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DEPARTMENT OF ECONOMICS

WHY IS THERE MONEY?  
ENDOGENOUS DERIVATION OF 'MONEY' AS THE MOST LIQUID  
ASSET: A CLASS OF EXAMPLES

BY

ROSS M. STARR

**DISCUSSION PAPER 2000-25R  
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**Why Is There Money?**  
**Endogenous Derivation of 'Money' as the Most Liquid Asset:**  
**A Class of Examples**

Revised October 30, 2001

Ross M. Starr  
University of California, San Diego

Abstract

The monetary character of trade, the existence of a common medium of exchange, is derived as an outcome of the economic general equilibrium in a class of examples. Two constructs are added to an Arrow-Debreu general equilibrium model: market segmentation with multiple budget constraints (one at each transaction) and transaction costs. The multiplicity of budget constraints creates a demand for a carrier of value between transactions. A common medium of exchange, money, arises endogenously as the most liquid (lowest transaction cost) asset. Government-issued fiat money has a positive equilibrium value due to its acceptability in payment of taxes. Scale economies in transaction cost account for uniqueness of the (fiat or commodity) money in equilibrium. The monetary structure of trade and the uniqueness of money in equilibrium can thus be derived from elementary price theory.

# **Why Is There Money?**

## **Endogenous Derivation of 'Money' as the Most Liquid Asset: A Class of Examples<sup>1</sup>**

Revised October 30, 2001

Ross M. Starr

University of California, San Diego

### I. Money in Walrasian General Equilibrium

Consider three commonplace observations on the character of trade in virtually all economies:

(i) Trade is monetary. One side of almost all transactions is the economy's common medium of exchange.

(ii) Money is (virtually) unique. Though each economy has a 'money' and the 'money' differs among economies, almost all the transactions in most places most of the time use a single common medium of exchange.

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<sup>1</sup> This paper has benefited from seminars and colleagues' helpful comments at the University of California - Santa Barbara, University of California - San Diego, NSF-NBER Conference on General Equilibrium Theory at Purdue University, Society for the Advancement of Behavioral Economics at San Diego State University, Econometric Society at the University of Wisconsin - Madison, SITE at Stanford University, Federal Reserve Bank of Kansas City, Federal Reserve Bank of Minneapolis, Midwest Economic Theory Conference at the University of Illinois - Urbana Champaign, University of Iowa, Southern California Economic Theory Conference at UC - Santa Barbara, Midwest Macroeconomics Conference at University of Iowa, University of California - Berkeley, European Workshop on General Equilibrium Theory at University of Paris I, Society for Economic Dynamics at San Jose Costa Rica, World Congress of the Econometric Society at University of Washington, Cowles Foundation at Yale University, and from comments of Meenakshi Rajeev. Remaining errors are the author's.

(iii) 'Money' is government-issued fiat money, trading at a positive value though it conveys directly no utility or production.

Where economic behavior displays such uniformity, a general elementary economic theory should be able to account for the universal usages. But each of these three observations contradicts the implications of a frictionless Walrasian general equilibrium model. This essay presents elementary structure in that model sufficient to derive points (i), (ii), and (iii) as outcomes. In doing so, this essay responds to a challenge expressed by Tobin (1980)

Barter would restrict transactions to "double coincidences of wants" ... [This] insight tells us why the social institution of money has been observed throughout history even in primitive societies. An insight is not a model, and it does not satisfy the trained scholarly consciences of modern theorists who require that all values be rooted, explicitly and mathematically, in the market valuations of maximizing agents...

Social institutions like money are public goods. Models of general equilibrium --- competitive markets and individual optimizing agents---are not well adapted to explaining the existence and quantity of public goods...

General equilibrium theory is not going to explain the institution of a monetary ... common means of payment.

Thus the examples below are intended to satisfy our 'trained scholarly consciences' and to show that a general equilibrium model can explain endogenously from price theory the institution of a common monetary means of payment<sup>2</sup>. The price system itself designates 'money' and guides transactors to trade using 'money.' The model emphasizes complete

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<sup>2</sup> A bibliography of the issues involved in this inquiry appears in Ostroy and Starr (1990). In addition, note particularly Banerjee and Maskin (1996), Hellwig (2000), Howitt (2000), Howitt and Clower (2000), Iwai (1996), Kiyotaki and Wright (1989), Marimon, McGrattan and Sargent (1990), Rey (2001) and Young (1998). The treatment of transaction costs in this essay (as opposed to the recent focus in the literature on search and random matching equilibria) resembles the general equilibrium models with transaction cost developed in Foley (1970), Hahn (1971), and Starrett (1973). The structure of bilateral trade here however is more detailed, with a budget constraint enforced on each transaction separately, so that the Foley, Hahn, and Starrett models do not immediately translate to the present setting.

markets and complete information. The examples --- as distinct from random matching models --- include no uncertainty in transactors searching for matching trades.

It is well known that a frictionless Arrow-Debreu model cannot accommodate a role for money. This essay is intended as a partial counterexample, demonstrating that minimal friction in trade is sufficient to induce the existence of money as a result, not an assumption. Indeed prices specify which good acts as 'money.' The monetary structure of the economy is derived from elementary price theory in a class of examples. Use of a common medium of exchange, a commodity money, is an outcome of the market equilibrium. Starting from a (non-monetary) Arrow-Debreu Walrasian model, the monetary quality of the economic equilibrium is derived through the addition of two constructs: market segmentation with multiple budget constraints (one at each transaction) and transaction costs. Transaction costs imply differing bid and ask prices for each good. Commodity money arises endogenously (without government intervention or designation as legal tender) as the most liquid (lowest transaction cost or narrowest bid/ask spread) asset. Fiat money --- issued by government --- derives its positive value from acceptability in payment of taxes; it becomes the common medium of exchange from its low transaction cost. Uniqueness of (fiat or commodity) money, uniqueness of the common medium of exchange in equilibrium, follows from scale economy in transaction costs.

There is a fourth, less commonplace, observation that turns out to be a significant guide to modeling:

(iv) In a monetary economy, even transactions displaying a double coincidence of wants are transacted with money.

Because transactions involving a double coincidence of wants are relatively rare, this characterization of trade is less obvious. Nevertheless, University of California faculty whose children are enrolled at the University pay the student fees in money, not in kind; Ford employees buying a Ford car pay for the car in money, not in kind; Albertson's supermarket checkout clerks acquiring groceries pay for their food in money, not in kind<sup>3</sup>.

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<sup>3</sup> Confirmed in telephone conversation with public relations offices at Ford and Albertson's. The public relations officers the author spoke to expressed some surprise at the notion that academic economists entertained the view that these trades would be made

This observation suggests that the focus on the absence of double coincidence of wants --- as distinct from transaction costs --- as an explanation for the monetization of trade may miss a significant part of the underlying causal mechanism.

Section III of the paper presents the model of segmented markets with linear transaction costs without double coincidence of wants, in a class of examples. Commodity money is endogenously chosen in market equilibrium as the lowest transaction cost (narrowest bid/ask spread) commodity. Section IV demonstrates that the absence of double coincidence of wants is essential to monetization of trade in a linear model by considering the same problem with full double coincidence of wants. The result is a nonmonetary equilibrium. Section VI considers a (nonconvex) transaction technology with scale economies. The examples there demonstrate that uniqueness of money (uniqueness of the endogenously chosen medium of exchange) results from scale economies in transaction costs. Further, Section VI demonstrates that scale economies in transaction cost account for monetization of trade with a unique 'money' even when there is full double coincidence of wants. Section VII considers government-issued fiat money whose value is supported by acceptability in payment of taxes. In a linear transaction cost model, fiat money's (assumed) low transaction cost makes it the common medium of exchange. Alternatively, in a nonlinear model, scale economies in transaction cost and government's large scale ensure that fiat money is the unique common medium of exchange.

#### Distinguishing Models of Money: Random Matching/Search versus General Equilibrium with Transaction Cost

Random matching/search models of monetary economies, typified by Kiyotaki and Wright (1989, 1991) and by Trejos and Wright (1993, 1995), endogenously generate a medium of exchange function. They have recently been the most prominent and successful formal models to do so. They display distinctive elements differing from monetary general equilibrium models, Starrett (1973), Ostroy and Starr (1974), Iwai (1996), including this essay. Random matching models do not represent organized markets. Agents meet to trade in isolation, recognizing that subsequent opportunities to

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in kind.

trade will occur, but unable to foresee them at the time of their meeting. These models represent an excellent formalization of Adam Smith (1776)'s notions,

when the division of labour first began to take place, this power of exchanging must frequently have been very much clogged and embarrassed in its operations...The butcher has more meat in his shop than he himself can consume, and the brewer and the baker would each of them be willing to purchase a part of it. But they have nothing to offer in exchange, except the different productions of their respective trades, and the butcher is already provided with all the bread and beer which he has immediate occasion for. No exchange can, in this case, be made between them. He cannot be their merchant, nor they his customers; and they are all of them thus mutually less serviceable to one another. In order to avoid the inconveniency of such situations, every prudent man in every period of society, after the first establishment of the division of labour, must naturally have endeavoured to manage his affairs in such a manner as to have at all times by him, besides the peculiar produce of his own industry, a certain quantity of some one commodity or other, such as he imagined few people would be likely to refuse in exchange for the produce of their industry. (v. I, book I, ch. 4)

The present study and its predecessors modeling general equilibrium with transaction cost, Foley(1970), Hahn (1971), Starrett (1973), reflect instead a formalization of Menger (1892)'s emphasis on asset liquidity as an explanation for monetization of trade.

The Kiyotaki and Wright (1989, 1991) models assume indivisible commodities held in unit quantity incurring a transaction cost on trade. One implication of indivisibility and unit quantity is that there is no meaningful price variation; all rates of exchange are unity. It is not possible to distinguish a retail price and a lower wholesale price or an asking price and a lower bid price. These are familiar elements of organized markets with transaction costs, and they enter essentially in the general equilibrium with transaction costs models of Foley (1970), Hahn (1971), Starrett (1973), and this essay. In a model with divisible goods (or with large quantities of indivisible goods), when two traders meet to trade it is possible to make a mutually advantageous trade with only a single coincidence of wants by discounting the undesired good. That is, suppose trader 1 has good A and wants good B, and trader 2 has good B and wants good C, and trader 3 has



good C and wants good A (a typical absence of double coincidence of wants). It is possible for 1 and 2 to exchange B for A to mutual benefit, even though 2 does not want A, by discounting A sufficiently so that 2 can advantageously retrade it. In this sense, sufficient price variation should allow several or all goods to act as media of exchange, Starr (1976) and Example III.2 below. Indivisibility and unit quantity in Kiyotaki and Wright (1989, 1991) limit price variation so that this is not possible.

Trejos and Wright (1993, 1995) consider divisible money and trade in *services*. Services are nondurable and hence not retradable (and there are no service IOU's) so they cannot act as commodity money. Again, since trade is in isolated pairs, and there is no retrade, there is no meaningful concept of bid and ask or wholesale versus retail price.

In order endogenously to generate a function for money or a medium of exchange, there have to be some frictions in the trading arrangements. Random matching/search, like transaction cost or overlapping generations, presents a friction. The random matching/search formalization of the friction in trade has a very classical implication: in the rare case where two agents have a double coincidence of wants and meet to trade, they will trade their goods or services directly for one another, Kiyotaki and Wright (1991), Trejos and Wright (1993). This is a distinctive feature, distinguishing the random matching/search models from general equilibrium with transaction cost models. In general equilibrium models with transaction cost, including the present model, trade takes place between individuals and the market or market maker, not directly between individuals. Hence, even in the rare instance of double coincidence of wants, general equilibrium models with transaction cost do not predict direct trade between parties with reciprocal demands and supplies.

In actual monetary economies, in those comparatively rare instances where double coincidence of wants occurs, it is seldom resolved by barter exchange. In a monetary economy, trade between agents --- even with a double coincidence of wants --- usually takes a monetary form. This is typified by the examples above of a University of California professor's child's University fees, a supermarket checkout clerk's payment for groceries, and an autoworker's purchase of a car. Even in those instances where double coincidence of wants occurs (the setting most propitious for barter), monetary trade

prevails. This usage contradicts the predictions of the random matching/search models. It is consistent however with Ostroy and Starr (1974) Theorem 4, and it is precisely the behavior Example VI.2 below would predict.

## II. Formalizing Menger's 'Origin of Money'

*Why is there money?* is one of the classic issues in the foundations of economic theory, with contributions extending from Smith's *Wealth of Nations*, to the present. *Money*, like *written language* and the *wheel*, is one of the fundamental discoveries of civilization. Nevertheless, despite the evident superiority of monetary trade over barter, there is a counterintuitive --- superficially irrational --- quality to monetary exchange. Monetary trade involves one party to a transaction giving up something desirable (labor, his production, a previous acquisition) for something useless (a fiduciary token or a commonly traded commodity for which he has no immediate use) in the hope of advantageously retrading this latest acquisition. An essential issue at the foundations of monetary theory is to articulate the elementary economic conditions that allow this paradox to be sustained as an individually rational market equilibrium.

Over a century ago, Carl Menger presented the paradox of monetary trade as a challenge to monetary theory and proposed an outline of its solution, a theory of market liquidity as the basis of monetary theory, Menger (1892):

It is obvious ... that a commodity should be given up by its owner ...for another more useful to him. But that every[one] ... should be ready to exchange his goods for little metal disks apparently useless as such...or for documents representing [them]...is...mysterious...

why...is...economic man ...ready to accept a certain kind of commodity, *even if he does not need it*, ... in exchange for all the goods he has brought to market[?]

The problem ... consists in giving an explanation of a general, homogeneous, course of action ...which ... makes for the common interest, and yet which seems to conflict with the ... interests of contracting individuals.

[Call] goods ... *more or less saleable*, according to the ... facility with which they can be disposed of ... at current purchasing prices or with less or more diminution... Men ... exchange goods ... for other goods ... more saleable...[which] become *generally* acceptable media of exchange [emphasis in original].

Menger is asking here why monetary trade is an equilibrium, the outcome of the interaction of agents' optimizing decisions. Further, why is the medium of exchange function concentrated on a unique (or small number of) medium(a) of exchange? It is not a sufficient answer to cite the inconvenience of barter. Inconvenience of barter is the reason why monetization of trade is efficient but it does not explain why monetary trade is a market equilibrium. No one can choose individually to monetize transactions. Monetization is the common outcome of the equilibrium of the trading process.

Menger's proposed solution to this puzzle focused on the liquidity of commodities. A good is very *saleable* (liquid) in Menger's definition above, if the price at which a household can sell it (the market's prevailing bid price) is very near the price at which it can buy (the market's prevailing ask price). Menger suggests that liquid goods, those with a narrow spread between bid and ask prices, become principal media of exchange, money. Liquidity creates monetization. This is the insight that will be formalized in the examples below.<sup>4</sup>

Starting from the non-monetary Arrow-Debreu model, two additional structures are sufficient to give endogenous monetization in equilibrium: multiple budget constraints (one at each transaction, not just on net trade) and transaction costs. Where do the multiple budget constraints come from? Each household may make several transactions for different goods, not just the single comprehensive transaction of the Arrow-Debreu model. One way of formalizing this multiplicity is as a trading post model. Walras (1875) describes the setting of trade in a market equilibrium as a complex of trading posts where goods trade pairwise against one another.

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<sup>4</sup> It is not clear whether Menger regards liquidity as an inherent quality of a commodity or as endogenously determined in the market. In the examples below both sources of liquidity arise --- some goods have naturally lower transaction costs than others *ceteris paribus*, but the bid/ask spread is the market price of liquidity, endogenously determined in equilibrium.

In order to fix our ideas, we shall imagine that the place which serves as a market for the exchange of all the commodities ... for one another is divided into as many sectors as there are pairs of commodities exchanged. We should then have  $m(m-1)/2$  special markets each identified by a signboard indicating the names of the two commodities exchanged there as well as their prices or rates of exchange... Thus, if there are  $m$  goods, Walras envisions a large number,  $m(m-1)/2$ , of trading posts. That is the starting point of the examples below. The choice of which trading posts a typical household will trade at is part of the household optimization. The determination of which trading posts are active in equilibrium is endogenous to the model and characterizes the monetary character of trade. The equilibrium is monetary with a unique money if only  $(m-1)$  trading posts are active, those trading all goods against 'money.'

In this setting, price theory includes a theory of liquidity. The segmented market creates a demand for a carrier of value between transactions. Separate bid and ask prices represent transaction costs and put a price on liquidity: a good's bid/ask spread is the price of using it as a medium of exchange. Hence, a good with a uniformly narrow bid/ask spread is highly liquid --- in Menger's word 'saleable' --- and constitutes a natural 'money.' Price theory here implies monetary theory.

In a monetary economy, almost all trade is of goods for money (or instruments denominated in money), a single (or few) distinguished instrument(s) entering almost all trades. "Money buys goods. Goods buy money. Goods do not buy goods," Clower (1967). Thus, in a monetary economy, most of the  $m(m-1)/2$  trading posts Walras posits will be inactive. Active trade will be concentrated (in the case of a unique money) on a narrow band of  $m-1$  posts, those trading in 'money' versus the  $m-1$  other nonmonetary commodities. The examples below derive this result as an outcome of the market equilibrium of optimizing agents based on elementary considerations of transaction cost. Household optimization includes deciding at which trading posts the household will trade. For a given mix of goods, trade is drawn to the lowest transaction cost trading posts. The question *Why is there money?* can then be answered by presenting sufficient conditions so that an equilibrium trading array has  $m-1$  active trading posts, those trading in a single good (the common medium of exchange) versus the  $m-1$  other goods.

Market segmentation and the multiplicity of budget constraints facing each household --- requiring that goods acquired be paid for by delivery of equal value at each trade separately --- creates a demand for media of exchange, carriers of value among successive trades. Transaction costs create a spread between public buying (ask) and public selling (bid) prices in equilibrium. Liquidity is priced: its price is the bid/ask spread. The most liquid instrument is the one with the narrowest bid/ask spread (as a proportion of bid price). The most liquid asset, the instrument that provides liquidity at lowest cost, will be chosen as the common medium of exchange. Thus, the choice of a (possibly unique) 'money' is the outcome of optimizing behavior of economic agents in a market equilibrium.

### III. Monetization Comes from Liquidity: Monetary Competitive Equilibrium with Linear Transaction Costs

This section describes a population of households, trading posts, and the definition of a competitive equilibrium. The distinctive features of the model are (i) transactions exchange pairs of goods, (ii) budget constraints are enforced at each transaction separately, generating a role for a carrier of value between transactions (a medium of exchange), and (iii) transaction costs are assumed to be linear. In the linear transaction cost case without double coincidence of wants, the most liquid (lowest transaction cost) good becomes the common medium of exchange. There may be multiple media of exchange when there is a tie for lowest cost.

Let there be  $N+1$  commodities, numbered  $0,1,2,\dots,N$ . They are traded in pairs --- good  $i$  for good  $j$  --- at specialized trading posts. The trading post for trade of good  $i$  versus good  $j$  (and vice versa) is designated  $\{i,j\}$ ; trading post  $\{i,j\}$  is identical to trading post  $\{j,i\}$ . Trading post  $\{i,j\}$  is a business firm, the market maker in trade between goods  $i$  and  $j$ .  $\{i,j\}$  actively buys and (re)sells both  $i$  and  $j$ . Trade as a resource using activity is modeled by describing the post's transaction costs. Idealizing trade as occurring at a trading post is a simple formalization of a much more complex reality. The essential element is that markets are segmented and resource-using. The practice of representing

transaction costs in a trading firm or an economy-wide transaction technology as in Foley (1970) and Hahn (1971) embodies both the notion that there are businesses specializing in the transaction function (retailers, wholesalers, etc.) and a convenient abbreviation.

Rather than depict the transaction costs incurred at the level of the individual transactor separately, those costs are thought to be bundled into the costs of the transacting firm and priced in the difference between buying and selling prices of goods. The alternative is to describe a transaction technology for each firm and household as in Kurz (1974) or in Heller and Starr (1976).

Specify a transaction cost function for these pairwise trading posts so that all transaction costs accrue in good 0. This is obviously a restrictive convention, but it simplifies accounting for transaction costs. Trading post  $\{i,j\}$  buys good 0 as an input to its transaction costs. The typical transactions of trading post  $\{i,j\}$  will consist of purchases  $y^{(i,j)B}_i, y^{(i,j)B}_j, y^{(i,j)B}_0 \geq 0$ , of  $i, j$ , and 0 respectively and sales  $y^{(i,j)S}_i, y^{(i,j)S}_j \geq 0$  of  $i$  and  $j$ . In this section, we use the further simplifying assumption of linear transaction costs. The cost structure is generalized to non-convex costs in sections V, VI, and VII.

The transaction cost function for trading post  $\{i,j\}$  is

$$C^{(i,j)} = y^{(i,j)B}_0 = \delta^i y^{(i,j)B}_i + \delta^j y^{(i,j)B}_j \quad (\text{TCL})^5$$

where  $\delta^i, \delta^j > 0$ . In words, the transaction technology looks like this: Trading post  $\{i,j\}$  makes a market in goods  $i$  and  $j$ , buying each good in order to resell it. It incurs transaction costs in good 0. These costs vary directly (in proportions  $\delta^i, \delta^j$ ) with volume of trade. The transaction cost structure is separable in the two principal traded goods. The trading post  $\{i,j\}$  buys good 0 to cover the transaction costs it incurs, paying for 0 in goods  $i$  and  $j$ .<sup>6</sup>

The population of households is denoted  $H$ . For the purposes of the following examples  $H$  will consist of a mix of subpopulations (with different tastes and endowments). A typical household  $h \in H$ , has an endowment  $r^h \in \mathbb{R}^N_+$ ;  $r^h_n$  is  $h$ 's endowment

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<sup>5</sup> (TCL) is intended as a mnemonic for linear transaction cost.

<sup>6</sup> The transaction technology posited here supposes that all trading posts for good  $j$  have the same transaction technology for  $j$ . This is in contrast to Banerjee and Maskin (1996) where various traders have differing transaction costs (difficulty in assessing product quality) for the same good.

of good  $n$ . For purposes of simplicity in the examples below, each household is endowed with only one commodity. This is obviously inessential.  $h$ 's utility function is

$$u^h(x) = u^h(x_0, x_1, \dots, x_N).$$

Good 0 is specialized as the input to the transactions process, though it can also be consumed. It is convenient to arrange a subpopulation  $H^0$  to provide good 0.  $H^0$ 's endowment of good 0 is characterized as  $\sum_{h \in H^0} r_0^h > \sum_{h \in H} \sum_{i=1}^N \delta^i r_i^h$ . For typical  $h \in H^0$ ,  $h$ 's utility function is

$$u^h(x) = \sum_{i=0}^N x_i. \quad (U0)$$

That is, a subpopulation  $H^0$  owns all of the good 0; they have it in sufficient quantity to cover all the transaction costs in the economy that are likely to be incurred; their tastes treat all goods as perfect substitutes with MRS equal to unity. This is obviously an unrealistic assumption. It is designed to make accounting for transaction costs particularly easy.

A typical household outside of  $H^0$  may be denoted  $h=[m,n]$  where  $m$  and  $n$  are integers between 1 and  $N$  (inclusive).  $m$  denotes the good with which  $h$  is endowed.  $n$  denotes the good  $h$  prefers.  $[m,n]$ 's utility function can then be taken to be

$$u^{[m,n]}(x) = \sum_{i=0, i \neq n}^N x_i + 3x_n. \quad (U1)$$

$[m,n]$ 's endowment,  $r^{[m,n]}_m$ , is specified as part of the description of the subpopulation.

Households formulate their trading plans deciding how much of each good to trade at each pairwise trading post. This leads to the rather messy notation:

$b^{[m,n]\{i,j\}}_\ell$  = planned purchase of good  $\ell$  by household  $[m,n]$  at trading post  $\{i,j\}$

$s^{[m,n]\{i,j\}}_\ell$  = planned sale of good  $\ell$  by household  $[m,n]$  at trading post  $\{i,j\}$

The bid prices (the prices at which the trading post will buy from households) at  $\{i,j\}$  are  $q^{[i,j]}_i, q^{[i,j]}_j$  for goods  $i$  and  $j$  respectively. The price of  $i$  is in units of  $j$ . The price of  $j$  is in units of  $i$ . The ask price (the price at which the trading post will sell to households) of  $j$  is the inverse of the bid price of  $i$  (and vice versa). That is,  $(q^{[i,j]}_i)^{-1}$  and  $(q^{[i,j]}_j)^{-1}$  are the ask prices of  $j$  and  $i$  at  $\{i,j\}$ . The trading post  $\{i,j\}$  covers its costs by the difference between the bid and ask prices of  $i$  and  $j$ , that is, by the spread  $(q^{[i,j]}_j)^{-1} - q^{[i,j]}_i$  and the spread  $(q^{[i,j]}_i)^{-1} - q^{[i,j]}_j$ .

Transaction costs at the trading post are incurred in good 0. Post  $\{i,j\}$  pays for 0 in  $i$  and  $j$ , acquired in trade through the difference in bid and ask prices. The bid price of 0 in terms of  $i$  is  $q^{(i,j)}_{(i)0}$ . The bid price of 0 in terms of  $j$  is  $q^{(i,j)}_{(j)0}$ .

Given  $q^{(i,j)}_i, q^{(i,j)}_j$ , for all  $\{i,j\}$  household  $h$  then forms its buying and selling plans.

Household  $h \in H$  faces the following constraints on its transaction plans:

(T.i)  $b^{h(i,j)}_n > 0$ , only if  $n=i,j$ ;  $s^{h(i,j)}_n > 0$ , only if  $n=i,j,0$ .

(T.ii)  $b^{h(i,j)}_i \leq q^{(i,j)}_j \cdot s^{h(i,j)}_j$ ,  $b^{h(i,j)}_j \leq q^{(i,j)}_i \cdot s^{h(i,j)}_i$  for each  $\{i,j\}$ .

There is a slightly distinct version of (T.ii), (T.ii'), applying to households in  $H^0$ .

(T.ii') For  $h \in H^0$ , decompose  $s^{h(i,j)}_0$  into nonnegative elements  $s^{h(i,j)}_{(i)0}$  and  $s^{h(i,j)}_{(j)0}$ , so that  $s^{h(i,j)}_{(i)0} + s^{h(i,j)}_{(j)0} = s^{h(i,j)}_0$ , then we have  $b^{h(i,j)}_i \leq q^{(i,j)}_{(i)0} \cdot s^{h(i,j)}_{(i)0}$ , and  $b^{h(i,j)}_j \leq q^{(i,j)}_{(j)0} \cdot s^{h(i,j)}_{(j)0}$  for each  $\{i,j\}$ .

(T.iii)  $x^h_n = r^h_n + \sum_{\{i,j\}} b^{h(i,j)}_n - \sum_{\{i,j\}} s^{h(i,j)}_n \geq 0$ ,  $0 \leq n \leq N$ .

Note that condition (T.ii)[and (T.ii')] defines a budget balance requirement at the transaction level, implying the decentralized character of trade. Since the budget constraint applies to each pairwise transaction separately, there may be a demand for a carrier of value to move purchasing power between distinct transactions.  $h$  faces the array of bid prices  $q^{(i,j)}_i, q^{(i,j)}_j$ , and chooses  $s^{h(i,j)}_n$  and  $b^{h(i,j)}_n$ ,  $n=i, j$  (and  $n=0$  for  $h \in H^0$ ), to maximize  $u^h(x^h)$  subject to (T.i), (T.ii), (T.iii). That is,  $h$  chooses which pairwise markets to transact in and a transaction plan to optimize utility, subject to a multiplicity of pairwise budget constraints.

The trading posts in this economy have linear transaction technologies. A competitive equilibrium is an appropriate solution concept resulting in zero profits for the typical trading post (this has the additional technical benefit that no account need be taken of distribution of profits). The threat of entry (by other similar trading post firms) rationalizes the competitive model, but for simplicity we take there to be a unique trading post firm making a market in goods  $i$  and  $j$ , denoted indiscriminately  $\{i,j\} = \{j,i\}$ .

A competitive equilibrium under (TCL) consists of  $q^{o(i,j)}_{(i)0}, q^{o(i,j)}_{(j)0}, q^{o(i,j)}_i, q^{o(i,j)}_j$ ,  $1 \leq i,j \leq N$ , so that :



• For each household  $h \in H$ , there is a utility optimizing plan  $b^{oh\{i,j\}}_n, s^{oh\{i,j\}}_n$ , (subject to T.i, T.ii [or T.ii' for  $h \in H^0$ ], T.iii) so that  $\sum_n b^{oh\{i,j\}}_n = y^{o\{i,j\}S}_n, \sum_h s^{oh\{i,j\}}_n = y^{o\{i,j\}B}_n$ , for each  $\{i,j\}$ , each  $n$ , where

- $y^{o\{i,j\}S}_n \leq y^{o\{i,j\}B}_n, n=i,j$ .
- $y^{o\{i,j\}B}_0$  can be divided into two parts,  $y^{o\{i,j\}B}_{(i)0} \geq 0, y^{o\{i,j\}B}_{(j)0} \geq 0$ , so that  $y^{o\{i,j\}B}_{(i)0} + y^{o\{i,j\}B}_{(j)0} = y^{o\{i,j\}B}_0 = C^{(i,j)}$ .
- $q^{o\{i,j\}}_{(i)0} y^{o\{i,j\}B}_{(i)0} \leq y^{o\{i,j\}B}_i - q^{o\{i,j\}}_j y^{o\{i,j\}B}_j, q^{o\{i,j\}}_{(j)0} y^{o\{i,j\}B}_{(j)0} \leq y^{o\{i,j\}B}_j - q^{o\{i,j\}}_i y^{o\{i,j\}B}_i$ .
- $\delta^i + \delta^i q^{o\{i,j\}}_i = (q^{o\{i,j\}}_{(i)0})^{-1} (1 - q^{o\{i,j\}}_i q^{o\{i,j\}}_j), \delta^j + \delta^j q^{o\{i,j\}}_j = (q^{o\{i,j\}}_{(j)0})^{-1} (1 - q^{o\{i,j\}}_i q^{o\{i,j\}}_j)$

The expression in the last bullet is a marginal cost pricing condition: the transaction cost (in good 0) of buying one unit of  $i$  and enough  $j$  to pay for it (pricing the 0 in good  $i$ ) is equal to the amount of  $i$  left over after completing the trade in  $i$  and  $j$ . Similarly for trade in  $j$ .

An equilibrium is said to be monetary with a unique money,  $\mu$ , if --- for all households --- good  $\mu$  is the only good that a household will both buy and sell. There are examples of monetary equilibria where there are several media of exchange. An equilibrium will be said to be monetary with multiple moneys,  $\mu^1, \mu^2, \dots$ , if --- for all households ---  $\mu^1, \mu^2, \dots$  are the only goods that a household will both buy and sell. All trading posts are priced and available for trade. The equilibrium structure of exchange is the array of trading posts that actually host active trade. This is illustrated in Figures 1-4. Each node in the figures represents a commodity. Active trade is represented by a chord between nodes. A barter economy will have chords among a wide variety of goods --- one for each pair of goods where there is a household with a matching demand and supply (e.g. Figures 3 and 4). A monetary economy with a unique money will be a sparser array. There will be one good so that the only chords are those linking that good to all others (e.g. Figure 1). The question *why is there money?* is then reduced to asking for sufficient conditions so that the array of active trading posts in equilibrium looks like figure 1 instead of figure 4.

Jevons (1875) reminds us that monetization of trade follows in part from the absence of a double coincidence of wants. In the present model, that logic is particularly powerful. Absence of coincidence of wants means that the typical traded good will be traded more than once in moving from endowment to consumption. Barter trade successfully rearranging the allocation to an equilibrium will transact an endowment first at the trading post where it is supplied and again at a distinct post where it is demanded. Hence monetary trade as an alternative (substituting retrade of money for the retrade of nonmonetary goods) can be undertaken without increasing total trading volume or transaction cost, even without scale economies. Conversely, when there is a full double coincidence of wants and linear transaction cost, equilibrium will be non-monetary even in the presence of a natural money (section III).

Generations of economists have noted that some goods are more suitable than others as media of exchange.<sup>7</sup> Some of the properties of money --- general acceptability and price predictability, for example --- are conferred as part of the monetary equilibrium. Others are the indigenous property of the commodity, represented here as transaction costs: durability, portability, recognizability, divisibility. The transaction cost function  $C^{(ij)}$  is sufficiently flexible to distinguish transaction costs differing among commodities, for example to distinguish the transactions costs of mint-standardized gold medallions from those of fresh fish.

We now formalize the classic notion of the absence of double coincidence of wants. Let  $N$  be an integer,  $N \geq 3$ . A bit of additional notation is helpful to characterize permutations of the  $N$  actively traded commodities. For  $m=1,2,\dots,N$ , and positive integers  $i$ ,  $1 \leq i \leq N-1$ , let

$$m \oplus i = \begin{cases} m+i & \text{if } m+i \leq N, \\ m+i-N & \text{if } m+i > N. \end{cases}$$

That is,  $m \oplus i$  denotes  $m+i \bmod N$ , skipping 0 (since good 0 is used primarily as an input to the transaction process). Recall that  $[m,n]$  denotes a household endowed with good  $m$ ,

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<sup>7</sup> I am indebted to several colleagues --- including Henning Bohn, Harold Cole, James Hamilton, Harry Markowitz, Chris Phelan and Bruce Smith --- for reminding me how foolish it is to ignore this point.

strongly preferring good  $n$ . Using the notation above, let  $H^1 = \{[m, m \oplus 1] \mid m=1, 2, \dots, N; r_m^{[m, m \oplus 1]} = A > 0\}$ .  $H^1$  characterizes a population of  $N$  households with the same size of initial endowment, so that no pair of them have reciprocal matching endowments and preferences but so that their endowments in aggregate can be reallocated to make each one significantly better off (roughly by arranging the households clockwise in a circle ordered by endowment good and having each household  $[m, m \oplus 1]$  send his endowment one place counterclockwise).

We will use  $H^1$  and the transaction cost function (TCL) to present a class of examples demonstrating that in the absence of double coincidence of wants, with linear transaction costs, and with a natural money (lowest transaction cost instrument) there is a monetary equilibrium. If there is only a single lowest transaction cost instrument then the 'money' is unique (Figure 1). If there are several equally low cost natural moneys then the monetary instrument need not be unique (Figure 2). These results are developed in Examples III.1 and III.2.

**Example III.1:** Let the population of households be  $H = H^0 \cup H^1$ . Let  $C^{(i,j)}$  be described by (TCL). Let  $0 < \delta^i < 1/3$  and  $0 < \delta^1 < \delta^i$ , for  $i=2, 3, \dots, N$ . Transaction costs are constant and non-trivial for all goods; they are significantly lower in good 1. Then there is a unique competitive equilibrium allocation (though a range of prices may support the unique real allocation of trades and consumptions). The equilibrium is a monetary equilibrium with good 1 as the unique money.

**Demonstration of Example III.1:** Using marginal cost pricing and market clearing, we

have for each  $\{i, j\}$ ,  $i \neq j$ ,  $1 \leq i, j \leq N$ ,  $q_{(i)0}^{(i,j)} = q_{(j)0}^{(i,j)} = 1$ ,  $q_{i \oplus 1}^{(i, i \oplus 1)} = 1$ ,  $q_i^{(i, i \oplus 1)} = \frac{1 - \delta^i}{1 + \delta^{i \oplus 1}}$ , and for  $j \neq 1, i \oplus 1$ ,  $q_i^{(i,j)} = 1 - \delta^i$ ,  $q_j^{(i,j)} = 1 - \delta^j$ ;  $q_i^{(i,1)} = \frac{1 - \delta^i}{1 + \delta^1}$ ,  $q_1^{(i,1)} = 1$ .  $s^{[i, i \oplus 1]\{i, 1\}} = A$ ,  
 $b^{[i, i \oplus 1]\{i, 1\}} = q_i^{(i,1)} A = s^{[i, i \oplus 1]\{i \oplus 1, 1\}}$ ,  $b^{[i, i \oplus 1]\{i \oplus 1, 1\}} = q_{i \oplus 1}^{(i,1)} q_1^{(i \oplus 1, 1)} A$ .

What's happening in Example III.1? Think of a price and quantity adjustment process. At first household  $[i, i \oplus 1]$  goes to trading post  $\{i, i \oplus 1\}$  offering  $i$  in exchange for  $i \oplus 1$ . But no one is coming to the trading post offering  $i \oplus 1$ . So good  $i$  is priced at a large

discount at the post, reflecting the transaction costs of both  $i$  and  $i \oplus 1$ . On all other markets  $\{i, j\}$  goods are priced to reflect their transaction costs,  $q^{(i,j)}_i = 1 - \delta^i$ . But at that pricing, since  $\delta^1 < \delta^i$ , it is advantageous for  $[i, i \oplus 1]$  to trade through 1 as an intermediary. This follows since  $(1 - \delta^i) \cdot (1 - \delta^1) > (1 - \delta^i) \cdot (1 - \delta^{i \oplus 1})$ . This pricing creates a small shortage of 1 at each trading post (since small quantities of 1 are being retained at the post to cover 1's transaction costs) so prices are readjusted so that all of the discount in bid prices at  $\{i, 1\}$  appears in the bid price of  $i$ . This results in  $q^{(i,1)}_i = \frac{1 - \delta^i}{1 + \delta^1}$ ,  $q^{(i,1)}_1 = 1$ . All trade of  $i$  for  $i \oplus 1$  now goes through 1.

In Example III.1 liquidity is priced; the bid/ask spread is the price of liquidity. Good 1 is the most liquid good (with the lowest transaction cost). Bid/ask spreads at posts  $\{i, 1\}$ , for  $i=2, 3, \dots, N$ , the posts of the low transaction cost good 1, are narrower than  $\{i, j\}$ ,  $j \neq 1$ , at those of the other (high transaction cost) goods. Households respond to this pricing by rearranging their planned trades to good 1's trading posts. Household  $[i, i \oplus 1]$  sells  $i$  for 1 at  $\{i, 1\}$  at a bid price discounted for the transaction cost of  $i$  and 1. The household then trades 1 for  $i \oplus 1$  at  $\{1, i \oplus 1\}$ . All trade in equilibrium goes through good 1's trading posts. Good 1 has become 'money,' the unique low transaction cost common medium of exchange.

This is a pure flow model, so even though the volume of good 1 trading through the trading posts  $\{i, 1\}$  is large, there are no stocks of 1 held at the trading posts. The inflows and outflows balance, so markets clear even though the underlying quantity of good 1 is merely comparable to that of all other goods in the economy.

In actual monetary economies we usually see a single 'money' as in Example III.1. We'll argue in section V that the reason for uniqueness of 'money' is scale economy. Does there have to be a reason for uniqueness? Yes. US dollars, pounds sterling, and euros, all have similar low transaction costs but in their separate markets they overwhelmingly dominate the mix of currencies used. Economic theory should have an explanation. Example III.2 below emphasizes, by counterexample, that the nonconvexity in section V is important. In Example III.2, absent the nonconvexity, when there's a tie for lowest transaction cost, there are many media of exchange in use. Is a tie realistic; isn't it a

singularity? The example of dollars, sterling, and euros suggests that on the contrary, the notion of a tie for lowest transaction cost is a non-trivial event, so that uniqueness requires an explanation.

Example III.2: Let the population of households be  $H = H^0 \cup H^1$ . Let  $C^{(i,j)}$  be described by (TCL). Let  $0 < \delta^1 = \delta^2 = \delta^3 < \delta^i < 1/3$ ,  $i=4,5,\dots,N$ . Then there is a continuum of competitive equilibrium allocations with 1,2,3 acting as 'money' in proportions from 0% to 100%. Consumptions and utilities of all households are the same as in the equilibrium of Example III.1.

Demonstration of Example III.2: The marginal cost market-clearing pricing is identical to that in Example III.1 with goods 2 and 3 priced similarly to good 1. The exception is trade between 'money' 's where  $q^{(1,2)}_1 = 1 - \delta^1$ , and similarly for 2,3, all of these bid prices being equal. The trading posts  $\{i,1\}$ ,  $\{i,2\}$ , and  $\{i,3\}$ ,  $i=4,5,\dots,N$ , (for trade in good  $i$  versus goods 1,2,3) are the trading posts with narrow bid/ask spreads since 1,2,3 have low transaction costs. Households can now divide their transactions among trading posts for goods 1, 2, and 3 versus all other goods in any proportion (though in equilibrium they will be the same proportions for all households). Markets clear.

The logic of Example III.2 is merely the multi-money version of III.1. Goods 1, 2, 3 are equally liquid and become common media of exchange. They can be used however in any proportionate combination from 0% to 100% since absent economies of scale there is no reason further to specialize.

#### IV. Absence of Double Coincidence of Wants is Essential to Monetization in a Linear Model

Let  $H^D = \{[m,n] \mid m,n = 1, 2, 3, \dots, N, m \neq n\}$ .  $H^D$  is distinctive in creating a population of households with fully complementary demands and supplies.  $H^D$  presents a full double coincidence of wants. For each household  $[m, n]$  there is a complementary

[n, m]. We can use this population  $H^D$  to illustrate the importance of the absence of double coincidence of wants to monetization in a linear model. Under the same conditions where monetary equilibria existed --- and indeed were the only equilibria --- in examples III.1 and III.2 in the absence of double coincidence of wants, we can show that for  $H^D$ , with full double coincidence of wants, a barter equilibrium is the unique competitive equilibrium. Hence the classical focus on the absence of double coincidence of wants is confirmed; it is essential to monetization in a linear model.

The formal reason for the necessity of the absence of double coincidence of wants to monetization in the linear model is a bit subtle. Consider the pricing at trading posts in Example III.1. Absence of double coincidence means that --- at symmetric bid prices  $q^{o(i,j)}_i = (1-\delta^i)$ ,  $q^{o(i,j)}_j = (1-\delta^j)$  --- supply and demand are unbalanced at the trading posts  $\{i, i\oplus 1\}$  for direct trade of a household's desired excess supply for its planned excess demand; there is no complementary demander to the household's supply. The equilibrium pricing at these trading posts reflects this imbalance. At post  $\{i, i\oplus 1\}$  equilibrium pricing requires suppliers of  $i$  to pay the transaction costs both of  $i$  and of  $i\oplus 1$ . If the households in in Example III.1 try to undertake non-monetary trade, because of the absence of double coincidence of wants, each good will typically be traded twice and incur two transaction costs, once at the post where it is supplied by its original endowed owner, once at the post where it is acquired by those who actually want it. Hence, the transition to monetary trade, by using the trading posts in good 1,  $\{i, 1\}$  and  $\{1, i\oplus 1\}$ , incurs no increase in transactions volume, and does so at lower transaction cost. Absence of double coincidence of wants means that rearranging trade to a monetary pattern (using a common medium of exchange) does not increase total trading volume.

With full double coincidence of wants, on the contrary, direct trade is advantageous since trading volume and transaction costs are reduced compared to monetary trade. The complete barter pattern of trade with full double coincidence of wants is represented in Figure 4. Trading households need to provide only one side of the transaction costs on a smaller volume of trade (Example IV.1). Note that this result depends on the linearity (or convexity) of transaction costs; if scale economies are

present, then even with full double coincidence of wants, it may be resource saving to use a common medium of exchange with resulting high trading volumes.

Example IV.1: Let the population of households be  $H=H^0 \cup H^D$ . Let  $C^{(i,j)}$  be described by (TCL). Let  $0 < \delta^i < 1/3$  and  $0 < \delta^j < \delta^i$ , for all  $i, j=0, 2, 3, \dots, N-1$ . Transaction costs are constant and non-trivial for all goods but 1. Then there is a unique competitive equilibrium allocation. The equilibrium is non-monetary with active trade in all trading posts  $\{i,j\}$ ,  $1 \leq i,j \leq N$ .

Demonstration of Example IV.1: For each  $i,j$ ,  $1 \leq i,j \leq N$ ,  $q^{(i,j)}_i = (1-\delta^i)$ ,  $q^{(i,j)}_j = (1-\delta^j)$ .  $s^{[i,j](i,j)}_i = A$ ,  $b^{[i,j](i,j)}_j = q^{(i,j)}_i A$ ,  $s^{[j,i](i,j)}_j = A$ ,  $b^{[j,i](i,j)}_i = q^{(i,j)}_j A$ . Markets clear. The allocation is an equilibrium.

What's happening in example IV.1? It couldn't be simpler. Direct barter trade works successfully in the presence of double coincidence of wants. For each household  $[i,j]$  with a supply of one good and a demand for another, there is a precise mirror image  $[j,i]$  in the population. They each go the trading post  $\{i,j\}$  where their common demands and supplies are traded. They trade, each incurring the cost of trading one good. Monetary trade is not advantageous since it requires twice the transactions volume --- with corresponding cost --- of direct barter trade (similar volumes for each non-monetary good and an equal volume of trade in the medium of exchange). Monetization of trade in equilibrium (in a linear model) depends on absence of double coincidence of wants.

## V. Uniqueness of the Medium of Exchange: Scale Economies in Transaction Cost

Monetary trade is typically characterized by a unique medium of exchange or a small number of related media. How does this come about? Prof. Tobin (1980) suggests that scale economies in transaction costs are essential:

The use of a particular language or a particular money by one individual increases its value to other actual or potential users. Increasing returns to scale, in this

sense, limits the number of languages or moneys in a society and indeed explains the tendency for one basic language or money to monopolize the field.

When monetization takes place, households supplying good  $i$  and demanding good  $j$  are induced to trade in a monetary fashion, first trading  $i$  for 'money' and then 'money' for  $j$ , by discovering that transaction costs are lower in this indirect trade than in direct trade of  $i$  for  $j$ . But as Example III.2 points out, monetization of trade is no guarantee of uniqueness of the medium of exchange; low transaction cost is consistent with a multiplicity of media of exchange. Specialization, the concentration of the medium of exchange function in a unique or small number of assets in the monetary trade arrangement, reflects the workings of scale economies. If scale economies are present, then the higher the volume of trade in a particular medium of exchange, the lower will be unit transaction costs. Hence scale economies in transaction costs induce specialization in the medium of exchange function.

In the case of scale economies, nonconvex transaction costs, competitive equilibria may not exist; but in the examples below, average cost pricing equilibria will exist. Recall that the notion of transaction technology in this model, embodied in a trading post, summarizes costs that in an actual economy are incurred by retailers, wholesalers, individual firms and households. The bid/ask spread summarizes these costs to the model's transactors. The concept of average cost pricing of the bid/ask spread then represents the notion that transaction costs in any market can be summarized by the comparative ease or difficulty (high or low cost) of trading on the market. Nonconvex transaction cost is intended here to convey the idea of a scale economy from which all transactors can benefit through diminishing average cost.

Equilibria with scale economies in transaction cost will be characterized as monetary with a unique money: a single good will be distinguished in equilibrium as the medium of exchange common to virtually all transactions. Uniqueness of the medium of exchange results from scale economies in the transaction technology. Rey (2001) develops the implications of scale economies in transaction costs as a thick markets externality emphasizing international currency markets. Starr and Stinchcombe (1999) characterizes monetary trade, with a unique money, as the cost minimizing outcome of a centralized programming problem with scale economies. In section VI below a similar



result is established in a decentralized model using the price system; monetary trade with a single money is a decentralized market equilibrium.

Scale economy is not a necessary condition for uniqueness of the medium of exchange in equilibrium (Example III.1), but scale economy helps to ensure uniqueness (Example VI.1, below). If there is a unique low transaction cost instrument in an economy with a linear transaction cost structure, that natural money will be the unique medium of exchange in equilibrium. If there are many equally low cost candidates for the medium of exchange, then scale economy in transaction costs will allow one to be endogenously chosen as the unique medium of exchange. Inherent low cost and market determined high volume combine to yield unique monetization. Menger (1892) describes this transition:

when any one has brought goods not highly saleable to market, the idea uppermost in his mind is to exchange them, not only for such as he happens to be in need of, but...for other goods...more saleable than his own...By...a mediate exchange, he gains the prospect of accomplishing his purpose more surely and economically than if he had confined himself to direct exchange...Men have been led...without convention, without legal compulsion,...to exchange...their wares...for other goods...more saleable...which ...have ...become generally acceptable media of exchange.

Example VI.1 formalizes this argument emphasizing that liquidity is endogenous, a result of scale economy in the transaction process.

## VI. Monetization Comes From Liquidity Again: Monetary General Equilibrium with Unique Money under Average Cost Pricing of Non-Convex Transaction Costs

This section describes a population of households and trading post firms and an average cost pricing equilibrium concept suitable for a non-convex economy. The model is like that of section III except that transaction costs include scale economies. There may be (as in Example III.2) multiple candidate media of exchange with similar low transaction cost. Nevertheless, scale economies in the transaction cost structure induce

uniqueness of the equilibrium medium of exchange. Conversely, a good may have naturally high transaction costs, but if it is adopted as the economy's common medium of exchange, then with high trading volume and scale economy, it will have low average transaction cost and become the endogenously chosen medium of exchange. Example VI.1 below demonstrates that with sufficient scale economy in transaction cost, any good can become the unique medium of exchange in equilibrium. As Prof. Tobin(1959) tells us, "Why are some assets selected by a society as generally acceptable media of exchange while others are not? This is not an easy question, because the selection is self-justifying." Thus gold and dollar bills may have low transaction costs and be excellent candidates for medium of exchange, but if (despite high transaction cost) Yap Island stones are already the commonly chosen medium of exchange with high trading volume, then stones may have the lowest average transaction cost. The choice of Yap Island stones as the common medium of exchange is then self-justifying.

The nonconvex (scale economy) cost function for trading post  $\{i,j\}$  is

$$C^{\{i,j\}} = y^{\{i,j\}B}_0 = \min[\delta^i y^{\{i,j\}B}_i, \gamma^i] + \min[\delta^j y^{\{i,j\}B}_j, \gamma^j] \quad (\text{TCNC})^8$$

where  $\delta^i, \delta^j, \gamma^i, \gamma^j > 0$ . In words, the transaction technology looks like this: Trading post  $\{i,j\}$  makes a market in goods  $i$  and  $j$ , buying each good in order to resell it. It incurs transaction costs in good  $0$ . These costs vary directly (in proportions  $\delta^i, \delta^j$ ) with volume of trade at low volume and then hit a ceiling after which they do not increase with trading volume. The specification in (TCNC) is an extreme case: zero marginal transaction cost beyond the ceiling. Adding additional linear terms would represent a more general case.

For  $\gamma^i, \gamma^j$  sufficiently small, there is a scale economy in the relevant range of usage. The transaction technology is nonconvex, displaying diminishing marginal costs. The transaction cost structure is separable in the two principal traded goods. The trading post  $\{i,j\}$  buys good  $0$  to cover the transaction costs it incurs, paying for  $0$  in goods  $i$  and  $j$ .

Since the trading posts in this economy have nonconvex transaction technologies, a competitive equilibrium is not an appropriate solution concept<sup>9</sup>. The equilibrium notion

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<sup>8</sup> (TCNC) is intended as a mnemonic for non-convex transaction cost.

<sup>9</sup> When trading volumes are sufficiently low, the nonconvexity is not apparent in costs incurred (but can still affect price-taking competitive behavior). Then an average cost pricing equilibrium is indistinguishable from a competitive equilibrium with the linear

used is an average cost pricing equilibrium resulting in zero profits for the typical trading post firm (this has the additional technical benefit that trading post firms make no net profit, so no account need be taken of distribution of profits to shareholders). The rationale for this choice of equilibrium concept is the threat of entry (by other similar firms) if any economic rent is actually earned. The presence of potential entrants and their actions is not explicitly modeled.

An average cost pricing equilibrium consists of  $q^{o\{i,j\}}_{(i)0}, q^{o\{i,j\}}_{(j)0}, q^{o\{i,j\}}_i, q^{o\{i,j\}}_j$ ,  $1 \leq i,j \leq N$ , so that :

- For each household  $h$ , there is a utility optimizing plan  $b^{oh\{i,j\}}_n, s^{oh\{i,j\}}_n$ , (subject to T.i, T.ii [or T.ii' for  $h \in H^0$ ], T.iii) so that  $\sum_n b^{oh\{i,j\}}_n = y^{o\{i,j\}S}_n, \sum_n s^{oh\{i,j\}}_n = y^{o\{i,j\}B}_n$ , for each  $\{i,j\}$ , each  $n$ , where
  - $y^{o\{i,j\}S}_n \leq y^{o\{i,j\}B}_n, n=i,j$ .
  - $y^{o\{i,j\}B}_0$  can be divided into two parts,  $y^{o\{i,j\}B}_{(i)0} \geq 0, y^{o\{i,j\}B}_{(j)0} \geq 0$ , so that  $y^{o\{i,j\}B}_{(i)0} + y^{o\{i,j\}B}_{(j)0} = y^{o\{i,j\}B}_0 = C^{\{i,j\}}$ .
  - $q^{o\{i,j\}}_{(i)0} y^{o\{i,j\}B}_{(i)0} = y^{o\{i,j\}B}_i - q^{o\{i,j\}}_j y^{o\{i,j\}B}_j, q^{o\{i,j\}}_{(j)0} y^{o\{i,j\}B}_{(j)0} = y^{o\{i,j\}B}_j - q^{o\{i,j\}}_i y^{o\{i,j\}B}_i$ .

Let  $\kappa$  be a positive integer,  $2 < \kappa < (N/2)$ . Let  $H^K = \{[m, m \oplus i] \mid m=1,2, \dots, N; i=1,2,\dots,\kappa; i^{[m, m \oplus i]}_m = A > 0\}$ .  $H^K$  is a set of  $\kappa N$  households without double coincidence of wants. One way to visualize  $H^K$ 's situation is to think of the households arrayed in a circle clockwise, each one's position designated by endowment. They can arrange a Pareto improving redistribution by each taking his endowment and sending it  $i$  places counterclockwise. However, reflecting the absence of double coincidence of wants, if each of the households in  $H^K$  goes to the trading post where his endowment is traded against his desired good, he finds himself alone. He's dealing on a thin market.

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cost function. Hence the competitive equilibria in examples III.1 and III.2 are also average cost pricing equilibria.

Example VI.1: Let the population of households be  $H=H^0 \cup H^K$ . Let  $C^{(i,j)}$  be described by (TCNC). Let  $0 < \delta^i < +\infty$  all  $i = 1, 2, \dots, N$ . Let  $\frac{\gamma^i + \gamma^j}{\kappa A} < \frac{2}{3}$  and  $(1 - \frac{\gamma^i + \gamma^j}{\kappa A}) > (1 - \delta^i)(1 - \delta^j)$  for all  $i \neq j, i, j = 1, 2, \dots, N$ . Then for each  $i = 1, 2, \dots, N$  there is a monetary average cost pricing equilibrium with good  $i$  as the unique 'money.'

Demonstration of Example VI.1: Choose an arbitrary  $i = 1, 2, \dots, N$  as 'money.' For all  $j \neq i, j = 1, 2, \dots, N$ , let  $q^{(i,j)}_i = 1, q^{(i,j)}_j = 1 - \frac{\gamma^i + \gamma^j}{\kappa A}$ . For all  $j$ , and  $k = 1, 2, \dots, N, j \neq k \neq i, q^{(j,k)}_j = 1 - \delta^j, q^{(j,k)}_k = 1 - \delta^k$ . For  $1 \leq \ell \leq \kappa$ , let  $s^{[m, m \oplus \ell]\{i, m\}}_m = A, b^{[m, m \oplus \ell]\{i, m\}}_i = q^{(i, m)}_i A, s^{[m, m \oplus \ell]\{i, m \oplus \ell\}}_i = q^{(i, m)}_i A, b^{[m, m \oplus \ell]\{i, m \oplus \ell\}}_{m \oplus \ell} = q^{(i, m)}_i A$ .

What's happening in Example VI.1? Monetization comes from liquidity and --- with scale economies --- liquidity comes from trading volume. The economy is focusing on good  $i$  as its common medium of exchange. Since there are scale economies in transaction costs, high trading volume means low average cost with concomitant narrow bid/ask spread. The narrow bid/ask spread is the way the price system confirms and reinforces the choice of  $i$  as the medium of exchange. Trader  $[m, m \oplus \ell]$  wants to trade good  $m$  for good  $m \oplus \ell$ . He could do so directly, but the transaction costs are heavy, reducing his return on the trade to  $A(1 - \delta^m)(1 - \delta^{m \oplus \ell})$  units of  $m \oplus \ell$  after starting with  $A$  units of good  $m$ . The alternative is to trade good  $m$  for good  $i$  and then trade  $i$  for  $m \oplus \ell$ . This results in  $A(1 - [(\gamma^i + \gamma^{m \oplus \ell}) / \kappa A])$  units of  $m \oplus \ell$ . When  $\kappa$  is sufficiently large, that's a much greater return. Because of the narrow bid/ask spread on trade through  $i$ , every market with good  $i$  on one side attracts high trading volume,  $\kappa$  traders on each side of the market, the high trading volume needed to maintain good  $i$ 's low bid/ask spreads. The scale economy means that the choice of good  $i$  as the common medium of exchange is self-confirming.

Example VI.1 demonstrates the following conception of monetization of the transactions process. Scale economies in the transactions technology mean that high volume trading posts will be low average cost trading posts. The difference between barter and monetary exchange is the contrast between a complex of many thin high transaction cost markets and an array of a smaller number of thick low transaction cost

markets dealing in each good versus a unique common medium of exchange. The notion that the choice of medium of exchange is self-justifying comes from scale economy. Any good  $i$  with sufficient scale economy in its transaction technology (with  $\gamma^i$ , the ceiling on its transaction costs, sufficiently low) can become the unique medium of exchange in equilibrium when trading volume  $\kappa A$  is sufficiently high. The consensus on what is 'money', once established, locks in the incumbent 'money' through resulting low average transaction cost compared to the high unit costs --- at low volume --- of possible alternatives. The alternatives locked out may be better (potential) money's. They may have generally lower cost transaction technologies. Their low current trading volume results in locally high unit costs, keeping them locked out. Thus either mint-standardized gold coins (with a low cost transaction technology) or Yap Island stones (high cost technology) may be money depending on which is well established. Sufficient trading volume can confirm either choice.

Recall  $H^D = \{[m,n] \mid m,n = 1, 2, 3, \dots, N, m \neq n, r_m^{[m,n]} = A > 0\}$ .  $H^D$  is a set of  $N(N-1)$  households with full double coincidence of wants. For each household  $[m,n]$  there is a second household with precisely complementary demands and supplies  $[n, m]$ . In this setting, if each household  $[m,n]$  goes to trading post  $\{m, n\}$  then they will find precisely enough counterparts to make a mutually advantageous deal. That's what happened in Example IV.1. The following Example VI.2 demonstrates that the absence of scale economies is essential to the results in Example IV.1. Even in the presence of double coincidence of wants, sufficient scale economies in transaction costs can lead to monetization of trade, the use of a common medium of exchange.

Example VI.2: Let the population of households be  $H = H^0 \cup H^D$ . Let  $C^{(i,j)}$  be described by (TCNC). Let  $0 < \delta^i < +\infty$  all  $i = 1, 2, \dots, N$ . For some  $i$  and all  $j, 1 \leq i, j \leq N, i \neq j$ , let  $\frac{\gamma^i + \gamma^j}{(N-1)A} < \frac{2}{3}$  and  $(1 - \frac{\gamma^i + \gamma^j}{(N-1)A}) > (1 - \delta^j)$ ,  $(1 - \frac{\gamma^i + \gamma^j}{(N-1)A}) > (1 - \delta^i)$ . Then there is a monetary average cost pricing equilibrium with good  $i$  as the unique 'money.'

Demonstration of Example VI.1: For all  $j \neq i$ ,  $j=1,2,\dots,N$ , let  $q^{(i,j)}_i=1$ ,  $q^{(i,j)}_j=1-\frac{\gamma^i+\gamma^j}{(N-1)A}$ .

For all  $j$ , and  $k=1,2,\dots,N$ ,  $j \neq k \neq i$ ,  $q^{(j,k)}_j=1-\delta^j$ ,  $q^{(j,k)}_k=1-\delta^k$ . Let  $s^{[m,n]\{i,m\}}_m=A$ ,

$b^{[m,n]\{i,m\}}_i=q^{(i,m)}_i A$ ,  $s^{[m,n]\{i,n\}}_i=q^{(i,m)}_i A$ ,  $b^{[m,n]\{i,n\}}_n=q^{(i,m)}_i A$ .

What's happening in Example VI.2? Monetization comes from liquidity and --- with scale economies --- liquidity comes from trading volume. The economy is focusing on good  $i$  as its common medium of exchange. Since there are scale economies in transaction costs, high trading volume means low average cost with concomitant narrow bid/ask spread. The narrow bid/ask spread is the way the price system confirms and reinforces the choice of  $i$  as the medium of exchange. Trader  $[m,n]$  wants to trade good  $m$  for good  $n$ . He could do so directly at post  $\{m,n\}$ , and he'd find a willing trading counterpart at the trading post, so he'd only have to pay for the transaction costs on one side of the trade. But the transaction costs are still substantial, reducing his return on the trade to  $A(1-\delta^m)$  units of  $n$  after starting with  $A$  units of good  $m$ . The alternative is to trade good  $m$  for good  $i$  and then trade  $i$  for  $n$ . This results in  $A(1-[(\gamma^i+\gamma^n)/(N-1)A])$  units of  $n$ . When  $N$  is sufficiently large, that's a much greater return. Because of the narrow bid/ask spread on trade through  $i$ , every market with good  $i$  on one side attracts high trading volume,  $N-1$  traders on each side of the market, the high trading volume needed to maintain good  $i$ 's low bid/ask spreads. The scale economy means that the choice of good  $i$  as the common medium of exchange is self-confirming.

But how can monetization of trade occur where there is double coincidence of wants? The answer is scale economies. Example VI.2 demonstrates the following conception of the monetization of the transactions process. Scale economies in the transactions technology mean that high volume trading posts will be low average cost trading posts. The difference between barter and monetary exchange is the contrast between a complex of many thin high transaction cost trading posts and an array of a smaller number of thick low transaction cost trading posts for each good versus a unique common medium of exchange. Barter trade of an endowed good for a desired good --- even when there is a reciprocating counterparty --- necessarily takes place in a thin high

transaction cost trading post. The thick trading post for trading each good against a common medium of exchange achieves lower cost and is hence more attractive. This is less surprising than it sounds. In actual economies we routinely see monetary trade used instead of barter even when there is a double coincidence of wants. When a University of California faculty member's child is enrolled at the University there is no barter transaction to repay the cost of the child's study (e.g. one course of lectures by the faculty member for a year's enrollment of his child); he pays the fees in dollars. When a supermarket checkout clerk acquires food from the supermarket, the transaction is not conducted in barter (a week's groceries for a day's work); the employee pays for his groceries in dollars. If money were used only to overcome the absence of double coincidence of wants, we would see barter used in those instances where double coincidence occurs. On the contrary, even in the presence of double coincidence of wants, monetary trade prevails.

It is useful to recognize the network externality present in this model. When good  $i$  is the common medium of exchange, all households want to acquire good  $i$  because it is the common medium of exchange. That is, everyone demands good  $i$  because everyone else demands  $i$ . This is perfectly rational, not because of bandwagon or herd effects but out of narrow rational calculation of costs. Since everyone else is using good  $i$  to buy all other goods, (average) transaction costs for  $i$  versus other goods are low, reflecting a scale economy, which means that each remaining household finds it advantageous to acquire good  $i$  and use it as medium of exchange<sup>10</sup>. Viewing the same issue from the viewpoint of a trading post  $\{i,n\}$ , the availability and low bid/ask spread of posts  $\{i,j\}$  trading good  $i$  (the common medium of exchange) for most other goods  $j \neq i, j \neq n$ , means that a trading post  $\{i,n\}$  faces high demand for its services. In  $H^K$ , all of the households  $[m, m \oplus \ell]$  so that  $m \oplus \ell = n$  sell their endowments on  $\{i,m\}$  and then seek to trade on  $\{i,n\}$ . The low transaction costs of the posts  $\{i,m\}$  create a high demand for the complementary services of  $\{i,n\}$ .

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<sup>10</sup> Hahn(1997) recognizes this externality, noting that in the presence of market set-up costs, each transactor in the market benefits from the participation of others. Young (1998) assumes the externality without additional explanation. Rey (2001) denotes this interaction the "thick markets externality."

## VII. Government-Issued Fiat Money

In order to study fiat money we introduce a government with the unique power to issue fiat money. Fiat money is intrinsically worthless; it enters no one's utility function. But the government is uniquely capable of declaring it acceptable in payment of taxes. Adam Smith(1776) notes “A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money...” (v. I, book II, ch. 2). Abba Lerner(1947) comments

The modern state can make anything it chooses generally acceptable as money and thus establish its value quite apart from any connection, even of the most formal kind, with gold or with backing of any kind. It is true that a simple declaration that such and such is money will not do, even if backed by the most convincing constitutional evidence of the state's absolute sovereignty. But if the state is willing to accept the proposed money in payment of taxes and other obligations to itself the trick is done. Everyone who has obligations to the state will be willing to accept the pieces of paper with which he can settle the obligations, and all other people will be willing to accept these pieces of paper because they know that the taxpayers, etc., will be willing to accept them in turn.

Taxation --- and fiat money's guaranteed value in payment of taxes --- explains the positive equilibrium value of fiat money<sup>11</sup>. Scale economies explain its uniqueness as the medium of exchange.

As an economic agent, government is denoted G. Government sells tax receipts, the N+1<sup>st</sup> good. It also sells good N+2, an intrinsically worthless instrument, (latent) fiat money, that government undertakes to accept in payment of taxes, that is, in exchange for N+1. The typical household [m,n] in H<sup>1</sup> or H<sup>K</sup> desires to purchase tax receipts to the extent it prefers not to have a quarrel with the government's tax authorities. Government sets a target tax receipt purchase by the taxpayer of  $\tau^{[m,n]}$ . Then we rewrite [m,n]'s utility function as

$$u^{[m,n]}(x) = \sum_{i=0, i \neq n}^N x_i + 3x_n - 10[\max[(\tau^{[m,n]} - x_{N+1}^{[m,n]}), 0]] \quad (\text{UT}).$$

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<sup>11</sup> See also Li and Wright (1998) and Starr (1974).



That is, household [m,n] values paying his taxes with a positive marginal utility up to his tax bill  $\tau^{[m,n]}$  and with zero marginal utility for tax payments thereafter. Government uses its revenue to purchase a variety of goods  $n=1,\dots,N$ , in the amount  $x_n^G$ .

Good N+2 good represents latent fiat money. Government, G, sells N+1 (tax receipts) for N+2 at a fixed ratio of one-for-one. The trading post  $\{N+1, N+2\}$  where tax receipts are traded for N+2 operates with zero transaction cost. Acceptability in payment of taxes ensures N+2's positive value. If, in addition, N+2 has sufficiently low transaction cost, then it becomes the common medium of exchange. Once it is in general use as the common medium of exchange, then it is 'money.'

Example VII.1 Let the population of households be  $H=H^0\cup H^1$ . Let  $u^{[m,n]}$  be described by (UT). Let  $\tau^0 > 0$  be a constant. Let  $0 < \tau^{[m,n]} = \tau^0 < A(1-\delta^{N+2})(1-\delta^m)$ , all  $[m,n] \in H^1$ . Let  $x_n^G = \tau^0 q_{N+2}^{\{N+2,n\}}$  all  $n=1,2,\dots,N$ . Let  $C^{[i,j]}$  be described by (TCL). Let  $\delta^{N+2} \leq \delta^i < 1/3$  all  $i = 1,2, \dots, N$ . Then there is a monetary equilibrium with taxation with good N+2 as 'money.'

Demonstration of Example VII.1: Set  $q_{N+2}^{\{N+2,n\}} = 1$ , all  $n = 1,2, \dots, N$ . Set

$$q_n^{\{N+2,n\}} = (1-\delta^{N+2})(1-\delta^n), q_{N+1}^{\{N+2,N+1\}} = q_{N+2}^{\{N+2,N+1\}} = 1. \text{ Let } s_{n}^{[n,n\oplus 1]\{N+2,n\}} = A,$$

$$b_{N+2}^{[n,n\oplus 1]\{N+2,n\}} = A q_n^{\{N+2,n\}}, s_{N+2}^{[n,n\oplus 1]\{N+2,N+1\}} = \tau^0 = b_{N+1}^{[n,n\oplus 1]\{N+2,N+1\}},$$

$$s_{N+2}^{[n,n\oplus 1]\{N+2,n\oplus 1\}} = A q_n^{\{N+2,n\}} - \tau^0, b_{n\oplus 1}^{[n,n\oplus 1]\{N+2,n\oplus 1\}} = (A q_n^{\{N+2,n\}} - \tau^0) q_{N+2}^{\{N+2,n\oplus 1\}}. \text{ For}$$

$$n=1,2,\dots,N, \text{ let } s_{N+2}^{G\{N+2,n\}} = \tau^0, b_n^{G\{N+2,n\}} = \tau^0 q_{N+2}^{\{N+2,n\}}.$$

What's happening in Example VII.1? Transaction costs are assumed linear, so scale economies do not come into play in this example. Good N+2 is latent fiat money. It is a low transaction cost instrument suitable for paying taxes. That is, the market for N+2 versus N+1 (tax receipts) operates without transaction cost at a price ratio of 1:1. That would be enough to give N+2 a nonzero equilibrium price on other  $\{m, N+2\}$  trading posts, but that doesn't make N+2 a common medium of exchange. For N+2 to be a common medium of exchange requires that N+2 have low transaction cost, that

$\delta^{N+2} \leq \delta^i < 1/3$  all  $i = 1, 2, \dots, N$ . The combination of positive price and low transaction cost does the trick. There is an equilibrium where  $N+2$ , a useless government-issued instrument, fiat money, is the common medium of exchange.

In real life fiat money economies, government-issued fiat money is typically the unique common medium of exchange: in the US virtually all transactions are denominated in US dollars; in the UK virtually all (nonfinancial) transactions are denominated in pounds sterling. The virtual uniqueness of the monetary instrument is not merely a possibility; it seems to be a general fact. Dollars, euros, pounds sterling, and other government-issued fiat money's all seem to have similar low transaction costs. But in any single market economy precisely one of these instruments is likely to be the unique common medium of exchange. Prof. Tobin(1980) reminds us, above, that uniqueness is likely to be the result of scale economy. Example VII.2 harnesses this observation to explain why fiat money is (almost universally) the unique common medium of exchange.

Particularly in the case of scale economies in the transactions technology, there is a strong tendency to multiple equilibria. This creates an interest in determining which of the several equilibria the economy will actually select. One solution to this problem is to posit an adjustment process to equilibrium that makes the choice. Hence we use the following Tatonnement adjustment process for average cost pricing equilibrium:

Prices will be adjusted by an average cost pricing auctioneer.

Specify the following adjustment process for prices.

STEP 0: The starting point is somewhat arbitrary. In each pairwise market the bid/ask spread is set at average cost for low trading volume.

CYCLE 1

STEP 1: Households compute their desired trades at the posted prices and report them for each pairwise market.

STEP 2: Average costs (and average cost prices) are computed for each pairwise market based on the outcome of STEP 1. Prices are adjusted upward for goods in excess demand at a trading post, downward for goods in excess supply, with the bid/ask spread adjusted to average cost.

## CYCLE 2

Repeat STEP 1 (at the new posted prices) and STEP 2.

CYCLE 3, CYCLE 4, .... repeat until the process converges.

This plausible adjustment process explains why government-issued fiat money becomes the unique common medium of exchange ---- and would do so even in the absence of legal tender rules. Government has two distinctive characteristics: it has the power to support the value of fiat money by making it acceptable in payment of taxes; it is a large economic presence undertaking a high volume of transactions in the economy. Government's size means that it operates on sufficient scale to achieve scale economies (when the transaction technology admits them). In particular, if government is active on the posts trading fiat money for other goods then these trading posts will have the benefit of scale economies and low average transaction cost. Hence, government can make its fiat money the common medium of exchange merely by using it as such. The scale economies implied will make fiat money the low transaction cost instrument and hence the most suitable medium of exchange, not just for government but for all transactors.

Example VII.2 Let the population of households be  $H=H^0 \cup H^K$ . Let  $u^{[m,n]}$  be described by (UT). Let  $\tau^o > 0$  be a constant. Let  $0 < \tau^{[m,n]} = \tau^o < A(1-\delta^{N+2})(1-\delta^m)$ , all  $[m,n] \in H^K$ . Let  $x_n^G = \kappa \tau^o q_{N+2}^{\{N+2,n\}}$  all  $n=1,2,\dots,N$ . Let  $C^{[i,j]}$  be described by (TCNC). Let  $(\gamma^{N+2}/\kappa \tau^o) < \delta^i < 1/3$  all  $i = 1,2, \dots, N$ . Then a monetary average cost pricing equilibrium with taxation with good  $N+2$  as 'money' is the unique limit point of the tatonnement adjustment.

### Demonstration of Example VII.2:

Step 0: For  $n \neq m$ , set  $q_n^{\{m,n\}} = (1-\delta^n)$ .

Cycle 1, Step 1:

For  $i=1,2,\dots,\kappa$ , let  $s_n^{\{n,n\}} = A - (\tau^o/q_n^{\{N+2,n\}})$ ,  $b_{n\oplus i}^{\{n,n\}} = (A - (\tau^o/q_n^{\{N+2,n\}})) q_n^{\{n,n\}}$ ,

$s_{N+2}^{\{N+2,N+1\}} = \tau^o = b_{N+1}^{\{N+2,N+1\}}$ ;  $b_{N+2}^{\{N+2,n\}} = \tau^o$ ,  $s_n^{\{N+2,n\}} = \tau^o/q_n^{\{N+2,n\}}$ . For

$n=1,2,\dots,N$ , let  $s_{N+2}^{G\{N+2,n\}} = \kappa \tau^o$ ,  $b_n^{G\{N+2,n\}} = \kappa \tau^o q_{N+2}^{\{N+2,n\}}$ .

Cycle 1, Step 2: For  $n, m \neq N+2$ ,  $n \neq m$ , set  $q^{(m,n)}_n = (1 - \delta^n)$ .

$$q^{(N+2,n)}_n = (1 - \min[\delta^n, \gamma^n / \kappa \tau^o]) (1 - \gamma^{N+2} / \kappa \tau^o), \quad q^{(N+2,n)}_{N+2} = 1.$$

Cycle 2, Step 1: For  $n=1, 2, \dots, N$ , let  $s^{G(N+2,n)}_{N+2} = \kappa \tau^o$ ,  $b^{G(N+2,n)}_n = \kappa \tau^o q^{(N+2,n)}_{N+2}$ ;

$$s^{G(N+1,N+2)}_{N+1} = N \kappa \tau^o, \quad b^{G(N+1,N+2)}_{N+2} = N \kappa \tau^o; \quad b^{[n,n \oplus i]\{N+2,N+1\}}_{N+1} = \tau^o, \quad s^{[n,n \oplus i]\{N+2,N+1\}}_{N+2} = \tau^o;$$

$$s^{[n,n \oplus i]\{N+2,n\}}_n = A, \quad b^{[n,n \oplus i]\{n,N+2\}}_{N+2} = A q^{(N+2,n)}_n; \quad s^{[n,n \oplus i]\{n \oplus i, N+2\}}_{N+2} = A q^{(N+2,n)}_n - \tau^o,$$

$$b^{[n,n \oplus i]\{n \oplus i, N+2\}}_{n \oplus i} = (A q^{(N+2,n)}_n - \tau^o) q^{(n \oplus i, N+2)}_{N+2}.$$

Cycle 2, Step 2: For  $n, m \neq N+2$ ,  $n \neq m$ , set  $q^{(m,n)}_n = (1 - \delta^n)$ .

$$q^{(N+2,n)}_n = (1 - \min[\delta^n, \gamma^n / \kappa A]) (1 - \gamma^{N+2} / \kappa A), \quad q^{(N+2,n)}_{N+2} = 1.$$

Cycle 3, Step 1: Repeat Cycle 2, Step 1.

Cycle 3, Step 2: Repeat Cycle 2, Step 2.

Convergence.

What's happening in Example VII.2? Scale economies are taking their course!

Government is a large economic agent trading good  $N+2$  (incipient fiat money) for all real goods  $1, \dots, N$ , and then accepting  $N+2$  in payment of taxes,  $N+1$ . At first households try to trade their endowments for their desired goods directly, while at the same time arranging to trade enough endowment for good  $N+2$  (incipient fiat money) to finance their individual tax bills.

Government expenditures in all goods markets in exchange for  $N+2$  (and large household demand to acquire  $N+2$  to finance tax payments) result in a large trading volume on the trading posts for good  $N+2$  versus  $n=1, \dots, N$ . Volume is large enough that scale economies kick in. The average cost pricing auctioneer adjusts prices, the bid/ask spread, to reflect the scale economies. The bid/ask spreads incurred on trading  $m$  for  $m \oplus i$  by way of good  $N+2$  become considerably narrower than on trading  $m$  for  $m \oplus i$  directly. The price system then directs each household to the market  $\{m, N+2\}$  where its endowment is traded against good  $N+2$ . The household sells all its endowment there for  $N+2$  and trades  $N+2$  subsequently for tax payments and desired consumption. Scale

economy has turned  $N+2$  from a mere tax payment coupon into 'money,' the unique universally used common medium of exchange.

## VII. Conclusion

The taxonomy of cases developed is depicted in the table.

### Equilibrium Monetary Structure

#### Returns to Scale in Transaction Technology

| <u>Demand Structure</u>                                       | <i>Linear Transaction Technology</i>   | <i>Increasing Returns Transaction Technology</i>   |
|---|--|--|
| <i>Absence of Double Coincidence of Wants</i>                 | Monetary Equilibrium where the low transaction cost instrument becomes 'money' (Example III.1); Possibly multiple 'moneys' (Example III.2) | Monetary Equilibrium with Unique 'Money' (Example VI.1)  |
| <i>Absence of Double Coincidence of Wants with Fiat Money</i> | Fiat Money Equilibrium if fiat money is the low transaction cost instrument (Example VII.1)  | Fiat Money Equilibrium ('money' is unique) when tax payments and government purchases are sufficiently large (Example VII.2) |
| <i>Full Double Coincidence of Wants</i>                       | Nonmonetary equilibrium (Example IV.1)   | Monetary Equilibrium with Unique 'Money' (Example VI.2)  |

Absent double coincidence of wants, with linear transaction costs, a low transaction cost instrument is endogenously chosen as a common medium of exchange. Absence of double coincidence of wants is essential to monetary equilibrium with linear transaction costs. Alternatively scale economies in transaction cost (nonconvex transaction technology) lead to a corner solution, uniqueness of the common medium of exchange. Fiat money derives its positive value from acceptability in payment of taxes. Fiat money becomes the unique common medium of exchange when government taxation and purchases are sufficiently large that scale economies in transaction costs make it the low (average) transaction cost instrument.

The monetary character of trade, the existence of a common medium of exchange in economic equilibrium, can be logically derived from a (non-monetary) Arrow-Debreu Walrasian model through the addition of two constructs: segmented markets with multiple budget constraints (one at each transaction) and transaction costs. The multiplicity of budget constraints creates a demand for a carrier of value (medium of exchange) between transactions. Money (the common medium of exchange) arises endogenously as the most liquid (lowest transaction cost) asset. Government-issued fiat money derives its value from acceptability in payment of taxes. Uniqueness of the monetary instrument (fiat or commodity money) in equilibrium comes from scale economies in transaction cost.

The monetary structure of trade in general equilibrium, the uniqueness of money, and the existence of a fiat money equilibrium can be derived from elementary price theory.

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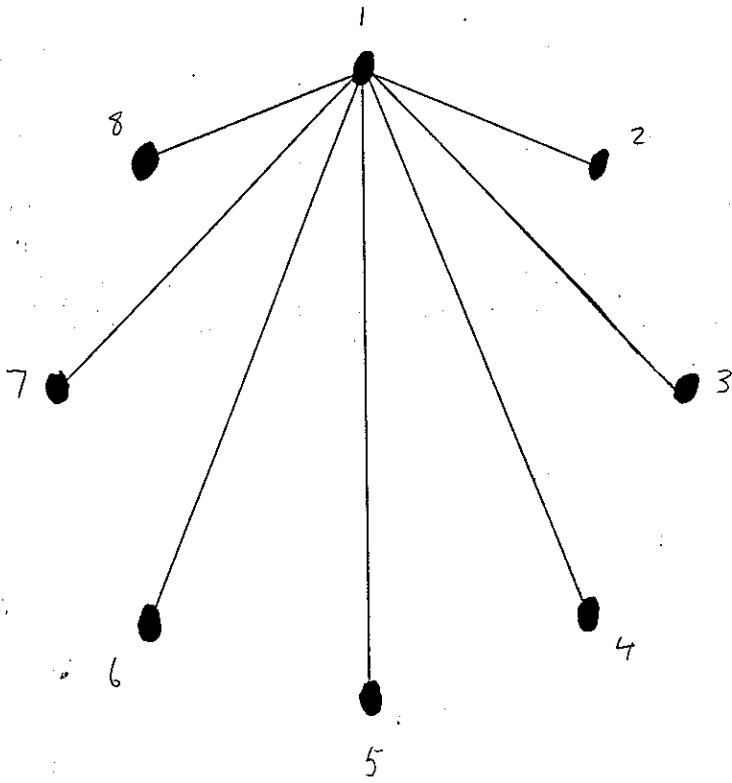


Figure 1: Monetary Equilibrium with Unique Money

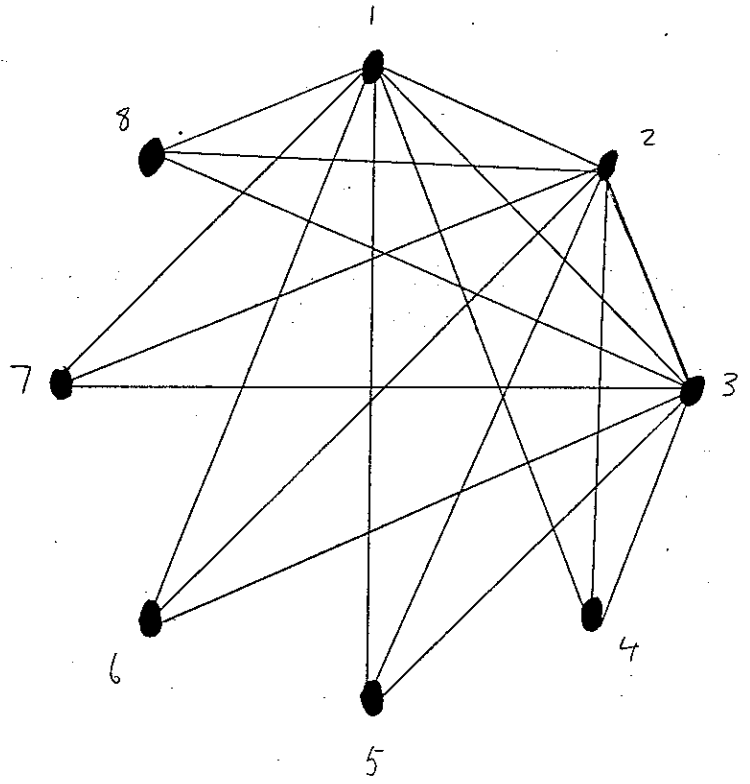


Figure 2: Monetary Equilibrium with Several Money's

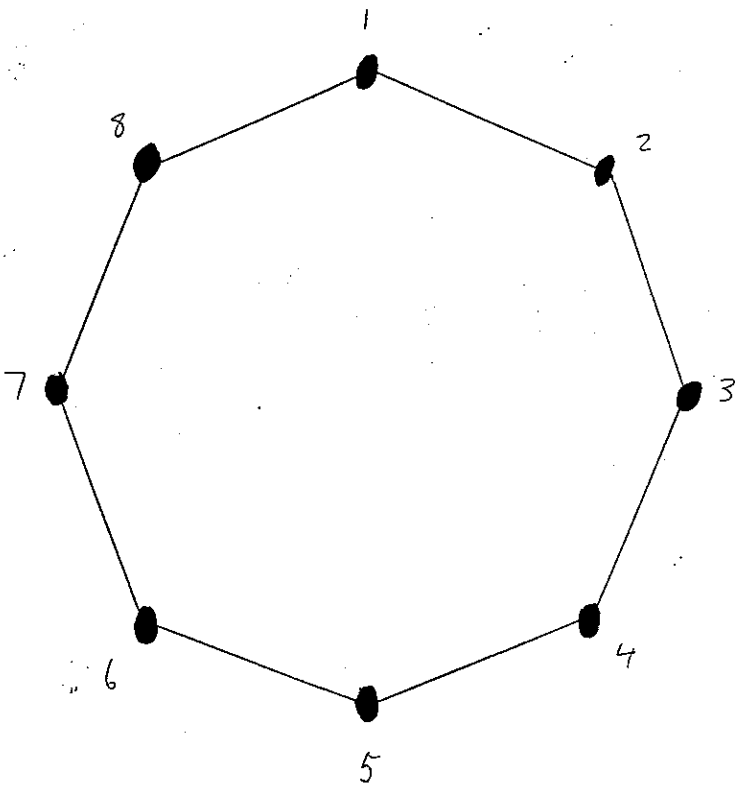


Figure 3: Barter Equilibrium for  $H^1$

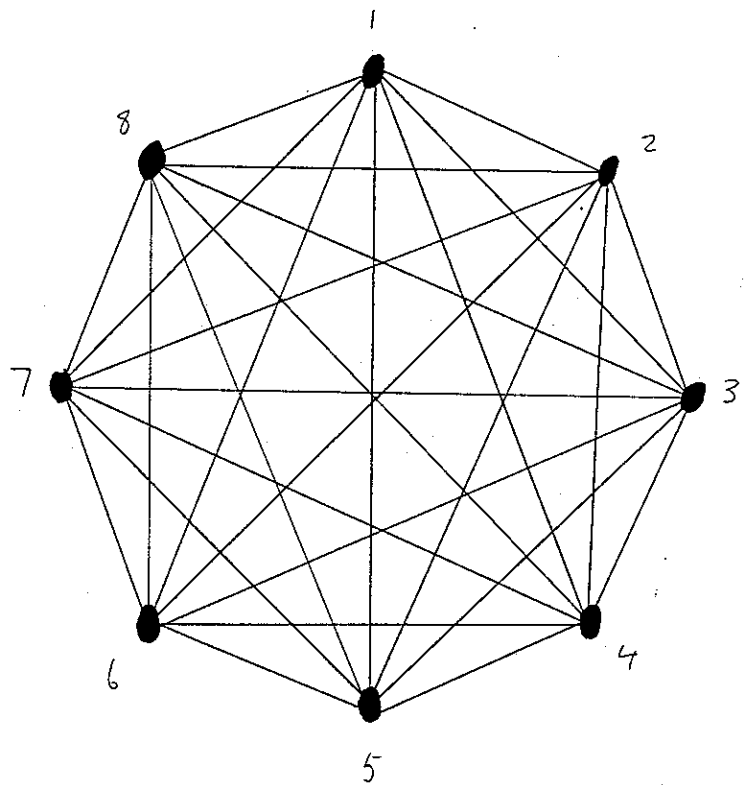


Figure 4: Barter Equilibrium for  $H^D$