Title

Flexible Culverts Under High Fills: Equilibrium Considerations

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FLEXIBLE CULVERTS UNDER HIGH FILLS: EQUILIBRIUM CONSIDERATIONS

A Report of an Investigation
by

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PREFACE

The work of Brown[1,2] on the subject of the loads on rigid culverts under high fills is extended to account for certain deformations of the culvert. However, the study considers only equilibrium states of the flexible culvert and the problem of buckling is ignored. The extensions of existing analytical solutions to provide for deformations of the culvert were carried out by D.R. Green. These additional matters were included in a computer program with the aid of S. Pawsey. The solution of special problems was carried out by the authors and comparisons were made with field test results and statements about the effects of construction methods, culvert rigidity and base material were possible.

Quantitative results were obtained by the use of the I.B.M. 7094 computer at the University of California, Berkeley. The field test results and material properties were provided by the Bridge Department of the Department of Highways, with the co-operation of the Materials and Research Department. Prof. J. Sackman provided remarkable liason facilities during the final stages of the work, after the departure of two of the authors from Berkeley.

The program of investigation was supported by the California State Division of Highways and The Bureau of Public Roads.

Introduction

Recently the loads on rigid culverts under high fills has been studied.[1,2] In particular, interest has been focussed on the possibility of predicting the load distribution on the culvert when the fill material, base material and method of fill were specified. Here the additional feature of the deformation of the culvert as the fill material is placed is taken into consideration. Equilibrium culvert modes are dealt with and no concern has been given to likelihood of the culvert buckling. After a brief review of the existing state of the problem the important analytical additions are developed and then employed in the solution of typical problems. These allow engineering statements to be made of the effects of culvert flexibility and comparisons with field tests.

The Problem of Incremental Effects

In the stress analysis of elastic bodies which reach their final dimensions by accretion of material the final stress and displacement fields due to gravitational loading depend upon the order and manner in which the body reaches its final state. This abscence of uniqueness is attributable to a dislocation of the Somigeliana type being formed between the added material and the existing body. With the addition of a subsequent layer of material this dislocation is healed but the final effect is that of incompatibility tensor occurring in the final description of the state of the completed body. The stresses and displacement at C(x,y) due to a layer of material added at A can be described for a plane strain case as

$$\bar{u}_i^{\,A} = \bar{u}_i^{\,A}(x,y) \qquad (1)$$
$$\bar{\sigma}_{ij}^{\,A} = \bar{\sigma}_{ij}^{\,A}(x,y)$$

A dislocation is formed at A which is healed when the next layer at A' is added. Thus an additional layer causes conditions at C as (1) except for A' instead of A. Fig. 1 illustrates this geometry. Further the construction to the final contour, F, can be similarly described. If the material addition process is continuous then the final state in the body at C may be obtained by the integration of such increments and

$$u_i = \int_c^F \bar{u}_i \, dy$$

$$\sigma_{ij} = \int_c^F \bar{\sigma}_{ij} \, dy \qquad (2)$$

These details have been fully formulated in references 3 and 4.

The presence of a culvert which is flexible also demands an incremental solution which takes account of the historical sequence of events. Consider Figure 2. As the fill is placed up the side of the culvert it takes on a shape of Fig. 2(a). The fill placed over the culvert tends to change the shape to that of Fig. 2(b). This tendency may be additionally affected by the base material under the culvert, the fill properties and the possible presence of organic back fill in the region over the crown. These suspectibilities and the proper description of the culvert deformation effects are the main considerations of this study. The extension to the previous papers (1,2) is in this element of the work. The main problem is the description of the deformation of the flexible culvert with load and the coupling of this response to the behaviour of the fill and foundation.

Modeling of Fill and Foundation

This part of the work followed the program developed previously for a rigid culvert.[1,2] Specifically the embankment with the included culvert was considered as a plane of unit thickness and the materials as being linear. From the

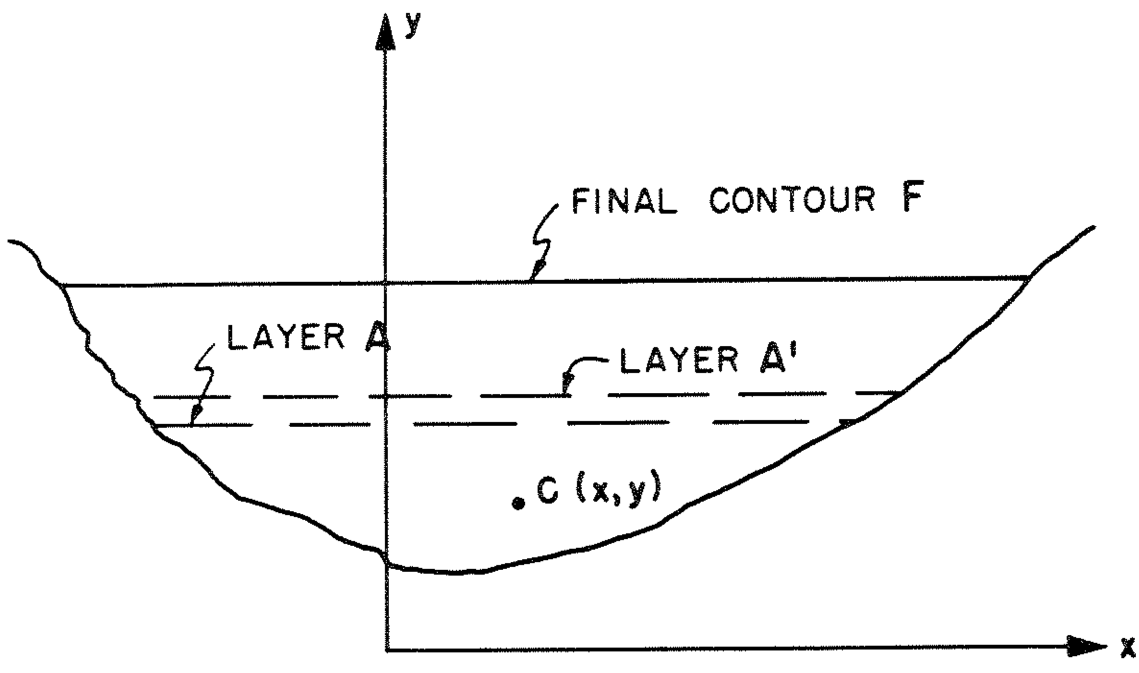


FIG. I

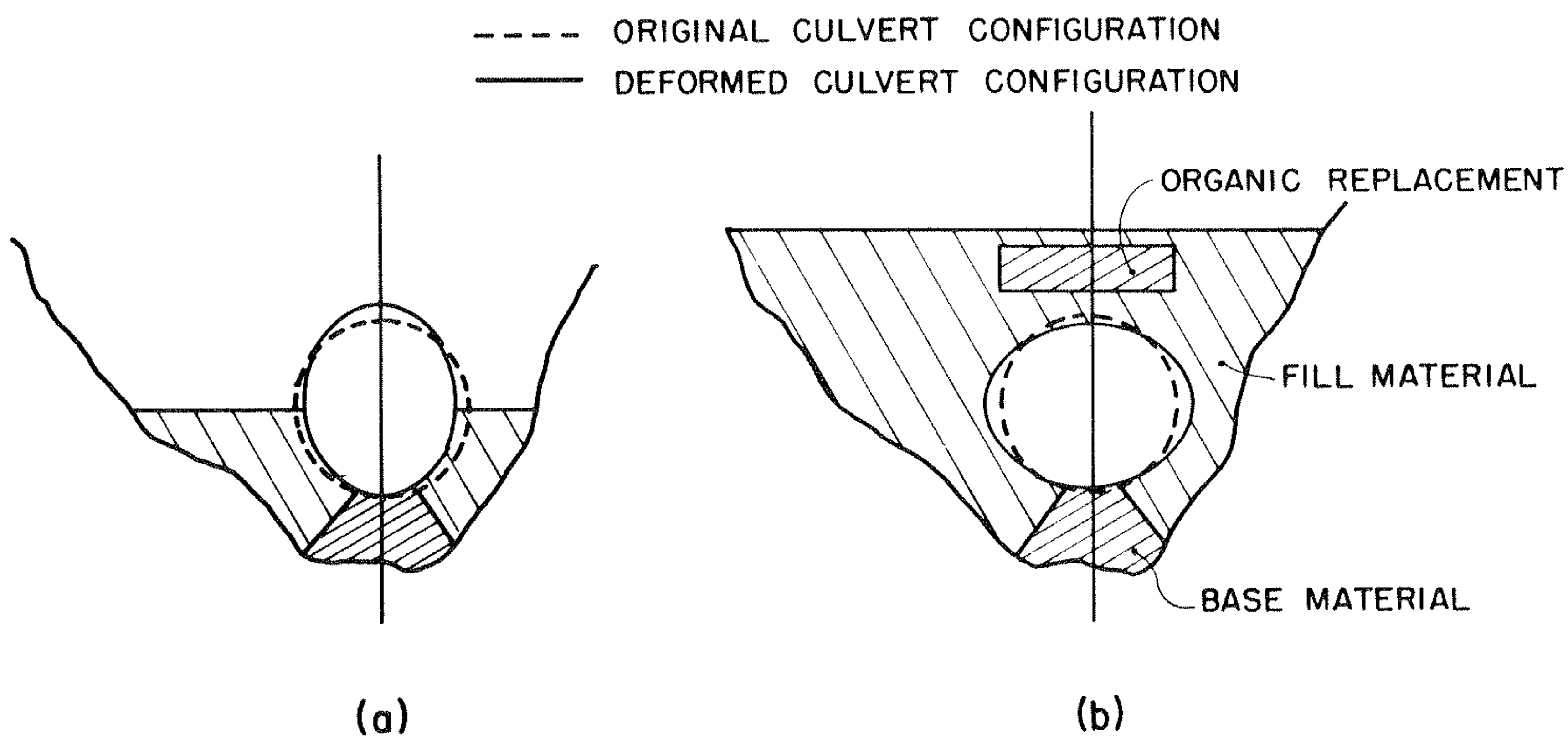

ORIGINAL CULVERT CONFIGURATION
DEFORMED CULVERT CONFIGURATION
ORGANIC REPLACEMENT
FILL MATERIAL
BASE MATERIAL
(a)
(b)

FIG. 2

experience of the analyses of references 1 and 2 the earth's surface was considered to be rigid. However, for the case when the material stiffness of the fill is of the same order as the earth on which it rests then the developments of these references allow for the involvement of this information. The common practice of placing a block of organic matter (hay) over the crown was included in the analysis. This required the consistent duplication of the construction sequence as follows:

1) construction of homogeneous fill up to the proposed top level of the organic block--determination of stresses due to this
2) trenching of the block region--the application of reverse normal and shear stresses, described in (1), on the perimeter of the block and the addition of the effects of their reversal to the stresses computed in (1)
3) addition of organic material--the dead load pressure of this material applied to the base of the organic block and the stresses from this added to those of (2).
4) continued fill above the level of the organic block--continued incremental analysis, but now on the heterogeneous existing body of fill
5) eventual rotting out of the organic block--reversal of all normal and shear stresses around the perimeter of the block due to the full height and the addition of these reversal effects to these stresses due to the full fill.

Fig. 3(a) shows the arrangement used for the analysis of the problem at Chadd Creek in Northern California. Fig. 3(b), (c) and (d) show the finite element array employed to model the fill with incremental effects at the eight levels.

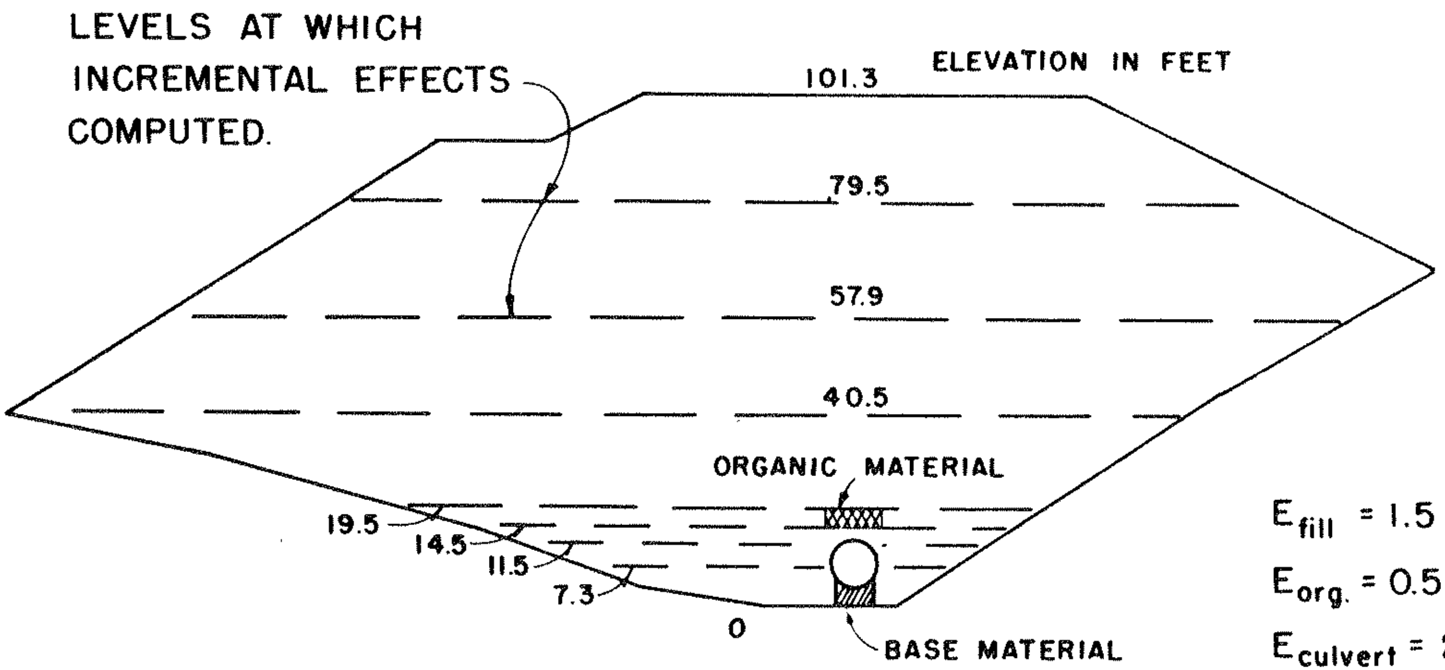


FIG. 3 (a)

$E_{fill} = 1.5 \times 10^5$ p.s.f.

$E_{org.} = 0.5 \times 10^5$ p.s.f.

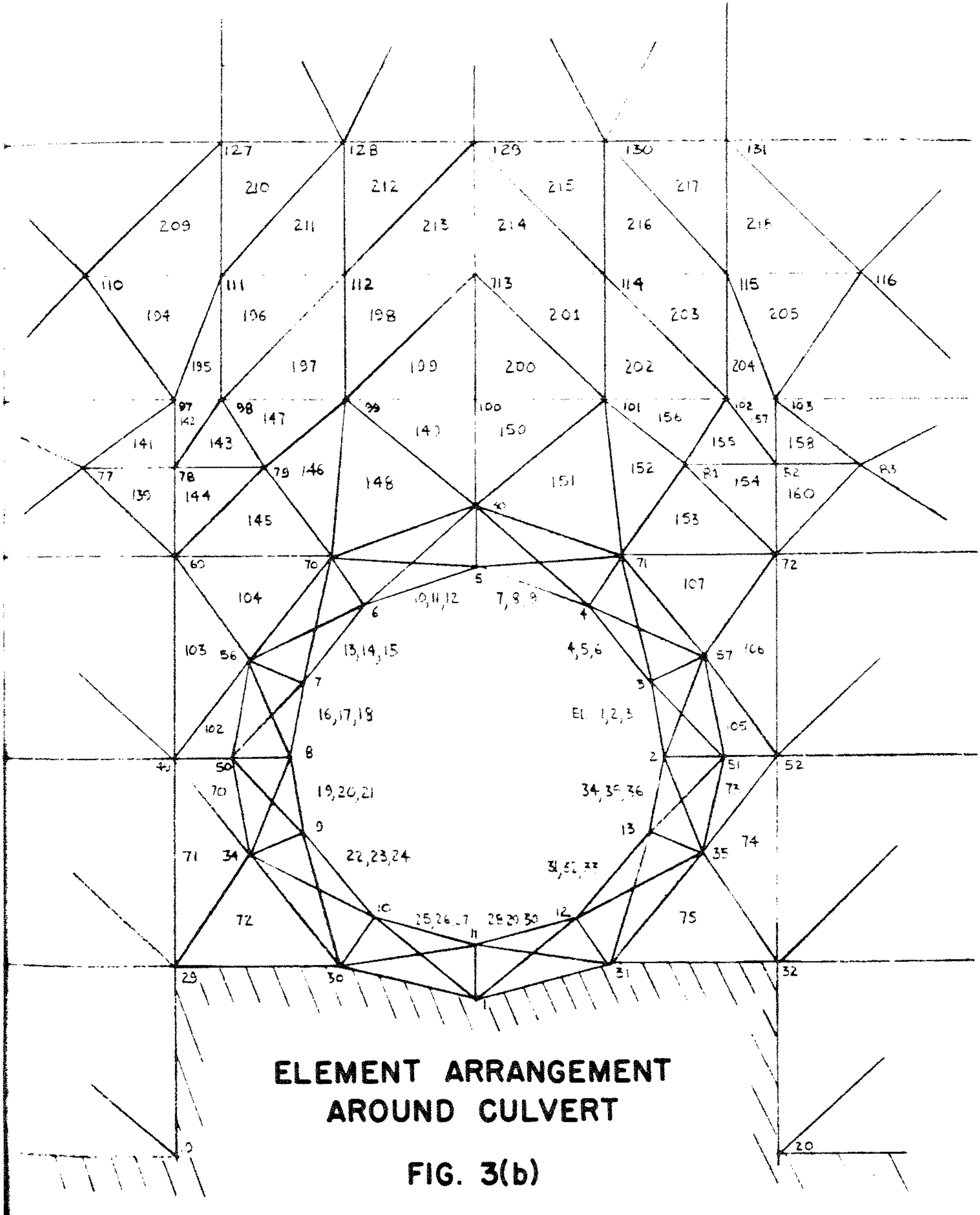
$E_{culvert} = 2.7 \times 10^7$ p.s.i.

$\nu_{fill} = \nu_{org} = 0.4$

$\nu_{culvert} = 0.3$

Culvert wall thickness = 0.2758"

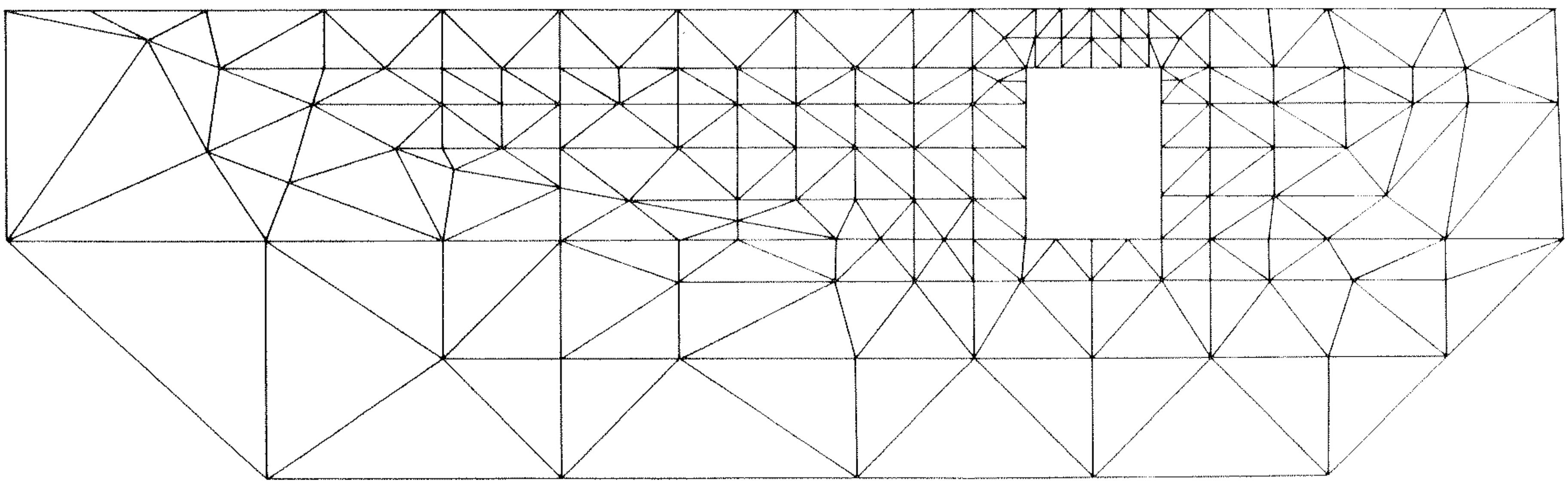
Culvert mean diameter = 9'-6"



ELEMENT ARRANGEMENT AROUND CULVERT

FIG. 3(b)

FIG. 3(c)



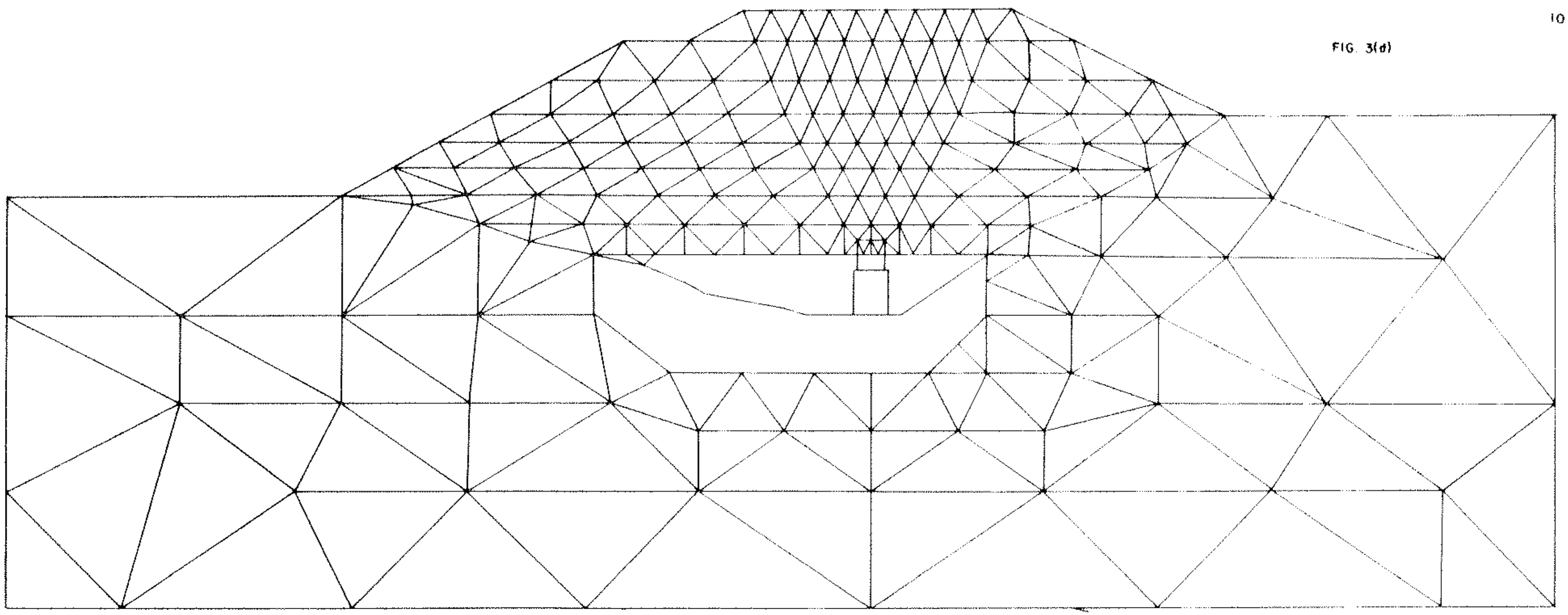


FIG. 3(d)

Modeling of the Flexible Culvert

One way to include for culvert flexibility is to replace the culvert region by a material of equivalent response. As indicated in Reference 1 this is not altogether successful. Here we choose to consider the analysis of a segment of the culvert boundary and then refer the results to a fixed coordinate system.

Consider the portion of the culvert indicated in Fig. 4(a); it is pinned at A and constrained from vertical motion with a horizontal slide at B. In the plane problem the six generalized forces F_i (i= 1 to 6) may act but

$$F_2 = f_1(F_3,F_4),\ F_5 = f_2(F_3,F_4) \tag{3}$$

and

$$F_1 = -F_6.$$

The independent forces may be chosen as F_3, F_4 and F_6. Similarly the six displacements δ_i (i = 1 to 6) may exist but

$$\delta_1 = \delta_2 = \delta_5 = 0 \tag{4}$$

leaving the interesting values δ_3, δ_4, δ_6.

Under unit loads at 3, 4 and 6 we have the nine responses at these points, namely Δ_{33}, Δ_{34}, Δ_{36}, Δ_{44}, Δ_{43}, Δ_{46}, Δ_{66}, Δ_{63}, Δ_{64}. If the internal forces are considered for this problem to be independent of the deformations then the reciprocity condition holds and

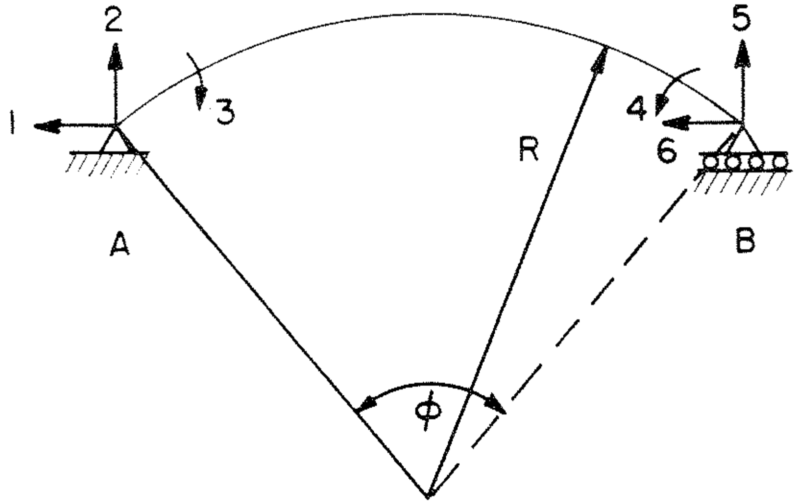
$$\Delta_{ij} = \Delta_{ji}. \tag{5}$$

With this in mind

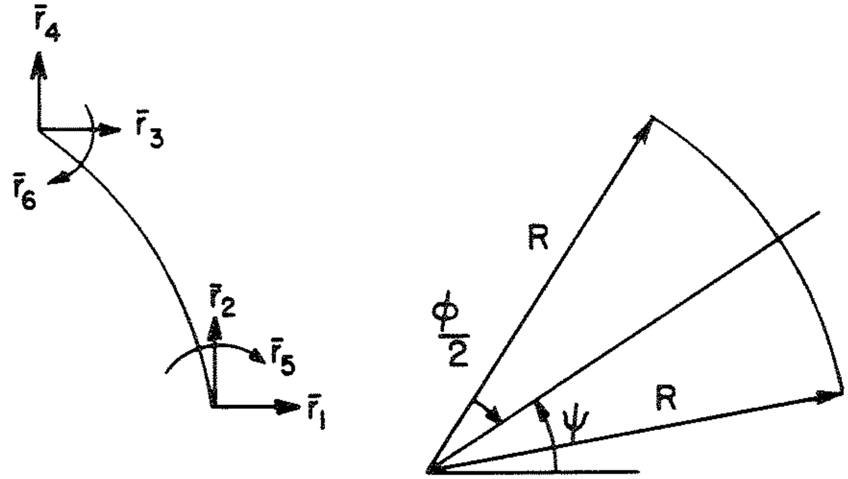
$$[\sigma] = [f][F] \tag{6}$$

where [f] is the flexibility matrix for a circular are expressed in terms of R, ϕ and EI. The inversion of (6) gives the member stiffness matrix [k];

$$[F] = [k][\sigma]. \tag{7}$$



(a)



(b)

FIG. 4

It is now necessary to refer (7) to a fixed coordinate system. Reference to Fig. -(b) indicates the manner in which the stiffness matrix may be written in the fixed U, F coordinate system in terms of six motions $\overline{r}_i$ ($i = 1$ to 6)

$$[\delta] = [a][\overline{r}] \qquad (8)$$

and therefore

$$[\overline{k}] = [a]^T[k][a] \qquad (9)$$

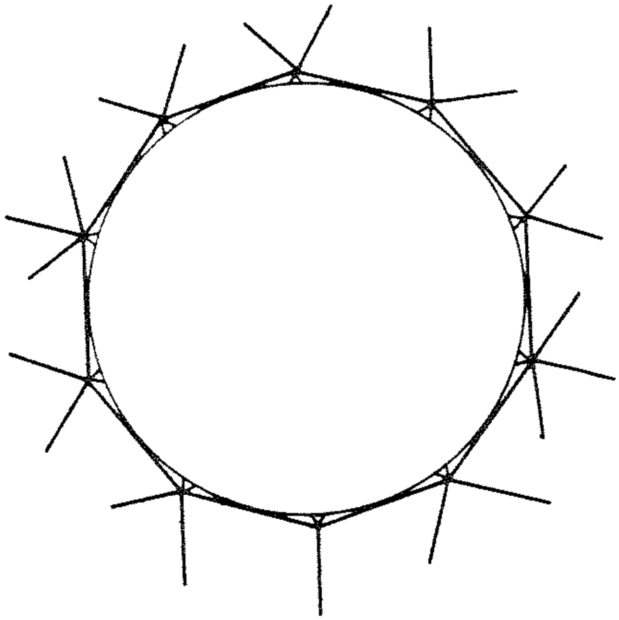
where $[\overline{k}]$ is the standard stiffness matrix in the fixed coordinate system. The details of the key equations (6) and (8) are given in the Appendix A.

Triangular Element Representation of Culvert

In order to be able to absorb the properties of the curved elements directly into a program designed for triangular two-dimensional elements, the curved member is represented by three overlapping triangular elements, whose stiffnesses are defined by suitably extracted terms of the stiffness of the curved element.

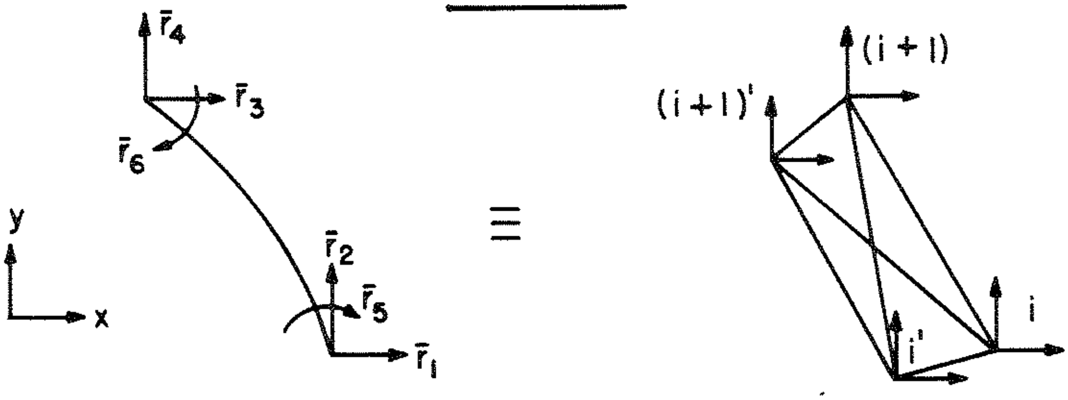
The x,y displacements of the points i, (i+1) are defined to represent those of the corresponding ends of the curved element and the x displacement of points i' and (i+1)' are defined to represent the rotations of the curved element at the respective ends. The y displacements of these latter two points have no physical meaning and for simplicity, the generalized forces associated with these degrees of freedom are set equal to zero.

The elements of the stiffness matrices of the triangles p, q and r are taken from $[\overline{k}]$ computed previously, and arranged so that the stiffness of the assemblage p+q+r matches exactly that of the curved element. Note that if only two triangles, e.g., p and r are included, then the assembled structure p+r could not reproduce the curved structure exactly, since there would be no connectivity between points i' and (i+1)' and thus the elements $\overline{k}_{16}$ and $\overline{k}_{26}$ would be zero, which is incorrect for the curved element.



MECHANICAL MODEL of FILL-CULVERT INTERACTION

FIG. 5(a)



CURVED MEMBER

THREE ELEMENT ARRAY

$p \equiv [\,i',\ i,\ (i+1)\,]$

$q \equiv [\,(i+1)',\ (i+1),\ i\,]$

$r \equiv [\,(i+1)',\ (i+1),\ i'\,]$

FIG. 5 (b)

Thus we define the stiffness of elements p,q,r as follows

Element p (i', i, i+1)

$$\begin{bmatrix} \bar{k}_{55}-1 & 0 & \bar{k}_{51} & \bar{k}_{52} & \bar{k}_{53} & \bar{k}_{54} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \bar{k}_{15} & 0 & \bar{k}_{11}-1 & \bar{k}_{12} & \bar{k}_{13} & \bar{k}_{14} \\ \bar{k}_{25} & 0 & \bar{k}_{21} & \bar{k}_{22}-1 & \bar{k}_{23} & \bar{k}_{24} \\ \bar{k}_{35} & 0 & \bar{k}_{31} & \bar{k}_{32} & \bar{k}_{33}-1 & \bar{k}_{34} \\ \bar{k}_{45} & 0 & \bar{k}_{41} & \bar{k}_{42} & \bar{k}_{43} & \bar{k}_{44}-1 \end{bmatrix}$$

Element q (i, i+1, [i+1]')

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \bar{k}_{16} & 0 \\ 0 & 1 & 0 & 0 & \bar{k}_{26} & 0 \\ 0 & 0 & 0 & 0 & \bar{k}_{36} & 0 \\ 0 & 0 & 0 & 0 & \bar{k}_{46} & 0 \\ \bar{k}_{61} & \bar{k}_{62} & \bar{k}_{63} & \bar{k}_{64} & \bar{k}_{66}-1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Element r (i', i+1, [i+1]')

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \bar{k}_{56} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \bar{k}_{65} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

then the assemblage p + q + r has stiffness

		i		(i+1)		i'		(i+1)'	
		x	y	x	y	x	y	x	y
i	x	$\bar{k}_{11}$	$\bar{k}_{12}$	$\bar{k}_{13}$	$\bar{k}_{14}$	$\bar{k}_{15}$	0	$\bar{k}_{16}$	0
	y	$\bar{k}_{21}$	$\bar{k}_{22}$	$\bar{k}_{23}$	$\bar{k}_{24}$	$\bar{k}_{25}$	0	$\bar{k}_{26}$	0
(i+1)	x	$\bar{k}_{31}$	$\bar{k}_{32}$	$\bar{k}_{33}$	$\bar{k}_{34}$	$\bar{k}_{35}$	0	$\bar{k}_{36}$	0
	y	$\bar{k}_{41}$	$\bar{k}_{42}$	$\bar{k}_{43}$	$\bar{k}_{44}$	$\bar{k}_{45}$	0	$\bar{k}_{46}$	0
i'	x	$\bar{k}_{51}$	$\bar{k}_{52}$	$\bar{k}_{53}$	$\bar{k}_{54}$	$\bar{k}_{55}$	0	$\bar{k}_{56}$	0
	y	0	0	0	0	0	2	0	0
(i+1)'	x	$\bar{k}_{61}$	$\bar{k}_{62}$	$\bar{k}_{63}$	$\bar{k}_{64}$	$\bar{k}_{65}$	0	$\bar{k}_{66}$	0
	y	0	0	0	0	0	0	0	2

When we extract rows and columns i'y and (i+1)'y which have no significance physically, we are left with the correct curved element stiffness. The computer program of refs. 1, 2 was altered to allow the culvert stiffness to be included in the analysis, as outlined above.

The full program is included in Appendix B.

Discussion of Various Analytical Results

Fig. 3(a) shows a schematic of the fill at Chadd Creek with a flexible included culvert. Fig. 6 plots the results for normal pressure on the culvert where the material directly under the culvert was considered rigid. The physical condition for each pressure line is given in the Table

Analyses	Fill	Interface Condition (Fill to Culvert)	Culvert
1	homogeneous	No slip	rigid
2	homogeneous	slip	rigid
3	hay replacement	slip	rigid
4	homogeneous	No slip	flexible
5	hay replacement	No slip	flexible

These pressure distributions allow comment on the effects of modifying the interface condition between the culvert and fill to allow for tangential slip, the effect of culvert flexibility and the effect of the hay replacement over the culvert crown.

Interface Condition

Analyses 1 and 2 allow the comparison of the extreme interface conditions of slip and no-slip. These lines are typical of those obtained in reference (1) and the sensitivity to this interface condition is apparently negligible in the rigid culvert case. It is this information which justifies the use of the analytically easier no-slip condition in the flexible culvert case.

Culvert Flexibility

Analyses 1 and 4 and analyses 3 and 5 may be compared to determine the effects of culvert flexibility. Both these comparisons indicate that the

SCALE:

1" = 10,000 p.s.f. = 70 p.s.i

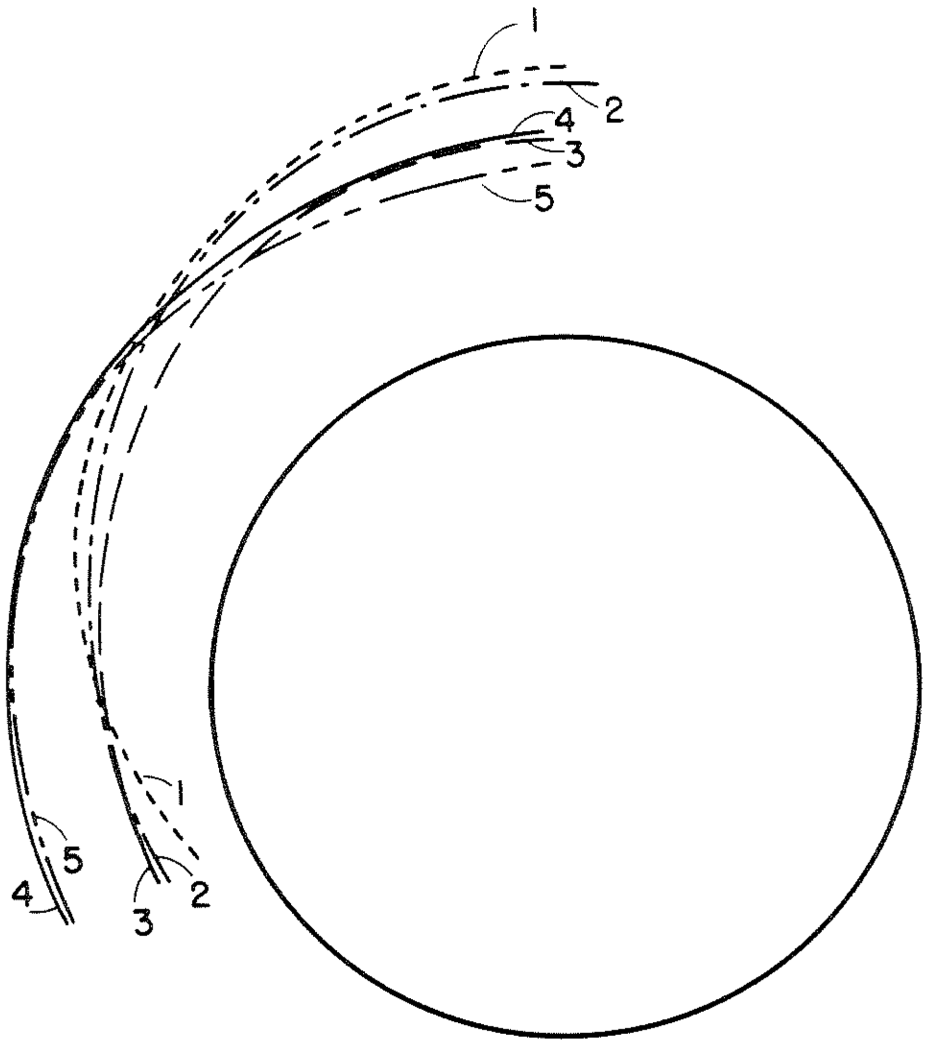


FIG. 6

arguments of Fig. 2 are correct. Essentially the deformation of the culvert results in reduction of the crown region pressures and the increase of wall pressures due to the additional inducement of passive pressures.

Hay Replacement on Flexible Culvert

Analyses 4 and 5 indicate the effect of hay replacement on a flexible culvert. The effect may be considered as increasing the local flexibility of the culvert over the crown and causing an additional reaction of the walls into the fill. The results are reduced crown pressures and increased wall pressures. It is of interest to compare this with the rigid case (Analyses 2 and 3) where the inability of the walls of the culvert to push into the fill results in minor alterations of the wall pressures. However, the hay effectively increases the crown flexibility of the rigid culvert and reduces the pressures in this region. These conclusions lead to a question concerning the effect of the stiffness of the base material directly beneath the culvert. To investigate this two cases were run on the arrangement of Fig. 3(a) with no hay inclusion. This comparison is indicated in Fig. 7 where the base material was

a) rigid (Line 1)

b) flexible with $\frac{E_{base}}{E_{fill}} = 2$ (Line 2)

The absence of a hay inclusion allows the full effect of the base stiffness to be appreciated. For regions away from the base a minor reduction in all pressures is evident. For regions involving the contact between the base and the foundation the results of analyses 2 would only be of interest; analysis 1, with a rigid base would involve an impractical pressure singularity.

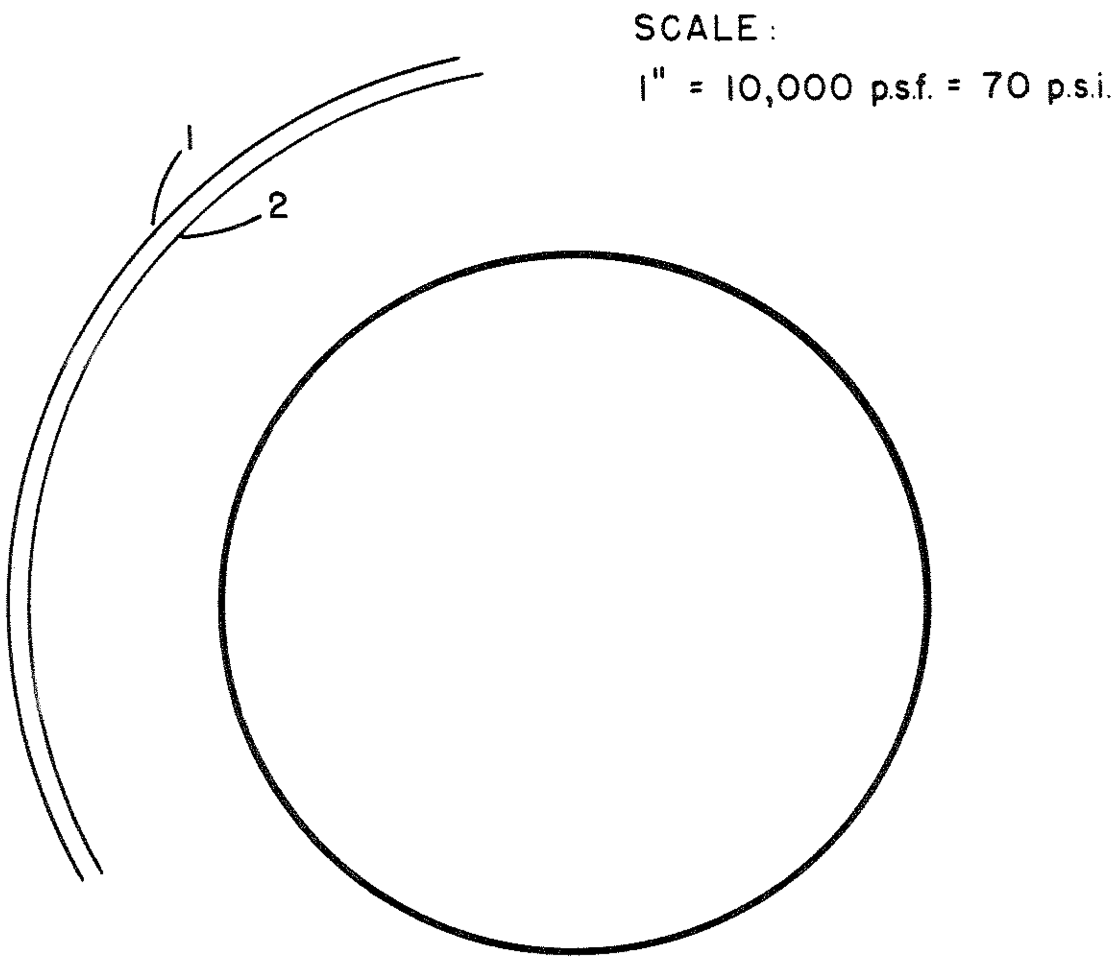


FIG. 7

A final point observed in References 1 and 2 involves the change in pressure distribution as the organic hay material rots away over a period of time. This amounts to the hay having zero stiffness and a dramatic reduction of crown pressures will be evident.

Synthesis of Analytical Results

The previous discussion indicates that the base, culvert and hay inclusion may be considered as a system the vertical flexibility of which is influenced by all the parts and the horizontal flexibility by the culvert alone. The pressures on the crown are reduced with the reduction of vertical stiffness and those on wall increased with reduced horizontal stiffness. This suggests that the effect of isotropic normal pressure may be attained by proper adjustment of the hay and base characteristics.

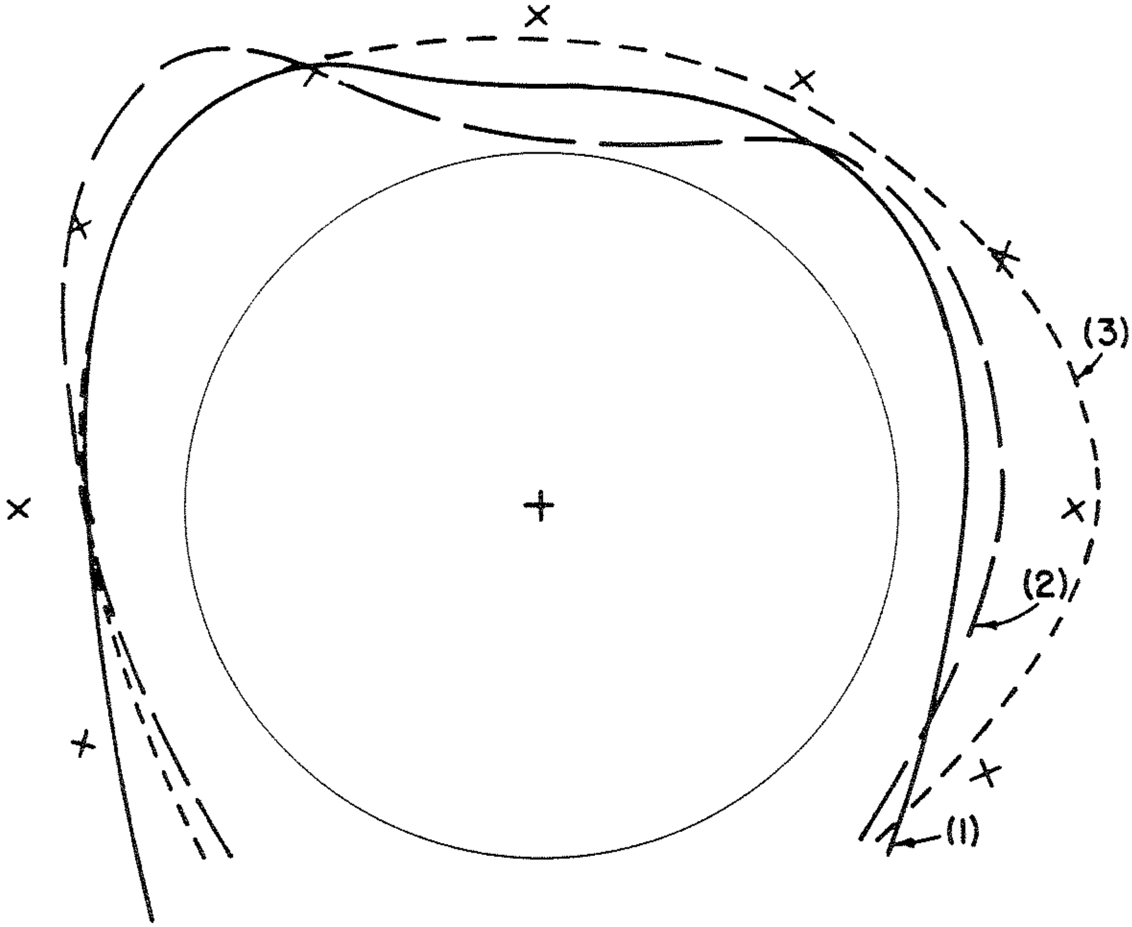
Comparison of Analysis with Field Measurements

Figure 8 shows measured pressures on the culvert at Chadd Creek with a total over-fill of 79'. The conditions above the crown are

Line 1	-1' earth	+	5' straw
Line 2	-2' earth	+	5' straw
Line 3	-2' earth	+	3' straw

Also plotted are the analytical results for a homogeneous over-fill with rigid base. A comparison with the straw inclusion and flexible base analysis may be obtained by the use of Figs. 6 and 7.

From a quantitative viewpoint it is clear that the analytical results are of the correct order of magnitude and portray the distribution of pressure fairly well. However, the most significant feature is that the analytical results are essentially symmetric about the center-line whereas the



SCALE 1" = 10,000 psf
= 70 psi

× – Computed values for homogeneous embankment with rigid base.

FIG. 8

pressure meter measurements indicate no such regularity. This would suggest that an initial absence of circularity in the culvert could lead to such results under gravity loading. For the case of an axial point load on the surface it has been shown that the pressure distribution is sensitive to the initial ellipticity of the culvert[5]. As the arguments developed here are based on a influence diagram made up of the integration of such surface effects, then it would not appear unreasonable to expect that the final pressure distribution would be gravely affected by the initial configuration and its subsequent motion. To obtain a complete picture it would be necessary to

a) know the initial configuration

b) follow the changing configuration with increase or fill.

From a practical design viewpoint it is apparent that pertinent pressure figures are obtainable from the analyses provided here. It is believed that more refined analyses would not be relevant to this work until the fill properties and behaviors are more reasonably understood.

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5. Scheffey, C. F., Private Communication

Appendix A

Equation (6) may written more fully in the form

$$\begin{bmatrix} \delta_3 \\ \delta_4 \\ \delta_6 \end{bmatrix} = \frac{R}{EI} \begin{bmatrix} \Delta_{33} & \Delta_{34} & \Delta_{36} \\ \Delta_{43} & \Delta_{44} & \Delta_{46} \\ \Delta_{63} & \Delta_{64} & \Delta_{66} \end{bmatrix} \begin{bmatrix} F_3 \\ F_4 \\ F_6 \end{bmatrix}$$

Where

$$\Delta_{33} = \Delta_{44} = \tfrac{1}{2}\alpha^2 \,(2\phi - \phi\cos\phi - \sin\phi)$$

$$\Delta_{34} = \Delta_{43} = \tfrac{1}{2}\alpha^2 \,(-\phi\cos\phi + \sin\phi)$$

$$\Delta_{36} = \Delta_{63} = \Delta_{46} = \Delta_{64} = \tfrac{1}{2} R\alpha \,(\phi\sin\phi + 2\cos\phi - 2)$$

$$\Delta_{66} = \tfrac{1}{2} R^2 \,(2\phi + \phi\cos\phi - 3\sin\phi)$$

and

$$\alpha = [2R \sin \frac{\phi}{2}]^{-1} \qquad \text{from Fig. 4.}$$

Equation (8) may be written more fully in the form

$$\begin{bmatrix} \delta_3 \\ \delta_4 \\ \delta_6 \end{bmatrix} = \begin{bmatrix} -\alpha\cos\psi & -\alpha\sin\psi & \alpha\cos\psi & \alpha\sin\psi & 0 & 1 \\ \alpha\cos\psi & \alpha\sin\psi & -\alpha\cos\psi & -\alpha\sin\psi & -1 & 0 \\ -\sin\psi & \cos\psi & \sin\psi & -\cos\psi & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{r}_1 \\ \bar{r}_2 \\ \bar{r}_3 \\ \bar{r}_4 \\ \bar{r}_5 \\ \bar{r}_6 \end{bmatrix}$$

```
$IBFTC PYPSIT  DECK
CPYPSIT LAYER ANALYSIS WITH PIPE AND FACILITY FOR REPLACING MATERIAL
C                           DESCRIPTION OF DATA FOR PYPSIT
C
C
C   INPUT
C
C
C   THE ELEMENTS MUST BE NUMBERED CONSECUTIVELY STARTING WITH THE
C   IN PLACE ELEMENTS AND CONTINUING ROW BY ROW
C
C   1ST CARD    COL.1-72     TITLE
C
C   2ND CARD                 PARAMETER ARRAY (1 CARD ONLY)
C               COL.1-4      TOTAL NUMBER OF ELEMENTS                          I4
C               COL.5-8      TOTAL NUMBER OF NODAL POINTS                      I4
C               COL.9-12     NUMBER OF ANALYSES TO PERFORM                     I4
C               COL.13-16    TOTAL NUMBER OF BOUNDARY POINTS
C               COL.17-20    FORCE UNBALANCE PRINT INTERVAL                    I4
C               COL.21-24    OUTPUT INTERVAL OF FULL RESULTS                   I4
C               COL.25-28    CYCLE LIMIT                                       I4
C               COL.29-32    FIRST NODAL POINT FOR INITIALIZATION
C                            OF DISPLACEMENTS                                  I4
C               COL.33-36    ELEMENT NO.-PRINT OF STRESSES FROM
C                            NEXT ELEMENT                                      I4
C               COL.37-38    DUMMY VARIABLE CONTROLLING OUTPUT.
C                            IF DUMMY =0 THEN PRINT INPUT MESH
C                            ARRAYS,OTHERWISE NOT                              I2
C               COL.39-40    NUMBER OF ANALYSES FOR WHICH PRINTOUT OF
C                            INTERMEDIATE CALCULATIONS DESIRED. (COMMENCING
C                            WITH FIRST ANALYSIS).                             I2
C
C   NEXT CARD                ELEMENT ARRAY (1 CARD FOR EACH ELEMENT)
C               COL.1-4      ELEMENT NUMBER                                    I4
C               COL.5-8      NUMBER OF NODAL POINT I                           I4
C               COL.9-12     NUMBER OF NODAL POINT J                           I4
C               COL.13-16    NUMBER OF NODAL POINT K                           I4
C               COL.17-28    MODULUS OF ELASTICITY                             E12.4
C                            EACH TRIANGULAR ELEMENT OF PIPE MUST
C                            HAVE NODES IN CORRECT ORDER.
C
C   NEXT CARD                NODAL POINT ARRAY (1 CARD FOR EACH NODAL PT.)
C               COL.1-4      NODAL POINT NUMBER                                I4
C               COL.5-12     X-COORDINATE OF POINT                             F8.1
C               COL.13-20    Y-COORDINATE OF POINT                             F8.1
C               COL.21-32    X-LOAD AT POINT                                   F12.2
C               COL.33-44    Y-LOAD AT POINT                                   F12.2
C               COL.45-56    INITIAL X-DISPLACEMENT                            F12.8
C               COL.57-68    INITIAL Y-DISPLACEMENT                            F12.8
C
C   NEXT CARD                BOUNDARY ARRAY (1 CARD FOR EACH POINT)
C               COL.1-4      NUMBER OF BOUNDARY POINT                          I4
C               COL.5-8      INDICATES RESTRAINT.  (0 FOR FIXED IN BOTH
C                            DIRECTIONS. 1 FOR FIXED IN X DIRECTION. 2 FOR
C                            FIXED IN Y DIRECTION.)                            I4
C               COL.9-16     SLOPE AT BOUNDARY POINT.WITH 2 IN COL.8           F8.3
C
```

```
C   NEXT CARD                      PIPE INTERIOR
C                COL.1-4           1ST ELEMENT ON INSIDE OF PIPE              I4
C                COL.5-8           LAST ELEMENT ON INSIDE OF PIPE             I4
C                COL.9-12          1ST NODAL POINT ON INSIDE OF PIP           I4
C                COL.13-16         LAST NODAL PO+NT ON INSIDE OF PIPE
C                                  NUMBER ELEMENTS AND NODAL POINTS CONSECUTIVELY
C
C   NEXT CARD                      PIPE GEOMETRY
C                COL.1-10          RADIUS OF PIPE                             F10.1
C                COL.11-20         ABSCISSA OF PIPE CENTRE                    F10.1
C                COL.21-30         ORDINATE OF PIPE CENTRE                    F10.1
C                COL.31-40         NUMBER OF INTERNAL NODES ON CULVERT        I10
C                COL.41-51         MODULUS OF ELASTICITY OF PIPE              E11.4
C                COL.52-61         THICKNESS OF PIPE WALL                     F10.5
C
C   NEXT CARD                      EXTERNAL NODES ON CULVERT LISTED FROM NODE
C                                  CORRESPONDING TO 1ST NODAL POINT INSIDE PIPE
C                                  (SEE ABOVE) AROUND CIRCUMFERENCE TO LAST
C                                  NODAL POINT INSIDE PIPE. THE 1ST POINT MUST BE
C                                  IN 1ST TWO QUADRANTS OF CIRCLE, BUT NOT AT PI
C                                                                            20I4
C
C   NEXT CARD                      MATERIAL REPLACEMENT CARD (1 CARD ONLY)
C                COL.1-4           NO. OF ELEMENTS TO TAKE OUT                I4
C                COL.5-8           NO. OF NODAL POINTS ALONG THE FREE
C                                  EDGES OF THE EXCAVATION                    I4
C                COL.9-12          ANALYSIS AT WHICH MATERIAL IS TO
C                                  BE REPLACED                                I4
C                COL.13-24         DENSITY OF REPLACEMENT MATERIAL            E12.4
C                COL.25-36         MODULUS OF ELASTICITY OF
C                                  REPLACEMENT MATERIAL                       E12.4
C                COL.37-48         ASSUMED INITIAL VERTICAL DISPLACEMENT
C                                  DUE TO REMOVAL OF MATERIAL                 E12.4
C
C   NEXT CARD                      ELEMENTS TO BE REMOVED.THE ELEMENTS ARE LISTED
C                                  20 ON EACH CARD WITH A FIXED POINT FORMAT
C                                  UNTIL ALL ELEMENTS WHICH ARE TO BE REMOVED
C                                  ARE LISTED                                 20I4
C
C   NEXT CARD                      NODAL POINTS ALONG THE FREE EDGES OF THE
C                                  EXCAVATION ARE LISTED  20 ON EACH CARD
C                                  WITH A FIXED POINT FORMAT UNTIL ALL THF FREE
C                                  NODAL POINTS ARE LISTED                    20I4
C
C   NEXT CARD                      ANALYSIS CONTROL CARD(1 CARD FOR EACH ANALYSIS)
C                COL.1-4           NUMBER OF ANALYSIS                         I4
C                COL.5-8           NO. OF LAST ELEMENT IN ANALYSIS            I4
C                                  THE ELEMENTS OF EACH ANALYSIS MUST BE NUMBERED
C                                  IN SEQUENCE
C                COL.9-12          NO.OF NP.S IN EACH ANALYSIS                I4
C                                  THE NP.S NEED NOT BE NUMBERED IN SEQUENCE FOR
C                                  EACH INDIVIDUAL ANALYSIS,BUT MUST BE IN
C                                  SEQUENCE FOR THE COMPLETE STRUCTURE
C                COL.13-16         NO. OF BOUNDARY POINTS IN ANALYSIS         I4
C                COL.17-24         POISSONS RATIO FOR ADDED LAYERS            F8.3
```

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               COL.25-34    DENSITY FOR ADDED LAYERS                          E10.4
               COL.35-44    INITIAL GUESS OF VERTICAL
                            DISPLACEMENT                                      E10.4
               COL.45-54    TOLERANCE LIMIT                                   E10.4
               COL.55-62    OVER RELAXATION FACTOR                            F8.4

  NEXT CARD                 LOAD APPLICATION CARD (1CARD FOR EACH ANAL.)
                            UNIFORMLY DISTRIBUTED LOAD IS APPLIED
                            BETWEEN THE FIRST AND THE LAST NODAL
                            POINT.TWO REGIONS OF DISTRIBUTED LOAD
                            ARE POSSIBLE ALONG A ROW OF NODAL POINTS
                            AT THE SPECIFIED ELEVATION.IF THE
                            CARD IS BLANK,GRAVITY LOAD IS APPLIED
                            TO THE STRUCTURE.
               COL.1-4      FIRST NP                                          I4
               COL.5-8      LAST NP.                                          I4
               COL.9-12     FIRST NP                                          I4
               COL.13-16    LAST NP.                                          I4
               COL.17-28    ELEVATION                                         E12.4

                            END OF DATA CARDS.

   OPTIONAL NEW PROBLEMS. REPEAT AS OFTEN AS DESIRED.

  NEXT CARD                 BLANK.  FOLLOW BY DATA CARDS FOR COMPLETELY
                            NEW PROBLEM, STARTING WITH TITLE CARD.
     DIMENSION AND COMMON STATEMENTS

      DIMENSION XORD(262), YORD(262), DSX(262), DSY(262), SIGXXT(472),
     1 MNP(265), XLOAD(262), YLOAD(262), NP(265,6), NPI(472), NPJ(472),
     2 NPK(472), NMEL(20), NMNP(20), NMRC(20), ET(472), XU(20),
     3 RO(20), SLOPE(50), NPB(50), NFIX(50), SXX(1048), SXY(1048),
     4 SYX(1048), SYY(1048), FRX(262), FRY(262), ELEV(20), SIGXX(472),
     5 SIGXY(472), SIGYY(472), LM(3), A(6,6), B(6,6), S(6,6),
     6 SIGYYT(472), SIGXYT(472), NELCUT(50), NPFREE(50), SIG(3)
      DIMENSION NRMPNT(20)
      COMMON        I2,NELP1,N,KIN,IN,NRMPNT,NRMPTS,YORD,YCENTR,XORD,
     1 XCENTR,THPIPE,RADIUS,EPIPE,S, NNPP1,NUMNP,NPP1,MNP,DSX,DSY,NPP2
      NDIM = 1048
C
C     FORMAT STATEMENTS
C
    1 FORMAT (72H1  BCD INFORMATION
     1                   )
    2 FORMAT (9I4,2I2)
    3 FORMAT (31H0TOTAL NUMBER OF ELEMENTS        =1I4/)
    4 FORMAT (31H TOTAL NUMBER OF NODAL POINTS =1I4/)
    5 FORMAT (31H NO. OF ANALYSES TO PERFORM     =1I4/)
    6 FORMAT (31H CYCLE PRINT INTERVAL           =1I4/)
    7 FORMAT (31H OUTPUT INTERVAL OF RESULTS     =1I4/)
    8 FORMAT (31H CYCLE LIMIT                    =1I4/)
    9 FORMAT (22H TOLERANCE LIMIT      =E10.3)
   10 FORMAT (22H RELAXATION FACTOR    =F6.3)
   11 FORMAT (27H1EL.    I    J    K          E     )
   12 FORMAT (4I4,E12.4)
```

```
   13 FORMAT (80H1       NP          X-ORD         Y-ORD        X-LOAD       Y-LOA
     1D        X-DISP        Y-DISP)
   14 FORMAT (1I4,2F8.1,2F12.2,2F12.8)
   15 FORMAT (40HONO. OF TERMS IN TOTAL STIFNESS ARRAYS =I6 )
   16 FORMAT (43H1RESULTS FOR ELEMENTS IN PLACE-ANALYSIS NO.I4/22HONO. O
     1F ELEMENTS      =I4/22H NO. OF NODAL POINTS =I4/22H NO. OF B.C.
     2      =I4)
   17 FORMAT (3I4,3E12.4)
   18 FORMAT (4I4,F8.3,3E10.4,F8.4)
   19 FORMAT (31H TOTAL NUMBER OF B.C.          =1I4/)
   20 FORMAT (1I8,4F12.1,2F12.8)
   21 FORMAT (2I4,1F8.3)
   22 FORMAT (20HOBOUNDARY CONDITIONS)
   23 FORMAT (32H0      CYCLE        FORCE UNBALANCE)
   24 FORMAT (1I12,1E20.6)
   25 FORMAT (42HONODAL POINT X-DISPLACEMENT Y-DISPLACEMENT)
   26 FORMAT (1I12,2E15.6)
   27 FORMAT(120H1  ELEMENT          X-STRESS         Y-STRESS        XY-STRESS
     1          MAX.STRESS      MIN.STRESS        MAX.SHEAR       DIRECTION)
   28 FORMAT (1I10,3F15.4,5X,4F15.2)
   29 FORMAT (25H1RESULTS FOR ANALYSIS NO. I4/22HONO. OF ELEMENTS      =
     1I4 /22H NO. OF NODAL POINTS =I4/ 22H NO. OF B.C.          =I4)
   30 FORMAT (1H1)
   31 FORMAT (37H-MODIFIED VERTICAL LOADS AT ELEVATION F8.2/13H NP.   YL
     1OAD )
   32 FORMAT (15H-TOTAL STRESSES /56H   ELEMENT         X-STRESS        Y-S
     1TRESS       XY-STRESS   )
   33 FORMAT (22H POISSONS RATIO      =F6.3/22H DENSITY               =E10
     1.3)
   34 FORMAT (22H1ANALYSIS OF THE FIRST I3,8H LAYERS )
   35 FORMAT (39H MATERIAL IS REMOVED AND LOADS MODIFIED /20H NP.   XLOA
     1D   YLOAD   )
   36 FORMAT (36H PIT IS FILLED WITH ORGANIC MATERIAL/9H DENSITY=E12.4/
     19H E       =E12.4)
   37 FORMAT (20I4)
   38 FORMAT (I8,2F12.4)
   39 FORMAT (32H-     NP.        XLOAD        YLOAD    )
   41 FORMAT (E12.4)
   42 FORMAT(1H0,24HELEMENT REPLACEMENT DATA)
   43 FORMAT(1I17,E15.6)
   44 FORMAT(4I4)
  497 FORMAT(24H NODES ON PIPE. (NRMPNT))
  499 FORMAT(59H1  RADIUS    XCENTR    YCENTR    NRMPTS    EPIPE     THP
     1IPE)
  501 FORMAT(20I4)
  711 FORMAT (32HOZERO OR NEGATIVE AREA, EL. NO.=1I4)
  712 FORMAT (36HOMORE THAN 7 POINTS CONNECTED TO NP. I4)
  800 FORMAT(3F10.1,I10,E11.4,F10.5)
  801 FORMAT(F4.2)
C
C     READ AND PRINT INPUT DATA
C
 1000 PRINT 30
      READ 1
      PRINT 1
```

```
      READ 2,NUMELT,NUMNPT,NANAL ,NUMBCT,NCPIN,NOPIN,NCYCM,NFROM,
     1 NBLOCK,I1,I2
      PRINT 3,NUMELT
      PRINT 4,NUMNPT
      PRINT 5,NANAL
      PRINT 19,NUMBCT
      PRINT 6,NCPIN
      PRINT 7,NOPIN
      PRINT 8,NCYCM
      READ 12,(N,NPI(N),NPJ(N),NPK(N),ET(N),              N=1,NUMELT)
  111 CONTINUE
      READ 14,(M,XORD(M),YORD(M),XLOAD(M),YLOAD(M),DSX(M),DSY(M),M=1,NUM
     1NPT)
      READ 21,(NPB(L),NFIX(L),SLOPE(L),L=1,NUMBCT)
      READ 44,NELP1,NELP2, NPP1, NPP2
      READ 800, RADIUS, XCENTR, YCENTR, NRMPTS,EPIPE, THPIPE
      READ 501, (NRMPNT(I), I=1,NRMPTS)
      IF(I1) 105,100,105
  100 PRINT 11
      PRINT 12,(N,NPI(N),NPJ(N),NPK(N),ET(N),              N=1,NUMELT)
      PRINT 13
      PRINT 20,(M,XORD(M),YORD(M),XLOAD(M),YLOAD(M),DSX(M),DSY(M),M=1,NU
     1MNPT)
      PRINT 22
      PRINT 21,(NPB(L),NFIX(L),SLOPE(L),L=1,NUMBCT)
      PRINT 499
      PRINT 800, RADIUS,XCENTR,YCENTR,NRMPTS,EPIPE,THPIPE
      PRINT 497
      PRINT 501,(NRMPNT(J),J=1,NRMPTS)
  105 READ 17,NOUT,NFREE,NTO,ROR,ETORG,DORG
      PRINT 17,NOUT,NFREE,NTO,ROR,ETORG,DORG
      READ 37,  (NELOUT(I),I=1,NOUT)
      PRINT 37, (NELOUT(I),I=1,NOUT)
      READ 37, (NPFREE(I),I=1,NFREE)
      PRINT 37,(NPFREE(I),I=1,NFREE)
C
C     INITIALIZATION OF ANALYSIS OF COMPLETE STRUCTURE
C
  110 NCTAG= 8
      IFLAG=0
      NAL=0
      NNN=0
      NID=0
      DO 101 I=1,NUMNPT
      DO 101 J=1,8
  101 NP(I,J) = 0
C
C     INITIALIZATION OF INDIVIDUAL ANALYSIS
C
  120 READ 18 ,(NAL),(NMEL(NAL),NMNP(NAL),NMBC(NAL),XU(NAL),RO(NAL),DISP
     1L,TOLER,XFAC)
      READ 12,NMNP1,NMNP2,NMNP3,NMNP4,ELEV(NAL)
      NUMEL=NMEL(NAL)
      NUMNP=NMNP(NAL)
      NUMBC=NMBC(NAL)
```

```
      IF (NAL-1) 125,122,125
  122 NNUMEL=1
      PRINT 16,NAL,NUMEL,NUMNP,NUMBC
      GO TO 127
  125 NNUMEL=NMEL(NAL-1)+1
      PRINT 29,NAL,NUMEL,NUMNP,NUMBC
  127 PRINT 10,XFAC
      PRINT 33,XU(NAL),RO(NAL)
      PRINT 9,TOLER
      IF (NMNP2) 145,145,139
C
C     UNIT LOAD MODIFICATION
C
  139 PRINT 31 ,ELEV(NAL)
  143 DO 142 N=NMNP1,NMNP2
      IF (N-NMNP2 ) 140,141,141
  140 UNITL=ABS ((XORD(N+1)-XORD(N)) /2.0)
      YLOAD(N)=YLOAD(N)-UNITL
      YLOAD(N+1)=YLOAD(N+1)-UNITL
  141 PRINT 14,N,YLOAD(N)
  142 CONTINUE
      IF (NMNP3) 145,145,144
  144 NMNP1=NMNP3
      NMNP2=NMNP4
      NMNP3=0
      GO TO 143
  145 IF (NMNP2) 146,146,129
C
C     DEAD LOAD MODIFICATION
C
  146 NFEL=NNUMEL
      NLEL=NUMEL
      RORG=RO(NAL)
  147 DO 161 M=NFEL,NLEL
      IF (NMNP2) 156,157,156
  156 N=NELOUT(M)
      GO TO 158
  157 N=M
      IF(N-NELP1)158,148,148
  148 IF(N-NELP2)161,161,158
  158 I=NPI(N)
      J=NPJ(N)
      K=NPK(N)
      AJ=XORD(J)-XORD(I)
      AK=XORD(K)-XORD(I)
      BJ=YORD(J)-YORD(I)
      BK=YORD(K)-YORD(I)
      AREA=(AJ*BK-AK*BJ)/2.0
  159 DL=AREA*RORG /3.0
  160 YLOAD(I)=YLOAD(I)-DL
      YLOAD(J)=YLOAD(J)-DL
      YLOAD(K)=YLOAD(K)-DL
  161 CONTINUE
      PRINT 39
      PRINT 38,(M,XLOAD(M),YLOAD(M),M=1,NUMNP)
```

```
  129 NCYCLE=0
      NUMPT=NCPIN
      NUMOPT=NOPIN
      DO 130 L=1,NDIM
      SXX(L)=0.0
      SXY(L)=0.0
      SYX(L)=0.0
  130 SYY(L)=0.0
C
C     THE TAG ARRAY FOR MAPPING OF TOTAL STIFNESSES IS FORMED
C
  162 DO 170 N=1,NUMEL
      LM(1)=NPI(N)
      LM(2)=NPJ(N)
      LM(3)=NPK(N)
      DO 170 II=1,3
      MS=LM(II)
      NP(MS,1)=MS
      DO 170 JJ=1,3
C
      IF (MS-LM(JJ)) 165,170,170
  165 DO 168 LS=2,NCTAG
  166 IF (NP(MS,LS)-LM(JJ)) 167,170,167
  167 IF (NP(MS,LS)) 163,169,163
  163 IF (LS-NCTAG) 168,164,164
  164 IFLAG=1
      PRINT 712,MS
  168 CONTINUE
C
C     IF NP IS ZERO,STORE LM(JJ)
  169 NP(MS,LS)=LM(JJ)
  170 CONTINUE
C**********************************************************************
      IF(I2)1001,1002,1001
 1001 PRINT 350
      PRINT 351,((NP(I,J),J=1,8),I=1,NUMNPT)
  350 FORMAT(13H11ST NP ARRAY//)
  351 FORMAT(8I6)
C**********************************************************************
 1002 IF (IFLAG) 442,153,442
C
C     CONSECUTIVE NUMBERING OF NODAL POINT LABELLING ARRAY
C
  153 L=0
      DO 155 N=1,NUMNPT
      IF (NP(N,1)) 155,155,154
  154 L=L+1
      MNP(L)=NP(N,1)
  155 CONTINUE
      MNP(NUMNP+1)=MNP(NUMNP)+1
C**********************************************************************
      IF(I2)1003,1004,1003
 1003 PRINT 352
      PRINT 353,(MNP(I),I=1,L),MNP(NUMNP+1)
  352 FORMAT(10H1MNP ARRAY//)
```

```
  353 FORMAT(I6)
C*********************************************************************************
C
C      COUNTING ADJACENT NODAL POINTS,THE COUNT IS STORED
C      IN THE FIRST COLUMN OF THE TAG ARRAY
C
 1004 NP(1,1)=1
      DO 175 M=1,NUMNP
      I=MNP(M)
      IN=MNP(M+1)
      N=1
  171 N=N+1
      IF (NP(I,N)) 174,174,172
  172 IF (N-NCTAG)   171,173,173
  173 NP(IN,1)=N+NP(I,1)
      GO TO 175
  174 NP(IN,1)=N+NP(I,1)-1
  175 CONTINUE
C*********************************************************************************
      IF(I2)1005,1006,1005
 1005 PRINT 354
      PRINT 351,((NP(I,J),J=1,8),I=1,NUMNPT)
  354 FORMAT(13H12ND NP ARRAY)
C*********************************************************************************
C
C      PRINTOUT OF NUMBER OF CONNECTED NODAL POINTS.THIS SHOULD EQUAL THE
C      NUMBER OF TERMS IN EACH OF THE TOTAL STIFNESS ARRAYS AND NOT
C      EXCEED NDIM.
C
 1006 N=MNP(NUMNP)
      NUMTOT=NP(N,1)
      PRINT 15,NUMTOT
      IF (NDIM-NUMTOT)  442,176,176
C
C      FORMATION OF STIFFNESS ARRAY
C
  176 M=0
      NNELP1 = NELP1
  178 M=M+1
      NLST=NMEL(M)
C INITIALIZATION FOR PIPE STIFFNESS CALCULATION
      NNPP1 = NPP1
      KIK=0
      IN = 0
      PSI=0.0
      IF (M  -1) 180,179,180
  179 NFST=1
      GO TO 181
  180 NFST=NMEL(M-1)+1
  181 DO 199 N=NFST,NLST
      N=N
      IF(NELP1-N) 521,522,521
  522 CALL PYP
      GO TO 531
  521 I=NPI(N)
```

```
      J=NPJ(N)
      K=NPK(N)
      AJ=XORD(J)-XORD(I)
      AK=XORD(K)-XORD(I)
      BJ=YORD(J)-YORD(I)
      BK=YORD(K)-YORD(I)
      AREA=(AJ*BK-AK*BJ)/2.0
C
C*************************************************************************
      IF(I2)1007,1008,1007
 1007 PRINT 507,N
  507 FORMAT( 9H0ELEMENT ,I3,31H   PROCESSED BY OLD PSIT ONLY  )
C*************************************************************************
C      PRINT OF ERRORS IN INPUT DATA
 1008 IF (AREA) 701,701,700
  700 IF (IFLAG) 702,702,199
  701 PRINT 711,N
      IFLAG=1
      GO TO 199
  702 COMM=0.25*ET(N)/((1.-XU(M)**2)*AREA)
      A(1,1)=BJ-BK
      A(1,2)=0.0
      A(1,3)=BK
      A(1,4)=0.0
      A(1,5)=-BJ
      A(1,6)=0.0
      A(2,1)=0.0
      A(2,2)=AK-AJ
      A(2,3)=0.0
      A(2,4)=-AK
      A(2,5)=0.0
      A(2,6)=AJ
      A(3,1)=AK-AJ
      A(3,2)=BJ-BK
      A(3,3)=-AK
      A(3,4)=BK
      A(3,5)=AJ
      A(3,6)=-BJ
      IF  (NFST-NLST) 703,520,703
  703 B(1,1)=COMM
      B(1,2)=COMM*XU(M)
      B(1,3)=0.0
      B(2,1)=COMM*XU(M)
      B(2,2)=COMM
      B(2,3)=0.0
      B(3,1)=0.0
      B(3,2)=0.0
      B(3,3)=COMM*(1.-XU(M))*.5
C
      DO 182 JJ=1,6
      DO 182 II=1,3
      S(II,JJ)=0.0
      DO 182 KK=1,3
  182 S(II,JJ)=S(II,JJ)+B( II,KK)*A(KK,JJ)
      DO 183 JJ=1,6
```

```
      DO 183 II=1,3
  183 B(JJ,II)=S(II,JJ)
      DO 184 JJ=1,6
      DO 184 II=1,6
      S(II,JJ)=0.0
      DO 184 KK=1,3
  184 S(II,JJ)=S(II,JJ )+B(II,KK)*A(KK,JJ)
  531 CONTINUE
C*********************************************************************
      IF(I2)1009,1010,1009
 1009 PRINT 506, N, ((S(K,J),J=1,6),K=1,6)
  506 FORMAT(26HOS MATRIX FOR        ELEMENT,I4/(6E12.2))
C*********************************************************************
C
C      SEARCHING FOR AND STORING NONZERO TERMS OF THE
C      TOTAL STIFNESS ARRAY
C
 1010 LM(1)=NPI(N)
      LM(2)=NPJ(N)
      LM(3)=NPK(N)
      DO 198 II=1,3
      KS=LM(II)
      DO 198 JJ=1,3
      NS=NP(KS,1)
      LS=LM(JJ)
      IF (KS-LS) 186,195,198
  186 DO 188 MS=2,NCTAG
  187 NS=NS+1
      IF (NP(KS,MS)-LS) 188,195,188
  188 CONTINUE
  195 SXX(NS)=SXX(NS)+S(2*II-1,2*JJ-1)
      SXY(NS)=SXY(NS)+S(2*II-1,2*JJ  )
      SYX(NS)=SYX(NS)+S(2*II  ,2*JJ-1)
      SYY(NS)=SYY(NS)+S(2*II  ,2*JJ  )
      IF(NELP1-NELP2) 198,198,532
  532 NELP1 = NELP1-1
  198 CONTINUE
  199 CONTINUE
C*********************************************************************
      IF(I2)1011,1012,1011
 1011 PRINT 802
  802 FORMAT(44H1    SXX          SXY          SYX          SYY)
      PRINT 803,(SXX(K),SXY(K),SYX(K),SYY(K),K=1,NUMTOT)
  803 FORMAT(4E12.2)
C*********************************************************************
 1012 IF (M-NAL) 178,200,200
  200 CONTINUE
      IF (IFLAG) 442,201,442
C
C      INITIALIZATION OF DISPLACEMENTS
C
  201 DO 205 I=NFROM,NUMNP
      N=MNP(I)
      IF (NID-1)  204,202,203
  202 DSY(N)=DORG
```

```
      GO TO 205
  203 DSY(N)=-DSY(N)
      GO TO 205
  204 DSY(N)=DISPL
  205 DSX(N)=0.0
C
C     INVERSION OF NODAL POINT STIFNESSES
C
      DO 210 I=1,NUMNP
      M=MNP(I)
      N=NP( M,1)
      COMM =SXX(N)*SYY(N)-SXY(N)*SYX(N)
      IF (COMM) 208,209,208
  208 TEM =SYY(N)/COMM
      SYY(N)=SXX(N)/COMM
      SXX(N)=TEM
      SXY(N)=-SXY(N)/COMM
      SYX(N)=-SYX(N)/COMM
  209 FRX(M)=XLOAD(M)
  210 FRY(M)=YLOAD(M)
C*****************************************************************
      IF(I2)1013,1014,1013
 1013 PRINT 802
      PRINT 803,(SXX(K),SXY(K),SYX(K),SYY(K),K=1,NUMTOT)
C*****************************************************************
C
C     MODIFICATION OF BOUNDARY FLEXIBILITIES
C
 1014 DO 240 L=1,NUMBC
      M=NPB(L)
      N=IABS (NP(M,1))
      NP(M,1)=-NP(M,1)
      IF(NFIX(L)-1) 225,220,215
  215 C=(SXX(N   )*SLOPE(L)-SXY(N   ))/(SYX(N   )*SLOPE(L)-SYY(N   ))
      R=1.-C*SLOPE(L)
      SXX(N   )=(SXX(N   )-C*SYX(N   ))/R
      SXY(N   )=(SXY(N   )-C*SYY(N   ))/R
      SYX(N   )=SXX(N   )*SLOPE(L)
       SYY(N   )=SXY(N   )*SLOPE(L)
      DSY(M)=0.0
      GO TO 240
  220 SYY(N   )=SYY(N   )-SYX(N   )*SXY(N   )/SXX(N   )
      DSX(M)=0.0
      GO TO 230
  225 SYY(N   )=0.0
      DSY(M)=0.0
      DSX(M)=0.0
  230 SXX(N   )=0.0
  235 SXY(N   )=0.0
      SYX(N   )=0.0
  240 CONTINUE
C*****************************************************************
      IF(I2)1015,1016,1015
 1015 PRINT 802
      PRINT 803,(SXX(K),SXY(K),SYX(K),SYY(K),K=1,NUMTOT)
```

```
 1016 IF(I2 .EQ. 0) GO TO 243
      I2=I2-1
C*********************************************************************
C
C     ITERATION OF NODAL POINT DISPLACEMENTS
C
  243 PRINT 23
  245 SUM=0.0
      DO 290 I=1,NUMNP
      N=MNP(I)
      NN=MNP(I+1)
C
C     NODAL POINT N IS RELAXED
      NM=IABS ( NP(N,1))
      IF  (SXX(NM)+SYY(NM)) 249,290,249
  249 NAP=IABS (NP(NN ,1))-IABS (NP(N,1))-1
      IF  (NAP) 260,260,250
  250 DO 255 LL=1,NAP
      NB=NP(N,LL+1 )
      M =LL+NM
      FRX(N)=FRX(N)-SXX(M)*DSX(NB)-  SXY(M)*DSY(NB)
  255 FRY(N)=FRY(N)-SYX(M)*DSX(NB)-  SYY(M)*DSY(NB)
  260 DX=SXX(NM)*FRX(N)-DSX(N)+SXY(NM)*FRY(N)
      DY=SYX(NM)*FRX(N)-DSY(N)+SYY(NM)*FRY(N)
      DSX(N) =DSX(N) +XFAC*DX
      DSY(N) =DSY(N) +XFAC*DY
      IF  (NP(N,1)) 265,262,262
  262 SUM=SUM+ABS (DX/SXX(NM))+ABS (DY/SYY(NM))
C
C     NODAL POINTS CONNECTED TO NODAL POINT N ARE RELAXED
      IF  (NAP) 280,280,265
  265 DO 275 LL=1,NAP
      NB=NP(N,LL+1)
      M=LL+NM
      FRX(NB)=      -SXX(M)*DSX(N )-  SYX(M)*DSY(N )+FRX(NB)
  275 FRY(NB)=      -SXY(M)*DSX(N )-  SYY(M)*DSY(N )+FRY(NB)
  280 FRX(N)=XLOAD(N)
      FRY(N)=YLOAD(N)
  290 CONTINUE
C
C     CYCLE COUNT AND PRINT CHECK
C
      NCHECK=0
      NCYCLE=NCYCLE +1
      IF  (NCYCLE-NUMPT)301,300,300
  300 NUMPT=NUMPT+NCPIN
      PRINT 24,NCYCLE,SUM
  301 IF  (SUM-TOLER)305,305,302
  302 IF  (NCYCM-NCYCLE)305,305,303
  303 NCHECK=1
      IF  (NCYCLE-NUMOPT)245,304,304
  304 NUMOPT=NUMOPT+NOPIN
C
C     PRINT OF DISPLACEMENTS AND STRESSES
C
```

```
305 PRINT 25
    DO 307 I=1,NUMNP
    M=MNP(I)
307 PRINT 26, M, DSX(M),DSY(M)
    PRINT 27
    DO 421 M=1,NAL
    NLST=NMEL(M)
    IF (M  -1) 311,310,311
310 NFST=1+NBLOCK
    GO TO 312
311 NFST=NMEL(M-1)+1
312 DO 421 N=NFST,NLST
    IF(N-NNELP1) 407,406,406
406 IF(N-NELP2) 421,421,407
407 I = NPI(N)
    J=NPJ(N)
    K=NPK(N)
    AJ=XORD(J)-XORD(I)
    AK=XORD(K)-XORD(I)
    BJ=YORD(J)-YORD(I)
    BK=YORD(K)-YORD(I)
    EPX=(BJ-BK)*DSX(I)+BK*DSX(J)-BJ*DSX(K)
    EPY=(AK-AJ)*DSY(I)-AK*DSY(J)+AJ*DSY(K)
    GAM=(AK-AJ)*DSX(I)-AK*DSX(J)+AJ*DSX(K)+(BJ-BK)*DSY(I)+BK*DSY(J)-BJ
   1*DSY(K)
    COMM=ET(N)/((1.-XU(M)**2)*(AJ*BK-AK*BJ))
    X=COMM*(EPX+XU(M)*EPY)
    Y=COMM*(EPY+XU(M)*EPX)
    XY=COMM*GAM*(1.-XU(M))*.5
    IF (NCHECK) 321,317,321
317 IF (NMNP2) 319,328,319
319 IF (NNN-1) 320,328,328
320 IF (NAL-M) 322,322,323
322 CG=(YORD(I)+YORD(J)+YORD(K))/3.0
    SIGYY(N)=-1.0
    GO TO 324
323 CG=ELEV(NAL-1)
324 TMUL=(ELEV(NAL)-CG)*RO(NAL)/2.0
    SIGYYT(N)=SIGYYT(N)+(SIGYY(N)+Y)*TMUL
    SIGXXT(N)=SIGXXT(N)+(SIGXX(N)+X)*TMUL
    SIGXYT(N)=SIGXYT(N)+(SIGXY(N)+XY)*TMUL
    SIGXX(N)=X
    SIGYY(N)=Y
    SIGXY(N)=XY
    GO TO 321
328 SIGXXT(N)=SIGXXT(N)+X
    SIGYYT(N)=SIGYYT(N)+Y
    SIGXYT(N)=SIGXYT(N)+XY
321 C=(X+Y)/2.0
    R=SQRT (((Y-X)/2.0)**2+XY**2)
    XMAX=C+R
    XMIN=C-R
    TMAX=(XMAX-XMIN)/2.0
    IF (Y-X) 325,326,325
325 PA=0.5*57.29578*ATAN  ( 2.* XY/(Y-X))
```

```
      GO TO 327
  326 PA=90.0
  327 IF (2.*X-XMAX-XMIN) 405,420,420
  405 IF (PA) 410,420,415
  410 PA=PA+90.0
      GO TO 420
  415 PA=PA-90.0
  420 L=N-NBLOCK
      PRINT 28,(L,X,Y,XY,XMAX,XMIN,TMAX,PA)
  421 CONTINUE
C
      IF (NCHECK) 431,431,243
  431 PRINT 32
      NFST=NBLOCK+1
      DO 433 N=NFST,NUMEL
      L=N-NBLOCK
      IF(N-NNELP1) 425,429,429
  429 IF(N-NELP2) 433,433,425
  425 PRINT 28, L,SIGXXT(N),SIGYYT(N),SIGXYT(N)
  433 CONTINUE
      NELP1 = NNELP1
      NPP1 = NNPP1
C
C     LOADS SET TO ZERO
C
      DO 435 I=1,NUMNP
      M=MNP(I)
      XLOAD(M)=0.0
      YLOAD(M)=0.0
      FRX(M)=0.0
      FRY(M)=0.0
  435 CONTINUE
      N=MNP(NUMNP+1)
      NP(N,1)=0
      IF (NAL -NTO) 440,500,440
C
C     MODIFICATION OF LOAD DUE TO REMOVAL OF MATERIAL
C
  500 NN=0
      IF (NNN-1) 505,580,502
  502 NNN=0
      NID=0
      DO 503 I=1,NFROM
      DSY(I)=0.0
  503 DSX(I)=0.0
      GO TO 440
  505 PRINT 34, NAL
      PRINT 35
  510 NN=NN+1
      LL=NELOUT(NN)
      ET(LL)=0.0
      NFST=LL
      NLST=LL
      M=NTO
      GO TO 181
```

```
  520 DL=AREA*RO(NTO)/3.0
      SIG(1)=SIGXXT(LL)
      SIG(2)=SIGYYT(LL)
      SIG(3)=SIGXYT(LL)
      LM(1)=NPI(LL)
      LM(2)=NPJ(LL)
      LM(3)=NPK(LL)
      DO 530 JJ=1,3
      M=LM(JJ)
      FRY(M)=FRY(M)+DL
      DO 530 II=1,3
      FRX(M)=A(II,2*JJ-1)*SIG(II)/2.0+FRX(M)
      FRY(M)=A(II,2*JJ)*SIG(II)/2.0+FRY(M)
  530 CONTINUE
      IF (NN-NOUT) 510,540,540
  540 DO 570 M=1,NFREE
      N=NPFREE(M)
      XLOAD(N)= FRX(N)
      YLOAD(N)= FRY(N)
  570 PRINT 14, N,XLOAD(N),YLOAD(N)
      NNN=1
      NID=1
      GO TO 129
C
C     REPLACING MATERIAL
C
  580 DO 590 N=1,NOUT
      L=NELOUT(N)
  590 ET(L)=ETORG
      RORG=ROR
      PRINT 34,NAL
      PRINT 36,RORG,ETORG
      NID=2
      NNN=NNN+1
      NFEL=1
      NLEL=NOUT
      GO TO 147
C
  440 IF(NANAL-NAL) 444,444,120
  444 READ 801, NEXT
      IF(NEXT .EQ. 0) GO TO 1000
  442 STOP
      END
```

```
$IBFTC PYPO     DECK
      SUBROUTINE PYP
      DIMENSION NRMPNT(20), YORD(262), XORD(262), ELFLEX(3,3), ELRIG(3,3
     1), ELA(3,6), ELATRN(6,3), ATK(6,3), BARDK(6,6), S(6,6), MNP(265),
     2 DSX(262), DSY(262)
      COMMON         I2,NLLP1,N,KIK,IN,NRMPNT,NRMPTS,YORD,YCENTR,XORD,
     1 XCENTR,THPIPE,RADIUS,EPIPE,S, NNPP1,NUMNP,NPP1,MNP,DSX,DSY,NPP2
      EQUIVALENCE (ELRIG,ELFLEX)
  522 IF(KIK)523,527,523
  527 IN = IN+1
C
C   FOR EACH ELEMENT OF PIPE
C
C
C   CALCULATE ANGLE PHI SUBTENDING EACH ELEMENT OF PIPE
C
C*********************************************************************
      IF(I2)2001,2002,2001
 2001 PRINT 492,IN
  492 FORMAT(14H1PIPE ELEMENT I2)
C*********************************************************************
 2002 NRI = NRMPNT(IN)
      IF(IN-NRMPTS) 618,617,618
  618 NRI1 = NRMPNT(IN+1)
      GO TO 615
  617 NRI1 = NRMPNT(1)
  615 YI1 = YORD(NRI) - YCENTR
      XI1 = XORD(NRI) - XCENTR
      FI1 = ATAN2 (YI1,XI1)
  619 YI2=YORD(NRI1)-YCENTR
      XI2 = XORD(NRI1) - XCENTR
      FI2 = ATAN2 (YI2, XI2)
  616 PHI=FI2-FI1
      IF (PHI) 620,621,621
  620 IF(FI1)623,622,623
  622 PHI=PHI+3.14159265
      GO TO 621
  623 IF(FI2)624,622,624
  624 PHI=PHI+6.2831853
  621 PHION2=PHI/2.0
      PHIBY2 = 2.0*PHI
C*********************************************************************
      IF(I2)2003,2004,2003
 2003 PRINT 1006,FI1
 1006 FORMAT( 8HFI       ,F10.4)
      PRINT 1006,FI2
      ANPHI=PHI*180.0/3.142
      PRINT 1496,ANPHI
 1496 FORMAT( 6H0ANPHI/F10.5)
C*********************************************************************
C
C   CALCULATE INCLINATION OF EACH PIPE ELEMENT
C
 2004 IF(IN-1) 650,651,650
  651 PSI = FI1
```

```
  650 PSI = PSI+PHION2
C*************************************************************************
      IF(I2)2005,2006,2005
 2005 ANPSI=PSI*180.0/3.142
      PRINT 1495,ANPSI
 1495 FORMAT( 6H0ANPSI/F10.5)
C*************************************************************************
C
C  CALCULATE MOMENT OF INERTIA OF PIPE, = PIPEI
C
C
 2006 PIPEI =(THPIPE**3.0)/12.0
C
C  COMPUTE PIPE ELEMENT FLEXIBILITY MATRIX ELFLEX
C
      ALFA = 1.0/(2.0*SIN (PHION2))
      ELFLEX(1,1)=PHI-PHI*COS (PHI)/2.0-SIN (PHION2)
      ELFLEX(1,1) = ELFLEX(1,1)*ALFA*ALFA
      ELFLEX(1,2)=-PHI*COS (PHI)/2.0+SIN (PHI)/2.0
      ELFLEX(1,2) = ELFLEX(1,2)*ALFA*ALFA
      ELFLEX(1,3)=SIN (PHI)*PHI/2.0-1.0+COS (PHI)
      ELFLEX(1,3) = ELFLEX(1,3)*RADIUS*ALFA
      ELFLEX(2,1) = ELFLEX(1,2)
      ELFLEX(2,2) = ELFLEX(1,1)
      ELFLEX(2,3) = ELFLEX(1,3)
      ELFLEX(3,1) = ELFLEX(1,3)
      ELFLEX(3,2) = ELFLEX(2,3)
      ELFLEX(3,3)=PHI+PHION2*COS (PHI)-1.5*SIN (PHI)
      ELFLEX(3,3) = ELFLEX(3,3)*RADIUS*RADIUS
      DO 604 J=1,3
      DO 604 K=1,3
  604 ELFLEX(J,K) = ELFLEX(J,K)*RADIUS/(EPIPE*PIPEI)
C*************************************************************************
      IF(I2)2007,2008,2007
 2007 PRINT 1494,((ELFLEX(J,K),K=1,3), J=1,3)
 1494 FORMAT( 7H0ELFLEX/(3E12.2))
C*************************************************************************
C
C  INVERT ELFLEX TO GET PIPE ELEMENT STIFFNESS, = ELPIG
 2008 CALL SYMINV(ELFLEX,3)
C*************************************************************************
      IF(I2)2009,2010,2009
 2009 PRINT 1491,((ELRIG(J,K),K=1,3),J=1,3)
 1491 FORMAT(6H0ELRIG/(3E12.2))
C*************************************************************************
C
C  CONSTRUCT DISPLACEMENT TRANSFORMATION MATRIX TO FIXED AXES, = ELA
C
 2010 ELA(1,1) = COS (PSI)/(2.0*RADIUS*SIN (PHION2))
      ELA(2,1) =-ELA(1,1)
      ELA(3,1) = -SIN (PSI)
      ELA(1,2) = SIN (PSI)/(2.0*RADIUS*SIN (PHION2))
      ELA(2,2)=-ELA(1,2)
      ELA(3,2) = COS (PSI)
      ELA(1,3) = -ELA(1,1)
```

```
      ELA(2,3) = -ELA(2,1)
      ELA(3,3) = -ELA(3,1)
      ELA(1,4) = -ELA(1,2)
      ELA(2,4) = -ELA(2,2)
      ELA(3,4) = -ELA(3,2)
      ELA(1,5) = 0.0
      ELA(2,5) = -1.0
      ELA(3,5) = 0.0
      ELA(1,6) = 1.0
      ELA(2,6) = 0.0
      ELA(3,6) = 0.0
C*************************************************************************
      IF(I2)2011,2012,2011
 2011 PRINT 1007, ((ELA(J,K),K=1,6),J=1,3)
 1007 FORMAT( 4H0ELA/(6E12.2))
C*************************************************************************
C
C  CONSTRUCT TRANSPOSE OF ELA, =ELATRN
C
 2012 DO 603 J=1,3
      DO 603 K=1,6
  603 ELATRN(K,J) = ELA(J,K)
C*************************************************************************
      IF(I2)2013,2014,2013
 2013 PRINT 493,((ELATRN(K,J),J=1,3),K=1,6)
  493 FORMAT( 7H0ELATRN/(3E12.2))
C*************************************************************************
C
C  CONSTRUCT PIPE ELEMENT STIFFNESS IN FIXED COORDINATES, = BARDK
C
 2014 DO 710 I=1,6
      DO 710 J=1,3
      ATK(I,J)=0.0
      DO 710 KK=1,3
  710 ATK(I,J) = ATK(I,J) + ELATRN(I,KK)*ELRIG(KK,J)
      DO 709 I=1,6
      DO 709 J=1,6
      BARDK(I,J) = 0.0
      DO 709 KK=1,3
  709 BARDK(I,J) = BARDK(I,J)+ATK(I,KK)*ELA(KK,J)
      PSI=PSI+PHION2
C*************************************************************************
      IF(I2)2015, 523,2015
 2015 PRINT 1008,((ATK(K,J),J=1,3),K=1,6)
 1008 FORMAT( 4H0ATK/(3E12.2))
      PRINT 503, ((BARDK(K,J),J=1,6),K=1,6)
  503 FORMAT(31H0BARDK MATRIX FOR PIPE ELEMENT ,/(6E12.2))
C*************************************************************************
C STORING CURVED BAR STIFFNESS IN ELEMENT FORM P, Q, R
  523 DO 533 JJ=1,6
      DO 533 II=1,6
  533 S(II,JJ)=0.0
      IF(KIK-1)524,525,526
  524 S(1,1)=BARDK(5,5)-1.0
      S(1,3)=BARDK(5,1)
```

```
      S(1,4)=BARDK(5,2)
      S(1,5)=BARDK(5,3)
      S(1,6)=BARDK(5,4)
      S(2,2)=1.0
      S(3,1)=BARDK(1,5)
      S(3,3)=BARDK(1,1)-1.0
      S(3,4)=BARDK(1,2)
      S(3,5)=BARDK(1,3)
      S(3,6)=BARDK(1,4)
      S(4,1)=BARDK(2,5)
      S(4,3)=BARDK(2,1)
      S(4,4)=BARDK(2,2)-1.0
      S(4,5)=BARDK(2,3)
      S(4,6)=BARDK(2,4)
      S(5,1)=BARDK(3,5)
      S(5,3)=BARDK(3,1)
      S(5,4)=BARDK(3,2)
      S(5,5)=BARDK(3,3)-2.0
      S(5,6)=BARDK(3,4)
      S(6,1)=BARDK(4,5)
      S(6,3)=BARDK(4,1)
      S(6,4)=BARDK(4,2)
      S(6,5)=BARDK(4,3)
      S(6,6)=BARDK(4,4)-2.0
      KIK=1
      NELP1=NELP1+1
      GO TO 531
  525 DO 537 II=1,6
  537 S(II,II)=1.0
      S(1,5)=BARDK(1,6)
      S(2,5)=BARDK(2,6)
      S(3,5)=BARDK(3,6)
      S(4,5)=BARDK(4,6)
      S(5,5)=BARDK(6,6)-1.
      S(5,4)=BARDK(6,4)
      S(5,3)=BARDK(6,3)
      S(5,2)=BARDK(6,2)
      S(5,1)=BARDK(6,1)
      KIK=2
      NELP1=NELP1+1
      GO TO 531
  526 DO 538 II=1,6
  538 S(II,II)=1.0
      S(1,5)=BARDK(5,6)
      S(5,1)=BARDK(6,5)
      KIK=0
      NELP1=NELP1+1
  531 CONTINUE
      RETURN
      END
```

```
$IBFTC SINV    DECK,LIST                                                  SINV
      SUBROUTINE SYMINV (A,NMAX,NDIM)                                     SINV
      DIMENSION A(NDIM,1)                                                 SINV
      DO 100  N = 1,NMAX                                                  SINV
  100 A(N,1) = A(1,N)                                                     SINV
      DO 200  N = 1,NMAX                                                  SINV
      PIVOT = A(N,N)                                                      SINV
      A(N,N) = -1.                                                        SINV
      DO 140  J = 1,NMAX                                                  SINV
  140 A(N,J) = A(N,J)/PIVOT                                               SINV
      DO 180  I = 1,NMAX                                                  SINV
      C = A(I,N)                                                          SINV
      IF (I.EQ.N.OR.C.EQ.0.)  GO TO 180                                   SINV
      DO 160  J = 1,NMAX                                                  SINV
      IF (J.EQ.N)   GO TO 160                                             SINV
      A(I,J) = A(I,J) - C*A(N,J)                                          SINV
      A(J,I) = A(I,J)                                                     SINV
  160 CONTINUE                                                            SINV
  180 CONTINUE                                                            SINV
      DO 200  I = 1,NMAX                                                  SINV
  200 A(I,N) = A(N,I)                                                     SINV
      DO 300  I = 1,NMAX                                                  SINV
      DO 300  J = 1,NMAX                                                  SINV
  300 A(I,J) = -A(I,J)                                                    SINV
      RETURN                                                              SINV
      END                                                                 SINV
```