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Wyrick, Joshua R
Pasternack, Gregory B

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Title: Modeling energy dissipation and hydraulic jump regime responses to channel non-uniformity at river steps

Running Title: River step hydraulics

Authors: Joshua R. Wyrick^{1,2} and Gregory B. Pasternack^{1,*}

Addresses:

¹Department of Land, Air, and Water Resources, University of California, One Shields Avenue, Davis, Ca 95616, USA

²Current Address: Civil and Environmental Engineering Department, Rowan University, 201 Mullica Hill Rd., Glassboro, NJ, 08071, USA

*Corresponding author

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1 **Abstract**

2

3 The river step is an important driver for geomorphic evolution in bedrock rivers, but the
4 effect that variations in channel geometry upstream and downstream of a river step have on
5 hydraulic jump regime and energy dissipation has not previously been investigated. The
6 associated hydraulic jump is inherent to a river step and its regime is a primary control on step
7 morphodynamics. In turn, the hydraulic jump regime is controlled by several variables as
8 detailed in a new conceptual framework herein. Also in this study, a parsimonious semi-
9 analytical numerical model of step hydraulics is developed to quantify energy dissipation and
10 delineate hydraulic jump regimes, accounting for discharge, jump submergence, and non-uniform
11 channel geometry through a step. Despite remaining limitations in step theory, the model
12 simulates how natural steps respond to a wide range of conditions. The model shows that
13 hydraulic jump regime and energy dissipation exhibit greater sensitivity to channel non-
14 uniformity as discharge increases and/or step height decreases. Also, channel conditions that
15 create greater jump submergence lead to decreased energy dissipation, regardless of the discharge
16 regime. The model also reinforces the common observation about gully erosion that downstream
17 channel widening enhances upstream knickpoint migration. The new algorithm may be used to
18 aid river engineering involving steps and could be useful for landscape evolution modeling.

19

20 **Keywords:** hydraulic jump, energy dissipation, river step, hydraulic geometry, channel evolution
21 **AGU Index Terms:** Geomorphology; fluvial (1825), Modeling (1847), River channels (1856)

22

23

1 **1. Introduction**

2

3 **1.1 Background**

4 Landscape evolution models are just beginning to account for knickpoint migration in
5 alluvial gullies [Flores-Cervantes *et al.*, 2006], but the mechanisms by which lift and drag
6 actually scour the bed below diverse river steps at all landscape positions from bedrock mountain
7 tops to large alluvial rivers remain poorly understood [Pasternack *et al.*, 2007]. A key challenge
8 for understanding natural systems arises because experimental studies of hydro-morphologic
9 processes at river steps have previously been simplified with 2-D flume or scale model studies
10 [e.g., McCarthy and O'Leary, 1978; Mason and Arumugam, 1985; Lenzi *et al.*, 2002; Frankel *et*
11 *al.*, 2007]. Natural river steps, however, typically exhibit complex 3-D flow processes [e.g., Valle
12 *and Pasternack*, 2002, 2006a; Pasternack *et al.*, 2006] and occur in non-uniformly shaped
13 channels [e.g., Sinha *et al.*, 1998]. A visual confirmation of the natural variability of process and
14 form at river steps is evident in the entries to the world waterfall database ([http://www.world-](http://www.world-waterfalls.com)
15 [waterfalls.com](http://www.world-waterfalls.com)), which documents 949 waterfalls between ~100-1000 m high and ranging in
16 estimated discharge from ~150-42500 m³/s. A river step is defined herein as a vertical or near-
17 vertical downstream drop in channel bed elevation, and may be referred to similarly as a
18 waterfall, cascade, knickpoint, headcut, or downstep. This study addresses the fluid mechanics of
19 steps relevant for eventual process-based simulation, regardless of their degree of complexity
20 using a control volume approach in which upstream and downstream conditions constrain internal
21 step processes.

22 Studies of knickpoint migration in uniform alluvial gullies presently provide the most
23 developed basis for proposed equations for use in landscape evolution modeling, but they only

1 address the subset of natural steps that have significant plunge pools and they do not include the
2 important role of the lift force exhibited by the flow on the substrate. The form of the shear-stress
3 equation that is typically assumed to govern step migration rate, and thus long-term channel
4 incision in such alluvial gullies, posits that the migration rate increases as discharge increases,
5 because a higher discharge is expected to yield a higher shear stress on the bed below a step
6 [Alonso *et al.*, 2002; Flores-Cervantes *et al.*, 2006]. Experimental flume studies do not
7 consistently confirm that expectation, though there is always experimental error to consider.
8 Slattery and Bryan [1992] reported a general increase in migration rate with discharge, but
9 Robinson and Hanson [1996] reported a lower rate at higher discharge. Bennett *et al.* [2000]
10 found no statistically significant relation among nine experimental runs, though the lowest
11 observed migration rate did occur at the highest discharge. Beyond gullies, researchers have also
12 applied shear stress models to bedrock rivers and determined that channel incision generally
13 occurs locally around steepened knickpoint faces [*e.g.*, Seidl and Dietrich, 1992; Stock and
14 Montgomery, 1999]. On the basis of geomorphic studies of non-vertical, sloped waterfalls in
15 Japan, Hayakawa and Matsukura [2003] proposed that the erosive stress on the face of a falls
16 should be proportional to the square of the discharge.

17 Two important mechanisms help explain why bed shear stress at the base of a step does
18 not necessarily have to increase as a function of discharge when considering any arbitrary river
19 step. First, steps with a 3-D plan-view brink geometry (*e.g.* horseshoe falls, oblique falls, and
20 labyrinth weirs) exhibit stage-dependent convergence and/or divergence of flow. Pasternack *et al.*
21 [2006, 2007] showed that at low discharge, flow over a horseshoe falls converges strongly
22 causing higher shear stress. As discharge increases, flow convergence decreases enough to yield
23 a net decrease in local shear stress. Pasternack *et al.* [2007] experimentally observed lower

1 downthrust stresses for correspondingly higher discharges while holding hydraulic jump regime
2 constant. They also noted that these values could not be accurately predicted using the
3 mathematical approaches suggested in the preceding paragraph to predict erosion in gullies.

4 Second, shear stress on the bed below a step may decrease even as discharge increases
5 because velocity at the bed is dependent on the hydraulic jump regime, and the latter may change
6 as discharge increases [*Pasternack et al.*, 2006], causing a decrease in shear stress at higher flows.
7 Some previous geomorphic research has discussed the role that hydraulic jumps have in flow
8 mechanics and channel evolution [e.g., *Kieffer*, 1987; *Carling*, 1995; *Grant*, 1997; *Montgomery*
9 *and Buffington*, 1997]. Hydraulic jumps occur as rapid transitions from supercritical to subcritical
10 flow [*Chanson*, 1999], and are recognized to be controlled by variations in channel geometry
11 and/or stream discharge [e.g., *Mossa et al.*, 2003], but the extent of this control is largely
12 unknown [*Balachandar et al.*, 2000]. In experimental flume studies, it is possible to manipulate
13 the hydraulic jump regime independently of discharge through the use of a sluice gate
14 downstream of the step. By lowering the gate, flow can be reduced, thereby increasing water
15 depth downstream of the step (i.e. “tailwater” depth). In nature the analogous mechanism for
16 tailwater control is the hydraulic geometry associated with channel size and shape as well as
17 discharge. A detailed characterization of hydraulic jump regimes at steps is presented below in
18 section 2.4, and an explanation of the role of hydraulic geometry at a step is presented in section
19 2.2. As of yet, no studies have systematically explored lift and drag forces below steps over the
20 full range of hydraulic jump regimes. Studies of hydraulic jumps at the toe of dam spillways have
21 shown that jumps are capable of creating hydraulic forces that can weaken and erode such
22 structures [e.g., *Smith*, 1976; *Fiorotto and Rinaldo*, 1992; *Vischer and Hager*, 1998]. Based on
23 experimental measurements, bed-material can be plucked by turbulent pressure fluctuations and

1 strong lift forces under hydraulic jumps [*Bollaert and Schleiss, 2003; Pasternack et al., 2007*].
2 Plucked material can then be exported by the high drag forces just downstream of jumps
3 [*Bormann and Julien, 1991; Pasternack et al., 2007*]. In terms of knickpoint migration in gullies,
4 no experimental studies have systematically manipulated hydraulic jump regime to ascertain its
5 effect on migration rate. It is known that as the plunge pool deepens, the force of the jet
6 impinging at the bottom of the pool decreases. Similarly, it is conjectured that as a hydraulic
7 jump or plunge pool becomes increasingly submerged with increasing discharge, the deceleration
8 of the impinging jet would dampen pressure fluctuations on the bed and lift fluctuations above it.
9 These effects provide another reason why the rate of knickpoint retreat would not necessarily
10 increase with discharge. Hydraulic jump regime is therefore likely to be an important aspect in
11 river step mechanics, but it is largely controlled by channel geometry. Lacking experimental
12 studies to clarify these issues, the opportunity exists for new theoretical developments.

13 In previous research on river steps, the effect of variability of channel geometry upstream
14 and downstream of a step on step hydraulics has not been investigated. A few studies of
15 engineered spillways have discussed the use of downstream channel widening as an energy
16 dissipater [e.g. *Ram and Prasad, 1998; Ohtsu et al., 1999*]. However, studies of man-made dams
17 and spillways address a very narrow range of channel conditions in which a single optimal design
18 is sought. In contrast, natural channels can expand or constrict through a step to any arbitrary
19 degree yielding diverse nappe trajectories (i.e. water profiles over the vertical drop) and hydraulic
20 jump conditions (including the absence of a jump) whose combined effects on energy dissipation,
21 bed scour, and step migration are presently unknown.

22 Based on the above analysis of past studies, a key limiting factor in understanding and
23 predicting scour at river steps is associated with understanding the stage-dependence of river-step

1 fluid mechanics. The focus of this study was to use a numerical model to heuristically investigate
2 channel hydraulic geometry and discharge in determining the hydraulic jump regime and energy
3 dissipation at a river step. Specific objectives included predicting the hydraulic jump regime and
4 energy dissipation as (1) discharge varies for a given channel geometry, (2) channel geometry
5 varies for a given discharge, and (3) channel geometry upstream of a step varies relative to that
6 downstream of it. The approach involved a purely theoretical framework in which available
7 analytical and empirical equations were coupled to yield a new parsimonious model formulation.
8 Admittedly, the resulting numerical model has several assumptions and limitations, but it does
9 provide a strong heuristic explanation of why erosion at river steps is not a direct function of only
10 discharge. It also elucidates the key scientific gaps that need to be addressed to promote further
11 advancement.

12 Although the study presents detailed fluid mechanics results, the general conclusions are
13 relevant to a variety of applied water resources problems involving natural and man-made river
14 steps. One value of this work is that it provides a new and different approach to predicting
15 erosion at river steps in channels with variable geometry in landscape evolution models. This
16 model does not yet predict scour directly yet, but it predicts hydraulic jump regime and energy
17 dissipation, which are both important factors controlling bed scour below steps. Another value is
18 that river rehabilitation and engineering project conceptual models, including those for dam
19 removal, fish passage, and whitewater parks, often employ steps, but do not consider the
20 importance of channel geometry in controlling the safety and functionality of these hydraulic
21 structures. This model provides a tool that would be of immediate value in improving the safety
22 of hydraulic structures.

23

1 **1.2 Step Conceptual Framework**

2 The role of hydraulic jump regime in scour at the bottom of a river step is not widely
3 understood. Very few experimental studies of river steps have varied the hydraulic jump regime
4 over the full range possible to explore this factor. To promote a better understanding of the
5 general relevance of hydraulic jumps associated with river steps for water resources management
6 and to guide research ultimately leading to prediction of river step morphodynamics, a conceptual
7 model encompassing independent variables and responding processes was developed in which the
8 key dynamics were grouped into five categories (Figure 1). Evolution of a river step can be
9 characterized by the processes of scour hole formation, upstream retreat, and change in step
10 geometry. These processes are driven by a complex, interdependent set of hydrologic and
11 geologic variables acting over multiple scales. Basin variables include the independent watershed
12 inputs of water and sediment discharges as well as channel geology (Figure 1). For example,
13 some steps may exist in various geologic conditions ranging from well-bedded sedimentary
14 bedrock to fractured homogeneous igneous bedrock. The role of sediment supply has been an
15 important recent addition to shear stress models [*e.g. Sklar and Dietrich, 2004; Gasparini et al.,*
16 *2007*]. The basin variables control the channel variables, which include the cross-sectional
17 geometries within the channel, the channel slope, and the longitudinal spacing between river
18 steps. In this framework, a sequence of cross-sections is used as an indicator of complex 3-D
19 channel morphology, since channel width and cross-sectional area often fluctuate down a
20 mountain river system, even as they generally increase downstream due to increasing discharge.
21 Basin variables also help shape the step morphology components of step height, planform shape
22 of the step, step slope, and step roughness. Step height can be dependent upon step spacing [*Wohl*
23 *and Grodek, 1994*].

1 All of the step morphology variables affect step hydraulics characterized by the nappe
2 trajectory and hydraulic jump regime, though step roughness only affects the jump regime
3 indirectly through nappe trajectory. Nappe trajectory and hydraulic jump regime are of central
4 importance in the conceptual model, because a) the former is a key variable controlling jet scour
5 of bedrock near the step toe [*Mason and Arumugam, 1985; Bormman and Julien, 1991; Stein et*
6 *al., 1993; Alonso et al., 2002*], where toe refers to the slope break at the bottom of the step, and b)
7 the latter controls turbulent lift forces that pluck and suspend bedrock downstream of the point of
8 jet impact [*Fiorotto and Rinaldo, 1992; Pasternack et al., 2007*]. Nappe trajectories for linear
9 overfalls have been thoroughly investigated [*USBR, 1948b; Vischer and Hager; 1998; Chanson,*
10 *2002*], while those for overfalls with 3-D brink configurations have only recently come under
11 some scrutiny [*Falvey, 2003; Pasternack et al., 2006*]. Hydraulic jump regimes for a free overfall
12 include: supercritical flow with no jump, pushed-off unsubmerged jump (defined later in Section
13 2.4), optimal jump, submerged jump, standing waves, and subcritical flow with no jump [*USBR,*
14 *1948b; Leutheusser and Birk, 1991*]. Hydraulic jump regime is strongly influenced by discharge
15 and tailwater depth [*USBR, 1948b; Pasternack et al., 2006*], with the latter in turn controlled by
16 upstream and downstream channel configuration. Step brink planform shapes that deviate from
17 linearity cause nappe interference and a shift in jump regime for a given discharge and channel
18 configuration [*Falvey, 2003; Pasternack et al., 2006*]. Additionally, the nappe regime has been
19 anecdotally observed to be affected by the direction and magnitude of wind impacting it, but no
20 scientific studies have yet explored nappe response to diverse wind regimes.

21 Step hydraulics such as jet impact, turbulent pressure fluctuations, drag, and lift drive
22 channel morphodynamics by changing the size and shape of the scour hole [*Lenzi et al., 2003;*
23 *Alonzo et al., 2002*], step geometry [*Pasternack et al., 2006*], and upstream retreat of the step

1 [Hanson *et al.*, 1997; Bennett *et al.*, 2000; Stein and LaTray, 2002]. Upstream retreat is
2 determined by the relative erodibility and erosional force on the step top versus that on the face
3 and toe of the step [*e.g.*, Stein and Julien, 1993; Flores-Cervantes *et al.*, 2006; Frankel *et al.*,
4 2007]. The shape of the scour hole and the regime of the associated hydraulic jump affect the
5 erosional ability of the flow below the step. Scour depth has previously been shown to be
6 dependent on step morphology [Alexandrowicz, 1994; Lenzi *et al.*, 2002] and sediment supply
7 [*e.g.*, Marion *et al.*, 2006]. Step morphodynamics, in turn, can affect channel geometry and step
8 morphology.

9 The conceptual framework described above serves to organize past research, promote
10 quantification of identified linkages, and highlight important gaps in the current understanding. A
11 casual observer of major waterfalls and whitewater rapids in mountain rivers will quickly take
12 note of the diversity and complexity of natural step morphologies. Addressing natural diversity
13 presents the most important gap in the scientific understanding of river steps. This study
14 addresses the problem of how channel expansions and constrictions through geomorphic units
15 with steps affect step fluid mechanics.

16

17 **2. Step Systematics**

18

19 Although it is impossible at this time to produce a numerical model that incorporates all of
20 the processes discussed in section 1.2, that conceptual framework can serve as a roadmap to guide
21 process-based research leading toward an eventual predictive capability. The first phase of
22 developing a suitable numerical model requires starting at the step itself and adequately
23 characterizing step hydraulics on the basis of step morphology and surrounding channel variables.

1 The quantitative formulation used below is very different from that proposed previously for
 2 gullies [*e.g.* *Alonso et al.*, 2002; *Flores-Cervantes et al.*, 2006]. Admittedly, this approach has
 3 limitations and uncertainties, but the results in Section 3 provide new insights that shed light on
 4 how river steps likely evolve over time as a result of natural spatial variation in channel geometry.

6 **2.1 Eulerian Governing Equations**

7 As an introduction to the broader 3-D problem, consider steady energy and momentum
 8 conservation for a control volume in a level rectangular channel with clear water including a
 9 broad-crested bed overfall and the region downstream of the step (Figure 2). There holds for
 10 average conditions the following classic hydraulic equations [*e.g.*, *USBR, 1948b; Ackers et al.*,
 11 *1978; Chaudry, 1993; Chanson, 1999; Munson et al.*, 2006]:

12 the overall energy conservation equation:

$$13 \quad E_{up} = (H + P) = (h_d + h_{tail}) = E_{tail} + h_L \quad (1)$$

14 a rearrangement of equation (1) that solves for the submergence variable, h_d :

$$15 \quad h_d = h_L + h_{v_tail} = h_L + \frac{q^2}{2gh_{tail}^2} \quad (2)$$

16 the mass conservation equation that is valid for any cross-section, i :

$$17 \quad q = v_i h_i \quad (3)$$

18 a special case of equation (1) that solves for the critical flow condition:

$$19 \quad h_c = \left(\frac{q^2}{g} \right)^{1/3} \quad (4)$$

20 the definition of Froude Number at location i :

$$21 \quad Fr_i = \sqrt{\frac{q^2}{gh^3}} \quad (5)$$

1 and the broad-crested weir equation derived from equations (1) and (4):

$$2 \quad q = (2/3)^{3/2} C_b \sqrt{g} H^{3/2} \quad (6)$$

3 where E_i and h_i are total energy and water depth at any location i as defined in Figure 2, H the
4 specific energy at the upstream location (weir crest as datum), P the broad-crested step height, q
5 the specific discharge, h_L the total energy loss in the control volume, g the gravitational constant,
6 v_i the velocity at location i , and $C_b=0.848$ the broad-crested weir discharge coefficient [Ackers *et*
7 *al.*, 1978; Leutheusser and Birk, 1991; Chanson, 1999]. For the equations presented above and
8 throughout this text, the subscript “up” refers to the channel section upstream of the step, and the
9 subscript “tail” refers to the controlling channel section downstream of the step and associated
10 hydraulic jump regime (Figure 2). Equations (4)-(6) assume a rectangular channel, and are
11 approximate for a wide channel of any shape. They are not, however, ultimately used in the new
12 numerical model presented herein, thereby representing a significant advance and eliminating that
13 concern.

14 In past studies momentum conservation was used to isolate the head loss solely due to the
15 hydraulic jump for the classic hydraulic jump [Chanson, 1999] and unsubmerged jumps at
16 overfalls [Henderson, 1966]. In those cases there exists an identifiable upstream supercritical
17 cross-section and downstream subcritical cross-section bounding the jump. The Belanger
18 Equation has also been modified to describe the unsubmerged hydraulic jump for non-prismatic
19 channels [Negm, 2000]. However, these approaches are invalid when the hydraulic jump is
20 submerged because the equation necessitates a measurable upstream supercritical depth value, but
21 the supercritical jet is underwater. As the model developed herein encompasses all hydraulic
22 jump regimes, including submerged jumps, the classic Belanger Equation is not used and it is thus
23 not possible to isolate the energy dissipation of just the jump.

1 Critical depth is often used to non-dimensionalize variables; however, locating the critical
2 point introduces error [Ackers *et al.*, 1978], whereas defining upstream specific and total energy is
3 more practical and certain. In addition, the variable $(H+P)/H$ is the non-dimensional energy
4 variable accounting for both discharge and step height [USBR, 1948b]. It shows that geometric
5 scaling to yield any energy condition is achievable by holding either step height or flow constant.
6 Higher $(H+P)/H$ values correspond with taller steps with relatively less flow over them. As
7 discharge increases for a given step height, so does the head on the step. Therefore decreasing
8 $(H+P)/H$ values represent conditions of increasing discharge and increasing energy input for a
9 given step height. In the lower limit of no step ($P=0$), the variable approaches unity.

10 In this study, “submergence” is defined as the condition when h_{tail} is deep enough to place
11 the leading edge of the jump upstream of the location of the free-falling nappe toe [Leutheusser
12 and Birk, 1991], where the nappe toe is defined as the slope break in the water surface profile
13 downstream of the step. According to classic equations (1)-(6), when $(H+P)/H$ and the non-
14 dimensional submergence variable (h_d/H) are specified, the resulting $h_L/(H+P)$ and flow
15 kinematics at upstream and tail cross-sections are independent of step geometry [USBR, 1948b;
16 Pasternack *et al.*, 2006]. However, the degree of submergence and hydraulic jump regime are
17 dependent on aspects of step geometry, notably planform brink shape and step slope.

18 Whereas engineers classically solved equations (1)-(6) to find optimal spillway and
19 overfall designs for engineered structures to minimize damage to such structures [e.g. Moore,
20 1943; White, 1943; Henderson, 1966], those studies did not tackle the broader water resources
21 problem facing natural channels in which step fluid mechanics responds to a very wide range of
22 non-optimal upstream energy and downstream submergence conditions. For example, there exist
23 many steps in bedrock rivers that exhibit supercritical jets impinging on exposed bedrock and

1 continuing on as supercritical flow; peruse the world waterfall database at [http://www.world-](http://www.world-waterfalls.com)
2 [waterfalls.com](http://www.world-waterfalls.com) to see many examples of this phenomenon. There are also many steps with
3 submerged jumps due to the presence of a plunge-pool. While transference of classic engineering
4 foundations to the problem of understanding natural 3D waterfalls has value, the unique aspects
5 of natural systems warrant further investigation. *Pasternack et al.* [2006] addressed this broader
6 problem by solving equations (1)-(6) for fractional energy dissipation $h_L/(H+P)$ for a larger non-
7 classical range of submergence h_d/H and energy $(H+P)/H$. Unlike in laboratory flumes with
8 sluiced inflows, the upstream Fr approaching a step in a natural channel is not an independent
9 variable, so it is not a governing variable for natural systems. The results showed that the
10 maximum $h_L/(H+P)$ for any $(H+P)/H$ occurs when h_{tail} is exactly critical with no hydraulic jump
11 present. This maximum involves a transition from supercritical to critical flow and $h_{toe} < h_{tail}$.
12 Also, as h_{tail} is decreased to less than critical depth, $h_L/(H+P)$ decreases and the flow increases its
13 efficiency until $h_{toe} = h_{tail}$. The primary conclusion from their analysis was that arbitrary h_{tail} must
14 be included in models of flow kinematics and energy dissipation at natural river steps.

15

16 **2.2 Channel Parameterization**

17 Even though *Pasternack et al.* [2006] used the classic equations (1)-(6) to expand the
18 breadth of the relevance of those equations into non-classical water resources problems, their
19 model assumed a uniform rectangular channel, because that was their flume set-up for
20 experiments on a broad-crested horseshoe waterfall. In this study, equations (4)-(6) were replaced
21 to further generalize the model to reduce or eliminate the assumptions of a rectangular channel, a
22 2-D overfall, and mutual independence of $(H+P)/H$ and h_d/H . Arbitrary upstream and
23 downstream cross-sectional morphologies are now enabled and characterized using another

1 classic principle from a different branch of river science, namely empirical at-a-station hydraulic
2 geometry relations originally stemming from channel regime theory [*Garde and Ranga Ragu,*
3 1985]. Even though such equations are empirically-derived geometric descriptors, they do satisfy
4 mass conservation requirements [*Leopold and Maddock, 1953*] and are widely used by fluvial
5 geomorphologists, who accept them as classic in their discipline. These equations do not assume
6 a rectangular channel. They are used herein to replace equations (4)-(6) to obtain discharge from
7 upstream channel conditions and add more equations to govern channel changes through the
8 geomorphic unit containing the step. As will be shown, they also are used to directly link
9 $(H+P)/H$ and h_d/H , so those two variables are not independent. Although brink geometry and
10 nappe trajectory play an important role in step dynamics, they are not necessary in quantifying the
11 net energy loss between the upstream and downstream cross-sections. Admittedly, both factors
12 affect the jump regime [*Falvey, 2003; Pasternack et al., 2006*] and that remains an important
13 limitation of the numerical model discussed further below. However, the model will clearly
14 illustrate the importance of variable cross-sectional geometry on hydraulic jump regime and
15 energy loss at river steps.

16 According to classic channel regime theory, at a given cross-section the flow width, depth,
17 and velocity can be related to the volumetric discharge (Q) by the power functions [*Leopold and*
18 *Maddock, 1953*]:

$$w = aQ^b, h = cQ^f, \text{ and } v = kQ^m \quad (7a,b,c)$$

19 where w is average channel width, h is average channel depth, v is average flow velocity, $\{a,c,k\}$
20 are empirical coefficients, and $\{b,f,m\}$ are empirical exponents that largely control cross-sectional
21 shape. Equation (3) governs that $b+f+m=1$ and $a \times c \times k=1$ [*Leopold and Maddock, 1953*]. In this
22 study, $b, f, m, a, c,$ and k are all termed “parameters” of the overall model. No general values for
23

1 these parameters have been published yet for bedrock rivers with steps, though such studies
2 appear pending. Several specific values have been published for alluvial rivers [Wilcock, 1971;
3 Drury, 1976; Betson, 1979; Andrews, 1984; Rhoads, 1991; Singh, 2003; Stewardson, 2005], with
4 many states in the United States now developing extensive databases stratified by physiographic
5 province. After a review, the published values of the b exponent were found to vary from 0.04 to
6 0.6, and those for the f exponent from 0.20 to 0.65. Similarly, those of the a coefficient ranged
7 from 3 to 19, with a mean value of 9.74, and those of the c coefficient from 0.474 to 0.73, with a
8 mean value of 0.51. In this study, hydraulic geometry relations are used to describe channel size
9 and shape at one cross-section located upstream of a step and one downstream of it. When
10 applying the numerical model developed in this study to a given real step, one can use monitoring
11 data to parameterize the coefficients and exponents for the cross-sections at that site.

12 To interpret the results of this study, it is critical to understand the geometric patterns
13 implied by combinations of exponents and coefficients. Dingman [2007] provides a detailed
14 mathematical analysis of how to interpret these parameters, and the key concepts are summarized
15 below. Channel shape changes predictably as a function of the exponents, assuming constant
16 coefficients (Figure 3). When $b=f$, the channel is triangular in cross-section. In the limit, as b
17 approaches 0, width does not change as discharge increases. Geometrically, that necessitates that
18 the channel is rectangular when $b=0$. For values of $b<f$, the channel is concave up. Similarly, as f
19 approaches 0, depth does not change as discharge increases. Geometrically, that necessitates that
20 the channel is infinitely wide when $f=0$. For values of $b>f$ (Figure 3, shaded region), the channel
21 is convex up. In summary, when $b\neq f$ and one of those exponents is allowed to change, then the
22 geometric effect is one of “bending” – for increasing f bending is inward and for increasing b ,
23 bending is outward. As both b and f approach 0 together, the velocity exponent, m , increases.

1 Geometrically, a condition of no change to depth or width as discharge increases requires an
2 increase in energy slope and/or a decrease in channel roughness with increasing discharge. It is
3 widely held that roughness decreases as discharge increases [*Smart, 1999; Dingman, 2007*].
4 Similarly, for a meandering channel, valley slope is higher than channel slope at the reach scale,
5 so the energy slope is higher for out-of-bank flows than in-channel flows.

6 For any given set of exponent values and corresponding channel shape, the values of the
7 coefficients affect the scaling of the channel. A higher value of the coefficient a stretches the
8 banks horizontally (i.e. for a triangular channel, the angle between the banks increases), which
9 makes the wetted channel flatter. A higher value of the coefficient c stretches the banks
10 vertically, which makes the channel higher. Finally, a higher value of the coefficient k is
11 associated with a steeper bed slope and/or a smoother bed at any given discharge.

12 Although the exact geometries associated with changes in exponents versus coefficients
13 are different (i.e. bending versus stretching, respectively), the effects of both types of changes on
14 the simpler variables of cross-sectionally averaged channel depth and width are similar. For
15 example, increasing either a or b alone in equation (7a) causes the width at any given discharge
16 greater than one to increase (Figure 4). Either change also causes the slope of the tangent line of
17 the relation to increase. Thus in terms of width and depth, changes to both types of parameters
18 yield similar outcomes, even though they do so in different ways geometrically.

19 Finally, while hydraulic geometry relations can characterize braided or multi-threaded
20 channels, which sometimes are associated with river steps, as exemplified by Great Falls on the
21 Potomac River between Maryland and Virginia (38°59'51 N, 77°15'10 W), the parameters for
22 multi-threaded channels are non-unique and thus difficult to interpret. Although hydraulic
23 geometry equations are empirically derived, they are well suited for parsimonious exploration of

1 channel non-uniformity at river steps. They are also easily quantifiable at real river steps by
 2 mapping the cross-sections upstream and downstream of the step when flow is low and then using
 3 automated stage recorders at those locations to record the local stage-discharge responses.
 4 Accurate physics-based 3-D hydrodynamics models of the multiphase flow through hydraulic
 5 jumps at natural river steps have yet to be achieved, limiting strict physics-based alternatives.

6

7 **2.3 Channel Non-uniformity**

8 For the independent variables of step height and upstream depth along with the upstream
 9 values of the geometric exponents (b_{up} and f_{up}) and coefficients (a_{up} and c_{up}) of equations (7a,b),
 10 the resulting relative submergence and head loss through the step (equations. 1 and 2) are
 11 determined from the following set of equations. For the equations presented here, the subscript
 12 “up” refers to the channel section upstream of the step, and the subscript “tail” refers to the
 13 controlling channel section downstream of the step and hydraulic jump regime (Figure 2).

14 Rearranging equation (7b), discharge is determined at the upstream cross-section as:

$$15 \quad Q = \left(\frac{h_H}{c_{up}} \right)^{\frac{1}{f_{up}}} \quad (8)$$

16 where h_H is the effective flow depth above the step crest. Again, this equation does not assume a
 17 rectangular channel. As shown in Figure 2, h_H can be determined by:

$$18 \quad h_H = h_{up} - P \quad (9)$$

19 For varying effective step heights for the upstream and downstream channel sections, the
 20 definition of h_H remains the same, but the calculation of depth may differ. This definition
 21 accounts for the physical reality that the water from $h_{up}=0$ to $h_{up}=P$ is “dead” storage that does not
 22 affect the dynamics of the step, i.e. there can be no tailwater until this volume is filled. Figure 2

1 represents the geometry of this dead storage with an arbitrary channel bottom, which means there
 2 could be a step up for the flow or the channel bottom could be flat and planar leading up to the
 3 step drop. In either instance, that geometry does not affect the equations used in this model. In
 4 practical application, one would develop an upstream hydraulic geometry relation referenced to
 5 the step crest elevation instead of the channel bottom as most are.

6 Next, the tail depth is determined for the controlling downstream cross-section at the exit
 7 of the tail pool as:

$$8 \quad h_{tail} = c_{tail} Q^{f_{tail}} = c_{tail} \left(\frac{h_H}{c_{up}} \right)^{\frac{f_{tail}}{f_{up}}} = c_{tail} \left(\frac{h_H}{c_{up}} \right)^{\frac{1}{fratio}} \quad (10)$$

9 where *fratio* is the ratio of the upstream *f* exponent (f_{up}) to the downstream *f* exponent (f_{tail}). An
 10 *fratio* of one describes a uniform channel consistent with the previous assumption of *Pasternack,*
 11 *et al.* [2006]. An *fratio*<1.0 refers to a system in which flow transitions from a wider, more
 12 triangular channel into a narrower, taller canyon such that the upstream depth rises slower than
 13 the downstream depth as discharge increases (Figure 5). Examples of this are ubiquitous in
 14 mountain channels (e.g. Figure 6). A similar effect on width and depth could be achieved by
 15 having a lower *c*-value upstream and a higher *c*-value downstream, though the exact geometric
 16 interpretation would be different, since the non-uniformity would be due to stretching, as
 17 described earlier. Also, c_{up} is scaled by the *fratio* in equation (10), and this indirect effect on h_{tail}
 18 is synergistic with the direct effect of the *fratio*, though the relative magnitude of the effect will
 19 be evaluated in the results later. For example, if $c_{up}=0.51$ and *fratio*=0.50, then $c_{up}^{1/fratio} = 0.26$.
 20 This smaller value in the denominator makes h_{tail} bigger for any given discharge, and thus is
 21 consistent with a deeper constricted channel downstream. An *fratio*>1.0 refers to the opposite
 22 effect as just described. In this case, flow transitions from a narrower, more triangular channel

1 into a wider, flatter channel (Figure 7). Many natural example of this type of transition can be
 2 found in mountain rivers, as well (e.g. Figure 8). If $c_{up}=0.51$ and $f_{ratio}=1.5$, then $c_{up}^{1/f_{ratio}} = 0.64$,
 3 so the coefficient effect is smaller than for an equivalent incremental change of f_{ratio} when f_{ratio}
 4 <1 . In both cases, the step is located in the transitional region, and is thus impacted by the non-
 5 uniformity. To fully understand the consequences of using at-a-station hydraulic geometry
 6 relations to characterize channel non-uniformity, sensitivity analyses were performed over a wide
 7 range of model parameters, as discussed further below in Section 2.5.

8 Upstream and downstream widths are determined from hydraulic geometry relations based
 9 on the discharge obtained in equation (8):

$$10 \quad w_{up} = a_{up} Q^{b_{up}} \quad \text{and} \quad w_{tail} = a_{tail} Q^{b_{tail}} \quad (11a,b)$$

11 Insufficient literature exists on the interactions of b , f , and m values for at-a-station
 12 conditions in the vicinity of river steps to precisely constrain b -values at this time. However, it is
 13 conceptually evident that as a channel's f -value increases a channel becomes more rectangular
 14 and thus the b -value must decrease (Figure 3). *Dingman [2007]* quantitatively demonstrated this
 15 for three different hydraulic equations. On this basis, the ratio of upstream and downstream b
 16 values was assumed to be equal to the inverse of the f_{ratio} . Future field investigation may enable
 17 revision of this assumption, but it is expected to only affect magnitudes of effect and not cause
 18 threshold effects.

19 The cross-sectionally averaged upstream and downstream velocities and corresponding
 20 averaged velocity heads were determined from equation (7c) and calculated as:

$$21 \quad \begin{aligned} v_{up} &= k_{up} Q^{m_{up}} & \text{and} & & h_{v_up} &= \frac{v_{up}^2}{2g} \\ v_{tail} &= k_{tail} Q^{m_{tail}} & & & h_{v_tail} &= \frac{v_{tail}^2}{2g} \end{aligned} \quad (12), (13)$$

1 where the coefficients, k , and exponents, m , are solved from mass conservation ($k=1/ac$ and $m=1-$
2 $(b+f)$) [Leopold and Maddock, 1953]. These equations do not assume a rectangular channel.

3 The upstream specific energy (H) is the effective head on the step (Figure 2), and is
4 calculated as:

$$5 \quad H = h_H + h_{v_up} \quad (14)$$

6 Thus the key variables of submergence, h_d , (equation 2) and head loss, h_L , (equation 1) can
7 now be non-dimensionalized as h_d/H , and $h_L/(H+P)$ in conjunction with the other key variable,
8 non-dimensional upstream energy, $(H+P)/H$, to define the Eulerian fluid mechanics of a river
9 step.

11 2.4 Hydraulic Jump Regime Equations

12 Previous researchers have developed threshold equations for hydraulic jump regimes.
13 Most such equations delineate jump regimes using only the Froude number on the basis of
14 observations of the classic hydraulic jump in a rectangular flume with no bed step, where variable
15 Fr is imposed using an upstream sluice gate [e.g. Chanson, 1999]. For the non-classical case of
16 non-uniform channels with overfalls, jump regime equations must account for jump submergence.
17 Of the published equations, only those of USBR [1948b] for an ogee-crested 2-D overfall and
18 those of Pasternack et al. [2006] for a broad-crested, rectangular overfall do that (Figure 9). The
19 thresholds of USBR [1948b] are empirically derived and, most importantly, non-dimensional.
20 Those of Pasternack et al. [2006] are semi-analytical and only address two regimes- the critical
21 Fr and optimal jump thresholds. Conceptually similar regimes for a sharp-crest weir were
22 described by Leuthesser and Birk [1991], but not quantified. It is possible to modify and use the
23 model of Pasternack et al., [2006] to establish the critical Fr threshold for a sharp-crested weir.

1 More investigations into regime equations for typical 3-D overfall brink configurations are
 2 needed.

3 Accepting the limited availability of relevant equations, Figure 9 shows the four empirical
 4 regime boundaries used in this study that were first described by *USBR* [1948b] for an ogee-
 5 crested weir. The top curve represents the threshold above which flow over the step remains
 6 supercritical and cascades past the tail cross-section. The threshold curve below that delineates
 7 the optimal jump condition with the jump occurring exactly at the step toe. The condition that
 8 lies between those thresholds represents emerged, or pushed-off, hydraulic jumps with a
 9 measurable supercritical zone and rapid transition to a subcritical zone within the control volume.
 10 The next curve below delineates jumps that are sufficiently submerged to be considered
 11 “drowned”, which is undefined by *USBR* [1948b]. Drowned jumps are interpreted to mean those
 12 submerged jumps with a strong upstream recirculation. The bottom line in Figure 9 represents the
 13 threshold below which no hydraulic jump occurs, with the flow having either standing waves or
 14 no near-critical surface expression at all. There is no regime uniquely delineated for a standing
 15 wave condition in the *USBR* [1948b] framework, which is a detriment. Overall, this quantitative
 16 framework serves to delineate well-recognized jump types, each with characteristic fluid
 17 mechanics in terms of jet impact force, turbulent pressure fluctuations, drag along the bed, and lift
 18 force.

19 The threshold equations of the ascribed conditions for $2 \leq (H+P)/H \leq 6$ (Figure 9), with the
 20 corresponding terminology of *Leutheusser and Birk* [1991] given in parentheses, are:

21 Pushed-off Jump (Swept-out Jump) $\frac{h_d}{H} = 0.998 \left(\frac{H+P}{H} \right) - 0.595$ (15)

22 Hydraulic Jump (Optimum Jump) $\frac{h_d}{H} = 0.918 \left(\frac{H+P}{H} \right) - 0.913$ (16)

1 Drowned Jump (Plunging Nappe)
$$\frac{h_d}{H} = 0.176 \left(\frac{H+P}{H} \right)^{1.679} \quad (17)$$

2 No Jump (Surface Nappe)
$$\frac{h_d}{H} = 0.1 \epsilon \quad (18)$$

3 The above equations provide a reasonable characterization of the hydraulic jump regimes at or
4 downstream of a step toe. Substituting the equations of *Pasternack et al.* [2006] for Equations
5 (15, 16) produce a very small change in the lines, with minimal conceptual difference.

6 One important limitation of these equations is that they do not account for the effects of
7 non-uniform step brink geometry on jump regime (i.e. whether in plan view the brink is horseshoe
8 shaped, perpendicular to the banks, oblique to the banks, etc). *Pasternack et al.* [2006] describe
9 the deviations from these conditions associated with horseshoe-shaped brinks, such as that of
10 Niagara Falls. Another limitation of the equations is that they do not account for regime
11 deviations associated with abrupt channel expansions or constrictions. Additional factors that
12 could affect the equations include step and plunge-pool roughness, plunge-pool depth, bedrock
13 resistance, and high winds. Because the jump regime is influenced by so many variables, it is not
14 apparent how to further generalize these equations other than by careful study of each possible
15 archetypal configuration. Nevertheless, this study provides a first assessment as to how the most
16 important forms of channel non-uniformity would affect hydraulic jump regimes.

18 **2.5 Model Implementation**

19 Equations (1), (2), and (8) – (14) were programmed into Excel 2003 [*Microsoft Corp,*
20 *Redmond, WA*] to investigate the effect of varying upstream and downstream channel geometry,
21 in conjunction with discharge and step height, on flow processes, hydraulic jump regime, and
22 energy dissipation associated with a river step. The key independent variables were $(H+P)/H$ and

1 *fratio*, with f_{up} , b_{up} , a_{up} , and c_{up} serving as key parameters. The non-dimensional response
2 variables evaluated were h_d/H , indicative of hydraulic jump regime, $h_L/(H+P)$, a measure of
3 energy loss, and $h_{v_tail}/(H+P)$, a measure of kinetic energy.

4 The computational procedure for the model begins with a set of starting values $\{a_{up}, c_{up},$
5 $b_{up}, f_{up}, fratio, h_H,$ and $P\}$. From the upstream depth and hydraulic geometry values, a discharge is
6 calculated (equation 8) for that depth. The upstream width and specific energy (H) are calculated
7 using equations (11a) and (14). Using the given *fratio*, the downstream hydraulic geometry
8 parameters are calculated, and then the model calculates the downstream depth, width, and
9 specific energy (Eqs 10 – 14). Equations (1) and (2) are then used to determine the head loss, h_L ,
10 and submergence, h_d . These variables and the total energy ($H+P$) are non-dimensionalized by the
11 upstream specific energy, H .

12 Plots of these response variables as a function of $(H+P)/H$ were made for nine
13 combinations of upstream f and b values from the set $\{0.2, 0.25, 0.4\}$ to explore the sensitivity of
14 the model to the absolute value of each parameter. Values of f or b higher than 0.4 did not yield
15 substantially different results than those shown for 0.4, and hence are not presented here. For
16 each combination of b and f , six values of *fratio* $\{0.5, 0.75, 0.9, 1.0, 1.25, 1.5\}$ were used in the
17 model to investigate its influence on river step response. For these ranges of f and b values that
18 were used, the model yielded ranges of discharge from 3 – 104,000, upstream width from 12 –
19 400, and upstream depth from 0.8 – 5.5 among the six combinations of *fratios*. These variables
20 have typical dimensions, but specific units are purposefully omitted to allow the model to be
21 applied to any channel system within any system of units. These analyses therefore capture the
22 essential functionalities and sensitivities of the model with respect to the exponents, and through
23 them natural steps.

1 To explore the effects of the coefficients of equations (7a, b, c), a sensitivity analysis was
2 performed in which the range of reported values for a and c were investigated. The analysis
3 consisted of holding the upstream b and f values constant, while varying the geometric
4 coefficients, a and c , for the range of modeled *fratios*. In these tests, $a_{up}=a_{tail}$ and $c_{up}=c_{tail}$. Two
5 combinations of f_{up} and b_{up} were evaluated, namely $f_{up}=b_{up}=0.2$ and $f_{up}=b_{up}=0.4$. Future versions of
6 the model could solve for the model parameters based on various theories that attempt to explain
7 their origin, but in this study the goal was to provide a characterization of the effects of their
8 values on river step hydraulics.

9 Currently, this model is purely theoretical. As with many models in hydrology and
10 geomorphology, including distributed hydrological models, 3D hydrodynamic models, and
11 landscape evolution models, this one is out in front of the experimental capability to validate all
12 of its components. No datasets presently exist to test the range of the model. To empirically
13 satisfy the physicality of the model would require laboratory equipment that varies upstream and
14 downstream channel geometries through a suite of combinations, or locating natural study sites
15 that fit the suite of combinations. It would then be necessary to carefully measure the hydraulic
16 terms and estimate energy dissipation, which is something that *Henderson* [1966] describes as
17 extremely challenging to do accurately. The model consists of a manipulation of classic hydraulic
18 and geomorphic equations, and thus should be technically sound. It provides first insights into
19 energy dissipation and hydraulic jump regimes for non-uniform channels and a roadmap of key
20 variables deserving of further experimental investigation. An attempt has been made to fully
21 express all assumptions and limitations upfront.

22

23 **3. Results**

1 To understand the effects of channel non-uniformity on hydraulic jump regime and energy
2 dissipation at river steps, it is first important to understand how width and depth vary through the
3 section in response to changes in discharge and *fratio* (Figure 10). As b_{up} increases for a given
4 *fratio* and constant f_{up} , the downstream width-to-depth ratio also increases (e.g. Figure 10g, h, i).
5 As f_{up} increases for a given *fratio* and constant b_{up} , the downstream width-to-depth ratio decreases
6 (e.g. Figure 10i, f, c). In general, however, the geometric width exponent, b_{up} , has a greater affect
7 on the downstream width-to-depth ratio than f_{up} (Figure 10).

8 For the cases of $f=b$ and an *fratio*=1.0, the model shows the expected results of the
9 downstream width-to-depth ratio remaining constant for the range of discharge evaluated (Figures
10 10c, e, g, diamonds), while increasing the *fratio* increases the downstream width-to-depth ratio
11 (e.g. Figure 10c,e, g, circles). Decreasing the *fratio*, however, decreases the downstream width-
12 to-depth ratio (e.g. Figure 10c, e, g, triangles). The responses to either case of varying *fratio*
13 become more dramatic as discharge increases.

14 These patterns as a function of *fratio* comprehensively confirm the interpretations
15 illustrated in Figures 5 and 7 over the full range of exponent values explored. When $f_{up}=b_{up}=0.4$
16 and *fratio*=0.5, the geometric interpretation is that the channel is bending and stretching to a more
17 concave up cross-section through the step (Figure 5). The hydraulic shape of the downstream
18 channel shows that depth will increase much faster than width as discharge increases (Figure 10c,
19 circles). For the same upstream model parameters, but an *fratio*=1.5, the geometric interpretation
20 is that the channel is bending and stretching to a more convex up cross-section through the step
21 (Figure 7). The downstream channel shape will create a greater response to flow width than flow
22 depth as discharge increases (Figure 10c, triangles). As an example, the specific values of widths
23 and depths for two flow stages shown in Figures 5 and 7 quantify the differences in the rates of

1 change. For the low *fratio* example, the ratio of the upstream widths between the two discharges
2 is about five, and this ratio decreases through the step to 2.2 at the downstream cross-section
3 (Figure 5). For the high *fratio* example however, those width ratios increase from five to 11
4 (Figure 7). In summary, the results demonstrate that manipulation of the *fratio* in the model
5 effectively simulates non-uniform channel constriction and expansion through a river step. In the
6 following subsections, the fundamental response of a river step to different flow magnitudes and
7 different ways of manipulating channel non-uniformity is explored with respect to downstream
8 hydraulic jump regime and energy loss at the step.

9

10 **3.1 Role of Energy Input**

11 This model re-captures the basic response of the river step to increasing discharge for a
12 uniform channel with relatively high values of f and b as previously demonstrated by *Pasternack*
13 *et al.* [2006]. Consider the case of the uniform channel ($fratio=1.0$) with f and b values of 0.4
14 each (Figure 11c, diamonds). For high $(H+P)/H$, the step is relatively high and thus there exists a
15 drop over the step yielding a hydraulic jump at the step toe. Model results show that the highest
16 fractional energy loss occurs at the lowest energy input regardless of geometric combination,
17 because a high fall yields a conversion of potential energy into kinetic energy, and this large
18 amount of converted energy cannot be recovered as potential energy through the hydraulic jump
19 at the base of a high drop (*e.g.*, Figure 12c). When the flow has reach the downstream cross-
20 section, the kinetic energy is greatly dissipated, representing $<0.1\%$ of the total energy (*e.g.*,
21 Figure 13c). As $(H+P)/H$ decreases, the tail depth increases and the water drop height decreases,
22 causing the jump to submerge. In the limit as $(H+P)/H \rightarrow 1$, the hydraulic jump gives way to a
23 standing wave and ultimately no surface expression at all. Correspondingly, there is less potential

1 energy to be dissipated as the jump submerges, so the submerged hydraulic jump helps reduce the
2 energy loss at the step (Figure 12c). Also, since the kinetic energy increases at the upstream
3 cross-section with increasing energy input, it also shows the same functionality at the downstream
4 cross-section (Figure 13c). In the limit as $h_d/H \rightarrow 0$, simple hydraulic geometry relations no longer
5 apply, as the step is fully submerged and the downstream depth becomes the control for the
6 upstream depth. The overall mechanics reported above represent the classic 2-D step behavior for
7 a uniform channel. However natural mountain rivers have strongly non-uniform channels and 3-D
8 planform brink geometries, thereby creating a wide array of conditions remaining to be
9 understood.

11 3.2 Channel Non-uniformity

12 To fully understand the dependence of the downstream geometry on the upstream
13 parameters and the *fratio* used in this model, consider a channel with the upstream channel
14 geometry is defined by the hydraulic exponents (f_{up} and b_{up}) are equal to 0.2 (Table 1g). For an
15 *fratio* of 1.5, the *bratio* is equal to the inverse, i.e. $1/1.5 = 0.67$. The definition of *fratio* is the
16 ratio of f_{up} to f_{tail} , therefore the value for the downstream f value decreases to 0.13. Since the
17 *bratio* is the inverse of the *fratio*, the downstream b value increases from 0.2 to 0.3. Table 1
18 presents the range of hydraulic exponents explicitly used in this model.

19 Consider again the case of the upstream channel geometry having f_{up} and b_{up} values of 0.4
20 each (Figure 11c). Using the results associated with an *fratio* of 1.0 described above as the
21 baseline, the model shows the effect of varying the downstream geometry on the hydraulic jump
22 regime for a given discharge and step height. As the *fratio* for the channel increases above one,
23 the downstream cross-section bends and stretches outward, so water depth decreases, width

1 increases, and the hydraulic jump becomes increasingly emergent. For an *fratio* of 1.5, the zone
2 of the supercritical flow regime increases until $(H+P)/H < 3$ and there is no hydraulic jump present
3 within the downstream control volume. In contrast, as the *fratio* for the channel decreases below
4 one, the downstream cross-section bends and compresses inward, so water depth increases, width
5 decreases, and the hydraulic jump increasingly submerges for constant $(H+P)/H$ to a point where
6 no jump or standing wave feature is present. For example, when *fratio*=0.5, there is no surface
7 expression of the step for $(H+P)/H < 3.76$. As both f_{up} and b_{up} decrease together, the upstream
8 velocity exponent, m , must increase. The associated acceleration of flow decreases depth and
9 width at any given discharge, and thus the model shows that as m increases, the hydraulic jump
10 regime becomes more emergent for all *fratios* (Figure 11c, e, g). For example, when a channel
11 with $f_{up}=b_{up}=0.4$ is compared to one with $f_{up}=b_{up}=0.25$ at an *fratio*=0.75 (Figure 11c, e, crosses),
12 the former shows a wide range of hydraulic jump regimes for $2 < (H+P)/H < 6$, while the latter only
13 exhibits the pushed-off and optimum-jump regimes.

14 A non-uniform channel with an *fratio*>1 exhibits more relative head loss than one with
15 *fratio*<1 for all geometric combinations (Figure 12). The dominant cause of head loss at river
16 steps is the difference in potential energy at the upstream and downstream cross-sections, since
17 kinetic energy is extremely difficult to recover as potential energy and tends to dissipate through
18 multiple scales of turbulence and bed interaction. For high values of f , the amount of kinetic
19 energy at the downstream cross-section is less than 1% of total energy (Figure 13a, b, c). These
20 new results demonstrate a key role for channel non-uniformity in controlling the diverse
21 conditions found in the vicinity of natural river steps, in contrast to the limited conditions reported
22 for engineered steps.

23

1 3.3 Varying Upstream Channel Geometry

2 A comparison among results for different sets of f_{up} and b_{up} combinations (Figures 11 and
3 12) shows that the standard concept of how steps function breaks down when at-a-station
4 hydraulic geometry is not dominated by depth-responsiveness, i.e., when the value of f is low.
5 Because the sensitivity of the system to f and b values is co-dependent on the f_{ratio} , the results are
6 explained for specified example f_{ratio} values. The most extreme condition examined in which
7 the standard conceptual model breaks down is the case when $f_{ratio} \geq 1.5$ (Figure 11, circles).
8 Under this condition, the hydraulic jump is pushed downstream of the step toe at high $(H+P)/H$
9 values for all channel geometries. As $(H+P)/H$ decreases, the jump becomes increasingly
10 emergent (i.e. the supercritical zone downstream of the step toe becomes longer) until no jump is
11 present and the flow remains supercritical throughout the downstream section. The threshold for
12 supercriticality depends on the channel geometry, occurring at lower $(H+P)/H$ ratios as either f or
13 b increases. Thus for $f_{ratio} \geq 1.5$, the hydraulic jump regime is insensitive to the absolute values of
14 f and b . It is sensitive, however, in terms of energy loss and kinetic energy (Figure 12, 13). For
15 $f_{ratio} \geq 1.5$, the system in general incurs more head loss for higher values of f and b ; except for
16 cases in which $b \geq 0.4$, then the system exhibits greater head loss for decreasing f values. This
17 pattern corresponds with the general decrease in kinetic energy as f and/or b increase.

18 The behavior of the hydraulic jump regime as a function of f and b values when
19 $f_{ratio} = 1.25$ (Figure 11, squares) appears to be similar to that described above, when both f and b
20 values are low. Under that condition, the flow becomes fully supercritical throughout the
21 downstream section for low $(H+P)/H$ values. At high f and b values, however, the jump exists as
22 a pushed-off jump throughout the range of $(H+P)/H$ values. The head loss for systems with
23 $f_{ratio} = 1.25$ responds similarly to $f_{ratio} \geq 1.5$ systems described above.

1 In contrast to the insensitivity of the jump emergence regime to upstream f and b values
2 when $f_{ratio} \geq 1.0$, the fundamental behavior of the hydraulic jump regime shows strong sensitivity
3 to those variables when $f_{ratio} \leq 1.0$. When $f=0.4$ and $f_{ratio}=0.75$, the hydraulic jump submerges
4 increasingly fast as a function of decreasing $(H+P)/H$, regardless of the value of b . A similar
5 trend is evident when $b=0.4$ and $f_{ratio}=0.75$, except when $f=0.2$. In that case the depth-response
6 is weak enough that he just transitions from a pushed-off to an optimum jump condition, but not
7 to a submerged jump (Figure 11i). Holding $f=0.2$ and $f_{ratio}=0.75$, as b decreases from 0.4 to 0.2,
8 the geometric effect is to bend the channel from convex-up to triangular, but also to strongly raise
9 the stage-dependence of channel slope and/or roughness. These changes enable the downstream
10 cross-section to transport water away faster, which lowers the tailwater depth and emerges of the
11 hydraulic jump for any given $(H+P)/H$. (Figure 11g-i). The head loss when $f_{ratio}=0.75$ tends to
12 decrease with increasing upstream b or f values (Figure 12), since the tailwater depth is higher at
13 those higher exponent values.

14 When $f_{ratio}=0.5$ (Figure 11, triangles), the channel choking (i.e. upstream submergence
15 due to downstream controls) is extreme enough that the nonlinear rate of submergence increases
16 for all values of b and f to the point that no surface feature is present when $(H+P)/H=2$ unless f
17 and b are both ≤ 0.2 (Figure 11g). In that case, the at-a-station hydraulic geometry requires that
18 most of the response to increasing discharge goes to increasing flow velocity, thereby holding off
19 complete drowning of the step. The head loss decreases rather quickly compared to the other
20 f_{ratio} systems with decreasing $(H+P)/H$ values. The response of the head loss is, however,
21 insensitive to varying b and f values, except for the extreme lower b and f values (≤ 2) (Figure
22 12g). The head loss response mimics the hydraulic jump regime response.

23

1 3.4 Model Sensitivity to Geometric Coefficients

2 The impact of the hydraulic geometry coefficients, a and c , on step mechanics is similar to
3 that reported for manipulations of the f_{ratio} (Figures 14-17), and is consistent with intuition based
4 on the geometric explanations provided in sections 2.2, 2.3, 3.1, and 3.2. The step response to
5 only variations in geometric coefficients is evident in the results for $f_{ratio}=1$ (Figures 14-17,
6 diamonds). For example, when $f_{up}=b_{up}=0.2$ and $f_{ratio}=1$, the hydraulic jump regimes become
7 more supercritical as either a or c decrease with increasing discharge (i.e. decreasing $(H+P)/H$)
8 (Figure 14). A similar effect was described as either b or f decreased, while coefficients were
9 held constant (Figure 11). A decrease in either coefficient while holding the other constant and
10 the exponents constant requires an increase in the coefficient k . Physically, that means that the
11 channel is steeper and/or smoother, which enables water to transport out of the step region faster,
12 lowering tailwater depth and emerging the jump regime. The same emergence response of
13 hydraulic jump regime to decreasing a and/or c is evident regardless of f_{ratio} (Figure 14). These
14 patterns were also observed when holding coefficients constant and varying exponents (Figure
15 11).

16 When $f_{up}=b_{up}=0.4$, the sensitivity of the model to the coefficients is quite dampened
17 (Figure 16), as compared to the case when $f_{up}=b_{up}=0.2$. For all combinations of a and c tested at
18 these higher exponent values, the hydraulic jump submerged with increasing discharge when
19 $f_{ratio}<0.9$. These results show that channels whose energy slope and bed roughness exhibit a
20 sensitive discharge-dependence will be more sensitive to the exact values of the coefficients than
21 channels whose primary discharge sensitivity is exhibited in depth and/or width.

22 The similarity of the response of the hydraulic jump regimes to changes in the coefficients
23 versus the exponents of equation (7) holds in the energy loss relations as well (e.g. compare

1 Figures 12, 15, and 17). When $f_{up}=b_{up}=0.2$, energy loss is lower for higher coefficient values, since
2 tailwater depth and velocity are lower (Figure 15). When $f_{up}=b_{up}=0.4$, the range of variability in
3 energy loss as a function of coefficient values is less than when $f_{up}=b_{up}=0.2$ (Figure 17). These
4 results confirm that either the geometric coefficients or exponents can affect the step response,
5 even though they do so differently – by stretching versus bending of the channel cross-section.

6

7 **4. Discussion**

8

9 **4.1 Energy, Velocity, and Erosion**

10 For any set of model parameters, the results revealed an inverse relation between velocity
11 at the downstream cross-section and energy loss through the step for a given *f*ratio (Figure 13).
12 This finding has ramifications for step erosion mechanisms. At low discharge, step height is
13 relatively high and energy dissipation is in the range of 80-95% of total energy. As water goes
14 over the brink, it accelerates and hits the bed downstream of the step toe. For a vertical step, the
15 angle of the nappe profile is steepest at the lowest discharge [Pasternack et al., 2006], which
16 facilitates direct hydraulic erosion [USBR, 1948a]. For the range of hydraulic geometry
17 coefficients and exponents explored, the hydraulic jump regime for low discharge is
18 predominantly pushed off or optimal jump, meaning that the falling jet can go through the
19 tailwater, impact the bed, and cause erosion, promoting plunge-pool formation. However, since
20 the specific energy (H) is low at low discharge, the velocity of the water as it goes over the brink
21 is relatively low, and even with acceleration it remains relatively low at the step toe compared to
22 that at higher H . When the water reaches the downstream cross-section, its velocity is very low,
23 since most energy was dissipated (Figure 13). As discharge increases, upstream velocity,

1 tailwater depth, and tailwater velocity all increase, while nappe profile angle becomes more
2 horizontal and energy loss decreases, all driven by higher specific energy. Depending on the
3 *f*ratio, the hydraulic jump regime may become more emergent or more submerged. Different
4 scour mechanisms appear to exist for the different hydraulic jump regimes [Pasternack *et al.*,
5 2007], but insufficient literature exists to compare and contrast the relative magnitude of scour
6 under pushed off versus optimum jump conditions. However, once a jump is drowned, both jet
7 impingement and lift-stress variation on the bed likely decrease. Higher energy loss, higher
8 velocity at the step toe, and jump emergence promote bed scour, while lower energy loss, jump
9 submergence, and a more horizontal nappe profile diminish it. Therefore, the mechanisms and
10 magnitude of scour vary substantially for a given channel configuration as discharge changes.

11 Compared to the velocity, energy loss, and implied scour mechanics at an individual step,
12 there is a highly significant effect of changing channel configuration. For all sets of model
13 parameters explored, an increase in the *f*ratio (or an equivalent change to geometric coefficients)
14 for a given level of tailwater kinetic energy causes a dramatic increase in energy loss and keeps
15 the tailwater velocity high (Figure 13). For example, when kinetic energy is held at 5% of total
16 energy and the other parameters are as given for Figures 9e and 10e, energy loss increases from
17 zero for a *f*ratio of 0.5 to 63% of total energy for a *f*ratio of one to 76% for a *f*ratio of 1.5 (Figure
18 18). As the *f*ratio and energy loss increase, the hydraulic jump regime emerges, while the nappe
19 profile angle remains the same, since *H* is the same. In this case, different factors and scour
20 mechanisms are working synergistically to promote bed scour. Overall, the model results
21 demonstrate that variation in channel configuration is a powerful variable controlling scour
22 mechanisms at river steps.

23

1 **4.2 Knickpoints at Canyon Entrances**

2 One physiographic domain in which knickpoints are common is the transitional canyon
3 between high-elevation plateaus and lowlands. In such regions, there exist observations of
4 knickpoints occurring at the upstream limit of the canyon, right at the transition point where
5 valley and channel width suddenly changes [e.g., *Snyder and Kammer, 2008*]. Landscape
6 evolution models of this domain that simulate plateau retreat by computing channel scour as a
7 function of local slope and basin area predict higher rates of retreat during higher flows.
8 However, the results of this study indicate that the exact opposite could hold, or scour rates could
9 just be independent of discharge. Specifically, a knickpoint at the transition from a high plateau
10 to a bedrock canyon exhibits valley constriction through the step unit. As discharge increases
11 during an overbank event, the tailwater depth would quickly increase and drown the step,
12 neutralizing the most effective scour mechanisms acting on the step toe. If the step is too high to
13 completely submerge, it would at least drown enough to protect the bed from both hydraulic jet
14 scour and bedload-impact scour. Thus, the geomorphically significant example of plateau-to-
15 canyon transitions illustrates the importance of the new model relative to the existing algorithms
16 used in landscape evolution modeling. Scour at knickpoints in bedrock channels is definitely not
17 a simple power function of slope and basin area.

19 **4.3 Model Applications**

20 Investigation of the fluid mechanics at river steps in non-uniform channels using a
21 parsimonious semi-analytical model revealed important non-classical behavior governing the
22 fundamental nature of river steps. Although the model was generalized for broad scientific
23 inquiry, model parameterization for a specific local site would enable its use for a wide range of

1 management and engineering applications. For a specified site, such parameterization would
2 entail observing channel width and depth upstream and downstream of the step over the range of
3 flows under the baseline condition. Once the local flow-geometry relations are known, then the
4 model becomes easily testable for any site since the only independent input variable needed is the
5 upstream flow depth. Theoretical jump regimes and energy losses could then be compared to
6 experimental or observed conditions. After satisfactory validation of the local model, alternative
7 modifications to the channel geometry could be tested with the model to evaluate the resulting
8 hydraulic jump and energy dissipation regimes relative to the desired goals. Common goals
9 typically involve reducing swimmer or boater drowning hazard, decreasing the rate of knickpoint
10 migration in gullies, and creating exciting recreational whitewater parks.

11 *Leutheusser and Birk* [1991] described the drowning hazard posed by overflow hydraulic
12 structures that have a drowned jump (a.k.a. “plunging nappe”) and they analyzed the engineering
13 requirements to avoid this hazard. The same hazard exists at many natural steps as well. The
14 design goal to avoid the hazard is to ensure that the step has any other hydraulic jump regime over
15 the range of discharge and tailwater depth conditions that occur there. The solution provided
16 by *Leutheusser and Birk* [1991] only allows for adjustment of step height. Using the new model
17 proposed herein, the same hazard avoidance can be achieved using a variety of geometric
18 adjustments. For situations where step height could not be modified, one solution would be
19 increase the *fratio* of the channel to keep the step toe in the optimum jump condition over the
20 range of discharges expected.

21 Although the model does not directly predict scour and knickpoint migration rates at this
22 time, it does provide the basis for inference and channel design. A key advantage of linking scour
23 to energy dissipation instead of drag force is that existing equations used to predict scour using

1 the latter approach have been shown to perform poorly [Pasternack *et al.*, 2007], because they do
2 not account for the mechanics and trade-offs discussed in section 4.1. In contrast, this new
3 numerical model predicting energy dissipation does account for different processes, including the
4 hydraulic jump regime. According to the model, the higher the *fratio* for a knickpoint, then the
5 more the channel morphology is promoting higher energy dissipation (Figure 12) and thus higher
6 rates of toe scour and knickpoint migration related to this process. To reduce the rate of
7 knickpoint migration in that case, the channel manipulation would be the exact opposite of that
8 for drowning hazard mitigation. By decreasing the *fratio* at a site, the magnitude of energy
9 dissipation at the knickpoint would be decreased for a given tailwater velocity (Figures 12, 18)
10 and the hydraulic jump would be more submerged (Figure 11), for any given discharge. The
11 utility of this solution depends on the actual *fratio* at a knickpoint and the degree to which a site
12 can be re-engineered.

13 A third example where this model would be of value is in the construction of increasingly
14 popular, recreational whitewater parks. The primary engineering feature used repetitively in such
15 parks is a channel constriction at the step to produce the desired standing wave or hydraulic jump
16 for canoe and kayak acrobatics (Figure 19). Presently such features are designed based on
17 practitioner experience, but the model could be used to accurately predict and control what the
18 jump regime will be in advance as a cost-saving measure. Similar applications are readily
19 apparent for dam removal, river rehabilitation, and fish passage – all situations in which human
20 safety is jeopardized by the unknown hydraulic jump regime and geomorphic unit instability
21 associated with step scour that is not accurately predicted by accounting for the scour associated
22 with energy dissipation at the step.

23

1 4.4 Roadmap for Step Theory Advancement

2 The underlying assumptions and limitations of the new model presented in this study have
3 been described in detail throughout. Although this study explored a wide range of model
4 parameters, there exists a dearth of data on the hydraulic geometry of channels in the vicinity of
5 knickpoints. The exact relation between *fratio* and *bratio* in bedrock channels is unknown. The
6 relative abundance of steps with different *fratios*, perhaps as a function of uplift rate and geology,
7 is unknown. Overall, a new examination and appreciation of the real diversity in natural river
8 steps needs to replace existing models based on processes in alluvial rivers, gullies, and man-
9 made spillways.

10 The biggest deficiency in the model at this time is in the hydraulic jump regime equations
11 (15)-(18), which are only strictly valid for an ogee-crested, rectangular 2-D overfall. The
12 delineation between optimum jump and drowned jump regimes suggested by *USBR* [1948b]
13 appears highly subjective. Also, the thresholds do not delineate the standing wave regime.
14 Consequently, this theoretical study indicates that future research into the hydraulic geometry of
15 step units in bedrock rivers and the dynamics of hydraulic jump regimes for different archetypal
16 steps should be prioritized.

17 Beyond the numerical model, key areas of future research are suggested by the conceptual
18 framework (Figure 1). Most important would be to quantify the role of hydraulic jump regime
19 and energy dissipation in step morphodynamics. In the past, studies have focused on the role of
20 the hydraulic jet in scouring the plunge-pool [*Mason and Arumugam*, 1985; *Bormann and Julien*,
21 1991]. Some studies have also demonstrated the importance of pressure fluctuations and lift in
22 scour below steps [*Fiorotto and Rinaldo*, 1992; *Fiorotto, V., Salandin*, 2000; *Pasternack et al.*,
23 2007], though not accounting for scour associated with bedload-impact. As airborne laser swath

1 mapping provides meter-scale resolution of topography in otherwise inaccessible canyons, we
2 conjecture that it will be possible for landscape evolution models to incorporate the knickpoint
3 mechanics associated with channel non-uniformity. Other factors in the conceptual framework
4 that could be addressed relatively easily include step roughness and wind.

6 **5. Conclusions**

7 The effect of varying channel geometry upstream and downstream of a river step on
8 hydraulic jump regime and energy dissipation has not previously been investigated. This research
9 has quantified the roles that hydraulic geometry and discharge have in determining the hydraulic
10 jump regime and energy dissipation at river steps using a parsimonious model. For a given
11 discharge and step morphology, an increase in the *f* ratio leads to an increasingly emergent
12 hydraulic jump. Channel non-uniformities that exhibit an *f* ratio > 1.0 tend to be relatively
13 insensitive to changes in the *f* or *b* parameters in terms of the hydraulic jump regime, existing as a
14 pushed-off jump or supercritical flow for the whole range of $(H+P)/H$ values. However, these
15 channel non-uniformities exhibit higher energy loss through a step as *f* or *b* increases. For
16 channel non-uniformities with an *f* ratio < 1.0, there exists a strong sensitivity to changes in both *f*
17 and *b* values in terms of the hydraulic jump regime ranging from a pushed-off jump to a
18 completely drowned-out jump. The head loss of these systems decreases with either increasing *f*
19 or *b* values. For non-uniformities with *f* ratio > 1.0, head losses are high over the whole $(H+P)/H$
20 range. For non-uniformities with *f* ratio < 1.0, flow becomes choked and head losses decrease
21 significantly as $(H+P)/H$ decreases. Regardless of channel geometry, head losses due to potential
22 energy losses dominate the flow regime at high relative step heights.

1 Limitations in the model show that as the ratio h_u/H approaches zero, the simple hydraulic
2 geometry equations no longer apply at the upstream cross-section, because the downstream depth
3 acts as a control on the upstream depth. Because of the interplay between flow, channel
4 geometry, and geological resistance, step toe emergence with high energy dissipation can induce
5 scour and plunge pool formations, limiting the abundance of natural emerged jump regimes at
6 bedrock steps. It may be that using a cross-section to characterize the downstream control on the
7 step toe is inadequate, necessitating a 3-D topographic and hydrodynamic modeling framework.
8 However, the parsimony of the analytical approach presented herein provides a reasonable
9 predictive foundation for further analytical investigation.

10 A better understanding of hydro-geomorphic processes in mountain rivers, particularly at
11 river steps, would provide advancement in the understanding of mountain river fluid mechanics,
12 aquatic ecology, and channel evolution. Such improved understanding can aid in determining the
13 impacts of channel engineering projects and can provide guidance to river restoration and urban
14 stream rehabilitation efforts. Previous research has typically ignored the relevance of hydraulic
15 jumps within channel evolution models. Since hydraulic jump regimes can be associated with
16 steps of any size, including those induced by large wood jams and gravel bars, a better
17 understanding of complex step processes can contribute to a better understanding of many
18 channel features.

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3

4 **Notation**

5	a	hydraulic geometry width coefficient
6	b_{up}	upstream hydraulic geometry width exponent
7	b_{tail}	downstream hydraulic geometry width exponent
8	c	hydraulic geometry depth coefficient
9	C_b	broad-crested weir discharge coefficient
10	E	total flow energy
11	E_{up}	upstream total flow energy
12	E_{tail}	downstream total flow energy
13	f_{up}	upstream hydraulic geometry depth exponent
14	f_{tail}	downstream hydraulic geometry depth exponent
15	f_{ratio}	ratio of f_{up} to f_{tail}
16	Fr	Froude number
17	g	gravitational constant
18	H	specific energy upstream of step
19	h_b	flow depth at brink of step
20	h_c	critical depth
21	h_d	submergence variable of hydraulic jump
22	h_H	flow depth above step crest datum
23	h_{up}	upstream flow depth
24	h_{tail}	downstream flow depth
25	h_{toe}	flow depth at toe of hydraulic jump
26	h_L	head loss
27	h_v	velocity head
28	h_{v_up}	upstream velocity head
29	h_{v_tail}	downstream velocity head
30	k_{up}	upstream hydraulic geometry velocity coefficient
31	k_{tail}	downstream hydraulic geometry velocity coefficient
32	m_{up}	upstream hydraulic geometry velocity exponent
33	m_{tail}	downstream hydraulic geometry velocity exponent
34	P	step height
35	Q	volumetric discharge
36	q	specific discharge
37	v	flow velocity
38	w	flow width
39		

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13 **Figure Captions**

14 Figure 1 - Flowchart showing interrelationships between dependent and independent processes at
15 river steps. Rhombi are external independent processes. Bold arrows indicate processes
16 discussed herein.

17 Figure 2 – Longitudinal schematic of an idealized step showing a submerged jump illustrating
18 parameters used in the model. Not to scale to permit notation (See notation list).

19 Figure 3 – Physical interpretation of hydraulic geometry exponent values for a cross-section in a
20 single-threaded stream. Shaded region is where $b > f$.

21 Figure 4 – Sensitivity analysis illustrating the similar effects on channel width of varying either
22 (A) the coefficient a or (B) the exponent b in equation (7a), while holding the other constant.

23 Figure 5 – Illustration of the channel non-uniformity through a reach with a river step where the
24 $f_{ratio} < 1$. Widths, depths, and shapes are roughly to scale, but are not mapped quantitatively.
25 Depth and width values for values of $(H+P)/H$ equal to 5.81 (i.e. low discharge) and 1.98 (i.e.
26 high discharge) are shown for $f_{up} = b_{up} = 0.4$ and $f_{ratio} = 0.5$.

1 Figure 6 – Example of a step unit with $f > b$ and $f_{ratio} < 1$ showing channel constriction through the
2 domain. Upper South Fork Snoqualmie River, California, looking upstream

3 Figure 7 – Illustration of the channel non-uniformity through a reach with a river step where the
4 $f_{ratio} > 1$. Widths, depths, and shapes are roughly to scale, but are not mapped quantitatively.
5 Depth and width values for values of $(H+P)/H$ equal to 5.81 (i.e. low discharge) and 1.98 (i.e.
6 high discharge) are shown for $f_{up}=b_{up}=0.4$ and $f_{ratio}=0.5$.

7 Figure 8 – Example of a step unit with $f > b$ and $f_{ratio} > 1$ showing channel expansion through the
8 domain. Upper South Fork Blackwood Creek, California, looking upstream.

9 Figure 9 - Generalized hydraulic jump regimes in reference to dimensionless upstream energy and
10 downstream submergence, as derived from *USBR [1948b]*.

11 Figure 10 – Width to depth ratios at the downstream cross section as a function of dimensionless
12 upstream energy illustrating that the f_{ratio} effectively controls channel constriction and
13 expansion as a function of discharge through a reach with a river step.

14 Figure 11 – Hydraulic jump regime model results and sensitivity analysis for the ranges of f_{up} , b_{up} ,
15 and f_{ratio} explored. The f and b values stated refer to upstream condition. Solid lines
16 delineate hydraulic jump regimes.

17 Figure 12 - Energy dissipation model results and sensitivity analysis for the ranges of f_{up} , b_{up} , and
18 f_{ratio} explored. The f and b values stated refer to upstream condition.

19 Figure 13 – Energy loss through the step and kinetic energy at the downstream cross-section had
20 an inverse relation for any given set of parameters explored.

21 Figure 14 – Hydraulic jump regime model results and sensitivity analysis for the ranges of a_{up} , c_{up} ,
22 and f_{ratio} explored with $f_{up}=b_{up}=0.2$. The a and c values stated refer to upstream condition.
23 Solid lines delineate hydraulic jump regimes.

1 Figure 15 – Energy dissipation model results and sensitivity analysis for the ranges of a_{up} , c_{up} , and
2 *fratio* explored with $f_{up}=b_{up}=0.2$. The a and c values stated refer to upstream condition.

3 Figure 16 – Hydraulic jump regime model results and sensitivity analysis for the ranges of a_{up} , c_{up} ,
4 and *fratio* explored with $f_{up}=b_{up}=0.4$. The a and c values stated refer to upstream condition.
5 Solid lines delineate hydraulic jump regimes.

6 Figure 17 – Energy dissipation model results and sensitivity analysis for the ranges of a_{up} , c_{up} , and
7 *fratio* explored with $f_{up}=b_{up}=0.4$. The a and c values stated refer to upstream condition.

8 Figure 18 – Head loss through a river step for a given magnitude of tailwater velocity is higher for
9 reaches with a higher *fratio*. Relation shown is for $f_{up}=b_{up}=0.25$, $a_{up}=a_{tail}=9.74$, $c_{up}=c_{tail}=0.51$,
10 and 5% of total energy at the downstream cross-section exists as kinetic energy.

11 Figure 19 – Example of engineered step with hydraulic jump in a whitewater park. Lyons Play
12 Park, St. Vrain River, Lyons, Colorado.

13 14 **Table Captions**

15 Table 1 - Values used in sensitivity analysis of the effect of hydraulic geometry exponents on step
16 hydraulics

Table 1. Values used in sensitivity analysis of the effect of hydraulic geometry exponents on step hydraulics

(a) $f_{up}=0.4, b_{up}=0.2$

fratio	bratio	f_{tail}	b_{tail}
1.5	0.667	0.267	0.3
1.25	0.800	0.320	0.25
1	1.000	0.400	0.2
0.9	1.111	0.444	0.18
0.75	1.333	0.533	0.15
0.5	2.000	0.800	0.1

(b) $f_{up}=0.4, b_{up}=0.25$

fratio	bratio	f_{tail}	b_{tail}
1.5	0.667	0.267	0.375
1.25	0.800	0.320	0.313
1	1.000	0.400	0.25
0.9	1.111	0.444	0.225
0.75	1.333	0.533	0.188
0.5	2.000	0.800	0.125

(c) $f_{up}=b_{up}=0.4$

fratio	bratio	f_{tail}	b_{tail}
1.5	0.667	0.267	0.6
1.25	0.800	0.320	0.5
1	1.000	0.400	0.4
0.9	1.111	0.444	0.36
0.75	1.333	0.533	0.3
0.5	2.000	0.800	0.2

(d) $f_{up}=0.25, b_{up}=0.2$

fratio	bratio	f_{tail}	b_{tail}
1.5	0.667	0.167	0.3
1.25	0.800	0.200	0.25
1	1.000	0.250	0.2
0.9	1.111	0.278	0.18
0.75	1.333	0.333	0.15
0.5	2.000	0.500	0.1

(e) $f_{up}=b_{up}=0.25$

fratio	bratio	f_{tail}	b_{tail}
1.5	0.667	0.167	0.375
1.25	0.800	0.200	0.313
1	1.000	0.250	0.25
0.9	1.111	0.278	0.225
0.75	1.333	0.333	0.188
0.5	2.000	0.500	0.125

(f) $f_{up}=0.25, b_{up}=0.4$

fratio	bratio	f_{tail}	b_{tail}
1.5	0.667	0.167	0.6
1.25	0.800	0.200	0.5
1	1.000	0.250	0.4
0.9	1.111	0.278	0.36
0.75	1.333	0.333	0.3
0.5	2.000	0.500	0.2

(g) $f_{up}=b_{up}=0.2$

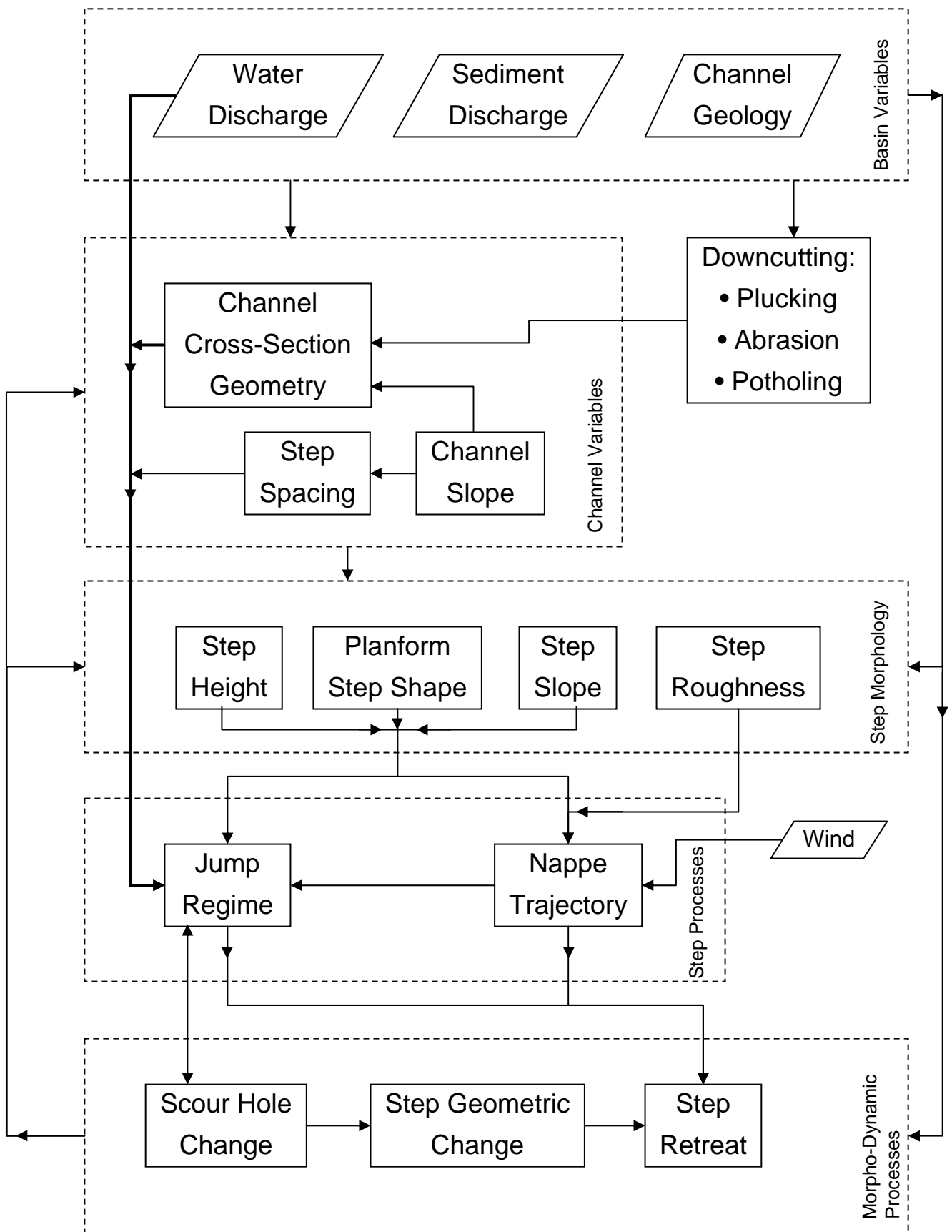
fratio	bratio	f_{tail}	b_{tail}
1.5	0.667	0.133	0.3
1.25	0.800	0.160	0.25
1	1.000	0.200	0.2
0.9	1.111	0.222	0.18
0.75	1.333	0.267	0.15
0.5	2.000	0.400	0.1

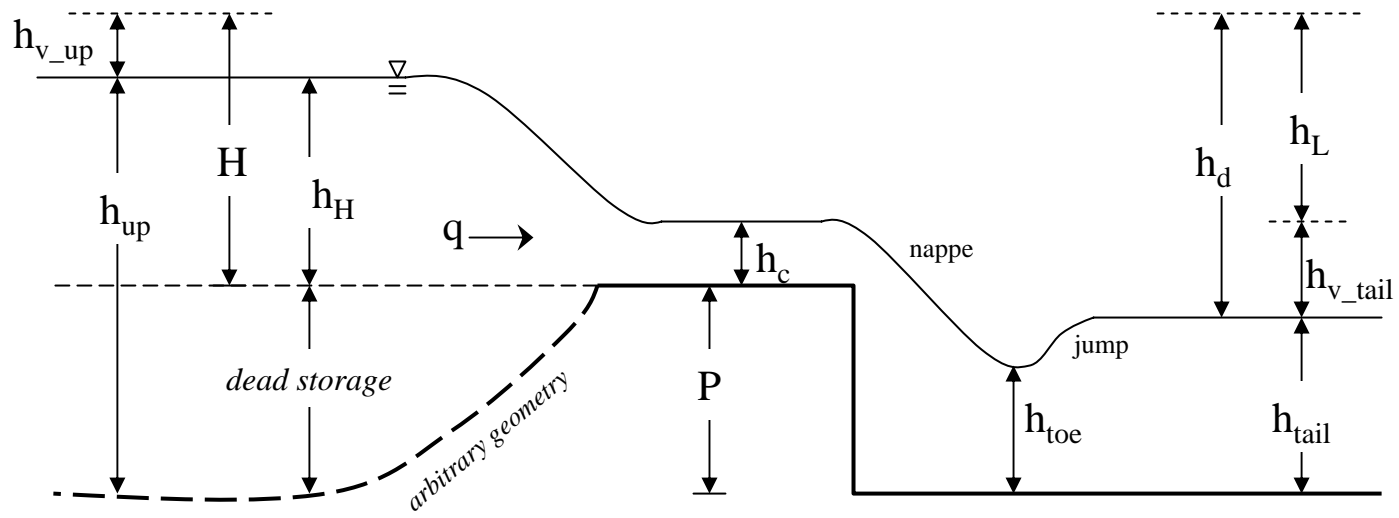
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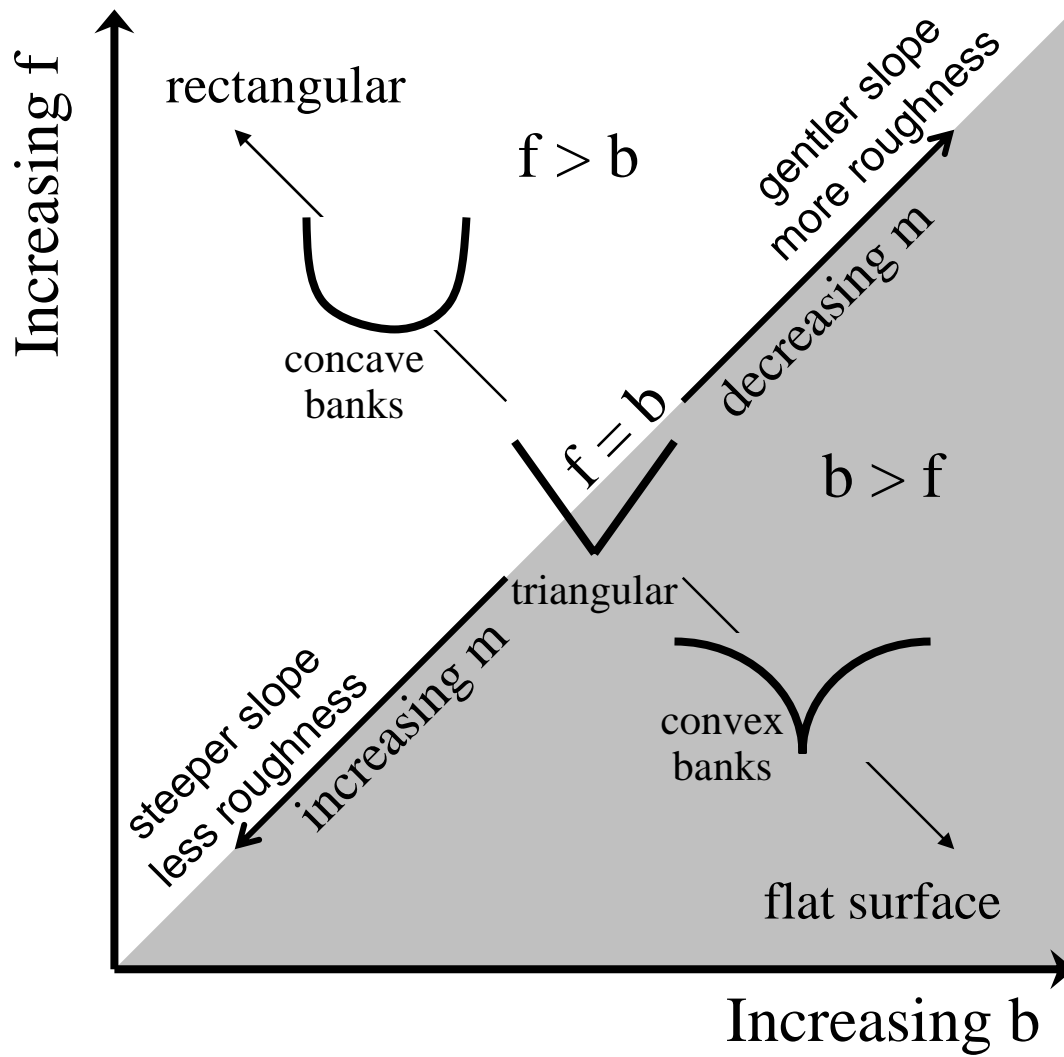
fratio	bratio	f_{tail}	b_{tail}
1.5	0.667	0.133	0.375
1.25	0.800	0.160	0.313
1	1.000	0.200	0.25
0.9	1.111	0.222	0.225
0.75	1.333	0.267	0.188
0.5	2.000	0.400	0.125

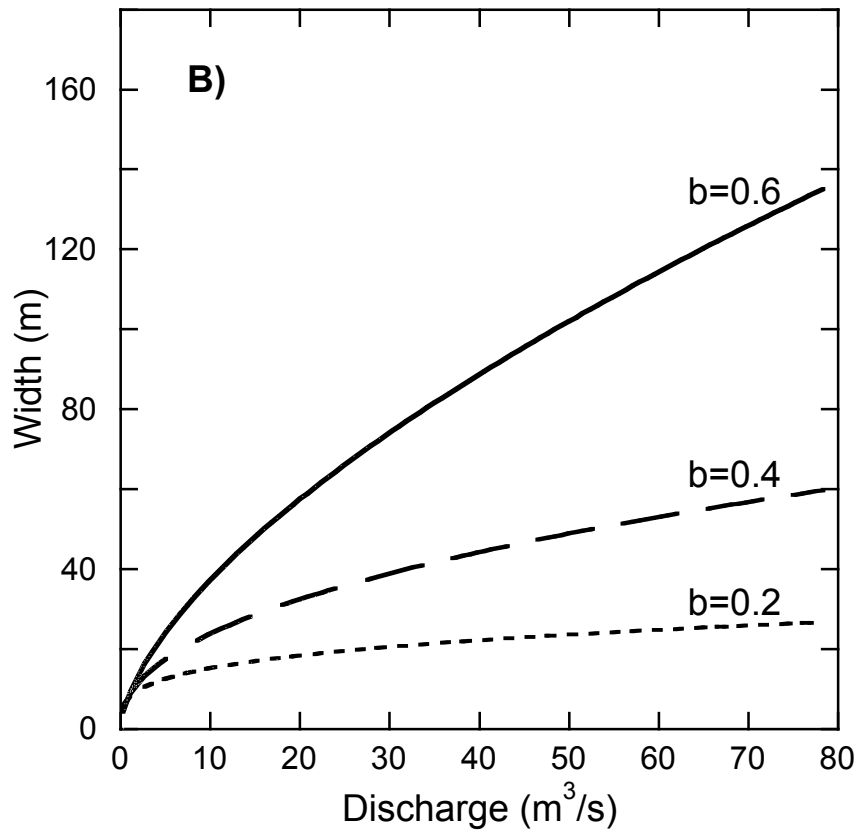
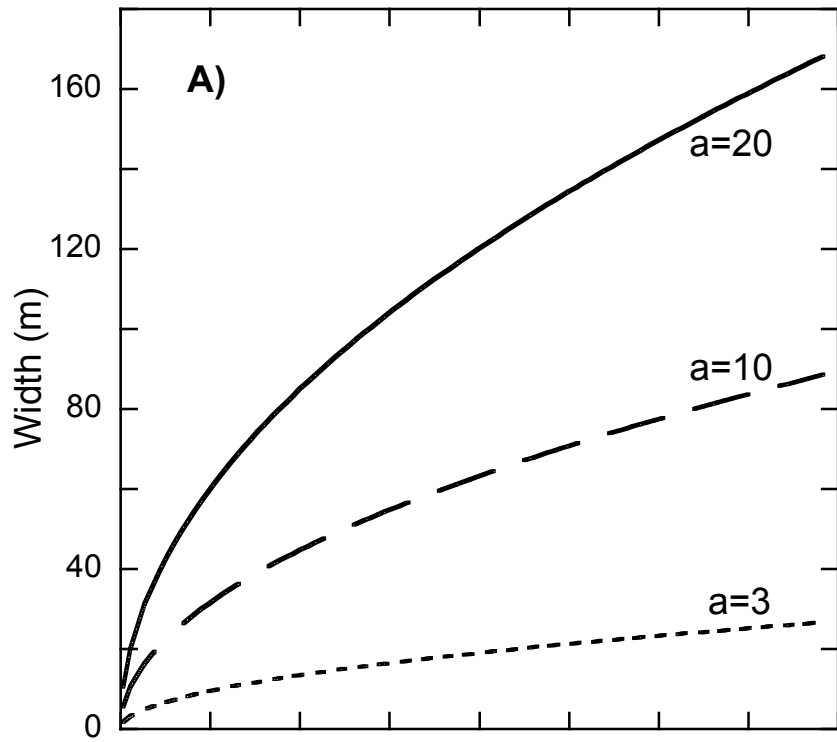
(i) $f_{up}=0.2, b_{up}=0.4$

fratio	bratio	f_{tail}	b_{tail}
1.5	0.667	0.133	0.6
1.25	0.800	0.160	0.5
1	1.000	0.200	0.4
0.9	1.111	0.222	0.36
0.75	1.333	0.267	0.3
0.5	2.000	0.400	0.2









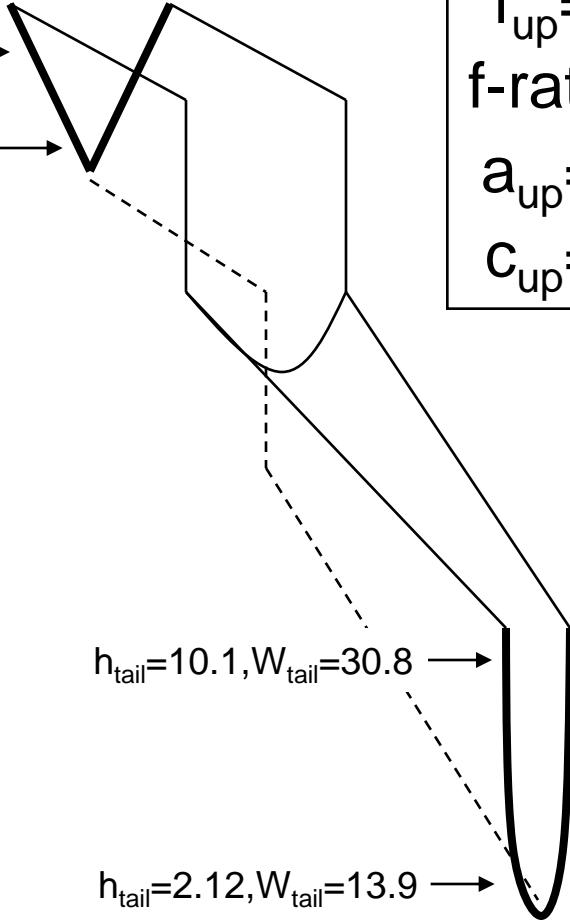
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$h_{up}=1.04, W_{up}=19.8$ →

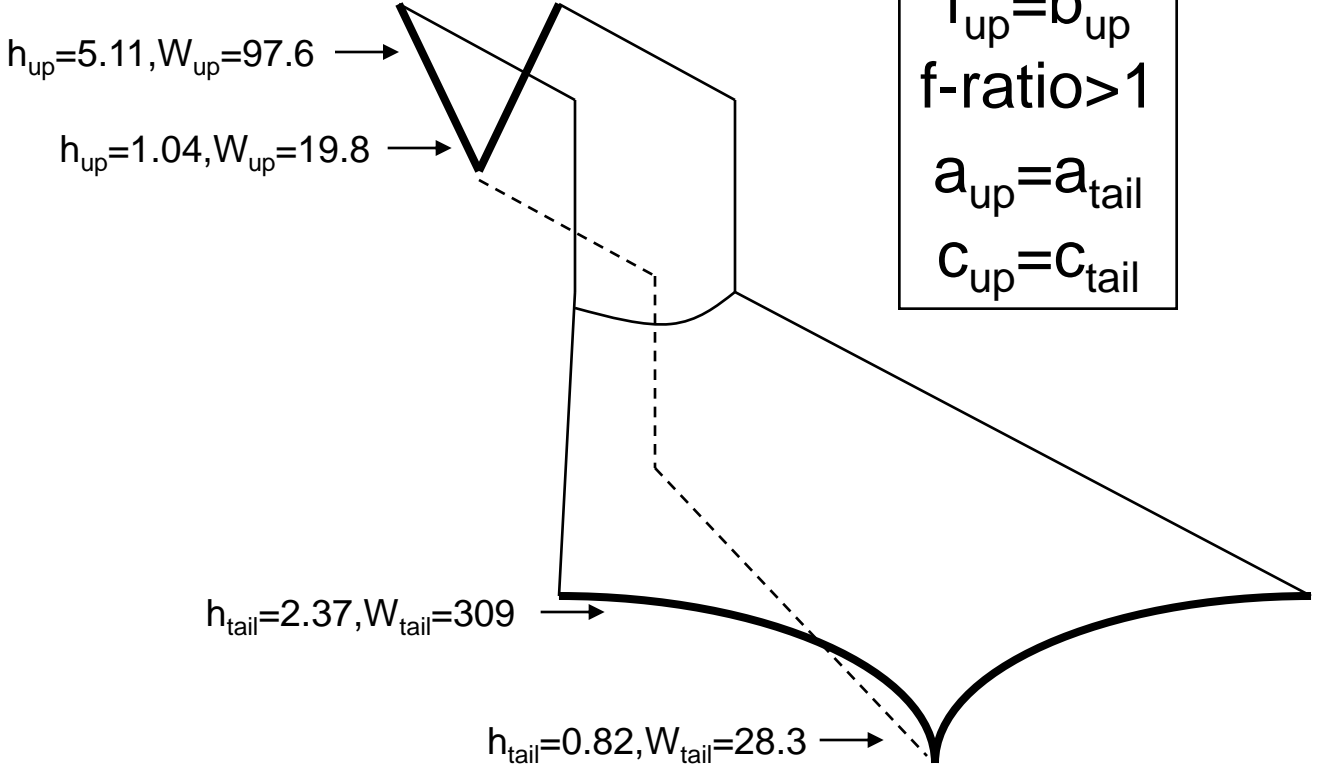
$f_{up}=b_{up}$
 $f\text{-ratio}<1$
 $a_{up}=a_{tail}$
 $C_{up}=C_{tail}$

$h_{tail}=10.1, W_{tail}=30.8$ →

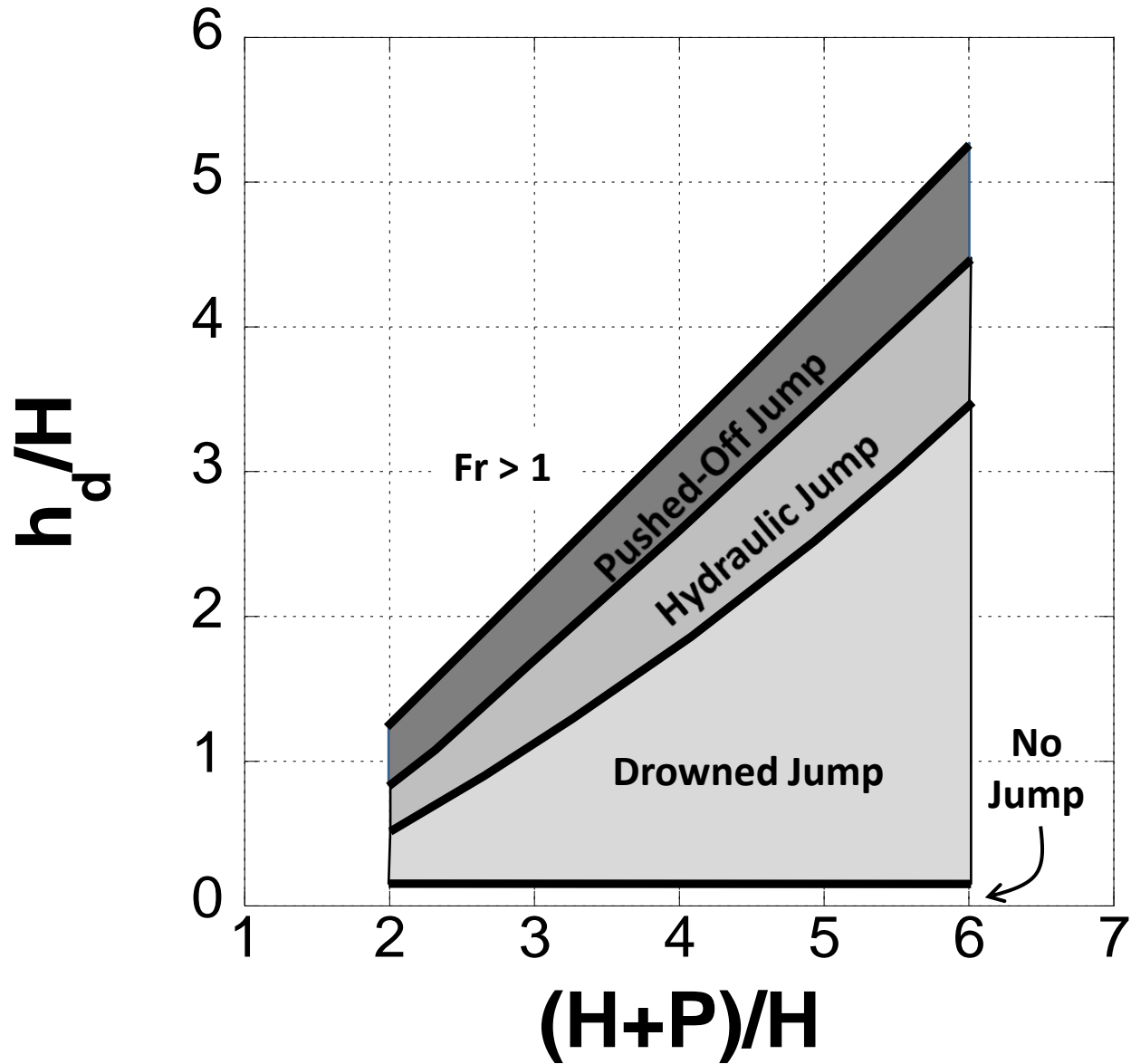
$h_{tail}=2.12, W_{tail}=13.9$ →





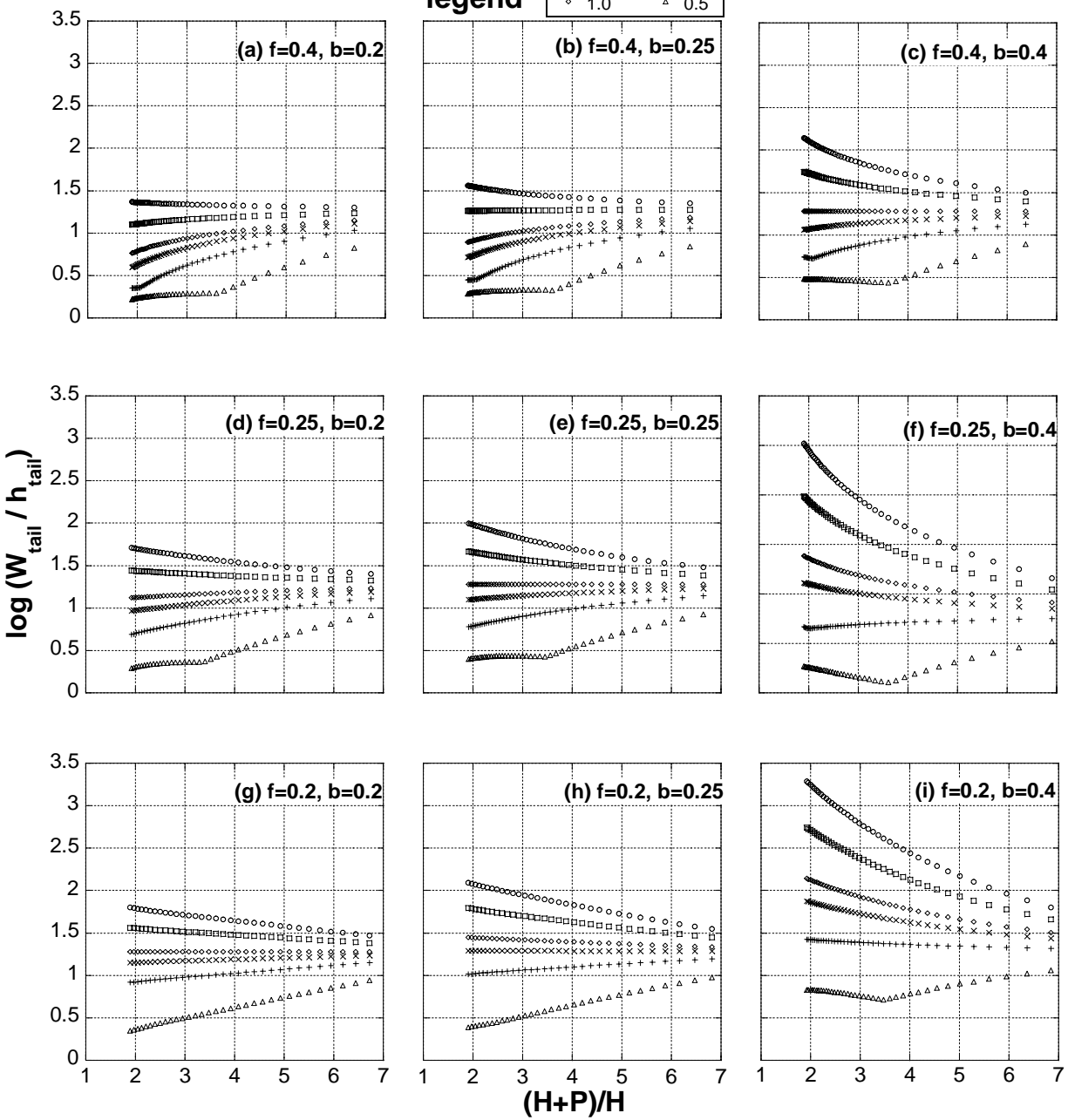






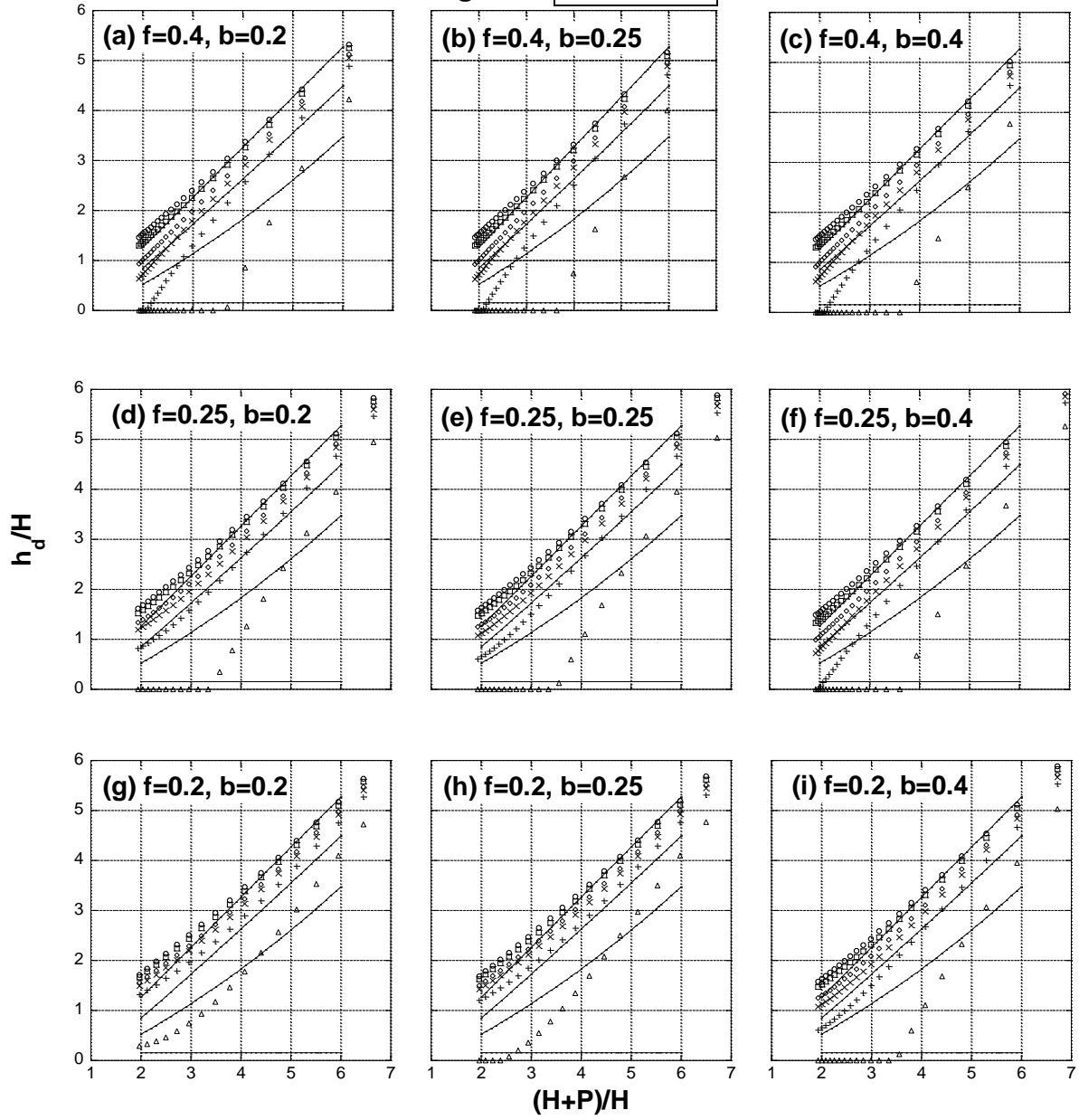
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○ 1.5	× 0.9
□ 1.25	+ 0.75
◇ 1.0	△ 0.5



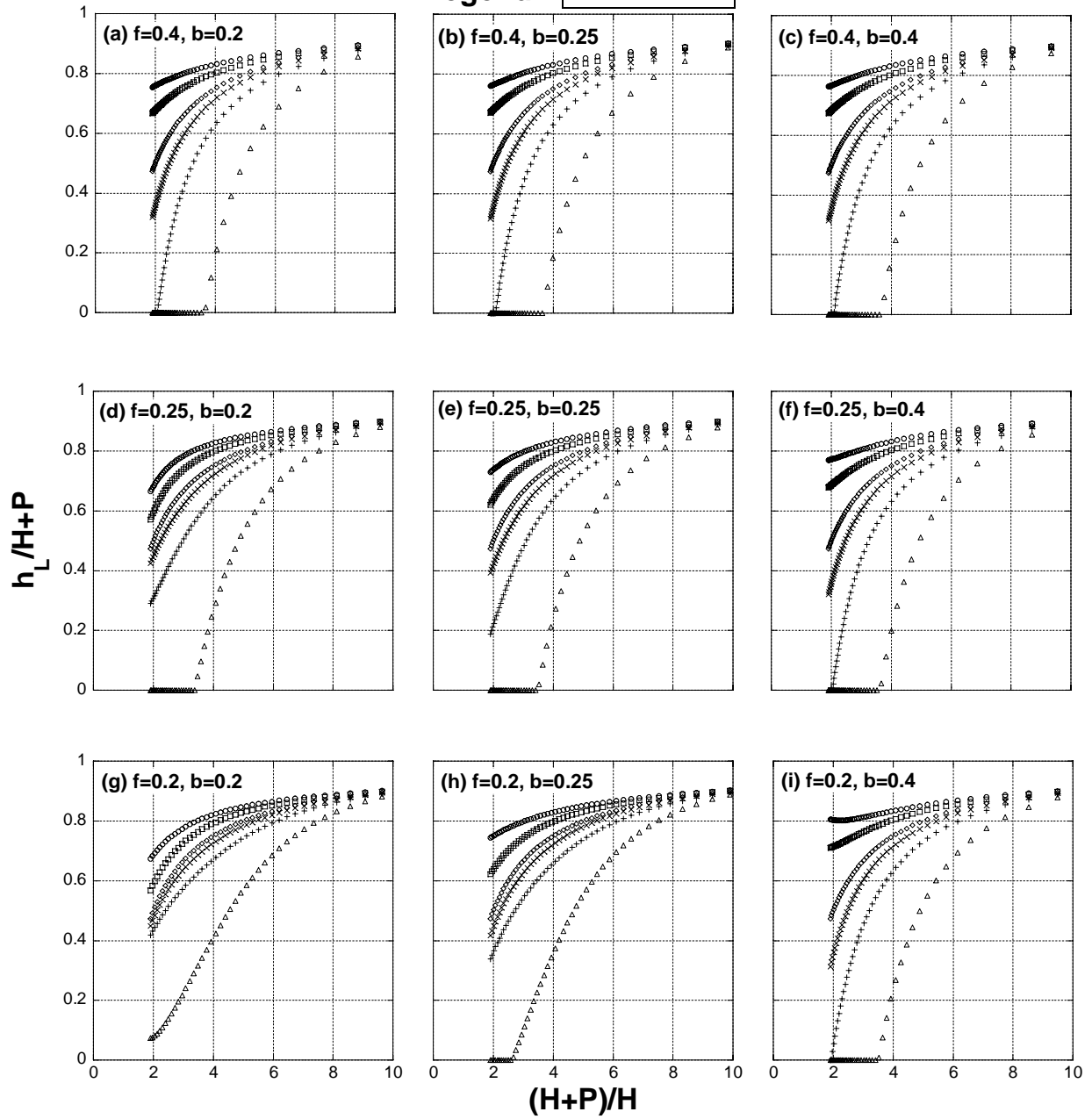
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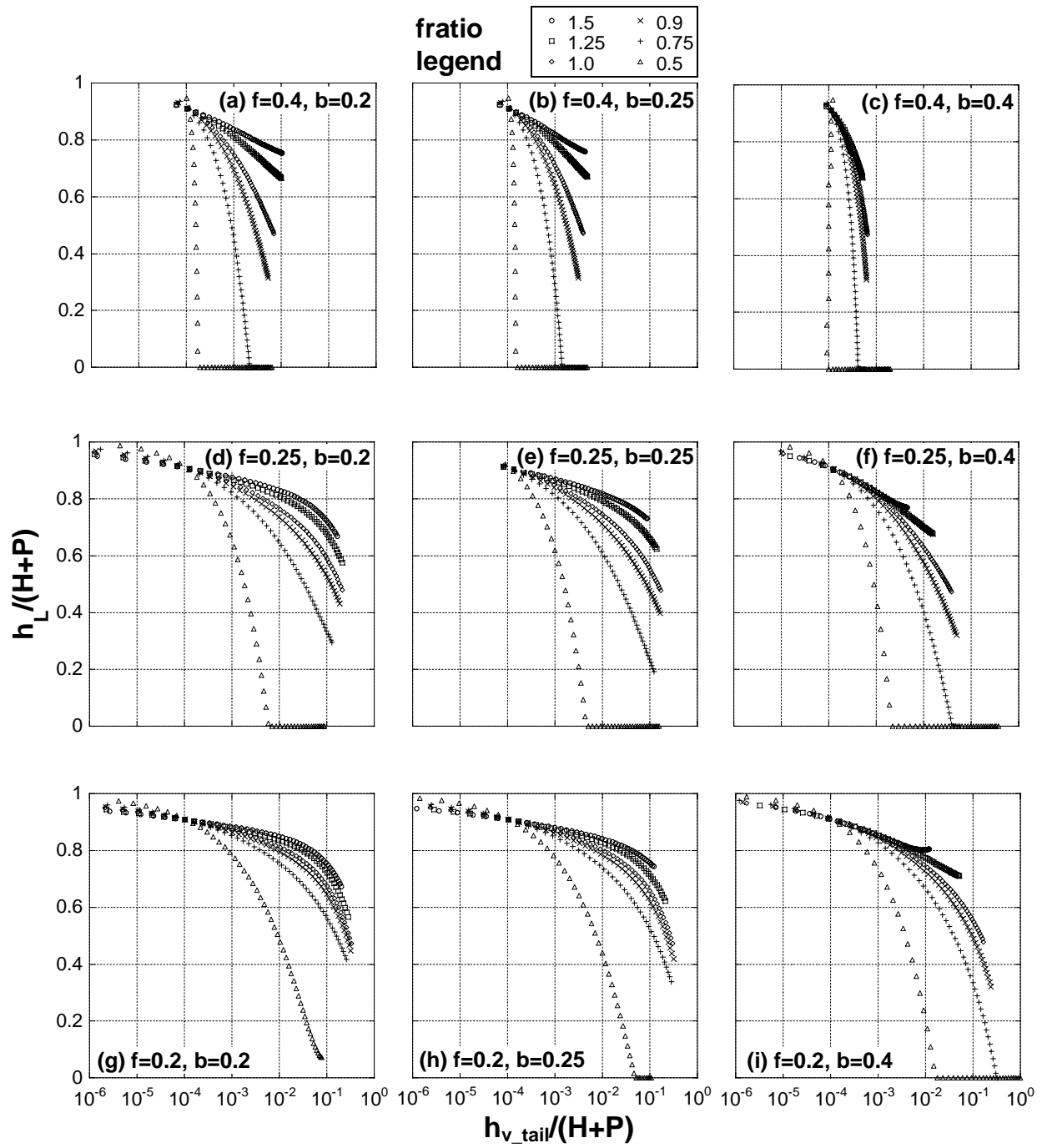
○	1.5	×	0.9
□	1.25	+	0.75
◊	1.0	△	0.5



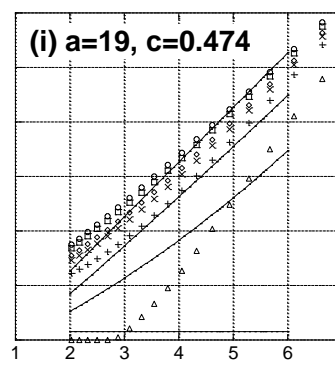
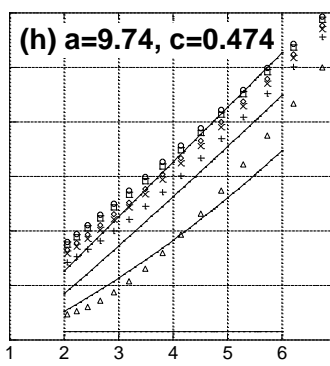
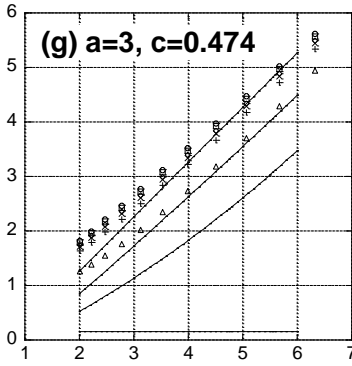
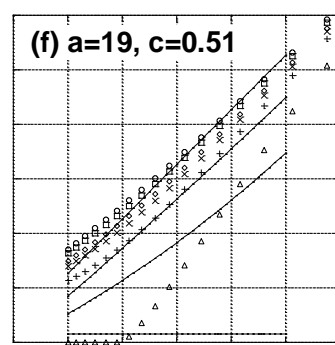
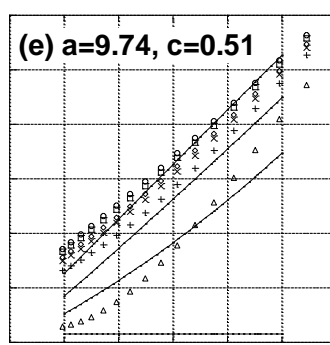
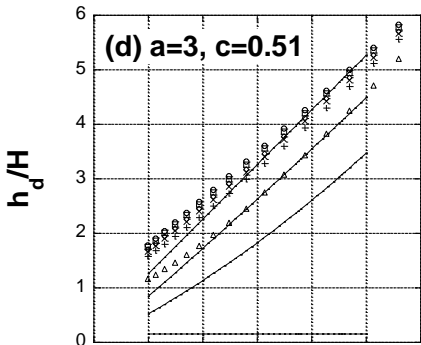
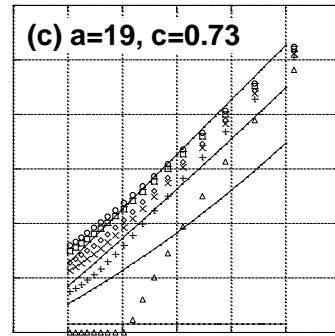
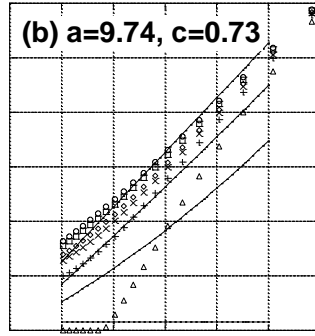
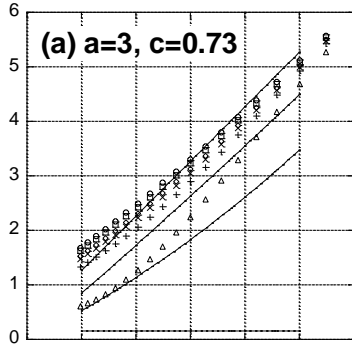
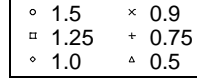
fratio
legend

○	1.5	×	0.9
□	1.25	+	0.75
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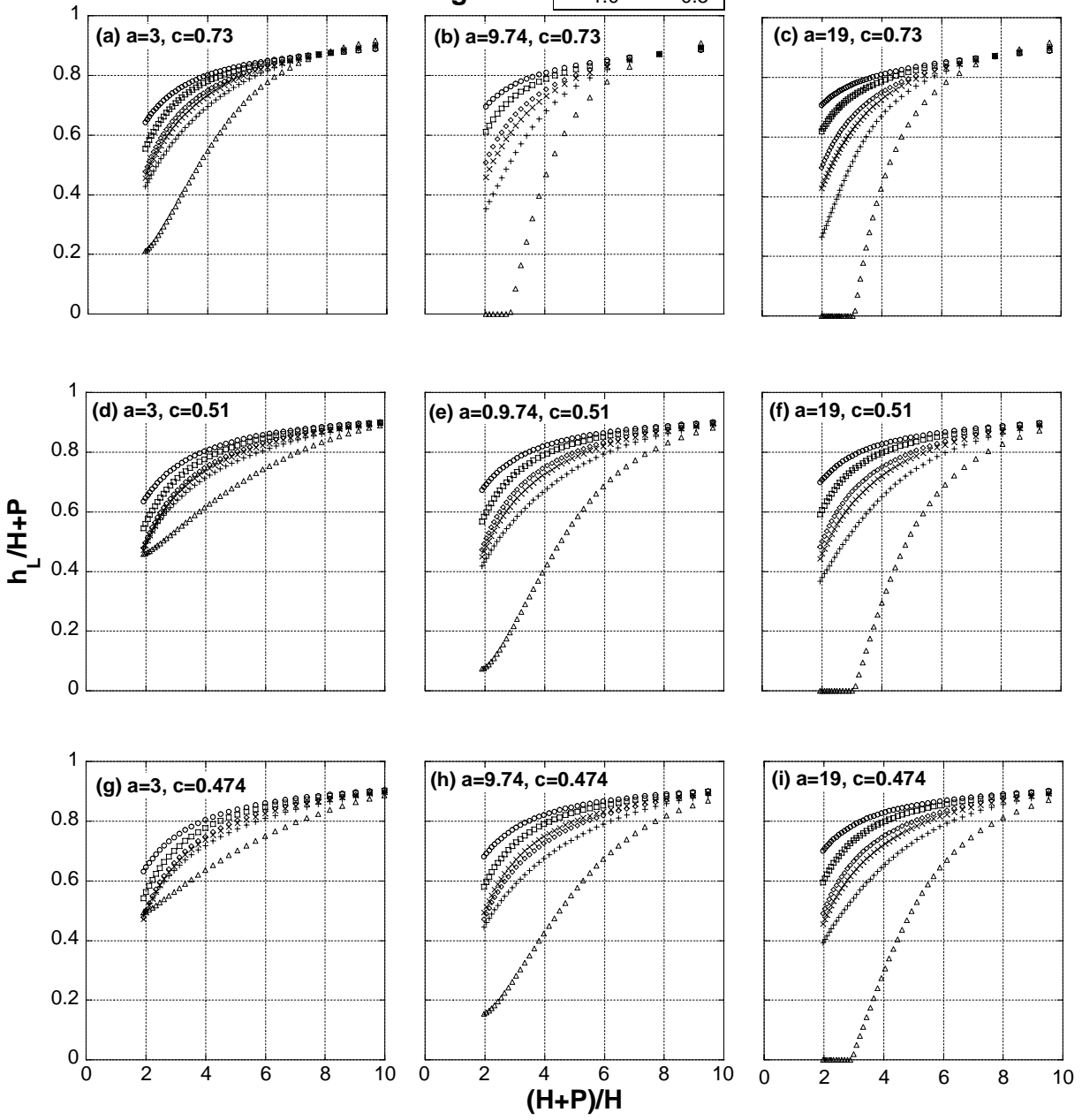
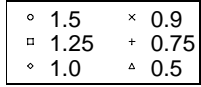
fratio
legend



$(H+P)/H$

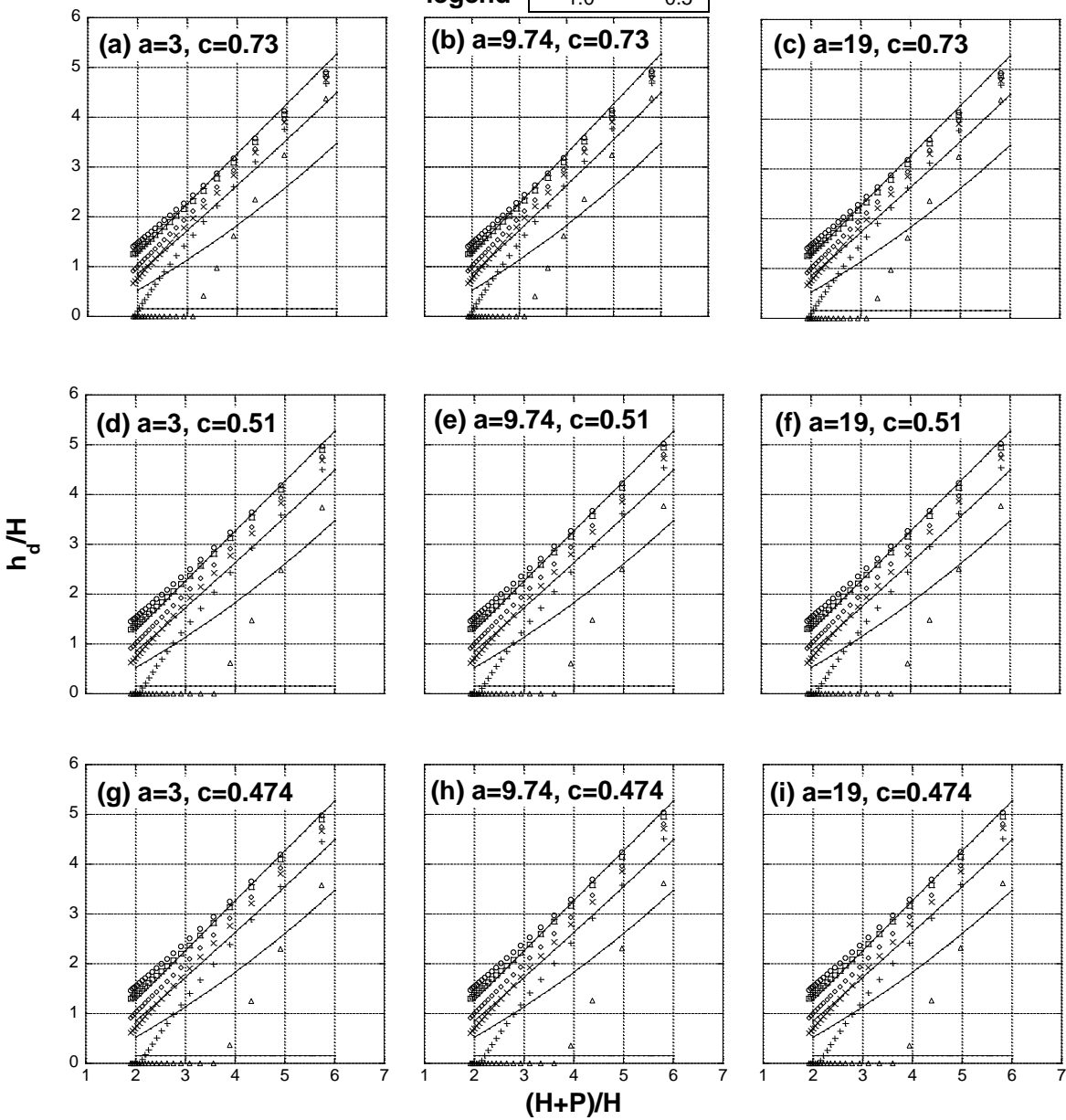
h_d/H

fratio
legend



fratio
legend

○	1.5	×	0.9
□	1.25	+	0.75
◇	1.0	△	0.5



fratio
legend

○	1.5	×	0.9
□	1.25	+	0.75
◇	1.0	△	0.5

