UNIVERSITY OF CALIFORNIA, SAN DIEGO

Everyday Equity for Latina/o Students:

Practices that Teachers and Students identify as Supporting Secondary Mathematics Learning

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Education

in

Teaching and Learning

by

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Committee in Charge:

Professor Mica Pollock, Chair
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Professor Jeff Rabin

2015

The Dissertation of Ivette Sanchez-Gutierrez is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Chair

University of California, San Diego

2015

Dedication

To my amazing husband, Dario Gutierrez; you are my greatest love. Your love and support has carried me through these four arduous years and I am fortunate to share my life with you. To my beautiful boys, you are my purpose, my greatest pride, and greatest success. To my mother, mami preciosa… de ti he aprendido lo que es ser una mujer fuerte que no se da por vencida. To all my students, colleagues, and professors; I am forever indebted for your inspiration, dedication, and the lessons you have taught me.

Epigraph

"We cannot seek achievement for ourselves and forget about progress and prosperity for our community… Our ambitions must be broad enough to include the aspirations and needs of others, for their sake and for ours."

Cesar Chavez

"For to be free is not merely to cast off one's chains, but to live in a way that respects and enhances the freedom of others."

Nelson Mandela

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Vita

Ivette Caribe Sanchez- Gutierrez

Education

University of California San Diego, June 2015
Doctor of Education in Teaching and Learning
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Master of Arts in Mathematics Emphasis in Secondary Education
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Bilingual - CLAD Credential
San Diego State University, December 1997
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Current Employment

Mathematics, Teacher on Special Assignment
State & Federal Programs
Sweetwater Union School District (July 2008 – Present)

- Synthesize critical data into district-wide placement lists for English Learners and strategic students
- Verify proper placement of English learners in Structured English Immersion and other support courses
- Design professional development specifically designed to facilitate instructional access for English Learners in all content area through the new California 2013 English Language Development (ELD) standards
- Deliver professional development on district initiatives that is reflective of district, site, and teacher needs
- Coach and support teachers of long-time English learners
- Develop resources for teachers, especially those to concurrently foster the development of academic language skills while implementing Common Core State Standards
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- Contribute Curriculum, Instruction, and Assessment (CIA)
- Assist teachers in regular analysis of student achievement data to guide curriculum, instruction, assessment, and professional development

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Professional Experience

Curriculum Development & Professional Development

- **Mathematics Resource Teacher.** Work with 20 mathematics teachers providing model lessons, assistance in planning lessons, assistance with student placement, data analysis, and CST/CAHSEE/ Placement testing. Part of the school leadership team, curriculum design, and PLC Lead. (July 2004- June 2008)
- **UCSD Professional Development Institute Curriculum Writer SB472 Mathematics McDougal & Prentice Hall.** Designed and wrote three 40-hour modules for state of CA that where approved by the CA state board for both the 2000 adopted texts and for the 2009 adopted texts with emphasis on proper lesson planning that addresses differentiated teaching and a balanced curriculum.(July 2006- September 2008)
- **Mathematics Consultant for UCSD Professional Development Institute & SDSU Math Project (LOS Project).** Developed tools for key 7th grade, Algebra Readiness, and Algebra I standards for teachers of English Learners. Wrote Trainer syllabi for math portion of LOS Project. (July 2005- present)
- **Mathematics Consultant for UCSD Professional Development Institute & SDSU Math Project (CAMSP Project).** Developed and trained teachers of English Learners in LAUSD local district 6 on content pedagogy for 7th grade, Algebra Readiness, and Algebra I standards. Wrote lessons and guided action research. (July 2009- present)
- **Mathematics Consultant for UCSD Professional Development Institute & SDSU Math Project (Access to the Core).** Design and deliver Algebra Readiness and Algebra I Access to the Core professional development for trainers in the Los Angeles Unified Schools District 6. Concentration on instructional strategies and addressing content knowledge in depth for all students but focusing on needs of English Learners. (July 2006- present)

- **Secondary Mathematics Project Specialist.** Work for the San Diego County Office of Education. Wrote curriculum, trained over 500 teachers, teacher leaders, and principals in mathematics content and pedagogy. Lead on the CAHSEE Compact "Closing the Achievement" Taskforce, addressing the needs of English Learners in mathematics, Curriculum and Instruction Math Sub-committee Lead, support and liaison for all 42 districts in San Diego and duties as assigned. (August 2003- May 2004)
- **Proyecto California.** Developed alignment of Mexico's online math resources and courses to California's standards focusing on 7th grade and Algebra key standards as a support system for Spanish speaking English Learners in California. Provided training and assistance San Ysidro Middle School teachers in using resources. (August 2003- June 2004)
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- **Daily Warm-ups: Exercising the CAHSEE Mathematics Standards Writer.** Wrote mathematics multiple choice problems that align and augment the 53 CAHSEE math tested standards, including test-wise functions, suggestion on how they may be used, vocabulary that needs to be addressed, and test taking strategies and tips. (Fall 2003)
- **Algebra for Teachers of English Learners Institute Trainer.** In collaboration with SDSU, trained 25 teachers to understand in detail how to incorporate the needs of English learners needs, pedagogy, English proficiencies, and rigorous math standards to create and assure greater access to mathematics for all students. (Summer 2004)
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- **AB 75/ AB 466.** Completed AB 466 McDougal -Littel and Prentice Hall AB466 certification. Completed AB 75 Module 1certification May 2004 (Fall 2003 - Present)
- **A.V.I.D. Presenter/Trainer.** Train over 500 math teachers to incorporate A.V.I.D. (Advancement Via Individual Determination) methodologies (writing, inquiry, collaboration, and reading) in the math classroom at Summer Institutes and *Write Path* trainings around the U.S. (Summer 2001 - Present)
- **CAPP-WAC Teacher Leader.** (WestED, California Academic Partnership Program-Western Assessment Collaborative) Create and teach standard-based lessons, analyze standards to ensure student mastery, train math department to write and teach standard-based lessons. Collect and analyze data pertaining to student mastery. (Fall 2000 - Present)

- **Consultant for CAPP Grant.** Consultant for changes in mathematics curriculum. Mar Vista High School (Fall 1996 - Spring 1997)
- **San Diego Math Project.** Participating teacher committed to seven years of exploring different techniques and technologies to enhance the teaching and learning of our students. Analyze and align curriculum to CA standards. Graduate of the Multimedia Academy in Chula Vista via SDMP. (Summer 1997 - Present)
- **Master Teacher.** Supervised and provided support for a new math teacher. Modeled lessons incorporating GSP. (Fall 2002)
- **Teacher.** Algebra , Formal Geometry, Intermediate Algebra, Math Analysis in English and Spanish, regular and Bilingual A.V.I.D. Saturday SAT-preparation course.

Professional Affiliations

American Educational Research Association (AERA)
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Abstract of the Dissertation

Everyday Equity for Latina/o Students:
Practices that Teachers and Students identify as Supporting Secondary Mathematics Learning

by

Ivette Sanchez-Gutierrez

Doctor of Education in Teaching and Learning

University of California, San Diego, 2015

Professor Mica Pollock, Chair

Student success in high school mathematics is recognized as being one of the most critical factors in determining access to post-secondary education and subsequent success. Currently, the American educational system is failing to prepare a mathematically proficient citizenry able to meet workforce demands. Hence, concerns regarding access to high-quality mathematics education for growing numbers of marginalized groups are increasingly being highlighted as a national issue. This dissertation identifies mechanisms for improving equity in mathematics education for the nation's fastest-growing demographic groups: Latin@s.

This qualitative research study examines the classroom experiences of both Latin@ students who are non-native English speakers and their teachers by looking at the instructional practices that students and teachers identify as being pivotal to student mastery of rigorous mathematics content. In addition, this study investigates the relationships between teachers' practices employed in daily lessons, "best practices" identified in research on Latin@s, and the practices students identify as fostering access to mathematics learning. Based on student and teacher interviews, study results indicate that effective teaching practices can be categorized into four major groups: 1) teacher overarching growth mindset practices, 2) teacher-student talk practices, 3) student processing time practices, and 4) partner processing time practices. Finally, the study confirmed that teachers consistently employed those methods that research has previously identified as "best practices" and which students had identified as being important to their learning and understanding; participants described such practices in more detail than prior research. Participants also named several beneficial practices not analyzed in detail by prior researchers.

Chapter 1: Introduction

"Mathematics education is a civil rights issue."

Civil rights leader Robert Moses

The United States of America is a world leader in many respects. It enjoys one of the most ethnically diverse populations in the world and boasts some of the greatest intellectual talent worldwide. It is a nation of incomparable wealth in terms of both human and material resources. Counted among those resources are some of the most prestigious universities and centers of research on earth. Creativity and invention, being nurtured by those institutions and the entire educational system, ensure that the United States remains a world leader and contribute to the country's ability to sustain the largest economy in the world.

Having a highly educated population is crucial to maintaining the country's position of prominence in the world. To that end, the United States provides free K-12 education to all of its youth and monitors the fitness of the nation's educational systems. Above all, the country promotes the notion of unencumbered opportunity for everyone, and education is the key activator of opportunity. Opportunity, in turn, is inextricably tied to freedom—which is regarded as the most essential feature of this country and everyone in it.

Yet at the same time as the country enjoys such unparalleled success, it also displays major shortcomings, such as significant gaps in educational attainment for large subgroups of the population, such as Latin@s. This does not bode well for a country that is quickly becoming majority non-White. Latin@s comprise an estimated 17% of the current U.S. population and forecasts predict that will grow to an estimated 29% by 2050 (Passel & Cohn, 2008; US. Census, 2014). For large states like California, continual economic stability is dependent on the success of Latin@s who currently account for 38% of the state's population and are estimated to be the majority of the state's population by 2050 (The Campaign for College Opportunity, 2013). Currently, half of all youths under 18 years of age in California are U.S. born Latin@s. Alarmingly, of California's 25 years and older population, only 11% of Latin@s have bachelors degrees compared to 23% of Blacks, 39.3% of Whites and 47.9% of Asians (The Campaign for College Opportunity, 2013). Overall, 30.3% of all California's 25 years of age and older population have a bachelors degree or higher. As the Latin@ population becomes a more significant percentage of the overall U.S. population, similar bleak trends can be expected throughout the nation if the Latin@ population is not supported more effectively.

One specific concern is that Latin@ children persistently score more than 20 points lower than their White counterparts in mathematics on the National Assessment of Educational Progress (NAEP) (National Center for Educational Statistics, 2013). Analyses of NAEP data show that as students get older, the gap widens in mathematics and reading achievement between Whites and marginalized or underserved groups such as Latin@s and Blacks

(National Center for Educational Statistics, 2013). In order to narrow gaps, all stakeholders need to specifically address and focus on current educational outcomes for Latin@ students. Lack of success in mathematic achievement is an urgent issue because it precludes students from scientific and professional opportunities and careers (Schoenfeld, 2002). High school mathematics coursework has consistently been linked to college attendance and completion rates (Tyson et al., 2007). Researchers from Harvard University and the University of Virginia found that rigorous high school mathematic coursework was the most important predictor of success in college science coursework such as biology, chemistry, and physics, more so than high school science coursework itself (Harvard University, 2007). This means that success in mathematics has life-long implications for students' lives, their future earning power, and the country's financial earnings. Latin@s' exclusion from higher education as a whole also excludes Latin@s from contributing to the country's financial stability and growth.

Furthermore, high school mathematics and science coursework prepares students for STEM (science, technology, engineering, and mathematics) careers (Tyson et al., 2007). Currently, the end of the pipeline in STEM looks particularly dire for the U.S. economy. Members of the *Rising Above the Gathering Storm* Committee found that while only 4% of the U.S. workforce is comprised of STEM careers; the Science, Technology, Engineering, and Mathematics field workforce generates 96% of all current jobs (Augustine et. al, 2010). Authors discuss the benefits that STEM progress produces for the workforce as a whole. They state,

> It is not simply the scientist, engineer and entrepreneur who benefit from progress in the laboratory or design center; it is also the factory worker who builds items such as those cited above, the advertiser who promotes them, the truck driver who delivers them, the salesperson who sells them, and the maintenance person who repairs them—not to mention the benefits realized by the user. Further, each job directly created in the chain of manufacturing activity generates, on average, another 2.5 jobs in such unrelated endeavors as operating restaurants, grocery stores, barber shops, filling stations and banks.

Members of the *Rising Above the Gathering Storm* Committee urge the United States to invest, recruit, and retain young people in STEM careers. Currently, 31% of China's bachelor's degrees are awarded in engineering while the U.S. awards a dismal 4% every year. As a result, much of STEM demand is met by other countries around the world (Augustine et. al, 2010). The National Academies *Gathering Storm* committee concluded that the U.S. future economic growth is predicated on job growth within STEM fields (2010). By 2018, 92% of STEM jobs will require postsecondary education with 65% of those jobs requiring at least a bachelor's degree (Carnevale, Smith, & Melton, 2011).

A Harvard study concluded that the U.S. could increase GDP growth per capita by enhancing its students' mathematic skills (Stem Education & Workforce, 2014). At present, the U.S. has had to look to foreigners to fill this gap in the workforce. Additionally, many foreigners are educated in the U.S. but do not stay to contribute to the U.S. workforce and instead return to their home countries to strengthen or develop companies (Kuenzi, 2008; US Dept. of State, 2009). Kuenzi (2008) states that foreign students earn approximately one-third of the doctorates issued in the United States. The National Science Foundation (NSF) reports that of the doctoral degrees awarded in the United States,

> foreign students earned more than half of those [awarded] in engineering, 44% of those in mathematics and computer science, and 35% of those in the physical sciences (Kuenzi, 2008, p.15).

The United States issues work visas and permanent residence to many mathematicians, scientists, and engineers from India, China and other countries to fill the gap left by U.S. citizens who lack the skills required by the workforce in science related fields. The tides of the global economy have shifted so drastically in the past few decades that the U.S. has resorted to outsourcing many of its coveted high-skilled jobs to other countries (US Dept. of State, 2009).

To counter this trend, the United States must look to the educational system to preserve its global ranking in the world market. Just as the United States has for the past century been the largest car manufacturer, a leader in technology and agriculture, it must now focus on producing a workforce that is proficient in mathematics, science, and technological fields in order to maintain its competitive edge. Although U.S. higher education institutions rank among the best in the world in their ability to prepare students, an inadequate number of students in the K-12 educational system are prepared to enter and excel in higher education, much less in the fields of mathematics and science. Of students who enter college as STEM majors, 38% never graduate with a degree in a STEM field (Carnevale, Smith, & Melton, 2011). Of concern, Latin@s and Blacks are amongst the least prepared to succeed in these fields. The current racial and ethnic distribution of the U.S. STEM workforce is 71% White, 15% Asian, 7% Hispanic, and 6% Black (Landivar, 2013). This is incongruent with national demographic data that show the general population consisting of 63% White, 16.9% Hispanic, 13.1% Black, and 5.1% Asian (U.S. Census, 2014).

Disturbingly, the National Math + Science Initiative reports that only 44% of U.S. high school students take the higher level coursework in mathematics necessary for collegiate mathematics. As of 2009, only 12% of Black and 17% of Latin@ students took Algebra I before high school, compared to 29% of White students and 48% of Asian (Stem Education & Workforce, 2014). Furthermore, only 9% of Latin@ students took advanced algebra or calculus in high school (Stem Education & Workforce, 2014). The disparity in Black and Latin@ students accessing those rigorous mathematics courses at the secondary level is alarming.

Guaranteeing that United States Latin@ students are mathematics proficient is thus vital to the future success of the country and California. Latin@s, the largest growing subgroup, will play a vital role in the workforce. It is evident that the United States requires a highly educated workforce especially within STEM fields (Augustine et. al, 2010). Therefore ensuring success for Latin@ students in mathematics is no longer simply a Latin@ issue but rather has more broad social consequences where the success and progress of society as a whole is dependent on the success of this critical subgroup. Indeed, American essentialism states that all students must be given access to a quality education and must be granted the right to pursue and develop their passions and talents. We must strive to ensure access to the full benefits of education, and quality mathematics education is at the center (Harvard University, 2007; The Campaign for College Opportunity, 2013; Tyson et al., 2007).

The United States is taking these statistics very seriously and is undergoing drastic changes in how students are educated as well as what they learn. To this end, in 2009, as part of the American Recovery and Reinvestment Act, President Obama and Secretary of Education Arne Duncan announced a $4.35 billion fund, named Race to the Top, focused on innovation and reform in state and local K-12 education school districts. As a result of the Race to the Top initiative, many state leaders, governors, and state commissioners of education joined forces to research and investigate best practices in our nation and globally. Under the guidance and leadership of the Council of Chief State School Officers (CCSSO), they commissioned working groups to generate new sets of standards for K-12 education in English/ Language Arts and Mathematics (National Governors Association

Center for Best Practices, & Council of Chief State School Officers, 2010). The Mathematics working groups, led by three experts in mathematics education, mathematics, and physics, investigated research, observed classrooms, and met with key stakeholders in order to develop a cohesive, coherent, and focused set of standards in mathematics. These standards provide grade level mastery expectations as well as mathematical habit of mind standards that they named the “Standards for Mathematical Practice.” The Standards for Mathematical Practice define the level of rigor, depth of knowledge, and focus for each grade level standard. They offer the lens through which each standard should be taught and specify the level of mastery expected for each grade-level standard. Rigor is a mathematical shift identified in the new Common Core standards and a goal of those standards (National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010). Specifically, the Common Core mathematics standards delineate that rigor is achieved when each major topic, pursues with equal intensity: 1) conceptual understanding, 2) procedural skill and fluency, and 3) application (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Furthermore, the Common Core State Standards Initiative specifically states that, ”rigor refers to deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). It aligns with the Rigor and Relevance Framework developed by the International Center for Leadership in Education to describe the levels of thinking and levels of application (Daggett, 2005). Therefore, each academic grade is expected to master the Standards for Mathematical Practice for each specifically identified grade level common core standard.

States had the option of adopting the proposed Common Core Standards with no more than 15% modification. Currently, forty-three states, including California, have adopted the standards. New assessments and textbook adoptions are underway, as well as training, and the incorporation of standards in classrooms throughout California. One of the most interesting differences between California's old achievement testing and the new proposed testing is the level of rigor between the two. For example, students are no longer asked to simply bubble in a multiple-choice answer. Rather, they must articulate in writing how they solved the mathematics problem, defend their selection, solve contextualized and real–world problems, and collaborate with others in the new assessments. In addition, the new assessments provide for multiple correct answers to a single question as well as multiple solution paths. These changes in mindset for both educators and students will take some time but the adjustments provide an additional element of hope that the statistics previously mentioned might be altered for the better.

Thus, this dissertation set forth to explore a core aspect of mathematics education in the United States: everyday instruction. Methodologically, this dissertation sought to listen to teachers’ and students’ perspectives on instruction that supports student learning and success. I turn now to a review of relevant literature.

Chapter 2: Review of Literature

Equity in mathematics education – generally defined as “the full range of opportunities [in mathematics education] that can stimulate each person to tap fully his or her interests and capabilities” – has been identified as a dire need that must be addressed in the United States (National Research Council, 1989). From as far back as twenty-five years, the National Research Council (1989) recognized that,

> Because mathematics holds the key to leadership in our information-based society, the widening gap between those who are mathematically literate and those who are not coincides, to a frightening degree, with racial and economic categories. We are at risk of becoming a divided nation in which knowledge supports a productive, technologically powerful elite while a dependent, semiliterate majority, disproportionately Hispanic and Black, find economic and political power beyond reach (p. 14).

In order to identify preparation gaps and to decipher problems in mathematics education, stakeholders need to examine how mathematics education is conducted in everyday classrooms across the nation.

Perspectives on Inequity in Mathematics Education

Research suggests that many factors contribute to preparation for mathematics success in school (Gandara, 2006). Most broadly, Gandara states that many Latin@ students are affected by numerous factors that have little or nothing to do with formal education, such as health care, nutrition, housing stability, and neighborhood environment. These factors have a strong influence on student academic outcomes. She cautions that although these factors are powerful, how “schools [currently] *do* affect student outcomes should not be confused with the extent to which schools *can* affect outcomes" (Gandara, 2006). Gandara argues that students' intellectual and academic preparation in school must be the focal point at the earliest grades with intensive emphasis on interventions; this focus must continue throughout their schooling program if gaps identified in the early years are to be closed. Pollock (2008; forthcoming), adapting the work of economist Rebecca Blank, further describes the ongoing widening or narrowing of the achievement gap due to accumulating student school experience as "cumulative advantage" for some and "cumulative disadvantage" for others. That is, initial small gaps can and do get wider if school system efforts and attitudes toward students are not focused on maximizing the potential of these students. The National Research Council (1989) argues relatedly that, "Differences in culture magnified by differential opportunities to learn imposed by twelve years of multipl[e] tracked classes produce vastly different evidence of mathematical power."

Researchers discuss other mechanisms related to mathematics achievement. The National Research Council suggests that in the United States, mathematics was part of the upper and middle class male culture (National Research Council, 1989); others argue this continues to be true today (Carnevale, Smith, & Melton, 2011). Ong reports that placement practices that use standardized test scores and teacher recommendations are often "racially biased and subjective" (as cited in Pollock, 2008). She argues that, as a result, half of all students are erroneously tracked to lower, remedial classes, even while 90% of those under-tracked students have the potential to successfully master the material of higher track coursework, provided they have sufficient support (as cited in Pollock, 2008). This statistic is particularly alarming, because it shows that we are slamming the door shut on students who could have succeeded.

Perhaps most important, however, is the everyday teaching that occurs in mathematics classrooms. More than two decades after the National Research Council published *Everybody Counts* (1989), a pivotal report in the field of mathematics education, typical mathematics teaching practices have been reevaluated to determine how they foster or impede learning for historically underserved groups in mathematics related fields. Research suggests that contributing causes of achievement gaps include: decontextualized mathematics curriculum, meaning that the purpose for learning mathematics is not made clear to students (Cahnmann & Remillard, 2002; Gutstein, 2003; Schoenfeld, 2002); negative teacher beliefs about students' abilities (Rousseau & Tate, 2003); lack of student participation in the learning process (Cahnmann & Remillard, 2002; Gutierrez, Willey, & Khisty, 2011); and teachers' ignorance of students' cultural assets, defined as the values, contributions, practices, language, and perspectives of the populations in their classrooms (Gutierrez, 2002; Gutierrez, Willey, and Khisty, 2011; Rousseau & Tate, 2003; Schoenfeld, 2002).

Most equity-oriented mathematics education research of the 1970's and 1980's focused on making mathematics content instruction accessible to all students by challenging the widely accepted societal belief that success in mathematics was predominantly due to differences in innate ability (National Research Council, 1989). As the National Research Council noted (1989), many researchers worked to disprove this notion by investigating topics such as the effects of: high expectations on outcomes, encouraging minority students and females to take high-level mathematics coursework, allowing students to collaborate in the problem-solving process, encouraging multiple ways of solving mathematical problems, and providing highly qualified mathematics teachers for all students.

Also, during the 1970's and 1980's, research in mathematics instructional practices had multiple foci. One such focus was allowing students to negotiate meaning, that is, to discuss how they understood a process. Such research recognized that students could take multiple paths to solve a problem and that problems were not necessarily solved quickly or in the same format privileged by the teacher (Schoenfeld, 2002). These were important findings in terms of equity because they validated students' sociocultural assets. That is, the research looked at mathematics not as facts to be memorized but as knowledge that needed to be constructed. Knowledge could be constructed in varying ways and teachers could enhance students' mathematics learning by validating alternate ways of thinking and problem solving.

A second major focus of research in the 1970's and 1980's encompassed classroom environment and interpersonal interactions conducive to learning. Findings during this era of mathematics education research highlighted that "one size does not fit all" regarding students' acquisition of mathematical knowledge. That is to say, there is no single way of learning mathematics, nor is there one single way of teaching mathematics that is effective for all students.

But equity researchers in the 1970's and 1980's, although attempting to level the playing field after the civil rights movement, were unable to eliminate student-deficiency models that were deeply embedded in the educational systems, nor were they able to fully influence academic outcomes for specific groups such as Latin@ and Black students. That is to say, researchers argued that teachers' long-standing beliefs about the character of student abilities, intelligence, and motivation to learn continued to influence the enacted opportunities to learn. Data showed that Latin@ and Black students were not faring well on standardized assessments, and therefore there was a call to provide opportunities to learn and access to rigorous mathematics for these subgroups (The National Research Council, 1989). The National Research Council, at the end of the 1980's, stated that "inadequate preparation in mathematics imposed a special economic handicap on minorities." This was an attempt to bring to light some of the racial disparities that negatively impacted minority groups.

Research during this time identified beneficial mathematics teaching practices that were advantageous for all students. For example, research found that mathematics is useful to students only when it is acquired through "personal intellectual engagement" and produces new understandings (National Research Council, 1989). *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* by the National Research Council (1989), found that students learn mathematics only when they construct their own mathematical understanding. This is accomplished by allowing students to examine, represent, transform, solve, apply, communicate, and prove mathematics in groups through discussions, and in other ways where they are in control of their own learning. The National Research Council (1989) further argued that "school mathematics" failed students by continuing traditional teaching practices where "most students cannot learn mathematics effectively by listening and imitating; yet teachers teach just this way" (p.57). They argued that mathematics needs to serve as a pump, not a filter in education, and that students cannot be blamed for their disinterest in mathematics when they rarely see the full power and richness of mathematics in the real world.

In attempting to connect more students to the purpose and value of mathematics, research identified a variety of equity-related factors in mathematics education. However, identifying equity-related factors specific to racial and ethnic group disparities of access and achievement in rigorous mathematics coursework did not become a focus until the mid-1990’s and 2000’s, when mathematics equity research incorporated a stronger focus on the socio-political aspects of mathematics education. The research findings of the 1970’s and 1980’s were particularly transformative for girls in terms of clarifying ways to secure their access to higher mathematics and college attainment. While many researchers investigated these educational practices during the 1990’s and 2000’s for underserved groups such as Latin@s and Blacks, research still did not do enough to increase marginalized groups’ achievement (Aliprantis, Dunne, & Fee, 2011; Gandara & Contreras, 2009). New research, which I call “Mathematics Equity Research” below, attempted to fill this gap.

Mathematics Equity Research with a Socio-political Perspective

“Mathematics Equity Research” with a “Socio-political” perspective (a term first coined by Mellin-Olsen in 1987) is a branch of mathematics education research, focused on *countering social injustices generated in past schooling practices within mathematics education*. This definition of “Mathematics Equity Research” is the one referenced and used in this dissertation. Its goal is to create a society in which women and various ethnic minorities enjoy equal opportunities and equitable treatment (NCTM, 1989; Schoenfeld, 2002). Socio-political mathematics equity research addresses the societal and political implications when disparities in access to rigorous mathematics continue in the K-12 pipeline as well as in university math-based careers for marginalized groups such as women, Blacks, Native Americans and Latin@s. Socio-political perspectives have a keen focus on social justice and are born from critical theory frameworks, but they also focus on everyday mathematics practices that can support teaching and learning. In sum, this body of research focuses specifically on mathematics instruction for marginalized students, including Latin@s.

Due to the strong connection between secondary school and college math achievement, some recent research questions in the field have focused on how mathematics education is experienced by marginalized groups in classrooms around the nation. Much of the equity research with a socio-political perspective uses theoretical frames such as Critical Mathematics Education, Critical Race Theory, Critical Latino/a Theory, and Post-Structuralism. These theoretical frameworks favor the perspectives of marginalized groups (Chapman & Hobbel, 2010; Gutierrez, Willey, & Khisty, 2011; Gutierrez, 2002; Gutstein, 2003; Rousseau & Tate, 2003). Some critical theories, such as LatCrit or Critical Race Theory, have been used to explore mathematics equity by privileging the voices of marginalized students through *testimonios* or counter-stories, some of which highlight student experiences in mathematics classrooms. These *testimonios* counter the socially accepted messages, also referred to as societal discourse, of marginalized students' underachievement. For example, a *testimonio* might demonstrate a school with a high population of Latin@ students exceling in Calculus, which counters the common assumption that Latin@s typically underachieve in mathematics.

Other research studies that will be explored further in this literature review investigated practices that support marginalized students' achievement (Gutierrez, 2008), how Latin@ students incorporate an academic mathematics identity into aspects of their current identity (Gutierrez & Irving, 2013), and the positive effects Latin@s experience in mathematics classrooms when instruction takes their identity and cultural resources into account (Gutstein, 2003; Schoenfeld, 2002). Other mathematics equity research with a socio-political perspective investigates practices that promote or discourage student learning based on teacher beliefs and the relationship to students' opportunity to learn in classrooms (Rousseau & Tate, 2003), the practices that benefit student learning when students see themselves as contributors to society using mathematics (Gutierrez, Willey, & Khisty, 2011), and the benefits of practices that encourage Latin@ students to use their evolving bilingualism to negotiate and co-construct mathematics knowledge (Gutierrez, 2008; Moschkovich, 1999). For purposes of this dissertation, the term "socio-political" is used to encompass these varied theoretical frames in order to explore themes and classroom practices that provide access to rigorous mathematics for underserved student populations. In addition, this dissertation defines classroom practice as a statement, action, or activity created by the teacher to engage students in learning mathematics.

For many in the mathematics education research field, research with a socio-political perspective is a means to inform educators and to remedy long existing inequities in the educational system. For as Schoenfeld (2002), a mathematician, states that, “to fail children in mathematics, or to let mathematics fail them, is to close off an important means of access to society’s resources (p.2).” Schoenfeld (2002) argues that the *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* Report (National Research Council, 1989) was the impetus for much of the research in the subsequent two decades. The report called for educators to develop “reform” mathematics curriculum that contextualizes mathematics and enables students to make sense of and solve problems in the world around them, specifically those they see in their own community and society. Therefore, many who research mathematics equity from the socio-political perspective encourage continued research that address one or more of the following facets for marginalized and underserved groups: achievement in mathematics, access to rigorous mathematics, awareness of student identity, and incorporation of student voice.

Since 1989, some mathematics equity research has focused on student involvement and inclusion as partners in the design of their own mathematics learning (Gutstein, 2003; Schoenfeld, 2002). Such research asks: How can we use the cultural resources that students have when they walk into the classroom to provide access to mathematics? How can we use learners’ community as a contextual link to the purpose of mathematics? How can mathematics become a viable tool that is useful in solving real problems and issues for students? And finally, how can we empower students to voice their ideas and perspectives, and to value their varied approaches to mathematics? For the purpose of this dissertation, I have organized the socio-political mathematics equity research into the following categories: 1) practices that offer opportunity to learn rigorous mathematics, 2) practices that enact positive teacher beliefs and high expectations, 3) practices that encourage students to talk about mathematics and use their experience to connect to new learning and, 4) practices that empower student identity in mathematics.

Practices that offer opportunity to learn rigorous mathematics.

Rigor is a mathematical shift identified in the new Common Core standards and a goal of the standards as mentioned in Chapter 1. As previously mentioned, it aligns with the Rigor and Relevance framework developed by the International Center for Leadership in Education to describe the levels of thinking and levels of application (Daggett, 2005). Therefore, if "what" students must learn is clearly defined, we must then discuss "how" or by what means students will access the defined learning objectives.

Access to learning can be defined as the availability of resources to students and teachers in and outside the classroom (Gutierrez, 2008) or can more simply be defined as the "opportunity to learn" (Cobb, 2002). It is important to highlight that opportunity to learn does not mean equal treatment.

Opportunity to learn is about equitable treatment –treatment designed to equalize outcomes (Pollock, 2008). Equal treatment is often defined as the same “input,” for example using the same books, having the same teacher, or being taught the same lesson. Equitable treatment is effort enacted to achieve the same “output,” for example the same degree of achievement across groups in mathematics assessments, differentiated supports that lead to college and career readiness for all students, or resources in native language that support cognition of content. In other words, equal treatment does not mean equitable treatment.

The foundational ideas of equity were established in the 1970’s through several court cases. One such case, Lau vs. Nichols (1974) established that for English Learners, “There is no equality of treatment merely by providing students with the same facilities, textbooks, teachers and curriculum; for students who do not understand English are effectively foreclosed from any meaningful education.”

According to R. Gutierrez (2008), “opportunity to learn” goes beyond access to the “right” curriculum or technology and must include access to quality teachers who have strong content knowledge, sound content pedagogical knowledge, an awareness of how learning manifests itself in the discipline, and awareness of how to meet students’ specific needs. She delineates the practice of strong pedagogical content knowledge that affects student learning includes coherent approaches to teaching mathematics focused on conceptual understanding, problem solving, and skills fluency (California Department of Education, 2006; 2014). Again, I define a classroom practice as “a statement, action, or activity created by the teacher to engage students.” Gutierrez’ realm of practice specifically discusses the more traditional aspects of teaching but includes teachers’ awareness of/response to student needs.

Practices that enact positive teacher beliefs and high expectations.

Research has shown that teachers’ beliefs about their students’ ability and future potential greatly affect the level of instructional rigor and the level of support granted to students when they struggle with content (Rousseau & Tate, 2003). Rousseau and Tate (2003) explored how teacher beliefs affected students’ opportunity to learn, that is, the enacted practices of additional time, explanations, clarifications, and encouragement needed in order to access rigorous mathematics opportunities in mathematics classrooms. They found that a teacher’s definition of equity is directly tied to what opportunities to learn are enacted in the classroom. In other words, the definition that teachers gave for equity was aligned to the opportunities to learn that were observable in classroom interactions between teacher and students. The authors argued that while “equal” treatment is based on the same input or treatment, equitable treatment seeks the same output or results for groups of students. They found that when teachers were able to reflect on equity, teachers’ assumptions and beliefs about marginalized groups were exposed.

The seven teachers of lower-track mathematics classes in the study, all teaching in the same high school with a substantial number of Black students in their classes, overwhelmingly defined equity as equal treatment. That is, they defined equity in teaching as providing the same lesson, treatment, and assistance for all students in the class.

Rousseau and Tate (2003) further found that when teachers defined equity as equal treatment, those teachers held color-blind mindsets where they blamed students' culture for the lack of understanding or achievement in mathematics. "It was not so much the teachers' refusal to acknowledge student race as it was their refusal to acknowledge race-related patterns in achievement and the potential role of racism in the underachievement of students of color (p. 213)." For example, the researcher noted in one classroom that if a student asked a fellow classmate for help, the teacher refused to allow the student to receive assistance and did not attempt to help the student herself. The teacher stated that the student needed to take "responsibility for their own learning" instead of trying to decipher the student's confusion and assist them in understanding. Many incidents of minimal interaction between teacher and students were documented. Rousseau and Tate (2003) characterize this pattern of action and inaction as "allowing students to fail." By blaming the student for his or her failure, and not providing resources, the teacher was thereby absolving his or her actions as a potential factor for the student's failure. Research that investigates the everyday experiences of underserved students has become more common as the socio-political perspective has gained acceptance and momentum in the field of mathematics equity research. Observing how opportunities to learn are or are not enacted in the classroom provides rich insights that contribute to the stories of students who succeed or fail in mathematics. This dissertation seeks to continue to learn from these types of classroom interactions.

Practices that encourage students to talk about mathematics and use their experience to connect to new learning.

Additional research studies with a socio-political lens zoomed in on the particular mechanisms in classrooms that offered students access to content and increased mathematics achievement. For example, Khisty and Chval's (2002) research found that modeling mathematical discourse in everyday mathematics lessons is crucial in providing students the opportunity to learn mathematics. In their study of two 5^{th}-grade teachers with sound pedagogical knowledge, both of whom had created a student-centered classroom environment that invited student participation, they found one of the teachers lacking *mathematical register*. Mathematical register is defined as the grammatical structures in which mathematics is discussed, as well as the specialized vocabulary that is often used in other content areas but which has very specific meanings in mathematics, such as “factors” or “difference” (Moschkovich, 2007). While one teacher provided scaffolding for the academic language within the discipline of mathematics, the other teacher avoided academic mathematics vocabulary and taught her lessons with limited use of the mathematical register. The teacher who used mathematical pedagogical discourse – that is, the mathematical register and embedded definitions –- was able to extend the mathematical discourse in her classroom by assisting students to appropriate the correct vocabulary and explicitly model the structure in which the vocabulary was used. The second teacher in the study effectively impeded student access to the mathematical community due to the absence of the mathematical pedagogical discourse. Thus, Khisty and Chval (2002) found that the type of teacher talk in the classroom plays a vital role in students' learning of mathematics.

In their research, Khisty and Chval (2002) further found that teacher beliefs regarding students' ability to comprehend the mathematical register, and teacher beliefs about how knowledge is acquired, were each directly linked to the type of teacher talk that was observed in the classroom lessons. That is, the first teacher believed that the students were capable of understanding the mathematics register and therefore used it in her lessons; the second teacher felt that the vocabulary was too confusing for the students and opted to avoid the specialized mathematics vocabulary. Both teachers had strong content knowledge of mathematics, however, their contrasting pedagogy yielded different opportunities to learn for students. In this case, Khisty and Chval (2002) identified the practice of encouraging and modeling the mathematics register in teaching and interacting with students as fruitful to learning and as a practice that promotes opportunity to learn.

Other studies with a sociopolitical perspective have focused on other aspects of mathematics pedagogy. Rochelle Gutierrez (2002) explored how pedagogical knowledge and teacher beliefs about students' mathematical learning manifested themselves in an urban high school. She looked at three high-school mathematics teachers who had advanced large numbers of Latin@ students (largely English-dominant) to higher mathematics courses. When analyzing data drawn from school and classroom observations over a 13-month period, and from teacher and student interviews, she found that several strategies used by elementary and middle school mathematics teachers and teachers of English Learners were also effective with high school Latin@s in their high-level mathematics classrooms. These practices included having students work in groups, allowing students to work in their primary language, supplementing textbook materials, and building on students' previous knowledge. Gutierrez argues that successful high school teachers allowed students to use their cultural resources, such as primary language, to negotiate rigorous mathematical concepts in the classroom. A key point of this research was that strategies that were identified as successful for mathematics instruction in elementary and middle school were equally successful at the high school level. The teachers employed a variety of strategies to provide an opportunity to learn. This study identified incorporating group or pair work and building on students' prior knowledge as practices that promote equity and opportunity to learn mathematics.

Research has defined "student voice" in mathematics classrooms as those efforts that encourage students to speak about their learning experience. A goal of research that incorporates student voice is to recognize that students are experts on their own learning. Jones and Yonezawa, in *Everyday Antiracism,* found that high school students were able to analyze their own classroom experiences and that their insights, when shared with staff, were extremely useful in helping teachers reconsider their assumptions and beliefs about students' motivation and capability (Pollock, 2008). Jones and Yonezawa (as cited in Pollock, 2008) found that teachers' negative preconceptions, often unintentional, were challenged and shifted when they were able to hear about students' classroom experiences. The new insights brought forth by exploring student voice generated useful suggestions for improving the motivation, learning, and engagement in everyday classroom lessons (Pollock, 2008). Many times, researchers have shown, giving students the opportunity to voice their experiences empowers them to develop a sense of agency: i.e. students who feel a sense of voice and belonging also believe in their capacity to learn (Gutierrez, 2007, 2010). Therefore, the practice of encouraging students to voice their experience to teachers who have a willingness to listen is framed as a precursor to being able to build a sense of agency within students in the mathematics classroom.

Research indicates that educators can value “student voice” in multiple ways. First, educators can create an environment where students feel that their voice matters in the mathematics classroom and where students feel they are able to contribute to their own learning and to that of others. Civil and Planas (2004) state that in order for mathematics students to be prolific mathematics learners, much more than the acquisition of concepts and skills is needed. They argue that a mathematics student must engage in “an active participation in the reconstruction of a specific kind of discourse.” That is, students must discuss mathematics, listen to different points of view, and build on other students’ ideas, in essence giving their own intellectual construction as much power as that of teachers and other mathematicians. This builds students’ self-confidence in their mathematics capabilities by giving their own intellectual construction as much power as that of teachers.

Another method that educators can use to value student voice is to allow and encourage students to use their native language and their bilingualism in academic settings. When it comes to first and second generation American students, many researchers have concluded that teachers valuing students’ primary language and allowing and encouraging students to use their primary language to negotiate and co-construct meaning in the mathematics classroom supports their cultural identity (Cobb & Hodge, 2002; Gutierrez, 2008; Moschkovich, 1999). Researchers (Civil & Planas, 2004; Cobb & Hodge, 2002; Gutierrez, 2008; Gutierrez, Willey, & Khisty, 2011; Khisty & Chval, 2002; Moschkovich, 1999) state that when teachers allow students to use their primary language in the context of the mathematics classroom**,** they are allowing students to use their cultural linguistic resources to construct new schemas based on prior knowledge. By allowing the use of primary language, teachers provide students with a link between their preferred language of thinking (including established prior knowledge) and the new content knowledge and language (Moschkovich, 1999; Sfard, 2001). Gutierrez, Willey, and Khisty (2011) state that,

> It is also true that Spanish has critical functions as a cognitive tool to help students make sense of complex mathematical ideas, as a means of maintaining family and community connections, as a marker of historical and present identity (p.37).

Teacher efforts that support students' use of all available languages in the mathematics classroom coincide with Sfard's (2001) finding in being able to use discourse and language as a means of extending thinking. If the native language is the more developed at the time of instruction or is the one that is enacted when retrieving prior knowledge, it makes sense to encourage the use of the native language in constructing and negotiating meaning of mathematics. Therefore, allowing students to use their cultural assets (such as their primary language) to discuss and negotiate their learning of mathematics is an effective practice for promoting learning and encouraging equity in the classroom.

In summary, research on providing access to rigorous mathematics content has identified specific mechanisms that matter in classrooms and clarifies that teacher content knowledge is not sufficient for access to the discipline of mathematics (Cobb et al., 2002; Gutierrez, 2002; Rousseau & Tate, 2003). Teaching and learning is based on complex student and teacher interactions in classrooms that build over time -- and greatly influence the outcomes of student success or failure (Pollock 2008).

Practices that empower student identity in mathematics.

Additionally, researchers identify a need for schools' and educators' awareness of and connection to student identity and culture. They argue that awareness and connectivity to students' lives are critical elements in mathematics success and provide an equitable education to Latin@ students in mathematics (Boaler, 2008; Gutierrez & Irving, 2012; Gutierrez, Willey, & Khisty, 2011). Teachers' awareness of student identity and culture is multi-faceted. Family, ethnic culture, pop culture, community, school experiences and everyday experience shape a student's identity (Pollock, forthcoming). Research suggests that the more such elements of students' identity are embedded in the opportunity to learn mathematics in school, the more students are engaged in critical thinking. Some examples of practices that address a teacher's awareness of student identity and culture can be found in the Gutierrez, Willey, and Khisty (2011) study, where they found that students were able to incorporate mathematics into their identity when they: 1) saw themselves as able to succeed in mathematics, 2) redefine the purpose and definition of mathematics, 3) engage in mathematics, and 4) contribute to their communities using mathematics as a tool.

R. Gutierrez (2002) argues that when teachers include elements of students' identity and culture in their mathematics teaching, students are more likely to incorporate mathematics into their identity. Gutierrez, Willey, and Khisty (2011) found that students were able to incorporate mathematics into their identity when they saw themselves as able to succeed in mathematics, redefine the purpose and definition of mathematics, engage in mathematics by solving open-ended problems, high challenge problems, using peers as resources to find valid strategies for problem solving, and contribute to their communities using mathematics as a tool.

In a number of publications, Gutierrez (2002, 2007, 2008, 2010, 2012) has argued that teachers give students an advantage in the classroom when they are aware of their students' culture and understand how cultural assets can leverage thinking and construction of new learning. This awareness and understanding of cultural assets promotes the incorporation of a mathematics academic identity for underserved students. When teachers are aware of the cultural and community resources and take those into consideration when they interact with students and parents, the teacher demonstrated that they value the students as participants and this in turn strengthens the student's identity (Gutierrez & Irving, 2012).

Awareness of identity can also take place amongst students in the classroom. Boaler (2008) explored how equity can be manifested in the mathematics classroom when students are explicitly taught how to act "equitably" with each other. She coined the term "relational equity" to denote the "equitable relations in the classroom": those relations that include "students treating each other with respect and considering different viewpoints fairly." Her research study (2008) focused on how students can be taught to approach mathematics interactions in the classroom setting with respect and responsibility to each other. She conducted a four-year study of approximately 700 students attending three high schools in which several cultures, languages, mathematics abilities and social classes were present. Boaler found that "relational equity" was achieved when students were taught to commit to reciprocity, that is the learning of others, and take responsibility when problems arose. In addition, she found that "relational equity" was achieved when students were taught to respect other's ideas and learned methods of communication that support learning. Boaler (2008) concludes that a teacher's ability to value students and have a commitment to helping students negotiate their similarities and differences were inherent attributes of teaching students to work in a mathematics classroom with "relational equity". By doing so, the teacher was showing how she valued students' identity and culture on an individual basis as well as collectively. Hence, the promising practice identified in this study was teaching students to respect each other and their perspectives and approaches in solving a mathematics problem.

Research also suggests that students need to talk about real issues in their lives that involve mathematics. M. Gutierrez et al. (2011) observed thirty-four third through sixth-grade students in a three-year after-school mathematics program in which students solved community-based problems using mathematical tools. Researchers noted that when students used mathematics to explore solutions to real problems and negotiate the mathematics problem solving process on issues that were meaningful to them, students could see mathematics as a tool for solving problems in society. M. Gutierrez et al. (2011) found that students began to challenge the instruction they received due to their changing perspectives of mathematics in their regular in-school classroom mathematics experiences. A new purpose for mathematics was established in the after-school program that linked their community concerns to mathematics content. In essence, by solving real-life problems, students developed their mathematical student voice, their solutions were valued, and therefore a purpose was established for mathematics. Hence, the practice that promoted students' access and equity consisted of incorporating the use of real-life, contextualized mathematics.

Such research suggests that understanding the context of students' lives, the issues they may face, and the assets that their respective homes and community provide, can be used as a resource for learning mathematics

(Gay, 2010 Gutstein, 2003). If teachers understand the history and the methods in which their students' culture approaches mathematics, then history and culture can be used as a connecting link between what is often very abstract mathematical ideas and a purpose that is valuable for students (Gutierrez & Irving, 2012). Schoenfeld (2002) further explored the type of curriculum that leverages access to rigorous mathematics. His findings, based on 40,000 students, showed that those students who experienced "reform" curriculum focused on contextualizing mathematics performed equally as well as those who experienced traditional curriculum focused on theoretical or abstract mathematics. For marginalized groups who experienced the contextualized "reform" curriculum, the "performance" gap between these groups and White students narrowed further.

Relatedly, researchers have focused on "culturally responsive teaching" as an asset-based perspective designed to connect mathematics to student identity and promote student voice. Gay (2010) defines culturally responsive teaching as,

> Developing a knowledge base about cultural diversity, including ethnic and cultural diversity content in the curriculum, demonstrating caring and building learning communities, communicating with ethnically diverse students, and responding to ethnic diversity in the delivery of instruction (p. 106).

Gutierrez and Irving (2012) highlight several projects that use culturally responsive teaching as a means of addressing student identity and culture in content curriculum in mathematics. These projects can take on the form of community-based projects, incorporating historical perspectives and methods on mathematics topics by a culture or people, and opportunities that may seek social justice such as investigating discrepancies in funding for a group.

Conclusion

As Schoenfeld (2002) argues, research has indicated that promising practices exist to address the needs of culturally, linguistically, and socially diverse students and likewise close the achievement gap for Latin@ and other marginalized groups, including English learners. Situating all of these findings together, we see that these practices engage students in rigorous, contextualized mathematics, and motivate them to participate. Some promising practices that researchers argue yield success for Latin@ children include: providing access to rigorous mathematics coursework, inculcating asset-based attitudes about their students, teaching students how to respect each other and others viewpoints, incorporating mathematics discourse in classrooms, heightening teachers' awareness and incorporation of student's identity and culture in mathematics teaching, encouraging students to talk about mathematics and the learning experience, teachers' encouraging students to use all linguistic resources, and contextualizing mathematics in real-world and real-life problems.

Embedded in the Common Core Content Standards that were fully implemented in the 2014-2015 school year is a call to: contextualize mathematics, engage students in solving problems by generating viable arguments that use mathematics as an analytic tool, and establish a purpose for the learning of mathematics in school for students. There are many implications for marginalized groups of students if we make such shifts versus continuing to do “business as usual” in classrooms across America. An extensive body of research has concluded that teachers must invest time and effort to develop and connect curriculum to the communities they serve, to listen to what students have to say about how they “see” mathematics and experience learning, to learn about students’ culture, and to connect to their identity (Civil & Planas, 2004; Cobb & Hodge, 2002; Gay, 2010; Gutierrez, 2007; Gutierrez & Irving, 2012; Gutstein, 2003; Pollock, 2008; Schoenfeld, 2002). In addition, research suggests that educators must recognize the cultural assets and perspectives that students can use to heighten learning experiences, including their language. Finally, developing teachers’ and students’ capacity to understand and discuss mathematics using the mathematical register is an invaluable tool for bolstering learning (Boaler, 2008, Gutierrez & Irving, 2012; Gutierrez, Willey, & Khisty, 2011; Moschkovich, 1999; Sfard, 2001). If these promising practices are not fully incorporated in daily lessons, Common Core will only serve to widen the achievement gap and further stratify America.

Mathematics education has been established as vital to America’s economic success (Augustine et. al, 2010; Obama, 2009). In order to meet the needs and goals of the nation, all aspects of mathematics education research need to be explored. The student voice perspective within mathematics education research, where students reflect on their own learning experience, is an important area of research still underutilized that can yield valuable ideas about how to improve the quality of mathematics education. Teachers too have insights on their own instruction. Each perspective: educators', students', and researchers', provide a piece of the puzzle that can serve to provide insight on maximizing the quality of education for young people.

Therefore, this dissertation contended that further research is needed to determine which practices *students and teachers* believe are most helpful to students' learning of mathematics. Thus far, research has suggested promising practices to support marginalized students' achievement of rigorous mathematics. *But do students believe those practices suggested by research actually support them?* And *do teachers' priorities align with researchers'?* Extensive reviews of research indicate that students in everyday classrooms (and to some degree, teachers) have not actually been consulted, nor have they weighed in on the efficacy of those practices. To date, no studies have examined what *high school students, particularly Latin@ students,* deem helpful in their daily learning experiences of rigorous mathematics.

A next step for research, then -- and the focus of this dissertation study – is to tap student and teacher perspectives in testing what research has determined as promising practices for the most marginalized groups. Allowing

Latin@ students themselves to analyze the elements that they say impede or promote their own learning in everyday lessons in rigorous mathematics can provide researchers additional insight into regarding effective instructional practices in mathematics classrooms. In essence, researchers can learn from triangulating what students identify as critical aspects of mathematics teaching – the moments when they experience new learning, the experiences of mathematics teaching that become barriers to their learning, and the supports that effective teachers build into their lessons – with what teachers say they are intending to do. This dissertation proposes that taking the time to listen to both students and teacher describe the specific aspects of learning mathematics on a day-to-day basis that encourage Latin@ students to persevere and engage in rigorous mathematics can provide insights for future professional development and future research. Although students (and teachers) may not be able to provide the complete picture of their learning experience, they can offer valuable insight that assessments alone fail to provide. It is important to value students' take on practices that help them gain access to rigorous content, even while they may not be able to name all the specific practices and nuances that have led to their learning. The more perspectives and data points that are analyzed within mathematics education research, the more complete a picture and understanding of the complexities of teaching and learning we gain.

Theoretical Framework

This study seeks to learn from perspectives of Latin@ students how everyday actions either provide access to higher mathematics, or stifle that pursuit in classrooms; this study also seeks to learn from the perspectives of those students' teachers. This research is grounded in a framework emphasizing everyday action for equity (Pollock, 2008, 2008b, forthcoming), a Mathematics Equity framework ((Gutierrez, 2008; Gutstein, 2003; NCTM, 1989; Schoenfeld, 2002), and the spirit of the *testimonios* tenet of Latino/a Critical Pedagogy, which calls for listening to student voices. In line with these perspectives, this study proposes that: (a) everyday lived experiences in schools snowball to grant success or failure (Pollock, forthcoming), (b) that the opportunity to learn mathematics is provided through interactions in classrooms that can counter social injustices generated in past schooling practices within mathematics education (Gutierrez, 2008; Gutstein, 2003; Khisty & Chval, 2002; NCTM, 1989; Pollock, forthcoming; Rousseau & Tate, 2003; Schoenfeld, 2002; Sfard, 2001), and (c) that student voices must be heard, valued, and incorporated into solution-oriented research (Civil & Planas, 2004; Gutierrez, Willey, & Khisty, 2011; Gutierrez & Irving, 2012; Jones & Yonezawa, 2009). These frameworks form the foundational underpinnings for this study and inform the research design.

Chapter 3: Methods

Purpose

The primary focus of this study was to examine the everyday classroom experiences of high school Latin@s on the cusp of success – students who will either replicate the statistics explored earlier, or negate them. At its essence, this study explores student voice as well as teacher voice, in order to identify those practices that Latin@ students and their teachers cite as supporting student learning in mathematics and promoting access to rigorous mathematics curriculum and coursework.

An array of research has identified many promising practices and essential components to Latin@ students' success in rigorous mathematics, but few studies have verified these practices from the students' perspective particularly. A key question remains: Can student voice be integrated as a means of strengthening the craft of teaching?

Additionally, this investigation explored the alignment of teachers' instructional planned decisions, that is, the activities meant to teach the content and ensure learning, with students' own experiences by exploring both teachers' sense of their choices and how students interpret those planned curricular and relational practices. Many teachers plan, anticipate, and organize their lessons with student understanding and potential misunderstanding in mind. The decisions of when to model, when to let students discover, grapple with cognitive disequilibrium, or explicitly teach students are all based on experience, belief in student abilities, and instructional time. Therefore, this study examined the instructional intentions and enacted practices of high school mathematics teachers and determined if students cited those practices as providing access.

The study focused specifically on the new Integrated Mathematics II and III, the second and third integrated mathematics high school courses respectively, that incorporate the new Common Core Mathematics California Standards. These last two mathematics courses fulfill the minimum entrance A-G requirements of California UC and CSU universities. The completion of these courses with a satisfactory grade determines post-secondary options and has been found to be the strongest predictor of success in college coursework (Adelman, 2006).

Mathematics educational research at the secondary level is limited and few research studies have investigated what teachers and students identify as helpful to their own education. Therefore, this study sought to add to current research by identifying characteristics of supportive mathematics educational practices that teachers and students specifically cited as encouraging them to persevere, explore, and learn.

As previously established in the literature review, research (Boaler, 2008; Civil & Planas, 2004; Cobb & Hodge, 2002; Gay, 2010; Gutierrez, 2007; Gutierrez & Irving, 2012; Gutierrez, Willey, & Khisty, 2011; Gutstein, 2003; Moschkovich, 1999; Pollock, 2008; Schoenfeld 2002; Sfard, 2001) suggests that teachers who provide needed opportunities to learn and increase achievement for Latin@ students are teachers who: provide access to rigorous mathematics coursework, have asset-based attitudes, teach students how to respect each other and others' viewpoints, incorporate pedagogical discourse in classrooms, encourage students to talk about mathematics and the learning experience, encourage students to use all linguistic resources, and contextualize mathematics with real-world problems (Civil & Planas, 2004; Cobb & Hodge, 2002; Gay, 2010; Gutierrez, 2007; Gutierrez & Irving, 2012; Gutstein, 2003; Pollock, 2008; Schoenfeld, 2002). This study allowed teachers and students to talk in their own words about all practices they found useful, while using a oral survey at the end to compare their perspectives on the practices cited as leveraging learning by research.

Research Questions

To address this purpose, I explored two primary research questions:

1. What can we learn from what students as well as teachers say about how students' everyday learning experiences are supported in the mathematics classroom?
2. How do these practices named as helpful by students and teachers compare to those highlighted in research as best practices?

More specifically, I aimed to answer these general questions, by exploring:

1. What practices do teachers intentionally build into everyday Integrated Mathematics classroom experiences that they believe promote achievement and access to rigorous mathematics curriculum for Latin@ students?
2. After a typical everyday classroom experience, what teaching practices do high school students in Integrated Mathematics identify as supporting their learning and what recommendations do they offer for improving their mathematics instruction?
3. How do students' identified practices compare to teachers' intended practices?
4. How do both students' and teachers' perspectives on beneficial mathematics practices compare to those identified in prior research?

Therefore, this study incorporated a mixed methods approach that prioritized student and teacher interviews and classroom lesson analysis and supplemented that data with a supplemental quantitative survey. The data collected and analyzed to answer the research questions were generated by the following: (1) student and teacher interviews, the primary tool used to gather participants' perspectives; 2) classroom observations, used to verify that practices named by participants were actually happening in classrooms; 3) a quantitative survey of participants' take on practices named in prior research; and (4) as context only, student assessment and records data.

Specifics on how each data were collected and analyzed will be discussed in detail later in this chapter.

Research Design

Taking center stage, student and teacher voice was a focal point of this study seeking to honor the spirit of *testimonios* identified as a tenet in Latin@ Critical Theory (Bernal, 2002). Fostering *voice* in research can take on many forms. Prior research states that encouraging students to speak about what they know or about their learning experiences is not only empowering to students, but also informative to student supporters (Civil & Planas, 2004; M. Gutierrez et al., 2011; Gutierrez, Willey, & Khisty, 2011; Pollock, 2008). Jones and Yonezawa found that high school students provided great insight into their own learning and were able to identify what helped or hindered their motivation, engagement, and understanding in school (as cited in Pollock, 2008). It is important that researchers as well as educators learn from students by allowing students to cite the specific moments in a lesson that gave them access to mathematics or those moments that required further supportive structures. Similarly, it is important students are asked to reflect on their learning to aid teachers in their own learning or to validate (or contradict) what research has cited as promising practices. Hence, a primarily qualitative research design was fitting so that close examination of student narratives, insights, and perspectives of everyday experience in the mathematics classroom could be captured and honored. In interviews and in data analysis, the individual classroom practices that participants said benefitted students' learning experiences were the units of analysis. The qualitative design allowed some flexibility in response to what participants said and invited commentary on what was observed in that day's lesson, rather than fixed rigid questioning. The goal was to highlight the lived experiences of participants, which could vary.

In the spirit of *testimonios*, the primary qualitative data collected and analyzed to answer the above questions were interview responses (where some portions were in Spanish). In order to fortify, elicit, and spark student lived experiences, video-recordings of the classroom lesson were available for reference during student interviews. In the end, lesson artifacts such as notes or packets were referenced more often than the video recording used during students or teachers interviews, because participants utilized and created these materials throughout the lesson. Ultimately, the video-recordings were used to extract specific observation data to substantiate what teachers and students highlighted during the interviews. As described further below in this chapter, students were asked to discuss the specific learning experiences of the mathematics lesson, cite specific moments that assisted or confused their understanding, discuss how the observed lesson compared to other lessons in their current mathematics class, and discuss what practices help them learn mathematics.

Also described in further detail in this chapter, participating teachers were asked how the planned lesson compared to the enacted lesson, to evaluate the lessons' success, and to reflect on how students were supported in the learning of the mathematics content. The interviews were audio-recorded and transcribed for data analysis. In addition, quantitative data in the form of student grades and formative assessments were collected to provide some context on how the practices teachers and students cited as beneficial may have affected more traditionally analyzed outcomes. Such data was not used to make any causal argument about the practices participants named.

As described in further detail later in this chapter, qualitative coding of student and teacher interview data facilitated segmenting the data into categories and determining patterns and themes. This initial top-down coding of the transcribed interview data was used for data analyses. As will be seen later, data analysis quantified the qualitative data by counting mentions of practices. Still, such quantification served to substantiate overall trends in the data, not to make detailed comparisons of "numbers of mentions."

Positionality

My personal experience as a Latina and a former English Learner, as well as my professional background and training in the field of mathematics education, have the potential to shape the interpretation of the data collected. I am currently an English Learner/ Mathematics instructional specialist who provides professional development for mathematics teachers in both content and language acquisition pedagogy. In addition, I provide support for schools and individual teachers as a mathematics / English learner coach by modeling lessons and strategies. Moreover, I research students' data who are of concern, create resources, and advise colleagues on placement. I have extensive knowledge in planning, teaching, and implementing content and linguistic support in the classroom. I received a bachelor's degree in Mathematics, Bilingual Cross-cultural Language Academic Development (CLAD) credential (Spanish), and Masters of Arts in Teaching Services in Mathematics (secondary teaching) from San Diego State University.

Due to my background, I am cognizant of my strong positive views towards linguistic-enhancing and bilingual pedagogical educational scaffolds. Nevertheless, my knowledge and experience in educating English Learners in mathematics assisted in the observations, data analysis, and interpretation, because it provides a lens for what is observed, specifically, how instructional decisions are supported by program requirements and research.

Context of Study

The case study was conducted in one secondary school district in California. In order to maintain study participants' anonymity, the district name, school names, and all participants' names have been replaced with pseudonyms. "Oceancrest Unified" serves a diverse population of over 42,000 students; over 75% of the students are Latin@. It is the largest secondary school district in California and is both ethnically and economically diverse.

Since 1921, when the first high school in the district was built, the district has grown to serve four cities bordering the most southwestern area of the continental United States.

This study was conducted in three comprehensive high schools and four Integrated Mathematics II and III classrooms (Appendix A, CCSS, 2009). Four tenured mathematics teachers and two Latin@ students from each teacher's classes were recruited to participate in the study. The research focused on the beneficial practices that Latin@ high school students in Integrated Mathematics II or III identified as important to their learning of and access to rigorous mathematics, as well as on the practices their teachers intentionally incorporated in order to ensure learning and understanding of rigorous mathematics content.

School context.

Oceancrest District is a Hispanic serving district with 77% of its student population identified as Hispanic and 21% identified as English Learners. Oceancrest District additionally serves a student population where 33% are reclassified fluent English proficient (RFEP), that is, these students started their schooling in California as English learners. This means that over 50% of students have traveled the academic journey where their content instruction was taught in a language in which they were yet not fully fluent. Furthermore, Oceancrest District is a Title I district; 57% of the student population identified as socioeconomically disadvantaged. Bocarios High School, Mustang High and Mesa High were participating sites of this study. These schools were selected in order to give a representation of teachers who serve in high and low poverty schools.

Bocarios High School serves approximately 2,700 students and is comprised of a teaching staff of 78 teachers and four full-time administrators. The school is considered low-income and consists of two predominant groups of students, nearly 80% Latin@ and 13% Filipino. The school reports 88% of families as socio-economically disadvantaged and 24% of the overall school population as English Learners. As a result, the school is considered a Title I school, receiving additional funds dedicated to improving the academic achievement of disadvantaged students and having one of the largest Title III programs, additional funding designated meet the needs of English learners, in the district. The school receives state funds dedicated to assist limited English proficient students to attain the same rigorous standards of English proficient students. Of the two students selected from this school, one was a current English learner and one was a former English learner (now a reclassified fluent English proficient (RFEP) student).

Mustang High School serves approximately 2,700 students and is comprised of a teaching staff of 85 teachers and four full-time administrators. The school is considered middle-income and consists of three predominant groups of students, nearly 62% Latin@, 20% Filipino, and 8% White. The school reports 29% of families as socio-economically disadvantaged and 10%

of the overall school population as English Learners. Mustang School is not a Title I school, but receives some additional district funds dedicated to improving the academic achievement of disadvantaged students and funds dedicated to assist limited English proficient students attain the same rigorous standards of English proficient students. Of the four students selected from this school, one was an English learner and three were former English learners (now reclassified fluent English proficient (RFEP) students).

Mesa High School serves approximately 2,400 students and is comprised of a teaching staff of 55 teachers and three full-time administrators. The school is considered low-income and consists of one predominant group of students; nearly 94% of students are Latin@. The school reports 82% of families as socio-economically disadvantaged and 37% of the overall school population as English learners. As a result, the schools is considered a Title I school, receiving additional funds dedicated to improving the academic achievement of disadvantaged students and as having one of the largest Title III programs in the district. Likewise, it receives state funds dedicated to assist limited English proficient students attain the same rigorous standards of English proficient students. Of the two students selected from this school, one was an English learner and one was a former English learner (now a reclassified fluent English proficient (RFEP) student).

Teacher selection criteria and procedures.

Teachers participating in the study did so voluntarily. They agreed to be observed and videotaped twice in their Integrated Mathematics II or III classes for two separate lessons. In addition, each teacher participant was interviewed after each lesson for a total of two times. I worked with principals to identify “good” and “effective” mathematics teachers (in principals’ assessment) who had a good rapport with students and taught Integrated Mathematics II or III. The classroom teachers I invited are those that have shown a commitment to their profession and to their own continual learning process. Furthermore, only teachers who were tenured and had bachelor degrees in mathematics were asked to participate in the study. This ensured that each participating teacher has at least two years experience and has been able to demonstrate practices that to some degree, as measured by administrators, support student learning.

Table 1: Teacher and School Data

Teacher	Gender	Course	Language Status	Years Teaching	# of Students /Class	School Population	Overall Latin@ population	School Overall EL Population
1	F	IM2	Bilingual	15	35	2751	79.9%	24.0%
2	M	IM3	Monolingual	16	36	2351	93.8%	37.2%
3	M	IM3	Bilingual	12	42	2685	61.9%	9.9%
4	F	IM2	Monolingual	14	34	2685	61.9%	9.9%

As can be noted from the Table 1, the participating teachers were a balanced between male and female and between monolingual and bilingual. The student population that they serve is overwhelmingly Latin@.

Student participant selection criteria and procedures.

This study reasoned that “A” students are in a sense outlier students who have found a way to succeed, many times with outside supports, or students who have a keen focus on the importance of education and are already well on their way to complete and succeed in the mathematics, science, and higher education pipeline. In the schools I studied, 16% of students are given A’s in mathematics coursework. I reasoned that more typical “B and below, but passing” students who have not succeeded with such ease in their higher-level mathematics class can provide perhaps more generalizable insight on what in-class practices worked to promote their success and perhaps what did not. If we are to encourage and promote more access to the mathematics, science, and higher education pipeline, I reasoned, it is these students that we must learn from. These students are within the realm of meeting the minimum UC/CSU eligibility application requirements yet they may be at risk of being knocked out of the pipeline altogether. Further, research has identified many practices that are beneficial for Latin@ students but implies that these promising practices may particularly engage students who may be struggling to succeed in rigorous mathematics. Thus, this study sought to offer insight into what specific practices may give access to the pipeline that so many students are currently being funneled out of. This study sought to learn from this group specifically, as content access for these students may determine the options available post-secondary.

Finally, all student participants needed to be former English learners (RFEP) or "reasonably fluent" English learners (EL) in the English language as determined by having an overall score of a Early Advanced or Advanced on the California English Language Development Test (CELDT) or having been recently reclassified English proficient (RFEP) (State Board of California, 2013). As mentioned previously over 50% of students in Oceancrest District are in this category. When comparing the percentage of combined English learners and former English learners served at each of three participating high schools; Bocarios, Mustang and Mesa High Schools numbers are 72%, 77%, and 37% respectively. Gandara and Contreras (2009) states that Latin@ students have historically been underserved by the American school system but that even more specifically, Latin@ "English Learners [EL] remain a grossly underserved segment of the student population." Research suggests that students who attempt to simultaneously learn rigorous curriculum while learning the English language experience a significant challenge (Gandara & Contreras, 2009). Therefore, it was important to learn and hear from students who had been recently mainstreamed into regular all-English mathematics coursework.

Hence, student participants were selected based on their willingness to participate in the study and enrollment in one of the three participating high schools. Each participant needed to be a Latin@ student and enrolled in Integrated Mathematics II or III. Due to the pivotal and important nature of Integrated Mathematics II and III to UC and CSU college entrance requirements and completion, students who had mathematics grades between a B- and D+ in their previous mathematics course were sought with the understanding that the practices studied could provide leverage to increase grades that would be eligible to meet the minimum UC/CSU eligibility application requirements. Even when students are not fully eligible for UC and CSU admission and opt to transfer in the future to a university via a Community College, students need to have acquired the knowledge necessary so that they can have access and pass transferable mathematics courses more quickly. Pollock, Yonezawa, and Edwards (2014, May 23) argue in a recent article that “statewide, every additional remedial mathematics course taken in community college lowers students’ chances of completing a certificate or degree by 20-plus percent.” They further state that students in this community do not seem to transfer from community colleges to UC or CSUs at acceptable rates because they often test into such low mathematics courses that it would take well beyond the two or three years of course work to transfer.

I looked through district records to identify students that fit the listed criteria based on previous grades in the prior year's course and the first progress report for the current year. I then approached some of the students to ask if they would be willing to participate in the study. A female and a male student were selected from every participating teacher's class in order to generate a representative sense of what is experienced in the classroom and to maintain a gender equitable perspective. Although some may argue that two students from each teacher does not generate a representative account, my goal was to gather a general sense of each class. The goal was to hear from students and determine if what they identify as supportive practices coincides with those identified by research. As it turned out, participants were strongly aligned in the specific practices they felt leveraged their learning the most.

The table that follows highlights student qualifying demographic data.

Table 2: Participating Student Data

Student Number	Gender	Grade Level	Socio-Economic Disadvantaged	Previous Mathematics Grade	Language Proficiency Status	Language Proficiency Level	Years in CA Schools	Native Language
1	F	9	Y	B-	RFEP	Proficient	9	Spanish
2	M	9	Y	C	EL	CELDT 4	6	Spanish
3	M	12	Y	C	RFEP	Proficient	11	Spanish
4	F	12	N	C-	EL	CELDT 4	13	Spanish
5	F	12	N	D+	RFEP	Proficient	13	Spanish
6	M	10	N	B-	RFEP	Proficient	7	Spanish
7	M	9	Y	B	RFEP	Proficient	11	Spanish
8	F	10	Y	D+	EL	CELDT 4	10	Spanish

I sought to keep a balance of male to female student participants throughout the study.   Eight students were interviewed so that a trend or lack thereof could be determined. Students ranged from 9th grade to 12th grade.

There is much overlap in content between the new Common Core Mathematics Standards and the old California Mathematics Standards (California Department of Education, 2006; 2014) but there is also much rearrangement of standards. Due to the current transition between old state course guidelines, where most 8th graders where in Algebra 1 and new state guidelines, where most 8th graders are in the new Integrated Mathematics Grade 8 course, several students appear to be in advanced courses but in fact their placement was a consequence of adopting new standards and courses. The district decided to have students move a course ahead and have embedded additional content instead of holding students back and requiring them to learn material they had already mastered. All students were either former English learners, as classified by their "RFEP" language proficiency status, or are still English learners. Additionally, all students are Latin@ and have a home language of Spanish.

Procedures

This study consisted of two types of data collection: 1) two interviews with each participating teacher and two interviews with each student participant all of whom returned parental consent forms and 2) a supplemental oral survey to all participants administered at the end of the second interview (so as not to bias the interviews), asking participants to rate practices identified by research as assisting Latin@ students when learning rigorous mathematics.

Primary Qualitative Data

Individual interviews were conducted with participating students and teachers after a 2-hour block lesson was observed and video-recorded. The goal was to determine which classroom practices students and teachers perceived as helpful. Interviews were conducted between October 6th, 2014 and December 12th, 2014, during the second quarter of the 2014-15 school year. Interviews generally lasted thirty minutes and were conducted in either during student lunch or directly after school. Students agreed to participate in the first interview individually and in the second interview in pairs. Interviews were audio recorded and students assented to participate in the interview and be audio recorded prior to beginning the interview. Interview questions for students and teachers are provided in Appendix A and B.

Supplemental Oral Survey

Oral surveys with participating students and teachers were conducted after the second interview was completed. Surveys included both quantitative (Likert-scale) and qualitative (open-ended) questions. The purpose of the oral survey was to determine how students and teachers rated all practices deemed important in accessing rigorous mathematics by research, including those that potentially did not arise during the interviews. A full copy of the survey can be found in Appendix C.

Classroom lesson video recording.

Eight classroom lessons were observed and video-recorded in the four classrooms (two for each teacher participating in the study) within a 60-day period. I placed a camera in the back of the room to capture the lesson, overall teacher-student interactions, and student-student interactions. The video-recording device was set up to capture as much of the specific activities and discourse of the class.

Students without formal consent and/or students who did not give formal parental consent to video recording were not filmed. Any student opting to not be video recorded maintained full access to all classroom activities. Teacher and student participants were notified that they could request that the video be stopped at any time during the session prior to every recording session.

Field notes during observation.

I collected field notes on the structure of the lesson, teacher-student interactions, and student-student interactions. Furthermore, during these observations, field notes were taken of classroom activities, teacher discourse, and non-verbal cues like “high-fives”, “thumbs up” and “fist bumps”. Field notes were time stamped during the observation to facilitate the alignment of the video recording to transcripts and post-lesson interviews with teacher and student participants. I also took notes on the classroom and school environment that might not have been a formal part of the lesson but seemed to contribute to interactions between teacher and students or student-to-student, such as organization of desks and notes on the board. Field notes of the lesson facilitated time stamping of key moments in the lesson as well as moments that may not have been captured in the video, due to side conversations between teacher and student or student and student. In the end, I did not cite these field notes for data on specific interactions in detail, but I used them to identify specific moments in the lesson that teachers or students referenced in the interviews.

Teacher and student interviews.

I conducted two semi-structured interviews with each teacher and student that ranged from 20-30 minutes in duration. These provided the majority of the data analyzed in this study. The predetermined interview questions were designed to elicit data that addressed practices that participants deemed important and helpful in learning rigorous mathematics. Follow-up questions and probes differed according to responses to the pre-determined questions. Each participant received a $20 gift card as compensation for his or her time.

The first sets of semi-structured interviews were with 1) individual teachers and 2) individual students after the lesson. The second sets of semi-structured interviews were with 1) individual teachers and 2) two students (classmates) together. Therefore, I conducted two student interviews for each student, one individually and one with their classmate. In all interviews, student and teacher participants were able to articulate practices that they said leveraged learning and understanding of the mathematics content. In the classmate interviews, students were able to build on their classmate's ideas and add further detail.

The purpose of the interviews was to elicit post-lesson information about which lesson practices participants felt supported student understanding, and the types of general supports students and teachers argued students needed in order to gain access to content. Furthermore, "barriers" that the practices needed to overcome surfaced during the interviews and will be discussed in detail in Chapter 4. The "barriers" that students and teachers identified were not the focus of this study, yet they did provide a richer understanding and context for how the practices participants identified were thought beneficial.

These interviews also sought to determine the routineness of structures that were incorporated in everyday lessons by asking students to describe a typical mathematics lesson and/or to compare the observed lesson to other mathematics lessons with the participating teacher up to that point. Pollock (2008, 2008b, forthcoming) suggests that every day, every person whom a young person encounters in school or community either provides advantageous or disadvantageous actions and opportunities. In this sense, practices that promote equity might best be *established and routine* -- actions and mindsets that are repeated in the classroom in everyday lessons and in the student's everyday community. Therefore, it was important to determine if, when a student or teacher identified a practice as assisting learning, that it was not a "fluke" but rather a recurring practice that could be found with some consistency. In order to determine the recurrence of the practice, two block lessons (90 to 120 minutes each) were observed and students were asked to describe a typical day in each class. My goal was to gain understanding of the types of supportive practices teachers intentionally embedded in everyday mathematics lessons and how students interpreted those and other supports.

Often, students used their notes to trigger their memory and reference specific moments that served as examples of supportive practices or of a need for further support. Interview questions therefore elicited specific information about practices in everyday mathematics lessons (including affective supports) that participants said facilitated understanding, access, and learning. In order to allow participants to name practices they considered valuable, I conducted both interviews with a focus on the lessons. In student interviews, the three major questions I asked were: “what was the lesson about?” “Were there areas of confusion?” And if so, “what allowed them or could have allowed them to get unconfused?” The specific questions are listed in Appendices A and B. For teacher interviews, the three major questions I asked were: “what were the lesson goal(s)?”, “what surprised you?”, and “what did you feel went well in the lesson?” The specific questions are listed in Appendix B. Follow up questions then stemmed from answers, to get more clarity on what specific practices were helpful. In short, both teacher and student interviews generated information about what in-class practices participants felt were needed in order to support access to rigorous mathematics instruction for Latin@ students.

Oral survey.

As described above, the qualitative data generated by interviews were the primary source of data. Furthermore, the primary focus of this study was to have participants share their perspectives on their learning and teaching experiences, yet there was a possibility that many of the practices cited in research would not be “called out” in the study. Since research had established a set of promising educational practices that are beneficial for

Latin@ and other marginalized groups, I wanted to know if participants, both teachers and students, also found these practices helpful. So, after all students and teachers had completed their interviews, the predominant source of data in this study, they were asked to rate practices identified by research in an oral survey using a Likert scale. The exact question was:

> I am going to read you a list of these practices. Researchers say these practices are helpful to students but you may not. Be honest in your rating because I am trying to learn from you. Tell me what you think about whether these practices are helpful to student learning higher-level mathematics by rating them on a scale of 1-not important, 2-a little important, 3- important, and 4-very important. Feel free to elaborate. Please ask me to clarify if you do not understand.

The goal of the survey was to determine if practices identified by research but that had not surfaced during interviews were valued by participants. Any additional comments offered by participants were coded separately from interview responses and served as a separate data source.

Formative assessments and records.

District assessments were collected so that a broader measure of the beneficial practices cited by students and teachers could, if needed, be compared to overall district success measures. District assessments are two-fold. At the end of every semester, all students take an end-of-course exam, which is a comprehensive 30-40 question exam addressing the standards established in the course instructional guides and aligned to the Common Core Mathematics Standards. The goal is to assess the written, taught and tested curriculums established by state standards, curriculum instructional specialist and teacher leaders. The district end-of-course exams were a conglomeration of released national standardized assessment, assessment questions selected from the adopted curriculum, and assessment questions selected from vetted Smarter Balanced Assessment Consortium (SBAC) resources. The exams include selected-response questions and constructed-response questions. In addition, the district requires all students to complete a performance task in each course where students work together for a portion of the problem to gather data and then must complete the second portion of the task on their own. This performance task is an open-ended and a reference to a real world application problem that is graded by the teacher with a holistic rubric with a scale of one through four. The performance task required students to show their work and allowed for partial credit. The overall scores on both the end-of-course exams, the performance task, and grades were provided as an additional measure to discuss the possible implications of practices on learning over time. I did not use these data to make causal claims about the effects of the practices cited, on student assessment outcomes; next studies can test practices' assessment effects.

In addition, demographic data was used and analyzed to gather additional academic information of each participating student's academic journey. In addition, I collected district consent to collect relevant academic, demographic, ethnic, and language proficiency data on students participating in the study. They assisted in identifying qualified students for the study and background data for the academic journey of each student. Electronic records were stored and protected on my computer.

Data Analysis

Classroom observations.

Field notes and video recording were analyzed to fully describe and corroborate the practices cited by the participants and identify any practices that were evident but not cited. Based on my analysis of the video and triangulation with field notes, I substantiated that practices cited by students or teachers were in place in the classrooms observed on the days observed. I reviewed all video-recording and transcribed sections using the computer software InqScribe (Inquirium, version 2.2) to determine emerging themes. In order to increase accuracy of the data, three colleagues were asked to look at portions of the video. These colleagues were given my codebook and used it to review and code the video portions and the field notes. The goal was to establish coding reliability to ensure that ideas and themes emerging from the data were accurate, that others reached similar conclusions, and validated the coding.

I used video in a couple of instances to spark participant identification of specific moments where they were confused in the lesson as well as specific moments that clarified understanding. In the past, researchers such as Tobin, Wu, and Davidson (1998) and Tobin, Hsueh, and Karasawa (2009) have used video-eliciting interview formats, in their case to study how culture influences preschool in three different countries. Their study videotaped everyday activities in average preschool classrooms to document everyday interactions that might unveil cultural values and early childhood goals. The video served as an additional data source that allowed the interviewee to use as evidence and generate dialog of how he/she was making sense of the lesson. Similarly, I used video recording to document lessons in the mathematics classrooms, and when lesson artifacts such as notes or packets did not suffice to jumpstart participants' reflections, I asked students to identify specific moments in the lesson video to discuss in interviews. I did not use the video to conduct a structured video-elicited interview, but rather as an additional source that the teachers, students or I could cite.

The lesson video-recordings were more valuable in coding practices that did not surface during the interviews but were evident during lesson and observation. The lessons were coded for practices cited by research in the literature review as promoting equity in mathematics (Boaler, 2008; Civil & Planas, 2004; Cobb & Hodge, 2002; Gay, 2010; Gutierrez, 2007; Gutierrez & Irving, 2012; Gutierrez, Willey, & Khisty, 2011; Gutstein, 2003; Moschkovich, 1999; Pollock, 2008; Schoenfeld 2002; Sfard, 2001). In most cases, participants were already citing these practices themselves. Portions of the video recording were transcribed using InqScribe transcription software during the data analysis and reduction phases of the study to support the data gathered in the interviews.

In order to gain a more thorough understanding of the data and engage in in-depth data analysis, I was the main transcriber of video-recording and interview data (Lapadat & Linsay, 1998; Tilley, 2003). Every attempt and care was taken in order to minimize personal biases in the transcription process.

Analysis of student interviews & teacher interviews.

The 24 interviews (16 student and 8 teacher) served as the majority of the data analyzed in this study. Students assented to participate in the interview and be audio recorded prior to beginning the interview. I transcribed all interviews within 1-4 days of when they took place. Computer software programs InqScribe and Dedoose were used in coding both student and teacher audio-recorded interviews to identify *a priori* codes and determine emerging themes. *A priori* codes were selected from the review of the literature available on equable practices in mathematics for marginalized students. Emergent codes were developed when a pattern was evident in participant responses. Each transcript was coded and recoded two to three times using a constant comparative analysis approach (Glaser, 1978; Swan et al., 2005). Transcripts were analyzed using a) a “top down” approach, where each transcript was coded with *a priori* practices (those practices previously identified by research to provide access to mathematics for marginalized groups of students). Transcripts were also analyzed using b) a “bottom up” approach, where *emergent* practices (those practices named by participants in this study, including those not yet mentioned by research) were coded. A code or tally was applied when a *full thought* pertaining to a practice was evident in the transcribed interviews. Codes were later analyzed for the number of “mentions” in interviews and emphasis in tone during participant statements, in order to identify overall trends in practices named. I went back and re-coded three transcripts from start to finish to verify intra-rater reliability and make sure that I was being consistent. The intra-rater reliability of the codes was further strengthened and clarified by graduate department faculty and students. Fellow graduate students, faculty, and work colleagues verified that codes that were being assigned made sense and were consistent. This generated a coherent codebook. Additionally, three colleagues were asked to independently code portions of interviews that I had coded to strengthen accuracy by helping clarify and define individual emergent codes and overall themes. These codes were triangulated with each other (teacher and student interview codes), observations, and formative assessments and records.

Due to the emphasis on student and teacher voice in the study, an attempt was made to capture the essence of the student’s experience by citing their own words and by putting their words alongside teachers’, and secondarily comparing to research’s expectations. Pollock (forthcoming) states that a person’s ability to succeed in “any realm is coproduced in everyday interactions” with others and the opportunity to learn. Therefore, success in a current opportunity to learn is co-constructed with other classmates and teachers (Pollock, forthcoming). To account for these issues that affect each student's opportunity to learn, an attempt was made to deconstruct how the interviewees made sense of the everyday mathematics classroom experiences -- that is, to understand their perspective and experience of everyday mathematics lessons.

Oral survey.

As mentioned previously, a oral survey followed the completion of interviews with all participants. Participant responses were tabulated in Microsoft Excel and averages were calculated.

Analysis of formative assessments and records.

A final data set was used to provide context for the *testimonios* of both students and teachers. The end-of-course assessment scores, the performance task scores and grades were retrieved from the district data management system. Results were used to provide context on the data collected from both teacher and student interviews, demonstrating simply that students were in fact doing well on the institutional measures that signal success in a course.

Chapter 4: Analysis and Interpretation

The purpose of this study was to identify what practices a group of high school Latin@ students, who are current or former English learners, and their teachers identify as classroom practices fostering rigorous mathematics learning. Relatedly, this study investigated the practices undertaken in four Latin@-serving classrooms to handle barriers students encountered when learning high-level mathematical concepts essential to access post-secondary opportunities. By "barriers," I mean moments when students struggle with, stifle, or pause their mathematics learning. By "practices undertaken to handle these barriers," I mean the actions and learning opportunities that teachers put in place on a consistent basis in their lessons to further learning and counter barriers. As discussed in previous chapters, I am interested both in practices identified in research and in how those practices triangulate with practices that teachers and students cite as helpful to student learning.

Many educators may argue that the practices to be described as beneficial here are "good" for every learner, since many are actually promoted as a basis for the newly adopted Common Core State Standards (National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010). However, a claim can perhaps be made that these practices are more <u>essential</u> to Latin@ students, many of whom are currently acquiring English while learning content, and to former English learners, many of whom may have residual gaps in understanding due to simultaneously learning foundational mathematics content while acquiring fluency in the English language. Gandara and Contreras (2009) found that regardless of when Latin@ students entered school, about half begin their schooling speaking primarily Spanish. They state that “half of Latin@s who must learn English while simultaneously attempting to learn the regular curriculum also experience a significant academic challenge” (pg. 32). This is after all what the U.S. Supreme Court found in the 1974 *Lau vs. Nichols* case. Gandara and Contreras argue that,

> In spite of the rather commonsense finding of the Court, that a student who does not speak English cannot make sense of what he or she is being taught and that it is unreasonable to expect that such a student would have to learn English before being taught the regular curriculum, there has nonetheless been a protracted debate in the United States about how best to educate these students. Bilingual education, a pedagogical strategy that employs two languages for instruction, has been a lightning rod for controversy in this debate.

An argument can be made that since going from not speaking English to speaking English is a process, students who have traveled this journey have had points of accumulating understanding as well points of gaps in understanding. These residual gaps therefore can impede future learning.

In 2013-2014, the California Language Census reported 1.413 million English learners accounting for 22.7% of all California public school students (CDE, 2014). Currently more than one in three English learners is being served in a secondary school setting accounting for 18% of all secondary learners (Olsen, 2010). Olsen has found that 59% of English learners in secondary schools are "long-term" English learners, that is, they have plateaus in their English language acquisition for more than two years and have been English learners for seven years or more. There are many reasons for this phenomenon. Olsen (2010) has found that after the passage of Proposition 227, English learners who do not progress overwhelmingly are those who do not receive any language development program in their primary grades despite the legal requirement mandated by *Lau vs. Nichols* (1974) and *Castañeda vs. Pickard* (1981). Furthermore, many of these students are taught with materials that were not designed to meet the needs of English learners and do not receive any primary language support in their schooling. Olsen argues that in order for English learners to "engage with academic demands of secondary school curriculum, they must learn more complex syntax, richer oral language, and the specialized vocabulary needed to understand the academic text and participate in classroom discussions" (p.23). Our current educational system continues to fixate on content being taught in English. Therefore, many Latin@ students whose home language is Spanish are asked to learn content while also facing language gaps, potentially amplifying typical content and conceptual gaps in mathematics learning (Goldenburg, 2013; Hakuta, 2011). These gaps can "snowball" into bigger conceptual gaps over time (Pollock, forthcoming) – potentially beyond those found in other students' educational journeys. Furthermore, many of these students come from low-income households where certain outside-school supports for mathematics learning may be limited. Such supports may include educated parents with knowledge of high-level mathematics concepts, people who have a command over academic English, and financial resources to access additional supports such as tutoring, web-based resources, and supplemental resources like books (Rothstein, 2000). Hence, the *classroom* practices that promote access to learning mathematics become more critical, as class-time may be the primary time when these supports are readily available.

We also know that many Latin@ students are stigmatized *as Latin@s* and fall victim to historical trends and beliefs about their capacity to do well in school and in rigorous mathematics coursework (Baron, Tom, & Cooper, 1985; Bol & Berry, 2005; Gutierrez & Irving, 2012). Additionally, researchers have found that when teachers were asked why disparities in mathematics success existed between minority students and white students, teachers did not contemplate structural barriers or oppressive actions by the school's staff as a possibility but located responsibility solely in students' lack of motivation, skill, work ethic, and family support (Bol & Berry, 2005; Ernest, 1991; Joseph, 1987). Furthermore, Latin@ students are statistically less likely to be in schools staffed by 'highly qualified teachers' mandated by the Elementary and Secondary Education Act (ESEA) (Darling-Hammond et al. 2005; Gandara & Contreras, 2009; Kozol, 2005). Classroom practices that students and teachers say counter these realities can be approached as potentially essential vehicles for students to access rigorous coursework, especially for Latin@ students who have been historically stigmatized and may be concurrently facing linguistic barriers in their pursuit of mathematical knowledge.

Interpretation

The following diagram shows the barriers that students and teachers in this study cited as barriers to mathematics learning and then, the practices students and teachers cited as assisting students to surmount those barriers. Both teacher and student participants discussed practices that leveraged their learning and understanding. Themes emerged from the data and organized around these categories. In interviews, participants named *nineteen* beneficial practices; *fourteen* of those practices were practices that research has already said are important to marginalized students' learning. Participants named *five* additional new practices as important that math equity research had not emphasized. These were: teacher anticipating student difficulty, schema building, having multiple opportunities to practice/solve math problems; ensuring understanding, and using student volunteers.

Both students and teachers identified specific practices as essential to an effective learning environment, practices I here call *growth mindset* practices (Dweck, 2006). Using words like “capable,” “believe,” and “encourages us,” participants framed growth mindset as overarching and manifested in all language used in supportive classrooms, with the entire class or with particular students, as well as with the organization of every lesson. Following from these overarching growth mindset practices, additional practices observed and cited by both student and teacher participants fell into three types. *Teacher-student talk* practices were practices the teachers used in facilitating learning and encouraging students during direct instruction. *Student processing time* practices were moments when the teacher encouraged students to take time to think and assess their understanding of the new learning – to solve mathematics problems, make connections, and review material. *Partner time* practices were practices where students were encouraged to use their peers as resources to further their own understanding or to clarify the understanding for their peers.

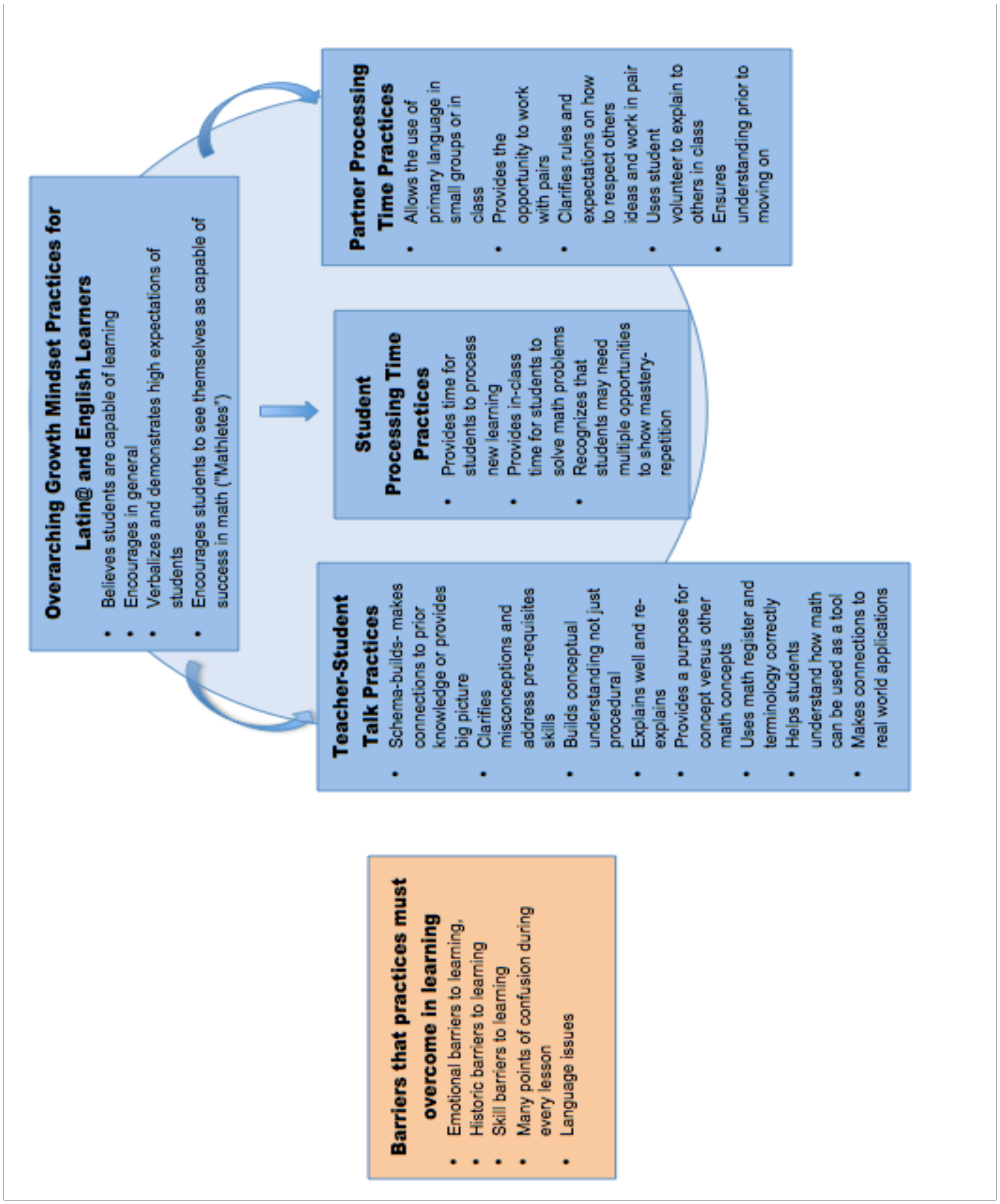


Figure 1: Mathematics classroom practices that promote equity according to study participants (students and teachers) and the barriers they countered

Students and teachers implied that growth mindset practices were necessary first. That is, if a teacher did not believe that her students were capable, she would not create opportunities for students to work together and teach each other; nor would she give them time to process.

When students and teachers cited practices as helpful and integral to their understanding of abstract concepts in rigorous mathematics, they identified barriers that students needed to overcome while learning. Interviews prompted that discussion by asking, “Were you confused at any point during the lesson? If so, when?” and, “Did you stay confused?” “Did you get unconfused?” “If you got unconfused, what practices helped you get unconfused?” “If you stayed confused, what could have helped you?”

In order for me to more thoroughly analyze the information that was provided by participants in this study, key learning theorists not reviewed in my pre-dissertation literature review became needed to think about what participants said in their interviews. Previous research has explored such processes of acquiring new knowledge. For example, Piaget (1952) offered a cognitive constructivist theoretical frame, a notion of organized knowledge in the brain as *schema.* Piaget conceptualized learning as the process of individual *adaptation* or adjustment. He theorized that learning was a process of: 1) assimilation, the use of the existing schema to address new information or experience, 2) accommodation, the process of changing the existing schema to embrace the new learning and organizing it in the pre-existing knowledge or schema, and finally, 3) equilibration, a state of balance which the brain seeks since it does not like to be frustrated or confused by new information. Therefore, if equilibrium is a state of balance, *disequilibrium* is the state in which confusion is encountered – the state that needs to be overcome in order for the new information to be added to the schema.

In this study, students who talked about struggles to understand an abstract concept such as long division or synthetic division of polynomials routinely talked about the process of experiencing initial confusion, what Piaget would have called disequilibrium, and then adding new knowledge to their existing schema. For example, one teacher, Mrs. G, spoke about her surprise and struggle at realizing in the middle of her lesson that students did not know how to rationalize denominators, a skill that students should have learned in the previous course. As another example, Max, a student, stated, "I was more confused in the long division when we have to like multiply them [a term by the binomial] together and put them [subtract] on the bottom. I just get numbers mixed up but other than that I was like pretty straight and then I got it." Additional examples shared by student and teacher participants will be cited further below in a discussion on barriers that existed and affected learning.

Piaget's cognitive theory is foundational to many learning theories. Yet, a significant gap in Piaget's theory was that he did not consider the effects of social interactions with teachers, peers, and parents on learning. In this study, students and teachers spoke of how students overcame moments of confusion when they interacted with other people in the classroom. Specifically, they described how important interactions among teachers and students and between students were to surmounting barriers to learning. The idea of learning from others is central to Vygotsky's (1962) foundational learning theory of social constructivism, which focuses on interactions that a learner has with others and self. His theory addresses the process through which a person speaks to him/herself to make connections, and the influence of other people's language (through interactions) on a person's inner thoughts and dialogue. The interactions teachers and students cited as helpful to breaking through barriers became the three types of talk cited in the graphic above. These were: teacher-student talk, student processing time (self talk), and partner processing time (peer talk). As discussed throughout this chapter and more in the final chapter, with only a couple of exceptions, each type of talk and more specific practice cited by participants also linked to prior ideas from research. That is, there were only a few practices that participants cited that research did not anticipate, and there were only a few practices that research would have anticipated that participants did not discuss much.

In each case described below, I comment first on practices participants named as important to surmounting specific barriers. I then triangulate participants' own commentary with research that does or does not name similar practices as essential to supporting Latin@ students to overcome barriers to learning. It is important to note that Vygotsky (1962) also discussed a learner’s capability of learning only when the new concept is within a “zone of proximal development,” meaning that there is not an enormous gap between what the learner knows and what is to be acquired. That means that if what a learner is attempting to learn is not in their “zone of proximal development” -if the gaps were too vast --the practices listed would not be sufficient to address during class time and additional resources and interventions would be necessary. Therefore, before I delve deeper into how participants cited these practices as helpful, it is necessary to understand what barriers to learning teachers and students named and that these three categories of talk practice in classrooms were dismantling. I explore the barriers to learning cited by participants, each of which is corroborated by research.

Barriers that Affect Learning

Research identifies various barriers to mathematical learning and thinking that exist inside and outside of the learner, and this study’s participants did as well. Therefore, further research about what is known about some of these barriers was required to better understand what specific issues the practices they cited as beneficial were countering.

The barriers listed below are examples of specific situations that participants said caused cognitive disequilibrium as well as affective obstacles that participants needed to counter. These barriers were all identified by teacher or student participants in interviews.

Points of confusion.

Students named various barriers to learning rigorous mathematics that teachers' practices had to counteract. One barrier to learning they named was common, inevitable confusion. That is to say, they described how new learning of a concept usually involved some state of confusion or disequilibrium as they tried to fit the new learning or concept into their existing schema of the subject. One student, Josue, described his point of confusion while trying to understand a definition that was the basis for the day's lesson. He said,

> **Josue:** No, I was still confused. Well, I knew it was some sort of function but not the same as *f* of *x*. That's why I still didn't know what we were doing.
>
> **Interviewer:** What do you mean?
>
> **Josue:** So he was just calling it. He was saying like "It's a function" but it's a …*p*. I got confused with the *p* that is referring to polynomial.
>
> **Interviewer:** um hum.
>
> **Josue:** Once I got that then I could focus… you know try to understand.

In this excerpt, Josue is describing how one word, the mathematics register and terminology, and its notation, of $p(x)$ was causing disequilibrium. The teacher introduced the process of dividing a polynomial function by another one but when he went to start dividing, the teacher switched to describing the process of dividing a function, which was a polynomial function. Josue was confused until he understood that “polynomial” was a qualifier and descriptor of a multi-termed function. As discussed in further detail later, students in this study identified the various points of disequilibrium they encountered as well as the specific practices that countered them.

Relatively new research has looked in more depth at “confusion” as an important part of the learning process. D’Mello (2012) states,

> One cognitive model emphasizes the importance of cognitive disequilibrium (Graesser & Olde, 2003; Piaget, 1952) and can be extended to provide some predictions regarding likely affective state transitions. According to this theory, deep comprehension is most likely to occur when learners confront contradictions, anomalous events, obstacles to goals, salient contrasts, perturbations, surprises, equivalent alternatives, and other stimuli or experiences that fail to match expectations (Jonassen, Peck, & Wilson, 1999; Mandler, 1976; Schank, 1986). Individuals in a state of cognitive disequilibrium have a high likelihood of activating conscious and effortful cognitive deliberation, questions, and inquiry that are directed to restore cognitive equilibrium and result in learning gains. Kort, Reilly, and Picard (2001) predicted that the affective states of confusion, and perhaps frustration, are likely to occur during cognitive disequilibrium, while affective states such as boredom and flow would typically occur during cognitive equilibrium.

Such research suggests that confusion is an integral part of learning complex ideas that is required in order for students to grow their knowledge. Students similarly could easily recount moments of confusion when discussing their most recent mathematics lesson. That is, confusion was a routine part of mathematics learning that needed to be addressed.

Another such example of confusion cited in an interview was when a student participant, Selina, described her experience in learning how to write a quadratic function in standard form, factored form, and vertex form. The lesson required that she understand how the algebraic manipulation of trinomials related to factoring a trinomial. The learning outcomes of the lesson then required her to understand how the different equivalent equations were related to the equivalent functions in factored form, vertex form, and standard form. As she stated,

> **Selina:** Today's lesson was about how to, how to go from standard form to this form.
>
> **Interviewer:** What is that?
>
> **Selina:** Vertex form
>
> **Interviewer:** Anything else you want to add?
>
> **Selina:** And, then, kind of like, like everything we've learned from like to standard form, vertex form and all that. We kind of, learned how to do it all together and graph it.
>
> **Interviewer:** How do you feel you understood the lesson?
>
> **Selina:** I got confused a lot today…I was confused when [my teacher was] going from standard form to factored form.
>
> **Interviewer:** Um hum
>
> **Selina:** Yeah. Um I didn't understand how [my teacher] got the two. The two factors. And then I asked Fatima and she helped me.

In this excerpt, Selina recalled the specific point of confusion where her teacher was factoring and writing polynomial as a product of two binomials. In her example, she cited how interacting with a peer helped her understand and reach the state of equilibrium, i.e. understanding. Her learning process related to Vygotsky's learning theory mentioned previously: an interaction (conversation) with a peer helped her move through her confusion.

Zadina (2014), an educational neuroscientist, further describes how when a student has an existing schema, any new learning must negotiate with the existing knowledge so that a connection can be made in the neural network, i.e. schema, that exists on the subject. For this reason, some confusion may exist simply because processing time is required to make the necessary connections to the existing schema. So, in Josue's example, he struggled to understand the connection between a polynomial and a function. Yet once he understood that when his teacher referenced a polynomial and then switched to calling it a function, these were one in the same, he was able to negotiate that information and then was able to "focus."

Skill barriers to learning.

Another common obstacle to students' mathematics learning identified by students and teachers occurred when pre-requisite skills or prior knowledge was missing from students' existing schema, such that new learning did not truly make sense. For example, as noted earlier, Mrs. G. discussed her shock at learning that her students had not acquired a pre-requisite skill that students should have learned in the previous course. Her concern was evident when she stated,

> What I wasn't expecting was how difficult it was for them to grasp the division part. And one of the things, I think, that kind of, you

> know, surprised me is that there was this expectation before we did, you know the complex numbers and the dividing that the kids had, which was radicals and rationalizing denominators. And then as I was teaching this to my first class, I noticed that very few of them actually had done that. So I tried to give them a mini lesson, and they did okay and I did that with the second period and it was my third period, struggles a little bit more. And um, I thought maybe because they were my SEI [Structured English Immersion] class, but then when I had the same struggles with my sixth period, I was like, man, maybe I should not teach division of complex numbers. Just held off for another time. You know but I didn't see that until I saw how much that class was struggling. So, you know, it's one of those where you just go back and reflect and you just, man it just didn't go as well as I hoped it would have. But, you know, you, you adjust and you try to go back and fix things and help them understand it and move on from there.

In this excerpt, the teacher describes the unanticipated "gap" in students' schema. Mrs. G knew that the pre-requisite concept was part of the previous year's coursework learning objectives, but she also had previously commented that this year -- as teachers were transitioning to the new Common Core standards -- certain standards from "before" were perhaps overlooked. Therefore, she adapted and created a mini lesson to counter the skill-based barrier that students needed to overcome in order to have access to the new learning and the lesson objective for her day's lesson. Her "adjustment" was to stop her planned lesson and show students how to rationalize a fraction with a square root in the denominator. Once students understood the concept of rationalizing the denominator with roots, then they could make a connection to rationalizing the denominator to exclude imaginary units. Rationalizing the denominator is a process done in mathematics to rewrite a fraction in a preferred standard form where roots and imaginary numbers are excluded from the denominator. As described later, she used teacher-student talk – explaining – to overcome this classic barrier to learning.

These types of barriers have been analyzed and substantiated for decades. As Radatz (1979) stated,

> Deficits in basic prerequisites include ignorance of algorithms, inadequate mastery of basic facts, incorrect procedures in applying mathematical techniques, and insufficient knowledge of necessary concepts and symbols. Bloom (1976) has emphasized the important role such variables play with respect to the great variation one ordinarily sees in learning outcomes. The pupil's history of learning in school yields large individual differences in the elements of previous knowledge available for a specific mathematical learning task.

Skill–based barriers also arise when a student does not yet understand how pre-requisite knowledge is connected to the new learning. That is, the connections between a pre-requisite skill and the new learning have not been made. This type of skill-based barrier was observed multiple times during lessons. One such example was when a student in class needed to be reminded how to subtract fractions of different denominators in the middle of lesson on dividing a polynomial by a binomial in an observation. In this case, the student was grasping the concept of synthetic division as well as its relationship to long division, but struggled when he had to subtract 1/3 from 2. The teacher, Mr. S, stopped and had the student think through how he could solve the problem. He asked the student if he could draw it. The student was able to walk the teacher through the method of dividing two “pies” in thirds and shading one of the thirds. A couple of minutes later, another student asked the teacher how a certain step in the solution process had resulted in 6 and the teacher asked, “Well, what were we doing to the -6?” and he pointed to a -1. The students response was “Uh, huh?” in an unconvincing tone. Mr. S proceeded to said, “What operation are we doing?” The student responded, “Multiplying.” And the teacher proceeded to ask, “What is a negative times a negative?” The student then said, “Oh, okay. Got it.” In the teacher interview that followed this lesson, the teacher highlighted this moment when he said, “Like today, I had to address Abel’s question about subtracting fractions. I can’t let that slide, you know. Cause, umm, I need them to use all the tools. ‘Using mathematical tools appropriately’ is one of the mathematical practices, right? All tools… drawing pies, counting on fingers, I’ll take it. I want them to know that they can go back as far as they need to.”

A student, therefore, needs someone to either teach them the pre-requisite skill in order to truly understand the new learning or be reminded of how the pre-requisite skill is a link to the new learning.   This ensures that the learner can make the connection in their schema and use that pre-requisite skill as a resource or basis to further the understanding of the new learning.

Other skill-based barriers described in research and by this study’s participants are not mathematical based skills but, rather, based on linguistic skills. Much like pre-requisite mathematics skills are needed to develop new mathematics learning, pre-requisite linguistic skills are required to comprehend the linguistic mode of instruction (Walqui, 2008). That is, a student may have the pre-requisite mathematical skills, but simply may not understand what the teacher is referring to because those concepts are being referenced in a language in which the student is not fully fluent. In such situations, terms that represent a concept cannot be "called up" or recalled in the learner's mind due to the language gap. In each classroom observation, students who were less fluent in English were observed clarifying teacher explanations, mathematical terminology, and their own thinking in Spanish.

Emotional barriers to learning.

Other barriers to learning that participants cited were emotional. Research suggests that some of the emotional barriers that can be mental blocks to learning mathematics are: mathematics anxiety, a learner's doubt in his/her capability to learn, a lack in mental maturity to address abstract concepts, or the lack of a safe environment to learn.

Hamid et. al. (2013) define mathematics anxiety as the "intense emotional and irrational fear of mathematics based on unrealistic feelings of frustration, hopelessness, and helplessness associated with repeated failure or lack of experience of success." This barrier was cited by teachers rather than by students themselves as a barrier that needed to be considered in their lesson organization. Mrs. E stated,

> They are struggling because of gaps in their knowledge and they have anxiety but as I work with them one-on-one on appointment books or whenever we're having pair work um, their questions are very thoughtful and even when they make errors, they're

> catching themselves on their errors and they are starting to see the connection that we're not just making up numbers to put in the x, that we're actually getting the numbers from the trinomial, so they are starting to see that connection. They're basically giving them a chance, giving themselves a chance to think. Once they start to give themselves a chance to think and think, "Oh, I can do this," then that's when they start to progress. But, overall, all of them, um, are very motivated and, and they are all very strong, um, I can see them, all of them, you know, reaching, PreCalc at least, or Calculus, AP stats.

This teacher is able to articulate the process that students must go through in order to fill in gaps and build their confidence as capable students. As we explore in depth later, in her “overarching growth mindset” practices, she spent time encouraging students and highlighting their effort and time that they give to thinking about mathematics critically.

Students said that when a teacher doubted their ability to learn, this had a profound effect on their performance and learning outcomes for a lesson or a course. During one interview, a student made a clear distinction between his current teacher’s practice of putting student’s in the “hot seat” after pair work, where the teacher answers questions with questions, versus the student’s previous math experience. Diego said with a smile,

> Yeah, sometimes he laughs but he's like, like in a good way. Like everybody knows he's not like picking strictly at you and he, he just… You don't get frustrated or nervous or you don't get upset like in other classes. Like in my past experiences. In my last year class, my teacher just picked on me and I was like, "No, I just need help. I don't really know what to do. Ahh.. like, "I'll just come back to you." And she never got back to me. Like, I totally got lost for the rest of the day. And that's not how [Mr. S] works. So he's just, I don't know. His level of teaching is way superior from like my other teachers back in the day.

In comments like this, students suggested that a teacher's investment of time (or lack thereof) to clarify or re-explain aspects of the new concepts being learned triggered positive or negative emotions for students about their ability and had direct implications on what new learning that student walked away with after that day's lesson. These barriers may at times be bigger impediments to learning than those addressed above. If students do not feel that they are capable of learning, or that they are safe taking the risks necessary when learning something new, the teacher may need to make significant efforts to challenge these negative mindsets and environments.

Historic barriers to learning.

Research suggests that yet other barriers to learning are historic. That is, some barriers to learning exist outside of the learner and exist in the environment in which learning is to take place. This type of barrier was not a barrier that teacher or student participants noted in the study, but one that research has established exists. It is mentioned here because, although not mentioned by teachers or students, it is one barrier that teacher practices deemed effective by researchers are countering and so, related to this dissertation's exploration of effective practice.

Research clarifies that many Latin@ students and other students of color face obstacles as they enter schools that have historically not accounted for nor integrated their culture, language, or perspectives (Gandara & Contreras, 2009). In these cases, it is what teachers and institutional actors believe about students and do that either counters or generates barriers. Research suggests that if teachers do not believe that Latin@ students can learn because historically and statistically they have not achieved, then teacher instruction will demonstrate that belief in how teachers answer points of confusion, how they handle student questions, and how they orchestrate their lessons. The same has been said for Latin@ students who are English learners as they at times are assigned to teachers who do not believe that they should be in the U.S. educational system (Gutierrez & Irving, 2012; Olivos & Ochoa, 2008). Some people believe that only U.S. born students should be taught in public schools and do not realize that the vast majority of English learners in secondary schools are U.S. born (Gandara & Contreras, 2009; Olsen, 2010). In these cases a teacher's negative beliefs can manifest themselves as inequitable practices in everyday classrooms even when a teacher's mathematical content skills are vast and he/she is able to employ other pedagogical skills with other students. These historical barriers and "fixed" mindsets about students' predetermined ability is exactly what teachers countered by employing overarching growth mindset practices. As Pollock (forthcoming) might note, even if teachers were not countering their own fixed mindsets, they were countering a mindset pervasive in the world.

Overarching Growth Mindset Practices

Prior research already cited suggests that to address many of the skill-based, emotional, and historic barriers described above and witnessed in the classrooms observed, a growth mindset may be particularly essential for stigmatized groups such as Latin@ students and English learners of all languages (Bol & Berry, 2005; Ernest, 1991; Gandara & Contreras, 2009; Gutierrez & Irving, 2012; Joseph, 1987). As shown in the work of Carol Dweck (2006), a growth mindset is a perspective on how intelligence is acquired and grows continually; it counters a "fixed" mindset. A "growth" mindset is a belief that intelligence can be grown or developed (Dweck, 2006). In this way, the brain is not seen as static but as dynamic, like a muscle that can be trained and developed.

Several student participants in this study stated emphatically in interviews that this sort of mindset fundamentally shaped teachers' approaches to the barriers discussed above. In fact, during interviews with the eight Latin@ students, growth mindset practices, further defined below, accounted for 25 percent of citations in the overall data of practices deemed helpful to accessing and learning rigorous mathematics. The four participating teachers also cited growth mindset practices as essential to their teaching. Specifically looking at teacher data, growth mindset practices were cited 37 percent of the time overall in interviews as essential to promote learning by the four teachers.

Research suggests that growth mindset practices are foundational in promoting math learning (Boaler, 2013; Dweck, 2006). In cases in which a student has a skill-based barrier, it can be addressed and remedied; if the student has an emotional mental block, he or she can be coached to see that they are capable of learning. If a student has internalized the historic stigmatization, the student can be mentored to see other realities. The growth mindset then becomes evident in visible teacher actions and practices in everyday mathematics lessons.

Therefore, in this study, participants cited growth mindset practices most often as the practices driving all of the other educator decisions and practices. Educators articulated their growth mindset beliefs often, as seen in the lesson videos and interviews. In the middle of class, many of these beliefs became evident in teachers' language and response to students. Below, I discuss five examples of Overarching Growth Mindset Teacher Talk practices.

Overarching Growth Mindset Practices: Teacher sees students as capable of learning.

In her book, Dweck (2006) discusses the two predominant mindsets towards intelligence and shares perspectives on the age-old debate of nature versus nurture and genes versus environment. She discusses how the two mindsets, fixed mindset (nature) and growth mindset (genes), drastically affect learning, motivation, and views toward others. Extending this to educators, a teacher's mindset determines his or her belief about a student's ability, capability or intelligence. Therefore, a teacher's belief about students' capability to learn is the foundational practice in the overarching growth mindset category. Khisty and Chval (2002) found that teachers' beliefs regarding students' ability to comprehend and teachers' beliefs about how knowledge is acquired, were each directly linked to the type of teacher talk that was observed in classroom lessons. Similarly, in this study, teachers would often explicitly convey their beliefs that their students were capable of learning by their language encouraging students to continue to try, their insistence on using their fellow peers to help them understand, and their expectations of student explanations and work. One such example was with Mrs. G. In her second lesson, she was teaching students operations with complex numbers. (A complex number is a number that is part real and part imaginary.) Students had been introduced to the concept of imaginary numbers in the prior lesson. After she introduced the notion of a complex number such as 6+4i, where the first term, 6, is the real number and the second term, 4i, is the imaginary number, she said,

> Now we are going to learn how to add, subtract, multiply and divide complex numbers. I believe that you already have the tools to figure this out without me so I am going to give you some problems and we are going to see what we get. Use what you know about the number system here and try your best. If you get it wrong, don't worry but try! Check with your partner and see if you can figure it out without me.

In this example, the teacher did not approach teaching operations on complex numbers as a blank slate situation or a situation with flawed learners. She had faith, based on previous lessons, that her students could apply what they knew about real numbers and basic operations on polynomials with minimal problems. But she stated that if they did get it wrong, it was not going to be "a big deal". She shared her belief in her students' ability to apply what they knew in this novel situation and figure it out. As she said in her interview,

> What we first did when we talked about those is that they should already know how to do it because an imaginary number is a just a unit just like the number 7, or just like the fraction one-half, or just like the variable, you know x, it's just a unit representing something, and so I talked to them about how with imaginary numbers, all those properties that they learned you know, in elementary school and middle school, you know, the commutative, the associative, the distributive property all apply in the same way, and so they did really, really good with um, adding and subtracting complex numbers, so you know, I thought about how you deal with *i* the same way. We went into multiplication so I, you know, I didn't really want to show them an example and okay, this is how you do it. I really wanted to see if they could just think about them a little bit.

She knew that this was a new situation but trusted that most students had the ability and could "apply in the same way" their existing schema to get basic operations such as addition, subtraction and multiplication right. Division was another story, since it involved the introduction of the concept of a "conjugate," but she anticipated what portions students were capable of solving and which would require further explanation. In so doing, she demonstrated a growth mindset consisting of her belief that students' knowledge could grow and that students had the ability to grow it by helping each other.

Students also were able to highlight this practice. When I asked Itzel to tell me a little bit about her teacher, she said, "I think he believes we're all capable of learning all that math that he's teaching us."

Overarching Growth Mindset Practice: Encouraging in general.

Research suggests that a teacher who encourages students to persist and recognizes student effort in mathematics classrooms have a positive impact on students' learning. Rousseau and Tate (2003) explored how teacher beliefs affected students' opportunity to learn and identified the practice (or lack thereof) of *providing encouragement* as promoting access to rigorous mathematics in the classrooms. This was evident in this study as well: all four teachers were seen encouraging students in each observed lesson multiple times. Students said, or conveyed in their actions, that the simple act of praise and acknowledgement of their work motivated them to keep trying with the remaining elements of the task they had been given.

In both lesson videos for Mrs. E, she was seen walking around the classroom and high-fiving students as she inspected their work. She would also say "Good job!" and "Check you out!" Students would high-five her in response and were observed smiling. As she passed by, many students were pushing their papers so that she could easily see them. She was also observed encouraging students to ask their peers about what they thought or if they agreed. She would say, "Does your partner agree with you?" "Did your partner understand your way of thinking?" "Ask your partner what they think." These actions and directions encouraged students to persist in seeking a deeper understanding of the material and to use each other as resources, thereby validating their constructed knowledge and their effort.

Itzel, one of the participating students, emphasized how her teacher encouraged them to seek understanding:

> He tells us to like if we're confused about something to ask questions cause he doesn't want us to like fail our test. In the beginning we would get like [get] very good like, average scores on our tests as a class. But like with the more confusing stuff, I feel like we all got confused….with the division I think it's going to be easier and he tells us to got to tutoring, you know, and he's like um helping us if we go to tutoring an we show him like the [tutoring and work] paper, we show that we tried, he like helps us like to do more stuff to like get our grade up.

In this quote Itzel explains how her teacher encourages students to keep learning and seek to understand, that is, to "ask questions" "cause he doesn't want [them] to fail," to attend after school tutoring offered by him or someone else, and that he gives them additional practice by giving them "more stuff" so they can get their "grade up." The message this conveys is that effort and time will yield success in mathematics; and that all students, even those who take more time, are capable of learning.

Similarly in her own interview, Sarai eagerly shared how her teacher also encouraged them to seek to understand even if they had done poorly on a test. Her teacher encouraged them to re-learn the material and attempt a parallel exam again:

> Yeah, it's just like. Just learn from the mistakes you make, like in the first test. And just know you're not going to make the same mistake and get like 100 percent. And he's going to count that, like the second grade. And he's just going to put the second best grade into your like grades and that like make your grades better and you won't be like failing his class.

These statements exemplify how encouraging students to persist and recognizing their efforts in learning further motivate students to learn from their mistakes and gain access to mathematics knowledge and skills. This practice is demonstrative of a growth mindset in that students are encouraged to see themselves as capable of learning.

Overarching Growth Mindset Practice: Verbalizes or otherwise demonstrates high expectations of students.

In this study, high expectations of students was defined as the expectation for students to work and achieve high-level, high quality work and was cited as important by both student and teacher participants. Based on teacher talk in classroom lessons and interviews, high-expectations were also denoted as the stated expectancy that students be thorough in their explanations of mathematics both in written and verbal forms. “High expectations” was an emerging code that surfaced during the study. During interviews, students would make statements that referred to their teacher’s high expectations, using phrases like “you have to do it, it doesn’t matter if you don’t want to”, “you have to explain your thinking”, and “it’s hard but he explains it like it’s going to be easy and he expects us to get it, he won’t let you go.” In one case, a student spoke about how his teacher would not move on to another student if he or she did not understand. He stated, “He does not let students off the hook because he believes that his students can learn.”

During one of the lessons, this same teacher reminded students that they would be taking a unit test in a week. He additionally stated, "If more than 10% of you get below a 90% on this exam, I know you can do this! You will ALL retake the test." In interviews, students conveyed that each of their teachers had had such high expectations of them and conveyed in words or actions that they were capable of learning. Diego emphatically stated,

> Um, well, like the number one thing I picked up on him was like, his discipline. Like, showing his professionalism…of his work. And like I said, everybody was like, "Oh, he's just going [to be] some other teacher whose going to be picking and picking on you. So you better get out and I don't know. Like the way you, you, what's it called, the way you bond with him throughout like any problem or like day by day. As the days goes by he's just like the same person. He doesn't like change. If you like get wrong a problem, he's not going to be like, "Oh, ok, I'll just call you later" and he won't call you back and he'll forget about you. He's like, "Nah, I'm going to help you. I'm going to be here today." He's like, he doesn't even say it like that way. But he shows it. Like, sometimes.

Diego notes how his friends told him to not take Mr. S' class because Mr. S was going to "pick" on him. In other words, be relentless about verbalizing what they knew. Yet Diego identifies this trait in his teacher as being "disciplined." He shares how Mr. S does not give up on them or judge them when they "get a problem wrong" by noting that his teacher does not "forget" about them. In this comment, he recognizes that learning is difficult. He equates "picking on you" as a sign of his teacher's belief in students, which translates to high expectations. He highlights how this teacher's behavior of "I'm going to help you" translates to "professionalism."

Overarching Growth Mindset Practice: Encourages students to see themselves as "in training" (Mathletes).

I've chosen one participating teacher's term, "Mathlete," to denote when a teacher encourages students to see themselves as capable of success in mathematics, by encouraging students to see themselves almost as a mathematical athlete who, through training, is capable of succeeding in mathematics. Furthermore, this practice relates to the ability to have students see themselves as mathematicians and incorporate mathematics into their identity. In other words, it fosters students seeing themselves as being able to do mathematics, succeeding in mathematics as a whole, and not just learning a specific concept. The practice instills a sense of being able to get "good" at mathematics. Gutierrez, Willey, and Khisty (2011) found that students who were able to incorporate mathematics into their identity – much like an athlete sees himself as a soccer player instead of a person who can play soccer -- saw themselves as able to succeed in mathematics.

This practice was manifested several times in this study. At times it was an organized scaffold that a teacher used such as the structured pair-work activity of "Coach" and "Mathlete" in which one student coaches another through a problem and the "Mathlete" follows and writes down the exact steps that the coach dictates. In other cases, it was what a teacher said to his students when they "tackled" a hard problem. One teacher would go around the class and say, "Come in! Beat up that problem. You got this!"

Mr. X commented on what he tells his students to keep them going when the mathematics is challenging. He said,

> This is something that takes a little bit of time. But my job is to make it as easy as possible. Using patterns, using whatever strategies I can, you know. And yeah, they do get frustrated. That's why I say, 'Look, don't worry. If you're understanding half' *Asi, ya se les quita de eso de anciedad* [Like that, they get rid of that anxiety]. You understand half. We're good. Keep going, *si puedes* [you can do it].

In each case, the teachers encouraged their students to persevere on the specifics within complex mathematics problems so that they see that their hard work is building their capacity.

Selina emphasized,

> If we get stuck, he makes you talk to your partner, look at your notes, you have to try. We just kept doing it and doing it until, like, it got into our head. She tells us we can do it.

Diego vigorously stated,

> but Mr. S he like encourages us to keep doing like more and more until we like get the topics super well. And other teachers just like, " Ok, were going to keep moving on with this and then your lost.

Diego's comment distinguishes the expectation between his current teacher and previous teachers, where he is now expected and encouraged to develop a sense of expertise in the content versus superficial understanding. This again connects back to the athlete analogy where an athlete develops their skill, while an amateur may simply understand some of the mechanics of the sport. Students articulated that their teacher pushing them to "try" helped them gain access to the learning. The metaphor of training is very evident in this practice and exemplifies the overarching mindset of the teacher acknowledging and encouraging perseverance and determination in mathematics. Participants suggested that perhaps when students experience success after being pushed to put in more effort and recognize that they are able to learn complex abstract concepts, then they will incorporate mathematics into their academic identity.

Overarching Growth Mindset Practice: Anticipates areas of student difficulty.

Burroughs & Luebeck (2010) found that one of the major differences between new pre-service teachers and experienced in-service teachers was the latter's ability to anticipate student misconceptions and areas of difficulty while learning certain mathematics topics. With experience, teachers are able to recognize what skills, concepts, and tasks are more cognitively demanding on their students' mathematical schema. Participants exhibited this practice during the classroom observations, demonstrating that they valued it, and mentioning it during interviews. As an example, Mrs. G anticipated that her students would have difficulty completing the square of a trinomial. She started the lesson with having students factor perfect square trinomials into two binomials. Then she had students determine the value of c in trinomials if the trinomial was a perfect square. So Mrs. G asked, "if $x^2 + 12x +$ ____ were a perfect square, what would the constant be in the blank?" Students solved about six of these problems in a row and re-wrote the equivalent trinomial as a binomial squared. In her interview she said,

> One of the things was, I wanted them to understand what it was to be a perfect square trinomial. I wanted them to understand what that was and why we were writing a perfect square trinomial so that they could use their Algebra skills because I think a lot of them, they don't understand, they can follow the steps when I give it to them, and I don't mind giving them the steps, but I want them to understand why they were doing the steps. So that was one of the things I had them do when I got them into groups, to factor only and notice a pattern, the pattern in the factoring, how it's a perfect square trinomial, and why the perfect square trinomials were special in, in helping us complete the square, solving them by completing the square. I wanted them to see it. I don't think they all saw it. I would say, maybe if I'm lucky, half of them saw it. But I'm hoping that, once that they notice the pattern of what they do, that they'll see, "Oh, ok, this is why we did this and this is why we did this.

In this case, Mrs. G not only anticipated that students would have trouble following all the steps necessary to complete the square of a trinomial, but she also wanted to build on the schema that students already had about factoring and now use that skill in completing the square. She anticipated students would struggle and planned her lesson accordingly, to minimize the struggle by having students work in groups and scaffold how students were introduced to the notion of completing the square. She “chunked” the information so that students would be able to focus on patterns and build their knowledge from those points. Her growth mindset is evident in this example because she knew that she could counter many of her students’ struggles with the environment she created in class and by nurturing student thinking by allowing them to work together.

Similarly, Mrs. E clarified,

> No, no, this is not an easy task. Um, I was hoping that they would have enough experience with factoring to understand and multiplying. So, in the multiplying lesson, to go from standard, from intercept to standard form, or from factor to standard form, um, we taught them how to multiply binomials in that context. So they were very confident with that.

She, too, anticipated what students' areas of difficulty might be from the unit standpoint with her Professional Learning Community (PLC). In doing so, she organized her lessons so that she could give students enough practice and build their skill set in preparation for having them navigate several forms of a quadratic functions to determine the essential elements to graph the function. From interviews and observations, I found that teachers with a growth mindset did not merely anticipate misconceptions, but they planned and orchestrated their lessons to maximize student understanding past these misconceptions. For those with a fixed mindset, misconceptions would be associated with a lack of ability and therefore minimal planning to counter the misconceptions would be evident. As mentioned by Diego earlier, some teachers simply say, "Ok, were going to keep moving on with this and then you're lost," not anticipating or addressing the barrier to learning.

In sum, growth mindset practices are continually recognizing that students face barriers in learning every day, yet these barriers can be countered by encouraging the right frame of mind is, i.e. that the student is capable, successful, effortful, and persistent. Mathematics is often viewed as a subject that is elite and attainable only to some (Gutierrez, 2002; Gutierrez &

Irving, 2012). Yet research has found that all students are wired to learn mathematics (Devlin 2001; Lipton & Spelke, 2003) and that knowledge can be grown (Dweck, 2006; Zadina, 2014). An overarching growth mindset is essential in teaching a subject that is considered difficult by society. The overarching growth mindset practices counter that accepted societal belief. They set the tone for the learning environment and communicate teachers' beliefs about their students' ability to learn and their own role as teachers in that learning process.

Students and teachers cited these practices as foundational to the practices named in "Teacher-Student Talk," "Student Processing Time," and "Partner Processing Time" categories. That is, the above overarching practices were, to participants, necessary precursors to the more specific interactions described next.

Teacher-Student Talk

Teacher-student talk practices are what I call those practices that are implemented and evident in the more traditional aspects of teaching – that is, all the moments a teacher talks while teaching in a whole class setting, as well as how a teacher responds to the student audience. Teacher-student talk takes many forms. It can be direct instruction as in a lecture with little or no student verbal participation, or it can be a verbal exchange between the teacher and the students. Teachers in this study, who were deemed “effective” and good” by administrators, focused on a two –way communication throughout their lessons. Mr. S highlighted,

> I’m always looking at their faces. You know that look they give you when they are lost. I stop. I have to have them go back and see what they’re confused with, explain it back to me.

One teacher strongly stressed,

> So, when I'm explaining, I have these layers of when I use stuff. So, I'm explaining how to do a problem that is brand new to them. Um, like completing the square, you know, show them the steps. And then I'll do one or two, depending on the complexity of the problem. While I'm doing those steps, I'm having them, I'm looking at their expressions and I'm checking to see, "Do they look confused? Do they look like…are their eyes glazing over?" You know, and, and I anticipate where also, things might be a little tricky. And so that's when I say, "Ok, turn to your partner and ask your partner, "Did Ms. [E] confuse you?" Even as I'm lecturing those two or three examples that I'm doing with them. And then, I'll have them try one and then they check with their partner.

Participants suggested that a teacher employing effective teacher-student talk practices responds to students’ verbal and non-verbal cues and adapts the teaching accordingly. At times, it is the teacher prompting students to interact with him or her, while at other times, it is the student or students who interject to gain clarity on the topic being taught and the teacher then responds. Or, as described in the second excerpt, the teacher allowed students to respond to one another and then addressed any lingering questions.

I will explore Teacher-Student Talk practices that schema-build, that address pre-requisite skills, that clarify misconceptions, that build conceptual knowledge, where the teacher explains and re-explains content, and provides a purpose for learning the content. These teacher-student talk practices were mentioned in participant interviews as helpful to learning, and research concurred.

Teacher-Student Talk that schema-builds.

Schema building as a teacher-student talk practice is a scaffold noted by researchers as necessary for English language learners. In fact, it is one of the six scaffolds identified in the conceptual framework for Specially Designed Academic Instruction in English (SDAIE) for English learners mandated by the state of California. Walqui (2006) states,

> Schema, or clusters of meaning that are interconnected, are how we organise knowledge and understanding. If building understanding, is a matter of weaving new information into pre-existing structures of meaning, then it becomes indispensible for teachers to help English Language Learners see these connections, through a variety of activities.

The goal, then, in schema building is to allow students to preview information, see how it is organized, and anticipate how what they are about to learn fits into their pre-existing knowledge. In the example given above in which Mrs. G had the students factor perfect square trinomials, she was not only anticipating that completing the square would be a difficult task for her students but she also organized her lesson so that students would practice a pre-requisite skill and build upon that skill. In her teacher-student talk at the front of the room, she was schema building so that students understood the relationship of one skill to the next and how these processes both related to trinomials and quadratic equations. In interviews, this practice was “called out” much more by teachers than students. This may be due to the fact that students classify this practice more generally – as “teacher explains well and is willing to explain” or “teacher clarifies misunderstanding.”

Sarai explained:

> um, well, in number three I did, I did get confused because everybody was um putting zero and then I just didn't know, I, I did get, I did get the part of *k*, which was zero, but I didn’t, I didn't get the part of *h*, which is then how I figured out it was the um, I saw in front of number 5 which the exponent had *x* minus one. Mr. S said that it was just like in, what's, um, rational functions, if we had, you know, like a trinomial or something, a trinomial *a* will be the biggest, the one with the biggest *x*, exponent, and then *h* will be the one with the, the next biggest exponent and then *k* will be the one, the number by itself. And that's how I applied my knowledge from the previous lesson to this one.

In this lesson, Sarai explains the point she was confused about. The teacher noted to the class that he was seeing a lot of “confused faces.” He reminded them of other functions they had graphed and once Sarai could see the connection to the previous lessons, she was able to access the content.

Jacob also commented on how his teacher helped him understand,

> It was different but it was similar at the same time because I mean, we're going over basics like division and multiplication. And um, the difference is we're, um, like the, with the conjugate.

His teacher had students recognize what was the same to what they already knew and build upon that knowledge to add new elements such as complex numbers and conjugate.

Three of the four teachers specifically discussed how they were trying to help students connect previous knowledge with the new content. The statement below serves as an example of how they also pointed out how student behavior in the class offered evidence that their approach was working:

> What surprised me? Um, I wouldn't say surprised me but I'll say that there were certain things that I, that I assumed that, truly believed that they were going to get but they didn't get in the beginning. So I had I had to go over back to what they knew about graphing. I knew they were going to have some difficulties, but I didn't know they were going to have difficulties in those areas. ….Since we've been using a, h, and k, umm, how we transform, and shift the curve. I didn't think that they were going to have such a huge problem as they did, so I had to go back to remind them how we worked with easier, you know shifting other graphs….I think …when I was talking to that one student, Manuel, Yeah he was like, "Oh, man" when he, he saw it. And as a class, like you know the ones that got it got it. And the ones that didn't had to, they were close to getting it. They could see the connection.

In this case, the teacher did not anticipate that students would not recognize the recurring features between graphing other functions and graphing exponential functions. Once he realized that students were not immediately making the connections and "calling up" their schema on graphing functions, he adapted the lesson to review what they already had learned about graphing and how the constants *a, h*, and *k* affected the parent function. Once he reminded them of this pre-existing schema, he was then able to help students extend their schema and apply what they knew to the new content of exponential functions. Thus, this teacher-student talk practice offered students an opportunity to tie new material to what students already knew and to extend their schema or make a connection within their schema.

Teacher-Student Talk that clarifies misconceptions and that addresses pre-requisite skills or gaps.

The Merriam-Webster dictionary defines Algebra as a "branch of mathematics; a generalization of arithmetic in which letters representing numbers are combined according to the rules of arithmetic". Much of high school mathematics is dependent on students' experience with the real number system in their elementary and middle school years. As a result, students often needed to "tap" into their schema to be reminded of a pre-requisite skill and how it fit to the new concept or skill. The previous practice of teacher-student talk that schema-builds discussed how important it was to ask students to "call up" their knowledge on a topic and then use that knowledge to build the new learning. What happens when the pre-requisite skill cannot be "called up" and confusion arises? In this case, a quick refresher on a concept could suffice to counter any confusion.

As noted earlier regarding barriers to learning, research notes that there are times when a student simply has not learned a pre-requisite skill or concept and a gap exists (Radatz, 1979). In order for any further learning to take place, and to counter the skill-based barrier, the student needs to gain understanding of that pre-requisite skill. It is in these situations, in the moment-to-moment interactions with a student, that a teacher must make the decision of altering the flow of the lesson to address or teach a prerequisite skill or to continue, knowing that not everyone in the class is furthering their understanding. These moments are often not anticipated by the teacher. Furthermore, there are times that students attach their knowledge to the wrong pre-requisite skill (Radatz, 1979), thereby necessitating that these misconceptions be addressed. The second example below serves to explore how attaching the incorrect pre-requisite skill manifests itself.

Earlier, an example was provided of a student who had needed to subtract 1/3 from 2, a pre-requisite skill to dividing a polynomial by a binomial involving fractions, that the teacher referenced in his interview. The teacher in this case did not have to re-teach subtracting fractions, but did have to backtrack to visually represent subtraction involving fractions. This teacher-student talk practice of addressing a pre-requisite skill served as a moment where the teacher pushed the student to tap into their schema and ignite the student's knowledge so that he could gain understanding. In this scenario, the student asked the question and the teacher could have said "well the answer is 1 2/3" or " I got the value from subtracting," but the communication between teacher and students caused the teacher to ask the student, "What is 2 -1/3? How do you know? Can you draw it out?" A teacher being able to read students, know their strengths and weaknesses, and know how to motivate students to think or reason is an aspect that make teacher- student talk practices such as this one effective. In the teacher interview, he emphatically stated that he wanted students to use all the tools they had available to them. As previously stated he said,

> Like today, I had to address Abel’s question about subtracting fractions. I can’t let that slide, you know. ‘Cause, umm, I need them to use all the tools. ‘Using mathematical tools appropriately’ is one of the mathematical practices, right? All tools… drawing pies, counting on fingers, I’ll take it. I want them to know that they can go back as far as they need to.

During student interviews, students discussed how their teacher addressed questions about pre-requisite skills. One student, Sarai, shared how some of her previous experiences were handled when she had a misconception that stemmed from a gap in pre-requisite skills. She was frustrated when she explained her experience and that some of her previous teachers would say,

> Sometimes you forget stuff from before. Like you're doing it wrong. Like other teachers do that. Like, "Oh, that's like kindergarten stuff and you shouldn't still be doing that and you have to move on but he doesn’t.

As Sarai points out, this teacher-student talk practice also conveys a message of “you should have learned this” or, “you can learn this” (a growth mindset). This student was able to articulate the struggles she faced in attempting to access rigorous mathematics in previous mathematics classes that simply noted that skills should have been learned in earlier levels of mathematics, and how these experiences had made her feel that she was “not good at math, like, I’m just not going to get it, you know.”

Fractions surfaced in Ms. E’s class as well as a skill that “should” have been learned earlier, and Josue said,

> Like the fractions that it comes out to sometimes like how doesn't it just come out to one solid number sometimes. Like, I get confused like "It's supposed to be a negative" or "It's supposed to be a positive". So I get confused on that but Mrs. E explains it so I get it.

Students described their feelings and how they gained clarity in understanding when their teacher took the time to re-teach or remind students about pre-requisite skills without belittling their mathematics ability. It is not simply the fact that teachers were willing to ensure that students had the required pre-requisite knowledge to latch their new knowledge to, but it was also the tone, the patience, and the belief in their ability that was transmitted by the way teachers talked to students. In essence addressing the pre-requisite skill eliminated the barrier and this facilitated learning.

Another student, Jacob, stressed how he felt when the teacher halted the lesson to clarify a peer's misconception during the lesson that I had observed,

> Because she takes her time. As an example, um, [CJ], he was struggling big time and I mean she didn't just keep on going. She wanted to help him out because she knows that we can all do it, if we at least try hard enough. And eventually he figured it out. He's like, "Oh, ok'. And then she was able to move on because she knew, "Ok, he finally understands it.

Jacob was referring to a classmate, CJ, who was struggling with exponents when multiplying two complex numbers. The student was multiplying $(5+3i^3)(6+i)$ and erroneously saying multiplying this was $30 + 5i + 18i^3 + 3i^3$.

This led to a very lengthy discussion because whereas the student got a final answer of $30 - 16i$, the teacher said that another student had the answer of

$33 - 13i$. The student was struggling because he did not remember his rules of exponents where $(3i^3)(i) = 3 \cdot i \cdot i \cdot i \cdot i = 3i^4$. He argued with the teacher that $(3i^3)(i) = 3i^3$ because the exponent of *i* was 1 and $3 \cdot 1 = 3$. The rule of exponents is a concept usually taught by sixth grade. Once again, the teacher could have dismissed the question by saying "that is 6th grade stuff," as Sarai had experienced, but Mrs. G sought to understand the source of CJ's misconception and then went back to an elementary example with real numbers, $(3 \cdot 5^3)(5)$. Once CJ was able to see the reasoning with real numbers, he was able to understand how the generalized rule applied in algebra.

Research suggests (Radatz, 1979) that in learning and understanding a new concept, learners at times make connections in their schema to the incorrect piece of information as in the above example. The student was multiplying most of the terms correctly, but was also multiplying the exponents of the variable terms, which is incorrect. Yet the time invested in this teacher-student talk practice was something that teachers worried about. Three of the teachers noted that backtracking to review pre-requisite skills takes time and that they struggle with giving up that time to review some of the basic mathematics skills. They felt that allowing students to work with other students (see Partner Talk, below), actually minimized the time they had to spend reviewing these "gaps." One teacher said,

> I just need to keep following up with those students. There's a huge separation in this class from the top students to the bottom

> students. And one of the things that I try to do in the seating chart is have a top student sitting next to a bottom student. So that way, when they do get into pairs, they can help each other out and then sometimes it doesn't happen because sometimes people are absent or whatever, but for the most part that's how it works. But, but that's you know, it's just one of those things you have keep following up, and following up, and following up with, what they're doing.

To avoid taking too much time addressing all of the possible gaps or pre-requisite skills using this teacher-student talk practice, this teacher organized her class and lesson so that she could use students themselves as human resources and she capitalized on the variation of student understanding in her class.

Additionally, when implementing new learning that is based on a series of steps, students may get confused on a portion that halts the understanding of the concept. The practice of clarifying these often unanticipated misconceptions serves to promote learning. When teachers recognize that abstract concepts such as synthetic division may be cumbersome for students, being patient when students struggle, employing the teacher-student practice of clarifying misconceptions and re-teaching pre-requisite skills, students are provided the opportunity to get back in the game of learning, versus being benched for messing up a play. It is the difference between being a facilitator of learning and a lecturer.

Teacher-Student Talk builds conceptual understanding, not just procedural.

Research has long established the need for mathematics knowledge beyond procedural skills (The National Research Council, 1989). Furthermore, for marginalized groups, the inclusion of conceptual understanding has been documented to close the achievement gap between groups (Gutierrez & Irving, 2012; Gutstein, 2003; Schoenfeld, 2002). Based on research in this area, the Mathematics Framework for California Public Schools (2013) stated that rigorous mathematics “requires conceptual understanding, procedural skills and fluency, and application be approached with equal intensity.” Although the two previous frameworks similarly define the goal of mathematics education as a balance between conceptual knowledge, procedural knowledge, and problem-solving skills, the California Standards Test (CST) for mathematics primarily focuses on assessing procedural knowledge. In doing so, many California mathematics teachers focused on preparing students for this high stakes assessment and conceptual understanding, while the application of mathematical skills in “real–world problems” was addressed only if time permitted.

The Mathematics Framework uses the word “understand” to explicitly denote conceptual understanding. Conceptual understanding, then, is defined as that beyond the answer to a question -- rather being able to solve mathematics problems using varying perspectives. “Students might demonstrate deep conceptual understanding of core mathematics concepts by solving short conceptual problems, applying the mathematics in new situations, and speaking and writing about their understanding" (California Mathematics Framework, 2013, p. 11).

All four teachers focused much of their lessons and lectures on developing conceptual understanding. They also incorporated a variety of activities and used instructional time to have students digest new learning and clarify understanding to their peers. Selected students were asked to explain their understanding to the class at varied points in the lesson. Two of the teachers incorporated writing tasks where students were asked to write about their conceptual understanding in lesson notes. During interviews, teachers were able to articulate how their planned lesson goals were an attempt to develop conceptual understanding for students. One teacher, Mrs. E., stressed,

> Um, the factoring is what we spent a lot of time on. And we were doing the multiplication, we emphasized *a*, **difference of squares and perfect square trinomials. And so, emphasizing those two helped** with today's lesson because you heard Fatima right away give out the answer when I was asking them to think about it. Um, so, the fact that they remembered what a perfect square trinomial was, was very helpful. And then that it showed up in the warm-up and so that lead up into **completing the square**, which I think facilitated the process. Um, we had examples where there was no *c* to just complete the square but I felt that was out of context and to me it didn't make sense to start there. To me, it made sense to start with the trinomial. What do you do with the *c*, you know, and handle, managing that *c* term, and so that's why I went ahead and started with example 2 instead of example 1. **Um, but it**

> **wasn't until we got to that point that I was comfortable with going to all three versions. You know, all three forms.**

This teacher discussed how she planned the unit so she could build on students' previous skills. Her goal was for students to understand how "completing the square" was tied to perfect square trinomials, how the leading coefficient, *a*, of the trinomial affected the perfect square trinomial, and how to manipulate the equation so that the problem became easier. In organizing the lesson, she felt that students' ability to adjust to a problem that may present new elements helps students deepen their understanding of algebra, in this case. Her "Teacher-Student Talk" asked students to gather information for graphing the quadratic function by re-writing the function in standards form, factored form, and vertex form, so they could extract the information needed from each equivalent function and discuss how the forms were similar and different. She gauged students skills and understanding regarding the process of completing the square and once she "was comfortable" with their skills she helped them understand how "all three versions" of quadratic functions fit into the schema or bigger concept of quadratics. In building and tying to previous skills, this teacher focused on both the answer (procedural skills) and helping students understand quadratic functions (conceptual knowledge). She intended for her lecture, questions, and explanations to help students tie specific elements related to quadratics to each other with their schema on quadratics. The topic of completing the square could be taught in isolation as a means of answering a textbook question, however, she chose to help students develop their understanding by organizing her lesson differently.

Students were also able to articulate their conceptual understanding beyond the procedural skills necessary to find the answer to a question, and they emphasized the importance of this ability. When discussing the lesson of dividing a polynomial by a binomial using synthetic division, Sarai was able to clearly describe the necessary procedural process with ease. When I asked her what she felt she understood well about the lesson, she answered:

> **Interviewer:** So, thinking about this lesson what do you think you really got or you really learned in this lesson? What do you feel you now understand?
>
> **Sarai:** I think he just showed us another way. Like I really understood how to set it up and how it connects to other stuff that he taught us like factoring. How that is just another simple way or like another resource you have. Because you forget factoring and to make it easier for you. To have options. Like to know what to do.
>
> **Interviewer:** So you felt he connected it to factoring and to anything else that you felt?
>
> **Sarai:** Grouping
>
> **Interviewer:** Grouping?
>
> **Sarai:** Yeah, that's where I thought he strongly connected the remainder theorem with the grouping.
>
> **Interviewer:** So can you tell me a little bit about the remainder theorem.
>
> **Sarai:** Umm
>
> **Interviewer:** Why is the remainder important? Or what does the remainder tell you?

> **Sarai:** And what I got is, is the solution to the function that they give you. It's another easy way to find the solution or in this case, the zeroes with the rational [root] theorem of the first linear equation they give you which is x minus a.
>
> **Interviewer:** So if your remainder is 16, what does that tell you?
>
> **Sarai:** It cannot be factored.
>
> **Interviewer:** It cannot be factored? So when can it be factored?
>
> **Sarai:** When it's 0.
>
> **Interviewer:** When it's 0? So if the remainder is 0..
>
> **Sarai:** …. you know it's a root.

This excerpt highlights the student's understanding of how synthetic division is connected to previously learned concepts, including rewriting polynomial functions in factored form (using factoring and grouping as methods), recognizing that if the remainder of the polynomial was 0 after being divided by a given first degree binomial or linear expression (or as she referred to it a linear equation), that the divisor yielded one of the roots of the polynomial, and that the binomial divisor accounted for one of the identified roots in the rational root theorem. Her teacher's teacher-student talk emphasized that synthetic division was a method that they could use to identify roots and that could use a "mix and match" approach of various methods to find all the roots to a given polynomial. Sarai not only articulated the procedural steps necessary to determine a root for a polynomial, but was further able to explain how the result fit into a much more complex schema of the properties and attributes of polynomials, an important concept. The excerpts discussed highlight the type of interaction between teacher and student to gain access to knowledge that goes beyond steps to solving. Accuracy in mathematics is important, but understanding how the bigger scheme of specific skills fit together is what building conceptual knowledge is all about. This teacher-student talk practice requires planning on the part of teachers to help students make the links within that schema. It requires teachers to emphasize certain features during their lessons, ask questions that help students see how one concept is related to another, and strategically encourage students to use all available mathematical resources. Participants suggested that effective interactive communication between students and teachers required planning, anticipation of student struggles, and verbal and non-verbal communication.

Teacher-Student Talk: Teacher is willing to explain and re-explain.

Among the practices identified under teacher-student talk, students cited their teacher's explanations and willingness to re-explain as the most helpful practice. In comparing this practice to others in Teacher-student Talk, this was cited most often as helpful and most emphasized during interviews. This particular practice refers to teacher explanations during students' initial learning and understanding of new skills and concepts, as opposed to addressing pre-requisite skills or clarifying misconceptions as described in previous sections. A teacher's ability to explain well and re-explain – to "go over some parts" and "spend the time" verbally as needed -- has been identified by Rousseau and Tate (2003) as essential practice for promoting access and equity in a mathematics classroom. In their study, they were able to document how students either engaged or disengaged from learning based on a teacher's willingness to re-explain. Not only do students need to be engaged in the lesson; paying attention, they also need to make sense of what they are learning. The state of disequilibrium, therefore, may surface at different points for different learners.

Teachers and students in this study spoke about this aspect of learning. They discussed how the process of learning requires that the teacher be willing to explain a process or concept multiple times and often in different ways. Students in this study reiterated that their teachers' explanations of a process or concept were extremely helpful to their learning. Even without prompting, many of the eight students gave specific examples of how their teachers' explanation during the guided lecture portion of the lesson—teacher-student talk-- furthered their learning and understanding of new concepts. Most participants cited re-explaining moments as the most beneficial portion of this practice, but the initial explanations were also cited. Teachers did re-explain to small groups, but would often wait for pair discussions to conclude prior to re-iterating explanations. They often started the discussion with words to the effect, "When I was walking around I noticed that some of you are still confused by… so let's discuss this again." Therefore, the explanations that were referenced during interviews were exclusively ones noted as whole class teacher-student talk.

Josue was able to articulate the specific explanation that his teacher gave the whole class that helped him get unconfused after stating that he had been confused a lot during the lesson,

> **Interviewer:** Where were you first confused?
>
> **Josue:** I was confused in that negative. Finding the axis of symmetry…she said it was the opposite of what the b was.
>
> **Interviewer:** the negative?
>
> Jo**sue:** yeah of the b. oh yeah so it was a little confusing at first but then she explained it by saying you just switch, you just if it's a negative 12 it would be uhh… positive 12.
>
> **Interviewer**: how did that help you?
>
> **Josue:** Because at first I was like "Why is she calling it.. the negative a opposite?" then she said that if you call it a negative then you might think it was always a negative.

In this excerpt, Josue articulated a common misunderstanding of when to call a quantity negative and when to call it the opposite of the quantity. He is referencing the formula for the axis of symmetry where $x = \frac{-b}{2a}$. His teacher, Mrs. E, had asked students to tell her what the formula for the axis of symmetry was, at which point several students yelled out "negative *b* over 2*a"*. This is how this formula is usually recited, but the teacher took the opportunity to re-explain the formula and why they should not call it *negative b* but rather the *opposite of b* since she had seen many students, such as Josue, evaluating the formula incorrectly. During her discussion with the whole class, Mrs. E said,

> If b is *12* what is the opposite of *b*? *-12*, but if *b* is *-17* what is negative -17 [she wrote -(-17) on the board) you might forget that other negative since its already negative…. Or you could think of it as the opposite of -17. Which is easier?

This re-explanation to the whole class is what Josue cites as helping him overcome cognitive disequilibrium and gaining understanding. The teacher's re-explanation of why she wanted students to view the *–b* as *opposite of b* was grounded in a simpler example and is also an example of how the terminology used in a re-explanation can help students understand a concept. Josue is still an English learner; the explanation of the vocabulary along with the example using basic numbers helped him clarify the concept and overcome his confusion.

Nuvia, a student in one of the other participating teacher's class, was emphatic about the benefits of a teacher re-explaining to the whole class. She stated,

> I feel like she does cause there's time where I do want to ask questions but I feel like I'm just not asking the right ones and there's other people that ask it, so I'm just like, um, you know like, cause when other people ask like the question that I would want to ask, she would normally demonstrate the whole thing again, like how to solve it, from the beginning to end.

Nuvia described herself as a "shy" student. She later discussed how she felt more comfortable asking peers for help or to re-explain aspects of the lesson she had not fully grasped. In this comment, she shared how she does at times ask her teacher questions, but that she also is able to gain understanding from her teacher re-explaining and answering other classmate's questions. Nuvia went on to say that her teacher was very patient because she was willing to re-explain from the beginning several times. When a student asks a question, the teacher has an opportunity to clarify a confusion for more than that one student--- that is the benefit of whole-class teacher-student talk as described by Nuvia. A variety of teacher-student talk practices that participants called important to student learning were embedded throughout the class time. Most often, the teacher would start with a teacher-student talk practice such as explaining, where they embedded the other teacher talk-student practices. Then they would instruct students to attempt a problem or two, as they monitored the types of student discussions, and then come back to re-explain and explain a more complex problem(s). This cycle was repeated multiple times in the two-hour blocks I observed.

Yet, another student, Max, underscored his teacher's willingness to explain and re-explain in broader terms,

> Um, I thought it helped because he was going over all the stuff that is going to be on the test, mostly, and um, I like how he shows us different ways to do it, instead of just one, so if I don't understand one, so if I don't understand one way I can always resort to a different path of solving that problem.

He went on to cite that his teacher's explanations qualified him as a "very good teacher" by saying,

> Um, I think that Mr. X is a really good teacher compared to a lot of other teachers I've had. And um… the way he explains things, and um, like talks to the class makes it easier. It seems like he wants to be here instead of.. with some of the teachers who come and teach and like just give us problems. Sometimes you don't understand it and then, and just stay misunderstood.

In his first comment, he shared how he values the varied approaches to solving a problem. He said he valued the flexibility that is offered by his teacher. Max seemed to equate a teacher's willingness to explain and re-explain with a teacher who enjoys teaching. He is able to compare his previous frustrated experience with a teacher who just "gives problems" – and implicitly, does not talk them through fully– with his current success. Max' teacher allowed students to use any method to solve a given problem and encouraged students to critically analyze the problem to determine their approach. During this lesson, a fellow student asked if they would be penalized for not using a certain method and Mr. X very clearly said, " Oh no!!! I want to see how you see it and I want to know how you think. Solve it, *como quieras* [how you want]!!!" Often when teachers re-explain, they may present another method or another perspective, but one is preferred or deemed more sophisticated. In observations, I noticed that in all four teachers' cases, they allowed and encouraged students to use the method or strategy that resonated with them so that the explanation or re-explanation was not just a means of helping students understand, but also became a new resource that students were allowed to use. As long as the method was mathematically sound and the teacher could follow the student's thinking, they allowed students to employ various methods. This was another important aspect of the teacher-student talk practice of re-explaining.

When another teacher was asked what had surprised him about the day's lesson, he said,

> That surprised me? Um, I wouldn't say surprised me but I'll say that there were certain things that I, that I assumed that I truly believed that they were going to get but they didn't get in the beginning. So I had to go over some parts. I knew they were going to have some difficulty, but I didn't know they were going to have difficulty in those areas. But it was good, I mean, they did really good cause some of them really started seeing, they started getting the deeper understanding of it. Umm, not just like a shallow understanding. So they got more depth. Which was good. I mean I didn't mind spending the time on it.

As Max noted earlier, his previous teacher's unwillingness to re-explain left him in a state of confusion. "Spending" valuable class time to explain or re-explain can affect how much material can be presented in the allotted time. Some teachers are torn by how often to enact this practice. This teacher's comment "I didn't mind spending the time" demonstrates his evaluation of that tension. Having said that, students seem to feel that the practice of explaining and re-explaining strongly affects their overall attainment of the material, as it was the most cited practice within the Teacher-Student Talk category by students.

In comments, students suggested that a teacher's routine investment of time to clarify or re-explain aspects of the new concepts being learned triggered emotions for students about their ability and had direct implications on how much the student learned during the lesson.

Teacher-Student Talk: Provides a purpose for learning a concept by connecting it to other mathematics concepts.

Gutierrez, Willey, and Khisty (2011) found that students were able to incorporate mathematics into their identity when they saw themselves as able to succeed in mathematics, redefine the purpose and definition of mathematics, and engage in mathematics. Their study specifically discussed the importance of having students use their mathematics -acquired knowledge in new situations and make connections among mathematics skills and concepts. Fifth grade students in their study were able to articulate how much more they enjoyed mathematics when topics were not taught in isolation and how understanding the connections helped them understand the purpose as well as build their own confidence in mathematics.

Similarly, this practice was cited and observed in my study. Teachers were the most concerned with this particular practice but students cited it as well. When teachers were asked what the goal of the day's lesson had been, they often discussed the connections that they were attempting to help students see among mathematics concepts and skills. One teacher, Mrs. E noted,

> We are, I hope you'll see we've been, you know, wrapping our minds around how to teach factoring and completing the square and the context of these three forms because that's what the common core standards call out for. And, if the big picture is how to convert from one to the other, get information to graph, then we struggled with details such as, so, when we take vertex form from standard form, can standard form have a negative *a*, can it have a 2, can it have an 8, you know, or are we always

> going to one. And so, looking at activities like the Mars activity the matching dominoes, the, um, where it has all three forms in the parabola pictures, they have to match them up, all of the vertex forms were either given to them if it had a negative or they all had a coefficient of one. And so we took that as a hint, we don't need to make it super hard. The goal is to understand these three, so let's just help the kids understand these three. So that's the background that I came in this lesson with after having discussed it with my PLC [professional learning community] and all of this is talking about what, how do we go with this and so I made a point to focus only on examples where *a* was one and *b* was even. Um, all the procedural stuff will come later. Right now, we really wanted to get this big picture across to them and that was my goal for, for today.

As this teacher notes, the new Common Core Mathematics standards identify some of the connections that students are to make among concepts and the purpose for connection. The new standards are much more explicit on the types of connections that students must understand by the end of a unit, but are not as clear on the depth of understanding. To determine the depth of knowledge, that is, what complexity to which they need to address the topic, Mrs. E., along with her colleagues, found tasks vetted by Common Core authors and other well respected mathematicians who have created websites that offer resources aligned to the Common Core mathematics goals and standards that helped determine the depth to which to teach this standard in her course. This is a shift in high school mathematics where previously procedural skills had taken on more importance. In order to achieve conceptual understanding, connections among concepts must be highlighted.

The students noted that making connections to other mathematics concepts really helped them understand and learn the new concept. When

Jacob was asked, “How do you feel about the concept of imaginary numbers or complex numbers?” Jacob replied,

> Um, I think it’s, it’s pretty simple from what we’ve been learning in the past. I think it’s the concept because for me I think it’s, it’s just like if you’re regularly multiplying or dividing or adding or subtracting. It just for me it all just flows together.”

In this case, Jacob understood how the operations on complex numbers were a slight adaptation to operations on whole numbers or operations on polynomial terms. His teacher had initially asked them to attempt the operations with no instructions—that is no initial explanation-- and to determine what skills students “called up.” After she saw that students’ intuition was for the most part accurate, she led a discussion on how the operations were similar to those on whole numbers and polynomial terms, but that they varied because they needed to take into account the imaginary component that could require further simplification.

Itzel also mentioned this aspect. The students had just finished the topic of long division of polynomials and also the rational root theorem, so the day’s lesson was on synthetic division. When I asked how her experience this year in mathematics compared to her experience in other years she said,

> **Itzel:** He needs to like, the teacher needs to be like, interested in what he's doing, like, he just can't be on the board talking, and then that's going to make us bored and we’re going to be like not be able to do our work. Like, today, doing both [long division and synthetic division]. And find the roots.
>
> **Interviewer:** What do you mean?
>
> **Itzel:** Well he did the two [long division and synthetic] on the board and we were like voting and stuff, and laughing.

Her teacher had had two students come to the board and do the same problem; one using long division and one using synthetic division. After the students sat down, the teacher had students vote on which method they preferred and why. He elaborated on student responses and rephrased what students were saying to accentuate connections between the two methods. Afterwards, he had a lengthy discussion with the students regarding the fact that these two methods could help them find the roots they needed to solve the polynomial. Using the rational root theorem, students could establish the number of rational roots. In contrast, long and synthetic division could establish the location of the real roots. To participants, this linkage of concepts was a teacher-student talk practice that seemed to foster student understanding.

Teacher-Student Talk: Uses mathematics register and terminology correctly.

Research suggests that teachers should speak in mathematics register, meaning the grammatical structures in which mathematics is discussed as well as the specialized vocabulary that is often used distinctly in other contexts (Moschkovich, 2007). Participants never cited teachers' use of mathematics register as a practice that facilitated learning, yet observations showed that both teachers and students used the mathematics register in every lesson. All four teachers used the correct mathematics register and explicitly taught mathematics terminology in one or both of the lessons observed. Students who were asked to explain their understanding or a mathematical process, often used common language. As such, several of the teachers would ask, “What is that called?”, “How do we say that?”, What is another way of saying that?” Interviewed students either expressed their understanding using the mathematics register or asked the interviewer for clarification on appropriate terminology. Although a direct causal effect cannot be determined as to how students were appropriating the mathematics register, research suggests that a teacher using mathematical pedagogical discourse (or mathematical register), that is, uses the mathematical register and embedded definitions, is able to extend the mathematical discourse in his or her class by assisting students to appropriate the correct use of the vocabulary and explicitly model the structure in which the vocabulary is used. Khisty and Chval (2002) found that such teacher talk in the classroom plays a vital role in students’ learning of mathematics, because it provided access to the mathematical community for students.

As previously mentioned, the practice of using the mathematical register and terminology correctly was never cited by students or teachers in interviews as specifically helpful, but was seen “in practice.” When Jacob was asked when he thought he should use the “completing the square method,” he stated,

> Oh, she, she prefers, well, Ms. G. , she prefers us to use it when, if its either if I recall a perfect square or if the *b*, if its even, not odd. And she, she wants us to, if we could bring it down to just x^2, and since they're all even, for this, you can divide two in all of

> this, so then you just get x squared, and when you have x squared it's a lot easier to do this kind of a problem. She doesn't want us doing it if the b they're [coefficient] odd numbers.

In this excerpt, Jacob is able to articulate when he thinks the "completing the square method" is best used to solve a quadratic. Although his mathematics register is not perfect or seamless, it is clear that he is appropriating it and using it to explain his understanding of the concept.

Similarly, Itzel attempted to use the mathematics register and correct terminology when she explained the difference between long and synthetic division,

> Well, we get the same answer. You can only use synthetic division when we have that the no degree and the constant or is that what it is? Coefficient something like that. And, so if you can't use synthetic division you have to use long division. They're like similar.

Itzel did not quite succeed in explaining the difference correctly, but was definitely on the right track. She referred to the first-degree binomial as a "no degree and the constant," then she confused the terminology with "coefficient." Coefficients, the number or parameter multiplied with a variable in an algebraic term, are definitely discussed in solving a problem using synthetic and long division, but "coefficients" are not synonymous with binomial terms or linear terms.

During interviews teachers also used mathematics terminology and the correct mathematics register as evident in Jacob's excerpt above. In a second example, a teacher discussed how multiple strategies offered by students may be beneficial to future topics. He emphasized the following story as a "good moment:"

> They get it. Or what's the other way? Synthetic, ok, and then, what I really liked is that they, they just followed me throughout that journey where we're just going to look for roots and keep on looking and if it doesn't work one way, grouping method, let's try the rational root theorem, and exhaust all possible roots, and what does that imply then, Well they're all complex then. And, you know, another thing was where [Zabala] was like, "No, I just want to do root theorem." "Go ahead, but what if they are not rational, you know they're complex, or irrational?" *Perdon* [sorry], you know, or square roots, yeah, you can't, from the rational. It just gives you fractions. So that's what? That's one of those moments where I have to tell them why and show them why. Then, "Oh, I see. Well, you don't see a *p*, you know, a factor of square root of 2. You don't see that, so you need the quadratic formula. So that was also one of those good moments. Yeah.

Mr. X discussed how he encouraged students to attempt all possible methods to determine the roots of a polynomial function. However, one of the problems did not have any real roots. A student insisted on using the rational root theorem and struggled to understand how the polynomial did not have real roots when the coefficients of the polynomial were all real numbers. This problem led to a discussion that Mr. X valued and previewed a future topic. Throughout this excerpt, the teacher referenced mathematics terminology correctly. His mathematics register was intermingled with everyday language and code-switching, "'Go ahead, but what if they are not rational, you know they're complex, or irrational?' *Perdon* [sorry], you know, or square roots, 'yeah, you can't, from the rational. It just gives you fractions." He demonstrated how multiple registers could be used to communicate. His comments also demonstrated how he has incorporated mathematics into his academic and cultural identity. As Khisty and Chval (2002) indicate, the mathematics register provide the tools to be able to communicate in the mathematics community with the correct lexicon whereas Mr. X demonstrated that he could be part of several communities at once. In this example mathematics register also served to facilitate understanding during the whole class discussion. Mr. X's discussion with Zabala was intended to help students understand what information and purpose the rational root theorem provided and that there were also other possible roots that could be irrational or complex roots. In the process he also referred to square roots, those that cannot be simplified into whole numbers, to remind students these numbers cannot be written as a fraction and are therefore irrational.

These examples, although not direct comments on the importance of the mathematics register, do serve as evidence that, to teachers particularly, mathematics register and terminology aid in explaining the learning that takes place in mathematics classrooms. During lessons, teachers were observed allowing students to use everyday language to describe mathematics but students were almost always asked to re-state what they had just said using the correct terminology. The correct mathematical register was used by teachers during the portions of their teacher-student talk when they were building conceptual understanding, explaining or re-explaining, schema-building and clarifying misconceptions. One such example was when students were asked to match a function with a quadratic function using three pieces of evidence. The teacher (while demonstrating to students what might be some acceptable evidence) asked the students, “What is this point?” A student raised her hand and said, “The lowest point of the ‘U’… the turning point”. The teacher said,

> Yes, this point is the ‘vertex’ of the ‘parabola’ that represents the graph of this [pointing] quadratic function. The vertex is the lowest point but it is also a point on the axis of symmetry that separated the parabola in half and mirrors that half [using her hand to model a reflection], right? Wow, that was a lot that I just said there [laughing]. Turn to you partner and explain what I just said, the youngest person in the pair starts first.

Teachers were observed not only using the mathematical register but trying to have students appropriate it as well. Academic Language has been a district-wide initiative in the Oceancrest District for the past four years. Teachers have been asked to assist students in appropriating the correct academic language and have been given, at a minimum, three hours of professional development. Academic language is also an indicator on “walk-through” forms that administrators use to provide feedback to teachers.

Teacher-Student Talk: Teacher makes connections to real world applications.

The newly adopted Common Core Mathematics Framework and standards, define “rigorous mathematics” as achieved when each major topic, “ pursue[s] with equal intensity: 1) conceptual understanding, 2) procedural skill and fluency and 3) application (National Governors Association Center for

Best Practices, & Council of Chief State School Officers, 2010).” The National Research Council (1989) and other researchers (Civil & Planas, 2004; Cobb & Hodge, 2002; Gay, 2010; Gutierrez, 2007; Gutierrez & Irving, 2012; Gutstein, 2003; Schoenfeld, 2002) have long argued that contextualized real-world applications motivate, encourage, and empower students to incorporate mathematics into their identity. Research suggests that a contributing cause of the achievement gap is decontextualized mathematics curriculum, meaning that the purpose for learning mathematics is not made clear to students (Cahnmann & Remillard, 2002; Gutstein, 2003; Schoenfeld, 2002).

None of the eight lessons that I observed addressed real world problems. For this reason perhaps, the practice was not cited as helpful. Chapter 5 will provide additional comments that were made during the oral survey and provide insight into how teachers and students felt about this specific practice.

Teacher-Student Talk: Helps students understand how mathematics can be used as a tool to solve problems in the real world.

The practice of “helping students understand how mathematics can be used as a tool” similarly is identified by research as important and is about *using* mathematics in a real problem or situation. Gutierrez, Willey, and Khisty (2011) found that students were able to incorporate mathematics into their identity when they contributed to their communities using mathematics as a tool. This is a practice that is related to the previously mentioned practice of teacher makes connections to real world applications. Research states that the goal is not only to highlight where mathematics is used in the real world but empower students to use mathematics as a tool in their community while learning to solve *real* problems that exist in the real world (Cahnmann & Remillard, 2002; Gutstein, 2003). The practice as described in research did not surface in this study perhaps because the “real world” applications were still decontextualized since they referenced real life application in a word problem but students were not applying the mathematics itself in the real world.

Yet what did surface was how teachers and students saw what they were teaching and learning as *tools* to future mathematics learning, other subjects, and as a tool for their future.

Mrs. E denoted how what students were currently learning would impact their access to future mathematics content by saying,

> so that helps um, when I'm giving lectures and I'm explaining to them, "This is going to be very helpful when you go and you move on. You know, factoring is a very, very important skill that you're going to be doing from now on" And so they've been taking it a lot more seriously. So, they're a very, very strong group of kids, they will end up in AP Calculus, Stats, they will go to college.

Again, because this was a practice that was minimally referred to in the interviews, the predominant data source for this study, I will reference quotes that were mentioned during the separate oral survey in Chapter 5.

Student comments focused on how they did use mathematics as a tool in other learning academic subjects or within mathematics itself. Max said,

> I'm not really like, super excited to do math. But I think it's more of like a, like an easier subject for me because I understand it a lot. And like sometimes in other classes, like I take chemistry, so like some of the same concepts apply. I'm like, Oh, I saw this in math class so it'll be, it's easier for me.

Max valued how what he was learning in “math” was applicable in other school subjects.

This practice was not cited as helpful during interviews when discussing what helped or could have helped students get “unconfused” but rather was addressed when the following interview questions was asked, “Do you think that math is important and why?” Students would make comments such as: “Like, I'm gonna say when you're doing like, bills, I think. I don't do bills so I don't know, but I'm pretty sure use like a calculator or something”, “I think math is important because like, it will help you in your future if you get a job”, “It like trains you to think differently… to break things down when you have a problem”, and “because there's a.... I've been looking at jobs for myself in the future, and I noticed that most of the jobs, like require like, a higher level of education, especially math. Like uh, there's different jobs. The math is just incorporated with a lot of things. So I think it's really important.” These comments signal that students made a connection between mathematics and the purpose it had to their future career and lives, yet these comments also signal that the purpose was global and perhaps abstract.

Connecting the classroom content and using it in the outside community was a practice less cited by participants as beneficial. Although research has identified it as beneficial, I hypothesize that teachers may see this practice of connecting to the community as an “if time permits” practice due to the resources and time they have available as well as the focus of the content. This issue is a question for further study. The Common Core mathematics standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), after all call for real world application of mathematics but not necessarily in the community in which students live.

I turn now to the third big category of Student Processing Time to be followed by Partner Processing Time. These last two categories are focused on how students used their own resources and their peers as resources to gain access to the learning and to deepen their understanding. While the previous two categories of Overarching Growth Mindset and Teacher- Student Talk were primary focused on what teachers said to students, the next two categories are about what students did with the time they were allocated to process all the talk that was previously mentioned.

Student Processing Time

As Vygotsky (1962) explained, the “Zone of Proximal Development” is important when learning a new concept. Students need to determine how the new learning is related to the pre-existing schema on the topic and be within a certain “zone” for it to be accessible. In trying to ground these connections, students often need time just to think or review a process, and need what Zadina refers to as enough practice to create a “pathway” in the brain so that automaticity or fluency can be established (2014). Participants cited various classroom practices that gave students time to process alone as critical to student understanding. I place these practices in the category of “Student Processing Time.” Since the teacher provided this time, I have “teacher” in each subheading.

Student Processing Time: Teacher provides processing time.

Processing time was accomplished in several ways in classrooms that participated in this study. At times it was allocated by asking all students to take a silent minute to think or asking students to stop and review a problem to gauge their understanding, or it was allocated by asking students to explain their thinking to a classmate. In whatever form this practice took place, the teacher built time for students to be metacognitive and determine their own understanding.

When students gave examples of getting unconfused, they often cited the opportunity to look back at the notes they had taken as beneficial to their understanding. This served as an example of individual processing time of the new topic. Nuvia said,

> I got unconfused. I would look back at like the examples that we did. Cause it says. This was the formula like it was like negative or like the opposite and then it was always multiply by two.

She had a moment to process the line of symmetry formula and understand where the final equation came from after looking at the notes she had taken during the lesson. In her case, she was struggling to understand where the numbers that were evaluated in the formula were coming from and then how they were manipulated within the formula. Nuvia noted that just having time to look "back at the example" allowed her to get unconfused.

Max emphatically commented on his teacher's routine,

> I think it was similar to most every day because every class we learn something, the next class, you notice he always goes back to it and just make sure we understand it. And he uh, applies the same concepts to different problems like, like I understand how, he gives us time to think about it you know, like going through the chapters, we used different methods for different types of problems or we could still use the first method we used for problems later on in the chapter. So he, kind of uses the same concept. So it makes it easy.

He shares how this "time to think about it" helps him gain access connections to other topics within the chapter.

Diego also explained how his teacher's routine for allocating time to process had an effect on his understanding. He said,

> Um, first I do the problem by myself and see if I can get it by myself. And like, either compare answers or if one of us don't get the answer, we try to figure it out, try different ways.

Having students attempt some problems alone and then work with partners provides an opportunity for students to process on their own prior to discussing it with peers; it offers an opportunity for students to gather their thoughts and establish what they know and do not know prior to interacting with others.

As one student said about her teacher's regular routine, "Yeah, he gives us a couple of minutes to like, look through it with our classmates and then if we're confused, we just like have somebody look to the board and work it out."

Vygotsky's (1962) learning theory analyses the link between language and thought. He argued that language was a type of manifested thought. In the excerpt above, the student stated that his teacher gave them "a couple of minutes" to look through with a partner, yet, I argue that this practice was put in place to initially give students time to process as that is a precursor to generating language and negotiating ideas with a partner. The practice of providing additional time (to reflect and process their learning) was also determined as a beneficial practice by Rousseau and Tate (2003), yet the participants in the study refined this practice. Study participants identified that this practice required individual think time first.

Similarly, teachers discussed in detail how they built additional processing time and how they managed this allocated time. Mrs. G discussed the decisions she often makes when teaching and giving students the time to process:

> And I think, stopping at every step and making sure that the students understood why I did that step and how I got the numbers was helpful for the students. I know that it's probably frustrating for the ones that get it right away. That they have to

> keep waiting, and waiting, and waiting. But I would rather them get frustrated because they're impatient than the other students get frustrated because they have no idea what they're doing. So that's, it's you know, it's always going to be one or the other. If I go too fast, I frustrate those students but I lose them. Or, I take it nice and slow the first time through and then the ones who really get it are frustrated because not it's too slow.

Mrs. E felt that time to think affected students' confidence in their own ability, saying,

> Once they start to give themselves a chance to think and think, "Oh, I can do this", then that's when they start to progress. But, overall, all of them, um, are very motivated and, and they are all very strong.

Time for students to process, sometimes as little as thirty seconds, gave them time to gather their thoughts before they were asked to discuss with a peer, yet this time was very valued by both teachers and students. The time allotted guided the conversations students later had with their classmates and, participants argued, made them more fruitful. Teachers argued that partner processing time alone was not enough time for students to process learning, and that time granted for students to individual think and time to process what they knew was just as important.

Student Processing Time: Teacher provides in-class time to solve mathematics problems.

Providing students in-class time to solve mathematics problems and "try out" their understanding of a concept or process was a practice that was identified as important by students and teachers in this study. Students referred to this practice as "solving math problems"; it is part of what researchers more generally term “engaging in mathematics”. To researchers, the opportunity to engage in mathematics during class time includes allowing students to engage in mathematics by solving open-ended, high challenge problems and using peers as resources to find valid strategies for problem solving (Gutierrez, Willey, & Khisty, 2011; Gutstein, 2003). In other words, class time is not spent simply taking notes on how mathematics is solved but allocating time to attempt to solve problems while still in class. Although the practice cited by researchers specifically “called out” the importance of students working on mathematics with peers, participants in this study clarified that they often needed time to attempt the problem on their own before they attempted to solve the problem with peers. Students in this study greatly valued the time to work on mathematics problems alone first and then with their peers. For example, Sarai described how her teacher structured time for students to work on their own and then yelled “switch” where she now worked with a partner. Sarai stated,

> **Interviewer:** What do you think about that?
>
> **Sarai:** Well, when I do it by myself, I know exactly what confuses me, so like I have an idea of like what are the problems my partner knows that I don't know
>
> **Interviewer:** Okay.
>
> **Sarai:** So it's kind of like a mixture of both. But I need to do it first on my own.
>
> **Interviewer:** You have to try it first?
>
> **Sarai:** So that I know what I can do and what I'm missing

Another student, Josue, similarly described this process. Josue was explaining how having time to solve a problem on his own helped him get unconfused. When asked if there was a point in the lesson where he was confused he said,

> **Josue:** The equation to find the axis of symmetry when you do x equals negative b over 2 times a.
>
> **Interviewer:** Um hum
>
> **Josue:** That was confusing.
>
> **Interviewer:** Did you stay confused?
>
> **Josue:** I got unconfused. I would look back at like the examples that we did. Cause it says. This was the formula like it was like negative or like the opposite and then it was always multiply by two. So I did it.
>
> **Interviewer:** Did what?
>
> **Josue:** Do a problem… try it on your own.
>
> **Interviewer:** Um hum..
>
> **Josue:** Did it first [by myself], because if she would have kept doing it I would have just probably messed up on the opposite or wouldn't know what to multiply the number by.
>
> **Interviewer:** How do you feel that that affects your learning?
>
> **Josue:** It like kind of makes me have to understand it because I have to explain it to them [his group].

They felt that this allocated time was valuable time to access their own understanding, to practice what they had learned and create stronger connections, to recognize any misconceptions and seek clarification, and to solve mathematics problems, building their confidence in preparation for discussion about the problem(s) with a peer. One student said that she needed both independent processing time and processing partner time when learning mathematics. So, although research focuses on processing time as fruitful when students engaged in mathematics with peers, student participants clarified that they often needed to attempt the problem on their own before having a discussion with their peers. They seemed to need time to attempt the problem and determine their own understanding first before they could ask for help or clarify for their partner. Three of the teachers were very structured in how they allocated time for students to attempt problems on their own before working with peers. As mentioned by Sarai, her teacher used a routine of yelling “Switch” to signal when students were to work in pairs after the teacher had given them problems and time to solve the problems on their own.

Another teacher, Mrs. G, emphasized,

> But I do, you know, and you noticed that I hope today, that I tried to transition them from different things. So they were by themselves, and then groups, and then partners, so you know, help release some of that energy but have them focus. It's still a lot. Like I feel tired when I'm done teaching.

All four teachers paused and embedded time in their lessons to have students attempt to solve mathematics problems. For the teacher who did not structure the time as individual and partner processing time, the students were observed doing it naturally on their own.

Student Processing Time: Teacher recognizes that students may need multiple opportunities to learn.

The practice of “teacher recognizes students may need multiple opportunities to learn” was one that emerged in interviews and was echoed by every interviewed student as something that helped student process the material they were learning. This emergent practice is really an extension of the practice “solving math problems” mentioned above but in a repetitive cycle. What students noted was that solving one problem was helpful but solving several problems allowed further time to process; each time they gained more equilibrium in their understanding. The most common pair work cycle observed was that students would solve a problem on their own, discuss it with a partner, attempt another problem on their own and then discuss the second problem with a partner. This cycle was repeated over and over in the four classes even when students were given the opportunity to initially start working with their partner. I would like to note that students would often go back and attempt to solve problems on their own after having gained some clarity from their notes or discussion with peers. One student said she got unconfused when, “I got it like, after doing it a couple of times”. To participants, this served the purpose of allowing students additional time to think and practice what they were learning. Participants also indicated that repetitive think and practice time built fluency with the material. For example, if students were learning how to complete the square of a trinomial, they were not only asked and given time to solve one problem, but rather were asked to solve and discuss two to four problems.

Jacob strongly stated,

> **Interviewer:** What do you think kind of helped you get unconfused with that?
>
> **Jacob:** Doing more problems
>
> **Interviewer:** Doing more problems?
>
> **Jacob:** Um,hum first. Then I checked just to make sure that I was right. But then I was teaching my partner how to, how to do these problems.
>
> **Interviewer:** When you teach somebody how do you feel about that?
>
> **Jacob:** It helps me understand it more because I mean I'm going over it again and I'm memorizing like, okay, those, this and this. Its just giving me more practice.

As mentioned before, students usually would repeat the cycle of working on a problem individually and then processing it with a peer. In the excerpt below, Selina described an activity called appointment book and how it helped her apply her knowledge on multiple problems (individually) and afterwards process them with different peers:

> **Selina:** Well, we get up and we have like, it's either like one through six or one through seven or like however many. And then we find partners that like, we want to work with and like, 1, 2, 3,4, whatever, and then we go sit with them when we have to work on the problem and stuff.
>
> **Interviewer:** How does that help you?
>
> **Selina:** Well like, you are doing lots of problems and like different people know how to do it different ways so you might learn an easier way.

Max explained the difference between his past experiences and this year. He urgently said,

> Oh, other math teachers, it's because like, it was kind of like, they wouldn't really go in depth what we were learning. They would just be like. They would just say, "This is how to do it" and like, "Here, here's the worksheet" and "If you have any questions you can ask." But it's different because now it's like we're really going in depth of how to learn what we’re doing in multiple problems so that we really understand um, we really can grasp this concept of what we're learning each day.

So in essence, the practice of allowing students multiple opportunities to show mastery as well as in-class repetition was a practice that students felt solidified their learning and, as Zadina (2014) stated, developed fluency. The way students participating in this study applied the multiple opportunities to learn was in a cyclical format where they attempted to solve the problem on their own each time to determine their level of understanding and then they would check with their partner. Similar to solving mathematics problems, students identified this practice as fruitful. They especially appreciated this practice when it was followed with peer discussions (discussed further in “partner processing time,” below).

Interestingly, every participating teacher extended the practice of “recognizes that students may need multiple opportunities” to testing and allowed and encouraged re-takes of test and/or quizzes. When students were asked “if they felt that their teacher cared about their learning?” and the follow-up question of, “How do you know?,” they all spoke at some point about how they were able to re-take an assessment if they did not do well the first time.

They also shared the process required in order to gain access to the “re-take”. All teachers asked students to do additional work, do quiz or test corrections, or go to tutoring, prior to being allowed to re-take a similar assessment. If the student’s grade was better in the second assessment, the student’s grade for that assessment was replaced with the better grade. Most teachers’ grades consist of 80 -90% assessments; so replacing a grade could significantly affect students’ grades. In addition, many students mentioned that they valued doing several mathematics problems of the same type in class. I view this grade-enhancing opportunity as an extension of “multiple opportunities to learn” since even when the time frame a topic had been allocated during class time had expired, teachers found ways to provide more practice outside of class and give students another opportunity to show mastery.

When students were asked about the opportunity to re-take an assessment, one student said,

> for someone who is having like an off day or whatever and just like completely flunks the test. Like you know that you're better than that I think that it's kind of cool that you can, you can retake test and get better than what you did before and have that as your, like your permanent grade for that test.

Students felt that it was possible that on testing days students could experience an “off day” or simply had not had enough time to process the learning. They indicated that allowing students extra practice, which provided additional individual processing time, and another opportunity to show that they had learned the material, was motivating. It served as an incentive to continue to seek to understand.

Partner Processing Time

Classroom partner or small group (3 or 4) discussions with peers have been identified by research as beneficial and as leveraging access to rigorous content for decades (Gutierrez, 2007; NCTM, 1989; Schoenfeld, 2002). The National Council of Mathematics Teachers has had communicating about mathematics with peers and teachers as a goal since 1989. Sadly, studies (Corden, 2001; Nystrand, 1997; Nystrand et al., 2003; Zwiers, 2011) have shown that in many classrooms serving diverse student populations, minimal dialogue among students is evident. Researchers (Arreaga-Mayer & Perdomo-Rivera,1996; Zwiers, 2011) have found English learners were discussing content with peers in class less than two percent of classroom time. New Common Core Mathematics State Standards and English Language Development Standards have addressed this issue and heavily emphasize both group and pair discussions. In the Common Core Mathematics standards one of the eight Standards for Mathematical Practice [these standards are consistent from kindergarten to twelfth grade and are overarching standards to the mathematical content standards] is to “construct viable arguments and critique the reasoning of others.” This standard for mathematical practice, although possible with a whole class of thirty-five to forty students, is recommended as a small group or pair practice as it would consume much of the instructional time to implement in a whole class setting if implemented with consistency. Therefore, teachers are encouraged by the authors of the Common Core standards to organize their lessons with embedded opportunities for students to discuss content.

In addition, some teacher participants felt that having students learn from each other lessens the amount of confusion when students are learning something new. When one of the participating teachers was asked how often she incorporated pair work, she adamantly stated,

> No, it's on a daily basis, um, all my lessons, I always think about, ok, which examples am I going to work out? But beyond that, how am I going to make sure that the kids are going to think about it, talk about it, and write about it. Because if they can't do those three things, then they didn't really learn it. And, I mean I could lecture all period long but talking at them is not going to get, is not going to guarantee me that they're going to think about it. It'll only be a few kids who think about it. But by forcing them to get into interactions with students in the class who are thinking about it, and can help them think about it, you know, then, then I can, I can make sure that I'm holding them accountable and that they're actually going to think about it and process what I'm telling them to process.

In her case, she also uses pair work to hold students accountable for their learning. She felt that if students were not able to talk about it and write about it then they had not thought about the new learning. All teachers stated that they incorporated both small group and partner work but they also clarified that they preferred partner work as it held students more accountable and students were less likely to get lost. District performance tasks require small groups of three or four students. On the days I observed, I consistently saw pair processing time with triads if there were an uneven number of students. Students also stated that they preferred pair work to small group work.

The practices below define the more specific partner processing time practices that both teachers and students cited as assisting and promoting learning in high-level mathematics. In each case, I name the specific elements of teacher support for partner-processing time that participants said were helpful to students' learning.

Partner Processing Time: Teacher provides the opportunity to work in pairs.

For decades, researchers have argued that teachers who support student learning by providing opportunities for students to work in pairs see more student engagement with the content and higher depth of understanding (NCTM, 1989; Zwiers, 2011). Furthermore, research has identified partner time as a best practice for Latin@ students in particular (Gutierrez, 2008; Gutierrez & Irving, 2012; Gutstein, 2003; Moschkovich, 2007; Schoenfeld, 2002; Walqui, 2006). Participants in this study agreed. When students were asked, "What helped you get unconfused?," an overwhelming response was "my partner." The following comments came from a variety of students when they described "how" their partner had helped them:

> "Well, I asked my classmate and he just kind of explained it to me."

> "It's different because we work in partners more and other teachers we don't work in partners as much. And I feel like it helps you more than just working alone."

> “Well… I am confident that my partner will teach me the stuff I don't know and I will teach him what he doesn't know.”

> “It’s easier when we are working together”.

Students implied a sense of mutual responsibility for their and their partners’ learning as well. Students said the following about partner work:

> “Like it affects my learning in a good way cause we help each other out and everybody at the end of the question gets it and they're like, "OH, ok."

> “And they have to explain it to me and I have to listen to them or like if they say something wrong like you have to catch it to make sure they understand it. It's like you have to really understand it whether you want to or not.”

> “So then you have to add two to each side, and I was only adding it to this side, and I kept on forgetting to add it to, to the other side but my partner noticed and helped me.”

During the four teacher interviews the role of partner work was noted often as an intended support they planned into their class time. Teachers also noted the benefits to allowing students to work in groups. They echoed some of the benefits that students cited. For example, Mrs. G spontaneously noted that especially on “higher level concepts,” “two minds” processing were better than one:

> Um, my thinking is for them to, if I know that it's a higher level concept, I like for them to get, help each other out. Cause I know they're going to have some parts where they're not going to understand but if they have two minds, two minds is better working than just one mind, so they're going to have somebody to bounce ideas off of. And that way the two can, can be at a specific level. So it's easier to address, sometimes, their concerns as pairs or as a group. Better than just individuals cause they have a tendency at least in that class they have a tendency to work better as pairs than have me working them

> individually. Cause some of the students haven't learned to overcome when you struggle. So it's still a learning process for them.

In Mr. S's interview, while describing why he always built in time for student to work alone and then work with a partner, Mr. S, without prompting, pointed out the human resource that students offer to each other:

> And it helps a lot. Like this girl right here [pointing to an empty seat in front of him]. She's one of my top students and she's one of my lower students [pointing to where another student sits]. But she's really good at explaining to her what's going on.

Mr. X stressed why he has students "pair up" and the positive consequences he had seen during that day's lesson,

> Yeah, and they can bring in their own experiences to add the conversation, to the question. Their own background knowledge. I think what really went well is that they were very focused. They were engaged. They were talking about it. They were motivated to get it right.

In Mr. X's comments it was evident that he valued student's "own experience" and their "background knowledge". He shared how he felt that the time allocated to partner processing time "engages" and "motivates" students to seek to understand.

Mrs. E is the teacher that asks students to turn to their partner and explain, "Where did Mrs. E confuse you?" The following comment followed the excerpt mentioned earlier where Mrs. E also described how she incorporated group work on a "daily basis" because if she were "forcing them to get into interactions with students" she was holding students "accountable and they're actually going to think about it and process what [she was] telling them to process." In response to a question my comment was "Um hum so that they aren't confused," Ms. E. forcefully said,

> Oh, no. I mean, it minimizes them [confusions], but it doesn't eradicate them, you know, there's still confusion, there's still kids who, especially my shy students who are not as outgoing as most of the class is, I have a few that are, that kind of keep to themselves or would rather work alone. Um, those kids, by forcing them to make appointments and forcing them to work with other people, it's still only that one-on-one interaction so, for the shy ones it works because there's only one person they have ask questions to, not raise their hand in the middle of this sea of students in the whole class. And admit that they don't understand something, you know, um, which I try to emphasize as much as possible that it's ok not to understand it, that it's ok that you have all these questions, that it's ok to be confused. It's still very intimidating to raise your hand in the middle of the class for my shy kids. So I feel that I'm supporting my shy students by pairing them up and you know, having these structured interactions, where they can have those conversations.

Both students and teachers noted without prompting that the investment in pair work to discuss and work alleviated many misconceptions. This practice also had the benefit of allowing students to fortify their depth of knowledge by explaining their thinking to others, build confidence in their learning, motivate and focus students on the material, and build a mutual responsibility for learning.

Pair work can be self-directed by each pair or can be structured by the teacher who guides how the pair will interact by assigning roles to students. In this study, an interesting version of the "practice of allowing students to work in pairs" was that several teachers employed both self-directed and structured pair work, that is times where students were allowed to manage their "pair time” as they wished and times where the teacher directed which partner was talking and the topic. The teachers varied these pair interactions throughout the lesson to maximize the quality of student interactions. Three of the four teachers created very specific routines that delineated student roles. They directed when processing time was individual and when it was with peers, as well as gave students the opportunity to work with different students in the class. Mrs. E clearly explained how complex problems were made manageable when students work in pairs when she said,

> So they have to understand how to match the graph and then how to complete the equations. Um, and there’s a lot going on. A lot of detail on those problems and so they have to have somebody else to talk it through and, you know, go over it with them. Um, but on a regular basis, I do pairing up a lot, and I use ‘Whiteboard drills’ a lot. I use ‘appointment books’ a lot, I use, um, the ‘coach and mathlete’ for the ‘sage and scribe’. Um, last, last class, with 6th period I tried the ‘hand up stand up pair up’ where they high five, and they liked it. It was the first time I tried it. I hadn't felt comfortable using it yet, but I thought, you know they've done ‘appointment books’, I think, this is very similar....And I always tell them, I always emphasize, make sure you pick somebody that's going to help you. You know, not somebody that's going to distract you….Again,[students] excited about the next time we do this so that motivates me to keep, keep them on their feet. But at the same time, that ensures me that they're thinking about the questions that I'm giving them. Because if I just give them. "Do number 5" They'll just sit there and wait until the notes, until they could just copy it from the notes and not just think about it. **But if I force them to think with somebody who is thinking about it,** then I have a better chance of all of them to understand the math.

The structured partner interactions that Mrs. E cited (“whiteboard drills”, appointment book”, coach and mathlete”, “hand-up stand-up pair-up”) in this excerpt have specific roles assigned for pair work, so that there is a clear sense of turn taking. For example in “whiteboard drills,” students pair up and work together on a problem, alternating writing out their work on an individual whiteboard tablet before time is called and all whiteboards have to be held up in the air for a “check”. Random teams are called to explain their work and the person who did not write is responsible for the explanation to the class. “Appointment book” is another way that pair work is structured so that students work with multiple students in a short period of time. Students make six appointments with six different students in the class. The teacher then assigns students to a certain appointment, for example “go to your appointment 3” and then gives those students a task to accomplish before she announces another appointment and students move and repeat the process. Mrs. E stated that changing the pairing “keeps them on their feet”. It allowed students the opportunities to work with their friends but also work with students they may not know very well.

Mr. S and Mrs. G also had very specific ways of structuring interactions with the “Switch” routine that was mentioned previously. Mrs. G commented on how she changes the seating chart often and gives students rules for who is to start the discussion such as “the person with the longest hair”, “the person on the left”, “the oldest person” and so forth. These variations ensured that students participated equally in interactions and students did not fall into the habit of letting their partner “take the lead”.

In interviews, teachers and students noted that having students work in pairs in general or structuring pair work allowed students to "help each other out", "ask questions", have aspects of the lesson "explained", "engage", "get comfortable", get over their "shyness", go beyond their own individual processing, and strengthen their own schema by incorporating the ideas of their peers, negotiating the ideas of other with their own, and clarifying their own understanding to others. The opportunity to work with peers exemplified Vygotsky's (1962) learning theory: students will learn most effectively when they are forced to think about a topic more deeply because they are accountable for talking about it. Participants noted that the process of talking about their understanding required students' depth of understanding to surface, while listening to another's ideas often provided an opportunity for a learner to add their own schema.

Partner Processing Time: Teacher allows the use of primary language in small groups or in class.

Gutierrez (2002) found that several strategies used by elementary and middle mathematics teachers and teachers of English Learners were effective with high school Latin@s in their high-level mathematics classrooms. Among these strategies were: having students work in groups and more specifically, allowing students to work in their primary language with partners. When students are allowed to use their primary language in the context of the mathematics classroom, researchers state that students can use their cultural linguistic resources to construct new schemas based on prior knowledge, providing a link between their preferred language of thinking and established prior knowledge and their the new content knowledge and language (Moschkovich, 1999; Sfard, 2001).

In this research, participants did demonstrate that they valued the practice of allowing students to use primary language in small groups to negotiate meaning of mathematical concepts because participants engaged frequently in this practice in class. This practice was less dependent on the teacher’s own talk, although two of the teachers were relatively fluent in Spanish; hence, primary language was predominantly a part of partner processing time. There were moments where the teacher did clarify concepts to the full class or to small groups in the students’ native language, and there were many more cases where the teacher was not able to clarify mathematics using a student's primary language due to lack of time or lack of fluency in the student’s primary language. Regardless of the teacher’s fluency with students’ primary languages, they allowed students to use their primary language with other students if they needed clarification or to negotiate thought. This was observed in every recorded lesson. There were cases when students who spoke other languages, aside from the dominant Spanish, were observed discussing mathematics in partners using their native language.

One of the teachers, Mrs. E, said during the interview that, “To speak in their own language to make sense of the content? Absolutely, cause then [otherwise] they might shut down and not talk about it at all so that's, you don't want to do that." This teacher realized that content understanding could and does occur in other languages. The teacher was giving students the opportunity to negotiate meaning using all their available resources, specifically their cultural resources.

During the second interview students were asked if they spoke another language and if so, if they used it in mathematics class. All students said Spanish although a couple said they were not very fluent and they overwhelmingly said that they "sometimes" used Spanish. This was expected as most of these students are now reclassified fluent English proficient (RFEP) and the three students who are still English learners are "reasonably fluent" and close to being reclassified. These questions were followed with, "should teachers allow students to use their other languages in groups?" They all agreed that it should be allowed. Jacob, a student, reflectively said,

> Um, well, if there like, if they only speak Spanish, then I guess that, it has a huge impact on them because they don't really understand what she's saying. They can only try to understand what she's like writing down. But um, for someone that can speak Spanish and English, I guess they can get a better, like they'll understand it the same, but I just think it'll be harder for um, for just um, people that only speak Spanish to understand this cause she's teaching the lesson in English not in Spanish.

Similarly, Nuvia discussed how there were a couple of students that her monolingual teacher tried to communicate with,

> Well, there's a couple [of students], but they don't really, well she tries explain it to them in Spanish when they like, talk to her, but at times, I don't know they have to learn right?

Nuvia's response to, "Should teachers allow students to use other languages?" was emphatically, "I don't know they have to learn right?" Although Jacob and Nuvia did not feel that they needed much primary language support, I would venture to say, based on their comments, that they did understand that language could become a barrier for some student's learning and that they understood that the primary goal was to learn the mathematics much more so than to exclusively speak in English. While Sarai explained how she worked with her partner during her interview, without prompting she voiced that her teacher Mr. S., "Like he lets us talk in Spanish."

In these excerpts, students noted some of the complexities that are involved in learning new content in a language that is not yet fully acquired. In the last case, the teacher was not fluent in Spanish, but does make efforts to communicate as best as she can with her limited Spanish. The teacher had placed the less proficient students with more fluent bilingual students so that during partner time they could negotiate their understanding and enhance their understanding of the content. For students, learning and being able to understand the content seemed to be the priority, not communicating in English. They understood that their grades were dependent on their mathematical knowledge and not necessarily on their level of English acquisition.

Yet students are asked to write in mathematics classes more and more as they transition to Common Core. This requires that students negotiate meaning in their native language and attempt to learn how to express their knowledge in English.

Three students and two teachers discussed at length the importance of allowing the use of primary language in partner time if needed or preferred by students. Some teachers were concerned about their own ability to explain the mathematics in students' primary language, leading them to prioritize partner processing time. For example, one teacher who was bilingual in Spanish and English was concerned about her other language speakers. Mrs. E explained her struggle,

> Umm, and see I have that a lot. I have students who speak but if, I feel bad for the kids who speak Tagalog over the kids who speak Spanish. I can speak Spanish so I can help them but the kids who speak Tagalog we have to struggle with the language and the content together but they're both being just as successful.

Mrs. E notes that that both Spanish and Tagalog students were "being just as successful". She attributes this to the pair processing time because she paired students with other students who speak their native language and who can explain both content and vocabulary. While observing partner processing time, I saw students interrupting their partner with clarifying questions or asking their partner to repeat something he or she had said in their native language. Such interruptions are neither customary nor acceptable during whole class discussions but were encouraged by the participating teachers during partner processing time.

Mr. S., a monolingual teacher, allowed students to use their primary language even though he was a little confused about the implications of Proposition 187 for instruction (in 1998, Proposition 187 outlawed formal bilingual instruction in favor of English-only instruction for English learners):

> **Mr. S:** I'm going to say that since the goal is math, I'm going to say, they should use whatever language while they're processing, but when they're talking to me only because it's the law. Then I make them practice English.
>
> **Interviewer:** Um hum
>
> **Mr. S:** I mean, I got to get them to learn the math so that's the most important thing but then you have the law
>
> **Interviewer:** What do you mean "the law"?
>
> **Mr. S.:** Well, isn't it Prop 187?

Proposition 187 actually decreed that instruction for English learners should be "overwhelmingly in English" (unless parents signed waivers for alternative programs), even as the teacher worried about student comprehension and access to the content. Even when teachers did not have the resources themselves to provide primary language or were misinformed about the role of primary language instruction in teaching and learning, they allowed students to use the cultural resources they needed and had at hand to try to gain access to the content by allowing students to process the intended learning together. Partner processing time became a time when students could discuss content in whatever language they deemed necessary for their own understanding. Therefore, students were able to fortify their understanding by employing

Vygotsky's theory of language and thought unconsciously. In this case, their understanding was transferring and extending in both languages.

Partner Processing Time: Teacher provides rules and expectations of how to respect other's ideas in groups, pairs and in class.

Jo Boaler (2008) states that "as the world becomes increasingly more globalized and communication across cultures an everyday part of life, schools need to renew their attention to the opportunities students receive to learn about effective communication, cultural appreciation, and respect." In order for partner processing time to have the maximized impact, she argues, students need to learn how to work together. This skill may not always be intuitive to people in general, much less adolescents who may be developing their own identities as people and scholars. Boaler's research is anchored in mathematics classrooms where students work in small groups (3 or 4 students) and pairs. Boaler argues that students must be taught how to "act equitably" in mathematics classrooms. That is: often, she argues, mathematics is seen as either right or wrong, but "equitable" interaction over mathematics allows varying view points and perspectives to be shared and valued. Boaler contends that students must learn how to respect each other, understand that mistakes are part of the learning process and further learning, and that alternative viewpoints deepen understanding.

Research like Boaler's suggests that if students felt safe to share their misconceptions, to understand alternative viewpoints, and to share their own perspectives, then the quality of student and partner's processing time would increase. Yet Boaler also notes that students don't automatically practice such respect; teachers may need to scaffold it.

For these reasons, observations and interviews in this research attended to how students interacted with each other; how they responded when they or their partner had misconceptions; whether students seemed able to incorporate alternative perspectives into their schema; and, whether and how teachers engaged varying student perspectives when students were discussing mathematics. All participants in this research cited as helpful to student learning moments when teachers *provided clear rules and expectations* on how to respect other's ideas in groups, pairs and in class. Such provision of expectations was evident in all participating classes.

If students cited pair work during the interview a follow-up question was asked: *Are there rules for how you work in the class or in pairs?* It was at these moments that participants noted the importance of teachers setting up norms for respectful dialogue. For example, Itzel emphasized,

> **Itzel:** You have to try your best. Well we can't be intellectual bullies. She has it on the board.
>
> **Interviewer:** What does academic bullies mean?
>
> **Itzel:** Um, kind of like, if you're explaining to them and you say something, like they just can't be like. "Oh, you're wrong". like, go off on you because you're wrong
>
> **Interviewer:** Um hum
>
> **Itzel:** That you cant like.. you have to be nice to each other and try to help each other and have to make sure we understand it

> and so just like… I don't know.. make sure listen and think about it not just your way.

Itzel was able to remember one of the rules that were on the board accurately. Her teacher in fact, did have three rules posted on the sideboard: 1) No intellectual bullies, 2) We don't make mistakes, we make adjustments. And 3) Be ready to contribute on your learning and to your classmates' understanding.

Diego similarly noted that his teacher would not "tolerate" student dialogue that was negative. As he put it in his interview,

> **Diego:** Well you have to be polite.
>
> **Interviewer:** What do you mean?
>
> **Diego:** Well like they can't be mean and say like "forget it your dumb". They have to try to help you.
>
> **Interviewer:** What would happen if you did say that?
>
> **Diego:** Oh, he wouldn't tolerate it. He would probably get picked on. Because he would think like, "Oh, so you're a know it all, so let's see. I'm going to pick on you to see if you can explain it to the whole class.

Both of these excerpts demonstrate the rules and expectations that teachers either explicitly or implicitly taught students about how they were to work together.

Sarai highlighted another benefit of trying to understand a peer's perspective. When asked how pair work helped her Sarai said,

> Yeah because most of the time. It is sometimes that we have the same doubt so we're like, "Well, so how can we help each other?" But sometimes it is that they get something that I don't and they try to explain me. Or the other way around, I explain it

> to them. Mr. S wants us to find as many different ways of explaining it.

In this case Sarai, highlighted that students were not seeking the “best” method or way to answer a question, but rather that her teacher valued and pushed students to understand and explain as many methods as possible. For this to be accomplished, students needed to truly seek to understand their partner’s perspectives. During observations, the participating teachers were heard saying, “What does your partner think?” “Does your partner understand it the way you do?” “What was your partner’s approach?” “Can you please share how your partner solved this problem?”

If teachers cited pair or group work during the interviews, I asked them why they thought pair or group work was important. Mr. S said that student perspectives “may be a big” idea in the future, noting,

> They have to learn how to respect each other. You know, respect each other's ideas, cause all those, they don't see the, see the relevance of their idea at that moment, but later on it may be a big idea.

Mrs. G similarly argued strongly that students needed to “take care of each other” and be responsible for each other’s understanding, pointing out that,

> Every time I explain, I'm making sure that they're understanding and that partners are taking care of each other. They have to understand how their partner is thinking about it too.

Both during observations and in the teacher interviews, teachers’ desire for students to assume a sense of responsibility for each other’s learning was evident. Teachers’ rules for partner interaction indicated that in order for students to be willing to learn from a peer, they needed to respect alternative view points, attempt to communicate clearly, and perhaps compare and contrast their perspective with others. These established rules and expectations deepened and maximized the partner processing time. Students not only gained clarity of their own understanding by having to explain it to a peer, but also reconciled other's perspectives with their own.

Partner Processing Time: Teacher uses student volunteers in class to explain to others in class.

Another emergent practice that all four teachers implemented after having students work in pairs was to ask volunteers to identify a problem that they may have been confused about and then ask a volunteer to explain the problem to the class while the teacher scribed what the student said, or have a volunteer come to the board and explain to the class how they solved the problem. This was not an explicit practice that was identified in research but it was an extension of having students discuss mathematics with peers and re-explain the mathematics content themselves. Four of the eight students specifically named the practice of having student volunteers explain to the class as being helpful. When I asked, why?, Diego explained his experience when his teacher "picks" on someone to explain.

> **Diego:** I'm actually following through the problem cause like, the first, I remember I was like his first victim. Like when the semester started. He just like, kept talking to me like I was trying to get him off and he was like, "No, I'm going to still pick on you. He made me read the DLT [Daily Learning Target], start the lesson, what do we need to take out and stuff like that and that's like… One gets nervous and like you think you don't know what you're doing but then once you finish, once you're finished with

> Mr. S and whatever the problem is, you look back to it and then your like, "Oh my god, like, I do get it.' It's just like the nervous pressure you have. But, I don't know, that's a good thing.
>
> **Interviewer**: Why?
>
> **Diego**: Like he cares
>
> **Interviewer:** That he cares?
>
> **Diego:** Yeah…of course. That he cares that you get it, learn it.

Nuvia said that she liked when her teacher asked students to volunteer because, "I don't think it's specifically about us. It's just like, she knows we're not talking our doubts you know. She's trying to in a way, forcefully "get your doubts out. Like in the open."

Mr. X excitedly explained how proud he was of his students, saying,

> I saw students coming up to the board. I saw David go up there and try a problem that even though he knew was probably going to be wrong. I like the fact that Janise helped him. And one student, I guess, had the same question and I had just helped one of them. And Davie "Go be the engineer and explain that". To help her, understand even better, she can explain to somebody else, which is ideal. So if they can coach themselves, that's where the learning takes on.

When asked why she had one of the students come up to the board Mrs. E voices her belief by saying,

> I can explain it everyday all I want but if they don't feel like they can teach it to others to the class then they don't know it, you know.

Participants indicated that the practice of using student volunteers to explain to the class also was demonstrative of student accountability. That is to say, these four teachers created a learning community in their classrooms where all students were responsible for the learning outcomes. Students were responsible to each other and to the class as a whole. Using volunteers to explain to the class served two purposes: 1) to voice a concern or misconception that requires attention after partner work, with the help of the students in the class, 2) to validate the work and thinking that took place in the pair work and allow students to share their ideas and perspective with the class and not just a partner. Using volunteers to explain was a student-to-student interaction which the teacher guided from the sidelines.

Partner Processing Time: Teacher tries to ensure understanding prior to moving on.

At different points of the lesson participating teachers would pause to ask students to turn to a partner and re-iterate what they had just learned. This was different from students trying to apply what they had just learned in a similar mathematics problem. Mrs. E would say, “Turn to your partner and say [Mrs. E] confused me when….” Mrs. G would say, “ Explain to your partner what we just did.” After giving students time to process and solve problems in pairs, Mr. X would say, “Are you still lost? Where? *Vamos* [come on] Give it to me?” This emergent practice of “ensuring learning before moving on” is related to the teacher-student talk practice of being willing to explain well and re-explain, but it goes beyond teacher re-explanation to asking *students* to assess their understanding and identify specifically what aspect was still confusing. To address any lingering confusion, teachers most often had volunteers re-explain and at times it was the teacher who was fine tuning his or her own explanation. Additionally, even after “student processing time” and “partner processing time”, teachers would ask if there were still any questions. Participants indicated that this practice allowed students to once again assess their understanding and request further clarification if the processing times did not address the confusion. Students made comments that highlighted how they felt that their teachers were willing to spend time to explain as many times as was necessary for learning to be achieved. This is not to say that students never left a lesson confused or that learning for each student was achieved but rather that the participating teachers “checked-in” with the class multiple times to see if there were misunderstandings they could clarify. This is different from answering questions that students took the initiative to ask. This practice also created pause points where students were asked to process together on what they had just learned.

Students cited this practice as being often helpful. For example, Jacob underscored how often his teacher explained however, his teacher, in observations, was seen answering student questions by having volunteers explain,

> I think that in this math class, it should be easier for students to comprehend what they're learning because um, the way she teaches, step by step, and she keeps on going over it again and again, and she'll give you time required and if you have questions. It's just like, anything you need help with, she'll be there and she'll help you out.

Jacob valued that his teacher did not let a question go unanswered. His teacher was willing to explain, but once students had pair work time, she often had students become the leads.

Sarai highlighted,

> I mean he makes sure. He asks us "May I proceed?" If we say no he helps. Yeah, he said a couple of times like, "May I proceed". He asked us for permission. If we say no he will explain it again or have a volunteer show where they are confused. That helps me a lot.

These two excerpts also signal the overarching growth mindset that the participating teachers demonstrated. Based on students' comments, I would argue that they feel that learning is a process and that learning how to solve a complex mathematics problem requires multiple opportunities to truly learn it. This is how teacher-student talk linked to partner processing time and individual processing time: teachers conveyed that learning required modeling by the teacher (i.e. teacher-student talk), time for students to process on their own, time for students to process with peers, and lastly time to for students to process as a class. Other than teacher explanation, all these components were necessary aspects of "processing" time and the teachers would not move forward with the lesson until they felt that enough "processing" had occurred to ensure understanding.

As Mr. X accentuated,

> We go back and I give them a new um *como se dice*[how do you say].. another refinish if you will, just to make sure that it's glossy. Another coat of paint. Another coat of knowledge. I always do that. Till I feel satisfied. Till, I see them getting more

> "Come on, ok, let's divide a trinomial in another way.. Ohhhh, I get it". I'm going to take it to that level.

Teachers in this study felt a sense of duty to give students multiple opportunities to process the learning and to achieve the learning objectives. They were patient with students when they did not understand and were willing to repeat themselves by restating what they were teaching in another way. Often they asked for permission to continue the lesson or the day's planned activities. They also conveyed a sense that students would ultimately understand the material that they were capable and that their learning was important enough to stop and make adjustments to their teaching.

Conclusion

Site administrators identified the four teachers selected in this study as "good" or "effective" teachers. Perhaps the key quality that administrators used to classify these four teachers as "good" and "effective" was their ability to embed multiple opportunities for students to think and talk about mathematics when organizing their content lessons and their time with students. Teachers in this study employed various practices to give students the time and opportunity to process learning and see another perspective on solving mathematics problems. Interactions with the teacher---Teacher-Student Talk, interaction time with self---Student Processing Time, and interactions with peer and in the whole class --Partner Processing Time, demonstrate how these teachers set up the process of learning with multiple opportunities to grow their knowledge and understanding.

By establishing these three key forms of practice, teachers conveyed their overarching beliefs about supporting student ability, mindset, and expectations. The four participating teachers conveyed a message to students that learning took time and was multi-faceted. By providing multiple opportunities and mechanisms for students to think and talk mathematics until they “got it,” teachers conveyed a learning mindset in which knowledge can be fostered and grown. Through the practices they employed, they expressed that learning took time to process, moments of confusion and disequilibrium were often inherent to learning, and that understanding required repetition and communication with various members of the class community. These teachers conveyed the notion that “we can all learn, we can all support each other, and we can all succeed together”.

“Effective” teaching may require that a relationship between teacher and students is flexible and that a teacher has established routines, yet are able to be flexible enough to adjust to student needs. Students in this study felt that their teachers listened to them, gave them multiple opportunities to process what they were learning, and that all perspectives, resources and tools were allowed if they helped them gain access to the mathematics. They did not hesitate to discuss what helped them learn and why. All of these practices were anchored in and required an overarching teacher mindset that encourages student to share with peers, the class, and their teacher their misconceptions, their confusions, and their perspectives. Ultimately, it is teachers who organize a learning environment that scaffolds and supports student to reflect on their learning and evaluate the depth of their understanding. These four teachers organized their day so that students walked away from their mathematics classes knowing that they could succeed, that tomorrow they would “get another coat of knowledge,” and that they had a team of peers and a coach who would help them succeed because they were willing to think and talk about mathematics until they all got it.

Chapter 5: Findings and Discussion

This purpose of this study was to analyze the practices that Latin@ students and teachers identify as supportive to everyday learning experiences in high school mathematics classrooms. Based on a review of literature, a study was designed to investigate what practices “good” and “effective” teachers of Latin@ students employed on a regular basis and how the practices articulated as helpful by students and teachers compared to those identified by research. This study is anchored in a framework emphasizing everyday action for equity (Pollock, 2008, 2008b, forthcoming), a Mathematics Equity framework (Gutierrez, 2007; Gutstein, 2003; Moses &Cobb, 2001; Schoenfeld, 2002) and the *testimonios* tenet of Latino/a Critical Pedagogy, which calls for listening to student voices particularly.

Specifically, this study sought to answer the following research questions:

1. What practices do teachers intentionally build into everyday Integrated Mathematics classroom experiences that they believe promote achievement and access to rigorous mathematics curriculum for Latin@ students?
2. After a typical everyday classroom experience, what teaching practices do high school students in Integrated Mathematics identify as supporting their learning and what recommendations do they offer for improving their mathematics instruction?

3. How do students' identified practices compare to teachers' intended practices?
4. How do both students' and teachers' perspectives on beneficial mathematics practices compare to those identified in prior research?

The participating mathematics classes included four high school teachers with large populations of Latin@ students and with significant populations of English learners (Table 2 in Chapter 3). The four participating high school teachers were purposefully selected through the recommendation of their site principal based their criteria and identification as a "good teacher." All of the participating teachers taught Integrated Mathematics II or Integrated Mathematics III, courses required with a satisfactory grade by the UC and CSU systems in order to qualify for admission. Of the participant teacher group, two were bilingual and two were monolingual, two were female, and all four had taught for over twelve years each. Additionally, a female and male Latin@ student from each teacher who are current or former English learners were asked to participate in the study. All eight students had a home language of Spanish and ranged from six to thirteen years of schooling in California schools according to district records.

The four teacher participants and eight Latin@ student participants cited practices that they deemed as helpful in furthering learning and understanding of higher-level rigorous mathematics topics in semi-structured interviews. From these citations, categories emerged that analytically organized the practices cited by participants. I have organized both a priori (research-expected) and emergent (named by participants) teacher practices under four categories (below) based on how participants discussed practices. In addition, I have created a category labeled "Issues practices must overcome in learning" to organize the types of *barriers* students and teachers identified. Participant commentary on these barriers was important data, as it clarified the struggles necessitating the practices they then identified as important.

As previously mentioned, practices cited as important to student learning were categorized as:

a. Overarching Growth Mindset

b. Teacher-Student Talk

c. Student Processing Time

d. Partner Processing Time

These four emerging categories were built on participants' identification of learning practices advantageous to the teaching process.

After all the qualitative data was collected through semi-structured interviews, the predominant source of data in this study, I asked participants to rate in an oral survey all practices identified in the literature review of this dissertation and established by research. There was a possibility that practices identified by research would not surface in interviews even while participants actually found them important, so an oral survey was added to the study to capture this specific information. Here, I wanted to know how participants' perspectives on beneficial practices explicitly compared to those established in research. I share that final data source below.

Findings and Discussion

Practices teachers intentionally build into everyday mathematics classroom experiences to help students access rigorous mathematics and promote achievement.

During interviews, as seen in the previous chapter, teachers spoke about their lesson objectives and goals. They described what considerations were taken in order to plan their lessons. They also described what surprised them about the lesson, what went well, and what were the next steps. From their responses and those of students, the previously mentioned four categories emerged. Chapter 4 provided 106 pages of qualitative data that details what teachers and students said about each practice. In interviews, participants named 19 practices; 14 of those practices were practices that research has already said are important to marginalized students' learning. Usefully, however, participants described such practices in more detail than had prior research, fleshing out *how* to "engage students in rigorous mathematics" or "provide access to mathematics."

Participants also named 5 additional new practices as important that research had not addressed as much. These were "schema building", "ensuring understanding", "using volunteers", "having multiple opportunities to practice/solve math problems" and "teacher anticipating student difficulty."

The qualitative data provided by the participant quotes are the most important data provided in this study. as they offer details of what participants said was beneficial about each practice and to learning overall. Yet it is important to understand how *each practice tallies up quantitatively*, that is the number of times a practice was named and emphasized. Transcripts were coded with *a priori* codes (those practices previously identified by research to provide access to mathematics for marginalized groups of students) or *emergent* practice codes (those practices named by participants in this study that were not mentioned by research often). A tally was made when a *full thought* pertaining that practice was evident in the transcribed interviews. I then tried to *quantify the qualitative results* in the graphs that follow by quantifying *'mentions'* of practices, but I still feel the pure qualitative data was the most exciting. The graphs that follow were generated by counting the number of times a practice or category was mentioned and then dividing that count by the total number of codes within that category or by the total number of codes applied in the study. Often a participant would name two practices in one statement and I would code that as an example of both practices being named. This "double counting" still allowed me to name trends in the data overall. Therefore, the graphs represent the practices that were mentioned the most. This is one way of analyzing the emphasis participants placed on the specific elements that were supportive to learning. Emphasis participants placed on practices when they named them also contributed to my understanding of trends in the data. This dissertation is not about specific percentages offered in each of the following graphs but rather about the general trends in the practices that people call important. Overall both student and teacher participants were aligned in what they cited as being supportive of learning.

The coding of teacher interviews revealed an overwhelming focus on Overarching Growth Mindset. When evaluating all the comments that were coded in transcripts, I found that teachers cited practices relating to Overarching Growth Mindset practices 34% of the time during interviews as seen in Figure 2.

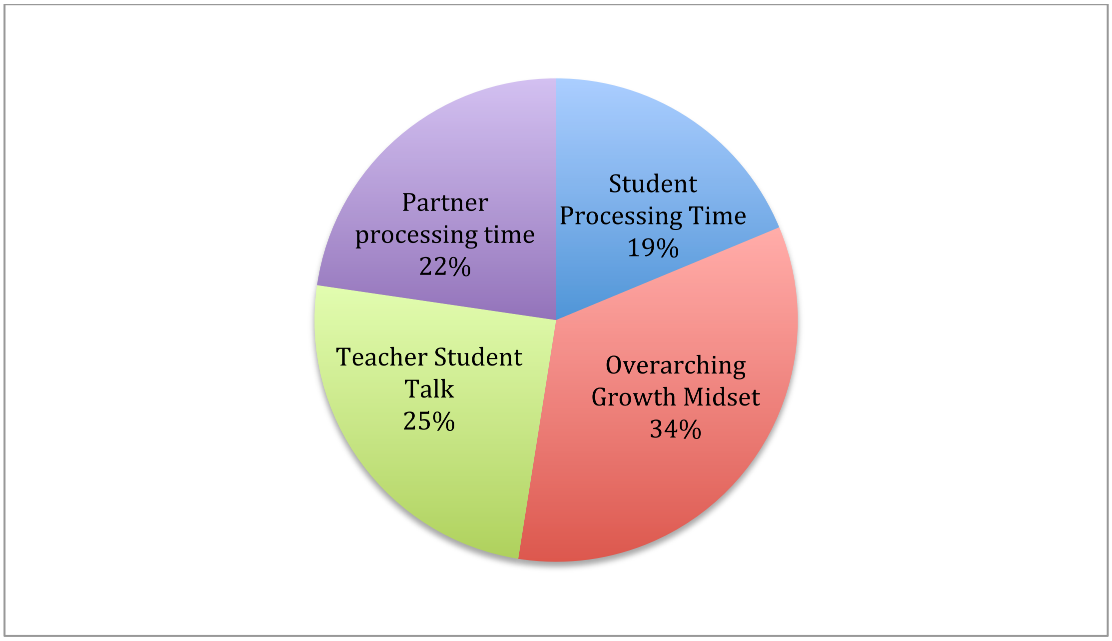


Figure 2: Teacher comments on categories of practices

The “Overarching Growth Mindset” category held the practices most heavily cited by teachers. Teachers overwhelmingly expressed that they anticipated that learning would take time and that their students were capable of learning and succeeding in mathematics. Their interview comments reflected a perspective of “we can do this together.” During observations, all four teachers made comments to students such as, “we don’t make mistakes we make adjustments”, ”keep trying”,” you have to explain your thinking”, “Come in! Beat up that problem. You got this!”, “Look carefully at how you are solving the problem, make sure it makes sense to your partner,” that were demonstrative of a growth mindset. These perspectives were re-iterated in interviews as previously mentioned in Chapter 4, when teachers said, for example,

> But my job is to make it as easy as possible. Using patterns, using whatever strategies I can, you know. And yeah, they do get frustrated. That's why I say, ‘Look, don't worry. If you're understanding half *Asi, ya se les quita de eso de anciedad* [Like that, they get rid of that anxiety]. You understand half. We're good. Keep going, *si puedes* [you can do it].

When discussing growth mindset practices, teachers identified the emergent practice of “anticipates area of student difficulty” to address student difficulty in the learning process and described how the planning of the lessons was orchestrated so that the sequence of knowledge grown facilitated learning. As noted by Mrs. G,

> I wanted them to understand what that was and why we were writing a perfect square trinomial so that they could use their Algebra skills because I think a lot of them, they don't understand, they can follow the steps when I give it to them, and I don't mind giving them the steps, but I want them to understand why we’re doing the steps. So that was one of the things I had them do when I got them into groups, to factor only and notice a pattern, the pattern in the factoring, how it's a perfect square trinomial, and why the perfect square trinomials were special in, in helping us complete the square, solving them by completing the square. I wanted them to see it.

What was clear was that to teacher participants, *employing practices related to growth mindset were precursors to employing practices in the other three categories*. Teachers had an asset-based perspective when discussing the negotiation of teaching and learning. When barriers to learning were discussed, the four teachers discussed many different strategies they employed on a day-to-day basis in order to counter any pitfalls in learning. As one teacher put it, they placed “nets” in the learning process to help all their students succeed. Figure 3 represents the frequency of comments coded with Overarching Growth Mindset Practices.

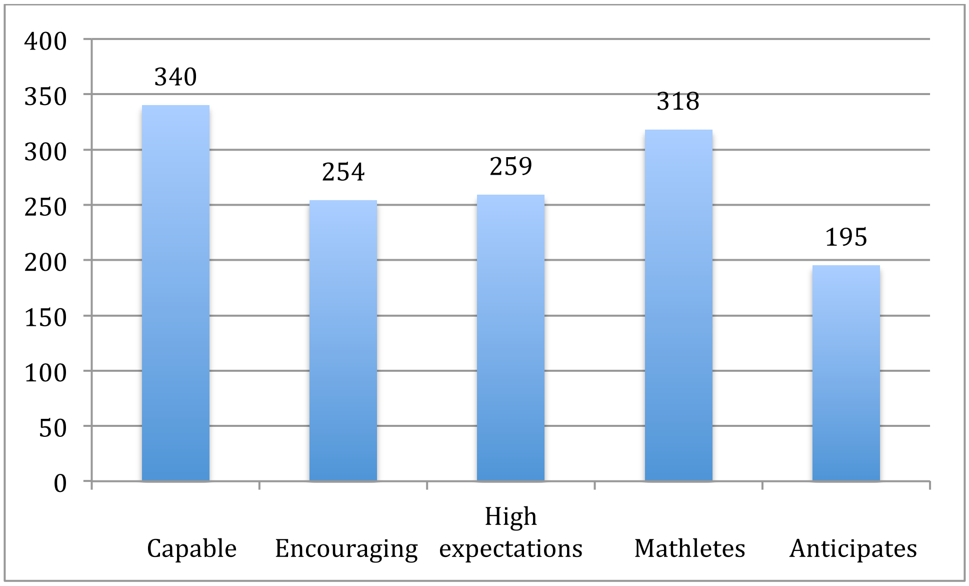


Figure 3: Overarching growth mindset- Comments on practices by teachers

As can be seen in Figure 3, within this category the practices all seemed to be beneficial precursors in order for students to learn, according to teacher comments. “Anticipating difficulty” was an emerging practice in this category (one named by participants, more than researchers) and one that teachers spoke about.

Teachers then cited many more specific practices that they employed while teaching a new topic that they felt would enhance learning. I categorize these practices as “Teacher-Student Talk”. In Figure 4 the distribution of practices cited by teachers can be noted.

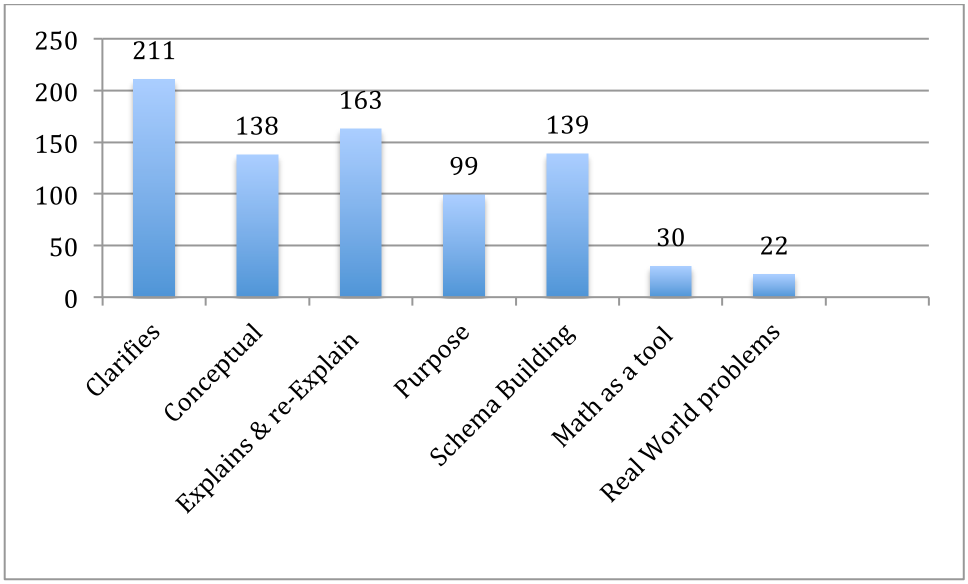


Figure 4: Teacher-student talk- Comments on practices by teachers

Teachers considered many practices when teaching. Within this category of "Teacher- Student Talk" it is evident that teachers were most concerned with how they would explain the material. As would be expected, teacher comments during interviews focused on aspects of the delivery of content during teacher talk. During the comments certain practices were emphasized over others. For example, their comments focused on their explanations and re-explanations of the content, building conceptual understanding, and clarifying misconceptions that were tied to other skills including pre-requisite skills. They identified the emergent practice of schema building as an essential component of their teacher-student talk where they made sure that they linked the new knowledge to pre-requisite knowledge.

It can also be noted that teachers minimally commented on how what they were teaching would fit with real-world applications (reference to), and on the research-expected practice of using mathematics as a tool for the real world (application of). Researchers have noted that these two practices of making connections to real- world applications (Cahnmann & Remillard, 2002; Gutstein, 2003; Schoenfeld, 2002) and using mathematics as a tool (Cahnmann & Remillard, 2002; Gutierrez, Willey, & Khisty, 2011; Gutstein, 2003) promote access to rigorous mathematics and enhance students' academic identity (Gutierrez & Irving, 2012). Researchers deem these practices important because they enable students to tie their in-class learning with contexts that extend the classroom walls. While teachers did not proactively name these practices as important in interviews, the primary source of data for this study, comments during the oral survey portion of the study provided additional information as you will see later in this chapter.

Teacher comments also pinpointed the importance of building in time for students to process what they were learning and giving students time to solve problems in class. As can be seen in Figure 5, teachers were concerned about building in time for students to process the content of what they were teaching.

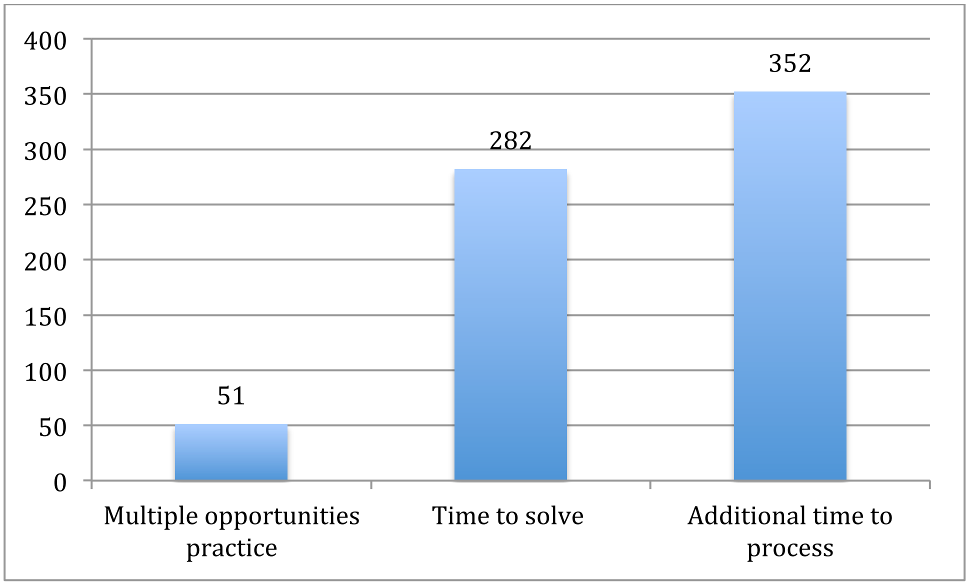


Figure 5: Student processing time- Comments on practices by teachers

Although teachers often asked and intended students to solve mathematics problems together, during interviews students stated that they usually needed to attempt to solve the problem on their own first. For this reason, time structured for individuals to solve mathematics problems was a practice placed in the category I came to call, "Student Processing Time".

Partner processing time was cited heavily and was observed as an integral aspect of every lesson observed. Teachers overwhelmingly cited pair work as a practice that led to ensuring understanding and that minimized the number of confusions. As can be seen in Figure 6, teachers considered the time they allocated to partners as beneficial and as an additional opportunity to embed many other practices.

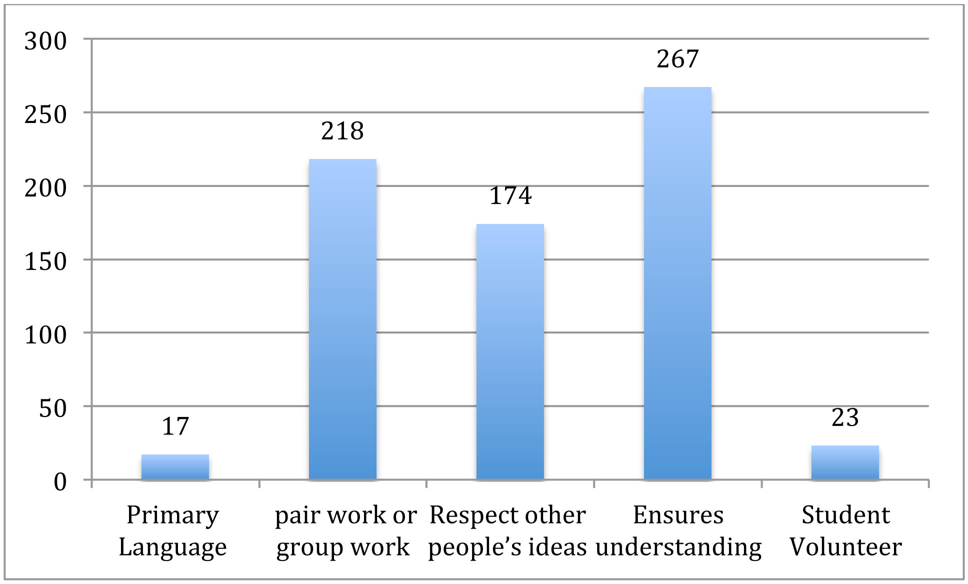


Figure 6: Pair processing time- Comments on practices by teachers

Furthermore, a by-product of "partner processing time" was the opportunity to teach students to listen to other ideas, to have the opportunity to explain their own thinking and gain clarity of their understanding, and the opportunity to be responsible for each other's learning. Teachers cited the emergent practice of "ensuring understanding" as a major goal of partner processing time. They felt that students needed this additional time to ensure that students were held accountable for thinking about the new content and also commented that by offering time for students to work on pairs they were minimizing misconceptions and misunderstanding. In order to maximize the quality of group work and harvest the most conducive environment for student learning and understanding, all four teachers had both implicit and explicit rules for how students were to work together. Some teachers had rules posted on the wall that stated "No intellectual bullies" while other teachers communicated that same expectation with consequences and other statements. Students were expected to seek out multiple methods of thinking for any and all mathematics concepts. Variations in approaches to solving mathematics were encouraged and valued.

As can be noted, the participating teachers in this study are thinking about much more than the content of the subject when planning their lessons. They are considering how they will motivate students to see themselves as capable, how they will explain the material, and orchestrate opportunities for students to process their learning alone and with peers so that students' own individual understanding is enhanced. They plan and incorporate practices that they feel help students understand the content of rigorous and often very abstract mathematical concepts. Teacher participants were able to identify emergent practices not noted in the research on supporting Latin@ students mathematically, mainly, "anticipating areas of difficulty", "the use of volunteers", "multiple opportunities to practice and build mastery", and "ensuring understanding prior to moving on". They recognize that learning is a demanding process and that students need multiple interactions and multiple opportunities to process in order for learning and understanding to be achieved. Mrs. E's final comment in her second interview was,

> It's kind of like, throwing a net out, you know, and trying to see, this will work for some kids but I need to have something else so that I can catch the other ones and so there's um, there's that differentiating piece in these strategies that I use um, but, I think we caught, we you know, we caught everything in the questions you asked me.

Teachers realize that each student comes to class with varying understanding of pre-requisite skills, different experiences and different views about mathematics learning. This requires maximizing the resources that they have, and it is evident that one of the biggest resources teachers value is the human capital that sits in desks everyday. As one teacher said, "Cada cabeza es un mundo," an idiom in Spanish which translate to, "Within every brain there is a world". These teachers structure class time so that student had the opportunity to interact and communicate with each other while still respecting their perspectives, ideas, and starting points. They exemplify that "all learners can learn."

Of course, cause and effect studies of each of these practices on students' overall outcomes (particularly, their overall understanding of required mathematics concepts) could be useful in future work. This study attempted to understand the varying day-to-day elements that participants say supported learning. Thus, this dissertation sought to understand which practices in "effective" and "good" teachers' classrooms, participants articulated and *emphasized* as particularly important to that success. Yet, as shown in Table 3, with just one exception, students' grades were in fact higher than in the immediately prior mathematics course after a semester in the participating teachers' classes. I offer this data purely as context for consideration.

Table 3: Participating Student Semester Academic Results

Student	Gender	Grade level	Language Proficiency Status	Semester Grade	EOC Grade	Performance Task
1	F	9	RFEP	B+	83.3	4
2	M	9	EL	A	85.7	4
3	M	12	RFEP	A	78	3
4	F	12	EL	C	63.33	3
5	F	12	RFEP	B-	73.3	3
6	M	10	RFEP	B+	82	3
7	M	9	RFEP	A	88.7	3
8	F	10	EL	D+	33.3	2

Overall end-of-course (EOC) exam scores also show that each student achieved a level of mastery on the district benchmark and scored well on the performance task (PT), where scores from 1-4 reference a quartile grade i.e. 1-0-25%, 2-26%-50%, etc. The Integrated Mathematics II and Integrated Mathematics III Performance Tasks and scoring rubrics are shown in Appendix D and E. These scores, along with student commentary during interviews, supported the notion that the intended practices were potentially advantageous to learning for the participating Latin@ students.

Teachers in this study were cognizant of student needs in order to gain access to the high-level rigorous mathematics content they were teaching. They recognized that students needed to have a procedural and conceptual understanding and integrated many practices to promote student learning and achievement. They also realized that students face varying barriers while attempting to learn abstract concepts. They broadcast that “all learners can learn” with the practices they instituted in their everyday lessons. They threw out “nets” and they gave several “coats of knowledge” every day to help students see themselves as able to succeed in mathematics and as “Mathletes”.

Teaching practices that Latin@ high school students in mathematics identify as supporting their learning.

Categories of practices cited by students in interviews were relatively evenly distributed. Students did comment slightly more than teachers did on partner processing time as helpful to their learning, as can be noted in Figure 7.

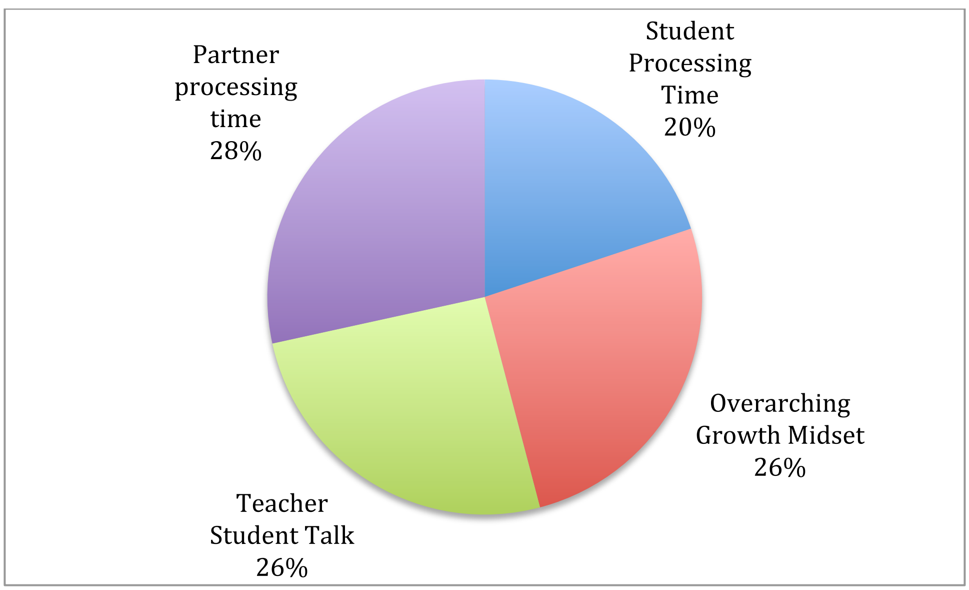


Figure 7: Categories of practices cited by students

Students cited many practices that teachers employed while teaching a new topic that were beneficial to their learning. As can be observed in Figure

8, in discussing what I've called "Overarching Growth Mindset" practices, students made an overwhelming amount of comments about how their teacher made them feel "capable of learning".

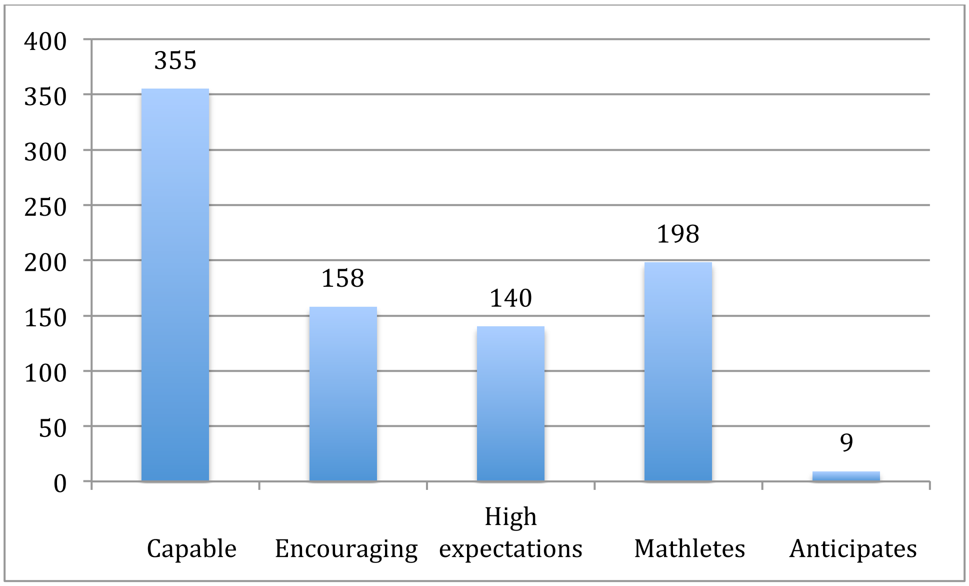


Figure 8: Overarching growth mindset- Practices student cited

In student interviews, participants also cited that they felt that their teachers felt they were able to succeed in mathematics; a practice that a teacher in this study calls treating students as "Mathletes." As mentioned in Chapter 4, student comments during interviews were informative about the affective practices that teachers employed to create a classroom environment conducive to learning. They described how their teachers required them to "explain your thinking", or seek agreement or understanding from peers. Students also emphasized that their teacher held high expectations and "does not let students off the hook because he believes that his students can learn" but also that they encouraged them "to ask questions cause he doesn't want us to like fail". Students underscored the positive effect of overarching mindset practices throughout their interviews. They were able to cite their teacher's words and actions throughout the interview process.

Students also cited the "Teacher-Student Talk" category as helpful to their learning 26% of the time in interviews. As Figure 9 demonstrates, students cited two practices over and over in this category as the most helpful.

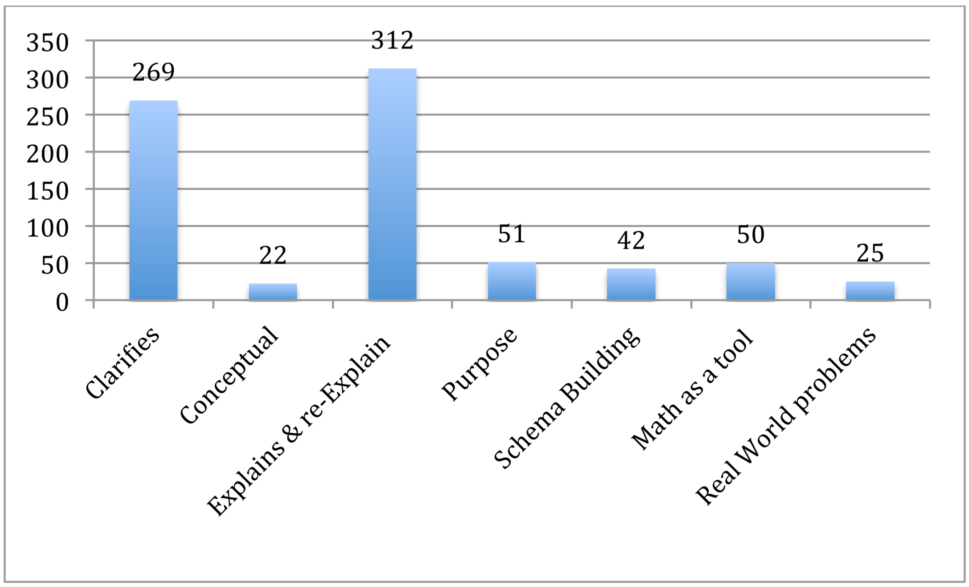


Figure 9: Teacher-student talk- Practices cited by students

The first was a teacher's willingness to clarify if they misunderstood (teacher clarifies confusing material to students by connecting the new learning to pre-requisite skills; or teachers reteach or remind students of a prerequisite skill that they needed to utilize to apply the new learning). Students were emotional as they compared their previous learning experiences to those in their current class. They discussed how their teachers would answer any questions they had and were very patient. In their previous experiences, some teachers discouraged some of their approaches towards problem solving and stated that the approaches were “kindergarten stuff “that they “shouldn't still be doing” so they would just “move on”.

The second practice students also cited as very helpful to their learning was a teacher’s willingness to explain well and re-explain multiple times.

> But the way he explains it, he explains it like,.. like it should be easy, and the we, we should like, we'll be able to get it, so like, it doesn't, it makes things, he makes it seem like it's easy, it's easier for me to do it. So like, I don't go into it thinking like, this is going to be really hard.

As noted in the data from teachers’ intended practices, teachers stressed how they planned lessons and anticipated misconceptions so that students would more easily see connections. This was an emergent practice that translated directly to the orchestrated discussions and talk that teachers provided in class. Students noted how their teacher’s explanations led to understanding and how patient their teachers’ were during re-explaining.

Additionally, student participants also accentuated the value of the practice “providing additional time to process” material individually and the practice of “time to solve mathematics problems” as the most beneficial practices within the category of “Student Processing Time”. Figure 10 shows the distribution of practices students cited as helpful for their own processing time.

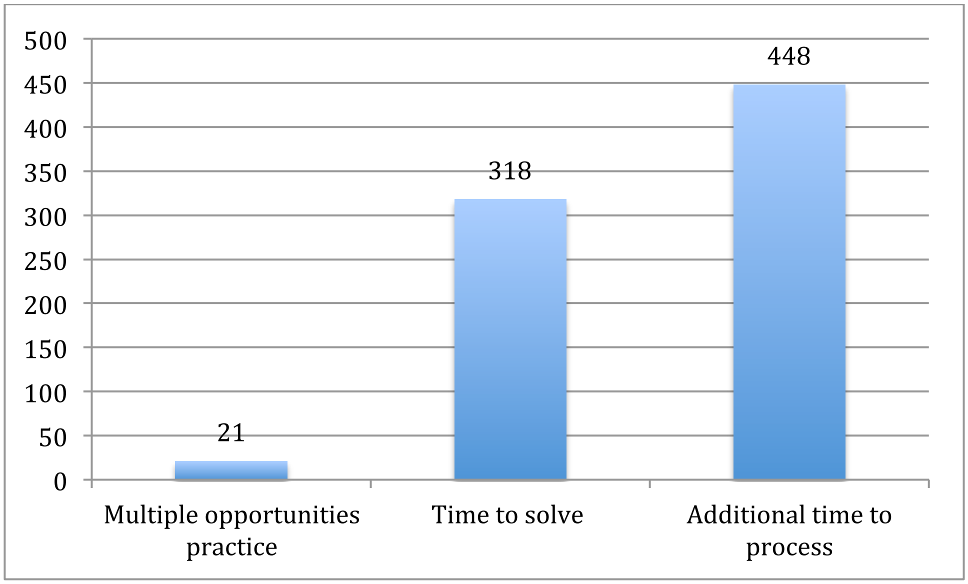


Figure 10: Student processing time- Practices cited by students

Researchers (Gutierrez, Willey, & Khisty, 2011; Gutstein, 2003) have cited as beneficial practices of "allowing students to engage in mathematics" in pairs where students solve open-ended problems and high challenge problems, using peers as resources to find valid strategies for problem solving. This practice is said to provide access to mathematical learning and strengthen students' academic identities. Although teachers did allow and encourage multiple approaches to problem solving, there was always one right answer. New Common Core Mathematics standards encourage teachers to also include mathematics problems that may have multiple correct responses with a valid justification (National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010). This last aspect of open-ended problems was not observed and therefore not referenced, most likely because these type of problems are not usually included in every single lesson and because the district and schools have yet not received the new Common Core aligned curriculum. These problems do exist as a portion of the curriculum in the form of performance tasks discussed earlier and shown Appendix D and E.

Students seemed to separate the larger practice of “processing time” into time to solve problems first on their own and then discuss strategies for problem solving with their peers. In other words, students valued the opportunity of practicing and applying what they had just learned during class time both on their own first and in interactions with their peers second. Several students contrasted their past experiences where the class time was primarily dedicated to the teacher teaching as less valuable. Students highlighted the two individually-focused practices of “additional time to process” and “time to solve” as very helpful so they could individually process what they had learned and apply what they had learned. Even when students were not given individual processing time, they took time from their "partner processing time" to do so. In addition students also highlighted the "practice of repetition"; that is having multiple opportunities to practice individually and attempt to show mastery as valuable. Practices in this category were cited as helpful by students; 24% of the comments in interviews addressed practices in this category.

The eight Latin@ students interviewed particularly valued the opportunity to work with partners on rigorous mathematics and cited this as helping them deepen their understanding of abstract and rigorous mathematics. I came to call this "Partner Processing Time". Practices in the category of “Partner Processing Time” were only slightly more cited than those in the other three categories (Figure 6), “Overarching Growth Mindset”, “Teacher-Student Talk” and “Student Processing Time”, as assisting their learning and understanding of the material. The specific practices that teachers integrated into their lesson and comprise this category can be observed in Figure 11.

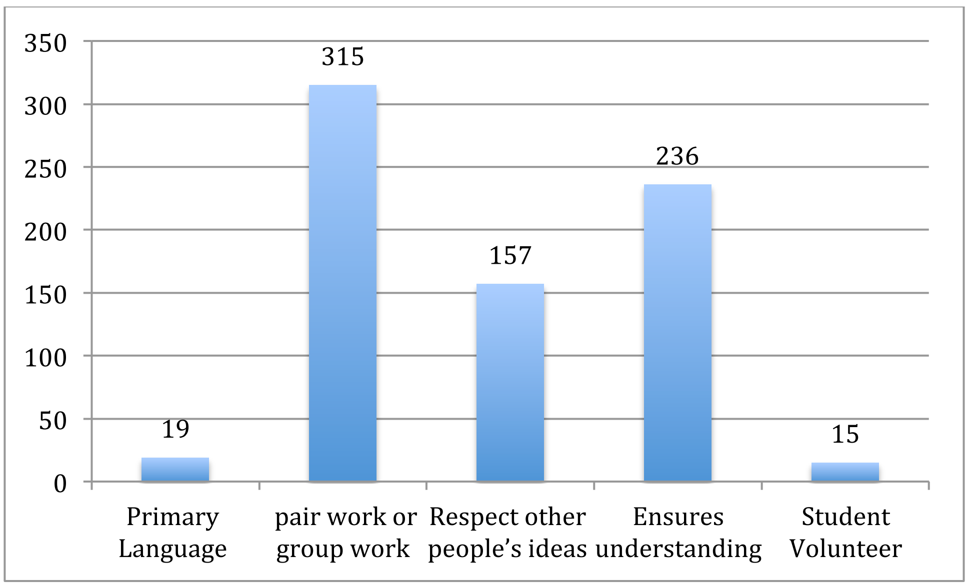


Figure 11: Partner processing time- Practices cited by students.

All eight students cited the practice of working with a peer(s) as helpful and “how” or “why” it helped. They explained that the practice assisted their learning because it helped them clarify their understanding, overcome confusion, understand a different perspective, or verify their thinking. This practice built confidence in their ability to grasp rigorous concepts. Research has identified the practices in this category as effective, but students were much more specific on “how” or “why” the practices helped than what was stated in the research; research tends to argue only that partner time is necessary. Students made statements such as: “I am confident that my partner will teach me the stuff I don't know and I will teach him what he doesn't know”, “it’s easier when we are working together”, and,

> Like it affects my learning in a good way cause we help each other out and everybody at the end of the question gets it and they're like, "OH, ok", and “they have to explain it to me and I have to listen to them or like if they say something wrong like you have to catch it to make sure they understand it.

These statements highlight how listening and speaking about what they and their peers understand helped to clarify their own understanding. In their statements, students essentially verified Vygotsky’s (1962) theory regarding the relationship between language and thought, as the opportunity to discuss what they knew actually fortified their own understanding. Relatedly, all eight student participants were current “reasonably fluent” English learners or were former English learners. In their interviews, they shared that they did not feel that they needed Spanish for their own learning, although most said they “sometimes” spoke Spanish in mathematics class in their pair work; but they all recognized that for the many students who primarily spoke Spanish and were in their mathematics class, being able to speak in Spanish was imperative to their learning and understanding of mathematical content.

The four categories of practices, Overarching Growth Mindset”, “Teacher-Student Talk”, “Student Processing Time” and “Partner Processing Time”, were relatively evenly distributed in the data.   Within those categories, students overwhelmingly cited practices that they said helped them feel capable of accessing the mathematical content and practices that they said helped them process the learning and self-assess their understanding. They clearly identified their teachers’ explanations, additional time to process, time to solve problems, and time to discuss mathematics with peers. References to processing time practices as a whole, both individual-“student processing time” and in partners- “partner processing time”, accounted for over 50% of comments students made, signifying how imperative they said these practices were to their learning and understanding. They continually referred back to those moments of processing as the most beneficial to their understanding of complex and abstract mathematics topics. So although it was extremely important that teachers “explained”, “schema built”, “built conceptual understanding”, students said they were able to ask the right questions to help them delve deeper into their understanding because processing time practices were in place. It is possible that because students knew that they were going to have to apply their knowledge and explain it to a partner, they were more invested in asking clarifying questions during the “Teacher-Student Talk”. Students said that these everyday practices encouraged and motivated students to say engaged in the lesson and it provided a safe environment to learn.

As can be noted from the graphs above, there was a strong alignment among students in terms of what practices were most heavily emphasized. In interviews, student voluntarily cited practices that were also aligned to those identified by research -- with a few exceptions. Student participants were able to identify more specific practices not noted in research on supporting Latin@ students, mainly, the “use of volunteers to explain”, “multiple opportunities to practice and build mastery”, and “ensuring understanding prior to moving on”. Student participants also were able to cite the need for “individual time” to solve problems before they worked with their peers, where research on supporting Latin@s only highlights the benefits of pair and group time. They discussed the important role this time to solve problems played in their individual processing time and learning.

Latin@ students' assessed practices compared to teachers' intended practices.

The data presented below was normalized by comparing percentages rather than counts so that data between the four participating teachers and eight participating Latin@ students could be compared. Looking at the four categories (Figure 12) the distribution of emphasis in individual interviews is clearly comparable. Teachers cited “overarching growth mindset” overwhelmingly highly; students also cited it very strongly, even while emphasizing “partner processing time” slightly more.

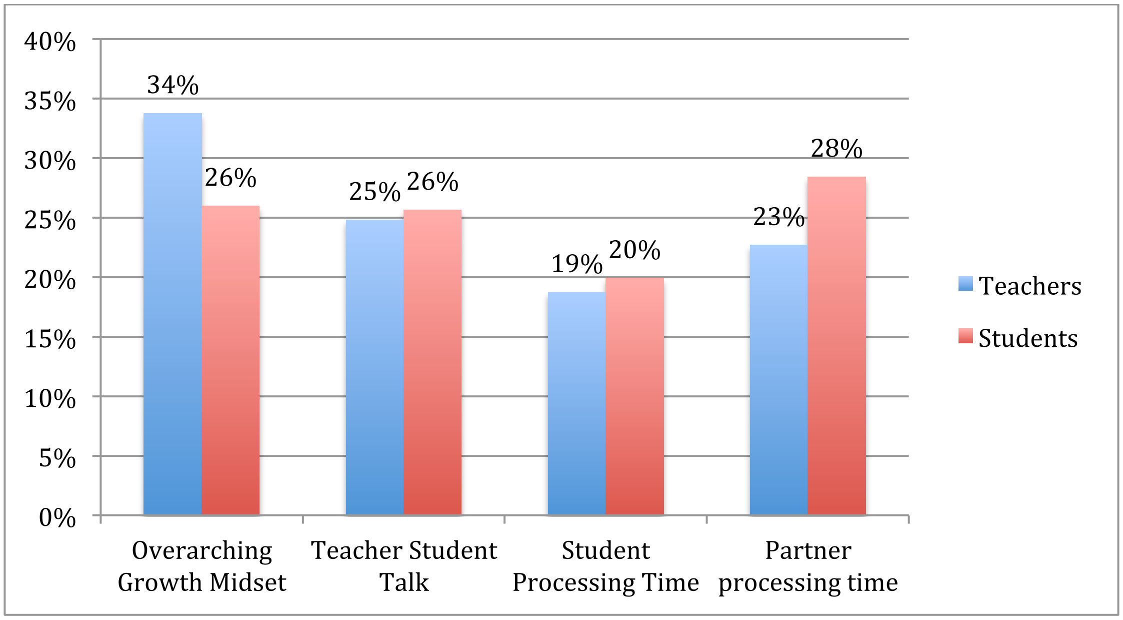


Figure 12: All categories- Teacher and student comparison

Participants indicated in interviews that the practices in “overarching growth mindset” were a pre-cursor to the practices in the other three categories that students cited as giving them access to rigorous mathematics content. It may be expected that teachers would focus on those practices that motivated students to learn and students would focus on the set of practices for which they applied their acquired learning. As teachers and students repeatedly emphasized each of these practices as important, their voices indicate that for them, every practice played a role in acquiring understanding and knowledge and therefore cannot be eliminated in the lesson.

If we look more closely at the “overarching growth mindset” category, we can see that students emphatically cited the practice of “communicating that student is capable of learning". Students identified this particular practice as the most important practice their teachers employed within the "overarching growth mindset" category. There is a strong relationship between the practices teachers named as beneficial throughout their individual interviews, those students cited as beneficial in their interviews, and the interactions actually observed during observations. Students felt that their teachers' belief in them was what gave them access to learning complex and abstract content, and observations showed such statements of belief routinely in classrooms.

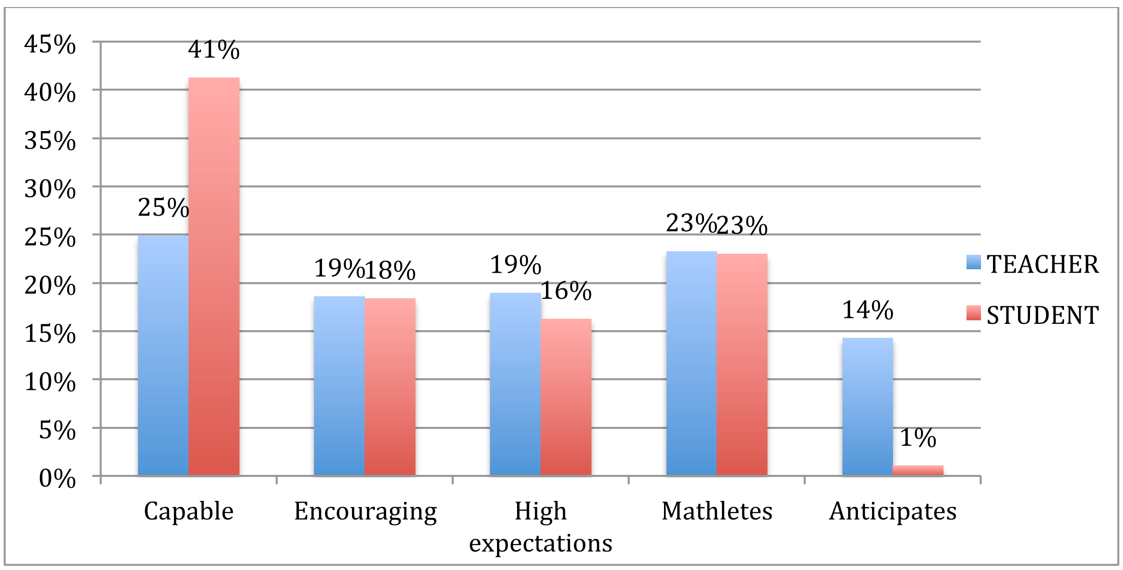


Figure 13:Overarching growth mindset- Teacher and student comparison

Teachers' cited practices were relatively evenly distributed. It is not surprising that "anticipates misconceptions" was so minimally cited by students but was significantly cited by teachers, as "anticipation" is an action by teachers that is difficult to see after the fact. Further, anticipation was a pre-cursor to the teacher talk practices. All practices anticipated the need for mentally preparing students to learn and to persist when they encountered barriers in their learning. Parallel emphasis during interviews can be seen with the practices of “encouraging students in general”, “has high expectations”, and “ encourages students to see themselves as succeeding in mathematics”, also termed treating students as “Mathletes”.

Citation of the “teacher-student talk” category by both students and teachers was virtually identical; all emphasized it as critically important. Yet when we delve into the practices participants named within the category we do see some variations. Students overwhelmingly cited practices that helped them build their own understanding, i.e. “clarified misconceptions” and “explains and re-explains” as those practices that provided leverage to their understanding.

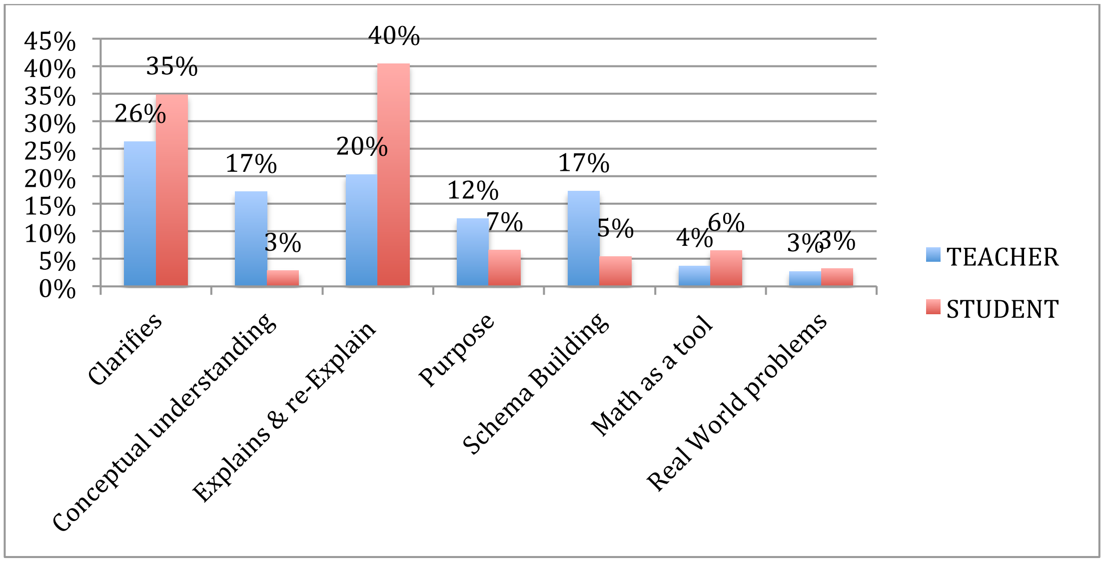


Figure 14:Teacher-student talk- Teacher and student comparison

As can be seen in the graph, 75% of practices identified by students in this category focused on the more traditional aspects of teaching (teacher talk that explains and clarifies). Teachers, on the other hand, cited a more comprehensive range of teacher talk practices as important, highlighting teacher talk for “conceptual understanding”, “purpose”, and “schema-building” (an emergent teacher-named practice) where students did not mention these in their interviews as much. This is not surprising as I think that students are not as savvy as teachers in discriminating the nuances of teacher talk.

In the category of “student processing time”, which was cited approximately 1/5 of the time during student and teacher interviews, teachers and students agreed that individual time to “solve math problems” and “additional time to process” were extremely important to the learning process.

As you can note in the graph below, these two practices comprised over 90% of student comments in interviews within this category.

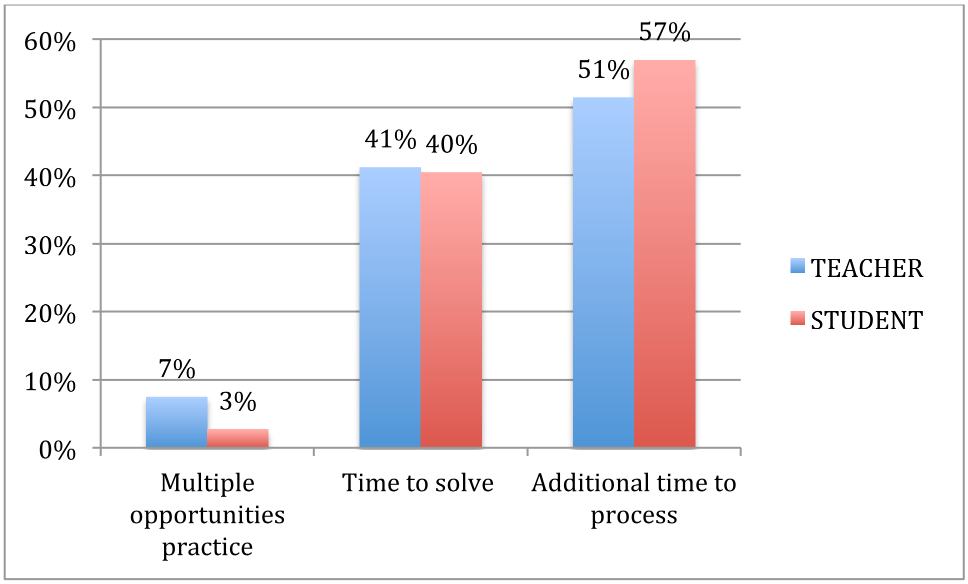


Figure 15: Student processing time- Teacher and student comparison

All teachers allocated time during their lessons to provide these two supports. Furthermore, in interviews, students and teachers cited this time to think and apply their understanding as practices that were extremely constructive. Additionally for students, allowing them time to solve math problems, which was an opportunity to determine whether they had learned the material, was motivating to them. It served as an incentive to continue to seek to understand of the content. The practice of offering students "multiple opportunities to practice" was an emergent practice that students and teachers cited and was evident in observations of lessons. The practice was cited enough to include in the data, yet it was not cited as one of the most essential practices.

Practices in the category “partner processing time” were the most mentioned in student interviews. Students and teachers emphasized “partner processing time” practices 28% and 22% respectively as aiding access to rigorous mathematics, again largely agreeing on the importance of such practices. As can be noted in Figure 15, and was argued in Chapter 4, all participants identified “pair work” and “ensuring understanding” as the most emphasized practices that participants said leverage learning and understanding. As you can see from this graph, students emphasized pair work as the most beneficial practice within this category and teachers cited ensuring understanding as the most beneficial. Teachers said that the goal of partner work was to have students walk away understanding and clarifying any misconceptions; students valued pair work because they walked away having their questions answered and often having helped their peers.

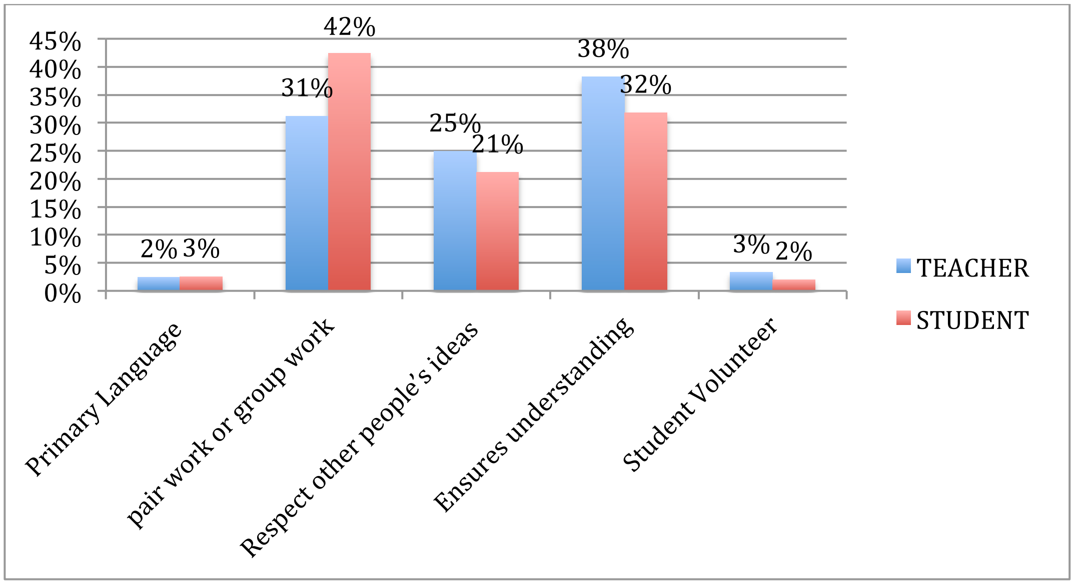


Figure 16:Partner processing time- Teacher and student comparison

Because all Latin@ students that participated in this study were former English learners or were identified as "reasonably fluent" English proficient, their primary language was a cultural resource they utilized but to them, not a necessity. This is to say that all students in this study had enough English acquired to be able to negotiate and discuss their ideas in English about the concepts they were learning. Several of the students code-switched during pair work but said it was more of a personal preference than a stipulation for understanding the content. However, students and teachers both agreed that students could and should use their primary language to negotiate and clarify meaning during pair discussions if needed or preferred.

Both "using student volunteers to teach or re-explain" and "ensuring understanding" were emergent practices in this category, named as important by both teachers and students. These practices were named by students and teachers and were evident in observations of lessons. As can be noted the emergent practice of "ensuring understanding", that is of restating and re-teaching until clarity was achieved by all students, was a high priority for both teachers and students. Students and teachers identified "volunteers" as a useful practice but it was less emphasized.

In the comparison of the four categories, it is interesting that students cited the most beneficial practices as being the ones in which they in essence became the "teacher." Although students were very often still in "learner" mode, they had to verbalize their understanding, which according to Vygotsky helps solidify the learning. But it is also imperative to note that they also cited how vital their teachers "explanations and re-explanations" and "clarification of misconceptions" were to their learning.

Students' and teachers' perspectives on necessary mathematics practices comparison to those identified by research.

All fourteen of the a priori practices (practices expected by research) did surface as practices participants considered important and five emergent practices (practices prior research did not expect) did as well.

All practices expected by research were cited by both the four teachers and eight Latin@ students interviewed as important to accessing rigorous mathematics, *except for three*. The practice of "helping students understand how mathematics can be used as a tool outside of the classroom and in your community", the practice of "making connections to real world applications", and the practice of “using the mathematics register and terminology correctly” were minimally cited as helpful practices. I describe this pattern in more detail momentarily.

As mentioned previously, a supplemental survey was added to the study to capture perceptions on practices that perhaps were not mentioned in the predominant portion of the study. To minimize the potential influence of this oral survey on data gathered in the qualitative interviews and as not to bias participants, this survey was conducted after all interviews were completed. The oral survey asked participants to rate the practices identified in research on a scale of 1-not important to 4-very important. The exact question was:

> I am going to read you a list of these practices. Researchers say these practices are helpful to students but you may not. Be honest in your rating because I am trying to learn from you. Tell me what you think about whether these practices are helpful to student learning higher-level mathematics by rating them on a scale of 1-not important, 2-a little important, 3- important, and 4-very important. Feel free to elaborate. Please ask me to clarify if you do not understand.

Based on this question, teacher and student participants rated each practice. I will note that some participants rated a practice as higher than a 4 because they felt it was “super important”; I allocated a 5 rating in those cases. I did allow people to go “off the scale” and could go back and adjust all 5’s as 4’s, but what’s most interesting about this data is actually the information they provided about practices that did not surface in the interviews often. Table 4 and Table 5 show the rating for each practice and the average by student and teacher participants respectively.

Table 4: Student Ratings of Practices

student	capable	encourage	Mathlete	additional time	solve mathematics problems	explains &re-explain	clarifies misunderstandings	mathematics register	pair work	respect ideas and approaches	primary language	purpose of mathematics	mathematics as a tool	real world app
1	3.0	3.0	4.0	4.0	4.0	4.0	4.0	4.0	5.0	5.0	4.0	3.0	3.0	3.0
2	4.0	3.0	4.0	4.0	5.0	5.0	4.0	4.0	5.0	4.0	4.0	4.0	3.0	3.0
3	3.0	4.0	3.0	5.0	5.0	5.0	4.0	3.0	5.0	3.0	5.0	4.0	4.0	4.0
4	4.0	4.0	5.0	5.0	5.0	4.0	5.0	4.0	5.0	3.0	4.0	3.0	3.0	3.0
5	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	3.0	3.0	3.0	3.0	3.0
6	4.0	4.0	4.0	4.0	4.0	4.0	3.0	4.0	4.0	2.0	3.0	3.0	3.0	4.0
7	4.0	4.0	4.0	3.0	4.0	4.0	3.0	4.0	2.0	4.0	4.0	4.0	4.0	4.0
8	4.0	3.0	4.0	4.0	3.0	4.0	3.0	4.0	4.0	3.0	4.0	4.0	4.0	4.0
Avg	3.8	3.6	4.0	4.1	4.3	4.3	3.8	3.9	4.3	3.4	3.9	3.5	3.4	3.5

Table 5: Teacher Rating of Practices

Teacher	capable	encourage	Mathlete	additional time	solve mathematics problems	explains &re-explain	clarifies misunderstandings	mathematics register	pair work	respect ideas and approaches	primary language	purpose of mathematics	mathematics as a tool	real world app
1	4	3	4	4	4	4	4	4	4	5	4	NA	5	4
2	4	4	4	4	NA	5	5	5	5	4	3.5	4	2.5	2.5
3	4	4	5	4	4	4	4	4	5	4	4	4	5	4
4	4	4	4	5	4	4	4	3.5	4	4	4	4	4	4
Avg	4	3.8	4.3	4.3	4	4.3	4.3	4.1	4.5	4.3	3.9	4	4.2	3.6

In interviews, as we have seen, participants named *nineteen* practices; *fourteen* of those practices were practices that research has already said are important to marginalized students' learning (across the top of the Table 4 and 5). We have also seen that participants named *five* additional new practices as important that research had not emphasized. As I mentioned earlier, the goal of this oral survey was to capture participants feelings toward practices that research had identified, but had the potential of not surfacing or being called out during the interviews. The supplemental survey also served as a final opportunity for participants to express their perceptions of practices that had surfaced during interviews. Hence, the survey did serve the purpose of allowing participants to name practices that had not surfaced. Furthermore, as can be noted from the survey results, teachers' and students' rating of the practices were pretty aligned, as in the interviews; they varied just in two practices. Teachers rated the practice of "teacher teaches you how to work with others on mathematics and how to respect other people's ideas and approaches in solving mathematics problems" much higher than students, 4.3 out of 4 compared to 3.4 out of 4, respectively. This may be a more important practice for teachers since they may be more cognizant of the type of learning environment they are trying to create in their classrooms.

What this survey showed most usefully was that there were indeed three practices from research that participants valued but only rarely mentioned in interviews: using mathematics register, using mathematics as a tool and real world applications. When prompted in the final oral survey to comment on research's expectations, participants argued that these three practices were in fact important. Although teachers did not often cite a need for using the mathematics register and terminology as a goal or objective of their lessons during interviews, I did observe that the mathematics register was used in all mathematics classrooms consistently, indicating the implicit importance of the practice to them. Furthermore, the variations in ratings between participants demonstrates that there was *not* a singular practice that was required in order for students to have access to rigorous mathematics but rather, that many practices were needed. Students and teachers spoke about the benefits to each practice, each meeting the needs of different individuals.

Additionally, on the survey teachers rated highly the research-expected practice of "Teacher helps you understand where and how mathematics is used as a tool outside of the classroom and in your community". In discussing this practice, which had not surfaced in interviews, Teachers related this practice to real world applications. They felt that this was an area of need for them. So although these are practices that they admitted they did not implement well, they were eager to learn and make mathematics more relevant for students. In their district, in fact, discussions around these two practices have been identified as an important need to address for all mathematics instruction. Relationships with the business community and with post-secondary higher education institutions are currently being fostered so that more authentic and community-based projects can be generated. Lastly, I would like to note that although real world applications was not cited as a helpful practice to learning during interviews, perhaps because the lessons I observed were more theoretical mathematics in nature, both teachers and students did cite this practice as “important” or “very important” in the oral survey. Thus, the results of the survey provide additional data to practices participants deem beneficial to teaching and learning and offered an opportunity for participants to share their perspective on practices that were not cited previously. Using mathematics as a tool and real world applications were not so visible in classrooms, but participants did say they valued them on the survey. Teachers’ overall rating of these two research-based practices respectively were 3.6 out of 4 and 4.1 out of 4. (As can be noted, “mathematics as a tool” rated higher than the allotted four-points. Clear directions were given on the rating scale but participants at times rated practices higher than the maximum to emphasize their perceived importance to teaching and learning.)

Table 5 later in this chapter provides ratings on all the practices. Still, I emphasize that the qualitative emphasis and comments participants provided when *discussing* this survey were the most informative aspect of the oral survey. For example, Mrs. G discussed how her student teacher had this knowledge because he came from an applied mathematics background and had taught at community college. She said,

> So when he was my student teacher, he was, he would explain to me how some of this stuff was used in real life. And I'm like, "Gee, that's, that's what we need. That's what we need as high school teaches is to know how all of this stuff. You know, not just the simple, you know, the rocket flew through the air type of things. Which those things are fine but there's so many more applications that we're just not aware of or even comfortable discussing.

Mr. X stated of the practice “helps students understand how mathematics can be used as a tool to solve problems in the real world,”

> I think that's a three. I'm not sure I do very well with that but I think that's important.

Another teacher commented, “That's definitely a four and that's something that we need more professional development on. It's definitely a big need.”

Similar to comments made by teachers, student participants did not highlight the practices of “real world application” and “using mathematics as a tool” often during the interviews but rated these practices highly on the final oral survey that took place after all interviews were completed. In the oral survey, participating students rated “real world application” practice 3.5 out of 4 overall and “mathematics as a tool” as 3.4 out of 4 overall. Again, the most valuable information that the survey yielded was an opportunity for participants to provide their additional thoughts on practices that may not have surfaced during the interviews, the primary portion of this study. During the survey, Diego made the comment,

> Um, a four. Like I said um, I want to study architecture and when we studied like, the dimensions of shapes, like area and perimeter, that really, um, helps, helps me, it helps me and

> motivates me to keep on having that idea of studying architecture and knowing that I can apply it later on in my life.

Josue said,

> Three. Like in construction, we learned that. Umm, when you have to put a ladder up so you don't fall. You have to see if the ladder is, is long enough. But I don't really think I would measure it.

Itzel reflected,

> Um, personally. I don't know when I'm going to have to use logs like at stores or something but my general math like adding. Like groceries if I have to go buy groceries when it tells like these 99 cents per pound… something like that, I think that kind of math you need, but I don't know about when I'm going to use logs at the store or something like that.

The fact that "real world applications" was not cited during the interview phase of the study is not to say that students never experienced lessons that incorporated word problems that represent "real world applications." This component of their learning simply was absent on the days of observation, and interviews about those days, as a result, did not capture this specific aspect of "rigorous mathematics." Therefore, what these comments may suggest is that the types of "real world" application that are available in the curriculum lack what students and teachers deem as "real world" context. Participants in this study hinted that real world application should be more authentic, moments when mathematics is being utilized to construct something or apply an idea in the real world, rather than simply make a verbal reference to the real world. They implied that word problems that *reference* a real world context are not as valuable as *using* mathematics in a real world context. With new textbook adoptions currently taking place in California, new curriculum may provide more authentic contexts to address this practice. In sum, these were practices teachers said they valued, but they felt they did not have adequate supports to put them into practice yet.

The data quantified in these tables above simply add another piece of information that the counts of comments emphasized in interviews in previous graphs did not. The goal of the study overall was to learn from those who are involved in the day-to-day work of teaching and learning. Therefore, the most valuable data was provided from the detailed things participants said about their experience as teachers and learners and the emphasis that they placed on particular practices.

Implications for Educators

In addition to naming specific pedagogical practices that supported learning, the participants in this study underscored the foundational importance of growth mindset perspectives and how teachers' ideas about students' potential to grow in mathematics influence the decisions they make day-to-day when they teach. The growth mindset perspective encourages teachers to accept that learning is an ever-evolving process and that knowledge can be fostered and germinates in classrooms were students are safe to ask questions and given the freedom to explore their own ideas and perspectives while learning. Participants indicated that learning mathematics is about layering another "coat of knowledge", searching for another method to solve, and linking ideas to another topic within the schemata of mathematics so that a network of understanding and knowledge can grow. Students in these classrooms were encouraged to use all their resources to think more deeply about concepts. With growth mindset perspectives, teachers can counter the barriers students face every day: language, gaps in knowledge, institutionalized and historic barriers.

The Latin@ population is quickly becoming the largest subgroup of our California school population. In addition to listening to successful teachers of Latin@ students, we need to learn to take in *students' own* perspectives into account when we organize our curriculum, lesson plans, and classroom experiences. The teachers in this study were very cognizant of the response students had daily to their Teacher-Student Talk; they created routines for student to process what they were trying to teach. They recognized their role was more than teaching-- it was also ensuring student understanding. They provided multiple opportunities for students to digest the concepts so that knowledge could grow and understanding could be incorporated in their existing schemas. These teachers who were deeemd successful with Latin@ students listened to and respected the verbal and non-verbal feedback they received from their students. With that information they adjusted and sought ways to re-explain, to make connections, and clarify misconceptions so that their students walked out of their class feeling capable and knowing that they did learn because they were able to explain their understanding to others. In order to help students know that they can succeed in mathematics, these teachers also tried to incorporate many affective practices that influence the outcomes of teaching. Days where students struggled were not held against them; they were encouraged to continue to seek to understand and to become "Mathletes".

Limitations and Implications for Future Research

This study was limited to four "effective" and "good" teachers and eight Latin@ students within three schools in one district. The study was focused on those practices that Latin@ students cited and excluded the perspectives of other student groups. The replication of this study to include a larger teacher and Latin@ student group may provide greater support for the findings of this study. In order to establish a better understanding of specific practices that promote access and success for English learners, other research studies are suggested that explore the learning experiences of "less than reasonably fluent" English learners. The replication of this study at additional school sites and with other marginalized groups may support the generalization of this study. Additionally, this study was limited to high school mathematics teachers and students. The replication of this study with students of middle school age may provide additional insight on specific practices that benefit learning earlier in the K-21 educational pipeline. Furthermore, and most obviously, this study investigated what students and teachers *said* helped them gain access to rigorous mathematics topics. A study that involved a controlled and experimental group may provide more definitive data pn the effect size each practice provides.

Further investigation of the more specific pedagogical practices that teachers with “growth mindset” perspectives employ in everyday lessons with their Latin@ students will strengthen the validity and reliability of this study’s findings. Such investigations may also provide additional practices not evident in this study and provide further insight into how teacher practices help students counter barriers they face every day in mathematics classrooms.

Conclusion

Mathematics education has large implications for the future opportunities of every student in the United States. Success in this pivotal subject determines whether a student has access to higher education and what career choices are available to them in the future (Harvard University, 2007; The Campaign for College Opportunity, 2013; Tyson et al., 2007). Mathematics must become a “pump” and not a “filter” to the opportunities granted to our young people. Life-long implications for young people and for our country are centered around success in this one subject. And as the largest growing subgroup, the success or failure of Latin@s will largely impact our state and country’s economy (Passel & Cohn, 2008; US. Census, 2014).

As a country we recognize that our educational system is not meeting the needs of all learners. The fact that our educational practices need to change is not in question. Our society is not content with the outcomes of our public education system and as a whole we recognize that we have left our Latin@ student population behind. What educators ask often is, “What can I do differently to ensure that more of our students succeed?” It is in this question that change blossoms. The purpose of this study was to examine what practices teachers said they employed in everyday lessons and what Latin@ students cited as giving them access to the rigorous mathematics they were attempting to learn. The findings suggest that “effective” and “good” teachers have growth mindset perspectives about learning. Teachers who help students learn complex mathematical concepts recognize that they face barriers every day and that learning is a continual process that requires many types of practices. They also are aware that in order for students to learn, they need to have time to process, practice, and discuss their thinking with others. When these practices were implemented with consistency, students started seeing themselves as capable of succeeding in the subject of mathematics and motivated to persist in seeking to understand abstract and complex material.

Nationwide, financial and human resources are being invested to revolutionize our K-12 educational system. National new standards in Mathematics and other core subjects have been written to affect the outcomes of public education. If we are to succeed with this new endeavor, we must not only pay attention to *what* is taught, but to *how* the subjects are taught. This means that we need to pay attention to the environment and the day-to-day interactions between teachers and students and students and their peers. In addition, to understand this "how," we must learn to listen to the learner and to what they say they need in order for them to succeed. I found that students in this study were very focused on learning. They wanted to succeed, and their teachers worked hard to employ an abundant amount of practices to ensure that mathematics comprehension and learning was achieved. Both teachers and students persevered and trusted that with enough and varying practices and strategies they would succeed together.

Therefore, this study found that *everyday actions for mathematics equity are possible*. Participants, both teachers and students, discussed the *routineness* of the practices they mentioned. It was in the routineness—this ongoing repetition of practices designed to help them -- that students found safety to persevere to understand and learn. They indicated that the everyday actions and practices in their mathematics classes built student confidence in their capacity for learning because they knew that safety nets would be provided. Students clearly discussed how the routineness of practices designed to help them differed from previous mathematics learning experiences, where they often felt "dumb" and "slow".

Additionally, this study found that there was a strong alignment overall between the practices that teachers and students named. That is, what teachers intended to do to help student learning was in fact what students cited as helping them learn. These "good" and effective," growth mindset teachers were very attuned to what students needed and paid attention to verbal and non-verbal cues, making adjustments to their lessons and anticipating as much as possible. These teachers also incorporated *many practices in one single lesson* to try to ensure learning. Teachers were more aware of some of the nuances of their teaching efforts (at supporting conceptual understanding and schema-building, anticipating misconceptions, etc.) and so they emphasized these more than students, but students were able to recognize these at times as well.

Lastly, according to participants, supporting learning requires not just growth mindset but detailed practices of teacher-student talk, student processing time, and partner processing time. These practices pushed students to see themselves for who they could be if they continued to strive, and they gave students multiple opportunities to comprehend the abstract concepts in higher level mathematics -- all of these practices participants called necessary to supporting learning. We have known that the art of teaching is that --*an "art"* -- but participant comments help us decipher what practices within this art students and teachers say make a difference and contribute to student success in this very pivotal subject of mathematics.

The importance of student voice

Perhaps some believe that students are not able to provide answers to "What helps them learn?" They may doubt that students can to be metacognitive about their own learning. But in this study, I found that students were able to identify the moments in learning that caused them difficulty and what specifically helped them overcome those barriers. Research had identified many of these empowering practices, but students were able to be much more descriptive of how these practices helped and what barriers to learning they countered. Students in this study were in fact able to identify practices they felt were helpful and provided insight into specifics about what was confusing for them and what helped them get "unconfused." Students were just as focused on their own comprehension of the content as were teachers. They worked to capitalize on all the practices provided to them during class time to further their own understanding of mathematics content.

Few studies have ventured to understand the learning process from the student perspective. As mathematics becomes perhaps the most vital subject in the STEM pipeline and continues to be the best predictor of post-secondary options, we should start investigating the specific nuances that make mathematics accessible to all learners, not just the small percentage that can earn an A. For if we can identify what practices help students become re-invigorated and re-engaged with mathematics, even when they have not excelled in the subject before, we can offer true life options to young people. Moreover, if we seek to understand what explicit practices and mindsets Latin@ students cite as beneficial, as a fast growing and alienated population in our school system, we can promote those practices that "effective" and "good" teachers employ.

Appendices

Appendix A: Student Interview Questions

NOTE: *All questions were asked during both interviews except those denoted with a .1 or .2, e.g. 7.1 and 7.2. .1 questions were asked in the first interview and .2 questions were asked in the second interview*

Student Interview Questions:

Hi,

My name is Mrs. Ivette Sanchez-Gutierrez and I am a doctoral student at UCSD and also a mathematics teacher in the district. I am doing a study to learn from students what they feel helps them have access and succeed in mathematics. I would like to ask you some questions but if at any time you do not want to answer a question please just let me know. My hope is that as teachers we can get better at what we do and really support students in the best way possible. Your voice and experience is very valuable and I hope to learn about what we need to work on or do more of. I am so grateful that you are giving me some of you time. Let's get started.

<u>Student questions:</u>

1) I observed the math lesson on _____________ day in (teachers name)'s class. **Can you tell me about the lesson?** What was the lesson about?? Did you understand the topic??

2) How well do you feel you understood the lesson? What about the lesson helped or needed more assistance?

3) Many say that learning is a process of being confused and then being unconfused. **Was there a moment in the lesson you were confused**? Let's look at that moment.

What helped you get unconfused?

4) Was there part of the lesson that **you felt helped you understand well**? Lets look at the video to find a moment. **Tell me more about why that helped you understand more.** What happened?

5) Lets look for another moment in the lesson. What about this part of the lesson helped you or confused you? What could have helped you more?

6) Thinking about this lesson **what do you think you really got, i.e. what do you think you learned?** What are you still confused about or fuzzy about??

7.1) **How do you learn best?** How do you feel this affects your learning? Does your teacher have rules about how you deal with other peoples' ideas?

7.2) Do you feel that learning math is important? Where do you see yourself using math outside of math class?

8) How is this lesson I observed **similar or different from other lessons** you have had with teacher x? Are there things that he/she always does?

9.1) Do you feel that your math **teacher believes that you can learn**? What does he/she do that makes you believe that?

9.2) Do you speak another language? What? Do you ever use that language in math class? Do you think your teacher should allow students to use other languages in pairs or small groups?

10.1) **Do you think your math teacher cares about your learning and your grade**? What kind of relationship do you have with your math teacher? How does this teacher compare to other math teachers?

10.2) Describe yourself as a math student? What helps you? What do you think makes a successful math student?

Appendix B: Teacher Interview Questions

Teacher questions:

1. Tell me a little about teaching Integrated Mathematics? Tell me a little about your students in that class?
2. What was the goal for this lesson? How did the lesson go?
3. Is there anything that surprised you about how the lesson went?
4. What do you think really worked or did not work in the lesson?
5. Lets look at that moment…. Looking back what helped or hindered that portion of the lesson? How did the reality of the lesson compare to what you planned?
6. What are things you consider when planning your lesson?
7. What are your next steps?

Appendix C: Oral Survey

Oral Survey

I am going to read you a list of these practices. Researchers say these practices are helpful to students but you may not. Be honest in your rating because I am trying to learn from you. Tell me what you think about whether these practices are helpful to student learning higher-level mathematics by rating them on a scale of 1-not important, 2-a little important, 3- important, and 4-very important. Feel free to elaborate. Please ask me to clarify if you do not understand.

(Asked in Oral survey format)

a. Teacher believes that his/her students are capable of learning
b. Teacher is encouraging in general
c. Teacher encourages you to see yourself capable of succeeding in math

d. Teacher gives or builds in additional time to process learning
e. Teacher gives you time to try to solve math in class

f. Teacher explains well and is willing to explain again if you do not understand
g. Teacher clarifies if you misunderstand
h. Teacher used math terminology and explains what it means
i. Teacher builds in pair and group work so that you have time to discuss math and learn from your classmates, getting different perspectives or approaches and sharing your own perspectives and approaches
j. Teacher teaches you how to work with others on math and how to respect other people's ideas and approaches in solving math problems
k. Teacher allows you to use your primary language (i.e. like Spanish) to discuss mathematics when you are in small groups or in class.
l. Teacher helps you understand the purpose of math
m. Teacher helps you understand where and how math is used as a tool outside of the classroom and in your community
n. Teacher has you solve problems that can or do exist in the real world.

Appendix D: Integrated Mathematics II Performance Task and Rubric

Performance Task Rationale

The Performance task that follows has been intentionally designed to elicit a higher level of thinking from students than traditional assessment items. The task will involve significant interaction of students with stimulus materials (e.g. readings, data, equations, graphs) and problem solving, ultimately leading to an exhibition of the students' application of knowledge and skills through mathematical reasoning and multiple representations.

The content of the performance task is intended to assess first semester content and skills. Feedback was solicited from site curriculum specialists and incorporated into the task. According to the Smarter Balanced Assessment Consortium,

> A key component of college and career readiness is the ability to integrate knowledge and skills across multiple content standards. Smarter Balanced will address this ability through performance tasks, because it cannot be adequately assessed with selected-response or constructed-response items.
>
> *Smarter Balanced Performance Task Specifications, page 1*

To the Teacher:
This task is designed to be administered in 60 to 120 minutes. In order to access prior knowledge about the **context of the task**, you may want to lead an informal discussion for 5 to 10 minutes with the whole class about the **context** (e.g. parking lots, egg-launch, shelves, speeding fines, spinners, or bamboo). After the discussion, pass out the task and have students complete it individually. If students have questions about vocabulary, directions, or intent of the problems, please use your professional judgment when providing assistance.

Students should be provided with the following tools for this task, if available:

- ☑ Calculator or calculator app
- ☑ Scratch paper
- ☑ Ruler/Straightedge

Student Directions (Please say to students)
Read all directions carefully, answer all parts of the task, and justify your thinking.

When we are scoring this task we value:

- ☐ Multiple representations (e.g. tables, graphs, equations, diagrams, and explanations)
- ☐ Accurate labeling (e.g. units of measure, axes, tables, variables, functions)
- ☐ Clear and thoughtful writing
- ☐ Logical mathematical reasoning
- ☐ Correct use of academic language

SWEETWATER
UNION HIGH SCHOOL DISTRICT

IM II Performance Task Fall 2014
Egg Launch Contest

NAME: ______________________
DATE: ______________________

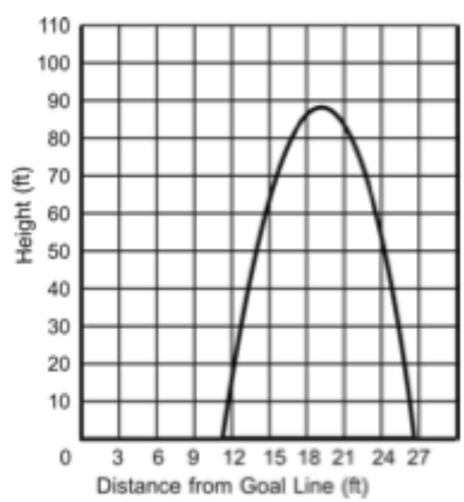
Mr. Rhodes' class is holding an egg launching contest on the football field. Teams of students have built catapults (see an image below) that will shoot an egg down the field. **Each team starts at a different distance from the goal line.** You are one of the judges who will decide which team wins the contest. You have various tools and ideas for measuring each launch and how to determine which team wins.

Team A used their catapult and shot an egg down the football field. Students used a motion detector to collect data while the egg was in the air. They came up with the table of data below.

DISTANCE FROM THE GOAL LINE (IN FEET)	HEIGHT (IN FEET)
7	51
9	75
14	100
19	75
21	51
24	0

Team B's egg flew through the air and landed down the field. The group of students tracking the path of the egg determined that the equation $y = -3x^2 + 48x - 45$ represents the path the egg took through the air, where x is the distance from the goal line and y is the height of the egg from the ground. (Both measures are in feet.)

When **Team C** launched an egg with their catapult, some of the judges found that the graph to the right shows the path of the egg.



Team A

1. Using the data from Team A, determine an equation that describes the path of the egg. Describe how you found your equation.

2. On the graph on the next page, graph the path of Team A's egg.
3. What is the maximum height that the egg reached? How far was the egg shot?

Team B

4. Using the equation from Team B, generate a table of values that shows different locations of the egg as it flew through the air. Be sure to include critical points.

x												
y												

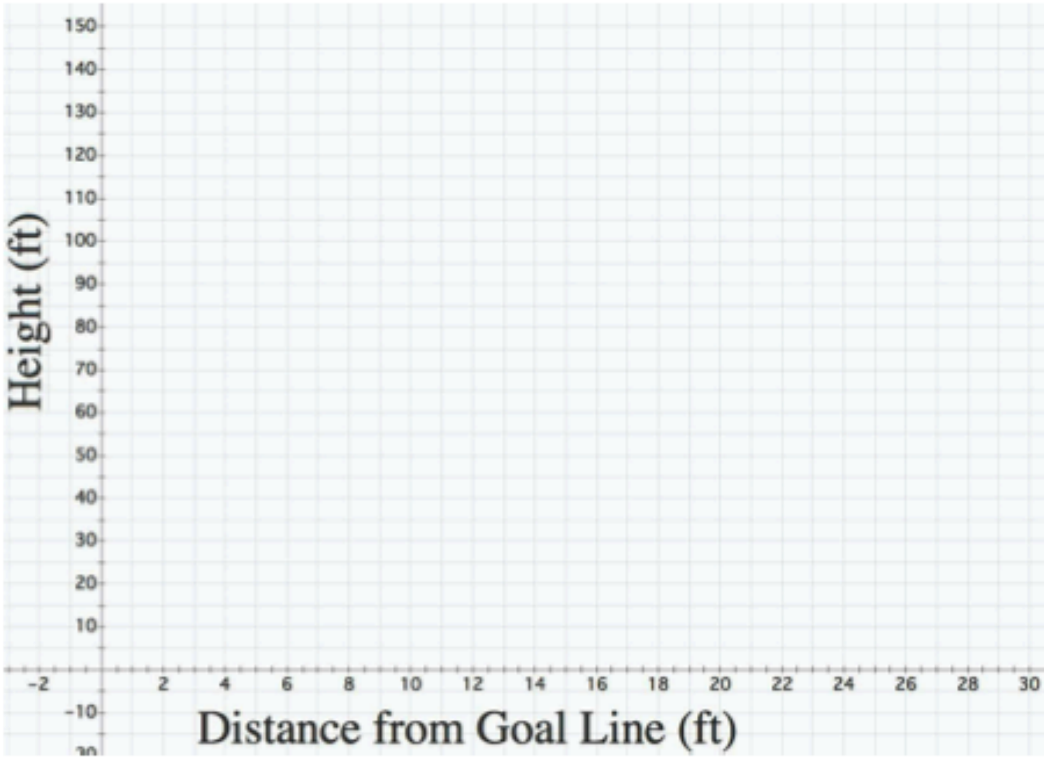
5. On the graph on the next page, graph the path of Team B's egg.
6. What is the maximum height that the egg reached? How far was the egg shot?

Team C

7. Using the data from Team C, generate a table of values that shows different locations of the egg as it flew through the air. Be sure to include critical points.

x												
y												

8. On the graph on the next page, re-graph the path of Team C's egg.
9. What is the maximum height that the egg reached? How far was the egg shot?



10. If it is a height contest, which team wins? How do you know?

11. If it is a distance contest, which team wins? How do you know?

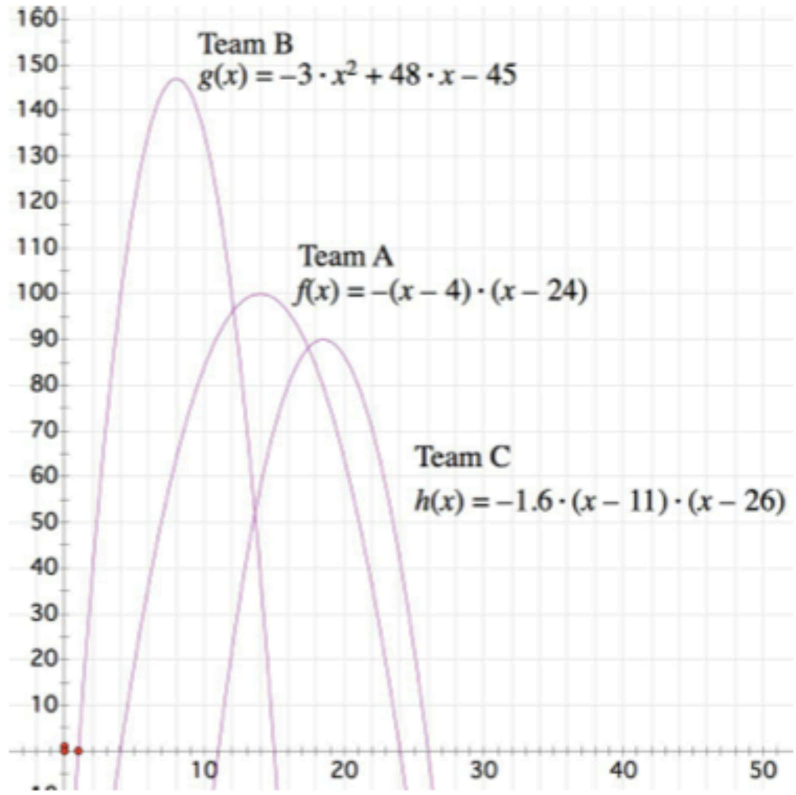
12. For the team you did not choose in questions 10 and 11, what type of contest could they have won? Explain.

IM II PT, Fall 2014: Egg Launch Contest

Scoring Criteria for the Egg Launch Task

Scorable Parts	Points	Claims
Team A 1a. Determine the equation 1b. Describe how you found the equation 2. Graph the path of the egg 3a. What is the maximum height? 3b. How far was the egg shot?	0-5 points Full credit for y = -(x-4)*(x-24) or equivalent equations. A student might say that they used the table to find the roots of the quadratic and then made each root into a binomial factor in the equation. The graph must contain the correct roots and vertex (shown on page 3). The maximum height is 100 feet and the egg was shot 20 feet.	Contributes evidence to Claim 1, concepts and procedures, Claim 3, Communicating Reasoning and Claim 4, Modeling
Team B 4. Generate a table of values 5. Graph the path of the egg 6a. What is the maximum height? 6b. How far was the egg shot?	0-4 points Full credit for a table of values that includes correct x and y values. For example, X Y 2.00 39.00 3.00 72.00 4.00 99.00 5.00 120.00 6.00 135.00 7.00 144.00 8.00 147.00 9.00 144.00 11.00 120.00 12.00 99.00 13.00 72.00 14.00 39.00 15.00 0.00 The graph is on page 3. The maximum height is 147 feet and the egg was shot 15 feet.	Contributes evidence to Claim 1, concepts and procedures, Claim 3, Communicating Reasoning and Claim 4, Modeling
Team C 7. Generate a table of values 8. Graph the path of the egg 9a. What is the maximum height? 9b. How far was the egg shot?	0-4 points Full credit for a table of values that includes correct x and y values. There may be some variance because the data is derived from a graph. For example, X Y 12.00 15.00 13.00 35.00 14.00 55.00 15.00 63.00 16.00 75.00 17.00 83.00 18.00 86.00	Contributes evidence to Claim 1, concepts and procedures, Claim 3, Communicating Reasoning and Claim 4, Modeling

	19.00 88.00 20.00 86.00 21.00 83.00 22.00 75.00 23.00 55.00 24.00 35.00 25.00 15.00 26.00 0.00 The graph is on page 3. The maximum height is approximately 88 feet and the egg was shot approximately 15 feet.	
10a. Which team wins? 10b. How do you know?	0-2 points Full credit includes naming Team B as the winner AND explaining that from the comparisons of the graphs, equations, or tables that team reached a maximum height of 147 feet.	Contributes evidence to Claim 2, Problem solving and Claim 3, Communicating Reasoning
11a. Which team wins? 11b. How do you know?	0-2 points Full credit includes naming Team A as the winner AND explaining that from the comparisons of the graphs, equations, or tables that team shot the egg a maximum 20 feet down the football field.	Contributes evidence to Claim 2, Problem solving and Claim 3, Communicating Reasoning
12. Contest that produces a new winner.	0-1 point Full credit creating a new contest for making team C the winner. For Example, "They could have won a contest that required the team to launch the egg closest to 90 feet in the air, without going over. Team C had maximum height of 88 feet so they would have won this contest."	Contributes evidence to Claim 2, Problem solving and Claim 3, Communicating Reasoning



Claim #1 – Concepts & Procedures

"Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency."

Claim #2 – Problem Solving

"Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies."

Claim #3 – Communicating Reasoning

"Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."

Claim #4 – Modeling and Data Analysis

"Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."

Appendix E: Integrated Mathematics III Performance Task and Rubric

Performance Task Rationale

The Performance task that follows has been intentionally designed to elicit a higher level of thinking from students than traditional assessment items. The task will involve significant interaction of students with stimulus materials (e.g. readings, data, equations, graphs) and problem solving, ultimately leading to an exhibition of the students' application of knowledge and skills through mathematical reasoning and multiple representations.

The content of the performance task is intended to assess first semester content and skills. Feedback was solicited from site curriculum specialists and incorporated into the task. According to the Smarter Balanced Assessment Consortium,

> A key component of college and career readiness is the ability to integrate knowledge and skills across multiple content standards. Smarter Balanced will address this ability through performance tasks, because it cannot be adequately assessed with selected-response or constructed-response items.
>
> *Smarter Balanced Performance Task Specifications, page 1*

To the Teacher:
This task is designed to be administered in 60 to 120 minutes. In order to access prior knowledge about the **context of the task**, you may want to lead an informal discussion for 5 to 10 minutes with the whole class about the **context** (e.g. parking lots, egg-launch, shelves, speeding fines, spinners, or bamboo). After the discussion, pass out the task and have students complete it individually. If students have questions about vocabulary, directions, or intent of the problems, please use your professional judgment when providing assistance.

Students should be provided with the following tools for this task, if available:

- ☑ Calculator or calculator app
- ☑ Scratch paper
- ☑ Ruler/Straightedge

Student Directions (Please say to students)
Read all directions carefully, answer all parts of the task, and justify your thinking.

When we are scoring this task we value:

- ☐ Multiple representations (e.g. tables, graphs, equations, diagrams, and explanations)
- ☐ Accurate labeling (e.g. units of measure, axes, tables, variables, functions)
- ☐ Clear and thoughtful writing
- ☐ Logical mathematical reasoning
- ☐ Correct use of academic language

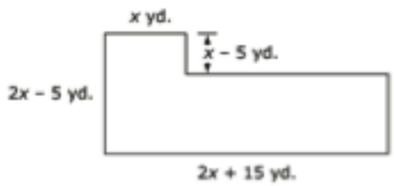
SWEETWATER
UNION HIGH SCHOOL DISTRICT

IM III Performance Task Fall 2014
Parking Lot

Name: ______________________
Date:____________Period:______

Part 1

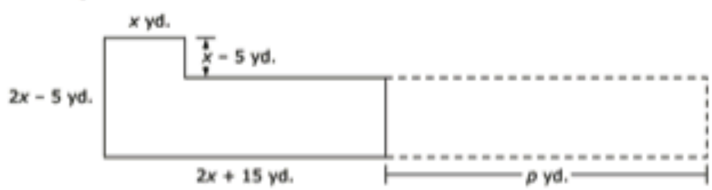
A town council plans to build a public parking lot. The outline below represents the proposed shape of the parking lot.



Write an expression for the area, in square yards, of this proposed parking lot. Explain the reasoning you used to find the expression.

Part 2

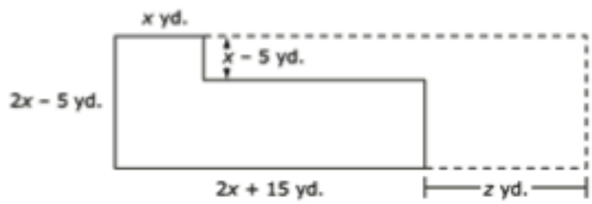
The town council has plans to double the area of the parking lot in a few years. They create two plans to do this. The first plan increases the length of the base of the parking lot by p yards, as shown in the diagram below.



Write an expression in terms of x to represent the value of p, in yards. Explain the reasoning you used to find the value of p.

Part 3

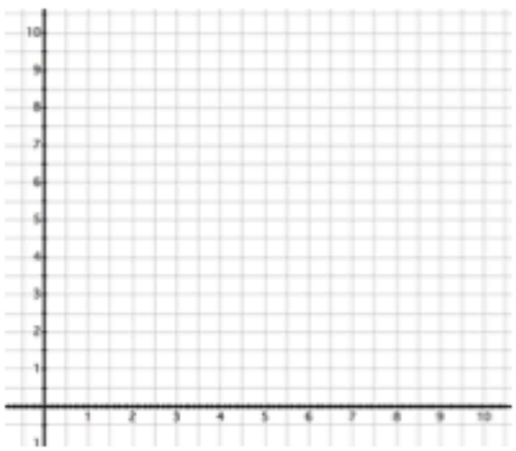
The town council's second plan to double the area changes the shape of the parking lot to a rectangle, as shown in the diagram below.



Can the value of z be represented as a polynomial with integer coefficients? Justify your reasoning.

Part 4

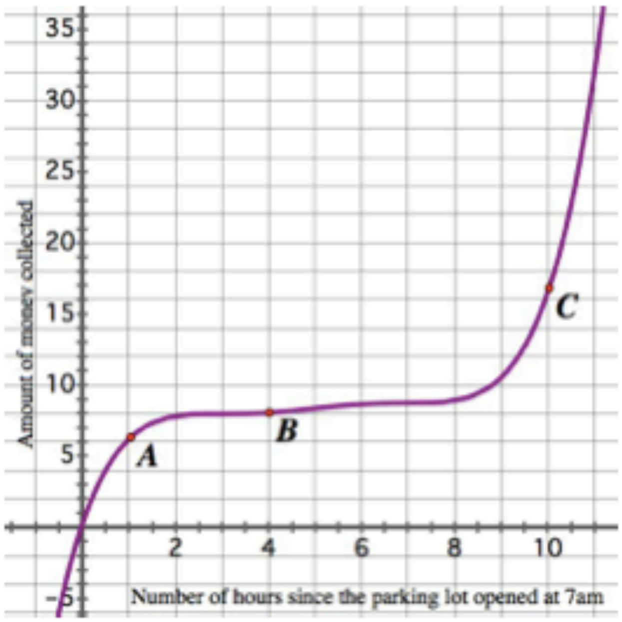
The number of cars using the parking lot throughout a given day can be modeled by a 4th degree polynomial function with real roots, t=3 (multiplicity 2) and t=7 (multiplicity 2), $0<t<10$. In the equation t represents the number of hours since 7am, when the parking lot opens. The parking lot closes at 5pm. If there are 10 cars in the lot when it opens and 10 cars left when it closes, sketch a graph of the function and write a possible equation for the function. Be sure to label your graph.



Does every point on your graph represent a legitimate time and number of cars in the context of the problem? Explain.

Part 5

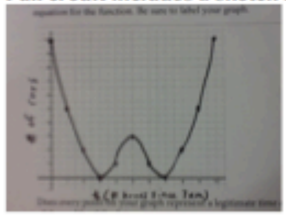
Drivers pay when they arrive at the parking lot. Below is a graph of the number of hours since the parking lot opened, at 7am, on a given day (x-axis) versus the amount of money collected by the attendant (y-axis). The parking lot closes at 5pm. Using the context of the problem, describe what is happening at points A, B, and C using as much detail as possible.



IM III PT, Fall 2014: Parking Lot

Scoring Criteria for the Parking Lot Task

Scorable Parts	Points	Claims
PART 1 1. Write an expression for the area. 2. Explain the reasoning you used.	0-2 points Full credit for finding the expression $3x^2 + 10x$ square yards. Students may do this using the addition or subtraction of areas method. Reasoning should indicate which method was used, for example, "I found the area of the big rectangle and subtracted the piece that was not part of the parking lot."	Contributes evidence to Claim 2, problem solving and Claim 3, Communicating Reasoning
PART 2 3. Write an expression in terms of x to represent the value of p. 4. Explain the reasoning you used.	0-2 points Full credit for writing the expression $3x + 10$ yards to represent p. Students may explain their reasoning by narrating the problem solving process, for example, "First I used the area that I found in Part 1, then I set it equal to the area of the new rectangle $p*(x)$. After dividing both sides by x I got my expression for the value of p."	Contributes evidence to Claim 2, problem solving and Claim 3, Communicating Reasoning
PART 3 5. Can the value of z be represented as a polynomial with integer coefficients? 6. Justify your reasoning.	0-2 points Full credit for solving for concluding that **NO, z cannot be written as a polynomial with integer coefficients.** Justification may be that the student solves for z as $(2x^2+75)/(2x-5)$ and concludes that the coefficients are not integer. Another justification may be "If z is a polynomial with integer coefficients, the length of the rectangle, $2x + 15 + z$, would be a factor of the doubled area. Likewise, $2x - 5$ would be a factor of the doubled area. But $2x - 5$ is not a factor of $6x^2 + 20x$. So, $2x + 15 + z$ is not a factor either. Therefore, z cannot be represented as a polynomial with integer coefficients."	Contributes evidence to Claim 1, concepts and procedures, Claim 3, Communicating Reasoning and Claim 4, Modeling

PART 4 7. Sketch a graph of the function. 8. Write a possible equation for the function. 9. Does every point on your graph represent a legitimate time and number of cars…?	0-3 points Full credit includes a sketch like,  with appropriate labels. A possible equation is $c(t) = 0.02(t - 3)^2(t - 7)^2$, accept reasonable equations. NO, not every point represents time/"reasonable number of cars" because although time is continuously changing the number of cars must be a whole number.	Contributes evidence to Claim 2, Problem solving, Claim 3, Communicating Reasoning and Claim 4, Modeling and Data Analysis
PART 5 10. Describe what is happening at points A, B, and C.	0-2 points Full credit includes a response that demonstrates a thorough interpretation of the problem at each of the three points. For example, "Point A represents 8am at which time the attendant has collected just over $6. Many people start work around 8 am so the number of cars entering the lot after point A slows down. Point B represents 11am at which time the attendant has collected $7. There is a lull in activity at the parking lot during this time period. Point C represents 5pm, the time the parking lot closes. The attendant has collected $17 at this point. Several cars arrived in the lot between 3pm and 5pm resulting in a spike in activity."	Claim 3, Communicating Reasoning and Claim 4, Modeling and Data Analysis

Claim #1 – Concepts & Procedures

"Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency."

Claim #2 – Problem Solving

"Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies."

Claim #3 – Communicating Reasoning

"Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."

Claim #4 – Modeling and Data Analysis

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