. \tag{2.7}$$

The second effect is the indirect *rent increase* due to sequential $(IC_t)_{t\geq 1}$. In period t, as retailer-θ^t increases sales by $\mathrm{d}q_t$, by incentive constraint IC_t, his higher type peers $(\theta^{t-1}, \theta_t')$ with condition $\theta_t' > \theta_t$ must each be paid an additional $\frac{\partial}{\partial q_t} R_\theta \, \mathrm{d}q_t = \mathrm{d}q_t$ to keep them honest. But then their predecessors $(\theta_1^{t-2}, \theta_{t-1}')$ with types $\theta_{t-1}' > \theta_{t-1}$ must also be paid $[F(\theta_t'|\theta_{t-1}) - F(\theta_t'|\theta_{t-1}')] \cdot \mathrm{d}q_t$ more, because retailer-$(\theta^{t-2}, \theta_{t-1}')$ is $[F(\theta_t'|\theta_{t-1}) - F(\theta_t'|\theta_{t-1}')]$ more likely than retailer-$(\theta^{t-2}, \theta_{t-1})$ to reach condition $\theta_t' > \theta_t$. Continuing this argument back to period 1, there are $\mathbb{P}(\theta_1' > \theta_1) = 1 - F(\theta_1)$ such retailers who have the misreporting options. Hence, the manufacturer must increase the rent payment by

$$\text{rent increase } = \delta^{t-1} \cdot [1 - F(\theta_1)] \cdot [F(\theta^{t\prime}|\theta_1) - F(\theta^{t\prime}|\theta_1')] \cdot \mathrm{d}q_t. \tag{2.8}$$

At optimum, the manufacturer must justify the rent increase (2.8) by the surplus gain (2.7), resulting in the first-order condition for optimal quantity:

$$q_t^n(\theta^t) = \underbrace{\tfrac{1}{2}(\theta_t - c)}_{\text{static first-best}} - \underbrace{\tfrac{1}{2} \cdot \underbrace{\frac{1 - F(\theta_1)}{f(\theta_1)}}_{\text{inv. hazard rate } h(\theta_1)} \cdot \underbrace{\frac{F(\theta^{t\prime}|\theta_1) - F(\theta^{t\prime}|\theta_1')}{f(\theta^t|\theta_1)}}_{\text{carryover effect } \alpha^{t-1}}}_{\text{downward distortion } \frac{1}{2} \cdot h(\theta_1) \cdot \alpha^{t-1}} = \tfrac{1}{2}(\theta_t - c) - \tfrac{1}{2} h(\theta_1) \alpha^{t-1}.$$

Intuitively, the manufacturer distorts quantity to make misreporting unprofitable for the retailer. To do it properly, she must resolve the tension between the surplus gain and rent increase. In a static context, this tension depends on the prior $f(\theta_1)$ alone; so the manufacturer only needs to decide the *size* of the distortion, $\frac{1}{2}h(\theta_1)$. In a dynamic context, the tension is far more complex—it spreads over multiple periods. So the manufacturer must decide both the *size* and *timing* of the distortion. In general, this would require complex tracking of the entire history $\theta^t = (\theta_1, \ldots, \theta_{t-1}, \theta_t)$. Yet I show this complex task has a simple solution: beyond initial θ_1, the manufacturer only needs to track current condition θ_t; then she can simply cut $\frac{1}{2}h(\theta_1)\alpha^{t-1}$ from the first-best quantity $\frac{1}{2}(\theta_t - c)$. This resolves the dynamic tension.

2.5.2 Regime $\mathcal{P}$: The Dual Role of Network Effects

In regime $\mathcal{P}$, network effects are salient ($\beta > 0$). So the retailer can use current sales q_t to manipulate future demand, $\theta_{t+1} = \alpha\theta_t + \beta q_t + \varepsilon_{t+1}$. To control him, the manufacturer must consider both information friction and network effects. The two forces in isolation put opposing pressures on output: network effects boost output while information friction depresses it, resulting in $q_t^n(\theta^t) \le \bar{q}_t^n(\theta_t) \le \bar{q}_t(\theta_t)$, $\forall \theta^t$. In regime $\mathcal{P}$ both forces are at play, how should the manufacturer control the retailer?

Network effects complicate channel contracting. They *endogenize* market process $(\theta_t)_{t\ge1}$, alter information asymmetry, and entail new manipulations. The retailer-θ_1 now can actively control his own future through sales q_t (and $F(\cdot|\theta_t, q_t)$). In response, the manufacturer must calibrate how much each retailer should sell, so that both revenue and rent payment are properly controlled. Let $\rho_t(\theta^t) \equiv \frac{\partial}{\partial q_t}\mathbb{E}\left[\sum_{\tau\ge t+1}\delta^{\tau-t}h(\theta_1)\alpha^{\tau-1}q_\tau^*(\theta^\tau)|\theta^t\right]$. I find:

Proposition 2.3 *(a) In regime $\mathcal{P}$, both parties benefit from better market condition:*

$$\frac{\partial}{\partial\theta_t}U_t(\theta^t) \geq 0, \qquad \frac{\partial}{\partial\theta_t}J_t(\theta^t) \geq 0.$$

(b) In regime $\mathcal{P}$, the optimal contract ϕ^* is

$$q_t^*(\theta^t) = \tfrac{1}{2}(\theta_t - c) - \tfrac{1}{2}h(\theta_1)\alpha^{t-1} + \tfrac{\delta}{2}\big(\lambda_t(\theta_t) - \rho_t(\theta_t)\big),$$

$$T_t^*(\theta^t) = R\big(\theta_t, q_t^*(\theta^t)\big) - U_t(\theta^t) + \delta\mathbb{E}[U_{t+1}(\theta^{t+1})|\theta_t],$$

where $U_t(\theta^t) = \int_{\underline{\theta}}^{\theta_t} \mathbb{E}\big[\sum_{\tau\geq t}\delta^{\tau-t}\alpha^{\tau-t}\cdot q_\tau^*(\theta^\tau)|\tilde{\theta}_t\big]\cdot \mathrm{d}\tilde{\theta}_t$ *is the rent payment.*[16]

(c) In regime $\mathcal{P}$, the optimal quantity for top retailer-$\bar{\theta}$ is always efficient, $q_t^*(\bar{\theta}, \theta_2^t) = \bar{q}_t(\bar{\theta}, \theta_2^t)$, $\forall\theta_2^t$; for other types $\theta_1 < \bar{\theta}$ it is bounded by $q_t^n(\theta^t) \leq q_t^*(\theta^t) \leq \bar{q}_t(\theta_t)$.

In regime $\mathcal{P}$, the optimal contract ϕ^* must reconcile two *countervailing incentives*—to exploit network effects and to control information friction—in a dynamic way. It works as follows. (i) Relative to the first-best $\bar{q}_t^n$ without network effects, the optimal quantity q_t^* employs two distortions. The *upward* adjustment $\frac{\delta}{2}\lambda_t(\theta_t)$ is productive, driven by the marginal gain from network effects, for the purpose of market growth; the *downward* distortion $\frac{1}{2}h(\theta_1)\alpha^{t-1}$ is unproductive, driven by pre-contract information θ_1, for the purpose of rent control; both distortions affect the distributions of future markets, and hence the additional adjustment $\frac{1}{2}\delta\rho_t(\theta_t)$ for ensuring *future payoff wedges.* (ii) In general, the upward adjustment persists over time, while the downward distortion vanishes in the long run, $\lim_{t\to\infty}\frac{1}{2}h(\theta_1)\alpha^{t-1} = 0$. Depending on their relative strength, the net distortion $(q_t^* - \bar{q}_t^n)$ can go either downward or upward, and the direction can change over time. It turns out,

[16] The optimal contract ϕ^* admits a closed-form solution, $q_t^*(\theta^t) = a_t\big(\theta_t - h(\theta_1)\alpha^{t-1}\big) - b_t c + d_t\mu$, where $a_t = \frac{1+[a_{t+1}(\delta\beta+2\alpha\delta)-\alpha\delta]\alpha}{2-[a_{t+1}(\delta\beta+2\alpha\delta)-\alpha\delta]\beta}$, $b_t = \frac{1+[b_{t+1}(\delta\beta+2\alpha\delta)-\alpha\delta]}{2-[a_{t+1}(\delta\beta+2\alpha\delta)-\alpha\delta]\beta}$, $d_t = \frac{[(a_{t+1}+d_{t+1})(\delta\beta+2\delta\alpha)-\delta\alpha]}{2-[a_{t+1}(\delta\beta+2\alpha\delta)-\alpha\delta]\beta}$, $a_T = \frac{1}{2}$, $b_T = \frac{1}{2}$, and $d_T = 0$. These constants decrease in t.

this dynamic interplay has a profound impact on channel performance and policies, a key property I explore in §2.6.

Proposition 2.3 reveals the dual role of network effects. (i) The first is the well-known *efficiency role*: by increasing market size, network effects can improve channel surplus. The key policy response is to make upward adjustment early in the relationship, so that market size will grow with cumulative sales. (ii) The second is the novel *agency role*: by changing the distribution of the future markets, network effects can either alleviate or exacerbate information friction. The policy response is more involved. The optimal quantity depends on how network effects shape the market condition: when network effects expand market gap $\Delta\theta_t$ between retailers, the manufacturer should limit market growth and distort sales downward; when network effects reduce the market size gap, she should encourage growth and distort sales upward. In both cases, to control rent, the manufacturer should make strategic use of network effects.

The strategic use of network effects can overturn conventional insights. One such insight is downward distortion for rent control. This insight hinges on the relation that raising output increases the value of private information, and hence inflates rent payment. The relation holds naturally in simplistic settings, with one-shot interaction and fixed private information. My results show the conventional insight may not work in dynamic settings with endogenous private information: network effects may entail upward distortion for rent control, even above the first-best level.

For channel managers, a key question is when they can benefit from network effects. The conventional view suggests they should always benefit, because network effects can enhance surplus. This is indeed the case under full information (Proposition 2.1). My view is more nuanced: under asymmetric information, network effects are a double-edged sword. Although they can improve channel surplus by expanding market size (the efficiency role), they can also exacerbate information friction by enhancing the retailer's ability to extract rent (the agency role). To control rent inflation, the manufacturer may moderate network effects, hurting channel surplus. This tends to occur in the early stage of the relationship, when the information friction is still severe. Ignoring the dark side of agency cost, however, previous studies may have overestimated the benefit of network effects. Conceptually, this insight echoes Cabral and Villas-Boas (2005): the strategic effect can overwhelm the direct effect.

2.6 Managerial Insights

I have shown that network effects can endogenize information asymmetry, induce countervailing incentives, and necessitate new contractual responses. I now examine three managerial implications of network effects.

2.6.1 The Role of Private Information and Advance Selling

By now one must have a good understanding of private information θ_1: it is a main barrier for achieving channel coordination and the *first-best* efficiency. The general insight is that it breeds opportunism, suppresses output, and reduces surplus (Mussa and Rosen, 1978).[17] This insight assumes all the private information θ_1 is endowed *before*

[17] The general insight that private information reduces efficiency has two versions (Bolton and Dewatripont, 2005): in standard screening, the loss comes from information rents; in standard signaling, the loss comes from the separation effort. There are a few exceptions. Jiang et al. (2016) and Wang et al. (2019)

contracting and remains fixed over time. In practice, private information can also arise after contracting and it can change over time. In regime $\mathcal{P}$, for example, network effects endogenize information asymmetry, allowing the retailer to learn *new information* ε_{t+1} after each round of selling q_t.[18] How does the new information $(\varepsilon_t)_{t\geq 2} = (\varepsilon_2, \ldots, \varepsilon_T)$ change the channel performance?

The question invites two opposing arguments. By the logic of adverse selection, one may argue the new information $(\varepsilon_t)_{t\geq 2}$ worsens information asymmetry: it will raise information rent, aggravate output distortion, and exacerbate efficiency loss. Hence, the new information should benefit the retailer, but hurt the manufacturer and the channel. On the other hand, the expected rent $\mathbb{E}U_1(\theta_1) = \mathbb{E}[U_1'(\theta_1) \cdot h(\theta_1)]$ depends on initial θ_1 only, independent of the new information $(\varepsilon_t)_{t\geq 2}$. This seems to implicate θ_1 but vindicate $(\varepsilon_t)_{t\geq 2}$: the new information should neither benefit the retailer, nor hurt the manufacturer or the channel—it is *innocuous*.

Is the new information truly innocuous? To have a definitive answer, I now consider a new regime $\mathcal{P}^r$, in which initial θ_1 is private, but post-contract information $(\varepsilon_t)_{t\geq 2}$ is public:

$$J_1^r = \max\{\, \tilde{J}(\phi) : IR, IC_1 \,\}. \tag{$\mathcal{P}^r$}$$

Regime $\mathcal{P}^r$ arises, e.g., when the manufacturer can mandate the disclosure of $(\varepsilon_t)_{t\geq 2}$, or the retailer can commit to sharing new information at the outset. It reduces information asymmetry to θ_1 only. So the manufacturer still needs to pay for screening θ_1, but she receives new information $(\varepsilon_t)_{t\geq 2}$ for free. Except the privacy of $(\varepsilon_t)_{t\geq 2}$, regime $\mathcal{P}^r$ is identical

show that information asymmetry can improve channel efficiency, through the offsetting mechanism of signaling and double marginalization. Since I do not restrict to wholesale contracts—the root cause of double marginalization—the model and insights are different.

[18] In period $t+1$, the market condition $\theta_{t+1} = \alpha\theta_t + \beta q_t + \varepsilon_{t+1}$ is the *cumulative information* that summarizes old information θ_t, fresh sales q_t, and *new information* ε_{t+1}.

to $\mathcal{P}$. If the conventional wisdom were correct, i.e., the privacy of $(\varepsilon_t)_{t\geq 2}$ indeed hurts, then in regime $\mathcal{P}^r$ with reduced information asymmetry, the retailer would extract less rent and the manufacturer would make more profit. Technically, the payoffs should differ because $\mathcal{P}^r$ *relaxes* future incentive constraints $(IC_t)_{t\geq 2}$. Yet I find:

Proposition 2.4 *All else equal, the retailer (manufacturer) makes the same profit in both regime $\mathcal{P}^r$ and $\mathcal{P}$.*

This is revealing: future incentive constraints $(IC_t)_{t\geq 2}$ have no bite—their *shadow price* is zero. As a result, (i) the retailer can extract the same amount of rent from initial information θ_1 alone (in $\mathcal{P}^r$), as he does from the entire process $(\theta_t)_{t\geq 1}$ (in $\mathcal{P}$); (ii) the privacy of future information $(\varepsilon_t)_{t\geq 2}$ in $\mathcal{P}$ does not reduce efficiency. Although both initial θ_1 and future information $(\varepsilon_t)_{t\geq 2}$ can be private, the initial piece is far more consequential. Why?

The answer lies in the *timing* of the private information. In $\mathcal{P}$, the retailer observes θ_1 *before* contracting; so he is certain about his advantage and rent at the contracting stage. To screen the pre-contract information θ_1, the manufacturer has no choice but to pay the rent and distort sales; hence the efficiency suffers. By contrast, the retailer observes future ε_t only *after* contracting; before period t he is also uncertain about his future shock ε_t and the exact rent it can bring; hence he enjoys no real advantage from ε_t at the contracting stage. At the outset, the retailer still knows θ_t better than the manufacturer—conditional distribution $f^{t-1}(\theta_t \,|\, \theta_1)$ vs. marginal distribution $f_t(\theta_t) \equiv \int f^{t-1}(\theta_t \,|\, \theta_1) \cdot f(\theta_1)\, \mathrm{d}\theta_1$; but that advantage flows from θ_1, not ε_t. When the future t arrives, the retailer can still gain after he observes ε_t; but the manufacturer can leverage his uncertainty of ε_t at time 1 to neutralize that gain, thereby screening future private information $(\varepsilon_t)_{t\geq 2}$ at no cost.

The screening device is *recursive advance selling*. In regime $\mathcal{P}$, to screen θ_t, the manufacturer must enforce IC_t with the payoff wedge between types θ_t and θ_t'—the technical origin of the information rent. In period 1, she has no choice but to pay the actual rent $U_1(\theta_1)$ for enforcing the wedge. Afterwards, however, she has additional time dimension to enforce the wedge: she can advance-sell future output q_{t+1} in period t, and refund later contingent on specific condition θ_{t+1}; the advance-selling price $\delta\mathbb{E}[U_{t+1}(\theta^{t+1})|\theta_t]$ is precisely the expected rent she will pay the retailer in period $t+1$. The resulting optimal payment can be decomposed into three terms (Proposition 2.3):

$$T_t^*(\theta^t) = \underbrace{R\big(\theta_t, q_t^*(\theta^t)\big)}_{\text{current sales}} + \underbrace{\delta\mathbb{E}[U_{t+1}(\theta^{t+1})|\theta_t]}_{\text{advance sales}} - \underbrace{U_t(\theta^t)}_{\text{refund}}. \tag{2.9}$$

Advance selling indeed enforces truthtelling IC_t: as a constant shift, it keeps the payoff wedge $U_t(\theta^{t-1}, \theta_t) - U_t(\theta^{t-1}, \theta_t')$ as required. Carrying out recursively, the manufacturer can extract all future information $(\varepsilon_t)_{t\geq 2}$ for free.

Proposition 2.5 *In regime $\mathcal{P}$, the manufacturer should pay rent for θ_1 only; she should use recursive advance selling to extract new information for free.*

These results sharpen the understanding of private information. In a static environment, the literature has a clear prediction: if the private information arises before contracting, it hurts efficiency; if it arises after contracting, it need not hurt efficiency. For example, Laffont and Martimort (2001, p. 58) show that, by making the retailer (agent) the "residual claimant", the manufacturer (principal) can achieve the first-best, extracting post-contract information at no cost. However, this static result rests on the simplistic assumption—*a single piece of exogenous* private information—which greatly restricts its applicability. I extend it to dynamic settings, with *multiple pieces of endogenous* private information. I show, in general, private information per se does not predict inefficiency; its effects depend on the timing and nature of the interactions it entails. Despite technical complexity, the dynamic result has much broader applications.

For example, my result helps explain a puzzling practice: leading manufacturers are willing to improve their retailers' forecasting capability, despite the threat imposed by information asymmetry. This practice is hard to explain in static models: because the private information is pre-contract, improving it can only hurt the manufacturer. The practice makes sense in the dynamic framework: the new information $(\varepsilon_t)_{t\geq 2}$ allows the manufacturer to better match supply with demand conditions, increasing surplus; because it is ex post, the manufacturer can use recursive advance selling to *tax away* all the surplus gain from $(\varepsilon_t)_{t\geq 2}$. Hence, the manufacturer is willing to improve retailer information capability, despite information asymmetry.

The results identify a new role of advance selling: screening. The extant literature rationalizes advance selling by price discrimination, demand uncertainty, and capacity constraints (Xie and Shugan, 2009). I discover a new rationale: advance selling can also serve as a screening device for eliciting private information sequentially. In my model, advance selling can achieve three objectives in one stroke: to coordinate the channel, to exploit network effects, and to screen private information. Although both output distortion and advance selling are screening devices, they differ in the efficiency and applicability: output distortion is less efficient but more applicable, as it can screen both pre- and post-contract information; by contrast, advance selling is more efficient but less applicable, as it can screen only post-contract private information. To my knowledge, the screening role of advance selling is new to the literature.

2.6.2 How Do Network Effects Affect Long-Run Performance?

Perhaps the most important prediction from the *c*lassic framework $\mathcal{P}^c$ is that, under information asymmetry, the first-best is unattainable (Mussa and Rosen, 1978).[19] Taking a static perspective, this prediction ignores *two-sided learning* in the relationship. It is true that knowing θ_1 gives the retailer information advantage initially. So information rent is imperative for truthtelling and sales distortion is necessary for rent control. Both measures reduce efficiency. Hence the first-best is unattainable in the one-shot, static world $\mathcal{P}^c$. But in the real world, a channel relationship involves multiple interactions, through which both parties can learn about each other. Given this reality, one may conjecture, the manufacturer-side learning should weaken the retailer's advantage, soften his temptation to manipulate, and dampen the distortion in the long run. When the relationship is sufficiently long, the manufacturer should be able to coordinate the channel, achieving the first-best. Therefore, perpetual distortion should be an exception ($\alpha = 1$), not the rule: the classic second-best ϕ^c is unstable in the long run. I formalize this conjecture as follows.

Proposition 2.6 *In regime $\mathcal{P}$, the optimal contract ϕ^* converges to the first-best $\bar{\phi}$ in the long run.*

The proposition bridges the classic first- and second-best solutions. It deepens the understanding of optimal channel performance (under ϕ^*). (i) In the short run, it

[19]Indeed, this prediction is the very reason *information asymmetry* is introduced in the first place (Stiglitz, 2002).

resembles the second-best (under ϕ^c), because the information friction is severe initially; in the long run, it converges to the first-best (under $\bar{\phi}$), because information friction vanishes eventually. (ii) The rate of convergence depends on market carryover rate α: the lower the carryover rate, the faster the convergence. The limit of convergence depends on the market growth rate β: the higher the growth rate, the higher the long-run limit. The extent of convergence depends on duration T: the longer the relationship, the smaller the distortion. When the market conditions are IID ($\alpha = 0$), the manufacturer can reach the first-best as early as period 2; when the market is fixed ($\alpha = 1$), however, the distortion perpetuates. (iii) The classic solution ϕ^c is optimal only for the extreme case with constant market conditions ($\alpha = 1, \beta = 0, \varepsilon_{t+1} \equiv 0$). To the extent normal markets can experience random shocks and fluctuate over time, the classic solution ϕ^c is suboptimal, unstable, even misleading for practical use. As I shall show in §2.6.3, the efficiency loss can be substantial.

One may argue this result is obvious: after all, the channel in $\mathcal{P}$ is a Markovian system, so the result can be explained by Markov convergence theorem alone. Specifically, in regime $\mathcal{P}$, given contract ϕ, retailer-θ_1 can predict future θ_{t+1} better than the manufacturer, $f^t(\theta_{t+1}|\theta_1)$ vs. $f_{t+1}(\theta_{t+1}) \equiv \int f^t(\theta_{t+1}|\theta_1) \cdot f(\theta_1)\,\mathrm{d}\theta_1$, but that advantage is limited to near future only. Irrespective of initial forecast (belief), both parties hold the same outlook f_∞ of the distant future θ_∞, because $\lim_{t\to\infty} f^t(\theta_{t+1}|\theta_1) = \lim_{t\to\infty} f_{t+1}(\theta_{t+1}) = f_\infty(\theta_\infty)$. Hence, the argument concludes, although the retailer has a better short-term forecast, he fares no better in the long run.

This argument, however, tells only half of the story. It cannot explain why sales distortion persists regardless of prior $f(\theta_1)$. Indeed, the underlying working is more subtle.

My result depends on the convergence of both forecast and sales: the former quantifies how the *forecasts* (beliefs) of market condition θ_t converge over time; the latter measures how the *residual effect* of initial condition θ_1 vanishes over time. As such, they are fundamentally different concepts, with different convergence rates and implications. Indeed, even if the channel begins with the steady-state $f(\theta_1) = f_\infty(\theta_1)$, the residual effect of initial ability θ_1, captured by α^t, will still persist. Hence, Markov convergence alone cannot explain the whole story. The mechanism for the long-run channel coordination is more nuanced: first, the *likelihood* of distortions declines over time, because $\lim_{t\to\infty} f^t(\theta_{t+1}|\theta_1) = f_\infty(\theta_\infty)$, $\forall\theta_1$; second, the *magnitude* of distortions fades away, because $\lim_{t\to\infty} \frac{1}{2}h(\theta_1)\alpha^t = 0$. Hence, it is sales convergence that holds the key to channel coordination.

Theoretically, the steady-state distribution f_∞ and equilibrium strategy ϕ^* are fundamentally different objects. Neither implies the other. (i) Steady-state distribution means that the stochastic process (dynamical system) has a stationary distribution that does not change over time (Stachurski, 2009). (ii) Equilibrium in the problem means PBE, in the sense that each party acts optimally given their posterior beliefs, that the posterior beliefs are consistent with others' best responses, and that posterior beliefs are updated with Bayes' rule whenever possible (Fudenberg and Tirole, 1991).

Proposition 2.6 has two policy implications. First, *when selecting a retailer, the manufacturer should prefer the incumbent.* This policy contrasts with the existing literature (Haucap et al., 2013), which often views the preference as entrenchment, a defect to correct.[20] I show the preference need not be defective; it has an efficiency justification:

[20] For example, (Haucap et al., 2013) finds that incumbent retailers tend to have higher bargaining power and squeeze manufacturers' profits; in response, the squeezed manufacturers set higher wholesale prices for new retailers (waterbed effect), which hurts efficiency.

the manufacturer-side learning reduces information asymmetry and distortion; for the same market condition, the incumbent retailer demands less rent, sells more, and hence is more preferable.

Second, *when the market evolves over time, the manufacturer should not pay rent or distort production indefinitely.* Rather, it should tailor the contract to the business-specific carryover rate α and duration T. When market carryover is weak and the relationship is long, the manufacturer should distort sales and pay rent only initially; she should phase out both measures and adopt the first-best eventually. The rationale is simple: both measures are meant to neutralize the retailer's information advantage at the time of contracting; they are most effective in the early stage of the relationship, when the channel is most responsive to initial condition θ_1. Next, I pinpoint the steady-state distribution and the drivers of optimal channel performance.

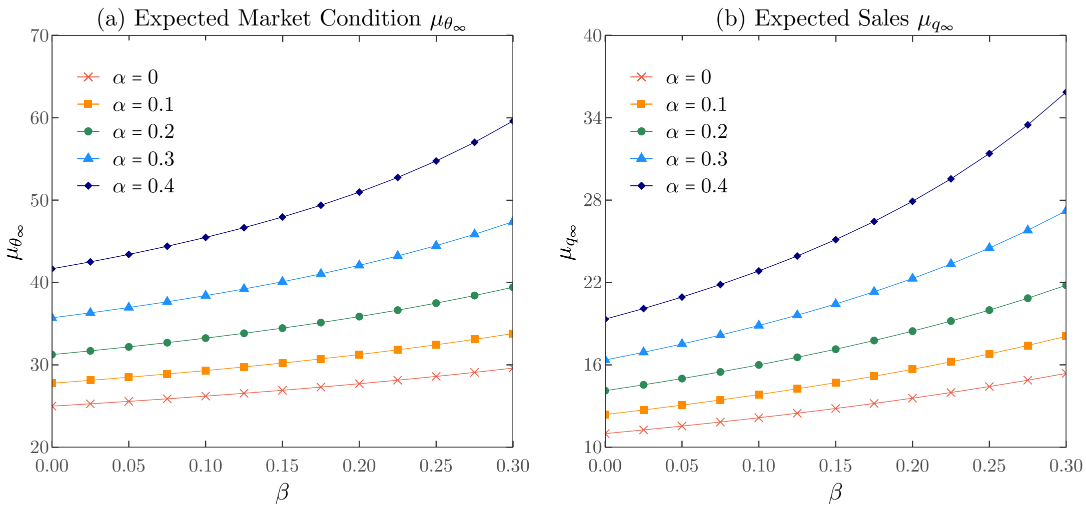
Proposition 2.7 (*a*) *In regime $\mathcal{P}$ under ϕ^*, the market condition θ_t and sales q_t^* converge to steady states θ_∞ and q_∞. If $\varepsilon_t \sim_{\text{IID}} \mathcal{N}(\mu, \sigma^2)$, then $\theta_t \to \theta_\infty \sim \mathcal{N}(\mu_{\theta_\infty}, \sigma^2_{\theta_\infty})$, and $q_t^* \to q_\infty \sim \mathcal{N}(\mu_{q_\infty}, \sigma^2_{q_\infty})$.*[21]

(*b*) *The steady-state market condition θ_∞ and sales q_∞ increase in carryover rate α and growth rate β.*

Proposition 2.7 characterizes the long-run trend (limit) of market condition and sales. It is driven by two key factors: the higher the market carryover α, the higher the growth rate β, the higher the long-run trend (Fig. 2.2). Tellingly, the prior $f(\theta_1)$—the key

[21] Specifically, $\mu_{\theta_\infty} = \frac{\beta d^* \mu - \beta b^* c + \mu}{1-(\alpha+\beta a^*)}$, $\mu_{q_\infty} = \frac{\beta d^* \mu - \beta b^* c + \mu}{1-(\alpha+\beta a^*)} a^* - b^* c + d^* \mu$, $\sigma^2_{q_\infty} = \frac{\sigma^2}{1-(\alpha+\beta a^*)^2}$, $\sigma^2_{\theta_\infty} = \frac{(a^*)^2 \sigma^2}{1-(\alpha+\beta a^*)^2}$, $a^* = \frac{1-\delta\alpha^2 - \sqrt{[1-\delta\alpha^2][1-\delta(\alpha+\beta)^2]}}{\delta\beta^2 + 2\delta\alpha\beta}$, $b^* = \frac{1-\delta\alpha}{2-a^*(\delta\beta^2+2\delta\alpha\beta)+\delta\alpha\beta-\delta\beta-2\delta\alpha}$, and $d^* = \frac{a^*(\delta\beta+2\delta\alpha)-\delta\alpha}{2-a^*(\delta\beta^2+2\delta\alpha\beta)+\delta\alpha\beta-\delta\beta-2\delta\alpha}$.

Figure 2.2: Steady-state μ_{θ_∞} and μ_{q_∞}

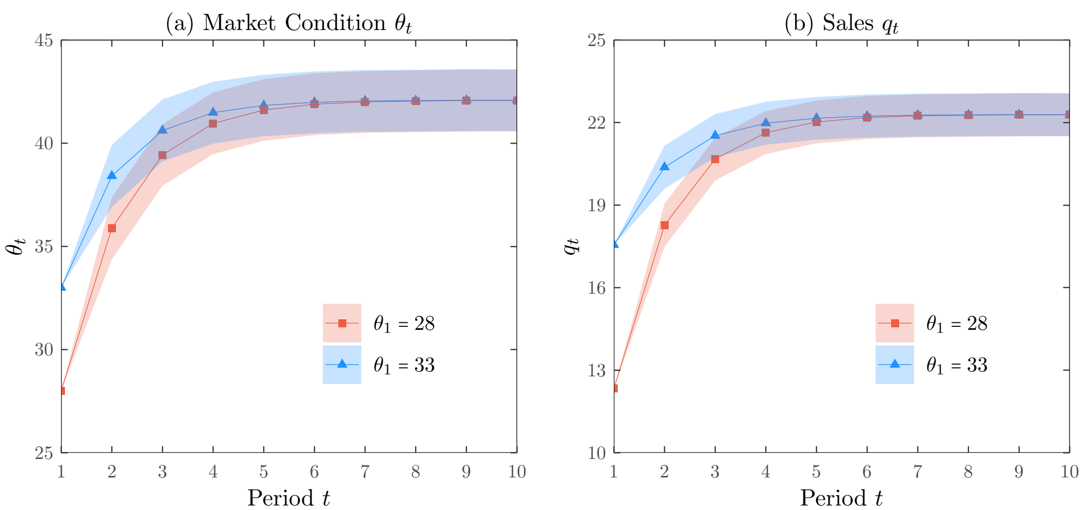


Note: $\alpha \in \{0, 0.1, 0.2, 0.3, 0.4\}$, $\beta \in [0, 0.3]$, $\delta = 0.9$, $\mu = 25$, $c = 3$.

driver of the classic solution ϕ^c—has no effect on the long-run trend; over time its influence is washed out by network effects and random shocks. Hence, the classic solution ϕ^c can be misleading in the long run.

Fig. 2.3 illustrates how the channel under ϕ^* evolves over time. It tracks 100 sample paths (envelope) $(\theta_t, q_t)_{t \geq 0}$ for two retailers, with initial conditions $\theta_1 = 28$ and 33. I find: (i) *Prior* $f(\theta_1)$ *is irrelevant in the long run.* Indeed, despite the huge gap 5 in initial market condition, the low-type retailer catches up quickly. The market gap shrinks to 1 by period 4; by period 7, the two retailers are stochastically indistinguishable—they both have market size θ_∞ and sell q_∞. (ii) *The convergence to the first-best is driven by market carryover and network effects.* As time goes by, the discriminatory treatment $\frac{1}{2}h(\theta_1)\alpha^{t-1}$ fades away; market carryover and network effects improve low-type retailers, but they are insufficient for keeping high-type retailer at his initial peak level. As a result, the retailers homogenize, and the channel coordinates, and the performance gravitates towards the first-best.

Figure 2.3: Evolution of $(\theta_t, q_t)_{t\geq 1}$ over time



Note: $\alpha = 0.3$, $\beta = 0.2$, $\delta = 0.9$, $\mu = 25$, $\underline{\theta} = 27$, $c = 3$, $T = 20$.

The result informs practice. In essence, network effects mean *increasing return* over time. To internalize such intertemporal externality, the manufacturer must take a long-term perspective. The short-term concern of information asymmetry—the heart of classic second-best ϕ^c—should not dictate her long-term goals of θ_∞ and q_∞. The long-term goals should focus on market fundamentals: when market carryover and growth potential are substantial, the manufacturer should strive for high market size with high sales performance.

2.6.3 Why Do Firms Prefer Long-Term Contracts?

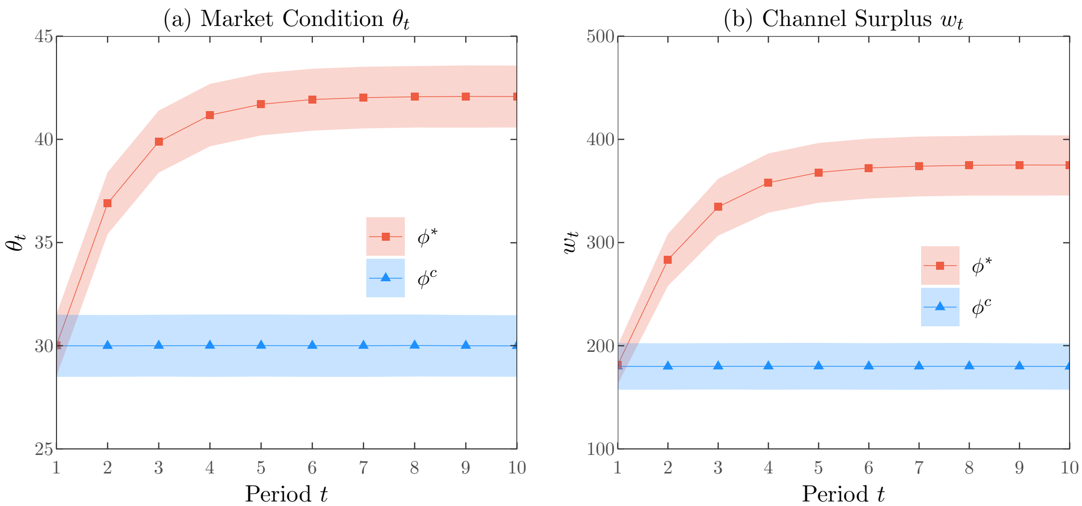
Channel contracts are often long-term, governing the relationships over multiple periods. This is puzzling, because during the long span of a relationship, supply and demand conditions can change substantially (He et al., 2008; Sudhir and Datta, 2008). As such, short-term contracts seem a better arrangement, because they allow more flexibility to manage uncertainty. Yet long-term contracts are prevalent (Lilien and Grewal, 2012). Why?

This question has attracted many explanations, e.g., to hedge the default risk, to mitigate the hold-up problem, and to reduce the transaction cost (Gibbons and Roberts, 2013). I will provide a new explanation. I argue that short-term contracts suffer both growth and information deficiencies. They ignore network effects, induce low sales, and pay high rents. These drawbacks conspire to undermine channel efficiency in the long run. By contrast, long-term contracts can address both deficiencies, and hence their prevalence.

I now elaborate. A short-term contract suffers growth deficiency. It focuses on the immediate gain, ignoring the impact of current sales on future market growth. Such myopic perspective sets low expectations for future markets, restricts sales, and thus discourages market growth. For example, the short-term contract ϕ^c prescribes sales $q^c(\theta_t)$, way below the optimal level $q_t^*(\theta^t)$. The low expectation creates a *vicious cycle*: the retailer produces low sales; the low sales limit network effects and market growth; the low market growth further depresses sales. As the vicious cycle stabilizes, the channel is trapped in a low-efficiency equilibrium, wasting market growth potential. See Fig. 2.4 for an illustration.

The short-term contract ϕ^c also suffers information deficiency. By definition, the contract ϕ^c must be renewed every period, *after* the retailer has obtained new information θ_t. Yet the new information is pre-contract, allowing the retailer to extract rent at every renewal opportunity. In response, the manufacturer must distort sales and pay rent in every period. Consequently, the optimal short-term contract is a sequence of the second-best ϕ^c: it prolongs rent payment and sales distortion, thereby perpetuating inefficiency.

Figure 2.4: 100 sample paths under ϕ^* and ϕ^c



Note: $\alpha = 0.3$, $\beta = 0.2$, $\delta = 0.9$, $\mu = 25$, $\underline{\theta} = 27$, $c = 3$, $T = 20$.

By contrast, the long-term contract ϕ^* can mitigate both deficiencies. First, it encourages market growth. Taking a long-term view, it sets aggressive sales targets $q_t^*(\theta^t)$. Using the upward adjustment $\frac{\delta}{2}(\lambda_t - \rho_t)$, it *internalizes* all future gains from network effects (Proposition 2.3). As such, the long-term contract creates a *virtuous cycle*: higher market size motivates the retailer to sell more; more sales accelerate market growth; market growth boosts sales still further. As the virtuous cycle stabilizes, the channel eventually sustains a high level of sales and market condition, thereby fulfilling the growth potential of the retailer.

Second, the long-term contract ϕ^* improves information efficiency. Except the initial information θ_1, it can eliminate all information advantages the retailer would obtain (under ϕ^c). At the outset it specifies foreseeable contingencies during the relationship, *before* the retailer can learn any new information beyond θ_1. This preemptive arrangement denies the retailer any new edge from learning new information $(\theta_t)_{t\geq 2}$ arising later, thereby limiting his advantage to θ_1 only. After the initial period, the retailer can still learn private information θ_t, but the manufacturer can use recursive advance selling to extract it, without paying rent or distorting sales (Proposition 2.5). Indeed, the sales distortion $\frac{1}{2}h(\theta_t)\alpha^{t-1}$ is entirely driven by θ_1, and it vanishes over time. In this sense, the long-term contract ϕ^* is information efficient for all but the initial period.

Table 2.3: Performance Comparison

Parameter	$\mathbb{E}J_1^*$	$\mathbb{E}U_1^*$	$\mathbb{E}W_1^*$	$\mathbb{E}J_1^c$	$\mathbb{E}U_1^c$	$\mathbb{E}W_1^c$	$\Delta J\%$	$\Delta U\%$	$\Delta W\%$
$T=1$	144	35	180	145	35	180	0	0	0
2	347	47	394	277	63	341	25	−25	15
5	919	53	969	599	136	736	53	−61	31
8	1,343	54	1,394	836	188	1,025	60	−71	36
12	1,737	52	1,788	1,054	237	1,291	64	−78	38
15	1,940	52	1,990	1,164	264	1,428	66	−80	39
$\alpha=0$	856	40	894	836	188	1,025	2	−78	−12
0.1	1,060	45	1,105	836	188	1,025	26	−76	7
0.2	1,343	54	1,394	836	188	1,025	60	−71	36
0.3	1,744	65	1,806	836	188	1,025	108	−65	76
0.4	2,343	84	2,427	836	188	1,025	179	−55	136
$\beta=0$	1,071	45	1,114	836	188	1,025	28	−76	9
0.1	1,192	48	1,237	836	188	1,025	42	−74	20
0.2	1,343	54	1,394	836	188	1,025	60	−71	36
0.3	1,542	58	1,599	836	188	1,025	84	−69	56
0.4	1,811	65	1,876	836	188	1,025	116	−65	83

Base case: $T=8$, $\alpha=0.2$, $\beta=0.2$, $\delta=0.9$, $\mu=25$, $\underline{\theta}=27$, $c=3$

Finally, I quantify *when* and *how much* the long-term contract ϕ^* can improve the short-term one ϕ^c. The improvement is measured by percentage changes in rent, profit, and surplus; e.g., $\Delta U\% = \frac{\mathbb{E}U_1^* - \mathbb{E}U_1^c}{\mathbb{E}U_1^c} \times 100\%$. Table 2.3 reveals: (i) The long-term contract can cut rent payment ($\Delta U\% \leq 0$), because it pays rent only for initial periods. (ii) It can increase channel surplus ($\Delta W\% > 0$), because it minimizes distortion and internalizes network effects. (iii) It can improve manufacturer profit ($\Delta J\% > 0$), because it eliminates the two deficiencies of the short-term contract. The improvement can be substantial, especially when the relationship is long and network effects are strong. To the extent these situations prevail, the manufacturer should favor the long-term contract.

2.7 Conclusion

Many products exhibit network effects. How should manufacturers sell them through retail channels? I study this new class of channel contracting problems. I find the optimal contract differs substantially from conventional ones. It resolves a dynamic tradeoff between exploiting network effects, screening new information, and maximizing channel efficiency. The contract structure is driven by the interplay of vanishing information friction and persistent network effects: to adapt to changing market conditions, it allows ex post adjustments for the best use of new information arriving over time; in the short run, it resembles the second-best, because information friction is still severe; in the long run, it converges to the first-best (adjust for network effects), because information friction vanishes but network effects persist. Taking a dynamic strategic perspective, I establish a deep connection between the classic first- and second-best channel policies.

I find network effects can change the channel relationship profoundly. They induce countervailing incentives: although network effects can improve channel efficiency by expanding market size, they can also exacerbate agency cost by enhancing the retailer's ability to manipulate markets. As a double-edged sword, the manufacturer should not promote network effects blindly: in the early stage of the relationship when the agency problem is still severe, the manufacturer should moderate network effects to control rent inflation. Ignoring the dark side of network effects, previous studies may have overestimated the benefits of network effects.

The results provide practical guidance. (i) I identify when and why manufacturers may moderate market growth, overproduce output, offer advance selling, favor incumbent retailers, and improve retailer information capability, despite information asymmetry. (ii) I show private information per se does not hurt efficiency; its effects depend on the timing and nature of the interactions it entails. Through repeated interactions, both parties can learn from each other, reducing information friction. Ignoring such two-sided learning, previous studies may have overestimated the harm of information asymmetry. (iii) I show the optimal long-term contract can improve the classic second-best, by alleviating both growth and information efficiencies. The improvement can be substantial, when the channel relationship is durable and network effects are strong. By highlighting the dual role of network effects in long-run channel performance, this study deepens the understanding of channel theory and practice.

Chapter 3

Inside the Subscription Box: Product Line Design with Consumer Habituation

3.1 Introduction

The rise of personalized subscriptions has transformed many retail markets, in categories such as food, beauty, fashion, and home decor (Bischof et al., 2020). As a new business model, *personalized subscriptions* (or curated subscriptions) allow firms to select and deliver curated products to match each customer's preference, in regular time intervals.[1] For example, Stitch Fix offers a monthly subscription service that ships the personalized

[1] Unlike traditional replenishment subscriptions, *curated subscriptions* offer variety, experience, and stimulation for novelty: they help consumers to discover new products with manageable surprise (Bischof et al., 2018). In 2018, curated subscriptions accounted for $15B in U.S. sales. Notable examples include ThreadBeast, IPSY, Glow Addict and Stitch Fix. For a book-length treatment, see Bischof and Rudolph (2021).

clothing items to customers, based on their purchase history and revealed style preferences; after trying on the clothes, the customers can then decide whether to purchase or return. In the meal-kit category, HelloFresh uses consumers' culinary preferences to personalize and deliver curated cooking subscription boxes on a weekly basis. In the beauty category, Glow Addict uses consumers' attractiveness preferences to personalize and deliver curated beauty subscription boxes monthly. The attraction of personalized subscriptions is obvious: for customers, they reduce search cost and improve product match; for firms, they help build long-term relationships and capitalize on customer lifetime values.

Despite the increasing popularity of personalized subscriptions, little is understood about their economics. A key challenge for a firm is how to learn consumers' evolving preferences and personalize its subscription box contents. Empirical studies show that, over the course of a subscription, a consumer's preferences can change stochastically over time (Iyengar et al., 2011; McCarthy et al., 2017; Datta et al., 2018; Bischof et al., 2020). The changes may come either exogenously from her idiosyncratic situations (Shin and Sudhir, 2010),[2] or endogenously from *consumer habituation* (Zhang et al., 2019). Indeed, because current experience shapes future valuations, repeated purchases can alter consumer preferences in a systematic way (Bronnenberg and Dubé, 2017).

Moreover, both the firm and the consumer are strategic and forward looking (Sun, 2005). To personalize the subscription service, the firm must have deep insight into consumer preference. Subscription facilitates such learning: from the consumer's purchase history, the firm can predict future consumer behavior with sufficient precision and deploy

[2] For example, when the vacation is coming, the consumer may buy more items shipped by Stitch Fix, Glow Addict.

behavior-based strategies. Anticipating the future discrimination, however, the consumer is reluctant to reveal her private information that affects her current purchase. As a result, the strategic reactions can eliminate the benefit of behavior-based strategies (Fudenberg and Villas-Boas, 2006). Therefore, it is unclear how the firm should design and price its subscription service.

In this paper, I seek to understand the economics of personalized subscriptions. Formally, I consider a monopolistic firm that serves a consumer (her) over two periods. The service in each period entails a menu of options that differ in price and quality. The consumer has private valuation that evolves endogenously over time: she knows her current need, but learns her future valuation only after current consumption; her future valuation is driven by preference persistence, current consumption, and random shocks. The firm cannot observe consumer valuations, but can infer them from her purchasing history. Importantly, the firm can use product offerings to influence consumer valuations.

This is a dynamic production line design problem with endogenous consumer preferences. Two factors complicate the problem. First, the firm must address the classic cannibalization effect (Moorthy, 1984): low-end products may tempt high-end consumers, thereby cannibalizing the sales of high-end items; in response, the firm must degrade the quality of low-end products. Second, the firm must manage the intertemporal dependence between successive product lines. This is driven by consumer habituation: because consumer preference can persist over time, the firm must internalize the persistence effect; because current consumption can shape future preferences, the firm must also internalize the habituation effect. Worse, all three effects conspire to further complicate the problem. How should the firm design and price its product lines?

I find consumer habituation can change the product line design profoundly, due to two effects. (i) The first is the familiar *welfare effect*: higher consumption can enhance future consumer valuation and social welfare. Therefore, the firm should distort initial quality upward, so that it can extract more surplus later. (ii) The second effect is the novel *strategic effect*: by changing future consumer heterogeneity, habituation can either alleviate or exacerbate cannibalization. Hence, it can induce new distortions beyond the classic second-degree discrimination.

The new distortions depend on the nature of consumer habituation. (i) When habituation is additive, high-end consumers benefit more in the future from higher initial offerings; hence habituation increases future consumer heterogeneity. In response, the firm should distort quality downward, below the classic second-best level, to control consumer rent. (ii) When habituation is satiating, however, a *homogenization mechanism* arises: low-end consumers benefit more in the future from higher initial offerings; hence habituation can reduce future consumer heterogeneity. This new mechanism is critical for rent control: the firm should leverage the homogenization mechanism to reduce consumer heterogeneity, by selectively accelerating habituation for certain types with upward distortion—even above and beyond the first-best level.

The results shed new lights on product line design. (i) The extant literature prescribes *downward distortion* to prevent cannibalization (Mussa and Rosen, 1978). This insight hinges on the relation that higher quality provision inflates the information rent. The relation holds naturally in traditional spot selling, with a one-shot transaction and known preferences. I show this insight may fail in new subscription models, where consumers have stochastic preferences that evolve endogenously over time. In these situations, upward distortion can be optimal for preventing cannibalization—even above the first-best level. (ii) The extant literature predicts that first-degree price discrimination is infeasible, when consumers have private, constant preferences—a key insight of Moorthy (1984). I show this insight does not carry over to subscription models: when subscription boxes are sufficiently surprising, the firm can leverage consumer uncertainty to practice first-degree discrimination after initial sales. (iii) The conventional view suggests that promotions encourage consumer consumption, intensify consumer habituation, and improve firm profit (Heerde and Neslin, 2017). This work calls for caution: in a subscription context, although promotions can enhance consumer valuation, they can also exacerbate cannibalization by enhancing the consumer's ability to extract the information rent. As such, excessive promotions can hurt firm profit and social welfare—a stark contrast to the conventional view.

The study helps explain the rising popularity of personalized subscriptions (*subscription* hereafter). First, subscription helps leverage consumer uncertainty. At the outset, the consumer knows her immediate needs, but is uncertain about her future wants. Leveraging this uncertainty, the firm can use the upfront subscription fee and price discount to craft an *offsetting mechanism*: they jointly ensure that the consumer has a stake in the future social welfare, reduce her current incentive to manipulate, and hence relax the cannibalization constraints. Using the offsetting mechanism, the firm can fully extract future consumer surplus: when the consumer has independent valuations over time, the firm after initial sales can even practice first-degree discrimination.

Second, subscription helps exploit both the welfare and strategic effects of consumer habituation. In traditional spot selling, the firm takes a transactional perspective and focuses on one-shot gain: unable to internalize the *intertemporal externality* of consumer habituation, it has no incentive to supply the first-best quality. With subscription, however, the firm takes a long-term perspective and focuses on customer lifetime value: because it can internalize the welfare gain from habituation, the firm has strong incentives to supply higher quality and improve social welfare. I show in a wide range of situations, the long-term welfare benefit of quality provision can dominate the short-term gain of price discrimination. Moreover, subscription helps exploit the strategic effect of consumer habituation. By optimizing sequential product offerings, the firm can make different consumers habituate at different rates, gradually reduce their valuation heterogeneity, and hence control the information rent. To my knowledge, both the homogenization mechanism and the rent control purpose of subscription are new to the literature.

This work builds on the habit formation literature (Becker et al., 1992). Depending on how current consumption affects future preferences, there are two types of habituation: *addictive* habituation has increasing marginal utility (Becker and Murphy, 1988), while *satiating* habituation has diminishing one (Baucells and Sarin, 2010). The empirical research is extensive, including studies on cigarettes (Chaloupka, 1991; Chen et al., 2009; Gordon and Sun, 2015; Wang et al., 2016; Chen, 2020), drugs (Olekalns and Bardsley, 1996; Grossman and Chaloupka, 1998; Liu et al., 1999), alcohol (Baltagi and Griffin, 2002; Arcidiacono et al., 2007), and gambling (Mobilia, 1993). For habit formation in social media consumption, internet browsing, and mobile app usage, see, e.g., Young (1998); Pelling and White (2009); Wan (2009); Kuss and Griffiths (2011); Kwon (2011) and Zhang et al. (2021). Despite the ubiquity of consumer habituation, there is no analytical work on how it affects product line design—the focus of this paper.

The product line design literature is extensive.[3] The main themes include market segmentation (Mussa and Rosen, 1978; Moorthy, 1984), distribution channels (Villas-Boas, 1998; Liu and Cui, 2010), advertising (Villas-Boas, 2004), search cost (Villas-Boas, 2009), and refund policies (Huang and Zhang, 2020); for a comprehensive review, see Chen (2009). This literature mainly focuses on the situations where consumers know their *exogenously fixed preferences*, and studies how firms can use product lines to extract consumer surplus.[4] By contrast, I study *stochastic preferences that evolve endogenously* over time; I show that firms can use the product lines not only to extract consumer surplus, but also to actively shape consumer preferences for profits.

I also contribute to the behavioral product line design literature. This literature studies how firms' strategies are shaped by behavioral factors, such as context-dependent preferences (Orhun, 2009), reference-group effects (Amaldoss and Jain, 2010), deliberation cost (Guo and Zhang, 2012), consumer learning (Xiong and Chen, 2014), limited attention (Dahremöller and Fels, 2015), loss aversion (Carbajal and Ely, 2016), and anticipated regret

[3] Methodologically, this literature builds on the mechanism design framework; see, e.g., Mirrlees (1971), Maskin and Riley (1984), Baron and Besanko (1984), Laffont and Tirole (1986), Laffont (1993), Courty and Hao (2000), Battaglini (2005), and Pavan et al. (2014). Mechanism design has broad applications; see, e.g., taxation (Kapička, 2013; Stantcheva, 2017), compensation design (Garrett and Pavan, 2015; Gao, 2022), auctions (Board, 2007), supply chain (Chen et al., 2016; Jian and Gao, 2020), and product line design (Carbajal and Ely, 2016; Xiong and Chen, 2014). For book-length treatments, see Laffont and Martimort (2001) and Bolton and Dewatripont (2005).

[4] A few exceptions include Orhun (2009), Guo and Zhang (2012), and Xu and Dukes (2019).

(Zou et al., 2020). I add to this literature by explicitly modeling two types of consumer habituation, and characterizing their differential impact on product line design. Moreover, I identify when and how the firm should leverage consumer habituation to improve profits. To my knowledge, this is the first analytical work on how consumer habituation affects dynamic product line design.

3.2 Model

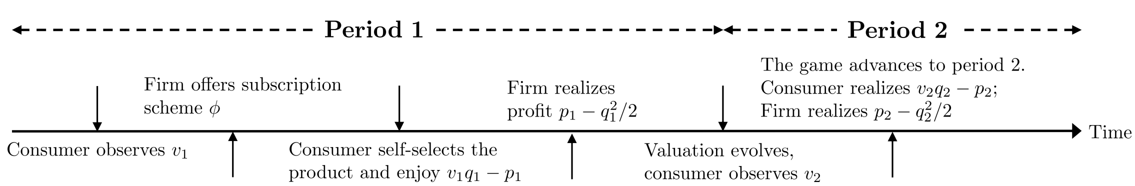
Table 3.1: Notation

Symbol	Description
$\equiv$	Equal by definition
α	Valuation persistence rate, $\alpha \in [0,1)$
$V_q(\cdot)$	Habituation intensity, $V_q(v_1) \equiv \beta + \gamma v_1 \geq 0$
$V_v(\cdot)$	Persistence intensity, $V_v(q_1) \equiv \alpha + \gamma q_1 \in [0,1]$
δ	Probability that the consumer stay in subscription
v_t	Consumer's valuation in period t, $v_t \in \mathcal{V} = [\underline{v}, \overline{v}] \subset \mathbb{R}_+$
v^2	Consumer's valuation history in period 2, $v^2 \equiv (\theta_1, \theta_2)$
$\hat{v}^2$	Consumer's revealed valuation history in period 2, $\hat{v}^t \equiv (\hat{v}_1, \hat{v}_2)$
ϕ	Subscription scheme, $\phi = (p_t, q_t)_{t=1}^2$ with price p_t and quality q_t
$F_1(\cdot)$	CDF of v_1, with density $f_1(\cdot)$
$F(\cdot\|v_1, q_1)$	Conditional CDF of v_2, with conditional density $f(\cdot\|v_1, q_1)$
ε	Random shock with mean $\mathbb{E}\varepsilon = \mu$, and distribution $G(\cdot)$
$\eta(v_1)$	Inverse hazard rate, $\eta(v_1) \equiv \frac{1-F_1(v_1)}{f_1(v_1)}$ decreases in v_1
$J_t(v^t)$	Firm's continuation profit for period t onward
$U_t(v^t)$	Consumer's continuation utility for period t onward
$\tilde{U}_t(\hat{v}^t; v_t)$	Consumer's continuation utility for period t onward
$w_t(v_t, q_t)$	Flow social welfare in period t, $w_t(v_t, q_t) \equiv v_t q_t - \frac{1}{2}[q_t]^2$
$W_t(v^t)$	Continuation social welfare for period t onward
$\mathbb{E}_t^\phi$	Expectation at time t under scheme ϕ

A monopolistic firm (it) serves a consumer (she) through a *subscription scheme* $\phi = \{\phi_t : t \leq 2\}$ over 2 periods. The stage game in period t follows that: the firm offers a menu ϕ_t of products that differ in price p_t and attribute q_t; from the menu ϕ_t, the consumer with private valuation v_t self-selects a product (p_t, q_t) and enjoys surplus $v_t q_t - p_t$; the firm incurs production cost $q_t^2/2$ and obtain profit $p_t - q_t^2/2$. After period 1, the consumer's valuation evolves from v_1 to v_2. She may continue the subscription service with probability δ, thereby the game proceeds to the period 2 with probability δ[5]. Table 3.1 defines all the notations. In particular, the attribute q_t has either quantity or quality interpretation; following the product line design literature (Moorthy, 1984), I call q_t *quality*.[6]

Both players are strategic and forward looking. The consumer has private *valuation (type)* $v_t \in \mathcal{V} \equiv [\underline{v}, \overline{v}]$ that evolves endogenously over time: she knows his initial valuation v_1 before subscription, and learns her new valuation v_{t+1} after each consumption q_t. However, the firm cannot observe v_t: it only knows the prior distribution F_1 of v_1 and must infer v_t from consumer choices. All other parameters are common knowledge. Following the literature, I assume decreasing (inverse) hazard rate $\eta(v_1) \equiv \frac{1-F_1(v_1)}{f_1(v_1)}$ of F_1.[7] To sharpen insights, I focus on the two-period setup, with the timing in Fig. 3.1.

Figure 3.1: Sequence of Events



[5] B2C subscription companies experience an overall 7.69% churn (https://www.gravysolutions.io/post/customer-churn-a-no-nonsense-guide-for-subscription-and-saas-businesses). With probability $(1-\delta)$, the consumer leaves at the beginning of period 2, and the whole game ends.

[6] For the quantity interpretation, see Maskin and Riley (1984); for the quality interpretation, see Mussa and Rosen (1978). Also, I use the terms *product* and *service* interchangeably.

[7] This monotone hazard rate assumption is standard in the mechanism design literature (Fudenberg and Tirole, 1991). It removes the case where multiple agents will select the same options. It admits commonly used distributions, such as uniform, normal, logistic, exponential, and Gamma (Bagnoli and Bergstrom, 2005).

Subscription is a long game. It entails repeated consumption (Janzer, 2020) and habit forming[8]. Following the literature, I model consumer habituation $v_2 = V(v_1, q_1, \varepsilon)$ by

$$v_2 \;=\; \alpha v_1 \;+\; \big(\beta + \gamma v_1\big) \cdot q_1 \;+\; \varepsilon. \tag{3.1}$$

In this model, (i) $\alpha \in [0,1)$ is the *persistence rate*: it captures how current preference affects the future valuation directly; e.g., fad products have lower α while brand loyalty entails higher α (Bronnenberg et al., 2012). Hence, $(1-\alpha)$ is "the rate of disappearance of the physical and mental effects of past consumption" (Becker and Murphy, 1988). (ii) $V_q(v_1) \equiv \frac{\partial}{\partial q} V = (\beta + \gamma v_1)$ is the *habituation intensity*: it measures how current consumption affects future valuation, with habituation rate β and cross rate γ (across type). I assume $V_q(v_1) \geq 0$: the higher the current consumption, the higher the *expected* future valuation.[9] (iii) ε is the random shock with distribution G and mean $\mathbb{E}\varepsilon = \mu$. It captures valuation uncertainty: given current preference v_1 and consumption q_1, the future valuation v_2 follows the conditional distribution $F(v_2|v_1, q_1) = G\left(v_2 - \alpha v_1 - \beta q_1 - \gamma v_1 q_1\right)$. (iv) This habituation

[8]The evidence for consumers to form a habit is extensive. First, empirically, by analyzing the digital-content consumption activity data Zhang et al. (2021) finds that consumers will stay in a high habit state under subscription plans, compared to the other spot-purchasing plans; the consumption level of consumers who choose subscription plans increases over time, indicating a habit formation through subscription plans. Second, a major source for consumers to develop a habit is the repetitive purchasing (Ji and Wood, 2007). Consumers form a habit through subscription given its repeated purchasing nature. Third, subscription works as a self-control mechanism to form consumer habits passively, e.g. membership (DellaVigna and Malmendier, 2006). Fourth, the new developed personalized subscription may reduce consumer churn and build consumer loyalty (Petro, 2019; Julien Boudet, 2017). In other words, consumers get "addicted" on subscription boxes. Psychologists have discovered consumers are addicted to subscription boxes (Barstow, 2016), especially in Beauty and Fashion (Teitell, 2014). One aspect is the principle of behavioral consistency — consumers are all creatures of habit to keep their subscription going. Another aspect is from the scarcity principle — the fear of missing out on curation by celebrities.

[9]This relation should be understood in the *first-order stochastic dominance* sense (Müller and Stoyan, 2002). Along each sample path, the higher initial consumption q_1 can still lead to lower *realization* of stochastic valuation v_2, due to the "bad" random shock ε.

model is quite general, allowing for linear and nonlinear habit formation.[10] To avoid negative valuation, I assume $v_t \geq 0$ almost surely.

The cross rate γ characterizes the nature of habituation. (i) When $\gamma > 0$, the habituation is *addictive*: as current valuation v_1 increases, the consumption q_1 has increasing larger effect on future preference v_2[11]. (ii) When $\gamma < 0$, the habituation is *satiating*: the higher type v_1 has lower consumption elasticity of future preference v_2[12]. (iii) When $\gamma = 0$, I recover the standard *linear* habit formation model (e.g., Rozen, 2010). (iv) It turns out, the nature of habituation plays a central role in subscription design: addictive habituation expands future consumer heterogeneity, while satiating habituation can either reduce or expand it. See Fig. 3.2 for an illustration.[13]

I now formulate the subscription design problem $\mathcal{P}$. Let $v^t \equiv (v_1, \ldots, v_t) \in \mathbb{R}^t$. I call the consumer with a valuation path v^t the consumer-v^t. Given a subscription scheme $\phi \equiv \{\big(p_t(v^t), q_t(v^t)\big) : v^t \in \mathcal{V}^t, t \leq 2\}$, consumer-$v^2$ enjoys surplus $\tilde{U}_2(\hat{v}^2; v_2) \equiv v_2 q_2(\hat{v}^2) - p(\hat{v}^2)$ from consuming product $\big(p_2(\hat{v}^2), q_2(\hat{v}^2)\big)$, while consumer-$v_1$ enjoys total surplus $\tilde{U}_1(\hat{v}_1; v_1) = v_1 q_1(\hat{v}_1) - p_1(\hat{v}_1) + \delta \mathbb{E}[\tilde{U}_2(\hat{v}_1, v_2; v_2) | v_1, q_1(\hat{v}_1)]$ from consuming prod-

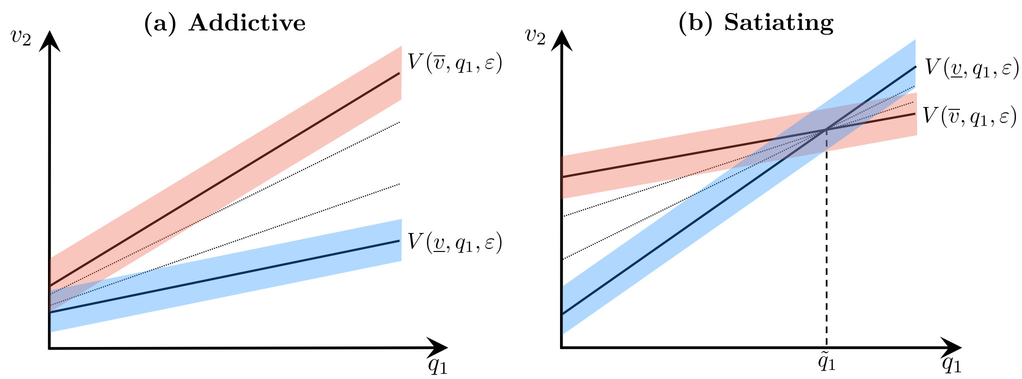
[10]The habituation model accommodates the specifications of Campbell and Cochrane (1999) and Otrok et al. (2002): $\ln(v_2) = \alpha + \phi \ln(v_1) + (1-\phi)\ln(q_1) + \varepsilon$.

[11]Goods, such as games, sugar, exhibit *addiction* behavior (Van Rooij et al., 2011; Avena et al., 2008), which is a strong habit (Becker et al., 1992) such that the habituation intensity is increasing over time. Consumers who purchase gaming subscription boxes, such as The BAM! Gamer Box, Loot Gaming, exhibits the addictive habituation.

[12]Goods with the diminishing habituation intensity incorporates the *satiation* behavior, e.g., diet sugar (May et al., 2020). One may sate over diet sugar, although he has formed a habit of consuming it. Baucells and Sarin (2010) characterize the consumer's utility under satiating habituation. They find a diminishing marginal utility over the past consumption level. The valuation dynamics under diminishing habituation intensity also relates to the compound effect of habit and variety-seeking (Thomadsen and Seetharaman, 2018). For example, in the fashion and beauty industry, consumers exhibit both habit formation (Mrad and Cui, 2017; Niazi and NOROWZI, 2019) and variety-seeking (Mandhachitara and Piamphongsan, 2011; Faust et al., 2018). Consumers who purchase popular fashion or beauty subscription boxes, such as ThreadBeast, Glow Addict, exhibit the satiating habituation.

[13]When $\gamma < 0$, there exists $\tilde{q}_1$ such that habituation reduces consumer heterogeneity for lower consumption $(q_1 < \tilde{q}_1)$, but expands it for higher consumption $(q_1 > \tilde{q}_1)$.

Figure 3.2: Addictive and Satiating Habituation



ucts $\big(p_1(\hat{v}_1), q_1(\hat{v}_1)\big)$ and $\big(p_2(\hat{v}_1, v_2), q_2(\hat{v}_1, v_2)\big)$ sequentially. Let $U_t(v^t) \equiv \tilde{U}_t(v^t; v_t)$. To maximize profit J, the firm must design subscription scheme ϕ, subject to three sets of constraints:

$$\max_{\phi} \; J_1 \equiv \mathbb{E}\Big\{ \Big(p_1(v_1) - \tfrac{1}{2}[q_1(v_1)]^2\Big) \; + \; \delta\Big(p_2(v^2) - \tfrac{1}{2}[q_2(v^2)]^2\Big) \Big\} \tag{$\mathcal{P}$}$$

$$\text{s.t.} \quad U_1(v_1) \geq 0, \quad \forall v_1 \in \mathcal{V}, \tag{IR_1}$$

$$U_2(v^2) \geq 0, \quad \forall v^2 \in \mathcal{V}^2, \tag{IR_2}$$

$$U_1(v_1) \geq \tilde{U}_1(\hat{v}_1; v_1), \quad \forall v_1, \hat{v}_1 \in \mathcal{V}, \tag{IC_1}$$

$$U_2(v^2) \geq \tilde{U}_2(\hat{v}^2; v_2), \quad \forall v^2, \hat{v}^2 \in \mathcal{V}^2. \tag{IC_2}$$

In ($\mathcal{P}$), the objective function J_1 is the expected *customer lifetime value* (CLV), where the term $\big(p_t(v^t) - \frac{1}{2}[q_t(v^t)]^2\big)$ is the firm profit from serving consumer-v^t in period t, and the expectation $\mathbb{E}$ is taken with respect to stochastic preferences (v_1, v_2), with $v_1 \sim F_1(\cdot)$ and $v_2 \sim F(\cdot|v_1, q_1(v_1))$; the participation constraint (IR_1) ensures that the consumer is willing to sign up, and (IR_2) ensures that the consumer won't cancel the service[14], while

[14] Free cancellation is an important feature of online subscription services. The contingent scheme design must incentivize the consumers to retain to earn their CLV; forcing (IR_2) to hold.

the self-selection (cannibalization) constraints (IC_1) and (IC_2) ensure that consumer-v^t should choose the product $\big(p_t(v^t), q_t(v^t)\big)$ intended for her preference v^t.

This formulation extends the classic model of Moorthy (1984) to dynamic settings. The key assumptions are consistent with the literature and practice. (i) I assume the firm can commit to the subscription scheme, a common practice in business (Bischof et al., 2020)[15]. (ii) I assume consumption affects preferences[16]. This assumption captures a key finding in the habit formation literature; see, e.g., Gordon and Sun (2015), Zhang et al. (2019), and Iyengar et al. (2021). It is also consistent with the behavioral science literature on *constructed preferences* (Tversky and Kahneman, 2000; Bettman et al., 2008): consumers develop new preferences over time, which are susceptible to the influences of past (consumption) experience and environmental shocks. (iii) I assume both the firm and consumer can engage in Bayesian learning: through repeated purchases, they can learn new information, update posterior beliefs, and forecast future valuation with increasing precision.[17] In particular, the firm can use product offerings for two purposes: to learn the consumer's current valuation, and to influence her future preference.

[15] For example, Glow Addict announces available subscription plans on its website (www.glowaddict.com), differentiate by quality including Beauty Bag (\$13.99), Beauty Box (\$18.99), Skincare Box (\$24.99), Luxe Beauty Box (\$34.99); ThreadBeast (www.threadbeast.com) offers Basic Plan (\$120), Essential Plan (\$190) and Premium Plan (\$300) on its website. Stitch Fix, as the other example, pre-announces the price range of the categories (www.stitchfix.com/women). In the same category, cloth price differs over brands (quality). Based on the preference over different brands, the consumer can estimate his acceptable price from the committed range.

[16] It seems that the consumers exhibit behavior biases of habituation, which is inconsistent compared to the traditional utility maximization setting. Once I incorporate the state dependence behavior into consideration, the Economic approach can internalize the psychological behavior limitations (See, e.g. Becker (1976) for comprehensive discussions.

[17] Formally, I fix a scheme ϕ. Let $f(v_2|v_1, q_1(v_1))$ be the conditional density of v_2 given v_1 and $q_1(v_1)$. Hence the marginal density $f_2(v_2) = \int f(v_2|v_1, q_1(v_1)) f_1(v_1)\, \mathrm{d}v_1$. In period 1, the firm forecasts the consumer's current valuation v_1 with prior density $f_1(v_1)$ and her future valuation v_2 with *marginal density* $f_2(v_2)$. By contrast, the consumer knows her current valuation v_1 precisely, and she can predict her future valuation v_2 with *conditional density* $f(v_2|v_1, q_1(v_1))$—a better prediction than the firm's.

3.3 Perfect Price Discrimination

I first establish a full-information benchmark: in regime $\bar{\mathcal{P}}$, the consumer has public preference v_t. The firm has the ability to observe v_t, and to isolate and address each consumer independently. Hence, it need not worry about the cannibalization problem among a line of products. Given consumer habituation, however, it must manage the *intertemporal dependence* between successive product offerings. The central notion is habituation intensity $V_q = (\beta + \gamma v_1)$, which measures the marginal impact of current consumption on future valuation.

Formally, the firm can ignore consumer self-selection and solve the dynamic product line design problem ($\bar{\mathcal{P}}$): $\max_\phi J_1$, s.t. $(IR_1), (IR_2)$. Without habituation ($V_q \equiv 0$), the problem reduces to *first-degree discrimination* ϕ^f in each period:

$$p^f(v_t) = v_t \cdot q^f(v_t), \qquad q^f(v_t) = v_t, \qquad \forall v_t \in \mathcal{V}. \tag{ϕ^f}$$

It charges consumer-v^t the reservation price $p^f(v_t) = v_t q^f(v_t)$ for quality $q^f(v_t) = v_t$, which equates marginal consumer surplus with marginal production cost. With habituation ($V_q \geq 0$), the optimal scheme $\bar{\phi}$ sets quality provisions[18]

$$\bar{q}_1(v_1) = v_1 \; + \; \delta(\beta + \gamma v_1) \cdot \mathbb{E}[\bar{q}_2 | v_1], \qquad \bar{q}_2(v_2) = v_2, \qquad \forall v_1, v_2 \in \mathcal{V}. \tag{$\bar{\phi}$}$$

I call $\bar{\phi}$ *perfect price discrimination.* Let $w_t = \mathbb{E}\left[v_t q_t - \frac{1}{2} q_t^2\right]$ be the social welfare in period t, and $W_1 = \mathbb{E}[w_1 + \delta w_2]$ the expected social welfare over two periods. I find:

[18]The explicit solution is $\bar{q}_1(v_1) = \frac{v_1 + \delta(\beta+\gamma v_1)(\alpha v_1 + \mu)}{1 - \delta(\beta + \gamma v_1)^2}$.

Proposition 3.1 (Welfare Effect) *Under full information, the firm can fully extract consumer surplus. Moreover, consumer habituation (weakly) increases quality provision, firm profit, and social welfare.*

This is the classic *full-information solution.* It reveals that consumer habituation has the welfare effect: under the scheme $\bar{\phi}$, higher current consumption $\bar{q}_1(v_1)$ shifts the distribution of v_2 upward, increases future consumer valuation $\mathbb{E}[v_2|v_1]$, thereby enhancing social welfare.[19] To internalize this *intertemporal externality*, the firm must distort quality upward:

$$\bar{q}_1(v_1) - q^f(v_1) \;=\; \delta(\beta + \gamma v_1) \cdot \mathbb{E}[v_2|v_1] \;\geq\; 0.$$

The higher the habituation intensity $V_q = (\beta + \gamma v_1)$, the larger the distortion. Although socially efficient, the scheme $\bar{\phi}$ is unkind to consumers: it allows the firm to capitalize on consumer habituation, engage in perfect price discrimination, and charge the *first-best* price of consumer utility $\bar{p}_t(v_t) = v_t\bar{q}_t(v_t)$, and extract the entire consumer surplus. Of course, few firms have the requisite ability to exercise perfect discrimination $\bar{\phi}$: to design feasible schemes, one must account for consumer self-selection.

3.4 Optimal Subscription Design with Consumer Habituation

Under spot selling $\mathcal{P}^s$, the consumer has private preference v_1 and purchases only once (Moorthy, 1984). The firm has to segment markets based on consumer self-selection. It must address the cannibalization problem: because high-end consumers have a valuation

[19] Technically, let $F_2(v_2) \equiv \int F\big(v_2|v_1, \bar{q}_1(v_1)\big)\,\mathrm{d}F_1(v_1)$ be the unconditional distribution of v_2. Then $V_q(v_1) \geq 0$ implies that random variable $v_2 \sim F_2(\cdot)$ is stochastically larger than $v_1 \sim F_1(\cdot)$, and hence $\partial\mathbb{E}[v_2|v_1, q_1]/\partial q_1 \geq 0$.

advantage, they may switch to low-end products, thereby cannibalizing the sales of high-end items. To internalize this contemporaneous externality, the firm must design the line of products jointly, resulting in *second-degree discrimination* ϕ^s:

$$q^s(v_1) = v_1 - \tfrac{1-F_1(v_1)}{f_1(v_1)}, \qquad p^s(v_1) = v_1 \cdot q^s(v_1) - U^s(v_1), \qquad \forall v_1 \in \mathcal{V}, \tag{ϕ^s}$$

where $U^s(v_1) = \int_{\underline{v}}^{v_1} q^s(v)\,\mathrm{d}v \geq 0$ is *consumer rent*. The rent is the total surplus the firm must concede, in the form of price discount or "sign-up bonus". Relative to first-degree discrimination ϕ^f, the firm should pay consumer rent $U^s(v_1)$ to prevent cannibalization, and use downward quality distortion $\frac{1-F_1(v_1)}{f_1(v_1)}$ to extra more surplus. As a result, both firm profit and social welfare suffer, but high-end consumers benefit from their valuation advance. Unlike ϕ^f, consumer heterogeneity is central to ϕ^s: the (inverse) hazard rate $\eta(v_1) = \frac{1-F_1(v_1)}{f_1(v_1)}$ of valuation v_1 measures the negative externality of cannibalization that type-v_1 imposes on the firm; for each type-v_1, the quality provision $q^s(v_1)$ is precisely her *virtual valuation* $\theta_1(v_1) \equiv v_1 - \eta(v_1)$, the maximal amount of consumer valuation that the firm can extract under the self-selection constraints.

However, the problem $\mathcal{P}$ is more complicated: as a dynamic version of $\mathcal{P}^s$, it features repeated purchases and consumer habituation. Besides the *contemporaneous externality* of cannibalization, the firm in $\mathcal{P}$ must also internalize two *intertemporal externalities*: (i) because consumer preference can persist over time, the firm must internalize the persistence effect $V_v(q_1) \equiv \frac{\partial}{\partial v_1} V = (\alpha + \gamma q_1)$; (ii) because current consumption can shape future preferences, it must internalize the externality of the habituation effect $V_q(v_1) = \frac{\partial}{\partial q} V = (\beta + \gamma v_1)$. Worse, the three effects conspire to further complicate the problem.

How should the firm design the subscription scheme? I first identify the optimal prices for a *given* product line design $\{q_1(v_1), q_2(v^2)\}$. By changing decision variables, I can equivalently identify optimal consumer rents $\{U_1(v_1), U_2(v^2)\}$. Consider type-v_1 who enjoys the *valuation advantage* $\mathrm{d}v$ over type-$(v_1 - \mathrm{d}v)$. Along each consumption path (q_1, q_2), type-v_1 enjoys $q_1 \cdot \mathrm{d}v$ more surplus in period 1, develops $(\alpha + \gamma q_1)\,\mathrm{d}v$ higher valuation in period 2, and hence enjoys $(\alpha + \gamma q_1) \cdot q_2 \cdot \mathrm{d}v$ more surplus from consuming q_2. To prevent type-v_1 from switching to type-$(v_1 - \mathrm{d}v)$'s plan, the firm in expectation must offer her two types of rents: *c*ontemporaneous rent $\mathrm{d}U_c = q_1\,\mathrm{d}v$, and *i*ntertemporal rent $\mathrm{d}U_i = \mathbb{E}[(\alpha + \gamma q_1) q_2]\,\mathrm{d}v$. Moreover, there are $[\underline{v}, v_1)$ such switching opportunities, so the total rent for consumer-v_1 is $U_1(v_1) = U_c(v_1) + \delta U_i(v_1)$, where[20]

$$U_1(v_1) = \underbrace{\int_{\underline{v}}^{v_1} q_1(v) \cdot \mathrm{d}v}_{\equiv U_c(v_1),\ \textit{c}\text{ontemporaneous rent}} + \delta \underbrace{\int_{\underline{v}}^{v_1} \mathbb{E}\big[(\alpha + \gamma q_1(v)) \cdot q_2(v, v_2) \;\big|\; v, q_1(v)\big] \cdot \mathrm{d}v}_{\equiv U_i(v_1),\ \textit{i}\text{ntertemporal rent}}. \tag{3.2}$$

Similarly, I can identify the rent for consumer-v^2 by

$$U_2(v^2) = \int_{\underline{v}}^{v_2} q_2(v_1, v) \cdot \mathrm{d}v. \tag{3.3}$$

Technically, the consumer rent is the "shadow price" for ensuring sequential self-selection, the price that the firm must pay to prevent the cannibalization across type and over time. In general, the higher the initial valuation v_1, the stronger the valuation persistency V_v, the higher the consumer rent $U_1(v_1)$. For the optimal design $\{q_1^*, q_2^*\}$, I denote the associated rents by $\{U_1^*, U_2^*\}$. Then the optimal prices follow immediately:[21]

$$p_1^*(v_1) = v_1 q_1^*(v_1) + \delta \mathbb{E}[U_2^*(v^2) | v_1, q_1^*(v_1)] - U_1^*(v_1), \qquad p_2^*(v^2) = v_2 q_2^*(v^2) - U_2^*(v^2). \tag{3.4}$$

[20] Without loss of generality, I normalize $U_1(\underline{v})$ and $U_2(v_1, \underline{v})$ to zero.

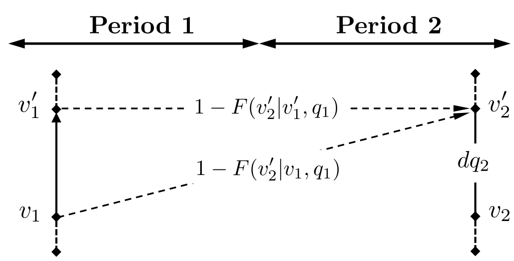
[21] To see why, note that the equilibrium consumer payoffs are $U_1 = v_1 q_1 - p_1 + \delta \mathbb{E}[U_2 | v_1, q_1]$ and $U_2 = v_2 q_2 - p_2$. Given $\{U_1, U_2\}$ and $\{q_1, q_2\}$, the equilibrium prices are $p_1 = v_1 q_1 + \delta \mathbb{E}[U_2 | v_1, q_1] - U_1$, and $p_2 = v_2 q_2 - U_2$.

Using consumer rents, I can reformulate the problem ($\mathcal{P}$) as $\max_{\{q_1,q_2\}} \mathbb{E}[J_1(v_1)]$, with

$$J_1(v_1) = \underbrace{\left[\theta_1(v_1)\cdot q_1(v_1) \;-\; \tfrac{1}{2}[q_1(v_1)]^2\right]}_{\text{profit from consumer-}v_1} \;+\; \delta\,\underbrace{\mathbb{E}\left[\theta_2\big(v^2, q_1(v_1)\big)\cdot q_2(v^2) \;-\; \tfrac{1}{2}[q_2(v^2)]^2 \big| v_1, q_1(v_1)\right]}_{\equiv\, \mathbb{E}[J_2(v^2)|v_1,q_1(v_1)],\text{ expected profit from consumer-}v^2},$$

where $\theta_1(v_1) = v_1 - \eta(v_1)$ and $\theta_2\big(v^2, q_1\big) = v_2 - \eta(v_1)\cdot(\alpha + \gamma q_1)$ are *virtual valuations* of the consumer-v_1 and v^2. Conceptually, $\theta_2\big(v^2, q_1\big)$ is the maximal amount the firm can extract from the consumer in period 2 without cannibalization: in particular, $(\alpha + \gamma q_1)$ measures the intertemporal externality of the persistent effect, and $\eta(v_1)\cdot(\alpha+\gamma q_1)$ is the deadweight loss caused by the firm to prevent consumer-v_1 from switching. Using the notion of virtual valuations, this reformulation incorporates all the externalities that each consumer imposes on others, *across type and over time*. Moreover, it simplifies the problem to the one of determining quality provisions $\{q_1^*, q_2^*\}$ only.

Figure 3.3: Two Countervailing Effects of Improve quality $q_2(v^2)$



I first identify the optimal quality $q_2^*(v^2)$. Consider increasing quality $q_2(v^2)$ by infinitesimal $\mathrm{d}q_2$. Two countervailing effects arise (see Fig. 3.3). (i) The first is the *direct* gain $(v_2 - q_2)\,\mathrm{d}q_2$ in welfare at consumer-v^2; from period 1's perspective, this amounts to extra welfare gain of $\delta f_1(v_1) f(v_2|v_1, q_1)\cdot(v_2 - q_2)\,\mathrm{d}q_2$. (ii) The second effect is the *indirect* rent increase for preventing cannibalization: in period 2, self-selection (IC_2) implies that, improving quality $q_2(v_1, v_2)$ by $\mathrm{d}q_2$ invites higher type $v_2' > v_2$ to switch, who demands extra rent $\frac{\partial}{\partial q_2}\left(\frac{\partial U_2}{\partial v_2}\right)\mathrm{d}q_2 = \mathrm{d}q_2$ for not switching; in period 1, self selection (IC_1) dictates that consumer-$v_1' > v_1$ must also get extra rent of $\delta[F(v_2'|v_1, q_1) - F(v_2'|v_1', q_1)] \cdot \mathrm{d}q_2$, because she is $[F(v_2'|v_1, q_1) - F(v_2'|v_1', q_1)]$ more likely than type-v_1 to develop higher valuation $v_2' > v_2$. There are $1 - F_1(v_1)$ such consumers; hence the loss from price discount is $\delta[1 - F_1(v_1)][F(v_2'|v_1, q_1) - F(v_2'|v_1', q_1)] \cdot \mathrm{d}q_2$. The optimal quality $q_2^*(v^2)$ then must balance these two effects, resulting in

$$q_2^*(v^2) = v_2 - \frac{1 - F_1(v_1)}{f_1(v_1)} \cdot \frac{F(v_2'|v_1, q_1^*) - F(v_2'|v_1', q_1^*)}{f(v_2|v_1, q_1^*)} = v_2 - \eta(v_1) \cdot (\alpha + \gamma q_1^*). \tag{3.5}$$

The optimal quality $q_1^*(v_1)$ is more involved. It entails both contemporaneous and intertemporal tradeoffs. (i) The contemporaneous tradeoff follows the familiar logic of cannibalization: improving quality $q_1(v_1)$ by $\mathrm{d}q_1$ will increase period-1 welfare by $\frac{\mathrm{d}w_1}{\mathrm{d}q_1}\mathrm{d}q_1 = (v_1 - q_1)\,\mathrm{d}q_1$, but to prevent cannibalization it must also increase the rent for higher type $v_1' > v_1$, by $\eta(v_1)\,\mathrm{d}q_1$. (ii) The intertemporal tradeoff is new and subtle.[22] Indeed, the improvement $\mathrm{d}q_1$ affects future profit via $\frac{\mathrm{d}\mathbb{E}_1 J_2}{\mathrm{d}q_1} \cdot \mathrm{d}q_1 = \mathbb{E}_1\big[\frac{\partial J_2}{\partial v_2} \cdot \frac{\partial V}{\partial q_1}\big]\mathrm{d}q_1 + \mathbb{E}_1\big[\frac{\partial}{\partial q_1} J_2\big]\mathrm{d}q_1$, producing both welfare and persistence effects: the welfare effect shifts the distribution of v_2 upward through $V_q = \frac{\partial V}{\partial q_1}$, resulting in profit gain $\delta\mathbb{E}_1\big[\frac{\partial J_2}{\partial v_2} \cdot \frac{\partial V}{\partial q_1}\big]\mathrm{d}q_1 = (\beta + \gamma v_1) \cdot \mathbb{E}_1[q_2]\,\mathrm{d}q_1$; the persistence effect $V_v = (\alpha + \gamma q_1)$ determines the size of future deadweight loss, resulting

[22] Recall $\mathbb{E}J_2 = \mathbb{E}\big[v_2 \cdot q_2 - \frac{1}{2}(q_2)^2 | v_1, q_1\big]$.

in $\delta\mathbb{E}_1[\frac{\partial}{\partial q_1}J_2]\,\mathrm{d}q_1 = \mathbb{E}_1[\frac{\partial}{\partial q_1}\{(v_2 - \eta\cdot(\alpha+\gamma q_1))q_2 - \frac{1}{2}q_2^2\}]\,\mathrm{d}q_1 = -\delta\cdot\gamma\eta\cdot\mathbb{E}_1[q_2]\,\mathrm{d}q_1$. (iii) The optimal quality $q_1^*(v_1)$ must internalize all these effects, resulting in

$$
\begin{aligned}
q_1^*(v_1) = &\underbrace{\underbrace{v_1}_{q^f(v_1)} - \underbrace{\eta(v_1)}_{\text{cannibalization distortion}}}_{\text{second-degree discrimination } q^s(v_1)} \\
&+ \underbrace{\delta(\beta+\gamma v_1)\cdot\mathbb{E}[q_2^*|v_1]}_{\text{distortion for the welfare effect}} - \underbrace{\delta\gamma\eta(v_1)\cdot\mathbb{E}[q_2^*|v_1]}_{\text{distortion for the persistent effect}} .
\end{aligned}
\tag{3.6}
$$

Taken together, Eqs. (3.2)–(3.6) characterize the optimal subscription scheme $\phi^* = \{(p_t^*, q_t^*)\}_{t=1}^2$. It reveals how the firm should reconcile three competing motives for subscription design—to exploit habituation, to prevent cannibalization, and to extract surplus. Relative to first-degree discrimination $q^f(v_1) = v_1$, the firm should deploy three distortions in initial offerings q_1^*. (i) The downward distortion η is driven by the classic cannibalization effect, for ensuring self-selection constraint IC_1. (ii) The upward distortion $\delta(\beta+\gamma v_1)\cdot\mathbb{E}[q_2^*|v_1]$ is driven by the welfare effect, for exploiting habituation to enhance future surplus. (iii) The distortion $\delta\gamma\eta(v_1)\cdot\mathbb{E}[q_2^*|v_1]$ is driven by the persistence effect, for ensuring self-selection constraint IC_1; it is downward if $\gamma > 0$, and upward if $\gamma < 0$.[23] As a result, the net distortion $(q_1^* - q^f)$ can go either downward or upward.

In summary, consumer habituation has two effects. (i) The first is the direct, *welfare effect*: higher consumption q_1 now can enhance future consumer valuation $v_2 = V(v_1, q_1, \varepsilon)$ and social welfare. The implication for product line design is straightforward: the firm should distort initial quality upward, so that it can extract more surplus later (see §3.3). (ii) The second effect is the indirect, *strategic effect*: by changing future consumer

[23] As shown in §3.5.1, this distortion is to ensure IC_1—not IC_2—by controlling the persistence effect of revealing v_1 on period 2.

heterogeneity, habituation can either alleviate or exacerbate the cannibalization problem. The optimal design depends on the cross rate $\gamma = \frac{\partial}{\partial q} V_v$, which measures the consumption elasticity of the persistent effect. When $\gamma > 0$, habituation increases future consumer heterogeneity. Relative to perfect discrimination $\bar{\phi}$, the firm should moderate consumer habituation and distort quality downward ($q_1^* \leq \bar{q}_1$). When $\gamma < 0$, however, habituation reduces future consumer heterogeneity. To capitalize on this homogenization mechanism, the firm may accelerate habituation for certain types and moderate habituation for others. As a result, the net distortion can be upward—even above and beyond the perfect discrimination quality $\bar{q}_1$—despite the deadweight loss.

Proposition 3.2 (Strategic Effect) *Under the subscription scheme ϕ^*, consumer habituation has both welfare effects and strategic effects. The strategic effect can either alleviate or exacerbate the cannibalization problem.*

3.5 Managerial Implications

Subscription can outperform spot selling, for at least three reasons: it can leverage consumer uncertainty, exploit consumer habituation, and homogenize consumer preferences. Understanding these economic forces can help avoid the pitfalls in subscription design. I now elaborate.

3.5.1 Leverage Consumer Uncertainty

In the product line design literature, a common assumption is that a consumer knows her preference v_1, while the firm knows only the prior F_1 of v_1. It implies that *first-degree discrimination* ϕ^f *is unattainable*—a key insight of Moorthy (1984). Indeed, under spot selling $\mathcal{P}^s$, the firm must cut price and quality to prevent cannibalization, which reduces the total surplus it can extract. Even under the optimal scheme ϕ^s, the firm still needs to compensate almost all types of consumers with positive rents: $U^s(v_1) > 0$, $\forall v_1 > \underline{v}$.

With the subscription scheme ϕ^*, however, the firm can do better. A key mechanism is to exploit consumer uncertainty with the subscription fee and price discount.[24] This is because subscription entails repeated purchases: at each purchase, the consumer knows her immediate needs v_1, but is uncertain about her future wants $v_2 \sim F(\cdot|v_1, q_1)$. Leveraging this uncertainty, the firm can extract more surplus, with the following *offsetting mechanism*:

$$p_1^*(v_1) = \underbrace{v_1 q_1^*(v_1)}_{\text{consumer utility}} + \underbrace{\delta\mathbb{E}[U_2^*(v^2)|v_1, q_1^*(v_1)]}_{\text{upfront subscription fee}} - \underbrace{U_1^*(v_1)}_{\text{sign-up bonus}},$$

$$p_2^*(v^2) = \underbrace{v_2 q_2^*(v^2)}_{\text{consumer utility}} - \underbrace{U_2^*(v^2)}_{\text{price discount}}.$$

This mechanism can relax the self-selection constraint (IC_2) in the following sense: by charging consumer-v_1 the subscription fee $\delta\mathbb{E}[U_2^*|v_1, q_1^*(v_1)]$ in period 1, the firm can offset the *price discount* $U_2^*(v^2)$ it must concede in period 2 (to ensure self-selection IC_2). Exploiting this offsetting mechanism, the firm can extract private valuation v_2 at no cost, thereby reducing its information disadvantage to initial valuation v_1 only. Although the consumer can gain discount $U_2^*(v^2)$ from private valuation v_2 in period 2, the firm can extract that gain through the upfront subscription fee.[25]

[24] For example, Glow Addict charges upfront: Beauty Bag (\$13.99), Beauty Box (\$18.99), Skincare Box (\$24.99), Luxe Beauty Box (\$34.99); ThreadBeast offers Basic Plan (\$120), Essential Plan (\$190) and Premium Plan (\$300) upfront.

[25] The offsetting mechanism in subscription ϕ^* echoes advance selling (Xie and Shugan, 2001): when paying subscription fee $\delta\mathbb{E}[U_2|v_1, q_1^*(v_1)]$ in period 1, the consumers of valuation v_1 are more homogeneous

Still, the firm needs to offer *sign-up bonus*—the consumer rent $U_1^*(v_1)$—for type-v_1 to revealing her preference v_1. However, the preference v_1 has the persistent effect V_v, generating intertemporal externalities: under scheme ϕ^*, the firm can learn v_1 from consumer choice q_1, and it can exploit it to infer v_2 for better price discrimination in period 2; in response, the consumer may manipulate her choice q_1 to thwart firm learning.[26] Unless the firm compensates her upfront for all the gains from the manipulation, she will not reveal her preference v_1 truthfully. Therefore, the rent $U_1^*(v_1)$ must internalize both contemporaneous and intertemporal externalities of revealing v_1.

At the time of signing up the subscription, what matters to rent $U_1(v_1)$ is: (i) how valuation v_1 determines current choice $q_1^*(v_1)$, and (ii) how informative v_1 is about future valuation v_2. The resulting rent $U_1^*(v_1) = U_c^*(v_1) + \delta U_i^*(v_1)$ entails both contemporaneous rent $U_c^*(v_1) = \int_{\underline{v}}^{v_1} q_1^* \, \mathrm{d}v$ and intertemporal rent $U_i^*(v_1) = \int_{\underline{v}}^{v_1} \mathbb{E}[(\alpha + \gamma q_1^*) \cdot q_2^* | v] \, \mathrm{d}v$. Clearly, the intertemporal rent $U_i^*(v_1)$ hinges on $V_v = (\alpha + \gamma q_1^*)$, the informativeness of v_1 about v_2. When v_1 is uninformative of v_2, I have $V_v = 0$ and hence $U_i^*(v_1) = 0$.

For example, in personalized subscription boxes, when the new purchase is sufficiently surprising, future valuation v_2 is largely determined by random shock ε, independent of v_1 (Bischof et al., 2020). This implies $V_v \approx 0$. In this case, by Eq. (3.5), the firm does not distort period-2 quality, $q_2^*(v_2) = v_2 = q^f(v_2)$. Also, it only pays one piece of rent for initial sales, $U_1^*(v_1) = U_c^*(v_1) = U^s(v_1)$, but pays no rent for period-2 sales. As such, the

than after they observe different valuations v_2 in period 2: the same v_1 vs. different $v^2 = (v_1, v_2)$. Two key differences are: (i) advance selling involves only one-off purchase, while subscription involves repeated, correlated purchases; (ii) advance selling is a pricing rule, while subscription involves both pricing and product line design.

[26] Ex ante, the firm forecasts period-1 valuation with prior distribution $f_1(v_1)$, and period-2 types with distribution $f_2(v_2) \equiv \int f(v_2|v_1, q_1^*(v_1)) f_1(v_1) \, \mathrm{d}v_1$. By contrast, consumer-$v_1$ knows her current valuation v_1 and forecasts future valuation v_2 with $f(v_2|v_1, q_1^*(v_1))$. Hence, revealing v_1 allows the firm to improve its forecast, from $f_2(v_2)$ to $f(v_2|v_1, q_1^*(v_1))$.

firm can leverage consumer uncertainty to achieve first-degree discrimination in period 2, despite cannibalization.

Proposition 3.3 (Rent Reduction) *With subscription scheme ϕ^*, the firm can use up-front subscription fee and price discount to extract future private valuation at no cost. Relative to spot selling ϕ^s, the firm can use ϕ^* to reduce consumer rent (per transaction).*[27] *When the consumer has independent valuations over time, the firm can practice first-degree discrimination after initial sales.*

3.5.2 Exploit Consumer Habituation

The strategic effect of habituation can fundamentally change the existing results on product line design. One such result is the *downward-distortion principle* for deterring cannibalization. Under spot selling $\mathcal{P}^s$, the firm must keep the quality of low-end products sufficiently low, so that high-end customers find switching unattractive. Because each type purchases only once, the tradeoff is *contemporaneous*, concerning only a single line $\{q^s(v_1)\}_{v_1 \in \mathcal{V}}$ of products.[28] Technically, this principle relies on the relation that downward distortion reduces consumer rent. The relation arises naturally under spot selling, with one-shot purchase and exogenously fixed consumer preference: under $\mathcal{P}^s$, cutting quality q^s indeed reduces rent $U^s(v_1) = \int_{\underline{v}}^{v_1} q^s(v)\,\mathrm{d}v$.

[27] For example, when $\gamma < 0$, $\gamma\left(\underline{v} - \eta(\underline{v})\right) + \beta \geq 0$, $4\alpha\gamma\left(\overline{v} - \eta(\overline{v})\right) + 2\alpha\beta\mu - \alpha^2\beta^2 - \gamma^2\mu^2 + 2\alpha\beta + 2\gamma\mu - 1 \geq 0$, the ratio of consumer rent between ϕ^* and ϕ^s (per transaction) is

$$\frac{1}{1+\delta} \leq \frac{U_1^*(v_1)/(1+\delta)}{U^s(v_1)} \leq 1.$$

[28] Hence, the price discrimination motive alone drives the optimal design ϕ^s . Under second degree price discrimination ϕ^s, the firm should reduce the quality of low-end products, and keep that of high-end product at the efficient level; i.e., "efficiency at the top" and "distortion at the bottom".

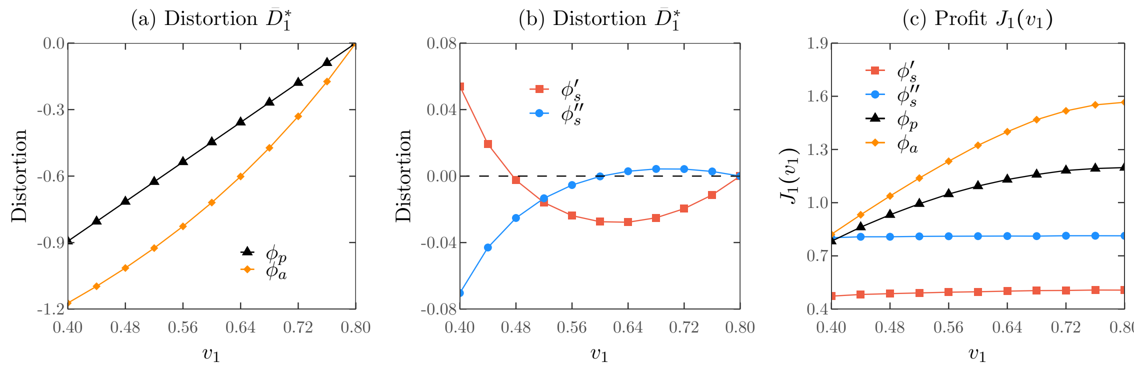
Under subscription $\mathcal{P}$, however, the downward-distortion principle may no longer work. This is because consumer habitation substantially complicates the cannibalization problem: as each type v_1 makes repeated purchases, the tradeoff becomes both *contemporaneous* and *interptemporal*, concerning two consecutive lines of products: $\{q_1^*(v_1)\}$ and $\{q_2^*(v^2)\}$. To prevent high type v_1 from switching to the products for lower type $v_1' < v_1$, the firm must design both lines jointly: not only current offering $q_1^*(v_1')$, but also future quality $q_2^*(v_1', v_2)$ should be distorted. In rent $U_i^*(v_1) = \int_{\underline{v}}^{v_1} \mathbb{E}[(\alpha + \gamma q_1^*) \cdot q_2^* | v] \, \mathrm{d}v$, the intensity of the intertemporal tradeoff is determined by the persistent effect $(\alpha + \gamma q_1^*)$. It implies that, when γ is negative and significant, it can be optimal to distort q_1^* upward—even beyond and above the full information level $\bar{q}_1$.

To appreciate the strategic effect of habituation, I now examine the distortion $\bar{D}_1^*(v_1) \equiv q_1^*(v_1) - \bar{q}_1(v_1)$ against the full-information benchmark $\bar{\phi}$. Since both solutions q_1^* and $\bar{q}_1$ internalize the welfare effect, the remaining distortion should be driven by price discrimination. Moreover, because downward distortion always reduces the contemporary rent $U_c^*(v_1)$, if the distortion $\bar{D}_1^*(v_1)$ turns out to be upward, then it must be driven by the intertemporal rent $U_i^*(v_1)$—the strategic effect of habituation.[29]

I find the distortion $\bar{D}_1^*$ depends on the nature of habituation. When habitation is addictive ($\gamma \geq 0$), the downward-distortion principle still applies ($\bar{D}_1^* \leq 0$): for low-end consumer $v_1' < v_1$, the downward distortion reduces not only current quality $q_1^*(v_1')$, but also the persistent effect $\big(\alpha + \gamma q_1^*(v_1')\big)$, and hence her future valuation v_2 and quality $q_2^*(v_1', v_2)$;

[29]Intuitively, both regimes $\mathcal{P}$ and $\bar{\mathcal{P}}$ consider consumer habituation. Their only difference is the privacy of consumer preferences and the resulting cannibalization problem in $\mathcal{P}$. Hence, both q_1^* and $\bar{q}_1$ internalize the direct welfare effect. The remaining distortion $\bar{D}_1^*(v_1) = q_1^*(v_1) - \bar{q}_1(v_1)$, if any, must serve the price discrimination motive, for the purpose of preventing cannibalization.

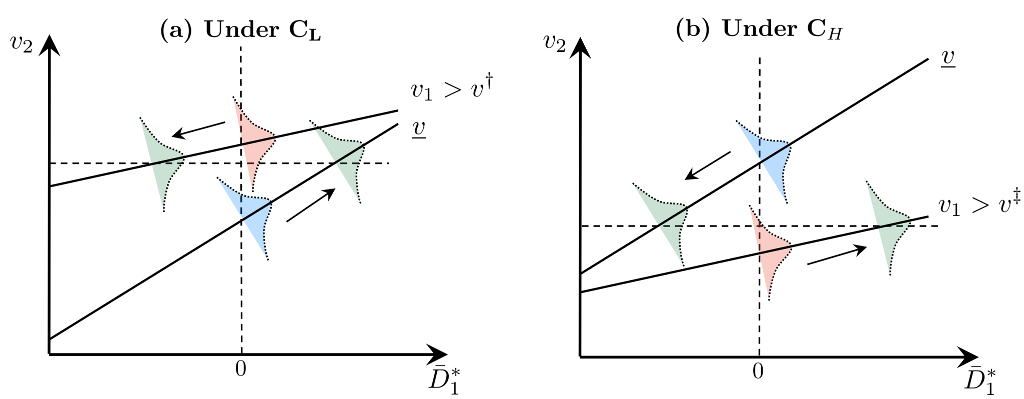
Figure 3.4: Performance comparison



Note: $V_q(v_1) = 0.7 + \gamma v_1$, $\underline{v}_1 = 0.4$, $\overline{v}_1 = 0.8$, $\mu = 0.6$, $\delta = 1$. ϕ_p: $\{(\alpha, \beta, \gamma_p) = (0.2, 0.7, 0)\}$; ϕ_a: $\{(\alpha, \beta, \gamma_a) = (0.2, 0.7, 0.1)\}$; ϕ_s^L: $\{(\alpha^L, \beta, \gamma_s) = (0.2, 0.7, -0.8)\}$; ϕ_s^H: $\{(\alpha^H, \beta, \gamma_s) = (0.9, 0.7, -0.8)\}$.

such a grim prospect reduces high-end consumers' incentive to switch, and hence their rent $U_1^*(v_1)$. Figs. 3.4.a and 3.4.c [30] illustrate the downward-distortion principle and its impact on firm profit $J_1^*(v_1)$, for $\gamma \in \{0, 0.1\}$: the higher the cross rate γ, the greater the distortion, and the higher the profit.

Figure 3.5: Taste Distribution Movement Under Satiating Habituation



When habituation is satiating ($\gamma < 0$), however, the downward-distortion principle may fail. Although downward distortion can reduce the contemporaneous rent $U_c^*(v_1)$, it

[30] ϕ_p is the optimal contract under plain habituation; ϕ_a is under additive habituation; ϕ_s^L is under satiating habituation satisfying $\mathbf{C}_L$; ϕ_s^H is under satiating habituation satisfying $\mathbf{C}_H$.

can also increase the intertemporal rent $U_i^*(v_1)$ and $U_1^*(v_1)$, making the firm worse off. As such, upward distortion can be optimal. Fig. 3.4.b depicts two such cases: (i) In case ϕ_s^L (– ■ –), low-end consumers v_1 have lower valuations in both periods. The simultaneous up-and-downward distortion then reduces future heterogeneity, making initial preference v_1 less informative of future valuation v_2, thereby reducing consumer rent $U_1^*(v_1)$. Fig. 3.5.a illustrates this homogenization process. (ii) In case ϕ_s^H (– • –), low-end consumers v_1 can have higher valuations in period 2. The simultaneous down-and-upward distortion is again to homogenize consumer preferences (see Fig. 3.5.b), and cut their rents. Because homogenization induces similar future preferences, the firm makes similar profits from serving ex-ante heterogenous consumers; see the flat curves ϕ_s^L and ϕ_s^H in 3.4.c. To formalize these findings, I define $\mathbf{C}_L \equiv \left\{ \bar{q}_1(\underline{v}) < \bar{q}_1\big(\theta(\underline{v})\big),\ \eta'(\bar{v})\bar{q}_1'(\bar{v}) < 0 \right\}$, and $\mathbf{C}_H \equiv \left\{ \bar{q}_1(\underline{v}) > \bar{q}_1\big(\theta(\underline{v})\big),\ \eta'(\bar{v})\bar{q}_1'(\bar{v}) > 0 \right\}$, where $\theta(v) \equiv v - \eta(v)$.

Proposition 3.4 (Simultaneous Up-and-Downward Distortion) *In regime $\mathcal{P}$, when habituation is addictive, the quality downward $\bar{D}_1^*(v_1) = q_1^*(v_1) - \bar{q}_1(v_1)$ is always downward. When habituation is satiating, under condition $\mathbf{C}_L$, the distortion is upward for low-end consumers, and downward for higher-end consumers; under condition $\mathbf{C}_H$, the distortion pattern is reversed .*[31]

To my knowledge, the simultaneous up-and-downward distortion is new to the production-line design literature. It has important implications for subscription design. For example, to segment markets and prevent cannibalization, the classic models recommend quality reduction. This model suggests a more nuanced view. For example, in digital goods

[31]Formally, under condition $\mathbf{C}_L$, there exists $v^\dagger$, such that $\bar{D}_1^*(v_1) > 0$, $\forall v_1 \in (\underline{v}, v^\dagger)$, and $\bar{D}_1^*(v_1) < 0$, $\forall v_1 \in (v^\dagger, \bar{v})$.

markets, habituation may reduce consumer heterogeneity and homogenize their tastes over time. In response, firms may use upward distortion, by offering excessive quality in the initial stage of the subscription. Besides the welfare effect of habituation, firms can also benefit from reducing future consumer rents—the strategic effect of habitation. The prediction is consistent with recent empirical findings; see, e.g., Zhang et al. (2019).

3.5.3 When Can Promotions Hurt the Firm?

Sales promotions can influence consumption, e.g., by changing habituation parameters (α, β, γ). The conventional view is that promotions increase consumer consumption, intensify consumer habituation, and hence increase firm profit (Heerde and Neslin, 2017). This work calls for caution: this is indeed the case under complete information, but the view is conceptually flawed when consumers have private and evolving preferences. Indeed, the conventional view considers only the bright side of consumer habituation—the welfare effect, ignoring its dark side of the strategic effect. As such, one may argue, consumer habituation is a "double-edged sword": although it can improve social welfare by enhancing consumer valuation, it can also exacerbate the cannibalization problem by enhancing the consumer's ability to extract surplus. In response, the firm may distort quality still further, to the extent that the surplus loss from preventing cannibalization overwhelms the welfare gain from exploiting habituation. As such, promotions may hurt firm profit and social welfare—a stark contrast to the conventional view.

Table 3.2 illustrates this key insight for $\gamma < 0$. When habituation is satiating, low-end consumers can have higher valuation in period 2. By jointly deploying up-and-downward

Table 3.2: Sensitive Analysis

α	$\mathbb{E}J_1^*$	$\mathbb{E}U_1^*$	$\mathbb{E}W_1^*$	β	$\mathbb{E}J_1^*$	$\mathbb{E}U_1^*$	$\mathbb{E}W_1^*$	γ	$\mathbb{E}J_1^*$	$\mathbb{E}U_1^*$	$\mathbb{E}W_1^*$
0.2	0.455	0.243	0.698	0.62	0.049	0.320	0.369	−0.70	0.455	0.243	0.698
0.3	0.414	0.298	0.712	0.67	0.245	0.276	0.521	−0.65	0.429	0.270	0.699
0.4	0.279	0.357	0.636	0.69	0.436	0.245	0.680	−0.60	0.343	0.300	0.643

Base case: $\alpha = 0.2,\ \beta = 0.7,\ \gamma = -0.7,\ \underline{v}_1 = 0,\ \overline{v}_1 = 1,\ \mu = 0.5,\ \delta = 1,\ F_1 = \mathcal{U}[0,1]$.

distortion, the firm can homogenize consumers in period 2, thereby reducing consumer rent $\mathbb{E}U_1^*$. As a result, promotions benefit the consumer, but they can hurt firm profit and social welfare (e.g., $\frac{\partial}{\partial\gamma}\mathbb{E}J_1^* < 0$, $\frac{\partial}{\partial\gamma}\mathbb{E}W_1^* < 0$). The former is driven by the welfare effect of habituation, while the latter is driven by the reduced effectiveness of upward distortion: when $\gamma < 0$, improving α and γ can reduce relative importance of increasing q_1 in changing $V_v = \alpha + \gamma q_1$, make upward distortion less effective for rent control, thereby hurt the firm profit and social welfare.

I now formalize this key insight. For consumer-v_1, let $\theta_1(v_1) \equiv v_1 - \eta(v_1)$ be her virtual valuation, and $Q_2^*(v_1) \equiv \mathbb{E}[q_2^*(v^2)|v_1]$ her expected quality assignment. Let tilde $\tilde{(\cdot)}$ denote random variables: ex ante, $\tilde{v}_1$, $\theta_1(\tilde{v}_1)$, $Q_2^*(\tilde{v}_1)$, and $q_1^*(\tilde{v}_1)$ are random variables. Let $\rho[\tilde{X},\tilde{Y}]$ be the correlation coefficient of $\tilde{X}$ and $\tilde{Y}$, and let $\rho^\dagger \equiv -\frac{\underline{v}}{\sqrt{\underline{v}^2+\mathbb{E}[\eta^2(v_1)]}}$.

Proposition 3.5 (Sensitivity Analysis) (a) *The firm always benefits from improving β.*

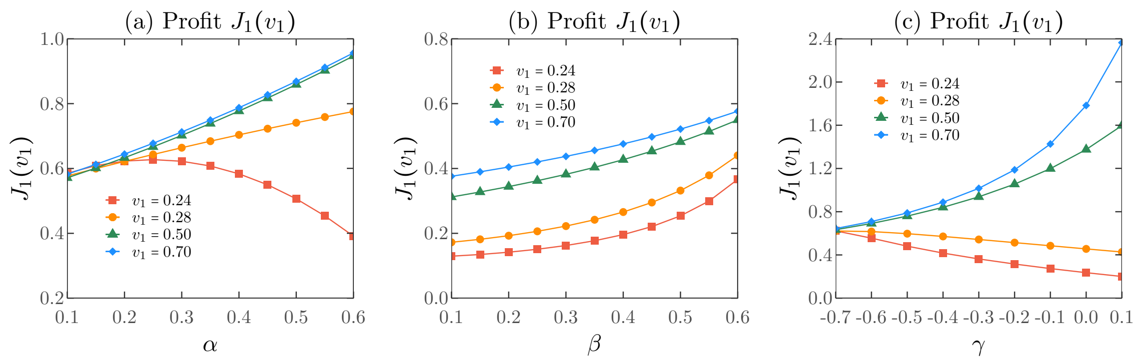
(b) *The firm benefits from improving α and γ for high-end consumers, but suffers for low-end consumers.*[32]

[32]Formally, there exists $v^\dagger \in \mathcal{V}$, such that

$$\tfrac{\partial}{\partial\alpha}J_1^*(v_1) \le 0,\quad \tfrac{\partial}{\partial\gamma}J_1^*(v_1) \le 0,\quad \forall v_1 \le v^\dagger;\qquad \tfrac{\partial}{\partial\alpha}J_1^*(v_1) \ge 0,\quad \tfrac{\partial}{\partial\gamma}J_1^*(v_1) \ge 0,\quad \forall v_1 \ge v^\dagger.$$

(c) *Ex ante, the firm suffers from improving* α *if* $\rho[\theta_1(\tilde{v}_1), Q_2^*(\tilde{v}_1)] \leq \rho^\dagger$, *and benefits if* $\rho[\theta_1(\tilde{v}_1), Q_2^*(\tilde{v}_1)] \geq 0$. *Moreover, the firm suffers from improving* γ *if* $\rho[\theta_1(\tilde{v}_1), q_1^*(\tilde{v}_1)Q_2^*(\tilde{v}_1)] \leq \rho^\dagger$, *and benefits if* $\rho[\theta_1(\tilde{v}_1), Q_2^*(\tilde{v}_1)] \geq 0$.

Figure 3.6: Sensitive Analysis



Note: Base case $\alpha = 0.2$, $\beta = 0.7$, $\gamma = -0.7$, $\underline{v}_1 = 0.24$, $\overline{v}_1 = 0.8$, $\mu = 0.52$.

Fig. 3.6 illustrates the proposition. I find: (i) The firm always benefits from improving habituation rate β. This is because larger β shifts the distribution of v_2 upward ($\frac{\partial}{\partial\beta}V = q_1^* \geq 0$), but has no direct effect on the intertemporal rent, and hence $\frac{\partial}{\partial\beta}J_1^*(v_1) = \delta\mathbb{E}[\frac{\partial}{\partial v_2}J_2^* \cdot \frac{\partial}{\partial\beta}V|v_1] = \delta\mathbb{E}[q_2^* \cdot q_1^*|v_1] \geq 0$, $\forall v_1 \in \mathcal{V}$. (ii) For low-end consumers with $\theta_1(v_1) < 0$, the firm can suffer from improving persistent rate α and cross rate γ.[33] This is because larger α can improve welfare by shifting the distribution of v_2 upward ($\frac{\partial}{\partial\alpha}V = v_1 \geq 0$), and inflate the intertemporal rent at the same time ($\frac{\partial}{\partial\alpha}\big((\alpha + \gamma q_1^*) \cdot q_2^*\big) = q_2^* \geq 0$). Whenever the rent inflation dominates, the firm suffers; i.e., $\frac{\partial}{\partial\alpha}J_1^*(v_1) = \delta\mathbb{E}\big[\frac{\partial}{\partial v_2}J_2^* \cdot \frac{\partial}{\partial\alpha}V - \frac{\partial}{\partial\alpha}\big(\eta \cdot (\alpha + \gamma q_1^*)q_2^*\big)\,\big|v_1\big] = \delta\mathbb{E}[q_2^* \cdot v_1 - \eta \cdot q_2^*|v_1] = \delta\theta_1(v_1) \cdot Q_2^*(v_1) \leq 0$, for $\theta_1(v_1) < 0$. The intuition for improving γ follows the same logic.

[33]Under subscription ϕ^*, the firm is willing to serve low-end consumers with negative virtual valuations, $\theta_1(v_1) < 0$, as long as long as they contribute more profit than the rents they demand from the two-period relationship.

In general, the firm can suffer from improving α and γ. To see why, let $v^\dagger$ be the threshold type with zero virtual valuation, $\theta_1(v^\dagger) = 0$. Because low- and high-end consumers have distinct virtual valuations ($\theta_1(v_1) \le 0$ for low-end consumers $v_1 \in [\underline{v}, v^\dagger)$), One can decompose the marginal profit of improving α into two terms:

$$\frac{\partial}{\partial \alpha}\mathbb{E}[J_1^*(\tilde{v}_1)] = \delta \int_{\underline{v}}^{v^\dagger} \theta_1(v_1) \cdot Q_2^*(v_1) \cdot \mathrm{d}F_1(v_1) + \delta \int_{v^\dagger}^{\overline{v}} \theta_1(v_1) \cdot Q_2^*(v_1) \cdot \mathrm{d}F_1(v_1). \tag{3.7}$$

Clearly, when low-end consumers have either sufficiently high mass $F(v_1^\dagger)$, or sufficiently high future quality $Q_2^*(v_1)$, the first term in Eq. (3.7) dominates, producing $\frac{\partial}{\partial \alpha}\mathbb{E}[J_1^*(\tilde{v}_1)] \le 0$. The intuition for improving γ is similar. See Table 3.2 for an example. The result has practical implications: when either low-end consumers have sufficient mass, or habituation is sufficiently satiating, excessive promotions can backfire, hurting both firm profit and social welfare.

3.6 Conclusion

The rise of personalized subscriptions has transformed the retail industry. A key challenge for a firm is how to learn consumers' evolving preferences and personalize its subscription service. I study this new class of product line design problems, where consumers' past purchases can influence their future valuations. I find the optimal design differs substantially from the classic solution: it internalizes both the contemporaneous and intertemporal externalities of consumer habituation and cannibalization. To control rent, the optimal design induces new distortions beyond second-degree discrimination: when habituation increases future consumer heterogeneity, the firm should distort quality downward, further below the second-best; when habituation reduces future consumer heterogeneity, the firm should leverage the homogenization mechanism with selective upward distortion—even above and beyond the first-first level.

My model helps explain the economics of personalized subscriptions. First, subscriptions help the firm to leverage consumer uncertainty: using the upfront subscription fee and price discount, the firm can craft an offsetting mechanism, relax the cannibalization constraints, and fully extract future surplus. When the consumer has independent valuations, it can even practice first-degree discrimination after initial sales. Second, subscriptions help the firm to capitalize on customer lifetime value. Because the firm can internalize the welfare gain from consumer habituation, it has strong incentives to improve product quality and enhance social welfare, resulting in Pareto improvement. Third, subscriptions help the firm to actively shape consumer preferences for profits. By optimizing sequential product offerings, the firm can make different consumers to habituate at different rates, compress their heterogeneity, and hence reduce deadweight loss.

This work refines the conventional wisdom: I demonstrate that the classic downward distortion principle may fail in new subscription models, that firms can practice first-degree price discrimination after initial sales, and that excessive promotions can hurt firm profit and social welfare. By providing a dynamic perspective with endogenous consumer preferences, this study advances the understanding of product line design.

Chapter 4

Salesforce Compensation with Self-Directed Training

4.1 Introduction

Firms' hiring practices have undergone significant changes in recent years. According to a survey by Robert Half, a staggering 84% of firms are now willing to hire workers even if they lack the required skills (Bolden-Barrett, 2019). In fact, 62% of workers were offered a position despite being under-qualified (Liu, 2019). However, firms still consider having the right *talent* (i.e., soft skills) to be essential:

> *"Workers can be trained on duties for a role, but individuals with the right soft skills are often harder to come by. Companies may need to re-evaluate their job requirements to hire the right talent."*,

stated by Paul McDonald, Robert Half's senior executive director (Half, 2019). He emphasizes a trend towards screening for talented workers and training them in the necessary skills — in other words, *hiring for talent and training for skills*. This has resulted in increased investment in training, particularly in sales. For instance, the use of CRM in small and medium-sized businesses (SMBs) has risen by 24% since 2019 (Malik, 2021). The importance of training is evident. Sales management research has long established that salesforce training is crucial for successful selling (Churchill Jr et al., 1985). Studies have shown that training increases salesforce productivity by imparting the skills required for effective sales (Román et al., 2002), thereby reducing selling costs and increasing the firm's profit (LaForge et al., 1997; Farrell and Hakstian, 2001). For example, a study by Klein (1997) found that every dollar invested in training resulted in a $122 increase in sales.

How do firms train their salesforce nowadays? According to LinkedIn's WorkPlace Learning Report 2021[1], 73% of L&D professionals expect to spend less on in-person-led training, and 79% plan to spend more on *self-directed training*. Why? One reason is that self-directed training allows for a more personalized approach to training, where firms can tailor programs to each salesperson's needs, including objectives, contents, timing (Zagada, 2018). This approach has proven to be more effective in engaging salespeople, enhancing their skills development, and improving their overall selling performance. Another reason for the rise of self-directed training is the shift towards remote work and virtual learning environments during the pandemic (Sven Smit and Govindarajan, 2020)[2]. Self-directed training refers to a learning approach where salespeople take ownership of their own learning and are responsible for setting and achieving their own goals. This approach is popular due to its flexibility and cost-effectiveness, allowing them to develop new selling skills more

[1] See `https://learning.linkedin.com/content/dam/me/business/en-us/amp/learning-solutions/images/wlr21/pdf/LinkedIn-Learning_Workplace-Learning-Report-2021-EN-1.pdf`

[2] During the pandemic, organizations have had to shift their training programs online, and self-directed training has become an effective way for salespeople to continue their professional development despite the disruptions caused by COVID-19. Some examples of self-directed training during the pandemic include online courses, webinars, virtual conferences, and self-paced learning modules.

efficiently (Herson, 2022; Melkonian, 2022). However, one drawback is that the firms lack the technology to monitor the salespeople during training. Without direct oversight, individuals may not receive the same level of feedback, guidance, and support that they would in a traditional training program (Stockton, 2016).

Theoretical literature primarily focuses on training policies, specifically how firms should schedule the level of training to achieve desired learning goals for their salespeople[3] (Krishnamoorthy et al., 2005). However, the literature is largely silent on how to schedule self-directed training and how to incentivize the salespeople on both learning and selling. In this paper, I aim to address this gap by exploring the optimal compensation design and training schedule when utilizing self-directed training. I address three questions: (i) What is the optimal schedule for self-directed training? (ii) What compensation scheme aligns with this optimal schedule? (iii) What are the managerial implications of using self-directed training in a firm's training program?

I present a theoretical model that captures the dynamics of a principal-agent relationship with adverse selection and moral hazard. The firm hires a salesperson to sell its products over multiple periods and offers a compensation plan that pays for performance. Prior to the selling season, the firm schedules training to improve the salesperson's skills. The salesperson has superior knowledge of their own skill and can invest in private skill development during training. In each period, the salesperson invests for himself, learns new skills during training, exerts unobservable effort, and receives performance pay based on their sales during the selling season. From the salesperson's performance and compensation

[3]The most important thing when scheduling a training is setting the learning goals, e.g., at what skills level the salespeople should reach. I refer to the level of training as the desired level of achievement from the salespeople (Knowles, 1975).

choice, the firm can infer his private skills, personalize the level of training and sales targets. Both parties are forward-looking and risk-neutral.

The firm faces several challenges in designing its compensation and scheduling the training policies. First, the firm must accurately learn about the salespeople's heterogeneous skills to conduct best personalization (Rao, 1990). However, salespeople have no incentive to reveal their true skills information to the firm, and may strategically game her by misreporting their skills. Second, training improves salespeople's selling skills in a systematic way. As time goes on, the salespeople gain new skills privately. The firm must adjust the incentives dynamically to ensure the salespeople can truthfully reveal their skills every time. Hence, the firm must screen information sequentially, and set contingent compensation. This dynamic information asymmetry problem requires sequential incentive constraints, involving dynamic private information arising stochastically. Third, self-directed training ignites unobservable self-investment; the salespeople can privately invest for themselves to boost selling skills. Without enough monitoring, the unobservable investment creates additional uncertainty even on how the private skills evolve. The extra agency costs exacerbate the dynamic information asymmetry problem. In response, the firm must provide contingent incentives for the salesperson: not only through the current compensation explicitly, but also through implicit incentives to encourage optimal investment by the salesperson. Technically, this game entails a mixing of dynamic adverse selection and intertemporal moral hazard—a non-trivial problem.

To overcome the challenges, the firm must first compensate for all the salesperson's opportunism: he can privately observe and control the evolving information advantage through unobservable self-investment when training; he can even mis-reveal skills and hide the lie through private selling effort. To dissuade him, the firm must pay the potential gain the salesperson expects from information advantage, compensate the salesperson's future gain to balance the current self-investment cost at the salesperson-optimal level, and provide enough commission to incentivize effort exertion. I find the optimal compensation scheme differs profoundly from the classical plan. First, in the selling season, the base salary and quota-commission structure helps the firm control the contemporaneous adverse selection problem and contemporaneous moral hazard accordingly, a classical device. Second, I pin down information rent as the sum of mis-revealing gains, which is precisely the option value of all the misreporting opportunities during the entire relationship. The optimal compensation involves deferred payment structure: the firm should withhold the expected future rent in the current period, and release it later contingent on future skills realization. The deferred information rent payment controls the dynamic adverse selection problem, by taking the salesperson as the residual claim to leverage his skills uncertainty, thereby screening new private skills for free. Third, the firm can use a front load training allowance to limit the salesperson's investment at the desired level. The salesperson's self-motivation restricts himself to invest at the level that maximizes all his future benefits, and implicit incentives. Giving him the allowance to match the salesperson's investment cost at that level, the firm ensures the salesperson won't under-invest to steal the money from the firm. Hence the intertemporal moral hazard is controlled.

The optimal quota and training schedules resolve a dynamic tradeoff between triggering implicit incentives, screening information, and maximizing efficiency. (i) To trigger implicit incentives, it sets an aggressive quota and pays over time. This incentivizes the salesperson to invest more to enhance skills and work hard to sell more. (ii) To screen information, despite the dynamic information rent payment, the firm downward adjusts the quota over time to prevent salesperson's gaming. The corresponding pay difference ensures that each salesperson is willing to perform the best of his ability. As a result, the salesperson's investment drops, thereby the firm downgrading the training. (iii) To maximize efficiency, the optimal scheme sets a higher quota and pays for higher skills, adjusting the quota and training level over time. The dynamic adjustments allow the firm to screen fresh information to alleviate dynamic adverse selection, create implicit incentives to trigger the best investment to mitigate intertemporal moral hazard, thereby maximizing efficiency.

My findings shed light on the impact of self-directed training. On one hand, self-directed training enhances the salesperson's skills, leading to increased sales and efficiency, as established in literature (Artis and Harris, 2007; Boyer et al., 2014a,b; Tuggle, 2014). In response, the firm should schedule the training in a skimming pattern: she aggressively upgrades the training in early periods, then declines the training level over time. That's the first-best level predicted by the literature.

On the other hand, self-directed training exacerbates the agency problem due to intertemporal moral hazard: the salesperson's unobservable investment influences his future private skills, providing additional opportunities to hide lies when revealing skills. In response, the firm should downgrade the training all the time relative to the first-best level to prevent this exacerbation. This reduction should be more aggressive in early periods when the salesperson has more opportunities to manipulate his skills, and it should be alleviated towards the end of the training period when such opportunities are reduced. Consequently, the firm should adopt an inverted U-shaped pattern for scheduling training, starting with a lower training level and gradually increasing it until some intermediate periods, after which it should be gradually decreased. It is worth noting that even in the long-run when the dynamic adverse selection problem fades away (Gao, 2022), the intertemporal moral hazard still persists and reduces efficiency.

My results have practical implications. First, I identify why the firms hire the young salespeople screening for their talents, but train their skills, despite the threat imposed by information asymmetry. With the feasibility of deferred compensation structure, the firm may screen the salesperson's future private skills for free, and encourage them to improve their skills. Second, my results suggest a growing trend towards self-directed training models in the industry: by providing an upfront self-investment allowance, the firm can indirectly monitor the salesperson to overcome the intertemporal moral hazard problem and benefit from the convenience of self-directed training. Third, I call two cautions: (i) In the short-run, the firm should not initially set the training level too high as it may over-incentivize the salesperson to increase their information rent, which can be detrimental. (ii) In the long-run, the firm shouldn't over emphasize the benefits of two-sided learning in their repeated interactions (Waldman, 2007). Both the firm and the salesperson have to learn about the future unknown skills; the salesperson loses his information advantage in the long-run, thereby reducing information friction. However, the two-sided learning only eliminates the dynamic adverse selection. The persistence of intertemporal moral hazard still hurts the firm's profit and efficiency. Even if the firms are familiar with the matured salespeople in the long-run, she shouldn't trust them indefinitely. By highlighting the role of self-directed training, this study sharpens my understanding of salesforce training and compensation theory, as well as the practice.

4.2 Related Literature

This paper stands at the intersection of several streams of marketing and economics literature. It connects salesforce compensation with training schedules.

My work contributes to the salesforce compensation literature. Classical salesforce compensation focuses on fundamental issue of moral hazard: the literature focuses on compensation design for profit maximizing when the salesperson's efforts are unobservable (Basu et al., 1985; Jain, 2012; Lal and Srinivasan, 1993; Hauser et al., 1994; Steenburgh, 2008; Mantrala et al., 2010; Rubel and Prasad, 2016; Bhargava and Rubel, 2019; Joseph and Thevaranjan, 1998; Jerath and Long, 2020). Meanwhile, literature also considers compensation design when salespeople are heterogeneous. Originating from the seminal work of Lal and Staelin (1986) and Rao (1990), literature assumes the salesperson has exogenously unobservable types (Albers, 1996; Daljord et al., 2014; Chen et al., 2021; Waiser, 2021). A core element of these studies is that the firm offers a menu of contracts, while the design requires full information about the distribution of the salesperson's type. Incorporating the salesperson's training stage behavior, by contrast, I study the new case where the salesperson can control the type distribution over time through unobservable actions, while the firm can infer the evolving type distribution through the prior. By incorporating the salesperson's training stage behavior, I contribute to the literature by exploring the interplay between compensation and training schedules, highlighting the role of self-directed training in salesforce management.

My work builds on the salesforce training literature. Previous studies have consistently emphasized the importance of training for successful selling (Krishnamoorthy et al., 2005). Studies have consistently expresses that training is vital to successful selling (Churchill Jr et al., 1985), and training has been shown to increase salesforce productivity by giving salespeople the skills to work effectively (Martin and Collins, 1991), thereby, it decreases selling cost to increase firm' profit (see Albers (2002) and Guenzi and Geiger (2011) Ch.12 for reviews). Analytical work has also captured the productivity-enhancing role of training (Krishnamoorthy et al., 2005; Chung et al., 2021). However, these studies assume that productivity is observable to both parties, overemphasizing the efficiency role of training. In contrast, my study models training program design in an environment of dynamic information asymmetry and intertemporal moral hazard, where the productivity-enhancing role of training must be balanced with the agency role of training. My result is consistent with the empirical finding of Magnotta et al. (2020) such that training also has inefficiency. I provide explanations about that inefficiency: it will exacerbate the agency problem to reduce efficiency.

My study also contributes to the self-directed training literature by providing the first analytical work on how the firm should schedule self-directed training and how well-applied training affects compensation design. Conceptual work in this area has emphasized that sales managers can use self-directed training as a supplement to traditional salesforce training to improve the performance of salespeople (Artis and Harris, 2007; Boyer et al., 2012; Tuggle, 2014). By modeling the impact of self-directed training on compensation design in an environment of dynamic information asymmetry and intertemporal moral hazard, my study provides important insights into how firms can use self-directed training to improve salesforce productivity while balancing the agency costs of training.

Technically, I incorporate the concept career concerns into the training schedule to capture the skill-enhancing property. Career concerns stands for the implicit incentives: individual's performance incentives are driven by a desire to shape external perceptions, i.e. firm's beliefs about private skills, thereby affect the compensation (Dewatripont et al., 1999; Holmström, 1999; Gibbons, 2005; Arya and Mittendorf, 2011). Previous literature has modeled career concerns through repeated games, assuming fixed exogenous types. The key insight is how the firm can use future incentives to motivate the salesperson to work hard. This framework arises naturally when the firm cannot commit to a compensation scheme. In contrast, I study how implicit incentives arise to resolve the intertemporal moral hazard problem when the firm can commit to the compensation scheme[4]. Moreover, I incorporate the career concerns in the stochastic type with endogeneity: salesperson's incentives are driven by elevating the distribution of private future skills, thereby affecting the compensation ex ante. I discuss how the firm can use these implicit incentives to shape the salesperson's hidden investment, thereby resolving the intertemporal moral hazard.

[4]I assume that the firm with more bargaining power commits to the training program and long-term compensation contract, which is a common assumption in compensation design (Moorthy, 1993; Ritz, 2008). In many career concerns research, the salesperson's wage is determined by the labor market assessment (Dewatripont et al., 1999; Arya and Mittendorf, 2011). I abstract away from that consideration and instead focus on the bilateral relationship between the firm and one salesperson to explore how salesforce training affects optimal compensation design.

4.3 Model

Table 4.1: Notation

Symbol	Description
$\equiv$	Equal by definition
α	Skills persistence rate, $\alpha \in [0, 1)$
β	Training intensity rate, $\beta \in [0, \infty)$
δ	Discount factor, $\delta \in (0, 1)$
θ_t	Salesperson's realized skills, $\theta \in \Theta \equiv [\underline{\theta}, \overline{\theta}]$
θ^t	Salesperson's realized skills history upon period t, $\theta^t \equiv (\theta_0, ..., \theta_t)$
$\hat{\theta}^t$	Salesperson's reported skills history upon period t, $\hat{\theta}^t \equiv (\hat{\theta}_0, ..., \hat{\theta}_t)$
m_t	Salesperson's self-investment in period t
ε_t	IID Random skills shock with mean $\mathbb{E}[\varepsilon_t] = \mu$, and distribution $G(\cdot)$
ω_t	IID Random sales shock with mean $\mathbb{E}[\varepsilon_t] = 0$, and distribution $H(\cdot)$
$\bar{\mathcal{P}}, \check{\mathcal{P}}, \mathcal{P}$	Compensation environment
ϕ	Contract, $\phi = \{\xi_t, (x_t, q_t)\}$ with training level ξ_t, compensation x_t and quota q_t
T	Number of periods in planning horizon
$F(\cdot)$	CDF of θ_0 with density $f(\cdot)$
$F_t(\cdot \mid \theta_{t-1}, m_t)$	Conditional CDF of θ_t with conditional density $f_t(\cdot \mid \theta_{t-1}, m_t)$
$\eta(\theta)$	Inverse hazard ratio of $F(\cdot)$: $\eta(\theta) = \frac{1-F(\theta)}{f(\theta)}$ decreases in θ
$\Pi_t(\theta^t)$	Firm's continuation profit from period t onward, after θ_t is realized
$U_t(\theta^t)$	Salesperson's continuation payoff from period t onward, after θ_t is realized

I consider that a firm sells products through a salespeople over T periods. In each period, before the selling season, the firm will conduct the self-directed training to help the salesperson boost his selling skills[5].

4.3.1 Selling Seasons

I apply the standard principal-agent formulation mixed with adverse selection and moral hazard in each selling season (Rao, 1990; Chen, 2005; Waiser, 2021; Gao, 2022).

[5] Each period is partitioned by new product release. For example, Apple provides continuous training for their salespeople before the release of new products such as iPhones and iPads (Francis, 2022). The training covers the features and benefits of the new products, as well as how to demonstrate them to customers. Apple's salespeople also learn how to handle customer objections and questions about the new products.

The salespeople are heterogeneous in skills (Rao, 1990): each salesperson privately knows his skills $\theta_t \in [\underline{\theta}, \overline{\theta}]$. In each period $t \geq 1$, by exerting selling effort e_t, the salesperson can generate sales $s_t = q_t + \omega_t = \theta_t + e_t + \omega_t$, subject to random shock $\omega_t \sim H(\cdot)$ with $\mathbb{E}[\omega_t] = 0$. The salesperson is restricted to a convex disutility function $\frac{1}{2}e_t^2$. I assume the salesperson's skill θ_t and effort e_t are unobservable; all the other parameters are common knowledge. To simplify the analysis, I assume each unit of good sold provides the firm a profit of 1. The firm offers the salesperson a compensation scheme $(x_t, q_t)_{t\geq 1}$ with x_t specifies the compensation x_t and quota q_t. I assume the compensation takes the form of quota-commission: $x_t = A_t + B_t \cdot (s_t - q_t)$. Table 4.1 defines all the notations.

4.3.2 Self-Directed Training

The firm schedules self-directed training at the beginning of each period t[6]. In the new era of self-directed training, the firm only assigns what the salesperson needs to learn; without the firm's monitoring, the salesperson is responsible for self-directing their learning. Formally, the firm posts the training level ξ_t, which is the expected effort the salesperson needs to invest[7]; the salesperson decides the actual investment m_t during training with a convex cost $\frac{1}{2}m_t^2$. Without enough monitoring, the salesperson's investment is unobservable. I conceptualize the salesperson's self-investment as increasing the salesperson's selling skills. Following Krishnamoorthy et al. (2005), I specify,

$$\theta_t = \alpha\theta_{t-1} + \beta m_t + \varepsilon_t.$$

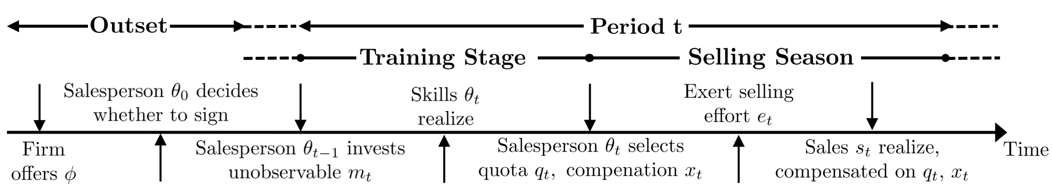
[6]Apple takes the advantage of digital resources to train its workforce with flexibility, under the continuous training practice (Francis, 2022).

[7]During the traing, firms often post learning resources. The training level is the desired investment level implied from the resources. In other words, that's the required investment for which the salesperson can fully digest the learning resources to improve his skills.

It determines the current period skills by past skill erosion $\alpha\theta_{t-1}$, effective investment βm_t and randomness ε_t. $\alpha \in [0,1]$ is the skills erosion rate. $\beta > 0$ is the training intensity, which captures how the salesperson's self-investment can transform into his selling skills. $\varepsilon_t \sim G(\cdot)$ with $\mathbb{E}\varepsilon_t = \mu$ drives the stochastic evolution in salesperson's skills, i.e. culture shocks (Guy et al., 1996). Given (θ_t, m_t), the future skill is distributed according to $F_t(\cdot \mid \theta_{t-1}, m_t)$ and $F_t(\theta_t \mid \theta_{t-1}, m_t) = G(\theta_t - \alpha\theta_{t-1} - \beta m_t)$.

I assume both parties are risk neutral, strategic and forward-looking. At the outset, the firm offers a contract $\phi \equiv \{\xi_t, (x_t, q_t)\}_{t\geq 1}$ over T periods specifying the training level ξ_t, quota q_t and compensation x_t in each period. The sequence of events are as follows, shown in Fig. 4.1. (i) The firm commits to the contract ϕ at the outset. (ii) Salesperson with private θ_0 decides whether to accept the contract. (iii) Upon acceptance, in each period t, the firm posts training level ξ_t at the beginning of the training stage. The salesperson self-invests an unobservable amount m_t to enhance his selling skills. (iv) The salesperson's skill θ_t is realized after training. He selects quota q_t and compensation x_t from ϕ. He then exerts selling effort e_t during the selling season. (v) Sales s_t realizes. The salesperson is paid based on the quota q_t and compensation x_t.

Figure 4.1: Sequence of Events



4.3.3 Contract Design

Following Rao (1990), I can formulate the contract design problem as one of mechanism design. The revelation principle simplifies the search for optimal contracts to direct truth-telling mechanisms (Myerson, 1986). Let $\theta^t = (\theta_0, \theta_1, ..., \theta_t)$ as the salesperson's skills history. Due to the salesperson's self-selection and revealed preference, the firm's contract reduces to a sequence of training level, quota and compensation functions: $\xi_t : \Theta^{t-1} \to \mathbb{R}_+$ $q_t : \Theta^t \to \mathbb{R}_+$ and $x_t : \Theta^t \times \mathbb{R}_+ \to \mathbb{R}_+$. Therefore, the firm must design the contract in such a way as to ensure that the salesperson truthfully reveals their skills in each period, provides enough incentives to ensure that the salesperson achieves the training level during training, and targets the quota in each selling season.

I begin by outlining how the salesperson responds to the contract ϕ. I first assume that the salesperson will aim to achieve the training level and deal with truthful skills revealing. I then propose how the firm can enforce obedience to ensure the salesperson will target the training level. In period t, after θ_t realizes, the salesperson-θ^t selects quota $q_t(\hat{\theta}^t)$ and compensation $x_t(\hat{\theta}^t, s_t)$ in selling season. He will then exert selling effort $q_t(\hat{\theta}^t) - \theta_t$ to maximize his expected payoff, taken the firm's training schedule $\xi_{t+1}(\hat{\theta}^t)$ into consideration:

$$\tilde{U}_t(\hat{\theta}^t; \theta^t) = \mathbb{E}[x_t(\hat{\theta}^t, s_t)] - \frac{1}{2}[q_t(\hat{\theta}^t) - \theta_t]^2 + \delta\Big\{ - \frac{1}{2}[\xi_{t+1}(\hat{\theta}^t)]^2 + \mathbb{E}[U_{t+1}(\hat{\theta}^t, \theta_{t+1}) \mid \theta_t, \xi_{t+1}(\hat{\theta}^t)]\Big\},$$

where $U_t(\theta^t) = \tilde{U}_t(\theta^t; \theta^t)$ denotes his continuation payoff under the truth-telling strategy ($\hat{\theta}_t = \theta_t, \ \forall t$). Note that $\tilde{U}_0(\hat{\theta}_0; \theta_0) = \delta\{-\frac{1}{2}[\xi_1(\hat{\theta}_0)]^2 + \mathbb{E}[U_1(\hat{\theta}_0, \theta_1) \mid \theta_0, \xi_1(\hat{\theta}_0)]\}$. The salesperson will truthfully reveal θ_t in each selling season if truthtelling is in his best interest:

$$U_t(\theta^t) = \max_{\hat{\theta}^t} \tilde{U}_t(\hat{\theta}^t; \theta^t), \quad \forall \theta^t. \qquad (IC_t)$$

The salesperson is sequential rational in the case he will never find it optimal to leave the firm when he has the option to stay:

$$U_t(\theta^t) \geq 0, \quad \forall \theta^t. \tag{IR_t}$$

I can now specify how the firm can enforce obedience. The main idea is that the training level posts must coincide with the salesperson's optimal action. Specifically, in period t during the training stage, the salesperson will target the training level if:

$$\xi_{t+1}(\theta^t) \in \arg\max_{m_{t+1}} \left\{-\tfrac{1}{2}m_{t+1}^2 + \mathbb{E}[U_{t+1}(\theta^{t+1}) \mid \theta_t, m_{t+1}]\right\}, \quad \forall \theta^t. \tag{OB_t}$$

Finally, the firm must provide sufficient incentives in each period to encourage the salesperson to achieve the quota during selling.

$$q_t(\theta^t) - \theta_t \in \arg\max_{e_t} \left\{\mathbb{E}[x_t(\theta^t, s_t)] - \tfrac{1}{2}e_t^2\right\}, \quad \forall \theta^t, t \geq 1. \tag{COM_t}$$

The firm will then maximize the expected profit:

$$\max_{\phi} \mathbb{E}[\Pi_0(\theta_0)] = \max_{\phi} \mathbb{E}\left[\sum_{t=1}^{T} \delta^t \Big(q_t(\theta^t) + \omega_t - x_t(\theta^t, s_t)\Big)\right],$$

subject to (IC_t), (IR_t), (OB_t) and (COM_t) for all t.

My model is compounded with the interactions of dynamic information asymmetry and intertemporal moral hazard. To explicate each part, I first analyze the problem $\bar{\mathcal{P}}$ by assuming the firm has full visibility on the salesperson's skills and can fully monitoring the training (both (both IC_t) and (OB_t) are dropped). Then, I analyze the problem $\check{\mathcal{P}}$ under a traditional training environment where the firm has no knowledge about the salesperson's skills but can fully monitor the training (only (OB_t) are dropped). Full monitoring is well characterized in traditional training environments, see e.g., (Krishnamoorthy et al., 2005; Chung et al., 2021). Lastly, I illustrate how the firm should manage both training and compensation in the new era with self-directed training. I use the perfect Bayesian equilibrium (PBE) solution concept for all game-theoretical analyses.

4.4 Full Visibility Benchmarks

I start from the case that the firm has full visibility and control over the salesperson: the firm can perfectly monitor the salesperson upon training and observe the salesperson's skills at each time. The benchmark $\bar{\mathcal{P}}$ is in the environment without dynamic information asymmetry, as discussed in (Krishnamoorthy et al., 2005). With full control, the firm doesn't need any incentive plans despite the selling season, instead, the firm needs to ensure the sequential rationality (IR_t) and compensation incentives (COM_t). The problem then reduces to a centralized control that maximizes the continuation of total efficiency inside the employment relationship. The firm's problem becomes

$$\Pi_t(\theta_t) = \max_{q_t, x_t, \xi_t} \Big\{ q_t(\theta_t) + \omega_t - \tfrac{1}{2}[q_t(\theta_t) - \theta_t]^2 + \delta\big\{ - \tfrac{1}{2}[\xi_{t+1}(\theta_t)]^2 + \mathbb{E}\big[\Pi_{t+1}(\theta_{t+1}) \big| \theta_t, \xi_{t+1}(\theta_t)\big]\big\}\Big\}. \tag{$\bar{\mathcal{P}}$}$$

On the equilibrium path, the quota and compensation have only a contemporaneous effect, allowing us to apply the classical static moral hazard framework to balance current revenue and compensation payout. However, determining the optimal training level presents a tradeoff between current cost overrun and future gains due to skills improvement. Increasing the training level can improve both current and future efficiency but also increases the learning cost in the employment relationship. My analysis reveals that the intertemporal tradeoff between these factors is crucial in setting the training level. I find

Proposition 4.1 *In regime $\bar{\mathcal{P}}$, the optimal contract $\bar{\phi}$ sets*

(*a*) *Quota and compensation:*

$$\bar{q}_t(\theta_t) = 1 + \theta_t, \qquad \bar{x}_t(\theta^t, s_t) = \bar{A}_t(\theta^t) + \bar{B}_t(\theta_t) \cdot (s_t - \bar{q}_t(\theta_t)), \quad \forall t \geq 1,$$

where base salary $\bar{A}_t(\theta^t) = \frac{1}{2} + \frac{1}{2}[\bar{\xi}_t(\theta_{t-1})]^2$ *and commission rate* $\bar{B}_t(\theta_t) = 1$.

(*b*) *Training levels:*

$$\bar{\xi}_{t+1}(\theta_t) = \beta \frac{1 - (\delta\alpha)^{T-t}}{1 - \delta\alpha}, \quad \forall t \geq 0.$$

Moreover, the training levels $\bar{\xi}_{t+1}(\theta_t)$ *are decreasing over time.*

Part (a) shows that the quota and compensation are determined based on classical contemporaneous moral hazard problem, since the quota doesn't offer any intertemporal dependence. The current quota $\bar{q}_t$ has only short-term consequence, thereby in each period, the firm sets it to equalize marginal revenue 1 with marginal cost $\frac{\partial}{\partial q_t}[\frac{1}{2}(q_t - \theta_t)^2] = q_t - \theta_t$. The firm decomposes the compensation into basic salary and commission. The commission rate is to incentivize the salesperson to exert efforts to target the quota: the marginal benefits from exerting efforts $\bar{B}_t$ should match the marginal cost of efforts $\frac{\partial}{\partial e_t}[\frac{1}{2}e_t^2] = e_t$ evaluated at where the quota is targeted: $\bar{q}_t - \theta_t$. The basic salary is designed to ensure the firm only pays actual selling cost and self-investment cost in the training stage, thereby extracting the entire efficiency.

Part (b) characterizes the optimal training schedules in regime $\bar{\mathcal{P}}$. With perfect visibility, the firm ensures the salesperson θ_t invests the exact amount $\bar{\xi}_t(\theta_{t-1})$ required by the training schedules. Note that the salesperson's skill is driven by past skill decay and the self-investment $\theta_t = \alpha\theta_{t-1} + \beta\xi_t + \varepsilon_t$. The self investment affects not only the salesperson's current skill θ_t directly with intensity β, but also his future skills indirectly through skills decay $\alpha\theta_t$. By scheduling more training today, the salesperson may suffer more cost on self-investment, but more training brightens the future such that the salesperson gains more skills on making more profit. Hence the firm must internalize both the contemporaneous and the intertemporal effects of training schedule to reach the highest profit: the firm should balance current marginal training cost $\frac{\partial}{\partial\xi_t}[\frac{1}{2}\xi_t^2]$ with marginal future benefits due to skill enhancement $\frac{\partial}{\partial\xi_t}\mathbb{E}[\Pi_t]$. The required learning investment is decreasing over time since the training is provided to bring the salesperson to the highest productivity as quickly as possible. The training is hence more aggressive in earlier times of recruitment and becomes more gentle when the salesperson becomes more resilient. This observation satisfies that firms usually begin with impact training through intensive periods to build salespeople's basic knowledge to an intermediate level (Singh, 2022).

4.5 Optimal Compensation and Self-Directed Training Schedule

The concept of full visibility and control over salespeople is well established in traditional training with general monitoring (Joseph and Thevaranjan, 1998). However, in the popular self-directed training approach, firms fail to monitor salespeople perfectly to observe their skills. This leads to information friction and salespeople with high skills can strategically select a lower quota to benefit from their information advantage. The firm has to pay information rent and distort sales quota and commission rate—the classical solution for a mixing of adverse selection and moral hazard (Rao, 1990). With self-directed training, the salesperson can strategically invest himself to enhance his skills hence his position of information advantage: by manipulating the period-t investment m_t, the salesperson can control the distribution $F(\cdot \mid \theta_{t-1}, m_t)$ of his selling skills θ_t, hence endogenize information asymmetry and increase his information advantage. As a consequence, the agency problem is even worse. The firm has a headache on how to incentivize the salesperson to target the training level, exert enough efforts to reach the sales quota and share accurate skills for personalization.

In response, besides the contemporaneous opportunism described by Rao (1990), the firm has to internalize three intertemporal effects. First, since the salesperson's current skills can persist into his future skills, the firm must internalize the decay $\frac{\partial}{\partial \theta_{t-1}} \theta_t = \alpha$ of information asymmetry, a dynamic adverse selection problem rises. Second, as the salesperson's self-investment increases his skills, the firm must internalize the impacts of it through $\frac{\partial}{\partial m_t} \theta_t = \beta$. Lastly, the fact that the salesperson's investment is unobservable to the firm gives the salesperson the flexibility to manipulate his information advantage in the selling season, exacerbating the intertemporal moral hazard problem. To better understand the effect of dynamic information asymmetry and intertemporal moral hazard, I examine $\check{\mathcal{P}}$ and $\mathcal{P}$ in sequel.

4.5.1 Dynamic Adverse Selection under Traditional Training

Regime $\check{\mathcal{P}}$ corresponds to the problem where the firm offers traditional training. For instance, the firm provides general classroom training where learning is strictly enforced, and the salespeople are closely monitored by their managers to ensure the effectiveness of training. Thus, the firm can fully monitor the salesperson's self-investment, m_t, and can set the training level as $\xi_t = m_t$. The only issue that remains is the problem of dynamic information asymmetry, and I can drop the term (OB_t) to enforce that the salesperson targets the training level costlessly. Since the salesperson privately observes his own skills due to proximity, after the period-t training, with public knowledge of ξ_t, the salesperson-θ_{t-1} observes ε_t and infers $\theta_t = \alpha\theta_{t-1} + \beta\xi_t + \varepsilon_t$ as his private skill in period t. However, the firm cannot observe any θ_t, so she must screen them for better quota selection. Sequential IC_t is necessary for screening to guarantee truthtelling in all periods. The central issue here is how to compensate for the salesperson's information advantage.

How Should the Firm Compensate the Salesperson to Learn His Skills?

I start by deriving the firm's compensation scheme backward given the training schedule. A key step is to change the analytic variable from the compensation $\{x_t(\theta^t, s_t)\}_{t\geq 1}$ to identify the salesperson's continuation rent $\{U_t(\theta^t)\}_{t\geq 0}$ in each period. In period t, consider salesperson-θ_t who enjoys skills advantage $\mathrm{d}\tilde{\theta}_t$ over salesperson-$(\theta_t - \tilde{\theta}_t)$. Along each quota path $(q_1, ..., q_t)$ and training level path $(\xi_1, ..., \xi_t)$, salesperson-θ_t enjoys $(q_t - \theta_t) \cdot \mathrm{d}\tilde{\theta}_t$ more efficiency in period t, develops $\mathbb{E}[\frac{\partial\theta_{t+1}}{\partial\theta_t}]\,\mathrm{d}\tilde{\theta}_t = \alpha\,\mathrm{d}\tilde{\theta}_t$ higher skills in period $t+1$, hence enjoys $\delta\mathbb{E}[\alpha\frac{\partial U_{t+1}(\theta^{t+1})}{\partial\theta_{t+1}}]\,\mathrm{d}\tilde{\theta}_t$ more expected efficiency in the future. Therefore, the marginal increases of the salesperson's efficiency in period t can be decomposed as

$$\begin{aligned}\frac{\partial}{\partial\theta_t}U_t(\theta_t)\cdot\mathrm{d}\tilde{\theta}_t &= \underbrace{(q_t(\theta^t)-\theta_t)\cdot\mathrm{d}\tilde{\theta}_t}_{\text{contemporaneous}} + \underbrace{\delta\,\mathbb{E}\left[\alpha\frac{\partial U_{t+1}(\theta^{t+1})}{\partial\theta_{t+1}}\right]\cdot\mathrm{d}\tilde{\theta}_t}_{\text{intertemporal}} \\ &= (q_t(\theta^t)-\theta_t)\cdot\mathrm{d}\tilde{\theta}_t + \sum_{k=t+1}^{T}\delta^{k-t}\alpha^{k-t}\mathbb{E}[q_k(\theta^k)-\theta_k]\cdot\mathrm{d}\tilde{\theta}_t.\end{aligned}$$

To prevent the salesperson-θ_t from pretending to be salesperson-$(\theta_t-\mathrm{d}\tilde{\theta}_t)$, the firm in expectation must reward him with both contemporaneous rent $(q_t-\theta_t)\cdot\mathrm{d}\tilde{\theta}_t$ and intertemporal rent $\sum_{k=t+1}^{T}\delta^{k-t}\alpha^{k-t}\mathbb{E}[q_k(\theta^k)-\theta_k]\cdot\mathrm{d}\tilde{\theta}_t$. Moreover, there are $[\underline{\theta},\theta_t)$ such switching opportunities, so the total rent for salesperson-θ_t is

$$U_t(\theta_t) = U_t(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_t}(q_t(\tilde{\theta}^t)-\tilde{\theta}_t)\,\mathrm{d}\tilde{\theta}_t + \int_{\underline{\theta}}^{\theta_t}\sum_{k=t+1}^{T}\delta^{k-t}\alpha^{k-t}\mathbb{E}[q_k(\tilde{\theta}^k)-\theta_k]\cdot\mathrm{d}\tilde{\theta}_t.$$

Therefore, at the time design the employment contract, the salesperon-θ_0 expects to obtain rents

$$U_0(\theta_0) = U_0(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_0}\sum_{t=1}^{T}\delta^t\alpha^t\mathbb{E}[q_t(\tilde{\theta}^t)-\theta_t]\cdot\mathrm{d}\tilde{\theta}_0.$$

The rent formulation facilitates the relaxation of (IC_t). Actually, the salesperson's rent is the shadow price for ensuring sequential self-selection, the price the firm must pay to prevent opportunism across skills and over time. The higher the initial skills, the less the skills decay, the higher the salesperson's rent $U_0(\theta_0)$. To ensure the participation and protect the salesperson to have nonnegative rent in each period, e.g., $U_t(\theta^t)\geq 0$, I can take $U_t(\theta^t)=0$ to relax all the (IR_t) constraints.

What Are the the Quotas and Training Schedules?

Up to this point, I have pinned down all the (IC_t) and (IR_t) for the firm to consider. Using the salesperson's rent $\{U_t(\theta^t)\}$, I can reformulate the firm's profit-maximizing problem as

$$\max_{\{q_t,\xi_t\}_{t\geq 1}} \mathbb{E}\left[\sum_{t\geq 1}^{T} \delta^t \Big(-\tfrac{1}{2}[\xi_t(\theta^{t-1})]^2 + q_t(\theta^t) + \omega_t - \tfrac{1}{2}[q_t(\theta^t) - \theta_t]^2 - \eta(\theta_0)\cdot \alpha^t \big(q_t(\theta^t) - \theta_t\big)\Big)\right], \tag{$\check{\mathcal{P}}$}$$

where $\eta(\theta_0) = \frac{1-F(\theta_0)}{f(\theta_0)}$ is the inverse hazard ratio of the salesperson's initial skills θ_0[8]. The firm's objective now shifts to maximizing virtual efficiency, which represents the maximum amount that the firm can extract from the salesperson without risking opportunism. In particular, α^t captures the intertemporal externality caused by skill decay and $\eta(\theta_0)\alpha^t$ is the deadweight loss from the firm to prevent the salesperson from gaming. The virtual efficiency formulation simplifies the problem to one of determining the quotas provision $\{q_t\}_{t\geq 1}$ and training level $\{\xi_t\}_{t\geq 1}$ optimize the objective. Applying the first order condition, I find

Proposition 4.2 *In regime $\check{\mathcal{P}}$, the optimal contract $\check{\phi}$ sets*

(*a*) *Quotas*

$$\check{q}_t(\theta^t) = \theta_t + 1 - \eta(\theta_0)\alpha^t.$$

[8]Note that $\mathbb{E}[U_0(\theta_0)] = \mathbb{E}[\eta(\theta_0)\cdot U_0'(\theta_0)]$ by integration by parts.

Moreover, $\check{q}_t(\theta)$ *is increasing in* t *for any* $\theta \in [\underline{\theta}, \overline{\theta}]$*. Compensation structures are* $\check{x}_t(\theta^t, s_t) = \check{A}_t(\theta^t) + \check{B}_t(\theta_t) \cdot (s_t - \check{q}_t(\theta_t))$ *where commission rate* $\check{B}(\theta^t) = \check{q}_t(\theta^t) - \theta_t$ *for all* t*, and base salary satisfies*

$$\check{A}_t(\theta^t) = \frac{1}{2}[\check{\xi}_t(\theta_{t-1})]^2 \cdot \mathbb{1}_{\{t=1\}} + \frac{1}{2}[\check{q}_t(\theta^t) - \theta_t]^2 + U_t(\theta^t) + \delta\frac{1}{2}[\check{\xi}_{t+1}(\theta^t)]^2 \\ - \delta\mathbb{E}[U_{t+1}(\theta^{t+1}) \mid \theta_t, \check{\xi}_{t+1}(\theta^t)], \quad \forall t < T$$

and $\check{A}_T(\theta^T) = \frac{1}{2}[\check{q}_T(\theta^T) - \theta_T]^2 + U_T(\theta^T)$.

(*b*) *Training level:*

$$\check{\xi}_{t+1}(\theta_t) = \beta\frac{1 - (\delta\alpha)^{T-t}}{1 - \delta\alpha}, \quad \forall t \geq 0.$$

Moreover, the training level $\check{\xi}_{t+1}(\theta_t)$ *are decreasing over time.*

The firm should pay information rent and distort quota to ensure the truthful revealing. Relative to the myopic salesperson case characterized by Rao (1990), the plans internalize information asymmetry over time: the salesperson not only enjoys the skills advantage today, but also enjoys it in the future due to skills persistence. Hence the firm must pay him for all the skills advantage in each period affected by the salesperson's initial skills θ_0 (Gao, 2022):

$$U_0(\theta_0) = U_0(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_0} \sum_{t=1}^{T} \delta^t \alpha^t \mathbb{E}[q_t(\tilde{\theta}^t) - \theta_t] \cdot \mathrm{d}\tilde{\theta}_0.$$

By buying out the salesperson's option to behave opportunistically, the salesperson is under the firm's control. These options stem from the firm's uncertainty (across salesforce heterogeneity θ_0) and salesperson's skills persistence α^t (over time). I can simplify the expected rent faced by the firm:

$$\mathbb{E}[U_0(\theta_0)] = \mathbb{E}[U_0'(\theta_0) \cdot \eta(\theta_0)] = \mathbb{E}\left[\sum_{t=1}^{T} \delta^t \alpha^t (q_t(\theta^t) - \theta_t) \cdot \eta(\theta_0)\right].$$

The greater the manufacturer's uncertainty (large $\eta(\theta_0)$), the greater the skills advantage $(q_t - \theta_t)$, the stronger the skills persistence α^t, the larger the expected rent payment.

Given the information rent structure, the firm will set the quota by trading off the contemporaneous marginal revenue $\frac{\partial}{\partial q} q_t = 1$, contemporaneous marginal effort cost $\frac{\partial}{\partial q}\frac{1}{2}(q_t - \theta_t)^2 = q_t - \theta_t$ and marginal cost due to dynamic adverse selection $\frac{\partial}{\partial q}\alpha^t \eta(\theta_0)(q_t - \theta_t)^2 = \alpha^t \eta(\theta_0)$. Intuitively, rent $\mathbb{E}[U_0(\theta_0)]$ increases in the quotas $q_t(\theta^t)$. To reduce rent, the firm should distort the quota dynamically, not only for current skills θ_0, but also the future quotas along each path θ^t. The magnitude of distortion is given by $\bar{q}_t - \check{q}_t = \eta(\theta_0)\alpha^t$: the stronger the skills persistence α, the earlier the relationship (small t), the deeper the quota distortion.

To achieve profit maximization, the firm must schedule the training by balancing the marginal cost $\frac{\partial}{\partial \xi_t}[\frac{1}{2}\xi_t^2] = \xi_t$ with marginal future profit due to skill enhancement $\frac{\partial}{\partial \xi_t}\mathbb{E}[\Pi_t] = \beta[\sum_{k=t}^{T} \delta^{k-t}\alpha^{k-t}\mathbb{E}[q_k(\theta^k) - \theta_k + \eta(\theta_0)\alpha^k]]$:

$$\check{\xi}_t = \beta\left[\sum_{k=t}^{T} \delta^{k-t}\alpha^{k-t}\mathbb{E}[\check{q}_k(\theta^k) - \theta_k + \eta(\theta_0)\alpha^k]\right].$$

Surprisingly, the firm schedules the training at the first-best level, a counterintuitive result. One may argue that training enhances the salesperson's private skills, thereby increasing his information advantage in selling seasons, and the adverse selection problem exacerbates (Gao, 2022). In response, the firm should distort the training investment to mitigate the adverse selection problem. Why doesn't the firm distort that? It is important to look at the salesperson's rent structure. Consider a perturbation $\mathrm{d}\xi_t$ in the training level, the expected increase in the salesperson's skill is $\beta \mathrm{d}\xi_t$ in period t. Given the optimal quota $\check{q}_t(\theta^t) = \theta + 1 - \eta(\theta_0)\alpha^t$, the firm should in response to increase the quota with amount of

$\beta \mathrm{d}\xi_t$. Therefore, the salesperson's information rents in period t don't increase; the firm is radical to intense training without being threatened by the potential adverse selection problem. Intuitively, what really determines the salesperson's rent is his effort. Anticipating the potential skills increase, the firm will adjust the quota accordingly to induce the salesperson's effort unchanged. By doing so, the firm can reinforce training up to the first-best level without exacerbating the adverse selection problem.

4.5.2 Intertemporal Moral Hazard under Self-Directed Training

With self-directed training, The firm lacks monitoring technology to fully capture what the salesperson invests during the training stage. To ensure the salesperson will invest at the desired level, the firm needs to provide enough incentives. The incentives rise both contemporaneously and intertemporally. Specifically, the salesperson's investment enhances his current selling skills through $\frac{\partial}{\partial \xi_t}\theta_t = \beta$, leading to a direct increase in selling compensation. Moreover, as the salesperson's skills can persist at rate $\frac{\partial}{\partial \theta_t}\theta_{t+1} = \alpha$, the investment indirectly boosts the future skills to raise the salesperson's intertemporal payoff.

How Should the Firm Incentivize the Salesperson to Mitigate Moral Hazard?

By unobservable self-investment, the salesperson can leverage his skills to gain information advantage when selling. Therefore, by elevating the salesperson's information rent in the selling season, the firm can incentivize him to invest more in the training stage, up to the optimal level where the marginal future information rent matches the salesperson's marginal investment cost. To achieve this, the firm recommends the optimal investment level $\xi^*_{t+1}(\theta^t)$ as the training level to the salesperson, who will follow the recommendation without hesitation since it is their utmost level to invest. By devoting an infinitesimal investment level $\xi^*_{t+1}(\theta^t) + \mathrm{d}\tilde{\xi}$, the salesperson will experience more disutility by amount of $\psi'(\xi^*_{t+1}(\theta^t))\cdot \mathrm{d}\tilde{\xi}$. However, the salesperson gain more information advantage $\frac{\partial}{\partial \xi}\mathbb{E}[U_{t+1}(\theta^{t+1}) \mid \theta_t, \xi^*_{t+1}] \cdot \mathrm{d}\tilde{\xi}$ in selling seasons. Note that

$$\begin{aligned}\frac{\partial}{\partial \xi}\mathbb{E}[U_{t+1}(\theta^{t+1}) \mid \theta_t, \xi^*_{t+1}] \cdot \mathrm{d}\tilde{\xi} &= \mathbb{E}[\frac{\partial}{\partial \theta_{t+1}} U_{t+1}(\theta^{t+1}) \cdot \frac{\partial \theta_{t+1}}{\partial \xi}] \cdot \mathrm{d}\tilde{\xi} \\ &= \beta\left[\sum_{k=t+1}^{T} \delta^{k-t-1}\alpha^{k-t-1}\mathbb{E}[q_k(\theta^k) - \theta_k]\right] \cdot \mathrm{d}\tilde{\xi}.\end{aligned}$$

Hence, the optimal training level must solve

$$\xi^*_{t+1}(\theta^t) = \beta\left[\sum_{k=t+1}^{T} \delta^{k-t-1}\alpha^{k-t-1}\mathbb{E}[q_k(\theta^k) - \theta_k]\right], \quad \forall t. \qquad (OB_t)$$

What Are the Best Policies?

Using the salesperson's rent $\{U_t(\theta^t)\}$ and the accommodated incentives constraints, I can reformulate the firm's profit-maximizing problem as

$$\max_{\{q_t\}_{t\geq 1}} \mathbb{E}\left[\sum_{t\geq 1}^{T} \delta^t\Big(-\tfrac{1}{2}[\xi_t(\theta^{t-1})]^2 + q_t(\theta^t) + \omega_t - \tfrac{1}{2}[q_t(\theta^t) - \theta_t]^2 - \eta(\theta_0)\cdot \alpha^t\big(q_t(\theta^t) - \theta_t\big)\Big)\right],$$

where $\xi_t(\theta^{t-1})$ is pinned down by (OB_t) and $\eta(\theta_0) = \frac{1-F(\theta_0)}{f(\theta_0)}$ is the inverse hazard ratio of the salesperson's initial skills θ_0. The firm's problem becomes the virtual efficiency maximization, which is the maximal amount the firm can extract from the salesperson without opportunism. In particular, α^t captures the intertemporal externality due to skills decay and $\eta(\theta_0)\alpha^t$ is the deadweight loss from the firm to prevent the salesperson from gaming. The virtual efficiency formulation simplifies the problem to one of determining the quotas provision $\{q_t\}_{t\geq 1}$ only. I find

Proposition 4.3 *In regime $\mathcal{P}$, the optimal contract ϕ^* sets* (*a*) *Quotas for* $t < T$ *satisfies*[9]

$$q_t^*(\theta^t) - \theta_t - \frac{1-\eta(\theta_0)\alpha^t}{1+\beta^2} = \left[\frac{q_{t+1}^*(\theta^{t+1}) - \theta_{t+1}}{1+\beta^2} - \frac{1-\eta(\theta_0)\alpha^{t+1}}{1+\beta^2}\right],$$

with $q_T^*(\theta^T) = \theta_t + \frac{1-\eta(\theta_0)\alpha^T}{1+\beta^2}$. *Moreover,* $q_t^*(\theta)$ *is increasing in* t *for any* $\theta \in [\underline{\theta}, \overline{\theta}]$. *Compensation structures are* $x_t^*(\theta^t, s_t) = A_t^*(\theta^t) + B_t^*(\theta_t) \cdot (s_t - q_t^*(\theta_t))$ *where commission rate* $B^*(\theta^t) = q_t^*(\theta^t) - \theta_t$ *for all* t, *and base salary satisfies*

$$A_t^*(\theta^t) = \tfrac{1}{2}[\xi_t^*(\theta_{t-1})]^2 \cdot \mathbb{1}_{\{t=1\}} + \tfrac{1}{2}[q_t^*(\theta^t) - \theta_t]^2 + U_t(\theta^t) + \delta\tfrac{1}{2}[\xi_{t+1}^*(\theta^t)]^2$$
$$- \delta\mathbb{E}[U_{t+1}(\theta^{t+1}) \mid \theta_t, \xi_{t+1}^*(\theta^t)], \quad \forall t < T$$

and $A_T^*(\theta^T) = \frac{1}{2}[q_T^*(\theta^T) - \theta_T]^2 + U_T(\theta^T)$.

(*b*) *Training level*[10]*:*

$$\xi_{t+1}^*(\theta_t) = \beta\left[\sum_{k=t+1}^{T} \delta^{k-t-1}\alpha^{k-t-1}\mathbb{E}[q_k^*(\theta^k) - \theta_k]\right].$$

Moreover, there exists some $T^\dagger$ *such that* $\xi_{t+1}^*(\theta)$ *is increasing in* t *when* $t < T^\dagger$ *and decreasing in* t *when* $t > T^\dagger$.

(*c*) *Self-directed training has agency impact:*

$$q_t^*(\theta^t) \le \check{q}_t(\theta^t), \quad \xi_{t+1}^*(\theta^t) \le \check{\xi}_{t+1}(\theta^t), \quad \forall\theta^t.$$

[9] q_t^* admits closed form solution:

$$q_t^*(\theta^t) = \theta_t + 1 - \eta(\theta_0)\alpha^t - \sum_{k=t}^{T}\left[\left(\frac{\delta\alpha}{1+\beta^2}\right)^{k-t}\frac{\beta^2}{1+\beta^2}\left(1-\eta(\theta_0)\alpha^t\right)\right].$$

[10] $\xi_t^*(\theta^{t-1})$ exhibits closed form solution:

$$\left[\frac{\delta\alpha(1+\eta(\theta_0)\alpha^{t+1})}{(1+\beta^2)^2 - \delta\alpha(1+\beta^2)(2+\beta^2)} + \frac{1+\eta(\theta_0)\alpha^t}{1+\beta^2}\right]\frac{1-(\delta\alpha)^{T-t}}{1-\delta\alpha}$$
$$-\left[\frac{\delta\alpha\left(\frac{\delta\alpha(2+\beta^2)}{1+\beta^2}\right)^{T-t}}{(1+\beta^2)^2 - \delta\alpha(1+\beta^2)(2+\beta^2)} + \frac{\delta\eta(\theta_0)\alpha^{t+2}\left(\frac{\delta\alpha^2(2+\beta^2)}{1+\beta^2}\right)^{T-t}}{(1+\beta^2)^2 - \delta\alpha^2(1+\beta^2)(2+\beta^2)}\right]\frac{1-\left(\frac{1+\beta^2}{2+\beta^2}\right)^{T-t}}{1-\left(\frac{1+\beta^2}{2+\beta^2}\right)}.$$

In regime $\mathcal{P}$, the optimal scheme ϕ^* reconciles additional pressures compared to the scheme $\check{\phi}$—exploit the best incentives for the optimal self-investment, since both parties have diverse interests. For the firm, Proposition 4.2 emphasizes the firm desires to schedule the training at the first-best level; however, it is too costly to do so: the salesperson's marginal cost $\check{\xi}_t$ at the first best level overruns his marginal rent benefits $\frac{\partial}{\partial \xi_t} U_t(\theta^t)$, enforced by the (OB_t) constraints. Even if the firm schedules training at the first-best level, the salesperson will only invest at most $\frac{\partial}{\partial \xi_t} U_t(\theta^t)$ resulting in the salesperson being under qualified to the level the firm desires. If the firm were to post the quota $\check{q}_t(\theta^t)$ along with the commission rate $\check{B}_t(\theta^t) = \check{q}_t(\theta^t) - \theta_t$, the commission is too high for the underqualified salesperson. While this does create extra incentives for the salesperson to exert more effort when selling. On one hand, the extra efforts ensure the salesperson can reach the quota and the firm can get the sales target. On the other hand, the firm must pay the salesperson with extra information rent $\mathbb{E}[\sum_t \delta^t \alpha^t (\check{q}_t - \theta_t)]$, an efficiency loss. Therefore, the optimal quota $q_t^*(\theta^t)$ includes addition downward distortion $\sum_{k=t}^{T}[(\frac{\delta\alpha}{1+\beta^2})^{k-t}\frac{\beta^2}{1+\beta^2}(1-\eta(\theta_0)\alpha^t)]$, driven by the intertemporal moral hazard problem for the purpose to control the future rents due to over-incentives. Note that as it approaches the period T, the additional distortion drops. On one aspect, the impact of pre-contract information θ_0 decreases over time; on the other aspect, the salesperson's potential benefits from misalignment shrinks since he won't have enough opportunities to hide his lie from mis-behaving when it is close to the period T. This "timing effect" limits the salesperson's opportunities to grasp more rents, thus alleviating the agency problem, and the firm will cut the distortions in response.

Anticipating the possible misalignments, the firm must schedule the training at the salesperson's optimal investment level enforced by the obedience constraints. Otherwise, the firm would be wasting resources due to over-incentivization. Observe that the desired training investment is also downward distorted. If the firm schedules too much training, it will raise the anticipated quota, which in turn increases the salesperson's benefits on information rent. This exacerbates the agency problem. As a response, the firm will downgrade the training level, which limits the salesperson's skills and reduces rent due to dynamic adverse selection. But a downgraded training level also lowers the desired quota, which limits the salesperson's efforts and benefits on information rent. This result highlights the darkside of training when the firm cannot observe the salesperson's self-investments, which contributes to the literature by arguing that training will not only increase efficiency, but also exacerbates the agency problem due to dynamic adverse selection and intertemporal moral hazard.

4.6 Managerial Implications

I now address three managerial questions: (i) What are the incentive implementations of the optimal policy? (ii) Is the training schedule pattern the same as classical predictions? (iii) How does the optimal policy perform in the long-run?

4.6.1 What Are the Incentive Implementations of The Optimal Policy?

In problem $\mathcal{P}$, the firm must address multiple challenges: dynamic adverse selection when screening the salesperson, contemporaneous moral hazard when targeting sales and intertemporal moral hazard when scheduling the training. While these challenges may appear daunting to address, the results demonstrate that the optimal scheme ϕ^* can be implemented simply and efficiently.

First, in each selling season, the firm can use base salary and quota to control the contemporaneous adverse selection problem and contemporaneous moral hazard. Actually, the firm offers a menu of compensation schemes contingent on the salesperson's skills θ^t. The salesperson's self-selection will reveal his skills. When the menu is well designed, the salesperson-θ^t will truthfully pick $q_t^*(\theta^t)$ matching his skills, thereby inferring his true skills. Anticipating the salesperson will choose the desired quota to reach, what only matters for the compensation is the type-dependent salary A_t^*, only by enough salary, the salesperson will target the quota exactly; that's what the firm will pay for an honest salesperson. In addition, the firm sets a commission rate B_t^* to motivate the salesperson to hit the quota. It is important to see that the salesperson's marginal cost to reach the quota matches his marginal gain from increasing sales. Therefore, targeting the quota will maximize the salesperson's payoff. This is the classic device from Rao (1990) to overcome the contemporaneous adverse selection and moral hazard problem. The firm can still apply those means to prevent the salesperson from manipulation in each selling season.

Second, the firm can use deferred information rent payment to control the dynamic adverse selection. The salesperson's private skills evolve stochastically over time, driven by skills decay and training boosting. Hence he receives fresh private skills information over time. It seems that the adverse selection problem is severe over time. However, I will argue that the firm can use the deferred rent payment structure inside the base salary $A_t^*(\theta^t)$

to screen the future private skills for free. As a consequence, the implementation ensures that the firm only suffers the initial piece of private skills θ_0 and its persistence impacts. Formally, I consider a new problem $\mathcal{P}^r$: it is under the same settings with the game $\check{\mathcal{P}}$, but initial θ_0 is private, new information $\{\theta_t\}_{t\geq 1}$ are known by both parties. The new game arises e.g. the firm can impose strict monitoring techniques to force the salesperson to reveal new information. To address the impact of dynamic adverse selection, I ignore the intertemporal moral hazard in $\mathcal{P}^r$. Hence $\mathcal{P}^r$ reduces the information asymmetry problem to θ_0 only. The firm still needs to pay to screen the θ_0, but receives $\{\theta_t\}_{t\geq 1}$ for free. Under the classical conjecture, the firm should earn more profits under $\mathcal{P}^r$. The profits differ since $\mathcal{P}^r$ relaxes all the future incentive compatibility constraints. I find that

Proposition 4.4 *The firm and the salesperson make the same expected payoff in both* $\mathcal{P}^r$ *and* $\check{\mathcal{P}}$. *The firm can use the deferred compensation to screen the new information for free.*

It reveals that the future incentive constraint is innocuous with zero shadow price. Although both initial θ_0 and future information $\{\theta_t\}_{t\geq 1}$ can be private, the initial piece is far more critical. The reason lies in the timing of the private information and the freedom of compensation transfers. In $\mathcal{P}$, the salesperson knows his basic skills θ_0 before contracting. Therefore, he is certain about his information advantage and rent at the time he signs the contract. To screen the pre-contract information θ_0, the firm has to pay the rent for the salesperson. However, the salesperson observes $\{\theta_t\}_{t\geq 1}$ only after contracting; he is also uncertain about $\{\theta_t\}_{t\geq 1}$ before contracting. Therefore, he holds no information advantage relative to the firm. What's more important, the firm can postpone the compensation into future periods with freedom. Recall that from Proposition 4.3, the base salary controlled for adverse selection problem is given by

$$A_t^*(\theta^t) = \frac{1}{2}[q_t^*(\theta^t) - \theta_t]^2 + \underbrace{U_t(\theta^t)}_{\text{Rent Payable}} + \underbrace{\delta\frac{1}{2}[\xi_{t+1}^*(\theta^t)]^2}_{\text{Allowance}} - \underbrace{\delta\mathbb{E}[U_{t+1}(\theta^{t+1}) \mid \theta_t, \xi_{t+1}^*(\theta^t)]}_{\text{Deferred Payment}}.$$

This form of deferred payment inforces (IC_t), thereby allowing the firm to extract the private $\{\theta_t\}_{t\geq 1}$ at no cost. The results sharpen the understanding of private information: leveraged by the residual uncertainty, the post contract private information is dynamically irrelevant with the efficiency suffering (Eső and Szentes, 2007, 2017).

Third, the firm can utilize a front-load allowance to leverage the salesperson's self-motivation to control the intertemporal moral hazard problem. On one hand, a salesperson's self-investment increases his skills, hence boosting the efficiency in selling season. However, by leveraging the salesperson's skills, the investment increases the salesperson's information advantage. The unobservable investment makes it even harder to track the salesperson's behavior hence limiting the firm's ability to control the rent. How should the firm overcome the difficulty? The answer lies in leveraging the salesperson's self-motivation: the future information advantage naturally creates an incentive for the salesperson to invest. The salesperson who has self-motivation will balance his marginal investment cost and the marginal future rent to decide the investment level. For each salesperson's skill, the firm can control how to compensate for the corresponding information advantages to control the salesperson's potential information rent, hence indicating the salesperson's optimal investment accordingly. In addition, the firm can post the front-load investment allowance $\delta\frac{1}{2}[\xi_{t+1}^*(\theta^t)]^2$ to reimburse the salesperson's self-investment cost at which the investment is at the optimal level. Given the allowance, the salesperson is motivated to invest at most $\xi^*_{t+1}(\theta^t)$ in the future to gain the utmost information rent.

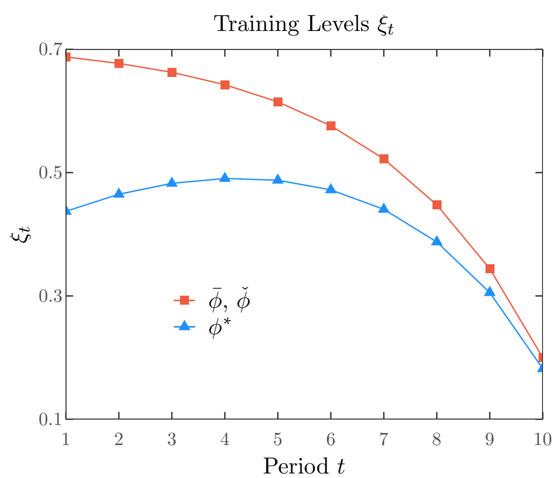
The simple implementation guide practice. First, the results explain a leading phenomenon when firms recruit salespeople: firms hire the young salespeople screening for their talents, but train their skills, despite the threat imposed by information asymmetry. Literature of training program design focuses on how to control the skills that can lead to higher salesforce productivity but lower salesforce bargaining abilities. It seems that classical research fails to explain the phenomenon. However, in my framework, the additional skills level improves salesforce productivity; because it is an ex post, the firm can induce the salesperson to reveal the skills at no cost. Hence the firm is willing to improve the salesperson's skills. He can extract the salesperson's skill information through deferred payment. Second, the results shed light on the increasing popularity of self-directed training in the industry. By providing a front-load self-investment allowance, the firm can monitor the salesperson indirectly and overcome the moral hazard problem while also benefiting from the convenience of self-directed training. This approach provides a way to incentivize salespeople to invest in their own skills, while also ensuring that the investment is aligned with the firm's interests.

4.6.2 New Patterns of Training Schedules

One of the primary reasons for salesforce training is to boost the skills and abilities of new and inexperienced salespeople. Consequently, firms schedule more intensive training to improve their skills and efficiency quickly in early times. Once the salesperson gains more experience and skills, firms reduce the training level over time, a pattern known as skimming

(Krishnamoorthy et al., 2005). This view assumes perfect monitoring of salesperson behavior, as is the case with traditional, in-person-led training. However, this view is flawed when the firm fails to monitor the salesperson, as is the case with modern self-directed training. In such cases, the conventional view ignores the potential dark side of training, which can exacerbate agency problems. To avoid this, firms should always downgrade training levels relative to the first-best level to prevent worsening of agency problems. Fig. 4.2 illustrates the new patterns of the self-directed training levels. Under $\bar{\phi}$ and $\check{\phi}$, the firm declines the training over time. While under ϕ^*, the firm sets lower training levels initially and elevates the level until some intermediate periods; she subsequently declines the training over time to the end, an inverted U-shaped pattern.

Figure 4.2: Training Schedules Comparison



Note: $\theta_0 \in \mathcal{U}[0,1]$, $\alpha = 0.8$, $\beta = 0.2$, $\delta = 0.9$, $\mu = 0$.

One key take away from Proposition 4.2 is that the dynamic adverse selection doesn't hurt the firm when scheduling the training; she can schedule the training for the highest efficiency. As a result, the firm can gradually reduce the training level over time as the additional benefits of increasing the salesperson's skills decrease. However, when intertemporal moral hazard is also a concern, the firm must consider agency costs when scheduling training. At the beginning of the salesperson's tenure, the firm faces a severe intertemporal moral hazard problem, as the salesperson has many opportunities to conceal their actions in the future. Therefore the firm has strong ncentives to downgrade the training aggressively in early periods to limit the agency problems. Over time, the agency cost due to moral hazard decreases as the salesperson has fewer chances to manipulate their skills, resulting in a reduction in the magnitude of the training level downgrade. Thus, while the training schedule may initially increase over time due to the high level of downgrade, it will eventually decrease as the nature of the declined training level dominates in the later periods.

What's the cost if the firm offers a skimming pattern of training schedules for the modern self-directed training? In the early times, more aggressive training $\beta[\sum_{k=t+1}^{T}(\delta\alpha)^{k-t-1}\mathbb{E}(q_k - \theta_k + \alpha^k\eta(\theta_0))]$ with high training investment allowance are offered by the firm, as well as the quota. However, this can lead to over-incentivizing the salesperson, as they may only invest themselves at a level $\beta[\sum_{k=t+1}^{T}(\delta\alpha)^{k-t-1}\mathbb{E}(q_k - \theta_k)]$ that maximizes their future information rent given the quota provision. This means that the additional self-investment $\beta[\sum_{k=t+1}^{T}(\delta\alpha)^{k-t-1}\alpha^k\eta(\theta_0)]$ is redundant to the firm, while the corresponding additional allowance increases the salesperson's rent. Additionally, the quota is higher than the optimal level, meaning the salesperson obtains higher information rent since $U_t(\theta^t)$ increases with q_t. This has practical implications for firms in the new era of self-directed training. They should be more conservative with their training schedules at the beginning to avoid over-incentivizing the salesperson.

Proposition 4.5 *In regime $\mathcal{P}$ with scheme $\check{\phi}$, the firm over-incentivizes the salesperson: $\check{\xi}_t \geq \xi_t^*$ and $\check{q}_t \geq q_t^*$. The over-incentives are harmful to the firm: $\check{U}_t \geq U_t^*$ and $\check{\Pi}_t \leq \Pi_t^*$.*

4.6.3 How Does the Optimal Scheme Perform in the Long-Run?

Perhaps the most important prediction under the classical training $\check{\mathcal{P}}$ is that, under the dynamic information asymmetry, the first-best is attainable in the long-run (Gao, 2022). Their results sharply alter the classical prediction that the first-best solution is unattainable (Rao, 1990). Why? The classical prediction takes a static perspective that it assumes one-shot interaction with exogenous given skills. It ignores the two-sided learning in the dynamic relationship. Clearly, the agent holds real information advantage initially, but the employment relationship evolves in multiple interactions where both parties learn about each other. Hence the firm-side learning should weaken the salesperson's information advantage and dampen the quota distortion in the long-run. When the relationship becomes sufficiently long, the firm can achieve the first-best level. Formally, on one hand, the likelihood of quota distortion declines over time. In regime $\mathcal{P}$, given the compensation scheme ϕ, the salesperson-θ_0 can predict future θ_t better than the firm, $f^t(\theta_t \mid \theta_0)$ vs. $f_t(\theta_t) \equiv \int f^t(\theta_t \mid \theta_0) \cdot f(\theta_0)\, \mathrm{d}\theta_0$. However, by Markov convergence theorem, $\lim_{t\to\infty} f^t(\theta_t \mid \theta_0) = \lim_{t\to\infty} f_t(\theta_t)$. The salesperson fares no better using his initial information advantage in the long-run. Moreover, the magnitude of distortions fades away over time. Note that the magnitude of distortions $\alpha^t \eta(\theta_0)$ are the residual effect of initial skill θ_0. As time goes on, the impact of the salesperson's initial information advantage gradually fades away, leading to $\lim_{t\to\infty} \alpha^t \eta(\theta_0) = 0$.

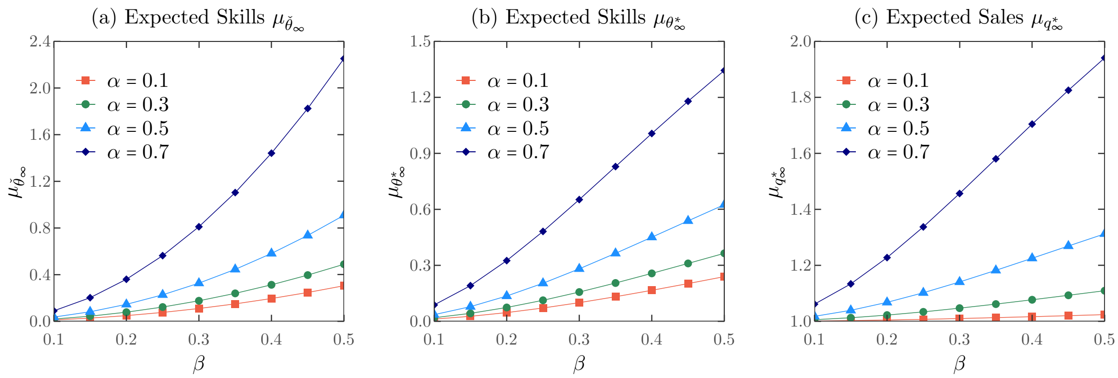
The sharp prediction is based on the assumption of perfect training monitoring, but will it still hold in the new environment of self-directed training where firms may not be able to monitor salespeople perfectly? The aforementioned intuition should still hold, as both parties are involved in two-sided learning. Therefore, as time goes on, the residual effect of initial skills fades away; the salesperson loses his position to better forecast his future skills. This means that the distortion caused by private information will converge to zero in terms of both its likelihood and magnitude. However, this prediction only applies partially to the self-directed training environment, as it only addresses the dynamic adverse selection aspect. Once intertemporal moral hazard is taken into account, additional agency costs may alter these predictions. I find

Proposition 4.6 *(a) In regime $\check{\mathcal{P}}$, the scheme $\check{\phi}$ converge to the first-best level under $\bar{\phi}$ in the long-run. In addition, the skills θ_t and quota $\check{q}_t$ converge to the steady state $\check{\theta}_\infty$ and $\check{q}_\infty$. Formally, if $\varepsilon_t \sim \mathcal{N}(\mu, \sigma^2)$, then $\theta_t \to \check{\theta}_\infty \sim \mathcal{N}(\mu_{\check{\theta}_\infty}, \sigma^2_{\check{\theta}_\infty})$ and $\check{q}_t \to \check{q}_\infty \sim \mathcal{N}(\mu_{\check{q}_\infty}, \sigma^2_{\check{q}_\infty})$, where $\mu_{\check{\theta}_\infty} = \frac{\mu}{1-\alpha} + \frac{\beta^2}{(1-\alpha)(1-\delta\alpha)}$, $\mu_{\check{q}_\infty} = 1 + \mu_{\check{\theta}_\infty}$ and $\sigma^2_{\check{\theta}_\infty} = \sigma^2_{\check{q}_\infty} = \frac{\sigma^2}{1-\alpha^2}$.*

(b) In regime $\mathcal{P}$ with scheme ϕ^*, the skills θ_t and quota q^*_t converge to the steady state θ^*_∞ and q^*_∞. Formally, if $\varepsilon_t \sim \mathcal{N}(\mu, \sigma^2)$, then $\theta_t \to \theta^*_\infty \sim \mathcal{N}(\mu_{\theta^*_\infty}, \sigma^2_{\theta^*_\infty})$ and $q^*_t \to q^*_\infty \sim \mathcal{N}(\mu_{q^*_\infty}, \sigma^2_{q^*_\infty})$, where $\mu_{\theta^*_\infty} = \frac{\mu}{1-\alpha} + \frac{\beta^2}{(1-\alpha)(1+\beta^2-\delta\alpha)}$, $\mu_{q^*_\infty} = \frac{1-\delta\alpha}{1+\beta^2-\delta\alpha} + \mu_{\theta^*_\infty}$ and $\sigma^2_{\theta^*_\infty} = \sigma^2_{q^*_\infty} = \frac{\sigma^2}{1-\alpha^2}$.

(c) $\mu_{\check{\theta}_\infty} > \mu_{\theta^*_\infty}$. Moreover, the steady state θ^*_∞ and q^*_∞ are increasing stochastically in persistence rate α and training intensity β.

Figure 4.3: Average Steady-state

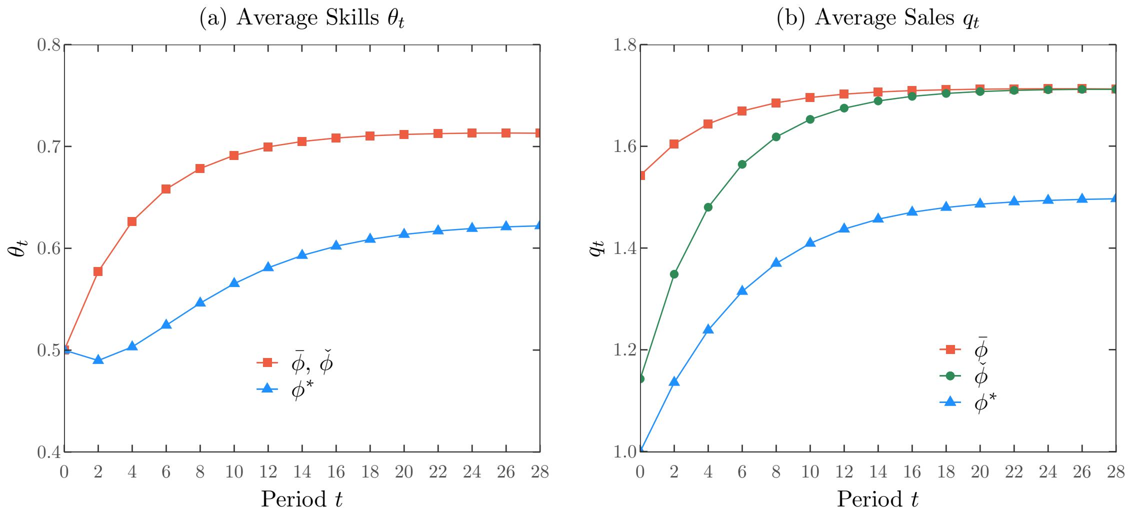


Note: $\mu_{\check{\theta}_\infty}$, $\mu_{\theta^*_\infty}$ and $\mu_{q^*_\infty}$: $\alpha \in \{0.1, 0.3, 0.5, 0.7\}$, $\beta \in [0.1, 0.5]$, $\delta = 0.9$, $\mu = 0$.

Proposition 4.6 reveals the long-run performance of ϕ^*. It is driven by two factors: the higher the skills persistence α, the higher the training intensity β, the higher the long-run performance (higher skill $\mu_{\theta^*_\infty}$ and sales $\mu_{q^*_\infty}$, see Fig. 4.3). Intuitively, the initial piece of private information θ_0, which is the key driver of the classical compensation plan, is irrelevant in the long-run. Its impact diminishes over time due to skill persistence $\alpha < 1$. Therefore, in an environment with dynamic adverse selection only, the first-best scheme can be achieved (Proposition 4.6 (a)). However, under the environment in the presence of intertemporal moral hazard, unobservable investments affect the firm's incentives in every period; the impacts persist perfectly even in the long-run. Hence the firm still has to distort the training and sales quota—both the skills development and the expected sales are undervalued in the long-run (Proposition 4.6 (c)).

Fig. 4.4 illustrates the long-run performance. I observe, (i) In the environment with dynamic adverse selection only, the average sales $\check{q}_t$ is downward distorted in the short-run but converges to the first-best level $\bar{q}_t$ in the long-run. The reason is that the

Figure 4.4: Average Skills and Sales Over Time



Note: $\theta_0 \in \mathcal{U}[0,1]$, $\delta = 0.9$, $\alpha = 0.8$, $\beta = 0.3$, $\mu = 0$.

initial piece of private information θ_0 has a severe impact in the short run, but this impact is gradually washed out over time. (ii) In an environment with both dynamic adverse selection and intertemporal moral hazard, I still observe the convergence of optimal policy and the salesperson's skills. As a result, the salesforce homogenizes toward θ^*_∞. However, the average sales q^*_t are downward distorted due to the intertemporal moral hazard in both the short-run and the long-run. With less training, the salesperson's skills are not developed well in the future, leading to lower expected skills in the long-run ($\mu_{\theta^*_\infty} < \mu_{\check{\theta}_\infty}$). (iii) Note that the distortion $\bar{q}_t - q^*_t$ is decreasing over time since the source of distortion comes from both the dynamic adverse selection and intertemporal moral hazard in the short run, but only from intertemporal moral hazard in the long run. (iv) The salesperson's expected skills can even decline in early periods due to the firm's fear of over-incentivizing. The firm offers severely distorted training ξ^{t+1} in early periods. Once the skills boosting due to training

$\beta\xi^*_{t+1}$ is less than the decay of the salesperson's skills $(1-\alpha)\theta_t$, the salesperson expects lower future skills θ_{t+1}.

4.7 Conclusion

Self-directed training is a popular practice to efficiently improve salespeople's selling skills. However, firms often lack technology to monitor salespeople's learning behavior. How should a firm schedule the self-directed training and design the compensation? I solve this joint design problem in the context of dynamic adverse selection and intertemporal moral hazard. My findings suggest that the optimal compensation scheme and training schedule differ from existing ones and can be implemented relatively simply: using a quota-commission structure to control both adverse selection and moral hazard, employing deferred compensation to mitigate adverse selection, and using front-loaded training allowances to alleviate intertemporal moral hazard.

I also highlight the negative consequences of self-directed training: despite the efficiency gain due to skills enhancing, training exacerbates the agency problem by enhancing the salesperson's ability to manipulate the skills. My recommendation for an inverted U-shaped training schedule further emphasizes this point, as the optimal training level initially rises and then declines.

The results inform practice on why firms "hires for talend and trains for skills", prefer self-directed training. I also call two cautions that firms shouldn't schedule training aggressively in the short-run, and trust the matured salespeople indefinitely in the long-run.

By highlighting the role of self-directed training, this study sharpens the understanding of salesforce training and compensation theory, as well as the practice.

Chapter 5

Conclusions

In this dissertation, I study the dynamic information and incentives management in marketing context. With the machinery of dynamic programming, game theory, and mechanism design, this dissertation seeks to answer what management can achieve, when the information necessary for decision making is dispersed, privately held, and evolving dynamically over time.

In Chapter 2, I study a new class of channel contracting problem, where the retailer privately observes and controls the evolving market conditions. The optimal contract resembles the classic second-best in the short run, but converges to the dynamic first-best in the long run. However, without considering network effects, classical contracts over emphasize the second-best contracts. The result highlights the dual role of network effects: although network effects can improve channel profit by expanding market size, they can also exacerbate the of information sharing by enhancing the retailers ability to manipulate the market. I provide new practical guidance: it sheds light on when and why manufacturers should overproduce supply, mitigate network effects, prefer long-term contracts, favor incumbent retailers, and improve retailer information capability, despite information friction. By highlighting how the manufacturer responses to the endogenous information friction, this chapter sharpens the understanding of channel theory and practice.

In Chapter 3, I study the product line design problem where the consumers' future preferences are determined endogenously by past purchases, current valuation, and random shocks. The optimal design differs substantially from the classic solution of second-degree price discrimination: it resolves a dynamic tradeoff between preventing cannibalization, extracting surplus, and exploiting consumer habituation; depending on the nature of consumer habituation, the optimal design may entail upward distortioneven beyond and above the first-best levelto homogenize future consumer preferences for rent control. The results shed new lights on product line design: in the subscription context, the classic downward distortion principle may no longer work, firms can practice first-degree price discrimination after initial sales, and excessive promotions can hurt firm profit and social welfare. More important, this chapter helps explain the rising popularity of personalized subscriptions: they can help firms to leverage consumer uncertainty to relax the cannibalization constraints, internalize the welfare gain from consumer habituation, and improve social welfare by reducing consumer heterogeneity. By providing a dynamic perspective with endogenous consumer preferences, this chapter deepens the understanding of product line design theory and practice.

In Chapter 4, I study a joint design of salesforce compensation and self-directed training schedule. The optimal solution highlights the dark side of self-directed training:

despite the efficiency gain due to skills enhancing, training exacerbates the agency problem by enhancing the salespersons ability to manipulate the skills. Hence I recommend an inverted U-shaped training schedule: the optimal training level elevates initially and then declines till the end. In addition, the intertemporal moral hazard problem can persist even in the long-run. Although I analyze a complicated problem, the optimal scheme has simple implementations: quota-commission structure controls the contemporaneous adverse selection and moral hazard; deferred compensation mitigates the dynamic adverse selection; front-load training allowance alleviates intertemporal moral hazard. By highlighting the role of self-directed training, this study sharpens the understanding of salesforce training and compensation theory, as well as the practice.

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Appendix A

Appendix for Chapter 2

For completeness, I detail all the proofs. Let $f_y(x, y) = \partial_y\ f(x, y) \equiv \frac{\partial}{\partial y}\ f(x, y)$ be the partial derivative of function f with respect to y. Let $\mathbb{E}^{\phi}[\cdot]$ be the expectation taken with respect to the stochastic process induced by contract ϕ. For other mathematical concepts I use, please see Stokey et al. (1989); Dudley (2002); Corbae et al. (2009); for economic concepts I use, please see Fudenberg and Tirole (1991); Laffont and Martimort (2001); Bolton and Dewatripont (2005); Mailath and Samuelson (2006).

Lemma A.1 and the Proof

I need the follow technical lemma for establishing the main results.

Lemma A.1 ***(a)*** *In regime $\mathcal{P}$ under contract $\phi \in \Phi$, the retailer's payoff $U_t(\theta^t)$ is increasing in θ_t, and differentiable in θ_t almost everywhere. At each differentiable point θ_t,*

$$\frac{\partial}{\partial \theta_t} U_t(\theta^t) = \mathbb{E}^{\phi}\left[\sum_{\tau \geq 0} \delta^{\tau} \alpha^{\tau} q_{t+\tau}(\theta^{\tau}) \middle| \theta^t\right]. \tag{A.1}$$

(b) *In regime $\mathcal{P}$, the manufacturer obtains the optimal payoff $J_1 = \max\{\tilde{J}^R(\phi) : IR\}$, where*

$$\tilde{J}^R(\phi) = \mathbb{E}^{\phi}\left\{\sum_{t=1}^{T} \delta^{t-1}\left[q_t\left(\theta_t - q_t\right) - h(\theta_1) \cdot \alpha^{t-1} \cdot q_t - cq_t\right]\right\}.$$

(c) *In regime $\mathcal{P}$, the optimal quantity $q_t^*(\theta^t)$ is nondecreasing in θ_t.*

Proof: To ease notation, let $R(\theta, q) \equiv (\theta - q)q$, and $Z(\theta, q, \varepsilon) \equiv \alpha\theta + \beta q + \varepsilon$.

(a) I prove by backward induction. For the last period T, I have the static screening problem (Fudenberg and Tirole, 1991). The IC_T constraint $U_T(\hat{\theta}^{T-1}, \theta_T) = \max_{\hat{\theta}_T} \tilde{U}_T(\hat{\theta}^T; \theta_T)$, where

$$\tilde{U}_T(\hat{\theta}^T; \theta_T) = R\big(\theta_T, q_T(\hat{\theta}^T)\big) - T_T(\hat{\theta}^T).$$

Clearly, $U_t(\hat{\theta}^T; \theta_T)$ increases in θ_T (since $R_\theta \le 0$); hence it is differentiable almost everywhere in θ_T, with

$$\tfrac{\partial}{\partial \theta_T} \tilde{U}_T(\hat{\theta}^T; \theta_T) = \ R_\theta(\theta_T, q_T(\hat{\theta}^T)) = q_T(\hat{\theta}^T).$$

By the envelope theorem, the value function $U_T(\theta^{T-1}, \theta_T)$ is also increasing in θ_t, and differentiable in θ_T almost everywhere. Thus, the claim holds for period T.

Suppose the claim holds for $t+1$. I now show it also holds for period t. Note

$$\tilde{U}_t(\hat{\theta}^t; \theta_t) = R(\theta_t, q_t(\hat{\theta}^t)) - T_t(\hat{\theta}^t) + \delta \mathbb{E}[U_{t+1}(\hat{\theta}^t, \theta_{t+1}) | \theta_t, q_t(\hat{\theta}^t)]. \tag{A.2}$$

Incentive compatibility implies that $U_t(\hat{\theta}^{t-1}, \theta_t) = \tilde{U}_t(\hat{\theta}^{t-1}, \theta_t; \theta_t) \ge \tilde{U}_t(\hat{\theta}^{t-1}, \hat{\theta}_t; \theta_t)$, and $U_t(\hat{\theta}^{t-1}, \hat{\theta}_t) = \tilde{U}_t(\hat{\theta}^{t-1}, \hat{\theta}_t; \hat{\theta}_t)$. Hence, for $\theta_t > \hat{\theta}_t$,

$$\begin{aligned}
& U_t(\hat{\theta}^{t-1}, \theta_t) - U_t(\hat{\theta}^{t-1}, \hat{\theta}_t) \\
& \ge \tilde{U}_t(\hat{\theta}^{t-1}, \hat{\theta}_t; \theta_t) - \tilde{U}_t(\hat{\theta}^{t-1}, \hat{\theta}_t; \hat{\theta}_t) \quad \text{by incentive compatibility} \\
& = \underbrace{[R(\theta_t, q_t(\hat{\theta}^t)) - R(\hat{\theta}_t, q_t(\hat{\theta}^t))]}_{\ge 0, \text{ since } R_\theta \ge 0} + \delta \underbrace{\Big(\mathbb{E}[U_{t+1}(\hat{\theta}^t, \theta_{t+1}) | \theta_t, q_t(\hat{\theta}^t)] - \mathbb{E}[U_{t+1}(\hat{\theta}^t, \theta_{t+1}) | \hat{\theta}_t, q_t(\hat{\theta}^t)]\Big)}_{\ge 0} \\
& \ge 0,
\end{aligned}$$

where the last inequality holds because $\theta_t > \hat{\theta}_t$, U_{t+1} increases in θ_{t+1}, and the state $\theta_{t+1} = \alpha\theta_t + \beta q_t(\hat{\theta}^t) + \varepsilon_{t+1}$ increases in θ_t. This complete the induction step of the monotonicity part.

For the differentiation part, at any differentiable point θ_t, by the hypothesis and the definition of $\tilde{U}_t$ in (A.2), I have

$$\begin{aligned}
\tfrac{\partial}{\partial \theta_t} \tilde{U}_t(\hat{\theta}^t; \theta_t) &= R_\theta(\theta_t, q_t(\hat{\theta}^t)) + \tfrac{\partial}{\partial \theta_t} \delta \mathbb{E}[U_{t+1}(\hat{\theta}^t, \theta_{t+1}) | \theta_t, q_t(\hat{\theta}^t)] \\
&= R_\theta(\theta_t, q_t(\hat{\theta}^t)) + \delta \mathbb{E}_\varepsilon \left[\alpha \cdot \tfrac{\partial}{\partial \theta_{t+1}} U_{t+1}\big(\hat{\theta}^t, Z(\theta_t, q_t(\hat{\theta}^t), \varepsilon)\big)\right] \\
&= R_\theta(\theta_t, q_t(\hat{\theta}^t)) + \delta \mathbb{E}[\alpha \cdot \tfrac{\partial}{\partial \theta_{t+1}} U_{t+1}(\hat{\theta}^t, \theta_{t+1}) \mid \theta_t, q_t(\hat{\theta}^t)].
\end{aligned} \tag{DU}$$

Applying the hypothesis and Eq. (DU) inductively to $t+1, t+2, \ldots$, I have

$$\begin{aligned}
\tfrac{\partial}{\partial\theta_t}\tilde{U}_t(\hat{\theta}^t;\theta_t) &= R_\theta(\theta_t, q_t(\hat{\theta}^t)) + \delta\mathbb{E}[\alpha \cdot \tfrac{\partial}{\partial\theta_{t+1}} U_{t+1}(\hat{\theta}^t, \theta_{t+1}) | \theta_t, q_t(\hat{\theta}^t)] \\
&= R(\theta_t, q_t(\hat{\theta}^t)) + \delta\mathbb{E}[\alpha \cdot R_\theta\big(\theta_{t+1}, q_{t+1}(\hat{\theta}^t, \theta_{t+1})\big) | \theta_t, q_t(\hat{\theta}^t)] \\
&\quad + \delta^2\mathbb{E}[\alpha\cdot\alpha\cdot \tfrac{\partial}{\partial\theta_{t+2}} U_{t+2}(\hat{\theta}^t, \theta_{t+1}^{t+2}) | \theta_t, q_t(\hat{\theta}^t)] \\
&\;\;\vdots \\
&= \delta^0\alpha^0 R_\theta(\theta_t, q_t(\hat{\theta}^t)) + \textstyle\sum_{\tau\geq 1}\delta^\tau \mathbb{E}\left[\alpha^\tau \cdot R_\theta\big(\theta_{t+\tau}, q_{t+\tau}(\hat{\theta}^t, \theta_{t+1}^{t+\tau})\big) \,\middle|\, \theta_t, q_t(\hat{\theta}^t)\right] \\
&= \mathbb{E}\left[\textstyle\sum_{\tau\geq 0}\delta^\tau\cdot\alpha^\tau\cdot q_{t+\tau}\big(\hat{\theta}^t, \theta_{t+1}^{t+\tau}\big) \,\middle|\, \theta_t, q_t(\hat{\theta}^t)\right].
\end{aligned}$$

By the envelope theorem (Corbae et al., 2009), I have

$$\tfrac{\partial}{\partial\theta_t}U_t(\theta^t) = \tfrac{\partial}{\partial\theta_t}\tilde{U}_t(\theta^{t-1}, \hat{\theta}_t;\theta_t)|_{\hat{\theta}_t=\theta_t} = \mathbb{E}^\phi\left[\textstyle\sum_{\tau\geq 0}\delta^\tau\cdot\alpha^\tau\cdot q_{t+\tau}\big(\theta^t, \theta_{t+1}^{t+\tau}\big) \,\middle|\, \theta_t\right].$$

(b) Given contract ϕ, the manufacturer's payoff is

$$\tilde{J}(\phi) = \mathbb{E}^\phi\left[\textstyle\sum_{t=1}^T\delta^{t-1}\big(T_t(\theta^t) - cq_t(\theta^t)\big)\right] = \mathbb{E}^\phi\left[\textstyle\sum_{t=1}^T\delta^{t-1}\big(R(\theta_t, q_t(\theta^t)) - cq_t(\theta^t)\big)\right] - \mathbb{E}U_1(\theta_1) \tag{A.3}$$

By part (a) and Fubini's Theorem, I have

$$\begin{aligned}
\mathbb{E}U_1(\theta_1) &= \mathbb{E}\left[\int_{\underline{\theta}_1}^{\theta_1}\tfrac{\partial}{\partial\theta_1}U_1(s)\,\mathrm{d}s + U_1(\underline{\theta}_1)\right] \\
&= \mathbb{E}\int_{\underline{\theta}_1}^{\theta_1}\left\{q_1(s) + \mathbb{E}^\phi\left[\textstyle\sum_{\tau=2}^T\delta^{\tau-1}\cdot\alpha^{\tau-1}\cdot q_\tau(s,\theta_2^\tau)\,\middle|\, s, q_1(s)\right]\right\}\mathrm{d}s + U_1(\underline{\theta}_1) \\
&= \int_{\underline{\theta}_1}^{\overline{\theta}_1}\int_{\underline{\theta}_1}^{\theta_1}\left\{q_1(s) + \mathbb{E}^\phi\left[\textstyle\sum_{\tau=2}^T\delta^{\tau-1}\cdot\alpha^{\tau-1}\cdot q_\tau(s,\theta_2^\tau)\,\middle|\, s, q_1(s)\right]\right\}\mathrm{d}s\cdot f(\theta_1)\,\mathrm{d}\theta_1 + U_1(\underline{\theta}_1) \\
&= \int_{\underline{\theta}_1}^{\overline{\theta}_1}\left\{q_1(s) + \mathbb{E}^\phi\left[\textstyle\sum_{\tau=2}^T\delta^{\tau-1}\cdot\alpha^{\tau-1}\cdot q_\tau(s,\theta_2^\tau)\,\middle|\, s, q_1(s)\right]\right\}\int_s^{\overline{\theta}_1}f(\theta_1)\,\mathrm{d}\theta_1\,\mathrm{d}s + U_1(\underline{\theta}_1) \\
&= \int_{\underline{\theta}_1}^{\overline{\theta}_1}\left\{\mathbb{E}^\phi\left[\textstyle\sum_{\tau=1}^T\delta^{\tau-1}\cdot\alpha^{\tau-1}\cdot q_\tau(s,\theta_2^\tau)\,\middle|\, s, q_1(s)\right]\right\}(1-F(s))\,\mathrm{d}s + U_1(\underline{\theta}_1) \\
&= \int_{\underline{\theta}_1}^{\overline{\theta}_1}\left\{\mathbb{E}^\phi\left[\textstyle\sum_{\tau=1}^T\delta^{\tau-1}\cdot\alpha^{\tau-1}\cdot q_\tau(\theta^\tau)\,\middle|\, \theta_1, q_1(\theta_1)\right]\right\}\underbrace{\frac{(1-F(\theta_1))}{f(\theta_1)}}_{=h(\theta_1)}\cdot f(\theta_1)\,\mathrm{d}\theta_1 + U_1(\underline{\theta}_1) \\
&= \mathbb{E}^\phi\left[\textstyle\sum_{\tau=1}^T\delta^{\tau-1}\cdot\alpha^{\tau-1}\cdot h(\theta_1)\cdot q_\tau(\theta^\tau)\right] + U_1(\underline{\theta}_1).
\end{aligned} \tag{EN}$$

At the optimum, IR for retailer-$\underline{\theta}_1$ is binding: $U_1(\underline{\theta}_1) = 0$. By Eqs. (EN) and (A.3), the manufacturer's payoff becomes

$$\tilde{J}^R(\phi) = \mathbb{E}^\phi\left\{\textstyle\sum_{t=1}^T\delta^{t-1}\left[q_t\,(\theta_t - q_t) - cq_t - h(\theta_1)\cdot\alpha^{t-1}\cdot q_t\right]\right\}.$$

Importantly, $\tilde{J}^R(\phi)$ is a function of quantities $(q_t)_{t\geq 1}$ only, independent of payments $(T_t)_{t\geq 1}$. It simplifies the problem $\mathcal{P}$ to optimizing the net present value of virtual surpluses, i.e.,

$$J_1 = \max_{\phi \in IR} \mathbb{E}^{\phi}\left[\sum_{t=1}^{T} \delta^{t-1} S_t(\theta_t, q_t)\right],$$

where $S_t(\theta_t, q_t) \equiv q_t(\theta_t - q_t) - cq_t - h(\theta_1)\cdot\alpha^{t-1}\cdot q_t$ is the flow virtual surplus in period t, adjusted for information friction by $h(\theta_1)\cdot\alpha^{t-1}\cdot q_t$. As such, I can use point optimization $\frac{\partial}{\partial q_t}\tilde{J}^R(\phi) = 0$ to find the optimal quantity, and then use participation constraint IR to find the optimal payment.

(c) It suffice to show that $\tilde{J}_t^R$ is supermodular in (θ_t, q_t). I prove it by backward induction. For period T, the claim follows from that $\tilde{J}_T^R = q_T(\theta_T - q_T) - h(\theta_1)\cdot\alpha^{T-1}\cdot q_T - cq_T$, and that

$$\frac{\partial^2 \tilde{J}_T^R}{\partial q_T \partial \theta_T} = 1 > 0.$$

Assume $\tilde{J}_{t+1}^R$ is supermodular in (θ_{t+1}, q_{t+1}). Then the supermodularity of $\tilde{J}_t^R$ follows from $\tilde{J}_t^R = q_t(\theta_t - q_t) - h(\theta_1)\cdot\alpha^{t-1}\cdot q_t - cq_t + \delta\mathbb{E}\tilde{J}_{t+1}^R$, and

$$\frac{\partial^2 \tilde{J}_t^R}{\partial q_t \partial \theta_t} = 1 + \delta\mathbb{E}\frac{\partial^2 \tilde{J}_{t+1}^R}{\partial q_{t+1}\partial\theta_{t+1}} > 0.$$

Following the same line of argument, I conclude $\tilde{J}_t^R$ is supermodular for all $t \geq 2$. For the period 1, I have

$$\frac{\partial^2 \tilde{J}_1^R}{\partial q_1 \partial \theta_1} = 1 - \frac{\partial h(\theta_1)}{\partial \theta_1} + \delta\mathbb{E}\frac{\partial^2 \tilde{J}_2^R}{\partial q_2 \partial \theta_2} > 0,$$

by the monotone hazard rate assumption $\frac{\partial h(\theta_1)}{\partial \theta_1} < 0$. Hence, $\tilde{J}_t^R$ is supermodular for all t. It follows from Topkis (1998) that $q_t(\theta^t) \in \arg\max \tilde{J}^R(\phi)$ is nondecreasing with θ_t.

■

Proof of Proposition 2.1

(a) Regime $\bar{\mathcal{P}}^n$ has no network effects ($\beta = 0$). The associated Bellman equations become

$$J_t(\theta^t) = \max_{q_t \geq 0}\left\{(\theta_t - q_t)q_t - cq_t + \delta\mathbb{E}[J_{t+1}(\theta^t, \alpha\theta_t + \varepsilon_{t+1})]\right\},$$

with $J_T(\theta^T) = \max_{q_t \geq 0}\{(\theta_T - q_T)q_T - cq_T\}$. By the first order condition (FOC), I have

$$\frac{\partial}{\partial q_t}R(\theta_t, \bar{q}_t^n) - c = \theta_t - 2\bar{q}_t^n - c = 0.$$

Hence,

$$\bar{q}_t^n(\theta_t) = \tfrac{1}{2}(\theta_t - c).$$

With full information, the manufacturer can extract all the retailer's surplus with the following payment:

$$\bar{T}_t^n(\theta_t) = R\big(\theta_t, \bar{q}_t^n(\theta_t)\big) = \bar{q}_t^n(\theta_t) = \tfrac{1}{2}(\theta_t + c)(\theta_t - c).$$

(*b*) Regime $\bar{\mathcal{P}}$ has network effects ($\beta > 0$). The associated Bellman equations become

$$J_t(\theta^t) = \max_{q_t \geq 0} \left\{(\theta_t - q_t)q_t - cq_t + \delta\mathbb{E}[J_{t+1}(\theta^t, \alpha\theta_t + \beta q_t + \varepsilon_{t+1})]\right\},$$

with $J_T(\theta^T) = \max_{q_t \geq 0}\{(\theta_T - q_T)q_T - cq_T\}$.

I solve the problem in three steps.

(*1*) By the first order condition, I have

$$R_q(\theta_t, q_t) - c + \delta\mathbb{E}_{\varepsilon_{t+1}}\left[\frac{\partial J_{t+1}(\theta^t, \theta_{t+1})}{\partial \theta_{t+1}}\frac{\partial \theta_{t+1}}{\partial q_t}\right] = R_q(\theta_t, q_t) - c + \delta\beta\mathbb{E}_{\varepsilon_{t+1}}\left[\frac{\partial J_{t+1}(\theta^t, \theta_{t+1})}{\partial \theta_{t+1}}\right] = 0.$$

Hence,

$$\delta\mathbb{E}_{\varepsilon_{t+1}}\left[\frac{\partial J_{t+1}(\theta^t, \theta_{t+1})}{\partial \theta_{t+1}}\right] = \frac{-R_q(\theta_t, q_t) + c}{\beta}. \tag{FOC}$$

(*2*) By the envelope theorem, I have

$$\frac{\partial J_t(\theta^t)}{\partial \theta_t} = R_\theta(\theta_t, q_t) + \delta\alpha\mathbb{E}_{\varepsilon_{t+1}}\left[\frac{\partial J_{t+1}(\theta^t, \theta_{t+1})}{\partial \theta_{t+1}}\right]. \tag{ENV}$$

Plug in (FOC), I have

$$\frac{\partial J_t(\theta^t)}{\partial \theta_t} = R_\theta(\theta_t, q_t) - \frac{\alpha}{\beta}\left[R_q(\theta_t, q_t) - c\right]. \tag{A.4}$$

Similarly, I have

$$\frac{\partial J_{t+1}(\theta^{t+1})}{\partial \theta_{t+1}} = R_\theta(\theta_{t+1}, q_{t+1}) - \frac{\alpha}{\beta}\left[R_q(\theta_{t+1}, q_{t+1}) - c\right]. \tag{A.5}$$

(*3*) Plug Eqs. (A.4) and (A.5) into (ENV), I obtain

$$\begin{aligned} R_\theta(\theta_t, q_t) - &\frac{\alpha}{\beta}\left[R_q(q_t, \theta_t) - c\right] \\ &= R_\theta(\theta_t, q_t) + \delta\alpha\mathbb{E}_{\varepsilon_{t+1}}\left\{R_\theta(\theta_{t+1}, q_{t+1}(\theta^{t+1})) - \frac{\alpha}{\beta}\left[R_q(q_{t+1}(\theta_{t+1}), \theta^{t+1}) - c\right]\right\}. \end{aligned}$$

Hence,

$$-\frac{1}{\beta}\left[R_q(q_t(\theta^t),\theta_t)-c\right]=\delta\mathbb{E}_{\varepsilon_{t+1}}\left\{R_\theta(\theta_{t+1},q_{t+1})-\frac{\alpha}{\beta}\left[R_q(\theta_{t+1},q_{t+1})-c\right]\right\}. \tag{A.6}$$

The optimal quantities $\bar{q}_t$ must satisfy the above Euler equation (A.6). The ending period T have no network effects. Hence,

$$\bar{q}_T(\theta_T)=\tfrac{1}{2}(\theta_T-c),$$

which coincides with $\bar{q}_T^n$. By Eq. (A.6) and $R(\theta_t,q_t)=(\theta_t-q_t)q_t$, I get

$$-\frac{1}{\beta}\left(\theta_t-2\bar{q}_t-c\right)=\delta\mathbb{E}_{\varepsilon_{t+1}}\left[\bar{q}_{t+1}-\frac{\alpha}{\beta}\left(\theta_{t+1}-2q_{t+1}-c\right)\right],$$

which yields the solution in the recursive form

$$\bar{q}_t(\theta_t)=\frac{\theta_t-c}{2}+\frac{\delta\beta}{2}\mathbb{E}_{\varepsilon_{t+1}}\left[(1+\frac{2\alpha}{\beta})\bar{q}_{t+1}(\theta_{t+1})-\frac{\alpha}{\beta}\theta_{t+1}+\frac{\alpha}{\beta}c\right]. \tag{A.7}$$

To derive the close-form solution, I use backward induction. For period T, I have

$$\bar{q}_T(\theta_T)=\tfrac{1}{2}(\theta_T-c).$$

For period $T-1$, I have

$$\begin{aligned}
\bar{q}_{T-1}&=\frac{\theta_{T-1}-c}{2}+\frac{\delta\beta}{2}\mathbb{E}_{\varepsilon_T}\left[\bar{q}_T-\frac{\alpha}{\beta}\left(\theta_T-2\bar{q}_T-c\right)\right]\\
&=\frac{\theta_{T-1}-c}{2}+\frac{\delta\beta}{2}\mathbb{E}_{\varepsilon_T}\left[\frac{\theta_T-c}{2}-\frac{\alpha}{\beta}\left(\theta_T-2\cdot\frac{\theta_T-c}{2}-c\right)\right]\\
&=\frac{\theta_{T-1}-c}{2}+\frac{\delta\beta}{2}\mathbb{E}_{\varepsilon_T}\left[\frac{\alpha\theta_{T-1}+\beta\bar{q}_{T-1}+\varepsilon_T-c}{2}\right]\\
&=\frac{\theta_{T-1}-c}{2}+\frac{\delta\beta}{2}\left[\frac{\alpha\theta_{T-1}+\beta\bar{q}_{T-1}+\mu-c}{2}\right],
\end{aligned}$$

which yields

$$\bar{q}_{T-1}(\theta_{T-1})=\frac{(1+\frac{\delta}{2}\beta\alpha)\theta_{T-1}}{2-\frac{\delta}{2}\beta^2}-\frac{(1+\frac{\delta}{2}\beta)c}{2-\frac{\delta}{2}\beta^2}+\frac{\frac{\delta}{2}\beta\mu}{2-\frac{\delta}{2}\beta^2}=a_{T-1}\theta_{T-1}-b_{T-1}c+d_{T-1}\mu, \tag{A.8}$$

as desired.

Assume $\bar{q}_{t+1}(\theta_{t+1})=a_{t+1}\theta_{t+1}-b_{t+1}c+d_{t+1}\mu$. I shall show $\bar{q}_t(\theta_t)=a_t\theta_t-b_tc+d_t\mu$. By Eq. (A.7), I have

$$\begin{aligned}
2\bar{q}_t&=\theta_t-c+(\delta\beta+2\delta\alpha)\left[a_{t+1}(\alpha\theta_t+\beta\bar{q}_t+\mu)-b_{t+1}c+d_{t+1}\mu\right]-\delta\alpha(\alpha\theta_t+\beta\bar{q}_t+\mu)+\delta\alpha c\\
&=\theta_t-c+\left[a_{t+1}(\delta\beta+2\delta\alpha)-\delta\alpha\right]\alpha\theta_t+\left[a_{t+1}(\delta\beta+2\delta\alpha)-\delta\alpha\right]\beta\bar{q}_t\\
&\quad-\left[b_{t+1}(\delta\beta+2\delta\alpha)-\delta\alpha\right]c+\left[(a_{t+1}+d_{t+1})(\delta\beta+2\delta\alpha)-\delta\alpha\right]\mu,
\end{aligned}$$

which yields

$$\begin{aligned}\bar{q}_t(\theta_t) &= a_t\theta_t - b_t c + d_t\mu \\ &= \frac{\{1+[a_{t+1}(\delta\beta+2\delta\alpha)-\delta\alpha]\alpha\}\theta_t - \{1+[b_{t+1}(\delta\beta+2\delta\alpha)-\delta\alpha]\}c + [(a_{t+1}+d_{t+1})(\delta\beta+2\delta\alpha)-\delta\alpha]\mu}{2-[a_{t+1}(\delta\beta+2\delta\alpha)-\delta\alpha]\beta},\end{aligned}$$

as desired. This completes the induction step.

With full information, the manufacturer can extract all the channel surplus, with payment

$$\bar{T}_t(\theta_t) = R\big(\theta_t, \bar{q}_t^n(\theta_t)\big) = \big(a_t\theta_t - b_t c + d_t\mu\big)\big[(1-a_t)\theta_t + b_t c - d_t\mu\big].$$

(c) By Eq. (A.7), I can pinpoint the network gain by

$$\lambda_t(\theta_t) \equiv \tfrac{\partial}{\partial q_t}\mathbb{E}[W_{t+1}|\theta_t] = \mathbb{E}\big[\bar{q}_{t+1}(\theta_{t+1}) - \tfrac{\alpha}{\beta}\left(\theta_{t+1} - 2\bar{q}_{t+1}(\theta_{t+1}) - c\right)\big|\theta_t\big].$$

Given the centralized control, I have $W_1 = J_1$. It suffices to show J_1 is nondecreasing in β. By the envelope theorem, I have

$$\frac{\partial J_1}{\partial \beta} = \delta\mathbb{E}\frac{\partial J_1}{\partial\theta_2}\frac{\partial\theta_2}{\partial\beta} = \delta\mathbb{E}\frac{\partial J_1}{\partial\theta_2}\bar{q}_1 = \tfrac{1}{\beta}\lambda_1(\theta_1)\bar{q}_1 \geq 0.$$

Thus $W_1 = J_1$ is nondecreasing with β. Since $W_1(\bar{\phi}^n)$ is the special case with $\beta = 0$, I conclude $W_1(\bar{\phi}) \geq W_1(\bar{\phi}^n)$.

■

Proof of Proposition 2.2

By Lemma B.2, the manufacturer maximizes the virtual surplus, with Bellman equations

$$J_t(\theta^t) = \max_{q_t\geq 0}\left\{q_t\left(\theta_t - q_t\right) - h(\theta_1)\alpha^{t-1}q_t - cq_t + \delta\mathbb{E}[J_{t+1}(\theta^t, \alpha\theta_t + \varepsilon_{t+1})]\right\}, \quad \forall t < T,$$

$$J_T(\theta^T) = \max_{q_T\geq 0}\left\{q_T\left(\theta_T - q_T\right) - h(\theta_1)\alpha^{T-1}q_T - cq_T\right\}.$$

I can find the optimal quantity q_t^n by the first order condition $\frac{\partial}{\partial q_t}R(\theta_t, q_t^n) - h(\theta_1)\alpha^{t-1} - c = 0$, which yields

$$q_t^n(\theta^t) = \tfrac{1}{2}(\theta_t - c) - \tfrac{1}{2}h(\theta_1)\alpha^{t-1}.$$

By the part (a) of Lemma B.2, the information rent in period t is

$$U_t^n(\theta^t) = U_t^n(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_t}\left\{q_t^n(\theta^{t-1}, s) + \mathbb{E}^{\phi^n}\left[\textstyle\sum_{\tau=t+1}^{T}\delta^{\tau-t}\cdot\alpha^{\tau-t}\cdot q_\tau^n(\theta^\tau)\,\middle|\, s, q_t^n(\theta^{t-1}, s)\right]\right\}\mathrm{d}s.$$

Without loss of generality, I can set $U_t^n(\underline{\theta}) = 0$. The optimal payment then follows from $U_t^n(\theta^t) = R(\theta_t, q_t^n(\theta^t)) - T_t^n(\theta^t) + \delta\mathbb{E}^{\phi^n}[U_{t+1}^n(\theta^{t+1})|\theta_t]$. ■

Proof of Proposition 2.3

(*a*) In regime $\mathcal{P}$ with network effects, the manufacturer's Bellman equations are

$$J_t(\theta^t) = \max_{q_t \geq 0} \left\{ q_t\,(\theta_t - q_t) - h(\theta_1)\alpha^{t-1} q_t - cq_t + \delta\mathbb{E}[J_{t+1}(\theta^t, \alpha\theta_t + \beta q_t + \varepsilon_{t+1})] \right\}, \quad \forall t < T,$$

$$J_T(\theta^T) = \max_{q_T \geq 0} \left\{ q_T\,(\theta_T - q_T) - h(\theta_1)\alpha^{T-1} q_T - cq_T \right\}.$$

Her rent-to-go from period t onward is $U_t(\theta^t) = \mathbb{E}^{\phi}\left[\sum_{\tau=t}^{T} h(\theta_1)\alpha^{\tau-1} \cdot q_\tau(\theta^\tau)|\theta^t\right]$. By Lemma B.2.(c), the optimal quantity $q_t^*(\theta^t)$ increases in θ_t. Hence,

$$\frac{\partial}{\partial\theta_t} U_t(\theta^t) \geq 0.$$

Next, I show $\frac{\partial}{\partial\theta_t} J_t(\theta^t) \geq 0$ by backward induction. For period T, by the envelope theorem, I have

$$\frac{\partial}{\partial\theta_T} J_T(\theta^T) = q_T \geq 0.$$

Assume $\frac{\partial}{\partial\theta_{t+1}} J_{t+1}(\theta^{t+1}) \geq 0$. In period t, the envelope theorem implies

$$\frac{\partial}{\partial\theta_t} J_t(\theta^t) = q_t + \delta\mathbb{E}\left[\frac{\partial}{\partial\theta_t} J_{t+1}(\theta^t, \alpha\theta_t + \beta q_t + \varepsilon_{t+1})\right] = q_t + \delta\alpha\mathbb{E}\left[\frac{\partial}{\partial\theta_{t+1}} J_{t+1}(\theta^{t+1})\right] \geq 0,$$

which proves $\frac{\partial}{\partial\theta_t} J_t(\theta^t) \geq 0$, $\forall t \geq 2$. In period 1, by assumption $\frac{\partial}{\partial\theta_1} h(\theta_1) \leq 0$, I have

$$\frac{\partial}{\partial\theta_1} J_1(\theta^1) = q_t - \frac{\partial}{\partial\theta_1} h(\theta_1) q_t + \delta\alpha\mathbb{E}\left[\frac{\partial}{\partial\theta_2} J_2(\theta^2)\right] \geq 0.$$

Hence, $\frac{\partial}{\partial\theta_t} J_t(\theta^t) \geq 0$, $\forall t$.

(*b*) I solve the dynamic program in three steps.

(*1*) By the first order condition, I have

$$\begin{aligned} & R_q(\theta_t, q_t) - c - h(\theta_1)\alpha^{t-1} + \delta\mathbb{E}_{\varepsilon_{t+1}}\left[\frac{\partial}{\partial\theta_{t+1}} J_{t+1}(\theta^t, \theta_{t+1}) \cdot \frac{\partial}{\partial q_t}\theta_{t+1}\right] \\ &= R_q(\theta_t, q_t) - c - h(\theta_1)\alpha^{t-1} + \delta\beta\mathbb{E}_{\varepsilon_{t+1}}\left[\frac{\partial}{\partial\theta_{t+1}} J_{t+1}(\theta^t, \theta_{t+1})\right] \\ &= 0. \end{aligned}$$

Hence,

$$\delta\mathbb{E}_{\varepsilon_{t+1}}\left[\frac{\partial J_{t+1}(\theta^t, \theta_{t+1})}{\partial\theta_{t+1}}\right] = \frac{-R_q(\theta_t, q_t) + c + h(\theta_1)\alpha^{t-1}}{\beta}. \tag{FOC}$$

(2) By the envelope theorem, I have

$$\frac{\partial J_t(\theta^t)}{\partial \theta_t} = R_\theta(\theta_t, q_t) + \delta\alpha\mathbb{E}_{\varepsilon_{t+1}}\left[\frac{\partial J_{t+1}(\theta^t, \theta_{t+1})}{\partial \theta_{t+1}}\right]. \tag{ENV}$$

Plugging in (FOC), I get

$$\frac{\partial J_t(\theta^t)}{\partial \theta_t} = R_\theta(\theta_t, q_t) - \frac{\alpha}{\beta}\left[R_q(\theta_t, q_t) - c - h(\theta_1)\alpha^{t-1}\right].$$

Similarly, I can obtain

$$\frac{\partial J_{t+1}(\theta^{t+1})}{\partial \theta_{t+1}} = R_\theta(\theta_{t+1}, q_{t+1}) - \frac{\alpha}{\beta}\left[R_q(\theta_{t+1}, q_{t+1}) - c - h(\theta_1)\alpha^{t}\right].$$

(3) Plugging in the above $\frac{\partial J_t(\theta^t)}{\partial \theta_t}$ and $\frac{\partial J_{t+1}(\theta^{t+1})}{\partial \theta_{t+1}}$ into (ENV), I get

$$\begin{aligned} R_\theta(\theta_t, q_t) &- \frac{\alpha}{\beta}\left[R_q(\theta_t, q_t) - c - h(\theta_1)\alpha^{t-1}\right] \\ &= R_\theta(\theta_t, q_t) + \delta\alpha\mathbb{E}_{\varepsilon_{t+1}}\left\{R_\theta(\theta_{t+1}, q_{t+1}) - \frac{\alpha}{\beta}\left[R_q(\theta_{t+1}, q_{t+1}) - c - h(\theta_1)\alpha^{t}\right]\right\}, \end{aligned}$$

which yields the Euler equations for the optimal solution

$$-\frac{1}{\beta}\left[R_q(\theta_t, q_t) - c - h(\theta_1)\alpha^{t-1}\right] = \delta\mathbb{E}_{\varepsilon_{t+1}}\left\{R_\theta(\theta_{t+1}, q_{t+1}) - \frac{\alpha}{\beta}\left[R_q(\theta_{t+1}, q_{t+1}) - c - h(\theta_1)\alpha^{t}\right]\right\}. \tag{A.9}$$

The final period T has no network effects. Hence, the solution is is identical to q_T^n:

$$q_T^* = \tfrac{1}{2}(\theta_T - c) - h(\theta_1)\alpha^{T-1}.$$

For period t, plugging the revenue function $R(\theta_t, q_t)$ into Eq. (A.9), I obtain

$$-\frac{1}{\beta}\left(\theta_t - 2q_t - c - h(\theta_1)\alpha^{t-1}\right) = \delta\mathbb{E}_{\varepsilon_{t+1}}\left[q_{t+1} - \tfrac{\alpha}{\beta}\left(\theta_{t+1} - 2q_{t+1} - c - h(\theta_1)\alpha^{t}\right)\right].$$

Hence,

$$\begin{aligned} q_t^*(\theta^t) &= \tfrac{1}{2}(\theta_t - c) - \tfrac{1}{2}h(\theta_1)\alpha^{t-1} + \tfrac{\delta\beta}{2}\mathbb{E}_{\varepsilon_{t+1}}\left[q_{t+1}^* - \tfrac{\alpha}{\beta}\left(\theta_{t+1} - c - 2q_{t+1}^* - h(\theta_1)\alpha^{t}\right)\right] \\ &= \tfrac{1}{2}(\theta_t - c) - \tfrac{1}{2}h(\theta_1)\alpha^{t-1} + \tfrac{\delta}{2}(\lambda_t(\theta_t) - \rho_t(\theta_t)). \end{aligned}$$

To derive the close-form solution, I use backward induction. For period T, I have

$$q_T^*(\theta^T) = \tfrac{1}{2}(\theta_T - c) - \tfrac{1}{2}h(\theta_1)\alpha^{T-1}.$$

For period $T-1$, I have

$$
\begin{aligned}
q^*_{T-1}(\theta^{T-1}) &= \frac{\theta_{T-1}-c}{2} - \frac{h(\theta_1)\alpha^{T-2}}{2} + \frac{\delta\beta}{2}\mathbb{E}_{\varepsilon_T}\left[q^*_T(\theta^T) - \frac{\alpha}{\beta}\left(\theta_T - 2q^*_T(\theta^T) - \frac{h(\theta_1)\alpha^{T-1}}{2} - c\right)\right] \\
&= \frac{\theta_{T-1}-c}{2} - \frac{h(\theta_1)\alpha^{T-2}}{2} \\
&\quad + \frac{\delta\beta}{2}\mathbb{E}_{\varepsilon_T}\left[\frac{\theta_T - c}{2} - \frac{h(\theta_1)\alpha^{T-1}}{2} - \frac{\alpha}{\beta}\left(\theta_T - 2\cdot\left(\frac{\theta_T-c}{2} - \frac{h(\theta_1)\alpha^{T-1}}{2}\right) - \frac{h(\theta_1)\alpha^{T-1}}{2} - c\right)\right] \\
&= \frac{\theta_{T-1}-c}{2} + \frac{\delta\beta}{2}\mathbb{E}_{\varepsilon_T}\left[\frac{\alpha\theta_{T-1} + \beta q_{T-1} + \varepsilon_T - c}{2} - \frac{h(\theta_1)\alpha^{T-1}}{2}\right] \\
&= \frac{\theta_{T-1}-c}{2} + \frac{\delta\beta}{2}\left[\frac{\alpha\theta_{T-1} + \beta q_{T-1} + \mu - c}{2} - \frac{h(\theta_1)\alpha^{T-1}}{2}\right],
\end{aligned}
$$

which yields the desired form

$$
q^*_{T-1}(\theta^{T-1}) = \frac{(1+\frac{\delta}{2}\beta\alpha)}{2-\frac{\delta}{2}\beta^2}\cdot\left[\theta_{T-1} - h(\theta_1)\alpha^{T-2}\right] - \frac{(1+\frac{\delta}{2}\beta)}{2-\frac{\delta}{2}\beta^2}\cdot c + \frac{\frac{\delta}{2}\beta}{2-\frac{\delta}{2}\beta^2}\cdot\mu.
$$

Assume $q^*_{t+1}(\theta^{t+1}) = a_{t+1}\left[\theta_{t+1} - h(\theta_1)\alpha^t\right] - b_{t+1}c + d_{t+1}\mu$. I shall show $q^*_t(\theta^t) = a_t\left[\theta_t - h(\theta_1)\alpha^{t-1}\right] - b_t c + d_t\mu$. By Eq. (A.9), I have

$$
q^*_t(\theta^t) = \frac{\theta_t - c}{2} - \frac{h(\theta_1)\alpha^{t-1}}{2} + \frac{\delta\beta}{2}\mathbb{E}_{\varepsilon_{t+1}}\left[\left(1+\frac{2\alpha}{\beta}\right)q^*_{t+1}(\theta^{t+1}) - \frac{\alpha}{\beta}\theta_{t+1} + \frac{\alpha}{\beta}c + \frac{\alpha}{\beta}h(\theta_1)\alpha^t\right].
$$

Then

$$
\begin{aligned}
2q^*_t(\theta^t) &= \theta_t - c - h(\theta_1)\alpha^{t-1} + (\delta\beta + 2\delta\alpha)\left[a_{t+1}(\alpha\theta_t + \beta q_t + \mu) - b_{t+1}c + d_{t+1}\mu -{}_{t+1} h(\theta_1)\alpha^t\right] \\
&\quad - \delta\alpha(\alpha\theta_t + \beta q_t + \mu) + \delta\alpha c + \delta\alpha h(\theta_1)\alpha^t \\
&= \theta_t - c - h(\theta_1)\alpha^{t-1} + [a_{t+1}(\delta\beta + 2\delta\alpha) - \delta\alpha]\alpha\theta_t + [a_{t+1}(\delta\beta + 2\delta\alpha) - \delta\alpha]\beta q_t \\
&\quad - [b_{t+1}(\delta\beta + 2\delta\alpha) - \delta\alpha]c + [(a_{t+1} + d_{t+1})(\delta\beta + 2\delta\alpha) - \delta\alpha]\mu - [a_{t+1}(\delta\beta + 2\delta\alpha) - \delta\alpha]h(\theta_1)\alpha^t.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
q^*_t(\theta^t) &= a_t\left[\theta_t - h(\theta_1)\alpha^{t-1}\right] - b_t c + d_t\mu \\
&= \frac{1 + [a_{t+1}(\delta\beta + 2\delta\alpha) - \delta\alpha]\alpha}{2 - [a_{t+1}(\delta\beta + 2\delta\alpha) - \delta\alpha]\beta}\cdot\left[\theta_t - h(\theta_1)\alpha^{t-1}\right] - \frac{1 + [b_{t+1}(\delta\beta + 2\delta\alpha) - \delta\alpha]}{2 - [a_{t+1}(\delta\beta + 2\delta\alpha) - \delta\alpha]\beta}\cdot c \\
&\quad + \frac{[(a_{t+1} + d_{t+1})(\delta\beta + 2\delta\alpha) - \delta\alpha]}{2 - [a_{t+1}(\delta\beta + 2\delta\alpha) - \delta\alpha]\beta}\cdot\mu,
\end{aligned}
$$

which concludes the induction step of the backward induction.

Follow the same line of argument in Lemma 2.2, the optimal payment and information rent are

$$T_t^*(\theta^t) = R(\theta_t, q_t^*(\theta^t)) - U_t^*(\theta^t) + \delta\mathbb{E}[U_{t+1}^*(\theta^{t+1})|\theta_t],$$
$$U_t^*(\theta^t) = \int_{\underline{\theta}}^{\theta_t} \mathbb{E}\left[\sum_{\tau=t}^{T} \delta^{\tau-t} \cdot \alpha^{\tau-t} \cdot q_\tau^*(\theta^\tau) \,\middle|\, \hat{\theta}_t\right] d\hat{\theta}_t.$$

(c) From Proposition 2.1 and Proposition 2.3 part (b), due to the identity of the sequence of coefficients,

$$\bar{q}_t(\theta_t) - q_t^*(\theta^t) = a_t h(\theta_1)\alpha^{t-1}.$$

For the top retailer-$\bar{\theta}$, $h(\bar{\theta}) = 0$ and hence $\bar{q}_t(\bar{\theta}, \theta_2^t) - q_t^*(\bar{\theta}, \theta_2^t) = 0$. For the retailer with $\theta_1 < \bar{\theta}$, as $h(\theta_1)$ is nondecreasing in θ_1, $h(\theta_1) \geq h(\bar{\theta}) = 0$. In addition with the positive property of a_t and α^{t-1}, $\bar{q}_t(\theta^t) - q_t^*(\theta^t) \geq 0$.

To show $q_t^n(\theta^t) \leq q_t^*(\theta^t)$, notice that for period T, $q_t^n(\theta^t) = q_t^*(\theta^t) = \frac{1}{2}(\theta_T - h(\theta_1)\alpha^{T-1}) - \frac{1}{2}c$. Then for period $t < T$,

$$q_t^*(\theta^t) - q_t^n(\theta^t) = \frac{\delta\beta}{2}\mathbb{E}_{\theta_{t+1}}\left[q_{t+1}^* - \frac{\alpha}{\beta}\left(\theta_{t+1} - c - 2q_{t+1}^* - h(\theta_1)\alpha^t\right)\right].$$

It suffices to show $q_{t+1}^* - \frac{\alpha}{\beta}\left(\theta_{t+1} - c - 2q_{t+1}^* - h(\theta_1)\alpha^t\right) \geq 0$, or equivalently,

$$q_{t+1}^* \geq \tfrac{\alpha}{2\alpha+\beta}(\theta_{t+1} - c - h(\theta_1)\alpha^t). \tag{A.10}$$

I show it by backward induction. First, for period $t = T-1$, since $\theta_T - h(\theta_1)\alpha^{T-1} - c \geq 0$, it is clear that

$$q_T^*(\theta^t) = \tfrac{1}{2}(\theta_T - h(\theta_1)\alpha^{T-1} - c) \geq \tfrac{\alpha}{2\alpha+\beta}(\theta_T - h(\theta_1)\alpha^{T-1} - c).$$

That is $q_{T-1}^n(\theta^{T-1}) \leq q_{T-1}^*(\theta^{T-1})$.

Second, assume for period t, inequality (A.10) holds, then $q_t^n(\theta^t) \leq q_t^*(\theta^t)$. That is

$$q_t^*(\theta^t) \geq \tfrac{1}{2}(\theta_t - h(\theta_1)\alpha^{t-1} - c) \geq \tfrac{\alpha}{2\alpha+\beta}(\theta_t - h(\theta_1)\alpha^{t-1} - c).$$

Thus it suffices to say

$$q_{t-1}^*(\theta^{t-1}) - q_{t-1}^n(\theta^{t-1}) \geq 0.$$

By the logic of induction, I conclude $q_t^n(\theta^t) \leq q_t^*(\theta^t)$ for all $t \leq T$.

■

Proof of Proposition 2.4

Under regime $\mathcal{P}^r$, only IC_1 and IR constraints matter. The problem is classic adverse selection, with fixed private information θ_1 and uncertain future states $(\varepsilon_t)_{t\geq 2}$. By the incentive constraint IC_1 and the envelope theorem, I have

$$\frac{\partial}{\partial\theta_1}U_1(\theta_1) = \mathbb{E}\left[\sum_{t=1}^T \delta^{t-1} q_t(\theta^t)\frac{\partial\theta_t}{\partial\theta_1} \mid \theta_1\right] = \mathbb{E}\left[\sum_{t=1}^T \delta^{t-1} q_t(\theta^t)\frac{\partial\theta_t}{\partial\theta_{t-1}}\cdots\frac{\partial\theta_2}{\partial\theta_1} \mid \theta_1\right] = \mathbb{E}\left[\sum_{t=1}^T \delta^{t-1} q_t(\theta^t)\alpha^{t-1} \mid \theta_1\right].$$

Integrating both sides of the above envelope formula, I obtain the payoff equivalence:

$$U_1(\theta_1) = U_1(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_1} \left\{q_1(s) + \mathbb{E}\left[\sum_{\tau=2}^T \delta^{\tau-1}\cdot\alpha^{\tau-1}\cdot q_\tau(s,\theta_2^\tau)\,\middle|\, s, q_1(s)\right]\right\} \mathrm{d}s. \tag{A.11}$$

At the optimum, the participation constraint IR for retailer-$\underline{\theta}$ is binding, i.e., $U_1(\underline{\theta}) = 0$. I then plug Eq. (C.1) into $\tilde{J}(\phi)$, integrate by part, and obtain the virtual surplus expression of the manufacturer's payoff:

$$\tilde{J}^r(\phi) = \sum_{t=1}^T \mathbb{E}\left\{\delta^{t-1}\left[q_t\left(\theta_t - q_t\right) - h(\theta_1)\cdot\alpha^{t-1}\cdot q_t - cq_t\right]\right\}.$$

The problem $\mathcal{P}^r$ reduces to $\max\{\tilde{J}^r(\phi) : IR\}$, which is identical to the formulation $\max\{\tilde{J}^R(\phi) : IR\}$ in regime $\mathcal{P}$ (see part (b) of Lemma B.2). Therefore, the manufacturer makes the same profit in $\mathcal{P}^r$ and $\mathcal{P}$; so does the retailer. ■

Proof of Proposition 2.5

By Proposition 2.3, the optimal contract $\phi^* = (q_t^*, T_t^*)_{t=1}^T$ ensures $(IC_t)_{t\geq 1}$. The total expected payment the manufacturer recieves is

$$\begin{aligned}
\mathbb{E}\left[\sum_{t=1}^T \delta^{t-1} T_t^*(\theta^t)|\, \theta_1\right] &= \mathbb{E}\left[\sum_{t=1}^T \delta^{t-1} R\big(\theta_t, q_t^*(\theta^t)\big)|\theta_1\right] \\
&\quad + \mathbb{E}\left\{\delta\mathbb{E}[U_2^*(\theta^2)|\theta_1] - U_1^*(\theta_1) + \delta^2\mathbb{E}[U_3^*(\theta^3)|\theta_2] - \delta U_2^*(\theta^2) + \cdots\right. \\
&\quad \left. + \delta^{T-1}\mathbb{E}[U_T^*(\theta^T)|\theta_{T-1}] - \delta^{T-2}U_{T-1}^*(\theta^{T-1}) - \delta^{T-1}U_T^*(\theta^T)|\theta_1\right\} \\
&= \mathbb{E}\left[\sum_{t=1}^T \delta^{t-1} R\big(\theta_t, q_t^*(\theta^t)\big) \mid \theta_1\right] - U_1^*(\theta_1).
\end{aligned}$$

It reveals that the manufacturer can extract all the retailer's profit except for the information rent on θ_1 through the recursive advance selling. The recursive advance selling ensures that the manufacturer can extract all the new information $(\varepsilon_t)_{t\geq 2}$ and $(\theta_t)_{t\geq 2}$ for free. ■

Proof of Proposition 2.6

I proceed in two steps.

Step 1: I show the convergence of contract coefficients.

I claim that, if the stability condition $\delta(\alpha+\beta)^2 \leq 1$ holds, the coefficients $(a_t, b_t, c_t)_{t\geq 1}$ of the optimal contract ϕ^* converge to a stable vector (a^*, b^*, d^*), where

$$a^* = \frac{1-\delta\alpha^2 - \sqrt{[1-\delta\alpha^2][1-\delta(\alpha+\beta)^2]}}{\delta\beta^2+2\delta\alpha\beta},$$
$$b^* = \frac{1-\delta\alpha}{2-a^*(\delta\beta^2+2\delta\alpha\beta)+\delta\alpha\beta-\delta\beta-2\delta\alpha},$$
$$d^* = \frac{a^*(\delta\beta+2\delta\alpha)-\delta\alpha}{2-a^*(\delta\beta^2+2\delta\alpha\beta)+\delta\alpha\beta-\delta\beta-2\delta\alpha}.$$

I begin with the observation that the coefficients are the well-known Riccati equation coefficients. Hence their limit exist with suitable convergence condition. If $\lim_{T\to\infty} a_T = a^*$, then dynamics for a_t has a fixed point; equivalently, the equation

$$a^* = \frac{1+[a^*(\delta\beta+2\delta\alpha)-\delta\alpha]\alpha}{2-[a^*(\delta\beta+2\delta\alpha)-\delta\alpha]\beta}$$

has an unique solution for a^*. This is a quadratic equation:

$$(\delta\beta^2+2\delta\alpha\beta)(a^*)^2+(2\delta\alpha^2-2)a^*+1-\delta\alpha^2=0.$$

To ensure the existence of fixed point, I must have

$$\Delta = (2\delta\alpha^2-2)^2-4(\delta\beta^2+2\delta\alpha\beta)(1-\delta\alpha^2) = 4(1-\delta\alpha^2)[1-\delta(\alpha+\beta)^2] \geq 0,$$

which yields the stability condition

$$\delta(\alpha+\beta)^2 \leq 1.$$

Under this condition, the roots of the quadratic equation are

$$a_1^* = \frac{1-\delta\alpha^2+\sqrt{[1-\delta\alpha^2][1-\delta(\alpha+\beta)^2]}}{\delta\beta^2+2\delta\alpha\beta}, \qquad a_2^* = \frac{1-\delta\alpha^2-\sqrt{[1-\delta\alpha^2][1-\delta(\alpha+\beta)^2]}}{\delta\beta^2+2\delta\alpha\beta}.$$

To have nonnegative retail price $P_\infty = \theta_\infty - q_\infty \geq 0$, I must have $q_\infty = a^*\theta_\infty - b^*c + d^*\mu \leq \theta_\infty$, and a^* cannot exceed 1. However, the root a_1^* is large for small δ, α and β. Hence, I can rule out a_1^*, and conclude

$$a^* = \frac{1-\delta\alpha^2-\sqrt{[1-\delta\alpha^2][1-\delta(\alpha+\beta)^2]}}{\delta\beta^2+2\delta\alpha\beta}$$

By the same logic, I obtain the claimed result

$$b^* = \frac{1 - \delta\alpha}{2 - a^*(\delta\beta^2 + 2\delta\alpha\beta) + \delta\alpha\beta - \delta\beta - 2\delta\alpha}, \qquad d^* = \frac{a^*(\delta\beta + 2\delta\alpha) - \delta\alpha}{2 - a^*(\delta\beta^2 + 2\delta\alpha\beta) + \delta\alpha\beta - \delta\beta - 2\delta\alpha}.$$

Step 2: I show the convergence of the optimal sales (quantity).

The optimal contract ϕ^* induces the dynamic market process

$$\begin{aligned}\theta_{t+1} &= \alpha\theta_t + \beta q_t^*(\theta^t) + \varepsilon_{t+1} \\ &= \alpha\theta_t + \beta[a_t(\theta_t - h(\theta_1)\alpha^{t-1}) - b_t c + d_t\mu] + \varepsilon_{t+1} \\ &= (\alpha + \beta a_t)\theta_t \;+\; \beta[d_t\mu - b_t c - h(\theta_1)\alpha^{t-1})] \;+\; \varepsilon_{t+1}.\end{aligned}$$

As $t \to \infty$, $h(\theta_1)\alpha^{t-1} \to 0$, and the market process converges to the linear first-order autoregressive process AR(1) (Bhattacharya and Majumdar, 2007):

$$\theta_{t+1} = (\alpha + \beta a^*)\theta_t \;+\; \beta[d^*\mu - b^*c] \;+\; \varepsilon_{t+1},$$

which is the long-run market process θ_∞ under $\bar{\phi}$ in $\bar{\mathcal{P}}$. Also, as $t \to \infty$, the optimal sales

$$\lim_{t\to\infty} q_t^*(\theta^t) = \lim_{t\to\infty} \left[a_t(\theta_t - h(\theta_1)\alpha^{t-1}) - b_t c + d_t\mu\right] = a^*\theta_\infty - b^*c + d^*\mu = \lim_{t\to\infty} \bar{q}_t(\theta_t).$$

Hence, the optimal contract ϕ^* converges to the first-best $\bar{\phi}$ in the long run. ■

Proof of Proposition 2.7

(a) By the proof of Proposition 2.6, the market process converges to an AR(1) process:

$$\theta_t = \beta d^*\mu + (\alpha + \beta a^*)\theta_{t-1} - \beta b^* c + \varepsilon_t.$$

Given $\alpha + \beta a^* < 1$, as the AR(1) process is stationary, I get $\theta_t \to \theta_\infty$. The unconditional mean $\mathbb{E}[\theta_\infty] = \mathbb{E}[\theta_t] = \mathbb{E}[\theta_{t-1}]$ can be solved by

$$\mathbb{E}[\theta_t] = \beta d^*\mu + (\alpha + \beta a^*)\mathbb{E}[\theta_{t-1}] - \beta b^* c + \mathbb{E}[\varepsilon_t].$$

Similarly, the unconditional variance $\mathrm{Var}[\theta_\infty] = \mathrm{Var}[\theta_t] = \mathrm{Var}[\theta_{t-1}]$ can be solved by

$$\mathrm{Var}[\theta_t] = (\alpha + \beta a^*)^2\,\mathrm{Var}[\theta_{t-1}] + \mathrm{Var}[\varepsilon_t].$$

If $\varepsilon_t \sim_{\text{IID}} \mathcal{N}(\mu, \sigma^2)$, then I have $\theta_t \to \theta_\infty \sim \mathcal{N}(\mu_{\theta_\infty}, \sigma^2_{\theta_\infty})$, where

$$\mu_{\theta_\infty} = \frac{\beta d^* \mu - \beta b^* c + \mu}{1 - (\alpha + \beta a^*)}, \qquad \sigma^2_{\theta_\infty} = \frac{\sigma^2}{1 - (\alpha + \beta a^*)^2}.$$

Similarly, due to the linerity of q_t^* over θ_t, I have $q_t^* = a_t(\theta_t - h(\theta_1)\alpha^{t-1}) - b_t c + d_t \mu \to a^* \theta_\infty - b^* c + d^* \mu = q_\infty \sim \mathcal{N}(\mu_{q_\infty}, \sigma^2_{q_\infty})$, where

$$\mu_{q_\infty} = \frac{\beta d^* \mu - \beta b^* c + \mu}{1 - (\alpha + \beta a^*)} a^* - b^* c + d^* \mu, \qquad \sigma^2_{q_\infty} = \frac{(a^*)^2 \sigma^2}{1 - (\alpha + \beta a^*)^2}.$$

(b) I first show μ_{θ_∞} is increasing in α and β, where $\alpha \in [0, 1)$ satisfies stationary condition $\alpha + \beta a^* < 1$ and $\delta(\alpha + \beta)^2 \leq 1$. I proceed in four steps.

Step 1: I first show that $a^* \geq \frac{1}{2}$.

To have

$$a^* = \frac{1 - \delta\alpha^2 - \sqrt{[1 - \delta\alpha^2][1 - \delta(\alpha + \beta)^2]}}{\delta\beta^2 + 2\delta\alpha\beta} \geq \frac{1}{2},$$

I need

$$\begin{aligned} & 2 - 2\delta\alpha^2 - 2\sqrt{[1 - \delta\alpha^2][1 - \delta(\alpha + \beta)^2]} \geq \delta\beta^2 + 2\delta\alpha\beta \\ \Leftrightarrow\ & 1 - \delta\alpha^2 + 1 - \delta(\alpha^2 + 2\alpha\beta + \beta^2) - 2\sqrt{[1 - \delta\alpha^2][1 - \delta(\alpha + \beta)^2]} \geq 0 \\ \Leftrightarrow\ & [\sqrt{1 - \delta\alpha^2} - \sqrt{1 - \delta(\alpha + \beta)^2}]^2 \geq 0, \end{aligned}$$

which is satisfied for any feasible δ, α, β triple. I conclude that $a^* \geq \frac{1}{2}$.

Step 2: I provide sufficient condition such that a^* is increasing in α and β.

The partial derivative of a^* with respect to α and β:

$$\begin{aligned} \frac{\partial}{\partial \alpha} a^* =& \frac{1}{\delta\beta^2 + 2\delta\alpha\beta} \cdot \left[-2\delta\alpha + \frac{2\delta\alpha[1 - \delta(\alpha + \beta)^2] + 2\delta(\alpha + \beta)[1 - \delta\alpha^2]}{2\sqrt{[1 - \delta\alpha^2][1 - \delta(\alpha + \beta)^2]}} \right] \\ &- \frac{2\delta\beta}{(\delta\beta^2 + 2\delta\alpha\beta)^2} \left[1 - \delta\alpha^2 - \sqrt{[1 - \delta\alpha^2][1 - \delta(\alpha + \beta)^2]} \right] \\ =& \frac{1}{\beta^2 + 2\alpha\beta} \cdot \left[-2(\alpha + \beta a^*) + \frac{\alpha[1 - \delta(\alpha + \beta)^2] + (\alpha + \beta)[1 - \delta\alpha^2]}{\sqrt{[1 - \delta\alpha^2][1 - \delta(\alpha + \beta)^2]}} \right] \\ =& \frac{1}{\beta^2 + 2\alpha\beta} \cdot \left[-2(\alpha + \beta a^*) + \alpha\sqrt{\frac{1 - \delta(\alpha + \beta)^2}{1 - \delta\alpha^2}} + (\alpha + \beta)\sqrt{\frac{1 - \delta\alpha^2}{1 - \delta(\alpha + \beta)^2}} \right], \end{aligned}$$

and

$$\begin{aligned}\frac{\partial}{\partial\beta}a^* =& \frac{1}{\delta\beta^2+2\delta\alpha\beta}\cdot\left[\frac{\delta(\alpha+\beta)[1-\delta\alpha^2]}{\sqrt{[1-\delta\alpha^2][1-\delta(\alpha+\beta)^2]}}\right]\\ &-\frac{2\delta(\beta+\alpha)}{(\delta\beta^2+2\delta\alpha\beta)^2}\left[1-\delta\alpha^2-\sqrt{[1-\delta\alpha^2][1-\delta(\alpha+\beta)^2]}\right]\\ =&\frac{\alpha+\beta}{\beta^2+2\alpha\beta}\cdot\left[-2a^*+\frac{1-\delta\alpha^2}{\sqrt{[1-\delta\alpha^2][1-\delta(\alpha+\beta)^2]}}\right]\\ =&\frac{\alpha+\beta}{\beta^2+2\alpha\beta}\cdot\left[-2a^*+\sqrt{\frac{1-\delta\alpha^2}{1-\delta(\alpha+\beta)^2}}\right].\end{aligned}$$

To have $\frac{\partial}{\partial\alpha}a^*$ and $\frac{\partial}{\partial\beta}a^*$ nonnegative, I shoule have

$$\sqrt{\frac{1-\delta\alpha^2}{1-\delta(\alpha+\beta)^2}}\geq 2a^*,\qquad \alpha\sqrt{\frac{1-\delta(\alpha+\beta)^2}{1-\delta\alpha^2}}+(\alpha+\beta)\sqrt{\frac{1-\delta\alpha^2}{1-\delta(\alpha+\beta)^2}}\geq 2(\alpha+\beta a^*).$$

The two inequality induces that

$$2a^*\leq\sqrt{\frac{1-\delta\alpha^2}{1-\delta(\alpha+\beta)^2}}\leq\frac{1}{2-2a^*}. \tag{MON}$$

To have above (MON) condition holds, I need

$$2a^*\leq\frac{1}{2-2a^*}\Rightarrow(2a^*-1)^2\geq 0,$$

which is satisfied for any feasible a^*.

Step 3: To show that $d^*\mu-b^*c$ is increasing in α and β.

Note that

$$d^*\mu-b^*c=\frac{\delta[a^*(\beta+2\alpha)-\alpha]\mu-(1-\delta\alpha)c}{2-a^*(\delta\beta^2+2\delta\alpha\beta)+\delta\alpha\beta-\delta\beta-2\delta\alpha}.$$

For the positive denominator:

$$\tfrac{\partial}{\partial\alpha}[2-a^*(\delta\beta^2+2\delta\alpha\beta)+\delta\alpha\beta-\delta\beta-2\delta\alpha]=-\tfrac{\partial}{\partial\alpha}a^*(\delta\beta^2+2\delta\alpha\beta)-2a^*\delta\beta+\delta\beta-2\delta<0,$$

and

$$\tfrac{\partial}{\partial\beta}[2-a^*(\delta\beta^2+2\delta\alpha\beta)+\delta\alpha\beta-\delta\beta-2\delta\alpha]=-\tfrac{\partial}{\partial\beta}a^*(\delta\beta^2+2\delta\alpha\beta)-2a^*(\delta\beta+\delta\alpha)+\delta\alpha-\delta<0,$$

since $a^*\geq\frac{1}{2}$. Thus the denominator is decreasing in α and β.

Then for the positive numerator,

$$\tfrac{\partial}{\partial\alpha}[\delta[a^*(\beta+2\alpha)-\alpha]\mu-(1-\delta\alpha)c]=\delta[\tfrac{\partial}{\partial\alpha}a^*(\beta+2\alpha)+2a^*-1]\mu+\delta c>0,$$

since $a^* \geq \frac{1}{2}$. In addition

$$\frac{\partial}{\partial\beta}[\delta[a^*(\beta + 2\alpha) - \alpha]\mu - (1 - \delta\alpha)c] = \delta[\frac{\partial}{\partial\beta}a^*(\beta + 2\alpha) + a^*]\mu > 0.$$

Thus the numerator is increasing in α and β and $d^*\mu - b^*c$ is increasing in α and β separately.

Step 4: I show that μ_{θ_∞} is increasing in α and β.

The numerator of μ_{θ_∞}: $\beta(d^*\mu - b^*c) + \mu$ is increasing in α and β. The denominator of μ_{θ_∞}: $1 - (\alpha + \beta a^*)$ is decreasing in α and β. I can conclude μ_{θ_∞} is increasing in α and β.

As μ_{θ_∞}, a^* and $d^*\mu - b^*c$ are all increasing in α and β, definitely $\mu_{q_\infty} = a^*\mu_{\theta_\infty} - b^*c + d^*\mu$ is increasing in α and β, which concludes the proof. ■

Appendix B

Appendix for Chapter 3

Lemma B.1 and the Proof

Lemma B.1 *Consider a random variable v with support $\mathcal{V} \equiv [\underline{v}, \overline{v}]$, distribution F, density f, and inverse hazard rate $\eta(v) = \frac{1-F(v)}{f(v)}$. If $\psi : \mathcal{V} \to \mathbb{R}$ is differentiable almost everywhere (a.e.), then*

$$\mathbb{E}[\psi(v)] = \psi(\underline{v}) + \mathbb{E}[\eta(v)\psi'(v)].$$

Proof: The differentiability of F and ψ implies

$$\frac{\mathrm{d}}{\mathrm{d}v}\big[(1-F(v))\cdot\psi(v)\big] = -f(v)\cdot\psi(v) + (1-F(v))\cdot\psi'(v), \quad a.e. \tag{B.1}$$

Integrating both sides of Eq. (B.1), I have

$$\int_{\mathcal{V}} \frac{\mathrm{d}}{\mathrm{d}v}\big[(1-F(v))\cdot\psi(v)\big]\,\mathrm{d}v = -\int_{\mathcal{V}} f(v)\psi(v)\,\mathrm{d}v + \int_{\mathcal{V}}(1-F(v))\psi'(v)\,\mathrm{d}v = -\mathbb{E}\left[\psi(v)\right] + \mathbb{E}\left[\tfrac{1-F(v)}{f(v)}\psi'(v)\right]. \tag{B.2}$$

The fundamental theorem of calculus implies

$$\int_{\mathcal{V}} \frac{\mathrm{d}}{\mathrm{d}v}\big[(1-F(v))\cdot\psi(v)\big]\,\mathrm{d}v = (1-F(v))\cdot\psi(v)\,\Big|_{\underline{v}}^{\overline{v}} = -\psi(\underline{v}). \tag{B.3}$$

The result then follows from Eqs. (B.2) and (B.3). ■

Lemma B.2 and the Proof

Lemma B.2 *(a) A scheme ϕ is sequential incentive compatibility, if and only if it satisfies revenue equivalence, the rent monotonicity condition, and quality monotonicity condition:*

$$U_1(v_1) = U_1(\underline{v}) + \int_{\underline{v}}^{v_1} \left\{q_1(s) + \delta\mathbb{E}\big[(\alpha + \gamma q_1(s)) \cdot q_2(s, v_2) \bigm| s, q_1(s)\big]\right\} \mathrm{d}s,$$

$$U_1(v_1) - U_1(\hat{v}_1) \geq \int_{\hat{v}_1}^{v_1} \{q_1(\hat{v}_1) + \delta\mathbb{E}[(\alpha + \gamma q_1(\hat{v}_1))q_2(\hat{v}_1, v_2) \mid s, q_1(\hat{v}_1)]\} \,\mathrm{d}s, \quad \forall v_1 \geq \hat{v}_1,$$

$$q_2(\cdot, v_2) \geq q_2(\cdot, \hat{v}_2), \qquad \forall v_2 \geq \hat{v}_2.$$

(b) The firm's problem $\mathcal{P}$ can be reformulated as:

$$\max_{q_1, q_2} \mathbb{E}\big\{(v_1 - \eta(v_1))q_1(v_1) - \tfrac{1}{2}[q_1(v_1)]^2$$
$$+\delta\mathbb{E}\left[\big(v_2 - \eta(v_1) \cdot (\alpha + \gamma q_1(v_1))\big) \cdot q_2(v_1, v_2) - \tfrac{1}{2}[q_2(v_1, v_2)]^2 \bigm| v_1, q_1(v_1)\right]\big\}.$$

(IR_1) pins down to $U_1(\underline{v}) = 0$ and (IR_2) pins down to $U_2(v_1, \underline{v}) = 0$ for all v_1.

Proof: I use backward induction.

(a) I first show necessity. In period 2, I have a static screening problem: IC_2 requires

$$U_2(v^2) = \max_{\hat{v}_2} \tilde{U}_2(v_1, \hat{v}_2; v_2),$$

which implies $\frac{\partial U_2(v^2)}{\partial v_2} = q_2(v^2)$, and hence the period-2 revenue equivalence:

$$U_2(v_1, v_2) = U_2(v_1, \underline{v}) + \int_{\underline{v}}^{v_2} q_2(v_1, s)\,\mathrm{d}s.$$

In period 1, IC_1 requires

$$U_1(v_1) = \max_{\hat{v}_1} \tilde{U}_1(\hat{v}_1; v_1).$$

By the envelope theorem, I have

$$\begin{aligned}
\frac{\mathrm{d}U_1(v_1)}{\mathrm{d}v_1} &= q_1(v_1) + \delta\mathbb{E}\left[\frac{\partial V}{\partial v_1} \cdot \frac{\partial U_2(v_1, v_2)}{\partial v_2}\middle| v_1, q_1\right] \\
&= q_1(v_1) + \delta\mathbb{E}\left[(\alpha + \gamma q_1) \cdot q_2 \middle| v_1, q_1\right].
\end{aligned}$$

Hence, the period-1 revenue equivalence is

$$U_1(v_1) = U_1(\underline{v}) + \int_{\underline{v}}^{v_1} \left\{q_1(s) + \delta\mathbb{E}\left[(\alpha + \gamma q_1) \cdot q_2 \middle| s, q_1(s)\right]\right\} \mathrm{d}s.$$

Next, I show sufficiency. In period 2, I shall show reporting the true type v_2 is optimal for the consumer. By the quality monotonicity condition, I have

$$[q_2(\hat{v}_1, v_2) - q_2(\hat{v}_1, \hat{v}_2)]\,(v_2 - \hat{v}_2) \geq 0.$$

By the envelope formula, the consumer rent satisfies

$$\left[\tfrac{\partial}{\partial v_2}\tilde{U}_2(\hat{v}_1, v_2; v_2) - \tfrac{\partial}{\partial v_2}\tilde{U}_2(\hat{v}_2, \hat{v}_2; v_2)\right](v_2 - \hat{v}_2) \geq 0.$$

It implies that truthtelling in period 2 gives the consumer higher, and hence IC_2 is satisfied.

In period 1, conditional on truthtelling in period 2, the consumer-v_1 by reporting $\hat{v}_1 \leq v_1$ can obtain the payoff

$$v_1 q_1(\hat{v}_1) - p_1(\hat{v}_1) + \delta\mathbb{E}[v_2 q_2(\hat{v}_1, v_2) - p(\hat{v}_1, v_2)|v_1, q_1(\hat{v}_1)],$$

where $v_2 = \alpha v_1 + \beta q_1(\hat{v}_1) + \gamma v_1 q_1(\hat{v}_1) + \varepsilon$; by contrast, her truthtelling payoff is

$$v_1 q_1(v_1) - p_1(v_1) + \delta\mathbb{E}[U_2(v_1, v_2)].$$

IC_1 then follows from

$$\begin{aligned}
&v_1 q_1(v_1) - p_1(v_1) + \delta\mathbb{E}[U_2(v_1, v_2)] - \Big(v_1 q_1(\hat{v}_1) - p_1(\hat{v}_1) + \delta\mathbb{E}[v_2 q_2(\hat{v}_1, v_2) - p(\hat{v}_1, v_2)|v_1, q_1(\hat{v}_1)]\Big)\\
=&v_1 q_1(v_1) - p_1(v_1) + \delta\mathbb{E}[U_2(v_1, v_2)] - \Big(\hat{v}_1 q_1(\hat{v}_1) - p_1(\hat{v}_1) + \delta\mathbb{E}[U_2(\hat{v}_1, v_2)]\Big)\\
&+ (\hat{v}_1 q_1(\hat{v}_1) - p_1(\hat{v}_1) + \delta\mathbb{E}[U_2(\hat{v}_1, v_2)]) - \Big(v_1 q_1(\hat{v}_1) - p_1(\hat{v}_1) + \delta\mathbb{E}[v_2 q_2(\hat{v}_1, v_2) - p(\hat{v}_1, v_2)|v_1, q_1(\hat{v}_1)]\Big)\\
=&U_1(v_1) - U_1(\hat{v}_1) - [(v_1 - \hat{v}_1)q_1(\hat{v}_1) + \delta\mathbb{E}[v_2 q_2(\hat{v}_1, v_2) - p(\hat{v}_1, v_2)|v_1, q_1(\hat{v}_1)] - \delta\mathbb{E}[U_2(\hat{v}_1, v_2)]]\\
=&U_1(v_1) - U_1(\hat{v}_1) - [(v_1 - \hat{v}_1)q_1(\hat{v}_1) + \delta\mathbb{E}[\alpha v_1 q_2(\hat{v}_1, v_2) + \gamma v_1 q_1(\hat{v}_1) q_2(\hat{v}_1, v_2)|v_1, q_1(\hat{v}_1)]\\
&- \delta\mathbb{E}[\alpha\hat{v}_1 q_2(\hat{v}_1, v_2) + \gamma\hat{v}_1 q_1(\hat{v}_1) q_2(\hat{v}_1, v_2)|\hat{v}_1, q_1(\hat{v}_1)]\\
=&U_1(v_1) - U_1(\hat{v}_1) - \int_{\hat{v}_1}^{v_1}\{q_1(\hat{v}_1) + \delta\mathbb{E}[(\alpha + \gamma q_1(\hat{v}_1))q_2(\hat{v}_1, v_2)\ |s, q_1(\hat{v}_1)]\}\,\mathrm{d}s \geq 0,
\end{aligned}$$

where the last inequality follows from the rent monotonicity and quality monotonicity. Therefore, revenue equivalence, rent monotonicity, and quality monotonicity are sufficient for truthtelling.

(b) By changing the decision variables from prices $(p_t)_t$ to rents $(U_t)_t$, I can rewrite firm profit as

$$\begin{aligned}
&\mathbb{E}\left\{p_1(v_1) - \tfrac{1}{2}[q_1(v_1)]^2 + \delta\big[p_2(v_1, v_2) - \tfrac{1}{2}[q_2(v_1, v_2)]^2\big]\right\}\\
=&\mathbb{E}\left\{v_1 q_1 - \tfrac{1}{2}[q_1(v_1)]^2 + \delta\big[v_2 q_2 - \tfrac{1}{2}[q_2(v_1, v_2)]^2\big]\right\} - \mathbb{E}U_1(v_1). \qquad \text{(A)}
\end{aligned}$$

By the revenue equivalence in part (a) and Fubini's Theorem, I have

$$\begin{aligned}
\mathbb{E}U_1(v_1) =& U_1(\underline{v}) + \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{v_1} \left\{q_1(s) + \delta\mathbb{E}\left[(\alpha+\gamma q_1)\cdot q_2 \middle| s, q_1(s)\right]\right\} \mathrm{d}s f_1(v_1)\,\mathrm{d}v_1 \\
=& U_1(\underline{v}) + \int_{\underline{v}}^{\overline{v}} \left\{q_1(s) + \delta\mathbb{E}\left[(\alpha+\gamma q_1)\cdot q_2 \middle| s, q_1(s)\right]\right\} \int_{s}^{\overline{v}} f_1(v_1)\,\mathrm{d}v_1\,\mathrm{d}s \\
=& U_1(\underline{v}) + \int_{\underline{v}}^{\overline{v}} \left\{q_1(v_1) + \delta\mathbb{E}\left[(\alpha+\gamma q_1)\cdot q_2 \middle| v_1, q_1(v_1)\right]\right\} (1-F_1(v_1))\,\mathrm{d}v_1 \\
=& U_1(\underline{v}) + \mathbb{E}\left\{\eta(v_1)q_1(v_1) + \delta\mathbb{E}\left[\eta(v_1)(\alpha+\gamma q_1)\cdot q_2 \middle| v_1, q_1(v_1)\right]\right\}. \qquad \text{(B)}
\end{aligned}$$

After pinning down (IC_1) and (IC_2), I then deal with (IR_1) and (IR_2). To maximize the profit, the firm can set $U_1(\underline{v}) = 0$. By Eqs. (A) and (B), the firm's problem becomes

$$\max_{q_1(\cdot),q_2(\cdot)} \mathbb{E}\left\{(v_1-\eta(v_1))q_1(v_1) - \tfrac{1}{2}[q_1(v_1)]^2 + \delta\mathbb{E}\left[\big(v_2-\eta(v_1)\cdot(\alpha+\gamma q_1(s))\big)q_2(v_1,v_2) - \tfrac{1}{2}[q_2(v_1,v_2)]^2 \mid v_1, q_1(v_1)\right]\right\}.$$

By Lemma B.2 (a), $q_2(v_1,v_2)$ is nondecreasing in v_2, I can take $U_2(v_1,\underline{v}) = 0$ to have $U_2(v^2) = U_2(v_1,\underline{v}) + \int_{\underline{v}}^{v_2} q_2(v_1,s)\,\mathrm{d}s \geq 0$. By doing that, the provider pays less consumer rent and ensures the (IR_2). Similary, by Lemma B.2 (a), $U_1(v_1)$ is nondecreasing in v_1, $U_1(\underline{v}) = 0$ ensures $U_1(v_1) \geq 0$ (IR_1). This finishes the proof.

■

Proof of Proposition 3.1

Let J_t be the firm's continuation payoff from period t onward. Under full information, the firm can charge the price $p_t = v_t q_t$, to fully extract consumer surplus. The problem becomes

$$\max_{q_1,q_2} \mathbb{E}J_1 = \mathbb{E}\left\{v_1q_1 - \tfrac{1}{2}[q_1]^2 \;+\; \delta\mathbb{E}[v_2q_2 - \tfrac{1}{2}[q_2]^2 | v_1, q_1]\right\}.$$

(a) Without habituation, $V_q \equiv 0$. Then the First Order Conditions (FOC) $\partial J_1/\partial q_1 = \partial J_2/\partial q_2 = 0$ yield the solution of first-degree discrimination:

$$p^f(v_t) = v_t q^f(v_t), \quad q^f(v_t) = v_t, \quad \forall v_t. \qquad (\phi^f)$$

(b) With habituation, $V_q \geq 0$. The FOC $\partial J_2/\partial q_2 = 0$ yields

$$\bar{q}_2(v_2) = v_2.$$

The FOC $\partial J_1/\partial q_1 = 0$ yields

$$0 = v_1 - q_1 + \delta \frac{\partial}{\partial q_1} \mathbb{E}\left[v_2 \bar{q}_2 - \tfrac{1}{2}\bar{q}_2^2 | v_1, q_1\right]$$

$$= v_1 - q_1 + \delta \mathbb{E}\left[(\beta + \gamma v_1)\bar{q}_2 | v_1, q_1\right].$$

Along with $\bar{q}_2 = \alpha v_1 + (\beta + \gamma v_1) q_1 + \varepsilon$, I obtain

$$\bar{q}_1 = \frac{v_1 + \delta(\beta + \gamma v_1)(\alpha v_1 + \mu)}{1 - \delta(\beta + \gamma v_1)^2}.$$

(*c*) I now compare $\bar{q}_t$ and q_t^f. By parts (a) and (b), I have

$$\bar{q}_2(v_2) = v_2 = q^f(v_2),$$

$$\bar{q}_1(v_1) - q^f(v_1) = \delta \mathbb{E}\left[(\beta + \gamma v_1)\bar{q}_2 | v_1, q_1\right] \geq 0.$$

Hence, consumer habituation (weakly) increases quality provision.

The full surplus extraction implies $J_1 = W_1$. It remains to show $J_1(\bar{\phi}) \geq J_1(\phi^f)$; i.e., the firm profit under scheme $\bar{\phi}$ is higher than that under ϕ^f. By the envelope theorem, I have

$$\frac{\partial J_1}{\partial V_q} = \frac{\partial J_1}{\partial v_2} \cdot \frac{\partial V}{\partial V_q} = \delta \bar{q}_2 \bar{q}_1 \geq 0.$$

Hence J_1 and W_1 are nondecreasing in V_q. Since $J_1(\bar{\phi})$ and $W_1(\bar{\phi})$ are associated with positive V_q, and $J_1(\phi^f)$ and $W_1(\phi^f)$ are associated with zero V_q, I have

$$J_1(\bar{\phi}) \geq J_1(\phi^f), \qquad W_1(\bar{\phi}) \geq W_1(\phi^f).$$

■

Proof of Proposition 3.2

I first determine the optimal qualities $(q_t^*)_t$. By Lemma B.2, the firm should solve:

$$\max_{q_1, q_2} \mathbb{E}\left\{(v_1 - \eta(v_1))q_1(v_1) - \tfrac{1}{2}[q_1(v_1)]^2 + \delta \mathbb{E}\left[\left(v_2 - \eta(v_1)\cdot(\alpha + \gamma q_1(s))\right)q_2(v_1, v_2) - \tfrac{1}{2}[q_2(v_1, v_2)]^2 \mid v_1, q_1(v_1)\right]\right\}.$$

The FOC $\partial J_2/\partial q_2 = 0$ yields

$$q_2^*(v^2) = v_2 - \eta(v_1)\left[\alpha + \gamma q_1^*(v_1)\right].$$

The FOC $\partial J_1/\partial q_1 = 0$ implies

$$\partial J_1/\partial q_1 = v_1 - \eta(v_1) - q_1^* + \delta \mathbb{E}\left[(\beta + \gamma v_1) q_1^*\right] - \delta \mathbb{E}\left[\eta(v_1)\gamma q_2^*\right] = 0.$$

Plugging in q_2^*, I get

$$q_1^* = v_1 - \eta(v_1) + \delta(\beta + \gamma v_1)\left[\alpha v_1 + (\beta + \gamma v_1)q_1^* + \mu\right] - \delta\eta(v_1)(\beta + \gamma v_1)\left[\alpha + \gamma q_1^*\right] - \delta\eta(v_1)\gamma\mathbb{E}[q_2^*].$$

Hence, the closed form solution for q_1^* is

$$q_1^*(v_1) = \frac{v_1 - \eta(v_1) + \delta\big((\beta + \gamma v_1) - \eta(v_1)\gamma\big)\big(\alpha(v_1 - \eta(v_1)) + \mu\big)}{1 - \delta\big((\beta + \gamma v_1) - \eta(v_1)\gamma\big)^2}.$$

I now determine the optimal pricing rule $(p_t^*)_t$. By the definition of consumer rent U_t, I have

$$\begin{aligned} U_2(v^2) &= v_2 q_2^* - p_2^*, \\ U_1(v_1) &= v_1 q_1^* - p_1^* + \delta\mathbb{E}\left[U(v^2)\middle|v_1, q_1^*\right]. \end{aligned}$$

Hence, the optimal pricing rule is

$$\begin{aligned} p_2^* &= v_2 q_2^* - U_2(v^2), \\ p_1^* &= v_1 q_1^* + \delta\mathbb{E}\big[U_2(v^2)\big|v_1, q_1^*\big] - U_1(v_1), \end{aligned}$$

where $(U_t)_t$ are the consumer rents determined by Lemma B.2:

$$\begin{aligned} U_1(v_1) &= \int_{\underline{v}}^{v_1} \left\{q_1^*(s) + \delta\mathbb{E}\big[(\alpha + \gamma q_1^*)q_2^*(s, v_2)\big|s, q_1^*(s)\big]\right\} \mathrm{d}s, \\ U_2(v^2) &= \int_{\underline{v}}^{v_2} q_2^*(v_1, s)\,\mathrm{d}s. \end{aligned}$$

Finally, I show the welfare and strategic effects of habituation. By the FOC of q_1^*, I have

$$q_1^*(v_1) = v_1 - \eta(v_1) + \delta\mathbb{E}_1\left[(\beta + \gamma v_1)q_2^*\right] - \delta\mathbb{E}_1\left[\eta(v_1)\gamma q_2^*\right].$$

The term $\delta\mathbb{E}_1\left[(\beta + \gamma v_2)q_1^*\right]$ measures the welfare effect, while the term $-\delta\mathbb{E}_1\left[\eta(v_1)\gamma q_2^*\right]$ measures the strategic effect. In particular, when $\gamma > 0$, the strategic effect is negative, and it exacerbates the cannibalization; when $\gamma < 0$, the strategic effect is positive, and it alleviates the cannibalization. ■

Proof of Proposition 3.3

(a) I first show the firm can fully extract future surplus, with the offsetting mechanism is the upfront subscription fee $\mathbb{E}[U_2^*|v_1]$ and price discount $U_2(v^2)$. Indeed, under scheme ϕ^*, the firm's total expected revenue from consumer-v_1 is

$$\begin{aligned}\mathbb{E}\left[p_1^*(v_1) + \delta p_2^*(v^2) \mid v_1\right] =& \mathbb{E}\left[v_1 q_1^*(v_1) + \delta v_2 q_2^*(v^2) \mid v_1\right] \; + \; \mathbb{E}\left\{\delta \mathbb{E}[U_2^*(v^2)|v_1] - U_1^*(v_1) - \delta U_2^*(v^2)|v_1\right\} \\ =& \mathbb{E}\left[v_1 q_1^*(v_1) + \delta v_2 q_2^*(v^2) - U_1^*(v_1) \mid v_1\right] \\ =& \mathbb{E}\left[v_1 q_1^*(v_1) + \delta v_2 q_2^*(v^2) \mid v_1\right] - U_1^*(v_1),\end{aligned} \tag{C}$$

where the equalities follow from the definition of ϕ^* and the properties of conditional expectation (Dudley, 2002). The relation (C) shows that, except the sign-up bonus $U_1^*(v_1)$, the firm can extract all consumer surplus.

(b) I show the subscription scheme ϕ^* can reduce consumer rent, relative to the spot selling scheme ϕ^s. To this end, I consider $\frac{U_1^*/(1+\delta)}{U^s}$, the rent ratio (per transaction) between ϕ^* and ϕ^s. I shall show

$$1/(1+\delta) \le \frac{U_1^*/(1+\delta)}{U^s} \le 1 \tag{B.4}$$

under condition $\mathbf{C}^R \equiv \left\{\gamma < 0,\ \gamma\left(\underline{v} - \eta(\underline{v})\right) + \beta \ge 0,\ 4\alpha\gamma\left(\overline{v} - \eta(\overline{v})\right) + 2\alpha\beta\mu - \alpha^2\beta^2 - \gamma^2\mu^2 + 2\alpha\beta + 2\gamma\mu - 1 \ge 0\right\}$.

The relation $1/(1+\delta) \le \frac{U_1^*/(1+\delta)}{U^s}$ follows from

$$\begin{aligned}U_1^*(v_1) &= \int_{\underline{v}}^{v_1} \left\{q_1^*(s) + \delta \mathbb{E}[V_v q_2^*(s, v_2)]\right\} \mathrm{d}s && \text{the definition of } U_t^* \\ &\ge \int_{\underline{v}}^{v_1} q_1^*(s)\,\mathrm{d}s && V_v = \alpha + \gamma q_1 \in [0,1] \\ &\ge \int_{\underline{v}}^{v_1} q^s(s)\,\mathrm{d}s && \text{the definition of } q_1^*,\ q^s, \text{ and condition } \mathbf{C}^R \\ &= U^s(v_1). && \text{the definition of } U^s\end{aligned}$$

I now show the relation $\frac{U_1^*/(1+\delta)}{U^s} \le 1$ holds. Let $\theta_1(v_1) \equiv v_1 - \eta(v_1)$, which increases in v_1. Condition $\mathbf{C}^R$ implies

$$\gamma^2\theta_1(v_1) + \beta\gamma \le 0, \qquad \forall v_1, \tag{$\mathbf{C}^R$-1}$$

and

$$4\alpha\gamma\theta_1(v_1) + 2\alpha\beta\mu - \alpha^2\beta^2 - \gamma^2\mu^2 + 2\alpha\beta + 2\gamma\mu - 1 \geq 0$$

$$\Leftrightarrow \alpha^2\theta_1(v_1) + \alpha\mu - \frac{(2\alpha\gamma\theta_1(v_1) + \alpha\beta + \gamma\mu - 1)^2}{4(\gamma^2\theta_1(v_1) + \beta\gamma)} \leq 0, \qquad \forall v_1. \qquad (\mathbf{C}^R\text{-}2)$$

The inequalities ($\mathbf{C}^R$-1) and ($\mathbf{C}^R$-2) jointly imply

$$(\gamma^2\theta_1(v_1) + \beta\gamma)q_1^2 + (2\alpha\gamma\theta_1(v_1) + \alpha\beta + \gamma\mu - 1)q_1 + \alpha^2\theta_1(v_1) + \alpha\mu \leq 0,$$

$$\Leftrightarrow (\alpha + \gamma q_1^*)[(\alpha + \gamma q_1^*)(v_1 - \eta(v_1)) + \beta q_1^* + \mu] \leq q_1^*,$$

$$\Leftrightarrow q_1^* + \delta(\alpha + \gamma q_1^*)[(\alpha + \gamma q_1^*)(v_1 - \eta(v_1)) + \beta q_1^* + \mu] \leq (1 + \delta)q_1^*,$$

which implies $U_1^*(v_1) \leq (1 + \delta)U^s(v_1)$, as desired.

(c) I now consider the case wherein the consumer has independent valuations over time. This means $\alpha = \beta = \gamma = 0$, and the future valuation $v_2 = \varepsilon$ is purely driven by random shock ε. By Proposition 3.2, I have

$$q_1^*(v_1) = v_1 - \eta(v_1), \qquad q_2^*(v^2) = v_2.$$

Hence, the firm only distorts the first period quality with $q_1^* = q^s$, but supplies the first-best quality in the second period with $q_2^* = q^f$. By the proof of Lemma B.2 part (a), the consumer rent is

$$U_1^*(v_1) = U_1^*(\underline{v}) + \int_{\underline{v}}^{v_1} q_1^*(s)\,\mathrm{d}s = U^s(v_1).$$

Hence the firm pays no rent for the privacy of v_2, and thus it can practice first-degree price discrimination in period 2.

■

Proof of Proposition 3.4

Let $\bar{D}_1^*(v_1) \equiv q_1^*(v_1) - \bar{q}_1(v_1)$. By the closed-form solutions of $q_1^*(v_1)$ and $\bar{q}_1(v_1)$, I can verify that $\bar{D}_1^*(v_1)$ is continuous, and

$$q_1^*(v_1) = \bar{q}_1(v_1 - \eta(v_1)). \qquad (\mathrm{D})$$

There are two cases:

(a) Additive habituation ($\gamma > 0$): to show downward distortion $\bar{D}_1^*(v_1) \le 0$, I recall

$$\bar{q}_1(v_1) = \frac{v_1 + \delta(\beta + \gamma v_1)(\alpha v_1 + \mu)}{1 - \delta(\beta + \gamma v_1)^2}.$$

Since $\gamma > 0$, the denominator of $\bar{q}_1(v_1)$ decreases in v_1, while its numerator increases in v_1, and hence $\bar{q}_1(v_1)$ increases in v_1:

$$\bar{q}_1(\theta_1) \le \bar{q}_1(v_1), \quad \forall \theta_1 \le v_1. \tag{E}$$

By relations (D), (E), and $\theta_1 = v_1 - \eta(v_1) \le v_1$ for $v_1 \in \mathcal{V}$, I conclude the downward distortion:

$$\bar{D}_1^*(v_1) = \bar{q}_1(v_1 - \eta(v_1)) - \bar{q}_1(v_1) \le 0.$$

(b) Satiating habituation ($\gamma < 0$): consider condition $\mathbf{C}_L \equiv \{\bar{q}_1(\underline{v}) < \bar{q}_1(\underline{v} - \eta(\underline{v})),\ \eta'(\overline{v}) \cdot \bar{q}_1'(\overline{v}) < 0\}$. I have: (i) the relation $\eta(\overline{v}) = 0$ implies no distortion at the top. (ii) The relation $\bar{q}_1(\underline{v}) < \bar{q}_1(\underline{v} - \eta(\underline{v}))$. implies the upward distortion at the bottom. (iii) The relation $\eta'(\overline{v}) \cdot \bar{q}_1'(\overline{v}) < 0$ implies the downward distortion at consumer-$(\overline{v} - \zeta)$, with infinitesimal $\zeta > 0$:

$$\begin{aligned}
\bar{D}_1^*(\overline{v} - \zeta) &= \bar{q}_1(\overline{v} - \zeta - \eta(\overline{v} - \zeta)) - \bar{q}_1(\overline{v} - \zeta) \\
&= \frac{d}{dv_1}\bar{q}_1(\overline{v} - \eta(\overline{v})) \cdot (-\zeta) - \frac{d}{dv_1}\bar{q}_1(\overline{v})) \cdot (-\zeta) \\
&= \frac{\partial}{\partial v_1}\bar{q}_1(\overline{v} - \eta(\overline{v})) \cdot (1 - \eta'(\overline{v})) \cdot (-\zeta) - \frac{\partial}{\partial v_1}\bar{q}_1(\overline{v})) \cdot (-\zeta) \\
&= \eta'(\overline{v}) \cdot \bar{q}_1'(\overline{v}) \cdot \zeta \\
&< 0.
\end{aligned}$$

By the facts (ii), (iii), and the intermediate value theorem (Corbae et al., 2009), there exists $v^\dagger \in (\underline{v}, \overline{v})$ such that $\bar{D}_1^*(v^\dagger) = 0$. Hence, the distortion is downward ($\bar{D}_1^*(v_1) < 0$) for low end types, and upward ($\bar{D}_1^*(v_1) > 0$) for high end types (except the top type $\overline{v}$).

Next consider the condition $\mathbf{C}_H \equiv \{\bar{q}_1(\underline{v}) > \bar{q}_1(\underline{v} - \eta(\underline{v})), \eta'(\overline{v}) \cdot \bar{q}_1'(\overline{v}) > 0\}$. Following the same argument, I have: (i) condition $\bar{q}_1(\underline{v}) > \bar{q}_1(\underline{v} - \eta(\underline{v}))$ implies downward distortion at the bottom; (ii) condition $\eta'(\overline{v}) \cdot \bar{q}_1'(\overline{v}) > 0$ ensures $\bar{D}_1^*(\overline{v} - \zeta) > 0$, the upward distorted for some type $\overline{v} - \zeta$ sufficiently close to the top $\overline{v}$. Hence, there exists $v^\ddagger \in (\underline{v}, \overline{v})$ such that $\bar{D}_1^*(v^\ddagger) = 0$: the distortion is downward for high end types, and upward for low end types.

■

Proof of Proposition 3.5

Recall the firm's problem ($\mathcal{P}$):

$$J_1^* = \max_{q_1,q_2} \mathbb{E}\left\{(v_1 - \eta(v_1))\, q_1(v_1) - \tfrac{1}{2}[q_1(v_1)]^2 \right.$$
$$\left. + \delta \mathbb{E}\left[(v_2 - \eta(v_1)\cdot(\alpha + \gamma q_1(v_1)))\cdot q_2(v_1, v_2) - \tfrac{1}{2}[q_2(v_1, v_2)]^2 \mid v_1, q_1(v_1)\right]\right\}.$$

By changing the probability distribution from $v_2 \sim F(\cdot|v_1, q_1)$ to $\varepsilon \sim G(\cdot)$, rewrite ($\mathcal{P}$) as

$$J_1^* = \max_{q_1,q_2} \mathbb{E}_{v_1,\varepsilon}\left\{(v_1 - \eta(v_1))\, q_1(v_1) - \tfrac{1}{2}[q_1(v_1)]^2 \right.$$
$$\left. + \delta\Big(\alpha v_1 + \beta q_1(v_1) + \gamma v_1 q_1(v_1) + \varepsilon - \eta(v_1)\cdot(\alpha + \gamma q_1(v_1))\Big)\cdot q_2(v_1, \varepsilon) - \tfrac{1}{2}[q_2(v_1, \varepsilon)]^2\right\},$$

where $\mathbb{E}_{v_1,\varepsilon}[\psi(v_1, \varepsilon)] \equiv \int\int \psi(v_1, \varepsilon)\, \mathrm{d}F_1(v_1)\, \mathrm{d}G(\varepsilon)$. One can find the optimal quality with type-wise optimization. The firm's payoff from type-v_1 is

$$J_1^*(v_1) = \max_{q_1,q_2}\left\{(v_1 - \eta(v_1))\, q_1(v_1) - \tfrac{1}{2}[q_1(v_1)]^2 \right.$$
$$\left. + \delta \mathbb{E}_\varepsilon\left[\Big(\alpha v_1 + \beta q_1(v_1) + \gamma v_1 q_1(v_1) + \varepsilon - \eta(v_1)\cdot(\alpha + \gamma q_1(v_1))\Big)\cdot q_2(v_1, \varepsilon) - \tfrac{1}{2}[q_2(v_1, \varepsilon)]^2\right]\right\}.$$

To see the impact of *improving* α, β, and γ, I apply the envelope theorem (Milgrom and Segal, 2002):

$$\begin{cases} \frac{\partial}{\partial \alpha} J_1^*(v_1) = \delta(v_1 - \eta(v_1)) \cdot \mathbb{E}_\varepsilon[q_2^*(v_1, \varepsilon)], \\ \frac{\partial}{\partial \beta} J_1^*(v_1) = \delta q_1^*(v_1) \cdot \mathbb{E}_\varepsilon[q_2^*(v_1, \varepsilon)], \\ \frac{\partial}{\partial \gamma} J_1^*(v_1) = \delta(v_1 - \eta(v_1)) \cdot \mathbb{E}_\varepsilon[q_1^*(v_1) \cdot q_2^*(v_1, \varepsilon)]. \end{cases} \tag{B.5}$$

Next I show the three results.

(a) Eq. (B.5) and $q_t^* \geq 0$ imply $\frac{\partial}{\partial \beta} J_1^*(v_1) \geq 0$, $\forall v_1$. Hence part (a) holds.

(b) Since the inverse hazard rate $\eta(v_1)$ decreases in v_1, the virtual valuation $\theta_1(v_1) = v_1 - \eta(v_1)$ increases on $\mathcal{V}$. Hence I can define the threshold

$$v^\dagger = \begin{cases} \inf\{v_1 \in \mathcal{V} : \theta_1(v_1) = 0\}, & \text{if } \underline{v} < \eta(\underline{v}) \\ \underline{v}, & \text{if } \underline{v} \geq \eta(\underline{v}). \end{cases} \tag{B.6}$$

Then by $q_t^* \geq 0$, Eqs. (B.5) and (B.6), I can conclude

$$\frac{\partial}{\partial \alpha} J_1^*(v_1) \leq 0, \quad , \frac{\partial}{\partial \gamma} J_1^*(v_1) \leq 0, \quad \forall v_1 \leq v^\dagger,$$
$$\frac{\partial}{\partial \alpha} J_1^*(v_1) \geq 0, \quad , \frac{\partial}{\partial \gamma} J_1^*(v_1) \geq 0, \quad \forall v_1 \geq v^\dagger.$$

(c) I first consider the impact of improve α on ex ante firm profit:

$$\tfrac{\partial}{\partial\alpha}\mathbb{E}[J_1^*(v_1)] \;=\; \delta\mathbb{E}_{v_1,\varepsilon}[\theta_1(v_1)\cdot q_2^*(v_1,\varepsilon)] \;=\; \mathbb{E}_{v_1}[\theta_1(v_1)\cdot Q_2^*(v_1)], \tag{B.7}$$

where $\theta_1(v_1) = v_1 - \eta(v_1)$ is the virtual valuation of customer-v_1, and $Q_2^*(v_1) = \mathbb{E}[q_2^*(v^2)|v_1]$ is her expected future quality assignment.

Applying Lemma B.1 to $\psi(v) = v$, I have $\mathbb{E}[v] = \underline{v} + \mathbb{E}[\eta(v)]$, which implies

$$\mathbb{E}[v_1 - \eta(v_1)] = \mathbb{E}[\theta_1(v_1)] = \underline{v}. \tag{B.8}$$

Applying Lemma B.1 to $\psi(v) = v^2$, I have $\mathbb{E}[v^2] = \underline{v}^2 + \mathbb{E}[\eta(v)\cdot 2v]$, which implies $\mathbb{E}[(v-\eta(v))^2] = \underline{v}^2 + \mathbb{E}[\eta^2(v)]$; hence

$$\left(\mathbb{E}[\theta_1(v_1)^2]\right)^{1/2} = \left(\underline{v}^2 + \mathbb{E}[\eta^2(v_1)]\right)^{1/2}. \tag{B.9}$$

Let $\sigma[X] = \sigma_X$ be the standard deviation of random variable X. I have two subcases:

(i) When $\rho[\theta_1(v_1), Q_2^*(v_1)] \geq 0$, I have

$$\begin{aligned}
\tfrac{\partial}{\partial\alpha}\mathbb{E}[J_1^*(v_1)] &= \delta\mathbb{E}[\theta_1(v_1)\cdot Q_2^*(v_1)] && \text{by Eq. (B.7)}\\
&= \delta\mathbb{E}[\theta_1(v_1)]\cdot\mathbb{E}[Q_2^*(v_1)]\\
&\qquad +\; \delta\rho[\theta_1(v_1), Q_2^*(v_1)]\cdot\sigma[\theta_1(v_1)]\cdot\sigma[Q_2^*(v_1)] && \text{by the covariance formula}\\
&\geq \delta\mathbb{E}[\theta_1(v_1)]\cdot\mathbb{E}[Q_2^*(v_1)] && \text{by } \rho[\theta_1(v_1), Q_2^*(v_1)] \geq 0\\
&= \delta\underline{v}\cdot\mathbb{E}[Q_2^*(v_1)] && \text{by Eq. (B.8)}\\
&\geq 0.
\end{aligned}$$

Hence, the firm benefits from improving α, when $\rho[\theta_1(v_1), Q_2^*(v_1)] \geq 0$.

(ii) When $\rho[\theta_1(v_1), Q_2^*(v_1)] \leq \rho^\dagger \equiv -\frac{\underline{v}}{\left(\underline{v}^2+\mathbb{E}[\eta^2(v_1)]\right)^{1/2}}$, I have

$$
\begin{aligned}
&\frac{\partial}{\partial\alpha}\mathbb{E}[J_1^*(v_1)] \\
&= \delta\mathbb{E}[\theta_1(v_1)\cdot Q_2^*(v_1)] && \text{by Eq. (B.7)} \\
&= \delta\mathbb{E}[\theta_1(v_1)]\cdot\mathbb{E}[Q_2^*(v_1)] \\
&\quad + \ \delta\rho[\theta_1(v_1), Q_2^*(v_1)]\cdot\sigma[\theta_1(v_1)]\cdot\sigma[Q_2^*(v_1)] && \text{by the covariance formula} \\
&\leq \delta\mathbb{E}[\theta_1(v_1)]\cdot\mathbb{E}[Q_2^*(v_1)] && \text{by } \sigma_X^2 = \mathbb{E}X^2 - (\mathbb{E}X)^2 \leq \mathbb{E}X^2 \\
&\quad + \ \delta\rho[\theta_1(v_1), Q_2^*(v_1)]\cdot\left(\mathbb{E}[\theta_1(v_1)^2]\right)^{1/2}\cdot\left(\mathbb{E}[Q_2^*(v_1)^2]\right)^{1/2} \\
&\leq \delta\mathbb{E}[\theta_1(v_1)]\cdot\mathbb{E}[Q_2^*(v_1)] \\
&\quad + \ \delta\frac{-\underline{v}}{\left(\underline{v}^2+\mathbb{E}[\eta^2(v_1)]\right)^{1/2}}\cdot\left(\mathbb{E}[\theta_1(v_1)^2]\right)^{1/2}\cdot\left(\mathbb{E}[Q_2^*(v_1)^2]\right)^{1/2} && \text{by the hypothesis} \\
&= \delta\mathbb{E}[\theta_1(v_1)]\cdot\mathbb{E}[Q_2^*(v_1)] \\
&\quad + \ \delta\frac{-\underline{v}}{\left(\mathbb{E}[\theta_1(v_1)^2]\right)^{1/2}}\cdot\left(\mathbb{E}[\theta_1(v_1)^2]\right)^{1/2}\cdot\left(\mathbb{E}[Q_2^*(v_1)^2]\right)^{1/2} && \text{by Eq. (B.9)} \\
&= \delta\underline{v}\cdot\mathbb{E}[Q_2^*(v_1)] - \underline{v}\cdot\left(\mathbb{E}[Q_2^*(v_1)^2]\right)^{1/2} && \text{by Eq. (B.8)} \\
&= \delta\underline{v}\cdot\left(\mathbb{E}\left[\sqrt{[Q_2^*(v_1)]^2}\right] - \sqrt{\mathbb{E}[Q_2^*(v_1)^2]}\right) \\
&\leq 0. && \text{by Jensen's inequality}
\end{aligned}
$$

Hence, the firm suffers from improving α, when $\rho[\theta_1(v_1), Q_2^*(v_1)] \leq \rho^\dagger$.

The case for γ follows the same line of argument. ■

Appendix C

Appendix for Chapter 4

Proof of Proposition 4.1

The firm can maximize and extract total efficiency for each salesperson

$$\max_{\phi} \mathbb{E}\Big\{ \sum_{t=1}^{T} \delta^t \Big[-\tfrac{1}{2}\big[\xi_t(\theta_{t-1})\big]^2 + q_t(\theta_t) + \omega_t - \tfrac{1}{2}\big[q_t(\theta_t) - \theta_t\big]^2 \Big] \Big\},$$

subject to (COM_t) for all t.

(a) There is no intertemporal tradesoff to determine the quota and compensation. For each θ_t in period t, first order condition ensures that

$$1 - (\bar{q}_t(\theta_t) - \theta_t) = 0.$$

Therefore, I can get $\bar{q}_t(\theta_t) = 1 + \theta_t$. Under (COM_t), given the quota-commission scheme, I can apply the envelope theorem (Milgrom and Segal, 2002) to have

$$\frac{\partial}{\partial e_t}\Big[\bar{A}_t(\theta^t) + \bar{B}_t(\theta_t)\cdot(\theta_t + e_t + \omega_t - \bar{q}_t(\theta_t)) - \tfrac{1}{2}e_t^2\Big]\Big|_{e_t = \bar{q}_t(\theta_t) - \theta_t} = \bar{B}_t(\theta_t) - (\bar{q}_t(\theta_t) - \theta_t) = 0.$$

Therefore, commission rate $\bar{B}_t(\theta_t) = 1$. In addition, the firm can extract all the efficiency in each period, given the salesperson's sequential individual rationality constraints (IR_t), the firm can just pay the salesperson his effort costs and learning investment to extract all the surplus and ensure the salesperson's acceptance. Hence

$$\mathbb{E}_{\omega}\Big[\bar{A}_t(\theta^t) - \bar{B}_t(\theta_t)\cdot(s_t - \bar{q}_t(\theta_t)) - \tfrac{1}{2}\big[\bar{q}_t(\theta_t) - \theta_t\big]^2 - \tfrac{1}{2}\big[\bar{\xi}_t(\theta_{t-1})\big]^2\Big] = \bar{A}_t(\theta^t) - \tfrac{1}{2} - \tfrac{1}{2}\big[\bar{\xi}_t(\theta_{t-1})\big]^2 = 0.$$

Therefore, the base salary $\bar{A}_t(\theta^t) = \frac{1}{2} + \frac{1}{2}[\bar{\xi}_t(\theta_{t-1})]^2$.

(b) The firm determines the training level by trading off the current skills investment cost and intertemporal continuation profit affected by skills investment. First order condition yields

$$
\begin{aligned}
\bar{\xi}_{t+1}(\theta_t) =& \frac{\partial}{\partial \xi_{t+1}} \mathbb{E}\left\{ \sum_{k=t+1}^{T} \delta^{k-t-1}\Big[\bar{q}_k(\theta_k) + \omega_k - \tfrac{1}{2}[\bar{q}_k(\theta_k) - \theta_k]^2 - \delta\frac{1}{2}[\xi_{k+1}(\theta_k)]^2\Big] \Big| \theta_t, \bar{\xi}_{t+1} \right\} \\
=& \frac{\partial \theta_{t+1}}{\partial \xi_{t+1}} \cdot \frac{\partial}{\partial \theta_{t+1}} \mathbb{E}\left\{ \sum_{k=t+1}^{T} \delta^{k-t-1}\Big[\bar{q}_k(\theta_k) + \omega_k - \tfrac{1}{2}[\bar{q}_k(\theta_k) - \theta_k]^2 - \delta\frac{1}{2}[\xi_{k+1}(\theta_k)]^2\Big] \Big| \theta_t, \bar{\xi}_{t+1} \right\} \\
=& \frac{\partial \theta_{t+1}}{\partial \xi_{t+1}} \cdot \frac{\partial}{\partial \theta_{t+1}} \mathbb{E}\Big[\bar{q}_{t+1}(\theta_{t+1}) + \omega_{t+1} - \tfrac{1}{2}[\bar{q}_{t+1}(\theta_{t+1}) - \theta_{t+1}]^2 - \delta\frac{1}{2}[\xi_{t+2}(\theta_{t+1})]^2 \Big| \theta_t, \bar{\xi}_{t+1}\Big] \\
&+ \frac{\partial \theta_{t+1}}{\partial \xi_{t+1}} \cdot \frac{\partial \theta_{t+2}}{\partial \theta_{t+1}} \cdot \frac{\partial}{\partial \theta_{t+2}} \delta \mathbb{E}\Big[\bar{q}_{t+2}(\theta_{t+2}) + \omega_{t+2} - \tfrac{1}{2}[\bar{q}_{t+2}(\theta_{t+2}) - \theta_{t+2}]^2 - \delta\frac{1}{2}[\xi_{t+3}(\theta_{t+2})]^2 \Big| \theta_t, \bar{\xi}_{t+1}\Big] \\
&\cdots \\
&+ \frac{\partial \theta_{t+1}}{\partial \xi_{t+1}} \cdot \frac{\partial \theta_{t+2}}{\partial \theta_{t+1}} \cdots \frac{\partial \theta_T}{\partial \theta_{T-1}} \cdot \frac{\partial}{\partial \theta_T} \delta^{T-t-1} \mathbb{E}\Big[\bar{q}_T(\theta_T) + \omega_T - \tfrac{1}{2}[\bar{q}_T(\theta_T) - \theta_T]^2 \Big| \theta_t, \bar{\xi}_{t+1}\Big] \\
=& \mathbb{E}\Big[\beta[\bar{q}_{t+1}(\theta_{t+1}) - \theta_{t+1}] + \beta\alpha\delta[\bar{q}_{t+2}(\theta_{t+2}) - \theta_{t+2}] + \cdots + \beta\alpha^{T-t-1}\delta^{T-t-1}[\bar{q}_T(\theta_T) - \theta_T]\Big] \\
=& \beta\mathbb{E} \sum_{k=t+1}^{T} \Big[\delta^{k-t-1} \cdot \alpha^{k-t-1}\Big] \\
=& \beta\frac{1-(\delta\alpha)^{T2-t}}{1-\delta\alpha}.
\end{aligned}
$$

Note that, $\bar{\xi}_{t+1}(\theta_t)$ is decreasing over time index t:

$$\frac{\partial}{\partial t}\beta\frac{1-(\delta\alpha)^{T-t}}{1-\delta\alpha} = \frac{\beta}{1-\delta\alpha}[(\delta\alpha)^{T-t} \cdot \ln(\delta\alpha)] < 0.$$

■

Lemma C.1 and the Proof

I need the following technical lemma for establishing main results

Lemma C.1 *Suppose random variable θ with the support $\Theta \equiv [\underline{\theta}, \overline{\theta}]$ has distribution F and density f. Denote the inverse hazard ratio $\eta(\theta) \equiv \frac{1-F(\theta)}{f(\theta)}$. If the function $V : \Theta \to \mathbb{R}$ is absolutely continuous, then*

$$\mathbb{E}^{\theta}[V(\theta)] = V(\underline{\theta}) + \mathbb{E}^{\theta}[\eta(\theta)V'(\theta)].$$

Proof:

Since both F and V are absolutely continuous on a compact set Θ, the function $(1-F)\cdot V : \Theta \to \mathbb{R}$ is also absolutely continuous and differentiable almost everywhere. I have

$$\begin{aligned}
\mathbb{E}^{\theta}[\eta(\theta)V'(\theta)] - \mathbb{E}^{\theta}[V(\theta)] &= \int_{\underline{\theta}}^{\overline{\theta}} [1 - F(\theta)]V' \, \mathrm{d}\theta - \int_{\underline{\theta}}^{\overline{\theta}} V(\theta) f(\theta) \, \mathrm{d}\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \big((1 - F(\theta) \cdot V(\theta))\big)' \, \mathrm{d}\theta && \text{(By Chain Rule)} \\
&= (1 - F(\theta) \cdot V(\theta))\Big|_{\underline{\theta}}^{\overline{\theta}} \\
&= -V(\underline{\theta}). && \text{(by } F(\overline{\theta}) = 1 \text{ and } F(\underline{\theta}) = 0\text{)}
\end{aligned}$$

■

Lemma C.2 and the Proof

Lemma C.2 *For any monotone quota $q_t(\theta^t)$, the sequantial incentive compatibility constraints (IC_t) implies*

$$\frac{\partial}{\partial \theta_t} U_t(\theta^t) = \sum_{k=t}^{T} \delta^{k-t} \alpha^{k-t} \mathbb{E}[q_k(\theta^k) - \theta_k].$$

Moreover, the conditions can be formulated as The two conditions can also be formulated as

$$U_t(\theta_t) = U_t(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_t} (q_t(\tilde{\theta}^t) - \tilde{\theta}_t) \, \mathrm{d}\tilde{\theta}_t + \int_{\underline{\theta}}^{\theta_t} \sum_{k=t+1}^{T} \delta^{k-t} \alpha^{k-t} \mathbb{E}[q_k(\tilde{\theta}^k) - \theta_k] \cdot \mathrm{d}\tilde{\theta}_t.$$

Proof:

I prove it by backward induction. In the last period, the problem reduces to a static one. (IC_T) requires that

$$U_T(\theta^{T-1}, \theta_T) = \max_{\hat{\theta}_T} \tilde{U}_T(\theta^{T-1}, \hat{\theta}_T; \theta_T) = \max_{\hat{\theta}_T} \{\mathbb{E}[x_T(\hat{\theta}^T), s_T)] - \tfrac{1}{2}[q_T(\hat{\theta}^T) - \theta_T]^2\},$$

which implies

$$\tfrac{\partial}{\partial \theta_T} U_T(\theta^T) = \tfrac{\partial}{\partial \theta_T} \tilde{U}_T(\theta^{T-1}, \hat{\theta}_T; \theta_T)\big|_{\hat{\theta}_T = \theta_T} = q_T(\theta^T) - \theta_T.$$

by envelope theorem (Milgrom and Segal, 2002). Assume $\frac{\partial}{\partial \theta_{t+1}} U_{t+1}(\theta^{t+1}) = \sum_{k=t+1}^{T} \delta^{k-t-1} \alpha^{k-t-1} \mathbb{E}[q_k(\theta^k) - \theta_k]$ holds for period $t+1$. Then in period t, IC_t requires that

$$U_t(\theta^t) = \max_{\hat{\theta}_t} \tilde{U}_t(\theta^{t-1}, \hat{\theta}_t; \theta_t) = \max_{\hat{\theta}_t} \{\mathbb{E}[x_t(\hat{\theta}^t), s_t)] - \tfrac{1}{2}[q_t(\hat{\theta}^t) - \theta_t]^2 + \delta[-\tfrac{1}{2}\xi_{t+1}^2(\hat{\theta}) + \mathbb{E}[U_{t+1}(\theta^{t+1}) \mid \theta_t, \xi_{t+1}(\hat{\theta}_t)]]\}.$$

The envelope theorem and chain rule imply that

$$\begin{aligned}
\frac{\partial}{\partial \theta_t} U_t(\theta^t) &= q_t(\theta^t) - \theta_t + \delta \frac{\partial}{\partial \theta_t} \mathbb{E}[U_{t+1}(\theta^{t+1})] && \text{(by envelope theorem)} \\
&= q_t(\theta^t) - \theta_t + \delta \mathbb{E}\left[\frac{\partial \theta_{t+1}}{\partial \theta_t} \cdot \frac{\partial}{\partial \theta_{t+1}} U_{t+1}(\theta^{t+1})\right] && \text{(by chain rule)} \\
&= q_t(\theta^t) - \theta_t + \delta\alpha \mathbb{E}\left[\sum_{k=t+1}^{T} \delta^{k-t-1} \alpha^{k-t-1} [q_k(\theta^k) - \theta_k]\right] \\
&= \sum_{k=t}^{T} \delta^{k-t} \alpha^{k-t} \mathbb{E}[q_k(\theta^k) - \theta_k].
\end{aligned}$$

This finishes the proof through backward induction. For all t, take integral from $\underline{\theta}$ to θ_t on both sides, I get

$$U_t(\theta_t) = U_t(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_t} (q_t(\tilde{\theta}^t) - \tilde{\theta}_t) \, \mathrm{d}\tilde{\theta}_t + \int_{\underline{\theta}}^{\theta_t} \sum_{k=t+1}^{T} \delta^{k-t} \alpha^{k-t} \mathbb{E}[q_k(\tilde{\theta}^k) - \theta_k] \cdot \mathrm{d}\tilde{\theta}_t.$$

■

Proof of Proposition 4.2

I first reformulate the firm's objective function under $\check{\mathcal{P}}$:

$$\begin{aligned}
\Pi_0 =& \mathbb{E}\left\{\sum_{t=1}^{T} \delta^t \Big[-\tfrac{1}{2}[\xi_t(\theta^{t-1})]^2 + q_t(\theta^t) + \omega_t - \tfrac{1}{2}[q_t(\theta^t) - \theta_t]^2\Big]\right\} - \mathbb{E}U_0(\theta_0) \\
=& \mathbb{E}\left\{\sum_{t=1}^{T} \delta^t \Big[-\tfrac{1}{2}[\xi_t(\theta^{t-1})]^2 + q_t(\theta^t) + \omega_t - \tfrac{1}{2}[q_t(\theta^t) - \theta_t]^2\Big]\right\} - \mathbb{E}[\eta(\theta_0) \tfrac{\partial}{\partial \theta_0} U_0(\theta_0)] - U_0(\underline{\theta}) \quad \text{(by Lemma C.1)} \\
=& \mathbb{E}\left\{\sum_{t=1}^{T} \delta^t \Big[-\tfrac{1}{2}[\xi_t(\theta^{t-1})]^2 + q_t(\theta^t) + \omega_t - \tfrac{1}{2}[q_t(\theta^t) - \theta_t]^2\Big]\right\} - \mathbb{E}\left[\eta(\theta_0) \sum_{t=1}^{T} \delta^t \alpha^t (q_t(\theta^t) - \theta_t)\right] - U_0(\underline{\theta}) \\
& \text{(by Lemma C.2)} \\
=& \mathbb{E}\left\{\sum_{t=1}^{T} \delta^t \Big[-\tfrac{1}{2}[\xi_t(\theta^{t-1})]^2 + q_t(\theta^t) - \eta(\theta_0)\alpha^t \cdot (q_t(\theta^t) - \theta_t) + \omega_t - \tfrac{1}{2}[q_t(\theta^t) - \theta_t]^2\Big]\right\} - U_0(\underline{\theta}).
\end{aligned}$$

The firm's problem becomes virtual efficiency maximization:

$$\max_{\xi_t, q_t, U_0(\underline{\theta})} \mathbb{E}\left\{\sum_{t=1}^{T} \delta^t \Big[-\tfrac{1}{2}[\xi_t(\theta^{t-1})]^2 + q_t(\theta^t) - \eta(\theta_0)\alpha^t \cdot (q_t(\theta^t) - \theta_t) + \omega_t - \tfrac{1}{2}[q_t(\theta^t) - \theta_t]^2\Big]\right\} - U_0(\underline{\theta}),$$

subject to (COM_t), (IR_t), where $\eta(\theta_0) = \frac{1-F(\theta_0)}{f(\theta_0)}$. I can take $U_t(\underline{\theta}) = 0$ for all the (IR_t) binds. The related decision variable in (COM_t) don't enter the objective function. I can enforce (COM_t) through appropriate design for x_t.

(a) I will use the pairwise maximization to solve the optimal policy for any given θ^t. The Hessian of the objective function is

$$\begin{pmatrix} \frac{\partial^2}{\partial (q_t)^2} & \frac{\partial^2}{\partial q_t \partial \xi_{t+1}} \\ \frac{\partial^2}{\partial q_t \partial \xi_{t+1}} & \frac{\partial^2}{\partial (\xi_{t+1})^2} \end{pmatrix} \Pi_0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Hence the objective function is strictly concave in (q_t, ξ_{t+1}). By first order condition on $q_t(\theta^t)$, I can have

$$1 - \eta(\theta_0)\alpha^t - (\check{q}_t(\theta^t) - \theta_t) = 0 \quad \Rightarrow \quad \check{q}_t(\theta^t) = \theta_t + 1 - \eta(\theta_0)\alpha^t.$$

I then derive the optimal commission rate $\check{B}_t(\theta^t)$ and base salary $\check{A}_t(\theta^t)$. Under (COM_t), I can apply the envelope theorem (Milgrom and Segal, 2002) to have

$$\frac{\partial}{\partial e_t}\big[\check{A}_t(\theta^t) + \check{B}_t(\theta^t)\cdot(\theta_t + e_t + \omega_t - \check{q}_t(\theta^t)) - \tfrac{1}{2}e_t^2\big]\Big|_{e_t = \check{q}_t(\theta_t) - \theta_t} = \check{B}_t(\theta^t) - (\check{q}_t(\theta^t) - \theta_t) = 0.$$

Therefore, commission rate $\check{B}_t(\theta^t) = \check{q}_t(\theta^t) - \theta_t$. In addition, ($IC_t$) requires that the salesperson's rent in period t as

$$U_t(\theta_t) = U_t(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_t} (q_t(\tilde{\theta}^t) - \tilde{\theta}_t)\, \mathrm{d}\tilde{\theta}_t + \int_{\underline{\theta}}^{\theta_t} \sum_{k=t+1}^{T} \delta^{k-t}\alpha^{k-t}\mathbb{E}[q_k(\tilde{\theta}^k) - \theta_k]\cdot \mathrm{d}\tilde{\theta}_t.$$

Hence in last period T,

$$\mathbb{E}_\omega\big[\check{A}_T(\theta^T) - \check{B}_T(\theta^T)\cdot(s_T - \check{q}_T(\theta^T)) - \tfrac{1}{2}[\check{q}_T(\theta^T) - \theta_T]^2 = U_T(\theta^T).$$

Therefore, the base salary $\check{A}_T(\theta^T) = U_T(\theta^T) + \frac{1}{2}[\check{q}_T(\theta^T) - \theta_T]^2$. For all $T > t \geq 2$,

$$\begin{aligned}\mathbb{E}_\omega\big[\check{A}_t(\theta^t) - \check{B}_t(\theta^t)\cdot(s_t - \check{q}_t(\theta^t)) - \tfrac{1}{2}[\check{q}_t(\theta^t) - \theta_t]^2 - \delta\tfrac{1}{2}[\check{\xi}_{t+1}(\theta_t)]^2 + \delta\mathbb{E}[U_{t+1}(\theta^{t+1}) \mid \theta_t, \check{\xi}_{t+1}(\theta^t)] \\ = \check{A}_t(\theta^t) - \tfrac{1}{2}[\check{q}_t(\theta^t) - \theta_t]^2 - \delta\tfrac{1}{2}[\check{\xi}_{t+1}(\theta_t)]^2 + \delta\mathbb{E}[U_{t+1}(\theta^{t+1}) \mid \theta_t, \check{\xi}_{t+1}(\theta^t)] = U_t(\theta^t).\end{aligned}$$

Therefore, the base salary $\check{A}_t(\theta^t) = U_t(\theta^t) + \frac{1}{2}[\check{q}_t(\theta^t) - \theta_t]^2 - \delta\frac{1}{2}[\check{\xi}_{t+1}(\theta_t)]^2 - \delta\mathbb{E}[U_{t+1}(\theta^{t+1}) \mid \theta_t, \check{\xi}_{t+1}(\theta^t)]$. Lastly, in initial period $t = 1$, the salary should accommodate the initial training investment. I should have

$$\begin{aligned}-\tfrac{1}{2}[\check{\xi}_1(\theta_0)]^2 + \mathbb{E}_\omega\big[\check{A}_1(\theta^1) - \check{B}_1(\theta^1)\cdot(s_1 - \check{q}_1(\theta^1)) - \tfrac{1}{2}[\check{q}_1(\theta^1) - \theta_1]^2 - \delta\tfrac{1}{2}[\check{\xi}_2(\theta_1)]^2 + \delta\mathbb{E}[U_2(\theta^2) \mid \theta_1, \check{\xi}_2(\theta^1)] \\ = -\tfrac{1}{2}[\check{\xi}_1(\theta_0)]^2 + \check{A}_1(\theta^1) - \tfrac{1}{2}[\check{q}_1(\theta^1) - \theta_1]^2 - \delta\tfrac{1}{2}[\check{\xi}_2(\theta_1)]^2 + \delta\mathbb{E}[U_2(\theta^2) \mid \theta_1, \check{\xi}_2(\theta^1)] = U_1(\theta^1).\end{aligned}$$

Therefore, the base salary $\check{A}_1(\theta^1) = \frac{1}{2}[\check{\xi}_1(\theta_0)]^2 + U_1(\theta^1) + \frac{1}{2}[\check{q}_1(\theta^1) - \theta_1]^2 - \delta\frac{1}{2}[\check{\xi}_2(\theta_1)]^2 - \delta\mathbb{E}[U_2(\theta^2) \mid \theta_1, \check{\xi}_2(\theta^1)]$.

(b) By the first order condition on $\xi_{t+1}(\theta_t)$, I have

$$
\begin{aligned}
\check{\xi}_{t+1}(\theta^t) =& \frac{\partial}{\partial \xi_{t+1}} \mathbb{E}\left\{ \sum_{k=t+1}^{T} \delta^{k-t-1}\Big[\check{q}_k(\theta^k) - \eta(\theta_0)\alpha^k(\check{q}_k(\theta^k) - \theta_k) - \tfrac{1}{2}[\check{q}_k(\theta^k) - \theta_k]^2 - \delta\frac{1}{2}[\xi_{k+1}(\theta^k)]^2\Big]\Big| \theta_t, \check{\xi}_{t+1} \right\} \\
=& \frac{\partial \theta_{t+1}}{\partial \xi_{t+1}} \cdot \frac{\partial}{\partial \theta_{t+1}} \mathbb{E}\left\{ \sum_{k=t+1}^{T} \delta^{k-t-1}\Big[\check{q}_k(\theta^k) - \eta(\theta_0)\alpha^k(\check{q}_k(\theta^k) - \theta_k) - \tfrac{1}{2}[\check{q}_k(\theta^k) - \theta_k]^2 - \delta\frac{1}{2}[\xi_{k+1}(\theta^k)]^2\Big]\Big| \theta_t, \check{\xi}_{t+1} \right\} \\
=& \frac{\partial \theta_{t+1}}{\partial \xi_{t+1}} \cdot \frac{\partial}{\partial \theta_{t+1}} \mathbb{E}\Big[\check{q}_{t+1}(\theta^{t+1}) - \eta(\theta_0)\alpha^{t+1}(\check{q}_{t+1}(\theta^{t+1}) - \theta_{t+1}) - \tfrac{1}{2}[\check{q}_{t+1}(\theta^{t+1}) - \theta_{t+1}]^2 - \delta\frac{1}{2}[\xi_{t+2}(\theta^{t+1})]^2 \Big| \theta_t, \check{\xi}_{t+1}\Big] \\
&+ \frac{\partial \theta_{t+1}}{\partial \xi_{t+1}} \cdot \frac{\partial \theta_{t+2}}{\partial \theta_{t+1}} \cdot \frac{\partial}{\partial \theta_{t+2}} \delta\mathbb{E}\Big[\check{q}_{t+2}(\theta^{t+2}) - \eta(\theta_0)\alpha^{t+2}(\check{q}_{t+2}(\theta^{t+2}) - \theta_{t+2}) - \tfrac{1}{2}[\check{q}_{t+2}(\theta^{t+2}) - \theta_{t+2}]^2 - \delta\frac{1}{2}[\xi_{t+3}(\theta^{t+2})]^2 \Big| \theta_t, \check{\xi}_{t+1}\Big] \\
&\ldots \\
&+ \frac{\partial \theta_{t+1}}{\partial \xi_{t+1}} \cdot \frac{\partial \theta_{t+2}}{\partial \theta_{t+1}} \cdots \frac{\partial \theta_T}{\partial \theta_{T-1}} \cdot \frac{\partial}{\partial \theta_T} \delta^{T-t-1}\mathbb{E}\Big[\check{q}_T(\theta^T) - \eta(\theta_0)\alpha^T(\check{q}_T(\theta^T) - \theta_T) - \tfrac{1}{2}[\check{q}_T(\theta^T) - \theta_T]^2 \Big| \theta_t, \check{\xi}_{t+1}\Big] \\
=& \mathbb{E}\Big[\beta[\check{q}_{t+1}(\theta^{t+1}) - \theta_{t+1} + \eta(\theta_0)\alpha^{t+1}] + \beta\alpha\delta[\check{q}_{t+2}(\theta^{t+2}) - \theta_{t+2} + \eta(\theta_0)\alpha^{t+2}] + \cdots + \beta\alpha^{T-t-1}\delta^{T-t-1}[\check{q}_T(\theta^T) - \theta_T + \eta(\theta_0)\alpha^T]\Big] \\
=& \beta\mathbb{E}\sum_{k=t+1}^{T}\left[\delta^{k-t-1}\cdot\alpha^{k-t-1}\right] \\
=& \beta\frac{1-(\delta\alpha)^{T-t}}{1-\delta\alpha}.
\end{aligned}
$$

■

Proof of Proposition 4.3

I first reformulate the firm's objective function under $\mathcal{P}$:

$$
\begin{aligned}
\Pi_0 =& \mathbb{E}\left\{ \sum_{t=1}^{T} \delta^t\Big[-\tfrac{1}{2}[\xi_t(\theta^{t-1})]^2 + q_t(\theta^t) + \omega_t - \tfrac{1}{2}[q_t(\theta^t) - \theta_t]^2\Big] \right\} - \mathbb{E}U_0(\theta_0) \\
=& \mathbb{E}\left\{ \sum_{t=1}^{T} \delta^t\Big[-\tfrac{1}{2}[\xi_t(\theta^{t-1})]^2 + q_t(\theta^t) + \omega_t - \tfrac{1}{2}[q_t(\theta^t) - \theta_t]^2\Big] \right\} - \mathbb{E}[\eta(\theta_0)\tfrac{\partial}{\partial\theta_0}U_0(\theta_0)] - U_0(\underline{\theta}) \quad \text{(by Lemma C.1)} \\
=& \mathbb{E}\left\{ \sum_{t=1}^{T} \delta^t\Big[-\tfrac{1}{2}[\xi_t(\theta^{t-1})]^2 + q_t(\theta^t) + \omega_t - \tfrac{1}{2}[q_t(\theta^t) - \theta_t]^2\Big] \right\} - \mathbb{E}\left[\eta(\theta_0)\sum_{t=1}^{T}\delta^t\alpha^t(q_t(\theta^t) - \theta_t)\right] - U_0(\underline{\theta}) \\
& \text{(by Lemma C.2)} \\
=& \mathbb{E}\left\{ \sum_{t=1}^{T} \delta^t\Big[-\tfrac{1}{2}[\xi_t(\theta^{t-1})]^2 + q_t(\theta^t) - \eta(\theta_0)\alpha^t\cdot(q_t(\theta^t) - \theta_t) + \omega_t - \tfrac{1}{2}[q_t(\theta^t) - \theta_t]^2\Big] \right\} - U_0(\underline{\theta}).
\end{aligned}
$$

Denote $e_t(\theta^t) = q_t(\theta^t) - \theta_t$. The linear transformation accommodates the search on optimal q_t to optimal e_t. The firm's problem becomes virtual efficiency maximization:

$$
\max_{\xi_t, e_t, U_0(\underline{\theta})} \mathbb{E}\left\{ \sum_{t=1}^{T} \delta^t\Big[-\tfrac{1}{2}[\xi_t(\theta^{t-1})]^2 + e_t(\theta^t) + \theta_t - \eta(\theta_0)\alpha^t\cdot e_t(\theta^t) + \omega_t - \tfrac{1}{2}[e_t(\theta^t)]^2\Big] \right\} - U_0(\underline{\theta}),
$$

subject to (COM_t), (OB_t), (IR_t), where $\eta(\theta_0) = \frac{1-F(\theta_0)}{f(\theta_0)}$. I can take $U_t(\underline{\theta}) = 0$ for all the (IR_t) binds. The related decision variable in (COM_t) don't enter the objective function. I can enforce (COM_t) through appropriate design for x_t.

(a) By changing of variables, I can express the (OB_t) as

$$\xi_t(\theta_{t-1}) = \beta\mathbb{E}\left[\sum_{k=t}^{T}\delta^{k-t}\alpha^{k-t}e_k(\theta^k)\right].$$

I then replace all the $\{\xi_t(\theta_{t-1})\}_{t\geq 1}$ in the objective function and solve for optimal $\{e_t(\theta_t)\}_{t\geq 1}$ backward. I claim that all $\{e_t(\theta^t)\}_{t\geq 1}$ only contingent on θ_0. I proceed it backward. In period T, firm's problem is

$$\max_{e_T}\mathbb{E}[-\tfrac{1}{2}(\mathbb{E}[\beta e_T(\theta^T)])^2 + e_T(\theta^T) + \theta_T - \eta(\theta_0)\alpha^T\cdot e_T(\theta^T) - \tfrac{1}{2}(e_T(\theta^T))^2].$$

By pointwise optimization over θ_T, first order condition delivers

$$-\tfrac{1}{2}\frac{\partial}{\partial e_T}(\mathbb{E}[\beta e_T^*(\theta^T)])^2 + 1 - \eta(\theta_0)\alpha^T - e_T^*(\theta^T) = 0.$$

Note that the equation in the first order condition is irrelevant to θ_T. I claim that the optimal $e_T(\theta^T)$ is irrelevant to θ_T: $\mathbb{E}[\beta e_T(\theta^T)] = \beta e_T(\theta^T)$. Hence the first order condition becomes

$$-\beta^2 e_T^*(\theta^T) + 1 - \eta(\theta_0)\alpha^T - e_T^*(\theta^T) = 0 \quad\Rightarrow\quad e_T^*(\theta^T) = \frac{1-\eta(\theta_0)\alpha^T}{1+\beta^2}.$$

I go back to period $T-1$. By point optimization over θ_{T-1}, first order condition delivers

$$-\tfrac{1}{2}\frac{\partial}{\partial e_{T-1}}(\beta\mathbb{E}[e_{T-1}^*(\theta^{T-1}) + \delta\alpha e_T^*(\theta^T)])^2 + 1 - \eta(\theta_0)\alpha^{T-1} - e_{T-1}^*(\theta^{T-1}) = 0.$$

Note that $e_T^*(\theta^T)$ is only contingent on θ_0, which is irrelevant to θ_{T-1}. Along with the first order condition equation is irrelevant to θ_{T-1}, $\beta\mathbb{E}[e_{T-1}^*(\theta^{T-1}) + \delta\alpha e_T^*(\theta^T)] = \beta[e_{T-1}^*(\theta^{T-1}) + \delta\alpha e_T^*(\theta^T)]$. Therefore, the first order condition reduces to

$$-\beta^2[e_{T-1}^*(\theta^{T-1}) + \delta\alpha e_T^*(\theta^T)] + 1 - \eta(\theta_0)\alpha^{T-1} - e_{T-1}^*(\theta^{T-1}) = 0 \quad\Rightarrow\quad e_{T-1}^*(\theta^t) = \frac{1-\eta(\theta_0)\alpha^{T-1} - \delta\alpha\beta^2 e_T^*(\theta^T)}{1+\beta^2}.$$

For any period t, the first order condition delivers

$$-\tfrac{1}{2}\frac{\partial}{\partial e_t}\left(\beta\mathbb{E}[e_t^*(\theta^t) + \sum_{k=t+1}^{T}(\delta\alpha)^{k-t}e_k^*(\theta^k)]\right)^2 + 1 - \eta(\theta_0)\alpha^t - e_t^*(\theta^t) = 0.$$

Suppose all the $\{e_k^*(\theta^k)\}_{k\geq t+1}$ are only contingent on θ_0. Along with the first order condition equation is irrelevant to θ_t, I have $\beta\mathbb{E}[e_t^*(\theta^t)+\sum_{k=t+1}^T(\delta\alpha)^{k-t}e_k^*(\theta^k)] = \beta[e_t^*(\theta^t)+\sum_{k=t+1}^T(\delta\alpha)^{k-t}e_k^*(\theta^k)]$. The first order condition is then

$$-\beta^2[e_t^*(\theta^t)+\sum_{k=t+1}^T(\delta\alpha)^{k-t}e_k^*(\theta^k)]+1-\eta(\theta_0)\alpha^t-e_t^*(\theta^t)=0 \quad\Rightarrow\quad e_t^*(\theta^t)=\frac{1-\eta(\theta_0)\alpha^t-\beta^2\sum_{k=t+1}^T(\delta\alpha)^{k-t}e_k^*(\theta^k)}{1+\beta^2}.$$

The same argument proceeds backward for all the e_t^*. I can write the Euler equation as

$$e_t^*(\theta^t)-\frac{1-\eta(\theta_0)\alpha^t}{1+\beta^2}=\delta\alpha\left[\frac{e_{t+1}^*(\theta^{t+1})}{1+\beta^2}-\frac{1-\eta(\theta_0)\alpha^{t+1}}{1+\beta^2}\right],\quad \forall t\geq 1.$$

In addition,

$$\begin{aligned}
e_T^*(\theta^T)&=1-\eta(\theta_0)\alpha^T-\frac{\beta^2}{1+\beta^2}[1-\eta(\theta_0)\alpha^T],\\
e_{T-1}^*(\theta^{T-1})&=1-\eta(\theta_0)\alpha^{T-1}-\left[\frac{\beta^2}{1+\beta^2}[1-\eta(\theta_0)\alpha^{T-1}]+\frac{\delta\alpha}{1+\beta^2}\frac{\beta^2}{1+\beta^2}[1-\eta(\theta_0)\alpha^T]\right],\\
e_{T-2}^*(\theta^{T-2})&=1-\eta(\theta_0)\alpha^{T-2}-\left[\frac{\beta^2}{1+\beta^2}[1-\eta(\theta_0)\alpha^{T-2}]+\frac{\delta\alpha}{1+\beta^2}\frac{\beta^2}{1+\beta^2}[1-\eta(\theta_0)\alpha^{T-1}]+\left(\frac{\delta\alpha}{1+\beta^2}\right)^2\frac{\beta^2}{1+\beta^2}[1-\eta(\theta_0)\alpha^T]\right],\\
&\dots\\
e_t^*(\theta^t)&=1-\eta(\theta_0)\alpha^t-\sum_{k=t}^T\left[\left(\frac{\delta\alpha}{1+\beta^2}\right)^{k-t}\frac{\beta^2}{1+\beta^2}(1-\eta(\theta_0)\alpha^t)\right].
\end{aligned}$$

I can then express

$$e_t^*(\theta^t)=1-\eta(\theta_0)\alpha^t-\left\{\frac{\beta^2}{1+\beta^2-\delta\alpha}\left[1-\left(\frac{\delta\alpha}{1+\beta^2}\right)^{T-t+1}\right]-\frac{\beta^2}{1+\beta^2-\delta\alpha^2}\left[1-\left(\frac{\delta\alpha^2}{1+\beta^2}\right)^{T-t+1}\right]\eta(\theta_0)\alpha^t\right\}.$$

Therefore, the quota

$$q_t^*(\theta^t)=\theta_t+1-\eta(\theta_0)\alpha^t-\left\{\frac{\beta^2}{1+\beta^2-\delta\alpha}\left[1-\left(\frac{\delta\alpha}{1+\beta^2}\right)^{T-t+1}\right]-\frac{\beta^2}{1+\beta^2-\delta\alpha^2}\left[1-\left(\frac{\delta\alpha^2}{1+\beta^2}\right)^{T-t+1}\right]\eta(\theta_0)\alpha^t\right\}.$$

Note that $q_t^*(\theta)$ is increasing in t for any $\theta\in[\underline{\theta},\overline{\theta}]$:

$$\begin{aligned}
\frac{\partial}{\partial t}q_t^*(\theta)=&-\eta(\theta_0)\alpha^t\ln\alpha\left[1-\frac{\beta^2}{1+\beta^2-\delta\alpha^2}\left[1-\left(\frac{\delta\alpha^2}{1+\beta^2}\right)^{T-t+1}\right]\right]-\frac{\beta^2}{1+\beta^2-\delta\alpha}\left(\frac{\delta\alpha}{1+\beta^2}\right)^{T-t+1}\ln\frac{\delta\alpha}{1+\beta^2}\\
&+\frac{\beta^2}{1+\beta^2-\delta\alpha^2}\left(\frac{\delta\alpha^2}{1+\beta^2}\right)^{T-t+1}\eta(\theta_0)\alpha^t\ln\frac{\delta\alpha^2}{1+\beta^2}\\
=&\underbrace{-\eta(\theta_0)\alpha^t\ln\alpha\frac{1-\delta\alpha^2}{1+\beta^2-\delta\alpha^2}}_{>0}-\left[\frac{\beta^2}{1+\beta^2-\delta\alpha}\left(\frac{\delta\alpha}{1+\beta^2}\right)^{T-t+1}-\frac{\beta^2}{1+\beta^2-\delta\alpha^2}\left(\frac{\delta\alpha^2}{1+\beta^2}\right)^{T-t+1}\eta(\theta_0)\alpha^t\right]\underbrace{\ln\frac{\delta\alpha^2}{1+\beta^2}}_{<0}\\
>&\underbrace{-\eta(\theta_0)\alpha^t\ln\alpha\frac{1-\delta\alpha^2}{1+\beta^2-\delta\alpha^2}}_{>0}-\underbrace{\frac{\beta^2}{1+\beta^2-\delta\alpha^2}\left(\frac{\delta\alpha^2}{1+\beta^2}\right)^{T-t+1}[1-\eta(\theta_0)\alpha^t]}_{>0}\underbrace{\ln\frac{\delta\alpha^2}{1+\beta^2}}_{<0}\\
>&0,
\end{aligned}$$

where the first inequality follows by

$$\frac{\beta^2}{1+\beta^2-\delta\alpha}\left(\frac{\delta\alpha}{1+\beta^2}\right)^{T-t+1} > \frac{\beta^2}{1+\beta^2-\delta\alpha^2}\left(\frac{\delta\alpha}{1+\beta^2}\right)^{T-t+1} > \frac{\beta^2}{1+\beta^2-\delta\alpha^2}\left(\frac{\delta\alpha^2}{1+\beta^2}\right)^{T-t+1}.$$

I then derive the optimal commission rate $B_t^*(\theta^t)$ and base salary $A_t^*(\theta^t)$. Under (COM_t), I can apply the envelope theorem (Milgrom and Segal, 2002) to have

$$\frac{\partial}{\partial e_t}\left[A_t^*(\theta^t) + B_t^*(\theta^t)\cdot(\theta_t + e_t + \omega_t - q_t^*(\theta^t)) - \tfrac{1}{2}e_t^2\right]\Big|_{e_t = q_t^*(\theta_t)-\theta_t} = B_t^*(\theta^t) - (q_t^*(\theta^t) - \theta_t) = 0.$$

Therefore, commission rate $B_t^*(\theta^t) = q_t^*(\theta^t) - \theta_t$. In addition, (IC_t) requires that the salesperson's rent in period t as

$$U_t(\theta_t) = U_t(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_t}(q_t(\tilde{\theta}^t) - \tilde{\theta}_t)\,\mathrm{d}\tilde{\theta}_t + \int_{\underline{\theta}}^{\theta_t}\sum_{k=t+1}^{T}\delta^{k-t}\alpha^{k-t}\mathbb{E}[q_k(\tilde{\theta}^k) - \theta_k]\cdot\mathrm{d}\tilde{\theta}_t.$$

Hence in last period T,

$$\mathbb{E}_\omega\left[A_T^*(\theta^T) - B_T^*(\theta^T)\cdot(s_T - q_T^*(\theta^T)) - \tfrac{1}{2}[q_T^*(\theta^T) - \theta_T]^2\right] = U_T(\theta^T).$$

Therefore, the base salary $A_T^*(\theta^T) = U_T(\theta^T) + \frac{1}{2}[q_T^*(\theta^T) - \theta_T]^2$. For all $T > t \geq 2$,

$$\mathbb{E}_\omega\left[A_t^*(\theta^t) - B_t^*(\theta^t)\cdot(s_t - q_t^*(\theta^t)) - \tfrac{1}{2}[q_t^*(\theta^t) - \theta_t]^2 - \delta\tfrac{1}{2}[\xi_{t+1}^*(\theta_t)]^2 + \delta\mathbb{E}[U_{t+1}(\theta^{t+1}) \mid \theta_t, \xi_{t+1}^*(\theta^t)]\right.$$
$$= A_t^*(\theta^t) - \tfrac{1}{2}[q_t^*(\theta^t) - \theta_t]^2 - \delta\tfrac{1}{2}[\xi_{t+1}^*(\theta_t)]^2 + \delta\mathbb{E}[U_{t+1}(\theta^{t+1}) \mid \theta_t, \xi_{t+1}^*(\theta^t)] = U_t(\theta^t).$$

Therefore, the base salary $A_t^*(\theta^t) = U_t(\theta^t) + \frac{1}{2}[q_t^*(\theta^t) - \theta_t]^2 - \delta\frac{1}{2}[\xi_{t+1}^*(\theta_t)]^2 - \delta\mathbb{E}[U_{t+1}(\theta^{t+1}) \mid \theta_t, \xi_{t+1}^*(\theta^t)]$.

Lastly, in initial period $t = 1$, the salary should accommodate the initial training investment. I should have

$$-\tfrac{1}{2}[\xi_1^*(\theta_0)]^2 + \mathbb{E}_\omega\left[A_1^*(\theta^1) - B_1^*(\theta^1)\cdot(s_1 - q_1^*(\theta^1)) - \tfrac{1}{2}[q_1^*(\theta^1) - \theta_1]^2 - \delta\tfrac{1}{2}[\xi_2^*(\theta_1)]^2 + \delta\mathbb{E}[U_2(\theta^2) \mid \theta_1, \xi_2^*(\theta^1)]\right.$$
$$= -\tfrac{1}{2}[\xi_1^*(\theta_0)]^2 + A_1^*(\theta^1) - \tfrac{1}{2}[q_1^*(\theta^1) - \theta_1]^2 - \delta\tfrac{1}{2}[\xi_2^*(\theta_1)]^2 + \delta\mathbb{E}[U_2(\theta^2) \mid \theta_1, \xi_2^*(\theta^1)] = U_1(\theta^1).$$

Therefore, the base salary $A_1^*(\theta^1) = \frac{1}{2}[\xi_1^*(\theta_0)]^2 + U_1(\theta^1) + \frac{1}{2}[q_1^*(\theta^1) - \theta_1]^2 - \delta\frac{1}{2}[\xi_2^*(\theta_1)]^2 - \delta\mathbb{E}[U_2(\theta^2) \mid \theta_1, \xi_2^*(\theta^1)]$.

(b) The optimal training level are driven by (OB_t) constraints

$$\begin{aligned}
\xi_{t+1}^*(\theta^t) =& \beta\left[\sum_{k=t+1}^{T}\delta^{k-t-1}\alpha^{k-t-1}(q_k^*(\theta^k) - \theta_k)\right] \\
=& \beta\left\{\frac{1-(\delta\alpha)^{T-t}}{1+\beta^2-\delta\alpha} - \frac{1-(\delta\alpha^2)^{T-t}}{1+\beta^2-\delta\alpha^2}\eta(\theta_0)\alpha^{t+1}\right. \\
&\left. + \left[\frac{1+\beta^2}{1+\beta^2-\delta\alpha}\left(\frac{\delta\alpha}{1+\beta^2}\right)^{T-t} - \frac{1+\beta^2}{1+\beta^2-\delta\alpha^2}\left(\frac{\delta\alpha^2}{1+\beta^2}\right)^{T-t}\eta(\theta_0)\alpha^{t+1}\right]\left[1 - \left(\frac{1}{1+\beta^2}\right)^{T-t}\right]\right\}.
\end{aligned}$$

■

Proof of Proposition 4.4

Under $\mathcal{P}^r$, all the $(IC_t)_{t\geq 1}$ are dropped. The problem is classic adverse selection, with fixed private information θ_0 and uncertain future states $(\varepsilon_t)_{t\geq 1}$. By the incentive constraint IC_0 and the envelope theorem, I have

$$\frac{\partial}{\partial\theta_0}U_0(\theta_0) = \mathbb{E}\left[\textstyle\sum_{t\geq 1}\delta^t\big(q_t(\theta^t)-\theta_t\big)\frac{\partial\theta_t}{\partial\theta_0}\;\Big|\;\theta_0\right] = \mathbb{E}\left[\textstyle\sum_{t\geq 1}\delta^t\big(q_t(\theta^t)-\theta_t\big)\frac{\partial\theta_t}{\partial\theta_{t-1}}\cdots\frac{\partial\theta_1}{\partial\theta_0}\;\Big|\;\theta_0\right] = \mathbb{E}\left[\textstyle\sum_{t\geq 1}\delta^t\big(q_t(\theta^t)-\theta_t\big)\alpha^t\;\Big|\;\theta_0\right].$$

Integrating both sides of the above envelope formula, I obtain the payoff equivalence:

$$U_0(\theta_0) = U_0(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_0}\mathbb{E}\left[\textstyle\sum_{t\geq 1}\delta^t\cdot\alpha^t\cdot\big(q_t(\tilde{\theta}^t)-\theta_t\big)\right]\mathrm{d}\tilde{\theta}_0. \tag{C.1}$$

At the optimum, the participation constraint IR_t for retailer-$\underline{\theta}$ is binding, i.e., $U_t(\underline{\theta}) = 0$. I then plug Eq. (C.1) into Π_0, integrate by part, and obtain the virtual surplus expression of the firm's payoff:

$$\mathbb{E}\left[\sum_{t\geq 1}^{T}\delta^t\Big(-\tfrac{1}{2}[\xi_t(\theta^{t-1})]^2 + q_t(\theta^t) + \omega_t - \tfrac{1}{2}[q_t(\theta^t)-\theta_t]^2 - \eta(\theta_0)\cdot\alpha^t\big(q_t(\theta^t)-\theta_t\big)\Big)\right], \tag{$\check{\mathcal{P}}$}$$

The problem $\mathcal{P}^r$ reduces to virtual surplus maximization, which is identical to that formulation in regime $\check{\mathcal{P}}$ (see the proof of Proposition 4.2). Therefore, the firm makes the same profit in $\mathcal{P}^r$ and $\check{\mathcal{P}}$; so does the salesperson. The firm in $\check{\mathcal{P}}$ can achieve the payoff as if she were able to observe future skills $(\theta_t)_{t\geq 1}$. This implies that the salesperson only receives information rents for his initial private information θ_0, and that the firm can extract future private information $(\theta_t)_{t\geq 1}$ at no cost.

■

Proof of Proposition 4.5

Note that

$$q_t^*(\theta^t) = \theta_t + 1 - \eta(\theta_0)\alpha^t - \sum_{k=t}^{T}\left[\left(\frac{\delta\alpha}{1+\beta^2}\right)^{k-t}\frac{\beta^2}{1+\beta^2}\left(1-\eta(\theta_0)\alpha^t\right)\right] < \theta_t + 1 - \eta(\theta_0)\alpha^t = \check{q}_t(\theta^t),$$

for all θ^t. Hence

$$\begin{aligned}\xi_{t+1}^*(\theta^t) = \beta\left[\sum_{k=t+1}^{T}\delta^{k-t-1}\alpha^{k-t-1}\mathbb{E}[q_k^*(\theta^k)-\theta_k]\right] &< \beta\left[\sum_{k=t+1}^{T}\delta^{k-t-1}\alpha^{k-t-1}\mathbb{E}[\check{q}_k(\theta^k)-\theta_k]\right]\\ &< \beta\left[\sum_{k=t+1}^{T}\delta^{k-t-1}\alpha^{k-t-1}\mathbb{E}[\check{q}_k(\theta^k)-\theta_k+\eta(\theta_0)\alpha^k]\right] = \check{\xi}_{t+1}(\theta^t).\end{aligned}$$

Given the rent structure that

$$U_t(\theta^t) = \int_{\underline{\theta}}^{\theta_t} \sum_{k=t}^{T} \delta^{k-t}\alpha^{k-t}\mathbb{E}[q_k(\tilde{\theta}^k) - \theta_k] \cdot \mathrm{d}\tilde{\theta}_t$$

is increasing in $q_t(\theta^t)$, the firm leaves more information rent to the salesperson when offing $\check{\phi}$ under $\mathcal{P}$.

■

Proof of Proposition 4.6

(*a*) In the infinite horizon, $\check{\xi}_\infty = \lim_{T\to\infty} \beta\frac{1-(\delta\alpha)^{T-t}}{1-\delta\alpha} = \frac{\beta}{1-\delta\alpha}$. The optimal scheme $\check{\phi}$ induces the skill process

$$\begin{aligned}\theta_{t+1} =& \alpha\theta_t + \beta\check{\xi}_\infty + \varepsilon_{t+1}\\ =& \alpha\theta_t + \frac{\beta^2}{1-\delta\alpha} + \varepsilon_{t+1}.\end{aligned}$$

As $t\to\infty$, the process has the steady-state distribution $\check{\theta}_\infty \sim \mathcal{N}(\mu_{\check{\theta}_\infty}, \sigma^2_{\check{\theta}_\infty})$. $\mu_{\check{\theta}_\infty}$ follows from

$$\mu_{\check{\theta}_\infty} = \alpha\mu_{\check{\theta}_\infty} + \frac{\beta^2}{1-\delta\alpha} + \mu \Rightarrow \mu_{\check{\theta}_\infty} = \frac{\mu}{1-\alpha} + \frac{\beta^2}{(1-\alpha)(1-\delta\alpha)},$$

and $\sigma^2_{\check{\theta}_\infty}$ follows from

$$\sigma^2_{\check{\theta}_\infty} = \alpha^2\sigma^2_{\check{\theta}_\infty} + \sigma^2 \Rightarrow \sigma^2_{\check{\theta}_\infty} = \frac{\sigma^2}{1-\alpha^2}.$$

Under the optimal scheme $\check{\phi}$, the quota process will converge

$$\check{q}_t = \theta_t + 1 \to \check{\theta}_\infty + 1 \sim \mathcal{N}(\mu_{\check{q}_\infty}, \sigma^2_{\check{q}_\infty}),$$

with $\mu_{\check{q}_\infty} = \mu_{\check{\theta}_\infty} + 1$ and $\sigma^2_{\check{q}_\infty} = \sigma^2_{\check{\theta}_\infty}$.

(*b*) In the infinite horizon,

$$\begin{aligned}\xi^*_{t+1}(\theta^t) =& \beta\Bigg\{\frac{1-(\delta\alpha)^{T-t}}{1+\beta^2-\delta\alpha} - \frac{1-(\delta\alpha^2)^{T-t}}{1+\beta^2-\delta\alpha^2}\eta(\theta_0)\alpha^{t+1}\\ &+ \left[\frac{1+\beta^2}{1+\beta^2-\delta\alpha}\left(\frac{\delta\alpha}{1+\beta^2}\right)^{T-t} - \frac{1+\beta^2}{1+\beta^2-\delta\alpha^2}\left(\frac{\delta\alpha^2}{1+\beta^2}\right)^{T-t}\eta(\theta_0)\alpha^{t+1}\right]\left[1-\left(\frac{1}{1+\beta^2}\right)^{T-t}\right]\Bigg\}\\ \to& \beta\left[\frac{1}{1+\beta^2-\delta\alpha} - \frac{1}{1+\beta^2-\delta\alpha^2}\eta(\theta_0)\alpha^{t+1}\right],\end{aligned}$$

as $T\to\infty$. The optimal scheme ϕ^* induces the skill process

$$\theta_{t+1} = \alpha\theta_t + \frac{\beta^2}{1+\beta^2-\delta\alpha} - \frac{\beta^2}{1+\beta^2-\delta\alpha^2}\eta(\theta_0)\alpha^{t+1} + \varepsilon_{t+1}.$$

As $t \to \infty$, $\eta(\theta_0)\alpha^{t+1} \to 0$, the process converges to the linear first-order autoregressive process AR(1) (Bhattacharya and Majumdar, 2007)

$$\theta_{t+1} = \alpha\theta_t + \frac{\beta^2}{1+\beta^2-\delta\alpha} + \varepsilon_{t+1},$$

which has the steady-state distribution $\theta^*_\infty \sim \mathcal{N}(\mu_{\theta^*_\infty}, \sigma^2_{\theta^*_\infty})$. $\mu_{\theta^*_\infty}$ follows from

$$\mu_{\theta^*_\infty} = \alpha\mu_{\theta^*_\infty} + \frac{\beta^2}{1+\beta^2-\delta\alpha} + \mu \Rightarrow \frac{\beta^2}{1+\beta^2-\delta\alpha} = \frac{\mu}{1-\alpha} + \frac{\beta^2}{(1-\alpha)(1+\beta^2-\delta\alpha)},$$

and $\sigma^2_{\theta^*_\infty}$ follows from

$$\sigma^2_{\theta^*_\infty} = \alpha^2\sigma^2_{\theta^*_\infty} + \sigma^2 \Rightarrow \sigma^2_{\theta^*_\infty} = \frac{\sigma^2}{1-\alpha^2}.$$

Under the optimal scheme ϕ^* in infinite horizon, the quota process becomes

$$q^*_t(\theta^t) = \theta_t + 1 - \eta(\theta_0)\alpha^t - \left[\frac{\beta^2}{1+\beta^2-\delta\alpha} - \frac{\beta^2}{1+\beta^2-\delta\alpha^2}\eta(\theta_0)\alpha^t\right] \to \theta^*_\infty + \frac{1-\delta\alpha}{1+\beta^2-\delta\alpha} \sim \mathcal{N}(\mu_{q^*_\infty}, \sigma^2_{q^*_\infty}),$$

with $\mu_{q^*_\infty} = \mu_{\theta^*_\infty} + \frac{1-\delta\alpha}{1+\beta^2-\delta\alpha}$ and $\sigma^2_{q^*_\infty} = \sigma^2_{\theta^*_\infty}$.

■