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A COMPUTATIONAL APPROACH TO THE COGNITION OF SPACE AND ITS LINGUISTIC EXPRESSIONS

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'The involutive algebra *A* corresponds to a given space *M* like in the classical duality Space Algebra in algebraic geometry'

(Alain Connes (1998) 'Noncommutative Geometry and Space-Time'. Ch. 4 *in* S. A. Huggett, *et al*., eds. *The Geometric Universe: Science, Geometry, and the Work of Roger Penrose*. Oxford: Oxford University Press. p. 50. underscoring added)

Abstract

To advance an algebraical-computational view of knowledge representation we examine the domain of space as an abstract relational system with wide cross-domain applicability with regard to relational properties generally. It is no accident that any coherent system of relations is understood as a 'space' of such relations. Jackendoff's work shows this. We have good grounds for considering spatial relations a universal 'modular' faculty of human cognition.

Just to set the stage for our paper and raise issues we need to address, we will first address certain fairly recent work done on the cognition of space by Levinson, Herskovits, and Talmy, and discuss their data and ours from languages like Italian, Burmese, Haka Chin, and Tongan and show that relativist conclusions follow directly from neo-behaviorist failure to be abstract enough in dealing with conceptual-relational structure.

Then we will introduce basic concepts of LOCUS, PLACE, MOTION, PATH and DIRECTION and will provide definitions of spatial prepositions like 'at', 'on' and

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1 No distinction is intended as between a junior and a senior author. Lehman's point of departure has been his formal-algebraic work on generalized relational spaces, and his work on the grammar of Burmese and other Tibeto-Burman languages, whilst Bennardo brings to this work his native Italian, his research on Tongan, and his study of the perceptual bases of vision. Our common ground is a computational-intensional approach to cognition.

This paper was originally presented to the Conference on The Relationship between Linguistic and Conceptual Representation. Annual Conference of the Linguistic Society of Belgium, at Antwerp, 26-28 November, 1992. and we thank the organisers and participants at that conference, especially Steve Levinson and Eric Pederson, for their many helpful comments. We acknowledge significant assistance from the following persons in the Cognitive Science Program with whom we worked during the production of this paper: Dr. Janet D. Keller (Anthropology), Dr. William Brewer (Cognitive Psychology), Dr. Jerry L. Morgan (Linguistics). Additional input from David Herdrich and Dr. Robert R. Sands (Anthropology) is also gratefully acknowledged.

'in', as well as discuss in detail prepositions like 'to', 'towards', 'from', 'away from', and 'via'. We will conclude our work by indicating a minimal universal content of the domain of space that will eventually become the axiomatizable component of the system from which the linguistic expressions derive as theorems of that same system.

0. Introduction.

0.1 *Notes on Spatial Categories as Computational: Computing Functions from Object-Positions.*

What we intend to do here is to develop an intensionalist cognitive theory of the spatial categories of natural language, computational in the sense of various papers in the book, Representation and Processing of Spatial Expressions (Olivier and Gapp 1998). In particular, we take our lead from Mukherjee (1998, esp. pp. 4-6) and suggestions in Landau and Munnich (1998) : our work defines object membership of spatial categories and relations as essentially computable functions from n-tuples of objects (N 1) — more properly, a function from subsets of points in the topologically well-defined neighbourhoods of n-tuples of objects — in 2- or 3-space. More particularly still, we attempt to bridge Mukherjee's distinction between 'neat'(uniquely idealised) and 'scruffy' (fuzzy) definitions (a) by means of a machinery that first defines the categories in terms of an algebraic topology of limits and (b) then computes a variable function from object positions within such categorial limits and salience of category membership relatively to competing category membership(s). This allows us to consider the spatial-conceptual categories as basically true sets rather than fuzzy sets, with category membership defined in an all-or-none fashion, whilst allowing the categories defined within such well-defined limits to overlap, so that such questions of relative saliency arise naturally. In this way, we argue that the scruffiness/fuzziness of such spatial category membership is a matter not of their definitions (as in the quantitative scalar approach of almost all proponents of spatial theories modelling fuzziness, whether AI theories as with Mukherjee 1998 or qualitative (discursive) approaches to fuzziness such as prototype theories, e.g., Lakoff 1987) but rather of the way objects and the like instantiate those definitions. The intuition is preserved, that conceptual categories are in general quite like ordinary sets or Proper Classes, definable semantically by necessary-and-sufficient conditions (which is to say defined intensionally and not in the first place extensionally), the fuzziness coming from the variable relevance of the way the position of an object fits the category definition. This is what makes the approach taken here radically intensionalist (say in the sense of Jackendoff 1983: 29ff; 1992: 56). A more general argument in favor of treating conceptual categories as Proper Classes, with fuzziness relegated to 'the instantiation problem' can be found in Keller and Lehman 1991 and in Lehman 1985). The basic approach can readily be understood in terms of a simple example.

Consider, say, (cf. fig. 1.1 in Mukherjee 1998) an object, o, suspended so that from some part of it (from some point on or properly contained within it) a perpendicular can be dropped directly to the surface (to some point on the surface) of some table, t, whilst from many other points on o a perpendicular can be projected down to the floor f

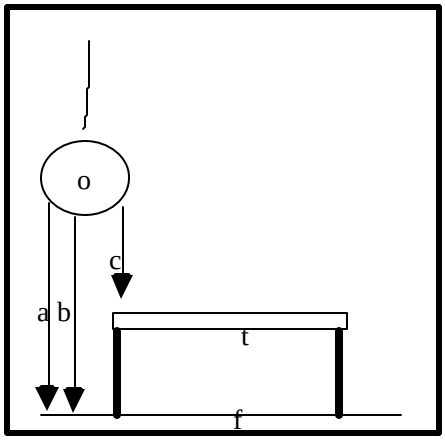

fig. 0.1

Now, is o suspended 'over/above' t or over/above the floor f ? We contend that this question, as usually posed, is simply misleading. It seems altogether clear to us that o is suspended *both* above the table and above the floor; and that the real question is rather about which truth is more importance, say pragmatically. Here we might contend that saying o is over the floor might seem more important (salient) because more points (cf. perpendiculars like a, b, as against c) project perpendicularly to f than to c; or we might contend, with equal good sense, depending upon one's perspective, that it is more important to say o is suspended above the table because in a plumb-fall o would hit the table first. Without apparent question, o is not, in any obvious sense, *more* above the one than above the other, so that the definition of, say, *above* is not at issue as to scruffiness/fuzziness. Rather, the question is more like '*how much* (in some non-uniquely specifiable pragmatic sense) is o above the one as against being above the other ?'[2]

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2 In Lehman (1985) an analogous case is made for conceptual categories not having to do with spatiality. For instance, (cf. Wierzbicka 1985), we realise that, in general at least, membership of conceptual categories is not taxonomic (save for 'natural kinds' categories in the sense of Atran (1990) and others). A knife, for instance, is not a *kind of* weapon, tool or tableware, where taxonomic, *kind-of* relations are understood as exclusive and unique an essentialistic. Thus, we cannot ask if a knife *is* the one or the other because it is all of them simultaneously, so that we have instead to ask, in any given pragmatic context of discussion, use, or whatever, which of the three larger categories is most salient as the category of a given knife.

It is also argued there, and more cogently in Keller and Lehman (1991) that the distinction between Set and Proper Class in the sense of a non Zermelo-Fraenkel version of axiomatic Set Theory has to be

Whilst it is clear that what we are doing in this paper is not without precedent (see references above to papers in Olivier and Gapp 1998) with regard to the theory of spatial concepts and the natural language words for them, we claim that our way of dealing with these matters is in fact novel and productive and, in that sense, an advance for the theory of such matters and in a real sense an improved idea of how such categories are represented and understood in the mind and how their meanings are represented in the mental lexicon of a natural language. Moreover, the distinction between the representation of knowledge in the mind (cognition proper, so to speak) and the representation of meanings in the mental lexicon (lexical Semantics) is a non trivial distinction. This is spelt out elsewhere (Keller and Lehman 1991, Lehman MS [CSRN]), but it can be outlined here by the proposition that, for instance, the cognitive domain concerned with space and spatiality has to be a mental object of considerable generative power, a theory-like object[3] in the sense of having axioms and formation rules that can produce indefinitely many surface theorems and representations with which to 'describe', i.e., to conceptualise relational properties quite generally, amongst which are the 'features' entering into the lexical semantic definitions of its categories, and the categories themselves.

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maintained. In the case of objects that can be weapons, tools, tableware and so on, the Set of objects is shared by the three categories, but a Proper Class of Weapons, or Tools, say, is understood as a pair {S, ID) [S, a set, ID an intensional definition], such that Sets can indeed be members of other Sets but Proper Classes cannot be members of one another (although, obviously, an ID pertinent to one Proper Class, taken as a matrix of minimal necessary and sufficient conditions, or 'features', can properly contain a subset of such features that is identical to some other ID of some other Proper Class — objects that are weapons are also in the larger class of material objects even though, technically at least, a weapon is not a *kind of* material object. In particular, with respect to the example under consideration here, though a knife can be (serve as) both weapon and tool, one cannot on this basis ask if, then, tools are sorts of weapons, or conversely, weapons are kinds of tools, and this is exactly what is meant by saying that these domains of conceptual categories are not taxonomic.

3 This is not the place to get into the persisting argument in cognitive theory generally as between a 'theory theory' of cognition and other, say more percept-like theories of cognition. Sufficient to say here, on this matter, that all we need to mean by our claim that cognitive domains are represented in the mind in theory-*like* form is that these representations have the sort of generative capacity, and machinery for it, indicated above. No claim is entailed that, like scientific theories, they are subject to systematic empirical testing, that they are used in more or less conscious and deliberate ways to 'explain' the world or anything else of the kind. So, for us, much of the contestable baggage of a 'theory theory' of cognition (in particular of the recognition of other minds) is beside the point (see now pp. 765-766, 838-841 *in* Wilson and Keil 1999 for a review of the literature on this matter and its pros and cons). More particularly, if one argues in favour of a 'simulation' theory of cognition, one must then have in mind a cognitive machinery for constructing such simulations and that machinery is inherently likely to be theory-like in just the intended sense — with the simulations as output theorems. . For instance, for simulating 'mind' in other persons on the foundation of self awareness one has to do what amounts to figuring out 'if I had the attributes I impute to him/her, how would I act/ if I acted in a given way what attributes would I need to impute to myself to account plausibly for such behaviour?', there being no computable means of inductively zeroing in on another person's mind, on solving the mutual knowledge problem (see Y. Wilks in Smith 1982) Similarly if one argues for cognition as a system of essentially analogue rather than 'symbolic' representation, the question automatically arises about the machinery for producing the analogue representations — indeed, as Keller and Lehman (1991) observe, for every analogue machine there is necessarily a digital ('symbolic') specification.

Note in this last connection that it then turns out to be far from accidental that 'space' is what one talks about, in technical discourse certainly, whenever one is talking about *any set of relations* describable algebraically , hence the notion of algebras themselves as algebraic *spaces*, and space itself as abstractly any set of relations coherently described by an algebra. For example, it is hardly an accident that one uses (a subset of) the same relational-spatial terms ('before', 'after' and so on) in talking about relations in two- or three-space and talking about relations in time. We deal with this well-attested fact, which has been noted in the semantic and other 'qualitative' literature time and time again, in the body of this paper, and the argument or demonstration can readily be extended to talking about even more abstract relational systems such as, for instance, social precedence or ranking — just as I can come after you in a progression in three-space, so also can I come after you in time (indeed the two progressions seem to be inherently conformal) or in social precedence or preferment.

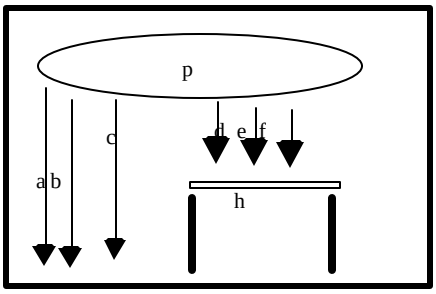
The remarks in the preceding paragraph clearly require some elaboration. In what ways can we claim that our methods and results are an advance? Well, for one thing, there is our use of the idea of fuzziness and its application. On the one hand, it is shown in Lehman (1985) that there are severe empirical problems with the way such notions as fuzzy set theory, Wittgensteinian 'family- resemblances' semantics and prototype semantics are used in the literature; we shall not review those demonstrations here. On the other hand, it is pretty clear (McCawley 1981) that, whilst it is both possible and productive to apply scalar ideas of class membership in robotics (in AI more generally) by means of implementations using direct quantitative numerical calculations (say of the kind referred to by Mukherjee 1998), ordinary colloquial human cognition is at best severely limited in its ability to perform such calculations or bring them on line, so to say, in run-of-the-mill cognitive life-tasks as against expert scientific work. Furthermore, it is also clear that much of the best of AI applications of fuzziness has turned out to lie not so much in the area of set computation as in the area of decision theory and especially information processing (see e.g., Lang, Carstens and Simmons 1991, Zadeh *in* Wilson and Keil 1999: 335-336, with references).

What we are attempting here in trying to model human cognitive reasoning about spatiality, is to apply fuzziness at the level of decision-making *about* class membership — about the salience of application *of* definitions of class membership, more specifically. Furthermore, we are employing for this task explicitly mathematical apparatus along the lines of topological algebras that seem intuitively more compatible with known colloquial cognitive capacities than are numerical calculation methods; and at the same time, since we are employing explicitly mathematical-computational machinery, we can avoid the inexplicitness of a lot of the non-AI, semantics literature that appeals to fuzziness/scruffiness, and thus, in principle at least, our work, though semantic and linguistic in its orientation and in its roots within cognitive

science, has, we claim, a better chance of linking up with the more rigorous or implementable (hence more explicitly testable) AI-type ways of modelling cognition.

Furthermore, we claim that what follows in the present paper captures a richer corpus of real life, real time, facts about the meaning, use and application of spatial terms and concepts over a considerable range of natural language types than does much of the literature that deals with spatiality, whether semantically (using this as a cover-term for non AI parts of cognitive science, including psychology, linguistics, anthropology and philosophy) or AI. One can refer here, once again, to Mukherjee's (1998, 6) reservations about the over-idealisation of the definition of spatial categories in much of the semantically literature (in this instances he refers to Herskovitz's 1986 work on spatiality in cognitive psychology). In fact we claim that much of the relevant literature seems to work nicely only because it uses only a more or less simplified, truncated data set! As a preliminary illustration of the claim of richer empirical results, let us for the moment consider a couple of extensions of our remarks on the example in figure 1, above — extensions in the spirit both of AI work as illustrated in Mukherjee's paper (1998: figures on 1-3) and the literature he refers to and of, for instance, the work of W. Levelt (in Nuyts and Pederson 1997).

Consider first what I might mean by saying the a certain park, p, is 'in front of/opposite' my house, h. Let us start with a modification of figure 0.1,



fig, 0.2.

Clearly the house is not entirely across from the park (more exactly, the converse is certainly not wholly so, as much of the park doe not face the house at all), but the more points there are like d, e, and from which perpendiculars may be drawn to the house, the more relevantly the house may be said to be opposite the park — rather than, say, 'diagonally' across from the park.

With the foregoing introductory remarks, we think we are ready to proceed with our substantive analysis of how to understand the terms in which natural languages deal with spatial location and spatial relations. It suffices that a fair amount of the park p is opposite my house in the sense of there being points within the closure of the park from which a perpendicular could be dropped to points within the closure of the front surface of my house. How many? Why an algebraical number whose lower limit is 1, so that the more there are the better the instantiation of 'opposite'. Indeed, we could say correctly about figure 0.3, below

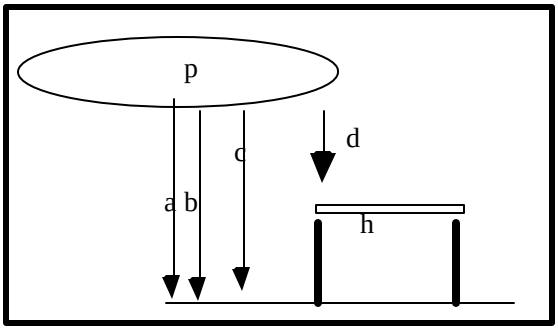


fig. 0.3

that while 'technically' the park is still opposite my house it makes more sense to say that it is, say, across the road but a bit down the street from the house.

And now consider a further extension. Imagine my house set in a certain plot of property, p, and imagine that the property contains a tree, t.

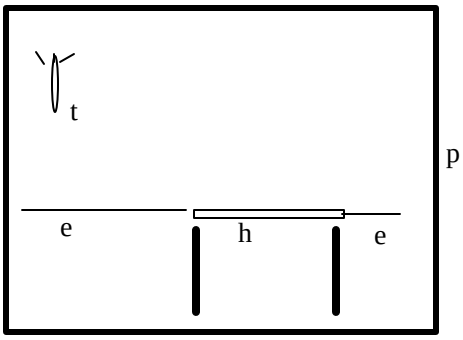


fig. 0.4

Can one say, in a proper application of colloquial English, that the tree is 'in front' of my house even though there is no point on t from which a perpendicular can be drawn to the front surface of the house? We claim the answer is yes. The reason seems to be as follows. We consider the area of property p defined in such a way that perpendiculars might be drawn from points in said area to an imaginary extension, e, of the front surface of my house — as in the case of my front yard extending left and right beyond the side of the house. Then, if tree t is in that front yard, it can be said to be in front of the house — just as, indeed, a tree were said to be 'across' from my house, in park p of figure 2, even if no perpendicular could be drawn from any part of the tree to my house at all.

It should now be clear that we are in many ways (probably not all ways) in agreement with Coventry (1998, 254-5 ff.) where he mentions the errors that much of the qualitative/discursive work on language tends to make. It will be best if we quote him selectively on this matter:

> ... there are three methodological errors ... . The first is that the meaning of a spatial preposition is not necessarily a direct reflection of the information the preposition brings to the sentence. ... the difference in meaning of the spatial expression may be a result of the lexical entries for [the kinds of nouns they are being used for] combined with the same lexicon entry [for the preposition]. ... If one does not recognise this distinction, then one can in principle recognize and infinite number of senses. ...
>
> The second error is that researchers confuse categories with lexical concepts. Catgorizing the world into different types of spatial relations does not necessarily map onto the lexical prepositions without a principled account of why this should be the case. In fact there are principled reasons why this is not the case. One must be able to categorize the world before one is able to map these onto language. However as Crangle and Suppes (1989) stated, "in spite of the spate of articles in the last decade or so on locative expressions, spatial prepositions, and the like, *detailed attention to the kinds of geometry needed to give a semantic analysis of the various locative expressions does not seem to have been previously attempted*" [our italics, because it is this task we set ourselves here, namely, the discovery of the 'geometries' that constitute the computational properties of this whole cognitive domain, even though, Coventry goes on to say here —] They went on to argue that a detailed understanding of the geometry is required before an adequate characterization of the meaning of spatial language can take place and Suppes (1991) outlines no less than seven different kinds of geometry that may need to be employed ... . Furthermore, as Suppes commented, there is no reason to believe that a full categorization of the

types of geometry that are required can be achieved in the first place. [Coventry continues —]

> ... the other objection to sense delineation through categorizing spatial relations is that there is not a one-to-one mapping between spatial relations and lexical entries. If the world were categorizable into distinct spatial geometric relations, then one would expect language to map onto these geometric relations monotonically, but they do not. For example, if an object is not in its canonical orientation the language used to talk about the same geometrical relations changes ...' [we take this to be an essential basis for the general idea of the distinction between semantics and underlying cognition seen as knowledge structure domains, as well as of our view that meaning and conceptualisation are fundamentally intensional in the first instance rather than being directly referential in the first place; and we take it that this entails the claim that indeed —]
>
> ... Functional relations clearly influence spatial language use, and furthermore involve information from general world knowledge about Newtonian laws as well as information regarding object functions that may reside in the lexicon.
>
> Before the decoding and encoding problems can be solved, one needs to construct an intermediate representation (mental model) between the input (e.g., a spatial scene) and information stored in memory (e.g., lexical semantics, general word knowledge).

With the foregoing introductory remarks, we think we are ready to proceed with our substantive analysis of how to understand the terms in which natural languages deal with spatial location and spatial relations.

0.2. On Varieties of Existing Accounts of Spatial Concepts and Relations.

There has been a great deal of work done with a view to providing a proper account of the way various peoples think about space and spatial relations, and the way various languages express these things. There are a number of reasons for the centrality of notions of space: a) it is possible to link it to the domain of perceptual universals (the neurobiology of vision); b) most of Jackendoff's work (1983, 1987, 1990. 1992) has already shown how much the linguistic representation of spatial relations underlies both thematic relations, in grammar, and conceptual structure quite generally; c) space is a wonderfully abstract relational domain in its own right, so that work on it can readily extend to the relational properties of other domains (it is no accident that in algebra any coherent system of relations is to be understood as a 'space' of such relations); and finally, d) spatial cognition being so relationally general, it provides good grounds for claiming universality for its underlying character. It is partly at least because of the fairly obvious way that it can be abstracted away for the representation of relation and orientation in

geographic 3-space and used (say analogically; some would say 'metaphorically') to deal with orientation, directionality and hence relational properties in general that space is a good domain to pursue here. It is common to talk of relations in terms of relational 'spaces' in a purely abstract, algebraic way, e.g., whatever the number of dimensions in the relational structure. The virtual universality of this generalization of the concept of space alone justifies this effort. Secondly, the very fact that the domain of space is in some sense or other perceptually embedded and universal because we are obliged to exist and get about in actual 3-space, makes it especially interesting to find the extent to which space may be differently conceptualized and/or differently expressed in different cultures and their respective languages. On all this, see now especially Jackendoff (1992, with its numerous citations), and also Keesing (1992), for an excellent treatment of the inescapable spatial 'metaphors' in which Time is represented.

Clearly, questions about the extent of cognitive and cultural relativism can be explored to advantage by looking into this matter, as can questions about linguistic relativism and such things as the Sapir-Whorf hypothesis that evaluate the extent of conceptual differences in terms of differences in their linguistic expression. It is therefore, we believe, especially important to take a new kind of look at the relation between spatial-relational concepts and their linguistic expression.

Our point of departure will be the aforementioned (near) universality of the essentially abstract and formal nature of the idea of space that motivates its extension to the expression of the relational properties of so many domains; more particularly intuitions derived from the way mathematics uses spatiality to deal with and express relationality in general. In this connection we intend to pursue the idea that relationality itself is fundamentally formal, abstract and algebraic in character. Whilst we argue (see §0.1) that our approach is properly called computational, we recognize the limitations on the extent to which one can base mathematics in general on computation, and leave the reader to look at the work of Penrose (1994) for this matter. When we speak of our theoretical stance as intensional, we have in mind the idea that cognition, and therewith meaning, are essentially mental objects and not objects subsisting in the first place on a ground of 'real world' reference, although certainly and necessarily adapted, in the first instance, for categorizing and otherwise dealing with reference to an objective world. We shall not go into the philosophical issues as to Fregean or extensional theories of meaning or objectivism see Lakoff 1987) versus radical intensionalism here; our position is best stated and explicated in such places as Keller and Lehman (1991) and Lehman (1985). In any case, we shall be treating the conceptual properties of space here as, in the first instance, abstract, formal, relational objects.

We have already said that we are using the word 'computational' to express amongst other things the formal properties of thought needed to account for its 'generative' properties. It seems to us that in so far as Knowledge-structures (hereafter K-structures, referring basically to the ordering characteristics of domains of knowledge) can be argued to be generative (for present purposes we can think of this as having to do with the ability to produce novel constructions and solutions to domain-relevant tasks), a K-structure has got to be represented as theory-like (Hirschfeld and Gelman 1995: Introduction and passim; cf. Hutchins 1995 and references in §0.1). That is to say it must be formalisable in principle axiomatically, with a generative engine capable of producing indefinitely many understandings, alternative construals of experience and so on; the argument is essentially the Chomskian one concerning linguistic knowledge (Chomsky 1986). But what is our conception of a K-structure? Where do its computational properties lie? More immediately of relevance to the present paper, how should one account for the fact that spatiality is so widely applied across K-structures, as Jackendoff has demonstrated?

We take the view (Hirschfeld and Gelman 1995: Introduction) that it is necessary to distinguish between domains and modules in a general theory of cognition in the following sense. We take cognition to be modular in the sense of a collection of innate, essentially computational engines; some of them highly specialized to a particular domain, as with the language capacity, some very broadly applicable, as in the case of spatiality (in spite of the fact that its prototypical field of application is the domain of knowledge about visible spaces — 2-space and 3-space), or perhaps a module having to do with our intuitions about number and quantity (perhaps arithmetic), or one concerned with what amounts to logical-propositional thought (see Johnson-Laird and Byrne 1991). A domain we conceive of as defined by some particular experiential or activity-oriented class of content. It is to be understood as a K-structure in that its structural, computational properties and capacities are provided by the application of one or more modules. It is for the time being an open question whether some domains are themselves essentially modules: arguably the domain of various sorts of sensory knowledge, and we have no reason to want to say anything about this. Likewise, for present purposes it is unnecessary to speculate about the source of encyclopedic knowledge in the sense of Sperber (1985; cf. French 1995), i.e., our capacity to associate widely across domains, possibly managed in part through the interaction of modular devices (but see Lehman MS: #14).

We take it as obvious that most K-structures are not themselves modules, and that for two reasons: first that they are often structured by more than one more or less general purpose computational engines, and second that it is impossible to imagine that culturally particular domains of knowledge (e.g., technical sciences, theories of art) are in any way wired in to the human organism's brain.

So, our position rests upon the premise that knowledge must be a rather abstract generative computational thing because it is not to be identified with any given individual view of anything, not even in a single, supposedly homogeneous cultural community. People are commonly able, even prone, to argue and debate on even the most ordinary topics of local knowledge, as any ethnographic field worker knows. It is not just that people disagree about facts, which in itself is likely to be trivial, but that they quite regularly draw opposing conclusions from shared premises and formulate contradictory arguments for their respective positions. That many such generative mechanisms are hardly reducible to the first order propositional calculus (augmented with set theory) is by now obvious from the results of formal theories about syntactic grammatical competence and other complex cultural domains (Keller and Lehman 1991); an example of a well understood and highly structured domain that has a domain specific algebraical structure quite distinct from the propositional is that of genealogically based systems of kinship categories (Lehman 1993, with references). Indeed it can be argued that the idea that all analytical knowledge (as opposed, say, to holistic imaging4) is in principle reducible to the cognitive machinery reduces to the propositional calculus is simply a holdover from the Logical Positivist program that, in the light of Gödel's incompleteness proof, is arguably inapplicable without residue even to the very sorts of algebraic systems that are themselves obvious candidates for theories of various sorts of K-structures (cf. Lakoff 1987 against the Fregean 'objectivist' heritage, but Lehman 1985 and Keller and Lehman 1991 for a radical intensionalist view of meaning and cognition very different from Lakoff's). Moreover, it may even be no accident that there is a long-standing connection between the Positivism that wishes to reduce thought to propositional form and the Behaviorism that wants to reduce thought to (possibly subliminal) speaking. Thus, in the final analysis, our arguments are not to be identified with those of the theory of thinking for speaking (Slobin, 1987) because it seems to us that analytical thought is not readily representable in propositional form, even if natural language is the privileged means for attempting the conscious articulation/expression of thought. Of course,

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4 Any work on the perception and cognition of vision (e.g., Churchland and Sejnowksi 1992) makes it clear that there is no disjunction between image representation and digital/symbolic representation. It is not possible in this paper to pursue this important issue. However, it is worth pointing out that for any image, and for any machinery that is to generate or recognise such an image, including any analogue machinery, there must necessarily be a description that is itself digital. From this it follows directly that the supposed strong opposition between analogue and compositional (commonly reduced to the formalism of the propositional calculus) theories of cognition collapses. For a good discussion of this issue of compositionality in the context of the work of J. A. Fodor (1983) see Bierwisch and Schreuder (1992: 27-28). A related issue that cannot be pursued here is whether (cf. Simon and Kaplan 1989) any cognitive representation of thought that cannot be formulated in the terms of the propositional calculus is necessarily an heuristic rather than a systematic K-structure or 'theory' of the domain in question. Once again, this turns upon whether one assumes that all systematic computational formalisms reduce in the final analysis to the propositional calculus. Since in actuality nobody seriously takes this strong position, it has to be the case that the first order propositional calculus is taken as the exclusive paradigm of thought and reasoning simply because it alone is 'complete' (Partee, ter Meulen & Wall 1990).

we do not mean to assert that all knowledge is reducible to computation without residue of, let us say, primitive perceptually-driven assumptions (see Penrose 1994).

In short, we argue here that a computational-intensional approach to cognition is to be preferred on both empirical and formal grounds. We shall show how this approach is especially apt for the sorts of complex K-structures that ethnography is required to deal with, in particular space and its linguistic expression.

We shall first review the literature pertaining to the relationship between language and spatial cognition and identify two schools of thought. Second, after proposing a computational approach to spatial cognition, we shall discuss the shortcomings of the two schools. Finally, we introduce a number of analytical concepts for the cognition of space and we demonstrate their productivity in the formal analysis of the meaning of some spatial prepositions.

1. *Two Approaches to the Relationship between Language and Cognition*.

We think it is possible to identify two different schools of thought that we label Neo-Whorfian Relativism (NWR) and Cognitive Semantics (CS).

1.1 *Neo-Whorfian Relativism (NWR).*

The tenets of NWR differ from classical Whorfian Relativism even if it is not certain exactly in what sense Whorf intended us to imagine that language constrains the way one is able to think as against the way one is most accustomed to think (see Gumperz and Levinson 1996: Introduction), but still assign to language a prominent role in helping to shape human cognition. Along these lines Levinson (1996, 1991b) proposes to divide human languages into three groups according to the system they adopt in dealing with spatial representations. He proposes three such systems: 'relative angles', 'absolute angles' and 'landmarks & place names' (Levinson 1991b:16).

The linguistic instantiations of these three systems have been and are being extensively investigated across a wide variety of languages and language families throughout the world (see Brown, Senft and Wheeldon 1993, Baayen and Danziger 1994, Pederson and Roelofs 1995) by members of the Cognitive Anthropology Research Group at the Max-Planck Institute for Psycholinguistics, Nijmegen, The Netherlands, directed by S. C. Levinson. The intent is to see if specific linguistic preferences for one of the three systems is reflected in cognitive preferences for that particular system, and, if so, in what ways and to what extent

Preliminary congruences between the linguistic and the cognitive realms in several languages, e.g. Tamil (a Dravidian language of South India), Tzeltal (a Mayan language of Meso

America), Guugu Yimithir (Australian Aboriginal), have been used to support the general approach to cognition as outlined by Levinson (1996). According to this view universal constraints on spatial representation, possibly to be identified with compatibility with visual and locomotor perception of space, act as filters "leaving open indefinite possible cultural variation within outer limits" and consequently "a moderately strong brand of linguistic relativity [our italics] in at least some domains" (Levinson 1996; ; but see also Bowerman 1996) is strongly suggested. Universal properties of the human mind are limited in their applicability to accounts of the general idea of space in favor of culturally and linguistically bound constructions of spatial representation. In other words, each language/culture constructs a specific representation of space that makes its world unique.

We do not reject Levinson's observations, nor the claim that they are of real cognitive significance. Contrariwise, we shall show that the distinctions he adduces between what he characterizes as, respectively, absolute angles, relative angles and landmarks etc., can be of real psychological significance, so that people used to using one or other of these strategies for orienting themselves in space (generally and relatively to other things and persons) may, without the perceptual or informational evidence necessary for the use of the preferred, e.g., most habitual means of representation, find themselves genuinely disoriented! We are, however, able to generalize formally over his somewhat disjunctive characterization of the phenomena, and we argue against his identifying this as a case of language influencing thought, if by thought one is intended to understand not simply the way one ordinarily thinks about something or registers something in retrievable memory, but rather the ability to think of, understand or otherwise perceive or conceptualize spatial relations in one of the modes for which one's language provides no means of direct expression. We accept a sense in which it can be said that they represent different ways of thinking, but we argue that this means only that different habits of usage may have deep consequences even though they cannot, being constructed on the ground of identical perceptual input, computationally disjoint, given an adequately explicit formulation of the general idea of space itself. Thus, we think that what Levinson is pointing to is real, but not any sort of Whorfian relativism of thought, where we define thought computationally. As a matter of fact, in discussion with one of the present authors, Levinson himself has once referred to his findings as perhaps instances of 'Whorfian effects for non-Whorfian reasons'. We shall return to these matters later on (on this whle controversy, see now Bowerman and Levinson 2001, passim).

We already have (see our treatment of 'from' in Burmese and Chin, below) a test case, a pure instance so to speak, of very different linguistic constructions for expressing a fundamental spatial relation, with, however, no evidence of any problem. Speakers of Burmese have no difficulty using the English 'from,' though they have no word in their language with that meaning, and English speakers at worst simply think of Burmese ka. (k)as meaning

'from,' but soon see what it means as they first encounter the longer form from which it is contracted. There is absolutely no evidence that either party thinks differently about the facts being expressed. This is the sort of 'pure' case against which the disjunctive-relativist hypothesis must be tested.

Consider some facts connected with Levinson's three categories of spatial orientation. In many languages there is a word that sometimes refers to a compass direction, sometimes to a notion we can call 'downstream, down an upland valley.' It turns out that this conflates Levinson's two categories of, respectively, relativeand absolute angular orientation, and maybe also the third, 'landmarks.' In Kayah, for instance, lja expressing a direction of something from somewhere can mean it is to the South or that it is down-valley from the reference place. Yet no Kayah feels disorientated when he is somewhere where he has no idea which way is up-valley. There is nothing like what is reported for, say, Bali, where people are said to feel utterly lost without knowing where their central mountain is. The Kayah just make the word refer to South by default. In fact the range of meanings comes from the fact that in their cosmology, well supported by local conditions, North, say, is defined (see Appendix) by a projection from the celestial zenith, whose most generalized pointer is the polestar; mountains just serve as local pointers upward to the imaginary zenith, so that the distinction between relative and absolute angles collapses, even if, in a sense, 'South' is determined relatively to a possibly arbitrary reference point, commonly identified with the speaker or the hearer and where he or she happens to be situated in fact or in imagination. Pragmatically, of course, one has to have a clue as to 'which zenith pointer' is the relevant one, but that's hardly a fundamental difference in thinking. Indeed in English there is a related phenomenon: 'up' can mean either North or up-hill, and sometimes one must simply ask which the speaker had in mind.

One can, for instance go 'up' from Oxford, and if the place be unknown to the hearer, the latter may not know if London is meant (which is not North of Oxford, but is 'up' in the sense of 'up to town') or, say, Edinburgh, which, though not 'town', is North. So we ask whether the speaker meant North or in the direction of London. The problem is not one of cognition but related to the mutual knowledge problem—knowing what is in someone else's mind. One must avoid confusing an inability to conceptualize with absence of relevant information.

Then, take the choice between orienting oneself by compass directions and orienting oneself by reference to landmarks or in any other way. These make a big difference, as we have said above. However, it seems to have nothing to do with different ways of conceptualizing space. Rather, it seems to have to do with available information once again. Even in an English-speaking society, there are persons, woodsmen and so on, who are habituated to orienting themselves compass-wise. They feel utterly at sea when they don't know which way is North,

and the first thing they do in a new place is to get this fixed in mind. That is what orientation is all about! The city person who has no idea of North gets lost in the country, where the landmarks and pointers are absent, especially where he (or she) makes nothing of rural landmarks anyhow. But notice, having regard to the Kayah example above, that it is not simply a matter of using such a feature but rather of how you compute directionality with it. A Kayah uses either a local or the general zenith marker, but the absence of the former creates no sense of disorientation. 'East' can be computed as right of any zenith marker equivalently, whilst 'to the right' of a fixed landmark, or to 'my' right, leaves no room for such substitution by default. A (possibly imaginary) Balinese, on the other hand, simply has the habit of computing angles of direction relatively to a unique reference mark. When displaced from Bali, he is disorientated like our city person; he lacks information useful to his habit of mind.

We do not dispute Levinson's facts, or the claim that these are deeply distinct cognitive styles; only that they represent different ways of thinking, where this may be taken to refer to distinct computational ways of processing perception to create K-structures, different ways of structuring knowledge as a space of relations on some substantive domain!

1.2 *Cognitive Semantics*.

The CS camp is mainly concerned with two central problems. One, the mental representation of the world and the relation of this representation to language; and two, the interaction of the visual system with other conceptual systems. Within CS there is a tendency to propose the collapse of linguistic and cognitive descriptions. For example, Jackendoff (1983, 1987, 1990, 1992) often seems to call for one of the classical components of language, semantics, to be coextensive with conceptual structures. It is at the level of conceptual structure that the integration of the information between the linguistic system (whose specific components are now only phonological and syntactic structures) and other cognitive systems (i.e., the visual system, motor system, etc.) takes place. However more recently (1994) Jackendoff distinguishes more sharply between semantics and conceptual structure

The CS camp is divided on the issue of the autonomy of syntactic representation. On one side, identified with the Cognitive Linguistics movement and also with what is called functionalist linguistics, the autonomy of syntax is abandoned so that well-formedness conditions of syntax are identified with conceptual well-formedness conditions (Fauconnier 1984, Langacker 1986, Herskovits 1986, Lakoff 1987). On the other side, work more rooted in the Chomskian school of linguistic theory, even though it does not completely represent the position

recently stated by Chomsky himself5 (see Chomsky 1993), keeps the autonomy of syntax alive (Jackendoff 1983, 1987, 1990; but see also Cienki 1989).

The view of cognition that this school of thought seems to adopt is one of a modular organization of mind, with internal modularity within the various modules as well (see Chomsky 1986 for the ennested modularity of language, modified and reduced considerably in Chomsky 1995). Specifically, spatial information is first organized by our perceptual apparatus via processes like edge detection, line orientation and texture segmentation (visual module 1), then acted upon by a variety of processes including binocular stereo, motion, color and others (visual module 2 — see Marr 1982). Finally, a complex aggregate of information is put together to build objects or concepts/categories (visual module 3 — see Biederman 1990). The output of visual module 3 has an internal structure that is compatible at the level of conceptual structures with the linguistic module via some correspondence rules.

Both schools seems to have some shortcomings that need discussion. The NWR school assigns a weak role to universal constraints (defined as 'mere filters'). On the contrary, the linguistic system is assigned a central role in shaping the conceptualization of spatial relations. However, in spite of the "indefinite possible cultural variation" that is hypothesized they have been able to define only three, and some languages, e.g., English, have at least two ('relative angles' and 'absolute angles') of the suggested three systems available to its speakers. In other words, the data they themselves present paradoxically support the hypothesis of a strong role for the universal constraints. The demonstration of this will be a central part of the present paper.6

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5 The position just referred to is very close to Chomsky's (1995). What Chomsky calls Conceptual Necessity, he always qualifies as Virtual, so that for him the conceptual system stands simply as motivation for, but not in identity with syntactic conditions of well-formedness; a functional similarity rather than a formal identity.

It must be pointed out here that Jackendoff's enterprise was never concerned with lexical content, but only with what he distinguishes as conceptual structure. Consequently, his project could easily accept this differentiation between semantics and conceptualisation just indicated. However, it must also be indicated that most of his work looks suspiciously more like lexical description than conceptual description, certainly in the sense that the notation employed seems to be chosen for the way it fits with a logico-propositional form of presentation rather than any other relational-computational system arguably more apt (see, e.g., Simon and Kaplan 1989 for a similar question in cognitive science generally) for one domain or other of knowledge. We shall see what limitations this places on Jackendoff's treatment.

It is beyond the scope of the present paper to pursue the question how a proposed correlation between modes of linguistic expression and cognitive styles and preferences is taken to mean that the latter influences the former. Perhaps it is because, if the influence were not seen as going in that direction, one might be forced to postulate that, after all, the various cognitive alternatives were themselves part of a set of universals — a position we in fact take here. The most recent and best discussion of these problems is, again, throughout Bowerman and Levinson 1991.

As regards the CS school, we question their frequent conflation of semantics with cognition. In fact, leaving aside the question of whether grammar must properly contain a language particular lexicon, any lexicon must be a highly reduced and specialized recoding of conceptual knowledge. Therefore, one must, after all, identify a semantics, as distinct from conceptual structure, with precisely that lexical recoding (cf. Lehman 1985).

Moreover, the CS school argues that visual data and linguistic data become compatible at the level of conceptual structure. Thus, we are led to suppose that there must be common processes and constraints on these processes, that bring this situation about. An integration of the descriptions of conceptual structures they offer with at least suggestions about the form of the perceptually grounded processes and constraints seems badly needed. The alternative, following surely from the poorly delineated distinction between semantics and cognition, seems to be a sort of infinite regress, in which propositional expressions are accounted for by a conceptual structures couched in the form of propositional structures, which therefore must be interpreted, again by propositional structures, and so on.

The foregoing problems justify the search for a different theoretical point of view that may overcome them.

2. *A Computational Approach to Cognition*.

In an attempt to resolve the tension between the two positions outlined we are grounding the present discussion in a corpus of work (Dougherty 1985, Dougherty and Keller 1985, Lehman 1985, Keller and Lehman 1991; but see also Keesing 1979, Talmy 1983, 1988, Bierwisch and Lang 1987) that suggests a different approach to these issues. Within this approach cognition is conceived as computational (cf. Ballim and Wilks 1991; see also Lang, Carstensen and Simmons 1991), and abstract. From this it follows that it is capable of generating a range of 'surface distinctions' (including linguistic and visual ones) that are not to be confused with the whole of 'thought' or cognition in that domain.

There is obviously some similarity between our contention that we must distinguish between cognition and semantics and, say, Slobin's (1987) notion of 'thinking for speaking'. It is not unprecedented to make some sort of distinction between underlying thought itself and its re coding for propositional-form expression. However, it is by no means clear that the similarity goes very far. In particular, we are not ready to assert that the lexico-semantic encoding is in any interesting sense a way of thinking, distinct from the 'deeper' level of thought, that people commonly resort to, by-passing cognition proper, as some sort of heuristic. It appears to us that this view, which may or may not be what Slobin really has in mind (for some common reservations about 'thinking for speaking,' see Levinson 1992:1-2), is insupportable and unnecessary. Others, too (cf. Bierwisch and Schreuder 1992) make a distinction between

(lexical) semantics and cognitive knowledge, but they do not characterize the latter in the sort of computational-intensional way we do here. Nevertheless, this distinction may after all be an instance of a general phenomenon noted by Jackendoff (e.g., 1992: Chapter 3), where there may be a sort of lexicon of thought: a place where one stores highly conventionalised outputs of thought that can be called up rapidly so as to avoid the need to compute one's construal of something from scratch' within the KS, just as one stores not just words but whole chunks of conventional constructions so as not to have to generate every sentence as a novelty.

In one sense, the 'thinking for speaking' view is a reversion to a non-trivial form of the Sapir-Whorf hypothesis (cf. Werth 1992, where it is argued in favor of 'thinking for speaking', that failure to allow for a distinction at least between thought and merely thinking for speaking amounts to making thought isomorphic with speech), against which we offer strong arguments in the present paper. On the matter of the arguments for and against heuristic mechanisms as against systematic logico-mathematical representations as a proper theory of cognition (e.g., Minsky 1985, Lave 1985, 1988), one of us (Lehman MS: #2) has argued elsewhere that heuristic solutions necessarily subsist on a base of more systematic conceptualizations, which are commonly resorted to at least to monitor and check on the results of heuristic calculations, even if it is frequently the case that people may find conscious reasoning with cognitive knowledge structures difficult, and be prone to error when doing such formal reasoning or calculation. After all, the argument that people are not generally very good at formal logical reasoning or even moderately complicated arithmetic or elementary algebra is no more an argument against these, or something like them, as underlying cognitive capacities than is an argument against grammatical competence as a theory of linguistic knowledge based on the fact that people have extremely imperfect conscious access to that knowledge.

Anyhow, the evidence that abstract knowledge structures are employed 'on line' not directly but rather through specific outputs, arguments, theorems, or whatever they generate is sufficient to take care of the distinction between abstract cognitive thought and the output thoughts we encode directly in speech. It does not follow that even these cognitive K-structure 'theorems' are themselves necessarily propositional in character (see Simon & Kaplan 1989 for a survey of the arguments for and against the idea that something on the order of the propositional calculus might plausibly serve as the computational formalism for theories of meaning and thought). The argument here is quite similar to the argument that the various ways we can speak of a situation or event is bound to be underlain by differing ways of perceiving and conceptualizing that event or state of affairs. To this extent, it is reasonably clear that lexico-semantic representation is computationally inadequate as a candidate for even this sort of 'surface' thinking. The reason is that it would then be necessary to suppose that lexico-semantic representation directly incorporates all those encyclopedic associations, possibly infinitely many, that lie behind any given understanding of a given state of affairs that is itself represented in

articulate speech7. Such a supposition is intolerable because it requires massive redundancy in the mental lexicon; ultimately, the same (possibly infinite) net of associations would have to be found in each relevant lexical representation (and possibly in all lexical entries, in so far as the encyclopedic knowledge of anything sooner or later extends to everything), which might indeed entail an infinite regress problem of the kind connected with the mutual knowledge paradox (cf. Ballim and Wilks 1991) and hence, arguably, the impossibility of finite lexical representation at all.

In order to exemplify and test the view of cognition just endorsed we choose to deal with the domain of space for the reasons set forth in the Introduction. The more abstract, mathematical treatment of all that here follows is left to the Appendix, in order that our argument may be made accessible to readers uninterested in, or put off by technicalities and even quasi-formal proofs.

3. *English Spatial Prepositions Revisited.*

We need to test the formal, abstract means of attacking this kind of problem by showing that it can be extended to related sets of problems for understanding spatial concepts and their language expressions. We shall deal with the prepositions because it has been held controversial whether or not the natural language evidence they provide favors or disfavors a universalistic theory of spatiality. We shall show, that our computational approach allows us to resolve this issue. We begin with some substantive issues, and then state our assumptions with especial regard to certain primitive concepts, e.g., PLACE, LOCUS and others, basing on, but going beyond Jackendoff[8].

Let us first introduce a set of sentences:

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7 On this Jackendoff (1992) has a good deal to say in connection with the observation that the 'implicit' arguments of a predication are likely to be far more numerous than the expressed arguments of the relevant proposition.

8 We wish to start from Jackendoff's treatment and see where we can take it by putting it in a certain kind of geometry-topology framework that both links it better to perception and to a system of concepts not inherently tied to propositional form and its logic, and allows one, to draw a richer body of empirically meaningful conclusions about spatiality. Where we take this is to make it clear that certain supposedly fundamentally different natural language systems for representing certain spatial concepts are really not all that different at a conceptual-perceptual level; that these phenomena, when thus analysed offer no support for any sort of deep conceptual/perceptual relativism. Note particularly that we start out solely in order to handle the English facts themselves and not in order to make English more like languages that seem to express space quite differently. It is, then, a considerable confirmatory bonus that the machinery we need for a *proper* treatment of the English, also in itself bridges the gap to those supposedly very different systems for the linguistic expression of spatiality. In fact, what the machinery does is effectually predict the other systems as possibilities, by quite direct theorems. And any good theory of a domain is necessarily, as Chomsky has said again and again, a theory of the class of *possible* phenomena in the domain.

(1) a. Maya is at the pool.
b. Maya is on the table.
c. Maya is in the car.

We have kept the syntax of (1) uniform so that specific features of the prepositions can be highlighted. We first notice that all sentences in (1) have two NP arguments, one a thematic subject and one the object of a prepositional phrase. What the prepositions are doing in (1) is expressing a relationship between the two objects[9] denoted by the two NPs. Talmy (1983: 230), before dealing specifically with this relationship, introduces the concepts of 'primary' and 'secondary' objects. He calls the primary object 'figure' and the secondary one 'ground' in Gestalt terms as follows:

> The Figure is a *moving* or conceptually move*able* object whose site, path, or orientation is conceived as a variable the particular value of which is the salient issue.
> The Ground is a reference object (itself having a stationary setting within a reference frame) with respect to which the Figure's site, path, or orientation receives characterization. (Talmy 1983: 232)

We will use the two concepts of 'figure' and 'ground' interchangeably with 'primary' and 'secondary' object in the following discussion.

The meaning of English spatial prepositions has been investigated by several scholars who have chosen to consider different ones as 'basic.' Clark (1973; but see also Lyons 1977: 694) says that 'at' is the simplest, but does not explain why (p.17 and 26). Fillmore (1975), similarly without explanation, indicates 'at' as the 'basic' one (p. 41). Miller and Johnson-Laird (1976) at least say that 'at' allows more spatial freedom (p. 387) than any other they analyze. Finally, Herskovits (1986) defines 'at', 'on' and 'in' as "the three basic topological prepositions".

Let us now look at the ideal meanings proposed by Herskovits (1986) for the spatial prepositions 'at', 'on' and 'in':

(2) a. 'at': for a point to coincide with another
b. 'on': for a geometrical construct X to be contiguous with a line or surface Y; if Y is the surface of an object Oy, and X is the space occupied by another object Ox, for Oy to support Ox
c. 'in': inclusion of a geometric construct in a one-, two-, or three-dimensional geometric construct

9 By the word 'object' we shall mean attributed existence in some universe of discourse existing in a possible world, either concrete or abstract, real or imaginary.

The first thing we notice in (2a) is that the concept of 'point' is introduced and left to the interpretation of the reader. We need the appropriate mathematical context (projective geometry and algebraic topology), assumptions, and theory to make 'point' explicitly meaningful. By taking 'point' seriously, we can connect it with other ideas "sufficiently abstract" (Lehman 1985), so that, taken together, they give us a truly deductive framework.

Secondly, the meaning of 'at' is literally translated by Herskovits as 'to coincide.' We are not, however, provided with any strict interpretation of the meaning of 'to coincide' either, so that the whole definition of 'at' is left floating on its interpretation by the reader. Furthermore, giving 'to coincide' a naï ve meaning10 has certain consequences.

If 'to coincide' means to 'occupy the same space,' how do we interpret sentence (1a)? Do all the points geometrically characterizing the pool coincide with all the points geometrically characterizing Maya, or vice versa ? Neither is necessarily, or even ordinarily the case. Moreover, if 'to coincide' means 'to be very close in space', then, this meaning is also part of the meanings of the other two prepositions, 'on' and 'in'. But this similarity of meaning is not indicated at all by the three definitions provided. Then, we definitely need to look further into the meaning of 'at' and try to clarify what Herskovits has left unexplained.

The definition of the preposition 'on' remains obscure. The definition of 'on' expresses two different relationships in two specific spatial configurations between different types of geometrical constructs (point, line, plane, solid). One infers that either 'on' has two meanings, or that a possible more general meaning (relationship expressed by 'on') is being left unaddressed. The former conclusion is unlikely (though possible) and the second one will later be seen to hold.

A similar comment to the one about the reduction of the meaning of 'at' to the verb 'to coincide' can be made about the proposed definition of the preposition 'in'. The meaning of 'in' is reduced to 'inclusion in' that contains a double repetition of 'in' (affix and preposition)! The reader is left to rely on intuitions instead of on formal clarity in the description. We will need, then, to look for a further analysis of the preposition 'in' as well.

10 Technically, folk 'meanings' are not necessarily meanings at all, in the sense of the way something is really understood in a full K-structure; they are in fact conventional ways of talking about cognitive material the 'folk' may not have conscious or articulate access to. It is only on a (neo-) behaviourist view (following the Flip Wilson principle for theorising about mental phenomena, that 'what you hear is what you get') that one comes to call these things 'meanings.'

3.1 *Objects and Points.*

Before looking into the relationship that the preposition establishes between the primary and secondary objects, we have first to look at the two individual objects, in order to justify their reduction to 'points.' We notice that whatever object we substitute in (1a) (without violating semantic constraints), for either argument position or both, the relationship expressed by 'at' does not change. In fact, a tree can be 'at the pool' as well as 'a bug' can, whilst Maya can be 'at the door' as well as 'at the desk'. The size, the shape, or indeed any geometrical/topological property of the two objects does not affect the type of spatial relationship 'at' is expressing. We are thus entitled to consider any and all objects entering this spatial relationship as generic geometric constructs and the choice of a 'point' as an appropriate substitution for any object is thus plausible. Support for that choice will ultimately come from the definition of LOCUS in projective geometry, where an arbitrary point is taken to represent the whole object.

> Another important feature of the two objects is noted by Talmy (1983):
> ... the second object can be used as a reference only by virtue, in a recursive manner, of its own known spatial disposition with respect to the remainder of the scene. This is to say that those spatial characterisations that are expressed overtly (as with prepositions) ultimately rest on certain unexpressed spatial understandings. (Talmy 1983: 230)

One of the 'unexpressed spatial understandings' is that they are always in relationship with some abstract spatial configurations in a particular, possible world[11]. Characterizing these spatial configurations is one goal of the present work.

3.2 *LOCUS and PLACE.*

What relationship is 'at' setting up? (on an intensionalist view, a relation is constructed and tested against a possible instantiation) First of all, 'at' indicates that we are dealing with LOCUS. The two objects are related to LOCI in the world. We must therefore clarify how LOCUS differs from PLACE[12]. The PLACE of an object is plausibly construed in terms of the actual amount of space that it occupies, thus, equivalent to, or properly containing the spatial

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11 We are here using the term 'possible' world just in the sense in which the 'real' world has a privileged character as the unique domain of truth-value instantiation tests. See Lehman (MS) and Ballim and Wilks (1991) for more on this issue, which is grounded in the distinction between a radically intensional as against a conventionally truth-functional, extensional view of semantics and meaning. Strictly speaking, from an intensional-computational point of view, all 'worlds' are 'imaginary,' even though in only one is defined as the one in which observational evidence is to be sought for the truth of propositions about it. We shall, however, not invoke this distinction below except where it is strictly necessary.

extension of the object itself. It is, in other words, a relationship between the object and space. In more precise mathematical language a PLACE is a possibly bounded neighborhood of interior points containing all the points defined by projecting into space (e.g., two- or three-space) all the points, say, on the surface of the object. That is the set of all points within the boundary of an object, or its projection (including the boundary points)[13].

The LOCUS of an object in projective geometry is defined at a more abstract level than is PLACE. LOCUS is the result of a projection, or collapsing, of a PLACE onto *any* one of its interior points, or the projection of that point. A LOCUS, then, is a neighborhood of possible projection points, the lower limit of which is one point (for an object that is itself a point) and any one is an algebraic 'ideal' of the whole. Thus, while PLACE is defined by the size, shape, and specific geometry of the object, LOCUS is not and, thus, can be arbitrarily reduced to a point by the application of a choice function for the ideal. Moreover, where PLACE is strictly related to the perceptual information that characterizes the objects in the world, LOCUS is free of perceptual constraints.

The concept of LOCUS and the concept of PLACE are rigidly and asymmetrically connected. A LOCUS can itself be taken as a PLACE, but the other way around is not

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12 In English it is possible to take advantage of the two lexical items just used, 'place' and 'location' (the colloquial correspondent to 'locus') to make a distinction (even though many speakers do not ordinarily make it in ordinary discourse). In other languages, e.g., Italian, the lexicon does not help. This, however, does not keep Italians from being able to conceptualise the distinction. We will return later to this difference between language (lexicon) and conceptualisation.

13 We could observe that (a) a 'place' is in fact a very special sub-instance of 'thing,' and that (b) whereas in the general case any 'thing' simply defines a PLACE, in the particular instance a PLACE, as a kind of 'thing,' doesn't need to define a PLACE, since it already is a place by its inherent conceptual-semantic nature, or features of representation. Or, in simple English, we would not put into this sub-formula anything like, say, *[Place$_X$] -> [Place $_X$], because the subscript index has no set-theoretical interpretation here independently of the conceptual semantic representation of the term in question. Again translating into plain English, 'a' PLACE has no meaning save as the place of, or relative to some THING or some one; abstract bits of space are not themselves places, from which it follows that there can be no set of places defined inherently.

It remains, nevertheless, somewhat puzzling that a word like English 'place' is not in the class of indexical shifters. It is a 'thing' that defines a point-in-space, i.e., a PLACE, so we can say 'at a place'. We think, however, that English 'place' refers to any PLACE defined by some, possibly unspecified 'thing'. If so, then 'here', 'there', 'now', and 'then' at any rate are purely derivative shifters, as ordinary usage suggests, boiling down to this place, that place, this time, that time. However, no equivalent reduction of the shifter class is possible for the personal pronouns and/or the demonstratives. For these we want the formula, given our general remarks about indexicality, [PLACE] -> [PLACE / 'thing'], where the slash (/) is made to indicate that the place is defined 'relatively to' a certain thing, or perspective. This needs lots more work, but it is surely in the right theoretical spirit. The notation is essentially taken from Jackendoff's work.

possible: where sets of points 'collapse' onto points, points cannot 'collapse' onto sets of points. The former relation is a function and the latter a one-to-many relation.

We start with the perception of a place, or a bounded space (of any object). All the points inside that boundary we address as a PLACE. We apply a choice function to the set of all points making up that PLACE and we derive a LOCUS (a point)[14].

The fine distinction between the two concepts is highlighted by the type of adjectives that the two nouns allow, respectively.

Thus,

(3) a. This is a big/long place
b. This is a beautiful place
c.? This is a big/long locus (location)[15]
d. This is a beautiful locus (location)
e.

It seems that while PLACE can be modified by adjectives entailing a specific geometrical description, LOCUS cannot readily do so except when we take 'location' to be a high-sounding substitute for PLACE.

The suggested distinction between PLACE and LOCUS points towards a separation of linguistic and conceptual representations, or between language and cognition. In fact, any Noun (N) represents an object (in our sense, see Section 3.1) and this latter occupies some space[16]. Thus, the possibility of treating any object as a LOCUS already indicates that some further level of representation distinct from the linguistic one has to be postulated. Further discussion along this line will be introduced in the investigation of the axiomatic contents of English spatial prepositions below.

Let us, however, go back to the preposition 'at' and its role as expressing a relation between two objects.

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14 It must be pointed out at this juncture that what just described is high congruent with the way in which the visual system is described to work in the literature about vision, e.g. see Marr (1982); Biederman (1990); and Churchland & Sejnowski (1992).

15 We are showing in parenthesis the word 'location' as an everyday discourse (American, though not necessarily British English) substitute for the more technical term 'locus'. .

16 In the case of abstract Nouns, the SPACE to be referred to is not physical but of a different nature.

3.3 *The Axiomatic Content of 'At'.*

The preposition 'at' expresses some relation between two objects unaffected by any geometrical feature of the objects themselves, and hence between the LOCI of the objects and not between their PLACEs.

We know that when an object A is conceived as 'at' another object B (where B may be a PLACE taken as a thing and projected to a LOCUS) it has to be in the neighborhood (including its interior) of B, that is, 'close' to B. How close? The answer is pragmatically constrained or determined. If we say that I am 'at the park,' I may be at an interior point of the park or, at a limit, near its border. This is because we know that a park has a spatial extension that one can go into. If we say that I am 'at my desk,' we know that I am not amongst the interior points of the desk but I am canonically very close to it and at a limit in contact with it. The possibility, then, of two objects' being in each others' vicinities, one inside the other, or just with their respective borders in contact is not determined by the preposition ''at'. It is the knowledge we have about the two objects and the constraints that this knowledge imposes on plausible relationships between them that provides the necessary construal. In other words, if we say that 'X is at Y', X may be inside Y, in contact with Y, or in the vicinity of Y that does not include its border or interior. But, when we substitute any two objects for X and Y, then, the knowledge about these objects will determine the possible types of relationship and the appropriate kinds of closeness..

Furthermore, since 'at' addresses only LOCI, we always have the possibility of conceiving the two objects as two points. Then, we conceive the area (*vicinity*) of the LOCUS (in any direction and including the LOCUS) of the secondary object (ground) as its *neighborhood*, again with boundaries pragmatically determined. Finally, we consider the LOCUS of the primary object (Figure) as a member of the set of points making up the secondary object's neighborhood. It is easy to see how, on this assumption, we can collapse into coincidence the 'contact' or 'in the interior' cases.

In our attempt to look into the meaning of the preposition 'at' we have been able to highlight the following:

- 'at' indicates a spatial relationship between two objects
- the relationship is between the LOCI of the two objects
- the primary object is in the neighborhood of the secondary object
- neighborhood includes vicinity (greater than zero distance) between the two objects
- neighborhood includes contact (zero distance) between the two objects
- neighborhood allows one object in the interior of (inside) the other

- neighborhood's border is pragmatically determined

We can now assign the following meaning to 'at':

(4) *'at': for LOCUS (of) A to be in the pragmatically determined neighborhood of LOCUS (of) B*

Thus, any two objects occupy two different PLACEs, even though, these PLACEs may be in a relation of closeness or inclusion or contiguity or partial overlap. The specific spatial relationship between those two PLACEs is simply not addressed. We consider just the uninstantiated LOCI of the two objects. In fact, it is only the LOCI of the two objects that may be conceived as points and consequently be at a limit in a coincidental relationship: so, Herskovits's and others' 'coincidence definition' turns out to be a limit instance only. PLACEs as instantiated LOCI cannot be put in that type of relationship. A PLACE entails an object (or its absence), and as such is unique in the world. There can be objects that are exactly the same in size, shape, appearance, etc., but each PLACE they define is nonetheless distinct, so that PLACEs may only be at a limit in an 'equivalency' relationship.

By keeping the concept of LOCUS distinct from that of PLACE we seem to have achieved significant clarity with a view towards understanding the meaning of 'at'. Note, too, if A is 'at' B, and at least has its LOCUS 'at' one of the interior points, including the boundary points, of the set of projections of the points of B, then we understand that we have chosen as the LOCUS of B the point coincident with the chosen LOCUS of A. This, we submit, is what makes the 'coincidence' case more or less prototypical for instantiations of 'at', where prototypes (cf. Lehman 1985) are not to be taken as defining their intensional categories.

Let us now look at the following sentences:

(5) a. Maya is at the desk
b. The desk is at the wall
c. Maya is at the wall

If (5a) and (5b) are true, it does not mean that (5c) is necessarily a plausible implicature pragmatically. This can be called the 'transitivity' problem. If A = B, and if B = C then A = C by the transitivity law, but this does not necessarily apply in (5), where 'at' is not the relationship '='.

For Maya to be 'at the desk' her LOCUS is in the pragmatically immediate neighborhood of the desk, at a limit as close as materially possible (contact in this case). The same is true of the relationship between the desk and the wall. However, the two pragmatically determined neighborhoods may not motivate transitivity. The distance of Maya from the desk (zero at a limit) does not put Maya automatically (transitively) into the pragmatic neighborhood of the wall

because the limit of her relation to the wall is contact and contact is precluded by the intervention of the desk. Only if we collapse the three LOCI of Maya, the desk and the wall onto three points, does the transitivity law apply. Consequently, for the preposition 'at' transitivity can always be posed as a limit possibility. The other cases have to be disambiguated by the pragmatic qualification of distance within the neighborhood expressed by 'at'.

In more formal terms we can say that where Maya's LOCUS is chosen from a set of the points making up the neighborhood of the desk, and where the desk's LOCUS is chosen from a set of the points making up the neighborhood of the wall, transitivity can apply because then the two sets above are equal. Note the independent evidence for this conclusion: let object 'c' be a 'region' and not a thing like the wall, e.g., 'the back of the room.' Now the implicature under examination follows because the 'back of the room' is itself a kind of neighborhood such that, anywhere the desk is in that neighborhood brings it about that the neighborhood of the desk is part of that of the room. Or, in somewhat other terms, 'walls' like 'rooms' have pragmatically limited, bounded neighborhoods — you can't get nearer the wall than right up against it, so that its neighborhood is one-sidedly bounded. neighborhoods themselves, of course, have no such properties.

The definition in (4) can be checked by looking into the consequences of its negation:

(6) Maya is not at the pool

This denies neither that there is a spatial relationship between Maya and the pool nor that the two LOCI (Maya's and the pool's) are in some type of relationship. Any LOCUS or its instantiation is in a spatial relationship with whatever is not itself, by simply the Law of the Excluded Middle. Maya is thus implicitly defined as being 'somewhere *other than* the pool.' Ideally, any LOCUS is 'at' another LOCUS. What is being denied is the membership of the LOCUS of Maya within the set of points that constitute the neighborhood of the pool with a pragmatically determined boundary. In other words, what a speaker would consider the appropriate distance between the two objects for one of them to be 'at' the other is not satisfied. (4) accurately entails what 'not at' indicates, thus, further confirming the correctness of (4).

We will now look at some controversial sentences. Let us start with the following sentence:

(7) The fish is at the ball

The definition in (4) allows this sentence even for a description that finds the fish 'in the ball' in the real world, but we know that, if this were the case, we would not use 'at' in our sentence but 'in'. Why?[17]

[17] We have to thank Alessandro Zucchi for pointing this sentence to our attention.

The first time we heard this sentence we thought that the fish being referred to was a wooden or metal one lying next to the ball or that it was a swimming fish that was now going by a ball (that was in the water as well). When we were told that after all our definition would allow that sentence as a description of 'a fish in the ball', we were somewhat puzzled. However, after a little thought it appeared clear to us that the phenomenon was not due to problems with our definition, but with language use or pragmatics.

If there is a lesson that structural linguistics has taught us, it is the fact that linguistic items as elements of the linguistic system never acquire meaning in isolation but always in relation to other elements. In this case the fact that English has a lexicalized form for one part of the meaning of 'at', that is, the spatial preposition 'in' for inclusion (see a later section of this paper for a more exact definition of 'in'), presents the speaker with a choice that adds new pragmatic meaning to the preposition eventually chosen.

In short, according to Grice's (1975) maxim of Quantity we would expect the speaker to choose 'in' if the fish is in the ball. If, however, he does not chooses 'in', then, we will expect the situation to be different (the fish is not in the ball). Consequently, the idea of a non canonical fish (as a wooden or metal object) can be investigated by the listener or that of a non canonical 'at' that is indicating a POINT in a PATH along which the fish is moving (see a later section of this paper on the definition of PATH for further clarification of this point).

Sentence (7), then, does not affect the content of the definition proposed in (4). On the contrary, we are starting to see how productive that definition can become in disambiguating sentences like (7) where pragmatic phenomena participate to the construction of the meaning of the sentence.

The following two sentences[18] need some further investigation:

(8) a. *X is at Maya
b. *X is at Alabama

In order to clarify the incorrectness of the first sentence, (8a), in spite of the definition provided of the preposition 'at' in (4), a digression on proper names and reflexivity as proposed by Lehman and Namtip (1985) and Namtip (1989) needs to be introduced.

Any name (like a reflexive) is an identity function on the set of persons (could be one). As such it is equal to the person, but not identical to that person. That is, a name is nothing but an identity element that is designated as a privileged member of the set of whatever you are talking

[18] We have to thank Paul Kay for pointing out to us these two sentences.

about (object, person, etc.). This identity function creates an element (that is, a name) whose level of abstraction makes it differ from some properties of the set onto which it is applied.

Now, if any person as an instance of an object (according to the definition adopted) defines a PLACE it must be possible to express this fact. The following sentence does exactly this:

(9) I am at the place where I am

However, if we use a reflexive, the sentence becomes odd:

(10) *I am at the place where myself is

In other words, what the two sentences in (9) and (10) show is the fact that reflexives (as well as names) are the result of the application of the identity function on the set referred to and as such they loose some properties of that set. They occupy a different abstract space than the set they derive from.

By reference to the distinction introduced above between the concepts of PLACE and LOCUS, a person is an object that defines a PLACE that can become a LOCUS via the application of a choice function. However, a name as the result of the application of an identity function seems to escape the possibility of a further application of a choice function so to become a LOCUS.

Put in a different language it can be said that a name is an abstraction on a person and as such cannot be abstracted again to become a LOCUS. This is because a name is not the canonical PLACE onto which the choice function can apply. Then, it retains as much characteristics of a PLACE to bar the possibility of the spatial preposition 'at' (defined as a single function onto two LOCI in the definition in (4) above) to apply.

The following figures may help to clarify the point:

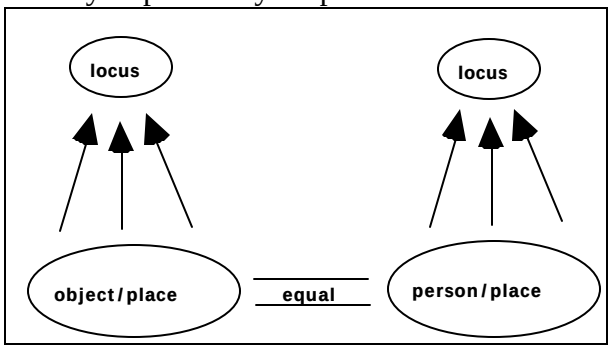

Figure #1: From object/person to LOCUS

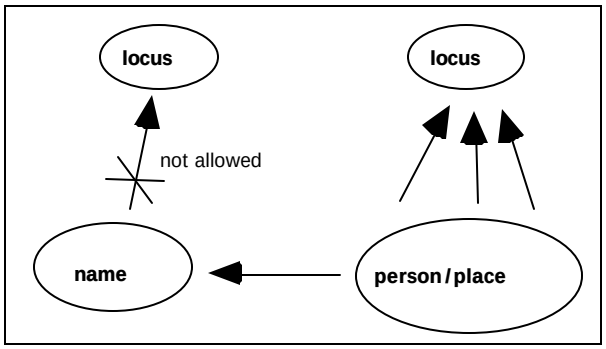

Figure #2: From name to LOCUS not allowed

What all of the above leads to is to the fact that the proposed definition of 'at' can be kept and the responsibility for certain expressions not being allowed can be assigned to characteristics of the nominal (proper versus common name).

There is an interesting sidelight on the foregoing. Persons themselves seem commonly to be understood, somewhat like names, as abstract entities, entities in a 'space' distinct from ordinary geographical 3-space. Persons have 'selves', identity elements, and these may be understood as occupying PLACEs, with attendant LOCI, in 3-space, much as do bodies; indeed, in some languages, e.g., Thai, the ordinary reflexive 'self' word is identical with that for 'body', although of course philosophically one may also understand the 'self' to be some enduring 'essence' transcending the temporally mutable body ('one's *true* self'). In any case, it is clear that a person (represented by a proper Name or a personal pronoun) is indeed 'elsewhere' than in 3-space in as much as it is common in many languages that a person cannot be spoken of as a physical place or location.

In English, for instance, something can be 'with' me, even 'on' me (understood as 'on my *person*' in the case meaning my physical body) but not *at* me. In some languages, e.g., Burmese, to say the equivalent of '[something is] *at* me' (*cun-do hma* ÄÙÿó–Œìæ –† ¿ – *hma*, the locative postposition) is understood as being something other than physical location, possession in fact. And while I may 'speak' *to* someone, and in general mark a person by the *to* indicating simply thematic goal-dative (give something *to* him [*thu. kou* ©... Ä¿»– *kou*, the oblique postposition] involving a transfer of abstract possession rather than of physical location) of an indirect object, and even throw something *to* him (*thu.kou*), I ought to 'send' something (purely physical transfer) *to* his *presence* (*thu. hsi kou* ©... Ü¡Ä¿»), a real PLACE in 3-space, the one his 'self' or body occupies, or may occupy, just as I can send it to his house, his office, and so on. Moreover, as mentioned above, whilst I can use the locative postposition after a personal name or personal pronoun (e.g., *cun-do.hma* — I-at), this can indicate only *existential* location, once again in some abstract space involving other than mere

physical presence. Burmese does not have a verb meaning 'have' and so to say that 'I have' something, I need to say 'It is *at* me' (*cun-do.hma hyi.te* ÄÙÿó—Œìæ———¢ ¿ì°—) — where *hyi.* is the existential locative verb signifying that something or someone exists, entailing that it exists somewhere, of course, in some space or other. So, if I say 'I have [some] money' (*cun-do.hma paishsan hyi.te* ÄÙÿó—Œæ—— †²æ–ö¿»Ü· ó——£ ¿ì°—— *paishsan* = 'money', *-te* = *realis* modal ending), I may add, somewhat sarcastically, '*da-bei-me.* (however) *mapa-hpu:*' ïøŒö†°— – – †öøü...ª meaning 'but it's not actually with me' (*pa* a verb meaning 'to be immediately present', and *ma ... hpu:* negation). This is a standard evasive reply to the begging question, 'Have you any cash?'. All this appears to involve some rather complicated conceptual mapping between 'spaces' of one kind and those of another, where only one is geographical/physical 3-space, a topic we cannot pursue further in the present paper.

Let us now look at sentence (8)b. We are immediately reminded of the fact that 'at Urbana[19]' would be acceptable in the same sentence. Why is 'at Urbana' acceptable and '*at Alabama' is not?. Well, first of all it is a problem of English and not of other languages like Tongan (Austronesian, Polynesian), where the only locative *'i* is used for sentence like 'at Alabama' and 'at Urbana'.

Second, in order to answer the question we need a little digression on the conceptualization of places vis à vis size. In English places are conceived in an hierarchy starting with the smaller non analyzable unit that will be called 'community' (whose size varies with discourse contexts) and building up to town, county, state, nation (this is a simplified hierarchy, but sufficient for our purposes). Each higher unit includes a number of the lower ones. In other words, a community is a set of places, and any other higher unit is a power set on the basic 'community' set.

Again, as for a Name, all the properties of PLACE are kept at the 'community' level and as such the choice function can apply and make the PLACE a LOCUS so that 'at' can be used. In all the other cases the abstraction already applied to the basic 'community' concept deprives the derived concept (e.g., nation) of some properties to be considered a proper PLACE. Then, the choice function does not apply and 'at' cannot be used insofar as no LOCUS is obtained. In cases where 'at' is used for say, Alabama, (take an astronaut looking down on earth and stating that an airplane is 'at Alabama') this is the result of the fact that the basic 'community' concept has been stretched by the context of the discourse so as to override the hierarchy. In other words, Alabama has become a plausible minimal unit of a new hierarchy dictated by the context (i.e. state, nation, continent, planet, etc.). As such the choice function applies to Alabama thus making it a LOCUS with the consequent acceptable use of the preposition 'at'.

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19 Urbana, Illinois is the town where University of Illinois is located.

It has become clear that this discussion needs further space, but it will be interrupted here for the time being. We have been able, however, to show how the definition of 'at' in (4) has stood up in the discussion of sentences that appeared to undermine its validity, and at the same time it has stimulated profitable speculations and interesting connections with other linguistic research. Furthermore, the explicit requirement that the preposition 'at' imposes on its two arguments (nouns, objects) as LOCI on one hand supports our suggestion of the distinction between PLACE and LOCUS, and on the other hand provides further evidence for the necessary separation of linguistic and conceptual representations.

3.4 *The Axiomatic Content of 'On'.*

Building upon our discussion of 'at', we now look into the meaning of the preposition 'on'. The first thing we notice is that LOCI (locations) are not usually 'on' each other (or 'in' each other). The locative relationship expressed by 'on' is between two instantiated LOCI, or more simply, two PLACEs. Let us recall that a PLACE is occupied by an object and that a PLACE (or an object) is a set of points within a boundary. What is then the specific spatial relationship that 'on' signifies?

In dealing with 'on' we cannot use the concept of neighborhood as we have used it for 'at', even though for two objects to be in the spatial relationship expressed by 'on' they must be in each other's neighborhood, or more exactly, one has to be in the neighborhood of the other. This neighborhood, however, is restricted to a specific form, or a limiting case. The two objects have to be in CONTACT. By 'in contact' is meant that at least a bounding interior point of the primary object has to be at the limit of closeness (zero distance) from a bounding interior point of the secondary object. The meaning we have just indicated for 'on' was, of course, one of the possible ones described for 'at'.

The necessity of lexicalizing this specific case may have arisen from the fact that the two objects now involved in the spatial relationship could not be treated as LOCI insofar as some of their geometrical characteristics are relevant. Minimally, in fact, a distinction between 'border' and 'interior' of the two objects is relevant. Notice that this distinction is not possible when an object is addressed as a LOCUS

In our brief excursus into the meaning of the preposition 'on' we have been able to highlight the following:

- 'on' indicates a spatial relationship between two objects
- the relationship is between the PLACEs of the two objects
- the primary object is in the neighborhood of the secondary object
- the two objects are in CONTACT with each other

Therefore, we can assign the following meaning to 'on':

(11) *'on': for two objects to be in CONTACT with each other.*

What is involved is in some sense gravitational contact: Maya, however much in contact with the desk is not 'on' it unless she is said to be, for instance, *leaning* on it, *stuck to* it, or else in non-suspended contact with its upper surface, and so on. Anything is 'on' the desk, if its contact with any possible (outer?) surface of the desk is gravitationally or otherwise adhesive contact with that surface.

We prefer 'in contact' to Herskovits's 'contiguous' because 'contiguous' seems to implicate rather contact extending over a linear area continuously.

A brief discussion in Clark (1996) of the different use of 'on' and 'in' in certain American and British expressions helped us grasp the reason that might have led Herskovits to maintaining her definition of 'on' in disjunctive format. The following paragraphs attempt to provide some further support for the suggested definition of 'on'.

(12) a. The house is on Vine Street (American)
b. The house is in Vine Street (British)

How does it happen that in American English the house is not in contact with the street and yet the language still allows the use of 'on'? Let us consider the street as a line and the house (at least a side of it) as defining a line segment. The British speaker will consider the house/segment as part of the line, that is to say any line of points running along and within the boundaries of the street, and in particular one chosen as running along the edge containing the relevant edge of the 'house,' and thus use the preposition 'in'. The fact that in Britain nowadays typical houses have pavements (American 'sidewalks') and even front yards separating them from the roadbed that ultimately defines something as a street becomes irrelevant insofar as a house is then conceived as including the front yard, and the pavement as a properly included extension of the street. Put more simply, a street, on this construal, 'properly contains' all the 'lines' just referred to.

Why then does the American speaker chooses 'on', and not 'in' as in British English? The answer lies in the chosen construal of the concept 'street'. Where the British speaker has been suggested to be reducing a street to a line, for the American this reduction is not appropriate (at least in this context). The street must be conceived as a ribbon with a house (including its yards) 'in contact' with it. Consequently, the choice of 'on' appears obvious and appropriate. Since Herskovits did not address this relevant issues of conceptualization she was led to the unnecessary disjunctive character of her definition of the preposition 'on'.

Before closing this discussion of the intension of the preposition 'on' it is worth looking into a couple of phrases pointed out to us by Alessandro Zucchi (personal communication to G.B.):

(13) a. the window on Vine Street
b. the bridge on the river Kwai

The first phrase, (13a), is common usage in both British and American English. According to the previous discussion we would have expected the British usage to be 'in Vine Street', but this is not the case. What has changed that is justifying the use of the preposition 'on' instead? Further discussion of the issue needs to be added below.

If we think of the discussion introduced above between the British and the American usages of 'in' versus 'on', we can comfortably say that the geometry involved was possibly within one plane. That is, we could think of a house as a two-dimensional object to be related to a line or a ribbon on the same plane.

When we introduce the 'window' concept a vertical axis and plane are added. Then, in the case of the British conceptualization of street, this latter becomes at least a plane, and in the American case it becomes a three-dimensional solid. It is the plane that the window belongs to that is put in a spatial relationship with the plane of the street. This is confirmed by the fact that, in such expressions 'on' can commute with 'onto', signifying that there exists conceptually a line-of-sight projecting from the window and passing to, or through the plane of the street. The American case follows from what just pointed out for the British case.

The second phrase, (13b), has a somewhat different account. We cannot say that the concept of river necessarily includes its banks so as to justify the possibility of 'contact' when the bridge does not touch the water. This reasoning would lead us towards a description of a person 'on a river bank' as being 'on the river' as well. And this is wrong in English.

What can be seen as a possible solution is to look back at the way in which bridges have historically come into being. Bridges have not always been the aerial structures we know today. They have often rested upon piers that themselves rested in the bed of the river. Consequently, it may be suggested that the possibility of lack of 'contact' in some of the bridges we experience nowadays does not (yet) affect the use of 'on' here. In any case, of course, we also have the expression 'over' a river; moreover, the use of 'on' in this instance seems rather specialized to one or two actual or imaginary bridges, and is not generally used. It may well be (the matter wants more work) that this is a specialized extension (metaphorical, perhaps) of the case in (13a): as when we say, instead of a house 'overlooking' the river', a house 'on' the river.

In concluding this discussion of the preposition 'on', we need to point out that at this point languages start to differ in lexicalization. Some languages do not make the distinction between

'at' and 'on' and have only 'at' as an available lexical item (Burmese and Tongan for instance). They none the less find other perfectly colloquial ways to express that distinction in meaning.

Finally, we underline a specific semantic function that the adding of the preposition 'on' to the English lexicon accomplishes. The relationship between LOCI as expressed by the preposition 'at' is now restricted to specific cases in which the PLACEs (and objects) instantiating those LOCI are in contact. The range of application of 'at' has been reduced by the introduction of 'on' that captures specific special cases of locative relationship otherwise left unaddressed or not distinguished by 'at'.

3.5 *The Axiomatic Content of 'In'.*

In a similar fashion as we have reasoned for the preposition 'on', it can be argued that the preposition 'in' introduces further constraints on the range of application of 'at' to locative relationships. We have already anticipated, in discussing the preposition 'on', that the spatial relationship expressed by 'in' cannot involve LOCI, but only PLACEs (or objects). Let us ask again, 'what is the specific spatial relationship that "in" expresses?'

For two objects to be in the spatial relationship expressed by 'in' one has to be in the neighborhood of the other. But neighborhood sets only a limit on what is being expressed; it is the interior neighborhood that is intended. One of the two objects or part of it must be *inside* the other one. In other words, at least some non-bounding interior points (with one of a limit) of one of the two objects must be in contact with some non-bounding ones (again with one as a limit) of the other object; note that, for a three dimensional second object, its interior points will, by definition of three-dimensionality, include all points of space properly enclosed by that object. Thus a knife can be stuck in (or into) a desk so shallowly as not to penetrate to any hollow inside that desk, whilst anything penetrating into, or properly contained within that enclosed hollow is also 'in' the desk (the first case in fact allows us to say that the knife is 'part way in' with respect to the desk[20]. The specific meaning we have just attributed to 'in' has already been described as one of the possibilities denoted by 'at'. However, as with 'on', it is not a spatial relationship between LOCI that is expressed, but one between PLACEs (or

[20] There is an interesting difference between closed and open volume-containing objects with hollows, i.e., non-solid, three dimensional objects. Something 'in' a desk, say, is, to that extent not merely in the desk's pragmatic vicinity: some part of it must meet the foregoing conditions. Something may, however be, say, 'in' the bend of the river, where we mean by that expression that part of it lies in a space bounded by the curvature of the riverbed and merely the reasonable vicinity of an imagined line drawn from one 'end' of the bend to the other. Open volumes seem to allow interior spaces to be construed as vicinities, which is not the case for closed volumes. We have not made this distinction precise.

objects). In fact, a minimal geometrical distinction between 'boundary' and 'interior' of the two objects must be made once again.

We can now briefly summarize our findings about the preposition 'in':

- 'in' indicates a spatial relationship between two objects
- the relationship is between the PLACEs of the two objects
- the primary object is in the neighborhood of the secondary object
- minimally one point of the boundary of an object is in contact with minimally one(not boundary) interior point of the other object

We then define 'in' as follows:

(14) *'in': for minimally one point of the boundary of an object to be in contact with minimally one (not boundary) interior point of another object*

There are some cases in which the definition just provided seems, on first view, inadequate to account for the spatial relationship described. Here are some examples:

(15) a. the flowers are in the vase
b. the bird is in the tree

Both the flowers and the bird in (15) can be in contact with the vase and the tree, respectively, but none of the points of their boundaries is in contact with an interior non-boundary point of either the vase or the tree. Herskovits (1986) appropriately addressed this issue and attributed the phenomena to 'geometric imagination', that is, the process by which 'geometric figures are imaginatively projected onto the real objects' (Herskovits, 1986, p. 43). Usually, concave or convex objects (or spiky ones like trees) are reduced to the geometry of the solids that most closely approximate their contours. Good examples are vases or glasses reduced to cylinders and top parts of trees reduced to sphere-like or conical solids[21].

Then, in (15) the vase might be conceived as a solid (cylinder-like) including its inside empty space bounded by the plane imaginatively passing through the rim. The tree, on the other hand, would be conceived as a volume bounded by the outline of the tree's branches (canonically a sphere-like one). This being the case, the definition (14) accurately captures the meaning of 'in'.

As in (12), it is a matter of conceptualization affecting the two objects that seemed to lead to problematical examples. Once we had clarified what these issues of object conceptualization were, the appropriate meanings for the prepositions under investigation could be clearly specified.

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21 This process is very similar to the one called for by Marr (1982) and Biederman (1990) for object recognition.

3.6 *'At' vs. 'On/In': Further evidence.*

We propose that the three prepositions under investigation be divided into two groups, 'at' in one group and 'on' and 'in' in another. They all share a basic axiomatic content that reduces to the meaning of 'at'. While 'at' expresses a spatial relationship between two LOCI, 'on' and 'in' express a spatial relationship between two PLACEs (or objects). In this sense, then, 'at' is the 'basic' preposition, while 'on' and 'in' are special cases where certain types of constraints have been imposed upon the range of application of 'at'.

A summary of the axiomatic contents of 'at', 'on', and 'in' so far discussed is introduced in Table #1 below.

Table #1: Axiomatic Contents of 'At', 'On', 'In'

PREPOSITION	LOCUS	PLACE	neighborhood		
At	x		x		
			INSIDE	CONTACT	VICINITY
On		x		x	
In		x	x		

The words in capital letters heading the columns in Table #1 indicate concepts. The concepts INSIDE, CONTACT, and VICINITY are axioms of the concept neighborhood. The way in which each English spatial preposition is related to a combination of these concepts is expressed by an 'x' appearing in the appropriate columns in each row. The lack of an 'x' in other columns in the same row stands for the absence of that concept in the axiomatic content of that specific spatial preposition.

The table clearly highlights the difference between 'at' on the one hand and 'on' and 'in' on the other as due to their relationship to the concepts of LOCUS and PLACE, respectively. Moreover, the proposed 'basic' nature of 'at' can be inferred from its simpler internal composition (only two concepts, LOCUS and neighborhood) and by its possible extensionally wider applications. Furthermore, it can be seen how 'on' and 'in' address only a specific part of the concept of neighborhood, that is, CONTACT and INSIDE, respectively.

Before starting an analysis of other locative prepositions we would like to point out that there is other independent evidence that supports our distinction between 'at', on the one hand, and 'on' and 'in', on the other. In fact, let us look at the following sentences:

(16) a. Maya is at the door
b. Maya threw a ball at the door
c. Maya is on the table

d. Maya walked on the table
e. Maya is in (the house)
f. Maya went in (the house)

It seems that all three prepositions, 'at', 'on' and 'in', can be used in expressions involving either a LOCUS or a PATH (motion — cf. Jackendoff 1983, 1990). The concept of PATH is an abstract one that needs to be explicitly defined (see Section #8.2 below for a discussion of the concept). A PATH is an ordered sequence of points (LOCI) with a beginning, a body and an end. Each of its parts (LOCI) can be instantiated to PLACEs (or objects).

In (16b, d, f) the notion of PATH is introduced by the verbs 'to throw,' 'to walk' and 'to go', respectively. In (16f) the PLACE instantiating the end (LOCUS) of the PATH comes into contact with the interior non-boundary points of a PLACE or object (the house), thus meeting the definition in (14). It is worth noticing here that 'to the house' would have left the distinction between being finally inside or outside of the house unspecified. This difference will become clear once we have dealt with the preposition 'to'.

In (16d) the line (or successive PLACEs) instantiating the series of points/LOCI constituting the PATH come into contact with some bounding, but not otherwise interior points of a PLACE or object (the table), thus complying with the definition in (11). In (16b) a PATH is expressed whose direction is the house, but we are not sure if the ball ever reached the door or not ('to the door' expresses that). Why this uncertainty? The answer comes from our proposed definition of 'at' in (4).

We know from (4) that the spatial relationship that 'at' expresses is between two LOCI, in this case the LOCUS of the end of the PATH and the LOCUS of the door. They are in each other's neighborhood, but nothing more is specified. Consequently, it is by definition impossible to establish if the ball ever reached the door in (16b). It seems to us that these examples have provided support for our classification of the prepositions 'at', 'on' and 'in' into two separate and distinct groups with 'at' being the 'basic' locative preposition.

This conclusion, besides being supported by the arguments and definitions we have provided, is nicely confirmed by linguistic data from languages of different families across the world. In Tibeto-Burman languages such as Haka Chin (Lai holh), or Lushai (Mizō ṭ àwng), only the locative preposition 'at' (a÷, spelt *ah*) is available; the same is true for many Polynesian languages, e.g., Tongan; in languages such as Burmese the only other preposition (postposition) is the directional *kou* (Ä), that also signifies the endpoint of the PATH constituted by the relation between the subject argument and its objects — it serves to mark the grammatical (oblique) case of direct and/or indirect objects. This same phenomenon is present in Tzeltal, a Mayan language. Some examples are to be found in the Appendix, showing that, as we have

argued above, languages of this supposedly very different type from English etc., in having a very different array of adpositions (prepositions /postpositions) in fact bear witness to the correctness of the *successively* derivative character of postpositions other than the generic 'at'.

Before closing this section we note that our analysis of 'at' throws light on a cloudy part of Clark's (1973) suggestion about the nature of 'at'. In that work he described the spatial preposition 'at' as the most simple (p. 41), but in his acquisition data he presented 'on'/'in' as first learned (p. 61). The analysis of 'at' that we have proposed avoids any suggestion of contradiction in this. Whereas 'at' is still the most basic of the locative prepositions, it requires the greatest amount of pragmatic knowledge. Consequently, it should be expected within the present analysis that 'at' would put a heavier burden on the learning capacity of children. The acquisition data are congruent with the formal semantic analysis. One needs specific instances before full generalization is available, especially for an Acquisition Device that arguably constructs its cognitive representations almost wholly upon the evidence of concrete, rule-governed, particular instances.

4. *The Axiomatic Contents of Prepositions Related to MOTION*.

We leave the realm of locative spatial relationship and devote this section to MOTION. Our formal approach will continue to make use of the analytical concepts already introduced, e.g. PLACE and LOCUS. At the same time new ones will be defined and used for the conceptual analysis of some prepositions involving motion like 'to', 'from', 'via' and 'towards'.

4.1 *MOTION, PATH, and Prepositions*.

Whenever we deal with objects in motion we know by definition (and experience) that their MOTION takes place along a PATH, indeed defines a PATH. We will clarify in the following discussion the specific relationship that exists between MOTION and PATH. Linguistically expressing MOTION of objects, then, entails being able to describe the PATHs that these moving objects determine. In order to describe a PATH we need minimally a *source*, that is the point from which the MOTION starts, and a *goal*, that is the (intended, intensional) limit of all the points where the MOTION is to end. All the intermediate points will be considered the actual PATH (or route) the object moves along; the source and goal are the limits, or closures of the PATH. However, let us recall that MOTION is a vector, a PATH with a *direction*. Since we are dealing with moving objects, then, the concept of direction will necessarily appear as an axiom of the concept of PATH.

Fillmore (1975: 26) proposed regarding Location, Source, Path, and Goal 'as expressing the basic static and dynamic positional notions' . And he added:

> 'Taking this point of view, we can regard the locational and directional notions associated with "at", "from", "via", and "to", as being basic, regarding all other locative or directional concepts as being conceptually more complex.' (Fillmore 1975:26)

We have already, in our previous discussion of 'at', 'on', and 'in', agreed with Fillmore about the basic nature of 'at' (we do not necessarily agree with him about 'at' being 'conceptually less complex'). In what follow we will address the remaining part of Fillmore's proposal, that is, the basic nature of 'from', 'via', and 'to', and try to see if and why his suggestion may still hold.

4.2 *Towards a Definition of PATH.*

Let us first introduce some sentences:

(17) a. Maya went to the zoo
b. Maya came from school
c. Maya flew (to Rome) via New York
d. Maya headed towards the park

All of the verbs of (17) express some kind of motion, defining a PATH. The prepositions following the verbs address some aspect of this PATH. In (17a) the *end* of the PATH (goal) is indicated by 'to'; in (17b) the B *beginning* of the PATH (source) is indicated by 'from'; in (17c) the *body* of the PATH (or route), or some part of it, is indicated by 'via'; in (17d) the *direction* of the motion and consequently of the PATH is indicated by 'towards'.

If all these prepositions are related to the notion of PATH, it is advisable to try to find a clear definition for this concept. In Jackendoff (1983, 1987, and 1990) we find some indications on how to deal with this issue. In his discussion of the spatial domain he proposes two major, and for him primitive (see Appendix) ontological categories, PLACE and PATH[22]. The conceptual structure of the latter he indicates as being made up of a path-function: either

[22] Note here that PATH needs to be specified, as PLACE already is, as a special sort of (pseudo-) THING, presumably a sub-instance of PLACE. One can after all be 'on,' or 'along' (cf. Jackendoff) a PATH in English and other languages; and it is thus proper to say that PATH is not only a pseudo-THING but more specifically a sub-instance of PLACE, defined with reference to an ordered pair of THINGS. This is a step Jackendoff fails to take, and because of this his treatment lacks adequate generality and accountability of observable language facts. We are certain that the correct account is to say that the predicate of motion itself acts as a PLACE-function in the sense of inducing the PATH as the relevant ordering; the adposition then is f, the functor, taking $THING_i$ (the thing in motion) to $THING_j$ (at least as a limit); and the ordering itself is in terms of a quasi-continuous succession of PLACEs of $THING_i$, such that, at the limit, $THING_i$ is 'at' the PLACE of THINGj, as above in the locative case.

'to', or 'from', or 'towards', or 'away-from', or 'via', "that map a reference Thing or Place into a related trajectory [Path]" (1990: 44).

He has added to our inventory of Path prepositions 'away-from', that stands for the directive counterpart of 'from'. However, we still do not know why he chooses to put directive ('towards', 'away-from') and PATH prepositions ('from', 'via', 'to') under the same heading. We do not know either why he chooses 'towards' and 'away-from' as the only path-function to stand for the directive ones (there are others), or why he chooses 'from', 'via', and 'to' as the only Path-functions to stand for all the non-directive path prepositions (here too there are others).

In Jackendoff (1983:165) we are introduced to a taxonomy of PATHs, bounded ones (the reference object or place is an end point of the path), directions (the reference object or place does not [necessarily] fall on the path), and routes (the reference object or Place is related to some point in the interior of the path). It is this taxonomy that led Jackendoff to include in the structure of the ontological category PATH the variety of path-functions mentioned above. However, in order to explain his other choices we need an analysis of those prepositions (or path-functions) coupled with a more precise definition of what a PATH is.

Before addressing these questions directly, let us look carefully at the proposed taxonomy of PATHs. Is it true that bounded PATHs are distinct from routes? A PATH always has a *beginning*, a *body* (or 'route') and an *end*. We may choose not to address any of those three parts, but this does not make Paths differ — what one expresses about a PATH is not coextensive, necessarily, with one's understanding of PATH (semantics as against cognition, again). Moreover, any PATH has a *direction*, though we may choose not to address it. What happens when we decide to address the *direction* of the PATH? What is probably confusing the issue is that we are not provided with a clear distinction between the concept of MOTION and that of PATH. In the following, therefore, we address this issue.

MOTION means that an object (or thing) is changing its PLACE in SPACE (going from one PLACE to another) in a specific direction for each successive pair of PLACEs (these directions can be the same for successive pairs, or differ for specific groups of PLACEs)[23]. This change happens over specific ordered instances of *time* that are unique EVENTs. That is,

[23] There are deep problems to be dealt with (see Lehman MS: #8) regarding the fact that this hopping is a series of events. The quasi-continuity of a temporal PATH (amounting to the notion that there is not imagined to be an infinite number of such successive PLACEs) can be made to follow from treating temporality not with the apparatus of tense logic but with that of the aspectual logic of the space of Events.

What we say in the sequel is merely convenient. Actually, we must generalise MOTION to the limiting case of OBJECTs, as it were, permanently in motion, in which case the bounding END-points are still defined, though infinitely far away in both directions.

each motion is never the same (in time) as any other motion we may experience or imagine. In fact, we can have a repetition of a MOTION of the same object along the same succession of PLACEs, with the same direction, i.e., in the same order, but we can never repeat the same instants of time. Moreover, we can 'bound' this MOTION, that is, assign a boundary by indicating a beginning and an end to it. In order to do so, we have to consider two PLACEs where the objects are not moving (actually, in the case of the goal at least, not intended to be moving) as the endpoints of the MOTION. In sum, MOTION is an ordered sequence of PLACEs in time with a direction, possibly bounded by two PLACEs without direction, i.e., the beginning is not a successor and the end is not a predecessor PLACE conceptually speaking. Or we can say, MOTION is an collection of vectors (successions of points with a direction) ordered in time and bounded by two points.

Now we can define the concept of PATH as a geometrical (purely spatial) description of MOTION 'abstracted' from MOTION itself. In fact, our focus is no longer on the moving object, but on the ordered sequence of PLACEs. These latter are addressed as LOCI (or points) in a specific order, that is, with a specific direction. In other words, what are regarded as relevant in the concept of PATH are the spatial characteristics of MOTION, as a species of EVENTs and not the fact that it is happening in actual time as given by the continuous time of tense logic (cf. Lehman MS: #8). We shall, in another paper, address the important issues in this connection, in particular the idea that EVENTs, as here understood, are indeed like PLACE's in a modal-cum-aspectual STATE SPACE, rather than LOCI, points on the everywhere dense real time line.

Having said this, we have already laid the ground for the unification of PATHs in space and PATHs in time, as in 'from now until Four O'clock, ' or 'from Monday to Thursday.' The concept of MOTION is inextricably tied to time, but the concept of PATH is kept free from it. In fact, we can indicate a PATH at one time and then indicate another PATH at a different time, and state that they are exactly the same without incurring a contradiction as would happen if the two parts of the comparison were two instances of MOTION in real time. The concept of TIME participates in the construction of a PATH, but the relations on instances of a PATH are not unique.

Another feature that the concept of PATH retains from that of MOTION is the combination of ordered sequence and boundedness. The interior points of a PATH are an ordered sequence of LOCI. In other words, they are LOCI with a direction, vectors of finite magnitude. The magnitude of a vector will be called its body and consists of a set of LOCI whose members may at a limit be one, thus, overlapping with the first constitutive LOCUS. The boundary of a PATH also consists of LOCI, one a vector that lacks left directionality (beginning), and another, a vector that lacks right directionality (end). In sum, then, a PATH is

an ordered collection of vectors[24] in SPACE that is bounded by two vectors disjunctively lacking either left or right directionality, respectively.[25]

Somewhat anticipating the spirit of our suggestion is the discussion that Miller and Johnson-Laird (1976) provide of the issue of the nature of PATH:

> 'The conceptual core of the system for indicating motion is the path, which usually has a distinctive beginning and end. As an object traverses a path it passes each successive location at a later moment in time, so time indices can be associated with each location. What the linguistic expressions must describe, therefore, is the logical equivalent of a sequence of AT relations.' (Miller and Johnson-Laird 1976: 406)

\ The proposal to start with MOTION seems to be in line with Miller and Johnson-Laird's approach. Also important is the suggestion that PATH need to be looked at as 'a sequence of AT relations.' By the definition of 'at' in (4), we may state that a PATH is just a sequence of LOCI, which we claimed in discussing (16), but without any justification.

What is missing in Miller and Johnson-Laird's (1976) discussion is the full exploitation of their introduction of the concept of time and how it participates differently in the definitions of MOTION and PATH, respectively. Furthermore, the lack of the clear distinction we have provided between the concepts of PLACE and LOCUS is also conducive to some obscurity in their discussion.

So far, then, we have found ample justification for Jackendoff's (1983) choice to include under the ontological category of PATH the variety of path-functions we have indicated, that is, both directed (possessing inherent direction in real time motion) and non directive (a path in space rather than in time). In fact, the concept of PATH contains within our definition two boundary points (beginning and end) and a vectoral body or route, and consequently a direction.

4.3 *The Axiomatic Content of 'To'*.

We must still address the second unexplained (but intuitively suggested) step that Jackendoff took, that is, the choice of the five prepositions 'from', 'to', 'via', 'away-from' and

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[24] Notice how the only difference between a PATH and a VECTOR is the fact that the latter lacks an end, though it has a specific magnitude.

[25] To say that a point is a vector is to say that it is a predecessor or a successor to some adjacent point in the ordered sequence; a beginning lacks a predecessor, and an end lacks a successor, and direction is given minimally by either.

'towards' to be the path- functions that realize a PATH. In order to account for this, we have to refer back to our definition of PATH and to our discussion of LOCUS and PLACE.

Let us first discuss the preposition 'to' by looking at sentence (17a), which we repeat here as —

(18) Maya went to the zoo

In (18) the primary object (figure) is Maya, whose LOCUS is by definition a point. She moves along a PATH, whose beginning and whose body (route) we are not informed about. What the preposition 'to' indicates is that the point/Maya traveled in a PATH whose end point was at the zoo. But, where was Maya when she stopped moving? Was she 'in' the zoo, 'on' the zoo, 'near' the zoo…? Nothing in (18) answers this. We know that both endpoints of a PATH are LOCI from the definition of PATH we have provided above. The LOCUS of the last vector will be the end of the PATH. Maya need not even be thought to have been traveling a straight PATH, and the successive vectors need not individually all point in the same direction. We define the end-point of the PATH, presumably, on the basis of some notion either that she may have intended ending up at the zoo, or that her estimated or known PATH, for perhaps pragmatic reasons (e.g., that it is a road, however circuitous) should have ended up at the zoo. The implicature that she did end up at the zoo is pragmatic, readily canceled, since we can easily append to (18), the tag, '... but she never got there.' Also, the zoo must be treated as a LOCUS because no geometrical characteristic of it would affect the type of relationship that is expressed by the preposition 'to'. In other words, it seems that the relationship between the end of the PATH (LOCUS/point) and the zoo (LOCUS/point) is one of LOCUS to LOCUS, i.e., an 'at' relation. Maya moved from an unspecified point, she followed a PATH, and then she reached the end of her MOTION, but, above all, it is the abstracted PATH that ends *at* the zoo.

If we want to be more discriminating in our spatial description of Maya (primary object/figure) we have available the choice of adding to the preposition 'to' the prepositions 'on' or 'in' and obtaining 'onto' and 'into'[26]. The resulting meaning of the resulting prepositions, however, will lead us from the realm of relationships between LOCI into the realm of relationships between PLACEs as our definitions of 'on' and 'in' in (11) and (14) above indicated.

In sum, the preposition 'to' expresses an 'at' relationship between the LOCUS of the secondary object (Ground) and the LOCUS of the end of a PATH. We can consequently start to see why the preposition 'to' is the one chosen by Jackendoff (1983) to be a Path-function and not, for example, 'onto' or 'into.

[26]The meaning of 'into' and 'onto' is not exactly the sum of 'in' and 'on' with 'to', but we are not addressing this point here. It is an issue that deserves to be fully addressed some time in later work.

The close relationship between the meaning of the prepositions 'at' and 'to' is also evident if we leave English and look very briefly at Italian[27]. Let us look at the following sentences:

(19) a. Maya andò *a* scuola
(Maya went *to* school)
b. Maya era *a* scuola
(Maya was *at* school)

In (19) the same preposition is used to indicate what in English is expressed by the prepositions 'at' and 'to', respectively. In Italian it seems that the meaning of the preposition 'a' (at/to) is reduced to the meaning of 'at' in English that we repeat here:

(4) *'at': for LOCUS (of) A to be in the pragmatically determined neighborhood of LOCUS (of) B*

The distinction that the preposition 'to' makes in English is not available in Italian. The fact, then, that one of the LOCI is the end of a PATH is not considered relevant enough to be lexicalized. MOTION is expressed by the verb and that is regarded as sufficient to indicate that one of the two LOCI considered in the relationship must be a point on a PATH (the contrast with 'da' /from and 'via' will provide the missing information to disambiguate this point as the end of the PATH[28]).

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27 We read in Lyons (1977) of the same phenomenon in French, German and Latin.

28 The English preposition 'from' translates with '*da*' in Italian. The phonological resemblance between the prepositions *'da'* (from) and *'a'* (to, at) may not be accidental. Possibly, it is the result of a fusion between '*di*' (of) and '*a*' (at).

It is interesting that most Tibeto-Burman, and many other South East Asian languages have no word, adpositional or otherwise, like the familiar 'from.' It might be thought that this expressive distinction represents a truly distinctive way of conceptualising space, but that is readily shown false. In Burmese, to take one example, the tail, or origin point of a PATH is necessarily expressed by a postposed particle, *ka*. (Ä), that marks noun phrases as having been picked out conceptually by a contrastive choice function — so that an NP with this particle postposed refers to it, so to say, '*rather than, or as against* anything else.' Any NP other than an object argument can be so marked, and this exception is no doubt due to the fact that notional PATHs are once-and-for all defined in the first instance, by the Goal argument, leaving only the tail or source with alternative options remaining to be defined (see the Appendix, where this is directly underlain by the relevant notion of ordering that defines a PATH). This is easily seen by paying attention to the fact that to say, e.g., that I went to Mandalay is true regardless of where I started, and to say that Rangoon is 350 miles away is true if, but only if, an appropriate starting place, such as Mandalay, is selected. Lest one imagine that *ka*. postposed to a source argument somehow means the same thing as English 'from,' it must be pointed out that this construction is just a contraction, whose full form is, say, '*Mandalay* ***ka.*** *nei-pi:*, (†òÍŒ§ª–Ä–ŒóöýI ª)' signifying 'having been [first at] Mandalay., where the usual phonologically empty pronoun represents whoever or whatever started from there, '*nei* (ŒÓ)' is the verb meaning 'to be, or remain at,' and *'pi* (ÖÃ¿ª)' marks the perfective aspect for the event of 'being at.' Much the same applies to a language like Haka Chin, or Lushai, where the source argument is marked simply

The external evidence just provided, however, was not strictly necessary. If we look carefully at just English we find this type of example:

(20) a. She launched *at* his neck.
b. She immediately went *at* it (some kind of activity).
c. She threw the stone *at* him (as against throwing it [less precipitously] *to* him)

Both sentences contain a verb of motion, and both contain the preposition 'at'. In (20a) it is clearly the end of the PATH that is addressed by 'at', thus, confirming once more our proposed analysis. In (20b) something slightly different is being expressed. We still have a verb of motion with a consequently defined PATH, but this time the preposition 'at' is not addressing any part of this PATH, but the PATH in its entirety. In fact these motions verbs, involving as they do something like precipitous motion, effectually presuppose that the motion is what we may tentatively call strongly intensional, targeted motion (Ngwei Thein, personal communication). In such cases this targeting apparently preempts what is entailed by motion over a PATH in favor of the targeted end point of that PATH. Such readings of verbs of motion induce the collapse of the PATH on its endpoint, with the consequence that the latter, as an object, is construed as less a dative-goal and more patient like, thematically. Note in particular that it must indeed be the PATH that is thus collapsed in as much as, once again, in her actual motion 'she' need not have actually arrived at that endpoint/LOCUS. Thus, 'she' in running, launching herself, etc., is perceived as simply being *at* a PATH collapsed on its end-point LOCUS. Along the same lines, we note that the lower limiting instance of a PATH is a point/LOCUS, and thus (if trivially) all LOCI are PATHs, but not conversely. We can use this observation below in understanding why it is that, if a PATH is actually mentioned, it is its endpoint that must be mentioned ('he went *to such-and-such a place*'), whilst its starting point need not be mentioned, though if mentioned must be highly marked ('from' and so on, whereas, the endpoint can, as in Italian be mentioned merely as a location/*at* , or in Burmese without any postposition at all —*yangon thwa:de* ¢ó—Ä»ó— ©ÿæªì°––, 'He went [to] Rangoon'[29]). It is claimed that this is because any PATH necessarily has a *length*, and the

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by taking the generalised adverbial postposition (often best glossed in English by the affix -ly) '-*in*, as in *Halkha-in* (literally Haka-wise/Haka-ly), here serving for what would in English be 'from Haka, 'i.e., considering Haka the starting point as against any other place.

29 It is worth observing that one cannot say that the absence of marking is due to the idea that the verb of motion already entails the fact of directionality or length. For, these verbs do not necessarily entail non-trivial directionality and length. Consider

i. He ran *in place*.

Running 'in place' takes one from where one is to that same PLACE, itself, hypothetically at least from an initial LOCUS to itself. Here the endpoint is indeed still mentioned ('in place'), signalling the special case of a (pseudo-) PATH, one lacking non-empty length and direction.

minimal *non-trivial* length of anything is, of course '1' unit of measurement. If, then, a PATH be collapsed upon (identified with) its endpoint, its length is minimized to 1, which is to say, a minimal difference between the starting point and the endpoint, where the former if of course necessarily (and as a matter of computational completeness) 0 [zero]. From this it follows directly that a PATH, in as much it entails *non-zero* length (motion being, by definition, a function of one or more non-zero vectors) does not include its own starting point (zero point). Conversely, a verb such as English 'to leave', does not necessarily mention the induced PATH, and one is free to mention the starting (exit) point 'from' which the PATH is induced without it being necessary to mark it especially ('he left home, left Chicago, etc.' — note that, in an expression such as 'He went from Chicago', there is zero-anaphoric reference to the endpoint because such a sentence is acceptable only if we understand it as equivalent to 'He went *there* from Chicago', the goal or endpoint having been effectually mentioned earlier on in the discourse). Similarly, 'he went out from [as against 'of'] the room' sounds at best odd, whilst 'he *came* out from the room' sounds better because the verb 'come' entails an end-point, which 'go' does not.

Now we know that a PATH as a collection of vectors is a PLACE, and thus can be treated as a LOCUS (by precisely its collapse, or perhaps its projection, onto its one distinguished LOCUS, its endpoint). Finally, the meaning of PATH is, at the level of conceptual structure (see Jackendoff, 1990), identified with an ACTIVITY.

The account just proposed is shown to be correct in its essentials through two observations. First, it is certainly not the precipitousness of the motion that is at work in (20) because more or less *any* motion verb can take 'at; instead of 'to', at least somewhat figuratively. One can 'toss' or 'lob' something 'at' someone or something; one can even 'walk' (possibly even 'crawl'?) 'right "at"' someone or something in appropriate contexts of deliberate intention, e.g., if one decides to go thence regardless of the lethal consequences sure to result for one. And that 'targeting' is enough to induce an 'at' reading is shown by the fact that the verb need not even be one or motion. For, one *aims* 'at' a target, and one shoots at it having first aimed, of course. Aiming, after all, conceptually induces a PATH in the abstract, namely, the one given by one's 'line of sight' (see Bennardo 1996 on lines of sight and associated PATHs), just as one 'looks' *at* someone or something. [30]

Each of the sentences in (20) assures us of the correctness of our proposed analysis, where the preposition 'to' is seen as a special case of the most basic spatial relationship we

30 It may well be the case that the *irrealiss* tense' marker of the English infinitive, 'to' may also have to do with an abstract PATH in modal state spaces, from the vantage point of the *realis* 'present' of the speaker-hearer 'to' the alternative, unrealised (*irrealis*) state of the event named by the infinitive verb. This deserves investigation elsewhere. On modal/aspectual state spaces and 'tense', see Lehman MS: #8).

indicated as the meaning of the preposition 'at' in (4). Basically, a further axiom has been added to the two axioms (LOCUS and neighborhood) making up the preposition 'at'. That is, the concept 'end of PATH' is now the third axiom in the make up of the axiomatic content of the preposition 'to'.

4.4 *The Axiomatic Content of 'From'.*

We now return to the preposition 'from' in (17b), which we reproduce here:

(21) Maya came from school

Before starting our analysis we would like to point out what Fillmore (1975) says about the preposition 'from':

> There are in English many directional complements of the preposition phrase type which have the same form as the underlying locative complement. This is true of the complements of the Path and Goal type, but it is not true of Source complements, because it seems that Source complements are essentially always marked with a Source-indicating preposition ['from']. (Fillmore, 1975: 26)

In other words, it seems that 'from' is somehow different from the other PATH prepositions. We are not informed, however, why this is so. The intuition, however will be shown to point in an important direction.

By looking at (21) we realize that the primary object (figure) of Maya (LOCUS/point) has moved on a PATH whose end (by default at least, the 'here' of the speaker) and route are not indicated. We infer that it is the endpoint of the PATH that we have called beginning that is addressed as one element of the spatial relationship that the preposition 'from' refers to. We know from our definition of PATH that the beginning of a PATH, being one of the two endpoints, is a LOCUS/vector without left directionality (in this case).

The geometrical characteristics of the secondary object addressed by 'from' are irrelevant; it can be of any size, shape, or dimensions. Consequently, the secondary object can be treated as a LOCUS as well. We can state, then, that the preposition 'from' is basically a sub-type of the preposition 'at' (see footnote 26 for evidence from other languages). In fact, it expresses a spatial relationship between two LOCI without any limitation on the meaning of 'in the neighborhood.'

Let us now look at two examples provided by Fillmore (1975: 26):

(22) a. The cat ran behind the sofa (Goal)
b. The cat ran behind the sofa out the window (Path)

and a third that he implies without explicitly providing:

d. The cat ran out from behind the sofa (Source)

In other words, the description of the MOTION of the cat in relationship with the PLACE 'behind the sofa' does not explicitly require that the prepositions 'to' and/or 'via' be expressed, while it needs the explicit presence of the preposition 'from'. No answer to this phenomenon is to be found in Fillmore (1975).

Furthermore, there are some languages like Burmese (Tibeto-Burman) where the presence of an adposition corresponding in meaning to the preposition 'to' in English does not entail others corresponding to 'via' and 'from'. In fact, only adpositions corresponding to 'at' and 'to' exist in Burmese. We will try now to address these language internal and cross-linguistic phenomena in light of what said so far about the nature of PATH.

Let us recall that a PATH is a collection of bounded vectors and that a vector is always defined by its characteristic value, that is, by its 'head' (graphically the point of an arrow) and never by its 'tail' (or beginning) or 'magnitude' (or body). In order to explain this we have to remember that the magnitude of a vector is measured in cardinal numbers and cardinality starts with quantity '0' and not quantity '1' as for ordinality. What needs to be expressed when addressing a vector is then its 'head' and 'magnitude' and not its 'tail', that is, necessarily '0'. What was just said might lead one to conclude that necessarily 'to' and 'from' both need to be linguistically expressed. This, however, is not the case. Here again the distinction between what is linguistically as against cognitively necessary allows us to suggest an explanation.

In sentence (22a) the concept of PATH is introduced by the motion verb 'to run'. Conceptually the whole PATH is introduced with its axiomatic content, that is, beginning, body, end, and direction. The phrase following the verb qualifies what part of the PATH the sentence is addressing. Had a prepositional phrase headed by 'to' followed the verb the end of the PATH would be addressed. But, the following prepositional phrase is headed by 'behind', a stative preposition. Why then is the sentence still interpreted as addressing the end of the PATH?

The vectoral nature of PATH as discussed above tells us that if a part of the PATH is expressed it has be necessarily either the 'head' (end) or the 'magnitude' (body). The former being a single LOCUS and the latter a collection of non-zero LOCI (vectors). The singular object (see our definition of object in Section 3.1 above) linguistically expressed by the prepositional phrase ('behind the sofa') lead us towards an interpretation of the sentence as addressing the end of a PATH. On the other end, the simultaneous presence of two prepositional phrases following the verb in (22b) lead us towards an interpretation of the sentence as addressing the body of a PATH as indicated by two LOCI (one point or collection

of points) in its body (i.e., 'behind the sofa', 'out [of] the window'). The cognitively necessary way of addressing a PATH, that is, a vector, by at least one of its two 'canonical' constituent parts makes optional their overt linguistic expression.

The opposite is true for the preposition 'from'. In fact, since no cognitive necessity or 'canonicity' is assigned to the beginning of a PATH (or cardinal number '0') it becomes compulsory to express the specific preposition that has the beginning of PATH in its axiomatic content. In other words, and in line with Fillmore's observation, 'from' needs to be linguistically expressed because it is not cognitively necessary. Notice also that had sentence (22c) lacked 'from' its interpretation would have overlapped the one for sentence (22a). Burmese lacks an adposition that can be interpreted as the English preposition 'from'. This cross-linguistic fact might lead to undermine the conclusion just reached in the previous discussion. However, this is not the case. In fact, what is happening in Burmese is not in contradiction with our suggestion, but simply in line with the full range of possibilities that human languages display around the world.

The linguistic solution that Burmese speakers have adopted is that of addressing the beginning of a PATH by means of a sentence that describes the beginning of the action. Here is an example:

(23a) *yangon* ka. *man:dalei:-kou* ¢ó–Ä»ó–Ä–†óÏŒ§ªÄ¿»
Rangoon ! Mandalay to

(23b) *yangon* ka. *nei-pi: man:dalei:-kou* ¢ó–Ä»ó–Ä–ŒóöÃ¡ª Ä–†óÏŒ§ªÄ¿»
Rangoon ! remain-finish Mandalay to
from Rangoon to Mandalay

where *ka*. is not 'from' but rather a particle indicating focal contrast (see fn. 26), and (a) is the colloquial contraction of (b), meaning 'having been [at] Rangoon!'

In other words, the Burmese solution could be described as sentential and not adpositional as in English. A sentence is used to address the beginning of the PATH and not a preposition like in English. Neither the English preposition 'from' nor the stative sentence in Burmese are optional, but must be expressed if the sentence needs to address the beginning of the PATH.

We have come a good way towards confirming the spirit of some of Fillmore's intuitions and also justifying the choice of 'from' (and 'to') as Path-functions for the ontological category of PATH.

4.5 *The Axiomatic Content of 'Via' (and 'By', 'Along', 'Through').*

We can now devote our attention to the prepositions addressing the body of the PATH (or route or path preposition 'via' in Fillmore's terms[31], or path-function 'via' in Jackendoff's terms). We need to introduce some sentences here:

(23) a. Maya flew (to Rome) via New York (same as (17c)
b. Maya walked by the store
c. Maya ran along the pool
d. Maya rushed through the door

In all the sentences in (23) the verbs express motion of the primary object (or figure) Maya. The PATH that this MOTION defines is addressed by the prepositions 'via', 'by', 'along', and 'through' not at its beginning or end, but at some point or points in its body or route. There is, however, an important distinction to be made between 'via' and the rest of these prepositions. The secondary object (New York) addressed by 'via' in (23a) is not geometrically defined, or better, its geometrical characteristics are not addressed by the preposition. We already know that when this is the case the preposition is addressing a LOCUS. Furthermore, since we have already seen that a PATH is a sequence of LOCI, or vectors, we can say that the preposition 'via' expresses a relationship between a vector in the PATH and the LOCUS of the secondary object (New York). This relationship can be one expressible by 'in the vicinity', 'in contact', or 'coincidence' (including 'containment').

Clearly, then, the conceptual content of the preposition 'via' is another subcase of the conceptual content of the preposition 'at'. In fact, it expresses a relationship between two LOCI in which one of the two LOCI addressed must be a vector. However, before we can justify Jackendoff's choice of 'via' over 'by', 'along' and 'through' we have to look carefully into the meanings of these last three.

In sentence (23b) the verb expresses MOTION, so a PATH is indicated. The primary object (Maya) traversed this PATH, and at some point on it there existed a spatial relationship between that point and some object (shop). This spatial relationship is expressed by the preposition 'by'. We know that, again, the geometrical features of the secondary object do not count, so we are dealing with its LOCUS. We also know that any point on the PATH is a vector and a LOCUS, and again we are bound to say that the spatial relationship expressed with 'by' is one between LOCI. But we cannot say that it is an 'at'-type relationship. In fact, the use of 'by' never includes the 'contact', 'coincidence' or 'containment' cases. It is always a

[31] It must be pointed out here that Fillmore's suggestion is based on the work done by Bennett (1970) whom Fillmore himself refers to in his 1975 work.

relationship of the 'in the vicinity' type. It is this last characteristic that makes 'by' less general and less basic than 'via', thus, providing some support for Jackendoff's choice.

The prepositions in sentences (23c) and (23d) ('along' and 'through') can be treated together. Both verbs in the two sentences express MOTION and thus define a PATH. Some part of this PATH, not a single point, but several, are addressed as being in a spatial relationship with an object. This time the geometrical characteristics of the object matter, consequently, one of the elements of the relationship is not a LOCUS, but a PLACE.

At this juncture the two meanings diverge. In fact, where for 'along', the PATH, or better the sub-PATH, is 'in contact' or 'at some pragmatically determined distance' from the object, for 'through' the sub-PATH is in contact with some non boundary points (at least one) of the object. In other words, 'along' is a sub-case of 'by' where one of the two partners in the spatial relationship is a PLACE. 'Through' is a sub-case of 'in' where both partners in the spatial relationship are PLACEs and one of these PLACEs is a collection of vectors (a sub-PATH).

Finally, we have found ample justification for the choosing the preposition 'via' as the preferred one to stand as representative for the prepositions addressing the body of a PATH. In fact, only 'via' is a sub-type of the preposition 'at', that is, expresses a relationship between two LOCI, including the full range of 'in the neighborhood' type of relationship available for 'at'. It seems to us, then, that good intuitions have to be trusted (see Bennett 1970, 1975 and Jackendoff 1983), but it is essential that one demonstrates that they are supported by the results of extensive analysis.

4.6 *The Axiomatic Content of 'Towards' (and 'Away-From').*

We are now going to take a closer look at the two directional prepositions 'towards' and 'away-from' that Jackendoff (1983) proposes as the last two path-functions for PATH. Let us first look at some sentences:

(24) a. Maya headed towards the park (same as 19d)
b. Maya drove away from the hotel

We know that the verb in (24a) indicates a MOTION, so we deduce that there must be a PATH traversed. We also realize that the geometrical characteristics of the secondary object are not relevant, thus we know we are dealing with a LOCUS. The preposition 'towards' addresses the PATH indicated by the verb, but it is neither the beginning, nor the end, nor yet the body of the PATH that is addressed. The presence of the preposition 'to' within 'towards' makes us think of the end of the PATH, but we know that the sentence does not say anything

about that. What is it that the preposition 'towards' is putting in a spatial relationship to the LOCUS of the secondary object ?

In order to try and find an answer we have to think back to the suggested definition of PATH. In that definition it was stated that a PATH is an ordered collection of vectors (LOCI/points with a direction) in SPACE, bounded by two LOCI (without left or right directionality). One of the most relevant characteristics of a PATH, then, is the fact that it has a direction. and we know that this derives from its strict relationship to MOTION. It is this latter characteristic of the PATH that 'towards' addresses. However, in order to do so, the whole PATH has to be taken into consideration. The PATH is regarded as an *object* (a vector or a collection of vectors) that possesses a direction.

We also know that each individual vector contained in the PATH can have a different direction. Thus, we are aware that there can be infinitely many routes for constructing a PATH joining two end points. How is it that this observation does not seem to come out from sentence (24a)? Let us consider that addressing the PATH as an object is not the end of the process that 'towards' leads us through. In fact, another step is needed, one we have become accustomed to in this chapter: the PATH/object is considered as a projected LOCUS/point, or better, as a vector, a point with a direction. We can now state that the preposition 'towards' addresses a relationship between two LOCI, a LOCUS with a direction (the PATH/vector), and a LOCUS without direction considered as the end of the PATH (the secondary object).

In the sense we have just delineated for the meaning of 'towards', it appears clear how the only content that it communicates is direction of the MOTION indicated by the verb that naturally takes place in a PATH whose only characteristic we are certain about is, again, nothing but its direction. In fact, this tells us the sense in which, etymologically, 'towards incorporates 'to'. For, 'towards' refers just to a direction which, if taken, leads 'to' some goal.

On very similar lines we can reason about the meaning of 'away-from'. The only relevant difference that this Path-function indicates is the fact that now we are not informed about the end/head of the PATH/vector, but about its tail. In other words, the prepositions 'away-from' is addressing, as for 'towards', a relationship between two LOCI, a LOCUS with a direction (the PATH/vector), and a LOCUS without direction, but this time considered as the beginning of the PATH (the secondary object).

It seems that both 'towards' and 'away-from' represent sub-cases of the prepositions 'to' and 'from', something that is also clearly indicated by the unequivocal presence of these latter in their morphology. 'Towards' and 'away-from' also share with the preposition 'at' the basic characteristic of addressing relationships between LOCI.

In sum, the five Path-functions suggested by Jackendoff (1983) have passed the scrutiny of the present analysis by clearly revealing their characteristic of being basic within the semantic domain they address. However, we have to point out that this conclusion has not been reached from a taxonomy of PATHs, whose validity remains still be considered, but whose potential to obscure some relevant spatial issues can no longer be denied. In fact, we have seen how abstract and explicit theoretical tools like the concept of PLACE together with that of LOCUS have taken us a long way in the investigation of those very issues. They have allowed us to suggest an axiomatic content for English spatial prepositions in such a way as to be able to differentiate among them and also arrive at a clearer specification of the relationships that hold amongst them.

Table 2 below contains the axiomatic content of the English spatial prepositions related to MOTION we have discussed in this section. The axiomatic content for the previously discussed prepositions ('at', 'on', and 'in') are reintroduced so that a complete picture of the results of the present work may be readily accessible.

The contents of Table 2 highlight the considerable complexity introduced by the axiomatic analysis of the English spatial prepositions related to MOTION. But, at the same time, the table provides a better insight into the way the system works. In fact, it is apparent by now how the whole content of a specific preposition like 'at' (already the result of the combination of the two concepts of LOCUS and neighborhood, that is, a theorem on those two axioms) can become in its turn the axiom for the construction (or derivation) of other prepositions (or theorems) like 'to', 'from', and 'via'.

Table #2: Axiomatic Contents of 'At', 'On', 'In' and of 'To', 'From', 'Via', 'Towards' and 'Away-From.'

PREPOSITION	LOCUS	PLACE	neighborhood						
At	x		x						
			INSIDE	CONTACT	VICINITY				
On		x		x					
In		x	x						
						MOTION*			
						(TIME) PATH (DIR)			
			neighborhood			BEG	body	end	DIR
To	x**		x					x	
From	x**		x			x			
Via	x**		x				x		
			INSIDE	CONTACT	VICINITY				
By	x			x	x		x		
Along		x		x	x		x		
Through		x	x	x			x		
Towards	x***		x					x	x
Away-From	x***		x			x			x

* The concept of TIME participates in making up the concept of MOTION and the concept of PATH. But this fact is here left aside for the concept of PATH since TIME does not contribute to the construction of the axiomatic content of any English spatial preposition. In fact 'spatiality' is their focus (see Section 4.2 for discussion). The other axiom for the concept of MOTION is direction, which also appears as an axiom of PATH

** Note that the three English spatial prepositions 'to', 'from', and 'via' address different parts of a PATH, but all of them fully contain the conceptual material that makes up 'at'. Thus, it might be argued that 'at' is an axiom in the construction of the theorems 'to', 'from', and 'via'.

*** One of the two LOCI involved is the LOCUS of a PATH, the other LOCUS is the end of a PATH for 'towards' or the beginning of a PATH for 'away-from'.

5. *Conclusions.*

5.1. Summing Up

The present research has presented data intended to support a computational approach to cognition in general and to spatial cognition in particular. First, two schools of thought have been identified in the available literature, Neo-Whorfian Relativism and Cognitive Semantics. Their shortcomings made the suggestion of a third approach to cognition, a computational approach, a justifiable alternative.

There followed a brief introduction to the major tenets of the computational approach proposed. Third, the central part of the research was introduced. In fact, the computational

approach to cognition proposed was tested and exemplified during the analyses of some English spatial prepositions.

We have introduced what we regard as some core analytical concepts for the investigation of spatial relations, LOCUS, PLACE (object/THING), MOTION (TIME, PATH [beginning, body, end], direction), but these are no longer to be taken as a simple list of primitive terms (though they may be that lexico-semantically — assuming that the feature composition of a lexical semantic item is an encoding of elements taken essentially from theorems, including axioms, of a K-structure). They are systematically interconnected by the abstract relational properties of a computation of spatiality itself. Employing these last concepts in the analysis of English spatial prepositions ('at', 'on', 'in', 'to', from', etc.) has proven highly productive in moving us towards a proper account of their axiomatic content.

We intend to use these concepts in the analysis of other expressions of spatial relations such as 'in front of', 'behind', 'below' and the like. Such an analysis will oblige us to look into the relationship of the domains of STATE, MOTION and projected SPACE, and, ultimately, existentiality itself; for it is well known that the various sense of the English copular verb, 'is,' (those of equation, set inclusion and existence —which some languages such as Siamese Thai keep pairwise lexically distinct) all firmly tie postulating existence for anything to implicitly at least postulating its location, in ordinary space, in the space of events, or in a properly quantified domain of sets and proper classes (cf. Lehman 1985, MS: #14 and *passim*, Freeze 1992; also Kamp and Reyle 1993: 270).

To conclude, we have been able to argue successfully for the following general points:
- cognition and language (semantics) have to be kept separate
- language lives on the form and content of cognition (it is one out of many cognitive systems)
- a computational approach to the cognition of space has been shown to be highly explanatory.

5.2 Setting the Paper in a More Comprehensive Context:

Having built and used our machinery in the body of this paper, we return to a survey, and see how our machinery allows us to deal usefully and insightfully and with new results and with other matters current in the literature, as follows:

Since the qualitative/discursive literature has not in general gone beyond the collection of relevant papers in Bloom *et al*. (1996) , save in its growth in size, we shall take that work as representative of the state of the literature up to the present. We shall refer to its contents to bring our present paper 'up to date' as of this writing. In as much as this is not a critique of that literature or an alternative to it but a complement to it, we do not feel it necessary undertake a

more comprehensive survey over and above what we have just envisioned to the effect that *semantics* has to do with *propositional* representation of knowledge and thought, and is to be held distinguishable from the latter in the sense of the more fundamental cognitive and conceptual knowledge Lehman has elsewhere called K-Structures, the theory-like representation of cognitive 'domains'. In this way we can agree with Levelt's use of Slobin's notion of 'thinking for speaking', namely, as the mapping (if we may so put it) from conceptual knowledge to its encoding for semantic-propositional representation, although we tend to feel that this is 'thinking' in a fairly secondary sense (here see especially pp. 77-78). Here we also find the evidence that much of the work is about deixis and location rather than about space itself.

Next we look at Levinson's paper about frames-of-reference (here see Bennardo 2002, now, as a commentary). We are particularly interested in what he calls Molyneaux's Question. That is, the question whether sensory modalities with regard to spatiality are or are not mutually convertible. In many ways, we are trying to argue for a basis for the affirmative answer that Levinson shows nicely has got to be correct, and we take it that such an answer has got to arise from a more abstract theory of spatiality itself rather than from a look at things like frame-of-reference, which presuppose spatiality. This has a lot to do, moreover, with whether, less deeply perhaps, the different types of frame-of-reference are themselves convertible, and again, in our paper we argue that, on the same basis, they have to be; this is consistent with Levinson's present paper, of course (see the appendix-note about this in our paper, having to do with the whole matter of a properly weakened version of the Whorf/Relativistic view of the relationship between language and thinking.

Then consider Leonard Talmy's paper, which we contend can be best resolved beyond thinking of 'fictive' spaces as essentially metaphors, if one takes spatiality as a matter of geometries and topologies in general, whereupon it becomes possible to define, as we do, 'motion' abstractly as [rapid, dense] sequence, so that 'veridical' motion in three-space (like, indeed, three-space itself) is seen as an experiential and perhaps default prototype (see Lehman 1985) of space but not the sole 'true' space. From this it is possible to argue that Talmy's varieties of 'fictive' motion are all generalisations of motion to other spaces!

There is a great deal of relationship between what we are doing here (but we are not working in experimental psychology) and what O'Keefe's paper is doing (*p. 278), for he claims, correctly we feel, much as we do, that motion and so one are indeed generalisations from a 3-space prototype and not in the usual sense a metaphorical extension ('fictive'). Actually, O'Keefe's paper can stand as an independent precedent for the line we are here pursuing. Like O'Keefe, we are considering a computational-algebraic view of spatial relationships and directions specified as vectors relating places. And like him we are considering

how to specify the meaning of relevant linguistic forms (prepositions chiefly); in particular, we are taking Paths as sequences of Places (p. 280), although, we are also concerned to think of this as a way of generalising the idea of Motion, and we find it necessary to distinguish between Places and Loci, in order to deal with the specifically topological properties of spaces and spatial relationships. Our only point of apparent difference with O'Keefe is that, given our approach, i.e., our attempt to deal in the first instance with Space and not merely spatial relations (relationships) — say, with generalised spaces within which vectors and paths are defined, we do not accept (p. 282) the notion that the extension from geographic 3-space to other dimensionalities is one of 'metaphor'. On the other hand, it seems to us that his work, and the work cited by him, suggests a proper grounding in brain function for the computational theory of these matters; and like O'Keefe, we take 'computational' to mean simply formal, mathematical specification and calculation, as justified in our Introduction; it is not to be understood as having to do with computer-based simulation or AI representation, although clearly it ought in principle to be implementable in such terms. Finally, it seems to us, somewhat tentatively still, that O'Keefe's distinction between a 'rectilinear' (say Cartesian co-ordinate) and a polar co-ordinate way of specifying relations on spaces (pp. 280 and *passim*) may well be related to (Lehman and Herdrich, 2002, and Lehman 1980) a fundamental distinction between to distinct ways of representing space itself and its relationship to the entities located in space, namely, as, respectively, something axiomatically edge-bounded (though possibly of indefinitely great extent), *containing* the entities and objects defined on it, and something defined *on* the entities or objects, namely, space as a point-field — another reason for distinguishing between Place and Locus (essentially a point-projection from Place).

The editors, in their final chapter of the book (at p. 555) take the view that 'space' is an absolute primitive, necessarily independent of the concept of entity-object. 'Empty space ..., exists...', as they say. Well, this is more complicated (see Lehman and Herdrich 2002). On the obvious interpretation of that view, it is correct only under the assumption that 'space' is container-like. However, on the point-field construction of space, it does not obviously hold. More precisely, if (and it is not clear) we have to say that 'space' exists independently of things, axiomatically, then it still remains the case that any such 'space'-in itself will need to be understood as collapsing on itself, so to speak, without any non-empty dimensionality or metricality. This is, of course, consistent with the Relativistic (Einsteinian) point-field view of things, which we have shown is also a view common to some cultures and natural languages also. This will prove a significant point to make in the revision, along the lines of our assertion that O'Keefe's and others' views need to be better grounded in a general formalism of space-as-such and its quasi-axiomatisation. From this perspective it is worth noting that our use of 'computational' is as close as needs be to O'Keefe's (in his various papers and the present chapter), namely, having to do not with the idea that cognition is utterly reducible to a computer programme or similar algorithms, but having rather to do with the idea that cognition, or some

important aspects of it in the brain, is usefully taken as having to do with formal, mathematical, relational constructions, calculations, and most particularly *derivations*. of categories, and the applicability of categories, from primitives, axioms and so on.

It is reasonably clear that we are able to refine, correct and extend O'Keefe's treatment of at least a number of spatial prepositions with regard to at least these considerations:

(1) We add a comparative perspective that allows us to tease out universals from language particular matters, and that is important because the computations he postulates have got, basically at least, to reside in universals. E.g., 'from' is not universal, and 'at' is more than what he says, being elsewhere a very general default locator.

(2) We distinguish PLACE from LOCUS, and the whole apparatus of point-projections, which reduces problems O'Keefe has with 'under ^{1}' and under^2'.

(3) By putting the computations into a framework of more explicit algebraic topology, we add neighbourhood functions in such a way as to reduce materially complexities in 'about' and other omnidirectional prepositions.

(4) We argue that O'Keefe's map of space onto temporality (time) is quite right but that there follows from this, assuming (see CRNS #8) that what is basic is Aspect-cum-Mode rather than the real time line, an interesting relationship between real and apparent motion, namely, that 'motion' is basically sequence with (possibly arbitrary) monotonic ordering), whilst real motion has this order dictated by the map to Time, where, as is well known (see now the discussion of this problem in relativity, by Penrose), the order is irreversible, and hence 'real'.

(5) This in turn is exactly what finally allows us to escape thinking of extensions of spatiality as simply metaphorical. Moreover, in the latter connection, we take as the basis of extension to 'other' spaces, replacement of the 'natural' gravity. (cf. now Coventry 1998:269-271 and Landau 1998 on L.Talmy's 1998 'force dynamics') 'basis' by other bases, e.g., other 'forces' such as influence, cause, and so on. Here we may compare O'Keefe's treatment of 'under' and other prepositions as applied in 'under the influence' and 'under the aegis'. In the former case, influence is taken as a force (in this case we may call it a pushing rather than an attracting force such as gravity, and in either case 'under' entails or implies the lower limit to which the force impels) to which the relevant something is *sub*ordinate, hence a scalar limit, whist the latter is taken as a 'cover' or 'ceiling' (upper limit, cf. *'over the widening sky'), that is to say, a reference-ground defining a relative relation/location, so that there is no necessary scale and one cannot be *'above the aegis of c'; more correctly and exactly, one takes 'aegis' or 'widening sky' as subsuming or conflating a whole implicit scale of degrees *up to* an arbitrary ceiling limit. Stated otherwise, something is under the aegis in the sense that the aegis is defined as maximality. Taking the two sorts of of cases together, one is able to generalise over them and say that they imply and entail one another, in as

> much as, if there is a maximal limit, the necessarily everything is somewhere between it and any lesser limit, and any location, in whatever space, less than maximal is by definition describable as 'under' that maximum, whether or not the location is 'forced' — in the case of gravity, for example, the maximum is understood as as far in the opposite direction that something might be when not made *sub*ject to the force and thus having to maintain a *lesser* position than it might otherwise have. This sort of thing again allows one to reduce the complexity of O'Keefe's treatment of 'under'.

Next, as to Barbara Landau's chapter about the way children seem to learn the way objects and grounds need to be encoded as to shape, axiality etc. for proper application of the lexicon (prepositions, verbs, count nouns) having to do with space and spatial relations, we see that our kind of treatment (adding a more algebraic version of geometry and topology and taking the latter as dealing not merely with 'shapes' but as well with neighbourhoods and boundary relations and fields) also applies usefully and productively to the way words encoding various different aspects of the geometries of objects with regard to spatial relations, allowing us to go beyond the idea that simply *alternative schematic representations* are invoked for the application of different such lexical terms. We can now get to integrated 'theories' of those classes of geometries and the like from which the 'local' representations are *derived* naturally. That is, we take *schemata* as relatively low-level derivations and constructions from more abstract knowledge-cognition structures (cf. Lehman MS *passim* and Keller and Lehman 1991), as in the introduction to the paper, which need to be mentioned as the starting ground for our whole enterprise), with good results. For example, we see that 'container' is not the proper idea for defining the object properties to which a preposition like 'in' is to apply. Instead, we need to invoke the idea of an *interior neighbourhood*, topologically, in order to account for 'dirt *in* a mat'. Or, e.g., 'on' can apply to a container *because,* 'if something is a container then necessarily it must have a containing part, hence a *surface*, to which 'on' properly applies. Or, again (cf. p. 323), we can look again at her problematical sentence ?*'the ball lay along the road', which may indeed seem awkward considering that a ball is configured as non-distributed axially in space. However, consider 'the ball lay *somewhere* along the road', which seems a perfectly fine sentence. Why? Well, it is, we maintain, because 'somewhere' entails a PATH, i.e., a set of points themselves distributed axially on the road, regardless of the object-shape of the ball; and the LOCATION of the ball is *disjointly* postulated 'along' this PATH. Thus, the algebraic view of geometries and topologies, allows us, with suitable capacity for abstractness of representation to get round such questions and problems.

Finally, perhaps, our 'algebraic' approach allows us even to make a couple of useful comments upon Melissa Bowerman's splendid survey of the differing ways in which languages apply or fail to apply spatial descriptions to objects and relations. For one thing, it would be useful to consider that where Bowerman refers to (386) the 'conceptual packaging' of space,

we would claim that these are low-level concepts that derive from cognitive 'theories' (K-structures), so that different languages can encode different parts or aspects of the K-structures in various ways, and it remains the case that the spatial cognitions are still largely universal. This goes along with the observation that lexical encoding hardy exhausts any language's ability to deal colloquially with the sort of thing that another language packages lexically. Thus, for instance, while (399) some languages may indeed not allow one to say the equivalent of English 'There is butter *on* the scissors' and require one to say something like 'the scissors *have* butter', in English one can also say 'there is butter *all over* the scissors'/ 'the scissors *have* butter all over them', showing that the distinction that, say, the other language makes by blocking use of locative *on* is in fact also understood by English speakers conceptually and made use of when necessary. Actually, of course (see her note 6), the distinction here is that between indefinite or distributed location and specific location, so that those other languages restrict the 'on'-type preposition to use for definite location only (as also with 'my cup has a crack *in it').

Really, the problem concerning the puzzle of the evidence for and against universality of spatial conceptualisation as reflected in differing linguistic usages is readily resolved by noting that 'concepts' as used here by Bowerman and others are far from obviously the basic components of cognition but rather only packagings, as stated, namely, in a sense derivations *from* cognitiive K-structures. Language, in its *lexicon*, but not necessarilly the rest of its apparatus, does indeed go far towards determining what aspects of the innate-universal conceptualisations of spatiality and spatial relationships are to be focussed upon differentially.

From this it follows directly that the errors children make in applying spatial *terms* are, as Alison Gopnik indeed suggests in her 1988 paper (cited in Bowerman), the consequence of the fact that their underlying 'theories' (K-structures) for spatiality are 'under development' at that stage (see Bowerman's discussion at pages 392-93). Note that Bowerman occasionally overstates her case in partial favour of language specific determination of spatial understanding precisely because she concentrates on the lexical aspect of the language only. Thus, whilst it is true that in Polish (399) one has to say 'the lid *of* the pickle jar' it remains possible to say something like 'the lid is firmly screwed *on* the jar' none the less. Again, in this latter connection, the distinction being drawn has to do with the difference between 'proper' location as against 'contingent' location. It is *not* the case that English here is simply treating part-whole relations *as if* they were spatial (see again her note 6).

Incidentally, there is a question of method, too, that arises in Bowerman's as in so many of the papers here and elsewhere in the literature about language and thought. That is, unless one uses a rather fine-grained formal (say, as with our work, algebraic) apparatus for one's theorising, one is perhaps apt to pay less than adequate attention to the full range of linguistic

evidence that might usefully bear upon one's conclusions, whereas the details of a fine-grained formal apparatus practically forces one's attention to details of usage and expression (one looks at one of the expressions in the formal apparatus and is bound to ask oneself, 'Now what could that correspond to?). Thus, for instance, whilst it is certainly true, for instance (pp. 401-2) that in Korean one at least prefers to say 'a fly has *entered* my cup, instead of the simplest, ***default*** English 'There's a fly in my cup', English is quite colloquially able to deal with the distinction involved, namely, saying' A fly has *got into* my cup', which, by the way, serves to show that the distinction has to do not with focus upon events leading to a presence, but instead with the fact of *intrusive* as against 'proper' locative presence. In plain fact the thing is that before one can safely assert things about the relationship between language and thought one really must look at the full run of the language evidence and not at just default expressions or lexicon alone.

Finally, we can refer again to a part of the most recent literature on language, thought and their relationships, and the Sapir-Whorf hypothesis, namely, Bowerman and Levinson, 20. We choose, for what must appear to be obvious reasons given the materials on spatial adpositions of the main body of the present paper, to deal only with one of the paper in that important book, i.e., Penelope Brown's 'Learning to Talk about Motion UP and Down in Tzeltal: Is there a Language Specific Bias for Verb Learning?' (Brown 2001)

It seems to us that two things emerge from this excellent study, though the author might not agree with what we say about it here.

We agree that (see especially pages 513), language does have a major role. It seems to force the construction of a *map from perception and elementary cognitions (up, down, and compass directions) to developed and lexical-semantic concepts.* Without a doubt there have to be prelinguistic percepts and even concepts having to do with up/down, or else there is no account available of the universality of such things — no doubt this has to do with the often cited fact of our gravitational field coupled with our up-right posture. And no doubt either that many, but hardly all peoples generalize their primitive perception-driven concepts of relative direction for spatial *relations* based upon non egocentric landmarks to so-called absolute directions (compass directions, call them). Moreover, as we show in this paper, the reason, ultimately, has to do with the demonstrable fact that compass orientation depends always upon culturally specific but widely available *projection* from an ultimate UP (whether, in the Northern hemisphere, the pole star or the celestial equator) to the 2-space perceptual surface of the land and its horizons.

Nevertheless, There remains the question why some peoples do what the Tzeltal, and many Southeast Asians do, equating both lexically and, as Brown has shown, conceptually, compass directions asymmetrically with local UP/DOWN directionality. Apparently it really is the case

that the language forces a *map* of the sort mentioned above, so that speakers end up, quite soon in fact (Brown again, * p. 515) literally 'thinking' or conceptualising the compass direction as UP, e.g., in effect if not literally, reversing the projectional source of the former.

One dare not, however, claim that the historical projection that 'create' an idea of, say, compass directions as such in many languages is available to speakers; that they have any way of recognizing that source or inherently recovering it cognitively unless by scholarly-scientific reasoning. Nor can without further work beyond mere lexical eliciting claim, save again 'historically' as to some discovery or invention — again so widespread there has got to be a prelinguistic cognitive capacity account available for the matter—, that the Tzeltal either have a 'primitive' direction concept in mind (N., S., etc.) independent of UP/DOWN, nor that they are synchronically aware of the projection principle or are themselves using it to equate UP/DOWN with S/N.

One needs to ask informants, under carefully controlled conditions, e.g., whether a certain direction they invariably label UP, say, is 'really' up, 'in the same way' as one literally climbs up a hill or a slope or a structure. This is a fundamental methodological principle for serious cognitive research generally that one of has written about elsewhere (e.g., Lehman 2000). Far too much of the work on to question of the Sapir-Whorf hypothesis and related matters of the influence of language on thought is severely weakened by apparent reliance upon mere observation and experiments that amount to un-probed eliciting. We cannot hope to go into this matter in the present beyond what we say in the appendix at some length regarding the Guguu-Yimithir and whether or not they can or not, if 'pushed', think of something as being in the direction of, say, one's right hand'. Easy conclusions about the pros and cons of whether or not language relativistically helps to determine thought cannot be answered with eliciting methods, however beautifully crafted with texts, even with lovely experiments.

APPENDIX

We have reserved to this Appendix the more detailed technical parts of our apparatus, in order to make our basic demonstration flow more smoothly in the body of the text, and so that readers not comfortable with the technicalities of the formalisms and range of languages on which our argument rests will not be put off.

PART I. In Samoan and Tongan (Polynesian languages, but also in Burmese, Siamese and other Asian languages) we have just one generalized preposition (or postposition) to indicate location. The 'relative' location is not given, as in English, by a preposition, but rather by a subordinate noun meaning things like 'space below,' 'space surrounding,' 'the [upper] surface,' 'the interior,' and so on, for, respectively, 'under,' 'near,' 'on,' 'inside.' They say 'at the below of the table,' 'at the neighborhood of the table,' at the [upper] surface of the table,' and 'at the interior of the house.' Sometimes there is no subordinate noun accompanying the one representing the 'thing' concept, when all we mean is general location: 'at.' Then there is no surface formative for the position of what Jackendoff has defined (1983) as a Place-function, and the 'thing' absorbs the generalized Place-function semantically; the principle is no doubt the rigid logico-semantic entailment that every 'thing' is somewhere, and if nowhere else, then 'in the place it defines currently.' This is in fact rather like what Herdrich (p.c. to Lehman) says when defining the particle *i*, that appears in certain significant locative expressions (below) in Samoan. It is more complicated, nonetheless, because not all nouns can take bare *i*. There are just some things that inherently contain in their lexical meaning specifications the subset of features defining them as a kind of 'place,' and only these can take bare *i*, i.e., absorb Place-function. This is so even in English. We can say 'at the door,' 'at a certain point,' and, of course, 'at a place.'

Two things follow from such considerations. First, the second, or alternative Jackendoff formulation (1983: 162) is to be preferred as capturing the Samoan as well as the English better. We prefer [$place_X$] -> [place PLACE-FUNCTION ([$thing_Y$]) because, even in English, it seems, we understand 'inside the house,' say, as equivalent to '(at) the inside of the house.' Moreover, Jackendoff's PLACE FUNCTION really comes down to precisely a 'function,' namely, one from things to places. For instance, 'under' (Samoan 'space-beneath') is a *pseudo- 'thing'* , conceptually constructed from the elementary notion of a place— a sub-space, actually, relativized by orientation to the thing (where, as above, the 'thing' itself defines a place, namely, the place the thing itself is at).

Secondly, the only true class of instances of the sub-formula [PLACE] -> [PLACE] (Jackendoff's first formulation) is the class of indexical shifters, words like English *here*, *there*, *now, I, you, he,* etc. The fact that they are indexical shifters is the same thing as the fact that, as

in the second Jackendoff formulation, there is nothing corresponding to the first half of his earlier formulation, namely, [PLACE] -> [PLACE]. That is because the PLACEs we are now looking at as it were define their own referential index, or 'entity.' To make much the same point, we could observe that (a) a 'place' is in fact a special sub-instance of 'thing,' and that (b) whereas in the general case any 'thing' simply defines a PLACE, in the particular instance a PLACE, as a kind of 'thing,' doesn't need to define a PLACE, since it already is a place inherently. In simple English, we would not put into this sub-formula anything like, say, *[Place$_X$] -> [Place $_X$] because the subscript index has no set-theoretical interpretation here independently of the conceptual semantic representation of the term in question. Again translating into plain English, 'a' PLACE has no meaning except as the place of, or relatively to, some THING, or some one; abstract bits of space are not themselves places, from which it follows that there can be no set of places defined inherently.

English *'place'* is a 'thing' that defines a point-in-space as an ideal of a region, i.e., a PLACE, so we can say 'at a (certain) place.' It refers, we guess, to any PLACE defined by some, possibly unspecified 'thing.' If so, then *here, there, now, then* at any rate are purely derivative shifters, as ordinary usage suggests, boiling down to *this place, that place, this time, that time.* However, no equivalent reduction of the shifter class is possible for the personal pronouns and/or the demonstratives. For these, we want the formula [PLACE]-> [PLACE] (roughly because these pronouns refer to 'this or that PERSON', a special instance of THING, which is always in its own shifted PLACE). So, given our general remarks about indexicality, [PLACE] -> [PLACE / 'thing$_Y$'], where the slash (/) indicates that the place is defined 'relatively to' a certain thing, or perspective.

This gives us a proper topological generalization over Jackendoff conceptual spatiality parameters.

The Samoan data, provided by Herdrich are:

(1)	'under the table'	*i lalo o le laulau*
	'near the table'	*i latalata o le laulau*
	'on the table'	*i luga o le laulau*
	'inside the table'	*i totonu o le laulau*

Here are the Burmese examples:

(2) Sa-ok Thitta-(a)hte: hma (hyi.thi). Öæ »ö––©Ö‡æîä–ª† ¿–¢ ¿©ä–\––

book box (its)inside at (existential verb)
[thing$_i$] [thing$_j$] [Place-f.] [f.]
[The book is in the box]

Burmese has basically just two lexical postpositions proper, the locative/stative one, and the PATH one, *kou* (Ä¿»), which, as a case-marker, serves also to mark the oblique case of

direct and / or indirect objects. Objects (stereotypically patients and goals) are also conceptually PATH notions, as in fact Jackendoff shows (especially 1990).

Note that Place-f. = Place-function; f. = a functor-operator (adposition, here a postposition). 'f' takes [thing$_i$] to [thing$_j$], in a way that it serves to 'locate' the former relatively to the later in spatial terms. Trivially, without an overt term of Place-f., we understand a generalized 'neighborhood' Place-f. Thus, in English, 'at the house' amounts uncontroversially to 'in the vicinity/neighborhood of the house.' Note, likewise, that the adpositions (English prepositions) 'in,' 'on,' 'under,' 'over,' 'before,' behind,' etc. are easily shown on independent evidence (regardless of any wish to make English conformal with what is needed for the Burmese) to work along the following lines:

f. [English neutral 'at,' where 'at' even works for directions, as in roughly the sense of 'to Chicago' = ' at PATH [cf. "go 'at' something or some 'place' "], where that is in turn defined by a directed arrow whose endpoint is 'at' Chicago itself], —> 'to' under conditions specified below. That is, we have here only a lexical suppletion rule to the effect that we use a lexically specific and specialized surface preposition sensitive to the feature of P-f, or incorporating those features in the absence of an overt P-f. (E.g., 'in(side) the house,' as against 'in/at the inside/interior of the house'). Note here that PATH needs to be specified, as PLACE already is, above, as a special sort of (pseudo-) THING, presumably a sub-instance of PLACE. One can after all be 'on,' or 'along' (cf. Jackendoff 1983) a PATH in English and other languages; and it is thus proper to say that PATH is not only a pseudo-THING but more specifically a sub-instance of PLACE, defined with reference to an *ordered* pair of THINGS. This is a step Jackendoff fails to take, possibly because of the way he has collapsed the implicit distinction between cognition and semantics. Because of this his treatment lacks adequate generality and adequate accountability of quite observable language facts.

We are certain that the correct account is to say that the predicate of motion itself acts as a PLACE-function in the sense of inducing the PATH as the relevant ordering; the adposition then is f, the functor, taking THING$_i$ (the thing in motion) to THING$_j$ (at east as a limit); and the ordering itself is in terms of a quasi-continuous succession of PLACEs of THING$_i$, such that, at the limit, THING$_i$ is 'at' the PLACE of THING$_j$, as above in the locative case.

(3) (Haka Chin)

(a) Inn-ā h a-um.
house-at he-is/stays.
(b) Inn-ā h a-kā l
house-to he-goes

where it is clear that the motion verb, kā l, induces a PATH feature interpretation on the covert P-f.

What follows directly is this:

In general, PLACE is just an ideal' of P-f under the condition adposition-f. *[THING$_j$ space] iff [THING$_j$ space] is phonologically null; equivalently, iff [THING$_j$ space] ->ø/ where Adposition-f absorbs the f-features of [THING$_j$ space] . This is the case of 'place' as a noun, and hence a sort of THING. Note that an apparent problem vanishes under close examination: if the THING$_j$ is phonologically empty, we have such expressions as 'at,' 'on,' 'to' THING$_j$, but this in no way precludes a separate dependent instance of THING$_j$, as in such expressions as 'the place *of* THING$_j$.

In general, this ideal (see, e.g., Birkhoff and MacLane 1963: 70, 349 — 'A nonvoid subset C of a commutative ring R is called an ideal when Ã ˜ C and b ˜ C imply (Ã±b) ˜ C, and Ã ˜ C, r˜ R imply rÃ ˜ C´ —p. 70. 'In any homomorphism H of a ring A, the subsets of all elements mapped on zero is an ideal in A' Ã p. 349; on the notion of ideals for rings and for algebras more generally, Takeuti & Zaring 1973: 19) exists because Adposition-f maps [THING$_j$ space] to points, as in the example where 'AT [THING$_j$ space /'interior neighborhood'],' = English 'in', treats [THING$_j$ space / 'interior neighborhood']' as if it were a point, even though it is, as shown, a neighborhood, or region with respect to [THING$_i$], which is to say as the point-ideal of the *region!* This amounts to motivating the distinction between PLACE and LOCUS. More precisely still, what makes PLACE the aforementioned special sort of THING seems to be the fact that any region or neighborhood, defined in the final analysis as the neighborhood of all point-projections of a THING itself, can take any of its interior points [= LOCUS in our treatment] as its ideal or representative entity!

That this is practically the correct view is easily seen when we take note of the fact that, say in English, as we show in the body of this paper, one can be 'at' something or some place, e.g., 'at the table,' when in fact all point projections of the THING one is AT, though in the interior neighborhood of the PLACE, are disjoint from any of the actual projection-points of the second, or reference, THING; in particular, the chosen point ideal of the neighborhood can very well be a point not amongst the point projections of the reference object.

Finally, therefore, we consider any THING's 'entity' to be represented as something unique, hence abstractly as any one of its points , and we go on to consider any neighborhood of points, in particular any neighborhood of projections of the points on any THING to have THINGness in exactly the same sense, i.e., in the sense that for any such collection-neighborhood of points there is some function that can collapse that collection onto one of its members. Note, by the way, that this also takes care, as above, of PATH, since a PATH is

already defined as a 'directed' neighborhood-collection of points. Thus, quite generally, we see that abstractly (also intuitively) the primitive notion of entity has got to be understood as equivalent to the notion 'point,' and SPACE itself is to be defined once and for all (relativistically in the sense of modern physics, a Whorfian sense) in terms of projections from THING-points to more abstract points still. Moreover, there can be no principled limit, then, to projections from projections, and hence to the recursive embedding of PLACES into THINGs, which in turn have PLACEs, and so on. Note, also, that no real problem arises in the case of abstract THINGs such as ' virtue,' democracy, 'curiosity,' an the like: such purely conceptual THINGs may be thought of as something very like unique points (hence their 'entity') that project only reflexively onto themselves. From this we can derive the fact that they have no non-empty PLACEs.

With this step, which we have arrived at only from a careful attempt to account for the full range of fact of English alone, we are already well on the way to having a complete and well motivated theory of adpositions across languages that disposes of any remaining suggestion of deep perceptual or conceptual or even expressive differences amongst languages in the way they deal with spatiality.

It ought also to be clear that there must be a proper extension of the foregoing analysis to the 'space' of events, and hence to temporality in general. In fact this extension has already been undertaken, and for this we refer the reader to Lehman (MS: #8). In spite of suggestions to the contrary in Jackendoff's (1983) work, where formal objects such as THING, PATH, PLACE, EVENT are taken as (at any rate semantic) primitives, EVENTs are, after all, a species of THINGs, as in model theory (see Kamp and Reyle 1993 on EVENTS), which is to say, that there has to be a SPACE in which they are located.

This is in the natural result of treating all formal-relational systems in terms of the perfectly abstract, algebraic generalization of SPACE. We close Section I of this Appendix by observing that the foregoing geometric-topological treatment of spatiality and entity is wholly consistent with what we now know about the neurobiology of perceptual processing of vision! For, if indeed we perceive entities (and hence places) in the first place through the path-like process of scanning, there can be no cognitive theory of space that treats THING/PLACE as primitives disjoint from PATH.

PART II. The following is an extended comment on the underlying cognitive computational basis of the four quadrants of directionality reliably reported by both Haviland (1992) and Levinson for the Guugu Yimithirr of Cape York Peninsula, Northern Territories, Australia.

Persons and objects are placed relatively to some reference point as in one or other of these quadrants, or rather as lying upon a line from the reference point into said quadrant. Our starting point is the intuition that somehow that these quadrants are underlain by something like unique directional points, North, East, South, and West. It is hard to pin this intuition down, admittedly, and furthermore, Levinson (p.c.) insists that his Summer, 1992 field investigations show clearly that the people do not recognize anything like ideal, or 'true' directionality in that sense. Of course, it is easily seen that people may have cognitive knowledge that they cannot access, or be aware of, and hence Levinson's data are less than wholly conclusive. That is enough to allow us to pursue the following formal demonstration. Indeed, should it go through, it would be easy enough to design an experiment to test whether the people could be made aware, as they are not now, that they are in fact using such a notion in computing directionality. All that would be needed would be to run through a simplified version of the demonstration that one cannot, for instance, be said to be going from, say, North farther into East beyond a certain imaginary point in the latter quadrant — a point, beyond which one must be said to be heading towards South, and then let the informants draw the necessary conclusions with little prompting.

Consider first that a quadrant, say North, is defined simply as the whole arc of points inclusively between its left and right boundaries, as in the diagram below, based on Levinson's diagram. Then, one is 'facing North,' as it were, when headed towards any such point; anything is, from any vantage point (reference point), to the North iff a line from the vantage point through the object extends to any point in question on the arc of the horizon. Yet even Levinson writes readily enough of these notions of directionality as a matter of 'angles,' and arcs subtended by such angles. But the notion of angle remains undefined unless one takes some such line as a starting point and rotates, in some sense, in one direction or other from it; any line coinciding with the starting point defines an angle of 'zero degrees.' If one combines Levinson's usages in describing the Guugu Yimithirr system of directionality, one ends up saying, for instance, that all directionality involves an 'angle' of zero degrees, and this is intolerable, possibly not even coherent.

One might consider taking either edge of a quadrant and rotating towards the other edge, but this would fail having regard to yet another difficulty. It is surely no accident that the Guugu Yimithirr quadrants are systematically related to 'our' compass directions and those of innumerable cultures everywhere and of all levels of development; that, geometrically at least, and without prejudice to anything the users actually 'know', each such quadrant 'centres' as closely as one pleases (correcting, as Levinson himself says, for the deviation from magnetic North as a basis) on 'our' 'true' compass directions! This is enough to motivate our searching for evidence, formal and/or substantive, however indirect, that, after all, the Guugu Yimithirr speakers are, however unaware of it, employing something very like 'true' directionality in computing actual direction in their system. Indeed, on the basis of any such evidence, it will then

be possible to design further experiments to test whether this is the case. For instance, on could simply point out to one's informants that as one rotates in any direction from, say, North, one is getting farther and farther towards East, or West, so that there can be only one point where one can say one is neither more East nor more West, having regard to a non-empty notion of angularity of measurement invoked by Levinson himself. Obviously, this experiment will make some psycholinguists uncomfortable in so far as their canons of experiment are still governed by neo-behaviorist/learning theoretic assumptions; but such 'probe' experiments are, and ought to be necessary and proper under assumptions consistent with a non-empty notion of cognition — probes carefully controlled, needless to say, so as to avoid simply putting ideas in one's subjects' mouths, though making people recognize what they have only 'known' unconsciously is not necessarily to be regarded as 'putting ideas in their heads.'

As a matter of fact the substantive evidence is there, albeit somewhat obscurely, in Levinson's own report (1992: 34-35). It is, once again, surely not a mere accident that expressions like both 'the right side,' and 'the *other* side' are naturally rendered in Guugu Yimithirr (a language that never employs right and left as directional descriptions), as in Bible translation, as 'the East side'. We submit, as a first approximation, the following account of this fact. Let the East-West 'axis,' or opposed pair of quadrants, be taken as primary. This is not uncommon; Burmese, does this, and maybe Levinson missed the evidence favoring an underlying Guugu Yimithirr true directionality by naï vely trying to find evidence for a Guugu Yimithirr perception of some substantive, rather than purely formal, *marker* of 'true' North, such as that defined in the Northern Hemisphere by the pole star. Let the obvious diurnal path of the sun's apparent motion define East-West as a starting point, possibly at some mid-point of the year, possibly just variably with the seasonal movement of the sun's 'path' towards and away from the equator. Then, if one takes the direction of that path as, in some sense, the way one might ideally be facing, East is 'back there,' and if one supposes, further, that one faces in a direction orthogonal to the path, or else in the direction towards which the sun appears to proceed, and granting that Guugu Yimithirr people (as both Haviland and Levinson admit) do distinguish the right from the left hand even though not having any way of describing directions by means of such words, 'right' indeed translates as East, and East, in its turn, as 'other,' *viz.*, not the direction of 'facing' of the path of the sun. If, however, the equating of 'right' and/or 'other' should turn out not to be consistently rendered in Guugu Yimithirr as East, then it is still likely that it is either East or West; that the sun's path is primary, in the sense of providing substantive markers of a computational starting point for 'absolute' rotational direction, but that one can 'face' in either direction on this path, without preference.

There is more still. Heaven, Levinson tells us (1992: 35), is said to be North. We suggest, again on the basis of wide comparative evidence (see Section I of this Appendix), that there is a

systematic projectional connection between Guugu Yimithirr North and 'up.' If so, then something like 'true' North may exist in the sense of the following.

Consider the imaginary zenith, and consider rotating from it in either direction along the sun's (East-West) path. Then project that rotation upon the horizon orthogonally to the sun's path. The result will necessarily be that the 'best' 'up' projected onto that quadrant of the horizon will correspond as nearly as one pleases, with 'true' North. That the zenith is recognized and distinguished generally amongst Aboriginal peoples in Australia is well known (e.g., Wierzbicka 1992: 8 for the Nyawaygi case).

The formal argument and evidence are again easy to find. One starts from the aforementioned observations about angular rotation as defining the computation of directionality. As one goes in one or other direction in the North quadrant, for instance, one proceeds increasingly towards the East or the West quadrant, respectively; there can be but a single point in that quadrant at which one is neither more towards the East nor more towards the West; and one can never be said to be in that quadrant but proceeding, or tending, more to the South. Is there any evidence that such considerations play a part in Guugu Yimithirr computations of directionality? Yes. The argument on the basis of this evidence parallels that of Lehman (1985) against the applicability of fuzzy sets theory to color categories. That is, it is an argument from relative closure of the categories two sidedly.

John Haviland (p.c. to Lehman) appears to share our intuition that his four cardinal 'quadrants' indeed presuppose some sort of mid-points underlyingly equivalent to actual directions in 'our' system. Thus, one can say in Guugu Yimithirr that something is somewhat to the North (quadrant) and yet also somewhat to the East. Yet what is absolutely impossible is that it be also, say, somewhat to the South. That is, the categories are closed in adjacent pairs, relatively to one another, such that for any adjacent pair, say North/East, they overlap in such a fashion that we must suppose that the right limit of North is just half way into East; and the left limit of East is just half-way into North. These are not, of course, necessarily conceptualized as 'true' compass-like directions, but rather as something very like focal instances (but see below — actually limit instances of a region) of the quadrant, rather in the sense of the well-known color foci. This predicts that, for instance, there is a central limit to the situation in which an object is, or can be described as being somewhat in either of just these two quadrants. Anything, say, to the right (clockwise) of focal East, will then tend also to the South Quadrant. The quadrants, then, are ultimately functions of this neighborhood (topological) relation between adjacent foci, and are therefore not underlying categories in the sense of internally two-sidedly closed *divisions* of space; otherwise no object could be 'in' more than one exclusively in any given instance. A quadrant, therefore is really a neighborhood relation on a triple of adjacent such foci; North quadrant, for instance, is then defined as being left bounded by an ideal mid-

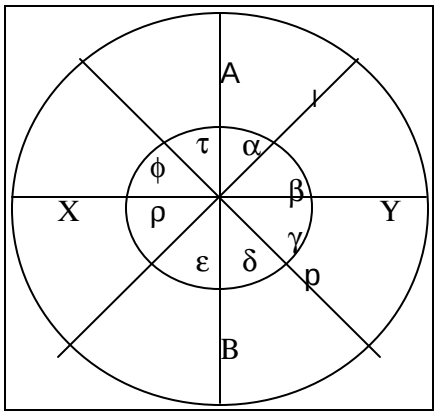
point of the neighborhood relation between focal North and focal West, right bounded by the ideal mid-point between focal North and focal East.

Furthermore, then, such 'boundaries' have no axiomatic cognitive status, or significance even, though they can obviously be computed, if that be called for, as immediately below. The real, i.e., categorial boundaries are the neighborhood limits on the two sides of any given 'direction' focus: namely, the adjacent foci themselves. With all of this there is, as with the color categories, an associated rule: as one proceeds beyond the hypothetical mid-point of any left/right neighborhood, the neighboring focus becomes more salient, and the focus one has 'left' in computing the angle becomes thus, as with ourselves, the difference between say, North-by-North-East an East-by-North-East.

After conversations with Levinson, it is still unclear whether we understand altogether whether or not an object not 'very near' the quadrant borders can ever be said to be somewhat in each of two adjacent quadrants. Let us explore this now. Assume, as a start, that we have read Haviland correctly. We accept, nevertheless, Levinson's experimental verification that the people in question have no overt conception of an 'ideal' or 'best' instance of, say, North (save, just perhaps when the projection of 'up' is brought into conscious play) or any other of the quadrants. Therefore, what we have constructed above cannot be a focal point in the quadrant's arc; it is just a limit of a set of points on an arc between adjacent boundary limits defining the arc, such that, e.g., if, in the diagram below, A,B,X,Y are the quadrants, then (a+b) subtends the point region »(A,Y). This depends somewhat on whether an object not altogether near the boundary line, 'l' can be said to be somewhat in each of the neighboring quadrants, even though to very unequal degrees, e.g., somewhat in both A and Y.

However, now assume Levinson is right, and this can only be if the object be very near a boundary. The above still holds generally because 'very near' is still obviously a neighborhood, i.e., an open and relative category, and therefore at least in principle, the quadrant mid-point is a limit for even the remotest possibility of any such joint membership.

Thus, in the following diagram, a quadrant is an arc of all points such that (i) it includes all points subtended by one of the angles (A,B,X,Y) formed by the intersections of diameters p and l, and (ii) any point in the neighborhood of p or l is in $\cup$ (A,Y),$\cup$ (Y,B) $\cup$ (B,X), or $\cup$ (X,A), respectively, and (iii) any such neighborhood of points has its left or right limit at diameters bisecting the angles (A,B,X,Y) defining the quadrants under (i). E.g., the limits of any such neighborhood are points subtended, respectively, by angles $(\alpha+\beta)$, $(\gamma,+\delta)$, $(\varepsilon+\rho)$, and $(\phi+\tau)$.



Analysis of the Angular Quadrants of
Guugu Yimithirr Directionality

It seem only proper to record at this place the outcome of a long conversation between Lehman and Levinson on the occasion of a panel on linguistic relativity chaired by Levinson at the November, 1996 annual meeting of the American Anthropological Association. Here is what we appeared to agree upon at that time, which is when Levinson said his results might well be called 'Whorfian effects for non-Whorfian reasons'.

(1) Clearly, there are languages with *only* absolute frame-of-reference expressions for spatial relationships, i.e., describing the place of some one or something never in terms of 'right' or 'left' but only in terms of compass directions from any reference point, and experiments prove the speakers do not directly map and recall their obvious perceptual processing of 'to the right of', having instead to say 'If I was in the chair and facing North, then the ball would be North of the chair.' or the like. So, we have to suppose that some time or other in the past the language developed only an encoding of absolute directionality. Presumably, this tends to put into disuse (not reinforce) appeal to the other perceptual computations (relative directionality), so that the latter is suppressed.

(2) There are, as stated, languages with only absolute directionality, but if a language has only one mode, it is always absolute; there are no languages with lexicalized expressions for *only* relative and not absolute directionality. So we need to infer that relative follows from, absolute directionality, computationally; this is a Guttman scale relationship, i.e., an implicational hierarchy. It entails that computationally there has got to be a common general module, and —

(3) This is what we are doing in the present paper: working up the general computational (and perceptual-topological) system from which Levinson's Frame-of-Reference types follow as particular sub-cases; and thence defining the Frame of Reference (FR, henceforward) categories computationally on this basis.

(4) The Absolute system has two forms of FR: (a) the locally variable (shifting referent) 'intrinsic' type (particular landmarks as reference points, with an object placed on imaginary sight lines from one to another such landmark or from a chosen vantage point such as that of the speaker and a landmark) and its generalization (fixed-implicit [polar-zenith]) referent, or true Absolute FR (with an object placed in a compass direction from a chosen vantage point); and, of course, combinations of the two, such that an object is located in a compass direction from a fixed landmark reference point (North of a central mountain, or whatever). The former (a) requires some extra *mapping* to real space, computational *processing*; shifting from one to another local landmark. The Relative system is typically the kind in which one locates and recalls objects as relative to an observer and the observers personal orientation, i.e., to the right, left, front, back, etc.

(5) The exclusive absolute FR also requires added cognitive processing because, in parallel, one must be recurrently computing orientation of perspective to the fixed (or shifting) reference point and therewith things like linear and angular distances as an observer changes or moves. Experiments conducted by Levinson and his Research Group at the Max Planck Institute for Psycholinguistic Research having the do with eliciting pointing responses, as well as Hutchins' account (1983) of the cognitive basis of traditional Micronesian Navigation make this clear. Thus, in the terms of the mapping computation from abstract topological space to real-world, real-time instantiations, these three FR types have very serious, even profound cognitive so-called Whorfian effects and consequences.

(6) There is no inconsistency between this finding and theoretical position and the task of trying to generalize the computational space of all three FRs in accounting for their common perceptual basis and the logic of the implicational hierarchical relationship between Absolute/Intrinsic and Relative.

(7) Moreover, our paper shows how it is that the Absolute/Intrinsic FRs are related as having Absolute as an abstract generalization of the Intrinsic, because with the former one need not compute shifting reference points. More significantly, we show that the Relative FR is in topological terms a very special case, living, as it does, upon an (at least) implicit conceptualization of the hypothetical observer's orientation to an Intrinsic or Absolute directional marker (a local landmark or an hypothetical point on the horizon defining a compass direction)

(8) Formally, using notions from algebraic topology once again, the following is a proper characterization of the Relative FR, which serves adequately to distinguish it from the Absolute FR: whereas the Absolute FR defines a direction *as such*, the Relative FR *induces* a direction. It specifies a LOCUS as being within a neighborhood of points of which the lower limit is the

left/right/front/back of the reference person, *viz*., a neighborhood the points of whose closure are points of the closure of the reference person. So, just as, in English, one implies, but does not directly establish, a PATH's direction in saying one has 'gone *from*' somewhere or other, so the Relative FR merely entails, implies or *induces* directionality because, as noted earlier on, a PATH has direction and direction involves a vector, which requires specification of a head, the foot not requiring to be specified. The Absolute FR, however, by specifying a 'line of sight' from a reference point, passing through a second object, *to* an independently chosen point (i.e., a compass point such as North), therewith specifies, in that latter point, a head and thus a vector and PATH, so that this FR directly defines a direction.

From this it follows that the Relative FR is computationally more complicated than the Absolute FR because an additional operation amounting to the induction of directionality is required. Therefore, one is led to predict that, if any language has only one or these two FRs, it will necessarily be the Absolute FR (an implicational hierarchy amongst FR, where the Relative entails the Absolute but not conversely). As far as we can determine this prediction is in fact empirically correct, thus tending to confirm the correctness of this whole line of analysis.

We may conveniently extend the foregoing arguments against a relativist view of cross-linguistic evidence about conceptualizations of space to a comment on recent relativist (Whorfian) arguments about language and thought more generally. John Lucy's (1992) general line of argument for anthropological evidence favoring a relativist position fails on the ground that such evidence, as he construes it, generally depends upon the importation of encyclopedic knowledge from the relevant K-structures — adding to the meaning of any such cognitive category all sorts of stuff that the people of a particular cultural tradition know 'about' the category in question. On the one hand, this is indeed going to be largely unique and specific for each culture; in fact that does mean that at some genuinely cognitive level of representation people's understanding of say, a cardinal-like direction will be culturally relative.

On the other hand, it is at least reasonably clear that the computational meaning, based upon perceptual-functional universals, is what enters directly into the lexico-semantic meaning, and it is quite clear that the potentially infinite encyclopedic knowledge cannot be part of lexical meaning, both because entries are finite (see Lehman MS: #14 for the way it is nevertheless able to access K-structures associationally) and because, in the final analysis, all entries would be the same, since the encyclopedic knowledge associations about any thing entails all knowledge about all things. In the final analysis, of course (see Lehman 1985 and Keller and Lehman 1991) that is what makes it necessary to postulate the distinction between cognition, as Knowledge structures (within which a class of possible well-formed computations are defined), and semantics.

PART III. It now seems to us that we can derive Jackendoff's point-and-path computational module from something more general, namely, from sets of points and functors and quantifiers. A stative verb may be taken to 'declare' or implicate a set of such points (LOCI, PLACEs and so forth), and then the 'locative' adpositions (English, prepositions) serve as logical quantifiers, which partition the set as a Choice Function (see Lehman, 1985), so that 'at' selects a particular ith member of the Set, whilst, for instance, 'near' selects, rather, an unspecified ith or jth member in the neighborhood of some specified point, p. These are, of course, second or higher order quantifiers, pairing one or more set point with another (a PLACE, projected as a LOCUS) with some object, the verb declaring the pairing.

A motion verb, declares, again as a functor from points to points, the Power Set P of such points. It pairs the object (possibly a singleton member of the P-Set) with another member of the P-Set, namely, an 'ordered' n-tuple of points, this principled order constituting the vector-PATH. The adposition, then, is again a higher order quantifier that executes the pairing in question; it actually chooses (partition/choice function, once again) the PATH, defined in the first instance by its head or endpoint. In the simplest case, that of 'to', it just places the moving entity on the PATH. In more highly marked cases (other such adpositions), it also pairs neighborhoods of points within the body of the PATH with the specified ENDpoint/head, and is thus at least a third order quantifier, pairing three points: the object moving, the PATH head and some point-neighborhood on the body of the PATH.

Note that, if any of this is on the right track, we are deriving Jackendoff's spatiality module (it is still a module if only because of the special connection it has with generalized visual perception) from more general computational material: quantifiable sets and the ordinary operators on these. What preserves Jackendoff's spatiality module as truly modular, or guarantees that it is so at least, is, we believe, the special constraint on the nature of the members of the Power Set of points, namely, everywhere continuously dense ordered subsets of points (construed as PATHS). I.e., differential subset cardinality (again see Lehman, 1985) is not made use of in the partition, although, of course, as a vector, any PATH has the two quantities of direction (obvious from the ordered relation between the head and foot of a PATH) and length, so that differential open interval 'size' is made use of, and this is badly in need of further explanation and explicit definition.

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